



**UNIVERSITI PUTRA MALAYSIA**

**SIMULATION AND CONTROL OF SALTWATER INTRUSION  
THROUGH NUMERICAL AND PHYSICAL MODELING TECHNIQUES**

**AHMED ABULOID GANFOUD**

**FK 1997 16**

**SIMULATION AND CONTROL OF SALTWATER INTRUSION THROUGH  
NUMERICAL AND PHYSICAL MODELING TECHNIQUES**

**By**

**AHMED ABULAIID GANFOUD**

**Dissertation Submitted in Fulfillment of the Requirements  
for the Degree of Doctor of Philosophy in the Faculty  
of Engineering Universiti Putra Malaysia**

**May 1997**



## ACKNOWLEDGMENTS

I would like to express my sincere thanks to the chairman of the supervisory committee Dr. Salim Said for his continuous help, support, friendship, and encouragement throughout this work. Hospitality of Dr. Salim's family has been very much appreciated. I also wish to thank Professor Dato M. Zohadie Bardaie and Dr. ShahNor Basri for serving as members of the supervisory committee and for their helpful, guidance, and valuable suggestions.

Special thanks are extended to Mr. Meh Awang, and Mr. Tajul Ariffin Tajuddin for their constant assistance during the construction of the physical model. I wish to express my gratitude to my friends in the faculty of engineering for their help and support, especially Dr. Mohamed Musadag El-Aawad, and Mr. Abdelhakim Othman.

I am grateful to the Peoples of the Libyan Arab Jamahiriya and Al-Fateh University who offer me the scholarship for pursuing the Ph.D degree at Universiti Putra Malaysia. Also, I would like to express my debt and gratitude to Universiti Putra Malaysia for financial support to the study.

Special thanks go to all the members of my family who without their encouragement and overwhelming support this work would not have been possible.

Finally and above all, much thanks and praise are directed to Allah (sbt), the Controller of the whole universe, who has provided me with health and makes this work comes true.



## TABLE OF CONTENTS

		<b>Page</b>
	ACKNOWLEDGEMENTS .....	ii
	LIST OF TABLES .....	vi
	LIST OF FIGURES .....	vii
	LIST OF PLATES .....	xi
	ABSTRACT .....	xii
	ABSTRAK .....	xv
 <b>CHAPTER</b>		
<b>I</b>	<b>INTRODUCTION</b> .....	1
	General .....	1
	Objective of The Study .....	9
	Thesis Organization .....	11
<b>II</b>	<b>LITERATURE REVIEW</b> .....	13
	Introduction .....	13
	Sharp Interface Approach .....	16
	Solute Transport Approach .....	22
	Physical Models .....	37
	Summary .....	42
<b>III</b>	<b>MATHEMATICAL FORMULATION</b> .....	43
	Introduction .....	43
	Mass Balance .....	45
	Equations of State .....	48
	Liquid Density .....	48
	Liquid Viscosity .....	50
	Medium Storage Properties .....	52
	Hydraulic Conductivity .....	55
	Hydrodynamic Dispersion .....	56
	Flow Equation .....	58
	Transport Equation .....	63
	Initial and Boundary Conditions .....	69
<b>IV</b>	<b>NUMERICAL APPROXIMATION</b> .....	77
	Introduction .....	77
	Principles of Methods of Weighted Residual .....	79
	Discretization of the Problem Domain .....	81



	Interpolation Functions .....	85
	Numerical Integration .....	93
	Basics of Galerkin Finite Element Approach .....	95
	Coefficient Representation in Space .....	97
	Numerical Formulation for Flow Equation .....	99
	Handling the Boundary Conditions .....	107
	Handling of Sources or Sink Terms .....	112
	Handling Time Derivative .....	113
	Numerical Formulation for Mass Transport.....	119
	Handling the Boundary Conditions .....	123
	Handling the Sources or Sink Terms .....	126
	Handling Time Derivative .....	127
<b>V</b>	<b>SOLUTION METHODS AND ERRORS .....</b>	<b>131</b>
	Introduction .....	131
	Review of the Solution Methods .....	132
	Assembling the Global Matrices and Vectors .....	139
	System Modification for the First Boundary Condition .....	141
	Pointer Matrix .....	143
	Numerical Accuracy and Stability .....	147
	Computation of additional requirements .....	152
	Program Optimization .....	155
<b>VI</b>	<b>PHYSICAL MODEL .....</b>	<b>159</b>
	Introduction .....	159
	Model Discription .....	161
	Sand Treatment .....	163
	Partical Size Distribution .....	165
	Determination of Aquifer Parameters .....	169
	Determination of Hydraulic Conductivity .....	169
	Determination of Porosity .....	173
	Determination of Dispersion Coefficient and Dispersivity .....	176
	Simulation of Seawater and Conductivity Measurements.....	186
	Simulation of Saltwater Intrusion Through the Physical Mode .....	188
<b>VII</b>	<b>MODEL VERIFICATION AND IMPEMINTATION .....</b>	<b>200</b>
	Introduction .....	200
	Solution Algorithm .....	200
	Henry's Problem .....	202
	Comparison with Physical Model .....	217
	Investigation of Recharge and Discharge Effect .....	234



<b>VIII</b>	<b>SUMMARY AND CONCLUSIONS .....</b>	<b>258</b>
	General .....	258
	Discussion and Conclusions .....	259
	Scope of Future Studies .....	263
<b>REFERENCES .....</b>		<b>266</b>
<b>APPENDIX</b>		
A	Algorithm for Incorporation of the First Boundary Conditions..	275
B	Generation of the Pointer Matrix .....	276
C	Some Plates for Physical Model .....	278
<b>VITA .....</b>		<b>280</b>



## LIST OF TABLES

Table		Page
1	Location of Gauss points and values of weights for exact integration of a polynomial function .....	94
2	Sieve analysis for the sand making up the aquifer medium in the physical model.....	167
3	Values of $\text{erf}(\beta)$ and $\text{eracf}(\beta)$ for positive values of $\beta$ .....	180



## LIST OF FIGURES

<b>Figure</b>		<b>Page</b>
1	Sharp interface in coastal aquifers .....	14
2	Effect of hydrodynamic dispersion on saltwater intrusion .....	14
3	Representative elementary volume .....	46
4	Water density as a function of salt concentration .....	51
5	Water viscosity as a function of salt concentration comparison of experimental data and empirical formula .....	53
6	Boundary conditions of flow domain between two bodies of water .....	71
7	Vertical cross-section in a confined leaky aquifer .....	74
8	Quadrilateral finite element mesh for two-dimensional vertical flow between two water bodies .....	82
9	Quadrilateral finite element in local coordinate system ( $\xi, \eta$ ) with Gauss points .....	88
10	Schematic diagram of the element conductance matrices and the global matrix for a quadrilateral element.....	105
11	Finite element treatment for specified flow boundary conditions .....	109
12	Example of using a pointer matrix to reduce the size of global matrix .....	146
13	Schematic diagram of the sand box model .....	162
14	Wells arrangement in the physical model .....	164
15	Grain size distribution curve for the sand of the physical model .....	168
16	A sand column apparatus designed to measure the hydraulic conductivity, porosity, and dispersivity .....	172
17	Breakthrough curve for sand column .....	181





18	Breakthrough curve in the physical model .....	184
19	Discharge variation with time for steady state conditions in the physical model .....	190
20	Head variation with time for unsteady state simulations in the physical model .....	191
21	Isochlor contours in the physical model at steady state .....	193
22	Isochlor contours after 30 minutes in the physical model .....	195
23	Isochlor contours after 60 minutes in the physical model .....	196
24	Isochlor contours after 90 minutes in the physical model .....	197
25	Advance of 0.5 isochlor contours in the physical model .....	198
26	Definitions of Henry's problem .....	203
27	Isochlor contours for Henry's problem using constant dispersion coefficient .....	207
28	Isochlor contours for Henry's problem using velocity-dependent dispersion coefficient .....	208
29	Velocity vector for Henry's problem using velocity dependent dispersion coefficient .....	210
30	Comparison of the 0.5 isochlor for Henry's problem using constant dispersion coefficient .....	211
31	Comparison of the 0.5 isochlor for Henry's problem using velocity-dependent dispersion coefficient .....	212
32	Advance of the 0.5 isochlor contour at the bottom of the aquifer using velocity-dependent dispersion coefficient .....	214
33	Hydraulic head distribution within the aquifer for Henry's problem ....	216
34	Isochlor contours at steady state for the physical model data using velocity-dependent dispersion coefficient with $\alpha = 1.29$ cm .....	220



35	Isochlor contours at steady state for the physical model data using velocity-dependent dispersion coefficient with $\alpha = 23.4$ cm .....	221
36	Isochlor contours at steady state for the physical model data using constant dependent dispersion coefficient with $D = 0.1434$ cm <sup>2</sup> /sec.....	222
37	Position of the 0.5 isochlor contour at steady state for physical model data using three methods dispersion values .....	224
38	Darcy's velocity vector in the physical model using velocity-dependent dispersion coefficient .....	225
39	Isochlor contours after 30 minutes using physical model data .....	227
40	Isochlor contours after 60 minutes using physical model data .....	228
41	Isochlor contours after 90 minutes using physical model data .....	229
42	Advance of the computed isochlor 0.5 contours in the physical model...	230
43	Comparison between the computed and measured isochlor contours at steady state conditions .....	232
44	Advance of the 0.5 isochlor contour toe at the bottom of the physical model for unsteady state .....	233
45	Hydraulic head distribution in the physical model .....	235
46	Measured and computed variation of the 0.5 isochlor due recharge in the physical model .....	237
47	Measured and computed variation of the 0.5 isochlor in the Physical model due to a well discharge rate of 10 cm <sup>3</sup> /sec .....	239
48	Hypothetical unconfined aquifer with the various boundary conditions.....	240
49	Hydraulic head distribution in the hypothetical model .....	243
50	Isochlor contours in the hypothetical model at steady state for no recharge or discharge conditions .....	244



51	Isochlor contour lines at steady state in the hypothetical aquifer with recharge rate of 0.06 m/day .....	246
52	Change of intruded length due to change of of the linear recharge rate .....	247
53	Change of the intruded length due to change of linear recharge location away from the sea side .....	249
54	Change of the intruded length due to change of point recharge rate .....	250
55	Change of intruded length due to change of the location of the point source recharge away from the sea .....	252
56	Isochlor contours in the hypothetical model at steady state with discharge rate of 4 m <sup>3</sup> /day from two wells .....	254
57	Change of intruded length due to change of discharge rate .....	255
58	Change of intruded length due to location change of discharge well .....	256



## List of Plates

<b>Plate</b>	<b>Page</b>
1 General view for the physical model .....	278
2 Side view showing the saltwater reservoir .....	278
3 View for the well's strainer used as discharge and recharge well .....	279
4 Photo for the C-S-T conductivity meter .....	279



Abstract of dissertation presented to the Senate of Universiti Putra Malaysia in fulfillment of the requirements for the degree of Doctor of Philosophy.

**SIMULATION AND CONTROL OF SALTWATER INTRUSION THROUGH  
NUMERICAL AND PHYSICAL MODELING TECHNIQUES**

By

**AHMED ABULAIID GANFOUD**

**MAY 1997**

**Chairman: Associate Professor Salim B. Said, Ph.D.**

**Faculty: Engineering**

Simulation and control of saltwater intrusion based on hydrodynamic dispersion has been investigated through numerical and physical modeling techniques. In the mathematical formulation, two equations were derived one for water flow and the other for solute transport that were coupled through Darcy's velocity and concentration. In the numerical model formulation, Galerkin finite element approach was applied for deriving the element matrix equation through quadrilateral elements. To save memory and computation time, a pointer matrix was used to avoid storage of most of the zeros in the resulting sparse matrix. The developed model was an efficient and a rather general one such that the aquifer can be of any types with different boundary conditions and unlimited number of sources and sinks. For model verification, Henry's problem was used to compare the model results with previous studies that applied constant and velocity-dependent dispersion coefficient. The comparison showed a good agreement



between the proposed model and the previous ones. Also, for simulation of saltwater intrusion the computed isochlor contours for the physical model were in good agreement with the experimental ones

By using the sandbox model and other apparatuses, the average values of porosity, hydraulic conductivity, dispersion coefficient, and saltwater density were found to be 0.36, 0.0855 cm/s, 14.34,  $10^{-2}$  cm<sup>2</sup>/s, and 1027.5 kg/m<sup>3</sup> respectively. These parameters were used in the numerical simulation. Saltwater intrusion was simulated experimentally using the physical model under steady and unsteady state conditions through the measurement of the sodium chloride distribution in the aquifer. According to physical simulation, the intruded length was changed from 48 cm at steady state to 79 cm at the end of 90 minutes from the steady state during which the freshwater head was changed every 30 minutes by 0.5 cm from 50 cm to 48.5 cm.

Both models were used to investigate the effect of aquifer recharge and discharge of saltwater from saltwater zone on the intruded length under steady state conditions with freshwater head and saltwater head as 48.5 cm and 47.5 cm respectively. The 0.5 isochlor contour toe was changed from 76 cm away from saltwater reservoir for no recharge to 71 and 70 cm for recharges of 10 and 12 cm<sup>3</sup>/sec respectively. For discharge case, it was changed from 76 cm to 72 cm for a discharge of 10 cm<sup>3</sup>/sec. Affect of rate and location of the recharge and discharge methods on the intruded length was investigated through the numerical model using a hypothetical anisotropic aquifer, with parameters very closed to reality. As a result of the

investigation, the rate in both methods has considerable effect on the intruded length of saltwater intrusion. For example, the intruded length was shifted from 136 m for no recharge to 62 m for a point recharge of 0.06 m/day. Whereas, for discharge the method the intruded length was changed from 136 m to 72 m for a discharge of 4 m<sup>3</sup>/day. On the other hand, the location has small effect especially when shifted away from the sea side.



Abstrak disertasi yang dikemukakan kepada Senat Universiti Putra Malaysia sebagai memenuhi keperluan untuk ijazah Doktor Falsafah.

**SIMULASI DAN KAWALAN TERHADAP PENEROBOSAN AIR MASIN  
MELALUI TEKNIK PEMODELAN FIZIKAL DAN BERANGKA**

Oleh

**AHMED ABULAIID GANFOUD**

**MEI 1997**

**Pengerusi: Profesor Madya Salim B. Said, Ph.D.**

**Fakulti: Kejuruteraan**

Kajian berdasarkan penyerakan hidrodinamik telah dijalankan dengan menggunakan teknik model fizikal dan berangka ke atas simulasi dan kawalan penerobosan air masin. Formulasi matematik telah menghasilkan dua persamaan iaitu untuk aliran air dan pengaliran larut yang kemudiannya dikembangkan mengikut kelajuan dan kepekatan formula Darcy. Formulasi elemen finit dengan menggunakan kaedah Galerkin telah digunakan untuk menghasilkan persamaan elemen matrik secara quadrilateral. Matrik penunjuk digunakan untuk mengurangkan penggunaan memori dan masa untuk pengiraan dan juga mengelakkan penyimpanan kebanyakan angka kosong dalam matrik ruang yang terhasil. Model yang terhasil adalah efisien dan lebih umum untuk diuji pada pelbagai akuifer dalam pelbagai keadaan sempadan serta pelbagai punca dan sink. Permasalahan Henry telah digunakan untuk membandingkan hasil keputusan model dengan kajian awal yang bergantung kepada koefisien penyerakan sekata dan





halaju. Perbandingan telah menunjukkan pertalian yang baik diantara model yang dicadangkan dengan model terdahulu. Pengiraan kontor isochlor untuk model fizikal adalah sebaik hasil eksperimen dalam simulasi penorobosan air masin.

Hasil kajian dari model fizikal yang merupakan model tangki berpasir mendapati nilai purata untuk keliangan, konduktiviti hidraulik, koefisien penyerakan dan ketumpatan air masin adalah 0.36, 0.0855 cm/s,  $14.24 \times 10^{-2}$  cm<sup>2</sup>/s dan 1027.5 kg/m<sup>3</sup>. Parameter ini telah digunakan dalam simulasi model berangka. Penyerakan air masin disimulasikan menggunakan model fizikal dibawah keadaan stabil dan tidak stabil melalui pengukuran pengagihan sodium klorida dalam akuifer. Mengikut simulasi fizikal jarak terobos berubah dari 48 cm pada keadaan stabil sehingga 79 cm selepas 90 minit dari keadaan stabil yang mana turus air tawar berubah setiap 30 minit dari 50 cm kepada 48.5 cm.

Kedua-dua model digunakan untuk mengkaji kesan kadar imbuh serta kadar luahan ke zon air masin yang mana jarak terobos didalam keadaan stabil pada turus 48.5 cm air tawar dan 47.5 cm air masin. Jari kontor 0.5 isochlor untuk model fizikal berubah dari takungan air masin daripada 76 cm kepada 71 dan 70 cm masing-masing untuk kadar imbuh 10 dan 12 cm<sup>3</sup>/s. Kadar luah 10 cm<sup>3</sup>/s telah mengubah jari kontor ke 72 cm . Kesan kadar dan lokasi untuk kaedah kadar luahan dan imbuhan keatas jarak terobos akuifer hipotesis takisotropik dengan meggunakan parameter yang menghampiri realiti telah diuji dengan menggunakan model berangka. Hasil kajian menunjukkan kedua-dua kaedah mempunyai kesan terhadap jarak terobos penorobosan air masin. Sebagai contoh, jarak terobos berubah

dari 136 cm bagi tanpa aliran kepada 62 m untuk titik aliran 0.06 m/hari manakala untuk kaedah kadar alir jarak terobos berubah dari 136 cm kepada 72 m untuk kadar alir 4 m<sup>3</sup>/hari. Lokasi cuma memberi kesan yang kecil terutama bila dianjak lari dari tepian pantai.

# **CHAPTER I**

## **INTRODUCTION**

### **General**

The term groundwater is usually referred to the water that occurs beneath the ground surface in a saturated zone of a geologic formation known as aquifers and is related to surface waters in rivers and lakes through the hydrologic cycle. It constitutes the largest available source of freshwater. Therefore, a proper development and utilization of this renewable natural resource are of interest for all water supply requirements. In any evaluation of groundwater resources, physical chemical, and biological characteristics of the water are of major importance in determining whether or not water is suitable for domestic, industrial, or agriculture use.

With increasing demand for water and with the intensification of water utilization, the quality problem becomes the limiting factor in the development of water resources, especially in coastal areas where the population is more condensed. Generally, groundwater contamination is traced back to environmental, agricultural, and domestic and industrial sources. Environmental that is a contamination due to environment through which the flow of groundwater takes place. Seawater intrusion



is an example of this pollution. Agricultural that is a pollution due to irrigation and rain water dissolving and carrying fertilizers and salts as they infiltrate through the ground surface. Domestic and industrial contamination that is caused by many ways such as percolation from septic tanks and artificial recharge of aquifers by sewage water. The scope of this study is concerned with the environmental pollution that is caused by the saltwater intrusion.

In coastal aquifers that have their boundaries in contact with saltwater bodies the fresh water is usually, under natural conditions, discharged into the ocean or at sea ward of the coastline. Because of sea water existence in the aquifer formations beneath the sea bottom, a zone of contact is formed between the lighter fresh water that flowing to the sea and the denser underlying sea water. A balance or equilibrium tends to become established between the fresh water and the salt water pressing in from the sea under natural undisturbed conditions in the coastal aquifer with a stationary interface and fresh water flow to the sea above it. When the water table (unconfined aquifer) or the piezometric head (confined aquifer) in the fresh water body becomes slightly above or less than the head in the adjacent seawater wedge, the interface starts to advance inland. This phenomenon is known as seawater intrusion. Almost, the same phenomenon can be found in most oceanic islands where fresh water forms a lens overlying the sea water. The extent of intrusion depends upon the climatic conditions, the characteristics of the natural flow within these aquifers, and the manner of groundwater usage.

Seawater intrusion represents displacement of miscible liquids in porous media since diffusion and hydrodynamic dispersion tends to mix the two fluids. Therefore, the interfacial surface becomes a transition zone with a variable concentration from fresh water at the top to seawater at the bottom. The thickness of the zone is highly variable; in steady flow the thickness is minimum, while unsteady flows such as pumping, recharge, and tide increases the thickness. Flow in the transition zone varies from that of fresh water at the upper surface to nearly zero at the lower surface. The movement in the transition zone transports the salt, that dispersing upward into the zone from the underlying seawater, to the sea. This flow tends to lessen the extent to which the saltwater occupies the aquifer.

Because of the deterioration of water quality, saltwater intrusion restricts the agricultural, industrial, and domestic use of groundwater from the invaded aquifers. Therefore, an assessment and control of saltwater intrusion in specific coastal area are needed before initiating a major groundwater development program. This can be achieved with proper management, considering the geological and hydrological conditions of the study area.

Management of groundwater system means making various decisions aimed at modifying the state of a considered system to achieve certain goals and objectives without violating specified constraints. Some examples of decision variables are pumpage distribution, quality of pumped water, and distribution and quality of artificial recharge. Examples of the state variables are water levels, concentration of specified species, and seawater intrusion. Some examples of objective functions are

maximizing the total net benefits during specified period of time, minimizing the cost of a unit volume of water supplied to the consumer, and minimizing the total consumption of energy. Examples of constraints are water levels that should not drop or rise specified minimum or maximum elevations, the concentrations of a certain species in pumping should not exceed specified threshold values, and the length of an intruding seawater wedge should not exceed specified value. To predict the outcome of implementing management decisions, a tool is required to forecast the aquifer's response to planned operations. The necessary information about the response of the system is provided by a model that describes the behavior of the groundwater system in response to some action. The model may be defined as a simplified version of the real system that approximately simulates the excitation response of the system. The simplification is introduced in the form of a set of assumptions that express the understanding of the nature of the system and its behavior.

Models that have been used in saltwater intrusion simulation can be classified as physical models, viscous analog, electric analogs, and numerical models. The physical model or sand box model is a reduced scale representation of the natural porous medium domain. Since both the prototype and the model involve flow through porous media, the sand box model is considered a true model. Because of its special features that permit studies of phenomena to the microscopic structure of the medium, it has been used extensively in groundwater investigations such as miscible and immiscible displacement and hydrodynamic dispersion. The drawback

of this model is presence of the trapped air in the voids and the effect of capillary fringe in case of phreatic aquifer.

The viscous flow analog or Hele-Shaw analog has been used extensively by many investigators for solving regional groundwater flow problems such as seawater intrusion and artificial recharge. The analogy is based on the similarity between the differential equations governing the saturated flow in a porous medium and those describing the flow of a viscous liquid in a narrow space between two parallel plates. In spite of the wide application, there are some factors that limit the model applications such as the effect of changes in temperature during the test, the flow measurement poses some problems, and sometimes over simplification are made to meet certain simulations. In addition, there are some situations where the model could not be used, for example, simulation of the hydrodynamic dispersion and simultaneous flow of two immiscible fluids.

The electric analogs that include electrolytic tank, RC networks, and ion motion are considered very powerful tools in studies involving flow in aquifers. The analogy is based on the similarity between the differential equations that govern the flow of a homogeneous fluid through a porous medium and those governing the flow of electricity through a conducting material. However, because of their analogy, their implementations for groundwater simulations are limited to certain situations. For instance, they could not be used to investigate simulation of phreatic aquifer, hydrodynamic dispersion, observations of streamlines and path lines, and simultaneous flow of two immiscible fluids.

Beside the physical models, various studies have been conducted based on analytical solutions of the governing equations, to study the interaction between freshwater and saltwater. These solutions are simple and take less time in computation. However, because of heterogeneity, irregular geometry, and boundary conditions, the analytical solutions may be impossible for most practical problems unless some drastic simplification assumptions are made. Therefore, only numerical models can provide the required forecast in saltwater intrusion simulations.

Generally, there are some factors such as the complexity of the geology, the manner groundwater flow, and the required applications of the results that control the classification of the numerical models. According to these factors numerical models can be classified as one-, two-, and three-dimensional models. Three-dimensional model is necessary where the geology is complex and the groundwater flow is considered important in all directions. However, its development is limited due to the requirement of large core storage and the complicated calculations involved. Therefore, most of three-dimensional flow models are multilayered models that mean the flow equations that govern the phenomena are actually one- or two-dimensional.

Commonly, the major numerical methods that have been used for modeling groundwater flow and solute transport are finite difference method(FDM), finite element method(FEM), method of characteristics(MOC), and boundary element method(BEM). Each of these methods is based on certain approach and has some advantages and disadvantages. The FDM is simple in concept, easy in programming,



and simple in data input. On the other hand, it is difficult to handle problems of irregular boundaries, irregular forms of sources and sinks, and irregular aquifer materials. The FEM is a very powerful and flexible method. It can handle any shape and any combination of boundary conditions. Also, it is more accurate than FDM if the same type and size of mesh are used. The drawback of this method is the difficulty in concept, programming, and data input. However; once the computer programs are developed, it can be easily adopted from one problem to another.

MOC is effective in dealing with the hyperbolic type of partial differential equations. It is widely used for simulating the transport of miscible compounds in groundwater, especially in one- and two-dimensional flow problems. BEM is effective in dealing with elliptic type of partial differential equations such as steady state flow. Both methods have difficulty to handle three-dimensional flow problems and they usually employed in homogeneous and isotropic aquifers.

Applying any of these numerical methods usually yields a set of linear or nonlinear algebraic equations. The solution procedure is either direct or iterative. Each one has advantages and disadvantages. In direct procedure initial estimates of the unknown variables, iteration parameters, and selections of error tolerance are not needed. Also, the matrix operations are performed only once. However, serious round-off errors may occur and it may take more computer storage. The iterative procedure saves computer storage and increases computation accuracy. On the other hand, the solution depends on the initial estimation of unknown variables, the choices of the iteration parameters, and the choices of the error tolerance values.