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ANALYTICAL APPROACH FOR LINEAR PROGRAMMING USING BARRIER AND PENALTY FUNCTION METHODS

PARWADI MOENGIN

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ANALYTICAL APPROACH FOR LINEAR PROGRAMMING USING BARRIER AND PENALTY FUNCTION METHODS

By

PARWADI MOENGIN

Thesis Submitted to the School of Graduate Studies, Universiti Putra Malaysia, in Fulfilment of the Requirements for the Degree of Doctor of Philosophy

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TO MY PARENTS
MY WIFE ENDANG SETIOWATI
MY SONS DITO RAMADHANI &
VITO RIHALDIJIRAN

Abstract of thesis presented to the Senate of Universiti Putra Malaysia in fulfilment of the requirements for the degree of Doctor of Philosophy.

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PARWADI MOENGIN

September 2003

Chairman : Associate Professor Noor Akma Ibrahim, Ph.D

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In order to solve the primal linear programming problems (and its dual) some methods have been used such as simplex method, geometric approach and interior points methods. None of these methods used Lagrangian function as a tool to solve the problem. This raises a question why are we not using this to solve the linear programming problems. Thus, in this research we study and analyze how the behavior and performance of barrier functions and penalty functions methods for solving the linear programming problems. All of these functions are in Lagrangian form.

With logarithmic barrier function methods we introduce three types of function; that is, primal logarithmic, dual logarithmic and

primal-dual logarithmic functions. There are two main results obtained from the logarithmic function method. First, we prove that for every value of the barrier parameter, the logarithmic barrier function for the problem has a unique minimizer; and then if the sequence of the values of barrier parameters tends to zero, then the sequence of the minimizers converges to a minimizer of the problem. From these properties, we construct an algorithm for solving the problem using the logarithmic barrier function methods. Second, we give the equivalences between the interior points set, the primal logarithmic barrier function, the dual logarithmic barrier function, the primal-dual logarithmic barrier function and the system of linear equations associated with these functions.

In this research we also investigate the behavior and performance of the penalty function methods for linear programming problems. It includes polynomial penalty functions (such as primal penalty and dual penalty functions) and exponential penalty function methods. The main result of these methods is that, we can solve the problem by taking a sequence of values of the penalty parameters that tends to infinity; and then the sequence of the minimizers of the penalty functions associated with the value of penalty parameter will tend to the minimizer for the problem.

With this, we can formulate an algorithm for solving the problem using penalty function methods. The study also includes the exponential barrier function. The result obtained is similar with the result of the exponential penalty function methods.

In this research we also investigate the higher-order derivatives of the linear programming and the system of linear equations associated with the problem. From this method we are able and successful in formulating an algorithm for solving the problem.

Finally, in this research we give an analysis of sensitivity for the linear programming using interior point approach introduced by Karmarkar and using logarithmic barrier function. It encompasses cases of changing the right-hand sides of the constrained, changing the cost vector of the objective function, upper bounds, lower bounds of variables and ranges of the constraints.

Abstrak tesis yang dikemukakan kepada Senat Universiti Putra Malaysia sebagai memenuhi keperluan untuk ijazah Doktor Falsafah.

ANALYTICAL APPROACH FOR LINEAR PROGRAMMING USING BARRIER AND PENALTY FUNCTION METHODS

Oleh

PARWADI MOENGIN

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Pengerusi : Profesor Madya Noor Akma Ibrahim, Ph.D

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Bagi menyelesaikan masalah pengaturcaraan linear primal (dan juga masalah dualnya) beberapa kaedah telah digunakan seperti kaedah simpleks, pendekatan geometri dan kaedah titik pedalaman. Tidak ada satupun kaedah tersebut yang menggunakan fungsi Lagrange sebagai alat untuk menyelesaikan masalah ini. Hal ini menimbulkan persoalan mengapa kita tidak menggunakannya untuk menyelesaikan masalah ini. Di dalam penyelidikan ini kami mengkaji dan menganalisis bagaimana perilaku dan pencapaian kaedah-kaedah fungsi sawar dan fungsi penalti untuk menyelesaikan masalah pengaturcaraan linear. Semua fungsi tersebut adalah dalam bentuk fungsi Lagrange.

Dengan kaedah fungsi sawar logaritma, kami memperkenalkan tiga jenis fungsi; iaitu, fungsi logaritma primal, fungsi logaritma dual dan fungsi logaritma primal-dual. Ada dua keputusan utama yang membabitkan kaedah fungsi sawar logaritma. Pertama, kami membuktikan bahawa setiap nilai parameter sawar, fungsi sawar logaritma bagi masalah ini mempunyai peminimum tunggal; dan kemudian jika siri nilai-nilai parameter sawar menumpu ke sifar, maka siri peminimum tersebut akan menumpu kepada peminimum masalah tersebut. Daripada sifat ini, kami membina suatu algoritma untuk menyelesaikan masalah tersebut menggunakan kaedah fungsi sawar logaritma. Kedua, kami memberikan kesetaraan diantara kumpulan titik-titik pedalaman, fungsi sawar logaritma primal, fungsi sawar logaritma dual, fungsi sawar logaritma primal-dual dan sistem persamaan linear yang berkaitan dengan fungsi-fungsi ini.

Di dalam penyelidikan ini kami juga mengkaji perilaku dan pencapaian dari kaedah-kaedah fungsi penalti bagi pengaturcaraan linear. Ini melibatkan fungsi-fungsi penalti polinomial (seperti fungsi penalti primal dan fungsi penalti dual) dan kaedah-kaedah fungsi penalti eksponen. Keputusan utama yang berkaitan dengan kaedah ini diberikan, iaitu, kami boleh menyelesaikan masalah tersebut dengan mengambil suatu siri daripada nilai parameter penalti yang menumpu ke infiniti; dan kemudian siri peminimum daripada fungsi penalti yang berkaitan dengan parameter penalti ini, akan menumpu ke peminimum masalah tersebut. Dengan demikian kami boleh merumuskan suatu algoritma untuk menyelesaikan masalah tersebut menggunakan kaedah fungsi penalti. Kajian ini juga melibatkan fungsi sawar eksponen. Keputusan yang diperoleh adalah serupa dengan keputusan daripada kaedah fungsi penalti eksponen.

Dalam penyelidikan ini kami juga mengkaji terbitan peringkat-tinggi bagi pengaturcaraan linear dan sistem persamaan linear yang berkaitan dengan masalah tersebut. Keputusan utama yang melibatkan kaedah ini adalah bahawa kami berjaya merumuskan suatu algoritma untuk menyelesaikan masalah pengaturcaraan linear.

Akhirnya, di dalam penyelidikan ini kami memberikan suatu analisis sensitiviti bagi pengaturcaraan linear menggunakan pendekatan titik pedalaman Karmarkar dan fungsi sawar logaritma. Ini merangkumi kes-kes mengubah ruas-kanan kekangan, mengubah vektor kos fungsi matlamat, batas atas dan batas bawah bagi pembolehubah, dan julat daripada kekangan.

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I certify that an Examination Committee met on 22 September 2003 to conduct the final examination of Parwadi Moengin, on his Doctor of Philosophy thesis entitled "Analytical Approach for Linear Programming Using Barrier and Penalty Function Methods" in accordance with Universiti Pertanian Malaysia (Higher Degree) Act 1980 and Universiti Pertanian Malaysia (Higher Degree) Regulations 1981. The Committee recommends that the candidate be awarded the relevant degree. Members of the Examination Committee are as follows:

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DECLARATION

I hereby declare that the thesis is based on my original work except for quotations and citations which have been acknowledged. I also declare that it has not been previously or concurrently submitted for any other degree at UPM or other institutions.

PARWADI MOENGIN

Date: 22 September 2003

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LIST OF NOTATIONS

The following notations will be used in this thesis:

Set Notation

$\in$	is an element of
$\notin$	is not an element of
ϕ	the empty set
$\cup$	union
$\cap$	intersection
$\subseteq$	is a subset of
$\supseteq$	is a superset of
R	the set of real numbers
R^+	the set of positive real numbers
bnd(D)	the boundary of set D
F_p	the region feasible of primal linear programming
F_d	the region feasible of dual linear programming
F_p^0	the (relative) interior set of F_p
F_d^0	the (relative) interior set of F_d

F_p^*	the minimizers set of primal linear programming
F_d^*	the maximizers set of dual linear programming
F	the feasible region of primal-dual problem
F^0	the (relative) interior set of F
$F_p(\sigma)$	the σ-level set of F_p
$F_d(\sigma)$	the σ-level set of F_d
$F(\sigma)$	the σ-level set of F

Miscellaneous Symbols

$=$	is equal to
$\neq$	is not equal to
$\equiv$	is identical to or is congruent to
$<$	is less than
$\leq$	is less than or equal to
$>$	is greater than
$\geq$	is greater than or equal to
∞	infinity
$\sum_{i=1}^{n} a_i$	$a_1 + a_2 + \ldots + a_n$
$\lvert a \rvert$	the modulus of a

Vectors

b, c, x	the vectors
c^T	the transpose of vector c
$c^T x$	the dot product of vectors c and x
$\|x\|$	the Euclidean norm of vector x
$\|x\|_\infty$	max $\{\lvert x_1\rvert, \lvert x_2\rvert, \ldots, \lvert x_n\rvert\}$
$\langle .,. \rangle$, $\|.\|$	the Euclidean scalar product
x^*	the minimizer of primal linear programming
f^*	the minimum value of primal linear programming
μ	a barrier parameter
σ	a penalty parameter
$\{\mu^k\}$	a decreasing sequence of positive barrier parameters
$\{\sigma^k\}$	an increasing sequence of positive barrier parameters
e	unit vector
$x(\mu)$	the value of x at the parameter μ
M_μ	the unconstrained minimizers set corresponding with barrier parameter μ

M_σ	the unconstrained minimizers set corresponding with penalty parameter σ

Matrices

A	a matrix A
A_i	the ith row vector of matrix A
A^{-1}	the inverse of the matrix A
A^{-2}	is equal to $(A^{-1})^2$
A^T	the transpose of the matrix A
X	$n \times n$ diagonal matrix with elements x_i
$X^{\frac{1}{2}}$	$n \times n$ diagonal matrix with elements $(x_i)^{\frac{1}{2}}$
Δx	increment of x
R^n	the set of all column vectors $n \times 1$ with elements in R
R^n_+	the set of all positive column vectors $n \times 1$ with elements in R
$R^{m \times n}$	the set of all matrices $m \times n$ with elements in R
Ax	the product of matrix A and vector x
rank (A)	the rank of matrix A

Functions

$f(x)$	the value of the function f at x
e^x, $\exp(x)$	exponential function of x
$\ln x$	natural logarithm of x
$g(t) \to \infty$	$g(t)$ tends to infinity
$\lim_{x \to a} f(x)$	the limit of $f(x)$ as x tends to a
$L(x, \mu)$	Lagrangian function
$B_l(x, \mu)$	the primal logarithm barrier function
$\nabla B_l(x, \mu)$	the gradient of $B_l(x, \mu)$
$\nabla^2 B_l(x, \mu)$	the Hessian of $B_l(x, \mu)$
$\min_{x \geq 0} B_l(x, \mu)$	$\min \{B_l(x, \mu) \mid x \in R^n, x \geq 0\}$
$B_a(x, \mu)$	the alternative primal logarithmic barrier function
$B_m(x, \mu)$	the multiplier primal logarithmic barrier function
$B_d(y, \mu)$	the dual logarithmic barrier function
$B_{pd}(x, s, \mu)$	the primal-dual logarithmic barrier function
$P(x, \sigma)$	the primal polynomial penalty function

$P_d(y, s, \sigma)$	the dual polynomial penalty function
$E(x, \sigma)$	the primal exponential penalty function
$E_b(x, \mu)$	the primal exponential barrier function
$\min_{x \geq 0} P(x, \sigma)$	$\min \{P(x, \sigma) \mid x \in R^n, x \geq 0\}$

CHAPTER I

INTRODUCTION

General Overview

The purpose of the mathematical programming, a branch of the optimization, is to minimize (or maximize) a function of several variables under a set of constraints. This is a very important problem arising in many real-world situations such as cost or duration minimization.

Linear programming means that we discuss about the function to be optimized whereby its associated set of constraints are linear. The simplex algorithm, first developed by Dantzig in 1947, is a very efficient method to solve this class of problems (Singiresu, 1996). It has been studied and improved since its first appearance, and is now widely used in commercial software to solve a great variety of problems such as production planning, transportation and scheduling (Singiresu, 1996).

However, an article by Karmarkar (Barnes, 1986; Ariayawansa et al., 2001) introduced in 1984 a new class of methods called interior point methods. Most of the ideas underlying these new methods originate from nonlinear optimization domain. These methods are