Determination of optimal values for the technical characteristics of construction machinery unit with application of fractional linear programming

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Abstract. Paper covers issues connected to the determination of the technical characteristics of construction machinery unit performing a certain category of work. On the basis of the results of the review and analysis of scientific works concerning the problem under consideration the key disadvantages have been identified for the existing techniques. Optimization model for determination of the technical characteristics of construction machinery unit has been created. Initial form of the model is not linear toward unknown variables but it can be transformed to the linear form that allows application of simplex method for efficient solution of the problem. The created model has been implemented on a practical case – the obtained results indicated the adequacy of the developed tool.

1 Introduction

Modern conditions of the construction industry's development are characterized by the large variety of corresponding machinery models with different technical characteristics and, as sequence, different rental costs and productivity indicators. That circumstance determines the high relevance of the problems connected to formation of optimal structure for construction machinery fleet on the basis of time and cost criteria in conditions of complex structure of the construction projects and large amount of variants for machines' design and productivity. The relatively sophisticated approach is based on the use of the modern simulation and optimization techniques that are based on mixed-integer linear programming [1–3], particle swarm optimization [4], ant colony optimization [5], genetic algorithms [6], etc. The corresponding scientific developments assume creation and implementation of complex mathematical models and algorithms to take into account the large number of important factors that have influence on the machinery fleet's efficiency indicators and obtain adequate results. Still, the corresponding calculations' conduction process require high qualification in the field of mathematics and programming for the person solving the task. The more simple (in terms of practical application) approach is based on the ranking of the available machinery variants (models) connected to the certain

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category of work with further selection of the appropriate number of the most preferred machinery models (in accordance with the rank and available number) to provide the required productivity of construction machinery complex for each category of work. To calculate the rank for machinery variants under consideration, it's necessary to justify the composition and values of the reference characteristics for the machinery unit – it will allow to calculate the deviations between actual and reference values of characteristics for each machinery model and determine the rank in accordance with the following principle: the less the value of generalized deviation for machinery model, the bigger the value of corresponding rank. However, the preliminary analysis of the scientific works connected to the justification of the machinery unit's characteristics showed the lack of effective tools providing calculation of reference values for machinery characteristics with full and objective taking into account of cost and productivity indicators. On the basis of the features indicated above the conclusion has been made about the necessity for conduction of the research aimed to the creation of the tools for the determination of the reference technical characteristics of construction machinery unit. The main tasks of the research have been formulated as follows:

1. Carrying out of the review and analysis of the modern scientific developments in the research field.
2. Creation of the optimization model for determination of optimal values for the technical characteristics of construction machinery unit with application of fractional linear programming.
3. Implementation of the created model on a practical example.

The object of the research is construction technological process that assumes performing of the certain category of work by cyclic-operating machinery unit within limited time interval. The subject of the research is the characteristics of the mentioned above process in terms of technical and cost characteristics of the machinery unit.

At the initial stages of the research the review and analysis of the scientific developments [1–17] in the field of determination of characteristics for the machinery unit within the technological (particularly, construction) process. Most scientific developments assume determination of the machinery's characteristics by selection of the most appropriate variant through the application of hierarchy analysis methods [7-16], fuzzy logic techniques [17, 18] and their combination [19]. The common disadvantage of the corresponding procedures is connected to the direct dependence of the solution search's laboriousness and the number of alternative variants of machinery. Additional disadvantages of the mentioned above categories of scientific developments are the following:

– dependence of the results on the expert opinion with a certain degree of subjectivity; the difficulty of providing an objective assessment of the objects of analysis [7-16,19];
– the difficulty of objective description of the interconnection between different categories of parameters (including qualitative and quantitative) [17, 18].

Mentioned above conclusions determined the necessity for creation of the optimization model which would provide determination of the technical characteristics of cyclic-operating construction machinery unit with adequate and objective account of the criteria of its productivity and economic efficiency. More detailed information is presented in the next sections of the work.

2 Methods

At the intermediate stage of the optimization model has been created on the basis of preliminarily formulated main provisions and supposed composition of initial data and

unknown variables. Detailed description of the model's structure is provided within the corresponding subsections.

2.1 Main provisions

Main provisions of the elaborated model are the following:
− the object under consideration is the technological process connected to the handling of cargo units (CU) by a unit of construction handling machinery (CHM) of cyclic action with battery power supply within the corresponding technological cycles, implemented within a limited time interval.
− the technological cycles implemented by the CHM unit correspond to a certain category of work and therefore have similar configuration parameters of the corresponding technological routes.
− the similarity of the configuration of technological routes implemented by the CHM unit within the corresponding cycles of CU processing makes it possible to form the most probable (median) technological route, the implementation of which within the considered time interval will ensure the productivity of the CHM unit, equal to the actual productivity of the CHM during the implementation of technological routes with similar, but not identical configuration.
− the general composition of the CHM technical characteristics includes the main technical characteristics that directly determine the rental cost for a CHM unit, and additional characteristics that do not significantly affect the rental cost. The list of the main characteristics for the considered CHM is presented in Table 1.

Table 1. The list of the main characteristics for the considered CHM.

No.	Name of the technical characteristic for CHM unit	Meas. unit	Designation in the expressions for calculated characteristics	
			general	customized
i	-	MSU	-	-
1	Load carrying capacity value	kg	x_1	G_c
2	Inverse value of stationary travel speed with CU	h/km	x_2	$1/\upsilon_{trv}^+$
3	CU lift height value	m	x_3	H_{lft}
4	Inverse value of the lifting speed for pickup device with CU	s/m	x_4	$1/\upsilon_{lft}^+$
5	Turning radius value	m	x_5	R_{trn}
6	Nominal energy value for accumulator storage battery (ASB)	J	x_6	A_{ASB}

− the dependence of the rental cost for a CHM unit on its main technical characteristics is described by a linear analytical model defined by the expression:

$$\widetilde{y} = a_0 + \sum_{i=1}^{z} a_i \cdot x_i \,, \tag{1}$$

where $\widetilde{y}$ – predictive value of the rental cost for a CHM unit, MU;

a_0 – cost constant, MU;

i – index of CHM technical characteristic; $i = 1, 2, ..., z$;

z – number of CHM technical characteristic, units;

a_i – model parameter - coefficient of proportionality of the value of the rental cost for a

CHM unit to the value of technical characteristic i ($i = 1, 2, ..., z$), MU/MSU;

MSU denotes a variable measure unit, the description of which is presented in Table 1);
x_i – value of technical characteristic i ($i = 1, 2, ..., z$), *MSU*.

- the configuration of the technological route describes the trajectory of the spatial movement of the CHM unit during the implementation of technological cycle and includes straight and curved (turns) sections;
- the passage of straight sections is carried out by a CHM unit at a stationary travel speed (for example, in the process of CU's transportation from the place of pickup to the place of release) or at a shunting travel speed (during operations of CU pickup or release).
- the passage of curved sections is carried out by a CHM unit only at shunting speed.
- there is a certain relationship between the travel speed of a CHM unit without CU and with it, the same for both stationary and shunting speeds.
- there is a certain ratio between the stationary and shunting travel speeds of a CHM unit, which is the same for the movement both with and without CU.
- there is a certain ratio between the speeds of the vertical movement of a CHM unit's pickup device with CU and without it, the same for the pickup device's lifting and lowering.
- there is a certain ratio between the speeds of lowering and lifting the load gripper, which is the same for the pickup device's movement both with and without CU.
- there is a certain ratio between the total mass of a CHM unit and its load carrying capacity.
- there is a certain ratio between the mass of a CHM unit's pickup device and its load carrying capacity.
- it is necessary to determine the values of the CHM technical characteristics, at which the maximum value of money saving for the rental of the CHM unit, referred to the average duration of the implemented technological cycle, is ensured.

2.2 Model description

Source data and unknown variables connected to the created optimization model are presented in Table 2 and Table 3 respectively.

Calculated characteristics used within the model and independent of its unknown variables, are presented in Table 4.

Table 2. Source data for the created optimization model.

No.	Name of the source data element	Meas. unit [1]	Designation / expression
1	Parameters of source data structure		
1.1	Number of CHM technical characteristics	unit	z
2	Indexes		
2.1	Index of CHM technical characteristics	-	$i = 1, 2, ..., z$
3	Source data specified for technological process		
3.1	Duration of the time interval of CHM continuous operation	h	T^{Σ}
3.2	CHM time utilization ratio	-	γ_T
3.3	Safety factor of the actual used energy volume for ASB [2]	-	δ_{ASB}
3.4	Maximal renting cost for a CHM unit	MU	C^{max}
3.5	Cost constant [3]	MU	a_0
3.6	Minimal (maximal) value of average cycle time [4]	s	$T_c^{min(max)}$

Table 2. Source data for the created optimization model (continued).

No.	Name of the source data element	Meas. unit [1]	Designation / expression
4	Source data specified for each CHM technical characteristic i ($i = 1, 2, ..., z$)		
4.1	Parameter of the analytical model, corresponding to the technical characteristic [3]	MU/*MSU*	a_i
4.2	Minimal (maximal) value of the characteristic [4]	*MSU*	$x_i^{\min(\max)}$
5	Source data specified for technological route		
5.1	Total length of CHM straight-line movements at a stationary speed with CU (without CU)	m	$L_{\mathrm{trv}}^{+(-)}$
5.2	Total length of CHM straight-line movements at a shunting speed with CU (without CU)	m	$L_{\mathrm{shn}}^{+(-)}$
5.3	Turning radius calculated value	m	R
5.4	Turning angle value	rad	φ
5.5	Number of turns with CU (without CU)	unit	$n_{\mathrm{trn}}^{+(-)}$
5.6	Height of CU lifting / lowering during pickup (release)	m	$H_{\mathrm{pick(rel)}}$
5.7	Average duration of additional operations during CU pickup (CU release)	s	$\Delta t_{\mathrm{pick(rel)}}$
6	Source data specified for construction handling machines' operation		
6.1	Mass of CU being processed	kg	m
6.2	Gravity acceleration	m/s²	g
6.3	Ratio of the CHM travel speed without CU to the CHM travel speed with CU	-	k_{trv}
6.4	Ratio of the CHM shunting speed of the CHM stationary speed	-	$k_{\mathrm{trv\,s}}$
6.5	Ratio of the lifting speed of the CHM pickup device without CU to the lifting speed of the CHM pickup device with CU	-	k_{lft}
6.6	Ratio of the CHM pickup device lowering speed to the CHM pickup device lifting speed	-	k_{lwr}
6.7	Ratio of the mass of a CHM unit to its carrying capacity	-	k_{mas}
6.8	Ratio of the mass of a CHM unit's pickup device to CHM unit's carrying capacity	-	k_{PD}
6.9	Coefficient of resistance to CHM movement with stationary speed	-	f_{trv}
6.10	Coefficient of resistance to CHM movement with shunting speed	-	$f_{\mathrm{trv\,s}}$
6.11	Efficiency of the CHM movement mechanism providing stationary speed	-	η_{trv}
6.12	Efficiency of the CHM movement mechanism providing shunting speed	-	$\eta_{\mathrm{trv\,s}}$
6.13	Efficiency of the lifting mechanism for CHM unit's pickup device	-	η_{lft}
6.14	ASB limit discharge ratio	-	μ_{ASB}

Note:

[1] the measure unit indicated for certain elements of source data and specified as *MSU* has a different meaning depending on the index of CHM technical characteristic (for more information, see Table 1);

[2] Value of source data element must correspond to the condition $\delta_{\mathrm{ASB}} \in [0;1]$

[3] source data elements can be calculated on the basis of the set of CHM samples with further solving of system of equations with application of Cramer's rule;

[4] source data elements can be calculated by the statistical processing of information connected of the set of CHM samples (in terms of values of technical characteristics under consideration) and

also must take into account features of the technological process's implementation – for example, CHM load carrying capacity G_c must be equal or higher the CU mass m, turning radius R_{trn} must be close to the corresponding calculated value of technological route R, etc.

Table 3. Unknown variables for the created optimization model.

No.	Name of the unknown variables' category	Meas. unit [1]	Designation / expression
1	Unknown variables of the base model (original unknown variables)		
1.1	Value of CHM technical characteristic [2] i ($i = 1, 2, ..., z$)	MSU	x_i
2	Unknown variables of the modified model (modified unknown variables)		
2.1	Inverse value of the average cycle time, implemented by a CHM unit during CU processing	s^{-1}	$z_0 = \dfrac{1}{T_c}$
2.2	The ratio of the value of CHM technical characteristic i ($i = 1, 2, ..., z$) to the value of the average cycle time	MSU/s	$z_i = x_i \cdot z_0 = \dfrac{x_i}{T_c}$

Note:

[1] the measure unit indicated for certain categories of unknown variables and specified as *MSU* has a different meaning depending on the index of CHM technical characteristic (for more information, see Table 1);

[2] alternative designations of unknown variables $\{x_i\}$ in expressions in items 1.1 and 2.2 are presented in Table 1.

Table 4. Calculated characteristics for the created optimization model.

No.	Name of calculated characteristic	Meas. unit	Expression
1	Characteristics of the duration of the technological cycle		
1.1	Specific average value of the distance of CU horizontal transportation provided by a CHM unit at a stationary speed	m	$\Lambda_{trv} = L_{trv}^+ + \dfrac{L_{trv}^-}{k_{trv}}$
1.2	Specific average value of the distance of CU horizontal transportation provided by a CHM unit at a shunting speed	m	$\Lambda_{trv\,s} = \varphi \cdot R \cdot \left(n_{trn}^+ + \dfrac{n_{trn}^-}{k_{trv}} \right) + L_{shn}^+ + \dfrac{L_{shn}^-}{k_{trv}}$
1.3	Specific average value of the distance of CU vertical transportation provided by a CHM unit's pickup device	m	$\Lambda_{lft} = H_{pick} \cdot \left(\dfrac{1}{k_{lft}} + \dfrac{1}{k_{lwr}} \right) +$ $+ H_{rel} \cdot \left(1 + \dfrac{1}{k_{lft} \cdot k_{lwr}} \right)$
1.4	Average value of the duration of a part of the cycle, which does not depend on the CHM technical characteristics	s	$\Delta T = \Delta t_{pick} + \Delta t_{rel}$
2	Characteristics connected to the energy consumption by a CHM unit during the implementation of technological cycle		
2.1	Ratio of the average value of the energy spent on CHM unit's movement at a stationary speed to the value of the load carrying capacity	J/kg	$\Theta_{trv} = \dfrac{g \cdot f_{trv}}{\eta_{trv}} \cdot k_{mas} \cdot \left(L_{trv}^+ + L_{trv}^- \right)$

Table 4. Calculated characteristics for the created optimization model.

No.	Name of calculated characteristic	Meas. unit	Expression
2.2	Ratio of the average value of the energy spent on CHM unit's movement at a shunting speed to the value of the load carrying capacity	J/kg	$\Theta_{\text{trv s}} = \dfrac{g \cdot f_{\text{trv s}}}{\eta_{\text{trv s}}} \cdot k_{\text{mas}} \times$ $\times \left(\varphi \cdot R \cdot \left(n_{\text{trn}}^{+} + n_{\text{trn}}^{-} \right) + L_{\text{shn}}^{+} + L_{\text{shn}}^{-} \right)$
2.3	Ratio of the average value of the energy spent on a CHM pickup device's lifting / lowering to the value of the load carrying capacity	J/kg	$\Theta_{\text{lft}} = \dfrac{g}{\eta_{\text{lft}}} \cdot k_{\text{mas}} \cdot k_{\text{PD}} \cdot 2 \cdot \left(H_{\text{pick}} + H_{\text{rel}} \right)$
2.4	Average value of the energy consumed by a CHM unit within the cycle, independent of the technical characteristics of the equipment	J	$\Delta A = m \cdot g \cdot \left(\dfrac{f_{\text{trv}}}{\eta_{\text{trv}}} \cdot L_{\text{trv}}^{+} + \dfrac{f_{\text{trv s}}}{\eta_{\text{trv s}}} \times \right.$ $\left. \times \left(\varphi \cdot R \cdot n_{\text{trn}}^{+} + L_{\text{shn}}^{+} \right) + \dfrac{1}{\eta_{\text{lft}}} \cdot \left(H_{\text{pick}} + H_{\text{rel}} \right) \right)$
2.5	Minimal permissible conditional value of ASB nominal energy	J	$A_{\text{ASB}}^{\text{cond min}} = x_{6}^{\text{cons min}} + \left(x_{6}^{\max} - x_{6}^{\min} \right) \cdot \delta_{\text{ASB}}$

Initial form of the optimization model corresponding to the main provisions specified in subsection 2.1 is described by the following expressions:

$$\left\{ \begin{array}{l} \dfrac{C^{\max} - \left(a_0 + \sum\limits_{i=1}^{z} a_i \cdot x_i \right)}{\left(\Lambda_{\text{trv}} + \dfrac{\Lambda_{\text{trv s}}}{k_{\text{trv s}}} \right) \cdot x_2 + \Lambda_{\text{lft}} \cdot x_4 + \Delta T} \rightarrow \max; \qquad (2) \\[4mm] \end{array} \right.$$

$$\dfrac{\left(\Theta_{\text{trv}} + \Theta_{\text{trv s}} + \Theta_{\text{lft}} \right) \cdot x_1 + \Delta A}{\left(\Lambda_{\text{trv}} + \dfrac{\Lambda_{\text{trv s}}}{k_{\text{trv s}}} \right) \cdot x_2 + \Lambda_{\text{lft}} \cdot x_4 + \Delta T} \cdot T^{\Sigma} \cdot \gamma_{\text{T}} \cdot \mu_{\text{ASB}} \leq x_6; \qquad (3)$$

$$x_i^{\min} \leq x_i \leq x_i^{\max}, \; i = 1, 2, ..., z. \qquad (4)$$

The expressions are based on the certain elements of the Table 4 with substitution of customized designations of technical characteristics for general designations in accordance with the information in Table 1.

Expression (2) represents objective function – money saving for the rental of the CHM unit, referred to the average cycle time – to be maximized. Expression (3) represents indirect constraints connected to the condition that total energy spent during the implementation of technological cycle by a CHM unit (taking into account ASB limit discharge ratio) must not exceed ASB nominal energy. Expression (4) describes the direct constraints for original unknown variables $\{x_i\}$.

It can be concluded that the model is not linear towards unknown variables $\{x_i\}$. However, expressions (2) – (4) include the elements corresponding to the fractional linear model and therefore can be transformed to the linear form by performing of the following changes:

1. Substitution of the inverse value of the denominator in expressions (2) and (3) for the unknown variable $z_0 = 1/T_c$ (item 2.1 in Table 3) with further replacement of original variables $\{x_i\}$ by modified ones $\{z_i\}$ in accordance with the expression

$$x_i = \frac{z_i}{z_0}; \tag{5}$$

corresponding results are described by the following expressions:

$$\frac{C^{\max} - \left(a_0 + \sum_{i=1}^{z} a_i \cdot x_i\right)}{T_c} = \left(C^{\max} - a_0\right) \cdot z_0 - \sum_{i=1}^{z} a_i \cdot z_i \to \max; \tag{6}$$

$$\left(\left(\Theta_{\text{trv}} + \Theta_{\text{trv s}} + \Theta_{\text{lft}}\right) \cdot z_1 + \Delta A \cdot z_0\right) \cdot T^{\Sigma} \cdot \gamma_{\text{T}} \cdot \mu_{\text{ASB}} \leq x_6; \tag{7}$$

it's necessary to note that expression (5) is linear toward the variables z_0, $\{z_i\}$; left part of the expression (7) is also linear towards the mentioned variables but the its right part is not (see expression (5).

2. Substitution of the expression (6) for two equivalent conditions defined by the following expressions

$$\begin{cases} \left(\left(\Theta_{\text{trv}} + \Theta_{\text{trv s}} + \Theta_{\text{lft}}\right) \cdot z_1 + \Delta A \cdot z_0\right) \cdot T^{\Sigma} \cdot \gamma_{\text{T}} \cdot \mu_{\text{ASB}} \leq A_{\text{ASB}}^{\text{condmin}}; & (8) \\ A_{\text{ASB}}^{\text{condmin}} \cdot z_0 \leq z_6. & (9) \end{cases}$$

3. Addition of integrity condition for the modified unknown variables z_0, $\{z_i\}$; the mentioned condition is connected to the following expression:

$$\left(\Lambda_{\text{trv}} + \frac{\Lambda_{\text{trv s}}}{k_{\text{trv s}}}\right) \cdot z_2 + \Lambda_{\text{lft}} \cdot z_4 + \Delta T \cdot z_0 = 1. \tag{10}$$

4. Transformation of the expression (4) through the replacement of the original variables in accordance with the expression (5) and further multiplication of all parts of the expression by z_0.

5. Addition of direct constraints for the variable z_0 with the use of inverse values of specific initial data elements (item 3.6 in Table 2)

As the result, the modified optimization model is defined by the following expressions

$$\begin{cases} \left(C^{\max} - a_0\right) \cdot z_0 - \sum_{i=1}^{z} a_i \cdot z_i \to \max; & (11) \\ \left(\left(\Theta_{\text{trv}} + \Theta_{\text{trv s}} + \Theta_{\text{lft}}\right) \cdot z_1 + \Delta A \cdot z_0\right) \cdot T^{\Sigma} \cdot \gamma_{\text{T}} \cdot \mu_{\text{ASB}} \leq A_{\text{ASB}}^{\text{condmin}}; & (12) \\ A_{\text{ASB}}^{\text{condmin}} \cdot z_0 \leq z_6; & (13) \\ \left(\Lambda_{\text{trv}} + \frac{\Lambda_{\text{trv s}}}{k_{\text{trv s}}}\right) \cdot z_2 + \Lambda_{\text{lft}} \cdot z_4 + \Delta T \cdot z_0 = 1; & (14) \\ \frac{1}{T_c^{\max}} \leq z_0 \leq \frac{1}{T_c^{\min}}; & (15) \\ x_i^{\min} \cdot z_0 \leq z_i \leq x_i^{\max} \cdot z_0, \ i = 1, 2, ..., z. & (16) \end{cases}$$

As it can be seen in expressions (11)–(16), the modified model is linear towards the variables z_0, $\{z_i\}$ and therefore can be effectively implemented with the use of simplex method (that guarantees identification of optimal solution in case the corresponding feasible region exists) in the modern software tools of optimization modeling.

3 Results and discussion

At the final stage of the research created model has been implemented on the practical case – determination of the optimal values for the technical characteristics of the reach-trucks operating within the storage zone for construction materials. Preparation of the model was carried out in "Microsoft Excel" software, implementation of the model was conducted with the use of "Solver" add-in. The general view of the "Microsoft Excel" worksheet is shown in Fig. 1. The settings of "Solver" add-in are presented in Table 5.

Table 1. Source data

No.	Name of the source data element	Sign	Meas. unit	Value
1	Source data specified for technological process			
1.1	Duration of the time interval of CHM continuous operation	T^{Σ}	h	8
1.2	CHM time utilization ratio	γ_T	-	0,9
1.3	Safety factor of the actual used energy volume for ASB	δ_{ASB}	-	0,75
1.4	Maximal renting cost for a CHM unit	C^{max}	ths.rub./unit	1737,6
1.5	Cost constant	a_0	ths.rub	2073,01
1.6	Minimal value of average cycle time	T_c^{min}	s	54,73
1.7	Maximal value of average cycle time	T_c^{max}	s	91,4
2	Source data specified for technological route			
2.1	Total length of CHM straight-line movements at a stationary speed with CU	L^+_{trv}	m	54,09
2.2	Total length of CHM straight-line movements at a stationary speed without CU	L^-_{trv}	m	59,68
2.3	Total length of CHM straight-line movements at a shunting speed with CU (without CU)	$L^{+(-)}_{shn}$	m	2,40
2.4	Turning radius calculated value	R	m	1,50
2.5	Turning angle value	φ	rad	1,57
2.6	Number of 90° turns with CU	n^+_{trn}	unit	5
2.7	Number of 90° turns without CU	n^-_{trn}	unit	3
2.8	Height of CU lifting / lowering during pickup	H_{pick}	m	0,12
2.9	Height of CU lifting / lowering during release	H_{rel}	m	5,10
2.10	Average duration of additional operations during CU pickup	Δt_{pick}	s	1
2.11	Average duration of additional operations during CU release	Δt_{rel}	s	1
3	Source data specified for construction handling machines' operation			
3.1	Mass of CU being processed	m	kg	1500
3.2	Gravity acceleration	g	m/s²	9,81
3.3	Ratio of the CHM travel speed without CU to the CHM travel speed with CU	k_{trv}	-	1,014
3.4	Ratio of the CHM shunting speed to the CHM stationary speed	$k_{trv\,s}$	-	0,45
3.5	Ratio of the lifting speed of the CHM pickup device without CU to the similar speed with CU	k_{lft}	-	1,625
3.6	Ratio of the CHM pickup device lowering speed to the CHM pickup device lifting speed	k_{lwr}	-	1,25
3.7	Ratio of the mass of a CHM unit to its carrying capacity	k_{mas}	-	2,36
3.8	Ratio of the mass of a CHM unit's pickup device to CHM unit's carrying capacity	k_{PD}	-	0,12
3.9	Coefficient of resistance to CHM movement with stationary speed	f_{trv}	-	0,02
3.10	Coefficient of resistance to CHM movement with shunting speed	$f_{trv\,s}$	-	0,03
3.11	Efficiency of the CHM movement mechanism providing stationary speed	η_{trv}	-	0,83
3.12	Efficiency of the CHM movement mechanism providing shunting speed	$\eta_{trv\,s}$	-	0,8
3.13	Efficiency of the lifting mechanism for CHM unit's pickup device	η_{lft}	-	0,65
3.14	ASB limit discharge ratio	μ_{ASB}	-	0,8

Table 2. Source data and unknown variables connected to CHM technical characteristics

№ п.п.	Name of the characteristic	Ед. изм.	Proportion coefficient	Value of original unknown variable			Value of original unknown variable		
				minimal	maximal	actual	minimal	maximal	actual
i	-	MSU	a_i	x_i^{min}	x_i^{max}	x_i	x_{i,z_0}^{min}	x_{i,z_0}^{max}	z_i
-	-	-	-	MSU	MSU	MSU	MSU/s	MSU/s	MSU/s
0	Inverse value of the average cycle time	s^{-1}	-	-	-	-	0,01094053	0,0182699	0,01323801
1	Load carrying capacity value	kg	0,24247587	1500	3000	1500	19,8570168	39,7140336	19,8570168
2	Inverse value of stationary travel speed with CU	h/km	8629,93	0,06752217	0,1	0,07583961	0,00077871	0,0013238	0,00100397
3	CU lift height value	m	0,06142835	2200	7495	2200	29,1236246	99,218894	29,1236246
4	Inverse value of the lifting speed for pickup device with CU	s/m	-496,54329	2	4	3,65425261	0,02647602	0,05295204	0,04837504
5	Turning radius value	mm	-0,6632182	1440	1750	1750	19,0627361	23,1665196	23,1665196
8	Nominal energy value for accumulator storage battery (ASB)	W·h	0,0101586	40680	50400	40680	538,522296	667,195765	538,522296

Table 3. Calculated characteristics

No.	Name of calculated characteristic	Sign	Meas. unit	Value
1	Characteristics of the duration of the technological cycle			
1.1	Specific average value of the distance of CU horizontal transportation at a stationary speed	Λ_{trv}	m	112,951988
1.2	Specific average value of the distance of CU horizontal transportation at a shunting speed	$\Lambda_{trv\,s}$	m	23,518826
1.3	Specific average value of the distance of CU vertical transportation provided by a CHM unit's pickup device	Λ_{lft}	m	7,78061538
1.4	Aver. value of the duration of a part of the cycle, which does not depend on the CHM tech. characteristics	ΔT	s	2
1.5	Ratio of the calculated value of average cycle time to the actual value	ψ_T	-	1
1.6	Actual value of average cycle duration	T_c	s	75,5400479
2	Characteristics connected to the energy consumption by a CHM unit during the implementation of technological cycle			
2.1	Ratio of the aver. value of the energy spent on CHM unit's movement at a stationary speed to the load carrying capacity value	Θ_{trv}	J/kg	63,4722034
2.2	Ratio of the aver. value of the energy spent on CHM unit's movement at a shunting speed to the load carrying capacity value	$\Theta_{trv\,s}$	J/kg	20,5321897
2.3	Ratio of the aver. value of the energy spent on a CHM pickup device's lifting / lowering to the load carrying capacity value	Θ_{lft}	J/kg	44,6220377
2.4	Average value of the energy consumed by a CHM unit within the cycle, independent of the technical characteristics	ΔA	kJ	145,178566
2.5	Minimal permissible conditional value of ASB nominal energy	$A^{cond\,min}_{ASB}$	W·h	40680
2.6	Actual value of the required energy of the ASB for the PHE unit's operation during the interval	A^{Σ}	W·h	40284,1142
3	Specific cost characteristics of the technological process			
3.2	Actual value of money saving for the rental of the CHM unit, referred to the average cycle time	dE	ths.rub./(unit·s)	14,2060137

Fig. 1. The general view of the "Microsoft Excel" worksheet.

As received results correspond to the assigned ranges of permissible values, the created model is adequate and can be used as the effective tool for solving of corresponding applied tasks. However, there are several important features that have significant influence on the practical significance of the optimization results. The mentioned-above features are the following:

− adequacy of the optimization results is directly connected to the adequacy of the analytical model describing the dependence of the rental cost for a CHM unit on its technical characteristics (see expression (1)). So, before the preparation of the optimization model, the adequacy of analytical model must be verified (for example, with the use of determination ratio R^2);

− configuration of the feasible region (and optimal solution as well) depends on the value of parameter δ_{ASB} (item 3.3 in Table 2) within the element $A_{ASB}^{condmin}$ (item 2.5 in Table 4) of the expressions (12) and (13) connected to the indirect constraints. Nevertheless, as the parameter δ_{ASB} varies from 0 to 1, it is possible to find the single value of parameter δ_{ASB} that corresponds to the most preferable variant of optimal solution of the model by sequential conduction of optimization experiments with different values of parameter δ_{ASB}.

Table 5. Settings for the "Solver" add-in used for implementation of the model.

Setting item	Value	Structural element of mathematical description
Target cell	O69	Expression (11)
Optimization type	maximize	
Variables cells	O44:O50	Table 3, items 1.1, 2.1, 2.2
Constraints	O44:O50 >= M44:M50	Left part of the expressions (15), (16)
	O44:O50 <= N44:N50	Right part of the expressions (15), (16), expression (13)
	O67 <= O66	Expression (12)
	O59 = 1	Expression (14)
Solver method	Simplex LP	-

4 Conclusion

On the basis of the conducted research, the following main conclusions have been made:

1. Review and analysis of the modern scientific developments in the research filed has been carried out. Corresponding results have shown the lack of the tools for determination of the construction machinery unit's characteristics that provide adequate taking into account of productivity and economic efficiency indicators.

2. Optimization model for determination of the construction machinery unit's technical characteristics with application of fractional linear programming has been created. On the basis of the results of the model's implementation on a practical case the conclusion has been made about the adequacy of the proposed tool.

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