Pertanika J. Trop. Agric. Sci. **35 (2)**: 319 - 337 (2012)
ISSN: 1511-3701

Design of an Object-oriented Framework for Modelling the Partitioning of Captured Solar Radiation and Evapotranspiration in Intercropping Systems

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ABSTRACT

x-library is a C++ object-oriented framework for modelling the partitioning of captured solar radiation and evapotranspiration in intercropping systems. The design and analysis of the x-library are done to ensure that the soil-plant-atmosphere system is categorised into classes, such as weather, microclimate, intercrop, crop, canopy, leaf, roots, soil, heat, and radiation. Meanwhile, x-library implements two kinds of solar radiation models; namely, one-dimensional (1-D), and two-dimensional (2-D) model, where irradiance varies in one dimension (vertical) and in two dimensions (vertical and horizontal), respectively. Radiation partitioning is based on weighting criteria so that a crop having the larger leaf area index and extinction coefficient would have greater share of captured radiation. Evapotranspiration partitioning is calculated using the Shuttleworth-Wallace equation. Model comparisons with a field experiment showed an overall good agreement between the simulated and measured solar radiation and transpiration values. A graphical user interface front-end for the x-library known as the x-model was also developed, primarily for non-modellers and non-programmers.

Keywords: Intercrop, model, object-oriented, reusability

INTRODUCTION

When two or more crops are grown together in an intercropping system, these crops may succeed in higher yields or they may fail. In order to understand the outcome of such crop combinations, it is necessary to be able to quantify the processes involved in the partitioning of resources in this particular system. This is because these resources, such as solar radiation and water, will determine not only the growth and development of individual crop species, but also the community as a whole (Wallace, 1997).

Consequently, capturing of solar radiation and evapotranspiration must be modelled together. It is important to note that plants rarely compete for solar radiation without simultaneously competing for water as well (Cannel & Grace, 1993; Wallace, 1995). The first modelling step is to quantify the partitioning of the captured radiation in mixed canopies, and this is followd by using this information in the evapotranspiration model to drive the processes

Received: 2 April 2010
Accepted: 24 May 2010
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of evaporation and transpiration which are strongly radiation-dependent (Wallace, 1997).

Although several such coupled radiation-evapotranspiration models exist (e.g., Kropff, 1993), most of them are implemented using procedural or structured programming (e.g., FORTRAN and BASIC), rather than using object-oriented programming (OOP). The primary advantage of having an object-oriented modelling framework is to achieve model reusability and extendibility, which are two highly desirable properties in model sharing and development. These two key properties facilitate: (i) the interchanging of component models within and between the whole-system models, (ii) incremental model development without having to rewrite existing code, and (iii) maintenance of more than one model of a component (van Evert and Campbell, 1994). Recognising these benefits, agricultural workers are using OOP more frequently in their modelling work (e.g. Pan *et al*., 2000; Salminen *et al*., 2005; Aumann, 2007; Martinez *et al*., 2008; Gocic & Trajkovic, 2010). The scenario was totally different a decade ago, when the production of high quality software was rarely a high priority in modelling work (McCown *et al*., 1996). This was because models were commonly built to be accurate and to meet immediate research objectives, but they were rarely built to be reusable or extendible. Moreover, models were also usually built and used in limited situations, and then discarded for newer ones. This has seriously hampered both the progress and the use of the existing models. Loomis (1985) and Seligman (1990) emphasise that modelling progress is best achieved by testing and improving the best of the existing models rather than writing new ones from scratch, which is akin to trying to "reinvent the wheel".

Consequently, the objective of this paper is to introduce a design of an object-oriented framework called x-library specifically for modelling the partitioning of captured radiation and evapotranspiration in intercropping systems. x-library has been designed following the principles of the object-oriented software design as specified by Meyer (1997) so that it would be reusable and extendible to achieve ease of model sharing and further development. While x-library is intended for modellers and programmers, x-model has also been developed as a graphical user interface front-end for x-library that is intended for non-modellers and non-programmers.

OBJECT-ORIENTED DESIGN AND ANALYSIS OF THE X-LIBRARY

x-library is written in C++ and conforms to the C++ standards set by the ISO (International Organization of Standardization). Thus, x-library is independent of hardware and type of operating system. x-library was primarily compiled and tested using Microsoft Visual C++ 2003.

Nonetheless, x-library can only be used as part of the developer's project. This means that the x-library cannot be compiled into an executable file by itself and then run. Developers must create a project, along with the x-library, to develop an executable program specific to the developer's intended hardware and operating system. There are two ways how x-library can be used in a project. The first is to compile all the x-library source files to produce a single library file. This library file is then linked with the developer's project. The second method is to include only the necessary x-library source and header files in the project, which are then compiled and linked together with the developer's own source files. The x-library can later be used to create a GUI (Graphical User Interface) Windows or Mac program, a command line driven DOS program, or a UNIX program.

The design and analysis of the x-library are done such a way that soil-plant-atmosphere system is categorised into several components (or classes), such as weather, microclimate, intercrop, crop, canopy, leaf, roots, soil, heat, and radiation (Table 1 and *Fig. 1*). The weather classes deal with meteorological conditions such as the daily and hourly weather properties, incoming solar radiation, and solar position. Meanwhile, the heat and solar radiation

TABLE 1
x-library classes

Group	Sub-group	Classes
Intercrop		XIntercrop, XICell
Crop		XCrop, XCrpLayer, XCrpCell
Canopy		XCanopy, XCnpyLayer, XCnpyCell
Leaf		XLeaf, XLeafCell, XLeafLin, XLeafPoly, XGrpLeaf
Root		XRootSys
Microclimate		XMicroclimate
Weather		XAstro, XDayWthr, XDiuWthr
Radiation		XRad, XRadComp, XRadLayer, XRadCell
Heat		XHeat, XSrc, XEvpSW
Soil		XMoistDist, XSoil, XSoilBasic, XSoilEnv
File		XFile
Support	Base	XObject
	Date	XDate
	Distributional	XDist, XCurveFit
	Directional	XDir
	Factory	XFactory, XFactoryLayer, XFactoryCell
	Exception	Exception, GeneralError, SpecificError, AccessEmpty, AccessNullPtr, WrongClass, OutOfRange, RangeTooSmall, AbnormalError, BadArg, BadState, DivideByZero, InitFail, NoLeaf

classes compute the evapotranspiration and the partitioning of captured radiation, respectively. A class that unifies the various components of the soil-plant-atmosphere system is the microclimate class, XMicroclimate. This microclimate class contains pointers to the heat base class (XHeat), radiation base class (XRad), weather base class (XAstro), soil base class (XSoil), and the intercrop base class (XIntercrop), in which the microclimate class acts as a link or common channel in which the various soil-plant-atmosphere classes can be used to call or use each other to obtain or share information. For example, the calculation of the crop roughness length and zero plane displacement is done in the heat classes (XHeat, XSrc and XEvpSW). However, these calculations require information such as crop height and leaf area index, which are obtained via the microclimate class from the crop classes (XCrop, XCrpLayer and XCrpCell).

The intercrop class is implemented to describe the cropping system as comprising one or more crop species, whereby each crop is in turn described as consisting of roots and canopy, where the latter comprises leaves. To implement this particular design, the intercrop base class XIntercrop has a container of crop objects; thus, if a cropping system has two crop species, for example, the container in XIntercrop will then have two crop objects. The crop base class XCrop, on the other hand, has a pointer to the canopy base class XCanopy, and another pointer to the root base class XRootSys. The canopy class, in turn, has a container of leaf objects. Each leaf class (XLeaf, XLeafCell, XLeafLin and XLeafPoly) represents the properties of a single leaf; however, the leaf class XGrpLeaf is unique because it represents the properties of a collection or group of leaves, rather than a single leaf in the canopy. Currently, x-library does not

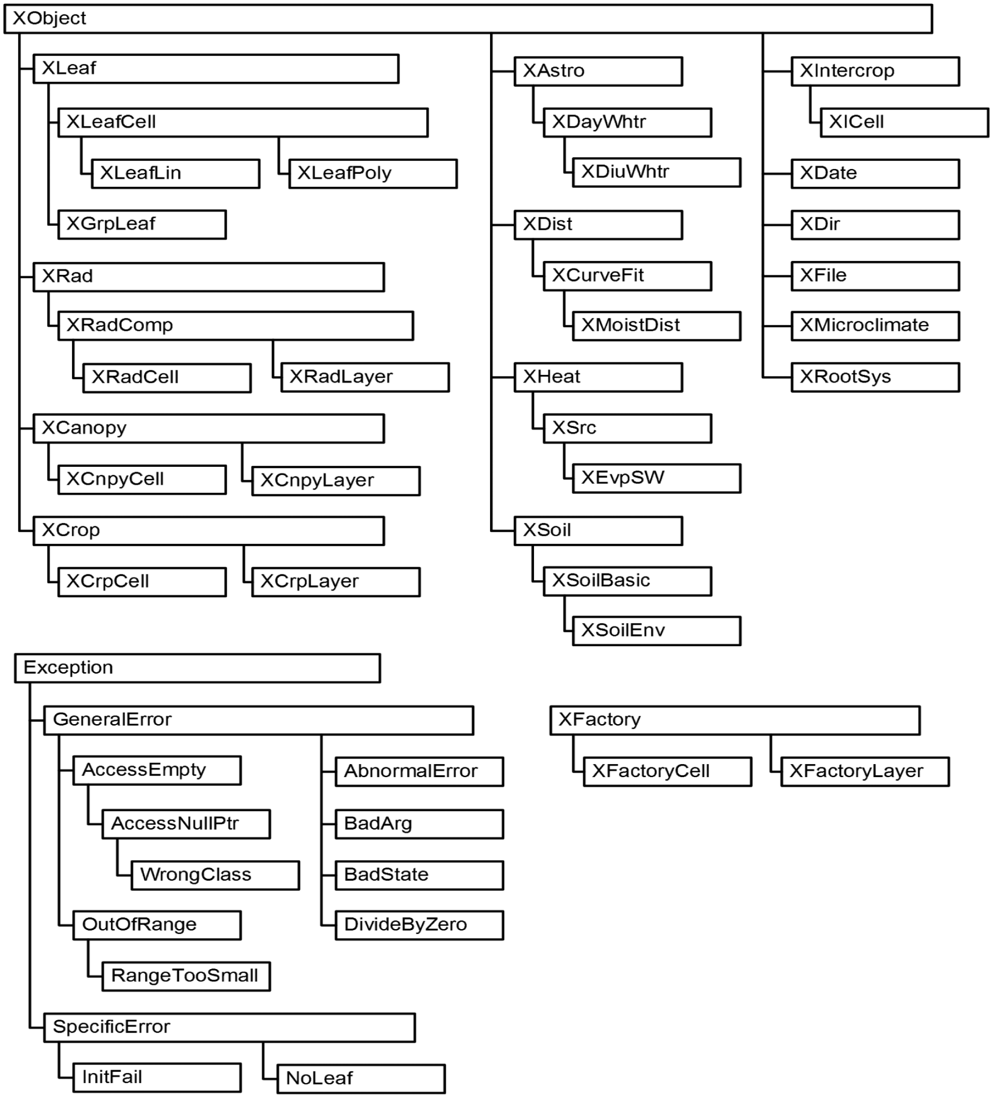

Fig. 1: Class hierarchy for the x-library

have a plant growth model; consequently, users must supply inputs, such as leaf area index (LAI), plant height, as well as the distributions of leaf azimuth and inclination density, for the canopy architecture. Nevertheless, users can implement their own plant growth model and incorporate it into x-library without having to modify the original code to achieve model compatibility because x-library is object-oriented and designed to be reusable and extendible.

The main x-library classes are derived from the abstract base class XObject. This particular base class is not useful by itself, but serves to support polymorphism and provide several common and basic services, such as object persistence, cloning and copying, to its derived classes.

TABLE 2
Classes used specifically by the (a) 1-D radiation model, and (b) 2-D radiation model

a) 1-D Radiation Model

Group	Sub-group	Classes
Intercrop		XIntercrop
Crop		XCrpLayer
Canopy		XCnpyLayer
Leaf		XGrpLeaf
Radiation		XRadLayer
Support	Factory	XFactoryLayer

b) 2-D Radiation Model

Group	Sub-group	Classes
Intercrop		XICell
Crop		XCrpCell
Canopy		XCnpyCell
Leaf		XLeafCell, XLeafLin, XLeafPoly
Radiation		XRadCell
Support	Factory	XFactoryCell

Weather

The weather classes handle daily and diurnal meteorological properties, which include solar radiation, air temperature, vapour pressure and wind speed. These weather data are usually stored in a pre-defined format in text files. Given the day and hour of simulation, as well as the latitude of the sites, these classes can calculate certain properties such as the solar position and day length.

Certain meteorological properties (such as solar position, solar declination, day length, time of sunrise and sunset, as well as total daily solar irradiance and its partitioning into direct and diffuse solar radiation components) are calculated according to the equations from Spitters (1986), Kropff (1993), and Campbell and Norman (1998).

Plant-radiation Regime

The x-library implements two kinds of plant-radiation regime models [a one-dimensional (1-D), and a two-dimensional (2-D) model], where the former models the plant-radiation regime in one dimension and irradiance varies only vertically, whereas the latter models the plant-radiation regime in two dimensions, where irradiance varies both vertically and horizontally. The 1-D and 2-D solar radiation models were specifically implemented in the XRadLayer and XRadCell radiation classes, respectively. However, both of these classes are supported by a specific set of other classes, as shown in Table 2. To perform its calculations, XRadLayer requires information from the intercrop base class (XIntercrop), as well as from the derived classes of crop (XCrpLayer), canopy (XCnpyLayer), and leaf (XGrpLeaf). Likewise, the radiation calculations in the XRadCell class require information that is only available from the derived classes of the intercrop (XICell), crop (XCrpCell), canopy (XCnpyCell), and leaf (XLeafCell, XLeafLin and XLeafPoly).

One-dimensional (1-D) Modelling

In the 1-D radiation modelling, the canopy architecture or foliage distribution must first be known. Information regarding canopy architecture is calculated in the crop, canopy and leaf classes, and passed on to the XRadLayer radiation class. The foliage distribution of a crop is characterised mathematically using the G-function defined by Ross and Nilson (1965), as:

$$G(\theta,\varphi) = \frac{1}{2\pi}\int_0^{2\pi}\int_0^{\pi/2} g(\theta_L,\varphi_L)\,|\cos r_L r|\,\sin\theta_L\, d\theta_L\, d\varphi_L \quad [1]$$

where cos $r_L r$ is the cosine angle between the leaf normal direction r_L and sun direction r, and is calculated by:

$$\cos r_L r = \cos\theta\cos\theta_L + \sin\theta\sin\theta_L\cos(\varphi - \varphi_L) \quad [2]$$

where (θ, ϕ) is the solar position (inclination and azimuth, respectively), and (θ_L, ϕ_L) is the leaf position (inclination and azimuth, respectively). Equation (1) is numerically integrated using the equation given by Lemeur (1973a, b):

$$G(\theta,\varphi) \approx \sum_{j=1}^{12}\left\{\sum_{i=1}^{16}\,[G'(\varphi_L)]_{\frac{\pi}{8}(i-1)}^{\frac{\pi}{8}i} \quad \theta \right. \quad [3]$$
$$\left. \cdot[G'(\theta_L)]_{\frac{\pi}{24}(j-1)}^{\frac{\pi}{24}j}\cdot[|\cos r_L r|]_{\frac{\pi}{24}(j-1),\frac{\pi}{8}(i-1)}^{\frac{\pi}{24}j,\frac{\pi}{8}i}\right\}$$

where π/2 (total inclination range) is divided into 12 equal successive parts, and 2π (total azimuth range) into 16 parts; G'(θ_L) is the cumulative distribution function of the leaf normal inclination, and G'(ϕ_L) is the cumulative distribution function of the leaf normal azimuth.

G(θ, ϕ) is corrected to account for the radiation scattering by leaves and when the leaves are not randomly distributed, but strongly clumped along planting rows, it is thus:

$$\hat{G}(\theta,\varphi) = G(\theta,\varphi)\cdot\sqrt{1-\sigma}\cdot\Omega(\theta) \quad [4]$$

where $\hat{G}(\theta,\varphi)$ is the corrected G-function; σ is the leaf scattering coefficient of radiation, and Ω(θ) is the clumping factor (Tournebize & Sinoquet, 1995; Campbell & Norman, 1998) which is determined empirically by:

$$\Omega(\theta) = \frac{\Omega_0}{\Omega_0 + [1-\Omega_0]\exp[-2.2\theta^{3.8-0.46\varepsilon}]} \quad [5]$$

where

$$\Omega_0 = \frac{\ln\left[f_c\exp\left(-k\frac{L}{f_c}\right) + (1-f_c)\right]}{-kL} \quad [6]$$

and ε is the ratio of plant height to width; and f_c is the fractional canopy cover which is the fraction per unit ground area occupied by canopy cover and is approximated by taking the ratio of canopy width to row spacing (Campbell & Norman, 1998). Finally, extinction coefficient k is related to the G-function by:

$$k = \frac{G(r)}{\cos\theta} \quad [7]$$

(Ross & Nilson, 1965; Lemeur, 1973a; Goudriaan, 1988).

Information on the G-function, leaf area index, extinction coefficient, leaf scattering coefficient for radiation, and clump factor is passed to the XRadLayer class. In the XRadLayer class, direct radiation within the mixed canopies with the n number of crops is calculated by:

$$I_{dr} = (1-\rho)I_{0,dr}\exp\left(-\sum_{j=1}^{n}\frac{\hat{G}(\theta,\varphi)_j}{\cos\theta}L_j\right) \quad [8]$$

where $I_{0,dr}$ is the amount of direct radiation above canopy; L_j is the leaf area index of the crop species j; and p is the mean canopy reflection coefficient calculated by:

$$\rho = \frac{(1-\sqrt{1-\bar{\sigma}})}{(1+\sqrt{1-\bar{\sigma}})}\times\frac{2}{(1+1.6\cos\theta)} \quad [9]$$

where $\bar{\sigma}$ is the mean leaf scattering coefficient of radiation for all crops (Goudriaan, 1977, 1988). Diffuse radiation within the mixed canopies is calculated by:

$$I_{df} = (1-p)\int_0^{2\pi}\int_0^{\pi/2} B(\theta,\varphi)\,\exp\left[-\sum_{j=1}^{n}\frac{\hat{G}(\theta,\varphi)_j}{\cos\theta}L_j\right]\cos\theta\,\sin\theta\,d\theta\,d\varphi \quad [10]$$

and integrated numerically using the 5-point Gaussian method (Mathews, 1987; Goudriaan, 1988).

The total amount of direct radiation A_{dr} and the diffuse radiation A_{df} intercepted by all crops are calculated by:

$$A_{dr} = I_{0,dr} - I_{dr}$$
$$A_{df} = I_{0,df} - I_{df} \quad [11]$$

The amount of radiation captured by crop species i is then calculated by:

$$A_{dr,c,i} = A_{dr}\cdot\omega_i$$
$$A_{df,c,i} = A_{df}\cdot\omega_i \quad [12]$$

where $A_{dr,c,i}$ and $A_{df,c,i}$ are the amount of direct and diffuse radiation captured by the crop species i, respectively, and ω_i is determined by:

$$\omega_i = \frac{k_i\,L_i\cdot\sqrt{1-\sigma_i}}{\sum_{j=1}^{n}\left[k_j\,L_j\cdot\sqrt{1-\sigma_j}\,\right]} \quad [13]$$

where k_i and σ_i are the leaf extinction coefficient and the leaf scattering coefficient of radiation, respectively, for crops species i (Tournebize & Sinoquet, 1995).

Two-dimensional (2-D) Modelling

XRadCell radiation class implements the radiation modelling in two dimensions according to the method by Sinoquet and Bonhomme (1992). However, XRadCell collaborates closely its support classes: XICell intercrop class, XCrpCell crop class, XCnpyCell canopy class, and the leaf classes XLeafLin and XLeafPoly. The XICell intercrop class will divide the canopy space into a set of contiguous rectangular cells, forming a two-dimensional grid network, perpendicular to the planting row direction (*Fig. 2*). The aerial space from the soil surface to the canopy top is divided into N_z horizontal layers of thickness E_z, and N_x vertical sections of thickness E_x, where only E_z and E_x are specified by users. N_x and N_z are then calculated by XICell depending on the size of the canopy space, and the given attributes E_z and E_x.

2-D radiation modelling is detailed and complex because the properties of individual leaves must be known (such as the leaf position, leaf inclination and azimuth, and leaf arch or curvature). The properties of each leaf in the crop canopy are consequently represented by the leaf classes XLeafLin and XLeafPoly. The difference between these two leaf classes is that XLeafPoly is for leaves that arch according a second-degree polynomial curve (such as the maize leaves in *Fig. 2*), and XLeafLin is for the leaves that incline at a constant angle (such as the sunflower leaves in *Fig. 2*; this leaf arch type is more common).

In the XRadCell class, the probability $\underline{P_k}$ of the total radiation intercepted within the k-th cell visited by a single beam is calculated by:

$$P_k = \left[\prod_{c=1}^{k-1}\exp\left(-\sum_{j=1}^{n}G_{jc}(r)\cdot\rho_{f,jc}\cdot s_c\cdot\sqrt{1-\sigma_j}\right)\right]\cdot\left[1-\exp\left(-\sum_{j=1}^{n}G_{jk}(r)\cdot\rho_{f,jk}\cdot s_k\cdot\sqrt{1-\sigma_j}\right)\right] \quad [14]$$

where the multiplicative series c=1 to ($\underline{k}$-1) represents every cell visited sequentially by the beam in reaching the target cell k; $G_{jc}(r)$ is the G-function for the j-th crop in the c-th cell; $\rho_{f,jc}$ is the leaf area density for the j-th crop in the c-th cell; s_c is the beam path length in the c-th cell; σ_j is the leaf scattering coefficient for crop species j; and n is the total number of crops (Tournebize & Sinoquet, 1995). Consequently, the fraction of the total radiation captured by crop species i in the k-th cell F_{ki} is determined by:

$$F_{ki} = P_k\cdot\omega_{ki} \quad [15]$$

where ω_{ki} is determined, similar to Equation (13), by:

$$\omega_{ki} = \frac{k_{ki}\,L_{ki}\cdot\sqrt{1-\sigma_i}}{\sum_{j=1}^{n}\left[k_{kj}\,L_{kj}\cdot\sqrt{1-\sigma_i}\right]} \quad [16]$$

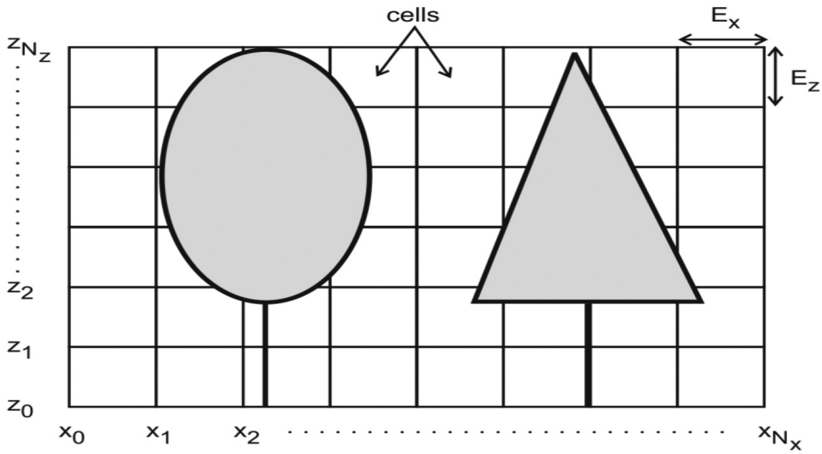

Fig. 2: The 2-D radiation model. The canopy space of an intercropping system is divided into a network of cells, perpendicular to the planting row direction.

where k_{ki} and L_{ki} are the extinction coefficient and the leaf area index of the crop species i in k-th cell, respectively.

Meanwhile, Eq. (14) shows that three properties must be calculated in each cell in the grid network: 1) the G-function, 2) the leaf area density, and 3) the distance travelled within the cell by a single beam. The plant profile method adapted from Stewart and Dwyer (1993) was used to mathematically project a three-dimensional plant architecture to a representative two-dimensional plane, perpendicular to the planting row direction. The plant profile method is implemented in the XCrpCell crop class and XCnpyCell canopy class. If a crop has large or long leaves (like sunflower or maize), a leaf may not lie entirely within a cell; instead, several cells may encompass a single leaf. Thus, XCrpCell and XCnpyCell classes are also to determine the leaf portion or section that is encompassed by a given cell. This is done so to ensure that the calculations on the G-function and leaf area density in a given cell are only based on that encompassed leaf section.

The XRadCell class calculates the distance travelled within a cell by a single beam based on the method discussed in Allen (1974), Gijzen and Goudriaan (1989), and Sinoquet and Bonhomme (1992). In *Fig. 3*, for example, a beam enters the cell at A (x_0, z_0, y_0) described by elevation angle $\underline{\beta}$ to horizontal and at $\underline{\alpha}$ angle from the planting row, and the beam reaches point C at the xz-plane, or (x, z, 0). Thus, it follows that:

$$\frac{\sin\beta}{\cos\beta\sin\alpha} = \frac{CD}{BD} = \frac{z - z_0}{x - x_0} \qquad [17]$$

(Sinoquet and Bonhomme, 1992). The 2-D model is able to track the course of the beam travel within the canopy space because the beam travel must satisfy Equation (17). The beam path length within a cell $\underline{s_c}$ is computed by calculating the intersections between the beam path and cell boundaries, i.e. by making x = vertical cell boundary, or z = horizontal cell boundary, and then computing the path length s_c in the c-th cell (Sinoquet & Bonhomme, 1992).

Note that a beam with the same inclination and azimuth angle can enter at any point on the cell. Thus, the limit on the number of beams to be "pushed" into each cell can be determined by several pre-trial runs to obtain the minimum number of beams that can be used without sacrificing accuracy. Each computed s_c is then substituted into Equation (14) to determine P_k so that the mean probability of the intercepted radiation for beams coming from (β, α) can finally be calculated ($\bar{P}_k$). Meanwhile, direct radiation intercepted by all the crops within cell k is calculated as:

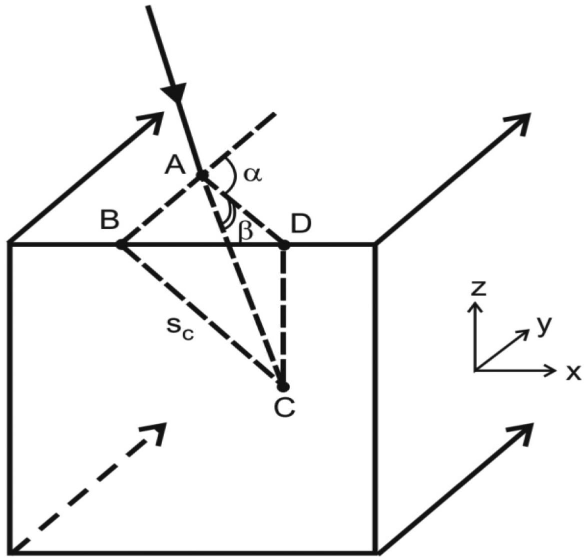

Fig. 3: Coordinate system to track the beam travel course within a cell

$$A_{dr,k} = (1-p)\cdot I_{0,dr}\cdot \bar{P}_k \quad [18]$$

The diffuse radiation intercepted by all the crops within cell k is calculated as:

$$A_{df,k} = (1-p)\sum_{\Omega=1}^{n} I_{0,df(\Omega)}\cdot \bar{P}_{k(\Omega)} \quad [19]$$

where $I_{0,df(\Omega)}$ is the incident diffuse radiation coming from direction $\underline{\Omega}$ under a UOC or SOC sky. To determine $I_{0,df(\Omega)}$, the sky is divided into x equal number of inclination intervals and y equal number azimuth intervals.

Evapotranspiration

The heat class XEvpSW, derived from XSrc and XHeat, determines various heat fluxes in the intercropping system according to the SW equation extended by Wallace (1997). XEvpSW calls the weather, radiation, intercrop and soil classes to retrieve the required information for the various heat flux calculations in the system. In the XEvpSW class, the total latent heat flux of the system with n crops is given by:

$$\lambda E = \sum_{i=1}^{n} C_{c,i}PM_{c,i} + C_s PM_s \quad [20]$$

where

$$PM_{c,i} = \frac{\Delta A + \{\rho c_p D - \Delta r_a^{c,i}(A - A_{c,i})\}/(r_a^a + r_a^{c,i})}{\Delta + \gamma\{1 + r_s^{c,i}/(r_a^a + r_a^{c,i})\}}$$

$$PM_s = \frac{\Delta A + \{\rho c_p D - \Delta r_a^s(A - A_s)\}/(r_a^a + r_a^s)}{\Delta + \gamma\{1 + r_s^s/(r_a^a + r_a^s)\}} \quad [21]$$

$$C_{c,i} = \left(1 + \frac{1/R_s + \sum_{j=1,j\neq i}^{n} 1/R_{c,j}}{\frac{1}{R_{c,i}} + \frac{1}{R_a}}\right)^{-1}$$

$$C_s = \{1 + R_s R_a / R_c(R_s + R_a)\}^{-1} \quad [22]$$

$$R_{c,i} = (\Delta + \gamma)\, r_a^{c,i} + \gamma\, r_s^{c,i}$$

$$R_a = (\Delta + \gamma) r_a^a \quad [23]$$

$$R_s = (\Delta + \gamma)\, r_a^s + \gamma\, r_s^s$$

where c_p is the specific heat of water at constant pressure (4182 J kg^{-1} K^{-1}); D is the vapour pressure deficit, or ; $e_s(T_r) - e_r$ Δ is the mean rate of change of saturated vapour pressure with temperature, or $[e_s(T_r) - e_s(T_0)]/(T_r - T_0)$; γ is the psychometric constant (0.658 mb K^{-1}); A and A_s are the total energy available to the system and soil, respectively, and $A_{c,i}$ is the amount of energy available to crop i, so that:

$$A_{c,i} = F_i R_n \quad [24]$$

where F_i is the fraction of radiation intercepted by crop species i (obtained from the radiation classes). Thus, F_i can be regarded as the link between radiation and evapotranspiration models. The energy available to the soil A_s is:

$$A_s = (R_n - G) \cdot \left(1 - \sum_{i=1}^{n} F_i\right) \quad [25]$$

And the heat flux into the soil G is calculated by:

$$G = 0.35 \cos\theta \cdot R_n^s \quad [26]$$

(Kustas & Norman, 1999a, 1999b).

The partitioning of the various latent heat fluxes is determined from the total latent heat flux λE which is the sum of all latent heat fluxes in the intercropping system, or in a two-crop intercropping system:

$$\lambda E = \lambda E_s + \lambda E_{c,1} + \lambda E_{c,2} = \frac{\Delta A_s + (\rho c_p D_0)/r_a^s}{\Delta + \gamma(1 + r_s^s/r_a^s)} + \frac{\Delta A_{c,1} + (\rho c_p D_0)/r_a^{c,1}}{\Delta + \gamma(1 + r_s^{c,1}/r_a^{c,1})} + \frac{\Delta A_{c,2} + (\rho c_p D_0)/r_a^{c,2}}{\Delta + \gamma(1 + r_s^{c,2}/r_a^{c,2})} \quad [27]$$

where D_0 is the vapour pressure deficit at the canopy source height, or

$$D_0 = D + \frac{r_a^a}{\rho c_p}\{\Delta A - (\Delta + \gamma)\lambda E\} \quad [28]$$

The SW model required several resistance components to be known. These include r^a_a (resistance between mean canopy flow and reference height); $r^{c,i}_s$ (bulk stomatal resistance); $r^{c,i}_a$ (bulk canopy boundary layer resistance); r^s_a (resistance between soil and mean canopy flow); and r^s_s (soil surface resistance). All components of these resistances are calculated in the heat classes, and they obtain information about the weather, crop and soil properties by calling the weather, crop and soil classes, respectively.

The aerodynamic resistance between the soil surface and the sink for momentum in the vegetation r^s_a is given by Shuttleworth and Gurney (1990), and Shuttleworth (1991) as:

$$r_a^s = \frac{h \cdot \exp(\eta)}{nK(h)} \cdot \left\{\exp\left[-\eta \cdot z_{s0}/h\right] - \exp\left[-\eta \cdot (z_0 + d)/h\right]\right\} \quad [29]$$

where h is the crop height; η is the attenuation coefficient for eddy diffusivity taken as 3.0, which is typical for most agricultural crops (Monteith, 1975); z_{s0} is the roughness length soil surface taken as 0.004 m for bare, tilled soil surface (Hansen, 1993); z_0 and d are the crop roughness length and zero displacement height, respectively; and K(h) is the eddy diffusivity at crop height h, and is calculated by:

$$K(h) = k^2 (h - d) u(z_x) / \ln\{(z_x - d)/z_0\} \quad [30]$$

where z_x is the reference height; k is the von Karman constant (0.41); and $u(z_x)$ is the wind speed at height z_x. Meanwhile, crop roughness length z_0 and zero displacement height d are calculated by:

$$d = 1.1h \ln(1 + X^{0.25}) \quad [31]$$

$$z_0 = \begin{cases} z_{s0} + 0.3hX^{0.5}; & 0 \le X \le 0.2 \\ 0.3h\left(1 - \frac{d}{h}\right); & 0.2 < X \le 1.5 \end{cases} \quad [32]$$

where $X = c_d L$; c_d is the mean drag coefficient for individual leaves (0.2); and L is the leaf area index (Choudhury & Monteith, 1988). The aerodynamic resistance between the mean canopy flow and reference height r_a^a is calculated by:

$$\hat{r}_a^a = \frac{1}{k^2 u(z_x)}\left(\ln\frac{z_x - d}{z_0}\right)^2 \quad [33]$$

$$r_a^a = \begin{cases} \hat{r}_a^a/(1 + \varepsilon)^2 & \text{stable, } \varepsilon < 0 \\ \hat{r}_a^a/(1 + \varepsilon)^{3/4} & \text{unstable, } \varepsilon > 0 \end{cases} \quad [34]$$

where

$\varepsilon = 5g(z - d) \cdot (T_0 - T_r)/u(z_x)^2 T_r$; and g is the acceleration due to gravity (9.81 ms^{-2}) (Choudhury & Monteith, 1988).

The mean boundary layer resistance of a crop $r_a^{c,i}$ over the total leaf area index L is calculated by:

$$r_a^{c,i} = \frac{50\alpha}{L} \cdot \left[\frac{w}{u(h)}\right]^{1/2} \cdot \left[1 - \exp\left(-\alpha/2\right)\right]^{-1} \quad [35]$$

where α is the wind speed attenuation coefficient within the canopy; and w is the mean leaf width (Choudhury & Monteith, 1988). The wind speed attenuation coefficient α is determined by:

$$\alpha \approx \frac{0.2Lh}{l_m} \quad [36]$$

where l_m is the canopy mixing length which is determined by assuming square leaves, or:

$$l_m = \left(\frac{3w^2}{2\pi\rho_f} \right)^{1/3} \quad [37]$$

where ρ_f is the leaf area density (Goudriaan, 1977).

A crop's stomatal resistance r_{st}^i is assumed to be related only to PAR (photosynthetically active radiation). This relationship can be described by:

$$\frac{1}{r_{st}^i} = \frac{a_1 \cdot I_{PAR}}{a_2 + I_{PAR}} \quad [38]$$

where I_{PAR} is the PAR irradiance, whereas a_1 and a_2 are the empirically-determined coefficients (Jarvis, 1976). Bulk stomatal resistance is simply determined by:

$$r_s^{c,i} = \overline{r_{st}^i} / L \quad [39]$$

(Thom, 1972; Shuttleworth, 1978; Stannard, 1993).

Soil resistance is assumed to be related only to soil moisture content according to the relationship described by:

$$r_s^s(\Theta) = r_s^s(0)\exp\left(-\varepsilon \cdot \Theta / \Theta_{sat} \right) \quad [40]$$

where $r_s^s(0)$ is the soil resistance when soil is totally dry, as determined by:

$$r_s^s(0) = \frac{\tau\, l}{\varphi_p D_v} \quad [41]$$

where D_v is the molecular diffusion coefficient (2.5×10^{-5} m^2 s^{-1}); τ is the soil tortuosity (2); l is the effective drying soil depth (0.15 m) (Choudhury & Monteith, 1988), and ϕ_p is the total soil porosity, which is calculated as:

$$\varphi_p = 1 - \frac{\text{bulk density}}{\text{particle density (2.65 Mg m}^{-3})} \quad [42]$$

and parameter ε is $1/\lambda$, where λ is the pore-size distribution index defined by Brooks and Corey (1964) as:

$$\left(\frac{\psi_e}{\psi} \right)^{\lambda} = \frac{\Theta - \Theta_r}{\Theta_{sat} - \Theta_r} \quad [43]$$

where ψ is the soil matric suction at volumetric water content Θ; ψ_e is the air entry suction; Θ_r is the residual volumetric water content; and Θ_{sat} is the volumetric water content at soil saturation.

Soil

The soil classes represent basic soil properties at a given soil depth, such as bulk density, particle-size distribution (texture), and soil water properties. Users will supply the data on the soil moisture characteristic curve, and based on this curve, properties such as porosity, air entry suction, pore-size distribution index (Brooks & Corey, 1964), and the water amount at a given suction, such as at saturation, field capacity, permanent wilting point, and air dry can be determined. Note that the x-library does not implement a soil water flow model, and consequently, users must always supply the current soil water amount. However, because the x-library is object-oriented and designed to be reusable and extendible, users can actually implement their own soil water flow model and incorporate it seamlessly into the x-library so that the current soil water amount does not have to be manually specified for each simulation run.

FILE INPUT AND OUTPUT

The x-library implements object persistence with the help from the XFile class, where the current state of an object can be stored to or retrieved from a text file. Classes supporting object persistence are those inherited from the base class XObject. Similarly, users can also create or edit the text file which can be used to enter or change an object's attributes (i.e., model parameters) because object persistence is implemented in the text form.

Data on weather are supplied in two forms; daily and hourly. The daily weather properties must include both minimum and maximum air temperatures, vapour pressure, irradiance and wind speed, and these data are stored in a pre-defined format in a text file. Meanwhile, the hourly weather properties are stored in a separate text file and listed in a pre-defined format as well, and these include the air temperature, irradiance, wind speed and vapour pressure.

Support Classes

The support classes (Table 1) are to provide minor, specialised services that are required by other classes. The XDate class, for example, deals with time, and date such as the calculation of the day of year, is used by the weather classes. The exception classes (e.g. Exception, GeneralError and SpecificError) are to support error-handling tasks, and the distributional classes XDist and XCurveFit are to store a two-dimensional array of (x, y) pairs of values. The last two classes can be used to interpolate between the stored pairs of values, where the curve-fitting in XDist is by the cubic spline method, whereas the XCurveFit uses linear interpolation. The XDir class is for directional properties, such as to store the leaf inclination and azimuth angles.

Finally, the factory classes (Gamma *et al.*, 1995) are used to ensure the correct object types are created, depending on the chosen type of radiation model. For the 1-D radiation model, the XRadLayer object must be created along with its support objects, as listed in Table 2a. Likewise for the 2-D radiation model, the XRadCell object must be created with its support objects as listed in Table 2b. Therefore, to ensure the correct object types are always created together, the XFactoryLayer and XFactoryCell objects are used for the 1-D and 2-D radiation model, respectively. These factory classes can thus be regarded as a template or mould to create proper object types. A factory object is used when an object is loaded from a file. For example, the C++ sample source code:

```
XFile fin("c:/data/input.dat");   // 1. file object
XFactoryLayer factory;            // 2. factory object
LOAD ar(&fin, &factory);          // 3. LOAD object
XMicroclimate mc;                 // microclimate object
mc.Load(ar);                      // load object now
```

Specifies loading of an object from file and requires three objects to be created first: the file object representing the data or input file, the factory object, and the LOAD object to hold both the file and factory objects. An object's Load method has two functions: 1) to create the proper objects using the factory object given, and 2) to assign values to the object's attributes from the given file object. Consequently, in the above example, after the call to its Load method, the microclimate object will have its pointers pointing to the correct type of objects, and its objects being pointed to will in turn point to the correct type of objects as well, and so on. The microclimate object will also have the proper values assigned to its attributes from the input file. After loading, the microclimate object in this example is now ready for use, and will implement the 1-D radiation model.

GUI FRONT-END FOR THE X-LIBRARY

As stated, the x-model is a graphical user interface (GUI) front-end for the x-library and it shields users from the implementation and programming details of x-library (*Fig. 4*). As a result, users of the x-model do not necessarily be programmers or they are also not required to understand the x-library framework. The x-model simplifies user input through a consistent and attractive graphical interface. The x-model runs only in the operating system Windows 95 and above, and was developed using MFC (Microsoft Foundation Classes) and compiled with Microsoft Visual C++ 2003. The x-model files occupy approximately 3 Mb disk space.

When users start the x-model, they have to choose to either use the 1-D or 2-D radiation model, after which the user will then input the

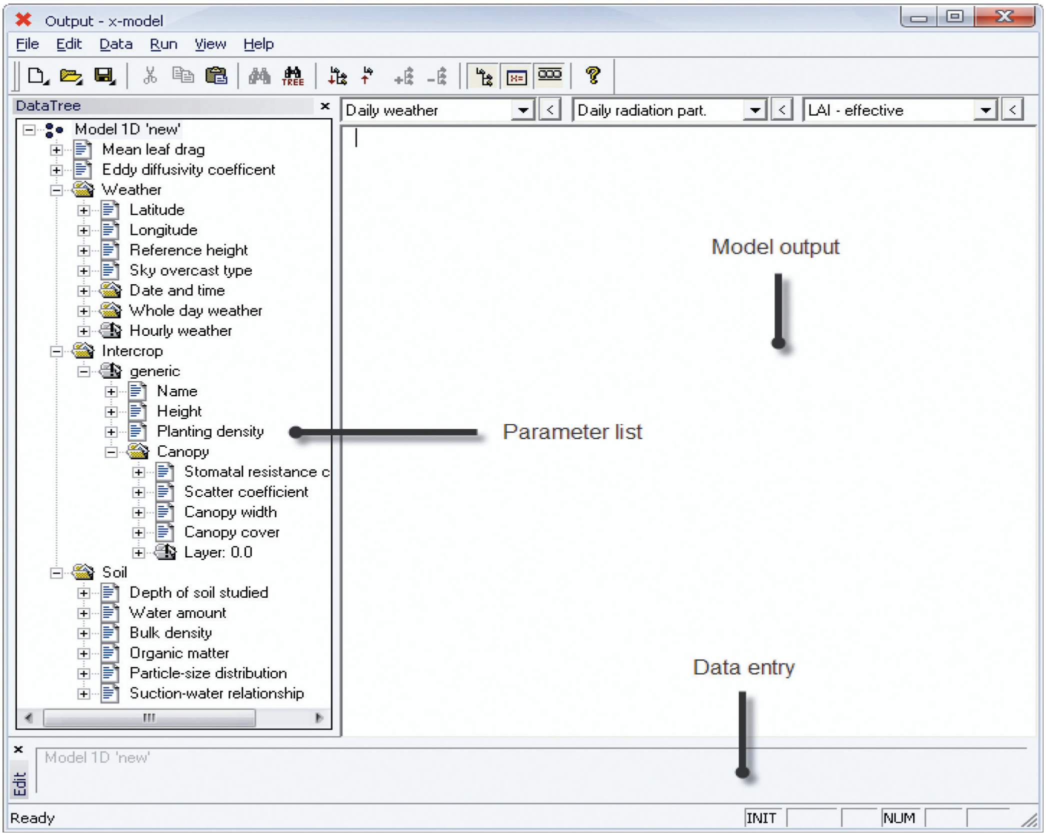

Fig. 4: x-model, the front-end for x-library

required model parameters. The model input can be further divided into four groups, which are referred to as follows: 1) weather, 2) soil, 3) intercrop, and 4) miscellaneous.

Weather Parameters

Weather inputs include site latitude, longitude, hour and day of year to determine the position of the sun, daylength, time of noon, and time of sunrise and sunset. For this purpose, two kinds of weather files are required: one containing daily weather data, and another for hourly weather data. These weather properties are needed to calculate the radiation captured and the various heat fluxes within the system. In addition, the reference height, where these weather properties are measured, is also required. Lastly, the type of sky is also required to determine the amount of diffuse radiation, where on cloudless days, the sky type is of UOC (isotropic), and on cloudy days, the sky type is assumed to be of SOC (Standard Overcast Sky). It is important to note that the UOC and SOC types are as defined by Anderson (1964).

Soil Parameters

Some examples of the soil parameters are bulk density, volumetric water content, and the soil moisture characteristic relationship (suction vs. volumetric water content). Bulk density is needed to determine the total porosity which is used to calculate soil surface resistance. The determination of the soil surface resistance also requires volumetric water content and the soil moisture characteristic relationship.

Intercrop Parameters

The intercrop parameters for the 1-D radiation model are different than the ones for the 2-D radiation model. In particular, the 1-D model requires less complex data on canopy architecture, whereas the 2-D model needs highly-refined data on the individual leaves in the canopy.

Using the 1-D Radiation Model

For the 1-D radiation model, crop height, mean leaf width, and planting density are required for each crop in the intercropping system. In particular, crop height and mean leaf width are needed mainly for aerodynamic resistance calculations. The model also requires input on the canopy properties, such as leaf area index, depth of canopy, and the stomatal resistance coefficients a_1 and a_2, as used in Equation (38). The leaf scattering coefficient is also required to account for the scattering of radiation by the canopy. Meanwhile, the leaf inclination and azimuth densities are required to calculate the G-function. Similarly, to account for the situation where leaves are not randomly distributed but clumped along rows, fractional canopy cover is required to calculate the clump factor.

Using the 2-D Radiation Model

For the 2-D radiation model, input on the planting row direction is needed so that the 2-D network of cells can be arranged perpendicular to the planting row direction. In addition, the 2-D model requires information on the width and height of a cell. For each crop in the intercropping system, crop height, planting density, and planting distance are also required. The planting distance refers to the distance the crop is planted from a reference point (*Fig. 5*). The planting distance enables the model to work out the order of the crop positions in the cell network. As shown in *Fig. 5*, the model assumes that all the individual plants of a given crop species will have identical properties. Just like the 1-D model, the 2-D model also requires information on the stomatal resistance coefficients a_1 and a_2, and the leaf scattering coefficient. Moreover, the model also requires the input on the leaf arch type, which must either be linear or second-degree polynomial

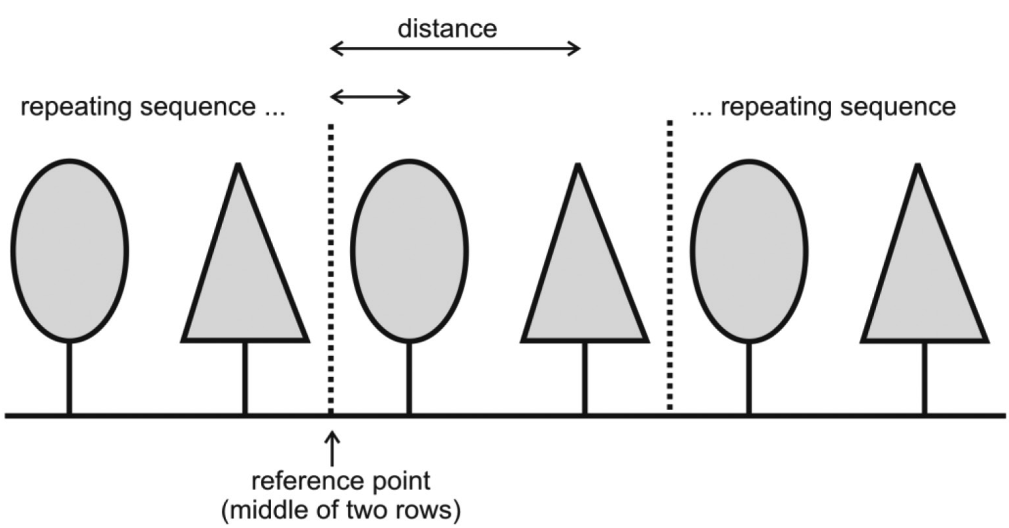

Fig. 5: Planting distance is the distance of a crop to the reference point which is the middle of two planting rows

only. Sunflower leaves, for example, can be set to a linear leaf arch, whereas maize leaves are represented as a polynomial leaf arch. Finally, the model requires information on every leaf. This means that if a plant has 20 leaves, the properties of each leaf must then be supplied as input. For each leaf, the parameters such as the leaf azimuth and inclination angle, and leaf position within the canopy are needed so that the model can determine which leaf section, if any, is encompassed by a given cell. Subsequently, the leaf area density and the G-function can be calculated for the cell based on the encompassed leaf section.

Miscellaneous Parameters

The inputs of the mean leaf drag coefficient and the eddy diffusivity coefficient are needed for heat flux calculations. In addition, the 2-D radiation model requires two other parameters, namely; the number of inclination and azimuth sky intervals, and the number of beams to be "pushed" into each cell in the network. The number of inclination and azimuth intervals correspond to how the whole sky, Ω, is divided to determine the amount of diffuse radiation coming from a particular sky region, as used in Equation (19).

Field Experiment

The simulations of the partitioning of the captured solar radiation and evapotranspiration were done for two types of crops, grown together as an intercrop, namely, maize (*Zea mays* L. cv. Hudson) and sunflower (*Helianthus annuus* L. cv. Sanluca). The simulations were then compared with the field data obtained by Teh *et al.* (2000) to test the accuracy of the solar radiation and evapotranspiration models.

A full description of the field experiment is given in Teh (2001), as only a brief description is presented in this paper. The maize and sunflower crops were sown together as an intercrop on Sonning Farm, UK (51°27' N and 0°58' W). The field size is 0.13 ha, and the planting densities of maize and sunflower were 30,000 and 15,000 plants ha^{-1}, respectively. Solar irradiance was measured using a sunfleck ceptometer (Decagon Devices Inc., Pullman, Washington, USA; Model SF-80) and plant sap flow using sap flow gauges based on the concept of stem heat balance (Kucera *et al.*, 1977). The field measurements started and ended approximately 30 and 90 days after sowing, respectively.

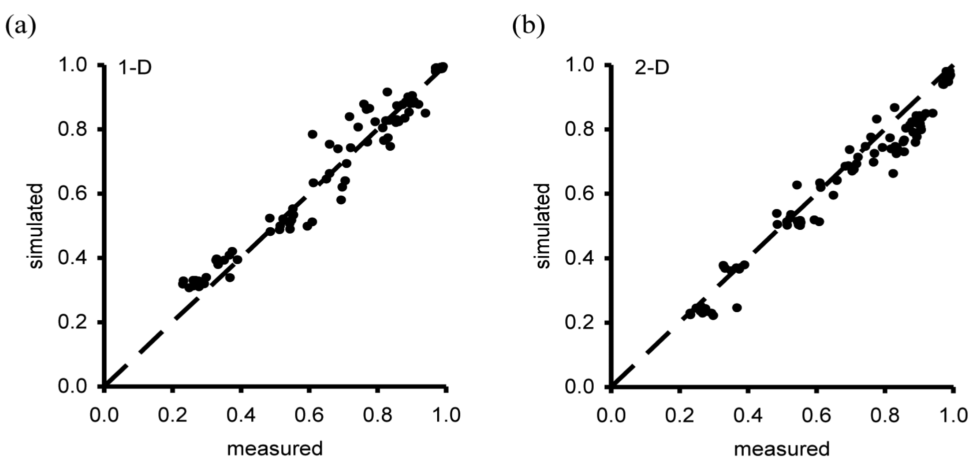

Fig. 6: Comparisons between simulated and measured fraction of total incident radiation intercepted by the maize-sunflower intercrop. Model simulations using the: (a) 1-D model and (b) 2-D model

RESULTS AND DISCUSSION

There was an overall good agreement between the simulated and the measured fraction of the total incident radiation intercepted (*Fig. 6*) for both the 1-D and 2-D radiation models. There was a close clustering of points along the 1:1 line of agreement for both the models. Meanwhile, the mean error for the 1-D radiation model was 0.01 with 95% of errors within -0.09 and 0.11. The mean error for the 2-D radiation model was -0.04 with 95% of errors within -0.13 and 0.06.

In addition, there was also an overall good agreement between the simulated and measured plant transpiration of maize and sunflower (*Fig. 7*). The mean prediction error of transpiration for both the crops was near zero (-0.01 mm h^{-1}) with 95% errors within -0.07 and 0.06 mm h^{-1}. The accuracy of the extended SW equation was not affected by the plant growth stages, but simulated transpiration during the high measured transpiration rates when the measured transpiration for maize and sunflower respectively exceeded 0.15 and 0.40 mm h^{-1}, and tended to be underestimated. This is probably because the x-library did not model the soil water processes rigorously.

Overall, the solar radiation and evapotranspiration models in the x-library showed a good simulation accuracy in modelling the following: 1) the total captured solar radiation, 2) the partitioning of total captured solar radiation between two crops of comparable heights (maize and sunflower), and 3) the partitioning of the transpiration between the two crops. As the x-library is object-oriented, modellers may find it easier to incorporate a plant growth or soil water model into the x-library.

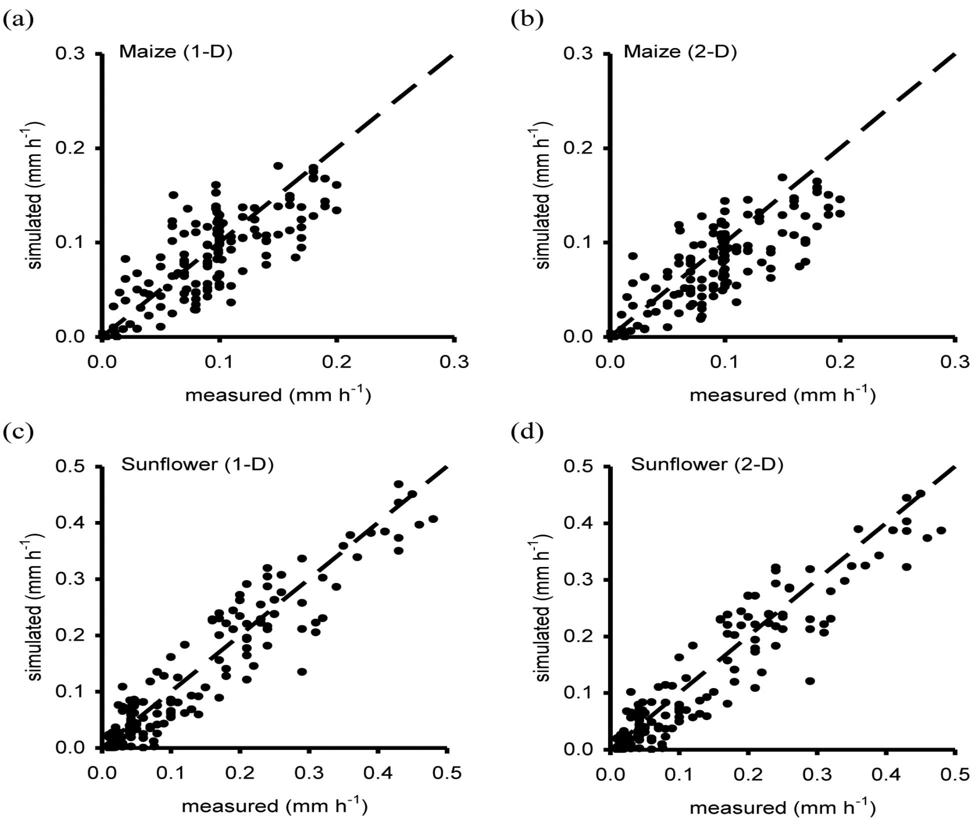

Fig.7: Comparison between simulated and measured transpiration for the maize-sunflower intercrop

CONCLUDING REMARKS

The x-library framework was designed to aid in the development of a solar radiation and evapotranspiration model because it was built using object-oriented principles. The framework simplifies the modification of the existing code and the addition of new code. The model has been found to simulate the partitioning of the captured solar radiation and evapotranspiration with an overall good accuracy. Finally, the x-model was developed as a front-end for the x-library, shielding and assisting users who are not programmers. Both the x-library and x-model are available upon request from the corresponding author.

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