



**UNIVERSITI PUTRA MALAYSIA**

**MODEL PREDICTIVE CONTROL DESIGN FOR A NONLINEAR  
FOUR-TANK SYSTEM**

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**MODEL PREDICTIVE CONTROL DESIGN FOR A NONLINEAR FOUR-TANK SYSTEM**

**By**

**SHADI ANSARPANAHI**

**Thesis Submitted to the School of Graduate Studies, Universiti Putra Malaysia, in  
Fulfilment of the Requirements for the Degree of Master of Science**

**December 2008**



## DEDICATION

I dedicate this dissertation to:

My Father; My Sun

and

My Mother; My Moon



Abstract of thesis to be presented to the Senate of Universiti Putra Malaysia in fulfilment of the requirements for the degree of Master of Science

**MODEL PREDICTIVE CONTROL DESIGN FOR A NONLINEAR FOUR-TANK SYSTEM**

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**SHADI ANSARPAHAHI**

**December 2008**

**Chairman: Samsul Bahari B. Mohd Noor, PhD**

**Faculty: Engineering**

In recent years, MPC has become a prominent advanced control technique, especially in large industrial processes. However, for enormous complexity, non-convexity and computational reasons, MPC practice and applications have been restricted to linear plants. During the last decade, many formulations have been developed for MPC formulation of linear and nonlinear, stable and unstable plants but still there remain some unsolved issues which depend on plant specifications. Instability, unfeasibility, non-convexity and lack of robustness are examples of unsolved issues.

In this thesis a control system has been designed for a highly nonlinear and non-minimum phase Four-Tank system. Then constrained optimization is employed in the MPC formulation to repair violation on boundaries. It also leads the system to work with the best performance. Additionally, the influence of most effective tuning parameters in MPC strategy has been investigated. In particular main part of the thesis has focused on performance criteria based on good reference tracking in model predictive control



domain. Regarding to investigate the performance of this algorithm and due to application of “nonlinear Four-Tank system” in control theory and industry, this system is considered as a plant to be examined under this method. The most attractive aspect of this system is; the time-varying movement of a right half plane transmission zeros across the imaginary axis. This system’s configuration makes the process difficult to control under the previous controllers. This problem appears to be one of the most important and practical designs of nonlinear system in process control.

In this thesis, besides good performance, the algorithm enjoys from relative simplicity and faster response in compare with the algorithms developed in other previous works. The problems of complexity of algorithm, non-convexity of the optimization, especially when working with nonlinear plants are the most common problems in the control design criteria. Since linear model predictive control is used instead of nonlinear model predictive control; these problems are avoided to be appeared in this work. All the results in this study show fast performance in controlling the Four-Tank system. Both of the weighting matrices are considered so that a system is fast enough smooth control signals and they are tuned till the desired performance is achieved.. Low value of prediction horizon and weighting matrices are more preferable to reduce number of free variable and avoid complexity of analysis.



Abstrak tesis yang dikemukakan kepada Senat Universiti Putra Malaysia sebagai memenuhi keperluan untuk ijazah Master Sains

**PELAKSANAAN KAWALAN PREDIKTIF MODEL LELURUS BAGI SISTEM  
EMPAT-TANGKI TAK LELURUS**

Oleh

**SHADI ANSARPAHAH**

**Disember 2008**

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Sejak kebelakangan ini, penggunaan MPC telah diperluaskan dengan teknik penggunaan kawalan termaju terutamanya di dalam industri penapisan dan juga petrokimia. Walaubagaimanapun, penggunaannya hanya terhad kepada model lurus sahaja kerana masalah kekompleksan, tidak cembung dan perkomputeran. Di dalam sistem dinamik aplikasi dunia nyata yang kebanyakannya bukan lurus, penggunaannya terpaksa dianggarkan supaya dapat digunakan ke atas konvensional MPC. Model-model seperti ini selalunya tidak mengambilkira kepada dinamik sistem sepenuhnya, terutamanya di bahagian yang tidak hampir dengan keadaan sasaran. Untuk kes-kes sebegini, model bukan lurus digunakan untuk menentukan kejituan sifat sesebuah loji. Dalam dekad terakhir ini, banyak formulasi telah dibangunkan untuk model lurus dan bukan lurus serta stabil dan tidak stabil. Akan tetapi, masih banyak lagi isu-isu yang belum diselesaikan.



Penyelidikan ini terbahagi kepada dua bahagian. Bahagian pertama meliputi gambaran keseluruhan keadaan semasa teknik kawalan prediktif model lurus, teknologi teknik kawalan prediktif model bukan lurus serta penerangan ringkas tentang prinsip kawalan prediktif dan garis panduan sebahagian teori, pengkomputeran dan juga aspek implementasi strategi kawalan. Dengan menggunakan kaedah yang diperjelaskan, model kawalan prediktif telah direka untuk sistem berterusan bukan lurus.

Dalam tesis ini kestabilan dan kekuatan algorithm yang dicadangkan dan pada kemudahan perhubungan dan kekurangan penaksiran masa berbanding dengan algorithm yang dihasilkan sebelum ini. Masalah penaksiran masa yang panjang, algorithm yang kompleks, optimum yang tidak sekata dan bahasan kestabilan yang terhad, terutama apabila pemprosesan plant yang tidak linear adalah. Masalah utama yang berlaku dalam kriteria merekabentuk kawalan. Masalah-masalah ini telah diatasi dalam projek ini. Kesimpulannya, kesemua keputusan menunjukkan pendekatan MPC dalam tesis ini akan memberi jaminan kepada kestabilan gelung tertutup. Ia menstabilkan sistem empat-tangki ketika menggunakan masalah paksaan optimum. Kamalan horizon yang rendah dan berat matrik adalah dipilih untuk mengelak berakunya masa penaksiran yang panjang dan kompleks.



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## APPROVAL

I certify that an Examination Committee met on 19<sup>th</sup> December 2008 to conduct the final examination of Shadi Ansarpanahi on her Master Degree thesis titled “Model Predictive Control Design for a Nonlinear Four-Tank System” in accordance with Universiti Pertanian Malaysia (Higher Degree) Act 1980 and Universiti Pertanian Malaysia (Higher Degree) Regulations 1981. The Committee recommends that the student be awarded the Master Degree.

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## **DECLARATION**

I declare that the thesis is my original work except for quotations and citations which have been duly acknowledged. I also declare that it has not been previously, and is not concurrently, submitted for any other degree at Universiti Putra Malaysia or at any other institutions.

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## TABLE OF CONTENTS

	<b>Page</b>
<b>DEDICATION</b>	ii
<b>ABSTRACT</b>	iii
<b>ABSTRAK</b>	v
<b>ACKNOWLEDGMENTS</b>	vii
<b>APPROVAL</b>	viii
<b>DECLARATION</b>	x
<b>LIST OF TABLES</b>	xiv
<b>LIST OF FIGURES</b>	xv
<b>LIST OF ABBREVIATION</b>	xviii
<b>CHAPTER</b>	
<b>1 INTRODUCTION</b>	1.1
<b>2 LITERATURE REVIEW</b>	2.1
2.1 Introduction	2.1
2.2 Model predictive control	2.2
2.2.1 Basic properties of MPC	2.2
2.2.2 Fundamental module	2.5
2.3 Advantages and limitation of model predictive control	2.7
2.3.1 Advantages of model predictive control	2.7
2.3.2 Limitation of model predictive control	2.8
2.4 Strategy of model predictive control	2.9
2.5 Formulation of linear model predictive control	2.10
2.5.1 CARIMA model	2.12
2.5.2 Generalized predictive control algorithm	2.12
2.5.3 Applications of model predictive control	2.14
2.6 Nonlinear model predictive control	2.14
2.7 Optimal control technique	2.16
2.8 Four-Tank system	2.17
2.9 Previous control techniques for Four-Tank system	2.24
2.9.1 The quadruple Tank process	2.25
2.9.2 Distributed MPC of an experimental four-tank	2.27
2.10 Conclusion	2.29



<b>3</b>	<b>METHODOLOGY</b>	3.1
3.1	Introduction	3.1
3.2	Algorithm overview	3.4
3.3	Utilization of the algorithm	3.6
3.3.1	Nonlinear system dynamic: modeling, estimation, constraints	3.7
3.3.2	Trajectory selection, system linearization, discretization	3.9
3.3.3	Deriving CARIMA model of the MIMO plant	3.13
3.3.4	Computing step response and free response of the plant	3.16
3.3.5	Tuning MPC parameter for QP optimization	3.21
3.3.6	Reference management	3.23
3.3.7	Computing optimal control law	3.24
3.4	Implementation in MATLAB/Simulink	3.26
3.4.1	Plant Simulation	3.27
3.4.2	Reference trajectory simulation	3.28
3.4.3	Complete block for simulation	3.28
<b>4</b>	<b>RESULTS AND DISCUSSION</b>	4.1
4.1	Introduction	4.1
4.2	Numerical analysis	4.1
4.2.1	Physical parameters	4.2
4.2.2	Matrices evaluation	4.3
4.3	Reference management	4.5
4.3.1	Reference vectors	4.5
4.3.2	Reference signals	4.6
4.4	Controller design	4.8
4.4.1	MPC design with unconstrained optimization and high value of weighting quote $\frac{\lambda_Q}{\lambda_R}$	4.8
4.4.2	MPC design with unconstrained optimization and lower value of weighting quote $\frac{\lambda_Q}{\lambda_R}$	4.11
4.4.3	MPC design with constrained optimization	4.14
4.4.4	MPC design with constrained optimization and longer Np	4.18
4.5	Performance comparison	4.19
4.5.1	Settling time comparison	4.19
4.5.2	Overshoot comparison	4.21
4.6	Summary	4.22



<b>5</b>	<b>CONCLUSION AND RECOMMENDATION FOR FUTURE WORK</b>	5.1
5.1	Conclusion	5.1
5.2	Major contribution	5.1
5.3	Accomplishments and limitations	5.2
5.4	Future work	5.3
	<b>REFERENCES</b>	R.1
	<b>APPENDICES</b>	A.1
	<b>BIODATA OF STUDENT</b>	B.1
	<b>LIST OF PUBLICATIONS</b>	P.1



## LIST OF TABLES

2.1	Location of zeros on the linearized system as a function of the flow ratios $\gamma_1$ and $\gamma_2$	2.24
2.2	Performance comparison of different control strategies	2.28
4.1	Numerical parameter used in control design of Four-Tank system	4.2
4.2	Desirable set-points for level of tanks	4.6
4.3	Settling time for different control algorithm in Tank 1	4.21
4.4	Settling time for different control algorithm in Tank 2	4.21
4.5	Percentage Overshoot for Different Control Algorithm in Tanks 1 & 2	4.22



## LIST OF FIGURES

2.1	Principle of Model Predictive Control	2.3
2.2	Hierarchy of Control System Functions in a Process Plant.	2.5
2.3	Basic MPC Module	2.6
2.4	Schematic of NMP- Control Loop	2.15
2.5	Physical Model of the Four-Tank System	2.18
2.6	The decentralized PI control for minimum phase setting	2.25
2.7	The decentralized PI control for non-minimum phase setting	2.25
2.8	Dynamic response of the Four-Tank system to a step disturbance.	2.28
3.1	The Research Flow Chart	3.3
3.2	Block Diagram of Algorithm 3.1	3.26
3.3	Plant Simulation	3.27
3.4	Reference Simulation	3.28
3.5	Complete Simulation Model	3.29
4.1	Vector of Reference 1	4.5
4.2	Vector of Reference 2	4.6
4.3	Reference Signal 1	4.7
4.4	Reference Signal 2	4.7
4.5	MPC Design with Unconstrained Optimization for Level Control in Tank 1 when $N_p=5$ and $\frac{\lambda_Q}{\lambda_R}=0.98$	4.8
4.6	MPC Design with Unconstrained Optimization for Level Control in Tank 2 when $N_p=5$ and $\frac{\lambda_Q}{\lambda_R}=0.98$	4.9





4.7	MPC Design with Unconstrained Optimization for Level Control in Tank 3 when $N_p=5$ and $\frac{\lambda_Q}{\lambda_R}=0.98$	4.9
4.8	MPC Design with Unconstrained Optimization for Level Control in Tank 4 when $N_p=5$ and $\frac{\lambda_Q}{\lambda_R}=0.98$	4.10
4.9	MPC Design with Unconstrained Optimization for Level Control in Tank 1 when $N_p=5$ and $\frac{\lambda_Q}{\lambda_R}=0.01$	4.12
4.10	MPC Design with Unconstrained Optimization for Level Control in Tank 2 when $N_p=5$ and $\frac{\lambda_Q}{\lambda_R}=0.01$	4.12
4.11	MPC Design with Unconstrained Optimization for Level Control in Tank 3 when $N_p=5$ and $\frac{\lambda_Q}{\lambda_R}=0.01$	4.13
4.12	MPC Design with Constrained Optimization for Level Control in Tank 4 when $N_p=5$ and $\frac{\lambda_Q}{\lambda_R}=0.01$	4.13
4.13	MPC Design with Constrained Optimization for Level Control in Tank 1 when $N_p=5$ and $\frac{\lambda_Q}{\lambda_R}=0.01$	4.16
4.14	MPC Design with Constrained Optimization for Level Control in Tank 2 when $N_p=5$ and $\frac{\lambda_Q}{\lambda_R}=0.01$	4.16
4.15	MPC Design with Constrained Optimization for Level Control in Tank 3 when $N_p=5$ and $\frac{\lambda_Q}{\lambda_R}=0.01$	4.17
4.16	MPC Design with Constrained Optimization for Level Control in Tank 4 when $N_p=5$ and $\frac{\lambda_Q}{\lambda_R}=0.01$	4.17
4.17	MPC Design with Constrained Optimization for Level Control in all Tank when $N_p=10$ and $\frac{\lambda_Q}{\lambda_R}=0.01$	4.18



4.18	Comparison Between Settling Time in Different Control Method	4.21
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## LIST OF ABBREVIATIONS

### Symbols

$s$	Complex variable used in Laplace transform
$z^{-1}$	Backward shift operator
$(M)_{ij}$	Element $ij$ of matrix $M$
$I_{n \times n}$ ( $n \times n$ )	Identify matrix
$I$	Identify matrix of appropriate dimension
$\widehat{x}(t+j   t)$	Expected the value of $\widehat{x}(t+j)$ with available information at time $t$
$\delta(P(\cdot))$	Degree of polynomial $P(\cdot)$
$\det(M)$	Determinant of matrix $M$
$\min_{x \in X} J(x)$	The minimum value of $J(x)$ for all value of $x \in X$

### Model Parameters and Variables

$m$	Number of input variables
$n$	Number of output variables
$u(t)$	Input variables at instant $t$
$y(t)$	Output variables at instant $t$
$x(t)$	State variables at instant $t$
$e(t)$	Discrete white noise zero mean
$d$	Dead time of the process expressed in sampling time unit
$A(z^{-1})$	Process left polynomial matrix for LMFD
$B(z^{-1})$	Process right polynomial matrix for LMFD



$C(z^{-1})$  Coloring polynomial matrix

### Controller Parameters and Variables

$N_{p_1}$	Lower value of prediction horizon
$N_{p_2}$	Higher value of prediction horizon
$N(N_p)$	Number of points of prediction horizon ( $N = N_{p_2} - N_{p_1}$ )
$N_u$	Control horizon ( $N_c$ )
$\lambda_Q$ & $\lambda_R$	Weighting factor for control increment
$\delta$	Weighting factor for predicted error
$u$	Vector of future control increments for the control horizon
$y$	Vector of predicted outputs for prediction horizon
$f$	Vector of predicted free response
$w$	Vector of future reference
$\overline{U}$	Vector of maximum allowed values of manipulated variables
$\underline{U}$	Vector of minimum allowed values of manipulated variables
$\overline{Y}$	Vector of maximum allowed values of output variables
$\underline{Y}$	Vector of minimum allowed values of output variables
$\overline{A}(z^{-1})$	Polynomial $A(z^{-1})$ manipulated by $\Delta$



## Acronyms

CARIMA	Controlled autoregressive integrated moving average
CARIMA	Controlled auto regressive moving average
CRHPC	Constrained receding horizon predictive control
DMC	Dynamic matrix control
EHAC	Extended horizon adaptive control
EPSAC	Extended prediction self-adaptive control
GMV	Generalized minimum variance
GPC	Generalized predictive control
LMFD	Left matrix fraction description
LMI	Linear matrix inequalities
LQ	Linear quadratic
LQG	Linear quadratic Gaussian
LP	Linear programming
MAC	Model algorithm control
MILP	Mixed integer linear programming
MIMO	Multi input multi output
MIP	Mixed integer programming
MIQP	Mixed integer quadratic programming
MPC	Model predictive control
NLP	Nonlinear programming
PFC	Predictive functional control
PID	Proportional integral derivative



QP	Quadratic programming
RMPCT	Robust model predictive control technology
SGPC	Stable generalized predictive control
SISO	Single input single output



## CHAPTER 1

### INTRODUCTION

#### 1.1 Preface

Due to increments of the world wide competition and major changes in marketing and business during the past decades, process industry has confronted a major change in the market. Therefore it is necessary to develop all the aspect of process control technologies such as: modeling, dynamic trajectory optimization, integrated software tools and high-performance industrial process control. Process dynamics tend to become too complex to be efficiently controlled by the current generation of control and optimization techniques. One of the advanced control technology which made a significant impact on industrial control engineering is model predictive control. The model predictive control technology is uses to steer processes closer to their physical limits in order to obtain a better economic production and to meet all the performances to attract the customers (Tyagunov, 2004). This is perhaps one of the most appealing and attractive approaches in industrial process control practice for our century. To see more examples of model predictive control application one can refer to (Clarke, 1988; Camacho and Bordons, 2004; Allgower and Zheng, 2004).



## 1.2 Model Predictive Control Technology

Model Predictive Control (MPC) , also known as moving or receding horizon control, is a feedback control scheme that has originated in industry as a real-time computer control algorithm to solve linear multi-variable problems that have constraints and time delays. It is a form of control in which the current control action is obtained by solving on-line, at each sampling instant, a finite horizon open-loop optimal control problem, using the current state of the plant as the initial state; the optimization yields an optimal control sequence and the first control in this sequence is applied to the plant (Tyagunov, 2004; Camacho and Bordons, 2004). Due to recent developments model predictive controller is suggested as a candidate to replace PID controller. It has received an ever growing interest and a great deal of attention for applications in industrial process control especially in chemical, oil and petrochemical industry. Main advantages of MPC are as follows (Tyagunov, 2004):

- 1) It can be used in most real world industry due to its ability in solving problems of multi-input multi-output (MIMO) system.
- 2) The concepts of MPC came from industry.
- 3) It is able to control a large number of processes, consist of those with non-minimum phase, long time delay, input / output constraints or open-loop unstable characteristics.
- 4) Able to handle the process constraints.





- 5) It can be applied to batch processes where the future reference signals are known.

Model predictive control of a system could be applied to process industries including chemical plants, oil refineries, petrochemical industries and etc. At first the model predictive control was offered for linear system then it is extended to nonlinear systems (Mayne and Michalska, 1990; Oliveira and M. Morari, 2000; Allgöwer et al., 1999). Linear MPC refers to a family of MPC schemes in which linear models are used to predict the system dynamics and considers linear constraints on the states and inputs and a quadratic cost function. Even if the system is linear, the closed-loop dynamics are in general nonlinear due to the presence of constraints. NMPC refers to MPC schemes that are based on nonlinear models and/or consider non-quadratic cost functional and general nonlinear constraints on the states and inputs (Camacho and Bordons, 2004).

### 1.2.1 Different Model Predictive Control Algorithms

In this section, the most popular methods of MPC are described. Model predictive control algorithms are just differing in the modeling of plant, noise and cost function for deriving optimal input. For example DMC<sup>1</sup> uses a step response to model a process. MAC<sup>2</sup> uses an impulse response of the process and etc. The most applicable and representative algorithm of model predictive control are as follows DMC, MAC, GPC<sup>3</sup>,

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<sup>1</sup> Dynamic Matrix Control

<sup>2</sup> Matrix Algorithm Control

<sup>3</sup> Generalized Predictive Control

