



**UNIVERSITI PUTRA MALAYSIA**

**EFFECTS OF THE USE OF CALCULATORS, THE POLYA  
HEURISTIC AND WORKED EXAMPLES ON PERFORMANCE IN  
LEARNING MATHEMATICS FROM A COGNITIVE LOAD  
PERSPECTIVE**

**LAWRENCE ALOYSIUS AERIA.**

**FPP 2004 27**

**EFFECTS OF THE USE OF CALCULATORS, THE POLYA HEURISTIC AND  
WORKED EXAMPLES ON PERFORMANCE IN LEARNING MATHEMATICS  
FROM A COGNITIVE LOAD PERSPECTIVE**

**By**

**LAWRENCE ALOYSIUS AERIA**

**Thesis Submitted to the School of Graduate Studies, Universiti Putra Malaysia,  
in Fulfilment of the Requirements for the Degree of Doctor of Philosophy**

**December 2004**



Abstract of thesis presented to the Senate of Universiti Putra Malaysia in fulfilment of the requirement for the degree of Doctor of Philosophy

**EFFECTS OF THE USE OF CALCULATORS, THE POLYA HEURISTIC  
AND WORKED EXAMPLES ON PERFORMANCE IN LEARNING  
MATHEMATICS FROM A COGNITIVE LOAD PERSPECTIVE**

By

**LAWRENCE ALOYSIUS AERIA**

**December 2004**

**Chairman: Associate Professor Rohani Ahmad Tarmizi, Ph.D.**

**Faculty: Educational Studies**

Cognitive load theory holds that if an instructional format reduces extraneous cognitive load and/or increases germane cognitive load during learning, as compared to another instructional format, then it will be more efficient in promoting learning, provided that the total cognitive load does not exceed the total mental resources. Based on this premise, a series of four experiments was conducted to investigate the use of calculators, the Polya heuristic and worked examples in an attempt to generate an alternative instructional format that is more instructionally efficient than the conventional one to teach Percentage to Form 1 students. It was hypothesized that the use of calculators and the use of worked examples can reduce cognitive load during learning and lead to better learning performance while the use of the Polya heuristic increases cognitive load and reduces learning performance. The results from the experiments



indicated that the use of calculators and worked examples reduced cognitive load and led to better learning performance but the use of the Polya heuristic produced zero effects. Based on the results from the experiments, an alternative instructional format, called the calcworked instructional format, that incorporated the use of calculators and the use of partial completion worked examples was generated. The calcworked instructional format was compared with the conventional instructional format and was found to be more instructionally efficient.

Abstrak tesis yang dikemukakan kepada Senat Universiti Putra Malaysia  
sebagai memenuhi keperluan untuk ijazah Doktor Falsafah

**KESAN PENGGUNAAN KALKULATOR, HEURISTIK POLYA  
DAN CONTOH PENYELESAIAN TERHADAP PRESTASI  
DALAM PEMBELAJARAN MATEMATIK DARIPADA  
SATU PERSPEKTIF BEBANAN KOGNITIF**

Oleh

**LAWRENCE ALOYSIUS AERIA**

**Disember 2004**

**Pengerusi:           Profesor Madya Rohani Ahmad Tarmizi, Ph.D.**

**Fakulti:             Pengajian Pendidikan**

Teori bebanan kognitif menegaskan bahawa jika sesuatu format pengajaran mengurangkan bebanan kognitif ekstraneous dan/atau menambahkan bebanan kognitif *germane* semasa pembelajaran, berbanding dengan sesuatu format pengajaran yang lain, maka format pengajaran tersebut akan lebih cekap dalam menggalakkan pembelajaran selagi jumlah bebanan kognitif tidak melebihi jumlah sumber mental. Berdasarkan pernyataan ini, satu siri yang mengandungi empat eksperimen telah dijalankan untuk mengkaji penggunaan kalkulator, heuristik Polya dan contoh penyelesaian supaya dapat diwujudkan satu format pengajaran alternatif yang lebih cekap berbanding format pengajaran konvensional, untuk mengajar tajuk Peratusan kepada pelajar Tingkatan 1. Hipotesis telah dibuat bahawa penggunaan kalkulator dan contoh penyelesaian dapat mengurangkan bebanan kognitif semasa



pembelajaran dan meningkatkan prestasi pembelajaran manakala penggunaan heuristik Polya akan menambahkan bebanan kognitif dan mengurangkan prestasi pembelajaran. Keputusan eksperimen menunjukkan bahawa penggunaan kalkulator dan contoh penyelesaian dapat mengurangkan bebanan kognitif dan meningkatkan prestasi pembelajaran tetapi penggunaan heuristik Polya menghasilkan kesan sifar. Berdasarkan keputusan eksperimen-eksperimen tersebut, satu format pengajaran alternatif, dinamakan format pengajaran *calcworked*, yang menggabungkan penggunaan kalkulator dan contoh penyelesaian separa lengkap telah diwujudkan. Jika dibandingkan dengan format pengajaran konvensional, format pengajaran *calcworked* didapati lebih cekap.

## ACKNOWLEDGEMENTS

Before and after I embarked on this study, I have been dependent on, beholden to and grateful for the kindness and efforts of many generous people who have helped me to make this study a reality. It is well past time to acknowledge my debts.

Like many post-graduate students who conduct studies in schools, I am first and foremost grateful to the many students, teachers, senior assistants and principals who tolerated my presence in their school and to the disruptions to their normal school routine. They were gracious in granting permission and extending their cooperation. I am overwhelmed by their generosity especially in the face of mounting work pressures and daily school assignments. I am especially grateful to the numerous students who willingly participated in the experiments.

I am grateful to the many lecturers and staff of the Faculty of Educational Studies, Universiti Putra Malaysia, without whom I would not have been able to complete this study. Foremost, I would like to thank members of my supervisory committee. Thank you Associate Professor Dr. Rohani Ahmad Tarmizi, Professor Dr. Kamariah Hj. Abu Bakar and Professor Dr. Turiman Hj. Suandi. Without your help, this study would never have been completed. I would also like to thank Associate Professor Dr. Ab. Rahim Bakar and Tuan Haji Azali Mahbar for giving me helpful pointers during my proposal defence and to Associate Professor Dr. Aida Suraya Haji Md.



Yunus for her encouragement during the presentation of my paper in the GREDEC Seminar in April 2001. I would also like to thank Puan Norkiah Mohd. Khalil and Puan Rafiah Maat both of whom have helped me in more ways than one.

Throughout this study I have solicited help from various sources outside UPM. Foremost, I would like to express my heartfelt gratitude to the many cognitive load theory researchers who have helped me. I especially would like to thank Professor Dr. John Sweller and Professor Dr. Fred Paas, both of whom not only gave me useful pointers but were also so patient in replying to my queries at various stages of my study. I am grateful to Dr. Paas for taking the time out to review the paper on this study that was presented at the National Seminar on Mathematics Education held at the De Palma Hotel in Kuala Lumpur in October 2002.

I am also grateful to members on the AERA-D Division D: Measurement and Research Methodology Listserv discussion group who responded to my queries on the methodology issues of this study. They have been extremely generous in helping me iron out some of the more thorny methodology issues.

This study would not have been possible without the validators and translators of the instruments, proof-readers and reviewers. I would like to thank all of them. I especially would like to thank Mr. Anthony Gomez,





Mr. Heng Fook Seng and Mr. Joseph Gomez for their proof-readings, reviews and comments. I am particularly indebted to Professor Margaret McLaren and Professor Ian McLaren who not only painstakingly proof-read and reviewed the many drafts of this thesis and made suggestions for improvements but also kept me believing in myself.

I have often been sustained or revived during the difficult periods of this study by many friends who listened, laughed, encouraged and offered advice. They have commented on drafts, debated key issues, dreamed up titles, suggested alternative explanations, and kept me believing that I was up to the task. Karen Lai Kuen Choo, Desmond Lee Chan Leong, Tan Aig Bee, Dr. Boon Pong Yin, Anne Minjoot, Mohanakrishnan, Dr. Wong Su Luan, Dr. Loh Sau Cheong, Gomes, Ms. Goh Soon Gaik, Dr. Chong Poh Wan, Dr. Linton Britten, Bernadette Soon Ah Nooi, Tuan Haji Habib Wagiman, Joseph Kulandai, Heng Fook Seng and all my colleagues at the International Languages Teacher Training Institute have been there for me and I thank all of them. I also would like to thank Jacqueline Lim, my cousin, for sourcing literature for me from the University of New South Wales at the initial stages of this study.

There have been countless others who have helped me somewhere along the way but whom I may have inadvertently omitted. I am grateful to all of them.



Finally to my family members, thank you for your patience and encouragement throughout this study. I especially would like to dedicate this study to the memory of my mum, Helena Lim, who provided me the most encouragement to complete this study.

I certify that an Examination Committee met on 15<sup>th</sup> December 2004 to conduct the final examination of Lawrence Aloysius Aeria on his Doctor of Philosophy thesis entitled "Effects of the Use of Calculators, the Polya Heuristic and Worked Examples on Performance in Learning Mathematics from a Cognitive Load Perspective" in accordance with Universiti Pertanian Malaysia (Higher Degree) Act 1980 and Universiti Pertanian Malaysia (Higher Degree) Regulations 1981. The Committee recommends that the candidate be awarded the relevant degree. Members of the Examination Committee are as follows:

**Othman Dato' Hj. Mohamed, Ph.D.**

Professor  
Faculty of Educational Studies  
Universiti Putra Malaysia  
(Chairman)

**Aida Suraya Haji Md. Yunus, Ph.D.**


Associate Professor  
Faculty of Educational Studies  
Universiti Putra Malaysia  
(Member)

**Habibah Elias, Ph.D.**

Professor  
Faculty of Educational Studies  
Universiti Putra Malaysia  
(Member)

**John Sweller, Ph.D.**

Professor  
Faculty of Education  
University of New South Wales  
(Independent Examiner)

  
\_\_\_\_\_  
**ZAKARIAH ABD. RASHID, Ph.D.**  
Professor/Deputy Dean  
School of Graduate Studies  
Universiti Putra Malaysia

Date: 17 MAR 2005



This thesis submitted to the Senate of Universiti Putra Malaysia and has been accepted as fulfilment of the requirement for the degree of Doctor of Philosophy. The members of the Supervisory Committee are as follows:

**Rohani Ahmad Tarmizi, Ph.D.**  
Associate Professor  
Faculty of Educational Studies  
Universiti Putra Malaysia  
(Chairperson)

**Kamariah Hj. Abu Bakar, Ph.D.**  
Professor  
Faculty of Educational Studies  
Universiti Putra Malaysia  
(Member)

**Turiman Hj. Suandi, Ph.D.**  
Professor  
Faculty of Educational Studies  
Universiti Putra Malaysia  
(Member)



---

**AINI IDERIS, Ph.D.**  
Professor/Dean  
School of Graduate Studies  
Universiti Putra Malaysia

Date: 14 APR 2005



## DECLARATION

I hereby declare that the thesis is based on my original work except for quotations and citations which have been duly acknowledged. I also declare that it has not been previously or concurrently submitted for any other degree at UPM or other institutions.

*Lawrence Aeria*

**LAWRENCE ALOYSIUS AERIA**

Date: 22<sup>nd</sup> April 2004

## TABLE OF CONTENTS

	Page
ABSTRACT	ii
ABSTRAK	iv
ACKNOWLEDGEMENT	vi
APPROVAL	x
DECLARATION	xii
LIST OF APPENDICES	xix
LIST OF TABLES	xx
LIST OF FIGURES	xxiii
<b>CHAPTER</b>	
<b>1 INTRODUCTION</b>	
Background of the Study	1
Introduction	1
Research in Mathematical Problem Solving	3
The Information Processing Theory	6
Cognitive Load Theory	8
Performance of Malaysian Lower Secondary School Students in Mathematical Problem Solving	12
The Use of Calculators	18
The Use of the Polya Heuristic	21
The Use of Worked Examples	24
Statement of the Problem	26
Purpose of the Study	29
Research Hypotheses	30
Significance of the Study	43
Limitations of the Study	46
Definition of Terms	48
<b>2 REVIEW OF RELATED LITERATURE</b>	
Introduction	55
Problem and Problem Solving in Mathematics	55
Meaning of Problem in Mathematics	55
Meaning of Problem Solving in Mathematics	59
The Information Processing Theory	62
The Modal Model of Memory	63
Sensory Buffer	64
Short Term Memory (STM)/ Working Memory	65
Long Term Memory	70
Encoding, Storage and Retrieval	72
Cognitive Load Theory	73
Overview of Cognitive Load Theory	73



Key Concepts in Cognitive Load Theory	74
Meaning of Cognitive Load	89
The Cognitive Load Construct	92
Measurement of Cognitive Load	97
Learning from the Perspective of Cognitive Load Theory	103
Expert-Novice Differences from the Perspective of Cognitive Load Theory	108
Principles of Cognitive Load Theory	112
Application of Cognitive Load Theory to Instructional Design	114
Conventional Problems and Conventional Problem Solving (Means-Ends Analysis)	118
The Instructional Techniques (Effects) Generated by Cognitive Load Theory	125
Methodology of Cognitive Load Studies	151
Cognitive Load Studies in Instructional Fields of Research	154
Recent Developments in Cognitive Load Theory Research	156
Calculators in Mathematics	164
Overview	164
Arguments For and Against the Use of Calculators	166
Research Findings On Calculators in Mathematics Education	168
Heuristics in Mathematics	173
What are Heuristics?	173
The Polya Heuristic	176
Research Findings on Heuristics	179
The Integrated Curriculum for Secondary Schools	183
Mathematics syllabus	184
Conceptual Framework of the Study	187
Summary	189
<b>3</b>	
<b>METHODOLOGY</b>	
Overview of Chapter	192
Introduction	192
Design of the Study	195
The Experimental Design	195
Overview of the Staggered Experimental Design	196
Justification for the Staggered Experimental Design	198
Design of Each Experiment	201
The Variables of the Study	206



Sample	210
Composition/Criteria	210
Sample Criteria	210
Sampling Procedure	211
Sample Size	212
Instrumentation	214
Pre-Experiment Preparation	216
Overview of Procedures for Experiments	217
Pilot Study	222
Preliminary Study (Experiment 1)	223
Development of Instrument 1 (The Selection Criteria Test)	224
Reliability of The Selection Criteria Test	227
Development of Instrument 3 (The General Guideline Lesson Plan)	229
Development of Instrument 4 (The Percentage Problems)	232
Reliability of Instrument 4 (The Percentage Problems)	235
Item Difficulty Index and Item Discrimination Index of Instrument 4	236
Development of Instrument 5 (The Worked Examples)	239
Development of Instrument 6 (The Paas Mental Effort Rating Scale)	240
Reliability of Instrument 6 (The Paas Mental Effort Rating Scale)	241
Details of the Procedures for the Experiments	243
Experiment 1	243
Experiment 2	253
Experiment 3	257
Experiment 4	260
Summary	267
<b>4 RESULTS</b>	
Introduction	269
Experiment 1	269
Effects of the Use of Calculators, the Polya Heuristic and Worked Examples on Performance and Mental Effort in the Acquisition Phase	271
Effects of the Use of Calculators, the Polya Heuristic and Worked Examples on Performance and Mental Effort in the Test Phase	277
Instructional Efficiency of the Conventional, Calculator, Heuristic and Worked Example Instructional Formats	281



	Correlation Between Cognitive Load and the Performance Variables	285
	Research Hypotheses	286
	Post-hoc Power Analyses	303
Experiment 2		304
	Effects of the Use of Calculators, the Polya Heuristic and Worked Examples on Performance and Mental Effort in the Acquisition Phase	305
	Effects of the Use of Calculators, the Polya Heuristic and Worked Examples on Performance and Mental Effort in the Test Phase	311
	Instructional Efficiency of the Conventional, Calculator, Heuristic and Worked Example Instructional Formats	320
	Correlation Between Cognitive Load and the Performance Variables	322
	Research Hypotheses	324
	Post-hoc Power Analyses	342
Experiment 3		342
	Effects of the Use of the Alternative (Calcworked) Instructional Format on Performance and Mental Effort in the Acquisition Phase	343
	Effects of the Use of the Alternative (Calcworked) Instructional Format on Performance and Mental Effort in the Test Phase	345
	Instructional Efficiency of the Conventional and Calcworked Instructional Formats	349
	Correlation Between Cognitive Load and the Performance Variables	350
	Research Hypotheses	352
	Post-hoc Power Analyses	367
Experiment 4		367
	Effects of the Use of the Alternative (Calcworked) Instructional Format on Performance and Mental Effort in the Acquisition Phase	368
	Effects of the Use of the Alternative (Calcworked) Instructional Format on Performance and Mental Effort in the Test Phase	370
	Instructional Efficiency of the Conventional and Calcworked Instructional Formats	374
	Correlation Between Cognitive Load and the Performance Variables	375



	Research Hypotheses	377
	Post-hoc Power Analyses	386
	Summary	387
<b>5</b>	<b>DISCUSSION</b>	
	Introduction	388
	Experiment 1	388
	Effects of the Use of Calculators on Performance in Learning	389
	Effects of the Use of the Polya Heuristic on Performance in Learning	390
	Effects of the Use of Worked Examples on Performance in Learning	391
	Summary of Research Findings of Experiment 1	392
	Experiment 2	393
	Effects of the Use of Calculators and Worked Examples on Performance in Learning	395
	Effects of the Use of the Polya Heuristic on Performance in Learning	396
	Summary of Research Findings of Experiment 2	398
	Experiment 3	399
	Effects of the Calworked Instructional Format Summary of Research Findings of Experiment 3	401
	Experiment 4	402
	Effects of the Calworked Instructional Format under Classroom Conditions Summary of Research Findings of Experiment 4	403
	Summary	404
	Summary	406
<b>6</b>	<b>SUMMARY, CONCLUSIONS, IMPLICATIONS AND RECOMMENDATIONS</b>	
	Introduction	409
	Overview of the Study	409
	Summary of the Purpose of the Study	409
	Summary of Research Methodology	410
	Summary of Research Findings	412
	Research Question No. 1	412
	Effects of the Use of Calculators	412
	Effects of the Use of the Polya Heuristic	417
	Effects of the Use of Worked Examples	418
	Research Question No. 2	422
	Effects of the Use of the Alternative Instructional Formats	422
	Conclusions of the Study	424



Implications of the Study	425
Theoretical Implications	425
Practical Implications	428
Recommendations	430
The Use of Calculators	430
The Use of the Polya Heuristic	432
The Use of Worked Examples	433
Recommendations for Further Research	434
<b>BIBLIOGRAPHY</b>	<b>436</b>
<b>APPENDICES</b>	<b>466</b>
<b>BIODATA OF THE AUTHOR</b>	<b>535</b>



## LIST OF APPENDICES

Appendix		Page
A	Ujian Pemilihan Kriteria (Instrumen 1) Selection Criteria Test	466
B	Record Keeping And Observational Form (Instrumen 2)	473
C1	Rancangan Mengajar untuk Eksperimen 1,2 & 3 (Instrumen 3) General Guideline Lesson Plan For Experiments 1, 2 & 3	475
C2	Rancangan Mengajar untuk Eksperimen 4 (Instrumen 3) General Guideline Lesson Plan For Experiments 4	485
C3	Rumus-Rumus	491
D1	Instrumen 4 bagi Eksperimen 1 (Soalan-Soalan Peratus) (The Percentage Questions)	492
D2	Instrumen 4 bagi Eksperimen 2 & 3 (Soalan-Soalan Peratus) (The Percentage Questions)	496
D3	Instrumen 4 bagi Eksperimen 4 (Soalan-Soalan Peratus) (The Percentage Questions)	500
E1	Instrumen 5 bagi Eksperimen 1 (Worked Examples)	501
E2	Instrumen 5 bagi Eksperimen 2 (Worked Examples)	505
E3	Instrumen 5 bagi Eksperimen 3 (Worked Examples)	511
E4	Instrumen 5 bagi Eksperimen 4 (Worked Examples)	517
F	Instrumen 6 (Skala Kadar Daya Mental) The Paas Mental Effort Rating Scale	525
G	Panel of Content Validators And Translators	527
H	Letters of Permission	531



## LIST OF TABLES

Table		Page
1.1	Differences between conventional instructional practices and cognitive load generated instructional practices	11
1.2	Performance of Malaysian students for PMR Mathematics Paper 1 (1993-1996)	12
1.3	Analysis of students' performance on percentage problems for PMR Mathematics Paper 1 (1993-1996)	14
1.4	The four-stage Polya heuristic	22
3.1	Rotation of control and treatment groups for Experiment 3 according to time allocation	205
3.2	The independent variable (instructional format) for Experiment 1 and Experiment 2	207
3.3	The independent variable (instructional format) for Experiment 3 and Experiment 4	208
3.4	Dependent variables (Measures of performance)	209
3.5	The recommended alpha range (DeVelli's Scale)	229
3.6	Difficulty and Discriminant indices of the acquisition and test items of instrument 4 in Experiment 1	237
3.7	Reliability estimates of the Paas Mental Effort Rating scale	243
3.8	Summary of Experiment 1	244
3.9	Summary of Experiment 2	255
3.10	Summary of Experiment 3	258
3.11	Summary of Experiment 4	261
4.1	Means and standard deviations of the dependent variables for the control and treatment groups for the acquisition phase of Experiment 1	273



4.2	One-way analysis of variance on the dependent variables for the acquisition phase of Experiment 1	274
4.3	Tukey post-hoc comparisons on the dependent variables for the acquisition phase of Experiment 1	275
4.4	Means and standard deviations of the dependent variables for the control and treatment groups for the test phase of Experiment 1	278
4.5	One-way analysis of variance on the dependent variables for the test phase of Experiment 1	279
4.6	Tukey post-hoc comparisons on time per test problem for the test phase of Experiment 1	280
4.7	Relative condition efficiency index of the control and treatment groups for Experiment 1	284
4.8	One-way analysis of variance on the relative condition efficiency index for Experiment 1	285
4.9	Pearson product moment correlation between mental effort and the performance variables of Experiment 1	286
4.10	Means and standard deviations of the dependent variables for the control and treatment groups for the acquisition phase of Experiment 2	306
4.11	One-way analysis of variance on the dependent variables for the acquisition phase of Experiment 2	307
4.12	Tukey post-hoc comparisons on the dependent variables for the acquisition phase of Experiment 2	308
4.13	Means and standard deviations of the dependent variables for the control and treatment groups for the test phase of Experiment 2	312
4.14	One-way analysis of variance on the dependent variables for the test phase of Experiment 2	313
4.15	Tukey post-hoc comparisons on the dependent variables for the test phase of Experiment 2	314
4.16	Relative condition efficiency index of the control and treatment groups for Experiment 2	321

4.17	One-way analysis of variance on the relative condition efficiency index for Experiment 2	321
4.18	Tukey post-hoc comparisons on the relative condition efficiency index for Experiment 2	321
4.19	Pearson product moment correlation between mental effort and the performance variables of Experiment 2	323
4.20	Means and standard deviations of the dependent variables for the conventional and calcworked groups and results of the <i>t</i> -test analyses for the acquisition phase of Experiment 3	344
4.21	Means and standard deviations of the dependent variables for the conventional and calcworked groups and results of the <i>t</i> -test analyses for the test phase of Experiment 3	346
4.22	Relative condition efficiency index of the conventional and calcworked groups and results of the <i>t</i> -test analyses for Experiment 3	350
4.23	Pearson product moment correlation between mental effort and the performance variables of Experiment 3	351
4.24	Means and standard deviations of the dependent variables for the conventional and calcworked groups and results of the <i>t</i> -test analyses for the acquisition phase of Experiment 4	369
4.25	Means and standard deviations of the dependent variables for the conventional and calcworked groups and results of the <i>t</i> -test analyses for the test phase of Experiment 4	371
4.26	Relative condition efficiency index of the conventional and calcworked groups and results of the <i>t</i> -test analyses for Experiment 4	374
4.27	Pearson product moment correlation between mental effort and the performance variables of Experiment 4	376



## LIST OF FIGURES

Figure		Page
2.1	The structure of memory	64
2.2	Schema of a car	76
2.3	Total cognitive load	92
2.4	Learning facilitated (mental resources exceed total cognitive load)	93
2.5	Learning fails (total cognitive load exceeds total mental resources)	93
2.6	Learning facilitated (total cognitive load made less than total mental resources by reducing extraneous cognitive load)	93
2.7	Cognitive load construct	95
2.8	Diagram showing the redundancy effect	147
2.9	Conceptual Framework of the Study	188
3.1	A progressive series of four experiments	196
3.2	Post-test only control group design for Experiments 1 and 2	202
3.3	Post-test only control group design for Experiments 3 and 4	202
3.4	Relative condition efficiency as a function of instructional format type	251





# CHAPTER 1

## INTRODUCTION

### Background of the Study

#### Introduction

In the last few decades, the mathematical community has struggled to define what mathematics is and what mathematical thinking means. One possible reason for this is the growing role of mathematics, science and technology in modern life (Organization of Economic Co-operation and Development, 2004). Consequently, there have been numerous changes made to the mathematics curriculum used in schools and institutions of higher learning.

The events in the United States typify the situation. According to Schoenfeld (1992), in 1957 when the Russians launched their space satellite *Sputnik*, it galvanized American educators, scientists and mathematicians to restructure the curriculum in line with their notion of what mathematics and mathematical thinking should be. Thus evolved *The New Maths*. When it was discovered that this new curriculum was not working, the pendulum swung to the other end and the *back-to-basics* movement grew in popularity. This too was eventually deemed to be inadequate as students were found wanting on measures of thinking. The dismal performance of the *back-to-basics* movement led

