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Modelling Classroom Space Allocation at University of Rwanda-A Linear Programming Approach

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Abstract

Education and training play a key role as the human capital function. This is especially true for tertiary education. However, infrastructure and equipment limitations are some factors that limits levels of students' enrollment in universities. This is moreso the case in developing countries where much of the infrastructure developments are donor-funded. For institutional managers and administrators, the allocating of the limited available classroom space is a constant problem that needs sophisticated approaches to deal with. Linear Optimization technique has shown promise in dealing with this problem. This research seeks to assess the Rwandan education system and highlight strides made to broaden access to tertiary education. Using data accessed from the College of Science and Technology for the 2019/2020 academic year, a linear programming model is formulated to assess the level of usage of the available classroom space at the College. The model is solved using the Dual Simplex algorithm via the Cplex solver implemented in AMPL. A solution analysis shows that, out of the 68 classrooms available on the Nyarugenge campus, only 18 with a seating capacity of 2,147 are being used to facilitate the learning of approximated 4,088 students, and that 50 classrooms with a seating capacity of 1,506 are being underutilized or not being used at all. Relevant recommendations including that the college explores the usage of virtual laboratory platforms to overcome space and material limitations associated with physical laboratories are presented.

Keywords: linear programming; classroom allocation; linear optimization; University of Rwanda

MSC 2020: 90C05, 90C90

1. Introduction

Assignment problems naturally occur in various situations where there is a need to determine how to optimally assign n objects to m subjects. Faudzi et al. (2018) identifies two main classes of assignment problems in the education domain, these are; allocation and timetabling problems. The earlier can be further split into course, school timetabling, and examination problems whilst the latter is further classified into new student allocation, space allocation, and student-subject allocation problems. In general, two kinds of constraints are involved in assignment problems, namely; soft and hard constraints. According to Ergul (1996), although soft constraints have a limited impact on the solution feasibility, however, they need to be adhered to as much as possible for a high-quality solution.

Several solution methods for allocation problems have been explored by researchers. In line with research findings in Bucco et al. (2017) and Phillips et al. (2015), these solution methods include; hybrid, heuristic, and metaheuristic, exact, etc. The exact method aims at finding an assessed optimal solution, although, for a complex problem, the exact solution methods tend to be more complicated than that of heuristic. Examples of these exact methods are; linear programming, integer programming, and dynamic programming. Metaheuristic methods can further be categorized into, local search and population search-based techniques. The local search solution techniques include; Tabu Search (TS), Great Deluge (GD), and Simulated Annealing (SA), while the population search techniques include; Genetic Algorithm (GA), Ant Colony Optimization (ACO), and Fly Algorithm (FA). The local search solution techniques iteratively consider single candidate solutions whilst the population search solution techniques employ a collection of candidate solutions throughout the solution search process for improvement. These solution techniques are further explored by Marti et al. (2012) who studies the heuristic techniques for the linear ordering scenarios. Whilst Lu (2008) applies heuristic approaches to solve the course timetabling problem. Below we briefly discuss the main allocation problems in the education domain.

2. Timetabling Problem

Consider the finite sets; T, a collection of time slots, R, a collection of resources, M, a collection of meetings and C, a collection of constants (Burke (2004)). Then, the timetabling problem involves assigning time slots and resources to the meetings while satisfying the constraints. The problem is further split into subproblems, namely; school timetabling, course timetabling and examinations timetabling problems. School timetabling involves generating a school timetable so that no single teacher attends two classes at the same time. The timetable follows a weekly cycle for all the classes. Here, students will be preassigned, but teachers and rooms need to be assigned. Below we briefly define these timetabling sub-problems.

- *The course timetabling problem:* involves the assigning of students, lecturers, courses, and classrooms, to a fixed period of time. Normally the fixed time is a working week and there will be a constraint that needs to be satisfied (Cote et al. (2005)). Although course and examination timetabling share some similarities, however, while it is possible to schedule more than one examination in the same room at the same timeslot, so long room capacity is not violated, it is impossible to have two courses scheduled in the same room at a given timeslot (Obit (2010)).

- *Examination timetabling problem:* the problem is how to assign a given set of examinations $E = \{e_1, \cdots, e_n\}$, to a fixed number of timeslots, $T = \{t_1, \cdots, t_n\}$ with respect to certain constraints. The problem is considered NP-hard.

3. Allocation problem

This problem is an assignment problem and it is considered a combinatorial optimization problem in the field of optimization or operation research. In the education domain, the problem is classified into three subproblems, namely; space allocation problem, new student allocation problem, and students' projects allocation problem. Below we briefly define these subproblems:

- *Space allocation problem:* this problem involves the allocation of resources to space areas, such as the allocation of classrooms to satisfy various demands and constraints (Frimpong and Owusu (2015)). Here, the goal is to ensure that there is no wastage or overuse of space.
- *New student allocation problem:* the problem involves the allocation of students into classes to meet the capacity in the respective classes. Examples of hard constraints to be met in this problem may include, the capacity of each class, and assigning students with same rankings to same classes (Zukhri and Omar (2008); Hassim et al. (2014)).
- *Student project allocation problem:* given sets of students, projects and lecturers, the problem is how to assign an individual to a case considering the interest(s) of both student and lecturer, without violating their capacity constraints.

4. Tertiary Education in Rwanda and Study Design

The Rwanda education system consists of pre-primary level, primary level, secondary level, and post-secondary level. Figure 1 shows the Rwandan Education system including the average number of years it takes students to complete each level (Stefan (2019); MINEDU (2018)). The College of Science and Technology (CST) is one of the six (6) subject-based colleges making up the University of Rwanda (UR). The others are the College of Arts and Social Science (CASS), the College of Animal Science and Veterinary Medicine (CAVM, the College of Business and Economics (CBE), the College of Education (CE), and the College of Medicine and Health Sciences (CMHS) (UR (2018)). With an enrolment of over 30,000 students, the University of Rwanda is the largest public University in Rwanda (UR (2020)). Private (or independent) higher learning institutions include; African Institute of Mathematical Sciences Rwanda (AIMS-Rwanda), African Leadership University (ALU), Carnegie Mellon University African (CMU-A) and others.

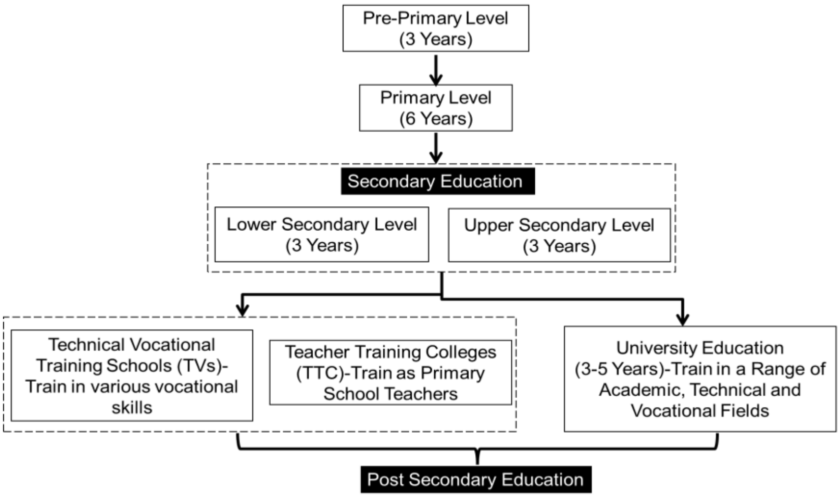

Figure 1. Rwanda education system

The College of Science and Technology has three campuses, one based in Nyarugenge at the former Kigali Institute of Science and Technology (officially known as the CST campus), another based in Huye and a third campus based at Musanze. The college has an average enrolment of over 5000 students in the various programs offered. Data used in this study was collected from the registry office of the CST campus. The data comprised details of classrooms and their respective sitting capacity at CST campus, names of programs offered at both undergraduate and postgraduate levels, and the number of students enrolled in each program at all levels for the 2019/2020 academic year (for all the three campuses). However, this work focuses on modeling classroom space allocation for the undergraduate level only at the CST Nyarugenge campus that hosts around 92% of all CST students and not Huye and Musanze campuses. Normally the postgraduate students are housed in labs in their respective departments, schools, or centers of excellencies. Figure 2 shows the general flow of this study, from the nature of data used to modeling and analysis of the feasible solution.

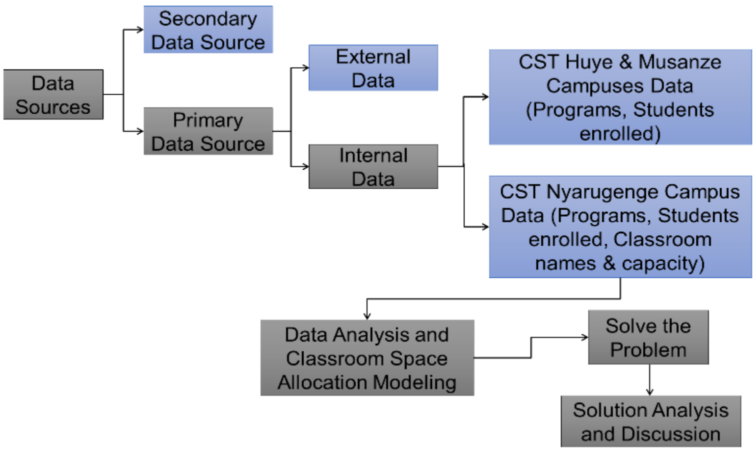

Figure 2. Data source and study flow

The students and the academic staff are the primary users of the college classroom resources. The college administrators ensure that the classroom resources are optimally assigned for use for academic activities and other activities that relate directly to the core functions of scholarly production, research, and teaching. In this work, we model the classroom space assignment at CST, Nyarugene campus as a linear programming problem. We consider classroom space allocation as the assignment of classrooms among several courses each with different student population sizes to achieve optimal utilization of available spaces while satisfying seating capacity constraints.

Table 1. Schools, programmes and students enrolled 2019/2020 academic year at CST

Schools	Student Enrollment					Campus
	Year 1	**Year 2**	**Year 3**	**Year 4**	**Year 5**	
Architecture & Built Environment	201	196	216	149	19	Nyarugenge
Engineering	601	450	380	510	167	Nyarugenge
Mining & Applied geology	72	50	34	72		Nyrarugenge
ICT	313	3021	230	301		Nyarugenge, Huye, Musanze
Science	383	272	194	179		Nyarugenge
Total	**2,295**	**1,270**	**1,054**	**1,211**	**186**	

Table 1 is a summary of the information regarding the schools, undergraduate programs and the number of students enrolled in each program at the various levels (or years) of study. Figure 3 shows the distribution of students in the three campuses of CST, namely; Huye, Musanze, and Nyarugenge. It is clear from Figure 3 that Nyarugenge is the largest campus among the three, motivating our choice to limit our model to this campus. The important condition during this assignment process is that the areas of usable space as required by the entities are not subject to modification. Hence, the ideal solution to the problem is the one that assigns space to all the entities without space wastage or overuse while simultaneously satisfying all the additional requirements and constraints. Although various studies exist on this problem, however, this work is the first to attempt modeling the classroom space allocation in Rwanda. Hence, we believe the results of this work will go a long way to inform decision making on space utilization within the University.

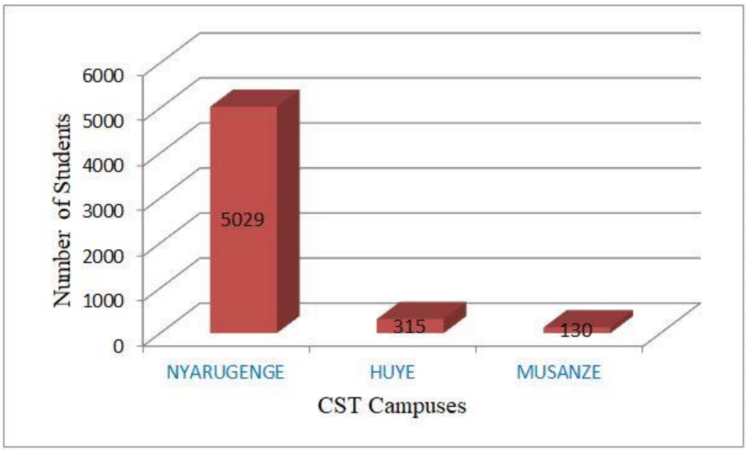

Figure 3. Students distribution in the three CST campuses

5. Classroom Space Allocation Model Formulation

In this section, a model of the classroom space allocation problem is presented. We first discuss the general assignment problem and the solution approach to the problem. The classroom space allocation problem belongs to the Educational Resource Allocation (ERA) (based on the resource type) and the ERA problem falls under the Constraint Satisfaction Problems (CSP) class. In general, an assignment problem aims at optimizing the assignment of resources against demand points (Faudzi et al. (2018)). Mathematically, the general assignment problem is defined as:

Maximize

$$\sum_{i=1}^{n} C_{ij} x_{ij}, \tag{1}$$

subject to

$$\sum_{i}^{n} x_{ij} = 1, \;\; j = 1, \cdots, n, \tag{2}$$

$$\sum_{j}^{n} x_{ij} = 1, \;\;\; i = 1, \cdots, n, \tag{3}$$

$$x_{ij} = 0{,}1, \;\; j = 1, \cdots, n, \qquad i = 1, \cdots, n, \tag{4}$$

and

$$x_{ij} = \begin{cases} 1, & \text{if a resource } i \text{ is assigned to demand } j, \\ 0, & \text{otherwise,} \end{cases} \tag{5}$$

where c_{ij} is the cost of assigning ith resource to jth demand, and n is the number of resource(s) or demand(s). Equation (2) and Equation (3) defines the constraints, where in (2) each demand j can only be assigned to one resource i and in (3), a resource i can only be assigned to one demand. Each assignment problem is associated with a matrix, C_{ij} known as the cost matrix, such that c_{ij} is the cost of assigning resource i to demand j. This matrix is also known as the assignment matrix where every resource can be assigned to only one demand. Matrix C_{ij} is defined as:

$$\begin{pmatrix} c_{11} & \cdots & c_{1n} \\ \vdots & \ddots & \vdots \\ c_{m1} & \cdots & c_{mn} \end{pmatrix}. \tag{6}$$

5.1. Linear Programming Model Formulation

Linear Programming (LP) modeling was coined in 1947 by George Dantzig, and has stood out as one of the widely used and effective optimization techniques. LP involves optimization of a linear objective in the decision variables $\{x_1, x_2, \dots, x_n\}$ subject to linear equality and/or inequality constraints on the $\boldsymbol{x}$. Normally, both the objective function and the constraints are provided. A LP problem in standard form is defined as follows:

Minimize

$$c^T \boldsymbol{x}, \tag{7}$$

subject to

$$\begin{gathered} A\boldsymbol{x} = b, \\ 0 \leq \boldsymbol{x} \leq u, \end{gathered} \tag{8}$$

where T denote transposition, $A \in R^{m \times n}$, $(\boldsymbol{c}, \boldsymbol{x}) \in R^n$, $b \in R^m$. Assuming that A has full rank, i.e., $A = m$, $m < n$, then the linear system $A\boldsymbol{x} = b$ is consistent. In the system above, A is the matrix of coefficients of the constraints (size $m \times n$), c is the vector of coefficients of the objective function (size $n \times 1$) and b is the vector of the right-hand side of the constraints (size $m \times 1$).

Note that the system above is also known as the primal problem. Adding slack variables transforms the system above to:

Minimize

$$c^T \boldsymbol{x}, \tag{9}$$

subject to

$$\begin{gathered} A\boldsymbol{x} = b, \\ \boldsymbol{x} + s = u, \\ \boldsymbol{x} \geq 0, s \geq 0, \end{gathered} \tag{10}$$

where the vectors $\boldsymbol{x}$ consists of the primal variables and s consists of the primal slack variables respectively.

Given a primal system, one can drive another LP system called the dual problem system. The dual problem corresponding to the above primal problem is:

Maximize

$$b^T \boldsymbol{y} - u^T \boldsymbol{w}, \tag{11}$$

subject to

$$\begin{gathered} A^T \boldsymbol{y} - \boldsymbol{w} + \boldsymbol{z} = c, \\ \boldsymbol{z} \geq \boldsymbol{0}, \boldsymbol{w} \geq \boldsymbol{0}, \end{gathered} \tag{12}$$

where vectors $\boldsymbol{y}$ and $\boldsymbol{w}$ consists of the simplex multiplier and vector $\boldsymbol{z}$ consists of the reduced costs.

Table 2. Grouping of classrooms according to seating capacity at CST Nyarugenge campus

Room Name	**Capacity**	**No. of Rooms**
P001	360	1
P008	100	1
P017	90	1
P009, P016	80	2
Room A,B, C,D,E,F,G,H,I	60	9
P106,P107,P206,P207,P306, P307,P406,P407,Room 03,Room 04, Room 05,Room 06,Room 11, Room 12, Room 13	40	15
Room 6,Room 7,Room 8,Room 9, P405, P404,P313, P312,P310,P305, P304, P212,P213,P205,P204,P113,P112	30	17
Auditorium 1, Auditorium 2,auditorium 3, Auditotium 4	150	4
Ground Floor 1	50	1
Area 4-B, Area 2-4	32	2
Area 4-C	28	1
Area 5-C, Area 2-2	22	2
Area 5-D	23	1
Area 2-C	25	1
Area 5-B	70	1
Area 5-A	44	1
Area 4-A	62	1
Area 2-B	56	1
Area 3-A	34	1

Area 2-A, Area 4-6	41	2
Area 2-6	31	1
Area 4-5	102	1
Area 2-1	63	1
Total	**3, 738**	**68**

5.2. Data Analysis for Model Formulation

The data collected from the registry of CST Nyarugenge campus comprised a list of undergraduate bachelors programs, list of students enrolled at the college in the 2019/2020 academic year and their level of studying (Year 1, Year 2, Year 3, Year 4 and Year 5), and the campus to which they are stationed. At Nyarugenge campus, the classrooms are housed in the building blocks, namely; Muhabura, Einstein, Ikazi, Muhazi, Camp Kigali, and Sabe. The names of classrooms and their respective seating capacities and the number of rooms with the specific seating capacity have been indicated in the Table 2. From Table 2 we note that the total seating capacity available for use is 3, 738 in the 68 available classrooms. Furthermore, Table 2 shows the number of rooms with a specific capacity. For example, only one classroom has a seating capacity of 360 while 17 classrooms have a seating capacity of 30, etc. Since our modeling is for the Nyarugenge campus, the details summarized in Table 2, and Table 3, excludes information for Huye and Musanze campuses.

Note that as summarized in Table 3, we make the following clarifications.

- As per the college's policy, programs such as Computer Science, Computer Engineering, Information Systems, and Information Technology are soon to be moved to Huye campus where all infrastructure including office space, computer labs, and classrooms have been completed. Hence, although from the data these programs appear to be offered at Nyarugenge campus, we have omitted them during the analysis since they are not in the long-term plans for this campus.
- Also, in recent past, the curriculum has been revised, leading to some programs being phased out while others have been replaced by new program, hence in instances where a program is being replaced by a new program, we have merged such programs into a single program because essentially we discovered that the phased programs have students only in the final year with the new program having students in the first and second year for example. Furthermore, some programs like Civil engineering: construction, building & construction, although different but students share over 95% of modules, hence they use the same classrooms. Hence, such programs have also been combined and treated as a single program.

Table 3. Programs and students enrolled for 2019/2020 academic year at CST

Programs	**Program Enrollment**					**Total**
	Y.1	**Y.2**	**Y.3**	**Y.4**	**Y. 5**	
Geography: Environmental Planning	27	24	37			**88**
Geography: Urban & Regional Planning	32	44	85	32		**193**
Estate management & valuation	39	47	33	41		**160**
Architecture	43	25	22	19	19	**128**
Electrical & Tele. Eng., Elect. Power Eng.	137	107	115	214		**573**
Surveying & Geomatics Eng., Quantity & Surveying	135	111	58	92		**396**
Civil Engineering	133	121	85	59	167	**565**
Mechanical Eng.: Plant Eng., Mechanical Eng.: production	76	72	43			**191**
Mechanical Energy Eng., Energy Eng.	30	2	85			**117**

Civil Eng.: Construction, Civil Eng.: Geotechnical, Building & Construction	145	95	116	37		**393**
Mining Engineering	36	24	24	37		**121**
Applied Geology	36	26	10	35		**107**
Biology: Zoology & Conservation	30	20	7			**57**
Biology: Biotechnology	45	52	49	15		**161**
Biology: Bio-Organic	47	28	16			**91**
Biology: Botany & Conservation	38	18				**56**
Biology: Biochemistry	57	44	42	22		**165**
Physics: Material Science	32	16	16	12		**76**
Physics:Atmospheric Climate	36	18	14			**68**
Applied Mathematics	29	26	9	23		**87**
Mathematics: Statistics	25	18	17			**60**
Chemistry: Environmental	44	32	24	40		**140**
Chemistry: Bio-Organic.	57	32	6			**95**
Total	**1309**	**1000**	**763**	**924**	**186**	**4088**

5.3. Classroom Allocation Model Formulation

We model the classroom space allocation as a LP problem. The classrooms are categorized into types according to the seating capacities, see Table 2. The students were grouped according to the Year of study as per the programmes in which they are enrolled, see Table 3. Below we define variables concerning the data that will later be put together into a LP model.

- Let c_i be the capacity of a classroom of type i for $i = 1, \ldots, n$.
- Let x_i be the type of classroom i, for $i = 1,2, \ldots, n$, i.e., classrooms categorized according to the seating capacity of the rooms. Hence, x_i is classroom of type i with seating capacity c_i.
- Let a_i be the number of classrooms of type i, (i.e., a_i is number of classrooms of classroom type i).
- Let k be the total available classroom space of all the classroom types. Hence;

$$k = \sum_{i=1}^{n} a_i c_i, \tag{13}$$

where a_i, c_i, k are as defined above.

Therefore, with reference to Table 2 and Table 3 we define the classroom space allocation linear problem as:

Maximize

$$\sum_{i=1}^{n} c_i x_j, \tag{14}$$

subject to

$$\sum_{i=1}^{n} a_i c_j \leq k, \quad i = 1,2, \cdots, n. \tag{15}$$

It is important to note that there are two conditions necessary for our model to be valid. 1) The number of students assigned to the room cannot be negative, such that; $x_i \geq 0$ for all i. 2) The total number of students assigned to various types of classrooms cannot exceed the total space available in each classroom. With these conditions in mind, we present the objective function of the classroom space allocation linear programming problem for CST, Nyarugenge campus as below.

6. Solution Approaches, Tools and Analysis

Linear programming is a mathematical programming method involving the optimal allocation of limited resources to competing needs (Rajgopal and Bricker (1990)). Various methods exist to solve linear programming problems, these methods include; the ellipsoid method, the interior-point method, and the simplex method (Raz and Bricker (1990); Abad and Banks (2011)). The ellipsoid method is faster and solves any linear program with a few steps which is a polynomial function of the amount of data defining the program. Instead of passing from vertex to vertex, the interior-point method finds a solution to a linear program by passing only through the interior of the feasible region. In practice, however, the interior-point method is costly due to its expensive execution time per iteration, and the possibility of numerical instability and its analysis remains a subtle subject, less easily understood. Despite the highlighted advantages of the ellipsoid and the interior-point methods, the simplex method is still considered superior in practice, it is well understood and also it is widely implemented in mathematical software packages. Since its introduction in 1947, over the years the simplex method has been strengthened computationally allowing it to handle large linear programming problems. This led to new improved algorithms such as the dual simplex algorithm and the interior point methods simplex algorithms (Dongarra and Sullivan (2000); Khachiyan (1980)). For large scale linear programming problems, the interior point methods have shown to computationally outperform the simplex method (Boxby (1992)). Hence, in recent years, the primal simplex and the dual simplex algorithm have become default methods for handling large scare LP problems. In this, work we apply the dual simplex method to find a feasible solution for the classroom space allocation linear program formulated.
Figure **4** shows the flow diagram of the major steps of the revised dual simplex method.

We used the AMPL software to solve the system. This is a powerful language designed specifically for mathematical programming and handles Linear, Integer and Nonlinear programming problems. In AMPL, one can easily algebraically represent a model in a model file and the parameter values in a data file. AMPL translates the model from the .mod file and the data from the .dat file into a format understandable by the solver. We summarize this process in Figure 5.

Maximize

$$Z = 360x_1 + 100x_2 + 90x_3 + 160x_4 + 540x_5 + 600x_6 + 510x_7 + 600x_8 + 50x_9 + 64x_{10} + 28x_{11} 44x_{12} + 23x_{13} + 25x_{14} + 70x_{15} + 44x_{16} + 62x_{17} + 56x_{18} + 34x_{19} + 82x_{20} + 31x_{21} + 102x_{22} + 63x_{23}$$

Subject to

$$27x_1 + 32x_2 + 39x_3 + 43x_4 + 137x_5 + 135x_6 + 133x_7 + 76x_8 + 30x_9 + 145x_{10} + 36x_{11} + 36x_{12} + 30x_{13} + 45x_{14} + 47x_{15} + 38x_{16} + 57x_{17} + 32x_{18} + 36x_{19} + 29x_{20} + 25x_{21} + 44x_{22} + 57x_{23} \leq 1309$$

$$24x_1 + 44x_2 + 47x_3 + 25x_4 + 107x_5 + 111x_6 + 121x_7 + 72x_8 + 95x_{10} + 24x_{11} + 26x_{12} + 20x_{13} + 52x_{14} + 28x_{15} + 18x_{16} + 44x_{17} + 16x_{18} + 18x_{19} + 26x_{20} + 18x_{21} + 32x_{22} + 32x_{23} \leq 1000$$

$$37x_1 + 85x_2 + 33x_3 + 22x_4 + 115x_5 + 58x_6 + 85x_7 + 43x_8 + 2x_9 + 116x_{10} + 24x_{11} + 10x_{12} + 7x_{13} + 49x_{14} + 16x_{15} + 42x_{16} + 16x_{17} + 14x_{18} + 9x_{19} + 17x_{20} + 24x_{21} + 6x_{22} + 7x_{23} \leq 83$$

$$32x_2 + 41x_3 + 19x_4 + 214x_5 + 92x_6 + 59x_7 + 85x_9 + 37x_{10} + 37x_{11} + 35x_{12} + 15x_{14} + 22x_{17} + 12x_{18} + 23x_{20} + 40x_{22} \leq 763$$

$$19x_4 + 167x_7 \leq 186$$

$$x_1 + x_2 + 2x_3 + 9x_4 + 15x_5 + 17x_6 + 4x_7 + x_8 + 2x_9 + x_{10} + 2x_{11} + x_{12} + 2x_{13} + x_{14} + x_{15} + x_{16} + x_{17} + x_{18} + 2x_{91} + x_{20} + x_{21} + x_{22} + x_{23} \leq 68$$

$$360x_1 + 100x_2 + 90x_3 + 80x_4 + 60x_5 + 40x_6 + 30x_7 + 150x_8 + 50x_9 + 32x_{10} + 28x_{11} + 22x_{12} + 23x_{13} + 25x_{14} + 70x_{15} + 44x_{16} + 62x_{17} + 56x_{18} + 34x_{19} + 41x_{20} + 31x_{21} + 102x_{22} + 63x_{23} \leq 3738$$

$$x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8, x_9, x_{10}, x_{11}, x_{12}, x_{13}, x_{14}, x_{15}, x_{16}, x_{17}, x_{18}, x_{19}, x_{20}, x_{21}, x_{22}, x_{23} \geq 0$$

$$
\begin{array}{l}
x_1 \le 1 \\
x_2 \le 1 \\
x_3 \le 1 \\
x_4 \le 2 \\
x_5 \le 9 \\
x_6 \le 15 \\
x_7 \le 17 \\
x_8 \le 4 \\
x_9 \le 1 \\
x_{10} \le 2 \\
x_{11} \le 1 \\
x_{12} \le 2 \\
x_{13} \le 1 \\
x_{14} \le 1 \\
x_{15} \le 1 \\
x_{16} \le 1 \\
x_{17} \le 1 \\
x_{18} \le 1 \\
x_{19} \le 1 \\
x_{20} \le 2 \\
x_{21} \le 1 \\
x_{22} \le 1 \\
x_{23} \le 1
\end{array}
$$

6.1. Solution Analysis and Discussion

Running the model in AMPL and solving using the dual simplex method via the CPLEX 12.10.0.0 solver, the optimal solution was found as presented in the **Error! Reference source not found.** below. From **Error! Reference source not found.** we note that $x_i = 0$ for i= 6, 7, 9, 10, 11, 12, 13, 14, 16, 18, 19, 20, 21, x_i=1 for i=1, 2, 3, 15, 17, 22, 23, x_8=4 and x_5=5 (rounded to whole number) respectively. To appreciate the implication of these results, we interpret them in line with the respective classrooms available at CST Nyarugenge campus. Recall that x_i represent classroom of type i with seating capacity c_i. Hence for x_i=0 it means that the classroom is not being used currently. We relate the solution to the classrooms in the **Error! Reference source not found.**.

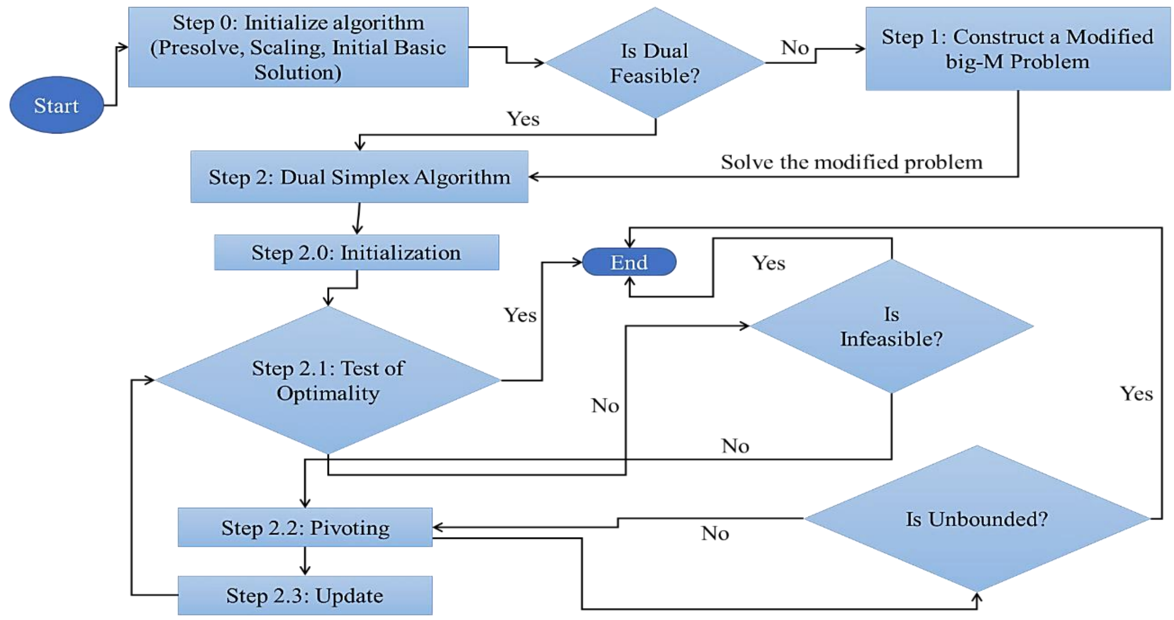

Figure 4. Revised dual simplex algorithm

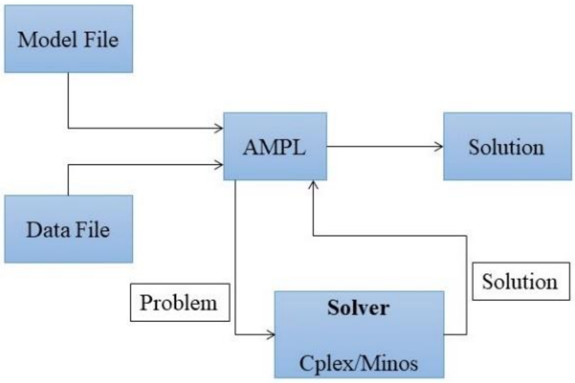

Figure 5. AMPL handling of a problem instance

Table 4. Model solution of the classroom space allocation found by AMPL

Objective Function Value	Parameter values											
	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9	x_{10}	x_{11}	x_{12}
6, 027	1	1	1	2	5	0	0	4	0	0	0	0
	x_{13}	x_{14}	x_{15}	x_{16}	x_{17}	x_{18}	x_{19}	x_{20}	x_{21}	x_{22}	x_{23}	
	0	0	1	0	1	0	0	0	0	1	1	

Table 5. Summary of classrooms being utilized and those not being utilized

Room Name	**Capacity**	**No. of Rooms**	**Rooms being utilized as per the model**
P001	360	1	1
P008	100	1	1
P017	90	1	1
P009, P016	80	2	2
Room A, B, C, D, E, F, G, H, I	60	9	5
P106, P107, P206, P207, P306, P307, P406, P407, Room 03, Room 04, Room 05, Room 06, Room 11, Room 12, Room 13	40	15	0
Room 6, Room 7, Room 8, Room 9, P405, 404, P313, P312, P310, P305, P304, P212, P213, P205, P204, P113, P112	30	17	0
Auditorium 1, Auditorium 2, Auditorium 3, Auditorium 4	150	4	4
Ground Floor 1	50	1	0
Area 4-B, Area 2-4	32	2	0
Area 4-C	28	1	0
Area 5-C, Area 2-2	22	2	0
Area 5-D	23	1	0
Area 2-C	25	1	0
Area 5-B	70	1	1
Area 5-A	44	1	0
Area 4-A	62	1	1
Area 2-B	56	1	0
Area 3-A	34	1	0
Area 2-A, Area 4-6	41	2	0
Area 2-6	31	1	0
Area 4-5	102	1	1
Area 2-1	63	1	1
Total	**3, 738**	**68**	**18**

We note from **Error! Reference source not found.** that out of the 68-rooms available on campus, only 18 with a seating capacity of 2,147 are being used to facilitate the learning of approximated 4,088 students, a scenario that constantly leads to massive congestion. Further, the model solution shows that the 50 classrooms with a seating capacity of 1,506 are being underutilized or not being used at all.

The objective function value implies that the available classrooms on campus can accommodate a student population of up to 6,027. However, currently, the campus has a projected student population of 4,088, implying that an additional student population of 1,939 can be enrolled, increasing access to education for those deserving. Considering that a lot of infrastructure development is on-going on campus, we note that the college can do more to put this infrastructure to optimal use, unlike the current scenario where a small percentage of the available infrastructure is being used for the intended purpose.

2. Recommendation on Laboratory Space Optimization

Two types of laboratories exist, namely; physical and virtual labs. Many teachers are faced with various subject specific challenges when it comes to conducting experiments in their teaching. Physical labs are faced with challenges including the damage to laboratory equipment and a lack of chemical solutions supply. Furthermore, with physical labs, students are not free to carry out experiments according to their own schedules since experiments may be limited to regular learning hours (Tiwari and Singh (2011); Muhamad et al. (2012)). We point out that, since CST is a science and technical college, the available laboratory space and equipment are important elements that can impact the level of enrollment at the institution.

Due to increased demand for high education, virtual laboratories are a cheaper and flexible strategy in removing the barriers to access to education and also can attract students. Extensive research continues to be carried out on the subject of virtual learning laboratories with various cloud computing platforms being explored to implement virtual laboratories (Dukhanov et al. (2014); Liu et al. (2015); Chan and Fok (2009); Damayanti et al. (2020)). Specifically, the advantages of virtual laboratory include;

1) allows experiments with too dangerous chemicals (or objects), global or those that takes much time,
2) eliminates need for expensive laboratory materials and equipment, hence overcoming the limitations or absence of materials,
3) more interactive and enables creation of more visually attractive laboratory,
4) supportive of slow learners, due to the ability of computers to display information needed, 5) eliminates time constraints problem, especially in situations where there may not be enough time to teach in the laboratory, and
6) enhance safety and security, since students to not interact with real chemicals and tools.

We therefore recommend that the college management looks into exploring moving some of the laboratories into virtual environments thereby harnessing the above highlighted benefits and more importantly allows increased enrollment.

3. Conclusion

An analysis of the impact of classroom utilization at the UR, CST, Nyarugenge campus has been presented. Considering that the survival of Universities rests heavily on the income generated and that the tuition fees forms part of a reliable constant income source for the institutions, available infrastructure must be put to optimal use to maximize income generation. This work has shown that currently, the classroom space at UR, CST, Nyarugenge campus is being underutilized leading to limited access to higher education by the deserving Rwandan youths. With more infrastructure being constructed at the campus, there a is room that new programs can be introduced or intake can be increased in the existing programs. Since our current work only focused on a single college campus, our future work will try to investigate further more sophisticated modelling techniques to enable a University-wide analysis of the classroom space allocation.

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