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Hall current and Radiation effects on Unsteady Natural Convection MHD flow with Inclined Magnetic field

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Abstract

In the present paper, Hall current and radiation effects on unsteady natural convection MHD flow with inclined magnetic field is studied. The viscous, incompressible and an electrically conducting fluid is considered. This model contains equations of motion, equation of energy and diffusion equation. The system of partial differential equations is transformed to dimensionless equations by using dimensionless variables. Exact solution of governing equations is obtained by Laplace Transform Technique. For analysing the solution of the model, desirable sets of the values of the parameters have been considered. The obtained results of velocity, concentration and temperature have been analysed with the help of graphs drawn for different parameters. The numerical values of Nusselt number have been tabulated. The results of the study may find applications in the field related to the solar physics dealing with the solar cycle, Magnetohydrodynamics sensors, rotating MHD induction machine energy generator, the sunspot development, the structure of rotating magnetic stars etc.

Keywords: MHD; Hall current; Radiation; Natural convection; Mass transfer

MSC 2010 No.: 76W05, 76D05, 80A20

1. Introduction

Problems related to unsteady MHD flow are thought to be important in the fluid dynamics. Magnetohydrodynamics sensors are used for precision measurements of angular velocities in inertial navigation system. Further, some important applications are seen in MHD generator, MHD pumps and accelerators, flow meters, planetary and solar plasma fluid dynamics systems, nuclear reactors using liquid metal coolants, rotating MHD induction machine energy generators etc.

Hall effect on free and forced convective flow in a rotating channel is considered by Prasada et al. (1982). Turkyilmazoglu (2019) investigated MHD natural convection in saturated porous media with heat generation/absorption and thermal radiation: closed-form solutions. Unsteady MHD flow past an impulsively started inclined plate with variable temperature and mass diffusion in the presence of Hall current is studied by Rajput and Kumar (2016). Seth et al. (2012) examined effects of Hall current and rotation on unsteady MHD Couette flow in the presence of an inclined magnetic field and found that the primary and secondary velocities decrease with increasing the value of Hartman number. Ghosh (2001) has analysed unsteady hydro-magnetic flow in a rotating channel permeated by an inclined magnetic field in the presence of an oscillator. Radiation effect on mixed convection along a vertical plate with uniform surface temperature is investigated by Hossain et al. (1996).

Azzam (2002) studied the radiation effects on the MHD mixed free-forced convection flow past a semi-infinite moving vertical plate for high temperature differences. Seth and Nandkeolya (2009) have investigated MHD Couette flow in a rotating system in the presence of an inclined magnetic field. MHD flow over a moving plate in a rotating fluid with magnetic field, Hall currents and free stream velocity was studied by Takhar et al. (2002). Ghosh et al. (2009) have studied Hall effects on MHD flow in a rotating system with heat transfer characteristics. Effect of rotation on MHD flow past an accelerated isothermal vertical plate with heat and mass diffusion was obtained by Muthucumaraswamy et al. (2010). Rajput and Kumar (2017) have studied chemical reaction effect on unsteady MHD flow past an impulsively started inclined plate with variable temperature and mass diffusion in the presence of hall current. Jana and Datta (1980) investigated Hall effects on MHD Couette flow in a rotating system. Radiation and mass transfer effects on MHD free convection flow past an exponentially accelerated vertical plate with variable temperature has been investigated by Rajesh and Varma (2009).

Garg (2012) has worked on combined effects of thermal radiations and Hall current on moving vertical porous plate in a rotating system with variable temperature. Further, Garg (2013) has worked on magnetohydrodynamics and radiation effects on the flow due to moving vertical porous plate with variable temperature. Attia (2003) analysed the effect of variable properties on the unsteady Hartmann flow with heat transfer considering the Hall effect. Seth et al. (2009) studied Hall effects on oscillatory hydro-magnetic Couette flow in rotating system. Study of Soret and ion slip effects on MHD flow near an oscillating vertical plate in a rotating system has been investigated by Rajput and Shareef (2018).

The present study is carried out to examine the Hall current and radiation effects on unsteady natural convection MHD flow with inclined magnetic field. The problem is solved by the Laplace transform method. Results of problem illustrating the effects of various parameters involved in the problem are presented and discussed. The numerical values of Nusselt number have been obtained for different parameters.

2. Mathematical analysis

The x - axis is taken along the vertical plane and z - axis is normal to it. Thus, the z - axis lies in the horizontal plane. The plate is taken along positive direction of x - axis. The fluid is permeated by uniform magnetic field B_0 imposed in the direction which makes an angle α from positive direction of horizontal xz - plane. Fluid is taken electrically conducting whose magnetic Reynolds number is very small, therefore the induced magnetic field produced by the fluid motion is negligible in comparison to the applied one. Initially it is considered that the temperature of plate and fluid is T_∞. Species concentration in the fluid is taken as C_∞. At time $t > 0$, the plate starts moving with a velocity u_0 in its own plane, and temperature of the plate is raised to T_w. The concentration C_w near the plate is raised linearly with respect to time.

So, under above assumptions, the governing equations are as follows:

$$\frac{\partial u}{\partial t} = \upsilon \frac{\partial^2 u}{\partial z^2} + g\beta (T - T_\infty) + g\beta^* (C - C_\infty) - \frac{\sigma B_0^2 Cos^2\alpha}{\rho(1 + m^2 Cos^2\alpha)}(u + vmCos\alpha), \quad (1)$$

$$\frac{\partial v}{\partial t} = \upsilon \frac{\partial^2 v}{\partial z^2} - \frac{\sigma B_0^2 Cos^2\alpha}{\rho(1 + m^2 Cos^2\alpha)}(v - umCos\alpha), \quad (2)$$

$$\rho C_p \frac{\partial T}{\partial t} = k \frac{\partial^2 T}{\partial z^2} - \frac{\partial q_r}{\partial z}, \quad (3)$$

$$\frac{\partial C}{\partial t} = D \frac{\partial^2 C}{\partial z^2}. \quad (4)$$

Equations (1) and (2) are called as momentum equations. Equation (3) and (4) are known as energy equation and diffusion equation respectively.

The following boundary conditions have been considered.

$$\left.\begin{aligned} & t \le 0: u = 0,\ v = 0,\ T = T_\infty,\ C = C_\infty, \text{ for every } z, \\ & t > 0: u = u_0,\ v = 0,\ T = T_\infty + (T_w - T_\infty)\frac{u_0^2 t}{\upsilon},\ C = C_\infty + (C_w - C_\infty)\frac{u_0^2 t}{\upsilon}, \text{ at } z = 0, \\ & u \to 0,\ v \to 0,\ T \to T_\infty,\ C \to C_\infty \text{ as } z \to \infty. \end{aligned}\right\} \quad (5)$$

The local radiant for the case of an optically thin gray gas is expressed as.

$$\frac{\partial q_r}{\partial z} = -4a^* \sigma^* (T_\infty^4 - T^4), \quad (6)$$

The temperature difference within the flow is taken sufficiently small, so T^4 can be expressed as the linear function of temperature. This is accomplished by expanding T^4 in a Taylor series about T_∞ and neglecting higher-order terms.

$$T^4 \cong 4T_\infty^3 T - 3T_\infty^4, \tag{7}$$

Using the values of equations (6) and (7) in equation (3), we get

$$\rho C_p \frac{\partial T}{\partial t} = k\frac{\partial^2 T}{\partial z^2} - 16a^* \sigma^* T_\infty^3 (T - T_\infty), \tag{8}$$

To obtain equations in dimensionless from, we introduce the following non-dimensional quantities

$$\left.\begin{aligned} &\bar{z} = \frac{z u_0}{\upsilon},\ \bar{u} = \frac{u}{u_0},\ \bar{v} = \frac{v}{u_0},\ \theta = \frac{(T - T_\infty)}{(T_w - T_\infty)},\ S_c = \frac{\upsilon}{D},\ \mu = \rho\upsilon,\ P_r = \frac{\mu c_p}{k}, R = \frac{16a^*\sigma^*\upsilon^2 T_\infty^3}{k u_0^2}, \\ &G_r = \frac{g\upsilon\beta(T_w - T_\infty)}{u_0^3},\ Ha^2 = \frac{\sigma B_0^2 \upsilon}{\rho u_0^2},\ G_m = \frac{g\beta^*\upsilon(C_w - C_\infty)}{u_0^3}, \bar{C} = \frac{(C - C_\infty)}{(C_w - C_\infty)},\ \bar{t} = \frac{t u_0^2}{\upsilon}. \end{aligned}\right\} \tag{9}$$

By using (9), the equations (1), (2), (8) and (4) become-

$$\frac{\partial \bar{u}}{\partial \bar{t}} = \frac{\partial^2 \bar{u}}{\partial \bar{z}^2} + G_r\,\theta + G_m\,\bar{C} - \frac{Ha^2 Cos^2\alpha}{(1 + m^2 Cos^2\alpha)}(\bar{u} + \bar{v} m Cos\alpha), \tag{10}$$

$$\frac{\partial \bar{v}}{\partial \bar{t}} = \frac{\partial^2 \bar{u}}{\partial \bar{z}^2} - \frac{Ha^2 Cos^2\alpha}{(1 + m^2 Cos^2\alpha)}(\bar{v} - \bar{u} m Cos\alpha), \tag{11}$$

$$\frac{\partial \theta}{\partial \bar{t}} = \frac{1}{P_r}\frac{\partial^2 \theta}{\partial \bar{z}^2} - \frac{R\theta}{P_r}, \tag{12}$$

$$\frac{\partial \bar{C}}{\partial \bar{t}} = \frac{1}{S_c}\frac{\partial^2 \bar{C}}{\partial \bar{z}^2}. \tag{13}$$

The corresponding boundary conditions (5) become-

$$\left.\begin{aligned} &\bar{t} \le 0: \bar{u} = 0,\ \bar{v} = 0,\ \theta = 0,\ \bar{C} = 0,\ \text{for every } \bar{z}, \\ &\bar{t} > 0: \bar{u} = 1,\ \bar{v} = 0,\ \theta = \bar{t},\ \bar{C} = \bar{t},\ \text{ at } \bar{z} = 0, \\ &\bar{u} \to 0,\ \bar{v} \to 0,\ \theta \to 0,\ \bar{C} \to 0,\ \text{ as } z \to \infty. \end{aligned}\right\} \tag{14}$$

Dropping bars in the above equations, we get

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial z^2} + G_r\,\theta + G_m\,C - \frac{Ha^2 Cos^2\alpha}{(1 + m^2 Cos^2\alpha)}(u + v m Cos\alpha), \tag{15}$$

$$\frac{\partial v}{\partial t}=\frac{\partial^2 v}{\partial z^2}-\frac{Ha^2Cos^2\alpha}{(1+m^2Cos^2\alpha)}(v-umCos\alpha), \tag{16}$$

$$\frac{\partial\theta}{\partial t}=\frac{1}{P_r}\frac{\partial^2\theta}{\partial z^2}-\frac{R\theta}{P_r}, \tag{17}$$

$$\frac{\partial C}{\partial t}=\frac{1}{S_c}\frac{\partial^2 C}{\partial z^2}. \tag{18}$$

The corresponding boundary conditions are-

$$\left.\begin{array}{l} t\le 0{:}\,u=0,\ v=0,\ \theta=0,\ C=0,\ \text{for every } z,\\ t>0{:}\,u=1,\ v=0,\ \ \theta=t,\ \ C=t,\ \text{at } z=0,\\ u\to 0,\ \ v\to 0,\ \ \theta\to 0,\ \ C\to 0,\ \text{as } z\to\infty. \end{array}\right\} \tag{19}$$

To solve above system, take $\eta=u+iv$, Combining the equations (15) and (16), we get

$$\frac{\partial\eta}{\partial \mathrm{t}}=\frac{\partial^2\eta}{\partial z^2}+G_r\,\theta+G_m\,C-\frac{Ha^2Cos^2\alpha}{1+m^2Cos^2\alpha}(1-imCos\alpha)\eta, \tag{20}$$

$$\frac{\partial\theta}{\partial t}=\frac{1}{P_r}\frac{\partial^2\theta}{\partial z^2}-\frac{R\theta}{P_r}, \tag{21}$$

$$\frac{\partial C}{\partial t}=\frac{1}{S_c}\frac{\partial^2 C}{\partial z^2}. \tag{22}$$

The corresponding boundary conditions become-

$$\left.\begin{array}{l} t\le 0{:}\,\eta=0,\ \theta=0,\ C=0,\ \text{for all } z,\\ t>0{:}\,\eta=1,\ \theta=t,\ C=t,\ \text{at } z=0,\\ \eta\to 0,\ \ \theta\to 0,\ \ C\to 0,\ \ \text{as } \ z\to\infty. \end{array}\right\} \tag{23}$$

The Laplace - transform technique is used for solving the dimensionless governing equations (20) to (22), subject to the boundary conditions (23). The solution obtained is as under:

$$\begin{aligned}\eta={}&\frac{1}{4\zeta^2}G_r[\,2e^{-\sqrt{az}}(A_1+P_rA_2)-zA_3e^{-\sqrt{az}}(\,\frac{R}{\sqrt{a}}-\sqrt{a}\,)+2tA_2e^{-\sqrt{az}}\zeta+2A_{12}A_4(1-P_r)]+\frac{1}{2}e^{-\sqrt{az}}A_{33}\\ &+\frac{1}{2a^2}G_m[e^{-\sqrt{az}}(A_1+\sqrt{a}A_3)+e^{-\sqrt{az}}A_2(S_c+at)+A_{13}A_5(1\ -S_c)]-\frac{1}{2\zeta^2A_{11}}P_rG_r[A_{14}A_7z(1-P_r)\\ &+A_{16}A_6z\{\,t\zeta-1+P_r\ \}+\frac{1}{2}\sqrt{\frac{P_r}{R}}A_{16}A_8A_{11}z\zeta\zeta]-\frac{G_m}{2a^2\sqrt{\pi}}[\,2az\sqrt{S_c}e^{-\frac{z^2S_c}{4t}}\sqrt{t}+A_{15}\sqrt{\pi}(az^2S_c\\ &+2at+2S_c-2)+A_{13}\sqrt{\pi}(A_9+A_{10}S_c)],\end{aligned} \tag{24}$$

$$\theta = \frac{1}{4\sqrt{R}} e^{-\sqrt{R}z}\{e^{2\sqrt{R}z} erfc[\frac{2\sqrt{R}t + zP_r}{\sqrt{P_r t}}](2\sqrt{R}t + zR) - erfc[\frac{-2\sqrt{R}t + zP_r}{\sqrt{P_r t}}](zR - 2\sqrt{R}t)\}, \quad (25)$$

$$C = t(1 + \frac{z^2 S_c}{2t}) erfc[\frac{\sqrt{S_c}}{2\sqrt{t}}] - t\frac{z\sqrt{S_c}}{\sqrt{\pi\sqrt{t}}} e^{-\frac{z^2}{4t}} S_c. \quad (26)$$

The expressions for the symbols involved in the above equations are given in the appendix.

2.1. Nusselt number

The dimensionless Nusselt number is given by the formula

$$Nu = -\left(\frac{\partial\theta}{\partial z}\right)_{z=0},$$

$$Nu = \frac{e^{-\frac{Rt}{P_r}}\sqrt{tP_r}}{\sqrt{\pi}} - erfc[\frac{\sqrt{Rt}}{\sqrt{tP_r}}](\sqrt{R}t - \frac{\sqrt{R}}{2}t + \frac{P_r}{4\sqrt{R}}) + erfc[-\frac{\sqrt{Rt}}{\sqrt{tP_r}}](\frac{\sqrt{R}}{2}t + \frac{P_r}{4\sqrt{R}}). \quad (27)$$

The numerical values of Nusselt number is obtained in table-1 for different parameters.

3. Result and discussions

The results are shown graphically in figures 1 to 8. The numerical values of Nusselt number are tabulated in table-1. It is evident from figures 3 and 7 that the primary velocity u and secondary velocity v increase on increasing the values of m. This shows that Hall current tends to accelerate fluid velocity in both the primary and secondary flow directions. It is also clear from figures 2, 6 and 4, 8 that the primary velocity u decreases and secondary velocity v increases on increasing the values of Ha and R. This implies that magnetic field and radiation tends to accelerate secondary velocity whereas it retards primary velocity in the boundary layer region. From figures 1 and 5 it is clear that the primary velocity u increases and secondary velocity v decreases with increasing the values of α. So inclination of magnetic field has an accelerating behavior on primary velocity and retarding influence on secondary velocity.

The numerical values of Nusselt number is given in table-1. It is seen that value of Nu increases with increase in Prandtl number, radiation parameter and time.

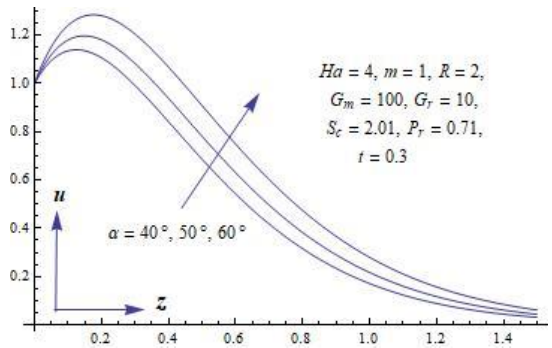

Figure 1. Velocity *u* for different values of α

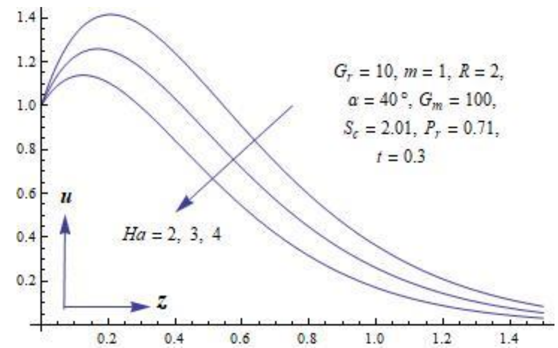

Figure 2. Velocity *u* for different values of *Ha*

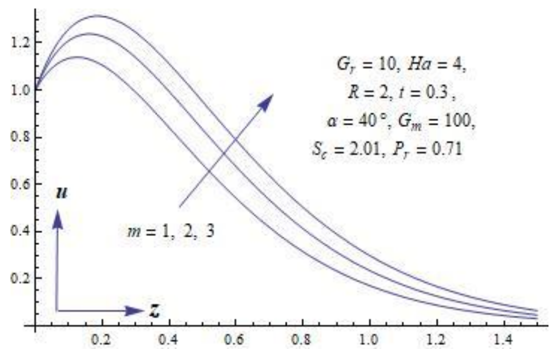

Figure 3. Velocity *u* for different values of *m*

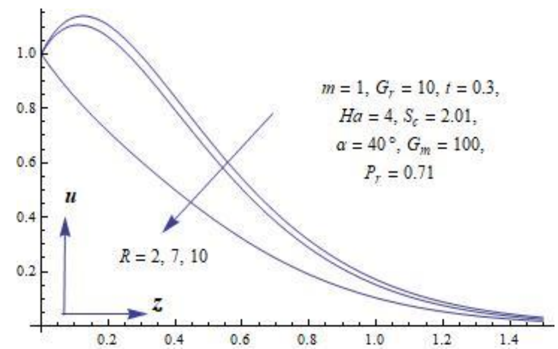

Figure 4. Velocity *u* for different values of *R*

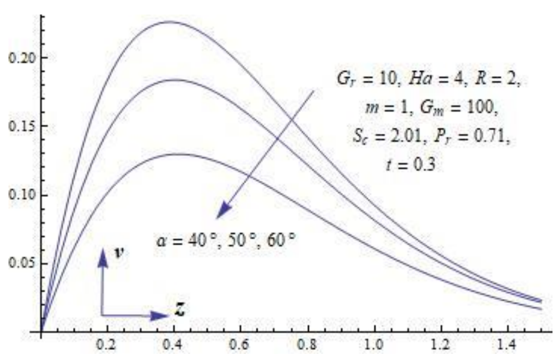

Figure 5. Velocity *v* for different values of α

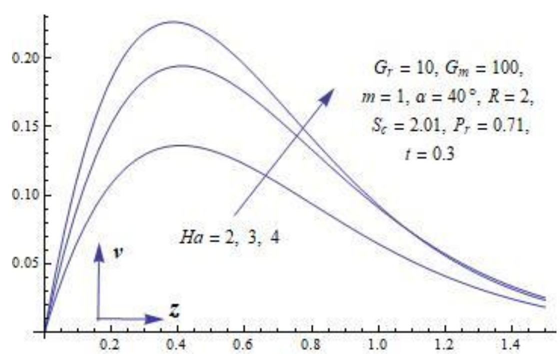

Figure 6. Velocity *v* for different values of *Ha*

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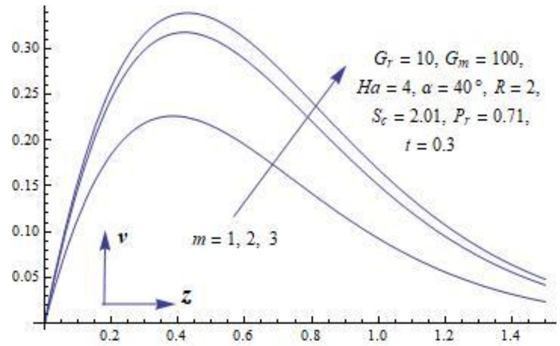

Figure 7. Velocity *v* for different values of *m*

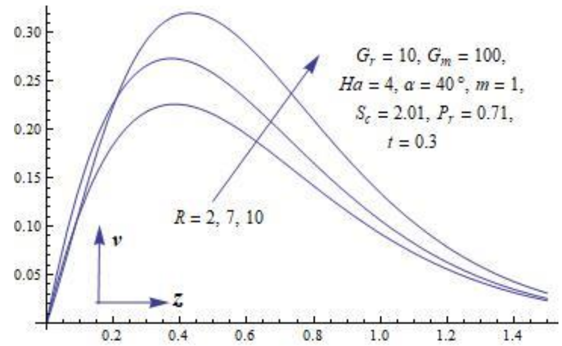

Figure 8. Velocity *v* for different values of *R*

Table 1. Nusselt number for different parameter

P_r	R	t	Nu
0.71	02	0.30	0.65640
7.00	02	0.30	1.68150
0.71	07	0.30	0.92750
0.71	10	0.30	1.06088
0.71	02	0.20	0.50089
0.71	02	0.25	0.57985

4. Conclusion

A theoretical analysis has been done to study the Hall current and radiation effects on unsteady natural convection MHD flow in the presence of an inclined magnetic field. It is found that the magnetic field, inclination of magnetic field, Hall current and radiation have significant effects on the flow. From the analysis, it is seen that Hall current tends to accelerate fluid velocity in both the primary and secondary flow directions. The effect of magnetic field and radiation on flow retards the primary velocity whereas it accelerates the secondary velocity. Inclination of magnetic field accelerates the primary velocity whereas it retards the secondary velocity. Nusselt number increases on increasing radiation parameter, Prandtl number and time. The results of this study may be applicable in the field related to the solar physics dealing with the solar cycle, MHD sensors, rotating MHD induction machine, energy generator, the sunspot development, the structure of rotating magnetic stars etc.

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APPENDIX

$$a=\frac{M(1-im)}{1+m^2},\ \zeta=a-R,\ A_0=\frac{u_0^2 t}{\upsilon},\ A_1=1+e^{2\sqrt{az}}(1-A_{18})-A_{17},\ A_2=-A_1,$$

$$A_3=1-e^{2\sqrt{az}}(1-A_{18})-A_{17},\ A_4=-1+A_{19}+A_{30}(A_{20}-1),\ A_5=-1+A_{21}+A_{28}(A_{22}-1),$$

$$A_6=-1+A_{23}+A_{26}(A_{31}-1),\ A_7=-1+A_{29}+A_{27}(A_{30}-1),\ A_8=-1+A_{23}+A_{26}(A_{31}-1),$$

$$A_9=-1-A_{24}-A_{28}(1-A_{25}),\ A_{10}=-A_9,\ A_{11}=Abs[z].Abs[P_r],\ A_{12}=e^{\frac{at}{P_r-1}-\frac{Rt}{P_r-1}-z\sqrt{\frac{aP_r-R}{P_r-1}}},$$

$$A_{13}=e^{\frac{at}{S_c-1}-z\sqrt{\frac{aS_c}{S_c-1}}},\ A_{14}=e^{\frac{at}{P_r-1}-\frac{Rt}{P_r-1}-Abs[z]\sqrt{\frac{P_r(aP_r-R)}{P_r-1}}},\ A_{15}=-1+erf[\frac{z\sqrt{S_c}}{2\sqrt{t}}],\quad A_{16}=e^{Abs[z]\sqrt{P_rR}},$$

$$A_{17}=erf[\frac{2\sqrt{a}t-z}{2\sqrt{t}}],\ A_{18}=erf[\frac{2\sqrt{a}t+z}{2\sqrt{t}}],\ A_{19}=erf[\frac{z-2t\sqrt{\frac{aP_r-R}{P_r-1}}}{2t}],\ A_{20}=erf[\frac{z+2t\sqrt{\frac{aP_r-R}{P_r-1}}}{2t}],$$

$$A_{21}=erf[\frac{z-2t\sqrt{\frac{aS_c}{S_c-1}}}{2t}],\ A_{22}=erf[\frac{z+2t\sqrt{\frac{aS_c}{S_c-1}}}{2t}],\ A_{23}=erf[\frac{Abs[z].Abs[P_r]}{2\sqrt{t}}-\sqrt{\frac{tR}{P_r}}],$$

$$A_{24}=erf[\frac{2t\sqrt{\frac{a}{S_c-1}}-2\sqrt{S_c}}{2t}],\ A_{25}=erf[\frac{2t\sqrt{\frac{a}{S_c-1}}+2\sqrt{S_c}}{2t}],\ A_{26}=e^{2Abs[z]\sqrt{P_rR}},\ A_{27}=e^{2Abs[z]\sqrt{\frac{P_r(aP_r-R)}{P_r-1}}},$$

$$A_{28}=e^{-2z\sqrt{\frac{aS_c}{S_c-1}}},\ A_{29}=erf[\frac{Abs[z].Abs[P_r]}{2\sqrt{t}}-\sqrt{\frac{t(R-aP_r)}{P_r(1-P_r)}}],\ A_{30}=e^{-2z\sqrt{\frac{aP_r-R}{P_r-1}}},$$

$$A_{31}=erf[\frac{Abs[z].Abs[P_r]}{2\sqrt{t}}+\sqrt{\frac{tR}{P_r}}],\ A_{32}=erf[\frac{Abs[z].Abs[P_r]}{2\sqrt{t}}+\sqrt{\frac{t(R-aP_r)}{P_r(1-P_r)}}],\ A_{33}=1+A_{17}+e^{2\sqrt{az}}A_{34},$$

$$A_{34}=erfc[\frac{2\sqrt{a}t+z}{2\sqrt{t}}].$$

Nomenclature

a^* absorption constant

β volumetric coefficient of thermal expansion

t time

T temperature of the fluid

g acceleration due to gravity

β^* volumetric coefficient of concentration expansion

μ the coefficient of viscosity

T_∞ temperature of the plate at $y\to\infty$

C species concentration in the fluid
υ kinematic viscosity
k thermal conductivity of the fluid
T_w temperature of the plate at $y=0$
C_w species concentration at the plate, at $y=0$
ρ density
Ha Hartmann number
m Hall current parameter
R Radiation parameter,
Sc Schmidt number
$\bar{u},\bar{v}$ dimensionless velocity in x & z direction
Gm mass Grashof number
$\bar{C}$ dimensionless concentration
C_∞ species concentration at $y \to \infty$
C_P specific heat at constant pressure
D mass diffusion coefficient
B_0 uniform magnetic field
σ electrical conductivity
Gr thermal Grashof number
α angle of inclination of magnetic field from horizontal
u,v velocities of the fluid in x & z-directions
$\bar{t}$ dimensionless time
Pr Prandtl number
θ dimensionless temperature
σ^* Stefan Boltzmann constant