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Abstract

In this paper, we investigate the effects of nonlinear behaviour of the dimensionless concentrations of the ethanol and acetaldehyde in a fixed bed laboratory reactor. The work is based on solving the nonlinear differential equation of concentration of the ethanol and acetaldehyde by means of the He's variational iteration method (VIM). Also, the numerical simulation (4^{th} order Runge – Kutta method) is reported using Matlab software. The analytical solutions are compared with numerical results in order to achieve conclusions based on not only for accuracy and efficiency of the solutions, but also the simplicity of the taken procedures which would have remarkable effects on the time devoted for solving process. The analytical result reported in this work is useful to understand the behaviour of the system. Furthermore, due to the accuracy and convergence of obtained solutions, it is proved that the VIM could be applied through other nonlinear problems even with high nonlinearity.

Keywords: VOCs; Catalytic combustion; Ethanol; Acetaldehyde; Non-linear equations; Variation iteration method; Numerical simulation

MSC 2020 No.: 34A45, 76D07, 76W05

1. Introduction

Many chemicals, petrochemical and allied industries are releasing volatile organic compounds (VOCs) (see Khan and Goshal (2000), Baker et al. (1987) and Deng et al. (1996)). Some common VOCs are as follows: Acetaldehyde, ethyl acetate, ethanol, methyl chloride, various chlorohydrocarbons and perfluorocarbons, which are emitted from several chemical industries (see Okumura et al. (2003), Morales et al (2008) and Sax et al. (2004)). In addition, ethyl acetate and ethanol are common VOC emitted in printing processes (Sax et al. (2004)). Also, it is necessary to limit and control VOCs because they affect the change of climate, the growth and decay of plants, the health of human beings (such as liver and kidney damage and cancer) and all animals (see Larsson and Andersson (2000) and Delimaris and Ioannides (2008)). Catalytic combustion is one important technology for eliminating VOC emissions. Agustina Campesi et al. (2011) reported the kinetic study of ethanol on an efficient catalyst identified as Mn/Cu catalyst. To study the kinetic parameters of a model, it is necessary to find the analytical solution for that model.

In this work, we apply the VIM (Agustina Campesi et al. (2011), He (1999), Wazwaz and Rach (2011), Wazwaz (2009) and Odibat and Momani (2006)) to systematically obtain a rapidly convergent analytic approximate solution of the concentrations of ethanol, acetaldehyde and ethyl acetate that is convenient for numerical simulations. Also, the obtained analytical expression is validated by graphs of the error analysis that features the error remainder functions and the maximal error remainder parameters instead of comparison to an alternate solution technique alone.

We remark that the VIM has been efficiently used to solve a wide variety of nonlinear problems in engineering and science (Soliman (2005), He and Wu (2006), Batiha (2009), Rafei et al (2007), Batiha et al. (2007), especially including several in theoretical chemistry (see Jamshidi and Ganji (2010), Bildik and Konuralp (2006), Mo et al. (20018), Ganji and Sadighi (2007), Tari et al. (2007), Sadighi and Ganji (2007) and Barari et al. (2008)).

2. Problem Description

In Agustina Campesi et al. (2011), a mathematical model that relates the molar concentrations of ethanol and acetaldehyde inside the catalyst particle is established as nonlinear differential equation. Consider nonlinear differential equations as the following:

$$D_{ef,Et}\left[\frac{1}{z^2}\frac{d}{dz}\left(z^2\frac{dC_{Et}}{dz}\right)\right]=r_1, \tag{1}$$

$$D_{ef,Ac}\left[\frac{1}{z^2}\frac{d}{dz}\left(z^2\frac{dC_{Ac}}{dz}\right)\right]=r_2-r_1. \tag{2}$$

The boundary conditions related Equations (1) and (2) are given by

$$C_{Et}=C_{Et}^b,\ \ C_{Ac}=C_{Ac}^b \quad \text{at } z=R, \tag{3}$$

$$\frac{dC_{Et}}{dz} = \frac{dC_{Ac}}{dz} = 0, \qquad \text{at } z = 0. \tag{4}$$

The reaction rates r_1 and r_2 are as follows:

$$r_1 = \frac{k_{ref_1} \exp[-(E_1/R_G)((1/T) - (1/T_{ref}))]\, C_{Et}}{1 + K_{C_{Et}} C_{Et} + K_{C_{Ac}} C_{Ac}}, \tag{5}$$

$$r_2 = \frac{k_{ref_2} \exp[-(E_2/R_G)((1/T) - (1/T_{ref}))] K_{C_{Ac}} C_{Ac}}{1 + K_{C_{Et}} C_{Et} + K_{C_{Ac}} C_{Ac}}, \tag{6}$$

where C_{Et} and C_{Ac} represent the molar concentrations of ethanol and acetaldehyde inside the catalyst particle, $D_{ef,Et}$ and $D_{ef,Ac}$ are the effective diffusivities of ethanol and acetaldehyde, T_{ref} is reference temperature, R_G is a gas constant, $K_{C_{Et}}$ and $K_{C_{Ac}}$ are the adsorption equilibrium constant of ethanol and constant of acetaldehyde, respectively, z is the axial length of the reactor, E_1 and E_2 are activation energies, respectively.
We introduce the following dimensionless variables:

$$U = \frac{C_{Et}}{C_{Et}^b};\ W = \frac{C_{Ac}}{C_{Ac}^b};\ x = \frac{z}{R};\ \Phi_1^2 = \frac{k_{ref_1} R^2}{D_{ef,Et}};\ \Phi_2^2 = \frac{k_{ref_2} R^2}{D_{ef,Et}};\ \Phi_3^2 = \frac{k_{ref_1} C_{Et}^b R^2}{D_{ef,Ac} C_{Ac}^b};$$

$$\beta_1 = k_{C_{Et}} C_{Et}^b;\ \beta_2 = k_{C_{Ac}} C_{Ac}^b;\ \gamma_1 = \frac{E_1}{R_G}\left(\frac{1}{T} - \frac{1}{T_{ref}}\right);\ \gamma_2 = \frac{E_2}{R_G}\left(\frac{1}{T} - \frac{1}{T_{ref}}\right).$$

If we view these dimensionless variables, then Equations (1) and (2) can be reduced to the following coupled system of nonlinear differential equations, respectively:

$$\frac{d^2U}{dx^2} + \frac{2}{x}\frac{dU}{dx} = F_1(U(x), W(x)), \tag{7}$$

$$\frac{d^2W}{dx^2} + \frac{2}{x}\frac{dW}{dx} = F_2(U(x), W(x)), \tag{8}$$

and the boundary conditions (3) and (4) are also listed as follows, respectively:

$$x = 0 \ \Rightarrow \ \frac{dU}{dx} = 0,\ \frac{dW}{dx} = 0, \tag{9}$$

$$x = 1 \ \Rightarrow \ U = 1, W = 1, \tag{10}$$

where the functions $U(x)$ and $W(x)$ are the concentration of the ethanol and acetaldehyde, respectively, x is axial length of the reactor inside the catalyst particle, and the system nonlinearities are

$$F_1\big(U(x), W(x)\big) = \Phi_1^2 \ \mathrm{e}^{-\gamma_1} \ f_1\big(U(x), W(x)\big),$$
$$F_2\big(U(x), W(x)\big) = \Phi_2^2 \ \mathrm{e}^{-\gamma_2} \ f_2\big(U(x), W(x)\big) - \Phi_3^2 \ \mathrm{e}^{-\gamma_1} \ f_1\big(U(x), W(x)\big),$$

where

$$f_1\big(U(x), W(x)\big) = \frac{U}{1+\beta_1 U + \beta_2 W}, \ f_2\big(U(x), W(x)\big) = \frac{W}{1+\beta_1 U + \beta_2 W}.$$

3. Mathematical Procedures and Default He's Variational Iteration Method

In this section, the VIM method has been investigated. We explain the basic ideas of He (1999) about the VIM. We now consider the following nonlinear differential equation:

$$Lu + Nu = g(t),$$

where L and N are linear and nonlinear operators, respectively, and $g(t)$ is the source inhomogeneous term. According to the VIM, we can write down a correction functional as follows:

$$u_{n+1}(x) = u_n(x) + \int_0^x \lambda \left(Lu_n(t) + N\tilde{u}_n(t) - g(t)\right) dt,$$

where λ is a general Lagrange multiplies. Some information about general Lagrange multiplies such as λ can also be find in He (1998; 2006) and Skeel et al. (1990). It should be noted that λ can be a constant or a function, and it can be identified optimally via the variation theory, the subscript n indicates the n^{th} approximation and $\tilde{u}_n$ is a restricted variation which means that $\delta\tilde{u}_n = 0$ .The successive approximations u_{n+1}, for $n \geq 0$, of the solution $u(t)$ can be readily obtained upon any selective function $u_0(t)$. Consequently, the exact solution, if it exists, will be given by

$$y(x) = \lim_{n\to\infty} u_n(x).$$

Furthermore, in particular cases, for some recent papers on soliton solutions to wave motion equation, solitary wave structures to fractional Schrodinger equation and Bogoyavlenskii equations, good Boussinesq equation, etc., we referee the readers to Alam and Tunç (2020a, 202b, 2020c), Almatrafi et al. (2020), Al-Asad et al. (2021) and Islam et al. (2021)).

4. Application of Described Manners in the Issue and He's Variational Iteration Method

In this section, we apply the VIM to nonlinear ordinary differential Equations (7), (8) with the boundary conditions (9), (10).
To use the VIM, we first determine the correction functional in the form:

$$U_{n+1}(x) = U_n(x) + \int_0^x \lambda \left(U_n''(t) + \frac{2}{t} U_n'(t) - F_1\big(U_n(t), W_n(t)\big) \right) dt,$$

$$W_{n+1}(x) = W_n(x) + \int_0^x \lambda\left(W_n''(t) + \frac{2}{t} W_n'(t) - F_2\left(U_n(t), W_n(t)\right)\right) dt .$$

The Lagrangian multiplier can be identified as $\lambda = \frac{t(t-x)}{x}$. As a result, we derive the following iteration formula:

$$U_{n+1}(x) = U_n(x) + \int_0^x \frac{t(t-x)}{x}\left(U_n''(t) + \frac{2}{t} U_n'(t) - F_1\left(U_n(t), W_n(t)\right)\right) dt,$$

$$W_{n+1}(x) = W_n(x) + \int_0^x \frac{t(t-x)}{x}\left(W_n''(t) + \frac{2}{t} W_n'(t) - F_2\left(U_n(t), W_n(t)\right)\right) dt.$$

The above formulation needs to start with the best choice for the zeroth approximations $U_0(x)$ and $W_0(x)$, which satisfy the boundary conditions, that accelerate the convergence of the successive approximations, and they have the following selections:

$$U_0(x) = U(0) + xU'(0) = \delta ,$$
$$W_0(x) = W(0) + xW'(0) = \delta' ,$$

where the undetermined constants δ and δ' are approximated by using the boundary conditions $U(1)$ and $W(1)$. This kind of selections has been proved to be effective to achieve convergent successive approximations. Moreover, one significant feature of the VIM is that it can be applied in a straightforward manner without any restrictive assumptions such as linearity and perturbation. Also, the VIM does not require the use of the Adomian polynomials. The obtained solutions for $U(x)$ and $W(x)$ will be provided in a convergent power series as proved in He (1999), Wazwaz and Rach (2011), Wazwaz (2009) and Odibat and Momani (2006)). We obtain the following approximations:

$$\begin{aligned}
&U_0(x) = \delta ,\\
&W_0(x) = \delta' ,\\
&U_1(x) = \delta + \left(\frac{\Phi_1^2 \ \mathrm{e}^{-\gamma_1} \delta}{1 + \beta_1 \delta + \beta_2 \delta'} \right) \frac{x^2}{6} ,\\
&W_1(x) = \delta' + \left(\frac{\Phi_2^2 \ \mathrm{e}^{-\gamma_2} \delta' - \Phi_3^2 \ \mathrm{e}^{-\gamma_1} \delta}{1 + \beta_1 \delta + \beta_2 \delta'} \right) \frac{x^2}{6} . \qquad (11)\\
&\quad \cdots
\end{aligned}$$

Substituting the boundary conditions $U(1) = 1$ and $W(1) = 1$ into (11) and solving the resulting equation, we can obtain the numerical values for δ and δ'. Also, it is obvious that the initial values $U_0(1) = \delta$ and $W_0(1) = \delta'$ are not fixed and depend mainly on the constants $\Phi_1, \Phi_2, \Phi_3, \gamma_1, \gamma_2, \beta_1$ and β_2 .Therefore, the approximate solution functions are as follows:

$$U(x) = \sum_{n=0}^{m} U_n(x), \qquad (12)$$

$$W(x) = \sum_{n=0}^{m} U_n(x) . \qquad (13)$$

5. Solution with MATLAB Software

The diffusion equations (Equations (7) and (8)) for the boundary conditions (9) and (10) are solved by numerical methods. The function pdex1 in MATLAB software is used to solve the initial-boundary value problems for the nonlinear differential equations. This numerical solution is compared with our analytical results in Figure 1 and Figure 2. Upon comparison, it yields a satisfactory agreement for all values of the dimensionless parameters $\Phi_1, \Phi_2, \gamma_1, \gamma_2, \beta_1$ and β_2.

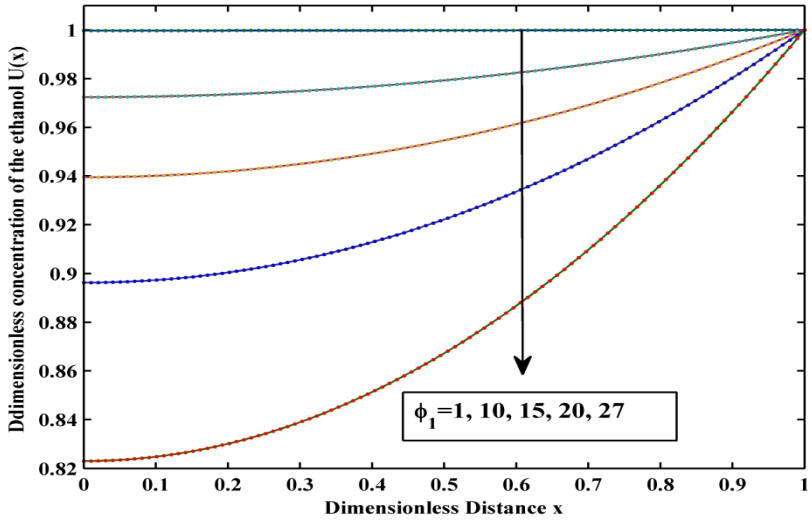

Figure 1. Comparison between numerical and the VIM solution results for dimensionless concentration of the ethanol $U(x)$ when $\beta_1 = \beta_2 = \Phi_1 = \Phi_2 = \Phi_3 = \gamma_2 = 1, \gamma_1 = 5$. Key to the plot: (—) represents Equation (12) and (•••) numerical simulation Equation (7)

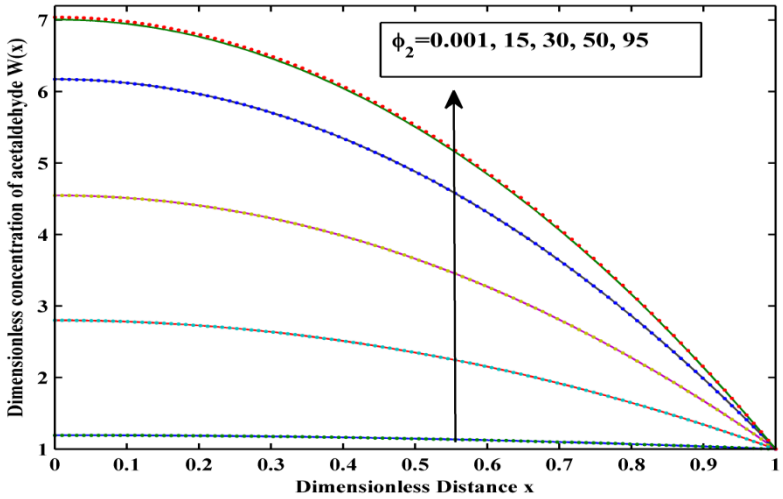

Figure 2. Comparison between numerical and VIM solution results for dimensionless concentration of the acetaldehyde $W(x)$ when $\beta_1 = \Phi_3 = 100, \Phi_2 = 0.001, \Phi_1 = \gamma_1 = \gamma_2 = 1$. Key to the plot: (—) represents Equation (13) and (•••) numerical simulation Equation (8)

6. Validation of the Model

The analytical results for the fixed bed laboratory reactor model using Equations (7), (8) with the boundary conditions (9), (10) are validated against the numerical results. Our analytical expression of dimensionless concentration of the ethanol $U(x)$ and acetaldehyde $W(x)$ are compared with simulation results in Tables 1 and 2 for various values of the dimensionless parameters $\Phi_1, \Phi_2, \gamma_1, \gamma_2, \beta_1$ and β_2, respectively. From these tables, it can be noticed that our analytical results match quite well with the numerical results. The relative error between our analytical and numerical result does not exceed 1.5% for all values of parameters considered in the simulation. In addition, the maximum relative errors between our results with numerical simulation results are 0.0949% and 0.7939 %, respectively.

7. Results and Discussions

In this paper, the VIM has been utilized in order to solve the strongly nonlinear differential equation of dimensionless concentration of the ethanol $U(x)$ and acetaldehyde $W(x)$ with in the fixed bed laboratory reactor model. We have shown the VIM efficiency and accuracy through proper figures and tables. Figures 3, 4 show the difference between obtained solution by the VIM and numerical simulation (4^{th} order Runge – Kutta method) in which we have introduced error percentage as follows:

$$\% \text{ Error} = \left| \frac{Z(x)_{NM} - Z(x)_{VIM}}{Z(x)_{NM}} \right| \times 100, \tag{14}$$

where $Z(x)_{NM}$ and $Z(x)_{VIM}$ are the values obtained by numerical method and the VIM. Here, Equation (14) has been applied through functions of Equations (12), (13) so the parameter Z has only been defined as symbol of data in this case.

Comparison between the VIM and numerical results for various values of the reaction parameters are shown in Figures 1, 2 and Tables 1, 2. The obtained results in comparison with numerical simulations show that the VIM has been enough accuracy and efficiency so it would be applicable for solving strongly nonlinear equations of coupled system. Subsequently, the effect of different parameters such as $\Phi_1, \Phi_2, \Phi_3, \gamma_1, \gamma_2, \beta_1$ and β_2 on the concentration characteristics is discussed. Figures 5, 6 show the set of figures which in each of these effects on the reaction parameter has been represented. The plots of a dimensionless concentration of the ethanol $U(x)$ versus dimensionless distance x are shown in Figures 5(a) - (d) using Equation (12). The dimensionless concentration of the ethanol $U(x)$ as attain maximum (i.e. $U \approx 1$) for the dimensionless reaction parameters $\Phi_1, \gamma_1, \beta_1$ and β_2 is greater than 0.001.

Figures 6 (a) - (d) show the dimensionless concentration of the acetaldehyde $W(x)$ versus dimensionless distance x using Equation (13). The graphs are plotted for various values of Φ_2, Φ_3, β_1 and β_2 and for some fixed values of other existing parameters. From Figure 6 (a), it is also inferred that the concentration is uniform (i.e. $W \approx 1$), when $\Phi_3 \geq 0.001$. Also, Figures 6 (b) – (d) represent that the concentration of the acetaldehyde $W(x)$ decreases when the value

of the reaction parameters Φ_2, β_1 and β_2 increase for all possible values of the other existing parameters.

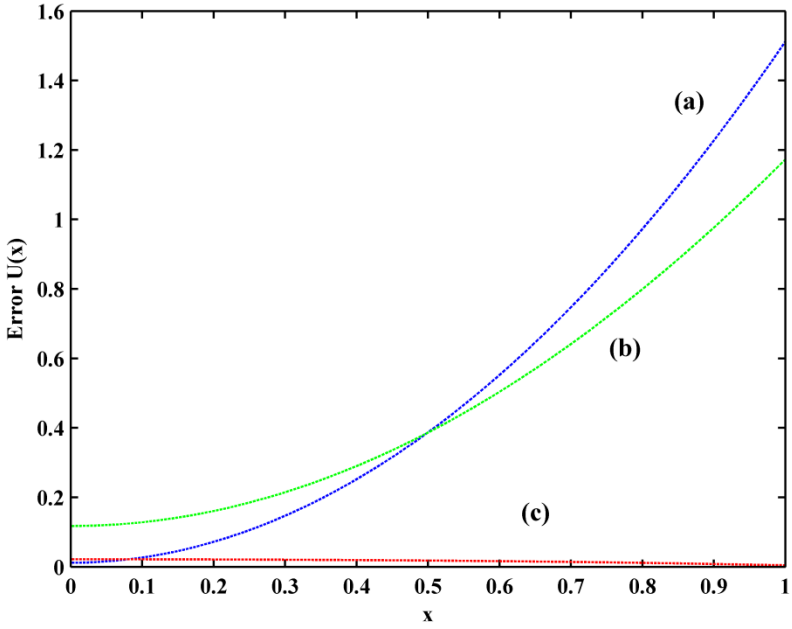

Figure 3. Obtained error for dimensionless concentration of the ethanol $U(x)$ when
(*a*) $\beta_1 = \beta_2 = \Phi_1 = \Phi_2 = \Phi_3 = \gamma_2 = 1,\ \gamma_1 = 5,$ (b)
$\beta_1 = \beta_2 = \Phi_1 = \Phi_2 = \Phi_3 = 1,\ \gamma_1 = 2.5,$
$\gamma_2 = 0.5,$ (c) $\beta_1 = \beta_2 = 10, \Phi_1 = \Phi_2 = \Phi_3 = \gamma_1 = \gamma_2 = 1$

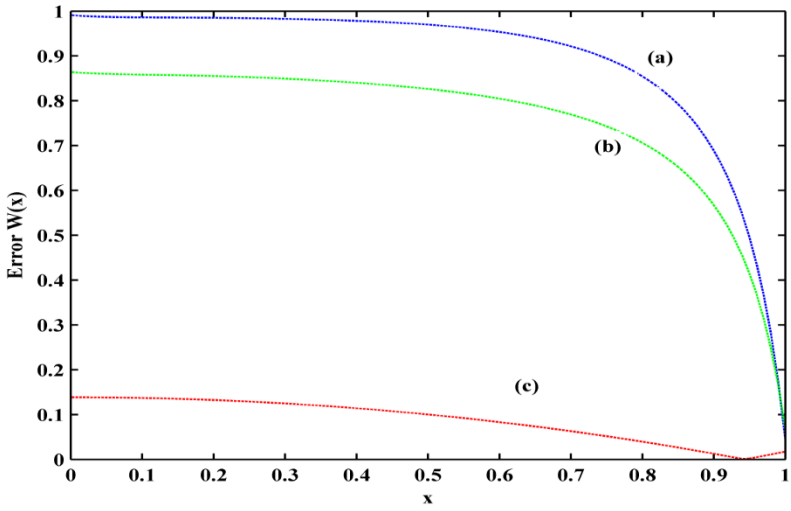

Figure 4. Obtained error for dimensionless concentration of the acetaldehyde $W(x)$ when
(*a*) $\beta_1 = \Phi_3 = 100, \beta_2 = 0.1, \Phi_1 = \gamma_1 = \gamma_2 = 1, \Phi_2 = 0.001,$ (b
$\beta_1 = \Phi_3 = 100, \beta_2 = \Phi_2 = 0.01,\ \Phi_1 = \gamma_1$ $= \gamma_2 = 1,$ (c)
$\beta_1 = \beta_2 = \Phi_1 = \Phi_2 = \gamma_2 = 1, \Phi_3 = 75, \gamma_1 = 10$

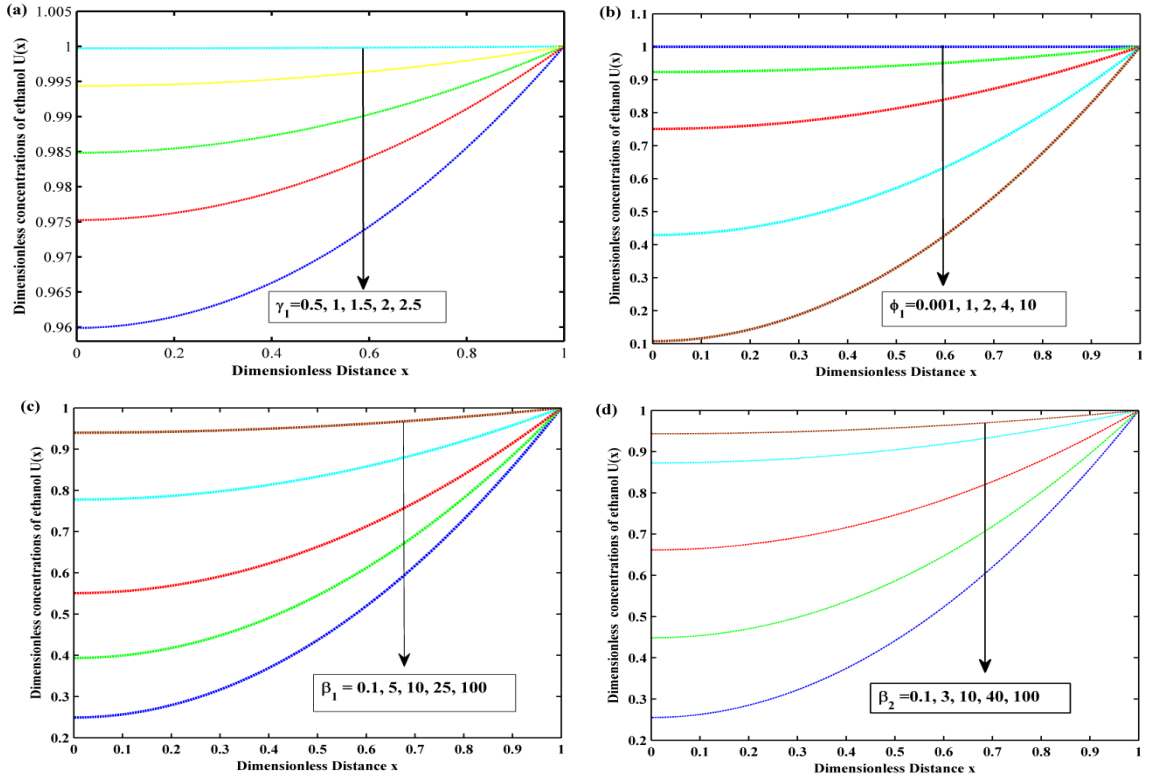

Figure 5. Effect of $\beta_1,\ \beta_2, \Phi_1, \gamma_1$ on dimensionless concentration of the ethanol $U(x)$ when (*a*) $\beta_1 = \beta_2 = \Phi_1 = \Phi_2 = \Phi_3 = \gamma_2 = 1,$ (b) $\beta_1 = \beta_2 = \Phi_2 = \gamma_1 = \gamma_2 = 0.001, \Phi_3 = 10,$ (c) $\beta_2 = 0.01,\ \Phi_1 = 10, \Phi_2 = \Phi_3 = \gamma_1 = \gamma_2 = 1,$ (d) $\beta_1 = 0.01,\ \Phi_1 = 10, \Phi_2 = \Phi_3 = \gamma_1 = \gamma_2 = 1$

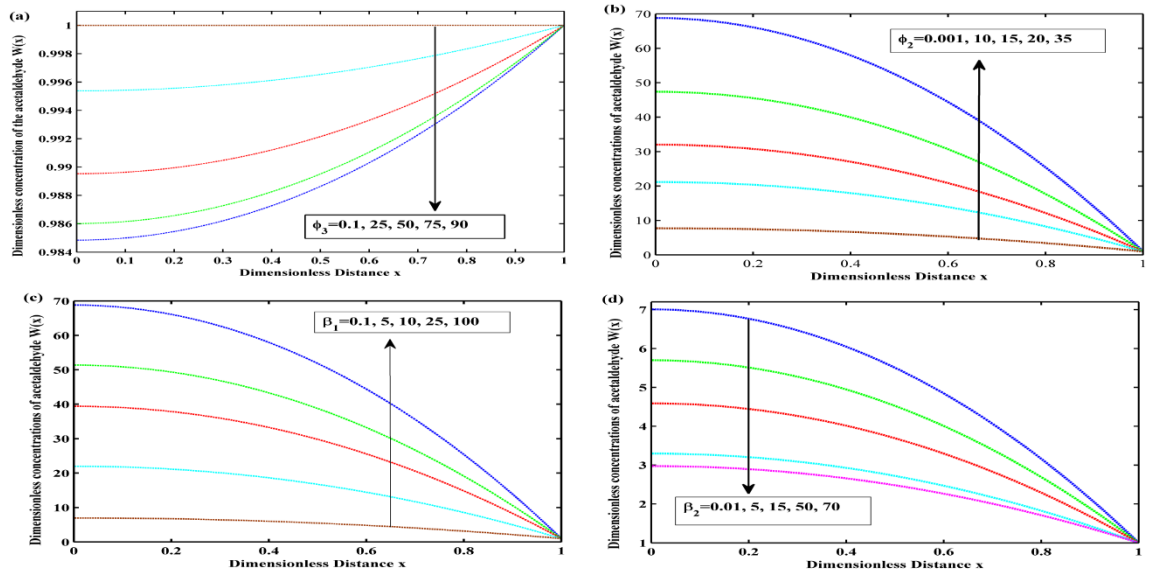

Figure 6. Effect of $\beta_1,\ \beta_2, \Phi_2, \Phi_3$ on dimensionless concentration of the acetaldehyde $W(x)$ when (*a*) $\beta_1 = \beta_2 = \Phi_1 = \Phi_2 = \gamma_2 = 1, \gamma_1 = 10,$ (b) $\beta_1 = \beta_2 = 0.1, \Phi_1 = \gamma_1 = \gamma_2 = 1, \Phi_3 = 100,$ (c) $\beta_2 = 0.1,\ \Phi_1 = 0.001, \Phi_3 = 100, \Phi_1 = \gamma_1 = \gamma_2 = 1,$ (d) $\beta_1 = \Phi_3 = 100,\ \Phi_2 = 0.001,\quad \Phi_1 = \gamma_1 = \gamma_2 = 1$

8. Conclusions

In this work, the VIM have been successfully applied to find the solution of the concentration of the ethanol $U(x)$ and acetaldehyde $W(x)$ with in the fixed bed laboratory reactor model. Comparisons have been done among the VIM and numerical method by different reaction parameters values. Data from error figures represents that obtained solutions with the VIM has minor differences with numerical simulations. Furthermore, according to achieved results, these works are useful to understand the behaviour of the system. Also, it is obvious that the VIM is convenient analytical method due to its accuracy, efficiency and convergence. It could be applicable for solving strongly nonlinear differential equations.

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Nomenclature

$D_{ef,Et}$ Effective diffusivities of ethanol (cm^2/s)

$D_{ef,Ac}$ Effective diffusivities of acetaldehyde (cm^2/s)

C_{Et} Concentration of ethanol (mol m^{-3})

C_{Ac} Concentration of acetaldehyde (mol m^{-3})

$K_{C_{Et}}$ Adsorption equilibrium constant of ethanol ($m^3\,mol^{-1}$)

$K_{C_{Ac}}$ Adsorption equilibrium constant of acetaldehyde ($m^3\,mol^{-1}$)

K_{ref_1} Reparameterized preexponential factor of step 1 at a reference temperature T_{ref} (s^{-1})

K_{ref_2} Reparameterized preexponential factor of step 2 at a reference temperature T_{ref} (mol $s^{-1}\,m^{-3}$)

E_1 Activation energy (J mol^{-1})

E_2 Activation energy (J mol^{-1})

T_{ref} Reference temperature (K)

T Temperature (K)

R_G Gas constant (J/ (mol k))

z Axial length of the reactor (cm)

C_{Et}^b Molar concentration of ethanol (mol m^{-3})

C_{Ac}^b Molar concentration of acetaldehyde (mol m^{-3})

R Particle radius (cm)

U Dimensionless molar concentration of ethanol (unitless)

W Dimensionless molar concentration of acetaldehyde (unitless)

x Dimensionless distance (unitless)

$\gamma_1, \gamma_2, \beta_1, \beta_2$ Dimensionless parameters (unitless)

Φ_i^2 Thiele modulus (unitless)

Table 1. Comparison of dimensionless concentration of the ethanol $U(x)$ with numerical results for various values of dimensionless distance x and some fixed values of dimensionless parameters $\Phi_1, \Phi_2, \Phi_3, \gamma_1, \gamma_2, \beta_1$ and β_2

X	Dimensionless concentration of the ethanol $U(x)$								
	$\Phi_1 = 1, \Phi_2 = 1, \Phi_3 = 1, \gamma_1 = 5,$ $\gamma_2 = 1, \beta_1 = 1$ and $\beta_2 = 1$			$\Phi_1 = 1, \Phi_2 = 1, \Phi_3 = 1, \gamma_1 = 2.5,$ $\gamma_2 = 0.5, \beta_1 = 1$ and $\beta_2 = 1$			$\Phi_1 = 1, \Phi_2 = 1, \Phi_3 = 1, \gamma_1 = 1,$ $\gamma_2 = 1, \beta_1 = 10$ and $\beta_2 = 10$		
	δ=0.4998591318, δ′ = 0.4925620054			δ= 0.4982850925, δ′ = 0.4892725513			δ=0.4986086302, δ′ =0.4999961282		
	Num.	VIM	% of Error	Num.	VIM	% of Error	Num.	VIM	% of Error
0	0.9996	0.9997	0.0100	0.9954	0.9966	0.1206	0.9970	0.9972	0.0201
0.2	0.9996	0.9997	0.0100	0.9956	0.9972	0.1607	0.9971	0.9973	0.0200
0.4	0.9997	0.9998	0.0100	0.9961	0.9990	0.2881	0.9975	0.9977	0.0201
0.6	0.9997	0.9998	0.0100	0.9983	0.9983	0.0000	0.9981	0.9982	0.0100
0.8	0.9998	0.9999	0.0100	0.9983	0.9983	0.0000	0.9989	0.9990	0.0100
1	1.0000	1.0000	0.0000	1.0000	1.0000	0.0000	1.0000	1.0000	0.0000
Average error %			8.34×10^{-03}	Average error %		0.0949	Average error %		0.0134

Table 2. Comparison of dimensionless concentration of the acetaldehyde $W(x)$ with numerical results for various values of dimensionless distance x and some fixed values of dimensionless parameters $\Phi_1, \Phi_2, \Phi_3, \gamma_1, \gamma_2, \beta_1$ and β_2

X	Dimensionless concentration of the acetaldehyde $W(x)$								
	$\Phi_1 = 1, \Phi_2 = 0.001, \Phi_3 = 100, \gamma_1 = 1, \gamma_2 = 1, \beta_1 = 100$ and $\beta_2 = 0.1$			$\Phi_1 = 1, \Phi_2 = 0.001, \Phi_3 = 100, \gamma_1 = 1, \gamma_2 = 1, \beta_1 = 100$ and $\beta_2 = 0.01$			$\Phi_1 = 1, \Phi_2 = 0.001, \Phi_3 = 75, \gamma_1 = 1, \gamma_2 = 1, \beta_1 = 100$ and $\beta_2 = 0.01$		
	δ=0.4997014895, δ′ = 3.485105005			δ= 0.4996996549, δ′ = 3.503451128			δ= 0.4999990531, δ′ = 0.4976888928		
	Num.	VIM	% of Error	Num.	VIM	% of Error	Num.	VIM	% of Error
0	7.0400	6.9702	0.9915	7.0680	7.0069	0.8645	0.9940	0.9954	0.1409
0.2	6.7984	6.7314	0.9855	6.8250	6.7666	0.8557	0.9941	0.9956	0.1507
0.4	6.0744	6.0150	0.9779	6.0458	6.0970	0.8469	0.9950	0.9961	0.1106
0.6	4.8674	4.8209	0.9553	4.8837	4.8444	0.8047	0.9962	0.9970	0.0803
0.8	3.1764	3.1493	0.8532	3.1850	3.1625	0.7064	0.9979	0.9983	0.0401
1	1.0000	1.0000	0.0000	1.0000	1.0000	0.0000	1.0000	1.0000	0.0000
Average error %			0.7939	Average error %		0.6797	Average error %		0.0871