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Ridwan Raheem Lekan
African Institute for Mathematical Sciences

Lord Clifford Kavi
University of Ottawa

Nancy Ann Neudauer
Pacific University

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[1*]**Ridwan Raheem Lekan,**[2]**Lord Clifford Kavi and** [3]**Nancy Ann Neudauer**

[1,2]African Institute for Mathematical Sciences
Summerhill Estates, East Legon Hills
Accra, Ghana
[1]ridwan@aims.edu.gh

[2]Department of Mathematics and Statistics
University of Ottawa
Ottawa, Ontario, Canada
[2]lkavi060@uottawa.ca

[3]Department of Mathematics and Computer Science
Pacific University
Forest Grove, Oregon, USA
[3]nancy@pacificu.edu

*Corresponding Author

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Abstract

A Transportation Problem can be modeled using Linear Programming to determine the best transportation schedule that will minimize the transportation cost. Solving a transportation problem requires finding the Initial Basic Feasible Solution (IBFS) before obtaining the optimal solution. We propose a new method for finding the IBFS called the Maximum Difference Extreme Difference Method (MDEDM) which yields an optimal or close to the optimal solution. We also investigate the computational time complexity of MDEDM, and show that it is $O(mn)$.

Keywords: Transportation models; Initial basic feasible solution; Optimal solution; Maximum difference extreme difference method

MSC 2010 No.: 90C35, 90C05

1. Introduction

The *Transportation Problem* was initiated by Hitchcock (Hitchcock (1941)) with the aim of minimizing the total shipping cost of a single commodity from m sources to n destinations subject to supply and demand constraints (Loch and da Silva (2014); Winston and Goldberg (2004); Murty (1983); Bazaraa et al. (2011)). This depends on the demand of the customers and supply available. It enables companies make adequate provisions in their budget for the transportation of goods. It has gained applications in many areas due to its efficiency and effectiveness. These areas include: operation scheduling, network design, and job assignment problems among others. Dantzig in (Dantzig (1951)) followed by Cooper et al. (1953) formulated effective methods for solving transportation problems.

The *Transportation Problem* is a special case of Linear Programming. A *Linear Programming* problem is an optimization problem whose objective function and constraints are linear (Pinedo and Chao (1999)). Optimization involves choosing the best solution that is most cost-effective or efficient (fastest, shortest distance) from a set of possible solutions (Edgar et al. (2001)). One of the possible ways to obtain the optimal solution to a transportation problem is to employ the optimization method. The problem is formulated such that the constraints and the objective functions are linear.

A *solution* to a transportation problem is the quantity of commodities that can be shipped from each of the sources to each of the destinations that will minimize the total cost of transportation.

A solution is said to be *feasible* if it satisfies all demand and supply constraints. A *basic feasible solution* is a feasible solution which has $m+n-1$ allocations, where m and n represent the number of sources and destinations respectively. An *optimal solution* is a feasible solution (that may not be basic) which cannot further generate a transportation route that will minimize the total cost of transportation.

Basic variables are nonzero values available in the basic solution. A variable which is added to the sources or destinations of a transportation problem but has zero effect on the unit cost of transportation is called a *dummy variable*.

The first approach to solving a transportation problem is finding an Initial Basic Feasible Solution (IBFS), and several methods have been proposed. These methods include North West Corner Method (NWCM) (Hamdy (2007)), Least Cost Method (LCM) (Ahmed et al. (2016b)), Row Minimum Method (RMM) (Anam et al. (2012)), Column Minimum Method (CMM) (Ahmed et al. (2016b)), Vogel's Approximation Method (VAM) (Soomro et al. (2014)), Maximum Difference Method (MDM) (Soomro et al. (2014)), Extreme Difference Method (EDM) (Kasana and Kumar (2005)), Allocation Table Method (ATM) (Ahmed et al. (2016b)) and the Incessant Allocation Table Method (IAM) (Ahmed et al. (2016a)).

In this paper, we propose a new method called Maximum Difference Extreme Difference Method (MDEDM) which usually yields optimal or close to optimal solutions. We compare the results obtained using MDEDM with the other traditional algorithms for finding the IBFS. We also show that our method can be useful in other areas of optimization such as a profit maximization problem.

In Section 2, we describe the network flow model of the transportation problem. In Section 3, we illustrate the transportation tableau. In Section 4, we formulate the mathematical representation of the transportation problem. The steps for solving a transportation problem are discussed in Section 5. In Section 6, we explain our proposed method. In Section 7, we apply the proposed method on some examples to illustrate it. In Section 8, we compare the results from the new method with other traditional methods. In Section 9, we analyse the results obtained in comparison with the results from other methods and represent them on plots for clarity and easy interpretation. In Section 10, we investigate the computational time complexity of MDEDM.

2. Network Flow Model of the Transportation Problem

The network flow model is an architecture which depicts the movement of commodities from the sources to the destinations with considerations given to the cost and the quantity of goods shipped across each route. Suppose a company has m production plants (sources) and n destinations (sinks) with daily capacities (supply) and daily demands of $s_1, s_2, \ldots, s_m$ and $d_1, d_2, \ldots, d_n$, respectively. Here a_i denotes the quantity of goods available at source s_i, where $i = 1, 2, \ldots, m$, and b_j denotes the quantity of goods demanded at destination d_j, where $j = 1, 2, \ldots, n$. Assume the cost of transportation from source i to destination j is c_{ij} and the units distributed from source i to destination j is x_{ij}. We can represent this information in the form of a network flow model as shown in Figure 1.

The problem is to determine the unknowns x_{ij} (decision variables) that will minimize the total cost of transportation without violating the demand and supply constraints.

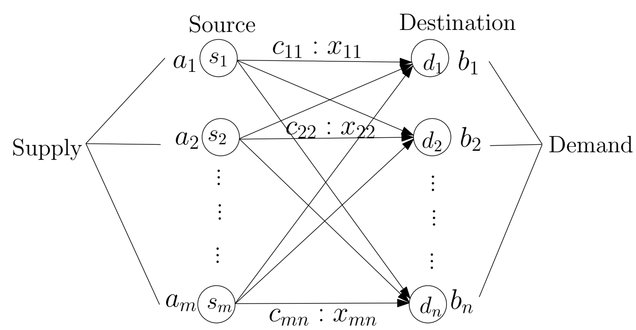

Figure 1. Network flow model of the transportation problem

In the next section, we represent the network flow model in a transportation tableau.

3. Transportation Tableau

A transportation tableau shows the summary of all the relevant parameters in a transportation problem. It is useful when dealing with large datasets that may be too complicated to be easily analysed with a network flow model. It can be used to gain a quick understanding of the transportation problem. Each cell in a transportation tableau is called a route. Each route contains a unit cost c_{ij} of shipping and a decision variable x_{ij}. Other parameters follow as in the network flow model given in Section 2.

Source	Destination				Supply
	d_1	d_2	$\cdots$	d_n	
s_1	x_{11} c_{11}	x_{12} c_{12}	$\cdots$	x_{1n} c_{1n}	a_1
s_2	x_{21} c_{21}	x_{22} c_{22}	$\cdots$	x_{2n} c_{2n}	a_2
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
s_m	x_{m1} c_{m1}	x_{m2} c_{m2}	$\cdots$	x_{mn} c_{mn}	a_m
Demand	b_1	b_2	$\cdots$	b_n	$\sum a_i$ $\sum b_j$

Figure 2. Transportation tableau

In the next section, we give the mathematical representation of the transportation problem.

4. Mathematical Formulation of the Transportation Problem

In this section, we formulate the mathematical representation of the transportation problem by using the parameters given in Figure 1. We consider a linear transportation model, and our goal is to

$$\text{Minimize } z = \sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij} x_{ij},$$

subject to the constraints:

$$\sum_{j=1}^{n} x_{ij} \leq a_i; \; i = 1, 2, \cdots, m,$$

$$\sum_{i=1}^{m} x_{ij} \geq b_j; \; j = 1, 2, \cdots, n,$$

$$x_{ij} \geq 0; \;\; i = 1, 2, \cdots, m \text{ and } j = 1, 2, \cdots, n.$$

A *balanced transportation problem* occurs if

$$\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j,$$

otherwise, it is unbalanced.

The necessary and sufficient condition for the existence of a feasible solution to the transportation problem is $\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j$, that is, the total capacity (supply) must equal total requirement (demand) (Singh (2015)).

In the next section, the general procedure (algorithm) for solving a transportation problem is discussed.

5. Steps for Solving a Transportation Problem

To obtain an optimal solution to a transportation problem, the following steps must be followed.

5.1. Algorithm for a Transportation Problem

Generally, the followings steps (Algorithm 1) must be followed sequentially to solve a transportation problem.

Algorithm 1: Steps for solving a transportation problem

1 Formulate the problem and represent it on a transportation tableau.
2 Check if the transportation problem is balanced; if not, add a dummy variable to either the supply or demand as appropriate.
3 Find the IBFS.
4 Keep performing optimality checks on the IBFS until an optimal solution is obtained.
5 Calculate the total transportation cost by evaluating the objective function.

The focus of this paper is on step 3 of Algorithm 1. We propose a new method called the Maximum Difference Extreme Difference Method (MDEDM) to find the initial basic feasible solution for a transportation problem. In the subsequent sections, we explain MDEDM and apply it to given numerical examples to illustrate how the algorithm works. We then show the effectiveness and accuracy of MDEDM by comparing it with other well known methods of finding the IBFS and also by comparing it with the optimal solutions.

6. Maximum Difference Extreme Difference Method (MDEDM)

The proposed algorithm is given below.

(1) (a) Find the *maximum differences* (MD_i), that is, the difference between the maximum unit cost and the immediate maximum unit cost along the rows.

(b) Find the *extreme differences* (ED_j), that is, the difference between the maximum unit cost and the minimum unit cost along the columns. If the maximum unit cost and the minimum unit cost are equal, the extreme difference is taken to be zero.

(2) (a) Among these costs, that is, the (MD_i)'s and (ED_j)'s, select the cell with the largest difference and find the smallest unit cost cell (c_{ij}) corresponding to it.

(b) If two or more cells of the (MD_i)'s or (ED_j)'s contain the largest difference, select the unit cost cell (c_{ij}) located at the topmost row and at the extreme left corner.

(3) Allocate to the current cell, the minimum between supply a_i and demand b_j, that is, min $\{a_i, b_j\}$.

(a) If $a_i < b_j$, then supplies in that row become zero (exhausted) and crossed-out from the table. The new value of the demand becomes $b_j - a_i$.

(b) If $b_j < a_i$, then demands in that column become zero (exhausted) and crossed-out from the table. The new value of the supply becomes $a_i - b_j$.

(c) If $a_i = b_j$, locate the least cost cell along the i-th row and j-th column and assign a value of zero to it, then cross-out the i-th row and j-th column from the table.

(4) Compute the new differences for the remaining cells as we have in Step 1 and allocate in the same manner. Continue the process until all the rows and columns are satisfied.

(5) Compute the minimum transportation cost by summing the product of cost c_{ij} and the corresponding quantity of goods shipped x_{ij} for the allocated cells.

7. Numerical Illustrations of MDEDM

7.1. Illustration 1-Balanced TP

Suppose Cavanot company has 3 plants located at A, B and C. The normal daily production (supply) of these plants is 50 for A, 70 for B and 45 for C. The company has 3 warehouses located at D, E and F with daily demands of $40, 65$ and 60, respectively. What shipping schedule should be adopted by the company to minimize the total transportation cost? The shipping cost per unit in US$ is given in Table 1.

Table 1. Cavanot company goods shipment (BTP-1)

Source	Destination			**Supply**
	D	E	F	
A	4	2	1	**50**
B	3	8	4	**70**
C	6	5	2	**45**
Demand	**40**	**65**	**60**	**165**

We use MDEDM to solve the problem given in Table 1.

First Iteration: The maximum differences for rows A, B and and C are $4-2=2,\ 8-4=4$ and $6-5=1$, respectively, as shown in the first column of the maximum difference table. Also,

Source	Destination			Supply
	D	E	F	
A	4	[50] 2	1	50
B	[40] 3	8	[30] 4	70
C	6	[15] 5	[30] 2	45
Demand	40	65	60	165

Maximum Difference

2	-	-	-	-
4	(4)	(4)	-	-
1	1	3	3	(2)

Extreme Difference

3	(6)	3
3	3	2
-	3	2
-	(5)	2
-	-	(2)

Figure 3. Solution: Applying Proposed Method to BTP-1

the extreme differences for columns $D,\ E$ and F are $6 - 3 = 3,\ 8 - 2 = 6$ and $4 - 1 = 3$, respectively, as shown in the first row of the extreme difference table. Among these numbers, $2, 4, 1, 3, 6$ and 3, the largest is 6 (circled) and it corresponds to the least unit cost cell AE. Allocate $50 = \min\,(50, 65)$ in cell AE. Cross out row A since it is satisfied and then the value for the demand for column E is reduced to $15 = (65 - 50)$.

Second Iteration: At the end of the first iteration, we have two rows left, rows B and C. The maximum differences for rows B and C are $8 - 4 = 4$ and $6 - 5 = 1$, respectively, as shown in the second column of the maximum difference table. Also, the extreme differences for columns $D,\ E$ and F are $6 - 3 = 3,\ 8 - 5 = 3$ and $4 - 2 = 2$, respectively, as shown in the second row of the extreme difference table. Among these numbers, $4, 1, 3, 3$ and 2, the largest is 4 (circled) and it corresponds to the least unit cost cell BD. Allocate $40 = \min\,(40, 70)$ in cell BD. Cross out column D since it is satisfied and then the value for the supply in row B is reduced to $30 = (70 - 40)$.

The same procedure is used to allocate x_{ij} to other cells as shown in Figure 3 until all the rows and columns are satisfied.

Therefore, the initial basic feasible solution occurs at cells AE, BD, BF, CE and CF. The number of allocated cells is $N = 5$; $m + n - 1 = 3 + 3 - 1 = 5$. Thus, $N = m + n - 1$ (is satisfied). Therefore, the transportation cost = $(40 \times 3) + (50 \times 2) + (15 \times 5) + (30 \times 4) + (30 \times 2) = \$\,475$.

7.2. Illustration 2-Balanced TP

Table 2. Table for BTP-6

Source	Destination				**Supply**
	D	E	F	G	
A	50	60	100	50	**20**
B	80	40	70	50	**38**
C	90	70	30	50	**16**
Demand	**10**	**18**	**22**	**24**	**74**

Consider another balanced transportation problem shown in Table 2. The same procedure explained in Figure 3 is employed in Figure 4 but the extreme difference among the unit costs $50, 50, 50$ is taken to be zero (as shown in the fourth column of the extreme difference table in Figure 4). This continues to be zero until just one cell is left with unit cost 50, where the extreme difference is taken to be 50. The number of allocated cells is $N = 6$; $m + n - 1 = 3 + 4 - 1 = 6$. Thus, $N = m + n - 1$ (is satisfied). Therefore, the transportation cost = $(10 \times 50) + (10 \times 50) + (18 \times 40) + (6 \times 70) + (14 \times 50) + (16 \times 30) =$ \$ 3320.

Source	Destination				Supply
	D	E	F	G	
A	[10] 50	60	100	[10] 50	20
B	80	[18] 40	[6] 70	[14] 50	38
C	90	70	[16] 30	50	16
Demand	10	18	22	24	74

Maximum Difference

40	(40)	(40)	-	-	-
10	10	20	20	10	(40)
20	-	-	-	-	-

Extreme Difference

40	30	(70)	0
30	20	30	0
-	20	30	0
-	40	(70)	50
-	40	-	(50)
-	(40)	-	-

Figure 4. Solution: Applying Proposed Method to BTP-6

7.3. Illustration 3-Unbalanced TP

Table 3. Table for UTP-3

Source	Destination					**Supply**
	E	F	G	H	I	
A	5	8	6	6	3	**800**
B	4	7	7	6	5	**500**
C	8	4	6	6	4	**900**
Demand	**400**	**400**	**500**	**400**	**800**	

Source	Destination					Supply
	E	F	G	H	I	
A	5	8	6	6	[800] 3	800
B	[400] 4	7	7	[100] 6	5	500
C	8	[400] 4	[500] 6	[0] 6	[0] 4	900
D	0	0	0	[300] 0	0	300
Demand	400	400	500	400	800	2500

Maximum Difference

2	2	(3)	-	-	-
1	1	1	1	1	-
2	2	2	(6)	-	-
0	0	0	0	0	(0)

Extreme Difference

(4)	4	1	0	2
-	(4)	1	0	2
-	-	1	0	2
-	-	1	0	-
-	-	-	(6)	-
-	-	-	(0)	-

Figure 5. Solution: Applying Proposed Method to UTP-3

Consider an unbalanced transportation problem shown in 3. In this case, we add a dummy supply to Table 3 and apply the same procedure as we have explained in Figure 3 to obtain the IBFS shown in Figure 5. The number of allocated cells is $N = 8$; $m + n - 1 = 4 + 5 - 1 = 8$. Thus, $N = m + n - 1$ (is satisfied). Therefore, the transportation cost = $(800 \times 3) + (400 \times 4) + (100 \times 6) + (400 \times 4) + (500 \times 6) + (0 \times 6) + (0 \times 4) + (300 \times 0) =$ \$ 9200.

8. Comparison with other Methods

In this section, we compute the initial basic feasible solutions for several problems. Most of the problems were taken from articles (Ahmed et al. (2016a); Soomro et al. (2014)) to test the effectiveness of our method.

8.1. Cost Minimization Problems

The various transportation problems we use for the cost minimization are summarized in Tables 4 and 5. The problems in Table 4 are balanced while those in Table 5 are unbalanced. The IBFS obtained using our method is written against each problem.

Table 4. Cost Minimization Problems (Balanced Problems)

Problem Number	Data of the Problem	IBFS
BTP-1	$[c_{ij}]_{3\times3} = [4\ 2\ 1;\ 3\ 8\ 4;\ 6\ 5\ 2]$ $[s_i]_{3\times1} = [50,\ 70,\ 45]$ $[d_j]_{1\times3} = [40,\ 65,\ 60]$	**475**
BTP-2	$[c_{ij}]_{3\times3} = [6\ 4\ 1;\ 3\ 8\ 7;\ 4\ 4\ 2]$ $[s_i]_{3\times1} = [50,\ 40,\ 60]$ $[d_j]_{1\times3} = [20,\ 95,\ 35]$	**555**
BTP-3	$[c_{ij}]_{3\times4} = [9\ 8\ 5\ 7;\ 4\ 6\ 8\ 7;\ 5\ 8\ 9\ 5]$ $[s_i]_{3\times1} = [12,\ 14,\ 16]$ $[d_j]_{1\times4} = [8,\ 18,\ 13,\ 3]$	**248**
BTP-4	$[c_{ij}]_{3\times4} = [3\ 1\ 7\ 4;\ 2\ 6\ 5\ 9;\ 8\ 3\ 3\ 2]$ $[s_i]_{3\times1} = [300,\ 400,\ 500]$ $[d_j]_{1\times4} = [250,\ 350,\ 400,\ 200]$	**2850**
BTP-5	$[c_{ij}]_{4\times4} = [7\ 5\ 9\ 11;\ 4\ 3\ 8\ 6;\ 3\ 8\ 10\ 5;\ 2\ 6\ 7\ 3]$ $[s_i]_{4\times1} = [30,\ 25,\ 20,\ 15]$ $[d_j]_{1\times4} = [30,\ 30,\ 20,\ 10]$	**415**
BTP-6	$[c_{ij}]_{3\times4} = [50\ 60\ 100\ 50;\ 80\ 40\ 70\ 50;\ 90\ 70\ 30\ 50]$ $[s_i]_{3\times1} = [20,\ 38,\ 16]$ $[d_j]_{1\times4} = [10,\ 18,\ 22,\ 24]$	**3320**
BTP-7	$[c_{ij}]_{3\times3} = [4\ 3\ 5;\ 6\ 5\ 4;\ 8\ 10\ 7]$ $[s_i]_{3\times1} = [90,\ 80,\ 100]$ $[d_j]_{1\times3} = [70,\ 120,\ 80]$	**1390**
BTP-8	$[c_{ij}]_{3\times3} = [5\ 7\ 8;\ 4\ 4\ 6;\ 6\ 7\ 7]$ $[s_i]_{3\times1} = [70,\ 30,\ 50]$ $[d_j]_{1\times3} = [65,\ 42,\ 43]$	**835**
BTP-9	$[c_{ij}]_{3\times3} = [1\ 8\ 6;\ 3\ 7\ 8;\ 4\ 9\ 10]$ $[s_i]_{3\times1} = [50,\ 45,\ 40]$ $[d_j]_{1\times3} = [35,\ 55,\ 45]$	**800**
BTP-10	$[c_{ij}]_{4\times6} = [1\ 2\ 1\ 4\ 5\ 2;\ 3\ 3\ 2\ 1\ 4\ 3;\ 4\ 2\ 5\ 9\ 6\ 2;\ 3\ 1\ 7\ 3\ 4\ 6]$ $[s_i]_{4\times1} = [30,\ 50,\ 75\ 20]$ $[d_j]_{1\times6} = [20,\ 40,\ 30,\ 10,\ 50,\ 25]$	**440**
BTP-11	$[c_{ij}]_{5\times7} = [12\ 7\ 3\ 8\ 10\ 6\ 6;\ 6\ 9\ 7\ 12\ 8\ 12\ 4;\ 10\ 12\ 8\ 4\ 9\ 9\ 3;$ $8\ 5\ 11\ 6\ 7\ 9\ 3;\ 7\ 6\ 8\ 11\ 9\ 5\ 6]$ $[s_i]_{5\times1} = [60,\ 80,\ 70\ 100,\ 90]$ $[d_j]_{1\times7} = [20,\ 30,\ 40,\ 70,\ 60,\ 80,\ 100]$	**1930**
BTP-12	$[c_{ij}]_{3\times4} = [2\ 2\ 2\ 1;\ 10\ 8\ 5\ 4;\ 7\ 6\ 6\ 8]$ $[s_i]_{3\times1} = [3,\ 7,\ 5]$ $[d_j]_{1\times7} = [4,\ 3,\ 4,\ 4]$	**68**

Table 5. Cost Minimization Problems (Unbalanced Problems)

Problem Number	Data of the Problem	IBFS
UTP-1	$[c_{ij}]_{3\times4} = [10\ 8\ 4\ 3;\ 12\ 14\ 20\ 2;\ 6\ 9\ 23\ 25]$ $[s_i]_{3\times1} = [500,\ 400,\ 300]$ $[d_j]_{1\times4} = [250,\ 350,\ 600,\ 150]$	**8350**
UTP-2	$[c_{ij}]_{4\times4} = [12\ 10\ 6\ 13;\ 19\ 8\ 16\ 25;\ 17\ 15\ 15\ 20;\ 23\ 22\ 26\ 12]$ $[s_i]_{4\times1} = [150,\ 200,\ 600,\ 225]$ $[d_j]_{1\times4} = [300,\ 500,\ 75,\ 100]$	**13225**
UTP-3	$[c_{ij}]_{3\times5} = [5\ 8\ 6\ 6\ 3;\ 4\ 7\ 7\ 6\ 5;\ 8\ 4\ 6\ 6\ 4]$ $[s_i]_{3\times1} = [800,\ 500,\ 900]$ $[d_j]_{1\times5} = [400,\ 400,\ 500,\ 400,\ 800]$	**9200**

8.2. Profit Maximization Problems

The transportation problems used for our profit maximization are summarized in Table 6 (Ahmed et al. (2016a)). These problems are balanced transportation problems.

Table 6. Profit Maximization Problems

Problem Number	Data of the Problem	IBFS
MTP-1	$[c_{ij}]_{3\times4} = [6\ 4\ 1\ 5;\ 8\ 9\ 2\ 7;\ 4\ 3\ 6\ 2]$ $[s_i]_{3\times1} = [14,\ 18,\ 7]$ $[d_j]_{1\times4} = [6,\ 10,\ 15,\ 8]$	**232**
MTP-2	$[c_{ij}]_{3\times4} = [14\ 19\ 7\ 5;\ 16\ 6\ 12\ 9;\ 6\ 16\ 5\ 20]$ $[s_i]_{3\times1} = [10,\ 12,\ 18]$ $[d_j]_{1\times4} = [9,\ 14,\ 7,\ 10]$	**654**
MTP-3	$[c_{ij}]_{3\times4} = [16\ 14\ 11\ 25;\ 18\ 29\ 12\ 27;\ 14\ 23\ 16\ 12]$ $[s_i]_{3\times1} = [140,\ 180,\ 70]$ $[d_j]_{1\times4} = [60,\ 100,\ 150,\ 80]$	**8020**
MTP-4	$[c_{ij}]_{5\times6} = [35\ 22\ 33\ 16\ 20\ 12;\ 14\ 21\ 28\ 30\ 15\ 24;\ 55\ 18\ 17\ 29\ 26\ 19;$ $21\ 16\ 15\ 17\ 31\ 28;\ 45\ 23\ 16\ 11\ 22\ 50]$ $[s_i]_{5\times1} = [320,\ 180,\ 200\ 300,\ 300]$ $[d_j]_{1\times6} = [225,\ 225,\ 200,\ 200,\ 275,\ 175]$	**44780**
MTP-5	$[c_{ij}]_{4\times3} = [10\ 18\ 2;\ 9\ 8\ 20;\ 14\ 21\ 7;\ 12\ 2\ 25]$ $[s_i]_{4\times1} = [500,\ 250,\ 350, 600]$ $[d_j]_{1\times3} = [300,\ 600,\ 800]$	**33800**

9. Analysis of Results

9.1. Cost Minimization

To test the effectiveness of our method (MDEDM), we compare it with 10 well-known methods, namely NWCM, LCM, RMM, CMM, MDSM, VAM, MDM, EDM, ATM and IAM. In total, we compute the IBFS for 15 transportation problems (12 balanced and 3 unbalanced).

We use the pulp module in Python to obtain the optimal solution. The results of our computations are summarized in Tables 7 and 8.

Table 7. Comparison of MDEDM with other Traditional Algorithms and Optimal Solution for Cost Minimization

S/N	NWCM	LCM	RMM	CMM	MDSM	VAM	MDM	EDM	ATM	IAM	MDEDM	Optimal
BTP-1	770	605	595	475	535	490	475	475	605	605	475	475
BTP-2	730	555	555	610	595	555	555	555	555	555	555	555
BTP-3	320	248	248	376	248	248	296	248	240	248	248	240
BTP-4	4400	2900	2850	3600	2850	2850	2850	3650	2850	2850	2850	2850
BTP-5	540	435	470	435	410	470	415	415	415	420	415	410
BTP-6	4160	3500	3320	3320	3880	3320	3620	3320	3320	3580	3320	3320
BTP-7	1500	1450	1450	1500	1660	1500	1390	1390	1390	1390	1390	1390
BTP-8	830	890	830	890	911	830	837	895	890	890	835	830
BTP-9	875	830	830	860	830	810	830	830	830	830	800	800
BTP-10	740	470	490	480	510	450	450	450	450	460	440	430
BTP-11	3180	2080	1970	1940	2170	1930	1960	2070	2300	1900	1930	1900
BTP-12	93	79	77	71	87	68	77	68	79	77	68	68
UTP-1	18800	8800	9250	16900	13150	8350	8350	8350	10000	8400	8350	7750
UTP-2	14725	14625	14625	12775	12850	13225	13225	13350	15875	13075	13225	12475
UTP-3	13100	9800	9200	9800	10300	9200	10300	10300	9200	9200	9200	9200

From the results shown in Table 7, we can conclude that MDEDM consistently performs better than the other methods, and the results obtained are optimal or close to the optimal solutions.

Table 8. The % of Correctness (PoCIR) of IBFS

S/N	NWCM	LCM	RMM	CMM	MDSM	VAM	MDM	EDM	ATM	IAM	MDEDM
BTP-1	61.69	78.51	79.83	100	88.79	96.94	100	100	78.51	78.51	100
BTP-2	76.03	100	100	90.98	93.28	100	100	100	100	100	100
BTP-3	75	96.77	96.77	63.83	96.77	96.77	81.08	96.77	100	96.77	96.77
BTP-4	64.77	98.28	100	79.17	100	100	100	78.08	100	100	100
BTP-5	75.93	94.25	87.23	94.25	100	87.23	98.8	98.8	98.8	97.62	98.8
BTP-6	79.81	94.86	100	100	85.57	100	91.71	100	100	92.74	100
BTP-7	92.67	95.86	95.86	92.67	83.73	92.67	100	100	100	100	100
BTP-8	100	93.26	100	93.26	91.11	100	99.16	92.74	93.26	93.26	99.4
BTP-9	91.43	96.39	96.39	93.02	96.39	98.77	96.39	96.39	96.39	96.39	100
BTP-10	58.11	91.49	87.76	89.58	84.31	95.56	95.56	95.56	95.56	93.48	97.73
BTP-11	59.75	91.35	96.45	97.94	87.56	98.45	96.94	91.79	82.61	100	98.45
BTP-12	73.12	86.08	88.31	95.77	78.16	100	88.31	100	86.08	88.31	100
UTP-1	41.22	88.07	83.78	45.86	58.94	92.81	92.81	92.81	77.5	92.26	92.81
UTP-2	84.72	85.3	85.3	97.65	97.08	94.33	94.33	93.45	78.58	95.41	94.33
UTP-3	70.23	93.88	100	93.88	89.32	100	89.32	89.32	100	100	100
Average of PoCIR	**73.63**	**92.29**	**93.18**	**88.52**	**88.73**	**96.9**	**94.96**	**95.05**	**92.49**	**94.98**	**98.55**
% of Error (PoEIR)	**26.37**	**7.71**	**6.82**	**11.48**	**11.27**	**3.1**	**5.04**	**4.95**	**7.51**	**5.02**	**1.45**

From the results shown in Table 7, we highlight the effectiveness of our algorithm in Table 8. The optimal solution for each problem was divided by its IBFS and multiplied by 100% to obtain percentage of correctness of IBFS shown in Table 8. That is,

$$\% \text{ of correctness of IBFS} = \frac{\text{optimal solution}}{IBFS} \times 100\%.$$

For the average of PoCIR shown on the same Table 8, we find the average of the problems, that is,

$$\text{Average of PoCIR} = \frac{\text{BTP-1} + \text{BTP-2} + \cdots + \text{UTP-3}}{15}.$$

The percentage of error was obtained by subtracting the average of PoCIR from 100%. That is, $\%$ of error $= 100\%$ - Average of PoCIR for each method. For example, $\%$ of error for NWCM $=$ $100\% - 73.63\% = 26.37\%$.

From Table 8, it is clear that our method, MDEDM outperforms the other methods, with average of PoCIR being 98.55% and just 1.45% of error. The average of PoCIR of our results is shown pictorially in Figure 6.

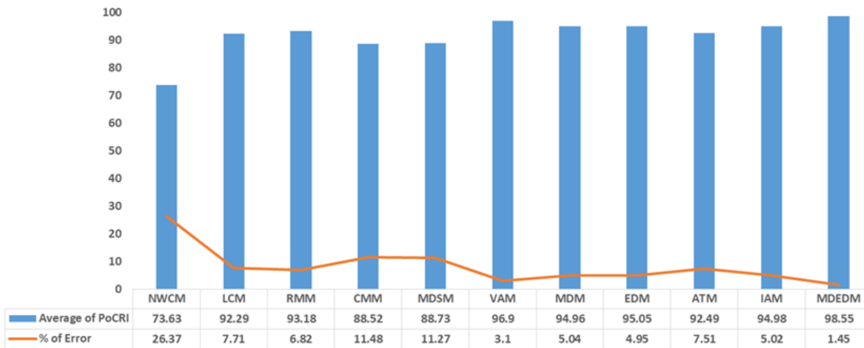

Figure 6. The Average of Correctness (PoCIR) of the various methods for finding the IBFS of Transportation Problems

9.2. Profit Maximization

Now we apply our method to solve the profit maximization problems shown in Table 6. We present the results of our computations in Table 9.

Table 9. Comparison of MDEDM with other Traditional Algorithms for Profit Maximization

S/N	NWCM	LCM	VAM	IAM	MDEDM	Optimal
MTP-1	137	232	232	232	232	234
MTP-2	468	654	662	662	654	662
MTP-3	5570	8020	8000	8020	8020	8020
MTP-4	36795	46760	46760	46700	44780	46760
MTP-5	28150	33800	34050	34050	33800	34050

From the results shown in Table 9, we can conclude that MDEDM performs quite well for the profit maximization problems. We highlight the effectiveness of our algorithm in Table 10.

Table 10. The % of Correctness of IBFS for Profit Maximization (PoCIR)

S/N	NWCM	LCM	VAM	IAM	MDEDM
MTP-1	58.55	99.15	99.15	99.15	99.15
MTP-2	70.69	98.79	100	100	98.79
MTP-3	69.45	100	99.75	100	100
MTP-4	78.69	100	100	99.87	95.77
MTP-5	82.67	99.27	100	100	99.27
Average of PoCIR	**72.01**	**99.44**	**99.78**	**99.8**	**98.6**
% of Error (PoEIR)	**27.99**	**0.56**	**0.22**	**0.2**	**1.4**

We apply similar procedures explained for Table 8 to obtain the percentage average of correctness (PoCIR) and percentage of error. This table also confirms the effectiveness of our method in solving profit maximization problems.

The average of PoCIR of our results is shown pictorially in Figure 7.

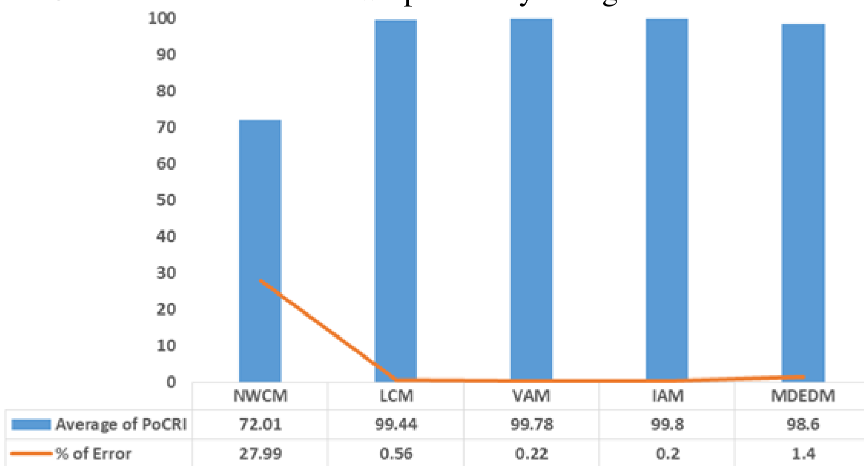

Figure 7. Plot showing the Average of Correctness (PoCIR) for solving the Profit Maximization Problems

10. Computational Complexity

To calculate the time complexity of MDEDM, we consider a transportation problem with m sources and n destinations. We denote the total computational time by $T(m, n)$. The time taken to calculate the maximum difference for a row and the extreme difference for a column is $n + 1$ and $m + 1$ respectively. Thus, the time taken to compute the maximum difference for m rows is $m(n + 1)$ and the time taken to compute the extreme difference for n columns is $n(m + 1)$.

The time taken to search for the largest difference is $\frac{m}{m+n}$ if found in a row (that is, among the maximum differences), but is $\frac{n}{m+n}$ if found in a column (that is, among the extreme differences). Therefore, the time taken to find the largest difference (if found among the maximum differences) and its corresponding cost is $\left(\frac{m}{m+n}\right) n$. Similarly, the time taken to find the largest difference (if found among the extreme differences) and its corresponding cost is $\left(\frac{n}{m+n}\right) m$. Thus, the time taken to obtain the largest difference for either row or column and its corresponding cost is the same and is given by $\frac{mn}{m+n}$.

To obtain the IBFS, we allocate $m + n - 1$ cells. Therefore, the time taken to compute the IBFS is $\left(\frac{mn}{m+n}\right)(m + n - 1)$.

Hence, the total time required is the sum of the time taken to calculate the extreme differences, maximum differences and the time taken to obtain the IBFS corresponding to the $m + n - 1$ allocations. That is,

$$\begin{aligned} T(m, n) &= O\left((n+1)m + (m+1)n + \left(\frac{mn}{m+n}\right)(m+n-1)\right) \\ &= O(nm + mn + mn) \\ T(m, n) &= O(3mn) = O(mn). \end{aligned}$$

Thus, the computational time complexity of MDEDM is $O(mn)$ which is equal to the computational time complexity of VAM (Chaudhuri et al. (2013)).

11. Conclusion

The Initial Basic Feasible Solution (IBFS) plays a vital role in obtaining the optimal solution to a transportation problem. In this paper, we proposed a new method called Maximum Difference Extreme Difference Method (MDEDM) for solving both cost minimization transportation problems and profit maximization transportation problems. We have compared and performed an analysis of our method to several well-known methods for finding an IBFS to a transportation problem and have arrived at the conclusion that MDEDM is an effective method and can be applied in solving other forms of transportation problems. This new method is a polynomial-time algorithm that yields an optimal solution or close to an optimal solution.

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