



**UNIVERSITI PUTRA MALAYSIA**

**DEVELOPMENT OF AN ADAPTIVE ALGORITHM FOR SOLVING THE  
INVERSE KINEMATICS PROBLEM FOR SERIAL ROBOT  
MANIPULATORS**

**ALI T. HASAN.**

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**By**

**ALI T. HASAN**

**Thesis Submitted to the School of Graduate Studies, Universiti Putra Malaysia, in  
Fulfilment of the Requirements for the Degree of Master of Science**

**July 2005**



## DEDICATION

*A Special Dedication To*

*My Family*

*To my bleeding country IRAQ, My **ALLAH** grant you peace*

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Abstract of thesis presented to the Senate of University Putra Malaysia in  
fulfilment of the requirements for the degree of Master Science

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**ALI T. HASAN**

**July 2005**

**Chairman: Professor A.M.S. Hamouda, PhD**

**Faculty: Engineering**

In order to overcome the drawbacks of some control schemes, which depends on modeling the system being controlled, and to overcome the problem of inverse kinematics which are mainly singularities and uncertainties in arm configuration.

Artificial Neural Networks (*ANN*) technique has been utilized where learning is done iteratively based only on observation of input-output relationship.

The proposed technique does not require any prior knowledge of the kinematics model of the system being controlled; the main idea of this approach is the use of an Artificial Neural Network to learn the robot system characteristics rather than having to specify an explicit robot system model.



Since one of the most important problems in using Artificial Neural Networks, is the choice of the appropriate networks' configuration, two different networks' configurations were designed and tested, they were trained to learn desired set of joint angles positions from a given set of end effector positions.

Experimental results have shown better response for the first configuration network in terms of precision and iteration.

The developed approach possesses several distinct advantages; these advantages can be listed as follows :(First) system model does not have to be known at the time of the controller design, (Second) any change in the physical setup of the system such as the addition of a new tool would only involve training and will not require any major system software modifications, and (Third) this scheme would work well in a typical industrial set-up where the controller of a robot could be taught the handful of paths depending on the task assigned to that robot.

The efficiency of the proposed algorithm is demonstrated through simulations of a general 6 D.O.F. serial robot manipulator.



Abstrak tesis yang dikemukakan kepada Senat Universiti Putra Malaysia  
sebagai memenuhi keperluan untuk ijazah Master Sains

**PEMBANGUNAN ALGORITHMMA ADAPTIF UNTUK MENYELESAIKAN  
MASALAH KINEMATIK SONGSANG ROBOT BERSIRI**

Oleh

**ALI T. HASAN**

**Julai 2005**

**Pengerusi: Profesor A.M.S. Hamouda, PhD**

**Fakulti: Kejuruteraan**

Dalam menyelesaikan permasalahan pada setengah skim kawalan yang bergantung kepada permodelan sistem yang dikawal dan juga menyelesaikan masalah kinematik songsang yang mana adalah secara tunggal dan tidak menentu dalam konfigurasi lengan.

Teknik Rangkaian Neural Buatan telah digunakan di mana pembelajaran telah dilakukan secara berulang berdasarkan kepada pemerhatian jalinan masukan-keluaran.

Teknik yang telah dicadangkan tidak memerlukan apa-apa pengetahuan berkenaan kinematik model sistem kawalan tersebut, di mana idea utama pendekatan ini adalah untuk menggunakan satu Rangkaian Neural Buatan bagi

mempelajari ciri-ciri sistem robot dari memperoleh model sistem robot secara eksplisit.

Salah satu daripada masalah yang terpenting dalam menggunakan Rangkaian Neural Buatan adalah pilihan konfigurasi. Salah satu daripada masalah yang terpentingnya, diuji, dan dilatih untuk mengajar set sambungan sudut yang diberikan daripada set posisi efektor.

Keputusan ujikaji menunjukkan keputusan yang lebih baik untuk konfigurasi pertama dalam terma kejituan dan pengulangan.

Pendekatan yang telah dihasilkan ini mempunyai beberapa kebaikan iaitu, pertama: model sistem tidak perlu diketahui sewaktu rekabentuk kawalan, kedua : apa-apa perubahan dalam persediaan fizikal sistem seperti penambahan alatan akan memerlukan latihan dan tidak memerlukan modifikasi perisian sistem. Ketiga, skim ini sepatutnya boleh beroperasi di dalam persediaan sesuatu industri yang memerlukan kawalan robot yang boleh dilatih pelbagai laluan bergantung kepada kerja yang diberikan kepada robot.

Kecekapan algorithm yang dicadangkan telah ditunjukkan melalui simulasi 6 darjah kebebasan bagi serial robot.

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I certify that an Examination Committee met on 29<sup>th</sup> July 2005 to conduct the final examination of Ali T. Hasan on his Master of Science thesis entitled "Development of an Adaptive Algorithm for Solving the Inverse Kinematics Problem for Serial Robot Manipulators" in accordance with Universiti Pertanian Malaysia (Higher Degree) Act 1980 and Universiti Pertanian Malaysia (Higher Degree) Regulations 1981. The Committee recommends that the candidate be awarded the relevant degree. Members of the Examination Committee are as follows:

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## DECLARATION

I hereby declare that the thesis is based on my original work except for quotations and citations which have been duly acknowledged. I also declare that it has not been previously or concurrently submitted for any other degree at UPM or other institutions.



**ALI T. HASAN**

Date: 10 AUGUST 2005

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## LIST OF ABBREVIATIONS

IK	Inverse Kinematics
D.O.F.	Degrees of Freedom
ANN	Artificial Neural Network
GDR	Generalized Delta learning Rule



## CHAPTER 1

### INTRODUCTION

Trajectory planning can be conducted either in the joint-variable space or in the Cartesian space, when planning of the manipulator trajectory is performed in joint coordinates, since no constraints are imposed on the manipulator hand; it is difficult for the user to trace the path that the manipulator hand traverses. Hence, the manipulator hand may hit obstacles with no prior warning. Then using the Cartesian space is much frequent because it is easier to visualize the correct end-effector configurations in Cartesian coordinates than in joint coordinates, in this way, the path constraints are specified in Cartesian coordinates, and the joint actuators are served in joint coordinates. Hence, to find a trajectory that approximates the desired path closely, an inverse kinematics ( *IK* ) solution routine should be called upon to make the necessary conversion [1].

Finding a solution to the *IK* problem analytically for serial manipulators is a difficult problem and a focus of many researches [2-14]. One source of difficulty is due to the fact that the inverse is not a true function; the set of solutions is infinite. In addition, the inverse mapping is nonlinear, and the nature of the inverse solution set for particular target locations undergoes a qualitative change as the manipulator moves between different regions of the workspace [15].



Control schemes used in most robots involve positions coordination in Cartesian space using a pre-specified model [16], Model-based approaches require high computational time due to the complexity of the model, if the specific model-structure selected does not properly reflect all the robot characteristics it can result in a poor control performance, Secondly the fact that the model is highly system-specific makes it very hard to accommodate physical changes such as the addition of a new tool [17].

For a robot to move in a predictable path it is necessary to recalculate joint variables many times a second. To make the robot follow a desired path, it is necessary to break this path into many small sections (segments) and make the robot follow these segments sequentially; this means that a new solution must be calculated for each segment.

If calculating a solution takes a long time (By using methods such as Gaussian elimination), the robot will lose accuracy or will not follow the specified path, the shorter the time it takes to calculate a new solution the more accurate the robot can be [18].

The overall complexity of robot control problem and the quest for a truly autonomous robot system has led to considerable interest being devoted to the application of neural network technology to robot control [19-23]. Artificial neural networks (ANNs) are widely accepted as a technology offering an alternative way to tackle complex and ill-defined problems. They can learn from examples, are fault tolerant in the sense that they are able to handle noisy and incomplete

data, are able to deal with nonlinear problems and, once trained, can perform prediction and generalization at high speed, They are particularly useful in system modelling such as implementing complex mappings [24].

To overcome the uncertainties and non linearity of the robot model, an (ANN) have been designed to learn the characteristics of a FANUC *M-710i* robot over particular region of the working space, the FANUC *M-710i* as can be seen in Figure 1.1, is a serial robot manipulator consisting of axes and arms driven by servomotors. The place at which arm is connected is a joint, or an axis. J1, J2, and J3 are main axes. The basic configuration of the robot depends on whether each main axis functions as a linear axis or rotation axis [51]. The wrist axes are used to move an end effector (tool) mounted on the wrist flange.

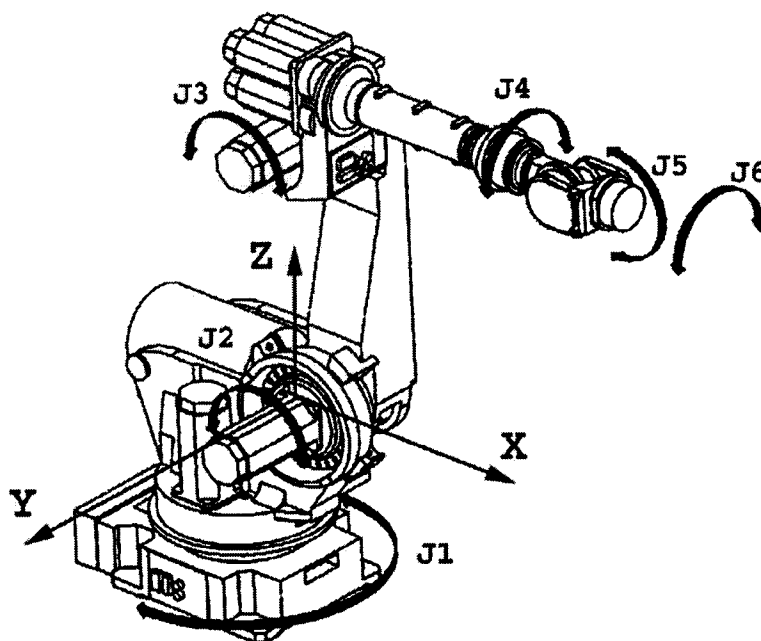


Figure 1.1: FANUC M710i Robot, Main axes, wrist axes and global coordinate system [51]

The wrist itself can be wagged about one wrist axis and the end effector rotated about the other wrist axis, this highly non-linear structure makes this robot very useful in typical industrial applications such as the material handling, assembly of parts, painting, etc.

One of the most important problems in using ANNs is the choice of appropriate network's configuration [16]. The current research is devoted to the development of an adaptive ANN controller to track IK control problem of a 6 D.O.F serial robot manipulator and to find the best configuration through different approaches, the learning algorithm is based on the adaptive updating of the weights of the network by minimizing the tracking error after each iteration process, this control scheme does not require any prior knowledge of the kinematics model of the system being controlled, the basic idea of this concept is the use of the ANN to learn the characteristics of the robot system rather than to specify an explicit robot system model. Using the ANNs have shown better results than using other known model-based control methods, two different configurations were used and the training data have been taken by driving the robot manually to follow a desired path.

## **1.1 Problem Statement**

The inverse kinematics problem is the problem of finding a vector of joint variables that produce a desired end effector location. If a unique vector of joint angles exists which attains the desired end-effector location, there is a well-

defined inverse to the forward kinematics function and the inverse kinematics problem is well posed. Unfortunately, the inverse kinematics problem can be ill posed because the solution for the forward kinematics is not unique; in many cases solving the inverse kinematics problem may result in infinite number of solutions [15].

The kinematics considerations for the manipulator shown in Figure 1.2 are based on the *forward kinematics equation*. The forward kinematics equation involves mapping of joint angle coordinates  $(\theta_1, \theta_2)$  to the end effector position  $(x, y)$ . The mapping expressions can be obtained by inspection of the figure as follows:

$$x = l_1 \cos \theta_1 + l_2 \cos (\theta_1 + \theta_2) \quad (1-1a)$$

$$y = l_1 \sin \theta_1 + l_2 \sin (\theta_1 + \theta_2) \quad (1-1b)$$

Where  $\theta_1$  and  $\theta_2$  are the joint angles of the first and second arm segments, respectively,  $l_1$  and  $l_2$  are respective arm segment lengths. Relation (1-1) expresses the forward kinematics problem and implements unique mapping from the joint angle space to the Cartesian space.

The inverse kinematics problem is described as follows [25]:

$$\theta_2 = \cos^{-1}[(x^2 + y^2 - l_1^2 - l_2^2)/(2l_1l_2)] \quad (1-2a)$$

$$\theta_1 = \tan^{-1}(y/x) - \tan^{-1}[l_2 \sin \theta_2 / (l_1 + l_2 \cos \theta_2)] \quad (1-2b)$$

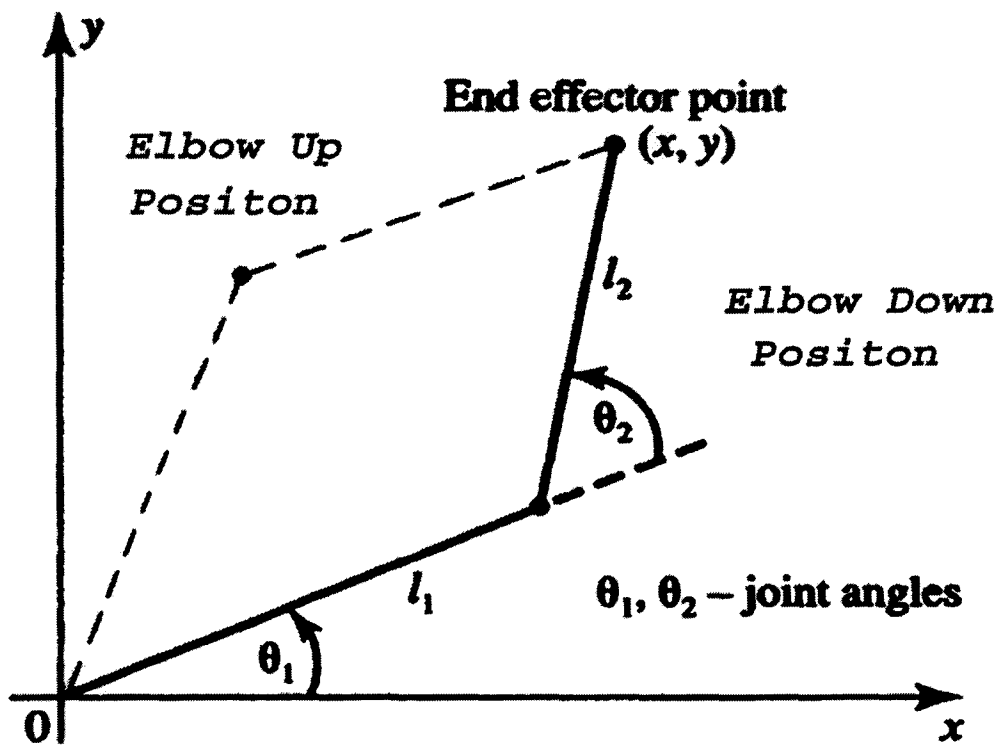


Figure 1.2: The two-arm configuration positioning at  $(x, y)$

Since  $(\text{Cos}^{-1})$  is not a single-valued function in the range of angles of interest, two possible orientations typically result from Relation (1-2) for the robot arm joint angles. The arm can be positioned with the elbow up or down, with the end effector still at the required  $(x, y)$  point. The inverse kinematics transformation (1-2) implementing mapping from Cartesian space to joint space is thus not unique.

Finding the *IK* solution for a six D.O.F manipulator is a problem. It is, in general, a non-linear algebraic computation requires the solution of a sixteenth-order polynomial equation with time consuming calculations [16].

## 1.2 Research Objectives

To overcome the non-linearity and uncertainties in the arm position which resulting from applying the robot model, and to fulfil the increased need for autonomous robot system, a considerable interest has been devoted to the application of ANN technology.

Main objectives of this study are:

- 1- To develop ANN algorithms to overcome the singularities and uncertainties in arm position configurations due to the Inverse Kinematics problem resulting from applying model-based approach.
- 2- To find the best network configuration for finding the inverse kinematics solution.

## 1.3 Research Scope

Kinematics deals with the aspects of motion without regard to the forces and/or torques that cause it. Hence kinematics is concerned only with the geometrical properties of a motion. The joint variables of a robot manipulator are related to the position and orientation of the end effector by the constraints imposed by the joints. These kinematics relations are the focal points of interest in a study of the kinematics of robot manipulators.



A simulation study will be the focus of this research using two different configurations of ANNs.

X, Y and Z coordinates and the relevant joint angles (training data) will be recorded directly from sensors fixed on the robot joints.

#### **1.4 Thesis Layout**

This thesis has been structured into six chapters. Following this introduction chapter, a review of literature concerning robot kinematics and neural networks as a theoretical background and review for the previous studies which have been carried out to solve the inverse Kinematics problem are illustrated in chapter two. The Kinematics model of the robot used in this study, which shows the uncertainties in arm configuration, is presented in chapter three; Chapter four presents the description of the research methodology adopted. Chapter five shows the simulation results obtained the neural networks configurations used and discusses the results obtained. Finally, the conclusions from the current study and the recommendations for the future studies are presented in chapter six.

