UNIVERSITI PUTRA MALAYSIA

HEURISTIC PLACEMENT ROUTINES FOR TWO-DIMENSIONAL RECTANGULAR BIN PACKING PROBLEMS

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HEURISTIC PLACEMENT ROUTINES FOR TWO-DIMENSIONAL RECTANGULAR BIN PACKING PROBLEMS

By

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HEURISTIC PLACEMENT ROUTINES FOR TWO-DIMENSIONAL RECTANGULAR BIN PACKING PROBLEMS

By

LILY WONG

October 2009

Chairman: **Lee Lai Soon, PhD**

Faculty: **Science**

In this study, we consider non-oriented and oriented cases of Two-Dimensional Rectangular Bin Packing Problems where a given set of small rectangles is packed without overlaps into a minimum number of identical large rectangles. In non-oriented case the rectangles are allowed to be rotated at 90° while the rectangles have fixed orientation in oriented case. We propose new heuristic placement routines called the Improved Lowest Gap Fill (LGFi) (for non-oriented case) and LGFi_{OF} (for oriented case) for solving the non-oriented and oriented cases of the problems respectively. These new approaches dynamically select the best rectangle for placement during the packing stage. Extensive computational experiments are conducted using benchmark problem instances proposed in the literature. The results show that the proposed routines are competitive when compared with other heuristic placement routines. The Two Factors Factorial Design Repeated on Both Factors is used to analyse the computational results using SAS package. The statistical result of the non-

oriented case shows that Floor Ceiling, Lowest Gap Fill, Touching Perimeter and LGFi which are not significantly difference and their performance are better than the Bottom-Left Fill. The statistical result of the oriented case indicates that Alternate Direction, Floor Ceiling and LGFi_{OF} are not significantly difference. This means that three of these heuristic placement routines are equally good. However, these results are not that efficient because the normality assumptions of the error of the model are not met. This maybe due to the present of the unexpected outliers in the error terms.

Abstrak tesis yang dikemukakan kepade Senat Universiti Putra Malaysia sebagai memenuhi keperluan untuk ijazah Master Sains

HEURISTIK-HEURISTIK PENEMPATAN RUTIN UNTUK MASALAH PENGISIAN BEKAS DUA-DIMENSI SEGI EMPAT

Oleh

LILY WONG

Oktober 2009

Pengerusi: **Lee Lai Soon, PhD**

Fakulti: **Sains**

Dalam kaji selidik ini, kami mempertimbangkan kes tidak orientasi dan orientasi untuk Masalah Pengisian Bekas Dua-Dimensi Segi Empat Saiz Bekas Tunggal di mana diberi satu set segi empat kecil diisi tanpa bertindih ke dalam segi empat besar secara minimum. Kes tidak orientasi membenarkan segi empat berputar pada sudut 90° manakala segi empat mempunyai orientasi yang tetap dalam kes orientasi. Kami mencadangkan rutin penempatan heuristik baru yang dinamakan Perbaikan Pengisisan Celahan Terendah (LGFi) (untuk kes tidak orientasi) dan LGFi_{OF} (untuk kes orientasi) untuk menyelesaikan kes tidak orientasi dan orientasi masing-masing. Pendekatan baru ini memilih segi empat yang paling sesuai untuk pengisian secara dinamik sepanjang peringkat pengisian. Eksperimen komputasi yang menyeluruh telah dijalankan dengan menggunakan contoh permasalahan yang dicadangkan dalam sorotan menunjukkan rutin yang dicadangkan adalah berdaya saing apabila berbanding dengan rutin penempatan heuristik yang lain. Rekabentuk Faktorial Dua Faktor Ulangan Ke Atas Kedua-

dua Faktor digunakan untuk menganalisis keputusan berkomputasi dengan menggunakan pakej SAS. Keputusan statistik bagi kes tidak orientasi menunjukkan Lantai Siling, Pengisian Celah Terbawah, Sentuhan Perimeter dan LGFi tidak mempunyai perbezaan yang signifikan dan prestasi mereka adalah lebih baik daripada Pengisian Bawah-Kiri. Keputusan statistik bagi kes orientasi menunjukkan Arah Berselang-seli, Lantai Siling dan LGFi_{OF} tidak mempunyai perbezaan yang ketara. Ini bermakna ketiga-tiga rutin penempatan heuristik ini adalah berprestasi sama baik. Walaubagaimanapun, keputusan ini adalah kurang cekap kerana andaian kenormalan ralat bagi model tidak dipenuhi. Ini mungkin disebabkan oleh kehadiran titik terpencil pada ralat yang tidat terduga.

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And last but not least, my appreciation goes to my dearest parents and beloved family for their greatest encouragement and unlimited support. I would like to share my enjoyment with them, they will always in my heart.

I certify that a Thesis Examination Committee has met on 19 October 2009 to conduct the final examination of Lily Wong on her thesis entitled "Heuristic Placement Routines For Two-Dimensional Rectangular Bin Packing Problems" in accordance with the Universities and University Colleges Act 1971 and the Constitution of the Universiti Putra Malaysia [P.U.(A) 106] 15 March 1998. The Committee recommends that the student be awarded the Master of Science.

Members of the Thesis Examination Committee were as follows:

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This thesis was submitted to the Senate of Universiti Putra Malaysia and has been accepted as fulfilment of the requirement for the degree of
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Date: 10 December 2009

DECLARATION

I declare that the thesis is my original work except for quotations and citations which have been duly acknowledged. I also declare that it has not been previously, and is not concurrently, submitted for any other degree at Universiti Putra Malaysia or at any other institution.

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LILY WONG

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CHAPTER 1

INTRODUCTION

1.1 Introduction

Cutting and Packing (C&P) problems are optimization problems that are concerned in finding a good arrangement of multiple small items into one or more larger object(s). Bin packing problem is a type of C&P problems where the general objective is to reduce the production costs by maximizing the utilization of the larger objects and minimizing the material used in term of reducing the wastage. In this study, we consider non-oriented and oriented cases of Two-Dimensional Rectangular Single Bin Size Bin Packing Problem (2DRSBSBPP). The objective of this problem is to allocate a set of *n* rectangular items, each characterised by a height, h_j and a width, w_j, without overlaps into a minimum number of identical bins defined by a height, *H* and a width, *W*. The non-oriented case of 2DRSBSBPP allowed the rectangular items to be rotated at 90˚ while the rectangular items have fixed orientation in the oriented case. This problem is classified as a class of NP-hard problem by Garey and Johnson [14].

In general, the 2DRSBSBPP contributes to many areas of application in business and industry such as in metal, wood, glass, and textile industries, newspaper paging, and cargo loading. The allocation process in the problem is essential. The objective of the allocation process is maximizing the usage of the larger objects and/or maximizing the value of the small items packed. For instance, the non-oriented case can be found in metal industry, where the pieces of the metal sheets are the bins (larger objects) while the different dimension of rectangular layout that needed to be cut out from the metal sheets are the items. The intention is to find a good arrangement of the layout which give the highest utilisation of the metal sheets. The process of newspaper paging can be illustrated as a oriented case where the pages of the newspaper are the bins and the news or the advertisements (with fixed orientation) is the items. The purpose is to arrange the maximum numbers of news (or advertisements) into minimum number of pages.

In manufacturing industry, the reduction of the cost is one of the important issues that the manufacturer concern with. The high material utilization is of particular interest to industries which are involved with mass-production, since a small improvement in layout or packing quality lead to huge savings of material used and reduce the production costs as well. The complexity of the problem and the solution approach depend on the geometry of the items to be placed and the constraints that are given.

To the best of our knowledge, there has been no published research material in the study of the statistical analysis on the computational results of C&P problems. This could be caused by one of the following possibilities:

1) some researchers may have tried and noticed that the error is not normally distributed;

2) they couldn’t find the best method to do the data transformation so that the error is normally distributed; or
3) there are unexpected outliers present in the data sets.

Due to these possibilities, the statistical design of experiment which is closer to the experimental design will be selected to analyze the computational results. Choosing an appropriate statistical design of experiment is necessary so that we can get a meaningful conclusion from the data. This also will lead to strengthen the conclusions obtained. In this research, we tried on an appropriate statistical design of experiments, namely, the two factors factorial design repeated on both factors.

In addition, model adequacy checking is needed to ascertain that certain assumptions of the model such as independence and normality of the errors have been met. Violations of these basic assumptions may produce invalid inferential statements. If there are significance differences between the treatment means, then the Duncan’s Multiple Range Test is used to identify which means differ.

1.2 Problem Statements

Generally, the problem of this study is to find a good arrangement of the small items in order to maximize the utilization of the large objects (bins) or minimize the number of bins used. The appropriate design of experiment is selected to analyze the computational results and get a meaningful conclusion to strengthen our results.

1.3 Scope of study

In this study, we concentrate on both non-oriented and oriented cases of 2DRSBSBPP. The design of experiment, namely, two factors factorial design repeated on both factors which closer to our study is selected to analyze the computational results.

1.4 Objectives

The objectives of this study are:

1. to develop a new heuristic placement routine for solving non-oriented case of Two-Dimensional Rectangular Single Bin Size Bin Packing Problem (2DRSBSBPP).
2. to design a new heuristic placement routine for solving the oriented case of 2DRSBSBPP by modifying the developed heuristic method for non-oriented case of 2DRSBSBPP
3. to conduct a study on the statistical analysis of the computational results for both oriented and non-oriented cases of 2DRSBSBPP.

1.5 Data sets used

In this study we consider ten different classes of benchmark problems instances proposed in the literature. The first six classes (I-VI) are proposed by Berkey and Wang [3]. In each class all the items are generated in the same interval. The items in each class are classified as follows:

Class I : w_j and h_j uniformly random in [1, 10], $W = H = 10$.

Class II : w_j and h_j uniformly random in [1, 10], $W = H = 30$.

Class III : w_j and h_j uniformly random in [1, 35], $W = H = 40$.

Class IV : w_j and h_j uniformly random in [1, 35], $W = H = 100$.

Class V : w_j and h_j uniformly random in [1, 100], $W = H = 100$.

Class VI : w_j and h_j uniformly random in [1, 100], $W = H = 300$.

The other four classes (VII- X) are introduced by Martello and Vigo [25] where a more realistic situation is considered. The items are classified into four types:

Type 1 : w_j uniformly random in $[\frac{2}{3}W, W]$, h_j uniformly random in $[1, \frac{1}{2}H]$.

Type 2 : w_j uniformly random in $[1, \frac{1}{2}W]$, h_j uniformly random in $[\frac{2}{3}H, H]$.

Type 3 : w_j uniformly random in $[\frac{1}{2}W, W]$, h_j uniformly random in $[\frac{1}{2}H, H]$.

Type 4 : w_j uniformly random in $[1, \frac{1}{2}W]$, h_j uniformly random in $[1, \frac{1}{2}H]$.

The bin size is $W = H = 100$ for all classes, while the items are as follow:

Class VII : *Type 1* with probability 70%, *Type 2, 3, 4* with probability 10% each.

Class VIII: *Type 2* with probability 70%, *Type 1, 3, 4* with probability 10% each.

Class IX : *Type 3* with probability 70%, *Type 1, 2, 4* with probability 10% each.

Class X : *Type 4* with probability 70%, *Type 1, 2, 3* with probability 10% each.

1.6 Overview

The remainder of the thesis is organized as follows. The literature review is presented in Chapter 2 where a brief introduction of the 2DRSBSBPP is given. The heuristic placement routines proposed in the literature are addressed. In addition, the descriptions of lower boundary schemes and time complexity will be discussed in this chapter. The statistical analysis will also discuss briefly in this chapter.

In Chapter 3, the methodology of the new heuristic placement routines for both oriented and non-oriented case will be discussed in details. The computational design and the statistical analysis tools will be discussed in this chapter. The computational results will be presented and discussed in Chapter 4. Finally, Chapter 5 highlights the conclusions of this study and some future works.

Chapter 2

Literature Review

2.1 Introduction

In this chapter, the existing literature covering the Cutting and Packing (C&P) Problem and definitions of different types of problems and solution approached will be investigated. Generally, Cutting and Packing (C&P) Problem can be summarized as follows (Wäscher et al. [33]):

> "Given two sets of elements, namely, a set of large objects (input, supply) and a set of small items (output, demand) which are defined in one, two, or an even larger number of geometric dimensions. Then some or all the small items will be grouped into one or more subsets and assign each of them into one of the larger objects with the conditions all small items of the subset lie entirely within the large object and the small items are not overlapping."

The time complexity will be discussed in the next section. In section 2.3, the typology of C&P problems will be discussed. The heuristic placement routines for 2DRSBSBPP proposed in the literature will be presented in Section 2.4. In Section 2.5, lower bounds for both oriented and non-oriented cases of 2DRSBSBPP are discussed.

2.2 Time complexity

In this section, the time complexity theory will be discussed. The definitions, as well as most of the theory presented in this section, are extracted from Tovey [32], and Whitley and Watson [34]. Details descriptions can be found in Garey and Johnson [14], Papadimitriou [30] and Sipser [31].

The term of computational complexity has two usages which must be distinguished. One of it refers to an *algorithm* for solving instances of a *problem*: broadly stated, the computational complexity of an algorithm is a measure of how many steps the algorithm will require in the worst case for an instance or input of a given size. The number of steps is measured as a function of that size. Another one is refer to a problem itself. The theory of computational complexity involves classifying problems according to their inherent tractability or intractability. Complexity theory is part of the theory of computation dealing with the resources required during computation to solve a given problem. The most common resources are time (how many steps it takes to solve a problem) and space (how much memory it takes).

The time complexity of a problem is the number of steps it takes to solve a problem as a function of the size of the input length using the most efficient algorithm. More formally, the *Big-O* notation is used: '*O(p(input length))*', where *p* is a function of the input length. A precise definition of *O( )* time bounds is that an algorithm has time bound *O(f(n))* if there exist constants *N* and *K* such that for every input of size $n \geq N$ the algorithm will not take more than *Kf(n)* processing time.

The idea of complexity theory is that of classifying problems into two main classes which called **P** and **NP**. A decision problem is a problem that takes an input some string and requires an output either **YES** or **NO**. If there is an algorithm which is able to produce the correct answer for any input string of length *n* in at most n^k steps, where *k* is some constant independent of the input string, then can be said that the problem can be solved in polynomial time and placed it in class **P**. So, the class **P** consists of all those decision problems that can be solved on a deterministic sequential machine in an amount of time that is polynomial in the size of the input. Meanwhile, the class **NP** consists of all those decision problems which positive solution can be verified in polynomial time given the right information, or can be said as which solution can be found in polynomial time on a non-deterministic machine. This class contains problems that people would like to be able to solve effectively such as the Boolean Satisfiability Problem and Travelling Salesman.

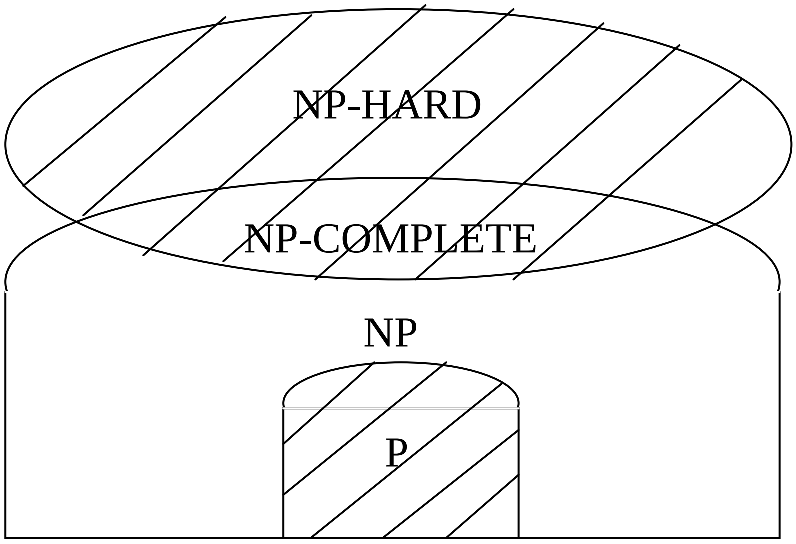

Figure 2.1: A Simple Diagram of **P** and **NP** (figure from Tovey [32])

It is clear that $\mathbf{P} \subseteq \mathbf{NP}$, and $\mathbf{P} \neq \mathbf{NP}$ is widely believed conjecture although no proof has been established to date. Figure 2.1 depicts that the class **P** is the set of easy problem. The **NP-hard** problems include the **NP-complete** problems and many hard problems that are not in **NP**. Further research has gained insight into the class **NP** by dividing the class into subclasses. **NP-complete** class is a subclass of **NP** which has a property that all **NP** problems can be reduced to the **NP-complete** problem in polynomial time. In other words, a *decision problem* is called **NP-complete** if it is polynomialy equivalent to the *satisfiability* problem, which is proved by Cook [10] in 1971 to be **NP-complete**. More formally, a problem R is **NP-complete** if R is in **NP** and R is **NP-hard**. An **NP-complete** problem has an important property, that is, if there is an efficient (i.e. polynomial) algorithm for some **NP-complete** problem, then there is an efficient algorithm for every problem in **NP**.

The term **NP-hard** is used to describe the corresponding optimization problem of a **NP-complete** decision problem. In computational complexity theory, **NP-hard** refers to the class of problems that contains all problem *H*, such that for every decision problem *L* in **NP** there exists a polynomial-time many-one reduction to *H*, written $L \leq H$. The **NP**-hardness of a problem suggest that it is impossible to find an optimal solution without the use of an essentially enumerative algorithm, for which computation times will increase exponentially with problem size. For this reason, heuristic methods have been developed to obtain good solutions for large problems in a reasonable amount of time. There is clearly a tradeoff between the computational investment in obtaining a solution and the quality of that solution.

2.3 Typology of Cutting and Packing Problems

2.3.1 Dyckhoff's Typology

Dyckhoff [12] published a typology of highlighting the common underlying structure of C&P problems. This typology supported the integration and cross-fertilisation of two largely separated research areas. As a result, he systematically classified packing problems into a 4-field representation of $\alpha \mid \beta \mid \gamma \mid \delta$ where,

α : Dimensionality.

β : Kind of Assignment.

γ : Assortment of Large Objects.