UNIVERSITI PUTRA MALAYSIA

CLASSIFICATION OF FIRST CLASS 9-DIMENSIONAL COMPLEX FILIFORM LEIBNIZ ALGEBRAS

SOZAN J. OBAIYS

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By

SOZAN J. OBAIYS

Thesis Submitted to the School of Graduate Studies, Universiti Putra Malaysia in Fulfilment of the Requirements for the Degree of Master in Pure Mathematics

August 2009

DEDICATION

To

My Father (Allah bless him) and my dear mother

--- and ----

For their great patience

My husband and lovely kids

For their encouragement

and

My lovely country Iraq

Abstract of thesis presented to the Senate of Universiti Putra Malaysia in fulfilment of the requirement for the degree of Master

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August 2009

Chair: Associate Prof. Dr. Isamiddin, PhD

Faculty: Science

Let V be a vector space of dimension n over an algebraically closed field K (char$K$=0). Bilinear maps $V \times V \rightarrow V$ form a vector space $Hom(V \otimes V, V)$ of dimensional n^3, which can be considered together with its natural structure of an affine algebraic variety over K and denoted by $Alg_n(K) \cong K^{n^3}$. An n-dimensional algebra L over K can be considered as an element $\lambda(L)$ of $Alg_n(K)$ via the bilinear mapping $\lambda : L \otimes L \rightarrow L$ defining a binary algebraic operation on L : let $\{e_1, e_2, \dots, e_n\}$ be a basis of the algebra L. Then the table of multiplication of L is represented by point (γ_{ij}^k) of this affine space as follows:

$$\lambda(e_i, e_j) = \sum_{k=1}^{n} \gamma_{ij}^k e_k.$$

Here γ_{ij}^k are called *structural constants* of L. The linear reductive group $GL_n(K)$ acts on $Alg_n(K)$ by $(g * \lambda)(x, y) = g(\lambda(g^{-1}(x), g^{-1}(y)))$("transport of structure"). Two algebra structures λ_1 and λ_2 on V are isomorphic if and only if they belong to the same orbit under this action.

Recall that an algebra L over a field K is called a *Leibniz algebra* if its binary operation satisfies the following Leibniz identity:

$$[x, [y, z]] = [[x, y], z] - [[x, z], y],$$

Leibniz algebras were introduced by J.-L.Loday. (For this reason, they have also been called "Loday algebras"). A skew-symmetric Leibniz algebra is a Lie algebra. In this case the Leibniz identity is just the Jacobi identity.

This research is devoted to the classification problem of $Leib_n$ in low dimensional cases. There are two sources to get such a classification. The first of them is naturally graded non Lie filiform Leibniz algebras and another one is naturally graded filiform Lie algebras. Here we consider Leibniz algebras appearing from the naturally graded non Lie filiform Leibniz algebras.

It is known that this class of algebras can be split into two subclasses. However, isomorphisms within each class have not been investigated yet. Recently U.D.Bekbaev and I.S.Rakhimov suggested an approach to the isomorphism problem of Leibniz algebras based on algebraic invariants.

This research presents an implementation of this invariant approach in 9-dimensional case. We give the list of all 9-dimensional non Lie filiform Leibniz algebras arising from the naturally graded non Lie filiform Leibniz algebras. The isomorphism criteria and the list of algebraic invariants will be given.

Abstrak tesis yang dikemukakan kepada Senat Universiti Putra Malaysia sebagai memenuhi keperluan untuk ijazah Master

PENGELASAN KELAS PERTAMA BERDIMENSI SEMBILAN ALJABAR LEIBNIZ FILIFORM KOMPLEKS

Oleh

SOZAN J. OBAIYS

July 2009

Pengerusi: Profesor Madya Dr. Isamiddin, PhD

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Biarkan V menjadi ruang vector berdimensi n merentasi suatu medan yang secara aljabarnya tertutup K (char$K$=0).. Pemetaan bilinear $V \times V \to V$ membentuk ruang vektor $Hom(V \otimes V, V)$ berdimensi n^3, yang mana ia boleh dipertimbangkan bersama kepelbagaian struktur semulajadi afin algebra merentas K dan dinyatakan sebagai $Alg_n(K) \cong K^{n^3}$. Satu algebra berdimensi-n L merentasi K boleh ditentukan sebagai satu unsur $\lambda(L)$ bagi $Alg_n(K)$ melalui pemetaan $\lambda : L \otimes L \to L$ menegaskan suatu operasi aljabar penduaan ke atas L : biarkan $\{e_1, e_2, \ldots, e_n\}$ menjadi asasi untuk algebra L. Maka jadual pendaraban bagi L diwakili oleh titik (γ_{ij}^k) bagi ruang afin ini seperti berikut:

$$\lambda(e_i, e_j) = \sum_{k=1}^{n} \gamma_{ij}^k e_k.$$

Disini γ_{ij}^k dipanggil pemalar berstruktur bagi L . Kumpulan penurunan linear $GL_n(K)$ bertindak ke atas $Alg_n(K)$ dengan $(g * \lambda)(x, y) = g(\lambda(g^{-1}(x), g^{-1}(y)))$("pengangkutan bagi unsur "). Dua struktur algebra λ_1

dan λ_2 atas V adalah isomorfik jika dan hanya jika mereka terkandung dalam orbit yang sama dibawah aksi ini.

Telah diketahui sebuah aljabar L merentasi sebuah medan K dipanggil Aljabar Leibniz jika operasi penduaannya memuaskan identiti Leibniz yang berikut:

$$[x,[y,z]] = [[x,y],z] - [[x,z],y],$$

Aljabar Leibniz mula diperkenalkan oleh J.-L Loday. (Oleh sebab itu ia juga dikenali sebagai 'Aljabar Loday'). Sebuah aljabar Leibniz simetri-pencong adalah aljabar Lie. Oleh sebab itulah identiti Leibniz ialah identiti Jacobi.

Kajian ini didedikasikan untuk mengelaskan masalah $Leib_n$ kes-kes berdimensi rendah. Ada dua punca untuk memperolehi sebarang pengelasan iaitu Aljabar Leibniz bukan-Lie filiform bergred semulajadi dan Aljabar Lie filiform bergred semulajadi. Disini, kita akan mempertimbangkan aljabar Leibniz yang muncul dari aljabar Liebniz tak-filiform bergred semulajadi. Secara umumnya kelas ini boleh dipisahkan menjadi dua sub-kelas.

Walaubagaimanapun, isomorfisma dalam setiap kelas belum lagi diketahui. Justeru itu muktakhir ini, U.D. Bakbaev dan I.S. Rakhimov mencadangkan satu langkah kepada masalah keisomorfisman bagi aljabar Leibniz berdasarkan aljabar tak-terubah.

Tesis ini membentangkan perlaksanaan kaedah ketakubahan dalam kes berdimensi sembilan. Kami telah menyenaraikan kesemua aljabar Leibniz bukan-Lie filiform berdimensi sembilan yang berpunca dari aljabar Liebniz bukan-Lie filiform bergred semulajadi. Kriteria isomorphisma dan senarai ketakubahan aljabar turut diberikan.

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I certify that a Thesis Examination Committee has met on 25 August 2009 to conduct the final examination of Sozan J. Obaiys on her thesis entitled "Classification of First Class 9-dimensional complex filiform Leibniz algebras" in accordance with the Universities and University Colleges Act 1971 and the Constitution of the Universiti Putra Malaysia [P.U.(A) 106] 15 March 1998. The Committee recommends that the student be awarded the Master of Science.

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Date: 16 November 2009

DECLARATION

I declare that the thesis is my original work except for quotations and citations which have been duly acknowledged. I also declare that it has not been previously, and is not concurrently, submitted for any other degree at Universiti Putra Malaysia or at any other institution.

Sozan J. Obaiys
Date: 7 October 2009

TABLE OF CONTENTS

CHAPTER 1

INTRODUCTION

1.1 Short history

Theory of Lie algebras is one of the most developed branches of modern algebra. It has been deeply investigated for many years by mathematicians. Active investigation of the properties of the Lie algebras led to the introduction of new and more general object called Leibniz algebra.

This work is concerned on studying Leibniz algebras, introduced by the French Mathematician J.-L.Loday. (That it is also have been called "Loday algebras"), and investigated later in [6, 22].

Leibniz algebras appear to be related in a natural way to several topics such as differential geometry, homological algebra, classical algebraic topology, algebraic K-theory, loop spaces, noncommutative geometry, quantum physics etc., as a generalization of the corresponding applications of Lie algebras to these topics.

Many papers concern to the study of homological problems of Leibniz algebras [9, 21, 24]. J.-L.Loday and T.Pirashvili described the free Leibniz algebras [20], A.A.Mikhalev and U.U.Umirbaev's results concern to solution of the non-commutative analogue of the Jacobian conjecture in the affirmative for free Leibniz algebras [22], in the spirit of the corresponding result of C.Reutenauer V.Shpilrain and U.U.Umirbaev.

The problems concerning Cartan subalgebras and solvability were studied by Sh.A.Ayupov and B.A.Omirov [1]. The notion of simple Leibniz algebra was suggested by A.Dzhumadil'daev [10], who obtained some results concerning special cases of simple Leibniz algebras.

Several classes of new algebras Loday introduced also, some of them have two generating operations and they are called dialgebras . The first motivation to introduce such algebraic structures (related with well known Lie and associative algebras) are problems in algebraic K -theory. The categories of these algebras over their operads can be assembled into the commutative diagram which reflects the Koszul duality of them. Leibniz algebras present a non-commutative (precisely, a non- antisymmetric) analogue of Lie algebras introduced as algebras that satisfy the following identity:

$$[x, [y, z]] = [[x, y], z] - [[x, z], y]$$

The structure theory of Leibniz algebras mostly remains unexplored and the extension of notions like simple, semisimple Leibniz algebras, radical, etc., have not been discussed. Some structural results concerning nilpotency, classification of low dimensional Leibniz algebras and related problems were, however, considered in [5], [2]. The reader may find similar results for Lie algebras in [15]. The classification, up to isomorphism, of any class of algebras is a fundamental and very difficult problem. It is one of the first problems that encountered when trying to understand the structure of some class of algebras.

From the geometrical point of view the classification of a class of algebras corresponds to a fiber of this class, that being the isomorphism classes. By "classification“we mean the algebraic classification, i.e., the determination of the types of isomorphic algebras, whereas geometric classification is the problem of finding generic structural constants in the sense of algebraic geometry. But the geometrical classification presupposes the algebraical classification.

1.2 The relation between Lie and Leibniz algebras

In investigation of the properties of cyclic homologies, Loday noted that, if one replace the external product in definition of $n-$th cochains, then it sufficed to show the validity of the Leibniz identity instead of the anti-commutativity and Jacobi equality.

To find the relation between Lie and Leibniz algebras, we begin by their definition.

Definition 1.2.1. Let L be an algebra over a field k, L is called Lie algebra if its multiplication operation $[\cdot,\cdot]$ has the following properties:
i. skew-symmetry, $[x,y]=-[y,x], \ \forall \ x,y \in L$
ii. Jacobi identity, $[[x,y],z]+[[y,z],x]+[[z,x],y]=0, \quad \forall \ x,y,z \in L.$

Example 1.2.1. *[15] Every vector space a with the bracket $[x,y]=0$, for all x and y in a, is a Lie algebra, called an Abelian Lie algebra.*

Example 1.2.2. *Let A be an associative algebra. The bracket on A define as follows:*

$$[a,b]=a.b-b.a \qquad a,b \in A,$$

then $(A,[\cdot,\cdot])-$ Lie algebra.

Definition 1.2.2. [14] An algebra L over a field K is called a Leibniz algebra if it satisfies the following Leibniz identity:

$$[x,[y,z]]=[[x,y],z]-[[x,z],y]$$

where $[\cdot,\cdot]$ denotes the multiplication in L.

Leibniz algebra is a generalization of Lie algebras, since

if a Leibniz algebra has the additional property of antisymmetry
$[x, y] = -[y, x]$ and substituting this property in Leibniz identity we obtain
$[[x, y], z] - [x, [y, z]] - [[x, z], y] = 0$,
then Leibniz identity can be easily reduced to the Jacobi identity, as illustrate in this formula

$$[[x, y], z] + [[y, z], x] + [[z, x], y] = 0$$

1.3 Nilpotent Leibniz Algebras

Some observations on non-associative Leibniz algebra are given in this section to assist readers less familiar with, to consider.

Let L be a complex Leibniz algebra. We put:

$$L^1 = L, \quad L^{k+1} = [L^k, L], \quad k \in N.$$

Definition 1.3.1. A Leibniz algebra L is said to be nilpotent if there exists an integer $s \in N$, such that

$$L^1 \supset L^2 \supset \ldots \supset L^s = \{0\}.$$

When studying a certain class of algebras it is important to describe at least the algebras of lower dimensions up to an isomorphism accuracy. In Leibniz algebras case difficulties arise even when considering nilpotent Leibniz algebras of dimension 3. For this reason a special class of nilpotent Leibniz algebras, called filiform algebras is considered in our work.

Definition 1.3.2. An n-dimensional Leibniz algebra L is said to be nulfiliform if $dimL^i = n - i + 1$, where $2 \leq i \leq n + 1$.

Up to isomorphism, there is only one n-dimensional non-Lie nulfiliform Leibniz algebra. It can be given by the following table of multiplications:

$$[e_i, e_1] = e_{i+1}, \qquad 1 \leq i \leq n - 1$$

where $\{e_1, e_2, ..., e_n\}$ is a basis of L and omitted products are supposed to be zero.

Definition 1.3.3. An n-dimensional Leibniz algebra L is said to be filiform if $dimL^i = n - i$, where $2 \leq i \leq n + 1$.

Note: It is clear that a filiform Leibniz algebra is nilpotent.

Let $Leib_n$ denote the class of all $n-$dimensional filiform Leibniz algebras. Given a nilpotent Leibniz algebra L with nilindex s, we put $L_i = L^i/L^{i+1}, \;\; 1 \leq i \leq s - 1$, and $grL = L_1 \oplus L_2 \oplus ... \oplus L_{s-1}$. Since $[L_i, L_j] \subseteq L_{i+j}$ the algebra grL is graded. grL is called the naturally graded Leibniz algebra.

Definition 1.3.4. If a Leibniz algebra G is isomorphic to a filiform naturally graded algebra grL, then G is said to be naturally graded filiform Leibniz algebra.

1.4 Research Objectives

The objectives of this research as follows:

1. To give the system of equations of 9-dimensional complex filiform non Lie Leibniz algebras and formulate the isomorphism criterion for 9-dimensional complex filiform Leibniz algebras, arising from naturally graded non-Lie filiform Leibniz algebras.

2. To represent the mentioned subclass as a disjoint union of subsets.

3. To give the isomorphism criteria for each disjoint subset contains parametric family of orbits.

4. To give the corresponding representatives for these subsets consisting of single orbit.

5. To prove the isomorphism of the system in α with Δ.

6. To provide the list of algebras of 9-dimensional complex filiform non Lie Leibniz algebras in adapted basis.

7. To give ideas and suggestions for future work.

1.5 Organization of thesis

In chapter 1, the history of Leibniz algebras are reviewed and the relation between Lie and Leibniz algebras with some examples are given. Theorem 2.5.1 is used to derive an isomorphism criterion. Our main results in this thesis are presented in chapters 3 and 4, where chapter 3, is concerned on classification of 9-dimensional filiform Leibniz algebras, and the list of all disjoint subset results from this classification are supported by proofs. The list of all algebras obtained from this work are performed in chapter 4.

CHAPTER 2

LITERATURE REVIEW

2.1 Chapter outline

This chapter is organized as follows:
A brief background on Leibniz algebras is given in section 2.2. In section 2.2, applications and relations on Leibniz algebras are demonstrated, while in section 2.4, the representations of the filiform Leibniz algebras via structural constants are performed. Finally the adapted changes of basis and the isomorphism criterion theorem of filiform Leibniz algebras are presented in section 2.4.

2.2 Literature review

Classification of complex filiform Leibniz algebras is obtained from two sources, the naturally graded non-Lie filiform Leibniz algebras, and the naturally graded filiform Lie algebras. In this work we deal with the complex filiform Leibniz algebras obtained from the naturally graded non-Lie filiform Leibniz algebras. This sort of algebras in dimension n is denoted by $Leib_n$.

In introducing several classes of algebras in [17], Loday's main motivation was the search of an "obstruction"to the periodicity of algebraic K-theory, that does not satisfy the Bott periodicity theorems valid for its topological version. Beside this purely algebraic motivation and development of Leibniz setting, some relationships with classical geometry have recently been discovered, which could lead us to investigate Leibniz homology in view of concrete applications in non-commutative geometry and its physical interpretations.

2.3 Applications and relations on Leibniz algebras

1. Structural problems

Leibniz algebra is a generalization of classical Lie algebra. It is known that a finite dimensional complex Lie algebra can be represented as a semidirect sum of its unique maximal solvable ideal (the radical) and semisimple subalgebra (the factor of Levi). This decomposition is called Levi-Malcev decomposition. Since a semisimple Lie subalgebra is a direct sum of simple Lie subalgebras (but all simple Lie algebras have been described by Dinkin's diagrams) thus the classification problem of such a class of algebras is reduced to the description of solvable part. Apparently, the same can be done for Leibniz algebras. The notion of simple Leibniz algebras was suggested by A.Dzhumadil'daev and S.Abdulkassymova [10], who obtained some results concerning special cases of simple Leibniz algebras. The problems concerning Cartan subalgebras and solvability were studied by S. Alberverio, Sh.A.Ayupov and B.A.Omirov [1].

For the classification of complex nilpotent Leibniz algebras in dimensions at most four see [3]. The isomorphism criteria for a subclass (called filiform) of finite-dimensional complex nilpotent Leibniz algebras was created in [7, 8, 14, 23].

2. Deformations and contractions

Deformation theory of Leibniz algebras and its related physical applications, was initiated by Fialowski, A., Mandal, A., Mukherjee, G. [13]. The algebraic variety of four-dimensional complex nilpotent Leibniz algebras was studied in [4].

3. (Co)homological problems

In [20] J.-L.Loday and T.Pirashvili have described the free Leibniz algebras. A.Mikhalev and U.U.Umirbaev [22] works were devoted to solution of the non-commutative analogue of the Jacobian conjecture in the affirmative for free Leibniz algebras, in the spirit of the corresponding result of C.Reutenauer [26], V.Shpilrain [27] and U.U.Umirbaev [28]. Loday J.-L.[17] used the analogy between algebraic K-theory and cyclic homology to build a program aiming to understand the algebraic K-theory of fields and the periodicity phenomena in algebraic K-theory. In particular, he conjectured the existence of a Leibniz K-theory which would play the role of Hochschild homology and proposed a motivated presentation for the Leibniz K-group of a field. In [24], the authors showed that the second Leibniz homology group $HL^2(S(n))$ of the Steinberg Leibniz algebra $S(n)$ is not necessarily trivial for $n = 3, 4$ without any assumption on the base ring and it is trivial for $n \geq 5$.

Lodder J.M. [21] constructed a natural homomorphism from Leibniz to Hochschild homology for an algebra R over a commutative ring k, and proved that it is surjective when $R = gl(A)$, A an algebra over a characteristic zero field.

4. Quantum mechanics

A nontrivial Leibniz analog of the Virasoro algebra does not exist [20]. Also, we can consider $2n-$dimensional space, and assume that a Poisson-type bracket for functions is given by a matrix M, so that $F, G = \bigtriangledown F^t M \bigtriangledown$. To verify the Leibniz identity, the following condition: $(\bigtriangledown F)^t M^t((HesH)M \bigtriangledown G + (\bigtriangledown F)^t M^t((HesH)M^t \bigtriangledown F = 0$, for all F, G, H is obtained, using the relation $\bigtriangledown F, G = (HesF)M \bigtriangledown G + (HesG)M^t \bigtriangledown F$, where Hes denotes the Hessian. Then it follows that $M^t = -M$, and so the Poisson bracket comes from a closed $2-$form which ensures that it is again the standard case.

5. Geometrical constructions

We would like to think of a Leibniz algebra as the tangent space at the identity, to some manifold possessing some algebraic structure (Loday's coquecigrue [19]) in a way similar to the relations between Lie algebras and Lie groups. There are good reasons to believe this is possible, since as shown in [20] and [18] there is a sort of universal enveloping algebra for a Leibniz algebra, so there is a candidate for the space of functions on the coquecigrue. Moreover, Kinyon and Weinstein in [16] have given a more concrete partial solution to the problem, by considering reductive homogeneous spaces with an invariant connection, which in a sense integrate the Leibniz algebra.

6. Mathematical Physics

It is well known that classical $R-$matrices in Mathematical Physics play an important role in the study of Integrable Systems. Recently, in [12] a generalization of classical $R-$matrices in Leibniz algebras case was discovered and the corresponding analogues of the classical Yang-Baxter-type equations for Leibniz algebras were studied. It can be used as a basis for a noncommutative extension of some integrable systems, such as the Toda lattice or the discrete Kadomtsev-Petviashvili type equations. (see [11] as well.)

2.4 Preliminaries

In this work, we use some concepts from algebraic geometry for example Zariski topology, therefore this brief section is devoted to Zariski topology and its properties.

Definition 2.4.1. Let X be a set of points in some space. A topology on X is a set τ of designated subsets of X, called the open sets of X, so that the following axioms are satisfied:
a) The union of any collection of open sets is open.
b) The intersection of any finite collection of open sets is open.
c) X and ϕ are open.
The closed sets in X are the complements of the open sets.

We define below the so-called Zariski topology on algebraic varieties. If we work over $\mathbb{C}$.

After this short information about Zariski topology, we give the representations of non-Lie filiform Leibniz algebras via structural constants.

Theorem 2.4.1. *[5] Any $(n+1)$-dimensional complex non-Lie filiform Leibniz algebra, is isomorphic to one of the following algebras:*

$$FLeib_{n+1} = \begin{cases} [e_0, e_0] = e_2 \ , \\ [e_i, e_0] = e_{i+1} \ , \ 1 \leq i \leq n-1 \\ [e_0, e_1] = \alpha_3 e_3 + \alpha_4 e_4 + ... + \alpha_{n-1} e_{n-1} + \theta e_n, \\ [e_j, e_1] = \alpha_3 e_{j+2} + \alpha_4 e_{j+3} + ... + \alpha_{n+1-j} e_n \ , \\ \qquad 1 \leq j \leq n-2 \\ \qquad \alpha_3, \alpha_4, ..., \alpha_n, \theta \in \mathbb{C}. \end{cases}$$