



UNIVERSITI PUTRA MALAYSIA

**FLUTTER ANALYSIS OF A SCALED MODEL OF AN EAGLE 150B/AC
WING**

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**FLUTTER ANALYSIS OF A SCALED MODEL OF AN EAGLE 150B/AC
WING**

By

AZMIN SHAKRINE BIN MOHD RAFIE

**Thesis Submitted to the School of Graduate Studies, Universiti Putra Malaysia,
in Fulfilment of the Requirements for the Degree of Doctor of Philosophy**

September 2007



DEDICATION

Alhamdulillah thanks to Allah the Almighty for the blessings and opportunities that He has provided the strength for me to accomplish this PhD study. This thesis is especially dedicated to;

My father and mother:

Mohd Rafie Hussain and Mariatul Kibtiah Mohd Yatim, thank you for your support, understanding and blessing to carry out this study.

My beloved wife:

Norazlinda Mohd Darby who constantly source of inspiration, motivation, encouragement and support throughout this study.

My son:

Muhammad Zunnurain, source of my inspiration.



Abstract of thesis presented to the Senate of Universiti Putra Malaysia in fulfilment
of the requirement for the degree of Doctor of Philosophy

**FLUTTER ANALYSIS OF A SCALED MODEL OF AN EAGLE 150B/AC
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September 2007

Chairman: Professor Ir. ShahNor Basri, PhD

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An investigation of the problem of the flutter condition of an Eagle 150B aircraft wing is undertaken. The research is largely devoted to investigating the adequacy of the ideal flutter theory that has been employed to predict flutter boundary for such wing. A series of panel flutter experiment carried out in UPM 1m × 1m wind tunnel at Mach number up to 0.132 are described in detail. Furthermore, an extensive parametric computational analysis has been conducted to improve flutter condition by reconfiguring the wing design specification. For experimental analysis, the ground test which includes static and dynamic tests of the wind model has been performed followed by the wind tunnel testing. The data gathered from the wind tunnel testing is analyzed using the logarithmic decrement method so that the flutter speed can be predicted. The wing model mounting system test rig has been designed and developed together with the data acquisition system which is used for data collection. In order to validate the experimental technique, wind tunnel testing using three different types of materials for rectangular flat plate has been conducted. The types of materials used are aluminum 6061, mild steel and stainless steel. The



agreement between experimental technique and computational analysis is acceptable since the error of difference is less than 6 percent.

MSC. Patran and Nastran software have been used to predict the flutter condition since it has the capability to carry out the aeroelasticity analysis of the actual wing and wing model. The PK-method and aerodynamic doublet lattice methods were selected for this analysis as it provides the eigenvalue solutions in the form of the V-g and V-f graphs. Validation of the computational analysis with two existing published results is performed to ensure the results are reliable. The parametric study produced the results on the effects of the mass, altitude, span length, stiffness and center of gravity position against the flutter speed condition. This research work may conclude that both techniques are reliable to investigate flutter speed since the validation results showed a good agreement. It was also found that through extensive parametric study, several suggestions have been made to reconfigure the wing in order to improve the flutter condition.

Abstrak tesis dikemukakan kepada Senat Universiti Putra Malaysia sebagai memenuhi keperluan untuk ijazah Doktor Falsafah

**ANALISIS FLUTER UNTUK MODEL BERSKALA BAGI SAYAP
PESAWAT EAGLE 150B**

Oleh

AZMIN SHAKRINE BIN MOHD RAFIE

September 2007

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Penyiasatan bagi masalah keadaan fluter untuk sayap pesawat Eagle 150B telah dilaksanakan. Penyelidikan ini menumpukan sepenuhnya untuk menyiasat kecekapan teori unggul fluter yang telah digunakan untuk meramal sempadan fluter sayap tersebut. Beberapa siri eksperimen panel fluter yang dijalankan di terowong angin $1\text{m} \times 1\text{m}$ UPM pada nombor Mach sehingga 0.132 telah dinyatakan dengan terperinci. Selanjutnya, kajian parameter yang mendalam menggunakan kaedah analisis berkomputer telah dilakukan untuk memperbaiki lagi keadaan fluter dengan mengubah spesifikasi rekabentuk sayap. Untuk analisis eksperimen, ujian di dataran yang mengandungi ujian statik dan dinamik untuk model sayap telah dilaksanakan diikuti oleh ujian terowong angin. Data yang diperolehi daripada ujian terowong angin akan dianalisa menggunakan kaedah pengurangan logaritma di mana kelajuan fluter akan dapat diramal. Sistem rig ujian pencagak model sayap telah direkabentuk dan dibangunkan bersama dengan sistem perolehan data yang digunakan untuk mengumpul data. Untuk tujuan pengesahan analisis eksperimen, ujian terowong angin dengan menggunakan tiga jenis bahan yang berbeza untuk plat rata segi empat tepat telah dilakukan. Jenis bahan yang digunakan adalah aluminium 6061, besi

lembut dan keluli tahan karat. Kesesuaian antara teknik eksperimen dan analisis berkomputer adalah diterima memandangkan perbezaan ralat adalah kurang daripada 6 peratus.

Perisian MSC. Patran dan Nastran telah digunakan untuk meramal keadaan flutter di mana ia mempunyai kebolehan untuk menjalankan analisis keanjalan udara untuk analisis sayap sebenar dan sayap model. Kaedah PK dan kaedah aerodinamik gandaan kekisi telah dipilih untuk analisis ini di mana ia memberikan penyelesaian nilai eigen di dalam bentuk graf V-g dan V-f. Pengesahan analisis berkomputer dengan dua keputusan penerbitan yang sedia ada telah dilakukan untuk memastikan keputusan yang boleh dipercayai. Kajian parameter memberikan hasil kesan jisim, ketinggian, panjang sayap, keanjalan dan kedudukan pusat gravity terhadap keadaan kelajuan flutter. Kerja penyelidikan ini dapat menyimpulkan bahawa kedua teknik berkebolehan untuk menyiasat keadaan flutter disebabkan keputusan pengesahan menunjukkan kesesuaian yang baik. Melalui kajian parameter yang mendalam, beberapa cadangan juga dapat dirumuskan untuk mengubah sayap pesawat dengan tujuan untuk memperbaiki lagi keadaan flutter.

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APPROVAL

I certify that an Examination Committee met on 14 September, 2007 to conduct the final examination of Azmin Shakrine Bin Mohd Rafie on his degree of Doctor of Philosophy thesis entitled “Flutter Analysis of A Scaled Model of an Eagle 150B/AC Wing” in accordance with Universiti Pertanian Malaysia (Higher Degree) Act 1980 and Universiti Pertanian Malaysia (Higher Degree) Regulation 1981. The Committee recommends that the candidate be awarded the relevant degree. Members of the Examination Committee are as follows:

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DECLARATION

I hereby declare that the thesis is based on my original work except for quotations and citations which have been duly acknowledged. I also declare that it has not been previously or concurrently submitted for any degree at UPM or other institutions.

AZMIN SHAKRINE BIN MOHD RAFIE

Date: 28 April 2008



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LIST OF ABBREVIATIONS

AGARD	Advisory Group for Aerospace Research and Development
AR	Aspect Ratio
ARES	Aeroelasticity Rotor Experimental System
ASTM	Standard Test Method
CAE	Computational Aeroelasticity
CAP-TSD	Computational Aeroelasticity Program – Transonic Small Disturbance
CFD	Computational Fluid Dynamic
CG	Center of Gravity
CSM	Computational Structural Mechanics
CTRM	Composites Technology Research Malaysia
DAQ	Data Acquisition
DLM	Doublet-Lattice Method
EA	Elastic Axis
EE	Euler equations
FE	Finite Element
FFT	Fast Fourier Transformation
FP	Full Potential
Hz	Hertz
ICW	Intermediate Complexity Wing
LCO	Limit Cycle Oscillation
LE	Leading Edge
MSC Nastran	MacNeal-Schwendler Corporation – NASA Structural Analysis



NASA	National Aeronautics and Space Administration
PAPA	Pitch and Plunge Apparatus
RAE	Royal Aeronautical Establishment
RBAR	Rigid Bar
RPM	Rotations per Minute
SDOF	Single Degree of Freedom
TDT	Transonic Dynamics Tunnel
TE	Trailing Edge
TLNS	Thin Layer Navier-Stokes Equations
TSD	Transonic Small Disturbance
TWG	Transonic Wind Tunnel Gottingen
VS	Virtual Surface

Nomenclature

$[A(t)]$	Matrix of unsteady aerodynamics operators
A	Constant
c	Chord
C	Cos (θ)
$C(k)$	Theodorsen's function
C_p	Pressure data
$[D]$	Damping matrix
e	Exponential
E	Modulus of elasticity
E_1	Longitudinal Young's modulus
E_2	Transverse Young's modulus



EI	Bending stiffness
f_f	Flutter frequency
g	Damping factor
G	Shear modulus of elasticity
G_{12}	In-plane shear modulus
GJ	Torsional stiffness
$H_n^{(2)}(k)$	Hankel functions of the second kind
h, α	Deflection in bending and torsional directions
\bar{i}, \bar{k}	A unit of Cartesian vector
i	imaginary number
I	Moment of inertia
I_α	Mass moment of inertia
J	Polar moment of inertia
$[K]$	Stiffness matrix
k_t	Constant thickness for each layers
k	Reduced frequency
K	Kernel function
K_α	Torsional stiffness
K_h	Bending stiffness
l, L	Span length
L	Vertical force per unit span
$[M]$	Mass matrix
m	Mass per unit length
M_∞	Mach number
M_y	Moment force about y axis per unit span



N_{lay}	Total number of layers
P_t	Load at tip
P	Pressure
P_∞	Pressure at free stream
$\{q\}$	Vector of generalized coordinates
\bar{Q}	Transformed reduced stiffness coefficients
q	Dynamic pressure
Q	Reduced stiffness coefficients
q_1, q_2	Generalized coordinate
Q_h, Q_α	Generalized forces
q_i	i th generalized coordinate
R	Gas constant for air
\vec{r}	Displacement of any point
r	Radius of gyration
S	Sin (θ)
S_α	Static moment
T°	Atmospheric temperature
T	Kinetic energy
t	Time
T_t	Torsional force at tip
T_d	Damped period
u	Horizontal displacement
U	Potential energy
V, V_∞	Free stream velocity
V_b	Vertical velocity vector due to bending



V_f	Flutter speed
V_r	Resulting velocity vector at wing
w	Vertical displacement
w_a	Vertical velocity component over airfoil contour
x	Deflection in x direction
X_1, X_2	Amplitudes
X_{cg}	Distance from center of gravity
$\{Z\}$	Force vector
Z	Distance from center line
z_a	Vertical displacement of airfoil at point x, z and t

Greek Symbols

α	Geometric angle of attack
ε_1	Longitudinal tensile strain
σ_2	Transverse tensile stress
α_{eff}	Effective angle of attack of wing
α_{ind}	Induced angle of attack due to bending
θ_t	Angle deflection at tip
γ_a, γ_w	Circulation along airfoil and wake
δ	Logarithmic decrement
δ_t	Deflection at tip
ε_2	Transverse tensile strain
ζ	Damping ratio
θ	Angle of lamina
μ	Mass ratio