UNIVERSITI PUTRA MALAYSIA

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By

MUHAFZAN

Thesis Submitted to the School of Graduate Studies, Universiti Putra Malaysia, in Fulfilment of the Requirement for the Degree of Doctor of Philosophy

March 2007

Dedication

Specially Dedicated to

My Wife, **Sri Mayang S. Ag**

and

My Sons, **Qori**, **Rahman** and **Ichwan**

Abstract of thesis presented to the Senate of Universiti Putra Malaysia in fulfillment of the requirement for the degree of Doctor of Philosophy

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MUHAFZAN

March 2007

Chairman : Professor Malik Bin Hj. Abu Hassan, PhD

Faculty : Science

The linear quadratic (LQ) control problem is a widely studied field in the area of control and optimization, particularly, in the area of optimal control. This problem is, in general, concerned with determining a controller such that the controller satisfies the dynamic constraint. On the other hand, the descriptor systems have received considerable interest over the last decade because it has some specificity in the structure of its solution. It is a natural mathematical model of many types of physical systems of which it appears frequently in the fields such as circuit systems, economics, power systems, robots and electric network. Therefore, the LQ control problem subject to the descriptor system has a great potential for the system modeling, because it can preserve the structure of physical systems and can include the non dynamic constraint and impulsive element.

In this thesis we consider the most general class of the LQ control problem subject to descriptor system of which the constraint is of the class of the nonregular descriptor system and rectangular descriptor system in the infinite horizon time. In addition, we allow the control weighting matrix in the cost functional to be positive semidefinite, but our results, however, hold for the constraint to be regular descriptor system as well as the control weighting matrix in the cost functional to be positive definite.

Our main aims are to find the optimal smooth solution of the LQ control problem subject to both nonregular and rectangular descriptor systems, respectively. For these purposes, we create the sufficient conditions that guarantee the existence, or existence and uniqueness if possible, of the smooth optimal solution of the problems.

In order to solve the considered problem we transform the LQ control problem subject to both nonregular and rectangular descriptor systems, respectively, into the standard LQ control problem. In fact, by utilizing the means of the restricted system equivalent of two descriptor systems and the equivalence principle of two optimal control problems, we can construct some bijections which show that there are equivalent relationship between the considered problem and the standard LQ control problem.

As a result of the transformation process, we have two kinds of standard LQ control problem, that are, the cases of which the control weighting matrix in the cost functional is positive definite and positive semidefinite. In the positive definite case, we utilize the available results of the standard LQ control problem. Otherwise, in the positive semidefinite case, the semidefinite programming approach is used in order to obtain the smooth solutions. As the ultimate results, the conditions that guarantee the existence of the smooth optimal solution are presented formally in several theorems.

Some testing problems are presented. The graphs of the trajectories are plotted to visualize the behavior of the optimal control-state pairs. The Maple 9 and Matlab 6.5 softwares are used for calculation and plotting the trajectories of the optimal solution.

Abstrak tesis yang dikemukakan kepada Senat Universiti Putra Malaysia
sebagai memenuhi keperluan untuk ijazah Doktor Falsafah

MASALAH KAWALAN KUADRATIK LINEAR TERTAKLUK KEPADA SISTEM PEMERIHALAN TAK SEKATA DAN SISTEM PEMERIHALAN SEGI EMPAT TEPAT

Oleh

MUHAFZAN

Mac 2007

Pengerusi : Profesor Malik Bin Hj. Abu Hassan, PhD

Fakulti : Sains

Masalah kawalan kuadratik linear (LQ) merupakan suatu bidang kajian yang luas dalam bidang kawalan dan pengoptimuman terutamanya dalam bidang kawalan optimum. Secara umumnya, masalah ini berkaitan dengan menentukan pengawal sedemikian hingga pengawal tersebut memenuhi kekangan dinamik. Sebaliknya, sistem pemerihalan telah menarik perhatian dalam beberapa dekad kebelakangan ini, disebabkan ia mempunyai struktur penyelesaian yang spesifik. Ianya merupakan suatu model matematik yang semula jadi daripada beberapa jenis sistem fizikal dan pada kebiasaannya ianya selalu muncul dalam beberapa bidang seperti sistem litar, ekonomi, sistem kuasa, robotik dan jaringan elektrik. Oleh kerana itu, masalah kawalan LQ tertakluk kepada sistem pemerihalan mempunyai potensi yang tinggi dalam pemodelan sistem kerana ianya boleh mengekalkan struktur sistem fizikal dan juga boleh memuatkan kekangan tak dinamik dan ciri impulsif.

Dalam tesis ini, kami mempertimbangkan kelas masalah kawalan LQ yang paling umum tertakluk kepada sistem pemerihalan yang mana kekangan adalah merupakan kelas sistem pemerihalan tak sekata dan sistem pemerihalan segiempat tepat dalam masa mengufuk takterhingga. Tambahan pula, kami membenarkan matriks pemberat kawalan dalam fungsi kos adalah semitentu positif, tetapi keputusan yang kami perolehi juga mengesahkan kekangan adalah merupakan sistem pemerihalan sekata mahupun matriks pemberat kawalan dalam fungsi kos adalah tentu positif.

Tujuan utama kami adalah untuk mencari penyelesaian optimum yang licin bagi masalah kawalan LQ tertakluk kepada sistem pemerihalan tak sekata dan sistem pemerihalan segiempat tepat. Bagi mencapai tujuan ini, kami membentuk syarat cukup yang menjaminkan kewujudan atau keunikan jika boleh bagi penyelesaian optimum yang licin.

Bagi menyelesaikan masalah tersebut, kami mentransformasikan masalah kawalan LQ tertakluk kepada kedua-dua sistem pemerihalan tak sekata dan sistem pemerihalan segiempat tepat, kepada masalah kawalan LQ piawai. Dengan menggunakan kesetaraan sistem tersekat bagi dua sistem pemerihalan dan prinsip kesetaraan bagi dua masalah kawalan optimum, kami boleh membina beberapa bijeksi yang menunjukkan bahawa wujud hubungan kesetaraan di antara masalah tersebut dan masalah kawalan LQ piawai.

Hasil daripada proses transformasi, kami mempunyai dua jenis masalah kawalan LQ piawai, iaitu kes matriks pemberat kawalan bagi fungsi kos adalah tentu positif dan kes matriks pemberat kawalan bagi fungsi kos adalah semitentu positif. Dalam kes tentu positif, kami menggunakan keputusan yang sedia ada bagi masalah kawalan LQ piawai. Sebaliknya, dalam kes semitentu positif, pendekatan pengaturcaraan semitentuan digunakan untuk memperolehi penyelesaian licin. Sebagai hasil akhir, syarat yang menjamin kewujudan bagi penyelesaian optimum licin dikemukakan secara formal dalam beberapa teorem.

Beberapa masalah pengujian juga dikemukakan. Graf trajektori diplotkan bagi menggambarkan telatah keadaan optimum dan kawalan optimum. Perisian Maple dan Matlab digunakan dalam proses pengiraan serta memplot trajektori bagi penyelesaian optimum.

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I certify that an Examination Committee has met on 15th March 2007 to conduct the final examination of Muhafzan on his Doctor of Philosophy thesis entitled "Linear Quadratic Control Problem Subject to Nonregular and Rectangular Descriptor Systems" in accordance with Universiti Pertanian Malaysia (Higher Degree) Act 1980 and Universiti Pertanian Malaysia (Higher Degree) Regulations 1981. The Committee recommends that the candidate be awarded the relevant degree. Members of the Examination Committee are as follows:

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This thesis submitted to the Senate of Universiti Putra Malaysia and has been accepted as fulfilment of the requirement for the degree of Doctor of Philosophy. The members of the Supervisory Committee are as follows:

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Date: 14 JUNE 2007

DECLARATION

I hereby declare that the thesis is based on my original work except for quotations and citations which have been duly acknowledged. I also declare that it has not been previously or concurrently submitted for any other degree at UPM or other institutions.

MUHAFZAN

Date: 04 MAY 2007

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CHAPTER 1

INTRODUCTION

1.1 Preliminaries

The linear quadratic (LQ) control problem is important in control and optimization theory and has been used in practice widely. This problem is, in general, concerned with determining a controller such that the controller satisfies the dynamic constraint. On the other hand, the descriptor systems have received considerable interest over the last decade because it has several specificity in the structure of its solution. Thus the LQ control problem subject to the descriptor systems has a great potential for the system modeling, because they can preserve the structure of physical systems and can include non dynamic constraint and impulsive element. A great number of applications occur in many different fields, such as economics, operation research, electrical engineering, power engineering, model and system design, biology and chemistry , see Mehrmann (1989), Avgerinos (1994) and Rehmn and Allgower (2002).

A great number of results on these topics have been appeared in the literatures. Nevertheless, almost all of these results consider the assumption that the descriptor system is regular and the control weighting matrix in the quadratic cost functional being positive definite. In this last case, *i.e.*, the quadratic cost being positive semidefinite, the existing LQ control problem theories always involve the impulse distributions (see Geerts, 1994). Thus it does not provide any answer to a basic question such as when the LQ control problem subject to descriptor system possesses an optimal solution in the form of a conventional control, in particular, one that does not involve impulse distribution. This question motivates our present work.

To the best of the author's knowledge, not much work has been done for LQ control problem subject to both the nonregular and rectangular descriptor system. Several authors, like Jiandong, *et al.* (1999 and 2002) have discussed this issue for a class of the problem by transforming the problem into a certain standard LQ control problem in which both are equivalent. Nevertheless, one major task still remains, that is, the new standard LQ control problem may be singular and this problem is not discussed yet in the above works, thus it is an open problem.

In this thesis, we generalized such problem by including the control term into the output equation such that the output equation does not solely depend on the state only, but it also depends on input. This makes the systems more realistic and dependable. In addition, the control weighting matrix in the quadratic cost is allowed being positive semidefinite. Likewise, our problems become the most general class of the LQ control problem subject to descriptor systems. Some preliminary results for this problem have been addressed in Muhafzan, *et al*. (2005a and 2005b) and Muhafzan, *et al*. (2006) where several new results are obtained.

In order to solve this problem, we transform the original LQ control problem into a new standard LQ control problem in which both of these LQ problems are equivalent. We also solve, as a new contribution, the singular parts of this new standard LQ control problem by using the semidefinite programming (SDP) approach in which several new criteria to explore the existence of the optimal state-control of the considered problem are obtained. By using this SDP approach means that we solve a dynamic optimization problem via a static optimization problem.

Notation: Throughout this thesis, the superscript "T" represents the transpose, I is the identity matrix with appropriate dimension, $\emptyset$ denotes the empty set, $\dot{x}(t)$ denotes the derivative of function x with respect to t, $\mathbb{R}^n$ denotes the *n*-dimensional Euclidean space, $\mathbb{R}^{m\times n}$ is the set of all $m \times n$ real matrices, $C_{\mathrm{p}}^{+}[\mathbb{R}^n]$ denotes the *n*-dimensional piecewise continuous functions space with domain in $[0, \infty]$, $C^k(\mathbb{I}, \mathbb{R}^n)$ denotes the set of k times continuously differentiable functions from $\mathbb{I} \subseteq \mathbb{R}$ to $\mathbb{R}^n$, and $\mathbb{C}$ denotes the set of complex number. In addition, $\det A$ denotes determinant of matrix A.

1.2 Statement of the Problems and Method of Solving

In what follows we formally define the LQ control problem subject to nonregular and rectangular descriptor systems and identify the classes of problems addressed in this thesis. Given a system of the form

$$E\,\dot{x}(t) = A\,x(t) + B\,u(t), \quad Ex(0) = x_0,\ t \geq 0$$
$$y(t) = Cx(t) + Du(t) \tag{1.1}$$

where $E, A \in \mathbb{R}^{m\times n}$, $B \in \mathbb{R}^{m\times r}$, $C \in \mathbb{R}^{q\times n}$, $D \in \mathbb{R}^{q\times r}$ are constant matrices, $x(t) \in \mathbb{R}^n$ denotes the state vector, $u(t) \in \mathbb{R}^r$ denotes the control vector, $y(t) \in \mathbb{R}^q$ denotes the output vector and $t \in [0, \infty)$. System (1.1) is often called *continuous linear time invariant descriptor system* (Bender and Laub, 1987).

If $m = n$ and the matrix E is nonsingular, then the system (1.1) is actually the usual standard state space system, because it can be written as

$$\dot{x}(t) = \hat{A}\,x(t) + \hat{B}\,u(t), \quad x(0) = \hat{x}_0,\ t \geq 0$$
$$y(t) = C\,x(t) + D\,u(t),$$

where $\hat{A} = E^{-1}A$, $\hat{B} = E^{-1}B$ and $\hat{x}_0 = E^{-1}x_0$. However, if $m = n$ and $\operatorname{rank} E < n$ then the system (1.1) becomes an interesting object. It is called as a *regular descriptor system* if $\det(sE - A) \neq 0$ for almost all $s \in \mathbb{C}$ (Yip and Sincovec, 1981). Otherwise, it is called as a *nonregular descriptor system* if $m = n$ and $\det(sE - A) = 0$ for each $s \in \mathbb{C}$ (Jiandong, *et al.* 1999). Especially, it is called *rectangular descriptor system* if E is a rectangular matrix $(m \neq n)$, see Ishihara and Terra (2001) and Zhang (2006).

Further, for a given admissible initial state $x_0 \in \mathbb{R}^n$, we consider the associated objective function (cost functional) as follows:

$$J(u(.),\ x_0) = \int_0^\infty y^T(t)\, y(t)\, dt. \tag{1.2}$$

In general, recall that the problem of determining the control $u(t) \in \mathbb{R}^r$ which minimizes the cost functional (1.2) and satisfies the system (1.1) for an admissible initial state $x_0 \in \mathbb{R}^n$, is often called LQ *control problem subject to descriptor system*. It is called the *singular* LQ *control problem subject to descriptor system*, if the control weighting matrix, *i.e.* $D^T D$, is positive semidefinite. By considering the infinite horizon nature of the problem, we further assume that the control to be stabilizing, it means that the corresponding state trajectory converges to zero as time goes to infinity.

Let us now consider the LQ control problem subject to the nonregular descriptor system (1.1) where $\operatorname{rank} E < n$. We further define the set of admissible control-state pairs for (1.1) and (1.2) by:

$$\mathbb{A}_{\text{ad}} \equiv \{(u(.), x(.)) \mid u(.) \in C_{\text{p}}^{+}[\mathbb{R}^r] \text{ and } x(.) \in C_{\text{p}}^{+}[\mathbb{R}^n] \text{ satisfy (1.1) and } J(u(.), x_0) < \infty\}.$$

The problem under consideration is to find $(u^*, x^*) \in \mathbb{A}_{\text{ad}}$, such that

$$J(u^*, x_0) = \underset{(u(.),\, x(.)) \in \mathbb{A}_{\text{ad}}}{\text{minimize}} \quad J(u(.),\ x_0), \tag{1.3}$$

under the assumption that the nonregular descriptor system is *impulse controllable* and the matrix $D^T D$ is allowed to be positive semidefinite. We denote, for simplicity, the LQ control problem with the set of admissible control-state pairs $\mathbb{A}_{\text{ad}}$, as Ω. The pair of optimal control-state $(u^*, x^*) \in \mathbb{A}_{\text{ad}}$ is called as the *optimal solution* of LQ control problem subject to nonregular descriptor system Ω.

Next, we consider the LQ control problem subject to rectangular descriptor. To distinguish notations between the objective functions, we shall put a hat over the cost functional J for rectangular descriptor system *i.e.*, $\widehat{J}$, whenever there is an ambiguity. For the rectangular descriptor system (1.1) where $\operatorname{rank} E < m$, we define the set of admissible control-state pairs for (1.1) and (1.2) by:

$$\widehat{\mathbb{A}}_{\text{ad}} \equiv \{(u(.), x(.)) \,|\, u(.) \in C_{\text{p}}^{+}[\mathbb{R}^{r}] \text{ and } x(.) \in C_{\text{p}}^{+}[\mathbb{R}^{n}] \text{ satisfy (1.1) and } \widehat{J}(u(.), x_0) < \infty\}.$$

The considered problem is to find $(u^*, x^*) \in \widehat{\mathbb{A}}_{\text{ad}}$, such that

$$\widehat{J}(u^*, x_0) = \underset{(u(.),\, x(.)) \in \widehat{\mathbb{A}}_{\text{ad}}}{\text{minimize}} \quad \widehat{J}(u(.), x_0), \tag{1.4}$$

under assumption that the rectangular descriptor systems is *impulse controllable* and the matrix $D^T D$ is allowed to be positive semidefinite. We denote, for simplicity, the LQ control problem with the set of admissible control-state pairs $\widehat{\mathbb{A}}_{\text{ad}}$, as $\widehat{\Omega}$. The pair of optimal control-state $(u^*, x^*) \in \widehat{\mathbb{A}}_{\text{ad}}$ is called as the *optimal solution* of LQ control problem subject to rectangular descriptor system $\widehat{\Omega}$.

In order to handle both of the above problems, we construct a bijective transformation which can be used to transforms the LQ control problem subject to