



UNIVERSITI PUTRA MALAYSIA

**RUNGE-KUTTA METHODS FOR SOLVING ORDINARY
AND DELAY DIFFERENTIAL EQUATIONS**

RAE'D ALI AHMED ALKHASAWNEH

FS 2006 63

**RUNGE-KUTTA METHODS FOR SOLVING ORDINARY
AND DELAY DIFFERENTIAL EQUATIONS**

By

RAE'D ALI AHMED ALKHASAWNEH

**Thesis Submitted to the School of Graduate Studies, Universiti Putra Malaysia,
in Fulfillment of the Requirements for the Degree of Doctor of Philosophy**

November 2006



DEDICATION

To my Great Father and Mother,

To my beloved Wife,

To my Brothers and Sisters.

Raed



Abstract of thesis presented to the Senate of Universiti Putra Malaysia in
fulfilment of the requirement for the degree of Doctor of Philosophy

**RUNGE-KUTTA METHODS FOR SOLVING ORDINARY AND DELAY
DIFFERENTIAL EQUATIONS**

By

RAE'D ALI AHMED ALKHASAWNEH

November 2006

Chairman: Associate Professor Fudziah bt Ismail, PhD

Faculty : Science

An introduction to Runge-Kutta methods for the solution of ordinary differential equations (ODEs) is introduced. The technique of using Singly Diagonally Implicit Runge-Kutta (SDIRK) method for the integration of stiff and non-stiff ODEs has been widely accepted, this is because SDIRK method is computationally efficient and stiffly stable. Consequently embedded SDIRK method of fourth-order six stage in fifth-order seven stage which has the property that the first row of the coefficient matrix is equal to zero and the last row of the coefficient matrix is equal to the vector output value is constructed. The stability region of the method when applied to linear ODE is given. Numerical results when stiff and non-stiff first order ODEs are solved using the method are tabulated and compared with the method in current use.

Introduction to delay differential equations (DDEs) and the areas where they arise are given. A brief discussion on Runge-Kutta method when adapted to delay differential equation is introduced. SDIRK method which has been derived previously is used to



solve delay differential equations; the delay term is approximated using divided difference interpolation. Numerical results are tabulated and compared with the existing methods. The stability aspects of SDIRK method when applied to DDEs using Lagrange interpolation are investigated and the region of stability is presented.

Runge-Kutta-Nyström (RKN) method for the solution of special second-order ordinary differential equations of the form $y'' = f(x, y)$ is discussed. Consequently, Singly Diagonally Implicit Runge-Kutta Nyström (SDIRKN) method of third-order three stage embedded in fourth-order four stage with small error coefficients is constructed. The stability region of the new method is presented. The method is then used to solve both stiff and non-stiff special second order ODEs and the numerical results suggest that the new method is more efficient compared to the current methods in use.

Finally, introduction to general Runge-Kutta-Nystrom (RKNG) method for the solution of second-order ordinary differential equations of the form $y'' = f(x, y, y')$ is given. A new embedded Singly Diagonally Implicit Runge-Kutta-Nyström General (SDIRKNG) method of third-order four stage embedded in fourth-order five stage is derived. Analysis on the stability aspects of the new method is given and numerical results when the method is used to solve both stiff and non-stiff second order ODEs are presented. The results indicate the superiority of the new method compared to the existing method.



Abstrak tesis dikemukakan kepada Senat Universiti Putra Malaysia
sebagai memenuhi keperluan untuk ijazah Doktor Falsafah

**KAEDAH RUNGE-KUTTA UNTUK MENYELESAIKAN PERSAMAAN
PEMBEZAAN BIASA DAN LENGAH**

Oleh

RAE'D ALI AHMED ALHASAWNEH

November 2006

Pengerusi: Profesor Madya Fudziah bt Ismail, PhD

Fakulti : Sains

Pengenalan kepada kaedah Runge-Kutta untuk menyelesaikan Persamaan Pembezaan Biasa (PPB) diperkenalkan. Teknik yang menggunakan kaedah Runge-Kutta Pепенjuru Tunggal Tersirat (RKPTT) untuk kamiran PPB kaku dan tak kaku telah diterima pakai kerana kaedah RKPTT ini cekap dan sangat stabil. Seterusnya, kaedah terbenam RKPTT bagi peringkat empat tahap enam dalam peringkat lima tahap tujuh dengan ciri-ciri baris pertama bagi matriks pekalinnya bersamaan dengan sifar dan baris terakhir bagi matriks pekalinnya bersamaan dengan nilai vektor outputnya dibina. Rantau kestabilan bagi kaedah ini apabila digunakan ke atas PPB linear diberikan. Keputusan berangka apabila PPB kaku dan tak kaku diselesaikan menggunakan kaedah itu dibentangkan dan dibandingkan dengan kaedah yang sedang digunakan sekarang.

Pengenalan kepada Persamaan Pembezaan Lengah (PBL) dan bidang di mana ianya timbul diberikan. Perbincangan ringkas mengenai kaedah Runge-Kutta apabila diadaptasikan kepada persamaan pembezaan lengah diperkenalkan. Kaedah RKPTT yang telah diperolehi sebelum ini digunakan untuk menyelesaikan PBL; sebutan



lengahnya dianggarkan menggunakan interpolasi beza bahagi. Keputusan berangka dibentangkan dan dibandingkan dengan kaedah yang sedia ada. Aspek kestabilan kaedah RKPTT apabila digunakan ke atas PPL menggunakan interpolasi Lagrange diselidik dan rantau kestabilannya dipersembahkan.

Kaedah Runge-Kutta-Nyström (RKN) bagi penyelesaian PBB khas peringkat kedua dalam bentuk $y'' = f(x, y)$ dibincangkan. Seterusnya, kaedah peringkat tiga tahap tiga terbenam dalam kaedah peringkat empat tahap empat Runge-Kutta Nystrom Pepenjuru Tunggal Tersirat (RKNPTT) dengan pekali ralat yang kecil dibina. Rantau kestabilan bagi kaedah yang baru ini dipersembahkan. Kaedah ini kemudiannya digunakan untuk menyelesaikan kedua-dua PBB kaku dan tak kaku peringkat kedua dan keputusan berangkanya menunjukkan bahawa kaedah ini lebih cekap berbanding kaedah yang sedang digunakan sekarang.

Akhir sekali, pengenalan kepada kaedah umum Runge-Kutta-Nystrom (RKNU) untuk menyelesaikan persamaan pembezaan biasa peringkat kedua dalam bentuk $y'' = f(x, y, y')$ diberikan. Kaedah umum Runge-Kutta Nystrom Pepenjuru Tunggal Tersirat (RKNUPTT) yang baru, peringkat tiga tahap empat terbenam dalam kaedah peringkat empat tahap lima diterbitkan. Analisis bagi aspek kestabilan kaedah baru ini diberikan dan keputusan berangka apabila kaedah ini digunakan untuk menyelesaikan kedua-dua PBB umum kaku dan tak kaku peringkat kedua dipersembahkan. Keputusannya menunjukkan kaedah baru tersebut adalah lebih baik berbanding dengan kaedah yang sedia ada.



ACKNOWLEDGEMENTS

In the Name of Allah, the Most Beneficent, the Most Merciful

First and foremost, I would like to express my sincere and deepest gratitude to the Chairman of the Supervisory Committee, Associate Professor Dr. Fudziah Ismail for her wise council, guidance, invaluable advice and constant encouragement, which always led me to the right research direction. This work could not have been carried out without both direct and indirect help and support from her.

Many thanks to my supervisory committee members, Prof. Dato' Dr. Mohamed Suleiman and Associate Professor Dr. Azmi Jaafar for their advice and motivation towards the completion of this thesis.

This research is partially supported by an IRPA fund. Thanks to University Putra Malaysia and the Malaysian government for the support.

I would like to thank Prof. Dr. John Butcher of Auckland University, New Zealand for finding the time to answer the many emails regarding the research.

My deepest appreciation goes to my father *ALI* and my mother *NUHA* for their love and prayers which are the essential ingredients towards the completion of the thesis.



My wife *SHEREEN*, deserve my unending gratitude for her patience and understanding that make the duration of the study easier for me. Without her encouragement, this thesis would have never been completed.

I am immensely grateful to my brothers and sisters for their continuous support, understanding and encouragement.

Last but not least, I would like to thank my friends especially, Dr. Feras Hanandeh, Dr. Anwar Fawakreh, Dr. Zeyad Al-Zhour, Mohammad El-Bashir, Dr. Majdi El-Qudah and Dr. Jayanthi, for their support and for sharing some light moments away from my work.

Raed Al-Khasawneh,

September 2006.



I certify that an Examination Committee has met on 24 November 2006 to conduct the final examination of Rae'd Ali Ahmed Alkhasawneh on his Doctor of Philosophy thesis entitled "Runge-Kutta Methods for Solving Ordinary and Delay Differential Equations" in accordance with Universiti Pertanian Malaysia (Higher Degree) Act 1980 and Universiti Pertanian Malaysia (Higher Degree) Regulations 1981. The Committee recommends that the candidate be awarded the relevant degree. Members of the Examination Committee are as follows:

Malik Hj Abu Hassan, PhD

Professor
Faculty of Science
Universiti Putra Malaysia
(Chairman)

Mohamad Rushdan Md. Said, PhD

Associate Professor
Faculty of Science
Universiti Putra Malaysia
(Internal Examiner)

Leong Wah June, PhD

Lecturer
Faculty of Science
Universiti Putra Malaysia
(Internal Examiner)

Abdul Razak Yaakub, PhD

Professor
Faculty of Technology Management,
Universiti Utara Malaysia
(External Examiner)

HASANAH MOHD. GHAZALI, PhD

Professor/Deputy Dean
School of Graduate Studies
Universiti Putra Malaysia

Date:



This thesis submitted to the Senate of Universiti Putra Malaysia and has been accepted as fulfilment of the requirement for the degree of Doctor of Philosophy. The members of the Supervisory Committee are as follows

Fudziah Ismail, PhD

Associate Professor
Faculty of Science
Universiti Putra Malaysia
(Chairman)

Mohamed bin Suleiman, PhD

Professor
Faculty of Science
Universiti Putra Malaysia
(Member)

Azmi Jaafar, PhD

Associate Professor
Faculty of Computer Science and Information Technology
Universiti Putra Malaysia
(Member)

AINI IDERIS, PhD

Professor/Dean
School of Graduate Studies,
Universiti Putra Malaysia

Date:



DECLARATION

I hereby declare that the thesis is based on my original work except for quotations and citations which have been duly acknowledged. I also declare that it has not been previously or concurrently submitted for any other degree at UPM or other institutions.

RAE'D ALI AHMED ALKHASAWNEH

Date:



TABLE OF CONTENTS

	Page
DEDICATION	ii
ABSTRACT	iii
ABSTRAK	v
ACKNOWLEDGEMENTS	vii
APPROVAL	ix
DECLARATION	xi
LIST OF TABLES	xv
LIST OF FIGURES	xviii
LIST OF ABBREVIATIONS	xx
 CHAPTER	
I	
INTRODUCTION AND OBJECTIVES	21
Introduction	21
Initial Value Problem	22
Numerical Methods for Solving Initial Value Problem	23
Runge-Kutta Method	25
Runge-Kutta Method and its Implementation	28
Objectives of the Study	30
Planning of the Thesis	31
 II	
LITERATURE REVIEW	33
Introduction	33
Ordinary Differential Equations	33
First-Order ODEs	34
Second-Order ODEs	35
Embedded Singly Diagonally Implicit Runge-Kutta Method for Solving ODE of first order	36
Stability of Runge-Kutta Method	38
Stiff Systems of Ordinary Differential Equations	40
Delay Differential Equations	41
Numerical Methods for Delay Differential Equations	43
Stability Analysis of Numerical Methods for solving Delay Differential Equations	46
Embedded Singly Diagonally Implicit Runge-Kutta-Nyström Method for Solving Special ODE of second-Order	48
Stability of Runge-Kutta-Nyström Method	51
Embedded Singly Diagonally Implicit Runge-Kutta-Nyström General Method for Solving ODE of second-Order	52
Stability of Runge-Kutta-Nyström General Method	53



III	SINGLY DIAGONALLY IMPLICIT RUNGE-KUTTA METHOD (4,6) EMBEDDED IN (5,7) FOR THE INTEGRATION OF STIFF SYSTEMS ORDINARY DIFFERENTIAL EQUATION	56
	Introduction	56
	Derivation of Embedded Singly Diagonally Implicit Runge-Kutta Method	59
	Using Simplifying Assumptions	62
	Choices of γ for the New Method	69
	Stability Analysis of the Method	70
	Stability Region of SDIRK Method Third-Order Embedded in Fourth-Order	73
	Implementation	75
	Problems Tested	84
	Numerical Results	86
	Discussion and Conclusion	93
IV	SOLVING DELAY DIFFERENTIAL EQUATIONS BY SINGLY DIAGONALLY IMPLICIT RUNGE-KUTTA METHOD AND ANALYSING THE P-STABILITY AND Q-STABILITY	94
	Introduction	94
	Solving Delay Differential Equations by Runge-Kutta Method	95
	P-Stability Analysis for SDIRK Method Using Lagrange interpolation	97
	Q-Stability Analysis for SDIRK Method Using Lagrange Interpolation	100
	Q-Stability Region of SDIRK Method Third-Order Embedded in Fourth-Order	101
	Implementation	102
	Problems Tested	108
	Numerical Results	111
	Discussion and Conclusion	118
V	SINGLY DIAGONALLY IMPLICIT RUNGE-KUTTA-NYSTRÓM METHOD (3,3) EMBEDDED IN (4,4) FOR THE INTEGRATION OF SPECIAL SECOND ORDER ORDINARY DIFFERENTIAL EQUATIONS	119
	Introduction	119
	Derivation of Embedded Singly Diagonally Implicit Runge-Kutta-Nystróm Method	122
	Stability of the Method	128



	Implementation	131
	Problems Tested	140
	Numerical Results	145
	Discussion and Conclusions	162
VI	SINGLY DIAGONALLY IMPLICIT RUNGE-KUTTA-NYSTRÓM GENERAL METHOD (3,4) EMBEDDED IN (4,5) FOR THE INTEGRATION OF SECOND ORDER ORDINARY DIFFERENTIAL EQUATIONS	
	Introduction	164
	Derivation of Embedded Singly Diagonally Implicit Runge-Kutta-Nystróm General Method	167
	Order Conditions of Runge-Kutta-Nystróm General Method	168
	Using Simplifying Assumptions	170
	Stability of the Method	174
	Implementation	177
	Problems Tested	187
	Numerical Results	192
	Discussion and Conclusions	210
VII	SUMMARY	211
	Conclusions	211
	Future Research	213
	REFERENCES	215
	APPENDICES	223
	BIODATA OF THE AUTHOR	272



LIST OF TABLES

Table		Page
3.1	The Coefficients of SDIRK Method (4,6) Embedded in (5,7)	59
3.2	Equations of Condition for orders 1 to 5	61
3.3	Equations of order condition left after using the simplifying Assumptions	68
3.4	SDIRK Method (4,6) Embedded in (5,7) with $\gamma = 0.28589$	68
3.5	SDIRK Method (3,5) Embedded in (4,6) with $\gamma = 0.25$	74
3.6	Numerical Results for Problem 3.1, Using Tolerances $10^{-2}, 10^{-4}, 10^{-6}, 10^{-8}$	87
3.7	Numerical Results for Problem 3.2, Using Tolerances $10^{-2}, 10^{-4}, 10^{-6}, 10^{-8}$	88
3.8	Numerical Results for Problem 3.3, Using Tolerances $10^{-2}, 10^{-4}, 10^{-6}, 10^{-8}$	89
3.9	Numerical Results for Problem 3.4, Using Tolerances $10^{-2}, 10^{-4}, 10^{-6}, 10^{-8}$	90
3.10	Numerical Results for Problem 3.5, Using Tolerances $10^{-2}, 10^{-4}, 10^{-6}, 10^{-8}$	91
3.11	Numerical Results for Problem 3.6, Using Tolerances $10^{-2}, 10^{-4}, 10^{-6}, 10^{-8}$	92
4.1	Numerical Results when Problem 4.1 is Solved by S1, S2 and S3 Methods	112
4.2	Numerical Results when Problem 4.2 is Solved by S1, S2 and S3 Methods	112
4.3	Numerical Results when Problem 4.3 is Solved by S1, S2 and S3 Methods	113



4.4	Numerical Results when Problem 4.4 is Solved by S1, S2 and S3 Methods	113
4.5	Numerical Results when Problem 4.5 is Solved by S1, S2 and S3 Methods	114
4.6	Numerical Results when Problem 4.6 is Solved by S1, S2 and S3 Methods	114
5.1	The coefficients of SDIRKN Method (3,3) Embedded in (4,4)	122
5.2	Equations of Condition for orders 1 to 5	124
5.3	SDIRKN Method (3,3) Embedded in (4,4) with $\gamma = 0.0021107859$	127
5.4	Numerical Results when Problem 5.1 is Solved by F1 and F2 Methods for Tolerances $10^{-2}, 10^{-4}, 10^{-6}, 10^{-8}, 10^{-10}$	147
5.5	Numerical Results when Problem 5.2 is Solved by F1 and F2 Methods for Tolerances $10^{-2}, 10^{-4}, 10^{-6}, 10^{-8}, 10^{-10}$	148
5.6	Numerical Results when Problem 5.3 is Solved by F1 and F2 Methods for Tolerances $10^{-2}, 10^{-4}, 10^{-6}, 10^{-8}, 10^{-10}$	149
5.7	Numerical Results when Problem 5.4 is Solved by F1 and F2 Methods for Tolerances $10^{-2}, 10^{-4}, 10^{-6}, 10^{-8}, 10^{-10}$	150
5.8	Numerical Results when Problem 5.5 is Solved by F1 and F2 Methods for Tolerances $10^{-2}, 10^{-4}, 10^{-6}, 10^{-8}, 10^{-10}$	151
5.9	Numerical Results when Problem 5.6 is Solved by F1 and F2 Methods for Tolerances $10^{-2}, 10^{-4}, 10^{-6}, 10^{-8}, 10^{-10}$	152
5.10	Numerical Results when Problem 5.7 is Solved by F1 and F2 Methods for Tolerances $10^{-2}, 10^{-4}, 10^{-6}, 10^{-8}, 10^{-10}$	153
5.11	Numerical Results when Problem 5.8 is Solved by F1 and F2 Methods for Tolerances $10^{-2}, 10^{-4}, 10^{-6}, 10^{-8}, 10^{-10}$	154



5.12	Numerical Results when Problem 5.9 is Solved by F1 and F2 Methods for Tolerances 10^{-2} , 10^{-4} , 10^{-6} , 10^{-8} , 10^{-10}	155
5.13	Numerical Results when Problem 5.10 is Solved by F1 and F2 Methods for Tolerances 10^{-2} , 10^{-4} , 10^{-6} , 10^{-8} , 10^{-10}	156
5.14	Total Time Taken to Solve Problems 5.1 Until 5.10 Over all Tolerances	161
6.1	The coefficients of SDIRKNG Method (3,4) Embedded in (4,5)	167
6.2	Equations of Condition for orders 1 to 4	169
6.3	Equations of order condition left for y' after using the simplifying Assumptions	172
6.4	SDIRK Method (3,4) Embedded in (4,5) with $\gamma = 0.25$	173
6.5	Numerical Results when Problem 6.1 is Solved by A1, A2, A3 and A4 Methods for Tolerances 10^{-2} , 10^{-4} , 10^{-6} , 10^{-8} , 10^{-10}	194
6.6	Numerical Results when Problem 6.2 is Solved by A1, A2, A3 and A4 Methods for Tolerances 10^{-2} , 10^{-4} , 10^{-6} , 10^{-8} , 10^{-10}	196
6.7	Numerical Results when Problem 6.3 is Solved by A1, A2, A3 and A4 Methods for Tolerances 10^{-2} , 10^{-4} , 10^{-6} , 10^{-8} , 10^{-10}	198
6.8	Numerical Results when Problem 6.4 is Solved by A1, A2, A3 and A4 Methods for Tolerances 10^{-2} , 10^{-4} , 10^{-6} , 10^{-8} , 10^{-10}	200
6.9	Numerical Results when Problem 6.5 is Solved by A1, A2, A3 and A4 Methods for Tolerances 10^{-2} , 10^{-4} , 10^{-6} , 10^{-8} , 10^{-10}	202
6.10	Numerical Results when Problem 6.6 is Solved by A1, A2, A3 and A4 Methods for Tolerances 10^{-2} , 10^{-4} , 10^{-6} , 10^{-8} , 10^{-10}	204
6.11	Numerical Results when Problem 6.7 is Solved by A1, A2, A3 and A4 Methods for Tolerances 10^{-2} , 10^{-4} , 10^{-6} , 10^{-8} , 10^{-10}	206
6.12	Total Time Taken to Solve Problems Over all Tolerances	208



LIST OF FIGURES

Figure		Page
3.1	Comparison between the Stability Region for SDIRK Method (3,5) Embedded in (4,6) and SDIRK Method (4,6) Embedded in (5,7).	75
4.1	The Comparison between the Q-Stability Region of SDIRK Method (3,5) Embedded in (4,6) and SDIRK Method (4,6) Embedded in (5,7)	102
4.2	Graphs of Problem 4.1 Show Comparison Between the S1, S2 and S3 Methods	115
4.3	Graphs of Problem 4.2 Show Comparison Between the S1, S2 and S3 Methods	115
4.4	Graphs of Problem 4.3 Show Comparison Between the S1, S2 and S3 Methods	116
4.5	Graphs of Problem 4.4 Show Comparison Between the S1, S2 and S3 Methods	116
4.6	Graphs of Problem 4.5 Show Comparison Between the S1, S2 and S3 Methods	117
4.7	Graphs of Problem 4.6 Show Comparison Between the S1, S2 and S3 Methods	117
5.1	P-Stability Region for SDIRKN Method	130
5.2	Graphs of Problem 5.1 Show Comparison between the F1 and F2 Methods	157
5.3	Graphs of Problem 5.2 Show Comparison between the F1 and F2 Methods	157
5.4	Graphs of Problem 5.3 Show Comparison between the F1 and F2 Methods	158
5.5	Graphs of Problem 5.7 Show Comparison between the F1 and F2 Methods	158



5.6	Graphs of Problem 5.8 Show Comparison between the F1 and F2 Methods	159
5.7	Graphs of Problem 5.9 Show Comparison between the F1 and F2 Methods	159
5.8	Graphs of Problem 5.10 Show Comparison between the F1 and F2 Methods	160
5.9	Graphs of Problems 5.1 until 5.10 Show Comparison between the Times for F1 and F2 Methods	162
6.1	Stability Region of SDIRKNG Method	177
6.2	Graphs of Problem 6.1 Show Comparison between the A1, A2, A3 and A4 Methods	195
6.3	Graphs of Problem 6.2 Show Comparison between the A1, A2, A3 and A4 Methods	197
6.4	Graphs of Problem 6.3 Show Comparison between the A1, A2, A3 and A4 Methods	199
6.5	Graphs of Problem 6.4 Show Comparison between the A1, A2, A3 and A4 Methods	201
6.6	Graphs of Problem 6.5 Show Comparison between the A1, A2, A3 and A4 Methods	203
6.7	Graphs of Problem 6.7 Show Comparison between the A1, A2, A3 and A4 Methods	207
6.8	Graphs for Problems 6.1 until 6.4 show Comparison Between the times for A1, A2, A3 and A4 Methods	209
6.9	Graphs for Problems 6.5 and 6.6 show Comparison Between the times for A1, A2, A3 and A4 Methods	209



LIST OF ABBREVIATIONS

IVP	Initial Value Problem
ODE	Ordinary Differential Equation
ODEs	Ordinary Differential Equations
IMEXRK	Implicit Explicit Runge-Kutta
DDE	Delay Differential Equation
DDEs	Delay Differential Equations
RDE	Retarded Delay Differential Equation
NDE	Neutral Delay Differential Equation
RKN	Runge-Kutta-Nyström
RKNG	Runge-Kutta-Nyström General
SDIRK	Singly Diagonally Implicit Runge-Kutta
DIRKN	Diagonally Implicit Runge-Kutta-Nyström
SDIRKN	Singly Diagonally Implicit Runge-Kutta-Nyström
SDIRKNG	Singly Diagonally Implicit Runge-Kutta-Nyström General
SODEs	Stiff Ordinary Differential Equations
CRK	Continuous Runge-Kutta
RKF	Runge-Kutta Fehlberg
FSAL	First Stage As Last



ABSTRACT

NUMERICAL SOLUTION OF ORDINARY AND DELAY DIFFERENTIAL EQUATIONS BY RUNGE-KUTTA METHODS

By

RAED ALI AHMED ALHASAWNEH

Chairman: Associated Professor Fudziah bt Ismail, Ph.D.

Faculty: Science.

An introduction to Runge-Kutta methods for the solution of ordinary differential equations (ODEs) is introduced. It is widely accepted that using Singly Diagonally Implicit Runge-Kutta (SDIRK) method become as an efficient technique for the integration of many stiff and non-stiff problems and at the same time, it can overcome the difficulties for using the fully implicit Runge-Kutta method and the limitations for explicit one. The derivation for embedded SDIRK method of fourth-order six stages in fifth-order seven stages is illustrated. The stability region is presented and the numerical results are compared with the other existing methods.

Introduction to delay differential equations (DDEs) and the areas where they arise are given. A brief discussion on Runge-Kutta method when adapted to delay differential equation is introduced. SDIRK method which derived in Chapter III is used to solve delay differential equations. The delay term is approximated using divided difference interpolation. Numerical results are tabulated and compared with the other existing methods. The stability properties of SDIRK method when applied to DDEs using Lagrange interpolation are investigated and their regions of stability are presented.



Runge-Kutta-Nyström (RKN) method for the solution of special second-order ordinary differential equations of the form $y'' = f(x, y)$ is described. Consequently, singly diagonally implicit Runge-Kutta Nyström (SDIRKN) of third-order three stages embedded in fourth-order four stages is constructed. The stability region of the new method is presented and numerical results are compared with the same method of lower order.

Finally, introduction to general Runge-Kutta-Nyström (RKNG) method for the solution of second-order ordinary differential equations of the form $y'' = f(x, y, y')$ is given. A new singly diagonally implicit Runge-Kutta-Nyström general (SDIRKNG) method of third-order embedded in fourth-order is derived. Analysis the stability region of the new method is discussed and numerical results are presented.



ABSTRAK

KAEDAH RUNGE-KUTTA UNTUK MENYELESAIKAN PERSAMAAN PEMBEZAAN BIASA DAN LENGAH

Oleh

RAE'D ALI AHMED ALHASAWNEH

September 2006

Pengerusi: Profesor Madya Fudziah bt Ismail, Ph.D.

Fakulti : Sains.

Pengenalan kepada kaedah Runge-Kutta untuk menyelesaikan Persamaan Pembezaan Biasa (PPB) diperkenalkan. Teknik yang menggunakan kaedah Runge-Kutta Pепенjuru Tunggal Tersirat (RKPTT) untuk kamiran PPB kaku dan tak kaku telah diterima pakai kerana kaedah RKPTT ini cekap dan sangat stabil. Seterusnya, kaedah terbenam RKPTT bagi peringkat empat tahap enam dalam peringkat lima tahap tujuh dengan ciri-ciri baris pertama bagi matriks pekalnya bersamaan dengan sifar dan baris terakhir bagi matriks pekalnya bersamaan dengan nilai vektor outputnya dibina. Rantau kestabilan bagi kaedah ini apabila digunakan ke atas PPB linear diberikan. Keputusan berangka apabila PPB kaku dan tak kaku diselesaikan menggunakan kaedah itu dibentangkan dan dibandingkan dengan kaedah yang sedang digunakan sekarang.

Pengenalan kepada Persamaan Pembezaan Lengah (PBL) dan bidang di mana ianya timbul diberikan. Perbincangan ringkas mengenai kaedah Runge-Kutta



apabila diadaptasikan kepada persamaan pembezaan lengah diperkenalkan. Kaedah RKPTT yang telah diperolehi sebelum ini digunakan untuk menyelesaikan PBL; sebutan lengahnya dianggarkan menggunakan interpolasi beza bahagi. Keputusan berangka dibentangkan dan dibandingkan dengan kaedah yang sedia ada. Aspek kestabilan kaedah RKPTT apabila digunakan ke atas PPL menggunakan interpolasi Lagrange diselidik dan rantau kestabilannya dipersembahkan.

Kaedah Runge-Kutta-Nyström (RKN) bagi penyelesaian PBB khas peringkat kedua dalam bentuk $y'' = f(x, y)$ dibincangkan. Seterusnya, kaedah peringkat tiga tahap tiga terbenam dalam kaedah peringkat empat tahap empat Runge-Kutta Nystrom Pepejuru Tunggal Tersirat (RKNPTT) dengan pekali ralat yang kecil dibina. Rantau kestabilan bagi kaedah yang baru ini dipersembahkan. Kaedah ini kemudiannya digunakan untuk menyelesaikan kedua-dua PPB kaku dan tak kaku peringkat kedua dan keputusan berangkanya menunjukkan bahawa kaedah ini lebih cekap berbanding kaedah yang sedang digunakan sekarang.

Akhir sekali, pengenalan kepada kaedah umum Runge-Kutta-Nystrom (RKNU) untuk menyelesaikan persamaan pembezaan biasa peringkat kedua dalam bentuk $y'' = f(x, y, y')$ diberikan. Kaedah umum Runge-Kutta Nystrom Pepenjuru Tunggal Tersirat (RKNUPPTT) yang baru, peringkat tiga tahap empat terbenam dalam kaedah peringkat empat tahap lima diterbitkan. Analisis bagi aspek kestabilan kaedah baru ini diberikan dan keputusan berangka apabila kaedah ini digunakan untuk menyelesaikan kedua-dua PBB umum kaku dan tak

