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Instability Analysis of the Jamuna River, Bangladesh

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ABSTRAK

Perubahan-perubahan besar yang berlaku pada topografi dasar dan planform Sungai Jamuna, Bangladesh, khususnya semasa musim banjir, dan kesan perubahan-perubahan tersebut diselidiki dalam kertas ini dalam bentuk satu analisis stabiliti oleh teknik penggangguan. Satu model pengangkutan keladak dan aliran dua dimensi di dalam sebuah sungai yang mengandungi aluvium dengan dasar yang terhakis dan tebingnya tidak terhakis dibangun dan digunakan di Sungai Jamuna, Bangladesh. Kesan gerakan berpilin disebabkan oleh kelengkungan saluran tegasan ricih dasar, dan kesan rintangan geseran akibat bentuk dasar diambil kira dalam model tersebut. Model 2-D boleh digunakan untuk memeriksa ketidakstabilan disebabkan oleh dasar mengandungi aluvium yang dilitupi bukit pasir. Satu teori kestabilan dibangunkan yang menentukan mod tak stabil, kepantasan dan rambatan bukit pasir, dan nombor gelombang sejajar dengan ketidakstabilan maksimum. Teori tersebut juga boleh membezakan antara fenomena gabungan dan berkelok-kelok. Beberapa implikasi model yang dicadangkan juga dibincangkan. Model yang dicadangkan digunakan untuk menganalisis kedua-dua paten sungai iaitu berkelok-kelok. Hasil daripada analisis Sungai Jamuna menunjukkan bahawa ketidakstabilan maksimum kerana aspek nisbahnya yang sangat rendah dan lebih daripada tiga gabungan.

ABSTRACT

Major changes take place in the planform and bed topography of the Jamuna River, Bangladesh, particularly during the flood season, and the effect of these changes are investigated in this paper in the form of a stability analysis by perturbation technique.

A two-dimensional model of flow and sediment transport in an alluvial river with erodible bed and non-erodible banks is developed and applied to the Jamuna River, Bangladesh. The effect of transverse slope resulting from the effect of secondary currents, the effect of spiral motion due to curvature of the channel on the bed shear stress, and the effect of frictional resistance occurring from pure skin friction and friction due to bedforms, are considered in the model. The 2-D model can be used to check the instability due to the dune-covered alluvial bed. A stability theory has been developed which determines the most unstable mode, the celerity and propagation of the sand dunes, and the wavenumber corresponds to maximum instability. The theory is also capable of differentiating between braiding and meandering phenomena.

Several implications of the proposed model are also discussed. The proposed model is used to analyze both the meandering and braided patterns of the river. The results from the analyses of the Jamuna River show that instability always exists in the Jamuna River under maximum instability conditions because of its very low aspect ratio (≈1/1000) and more than three braids.

Keywords: Instability, perturbation technique, dune-covered bed, 2-D model, aspect ratio

INTRODUCTION

The Jamuna River is the lowest reach of the Brahmaputra River, a large braided sand-bed river. The numbers of braids during low flow vary between 2 and 3 and the total width of the channel patterns ranges from 5 to 17 km. Leopold and Wolman (1957) made clear that the slope and discharge characterize the braiding. Consequently, the presence of a sequence of bifurcation and confluence is considered to be an essential feature for braided rivers. Sedimentation also plays an important role in developing braiding.

River channels possess three characteristic fluvial morphologies: straight, meandering and braiding. Braided rivers are defined as rivers containing more than one (low-flow) channel and bar/island in between the channels. A braided pattern probably brings to the minds of many authors, the concept of an aggrading stream. It has been observed that the banks of braided rivers are generally straighter than that of meandering rivers. Leopold and Wolman (1957) plotted bankfull discharge against channel slope and derived an equation for separation between braiding and meandering, i.e. $i = 0.0116Q_b^{-0.44}$, where Q_b is the bankfull discharge in m^3/s and i is the channel slope. A river would be braided if its slope is above the threshold slope. Parker (1976) demonstrated that sediment transport and channel frictions are the essential features for the occurrence of fluvial instability. According to him, a river would follow meandering, transition from meandering to braiding, and braiding, depending on the degree of instability and the relationship between $\frac{S}{F}$ and $\frac{d_0}{B}$, where S is the local energy slope, F is the Froude number, d_0 is the mean water depth, and B is the channel width. He confirmed that meandering streams usually have gentle slopes and rather narrow channels, while braided streams generally have steep slopes and wide channels. He introduced bed perturbations into the balance equations and concluded that instability requires $\phi_i > 0$, where ϕ_i is the imaginary part of the complex celerity, ϕ. He deduced the bed patterns corresponding to various values of number of braids m and presented that for $m = 1$, the bed pattern consisting of a single braid of submerged alternating bars characteristic of the early stages of meandering. Increased values of m imply an increased tendency towards incipient braiding, with m equaling the number of braids. Thus, a range of wavenumbers for which ϕ_i is positive, always exists and instability characteristics of meandering or braiding always occur. Engelund and Skovgaard (1973) introduced the effect of a transverse bed slope on the sediment transport and found that this effect is of great significance, because the theory predicts that the river will braid into an infinite number of branches if it is not included.

In this paper, a linear stability analysis of the Jamuna River by perturbation technique, investigation of fluvial instability, and differentiation between braiding and meandering regimes, will be presented in the following sections.

GOVERNING EQUATIONS

The flow in an alluvial channel with erodible bed and impermeable banks is considered. It is further considered that the flow is basic (undisturbed) on which a bed perturbation is superimposed. The momentum and mass balance equations in the x and y directions over this periodic bed (*Fig. 1*) are described as follows:

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=-g\frac{\partial H}{\partial x}-\frac{\tau_x}{\rho d} \tag{1}$$

$$\frac{\partial v}{\partial t}+u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}=-g\frac{\partial H}{\partial y}-\frac{\tau_y}{\rho d} \tag{2}$$

$$\frac{\partial}{\partial x}(ud)+\frac{\partial}{\partial y}(vd)+\frac{\partial d}{\partial t}=0 \tag{3}$$

$$\frac{\partial(H-d)}{\partial t}+\frac{1}{1-\lambda_p}\left(\frac{\partial q_x}{\partial x}+\frac{\partial q_y}{\partial y}\right)=0 \tag{4}$$

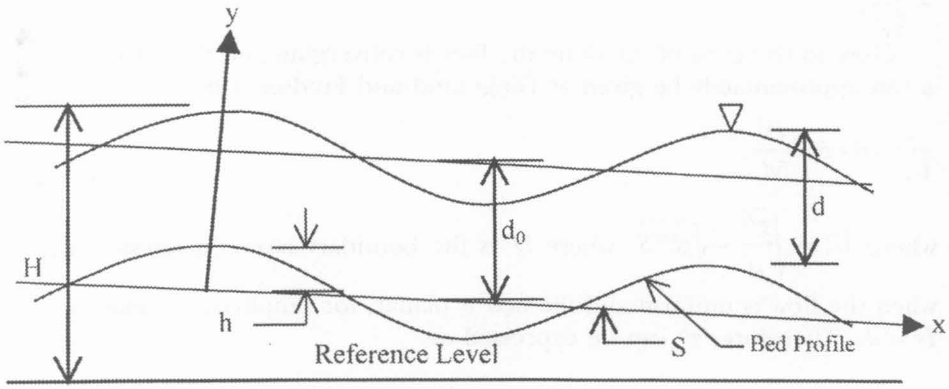


Fig. 1: Definition sketch

where u and v are the velocity components in the x and y directions, d is the local water depth, H is the water surface elevation above reference level, g is the acceleration due to gravity, ρ is the density of water, τ_x and τ_y are the local bed shear stress components in the x and y directions, q_x and q_y are the sediment transports per unit width of the channel in the x and y directions, t is time, and λ_p is the bed-material porosity. Eqns. (1), and (2) are valid if the vertical accelerations of the fluid are ignored (hydrostatic pressure), if vertical variations of the velocity are neglected and if the tractive force on the bed has the direction of the velocity vector.

Constitutive Relations

The variation of the friction factor with flow velocity in a river channel is rather complicated because different bedforms exist for different flow conditions. Perturbations about a steady flow with dune covered bed are to be considered. In this study, this effect is considered based on the drag force coefficient and the shear stress due to bedforms is considered to change with time and space.

The total bed shear stress τ is equal to the sum of pure skin friction $\tau^\wedge$ and friction due to bedforms $\tau^{\wedge\wedge}$, i.e.

$$\tau = \tau^\wedge + \tau^{\wedge\wedge} \tag{5}$$

The Shields parameter can be given by

$$\theta = \frac{U_*^2}{(s-1)gd_s} = \frac{\tau}{\rho(s-1)gd_s} \tag{6}$$

where θ is the Shields parameter, s is the relative density, d_s is the size of the sediment grain, assumed to be constant, and U_* is the friction velocity = $\sqrt{gd_0S}$ = $\sqrt{\frac{\tau}{\rho}}$.

Close to the crest of the dune the flow is converging and the mean velocity u can approximately be given as (Engelund and Fredsoe 1982)

$$\frac{u}{U_*'} = 6 + 5\ln\frac{D'}{5d_s},$$

where $U_*' = \sqrt{\frac{\tau^\wedge}{\rho}} = \sqrt{gD'S}$, where D' is the boundary layer thickness. Initially, when the flow is uniform and the bed is planar, for simplicity, we may assume $D' = d$. Therefore, $\tau^\wedge$ can be expressed as

$$\tau^\wedge = \rho(U_*')^2 = \rho\left(\frac{u}{6 + 2.5\ln\frac{d}{2.5d_s}}\right)^2 = f(u,d) \tag{7}$$

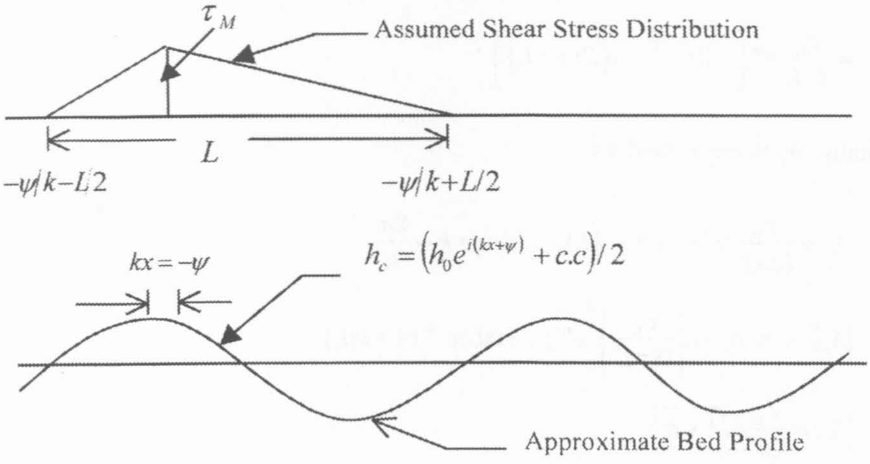


Fig. 2: Approximate bed profile and assumed shear stress distribution

For simplicity, the shear stress distribution can be approximated by *Fig. 2* and its peak is assumed to occur at lag $kx = -\psi$ from the peak of the bedform. The maximum stress generally occurs at flow separation point and is assumed to be τ_M. In *Fig. 2*, c.c is the abbreviation of complex conjugate. Applying Fourier series expansion, $\tau^{\wedge\wedge}$ can be expressed as

$$\tau^{\wedge\wedge} = \bar{\tau}^{\wedge\wedge} + A_\tau e^{ikx} + A_\tau^* e^{-ikx}$$

where $\bar{\tau}^{\wedge\wedge}$ is the average unit shear stress due to the friction of the bedforms, A_τ is the amplitude of τ, and $A_\tau^* e^{-ikx}$ is the complex conjugate of $A_\tau e^{ikx}$.

The total drag is given by

$$\int_{-\frac{\psi}{k}-\frac{L}{2}}^{-\frac{\psi}{k}+\frac{L}{2}} \tau^{\wedge\wedge} dx B = LB\bar{\tau}^{\wedge\wedge} = \frac{L}{2}\frac{\tau_M}{2}B$$

$$\therefore \bar{\tau}^{\wedge\wedge} = \frac{\tau_M}{4} = \frac{\rho}{2L} C_D u|u||h_0|$$

where τ_M is the maximum amplitude of shear stress, C_D is the drag coefficient, B is the channel width, and L is the dune wavelength. In order to obtain the amplitude A_τ, the integration is considered as follows:

$$\int_{-\frac{\psi}{k}-\frac{L}{2}}^{-\frac{\psi}{k}+\frac{L}{2}} \tau^{\wedge\wedge} e^{-ikx} dx = LA_\tau = \int_{-\frac{\psi}{k}-\frac{L}{2}}^{-\frac{\psi}{k}+\frac{L}{2}} \{\tau_M - \frac{\tau_M}{L/2}\left(x + \frac{\psi}{k}\right) e^{-ikx}\} dx$$

$$= \frac{\tau_M}{k^2 L} e^{i\psi} \left(-2e^{-\frac{ikL}{2}} - i\{2i + kL\} \right)$$

Finally, A_τ is expressed as

$$A_\tau = \frac{\tau_M}{(2\pi)^2} e^{i\psi} (2 + 2 - ikL), \qquad \text{since } k = \frac{2\pi}{L}$$

$$|A_\tau|^2 = A_\tau A_\tau^* = \left\{ \frac{\tau_M}{(2\pi)^2} \right\}^2 e^{i\psi} (4 - ikL) e^{-i\psi} (4 + ikL)$$

$$|A_\tau| = \frac{\tau_M}{2\pi^2} \sqrt{4 + \pi^2}$$

$$\therefore \tau^{\wedge\wedge} = \tau^{\bar{\wedge}\wedge} + A_\tau e^{ikx} + A_\tau^* e^{-ikx}$$

$$= \tau^{\bar{\wedge}\wedge} + \frac{2\tau_M}{\pi^2} \cos(kx + \psi) \tag{8}$$

To express $\tau^{\wedge\wedge}$ by bed profile, the following expression is considered:

$$e^{ikx} A_\tau = \frac{\tau_m}{(2\pi)^2} (4 - ikL) e^{i(kx+\psi)} = \frac{2\rho}{L} C_D u|u| |h_0| \left(\frac{4 - ikL}{(2\pi)^2} \right) e^{i(kx+\psi)}$$

$$= \frac{2\rho}{L} C_D u|u| \left(\frac{4 - ikL}{(2\pi)^2} \right) h_c, \ \textit{since } h_c = |h_0| e^{i(kx+\psi)}$$

and $$e^{-ikx} A_\tau^* = \frac{\tau_M}{(2\pi)^2} (4 - ikL) \bar{e}^{i(kx+\psi)} = \frac{2\rho}{L} C_D u|u| \, |h_0| \left(\frac{4 + ikL}{(2\pi)^2} \right) e^{-i(kx+\psi)}$$

$$= \frac{2\rho}{L} C_D u|u| \left(\frac{4 + ikL}{(2\pi)^2} \right) h_c^*,$$

where, $h_c^* = |h_0| e^{-i(kx+\psi)}$ is the complex conjugate of h_c.

Finally, $\tau^{\wedge\wedge}$ can be expressed as,

$$\tau^{\wedge\wedge} = \frac{\rho}{2L} C_D u|u||h_0| + \frac{2\rho}{\pi^2 L} C_D u|u| (h_c + h_c^*) + \frac{i\rho}{\pi L} C_D u|u| (h_c^* - h_c) \tag{9}$$

$$\tau = \rho\left(\frac{u}{6 + 2.5\ln\left(\frac{d}{2.5d_s}\right)}\right)^2 + \frac{\rho}{2L}C_D u|u||h_0| + \frac{2\rho}{\pi^2 L}C_D u|u|(h_c + h_c^*)$$
$$+ \frac{i\rho}{\pi L}C_D u|u|(h_c^* - h_c)$$

The Chezy's friction coefficient F_c can be expressed as

$$F_C = \left(\frac{1}{6 + 2.5\ln\left(\frac{d}{2.5d_s}\right)}\right)^2 + \frac{1}{2L}C_D|h_0| + \frac{2}{\pi^2 L}C_D(h_c + h_c^*)$$
$$+ \frac{i}{\pi L}C_D(h_c^* - h_c) \tag{10}$$

Since $\tau = \rho F_C u|u|$

The real component of the bed height fluctuation can be given as

$$h_c = |h_0|\cos(kx + \psi)$$

The most general possible constitutive relation for Chezy's resistance can be of the form:

$$F_C = \phi_c(u, v, d, h_c, h_c^*, L) \tag{11}$$

The total sediment transport is traditionally divided into (i) bed-material load, consisting of bed load and suspended load and (ii) wash load. Ignoring wash load, the total sediment transport can be expressed in dimensionless form as

$$\phi = \frac{q}{\sqrt{(s-1)gd_s^3}} \tag{12}$$

Engelund and Hansen (1967) established a relation between the dimensionless transport parameter ϕ and the dimensionless flow parameter θ as follows:

$$F_C\phi = 0.05\theta^{5/2} \tag{13}$$

where $\phi = \dfrac{q}{\sqrt{\Delta g d_s^3}}$ (14)

where Δ is the relative density difference, and

$$\theta = \frac{\tau}{(\rho_s - \rho) g d_s} = \frac{\tau}{\rho \Delta g d_s}$$

$$= \frac{1}{\rho \Delta g d_s} \left[\begin{array}{l} \rho \left(\dfrac{u}{6 + 2.5 \ln\left(\dfrac{d}{2.5 d_s} \right)} \right)^2 + \dfrac{\rho}{2L} C_D u |u| |h_0| \\ + \dfrac{2\rho}{\pi^2 L} C_D u |u| (h_c + h_c^*) + \dfrac{i\rho}{\pi L} C_D u |u| (h_c^* - h_c) \end{array} \right]$$

Rearranging terms and simplifying, ϕ can be expressed as

$$\phi = \frac{0.05}{F_c} \left(\frac{1}{\rho \Delta g d_s} \left[\begin{array}{l} \rho \left(\dfrac{u}{6 + 2.5 \ln\left(\dfrac{d}{2.5 d_s} \right)} \right)^2 + \dfrac{\rho}{2L} C_D u |u| |h_0| \\ + \dfrac{4\rho}{\pi^2 L} C_D u |u| (h_c + h_c^*) + \dfrac{2i\rho}{\pi L} C_D u |u| (h_c^* - h_c) \end{array} \right] \right)^{\frac{5}{2}} \tag{15}$$

Combining Eqns. (14), and (15), one can obtain

$$q = \frac{K}{F_c} \left(\left[\begin{array}{l} \rho \left(\dfrac{u}{6 + 2.5 \ln\left(\dfrac{d}{2.5 d_s} \right)} \right)^2 + \dfrac{\rho}{2L} C_D u |u| |h_0| \\ + \dfrac{4\rho}{\pi^2 L} C_D u |u| (h_c + h_c^*) + \dfrac{2i\rho}{\pi L} C_D u |u| (h_c^* - h_c) \end{array} \right] \right)^{\frac{5}{2}} \tag{16}$$

where $K = \dfrac{0.05}{\rho^{\frac{5}{2}} \Delta^2 g^2 d_s}$.

Influence of Transverse Slope on Sediment Transport

Let us consider the local direction of sediment transport with an average direction of particles trajectory that deviates an angle δ from the direction of average shear stresses under the action of gravity. Thus, one can write

$$q^* = \left(q_x, q_y\right) = (\cos\ \delta,\ \sin\ \delta)q$$

where q^* is the sediment transport parameter.

$$q_x = \frac{K}{F_C}\left(\left[\rho\left(\frac{u}{6 + 2.5\ln\left(\frac{d}{2.5d_s}\right)}\right)^2 + \frac{\rho}{2L}C_D u|u||h_0| + \frac{4\rho}{\pi^2 L}C_D u|u|\left(h_c + h_c^*\right) + \frac{2i\rho}{\pi L}C_D u|u|\left(h_c^* - h_c\right)\right]\right)^{\frac{5}{2}} \cos\ \delta, \text{ and}$$

$$q_y = \frac{K}{F_C}\left(\left[\rho\left(\frac{u}{6 + 2.5\ln\left(\frac{d}{2.5d_s}\right)}\right)^2 + \frac{\rho}{2L}C_D u|u||h_0| + \frac{4\rho}{\pi^2 L}C_D u|u|\left(h_c + h_c^*\right) + \frac{2i\rho}{\pi L}C_D u|u|\left(h_c^* - h_c\right)\right]\right)^{\frac{5}{2}} sin\ \delta$$

For relatively small values of δ, the following formula can be adapted following the equation of Engelund (1981):

$$\sin\ \delta = v\left(u^2 + v^2\right)^{-\frac{1}{2}} - \frac{r}{B\theta^{1/2}}\frac{\partial}{\partial y}(H - d)$$

where r is a constant which Engelund (1981) suggested to assume the value = (0.5-0.6). In accordance with Olesen's (1983) results, lower value of r (around 0.3) may lead to more satisfactory prediction of alternate-bar formation. Hence, the most general possible constitutive relation for sediment transport can be of the form:

$$q^* = \phi_q\left(u, d, L, h_c, \delta\right) \tag{17}$$

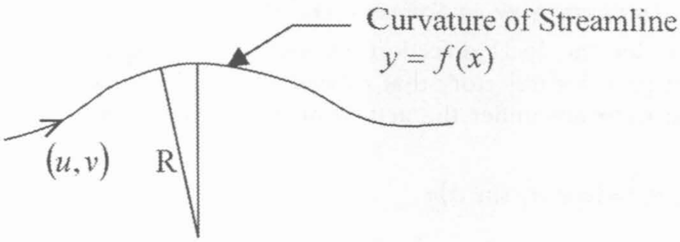


Fig. 3: Sketch of assumed curvature of streamline

The Effect of Secondary Flow Due to Curvature of River Bank

The magnitude of the secondary flow effect on the bed shear stress can be approximated as $\frac{A_1 d_0}{R}$, where A_1 is a constant, and R is the radius of curvature. R can be approximated as

$$\frac{1}{R} \cong f^{11} = \frac{d}{dx}\left(\frac{v}{u}\right) = \frac{\frac{dv}{dx}u - v\frac{du}{dx}}{u^2}$$

Now, $\frac{dv}{dx} = \frac{\partial v}{\partial x} + \frac{\partial v}{\partial y}\frac{\partial y}{\partial x} = \frac{\partial v}{\partial x} + \frac{v}{u}\frac{\partial v}{\partial y}$

Similarly, $\frac{du}{dx} = \frac{\partial u}{\partial x} + \frac{v}{u}\frac{\partial u}{\partial y}$

$$\therefore \frac{1}{R} = \frac{1}{u^2}\left(u\frac{\partial v}{\partial x} - v\frac{\partial u}{\partial x}\right)$$

Since $u = U + u'$, and $v = v'$

$$\frac{1}{R} \cong \frac{1}{U^2} = \left(U\frac{\partial v'}{\partial x}\right) \cong \frac{1}{U}\frac{\partial v'}{\partial x} \tag{18}$$

The Perturbed Equations of Motion

The balance equations are considered for slight perturbations about steady uniform flow, i.e.

$u = U + \varepsilon u'(t, x, y),$ $\quad v = \varepsilon v'(t, x, y),$

$d = d_0 + \varepsilon d'(t, x, y),$ $\quad h_c = 2\varepsilon h'(t, x, y),$

$$|u| = \sqrt{u^2 + v^2}, \qquad H = d_0 - Sx + \varepsilon h'(t, x, y) + \varepsilon d'(t, x, y),$$

$$F_C = \left(\frac{1}{6 + 2.5\ln\left(\frac{d}{2.5d_s}\right)} \right)^2 + \frac{1}{2L} C_D |h_0| + \frac{2}{\pi^2 L} C_D \left(h_c + h_c^*\right) + \frac{i}{\pi L} C_D \left(h_c^* - h_c\right),$$

$$\tau_x = \rho F_C \left(\sqrt{u^2 + v^2}\right) u, \qquad \tau_y = \rho F_C \left(\sqrt{u^2 + v^2}\right) v,$$

$$\theta = \frac{\tau_x}{\rho \Delta g d_s}, \qquad q = \frac{K \tau_x^{\frac{5}{2}}}{F_c},$$

$$q_x = q \cos \delta, \qquad q_y = q \sin \delta,$$

$$\delta = \varepsilon \delta'(t, x, y),$$

$$\delta'(t, x, y) = Lim_{\varepsilon \to 0} \frac{\partial}{\partial \varepsilon} \left[v\left(u^2 + v^2\right)^{-\frac{1}{2}} - \frac{r}{B\theta^{\frac{1}{2}}} \frac{\partial}{\partial y}(H - d) - \frac{A_1 d_0}{U} \frac{\partial}{\partial x} v'(t, x, y) \right]$$

where $\frac{\left(u'^2 + v'^2\right)^{\frac{1}{2}}}{U} << 1$. In the equation of $\delta'(t, x, y)$, the last term $\frac{A_1 d_0}{U} \frac{\partial}{\partial x} v'(t, x, y)$ is the effect of spiral motion on the bed shear stress due to the effect of the curvature of the river banks and the first two terms are due to the effect of the transverse slope of the sediment transport as explained before. If these perturbations are inserted into the steady uniform flow and put $\varepsilon \to 0$, zero-order perturbed equations will be obtained. Similarly, if these perturbations are inserted into the steady uniform flow, thereafter differentiate with respect to ε and set $\varepsilon \to 0$ gives first-order perturbed equations.

Stability of Spatially Periodic Disturbance

In this section, the stability of the river meandering and braiding will be discussed. To conduct a stability analysis, the following generalized sinusoidal perturbations are introduced into the balance equations:

$$\left.\begin{aligned} u' &= \hat{u}(y) \exp i\left(kx - \phi t\right)/2 \qquad & v' &= \hat{v}(y) \exp i\left(kx - \phi t\right)/2 \\ h' &= \hat{h}(y) \exp i\left(kx - \phi t\right)/2 \qquad & d' &= \hat{d}(y) \exp i\left(kx - \phi t\right)/2 \end{aligned}\right\} \quad (19)$$

Here, k is the dimensionless fluvial instability wavenumber, related to dimensional wavelength L by the relation $k = 2\pi/L$, and ϕ is the complex celerity, its real part

ϕ_r being related to the disturbance wave speed c by $c = \phi_r/L$, and its imaginary part ϕ_i being the temporal exponential growth rate. Instability requires that $\phi_r > 0$.

These perturbations will be introduced into the balance equations and are reduced with the aid of the boundary condition of impermeable banks; i.e. $v = 0$ at $y = \pm B/2$. The sidewalls will suppress the growth of the lateral disturbance.

As a result, for odd number of braids (m = 1, 3, 5, ...), it is found that $\widehat{u}, \widehat{v}, \widehat{d}, \widehat{h}$ can take the following forms:

$$\widehat{u}(y) = \widehat{u} \sin(k_B y), \qquad \widehat{v}(y) = \widehat{v} \cos(k_B y),$$
$$\widehat{d}(y) = \widehat{d} \sin(k_B y), \qquad \widehat{h}(y) = \widehat{h} \sin(k_B y)$$

In these equations, $k_B = m\pi/B$, and m is the number of braids. For $m = 1$, the bed pattern consists of a single braid of submerged alternating bars, i.e., the case of meandering river. Increasing values of m imply an increased tendency towards incipient braiding and increased instability.

After inserting sinusoidal perturbations and their forms into the basic equations, the resulting nonlinear algebraic equation (Eigen equation) for the complex celerity ϕ is obtained as follows:

The complex celerity ϕ must satisfy the dispersion equation (Eigen equation), i.e., Eqn. 20.

$$0.0625\phi^4 + \frac{10678.8k^2Km^2U^5}{B^3} + \frac{109375ik^3Km^2U^5}{B^3} + \frac{210791Km^4U^5}{B^5}$$
$$+ \frac{1.07949\times10^6 ikKm^4U^5}{B^5} + 570.457ik^3KU^6 + (163489 + 107800i)k^4KU^6$$
$$+(-1.10412\times10^6 + 1.73435\times10^6 i)k^5KU^6 - \frac{2321.96ikKm^2U^6}{B^2}$$
$$+\frac{(-1.12784\times10^6 - 709297i)k^2Km^2U^6}{B^2}$$
$$+\frac{(-1.08972\times10^7 + 1.71173\times10^7 i)k^3Km^2U^6}{B^2} + \frac{30.9806ikKm^2U^7}{B^3}$$
$$-\frac{417.64k^2Km^2U^7}{B^3} - \frac{1027.59ik^3Km^2U^7}{B^3} + (32.9612 - 51.7753i)k^3KU^8$$
$$+(-1060.59 - 675.195i)k^4KU^8 + (10373.3 - 16294i)k^5KU^8$$
$$+\phi\left(0.0183065iU - 0.1875kU + \frac{1027.59iKm^2U^4}{B^3} + (-10373.3 + 16294.3i)k^2KU^5\right)$$

$$+\phi^2\begin{pmatrix}-6.65241k^2-\dfrac{65.6566m^2}{B^2}-0.00119156U^2-0.0437082ikU^2+0.1875k^2U^2\\-5570.8k^2KU^4-\dfrac{10996.3Km^2U^4}{B^2}-\dfrac{300.985Km^2U^5}{B^3}-\dfrac{3082.77ikKm^2U^5}{B^3}\\+(530.297+337.598i)k^2KU^6+(31119.9-48883i)k^3KU^6\\\end{pmatrix}$$

$$+\phi\begin{pmatrix}-0.649505ik^2U+6.6524k^3U-\dfrac{12.8207im^2U}{B^2}+\dfrac{65.6566km^2U}{B^2}\\+0.0018843kU^3+0.0254017ik^2U^3-0.0625k^3U^3-\dfrac{109375ik^2Km^2U^4}{B^3}\\-\dfrac{1.07949\times10^6iKm^4U^4}{B^5}-543.903ik^2KU^5+11413.6k^3KU^5\\+(1.10412\times10^6-1.73435\times10^{6i})k^4KU^5-\dfrac{2147.24iKm^2U^5}{B^2}\\+\dfrac{24676.9kKm^2U^5}{B^2}+\dfrac{(1.08972\times10^7-1.71173\times10^7i)k^2Km^2U^5}{B^2}\\-\dfrac{19.591iKm^2U^6}{B^3}+\dfrac{718.625kKm^2U^6}{B^3}+\dfrac{3082.77ik^2Km^2U^6}{B^3}\\(-131.845+207.101i)k^2KU^7+(530.297+337.598i)k^3KU^7\\+(-31119.9+48883i)k^4KU^7\end{pmatrix}$$

$$=0 \qquad (20)$$

Eqn. (20) contains four Eigen values for ϕ. The number of parameters is large, and a general solution for ϕ is tedious. Here asymptotic expansions in the small parameter K are considered, i.e. $\phi = \phi_0 + K\phi_1 + K_2\phi_2 + \ldots$. To investigate instability, it is necessary to obtain the Eigen values of this polynomial equation for the complex celerity. We need to distinguish the physical phenomena, i.e. free surface displacement, velocity disturbance, and bed profile change, involved in river morphology. In order to do this, one needs to consider the following cases:

(i) Dune celerity c computed by $\dfrac{\mathrm{Re}[\phi]}{k}$, (ii) Wavenumber k, and (iii) Amplification rate $\mathrm{Im}[\phi]$.

In order to study river morphology, the last Eigen value will be taken into consideration and the flow is assumed to be quasi-steady. The equation for the lowest-order term in the expansion for the fourth root (proportional to sediment transport), vanishing as $K \to 0$, has implications to the morphological processes and can be written in the form:

$$\phi_4 \begin{pmatrix} -0.00129901ik^2U + 13.3048k^3U - 2.04887 \times 10^{-10} im^2U \\ +1.04926 \times 10^{-6} km^2U + \left(-3.93608 \times 10^{-7} + 0.000051422i\right)k^2U^3 \\ +(-0.131333 - 0.0040314i)k^3U^3 \end{pmatrix}$$

$$+K \begin{pmatrix} 3.50675 \times 10^{-13} k^2 m^2 U^5 + 3.5917 \times 10^{-9} ik^3 m^2 U^5 + 5.53104 \times 10^{-20} m^4 U^5 \\ +2.83252 \times 10^{-16} ikm^4 U^5 + 0.00124612ik^3 U^6 - 12.7631k^4 U^6 \\ -3.33932 \times 10^{-11} ikm^2 U^6 - 2.61896 \times 10^{-7} k^2 m^2 U^6 \\ +\left(1.79261 \times 10^{-16} - 1.06257 \times 1.06257 \times 10^{-16} i\right)k^2 m^2 U^7 \\ +\left(1.08831 \times 10^{-12} - 1.83604 \times 10^{-12} i\right)k^3 m^2 U^7 - 1.94852 \times 10^{-24} m^4 U^7 \\ -9.97864 \times 10^{-21} ikm^4 U^7 + \left(-1.74585 \times 10^{-7} + 2.74237 \times 10^{-7} i\right)k^3 U^8 \\ +(-0.00561762 - 0.00357629i)k^4 U^8 + (18.3147 - 28.7686i)k^5 U^8 \\ +9.55814 \times 10^{-21} ikm^2 U^9 + \left(-1.28845 \times 10^{-16} + 3.7433 \times 10^{-21} i\right)k^2 m^2 U^9 \\ +\left(-3.83399 \times 10^{-17} - 3.16972 \times 10^{-13} i\right)k^3 m^2 U^9 \end{pmatrix}$$

$$=0 \qquad (21)$$

Properties of the Total Load-Dependent Solution

The imaginary part $\mathrm{Im}[\phi_4]$ of the solution to Eqn. (21) is:

$$
\frac{2.7431\times10^{-8}K\left(\left(\left(\begin{array}{l}6.65241k^3U+\dfrac{65.6566km^2U}{B^2}+\\0.0018843kU^3-0.0625k^3U^3\end{array}\right)-\left(\begin{array}{l}\dfrac{109375k^3m^2U^5}{B^3}+\dfrac{1.07949\times10^6km^4U^5}{B^3}\\+570.457k^3U^6+107800k^4U^6+1.73435\times10^6k^5U^6\\-\dfrac{2321.96km^2U^6}{B^2}-\dfrac{709297k^2m^2U^6}{B^2}+\dfrac{1.71173\times10^7k^3m^2U^6}{B^2}\\+\dfrac{30.9806km^2U^7}{B^3}-\dfrac{1027.59k^3m^2U^7}{B^3}-51.7753k^3U^8\\-675.795k^4U^8-16294.3k^5U^8\end{array}\right)\right)+\left(-0.649505k^2U-\dfrac{12.8207m^2U}{B^2}+0.0254017k^2U^3\right)\left(\begin{array}{l}\dfrac{10678.8k^2m^2U^5}{B^3}+\dfrac{210791m^4U^5}{B^5}+163489k^4U^6\\-1.10412\times10^6k^5U^6-\dfrac{1.12784\times10^6k^2m^2U^6}{B^2}\\-\dfrac{1.08972\times10^7k^3m^2U^2}{B^2}-\dfrac{417.64k^2m^2U^7}{B^3}\\+32.9612k^3U^8-1060.59k^4U^8+10373.3k^5U^8\end{array}\right)\right)}{\left(\left(-0.649505k^2U-\dfrac{12.8207m^2U}{B^2}+0.0254017k^2U^3\right)^2+\left(\begin{array}{l}6.65241k^3U+\dfrac{65.6566km^2U}{B^2}+0.0018843kU^3\\-0.0625k^3U^3\end{array}\right)^2\right)}
\tag{22}
$$

In order to find the instability wavelength that can be expected to be one at which $\mathrm{Im}[\phi_4]$ is positive and maximum, i.e.,

$$
\frac{d\,\mathrm{Im}[\phi_4]}{dk}=0 \tag{23}
$$

We need to check the number of positive roots, which might be related to physical phenomena, such as, ripples, dunes or antidunes, mega-dunes and sand-bars or islands. If we consider the largest scale, the smallest value of k will be investigated. Under the indicated restrictions, a range of wavenumbers for which $\mathrm{Im}[\phi_4]$ are positive always exist and instability characteristic of meandering or braiding always occurs.

After incorporating the real and maximum positive root of k which is an implicit function of the number of braids m in Eqn. (22), we can find the maximum instability that is also a function of m. Let us assume one value of m, say $m = 1$, and calculate $\mathrm{Im}[\phi_4]$. Consider again higher values of m, say $m = 3$, and calculate $\mathrm{Im}[\phi_4]$. Comparing two values of $\mathrm{Im}[\phi_4]$, one can differentiate whether the river is braided or meandering corresponds to maximum instability.

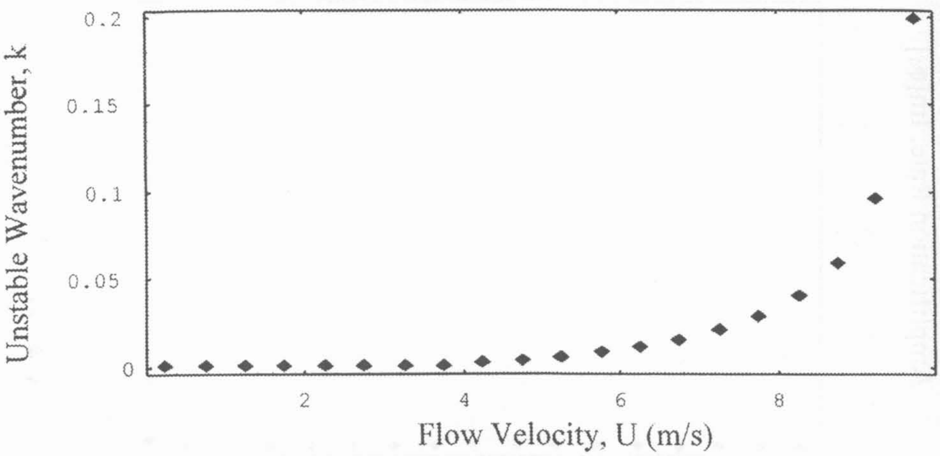
RESULTS AND DISCUSSION

The agreement between the theoretical stability analysis and the observation does not necessarily mean that the model is in accordance with the actual mechanism of instability. Some numerical examples on the stability of alluvial rivers will be discussed in order to figure out the extent to which the 2-D mathematical model describes the features known from other authors' works or from the nature. The effect of secondary currents on sediment transport, the effects of spiral motion and drag force on the bed shear stress are considered in the present model. The flow is treated as unsteady, the dunes will change their forms with time, because the dune dimensions vary with the hydraulic conditions. If one neglects C_d, the effect of spiral motion, and the transverse slope δ, the imaginary part of the fourth root of the Eigen equation, e.g., $\mathrm{Im}[\phi_4]$ will be similar with respect to power of the wavenumber k of the equation obtained by Parker (1976).

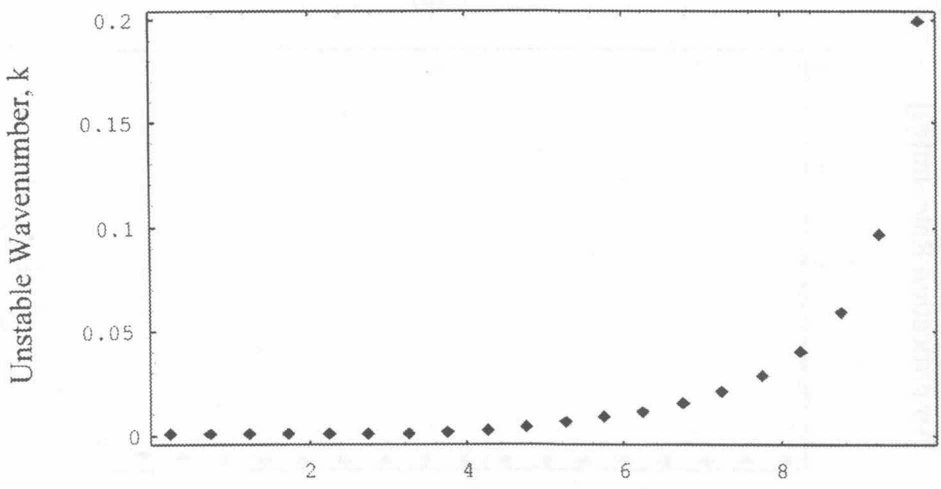
It is needed to choose some representative data on Jamuna River. In order to reduce the model parameters, some numerical values are taken into considerations, which will also be of great importance in reducing the memory needed by the computer during programming. Such numerical values are the fluid density, $\rho = 1000$ kg/m^3; the representative grain size, $d_s = 0.22 \times 10^{-3} m$; the depth-averaged water depth, $d_0 = 10.85$ m (PWD); the width of the river at J 6-1, $B = 11187.00$ m; the porosity of the sediment grains, $\lambda p = 0.4$; the relative density difference, $\Delta = 1.65$; the Von Karman constant, $k_1 = 0.4$; $r = 0.5$ (the constant appeared in the evaluation of transverse slope); and the acceleration due to gravity, $g = 9.81$ m/s^2.

The imaginary part of ϕ of the fourth root of the Eigen equation , i.e., $\mathrm{Im}[\phi_4]$, the amplification factor, has been evaluated as the functions of the number of braids m, the channel width B, the uniform flow velocity U, and the wavenumber k.

As instability is observed to occur at coherent finite wavelengths, the observed instability wavelength can be expected to be one at which $\mathrm{Im}[\phi_4]$ is positive and a maximum. If the roots of Eqn. (23) are to represent the characteristic meander or braid wavelengths, they must be real and correspond to positive, maximum instability for maximum real and positive root among all the roots of k. The unstable wavenumber against flow velocity for maximum instability condition is shown in *Fig. 4* for $m = 3$ and $m = 1$, respectively. It is seen



(a)



(b)

Fig. 4. Variations of unstable wavenumbers with flow velocity under maximum instability condition for (a) m = 3 and (b) m = 1

Parameter values considered are: $k_B = m\pi/B$; $\rho = 1000$ kg/m^3; $\lambda_p = 0.4$; $\Delta = 1.65$; $r = 0.5$; $g = 9.81$ m/s^2; $L = 2\pi/k$ m; $C_d = 1$; $k_1 = 0.4$; $d_0 = 10.85$ m; $d_s = 0.22*10^{-3}$ m; and B = 11187 m that the wavenumber exponentially increases with the increment of the flow velocity for both the cases. Although it is difficult to guess, the maximum instability is found to occur in the case of meandering stream (*Fig. 5*). The

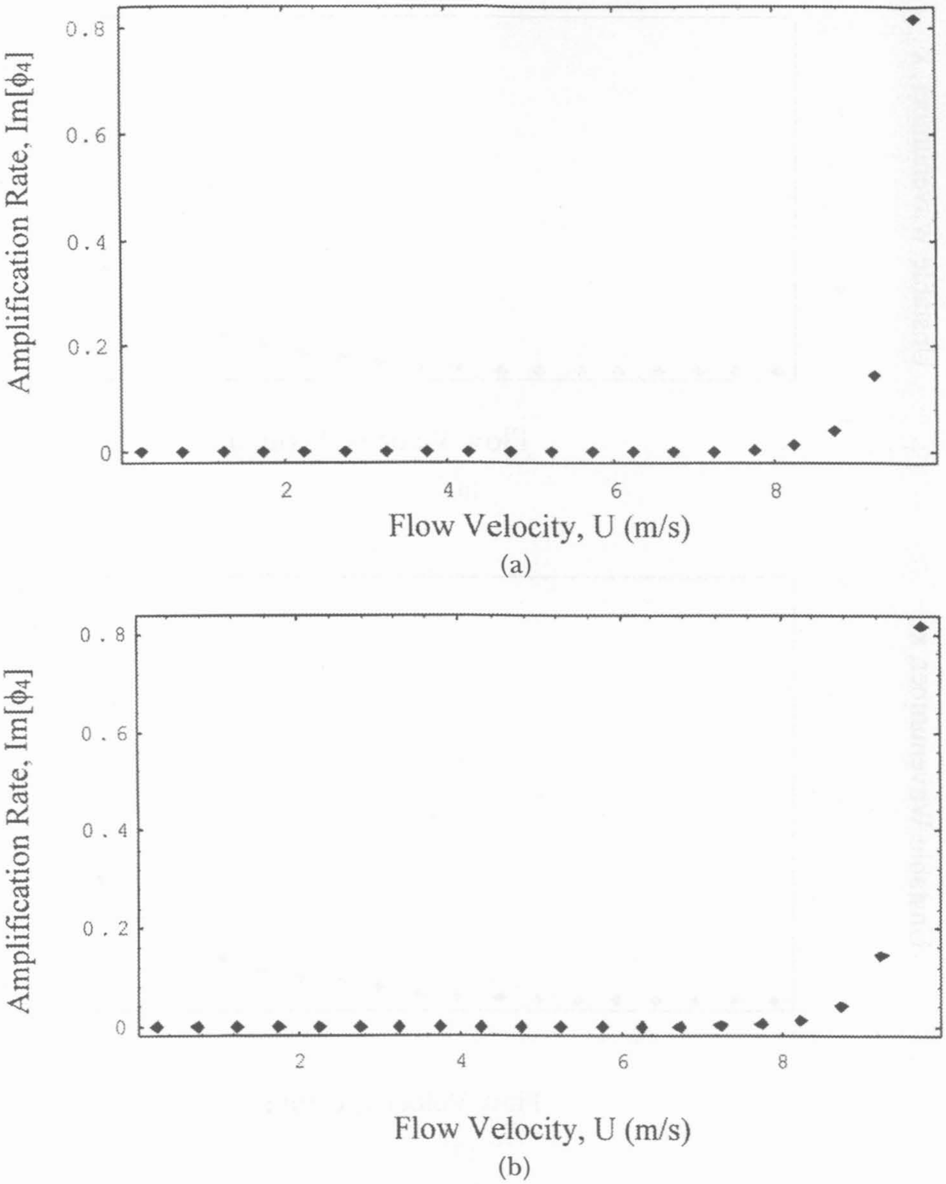


Fig. 5. Amplification factor against flow velocity under maximum instability condition for (a) m = 3 and (b) m = 1

Parameter values considered are: $k_B = m\pi/B$; ρ = 1000 kg/m^3; λ_p = 0.4; Δ = 1.65; r = 0.5; g = 9.81 m/s^2; $L = 2\pi/k$ m; C_d = 1; k_1 = 0.4; d_0 = 10.85 m; d_s = 0.22*10^{-3} m; and B = 11187 m amplification rate and the unstable wavenumber are directly proportional to the flow velocity. It is also found that Im[ϕ_4] always takes positive values, meaning there is no critical velocity for which instability appears. This could be due to the very large width of the Jamuna River and its very small aspect ratio (*Depth/Width*), 1/1000. Thus, to predict the stability is rather difficult and instability always exists in the Jamuna River.

Fig. 6 shows the amplification rate with the number of braids for the flow velocities of 9.9 m/s (solid line) and 10 m/s (dotted line), respectively. The dune wavelength is considered as 350m, which is the maximum possible wavelength in the Jamuna River (River Survey Project Bangladesh, Special Report No. 9, July 1996). Such large velocities are chosen in order to observe the variation more clearly, although these high velocities are rare, especially in the case of Jamuna River. The flow velocity seems to be the controlling factor in the calculation of Im[ϕ_4]. From this figure, it is found that Im[ϕ_4] increases with the increment of *m*, from which one can conclude that instability increases towards incipient braiding and this result is found to be coincident with that of the findings of Parker (1976). Appropriate velocity data as well as different dune wavelengths need to be considered in order to investigate the amplification rate properly.

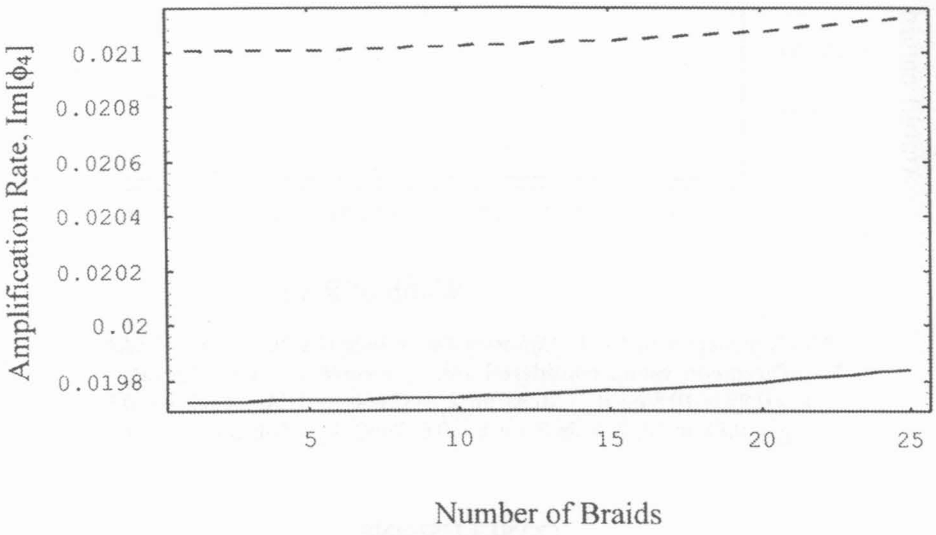


Fig. 6. Variation in the Amplification Factor with the Number of Braids
Parameter values considered are: $k_B = m\pi/B$; $\rho = 1000$ kg/m^3; $d_s = 0.22 \times 10^{-3}$ m; $\lambda_p = 0.4$; $\Delta = 1.65$; r = 0.5; g = 9.81 m/s^2; $L = 2\pi/k$ m; $C_d = 1$; $k_1 = 0.4$; k = 0.017952; and B = 11187 m

Referring to Eqn. (22), the effect of river width B on the amplification rate is shown in *Fig. 7*. The amplification rate increases towards increasing B, but its variation is almost negligible. It can be said that the larger the channel width, the larger will be the aspect ratio, which could be the case of braided stream. If the dotted line is plotted separately, it can be guessed that $Im[\phi_4]$ is more pronounced up to a certain value of B; thereafter its increment is quite negligible, indicating instability is on the state of nearly constant. The authors have tried to find the peak of the instability, but no peak is found. However, critical value could be between these two values to initiate instability.

The aspect ratio of the Jamuna River is very low (1/1000), which means that the instability always exists under maximum instability conditions and the prototype experiment is rather difficult to perform. Also, this fact indicates that it might have many braids, i.e., the number of braids will be more than 3.

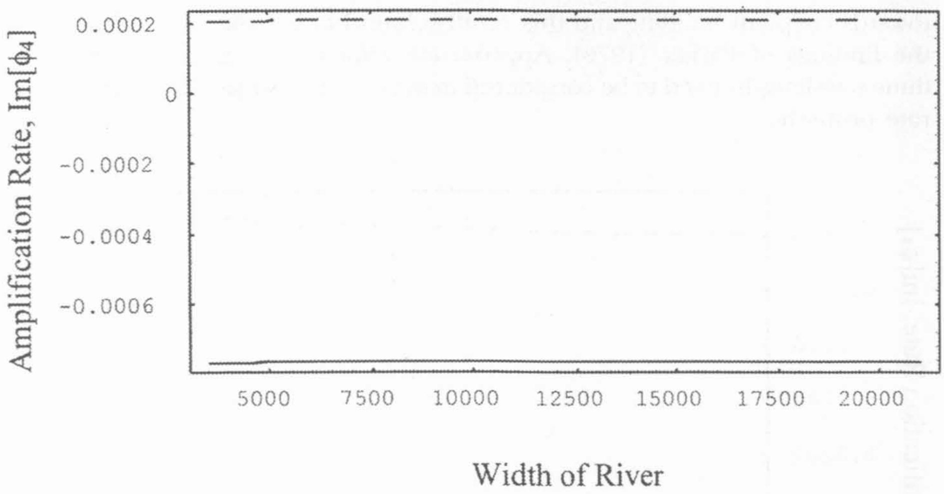


Fig. 7. Variation in the Amplification Factor Im[φ4] with the River Width B
Parameter values considered are: $k_B = m\pi/B$; $\rho = 1000$ kg/m^3; $d_s = 0.22 \times 10^{-3}$ m; $d_0 = 10.85$ m; $\lambda_p = 0.4$; $C_d = 1$; $\Delta = 1.65$; $r = 0.5$; $g = 9.81$ m/s^2; $L = 2\pi/k$ m; $k = 0.017952$; $k_1 = 0.4$; and $m = 3$

CONCLUSIONS

The developed 2-D model appears to explain some physical features satisfactorily: instability increases towards incipient braiding; instability is proportional to the width-depth ratio (aspect ratio); and the instability wavelength corresponds to maximum instability. A stability theory has been developed by applying perturbation technique which determines the real and positive roots of k corresponding to maximum instability and hence the maximum instability wavelength. Several implications of the proposed model based on the data of the Jamuna River, are discussed. The approximated ϕ value is also compared with the actual ϕ value. The theory may yield better results, if (i) the phase shift of the bedforms; (ii) the accurate description of the behavior of the sand dune on the river bed; and (iii) the proper estimate of the frictional resistance due to pure skin friction and bedforms; are properly included in the model. The purpose of including the effect of the transverse bed slope is that the river will braid into an infinite number of branches if it is not included. The aim of including the spiral motion on the bed shear is that the lateral bed shear stress will adapt faster to changing curvature of the channel than the intensity of the spiral motion. The choice of a specific sediment transport formula may also have influence on the model. The linear aspects of the present theory differs from that of Parker (1976) because of the consideration of transverse slope effect on the sediment transport, the effect of spiral motion on bed shear stress and the estimate of frictional resistance from pure skin friction and friction due to bedforms. Furthermore, the major inadequacy of the linear stability theory is that it cannot explain the longitudinal asymmetry embodied in the sand dunes or bars properly which may appear to be associated with nonlinear effects. From the analyses, it can be concluded that the instability increases towards incipient braiding, increased channel width, and dune wavelength. It is further concluded that stability will never be achieved in the Jamuna River under maximum instability condition because of the very small aspect ratio of the Jamuna River ($\approx 1/1000$).

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