Thermal Wave Resonant Cavity Technique in Measuring Thermal Diffusivity of Sucrose Solution

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ABSTRAK

Teknik 'rongga resonan gelombang terma' telah digunakan untuk mengukur keresapan terma bagi larutan sukrosa dengan kepersisan tinggi sehingga tiga angka bererti. Keresapan terma dari larutan adalah menurun dengan peningkatan peratusan berat sukrosa dan bersetuju dengan peraturan campuran mudah dalam renj sehingga ke suatu takat tepu. Juga keresapan terma larutan berubah secara linear terhadap kuasa dua FWHM. Nilai keresapan terma bagi air adalah amat hampir dengan nilai literatur.

ABSTRACT

The thermal-wave resonant cavity technique has been used to measure thermal diffusivity of sucrose solution with precision up to three-significant-figure. The thermal diffusivity of the solution decreases linearly with the increase of sucrose weight percentage and agrees to the simple mixture rule in the range up to a saturated point. Also the solution thermal diffusivity varies linearly with square FWHM. The thermal diffusivity value of water is very close to the literature value.

INTRODUCTION

Recent developments in monitoring and measuring the thermal properties of liquids by using photothermal techniques have attracted much attention. One of the technique is thermal-wave resonant cavity (TWRC) and has been applied successfully to acetone in water to determine thermal effusivity (Balderas-Lopez *et al.* 2000), to various liquids to determine thermal diffusivity (Balderas-Lopez *et al.* 2001), specific heat capacity and thermal conductivity (Caerels *et al.* 1998), to air and gas to measure thermodynamic equation of state (Pan and Mandelis, 1998), thermal diffusivity (Wang and Mandelis 1999), absolute infrared emissivity (Shen and Mandelis 1995; Shen *et al.* 1998) and monitor hydrocarbon vapours (Lima and Marin 2000).

The photothermal detector used in TWRC technique is a thin pyroelectric (PE) transducer such as polyvinylidene diflouride (PVDF) film that has strong PE effect (Kawai 1969; Bergman *et al.* 1971; Mandelis and Zver 1985; Xiao and Lang 1989; Mandelis *et al.* 1993) to detect the modulated or pulsed temperature rise induced in the material. The technique uses a modulated optical source that is a common feature of all photothermal techniques to generate thermal waves. The length and propagation characteristics of a thermal wave are simply

controlled by the frequency of the light source modulator. The spatial behaviour of the thermal wave in TWRC technique is monitored by varying the cavity length at one modulation frequency.

In this paper the TWRC technique application in monitoring the thermal diffusivity of sucrose solution is described as the saturated sucrose solution is diluted to low concentration at room temperature.

THEORY AND METHOD

The development of a photopyroelectric (PPE) detector with a various sample-to-source distance has successfully introduced the variability of the thermal-wave cavity length at single modulation frequency as another equally powerful control parameter. In order to analyse quantitatively PE quadrature data from the materials which are very weak sources of thermal waves, a general one dimensional treatment which takes into account all the energy pathways and determines explicitly the dependences of the PE signal on each and all system parameters (Mandelis *et al.* 1993). The one-dimensional theoretical geometry is shown in *Fig. 1.* When intensity modulated of light is illuminated onto a metal foil a thermal wave will be generated and then propagates to a medium next to it. By placing a PE sensor physically in contact with the medium the PE voltage will be detected. At a fixed thermal-wave oscillation frequency, $f = \omega / 2\pi$, the PE voltage signal across the detector is given by (Balderas-Lopez *et al.* 2000; Shen and Mandelis 1995).

$$V(L,\alpha_1,\omega) = Const(\omega)\frac{e^{-\sigma_1 L}}{1-\gamma_{ls}\gamma_{lp}e^{-2\sigma_1 L}} \tag{1}$$

where ω is the angular frequency of light chopper, L is the cavity length, and σ_1 is the complex thermal diffusion coefficient, defined by

$$\sigma_l = (1+i)\sqrt{\frac{\omega}{2\alpha_l}} \tag{2}$$

α_l is the thermal diffusivity of the liquid sample. The interfacial thermal coefficients γ_{jk} are defined as

$$\gamma_{jk} = \frac{\left(1-b_{jk}\right)}{\left(1+b_{jk}\right)} \tag{3}$$

where $b_{jk} = e_j / e_k$ is the thermal coupling coefficient, the ratio of thermal effusivities of media j and k, the subscripts s, p, and l refer to the thermal-wave source, the PE material and the liquid sample, respectively.

The magnitude of the complex expression given in Eqn.(1) can be written as

$$| V(L,\alpha_p,\omega)| = Const\ (\omega)\ e^{-A_l L},\tag{4}$$

and

$$A_l = \left(\frac{\pi f}{\alpha_l}\right)^{\frac{1}{2}}.\tag{5}$$

where f is the light chopping frequency.

The plot of the in-phase signal versus cavity length from Eqn.(4) is a curve that has a deep valley of depth IP_{min} followed by a small peak of height IP_{max}. From this, a 'full width at half maximum' (FWHM) is defined as the width of the curve in unit length at half maximum of the in-phase signal. The half maximum signal here is the half maximum of the curve height or $\frac{1}{2}(IP_{max} + IP_{min})$.

The TWRC cell was constructed by using a thin copper foil attached to one end of a glass tube and the intensity modulated laser beam was illuminated on the foil to induce thermal wave, *Fig. 1*. Since the foil is in direct contact with the liquid sample, the thermal wave generated propagates across the liquid to reach the PVDF film PE sensor. The tube is attached to a micropositioner to alter the foil position thus altering the cavity length, L. The small PE voltage signal was fed into a pre-amplifier for signal amplification and then into a lock-in amplifier for signal analysis. The cavity length was increased by 20-micron step, and at each step the lock-in magnitude and phase signals were recorded. By plotting ln(signal) versus cavity length L, the thermal diffusivity can be obtained from the plot gradient A_l using Eqn.(5). Also by plotting in-phase signal versus L the FWHM can be obtained for the particular liquid sample.

In the following this technique is used to test a 'simple mixture rule' on thermal diffusivity in liquid media mixture. For a mixture containing x w% of medium A of thermal diffusivity α_A and $(1 - x)$ w% of medium B of thermal diffusivity then the mixture thermal diffusivity of the media α_B can be written as

$$\alpha' = \frac{(\alpha_A - \alpha_B)}{100} x + \alpha_B\tag{6}$$

RESULTS AND DISCUSSION

A saturated sucrose solution at room temperature (about 23°C) was prepared by desolving 10.5 g sucrose in 10.0 ml distilled water, i.e. the sucrose weight percentage in water is about 51.2w%. The other lower w%'s sucrose solution were made by adding proportionate weight of water in the saturated solution. In the experiment the light chopping frequency was set at 7.60Hz and care has
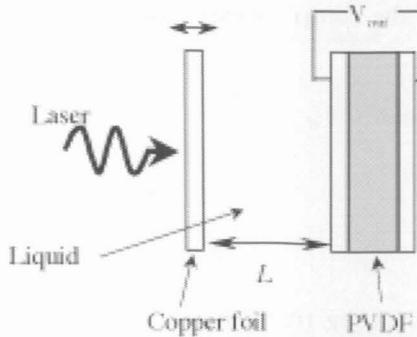
Fig. 1: Schematic diagram of thermak-wave resonant cavity cell

been taken to ensure no stray light off the glass tube hitting the PE sensor as this will introduce unwanted noise produced by the sensor.

From 9 data sets of 8 sucrose solutions of various w%'s and one of distilled water, the overal variation of PE voltage signal versus cavity length L is displayed in a 3-D spline surface plot in *Fig. 2*. The signal decreases exponentialy with cavity length for all w%'s but with different exponential constants. Here, the plain distilled water produces the highest PE signal of all sucrose solutions.

From Eqn.(4), the thermal diffusivity is obtained by plotting ln(signal) versus cavity length and for a particular case of 6.4w% sucrose solution is as can be seen in *Fig. 3*. The plot gradient is an exponential constant and the plot
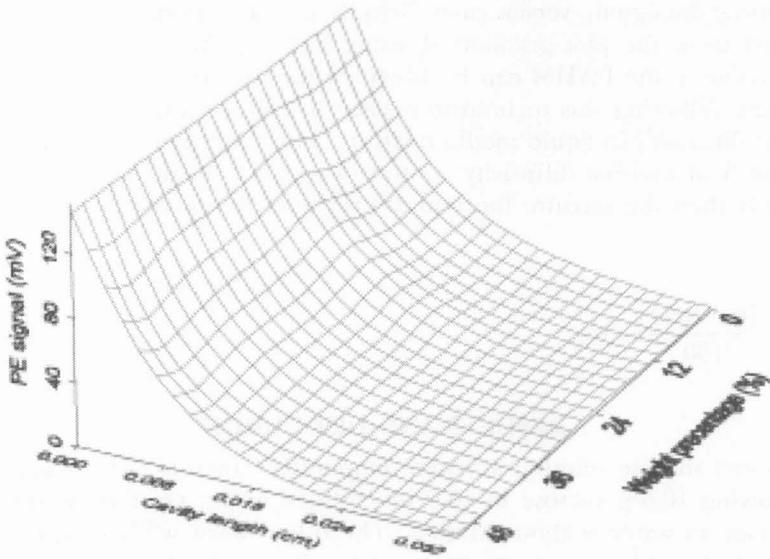

Fig. 2: Plot of PE voltage versus cavity length at various sucrose w%

is linear fitted by using Microsoft Origin software. The gradient and the thermal diffusivity are 130.860cm^{-1} and $0.139 \ 10^{-2} \text{cm}^2/\text{s}$, respectively. Here, the gradient is obtained by fitting only the first 15 data points that contribute to the good fit because at very large L they deviate from the line as the noise dominates. The pre-amplifier gain was set at 100 and the data points from the lock-in amplifier were read in millivolt. The thermal diffusivity error is mainly due to the gradient and is about 4%.
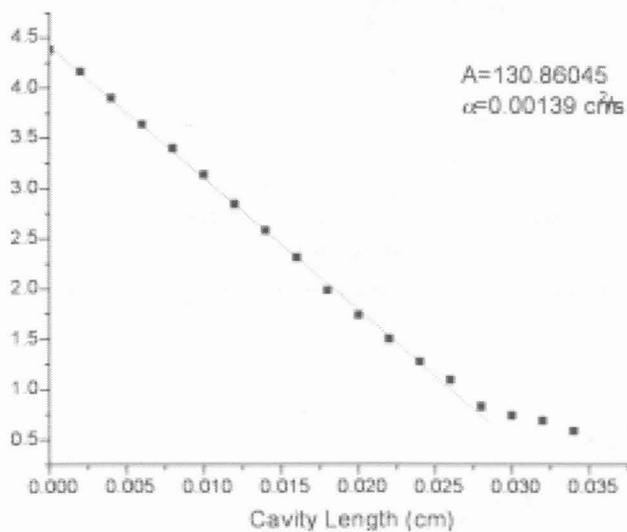

Fig. 3: 1n(PE signal) versus cavity length for 6.4w% of sucrose in water

The FWHM was obtained by fitting the inphase signal by using the value obtained from the experiment. This is because the magnitude and phase data obtained were limited up to certain value of L immediately after the minimum valley before the noise starts to dominates at large L. The fitting constant was then used to extrapolate the curve slightly beyond the small peak that is not actually covered by the experimental data points (Fig. 4). This is acceptable because it is merely extending the data points by fitting the existing experimental data so that the FWHM of the curve can be determined.

The plot of sucrose solution thermal diffusivity α' versus w% sucrose, Fig. 5, shows that α' decreases linearly with w% starting from the value for the plain water or for 0w% sucrose. This corresponds very well with the simple mixture rule of Eqn.(6) for sucrose solution up to about 51.2w%. From the plot, the water thermal diffusivity obtained from the intersection of the fitted line to the verticle axis is $0.144 \times 10^{-2} \text{cm}^2/\text{s}$, that is similar to the one obtained by Balderas-Lopez et al. (2000) by using the technique. From this linear relationship, the thermal diffusivity of the saturated solution (51.2w% sucrose) is $0.102 \times 10^{-2} \text{cm}^2/\text{s}$. It is accepted that, for liquids, the thermal diffusivity decreases with
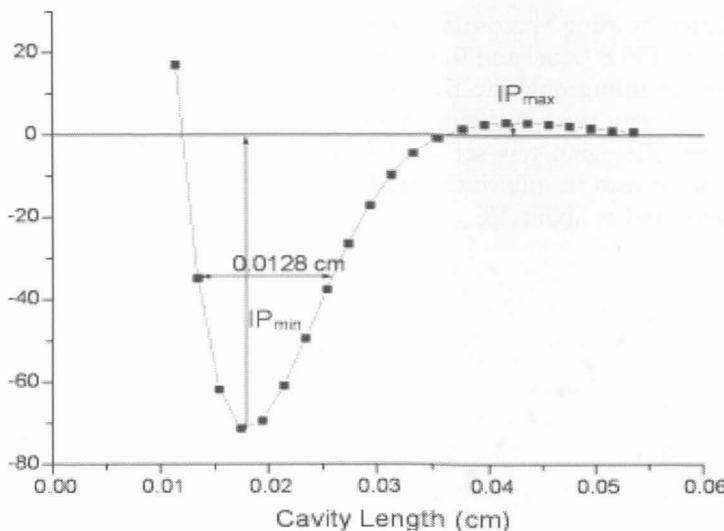
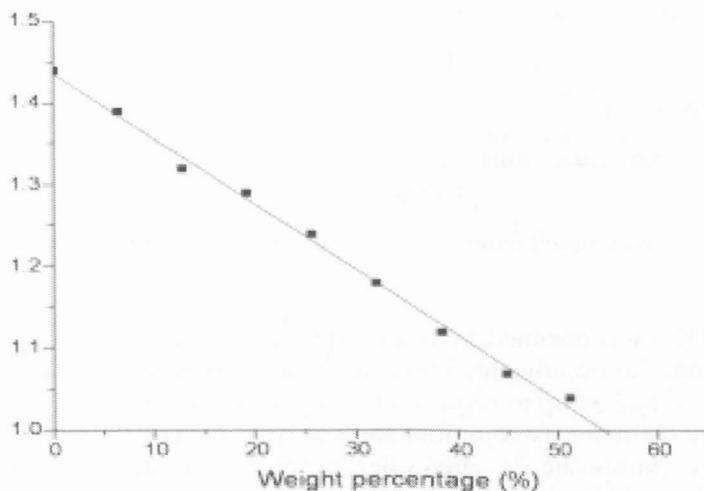*Fig. 4: In-phase signal versus cavity length at 6.4w% of sucrose in water*

Fig. 5: Plot of thermal diffusivity versus sucrose w%

increasing molecular size. Here, the sucrose molecule is bigger in dimension compared to water, hence increasing the sucrose w% decreases the thermal diffusivity of the solution. For the very high concentration value, i.e. higher than 51.2w%, the thermal diffusivity would be constant and has the value of that of 51.2w% because the undilluted sucrose crystal settles at the bottom of the cavity instead of suspended homogenously in the solution.

The plot of mixture thermal diffusivity α' of sucrose solution versus $(FWHM)^2$, *Fig. 6*, shows that α' increases linearly with $(FWHM)^2$. This indicates that the relationship of α' and FWHM can takes the following form;

$$\alpha' = M \times (FWHM)^2 + N,$$

where M is the gradient (8.099 ± 0.527 s^{-1}) and N is the intersection with verticle axis α' ($0.092 \times 10^{-3} \pm 0.075 \times 10^{-3}$ cm.s^{-1}). A few scattered points are due to erors in determining FWHM of a particular w% sucrose. This implies that FWHM can be a sensitive indicator to liquid thermal diffusivitiy changes.
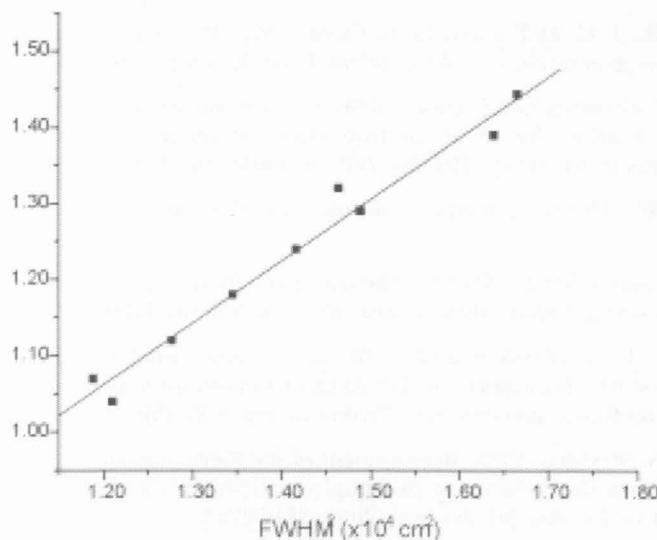

Fig. 6: Liquid mixture thermal diffusivity versus (FWHM)^2

CONCLUSION

Thermal diffusivity values for sucrose solution up to saturated solution can be expressed using a simple mixture rule. The thermal diffusivity value of water is very close to the literature value. The square of FWHM varies linearly with thermal diffusivity of sucrose solution up to sucrose saturated solution.

ACKNOWLEDGEMENT

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