Pertanika J. Sci. & Technol. 7(1): 33-37 (1999)
ISSN: 0128-7680

The Role of the Available Potential Energy in the Atmosphere

Alejandro Livio Camerlengo and Mohd. Nasir Saadon
Faculty of Applied Science and Technology
Universiti Putra Malaysia Terengganu
Mengabang Telipot
21030 Kuala Terengganu, Malaysia
e-mail: alex@upmt.edu.my

Received 9 May 1997

ABSTRAK

Suatu masalah utama (pokok) dalam dinamik tenaga atmosfera di bicarakan. Dengan menggunakan ungkapan tertentu formula matematik, didapati bahawa 0.5% dari tenaga potensi keseluruhan yang tersedia untuh konversi berubah menjadi tenaga kinetik.

ABSTRACT

A major problem in atmospheric energetic dynamics is addressed. In using a particular mathematical derivation it is verified that 0.5% of the total potential energy is available for conversion into kinetic energy.

Keywords: **atmospheric energetics, available potential energy, kinetic energy, internal energy**

INTRODUCTION

Strengthening or weakening of atmospheric systems which form the weather pattern is often measured in terms of the kinetic energy they possess. Under adiabatic motion, however, the total energy of the whole atmosphere remains constant.

In general, the motion of the atmosphere is not adiabatic. In particular, this is true in the lower layer of the atmosphere.

Friction is the only non-adiabatic process which ordinarily generates internal energy and destroys kinetic energy (Lindzen 1967). The remaining non-adiabatic processes, including the release of latent heat, alter only the internal energy directly. Thus, the only possible sources for the kinetic energy of the whole atmosphere are both the potential energy and the internal energy.

It has been shown that the net gain of kinetic energy of the whole atmosphere occurs at the expense of both the kinetic energy and the internal energy (Haurwitz 1941). Following Margules (1903), the sum of the potential energy and internal energy is known as total potential energy,

The total potential energy is not a good measure of the amount of energy available for conversion into kinetic energy under adiabatic flow. A quantity that measures the energy available for conversion into kinetic energy (KE), under adiabatic flow, is generally used. Such a quantity is known as the available potential energy (APE). The available potential energy of the atmosphere may be defined as the difference between the total potential energy and the minimum total potential energy, which may result from any adiabatic redistribution of mass.

Following Lorenz (1967), the major properties of the APE are:

(i) the sum of the APE and the KE is conserved under adiabatic flow,

(ii) the APE is completely determined by the distribution of mass.

(iii) the APE > 0 if the density stratification is not both horizontal and statistically stable.

THE PROBLEM

Both the atmospheric potential, P, and the internal, I, energy per unit mass of the whole atmosphere are defined as g z and cv T, respectively. Because P and I of a vertical column above a unit area bear the ratio:

$$P/I = (c_p - c_v)/ c_v \tag{1}$$

and since an element of mass per unit area is :

$$d\, m/ dx\, dy = dp/ g, \tag{2}$$

it yields:

$$P + I = \left(c_p/g\right)\int_0^{P_0} T\, dp \tag{3}$$

Upon substitution in equation (3) of

$$T = \Theta\, p^k\, p_{oo}^{-k} \tag{4}$$

where p_{oo} = 1000 hpa, it follows that:

$$P + I = \left(c_p/gp_{oo}{}^k\right)\int_0^{P_0}\Theta\, p^k\, dp = \left[c_p/g(k+1)p_{oo}{}^k\right]\int_0^{P_0}\Theta\, dp^{k+1} \tag{5}$$

Upon integration by parts of equation (5), it yields:

$$P + I = \left[c_p/g(k+1)p_{oo}{}^k\right]\left\{\int_t^b d\left(\Theta\, p^{k+1}\right) - \int_{\Theta_t(=\infty)}^{\Theta_b(=\Theta_o)} p^{k+1}\, d\Theta\right\} \tag{6}$$

where the subindex b (t) represents the bottom (top). Therefore:

$$P + I = \left[c_p/g(k+1)p_{oo}{}^k\right]\left\{\left(\Theta p^{k+1}\right)_b - \left(\Theta p^{k+1}\right)_t + \int_{\Theta_o}^{\infty} p^{k+1} d\Theta\right\} \tag{7}$$

Upon usage of equation (4), it follows that:

$$\Theta\, p^{k+1} = p\, T\, p_{oo}^{\,k}. \tag{8}$$

Because the pressure at the top of the atmosphere is zero, the second term in the RHS of equation (7) vanishes. Thus,

$$P + I = \left[c_p/g(k+1)p_{oo}^{\,k}\right] \int_0^{\Theta o} p^{k+1} d\Theta + \int_{\Theta o}^{\infty} p^{k+1} d\Theta$$

$$= \left[c_p/g(k+1)p_{oo}^{\,k}\right] \int_{\Theta o}^{\infty} p^{k+1} d\Theta \tag{9}$$

DISCUSSION

The average over an isentropic surface is denoted by a bar. It follows that:

$$\overline{p^{k+1}} = \left[\int\left(p^{k+1}\right)dx\, dy\right] / \left(\int dx\, dy\right) \tag{10}$$

The minimum total potential energy which may result from adiabatic rearrangement occurs whenever $\overline{p} = p$ everywhere. It is obtained by setting $\overline{p} = p$ in equation (9). Thus, the average potential energy per unit area of the earth's surface is:

$$\overline{A} = \left[c_p/g(k+1)p_{oo}^{\,k}\right] \int_0^{\infty} \left(\overline{p^{k+1}} - \overline{p}^{\,k+1}\right) d\,\Theta \tag{11}$$

Upon expansion, by the binomial theorem, of $p = \overline{p} + p'$, it yields:

$$p^{k+1} = \left(\overline{p} + p'\right)^{k+1} = \overline{p}^{\,k+1} + (1+k)p'\,\overline{p}^{\,k} + \left[(1+k)/2\right](p')^2\,\overline{p^{k-1}} + \ldots \tag{12}$$

The average of equation (12) is taken. Bearing in mind that $\overline{p}' = 0$, it yields:

$$\overline{p^{k+1}} = \overline{p}^{\,k+1} + \left[(k/2)(1+k)\overline{p}^{\,k-1}\overline{(p')}^2 + \left[(1+k)\,k\,(k-1)/3!\right]\overline{p^{k-2}\,(p')^3}\right] \tag{13}$$

Upon integration, it yields:

$$\int\left(\overline{p^{k+1}} - \overline{p}^{\,k+1}\right) d\,\Theta = \left[k\,(k+1)/2\right]\int \overline{p}^{\,k-1}\overline{(p')}^2\, d\,\Theta + \left[(1+k)\,k\,(k-1)/3!\right]\int \overline{p^{k-2}\,(p')^3}\, d\Theta \tag{14}$$

It follows that:

$$\overline{A} = \left[c_p/g(k+1)p_{oo}^{\,k}\right]\int_0^{\infty} \overline{p}^{\,k+1}\left\{\left[k(k+1)/2\right](p'/\chi)^2 - \left[(1+k)\,k\,(k-1)/3!\right]\overline{(p'/\chi)^3} + \ldots\right\} d\Theta \tag{15}$$

where $\chi = \overline{p}$. The power series (15) must converge if $p' < \chi$ everywhere. However, the rapidity of its convergence depends upon typical values of p'/χ.

Assume, for example, that on a particular isentropic surface p = 1000 hpa over half of the area. Over the other half of the area, p decreases linearly from 1000 hpa to 300 hpa. In this case, $\bar{p}$ = 825 hpa, $\overline{(p'^2)}/\chi^2 = 0.075$ and $\overline{(p'^3)}/\chi^3 = -0.019$. The ratio of these last two quantities is approximately 0.06. It may be concluded that this series is very well represented by its leading term. It follows naturally that:

$$\bar{A} = \left[(k\, c_p)/(2\, g\, p_{\infty}^{\,k})\right] \int_0^{\infty} \bar{p}^{k+1} \overline{(p'/\chi)^2}\, d\Theta \tag{16}$$

Therefore, $\bar{A}$ depends upon the variance of pressure over the isentropic surfaces. This variance is related to the variance of Θ on an isobaric surface. while $\bar{\Theta}$ and $\bar{T}$ represent the average values of Θ and T; and Θ' and T' represent the departures of Θ and T from their respective average values ($\bar{\Theta}$ and $\bar{T}$).

The function $\bar{\Theta}$ (p) is approximately determined by $\bar{p} = \bar{p}\,(\bar{\Theta}(p))$, in such a way that:

$$p' \approx \bar{p}\,(\Theta - \Theta') - \bar{p}(\Theta) \approx -\Theta'\,\partial\bar{p}/\partial\Theta \tag{17}$$

Therefore, it follows that:

$$\bar{A} = \left[(k\, c_p)/(2\, g\, p_{\infty}^{\,k})\right] \int_0^{\infty} \left\{\bar{p}^{k+1}\, \overline{(\Theta')^2} (\partial\,\bar{p}/\partial\,\Theta)/\left[(\partial\,\Theta/\partial\,\chi)\,\chi^2\right]\right\} d\,\Theta \tag{18}$$

But $d\,p = \left(\bar{\partial}\,p/\partial\Theta\right) \overline{(\Theta')^2}\, d\Theta$ Thus, it yields:

$$\bar{A} = \left[(k\, c_p)/(2\, g\, p_{\infty}^{\,k})\right] \int_0^{Po} \left\{\bar{p}^{k-1}\, \overline{(\bar{\Theta}')^2}/(-\partial\,\Theta/\partial\,\chi)\right\} d\,p \tag{19}$$

From the hydrostatic approximation, it follows that:

$$\partial\,\Theta/\partial\,p = -(k\,\Theta/p)(\gamma_d - \gamma)/\gamma_d, \tag{20}$$

where $\gamma_d = -\partial\,T/\partial\,z\big|_d = g/c_p$. Because of that fact that:

$$\Theta'/T' = \Theta/T, \tag{21}$$

it follows naturally that:

$$\bar{A} = (1/2) \int_0^{Po} \left\{\bar{T}_o\,\right)(\gamma_d - \gamma)^{-1} \overline{(T'/H)^2}\, d\,p \tag{22}$$

where $H = \bar{T}$.

CONCLUSION

Equation (22) is suitable for estimating the ratio $\overline{A}/(\overline{P} + \overline{I})$. This ratio equals a suitable average value of $(\gamma_d \overline{T}'^2)/[2(\gamma_d - \gamma)\overline{T}^2]$. If $\overline{\gamma} = 0.66\ \gamma_d$ and $\overline{T}'^2 = (15°C)^2$ are taken as typical values of the atmosphere, then it follows that:

$$\overline{A}/[\overline{P} + \overline{I}] \approx 1/200 \tag{23}$$

It is verified that less than 1% of the total potential energy is available for conversion into kinetic energy. As pointed out by Holton (1972), the atmosphere is a rather inefficient heat engine.

ACKNOWLEDGEMENT

This study was supported by a short grant of Universiti Putra Malaysia. The authors gratefully acknowledge this support.

REFERENCES

HAURWITZ, B. 1941. *Dynamic Meteorology*. New York: McGraw-Hill.

HOLTON, J.R. 1972. *An Introduction to Dynamic Meteorology*. New York: Academic Press.

LINDZEN, R.S. 1967. Planetary waves on beta-planes. *Monthly Weather Review* **95**: 441-451.

LORENZ, E.W. 1967. *The Nature and Theory of the General Circulation of the Atmosphere*. World Meteorological Organization.

MARGULES, M. 1903. Uber die Energie der Sturme. *Jahrb. Zentralanst. Meteor. Wien*, p. 1-26. Translation in C. Abbe (1910). *Smithsonian Inst. Meteorol. Coll.* **54**: 533-595.