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The Relative Growth Rates of the Two-dimensional and Three-dimensional Waves in the Atmosphere

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ABSTRAK

Ini dijelaskan bahawa suatu gelombang dua dimensi lahir (tumbuh) lebih cepat dari pada gelombang tiga dimensi di atmosfera.

ABSTRACT

Using a simple analysis the growth rate of two-dimensional and three-dimensional waves in the atmosphere is addressed. It is verified that a two-dimensional wave grows faster than a three-dimensional wave.

Keywords: relative growth rates, two-dimensional and three-dimensional waves

INTRODUCTION

Due to the fact that analytical solutions are difficult to come by, the mathematical treatment of atmospheric problems is rather complex. To simplify the problem under consideration, hypotheses are usually made. The aim of the atmospheric modeller is to make the right assumptions while at the same time preserving the necessary physics of the problem under consideration. In doing so, the partial differential equations that govern the primitive equations, most commonly used in atmospheric sciences, may have an analytical solution.

We seek to gain some understanding of the behaviour of two-dimensional as well as three-dimensional waves in the atmosphere. For this purpose a homogeneous, incompressible model of the atmosphere where the basic state, Uo (y), parameterized by the east-west velocity, is used. In doing so, the growth rates of the two-dimensional and three-dimensional waves in the atmosphere are compared.

THE PROBLEM

Consider a homogeneous, incompressible fluid, with the conditions Uo = Uo (y) only and $\pi = p/\rho$, where U represents the basic state east-west (zonal) velocity; y, the latitude (where y = 0, represents the Equator); p, the atmospheric pressure; and ρ, the atmospheric density.

The governing equations are:

$$\partial u/\partial t + Uo\, \partial u/\partial x + v\, \partial Uo/\partial y = -\partial \pi/\partial x \qquad (1)$$

$$] v/] t + Uo \;] v/] x \quad = -] \pi/] y \tag{2}$$
$$] w/] t + Uo \;] w/] x \quad = -] \pi/] z \tag{3}$$
$$] u/] x + \;] v/] y +] w/] z \quad = 0 \tag{4}$$

where v] Uo/] y represents the linearized version of the latitudinal variation of the basic state east-west velocity; (u,v,w) represents the east-west, the north-south and the vertical velocity components to the coordinates x, y, z (positive in the east, the north and the vertical direction, respectively). The first three equations represent the u, v and w momentum equations, while the last equation represents the continuity equation.

Following Holton (1972), the boundary conditions for this problem are:

$$v \; (y = y_1) = v \; (y = y_2) = 0 \tag{5}$$

Following Necco (1980), the solution:

$$(u,v,w,\pi) = (u^*,v^*,w^*,\pi^*) \; \exp \; [i(k \, x + \gamma \, y - \sigma \, t)] \tag{6}$$

where k and γ represent the zonal and the vertical wavenumbers; respectively; and σ, the frequency, is assumed. The following set of equations is obtained:

$$i \, k \; (Uo - c)u^* + v^*(] \; Uo/] \; y) + i \, k \, \pi^* \quad = 0 \tag{7}$$
$$i \, k \; (Uo - c) \; v^* \quad = - \;] \; \pi/] \; y \tag{8}$$
$$i \, \gamma \;\; \pi^* + i \, k \; (Uo - c) \;\; w^* = 0 \tag{9}$$
$$i \, k \, u^* + \;\;] \; v^*/] \; y \; + \; i \, \sigma \, w^* \quad = 0 \tag{10}$$

where c (= σ/k) represents the phase speed of the wave (Oort 1964).
Let

$$\alpha^2 + = k^2 \; + \gamma^2 \tag{11}$$
$$u^+ = (k \, u^* + \gamma \, v^*)/\alpha^+ \tag{12}$$

where

$$\pi^+/\alpha^+ = \pi^*/k \tag{13a}$$
$$v^+ = v^* \tag{13b}$$
$$c^+ = c^* \tag{13c}$$

Upon multiplication of equation (9) by γ/k, it yields:

$$i \, \gamma \; (Uo - c) \; w^* = i \, \gamma^2 \;\; \pi^+/\alpha^+ \tag{14}$$

Addition of equations (14) and (7) yields:

$$i \, k^+ \, \alpha^+ \; (Uo - c^+) \; u^+ \;\; + v^+ \; (d \; Uo/dy) \;\; =$$
$$- \, i \, \pi^* \; (\gamma^2/k + k) = i \; (\gamma^2 + k^2) \;\; \pi^+/\alpha^+ = \;\; i \; \pi^+\alpha^+ \tag{15}$$

Therefore, equation (8) becomes:

$$i\ k^+\ (Uo - c^+)\ v^+ = -\ (k^+/\ \alpha^+)\ ]\ \pi^+/]\ y \qquad (16)$$

or its equivalent equation:

$$i\ \alpha^+\ (Uo - c^+)\ v^+ = -\ ]\ \pi^+/]\ y \qquad (17)$$

Proceeding in the same fashion, equation (10) becomes:

$$i\ u^+\ \alpha^+ +]\ v^+/]\ y = 0 \qquad (18)$$

A new set of equations in the new variables is attained (Kuo 1956). The correspondent boundary conditions are:

$$v^+\ (y = y_1) = v^+\ (y = y_2) = 0 \qquad (19)$$

It is interesting to note that there is no vertical scale, nor there is any vertical motion in the new set of equations.

The phase speed, c, is decomposed as:

$$c = c_r + i\ c_I \qquad (20)$$

where c_r (c_I) denotes the real (imaginary) component.

For a three-dimensional wave, where $\gamma = 0$, it is obtained:

$$\exp\ (-\ i\ k\ c\ t) = \exp\ (k\ c_I\ t)\ \exp(-\ i\ c_r\ k\ t) \qquad (21)$$

It should be noted that the growth rate of three-dimensional waves is given by exp ($k\ c_I$ t); whereas the other factor, exp(- i cr k t) represents an oscillatory solution (Wiin-Nielsen 1959). We seek to know the growth rate. For this purpose, the factor exp (k cI t) is examined.

For a two-dimensional wave, for $g \neq 0$, from:

$$\alpha^{2+} = k^2 + \gamma^2$$

it follows that:

$$\alpha^+ > k \qquad (22)$$

Therefore, it is obtained:

$$\exp\ (\alpha^+\ c_I\ t) > \exp\ (k\ c_I\ t) \qquad (23)$$

CONCLUSION

Using a simple analysis it is verified that a two-dimensional wave grows faster than a three-dimensional wave in the atmosphere.

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