

Evaluation of Infiltration in Furrow Irrigation Part 1: Recession Flow

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ABSTRAK

Aliran susutan didalam ladang besar adalah bererti dari segi pandangan kecekapan penggunaan air. Ianya amat penting dalam kes di mana ladang furrow adalah panjang di mana aliran susutan ada kesan kepada pengairan tumbuhan. Aliran susutan dalam pengairan furrow adalah sangat bererti dengan cara pengairan pusuan. Dengan mod operasi lanjutan di dalam furrow yang panjang, jumlah aliran susutan amat bererti. Dikemukakan di sini satu teknik pemodelan matematik aliran susutan. Tatacara analisis ini berasaskan pendekatan gelombang kinematik yang telah diguna dalam simulasi aliran lajukan untuk furrow. Simulasi susutan yang dihasilkan didapati berguna demi untuk menilaikan kadar asas penyusupan di dalam furrow.

ABSTRACT

Recession flow in a large field can be significant from the point of view of water-use efficiency. This is particularly important in the case where the furrow field is long, thus the receding flow can contribute to the irrigation of crops. Indeed, recession flow is very significant in the surge method of furrow irrigation. In the continuous mode of operation in very long furrows the amount of recession flow can be significant. Herein is presented the mathematical modelling of the recession flow. The analytical procedure is based on the kinematic wave approach used for the simulation of the advance-flow trajectory for furrows. This resulting recession simulation was found to be very useful for the evaluation of the basic infiltration rate in a furrow.

INTRODUCTION

Recession flow in surface irrigation is considered as the stage of flow when irrigation water is shut off and the depletion of surface storage follows. Recession in furrows is important in the evaluation of irrigation efficiencies, especially when the fields are long. The importance of recession is highly significant in surge flow considerations since a substantial part of advance along the field occurs during recession. The surge flow technique of irrigation requires water to be supplied and cut off according to prescribed on/off time ratios in order to achieve complete irrigation of the whole length of fields. It is particularly useful in light soils and long furrows (Lee 1982). Recession flow is also important in determining the infiltration characteristics of the furrow and this is

what the paper and its following part attempt to show.

The objective of this paper is to develop the simulation of the recession phase of furrow irrigation using principles used for the development of the advance phase simulation.

MATHEMATICAL DEVELOPMENT

There have been studies simulating recession flow in open channel flows, e.g. the work by Cunge and Woolhiser (1975). However, the simulation is for border irrigation and thus would have to be modified for use with furrows.

The method of characteristics is used for the development of the simulation. *Fig. 1 and 2* show a family of characteristics emanating from the point at the time of cut-off of irrigation at the

inlet end, where Cunge and Woolhiser (1975) described the curve as the locus of the points where velocity and depth are zero. They mentioned that the forward and backward characteristics will be tangential to the vertical lines. The simulation of the advance-flow trajectory was reported earlier (Lee 1982) (it is shown here only for the purpose of illustrating the simulation of another important phase of irrigation) and the use of the kinematic wave theory to simulate field conditions was reported (Walker and Lee 1982; Walker and Humphreys 1983). The simulation of the recession phase is presented here. It is based on the same principles as for the advance simulation, only with the exception that furrow irrigation has been continued long enough for the basic infiltration rate f_0 to be reached.

For the case of furrow advance, where the time-varying infiltration rate holds true, numerical solutions can be obtained (Lee 1982). However, analytical solutions for irrigation recession can be obtained when infiltration rate is a constant and this is true for long-term irrigation, where the infiltration rate can be assumed to be constant and equals the basic infiltration rate. Thus, when the rate of infiltration is constant

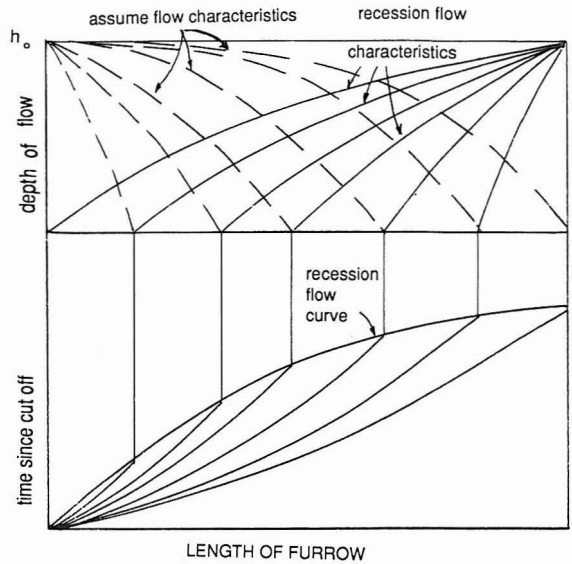


Fig. 2: Concept of characteristics of recession flow and characteristics of the assumed advance flow

throughout the furrow the rate of advance of the assumed advance phase would be the same as the rate water recedes down the same furrow. As mentioned before, this analytical solution can be found only if the rate of infiltration is constant. The following assumptions are made. Firstly, the channel geometry remains constant throughout the length of the furrow and the irrigation event. Secondly, there are no multiple recessions occurring in the furrow. Thirdly, once irrigation water is cut off, the depth of the flow at the inlet end is instantaneously zero. The depth of the flow at the inlet end of the imaginary advance phase is h_0 , equivalent to the depth of the recession flow at the outlet end. The continuity and stage discharge equations used in advance simulation and now employed for recession are

$$\frac{\delta A}{\delta t} + \frac{\delta Q}{\delta x} + W P f_0 = 0 \tag{1}$$

and

$$Q = \alpha y^{m+1} \tag{2}$$

The rate of cross-sectional area change for a prismatic channel is given by

$$\frac{\delta A}{\delta t} = \frac{A}{y} \frac{\delta y}{\delta t} = B \frac{\delta y}{\delta t} \tag{3}$$

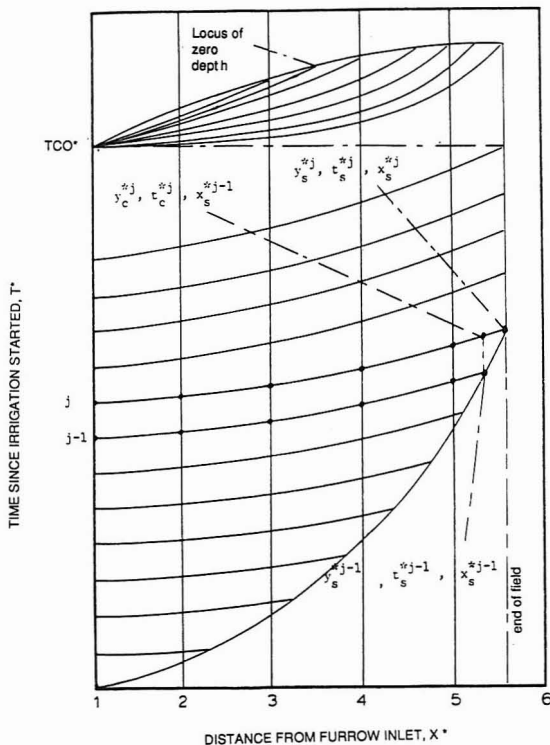


Fig. 1: Method of characteristics grid for kinematic-wave advance and recession.

where B is the surface width of flow and

$$B = \alpha_1 y^{\alpha_2-1} \tag{4}$$

Equation 1 can be nondimensionalized as follows (The performance of an irrigation system is governed by a large number of variables, singling out each variable and examining behavioural trends due to its variation would encompass an enormous list of combinations. By nondimensionalizing these governing equations the number of individual variables can be reduced and provide for an examination of the general behaviour of model responses to parameter variation),

$$\frac{A'\delta A^*}{T'\delta t^*} + \frac{Q'\delta Q^*}{X'\delta x^*} + WP'w p^* f_0 = 0 \tag{5}$$

Rearranging,

$$\frac{\delta A^*}{\delta t^*} + \frac{T'Q'}{A'X'} \frac{\delta Q^*}{\delta x^*} + f_0 \frac{T'}{A'} WP^* = 0 \tag{6}$$

Substituting relevant coefficients,

$$\frac{\delta A^*}{\delta t^*} + V^* \frac{\delta Q^*}{\delta X^*} + WP^* f_0^* = 0 \tag{7}$$

where

$$f_0^* = \frac{f_0 T'}{A'} = \frac{f_0 T'}{R'} \tag{8}$$

$$V^* = \frac{Q'T'}{A'X'} \tag{9}$$

We define f_0^* and V^* as unity since $f_0 T'$ is the infiltrated depth equivalent to the hydraulic depth of water to be infiltrated, and $Q'T'$ is the volume of water equal to the volume of water body $A'X'$. Therefore,

$$\frac{\delta A^*}{\delta t^*} + \frac{\delta Q}{\delta x} + WP^* = 0 \tag{10}$$

$$\frac{\delta A^*}{\delta t^*} = \frac{\delta \frac{A}{A'}}{\delta \frac{t}{T'}} = \frac{B}{B'} \frac{\delta \frac{y}{Y'}}{\delta \frac{t}{T'}} = y^{*\sigma_2-1} \frac{\delta y^*}{\delta t^*} \tag{11}$$

Hence,

$$y^{*\sigma_2-1} \frac{\delta y^*}{\delta t^*} + (m+1)y^{*m} \frac{\delta y^*}{\delta x^*} + \gamma_1 y^{*\gamma_2} = 0 \tag{12}$$

Rearranging,

$$\frac{\delta y^*}{\delta t^*} + (m+1)y^{*m+1-\sigma_2} \frac{\delta y^*}{\delta x^*} + \gamma_1 y^{*\gamma_2-\sigma_2+1} = 0 \tag{13}$$

The total differential of the dependent variable y , with respect to the two independent variables x and t , can be written as

$$\frac{dy}{dt} = \frac{\delta y}{\delta x} \frac{dx}{dt} + \frac{\delta y}{\delta t} \tag{14}$$

Comparing Equations 13 and 14, we have

$$\frac{dy^*}{dt^*} = -\gamma_1 y^{*\gamma_2-\sigma_2+1} \tag{15}$$

$$\frac{dx^*}{dt^*} = (m+1)y^{*m+1-\sigma_2} \tag{16}$$

EVALUATION PROCEDURE

Equations 15 and 16 are the characteristic equation solution for the simulation of recession and can be solved analytically as follows. On integrating Equation 15 we have

$$\int \frac{y^{*\sigma_2-\gamma_2-1}}{\gamma_1} dy^* = - \int dt^* \tag{17}$$

$$\frac{y^{*\sigma_2-\gamma_2}}{\gamma_1(\sigma_2-\gamma_2)} = -t_r^* + M \tag{18}$$

where M is the constant of integration and t_r^* is recession time. At the inlet end the boundary conditions are $t_r^* = 0$ and $y^* = h_0^*$, thus

$$M = \frac{h_0^{*\sigma_2-\gamma_2}}{\gamma_1(\sigma_2-\gamma_2)} \tag{19}$$

Thus, we have

$$y^{*\sigma_2-\gamma_2} = \left[h_0^{*\sigma_2-\gamma_2} - \gamma_1(\sigma_2-\gamma_2)t_r^* \right] \tag{20}$$

The locus of $y^* = 0$ results in

$$t_r^* = \frac{h_0^{\sigma_2 - \gamma_2}}{\gamma_1(\sigma_2 - \gamma_2)} \quad (21)$$

Substituting Equation 12 for Equation 11 gives

$$\int dx^* = \int (m + 1) \left[h_0^{\sigma_2 - \gamma_2} - \gamma_1(\sigma_2 - \gamma_2)t_r^* \right]^{\frac{m+1-\sigma_2}{\sigma_2 - \gamma_2}} dt^* \quad (22)$$

and upon integrating gives

$$X^* = \frac{(m + 1) \left[h_0^{\sigma_2 - \gamma_2} - \gamma_1(\sigma_2 - \gamma_2)t_r^* \right]^{\frac{m+1-\gamma_2}{\sigma_2 - \gamma_2}}}{-\gamma_1(m + 1 - \gamma_2)} \quad (23)$$

where N is the integration constant. For the initial conditions $t_r^* = 0$ and $x^* = 0$ we have

$$N = \frac{(m + 1)h_0^{\sigma_2 - \gamma_2}}{\gamma_1(m + 1 - \gamma_2)} \quad (24)$$

With Equation 21 and 24 substituted for Equation 23 and rearranging gives

$$h_0^* = \frac{\gamma_1(m + 1 - \gamma_2)x^*}{m + 1} \quad (25)$$

The analytical solution of the characteristic equations 22 and 25 for a given range of x^* is a set of (x^*, t_r^*) coordinates describing recession flow. Once the dimensionless set has been determined, the actual distances can be evaluated with the use of the characteristics time T' and characteristic distance X' .

RESULTS AND CONCLUSIONS

Field test results were extensively compiled by Elliott (1980) and were used to evaluate the recession flow. Only two cases were shown here. For the Benson farm (Colorado) the soil is loamy sand, while for the Matchett farm (Colorado) the soil is loam to clayey loam. Fig. 3 and 4 show the model simulation, indicating a good model prediction in evaluating the recession flow.

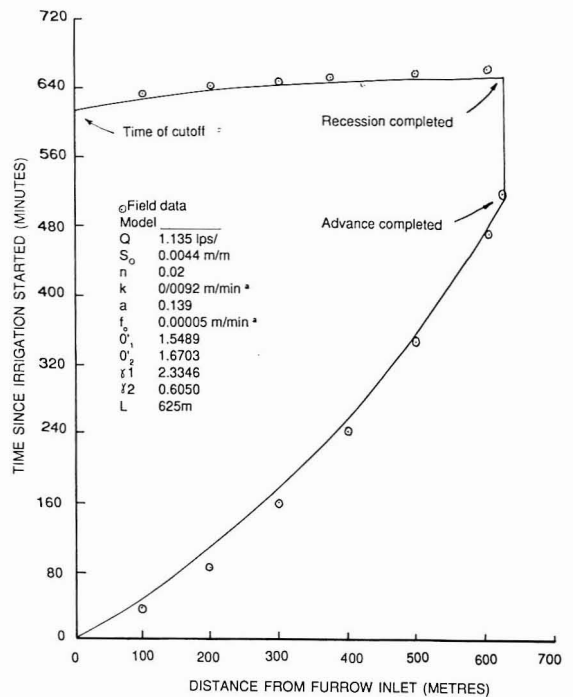


Fig. 3: Predicted and observed furrow advance and recession for irrigation event B 2-2-1, Colorado. [Field data source: Elliott 1980]

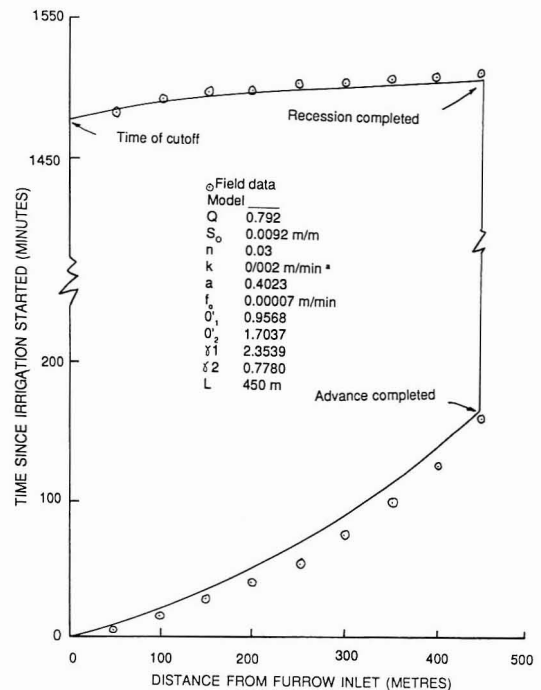


Fig. 4: Predicted and observed furrow advance and recession for irrigation event M 1-4-5, Colorado. [Field data source: Elliott 1980]

The following conclusions were made from this study. The two-point advance-volume balance method of evaluating infiltration in continuous furrow irrigation is confirmed to give a good indication of infiltration together with the Kostiakov Lewis equation. The method and the equation were used to derive the infiltration parameters (from field data collected) required in the modelling. These two details can be found in most texts written on irrigation. This follows from the use of the infiltration characteristics so derived in the kinematic wave analysis of advance (Lee 1982) in which the simulation was solved using finite differences with a computer program, and now the recession phase which was separately derived but based on the same principles. The kinematic wave analysis for furrow advance can similarly be pursued for the analysis of furrow recession with the accompanying assumptions mentioned previously.

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APPENDIX

LIST OF SYMBOLS

f_0	-	basic infiltration or intake rate
A	-	cross-sectional area of flow
Q	-	discharge
WP	-	wetted perimeter of flow
B	-	surface width of flow
y	-	depth of flow
t	-	advance time
σ_1, σ_2	-	parameter from $A = \sigma_1 y^{\sigma_2}$
γ_1, γ_2	-	parameter from $WP = \gamma_1 y^{\gamma_2}$
x	-	distance of flow
t_r	-	recession time
α, m	-	constants of stage discharge relationship
S_0	-	slope of furrow
n	-	Mannings number
a, k	-	soil infiltration parameters
L	-	length of furrow
R_0	-	hydraulic radius or depth
j	-	number count of characteristics
c	-	subscript of parameters at nodes
s	-	subscript at shock wave advance
h_0	-	inlet flow depth at recession
A'	-	characteristic area of flow
A*	-	dimensionless area of flow
Q'	-	characteristic discharge
Q*	-	dimensionless discharge
WP'	-	characteristic wetted perimeter
WP*	-	dimensionless wetted perimeter
T'	-	characteristic time
t*	-	dimensionless time
X'	-	characteristic distance
X*	-	dimensionless distance of flow