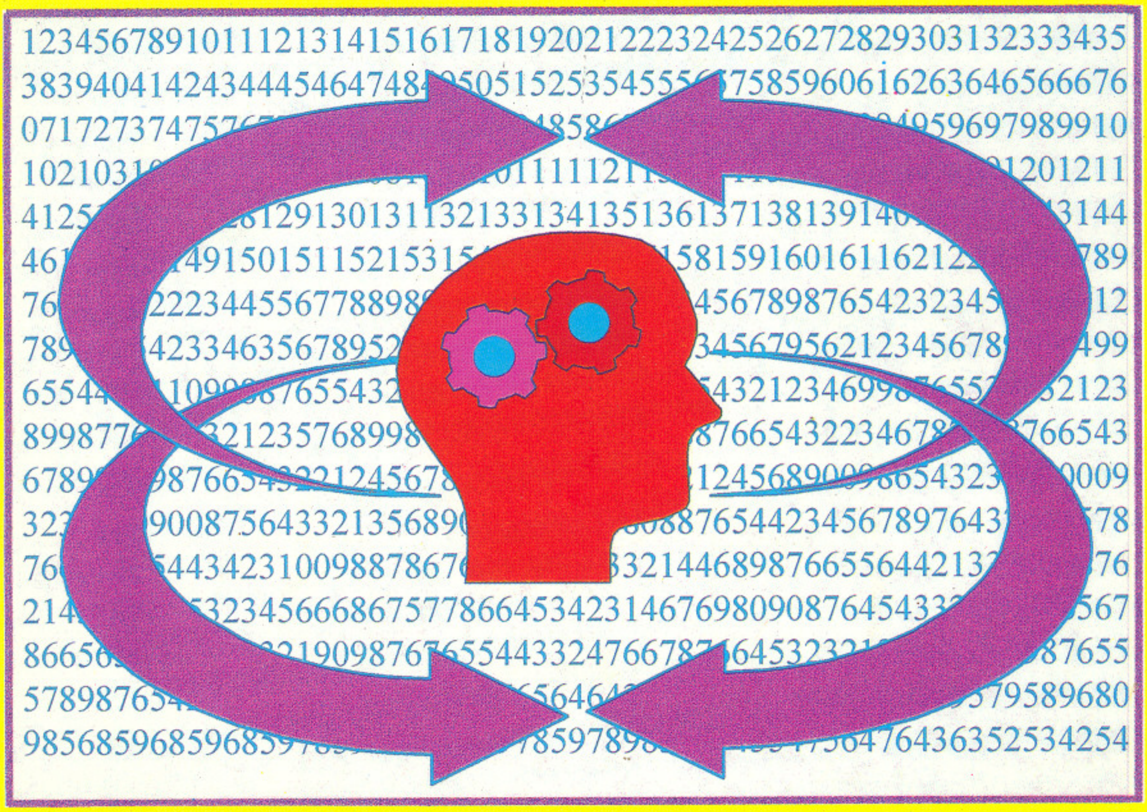

Syarahan INAUGURAL

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THE FASCINATING NUMBERS

Introduction

We see numbers almost everywhere and every day in our lives. Numbers such as telephone numbers, car plate numbers and IC numbers serve to organise our daily activities into some order. Numbers are money-spinners for TNB, Telekom and other business concerns and more recently IWK despite the ignorance of its activities by some quarters.

Numbers, especially big ones seem to be the nightmares of politicians and economists although the empty one (zero) is an elusive dream for consumers in this age of concern over the threat of inflation. Numbers are conveniently employed to determine the status of salary increments under the SSB.

Numbers make charming combinations. 69 is a pretty and matching combination of two numbers of opposite parity. The RIMV certainly knows this and turns them into good bank accounts. The first 10 natural numbers attain the status of royalty in this country for they are reserved for persons of revered status. Also no vision is as perfect as that of 2020.

Numbers have mystical properties. 6 is a perfect number whilst 7 is a nice friendly number and 13 is a foreboding one especially on Fridays. Although we would not get anything significant from the licence numbers of cars involved in an accident it is still a popular method to predict the outcome of horse races by a section of the populace in the country.

Birth dates play a similar role. In some communities plans of marriages have known to be aborted because of incompatible combination of birth dates of the courting couples that predict chaos if the plan is carried out.

Such is the overpowering influence of numbers in the lives of men. Infact numbers have been omnipresent in the organization of activities of men since time immemorial. The footprints of mathematical ingenuity can be seen in the construction of pyramids of the Pharoahs in Egypt and the Mayas in South America. In modern times the versatility of numbers have led men to invent computers and send rockets to outer space.

The role played by numbers is so extensive that they have become the fundamental elements of activities of our civilisation since ancient times. As a result it is inevitable that numbers become the object of scrutiny and study of scholars throughout the ages. Out of these activities has grown up the study of the theory of numbers.

Numbers (nowadays denoted by 1,2,3,...) have been in existence since men learn to count. They have been known to us for so long that the mathematician Leopold Kronecker (1823-1891) once remarked "God created the natural numbers, and all the rest is work of man". For this reason the numbers are also called the natural numbers. Symbols for these numbers were unique to each civilisation since the ancient times. So were the methods of operations involving these numbers. Although most of the time numbers were employed to simplify arguments and problems, in the course of time they had attracted the attention of men, mathematicians professionals and amateurs alike to their intrinsic properties.

It is found that numbers possess characteristics of their own and display interesting properties worthy of study. Hence through out history we find individuals and groups of them who would devote their time to studying of numbers for their own sake. A personality no less colourful than Napolean Bonaparte himself was often caught pondering over numbers when he was not planning any military strategies. In his book bearing the name of the French general, J. Abbot wrote "When he had a few moments for diversion, he not unfrequently employed them over a book of logarithms, in which he always found recreation".

The activities of these number theorists as they are called are akin to the linguists who study words and their properties independent of their meanings.

A Brief Historical Survey

The exact point in time when 'formal' studies on numbers were started is lost in antiquity. Written historical records show that the ancient Egyptians and Babylonians were experts in the use of numbers in dealing with matters pertaining to their daily activities. In the ensuing centuries numbers began to become the object of study by scholars especially during the time of the Chinese and Hindu civilisations and also during the high points of intellectual pursuits of the Greek, Roman and Islamic empires.

Mathematical activities of the Egyptian civilisation began as early as 2100 B.C. Records came down to us through the discovery of the heiroglyphical writings of the Egyptians on stone walls, skins of animals and especially on papyrus leaves. Two types of numbers were used by the Egyptians. One known as the demotic was used for daily purposes and the other as heiratic for religious activities.

As is known the Babylonian civilisation flourished on the banks of the Euphrates and Tigris rivers. As early as 2250 B.C. the Babylonian civilisation reached its peak. The discovery made by the archaelogist Otto Neugebauer in the 1920's showed that their number system was already in existence since the time of the emperor Hammarubi. The Babylonian numbers basically used the wedge symbols V to represent 1 and < representing 10.

In the development of world civilisation, western scholars were quite unanimous in choosing the Greek civilisation as the most important foundation in the development of civilisation in the west. They attribute the growth of ancient civilisation to the vigour and influence of the Greeks. This is especially apparent since it was during the time of the Greek civilisation that written works of scientific importance of their scholars were properly recorded. Because of the absence of such records previously it appears that none of the previous civilisations had any scholar of good standing to boast of.

The Greeks had employed various versions of number systems in the entire period of their civilisation, which are rather complicated and difficult to apply. For example they used what is known as the Graeco-Athic number system mainly centred in Athens. At the same time they used the Graeco-Ionia number system which employs the Greek alphabets to represent numbers.

Among the Greek mathematicians Pythagoras (580-500 B.C.) stood head and shoulders above the rest in making contributions to the study of numbers. His influence extends even to this day. His teachings to his disciples known as the Pythagoreans placed great emphasis on the role of numbers in 'cleansing' the souls and elevating them to a blissful plane. Thus numbers from his time on are known to have 'mystical' properties and whatever teachings he gave became closely guarded secrets known only to them.

Another giant among the Greek mathematicians is Euclid (320-260 B.C.). He made Alexandria his centre for expounding his ideas. His greatest contribution was his famous treatise known as the Elements, which remains to this day an often-quoted source in mathematics. According to Gittlemen (1975) "the Element is the most successful book in the world, second only to the Bible". The Elements records in it Euclid's mathematical ideas and also contains works of previous Greek scholars, including those of Pythagoras.

Archimedes (287-212 B.C.) and Erastosthenes (275-195 B.C.) are two other great scholars in the Greek civilisation who had contributed much to the study of numbers. Diophantus (c.250 B.C.) whose name has been immortalised in an area of active research in number theory in recognition of his outstanding contributions, the diophantine equations, was no less a giant in the Greeks field of mathematics.

Soon after the collapse of Alexandrian civilisation the Romans which had been wallowing in the shadows of the Greeks took centre stage. As early as 2000 B.C. on the banks of the Tyber in Rome the Roman civilisation grew. They were greatly influenced by the Greeks whom they conquered, in their scholarly works. They did not have many outstanding mathematics scholar to boast of. One of them Boethius (480-524 B.C.)

wrote on the Roman mathematics which was of a very inferior standard compared to those of the Greeks. The Romans however were instrumental in preserving and transmitting the works of Greek scholars to the subsequent civilisations.

The history of mathematics of India began in about 800 B.C. The period between 800 B.C. and 500 B.C. is known as the era of Sulvasutra ("strings arrangements") since during this time mathematical calculations were carried out by using strings. The calculations dealt mainly with problems related to religious purposes. This period is followed by the Siddhanta era, known to be an era of Renaissance for India, since during this period interests in studying the Hindu sciences grew. It is during this period that Sanskrit, the language of Hindu scholars, found its supremacy.

The Hindus had their own number systems especially since the days of Asoka the Great. They employed two types of number system known as the Brahmi and Kharosti. These numbers display some influence from the Greek and Roman numerical system. It is the Brahmi system that gives us a hint to the development of the present day system especially in giving different symbols for the first nine numbers. Whether the Hindu number system gave us the number zero (sunya) or not is debatable. The record by Gwalior (876 A.D.) was the earliest that mentions the use of this number by the Hindus. This however is subsequent to the use by the Muslims.

The Hindu civilisation contributed many mathematical scholars of its own. Between the 6th and 11th centuries Hindu mathematicians of the likes of Aryabhata, Varamahira, Brahmagupta and Mahavira were very active in their respective fields.

One of their eminent scholars, Sridhara (born 991 A.D.) also known as Sridharakarya (Sridhara the wise) contributed much to the Hindu mathematics through his work Gavita-Sara also known as Trisatika. In it number system is discussed at length and also its applications in measurements, arrangements and other problems in mathematics, and it gives instructions on proper use of mathematical operations. In the discussion on the number system Sridhara dwells on the topics of division and multiplications, square roots, fractions and gives an insight into the meanings of sunya (zero).

Bhaskara (1114-1185) a renown mathematician and astronomer wrote the treatise Lilavati in which are discussed basic properties of numbers and their roles in finding solutions to equations in more than one unknown.

The period of the Hindu civilisation saw studies on mathematics in general and number system in particular being carried out in earnest by their scholars. Much work that was produced during this period left an indelible mark on mathematics and especially in the present day number system. A treatise on astronomy which was written in the 5th century called Surya Siddhanta and contains some influence of Greek mathematics reached the courts of the Abbasid caliphs in Baghdad and was translated into Arabic which later found its way into the west.

Among the eastern civilisations the Chinese stands quite prominently as a contributor to the advancement of the studies on numbers. As early as 1105 B.C. the oldest known manuscript of Chinese mathematics was written. Known as Chou Pei Suan King it contains numerous mathematical notes of that era. Lia Hin (C.250 B.C.) wrote an important book called Chiu Chang Suan Shu (The Arithmetics of Nine Chapters). It contains discussions on the applications of mathematics in solving problems related to farming activities.

One of the earliest topics discussed in the Chinese mathematics is the magic squares. It was said to have been written in 2200 B.C. during the reign of Emperor Yu. The magic squares also became an engrossing topic of number theory in the Islamic and later the western civilisation.

In the 12th century contacts were made between Muslim scholars of mathematics and the Chinese in Samarqand. Prominent in expounding the Islamic mathematical ideas then was AlKashani who had many under his wings. The Chinese mathematics flourished soon after. In the 13th century a writer by the name of Chin Chiu Shao (1202-1261) wrote his master piece Shu-shu Chiu-Chang (Mathematics in nine chapters) in 1247. His book records for the first time the use of zero as a recognized entity, and deals with finding solutions to one-variable equations. Another prominent scholar Chu Shih-Chieh (1280-1303) discusses among others a triangle which later in the west became to be known as Pascal's triangle, in his book Ssu-yuan yu-chieu (The method of four elements) written in 1303.

Interest in the science of numbers and reckoning among Muslims goes back to the earliest Islamic centuries. At the beginning the Muslims distinguished between 'ilm al-adad (science of numbers) and 'ilm al-hisab (science of reckoning) following the Greeks. During the later centuries the two names were used almost interchangeably while the name arithmatiqi derived from the Greek was also employed by certain authors.

The interest with the science of numbers among Muslims was closely connected with the study of magic squares and amicable numbers which were also applied to various occult sciences from alchemy to magic. Two numbers are said to be amicable if the sum of their divisors are equal. For example 220 and 284 are a pair of amicable numbers. It used to be thought by some that if one person carries a talisman of some sort containing the number 220 and another with 284 then they would be favourably disposed to each other. Thabit ibn Qurrah had proposed a general rule for these types of numbers. Amicable numbers attracted the attention of respectable mathematicians even as late as the early twentieth century. Euler found many such pairs, and long lists of them exist. As far as magic squares are concerned they appeared in the writings of Jabir ibn Hayyan and were further studied by Ikhwan al-Safa. Studies on them were also carried out by Sham al-Din al-Buni. From such preoccupations came the study of numerical series. In the 10th century alKaraji in his Kitab al-Fakhri devoted a sizeable section to them while at-Biruni later wrote numerous studies on them.

In Persia the Islamic civilisation produced one of its outstranding and influential mathematician in the person of Shaykh Baha'al-Din Amili. He wrote his master piece Khulasat al-hisab which contains extensive discussions on number theory.

The influence of Islamic mathematics on the world today is extensive and profound. Whenever westerners think of Islamic civilisation, one of the first elements which comes to their minds is the Arabic numerals which reached the west in the 10th century. It brought about such a transformation in the west that some historians have compared their far-reaching significance to that of the new methods of harnessing the power and speed of the horse and the settling of the northern regions of Europe.

In the eastern lands of Islam extending as far as Egypt the numerals used are, with slight variations, as follows:

٩ ٨ ٧ ٦ ٥ ٤ ٣ ٢ ١ ٠

In North Africa the numerals are the same as those which westerners called Arabic numerals.

These number systems as practised today is born out of the marriage between the Islamic and Hindu number systems. According to Abraham ben Ezra, a Jewish mathematician of the 12th century, a Hindu scholar by the name of Kankah visited the courts of the caliph al-Mansur and introduced the first nine numbers to Muslim mathematicians. Al-Qifti a 13th century Muslim mathematician gave the date of the visit to be in the year 156H (778 A.D.). Abu Jaafar Mohammed Musa al-Khwarizmi was the scholar who worked with the system of Kankah's to arrive at the present system. His works were translated into Latin by the name of Algoritmi de numers Indorum and Liber Ysagogarum Alchorismi in artem astronomicum a magistro A compositus which were used in Europe in the teaching of mathematics and astronomy in the ensuing centuries. The Arabic numeral system win complete ascendancy over the then practised Roman one in Europe in the late 16th century.

With the adoption of the new number system works in number theory were greatly simplified and gained momentum in the hands of

Muslim and later western scholars. The history of number theory is then laden with new discoveries, conjectures and open questions. Research in this field has also led to the birth of new mathematical disciplines in their own right.

The word "algorithm" which we use widely in mathematical operations and more recently in connection with computing machines originates from the term "algorism" which originally comes from the name of one of Islamic civilisation's giant in mathematics Abu Jaafar Mohammed ibn Musa al-Khwarizmi (9th cent.) who wrote his master piece Kitab al jabr w'al-muqabala (Rules of Restoration and Reduction) from which the name algebra is derived. This book was translated into Latin by Robert de Chester in 1140 A.D.

Works by AlKhwarizmi, AlKhashani, AlKhazin, AlUqlidsi and many others had had such a great influence on the developments of number theory that their contributions remain to the present time a naturally quoted text in written materials on number theory.

Appeal of Number Theory

Due to the unquestioned historical importance of the subject, the theory of numbers has always occupied a unique position in the world of mathematics. It is one of the few disciplines that have demonstrable results which predates the very idea of a university or an academy. Nearly every century since classical antiquity has witnessed new and fascinating discoveries relating to the property of numbers. And at some point in their careers, most of the great masters of mathematical sciences have contributed to this body of knowledge. Because of its importance and commanding presence the great German mathematician C.F. Gauss (1777-1855) once said "Mathematics is the queen of science and number theory the queen of mathematics".

Why has number theory held such an irresistible appeal for the leading mathematicians and for thousands of amateurs? One answer lies in the basic nature of its problems. While many questions in the field are extremely hard to decide, they can be formulated in terms simple enough to arouse the interest and curiousity of those with little mathematical training. Some of the simplest sounding questions have withstood the intellectual assaults for ages and remain among the most elusive unsolved problems in the whole of mathematics.

Among the unsolved problems in number theory that have eluded the prowess of the human minds for centuries is the problem of showing that there do not exist triplets of positive integers (x,y,z) satisfying the equation.

$$x^n + y^n = z^n$$

with n a positive integer greater than 2. The solutions for (x,y,z) is infinitely many for n = 1, 2 as shown much earlier in history by AlKhazin (10th century), who generalised the work of the Greek Diophantus. Much work has been devoted to finding the proof of the above assertion for $n \geq 3$ since the days of the great French mathematician Pierre de Fermat (17th cent.) who wrote in his own copy of the book Arithmatecae of Diophantus:

"To resolve a cube into the sum of two cubes, a fourth power into fourth powers, or in general any power higher than the second into two of the same kind, is impossible; of which fact I have found a remarkable proof. The margin is too small to contain it".

Fermat died before he could publicise his proof. And if ever there was one it went with him to the grave. The above problem is then known as Fermat's Last Theorem.

Spurred by this incident many a mathematician, well-known and of lesser fame, had embarked on the search for the proof ever since. For example the mathematics giant of the 18th century, Leonard Euler had shown that the case is true for n=3. Later Dirichlet (1828) and Legendre (1830) demonstrated the truth of the assertion for n=5 and in 1832 Dirichlet proved the case for n=14. This was followed by Heath-Brown who showed that the percentage of n satisfying Fermat's Last Theorem as it is called would approach 100% as n assumes larger and larger values. Many claimed to have found the proof for the general case but all them contained flaws in their arguments. Before the first world war there was even a large prize offered in Germany for a correct proof. More than three centuries after Fermat precisely on 23rd of June 1993 in the quiet hall of Isaac Newton Institute of Mathematics, Cambridge University, Andrew Wiles of Princeton University presented a talk that gave more than a hint to the proof. In August 1995 at Boston University a complete proof was given by him assisted by other renown mathematicians on whose work Wiles had depended to obtain his proof.

The proof of Wiles contained in his paper "Modular Elliptic Curves and Fermat's Last Theorem" is considered to be correct up until now. It was a momentous occasion for the community of mathematicians through out the world on that day in June 1993. It was also a solemn moment for here at last is the proof that had escaped the human minds including those of great mathematicians more than three centuries since the time of Fermat.

The Number Builders

Among the community of numbers the most delightful, intriguing but frustrating are the prime numbers. These are numbers which cannot be expressed as product of other numbers. Examples of prime numbers are

$$2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, \ldots$$

The prime numbers are the 'building blocks' of other numbers as was known to the ancient Greeks. This is because every number can be written as a product of distinct powers of prime numbers. For example

$$2520 = 2^3 \times 3^2 \times 5 \times 7$$

$2^3, 3^2, 5, 7$ are called factors of 2520

The problem immediately arises of finding these primes for any given number. It is not hard to think of 'theoretical methods'. The "Sieve of Erathosthenes" developed by the Greek Erathosthenes (276 -196 BC) for example gave us a method for identifying primes. The method boils down to working out all possible products of primes and checking whether the number you want occurs anywhere. For practical purposes however this method is hopelessly inefficient. There remains a great deal of room for improvement and basic questions are still unanswered.

One of the sources of difficulty lies in the fact that there are infinitely many primes, which fact was also discovered by the Greeks. The argument to show that they are so is rather simple, elegant and compelling. Suppose 2, 3, 5,..., p be a list of all primes up to a certain prime p. We construct the number

$$N = 2 \times 3 \times 5 \times \ldots \times p + 1$$

This number is not divisible by any prime in our list since the division will always leaves a remainder of 1. However N has a prime factor which might be N itself if N is a prime. If it is not N then the prime has to be a number bigger than p. As a result, the list of prime numbers is never-ending.

The fact that prime numbers are infinite in number has driven many a number theorist to look for them, since antiquity. For example Kamaluddin al-Farisi a 13th century scholar studied these numbers (called al-asamm) critically based on his examinations of the works of Ikhwan-al-Safa, a group of Muslim scholars, and those of alKhwarizmi's. His study led him to hypothesise the existence of prime numbers of the form 2^n-1 where n is a positive integer. This is followed by other scholars of the likes of Thabit ibn Qurrah, and later Leonardo Fibonacci, Fermat and others. The prime numbers throughout the centuries increase in sizes as more and more of them are found. Since they are infinite in number the quest for finding bigger ones will continue as long as mathematicians are interested in them. The big prime numbers are all Mersenne primes, that is primes which are 1 less than a power of 2. In modern times most of the big primes are found through the use of computer technology. As the computing power increases more and more big primes are found. Up until 1992 the biggest prime number found is $2^{756839}-1$.

To our present knowledge there is no useful formula for calculating primes. This is part of the fascination one experiences in taking up the challenge to find bigger and bigger prime numbers. There is no certain knowledge of where or when the next Mersenne prime will make its appearance, or even if there is any more of them at all.

The second source of the difficulty lies in the schizophrenic but fascinating character of the primes. An expert number theorist, Don Zagler describes it thus:

"There are two facts about the prime numbers of which I hope to convince you so overwhelmingly that they will be permanently engraved in your hearts. The first is that they are the most arbitrary objects studied by mathematicians. They grow like weeds among the natural numbers, seeming to obey no other law than that of chance, and nobody can predict where the next one will sprout. The second fact states just the opposite that the prime numbers exhibit stunning regularity, and there are laws governing their behaviour and that they obey these laws with almost military precision".

Zagier is referring in particular to the extensive theories developed during the past century on the distribution of prime numbers. The Prime Number Theorem for example is one. It was first guessed by C.F. Gauss in 1792 based on numerical evidence. Later it was proved independently by J. Hadamard and Poussin in 1896. The theorem asserts that for a large number x the number of primes less than x denoted by $\pi(x)$ is approximately close to x/log x. This can be observed from the ratio $\frac{\Pi(x)}{x/\log x}$ obtained for some values of x as in Table 1.

x	$\pi(x)$	$\frac{\Pi(x)}{x/\log x}$
10	4	0.92
100	25	1.15
1000	168	1.15
10 000	1 229	1.14
100 000	9 592	1.11
1 000 000	78 498	1.09
10 000 000	664 57	1.08
100 000 000	5 761 455	1.06
1 000 000 000	50 847 534	1.05
10 000 000 000	455 052 512	1.04

Table 1

But this is only a good approximation for $\pi(x)$. In the quest for better approximation Riemann in 1860 saw that the prime numbers are intimately connected with a function now known as Riemann Zeta function. An exact formula for $\pi(x)$ is obtained in terms of the roots of this function. This led to the search for the whereabouts of these zeros. This task of locating roots of the Riemann Zeta function gives birth to the infamous Riemann hypothesis. Although this hypothesis is able to predict correctly the location of these zeros that is if we could list the zeros the hypothesis is still unable to help us to resolve our problem, as there are infinitely many zeros every one of which is important. Hence the ultimate answer to the question of the distribution of primes is still a long way off.

Of Rabbits Golden Rectangles and Sunflowers

Leonardo Fibonacci of Pisa Italy (born 1170) who was regarded as the first great mathematician of the Christian west studied an earlier work of Abu Kamil entitled Taarif al-Hisab (10th cent.) and based on this study wrote his famous work on mathematics called Liber Abacci (which means 'a book on abacus').

The book written in 1202 poses a question: how many pairs of rabbits can be produced from a single pair in one year if every month each pair produces one new pair and new pairs begin to bear young two months after their own birth? We can derive the build-up of the rabbit population to obtain the following sequence of numbers which counts the numbers of rabbit pairs in each of the calender month between January (when the first infant pair was introduced) and December as follows:

1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144

As we can observe each number, except for the first two, is the sum of the two previous ones. This sequence of numbers called "Fibonacci numbers" can be extended ~~un~~ in definitely. If we let F_1 to denote the first number F_2 the second and F_n the n^{th} number in the sequence we can arrive at the formula

$$F_n = F_{n-1} + F_{n-2}.$$

Using this formula we can always find the number at any position in the sequence, provided of course we already know the first two numbers in the sequence.

As we can see the numbers will grow rather rapidly in size. For example the 25th number in the the above series of numbers is already 75,025 while the 100th number is

$$F_{100} = 354, 224, 848, 179, 261, 915, 075$$

with 21 digits.

As these numbers grow they settle down into a simpler pattern. This we can observe when each Fibonacci number is divided by its next larger neighbour. Starting at the beginning with the first two ratios

$$\frac{F_1}{F_2} = 1, \frac{F_2}{F_3} = 0.5$$

we obtain the sequence:

1.000 000
0.500 000
0.666 666
0.600 000
0.625 000
0.615 385
0.619 048

0.617 647

0.618 182

0.617 978

0.618 056

0.618 026

0.618 037

0.618 033

0.618 034

0.618 034

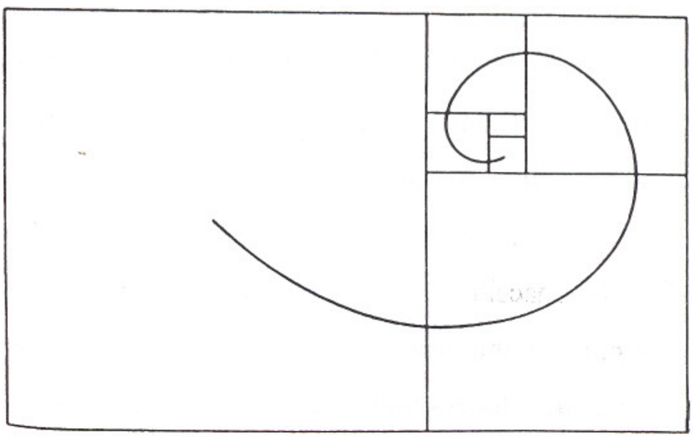
which settle down to the strange value 0.618 034...where the dots indicate the existence of more decimal places than the six decimal places given in the numbers above. In fact, in the limit of taking these "Fibonacci ratios" endlessly the number generated would approach ever closer to $(\sqrt{5}-1)/2$, an irrational number the root of the equation $x^2 + x - 1 = 0$, which in its decimal form is 0.618 033 989. This limiting ratio is often referred to as the "golden ratio".

Fascination with this particular number goes back for more than 2000 years. The ancients may not understand the mathematical aspects of this number the way we look at it but they know that art and architecture based on the golden ratio is pleasing to the eye. They defined the golden ratio as the point which divides a straight time into two parts in such a way that the ratio of the smaller to the larger is exactly equal to the ratio of the larger to the whole line. If we label the smaller part as x and the larger as 1 as below:

← x → ← 1 →

we will obtain the ratio $\frac{x}{1} = \frac{1}{1+x}$, from which we obtain the solution of x as 0.618033989 as above.

If a rectangle is drawn in which the ratio of the shorter to the longer sides is the golden ratic we will obtain an extremely famous piece of artwork known as the golden rectangle. If this rectangle is divided into a square and a smaller rectangle and we repeat the dissections over each rectangle produced ad infinitum then will obtain the figure below:



This infinite divisions will create an endless sequence of smaller and smaller squares and golden rectangles which spiral inward eventually to a point. If we connect the centres of these squares by a smooth curve we generate a spiral called the golden spiral. And this in turn will create a system of whirling squares. When incorporated into works of art this principle can produce illusions of movement. The term 'dynamic symmetry' has been used to describe this.

The golden ratio and the golden rectangle are frequently observed in Greek architecture as well as in pottery, sculpture, painting, furniture design and artistic design. The Greeks called the golden rectangle as the Divine Section. Fascinated by the golden ratio Leonardo da Vinci was known to have co-authored a book about them.

Fibonacci's spiral can be found extensively in nature, from the great galaxies of outer space that have arms which whirl outward in gigantic equianglular spirals to the shells of snails and cones and flowers found on earth.

The Fibonacci family of numbers has been the subject of study and intense interest over the centuries. One of the main reasons these numbers have been the centre of interest is the unexpected places these numbers occur in nature relating to plants, insects, flowers and the like. We can nearly always find the Fibonacci numbers in the arrangement of leaves on the stem of a plant or on the twigs of a tree. Perhaps the most well known appearances of the Fibonacci numbers is associated with the sunflower. In the head of a sunflower the seeds are found in small diamond-shaped pockets whose boundaries form spiral curves radiating out from the centre to the outside edge of the flower. If the number of clockwise and anticlockwise spirals in the pattern is counted you will almost always be rewarded with consecutive numbers of the Fibonacci sequence. In the figure below there are 13 clockwise and 21 anticlockwise spirals. Most real sunflower heads seem to have spirals of 34 and 55, although some smaller ones do have 21 and 34, while larger ones often contain 55 and 89. Examples with 89 and 144 have also been reported. The seed heads of most flowers and many other plant forms such as the leaves of the head of a lettuce, the layers of an onion, and the scale patterns of pineapples all contain the Fibonacci spirals.

Plato (427-347 B.C.) the Greek philosopher expounded among other profound thoughts of his, that the world was created on a mathematical basis. What is seen and felt by our senses are but approximations to reality hidden behind this physical world which only can be understood through mathematics. Are the above examples a reflection of the truth of Plato's thoughts?

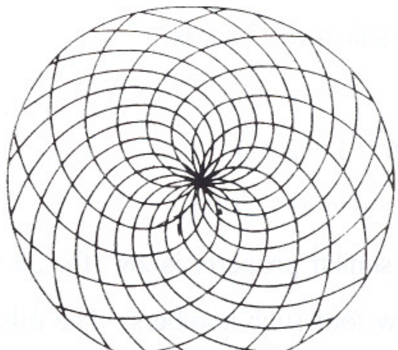


Figure of spirals in a sunflower head

Fibonacci numbers have remained to this day a fascinating object of research in its own right. In fact the literature on these numbers have become so large that a special journal called the Fibonacci Quarterly is devoted entirely to its properties, and papers of research on them have been presented in international conferences.

Perfect Numbers

The history of the theory of numbers abounds with famous conjectures and open questions. A number of the intriguing conjectures are associated with numbers that equal the sum of their factors excluding themselves. A few of these conjectures have been satisfactorily answered, but most remain unresolved. The study of these numbers was begun by the Greek Euclid (c.350BC) who wrote the great mathematical work the Elements. At the peak of the Greek-Egyptian civilisation the Pythagorean School devoted their attention to the study of these numbers. Nicomachus (c.100AD) a scholar of this school discussed this subject in his work Introductio Arithmeticae.

The Pythagoreans considered it rather remarkable that the number 6 is equal to the sum of its factors. That is

$$6 = 1 + 2 + 3$$

The next number with similar property is 28, for $28 = 1+2+4+7+14$. The Pythagoreans only knew four such numbers. The other two being 496 and 8198. In line with their philosophy of attributing mystical qualities to numbers, the Pythagoreans called such numbers "perfect".

For many centuries western philosophers in their beliefs were more concerned with the mystical or religious significance of perfect numbers than with their mathematical properties. For example St. Augustine explained that although God could have created the world all at once, He preferred to take six days because of the perfection of work is symbolized by the perfect number 6. Also early commentators on the Old Testament argued that the perfection of the Universe is represented by 28, the number of days it takes the moon to circle the earth. Similarly, the 8th century theologian, Alcuin of York, observed that the whole human race is descended from the 8 souls on Noah's Ark and that this second creation is less perfect than the first, 8 being an imperfect number.

Latter day scholars in the likes of Thabit ibn Qurrah, Ikhwan al-Safa, Ibn Sina, Fibonacci, Euler, Gauss and many others had these numbers studied and scrutinized their properties and made it their vocations to search for more of these numbers. This search has continued to this day.

In his Book IX of the Elements, Euclid showed that if the sum

$$p = 1 + 2 + 2^2 + \dots + 2^{k-1} = 2^k - 1$$

is a prime number, then 2^{k-1} p is a perfect number. In 1536 Regius in a work entitled Utrinsque Arithmetices showed that $2^{12}(2^{13}-1) = 33, 550, 336$ is a perfect number. In 1603, Cataldi found the sixth perfect number:

$$2^{16}(2^{17}-1) = 8, 589, 869, 056.$$

200 years after Euclid, Euler took a decisive step in proving that all even perfect numbers must be of the form

$$2^{k-1}\,(2^k-1) \text{ with } k>1.$$

We can show that 2^n-1 is composite if n is composite because if n=ab then

$$2^n-1 = (2^a-1)\,(2^{a(b-1)} + 2^{a(b-2)} + \dots + 1).$$

Hence 2^n-1 is prime only if n is prime. This follows that in the search for perfect numbers of the Euler form we only have to consider prime values of n. Every time a prime p is found such that 2^p-1 is prime, we can construct a perfect number.

The first few values of 2^p-1 are as follows

p	2	3	5	7	11	13	17
2^p-1	3	7	31	127	2047	8191	131,071

and all of these are prime except 2047 = 23.89. Thus we have 6 perfect numbers:

$$2(2^2-1) = 6$$
$$2^2(2^3-1) = 28$$
$$2^4(2^5-1) = 496$$
$$2^6(2^7-1) = 8,128$$
$$2^{12}(2^{13}-1) = 33,550,336$$
$$2^{16}(2^{17}-1) = 8,589,869,056$$

A question which immediately springs to our minds is whether there are infinitely many primes of the form 2^p-1 with p a prime. Unfortunately to answer this question is another famous unresolved problem.

Thus far the even perfect numbers determined by the formula $2^{p-1}(2^p-1)$ where p and 2^p-1 are primes are the only even perfect numbers known. As for odd perfect numbers no one knows if there are any and no one has proved that none can exist. It is known that if there is one it must be quite large. It was shown in 1967 if it exists it has to be bigger than 10^{36}.

The Theory of Congruences and Cryptography

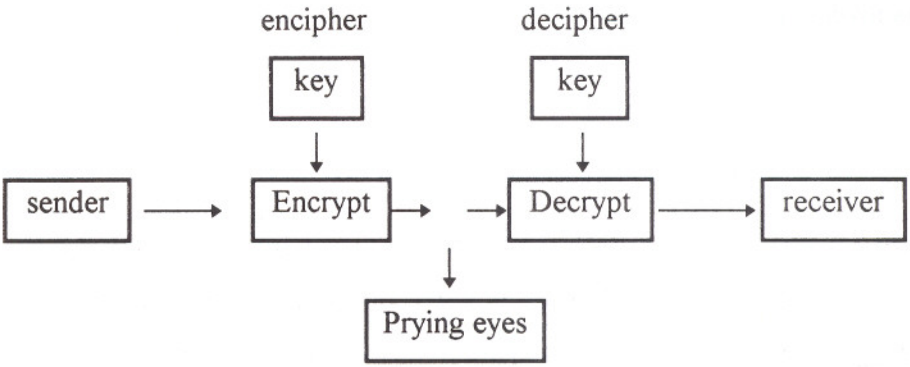
Any discussion on the development of number theory will not be complete if it does not include the theory of congruences. The theory deals with arithmetic of remainders or modular arithmetic. Two numbers a,b are said to be congruent moduls n, written as $a \equiv b \pmod{n}$ if a leaves a remainder b upon division by n. This means n divides exactly a-b with no remainder. Its concepts and notation are powerful tools in simplifying works in number theory. Although it is claimed that the German mathematician C.F. Gauss (1777-1855) is the founder of the theory, through his treatise Disquisitiones Arithmeticae, thus laying the foundations of modern number theory, the congruence theory was known to many earlier mathematician. According to Smith (1953), this method known as al-mizan was employed by al-Khwarizmi (died 825 AD). Datta and Singh (1938) wrote that this theory can be found in a Hindu text Mahasiddhanta written in 950 AD. Woepecke (1863) claimed that the theory was referred to by early Arab mathematicians as Tariq al-harasi. The Jewish mathematician Rabbi ben Ezra (12th cent.) had also studied the concept of al-mizan. In fact the Italian mathematician Leonardo Fibonacci was ahead of Gauss in discussing the theory of congruences which Fibonacci called pensa or portio in his Liber Abaci.

Fibonacci (1180-1250) had lived in the era of great Islamic intellectual activities. He had close relations with Muslim scholars of his time, which he maintained by visiting the then Muslim nation as a trader. He mastered the treatises Hisab-al Hind and al-Jabr Wa'lMuqabalah, in particular the commentaries made by Abu Kamil on the works of al-Khwarizmi. Fibonacci's works Liber Abaci, Liber Quadratorum, Flos and his other works played important roles in transmitting Islamic mathematical ideas into Europe.

On the other hand, Gauss's Disquisitiones Arithmeticae, a monumental work, appeared in 1801 when Gauss was only 24 years old. On Gauss's work the number theorist Leopold Kronecker remarked that, "It is really astonishing to think that a single man of such young years was able to bring to light such a wealth of results, and above all to present such a profound and well-organized treatment of an entirely new discipline". Is it possible that Gauss had in his possessions works of earlier mathematicians unknown to others? History is silent on this matter.

In recent times the hazards in commercial activities steeped in competition such as space travel and electronic banking and also the cold war between nations have generated exciting developments in secret codes. Central to this is the theory of numbers. Defence Establishments in some western countries consider it to be so important that they have the study of certain chosen aspects of number theory classified.

The public-key code is a species of code which makes it possible to transmit signed electronic mail. In conventional cryptography a secret key known only to the sender and receiver is employed but not known to any prying eyes as indicated below:



The main job here is to exchange just enough amount of key between sender and receiver and to keep it secret, for if the key is compromised at either end the prying eyes can have the message deciphered, by using a small amount of computer time. Consequently to maintain secrecy the enciphering keys must themselves be transmitted over a channel for secure communications.

To avoid assigning a key to each pair of individuals that must be kept secret from the rest of the network, a new type of cipher system, called a public-key system has been introduced recently. In this type of cipher system, enciphering keys can be made public, since an unrealistically large amount of computer time is required to find a deciphering transformations for an eciphering one.

Under this system the code requires two different keys one each for encrypting and decrypting. The code ensures that no knowledge of the encrypting key will help any 'outsider' in decrypting. The mechanism involved is a trap-door function. This is a function E for encryption

$$\text{message } x \xrightarrow{E} \text{cipher } E(x)$$

and its inverse D for decryption

$$\text{cipher } E(x) \xrightarrow{D} \text{message } D(E(x)) = x$$

with the important property that D cannot be discovered by examining E. Thus E is a 'trap-door' function which keeps the messages sent hidden and which only reveals them on the application of the special key D.

It is in the construction of the 'trap-door' functions that ingredients of number theory are applied, the first of which is the modular arithmetic and the second is Fermat's little theorem.

In 1640 Fermat discovered that if p ia a prime and b any integer not divisible by p then

$$b^{p-1} \equiv 1(\text{mod } p)$$

that is p divides exactly b^{p-1}-1. It can be shown from this theorem that if p and q are distinct primes and b any integer of which neither p or q is a factor then

$$b^{(p-1)(q-1)} \equiv 1(\text{mod } pq)$$

The RSA cipher system invented by Rivest, Ade Shamir, and Adleman in 1977 is a public key cipher system based on modular exponentiation where the keys are pairs of (e,n), consisting of an exponent e and a modulus n which is a product of two very large primes p,q i.e. n = pq so that e and (p-1)(q-1) do not have a common factor.

To encipher a message, we first translate the letters into their numerical equivalents, say A = 01, B = 02, C = 03,... so that we obtain a string of digits in place of letters in the message. The resulting string is then broken into blocks of 100 digits, each of which is then encrypted by the function

$$E(x) \equiv x^e (\bmod n)$$

The deciphering procedure requires knowledge of an inverse d of e modulo (p-1)(q-1). That is we look for the number d such that

$$ed \equiv 1 \ (\bmod\ (p-1)(q-1));$$

and then we use the decrypting function D given by

$$D(y) \equiv y^d (\bmod n).$$

This works because ed = 1 + k(p-1)(q-1) for some number k. Hence $D(E(X)) \equiv E(x)^d \equiv x^{ed} \equiv x^{1+k(p-1)(q-1)} \equiv x.x^{k(p-1)(q-1)} \equiv x (\bmod n)$.

The above calculations can be easily carried out by the computer. The point to notice is that E is a really trap door because the decrypting exponent d cannot be determined from the public information on n and e. n = pq has to be factorised first before d can be calculated. But n is a 100 digit number and it appears to be essentialy impossible to factorize such large numbers. The strength of the scheme depends on the fact that it is easy to find large primes but it is very difficult to factorize large numbers, for the moment at least. Perhaps the time will arrive when a new machine or method will be invented to carry out this task in future. Until such time we can rest assured that secret messages are safe coded this way.

G.H. Hardy perhaps the best known figure of British mathematician of twentieth century once wrote, "Both Gauss and lesser mathematicians may be justified in rejoicing that there is one science at any rate, and that their own, whose very remoteness from human activities should keep it clean and gentle". The prominent role that this "clean and gentle" science played in the newly invented public-key cryptosystems may serve as something of a reply to Hardy.

Exponential Sums

In the volume "Method of Fluxions" published by Colson in 1736, Newton introduced a diagram, later called Newton's parallelogram which he employed to find power series expansion for algebraic expansions. In the p-adic field Koblitz (1977) introduced the Newton polygon associated with a one-variable polynomial. p-adic field is a set of numbers written to the base p a prime number obeying certain algebraic rules. The Newton polygon is a p-adic analogue of the Newton's parallelogram. Fig. 1 gives an example. The polygon is a tool for Koblitz to ascertain the existence of roots of the polynomial with certain p-adic order. That is the highest power of a prime p dividing the zeros.

Earlier works on finding the estimates to the exponential sums $S=\sum \frac{2\pi i f(x,y)}{p^{\alpha}}$ were carried out by numerous researchers. These estimates are useful in finding solutions to problems in analytic number theory. The estimates are given in terms of quantities called 'discriminants'. These are numerical quantities derived from certain properties of variables associated with the sum. A new approach was undertaken by Deligne in giving the estimates in terms of some constants related indirectly to the polynomials. This is as a result of his proof of the Weil conjectures. He showed that if $S=\sum \exp\frac{2\pi i f(x)}{p}$, then $|S| \leq (m-1)^n p^{n/2}$. His work paved the way for other mathematicians but mainly restricted to one-variable polynomials.

In his work to find the estimates of an exponential sum of the form associated with a two-variable polynomial $S = \sum \exp\frac{2\pi i f(x,y)}{p^\alpha}$ Kamel Atan introduced the two-variables analogue of the Newton polygon called the Newton polyhedron from which he obtained the p-adic orders of common zeros of two polynomials with coefficients in the set of p-adic integers by using the 'indicator diagram' associated with the polyhedron.

Fig.2 is an example of the Newton polyhedron associated with the polynomial $f(x,y) = 3x^2y + xy + 3xy^2 + 9$ with the prime number $p = 3$. Fig. 3 gives its associated indicator diagram.

In order to estimate the exponential sum associated with a two-variable polynomial $f(x,y)$ it is necessary to know the cardinality V of the number of common solutions in the congruence equations of the polynomials $f_x(x,y)$ and $f_y(x,y)$ the partial derivatives of f with respect to x and y respectively modulo p^α.

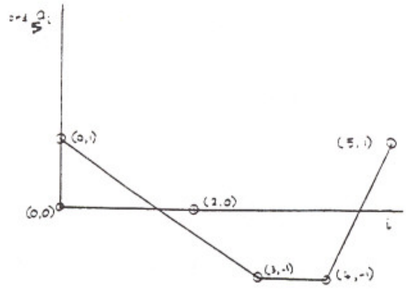


Fig. 1. Newton polygon of $f(x) = 5 + 3x^2 + \frac{1}{5}x^x + \frac{1}{5}x^4 + 5x^5$ with p=5.

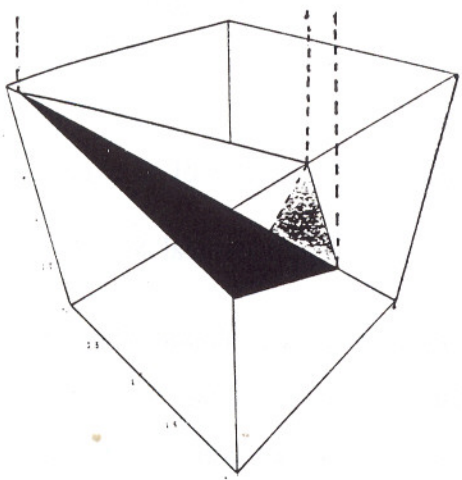


Fig. 2. The Newton polyhedron of polynomial $f(x,y) = 3x^2y+xy+3xy^2+9$ with $p = 3$.

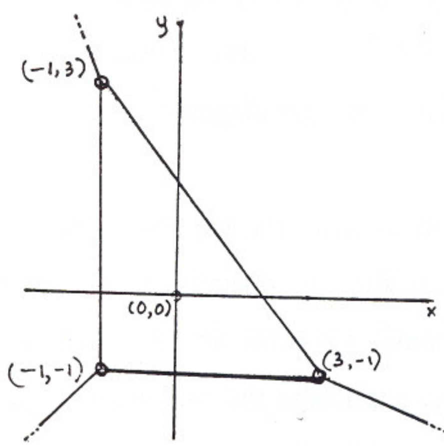


Fig. 3. Indicator diagram associated with the Newton polyhedron of $f(x,y) = 3x^2y + xy + 3xy^2 + 9$ with $p = 3$.

Kamel Atan showed that for a prime $p>3$ and the polynomial

$$f(x,y) = ax^3 + bx^2y + cxy^2 + dy^3 + ex + my + n$$ with coefficients in the ring of p-adic integers Z_p.

$$V \leq \min\{p^{2\alpha}, 4p^{\alpha+\delta}\}$$

where $\delta = \max\{ord_p a, ord_p b, ord_p c, ord_p d\}$. This result is arrived at by examining the combination of indicator diagrams associated with the Newton polyhedrons of $f_x(x,y)$ and $f_y(x,y)$.

By using the above result an estimation of the exponential sum associated with the polynomial above in terms of its coefficients can be arrived at. It is shown that

$$|S(f,p^\alpha)| \leq \min\{p^{2\alpha}, 4p^{\frac{3\alpha}{2}+\delta}\}$$

This result is very explicit and related closely to the coefficients of the homogeneous part of the polynomial in question. The search for better explicit estimates of the above is still being continued.

Epilogue

The development and evolution of number theory have come a long way since the days of the ancient civilisations. Contributions made by each civilisation towards enriching the knowledge in the study of numbers are immense. These contributions which came from different backgrounds and civilisations merge and become common property of every civilisation. The contributions made by the likes of Pythagoras, Diophantus, Brahmagupta, Al Uqlidisi, AlKhwarizmi, AlKhazin, AlTusi, Fibonacci, Chu Shih Chien, Kou Shou King, Pierre de Fermat, Goldbach, Gauss, Ramanujan, Erdos and numerous others will go down in history as flag bearers of number theoretical activities.

Many a problem in number theory have been studied and remained for many centuries to this day an engrossing and fascinating area of mathematical studies and many are unpenetrable to the intellectual assaults of mathematicians. In the process new strategies and ideas were born which become a field of study of their own and become the means to assist mathematicians in solving these old and other problems. For example to solve the so called Fermat's Last Theorem the methods employed by mathematicians have developed from a simple application of aspects of number theory to those of theories in modular elliptic curves in algebraic geometry. Along the way many a new concepts have sprung up. Concepts such as fields, algebraic numbers, cyclotomic arithmetic, ideal numbers, pseudoprimes, p-adic numbers, partition theory and many others are born through such works and become rich areas of mathematical research.

The tremendous enrichment of the field of mathematics was due to the sheer determination of earlier mathematicians in finding solutions to number theoretical problems. This is especially true in this century. Such sheer tenacity by the forebearers of active research in mathematics has set a standard to latter day scholars in this field in pursuing and opening up new frontiers of knowledge.

The examples that have been set by the mathematicians of the past must be made known to our young and budding mathematicians. It will provide a fertile background in which the interest in studying mathematics will grow and flourish. This is especially true since the birth of an idea, the beginning of a reaction or their appearances are usually the result of factors that create the fertile environment in which ideas germinate, grow and multiply. A true scholar's hunger for knowledge will not be satisfied and for some will diminish if he fails to appreciate the background and the origins of an idea. His understanding on a subject matter will be heightened and his appreciation of its beauty will be deepened if he could unravel the mysteries surrounding the birth of the area being studied. The soul of any man who only absorbs facts exuding from an already established area of study will be devoid of any such appreciation if he is ignorant of the origin and the causes that lead to the appearance of the subject, very similar to the soul of a person unaffected by the sight of the bare stone walls from the distant past uncovered by archaeologists. On the other hand the thoughts of a knowledgeable person will go beyond the image of the stone walls standing before him and he would 'see' the eventful and historic episodes that took place which lead to the construction of the walls. Through the walls he feels the presence of the past, and that makes him more knowledgeable and wiser.

Likewise in any field of knowledge appreciation of the 'experience' that the field has undergone to reach its present stage of development is important, as this will enable a scholar to be very familiar with and hence develop a true understanding of the nature of the subject being studied besides enriching his understanding of the development of the subject. A student's appreciation of the subject will undoubtedly grow when his mind is opened up and shown the journey that a topic in number theory say has travelled since ancient times through the ages to its present form. The development in number theory since antiquity witnesses the growth of the subject and also establishment of invisible network uniting scholars of opposing creeds, cultures and religions in their common pursuit to seek the hidden secrets of numbers and to unravel the mysteries enshrouding them. These pursuits are carried out in each civilisation either independent of each other or as an extension and continuations of activities of the previous cultures. The thread whose beginning is lost in antiquity runs through the ages transcending boundaries of every civilisation continues to this day attaining different face at every turn and phase in the journey. A student will appreciate the contributions made by scholars at every stage from every civilisation and generation in opening up new frontiers in the subject matter however difficult the task may be. Hence the role of history of mathematics in particular in ensuring such an appreciation grow among the students of mathematics is quite imperative.

Most universities in the west have this subject taught at undergraduate levels and become topics for research at postgraduate levels. Malaysian universities are yet to encourage earnestly the development of this subject at even undergraduate levels. If mathematics are to be taught effectively to our students the teaching of history of mathematics is a necessity and should be included in their curriculum.

In this limited time and space I have tried to present my science as best as I could, taking my audience first to a journey into the past into the age when number theory is born and nurtured and into the present where it has mushroomed into other areas and disciplines of study. In the last 100 years number theory has branched out into many areas in their own right. In this limited space it is not possible to discuss all of them.

I have also attempted to present the private face, the virgin beauty and fascination of the subject and its public use. To explain how number theory influences the growth of the whole of science would require a separate forum and more knowlegeable scholar. Suffice for me to requote from Loxton's "The love of numbers" the great Russion scholar Yu. I. Manin from Ann and Koblitz's text of their book "Mathematics and Physics":

"It is remarkable that the deepest ideas of number theory reveal a far-reaching resemblance to the ideas of modern theoretical physics. Like quantum mechanics, the theory of numbers furnishes completely non obvious patterns of relationship between the continuous and the discrete (the technique of Dirichlet series and trigonometric sums, p-adic numbers, non-archimedean analysis) and emphasises the role of hidden symmetries (class field theory, which describes the relationship between prime numbers and the Galois groups of algebraic number fields). One would hope that this resemblance is no accident, and that we are already learning new words about the world in which we live, but we do not yet understand their meaning".

A quote from the physicist Freeman Dyson: "One factor that has remained through out all the twists and turns of the history of physical science is the decisive importance of mathematical imagination. In every century in which major advances were achieved the growth in physical understanding was guided by a combination of empirical observation with purely mathematical intuition. For a physicist mathematics is not just a tool by means of which phenomena can be calculated; it is the main source of concepts and principles by means of which new theories can be created".

Numbers then hold the key to the doors of knowledge, befitting its position as the queen of mathematics which in turn is the queen of science.

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