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A POWERFUL ITERATIVE APPROACH FOR QUINTIC COMPLEX GINZBURG–LANDAU EQUATION WITHIN THE FRAME OF FRACTIONAL OPERATOR

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Abstract

The study of nonlinear phenomena associated with physical phenomena is a hot topic in the present era. The fundamental aim of this paper is to find the iterative solution for generalized quintic complex Ginzburg–Landau (GCGL) equation using fractional natural decomposition method (FNDM) within the frame of fractional calculus. We consider the projected equations by incorporating the Caputo fractional operator and investigate two examples for different initial values to present the efficiency and applicability of the FNDM. We presented the nature of the obtained results defined in three distinct cases and illustrated with the help of surfaces and contour plots for the particular value with respect to fractional order. Moreover, to present the accuracy and capture the nature of the obtained results, we present plots with different fractional order, and these plots show the essence of incorporating the fractional concept into the system exemplifying nonlinear complex phenomena. The present investigation confirms the efficiency and applicability of the considered method and fractional operators while analyzing phenomena in science and technology.

Keywords: Fractional Natural Decomposition Method; Caputo Derivative; Ginzburg–Landau Equation.

1. INTRODUCTION

Integral and differential calculus is the pair of pivotal tools employed to describe nature by variation and comparison. Differential equations (DEs) play a vital role in today's industrial needs. These are essential and stimulating because of most of the processes are associated to rates of change and which are meritoriously illustrated by them. Specifically, DEs deliver the notions for studying phenomena and developing ideas in economics, engineering, finance, medicine, and other associated areas of sciences. The analysis and investigation of these classes of equations are rooted in the study of rules which administrate most of the physical phenomena. Further, the analysis of nonlinear systems with integral and differential operators plays an important role while studying the phenomena related day-to-day life. While exemplifying real-world problems associated with complexity, when we examined its properties deeper and expanded it to study for the supplementary processes, each concept has its own limitations. In this regard, many researchers pointed out limitations of integer order calculus while studying the systems related to non-Markovian mechanisms, hereditary properties and others. In this connection, fractional calculus (FC) is also playing these types of tools, which is a generalization of classical calculus. It was rooted as soon as a classical concept as the meaning of extension. But it recently attracted scholars due to some stimulating and interesting consequences demonstrated by many authors to study and capture the behaviors of nonlinear real-world problems.[1–7]

The fundamental notions and associated rules of FC are extensively hired by many scholars to exemplify their viewpoints about the numerous classes of nonlinear phenomena. Specifically, the soliton solutions for optic problems, the mechanisms and processes exist in nanotechnology to study the behavior of human diseases with mathematical models to examine the behavior of systems related to chaotic behavior and many other activities and phenomena.[8–28] By incorporating this concept to any system, we get more degrees of freedom to examine the corresponding phenomena. However, day by day, new operators with respect to fractional calculus have been proposed by many researchers to overcome formerly acknowledged limitations and drawbacks.

The study of the Schrödinger equation has attracted great attention of researchers due to its wide applicability in diverse physical phenomena. It plays a vital role in quantum mechanics, particularly, state function or wave function of a quantum-mechanical system and governs the optical soliton propagation. In this study, the GCGL equation is reduced to the Schrödinger equation if the dissipative terms are neglected in this equation. In the model, quintic terms designate significant and essential physical properties, which is absent in other systems. The GCGL equation helps to model the non-equilibrium phenomena in physics. Specifically, it exemplifies erbium-doped fiber amplifiers and ultra-short pulse transmission with spectral filtering in optical transmission lines. The GCGL equation is a continuous approximation to the dynamics of the field in a passively mode-locked laser. It ascends in numerous phenomena, including statistical mechanics, condensed matter physics, mathematical biology, chemical physics, fluid dynamics, and more importantly in nonlinear optics.[29–32]

The GCGL equation aids us to analyze diverse practical and real-world models and it demonstrates rich dynamics and exemplar for the conversion to the chaos of spatio-temporal. Moreover, it helps to study hydro-dynamical stability models, nonlinear optics, reaction–diffusion systems, Rayleigh–Benard convection, Taylor–Coutte flow, Poiseuille flow, chemical turbulence and others.[29–44] In this paper, we consider the following GCGL equation with complex function $u(x,t)$:

$$u_t(x,t) = (1+i\beta)u_{xx} + \gamma u - (1+i\delta)|u|^{2n}u - (1+i\rho)|u|^{4n}u, \quad n \geq 1, \tag{1}$$

where β, γ, δ and ρ are real numbers. Here, γ denotes instability parameter and β represents a dispersive parameter.

On the other hand, the study of nonlinear analysis associated to daily life needs of living beings attracted all researchers due to its significance in modernization. As much as modeling with mathematical tools, finding the solution for the corresponding system is also vital. In this regard, there are numerous procedures available in the literature. Moreover, each algorithm essentially has its own needs and each one has its particular limitations. However, scholars are nurturing new methods by overcoming the limitations like huge computation, less accuracy, complex procedure, time taken for calculation and others. There are diverse schemes accessible in the literature with high accuracy among them. Adomian decomposition method (ADM) is one of the methods with great reliability and high accuracy.[45] Further, researchers are always experimenting and trying to propose new methods by modifying, nurturing, mixing or improving existing methods. In this connection, Rawashdeh and Maitama proposed a method with a mixture of ADM and natural transform (NT) in Ref. 46. Since the last two years, it has been comprehensively employed by many authors to study the diverse family of problems, for instance, authors in Ref. 47 wrote an equation to study the falling film phenomena and presented some numerical behaviors in comparison with another scheme, the numerical study has been illustrated by authors to show the accuracy of the projected scheme by authors in Ref. 48 for special cases of Korteweg–de Vries equations. The system of equation exemplifying 2019-nCoV is analyzed by researchers in Ref. 49, for the biological model, authors in Ref. 50 found the solution using FNDM within the frame of fractional calculus, the efficiency of the hired method is confirmed by solving the ordinary equation of fractional order by the author in Ref. 51, and many others hired this algorithm and ensured the accuracy and reliability.[52–56]

The fractional case of Eq. (1) is hired in the present framework by changing the time derivative with fractional derivative as follows:

$$D_t^{\alpha}u(x,t) = (1+i\beta)u_{xx} + \gamma u - (1+i\delta)|u|^{2n}u - (1+i\rho)|u|^{4n}u, \quad 0 < \alpha \leq 1, \tag{2}$$

where α is fractional order and which is defined with Caputo operator.

Recently, many researchers analyzed and derived some interesting results for the Ginzburg–Landau equation using distinct algorithms. For instance, the approximated solution is evaluated by authors in Ref. 33 using decomposition and homotopy perturbation schemes, the pulse-like solutions with stability are illustrated by researchers in Ref. 34, the global existence is examined by authors in Ref. 35 for the GL equation and they also discussed dispersion limit for the corresponding equation, for the one-dimensional case of the considered equation, the exact solution is derived in Ref. 36 and some interesting results have been derived, the series solution is derived by author in Ref. 37 using homotopy analysis scheme. Authors in Ref. 38 derived the bifurcation and stability, some stimulating results related to patterns of sources and sinks of the complex model are presented in Ref. 39, the physical behavior and effect of boundary in the hired model are analyzed by authors in Ref. 40, the hired model is analyzed and the solution found by using homotopy analysis algorithm and presented along with numerical stimulation in Ref. 40, and many other researchers investigated stimulating results for the projected system.[42–44]

In this paper, we find the solution for fractional order differential equation cited in Eq. (2) with the help of FNDM. Further, the behavior of the outcomes is captured with respect to fractional order. Particularly, the complex behavior has been captured. The rest of the work is arranged as follows: in the following section, we present the basic notions of FC and FNDM, which are recalled to derive the required results. The fundamental solution procedure of the projected algorithm is illustrated in Sec. 3 with the Caputo fractional operator. With the help of the basic procedure of FNDM, we find the solution for FGCGL equation in Sec. 4 with two different examples. Further, with respect to the obtained solution, we present the numerical results and discussion in Sec. 5 and finally, we conclude the archived results with respect to the considered method, fractional operator and model.

2. PRELIMINARIES

In this segment, we present some basic and essentials notions of FC.

Definition 1. The integral of a function $f(t) \in C_\delta(\delta \geq -1)$ with respect to fractional Riemann–Liouville is presented[1–6] as

$$J^\alpha f(t) = \frac{1}{\Gamma(\mu)}\int_0^t (t-\vartheta)^{\mu-1} f(\vartheta)d\vartheta. \tag{3}$$

Definition 2. The Caputo fractional derivative of $f \in C_{-1}^n$ is presented[2–7] as

$$D_t^\alpha f(t) = \begin{cases} \dfrac{d^n f(t)}{dt^n}, & \alpha = n \in \mathbb{N}, \\ \dfrac{1}{\Gamma(n-\alpha)}\displaystyle\int_0^t (t-\vartheta)^{n-\alpha-1} f^{(n)}(\vartheta)d\vartheta, & n-1<\alpha<n, \\ & n \in \mathbb{N}. \end{cases} \tag{4}$$

Definition 3. For the one-parameter, the Mittag-Leffler type function is presented[57] as

$$E_\alpha(z) = \sum_{k=0}^{\infty} \frac{z^k}{\Gamma(\alpha k+1)}, \quad \alpha > 0, \quad z \in \mathbb{C}. \tag{5}$$

Definition 4. For the function $f(t)$, the natural transform (NT) is denoted by $\mathbb{N}[f(t)]$ for $t \in \mathbb{R}$ and presented with the NT variables s and ω by[50]

$$\mathbb{N}[f(t)] = R(s,\omega) = \int_{-\infty}^{\infty} e^{-st} f(\omega t)dt; \quad s,\omega \in (-\infty,\infty).$$

Now, using Heaviside function $H(t)$, we define the NT as

$$\mathbb{N}[f(t)H(t)] = \mathbb{N}^+[f(t)] = R^+(s,\omega) = \int_0^\infty e^{-st} f(\omega t)dt; \quad s,\omega \in (0,\infty) \quad \text{and} \quad t \in \mathbb{R}. \tag{6}$$

Further, for $\omega = 1$, Eq. (6) is reduced to the Laplace transform, for $s = 1$. Eq. (6) denotes the Sumudu transform.

Theorem 1 (Ref. 58). *In Riemann–Liouville sense, the NT $R_\alpha(s,\omega)$ of the fractional derivative of $f(t)$ is presented as*

$$\mathbb{N}^+[D^\alpha f(t)] = R_\alpha(s,\omega) = \frac{s^\alpha}{\omega^\alpha}R(s,\omega) - \sum_{k=0}^{n-1} \frac{s^k}{\omega^{\alpha-k}}[D^{\alpha-k-1}f(t)]_{t=0}, \tag{7}$$

where $R(s,\omega)$ is NT of $f(t)$, α is the order and n is any positive integer. Further $n-1 \leq \alpha < n$.

Theorem 2 (Ref. 59). *In Caputo sense, the NT of the fractional derivative of $f(t)$ symbolizes $R_\alpha(s,\omega)$ and is defined as*

$$\mathbb{N}^+[{}^cD^\alpha f(t)] = R_\alpha^c(s,\omega) = \frac{s^\alpha}{\omega^\alpha}R(s,\omega) - \sum_{k=0}^{n-1} \frac{s^{\alpha-(k+1)}}{\omega^{\alpha-k}}[D^k f(t)]_{t=0}. \tag{8}$$

3. BASIC ROLE OF THE CONSIDERED METHOD

In order to demonstrate the fundamental theory and solution procedure of FNDM, we consider

$$D_t^\alpha v(x,t) + Rv(x,t) + Fv(x,t) = h(x,t). \tag{9}$$

With the initial condition

$$v(x,0) = g(x), \tag{10}$$

where $D^\alpha v(x,t)$ signifies the fractional Caputo derivative of $v(x,t)$, R and F, respectively, are the linear and nonlinear differential operator, $h(x,t)$ is the source term. On employing NT and by the aid of Theorem 2, Eq. (9) gives

$$V(x,s,\omega) = \frac{v^\alpha}{s^\alpha}\sum_{k=0}^{n-1}\frac{s^{\alpha-(k+1)}}{\omega^{\alpha-k}}[D^k v(x,t)]_{t=0} + \frac{\omega^\alpha}{s^\alpha}\mathbb{N}^+[h(x,t)] - \frac{\omega^\alpha}{s^\alpha}\mathbb{N}^+[Rv(x,t)+Fv(x,t)]. \tag{11}$$

Apply the inverse NT on Eq. (11) to get

$$v(x,t) = H(x,t) - \mathbb{N}^{-1} \times \left[\frac{\omega^\alpha}{s^\alpha}\mathbb{N}^+[Rv(x,t)+Fv(x,t)]\right]. \tag{12}$$

From non-homogeneous terms and the given initial condition, $H(x,t)$ exists. The infinite series solution is presented as

$$v(x,t) = \sum_{n=0}^{\infty} v_n(x,t), \quad Fv(x,t) = \sum_{n=0}^{\infty} A_n, \tag{13}$$

where the A_n indicates the nonlinear terms of $Fv(x,t)$. By using Eqs. (12) and (13), we have

$$\sum_{n=0}^{\infty} v_n(x,t) = H(x,t) - \mathbb{N}^{-1}\left[\frac{\omega^\alpha}{s^\alpha}\mathbb{N}^+ \times \left[R\sum_{n=0}^{\infty} v_n(x,t)\right] + \sum_{n=0}^{\infty} A_n\right]. \tag{14}$$

On relating both sides of Eq. (14), we obtain

$$v_0(x,t) = H(x,t),$$

$$v_1(x,t) = -\mathbb{N}^{-1}\left[\frac{\omega^\alpha}{s^\alpha}\mathbb{N}^+[Rv_0(x,t)] + A_0\right],$$

$$v_2(x,t) = -\mathbb{N}^{-1}\left[\frac{\omega^\alpha}{s^\alpha}\mathbb{N}^+[Rv_1(x,t)] + A_1\right].$$

$$\vdots$$

For $n \geq 1$, we can similarly achieve the recursive relation in general form and present it as

$$v_{n+1}(x,t) = -\mathbb{N}^{-1}\left[\frac{\omega^\alpha}{s^\alpha}\mathbb{N}^+[Rv_n(x,t)] + A_n\right]. \tag{15}$$

Finally, the approximate solutions are presented as

$$v(x,t) = \sum_{n=0}^{\infty} v_n(x,t).$$

4. SOLUTION FOR GCGL EQUATION

Here, we hired the fractional GCGL equation to present the solution for the corresponding equation. We consider two examples to illustrate the reliability of the projected scheme.

Example 4.1. Consider the following time-fractional GCGL equation:

$$D_t^\alpha u(x,t) - (1+i)u_{xx} - 3u + (1+2i)|u|^2 u + (1-4i)|u|^4 u = 0, \quad 0 < \alpha \leq 1, \tag{16}$$

subjected to the initial condition

$$u(x,0) = e^{ix}. \tag{17}$$

By applying NT on Eq. (16), one can get

$$\mathbb{N}^+[D_t^\alpha u(x,t)] = \mathbb{N}^+\left[(1+i)\frac{\partial^2 u}{\partial x^2} + 3u - (1+2i)|u|^2 u - (1-4i)|u|^4 u\right]. \tag{18}$$

The nonlinear operator is defined as

$$\frac{s^\alpha}{w^\alpha}\mathbb{N}^+[u(x,t)] - \sum_{k=0}^{n-1}\frac{w^{k-\alpha}}{s^{k+1-\alpha}}[D^k u]_{t=0} = \mathbb{N}^+\left[(1+i)\frac{\partial^2 u}{\partial x^2} + 3u - (1+2i)|u|^2 u - (1-4i)|u|^4 u\right]. \tag{19}$$

By Eqs. (18) and (19), we get

$$\mathbb{N}^+[u(x,t)] = e^{ix} + \frac{w^\alpha}{s^\alpha}\mathbb{N}^+\left[(1+i)\frac{\partial^2 u}{\partial x^2} + 3u - (1+2i)|u|^2 u - (1-4i)|u|^4 u\right]. \tag{20}$$

By plugging inverse NT to Eq. (20), we have

$$u(x,t) = e^{ix} + \mathbb{N}^{-1}\left[\frac{w^\alpha}{s^\alpha}\mathbb{N}^+\left[(1+i)\frac{\partial^2 u}{\partial x^2} + 3u - (1+2i)|u|^2u - (1-4i)|u|^4u\right]\right]. \quad (21)$$

Let $u(x,t) = \sum_{n=0}^{\infty} u_n(x,t)$ be the infinite series solution of $u(x,t)$. Note that $|u|^2u = \sum_{n=0}^{\infty} A_n$ and $|u|^4u = \sum_{n=0}^{\infty} B_n$ are the Adomian polynomials and signify the nonlinear terms. With the assistance of these, Eq. (21) becomes

$$\sum_{n=0}^{\infty} u_n(r,t) = e^{ix} + \mathbb{N}^{-1}\left[\frac{w^\alpha}{s^\alpha}\mathbb{N}^+\left[(1+i)\frac{\partial^2}{\partial x^2} \times \sum_{n=0}^{\infty} u_n + 3\sum_{n=0}^{\infty} u_n - (1+2i)\sum_{n=0}^{\infty} A_n - (1-4i)\sum_{n=0}^{\infty} B_n\right]\right]. \quad (22)$$

On solving the following equations with the help of $u_0(x,t)$ and the above system, we can evaluate the series terms

$$u(x,t) = \sum_{n=0}^{\infty} u_n(x,t) = u_0(x,t) + u_1(x,t) + u_2(x,t) + \cdots$$

The analytical solution for the above equation is

$$u(x,t) = e^{i(x+t)}.$$

Example 4.2. Consider the following time-fractional GCGL equation

$$D_t^\alpha u(x,t) - (1+3i)u_{xx} - 3u + (1+i)|u|^4u + (1-6i)|u|^8u = 0, \quad 0 < \alpha \le 1, \quad (23)$$

subjected to the initial condition

$$u(x,0) = e^{-ix}. \quad (24)$$

By applying NT on Eq. (23), we obtain

$$\mathbb{N}^+[D_t^\alpha u(x,t)] = \mathbb{N}^+\left[(1+3i)\frac{\partial^2 u}{\partial x^2} + 3u - (1+i)|u|^4u - (1-6i)|u|^8u\right]. \quad (25)$$

The nonlinear operator is defined as

$$\frac{s^\alpha}{w^\alpha}\mathbb{N}^+[u(x,t)] - \sum_{k=0}^{n-1}\frac{w^{k-\alpha}}{s^{k+1-\alpha}}[D^k u]_{t=0} = \mathbb{N}^+\left[(1+3i)\frac{\partial^2 u}{\partial x^2} + 3u - (1+i)|u|^4u - (1-6i)|u|^8u\right]. \quad (26)$$

By Eqs. (25) and (26), we get

$$\mathbb{N}^+[u(x,t)] = e^{-ix} + \frac{w^\alpha}{s^\alpha}\mathbb{N}^+ \times \left[(1+3i)\frac{\partial^2 u}{\partial x^2} + 3u - (1+i)|u|^4u - (1-6i)|u|^8u\right]. \quad (27)$$

By plugging inverse NT to Eq. (27), we have

$$u(x,t) = e^{-ix} + \mathbb{N}^{-1}\left[\frac{w^\alpha}{s^\alpha}\mathbb{N}^+\left[(1+3i)\frac{\partial^2 u}{\partial x^2} + 3u - (1+i)|u|^4u - (1-6i)|u|^8u\right]\right]. \quad (28)$$

Let $u(x,t) = \sum_{n=0}^{\infty} u_n(x,t)$ be the infinite series solution of $u(x,t)$. Note that $|u|^4u = \sum_{n=0}^{\infty} A_n$ and $|u|^8u = \sum_{n=0}^{\infty} B_n$ are the Adomian polynomials and signifying the nonlinear terms. With the assistance of these, Eq. (28) becomes

$$\sum_{n=0}^{\infty} u_n(r,t) = e^{ix} + \mathbb{N}^{-1}\left[\frac{w^\alpha}{s^\alpha}\mathbb{N}^+\left[(1+3i)\frac{\partial^2}{\partial x^2}\sum_{n=0}^{\infty} u_n + 3\sum_{n=0}^{\infty} u_n - (1+i)\sum_{n=0}^{\infty} A_n - (1-6i) \times \sum_{n=0}^{\infty} B_n\right]\right]. \quad (29)$$

On solving the following equations with the help of $u_0(x,t)$ and above system, we can find the series

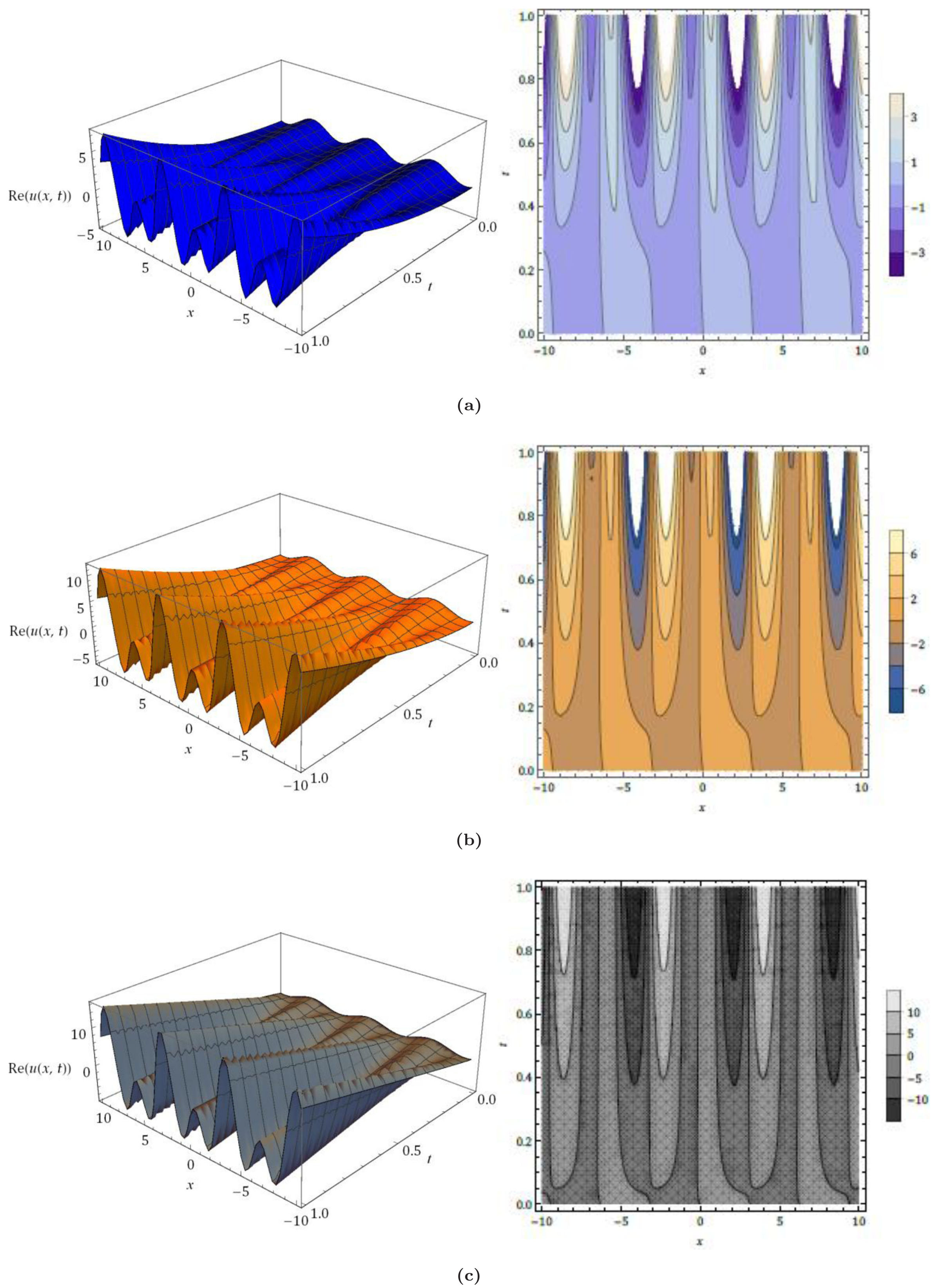

Fig. 1 Nature of obtained solution in surface and contour plots at **(a)** $\alpha = 1$, **(b)** $\alpha = 0.75$ and **(c)** $\alpha = 0.50$ for Example 4.1.

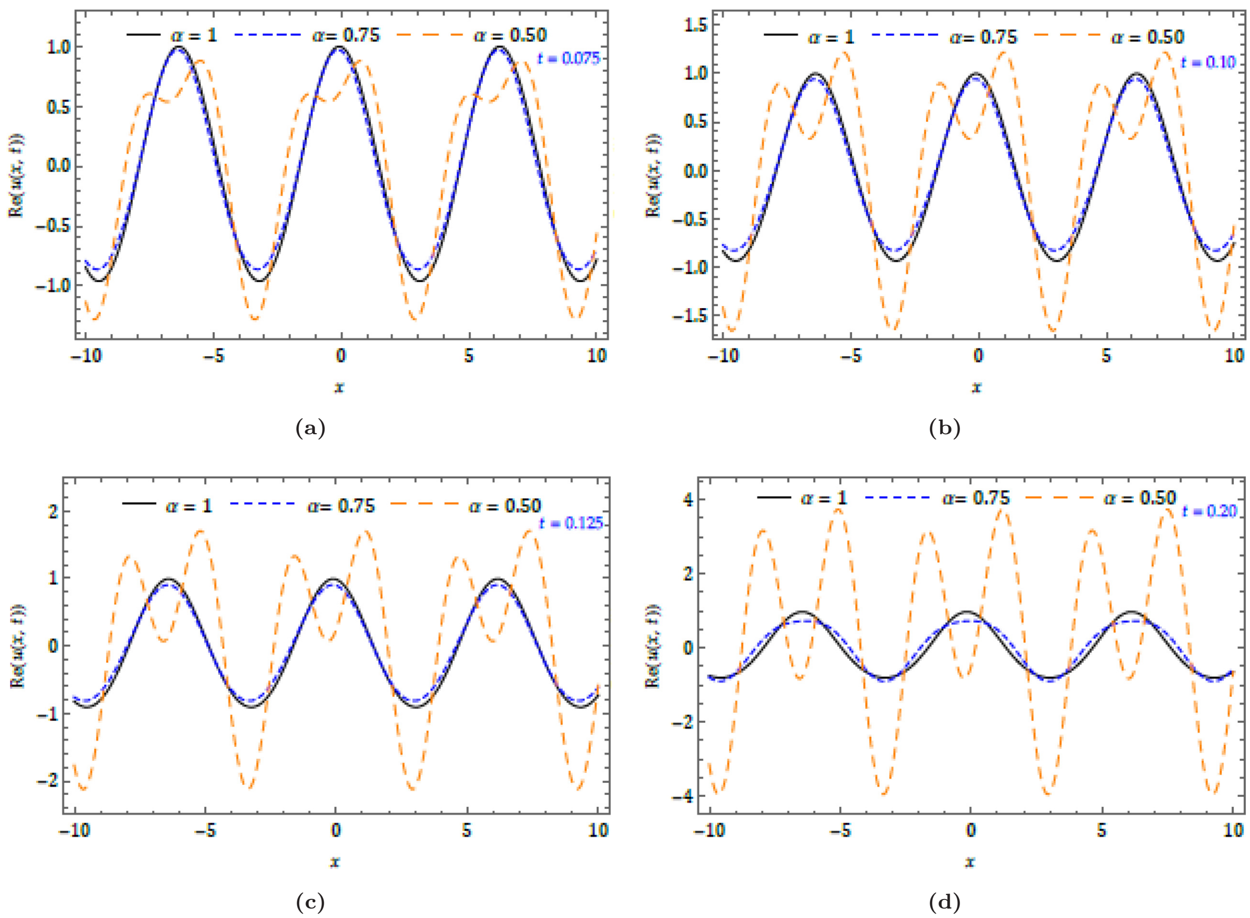

Fig. 2 Behavior of real part of the FNDM results for Example 4.1 with different α at **(a)** $t = 0.075$, **(b)** $t = 0.10$, **(c)** $t = 0.125$ and **(d)** $t = 0.20$.

terms

$$u(x,t) = \sum_{n=0}^{\infty} u_n(x,t) = u_0(x,t) + u_1(x,t) + u_2(x,t) + \cdots$$

The analytical solution for the above equation is

$$u(x,t) = e^{i(2t-x)}.$$

5. NUMERICAL RESULTS AND DISCUSSION

Owing to difficulty in modeling of the real-world problems related to sciences and engineering, achieving the solution for the associated equations or systems is also challenging and important. In this work, we analyzed the complex model which exemplifies the non-equilibrium phenomena in physics, called the GCGL equation, with arbitrary order using the reliable and accurate scheme, namely, FNDM. To confirm the exactness, we consider two examples with fractional order in Caputo sense. Here, we analyze and illustrate the behavior of the results achieved by the projected solution procedure. For different order, its nature has been presented in Figs. 1 and 2 with respect to Example 4.1. In Fig. 1, we presented the nature of real part in 3D plots and the behavior of the imaginary part in contour plots at $\alpha = 0.50, 0.75$ and 1 for Example 4.1 and from these we can observe that when we generalize derivative with arbitrary order we get stimulating changes in the behavior. These types of study can open the doors for new understanding of the corresponding models. In Fig. 2, we illustrated the nature with different time at distinct order to exemplify the essence fractional order and time. Similarly, for Example 4.2, we have presented it in Figs. 3 and 4. Moreover, these plots exemplify the essence of generalizing classical concept with the fractional operator to obtain more corresponding consequences and degree of freedom in a systematic and methodical way.

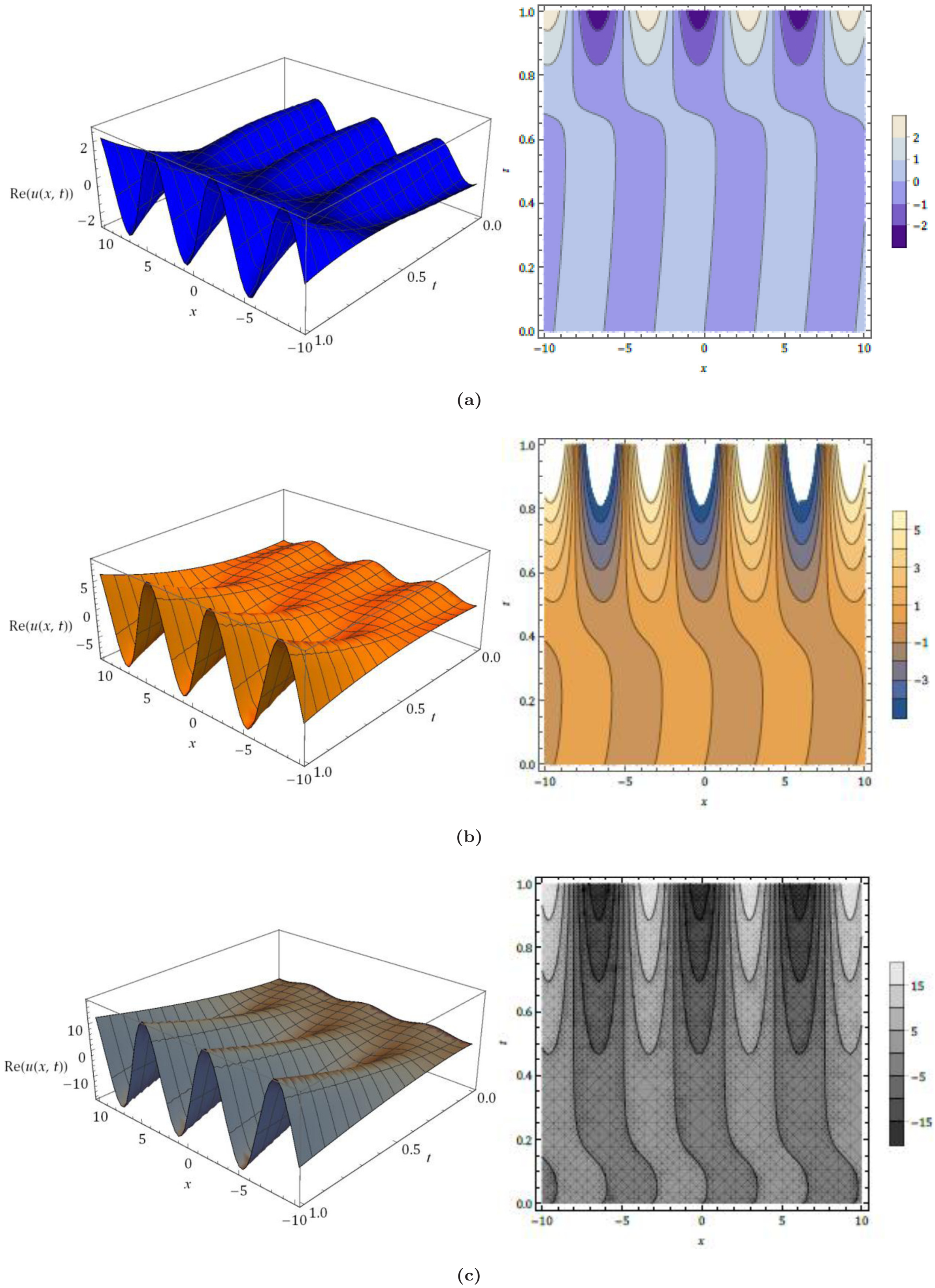

Fig. 3 Nature of obtained solution in surface and contour plots at **(a)** $\alpha = 1$, **(b)** $\alpha = 0.75$ and **(c)** $\alpha = 0.50$ for Example 4.2.

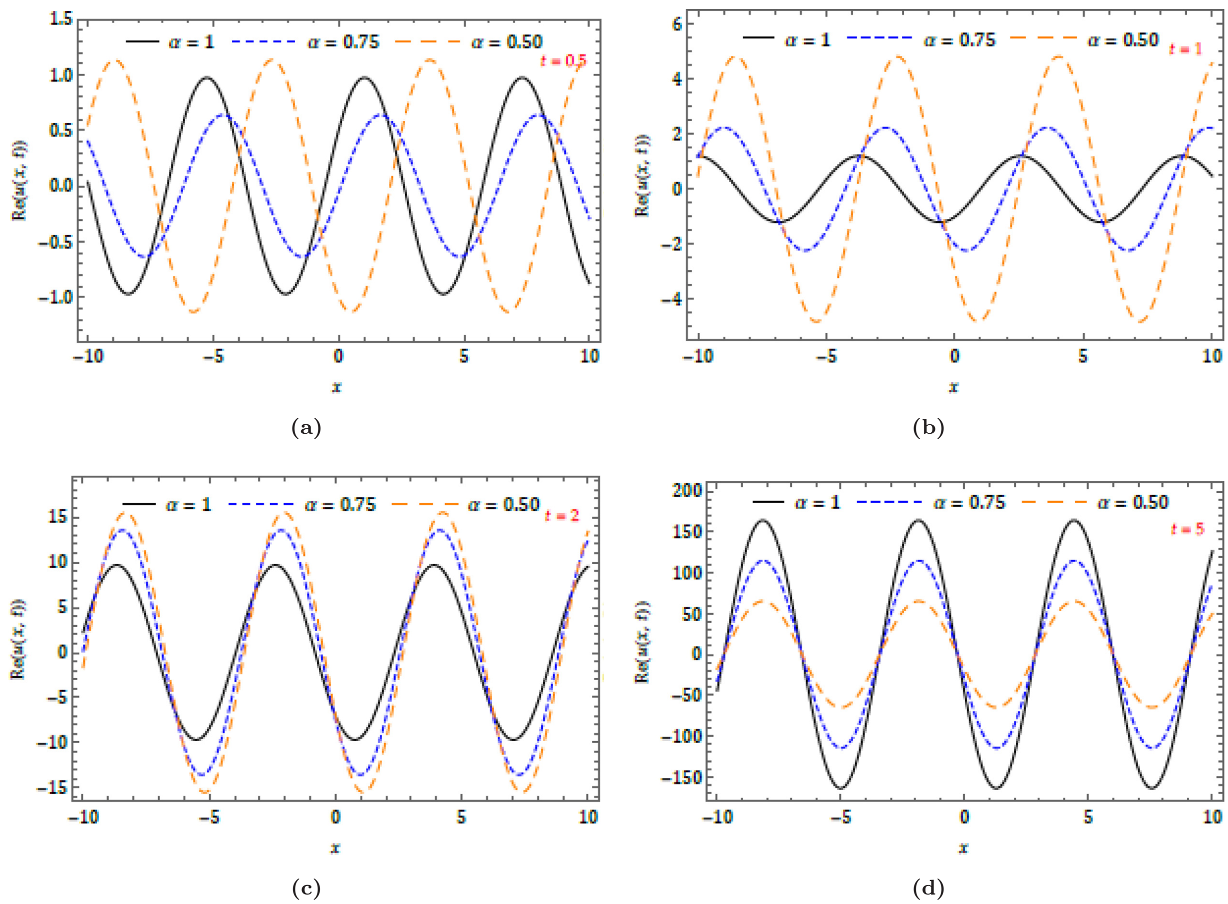

Fig. 4 Behavior of real part of the FNDM results for Example 4.2 with different α at **(a)** $t = 0.075$, **(b)** $t = 0.10$, **(c)** $t = 0.125$ and **(d)** $t = 0.20$.

6. CONCLUSION

The analysis and investigation of nonlinear physical models with new tools always aid us to moderate and for further development in science and technology. With the help of FNDM, we analyzed the GCGL equation with fractional order in the present framework. The reliability and employability of projected method are ensured by illustrating two examples. For the achieved results, the behaviors are presented in 3D and contour plots for distinct fractional order. These plots help to derive stimulating behaviors of the corresponding models. Moreover, FNDM is not required perturbation, conversion and consideration of any additional parameters or polynomials while finding the solution for nonlinear problems. The analysis of these categories of phenomena can offer novel notions to examine more real-world phenomena and it can release notions for hiring an accurate technique to examine nonlinear models associated with science and technology. This study elucidates the projected model, which remarkably depends on time instant and its history, and which can be persuasively exemplified using fractional concept. The present framework can help the young scholars to analyze the behavior of the various models using the considered method and fractional operator, and also gives very useful and interesting consequences.

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