A Computational Performance Comparison of MILP vs. MINLP Formulations for Oil Production Optimisation

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ABSTRACT

Model-based oil production systems optimisation under pressure and facility routing constraints is a testing challenge, especially in presence of complex downhole wellbore phenomena (water coning, slugging, phase separation). Nonlinearities and nonconvexities from underlying physics and binary decisions exacerbate model complexity, yielding Mixed Integer Nonlinear Programs (MINLP). To guarantee solvability of optimisation formulations and reduce MINLP complexity, piecewise linearisation techniques based on Special Ordered Sets of type 2 (SOS2) constraints are developed towards approximating nonlinear functions and transforming models to Mixed Integer Linear Programs (MILP). Nevertheless, computational analyses of MILP vs. MINLP formulations for oil production optimisation are scarce. This study explores the benefits of an MILP reformulation applied to three case studies of varying complexity. We compare MILP model results to original MINLP formulation solutions with multiple solvers, evaluating the impact of the number of linearisation breakpoints used on solution time, accuracy, robustness, model development effort and ease of automation.

Keywords: Real-Time Production Optimisation (RTPO); Mixed Integer Linear Programming (MILP); Mixed-Integer Nonlinear Programming (MINLP); Well routing; Electrical Submersible Pumps (ESP); Progressive Cavity Pumps (PCP)

1. Introduction

Oil and gas exploration and management activities are key to meeting the world's global energy demands. In response to this, petroleum exploration companies constantly seek innovative technologies that aid the hydrocarbon recovery process, with relatively cheap implementation costs (Redutskiy, 2017). Although several visualisation and analysis tools have continued to experience increased development, heuristic approaches currently dominate the production decisions in the upstream sector of the oil and gas industry (Grimstad, 2015). Compared to the downstream industry which has received significant attention from the process systems engineering (PSE) community, the upstream industry has hardly been penetrated by PSE tools, especially from the standpoint of real-time optimisation (Epelle and Gerogiorgis, 2019a; 2019b). Hence, there has been an increased incentive to apply mathematical optimisation for the recovery improvement of hydrocarbon reserves (Gunnerud and Foss, 2010).

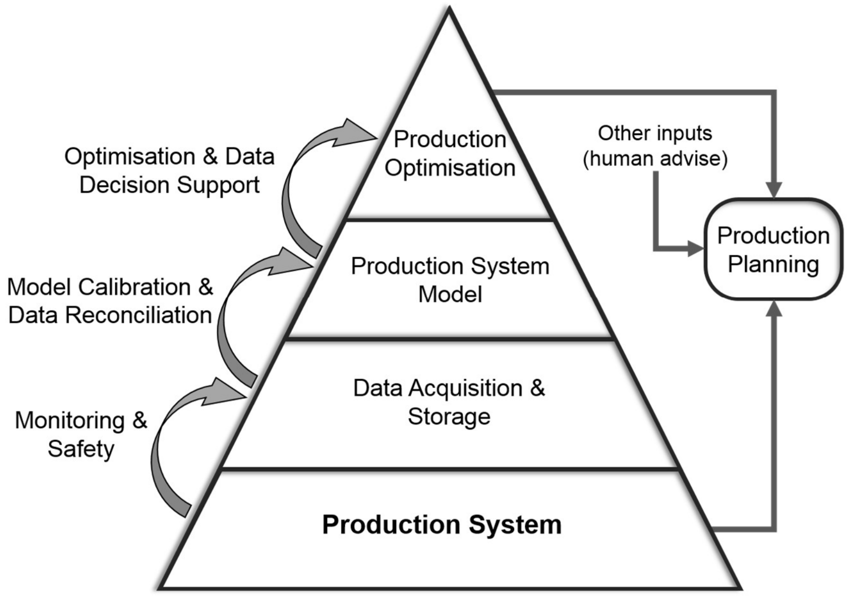

Figure 1: Technology pyramid of production optimisation (Grimstad et al., 2015).

Several factors generally constrain a production system (Fig. 1). For example, the inflow performance from the reservoir to the wellbore may vary over a short time horizon due to complex multiphase flow phenomena such as slugging or water and gas coning and over a long timeframe, due to the prevalent flooding and drainage conditions of production and injection wells. Processing facilitates also impose gas and water capacity constraints and a fixed separator pressure. Constraints relating to well drawdown-down pressure, choke settings, flow routing and flow assurance (hydrate formation and wax deposition) further complicate an optimisation formulation for such systems (Epelle et al., 2020). Hence, novel algorithmic advancements have enabled engineers model, simulate and optimise complex phenomena characterising the production activities in the petroleum industry (Codas et al., 2012).

An aggregate production system model consists of several models corresponding to the production system components (reservoir model, well model and surface facility models for the pipelines and separators). These models must be maintained by frequent calibration using acquired data of the system (Fig. 1). With these information, production optimisation can be performed to obtain the system's optimal operating conditions. Mixed-Integer Nonlinear Programming (MINLP) which combines the modelling capabilities of integer and nonlinear programming into a flexible and multifaceted framework, could result in formulations which are difficult to solve. Typical sources of nonlinearity in the production system include the production wells' pressure-rate response, the pipeline and valve pressure drop and multiphase flow rate relationships (Epelle and Gerogiorgis, 2019b). These complex relationships are usually not explicitly known and are dependent on several operational parameters estimated via high fidelity simulators. Piecewise linear models have the advantage of establishing linear relationships directly from the simulator sample points; a property that significantly reduces problem complexity. However, a frequently adopted simplification approach is to convert the MINLP to a Mixed Integer Linear Program (MILP) via piecewise linear approximations (Gerogiorgis et al., 2006; Silva and Camponogara, 2014; Kronqvist et al., 2018). A computational performance comparison of the trade-offs to be made when deciding the structure of the optimisation formulation is scarce in literature. This work provides some insights via a detailed analysis of the computational performance of both formulations.

2. Relevant Literature

Optimisation problems of production systems over short and long term horizons have been formulated as MINLPs and MILPs in several published studies (Tavallali et al., 2016). These studies incorporate the flow dynamics of certain elements of the entire production system, thus making them differ significantly in complexity.

A multiperiod MINLP formulation was established by Gupta and Grossmann (2012) for the optimal planning of oilfield (offshore) infrastructural development over a 20-year production horizon. The resulting formulation yielded good solutions when solved with MINLP solvers, DICOPT, SBB and BARON. They also presented a reformulation strategy of their MINLP problem into an MILP via binary reduction and elimination of bilinear terms; thus obtaining globally optimal solutions. Epelle and Gerogiorgis (2019a) presented an MINLP formulation for real-time optimisation of a production system with wells of different types. They considered gas lift, ESP-assisted and naturally flowing wells of complex geometries with the potential for sand production. They obtained rapid computations using the BONMIN solver. Hoffman and Stanko (2016) formulated a Real-Time Production Optimisation (RTPO) problem of a network consisting of pump-assisted wells (with Electrical Submersible Pumps – ESPs). An integrated approach for production optimisation from multiple offshore reservoirs in the Santos Basin of Brazil was presented by Camponogara et al. (2017). Their approach relied on a reformulation of an MINLP problem to an MILP via piecewise linear approximations which serve as proxy models for each production unit. They further demonstrated the robustness and scalability of their formulation for large heterogeneous oilfields. Silva et al. (2015) modelled flow splitting of well fluids to multiple headers and embedded this model in an MILP formulation via piecewise linearisation for production optimisation purposes. Piecewise reformulations of MINLPs have also been applied in many recent publications (Kosmidis et al., 2005; Gerogiorgis et al., 2009; Aguiar et al., 2012; Codas et al., 2012; Gunnerud et al., 2012; Silva and Camponogara, 2014).

Rodrigues et al. (2016) developed a new formulation called the Multicapacitated Platforms and Wells Location Problem (MPWLP) and solved it as an MILP using the CPLEX solver. The solution to this model provides the number and location of production platforms, wells and manifolds, the capacities of the production platforms, the interconnections between platforms, manifolds and wells, and which sections of each well should be vertical or horizontal. Zhang et al. (2017) developed a unified MILP formulation for obtaining the best possible topological structure in a production gathering network consisting of wells and pipelines under operational constraints. Their model (solved using MINLP solvers in GUROBI) facilitates the development of field planning schedules.

Other studies which directly incorporate details of the reservoir's conditions in their model apply both gradient-based and derivative-free methods for optimising water flooding operations, especially in the context of closed-loop reservoir management (Chen et al., 2010; Asadollahi et al., 2012; Epelle and

Gerogiorgis, 2020). However, the focus herein is the fast-paced dynamics of the wells, manifolds, pipelines and separator components of the production system. The objective of this paper is to develop a surrogate model-based production optimisation formulation which when solved, is capable of providing optimal operational settings that maximise a field's Net Present Value (NPV) while satisfying imposed operational constraints. To achieve this, this study incorporates the benefits of MILP on a synthetic but realistic case study. The obtained solutions of the MILP and that of the original MINLP are compared and an evaluation of the impact of the number of linearisation breakpoints on the solution time, accuracy, modelling effort and ease of automation is performed. It is ensured that high accuracy in the nonlinear models and piecewise approximations in comparison to the simulation data is maintained while carrying out optimisation calculations.

The novelty of this work lies in the comparative analysis of two linearisation strategies, using an already established linearisation technique in literature (Special Ordered Sets of type 2 – SOS2). The first strategy is to linearise nonlinear terms that appear in the MINLP formulation; whereas, the second strategy is to use data points in a generated look-up table (LKT). To the best of our knowledge, no previous study has quantified the performance of these two strategies. Rather than directly applying LKTs which result in a large number of variables (as many as 100,000 in the work of Silva and Camponogara, 2014, for a similar production system size to ours, cf. Case Study 3/CS3 here), we show that the first strategy yields significantly fewer variables (with a shorter implementation time) and good optimal solutions. The work of Silva and Camponogara (2014) comparatively analysed mathematical techniques of piecewise linearization on a broad spectrum using Disaggregated Convex Combination (DCC), Logarithmic Disaggregated Convex Combination (DLog), Aggregated Convex Combination (CC), Logarithmic Convex Combination (Log) and the SOS2 technique. From their analysis, they showed superior performance of the SOS2 type formulation compared to other linearisation techniques. Based on this finding, we delve deeper into the SOS2 type technique and explore the performances of different strategies of performing this linearisation in comparison to the original MINLP. Thus, the analysis presented herein is an extension of their work, with emphasis placed on the best performing linearisation technique. This novel analysis presented herein enables quality assessment of the relative performances of the respective MILP formulations and their impact on the overall oil production. We also address and discuss the impact of enlarging the optimisation search space on the quality of solutions of MILP reformulations compared to those computed via a global solver (SCIP) for the original MINLP.

Another novel element of this study is the combination of operationally distinct well behaviours with complex flow physics within an optimisation formulation. Flow routings at two levels (well to manifolds and pipelines to separators) are also modelled as shown in Fig. 4. Previous production optimisation studies have mainly considered well-to-manifold routing constraints within their optimisation formulations while assuming fixed pipeline-to-separator connections. However, determining the optimal connections between pipelines and separators is an operationally relevant decision, because of the interplay of several interconnected factors (separator capacity, internal pipeline roughness, pipeline diameter, resultant pipeline pressure drop and the operating pressure of the separators). To the best of our knowledge, only the work of Kosmidis et al. (2005) took this into consideration with gas lift and naturally flowing wells. In this work, we include more complex well pressure responses by utilising ESP- and PCP-assisted wells in our formulation. Comparisons with the newly launched PIPESIM's® network optimiser are made to demonstrate the robustness of the proposed formulation (solved with both local and global optimisation solvers); thus strengthening our contribution. This study also incorporates water coning behaviour which is a key operational problem in upstream systems, but hardly accounted for in production optimisation literature (Hasan et al., 2013).

3. Methodology

The surface network model is first constructed in a steady-state multiphase flow simulator (PIPESIM® v 2019. 3). As shown in Fig. 4, the model consists of the wells, chokes, flowlines, manifolds, pipelines and separators, which are all connected. Robust multiphase flow correlations are adopted to capture complex flow physics in the respective network components. Some of these phenomena include water coning behaviour, non-vertical/deviated well trajectories and downhole pressure assistance to maintain production by means of Progressive Cavity Pumps (PCPs) and Electrical Submersible Pumps (ESPs).

3.1 Steady-state model development

Water coning is a common problem in the oil and gas industry; it involves the upward movement of water into the perforations of a producing well due to drawdown pressure fluctuations and changes in the oil-water contact (to a bell-shaped form) in the reservoir (Fig. 2a). Although water coning is a transient process, the steady-state simulator (PIPESIM®) is capable of modelling this process using data tables (implemented herein) that describe oil production rate as a function of the water cut. This could cause changes to the Vertical Flow Performance (VFP) curves of a well and eventually reduce the oil production rate (Figs. 2b and c). In addition, several design considerations are also made during the selection of the PCPs and ESPs for optimal oil delivery from the wells. Some of the considered factors include: the depth at which the pumps should be placed in the well, the main influencing parameter on pump performance (to be used in the optimisation formulation), viscosity correction factors (due to oil-water emulsions downhole) and the downhole clearance for the pump's liquid intake.

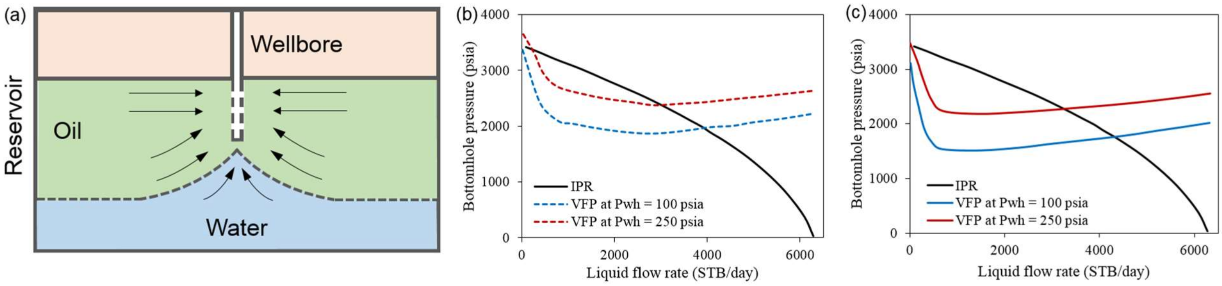

Figure 2: Water coning in a vertical production well (a) VFP curves of a well with water coning (b) VFPs of a well without water coning; IPR represents the Inflow Performance Relationship curve.

To model the flow behaviour in the wells and pipelines, the Hagedorn and Brown (1965) correlation is adopted (for the vertical multiphase flow), whereas the revised Beggs and Brill (1973) correlation is utilised for horizontal multiphase flow calculations in PIPESIM®. The vertical wells' performance are modelled in the simulator by supplying a productivity index value, while, the Joshi inflow performance relationship (IPR) is employed for horizontal wells.

3.2 Optimisation formulation

The surface network design procedure was followed by the generation of large data tables. This involved performing several simulations at different well and pipeline conditions, which correspond to different wellhead pressures and liquid production rates. Using these data, algebraic (polynomial) proxy models are developed for each network component. These proxy models are then utilised together with an objective function (Eq. 5) to optimise the oil production rate of the surface network. The performance of these proxy models is dependent on the data range used in their development. Once the network's operating conditions significantly change, the model parameters are recalibrated. This methodology takes advantage of the decomposable nature of the production network, in that separate equations can be written for each component, and these constitute the optimisation constraints. The complexity of the optimisation problem herein stems from the presence of discrete routing variables at different levels:

the well to manifold level and the pipeline to separator level. The nonlinear pressure-rate responses of the wells and pipelines coupled with these routing decisions inevitably result in an MINLP, (Bussieck and Pruessner, 2003; Wächter and Biegler, 2006; Gunnerud and Foss, 2010) described in Table 1.

3.2.1 Piecewise linearization

Piecewise linear formulations split the domain of a nonlinear function into a set of polytopes $\boldsymbol{P} \in \wp$; where each polytope has a set of vertices $V(\boldsymbol{P})$, and for each vertex $v \in V(\boldsymbol{P})$ of a polytope, there exists a corresponding continuous variable, $\lambda_{\boldsymbol{P},v}$ (Silva and Camponogara, 2014) as shown in Fig. 3. According to Vilema (2010), if $D \subseteq \mathbb{R}^n$ is a compact set, a continuous function $f: D \subset \mathbb{R}^n \to \mathbb{R}$ is piecewise linear if and only if there exists $\{m_P\}_{P\in\wp} \subseteq \mathbb{R}^n, \{c_P\}_{P\in\wp} \subseteq \mathbb{R}$, and a finite family of polytopes $\wp$ such that $D = \bigcup_{P\in\wp} \boldsymbol{P}$ and $f(x) = \{m_P x + c_P, \quad x \in \boldsymbol{P}\ \forall\ \boldsymbol{P} \in \wp$. If $x \in \boldsymbol{P_1} \cap \boldsymbol{P_2}$ for 2 polytopes $\boldsymbol{P_1}, \boldsymbol{P_2} \in \wp$, the above definition infers that $m_{P_1}x + c_{P_1} = m_{P_2}x + c_{P_2}$; this ensures the continuity of f on D. Thus, a convex combination of the vertices of each polytope represents a point (on a graph) of the function. A disaggregated convex combination of polytopes is given by (Eqs. 1–4):

$$\sum_{\boldsymbol{P}\in\wp}\sum_{v\in V(\boldsymbol{P})} \lambda_{P,v} v = x, \quad \sum_{\boldsymbol{P}\in\wp}\sum_{v\in V(\boldsymbol{P})} \lambda_{\boldsymbol{P},v}(m_{\boldsymbol{P}}x + c_{\boldsymbol{P}}) \le f(x) \tag{1}$$

$$\lambda_{\boldsymbol{P},v} \ge 0 \quad \forall \boldsymbol{P} \in \wp,\ v \in V(\boldsymbol{P}) \tag{2}$$

$$\sum_{v\in V(P)} \lambda_{\boldsymbol{P},v} = y_{\boldsymbol{P}} \quad \forall \boldsymbol{P} \in \wp \tag{3}$$

$$\sum_{P\in\wp} y_{\boldsymbol{P}} = 1 \quad y_{\boldsymbol{P}} \in \{0,1\}\ \forall \boldsymbol{P} \in \wp \tag{4}$$

A better representation of piecewise linear functions was proposed by Beale and Tomlin (1970) based on the convex combination of $\lambda_{\boldsymbol{P},v}$. They proposed that only 2 consecutive weighting variables can be non-zero in the branch and bound (BB) algorithm. These sets are named Special Ordered Sets of Type II (SOS2) and are implemented in this study. This MINLP formulation is linearised in 3 ways to generate MILPs; the computational performance of these 4 formulations (including the MINLP) are compared. The first MILP formulation (MILP-3) applies standard algebraic transformation and SOS2 constraints to linearise nonlinear terms (quadratic and bilinear terms – products of 2 continuous variables and products of a continuous and binary variable) in the MINLP formulation using 3 breakpoints.

The second MILP formulation (MILP-5) uses 5 breakpoints; whereas the third (MILP-LKT) directly utilises the look-up data tables for linear interpolation in 1 and 2 dimensions. Table 1 presents the detailed formulations for the MINLP and MILP, respectively. The aim is to maximise the objection function (in terms of the Net Present Value – *NPV*, Eq. 5); where the Revenue from Oil Production (ROP) and Cost of Water Production (CWP) are given by Eqs. 6 and 7 respectively; r_o is the oil price (USD/STB), r_{wt} denotes the water production unit cost (USD/STB) and N_{prod} is the number of wells. Eq. 8 ensures that the wellhead pressure (P^{wh}) is tightly bounded. The proxy models for the Naturally Flowing (NF) well, ESP well, PCP well and pipelines are given by Eqs. 8–11 respectively; q represents the flowrate, f_{ESP} the ESP frequency, Ω, the PCP impeller rotation speed and ΔP_p, the pipeline pressure drop. The indices o, wt, i, w, p, wh, m, s represent the oil phase, water phase, all phases, wells, pipelines, wellheads, manifolds, and separators respectively. Whereas, P^m and P^s denote the manifold and separator pressure.

Table 1: Optimisation formulations (MINLP and MILP)

Objective function

$$\text{Max}\,(NPV) = ROP - CWP \tag{5}$$

$$ROP = r_o \times \sum_{w=1}^{Nprod} q_o \tag{6}$$

$$CWP = r_{wt} \times \sum_{w=1}^{Nprod} q_w \tag{7}$$

Constraints of the MINLP formulation

$$P_{w,min}^{wh} \le P_w^{wh} \le P_{w,max}^{wh} \quad \forall\, w \tag{8}$$

$$q_{i,w,NF} = f(P_w^{wh}) \;\; \forall\, i, \forall\, w \tag{9}$$

$$q_{i,w,ESP} = f\left(P_w^{wh}, f_{i,w,ESP}\right) \quad \forall\, i, \forall\, w \tag{10}$$

$$q_{i,w,PCP} = f\left(P_w^{wh}, \Omega_{i,w,PCP}\right) \;\; \forall\, i, \forall\, w \tag{11}$$

$$\Delta P_p = f\left(q_{p,o}, q_{p,wt}\right) \quad \forall\, p \tag{12}$$

$$y_{w,p} P^m \le P_w^{wh} \qquad \forall\, w, \forall\, p \tag{13}$$

$$z_{p,s} P^s \le P^m \quad \forall\, p, \forall\, s \tag{14}$$

$$Q_{i,p} = \sum_w (y_{w,p} \times q_{i,w}) \;\; \forall\, i, \forall\, p \tag{15}$$

$$LC^s = \sum_p (z_{p,s} \times Q_{i,p}) \;\; \forall\, i, \forall\, s \tag{16}$$

$$\sum_p y_{w,p} = 1 \tag{17}$$

$$\sum_s z_{p,s} = 1 \tag{18}$$

$$P^s = P^m - \Delta P \tag{19}$$

$$\sum_p q_p \le LC^s \tag{20}$$

Piecewise linearization in 2 dimensions

$$P_w^{wh} = \sum_{j \in J} \sum_{k \in K} \lambda_{j,k} P_{(w)j}^{wh} \tag{21}$$

$$f_{w,ESP} = \sum_{j \in J} \sum_{k \in K} \lambda_{j,k} f_{(w,ESP)k} \tag{22}$$

$$q_{w,ESP} = \sum_{j \in J} \sum_{k \in K} \lambda_{j,k} q_{(w,ESP)j,k} \tag{23}$$

$$\sum_{j \in J} \sum_{k \in K} \lambda_{j,k} = 1 \tag{24}$$

$$\delta_j = \sum_{k \in K} \lambda_{j,k} \quad \forall\, j \tag{25}$$

$$\delta_k = \sum_{j \in J} \lambda_{j,k} \quad \forall\, k \tag{26}$$

$$\lambda_{j,k}, \delta_j, \delta_k \ge 0 \tag{27}$$

$$\delta_j \text{ and } \delta_k \text{ are } SOS2 \tag{28}$$

Piecewise linearization in 1 dimension

$$P_w^{wh} = \sum_{j \in J} \lambda_j \, P_{(w)j}^{wh} \tag{29}$$

$$q_w = \sum_{j \in J} \lambda_j \, q_{(w)j} \tag{30}$$

$$\sum_{j \in J} \lambda_j = 1 \tag{31}$$

$$\lambda_j \text{ is } SOS2 \tag{32}$$

Linearizing bilinear terms of type ($C_1 \cdot C_2$ and $B \cdot C$)

$$C_1 \cdot C_2 = \xi_1^2 - \xi_2^2 \tag{33}$$

$$L_1 \le C_1 \le U_1; \quad L_2 \le C_2 \le U_2 \tag{34}$$

$$\xi_1 = 0.5(C_1 + C_2); \quad 0.5(L_1 + L_2) \le \xi_1 \le 0.5(U_1 + U_2) \tag{35}$$

$$\xi_1 = 0.5(C_1 - C_2); \quad 0.5(L_1 - U_2) \le \xi_2 \le 0.5(U_1 - L_2) \tag{36}$$

$$\tau = B \cdot C; \;\; 0 \le C \le U \tag{37}$$

$$\tau \le U \cdot B \tag{38}$$

$$\tau \ge C - U(1 - B) \tag{39}$$

$$\tau \ge 0; \quad \tau \le C \tag{40}$$

Proxy model structure

$$q_{o,ESP} = \alpha_0 + \alpha_1 P_{wh} + \alpha_2 f_{ESP} + \alpha_3 P_{wh}^2 + \alpha_4 f_{ESP}^2 + \alpha_5 P_{wh} f_{ESP} \tag{41}$$

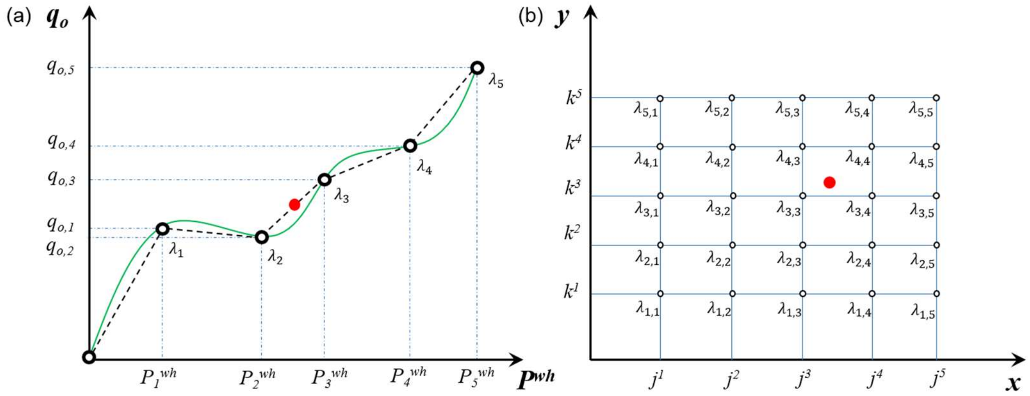

Figure 3: Piecewise linearization in 1 (a) and 2 (b) dimensions.

Binary variables $y_{w,p}$ assigned to each well ensure that the produced fluids from a well are routed by the choke (Eq. 13) to one of the pipelines. Similarly, $z_{p,s}$ in Eq. 14 ensures that the fluids in the pipelines are routed to the separator. The mass balance constraint between wells and pipelines is represented by Eq. 15; whereas, Eq. 16 ensures material balance between the pipelines and the separators (which operate at a fixed pressure). The selection of only 1 binary variable is enforced using Eqs. 17–18. The constraint defined by Eq. 19 ensures the target separator pressure is met, while the liquid capacity constraints of the separators are represented by Eqs. 20. The procedure for linearising functions in 2D and 1D are shown in Eqs. 21–32, respectively; where j and k represent the breakpoints associated with the different variables. Bilinear terms which occur in the MINLP formulation as shown in the typical proxy model structure (Eq. 41) are linearised using Eqs. 33–40. In these equations, C represents, a continuous variable, and B a binary variable; L and U denote the lower and upper bounds of a continuous variable. ξ and τ are additional variables introduced in the linearisation procedure. BONMIN (v.1.8.6), CBC (v.2.9.8), SCIP (v.3.2.1 & 5.0.1) and CPLEX (v.12.8.0.0) are adopted for solving the MINLP and MILP formulations respectively. We present 3 case studies, which are all solved using the formulation described in Table 1. Concise definitions of all variables and parameters are also provided in Table A1.

3.2.2 Case Study 1 (CS1)

In this case study (Fig. 4), a production network consisting of 4 production wells, 4 choke valves, 2 manifolds, 2 pipelines and 3 separators is optimised. Flow routing constraints at 2 levels (from wells to manifolds and from pipelines to separators) are applied to the MINLP and MILP formulations respectively. A wellhead pressure range of 300 – 380 psia is employed for all wells in the proxy model development phase. Further details of the production network, which were the input parameters for the modelling phase of the procedure are given in Table 2.

Table 2: Reservoir, well and pipeline parameters for CS1 and CS2.

Parameter	W–1	W–2	W–3	W–4	P–1	P–2
Reservoir pressure (psia)	3,800	3,800	3,800	3,800	–	–
Well type	Vertical	Deviated	Vertical	Deviated	–	–
Well PI (STB/day/psi)	1.7	3.7	2.5	3.3	–	–
GOR (SCF/STB)	500	500	500	500	–	–
WC (%)	20-32	20-32	20-32	20-32	–	–
TVD (ft)	10,000	10,000	10,000	10,000	–	–
Tubing diameter (in)	3.5	3.5	4.5	4.5	–	–
Pipeline length (ft)	–	–	–	–	6,000	4,000
P-ID (in)	–	–	–	–	10	11
P-IR (in)	–	–	–	–	0.001	0.001

RP: Reservoir Pressure; P-ID: Pipeline Internal Diameter; P-IR: Pipeline Internal Roughness; PI: Productivity Index

3.2.3 Case Study 2 (CS2)

This case study is similar to CS1 in terms of size and the parameters shown in Table1. However, we increase the optimisation search space in this case study by widening the wellhead pressure range (50 – 500 psia). We then examine the performance of this case study in comparison to CS1, for differences in the optimal routing structures, NPV and computation time.

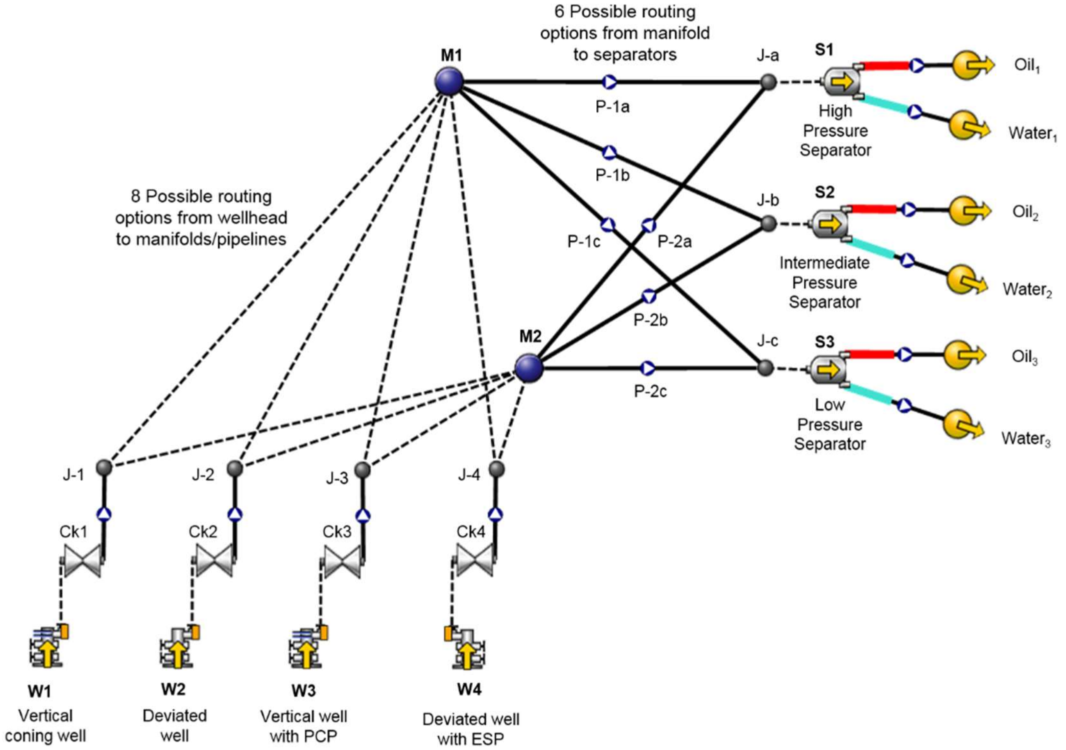

Figure 4: Surface production network and routing superstructure for CS1 and CS2.

3.2.4 Case Study 3 (CS3)

In this case study (Fig. 5), a larger production network consisting of 12 production wells with varying operating modes is solved. CS3 demonstrates the scalability and adaptability of the proposed formulation to bigger production systems. The wider wellhead pressure range is adopted here, and the parameters for each well are shown in Table 3.

Table 3: Reservoir, well and pipeline parameters for CS3.

Parameter	**W–1**	**W–2**	**W–3**	**W–4**	**W–5**	**W–6**	**P–1**
Reservoir pressure (psia)	3,800	3,800	3,800	3,800	3,800	–	–
Well type	Vertical	Deviated	Vertical	Vertical	Deviated	Deviated	–
Well PI (STB/day/psi)	1.5	3.7	2	2.5	2.5	2.5	–
GOR (SCF/STB)	500	500	500	500	500	500	–
WC (%)	20-32	20-32	20-32	20-32	20-32	20-32	–
TVD (ft)	10,000	10,000	10,000	10,000	10,000	10,000	–
Tubing diameter (in)	3.5	3.5	4.5	4.5	4.5	4.5	–
Pipeline length (ft)	–	–	–	–	–	–	6000
P-ID (in)	–	–	–	–	–	–	10
P-IR (in)	–	–	–	–	–	–	0.001
Parameter	**W–7**	**W–8**	**W–9**	**W–10**	**W–11**	**W–12**	**P–2**
Reservoir pressure (psia)	3800	3800	3800	3800	3800	–	–
Well type	Vertical	Deviated	Vertical	Deviated	Vertical	Deviated	–
Well PI (STB/day/psi)	1	3.1	1.2	4.5	1.8	1.9	–
GOR (SCF/STB)	500	500	500	500	500	500	–

WC (%)	20-32	20-32	20-32	20-32	20-32	20-32	–
TVD (ft)	10,000	10,000	10,000	10,000	10,000	10,000	–
Tubing diameter (in)	3.5	3.5	3.5	3.5	3.5	3.5	–
Pipeline length (ft)	–	–	–	–	–	–	4000
P-ID (in)	–	–	–	–	–	–	11
P-IR (in)	–	–	–	–	–	–	0.001

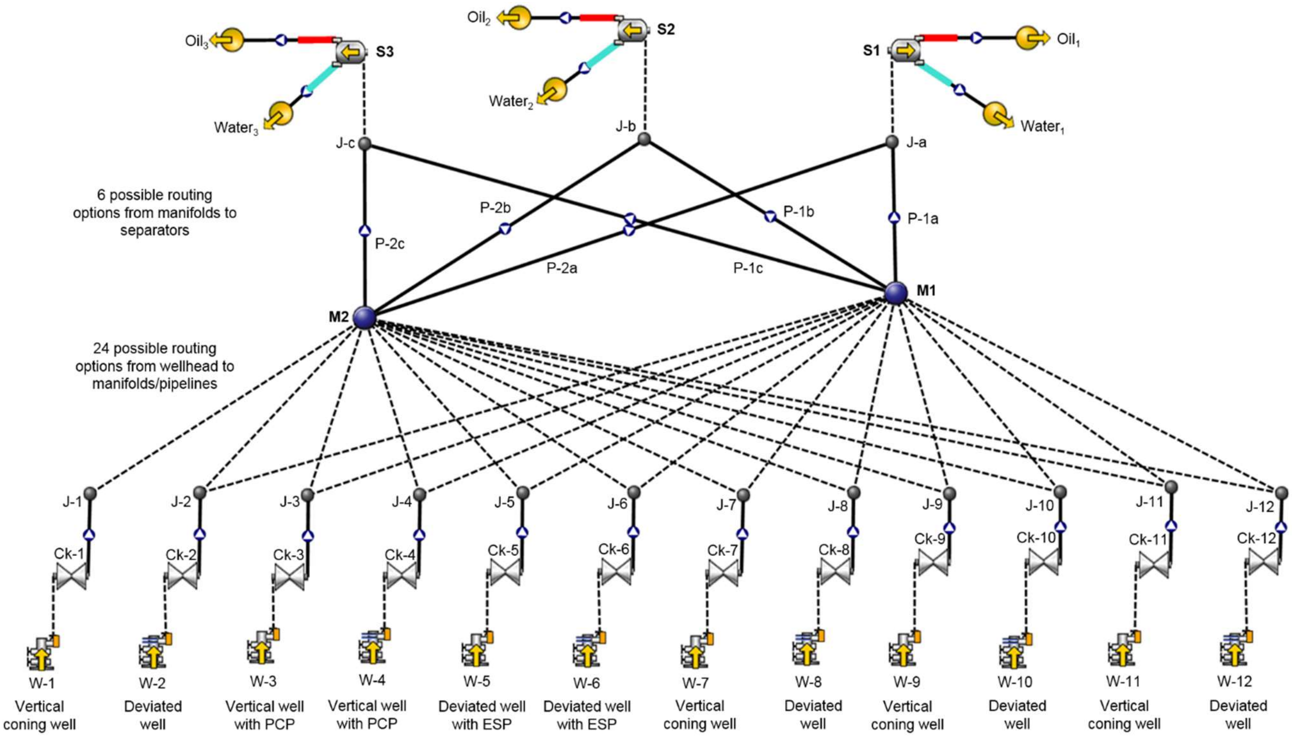

Figure 5: Surface production network and routing superstructure for CS3.

Other input data applied in the surface network simulator for data generation include the oil specific gravity (45 API) and the pipeline temperature (100ºF). The separators liquid handling capacities and operating pressures are also shown in Table 4.

Table 4: Separator capacities and operating pressures for all cases studies.

Case study	**Separator**	**Operating pressure (psia)**	**Liquid capacity (STB/day)**
Case Study 1 (CS1)	Sep – 1	45	8,000
	Sep – 2	35	10,000
	Sep – 3	25	12,000
Case Study 2 (CS2)	Sep – 1	35	12,000
	Sep – 2	25	10,000
	Sep – 3	20	8,000
Case Study 3 (CS3)	Sep – 1	35	25,000
	Sep – 2	25	18,000
	Sep – 3	20	15,000

4. Results and Discussion

The proposed formulations were programmed in MATLAB® R2016a (using OptiToolbox v2.28) and solved on an Intel Core i7-6700 processor at 3.40 GHz running on a 64 bit Windows workstation with

16GB of RAM. By implementing the proposed formulations, repeated calls to the simulator are avoided, thus enabling faster computations. The three separate MILP formulations and the MINLP problem are solved to optimise the production networks of CS1, CS2 and CS3 respectively.

4.1 Case Study 1

The MILP-LKT formulation consisted of 25 polytopes (squares) for the well performance function (5 breakpoints for the ESP frequency/rotational speed and 5 breakpoints for the wellhead pressure). For the pipelines, 144 polytopes (squares) were adopted (12 breakpoints for the oil and water phases respectively). This resulted in a total of 8,702 variables; this is significantly larger than the number of variables required in the other formulations (as shown in Table 5). Despite this number of variables, it is solved in a shorter time compared to the MINLP formulation with only 36 variables. This increase in problem size (number of constraints and variables) that ensues with an increasing number of data points, makes the implementation of the SOS formulation laborious; this is a significant drawback of this formulation; hence, it is only suitable for low dimensional problems. With the MINLP, the increase in the number of data points would hardly affect the approximations of the simulator output. In this regard, the MINLP formulation can be regarded as more scalable compared to the MILP.

The convergence of the proposed formulations to different optimal routing configurations (Table 5) is an indication of the non-convexity of the optimisation problem. However, good quality solutions were obtained from all formulations, as demonstrated in the relative gap obtained (Table 5). The MINLP formulation gave the best solution in terms of the NPV. Our computational analysis has also shown that the improvement in resolution quality affects the solution quality of the MILP-5 and MILP-3 formulations. With 5 breakpoints (MILP-5), the NPV obtained is closer to that of the MINLP compared to the lower resolution formulation, consisting of 3 breakpoints (MILP-3). The MILP-LKT formulation gave the lowest NPV (1% lower than the MINLP). This may be attributed to the fact the solutions are always approximated by linear segments (in the SOS formulation), which are generated between nonlinear data points. Water coning behaviour is a known source of nonlinearity in the wellbore model. The complex multiphase flows between each network component, and also the system pressures within a narrower search space is another possible explanation. However, as will be presented in CS2 and CS3 (with a broader optimisation search space), the MILPs show improved results.

Table 5: Computational performance of optimisation formulations for CS1.

Optimisation formulation	MINLP	MILP-3 (SOS2)	MILP-5 (SOS2)	MILP-LKT (SOS2)
Solver used	BONMIN	CPLEX	CPLEX	CPLEX
Number of constraints	34	184	184	340
Number of variables	36	134	170	8702
Relative optimality gap	0.00	0.00	0.00	0.00
Solution time (s)	0.536	0.111	0.152	0.287
Number of nodes	0	229	253	292
NPV (USD)	989,228	979,934	986,832	979,261
Total oil production rate (STB/day)	15,219	15,076	15,182	15,066
Total water production rate (STB/day)	3,803	3,767	3,794	3,766

It can also be observed that the high-pressure separator with lower capacity (S1) is the least preferred option for routing fluids from the manifolds (Fig.6). The pipeline diameter and length, and the high-pressure drop that ensues, make it difficult for fluids to be delivered to S1, which operates at 45 psia compared to S2 and S3 at 35 psia and 25 psia respectively.

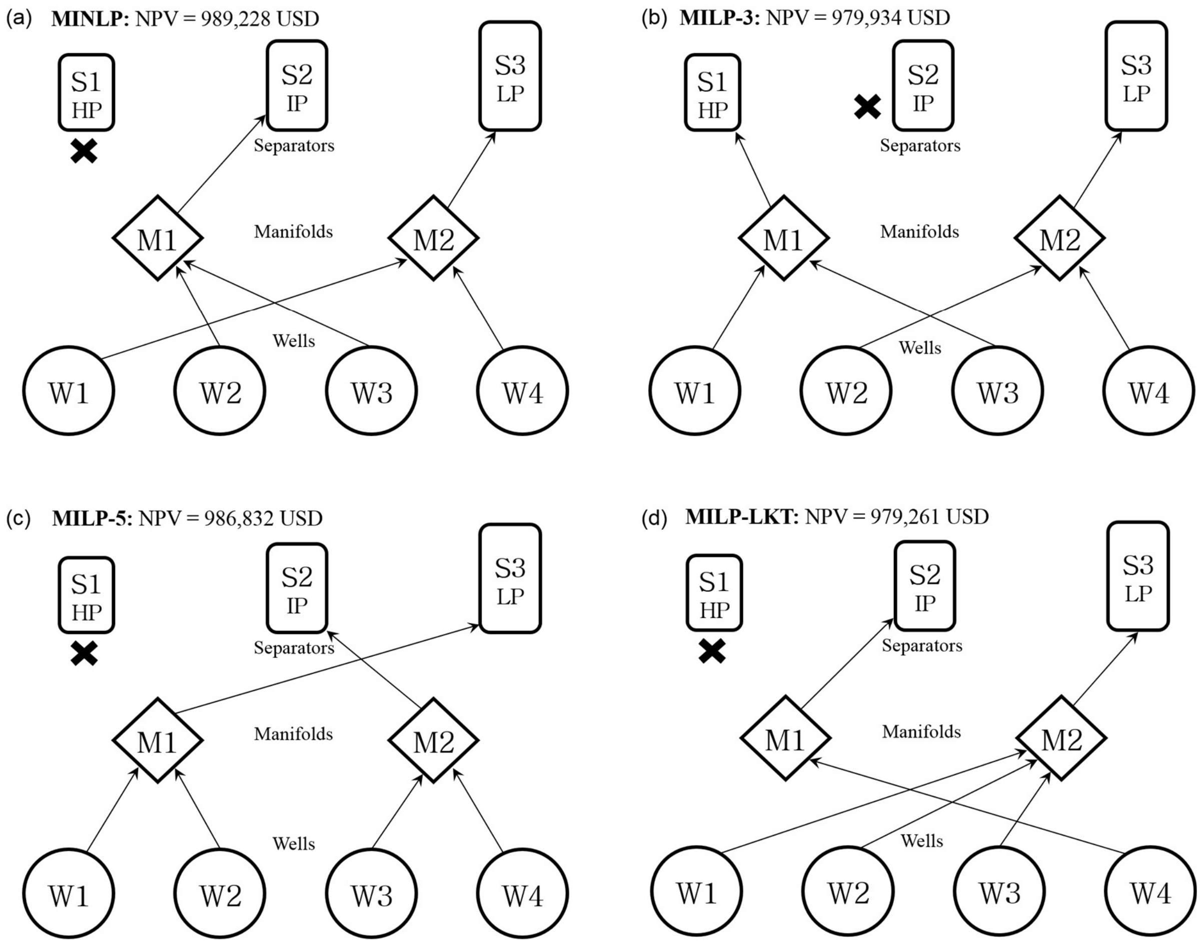

Figure 6: Optimal discrete routing structure for all formulations of CS1; MINLP (a) solved by BONMIN and MILP (b-d) by CPLEX.

The algorithm has shown good utilisation of separator capacities for routing the fluids. As shown in Table 5, the MILP formulations converge faster than the MINLP. Furthermore, the time required for proxy model development if incorporated, will further make the MINLP slower compared to the MILP (which directly use the table data points). It can also be observed that the oil and water production rates of the respective formulations are similar, despite the different optimal routing structures obtained. This indicates that the algorithmic treatment of discrete variables is complicated, especially when they exist at different levels. However, the number of nodes utilised for finding the optimal solution in all formulations reflects the efficiency of the CPLEX solver (which uses the Branch and Bound algorithm). However, on applying the CBC solver (based on the Branch and Cut algorithm) to our problem, the number of nodes reduces by an order of magnitude, although with a higher relative gap and a longer computational time. A detailed comparison of solver performances is presented in Section 4.4.

4.2 Case Study 2

Fig. 7 shows the optimal routing structure for CS2. Although different optimal routing strategies are obtained between the formulations, the MILP-5 and MINLP formulations yield exactly the same optimal configuration. Furthermore, the MILP-3 and MILP-5 formulations yield improved oil production rates (Table 6) and correspondingly increased NPVs compared to the MINLP formulation. Thus, it can be stated that widening the optimisation search space causes the local trapping of the MINLP formulation at suboptimal solutions; whereas, MILP-3 and MILP-5 are able to further explore the optimisation search space for improved results. However, the MILP-LKT formulation does not yield improved results compared to the MINLP. This may be attributed to the adopted table resolution, which will affect the accuracy since the obtained solutions lie on linear segments generated by the SOS2 formulation. The MILP-LKT formulation is also unable to extrapolate beyond the sampled region in which the quadratic approximation of the MINLP was fitted. Thus the MINLP, MILP-3 and MILP-5 formulations are able to check for the existence of better-operating conditions, despite their significantly smaller problem size. The work of Gupta and Grossmann (2012) illustrates a similar observation.

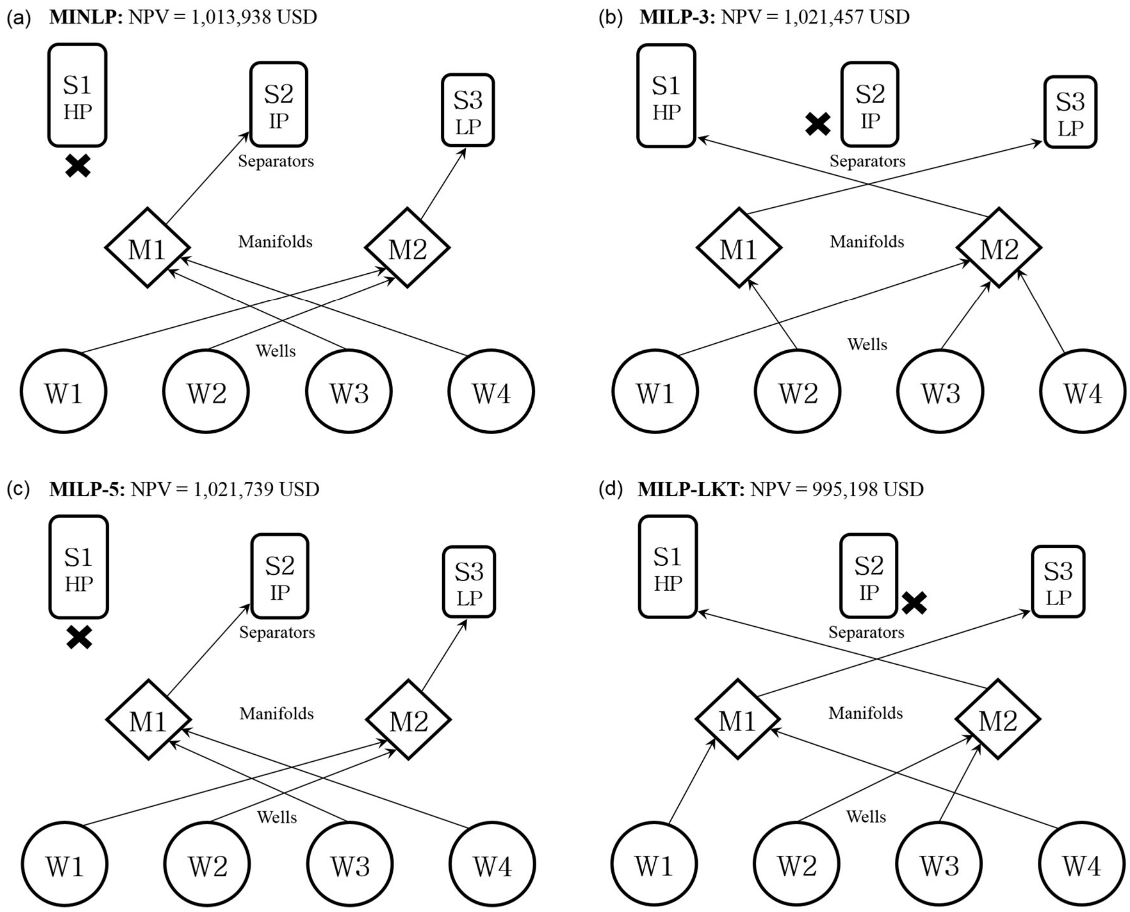

Figure 7: Optimal discrete routing structure for all formulations of CS2. MINLP (a) solved by BONMIN and MILP (b-d) by CPLEX.

It is also observed that improving the resolution quality (i.e. when the number of sampled breakpoints is increased from MILP-3 to MILP-5) positively impacts the performance of the MILP formulation. The MILP-5 formulation reached a better solution, yielding a slight increase in the NPV.

Table 6: Computational performance of optimisation formulations for CS2.

Optimisation formulation	MINLP	MILP-3 (SOS2)	MILP-5 (SOS2)	MILP-LKT (SOS2)
Solver used	BONMIN	CPLEX	CPLEX	CPLEX
Number of constraints	34	184	184	340
Number of variables	36	134	170	8702
Relative optimality gap	0.0057	0.00	0.00	0.00
Solution time (s)	23.4	0.182	0.188	0.460
Number of nodes	46	316	388	514
NPV (USD)	1,013,938	979,934	986,832	979,261
Total oil production rate (STB/day)	16,083	16,202	16,207	15,787
Total water production rate (STB/day)	5,593	5,635	5,637	5,494

Compared to the MILP formulations which are solved in less than a second, the MINLP solution to CS2 was obtained in 23.4 s (without exploring the possibility of parallel computing). These rapid computational times are due to the decomposable nature of the entire production network with proxy models developed for each component. Hence the simulator search space is considerably reduced (fewer dimensions) compared to the scenario in which an optimisation search occurs over the entire network. Furthermore, as shown in Fig. 7, S3 is the most preferred separator; since fluids are routed to this vessel after solving all formulations. This is attributable to its lower operating pressure, which in turn accommodates a lower operating wellhead pressure and thus increased production. This was also observed in CS1.

4.3 Case Study 3

In this case study, we increase the number of production wells in the network (from 4 to 12) and evaluated the performance of the MINLP, MILP-3 and MILP-5 formulations. As a result of the number of variables involved in the MILP-LKT formulation and the lower NPV's obtained compared to the other formulations in CS1 and CS2, the MILP-LKT formulation was not applied in CS3. Just as previously observed, improved solutions (in terms of the total oil production and NPV) are obtained for the MILPs in comparison to the MINLP. It is thus illustrated via CS2 and CS3 that, increasing the optimisation search space favours the MILPs over the MINLPs.

Table 7: Computational performance of optimisation formulations for CS3.

Optimisation formulation	MINLP	MILP-3 (SOS2)	MILP-5 (SOS2)
Solver used	BONMIN	CPLEX	CPLEX
Number of constraints	80	396	396
Number of variables	78	270	334
Relative optimality gap	0.0327	0.00	0.00
Solution time (s)	607.26	0.498	0.438
Number of nodes	758	25,644	22,514
NPV (USD)	2,585,310	2,609,320	2,612,370
Total oil production rate (STB/day)	41,145	41,520	41,571
Total water production rate (STB/day)	14,742	14,854	14,880

While the MILPs are solved in less than a second (just as in CS1 and CS2) for this case study, the MINLP takes approximately 10 mins to solve with a higher optimality gap compared to the MILP solutions; thus demonstrating the increased computational efficiency of the proposed MILPs (Table 7).

Compared to CS1 and CS2, the optimal routing configurations obtained in this case study (CS3) are rather similar. Fig. 8 shows exactly the same routing structure from wells to manifolds for all formulations. However, the manifold to separator routings are different for the MILPs and MINLPs. In this case study, S2 is the most utilised, as seen in Fig. 8.

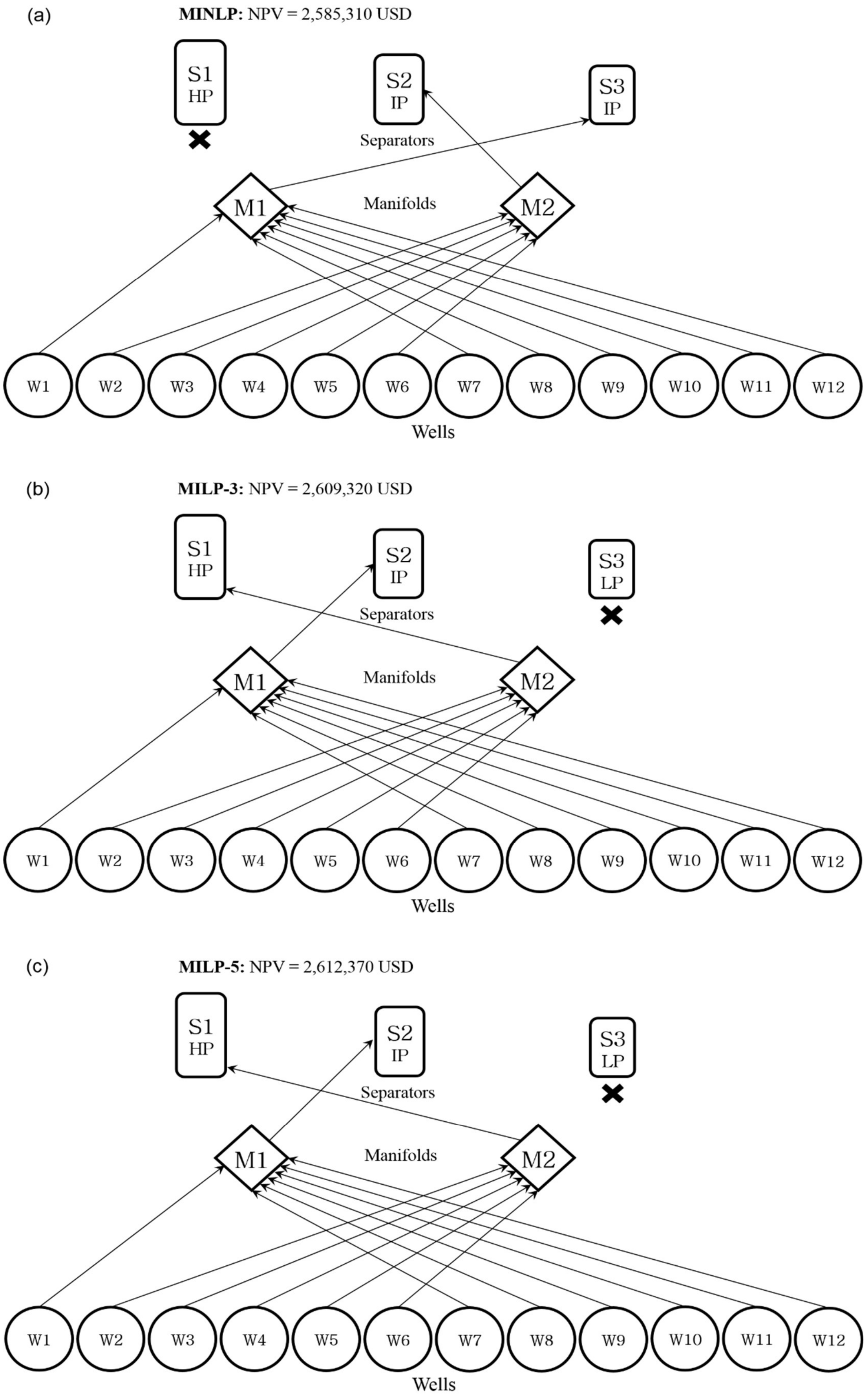

Figure 8: Optimal discrete routing structure for all formulations of CS3. MINLP (a) solved by BONMIN and MILP (b-c) solved by CPLEX.

4.4 Solver performance and comparison with PIPESIM's® network Optimiser

Table 8 summarises the computational performance in terms of the number of nodes, relative optimality gap and the solution times for the different optimisation solvers (CBC, CPLEX, SCIP and BONMIN) applied to the respective formulations. Table 8 shows that the fewest number of nodes are utilized by the CBC solver, although with the highest relative optimality gap compared to the other solvers. However, the overall solution quality obtained from all solvers is good, as illustrated by the obtained optimality gaps. As expected, longer simulation times are observed with the larger case study (CS3); however, CPLEX still solves this problem in less than a second; thus making it the best performing solver for our presented case studies compared to CBC and SCIP. In this section, the performance of the global optimisation solver (SCIP) in solving the MINLPs for the three case studies is also reported in comparison to the local MINLP solver BONMIN. It is observed that SCIP slightly outperforms BONMIN in terms of the optimality gap and the NPVs obtained, but is still inferior to the MILP solutions. However, the computational time required by the SCIP solver is significantly higher than the other solvers (Tables 8).

One of the case studies presented in a recent study of Gupta and Grossmann (2012) discusses a similar observation of inferior performance of a global MINLP solver (in their case, BARON) compared to solutions obtained from an equivalent MILP formulation (solved by CPLEX). Computational experiments performed and published by Klanšek (2015) echo and corroborate this observation. This phenomenon may be attributed to several factors: model complexity, nonconvexities, model sensitivity to perturbed inputs, optimisation search space and, most importantly, the reformulation strategy applied. By reformulating the problem as an MILP, we address the nonconvexity challenges of MINLPs and the difficulties they pose to global MINLP solvers (we convexify the problem, provided all functions are separable); thus better performance can be achieved. This is why all 3 solvers applied to the MILP problem (SCIP, CPLEX, CBC) produce the same NPV, except CS3 (SCIP fails to converge, Table 8). Besides applying the standard Branch-and-Bound or Branch-and-Cut or decomposition algorithms, solvers may also include the implementation of special presolving/preprocessing procedures to which the disparities in performance can be attributed. A more detailed investigation exceeds our scope but seems in order, considering the two cited precedents which feature similar performance observations.

We also compare the performance of PIPESIM's® (v. 2019.3) new optimisation toolbox with our optimisation methodology. In comparison to this toolbox, we generally observe faster solution times for all solvers with the presented formulations (except in CS3, MINLP-BONMIN); thus demonstrating the superior performance of our implemented formulation (Fig. 9). Although the computational time requirement is expected to increase with larger production networks, the fast solution times obtained herein are more attributable to the nature of our optimisation formulation than the size of the production network. The network size considered herein is comparable to those utilised previous comparative studies such as Silva and Camponogara (2014). Furthermore, by using the SCIP solver for the MINLP problem, a long solution time of up to 36 mins is obtained as reported in Table 8; thus demonstrating that the run times achieved are also dependent on the type of optimisation solver implemented.

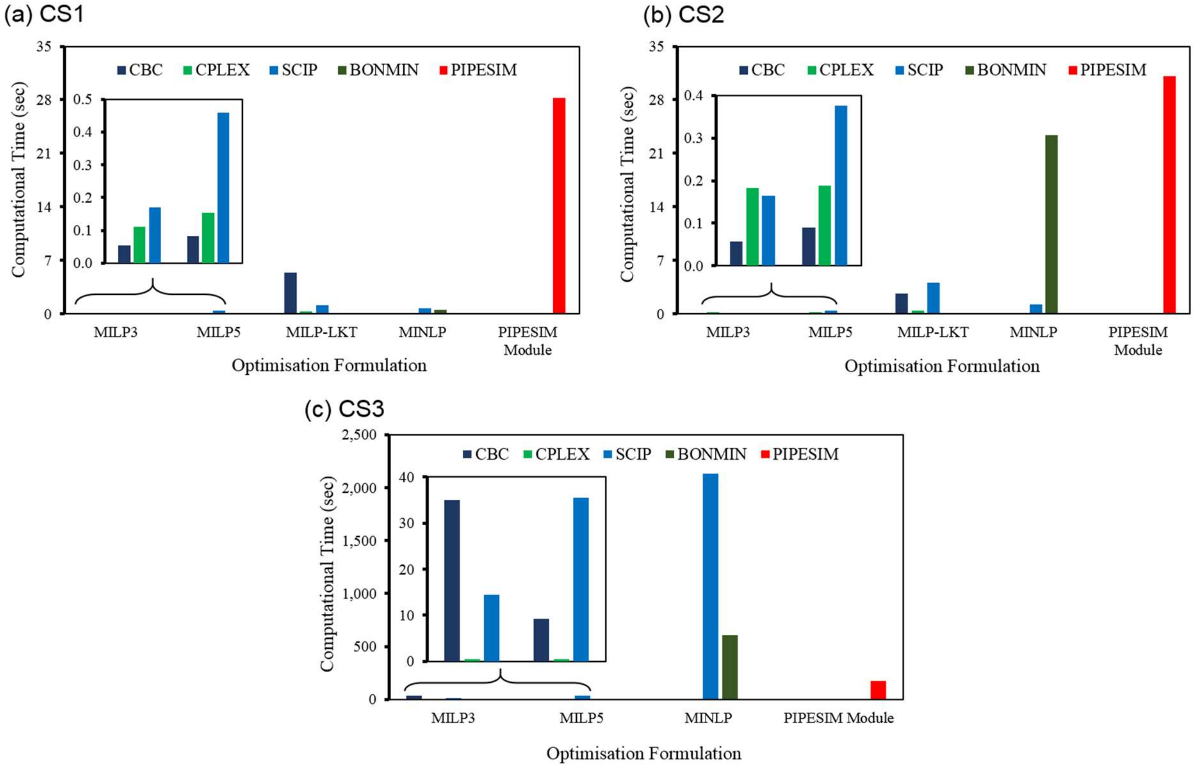

Figure 9: Comparison of computational times for the different optimisation solvers and case studies.

Table 8: Summary of optimal objective function values (NPV) and computational performance achieved by different optimisation solvers.

Case Studies	Solvers	OBJECTIVE FUNCTION: NET PRESENT VALUE NPV (USD)							COMPUTATIONAL PERFORMANCE															
									Number of Nodes					Relative Optimality Gap					Solution Time (sec)					
		MILP3	MILP5	MILP-LKT	MINLP (BONMIN)	MINLP (SCIP)	PIPESIM	NPV Increment (%)	MILP3	MILP5	MILP-LKT	MINLP (BONMIN)	MINLP (SCIP)	MILP3	MILP5	MILP-LKT	MINLP (BONMIN)	MINLP (SCIP)	MILP3	MILP5	MILP-LKT	MINLP (BONMIN)	MINLP (SCIP)	PIPESIM
CS1 (4 Wells)	CBC	979,934	986,832	979,261					2	24	150			0.029	0.022	0.02			0.05	0.08	5.42			
	CPLEX	979,934	986,832	979,261	989,228	**989,230**	***912,000***	8.47	229	253	292	0	39	0.000	0.000	0.00	0.000	0.000	0.11	0.15	0.29	0.54	0.73	**28.21**
	SCIP	979,934	986,832	979,261					87	347	7			0.000	0.000	0.00			0.17	0.46	1.14			
CS2 (4 Wells)	CBC	1,021,457	1,021,739	995,198					6	7	117			0.032	0.032	0.08			0.06	0.09	2.65			
	CPLEX	1,021,457	**1,021,739**	995,198	1,013,938	1,013,944	***912,000***	12.03	316	388	514	46	85	0.000	0.000	0.00	0.006	0.000	0.18	0.19	0.40	23.41	1.27	**31.11**
	SCIP	1,021,457	1,021,739	995,198					28	222	114			0.000	0.000	0.00			0.16	0.38	4.13			
CS3 (12 Wells)	CBC	2,609,320	2,612,370	–					8,563	1,734	–			0.018	0.016	–			35.04	9.19	–			
	CPLEX	2,609,320	**2,612,370**	–	2,585,310	2,585,350	***2,437,896***	7.16	25,644	22,514	–	758	4,222,889	0.000	0.000	–	0.003	0.000	0.50	0.44	–	607.26	2134.10	**178.12**
	SCIP	2,609,320	2,596,330†	–					6,517	10,000	–			0.000	0.021†	–			14.46	35.50	–			

†: SCIP terminated after reaching the maximum number of nodes.

The implemented optimisation methods see decreased speed when the network components increase. Furthermore, the SOS2 formulations are significantly affected because of the drastic increase in the number of data points and variables with bigger-sized production systems (more wells, manifolds, pipelines, and separators). However, this will hardly be a problem for the MINLP (when evaluating the proxy models). Hence there is always a trade-off between the quality of obtained solutions, the solution time and the model development time. Thus, the increased difficulty of handling SOS2-based formulations is reflected in the number of variables involved, which require significant effort for their implementation/development compared to the MINLP with considerably fewer variables.

It is particularly observed in Table 8 that the time taken by the CBC solver to find a solution for MILP3 is higher than that for MILP5. A possible explanation to this is that stronger/more rigorous branching and interpolation are required for the optimisation problem with just 3 breakpoints (MILP3) compared to the problem with 5 (MILP5), as reported in the iteration history of the CBC solver (for a good solution to be found). It may be argued that the time required for this strong branching supersedes the extra computational effort resulting from an increased number of variables in the MILP5 formulation. However, this is not the case with other solvers.

Table 8 and Fig. 10 summarize the obtained NPVs using all solvers for the different formulations in comparison to those obtained from PIPESIM's® network optimisation module (The NPVs in PIPESIM were calculated by optimising the oil production rate while constraining the water production rate according to the objective function) presented in Eq. 5. For all formulations, the obtained NPVs for CS1 and CS2 are the same. However, when the SCIP solver was applied to the MILP-5 formulation of CS3, a lower NPV was obtained compared to CBC and CPLEX. The reason for this is that the SCIP solver timed out when the maximum number of nodes was reached for the MILP5 problem of CS3. Regardless of the initial settings for the optimisation run, the solver did not converge to the global solution. This effect is also clearly observed in Table 8, where the relative optimality gaps are reported. However, as CBC is not a global solver, it fully converged within the default setting of the relative optimality gap.

Compared to CS1, the NPVs obtained in CS2 are generally higher. By widening the optimisation search space (in terms of the operating wellhead pressures), increased oil production can thus be attained. However, depending on the field's operation, this range is often bound by constraints that prevent sand production due to excessively high drawdown pressures and poor fluid lifting capacity in the well tubing due to liquid loading. Furthermore, it is observed that the NPVs obtained by applying PIPESIM's optimisation toolbox are 8%, 12% and 7% lower than our best-case scenario (in bold) for CS1, CS2 and CS3 respectively. This also demonstrates the computational efficiency of the proposed formulation.

The presented approach is thus useful for real-time decision support in the oil and gas industry considering its competing performance with an industry applied simulator and optimiser. However, it is important for further implementations of this method to incorporate network changes that extend the boundaries of applicability. For example, the change in the physical properties of the system, reservoir dynamics, network expansion (addition of new wells and pipelines) and flow regime changes are possible occurrences that may significantly change the production data; thus, necessitating proxy model reconstruction and updating the respective optimisation formulations (Ursin-Holm et al., 2014). However, the presented formulations are easily adaptable to these changes with some considerable effort required.

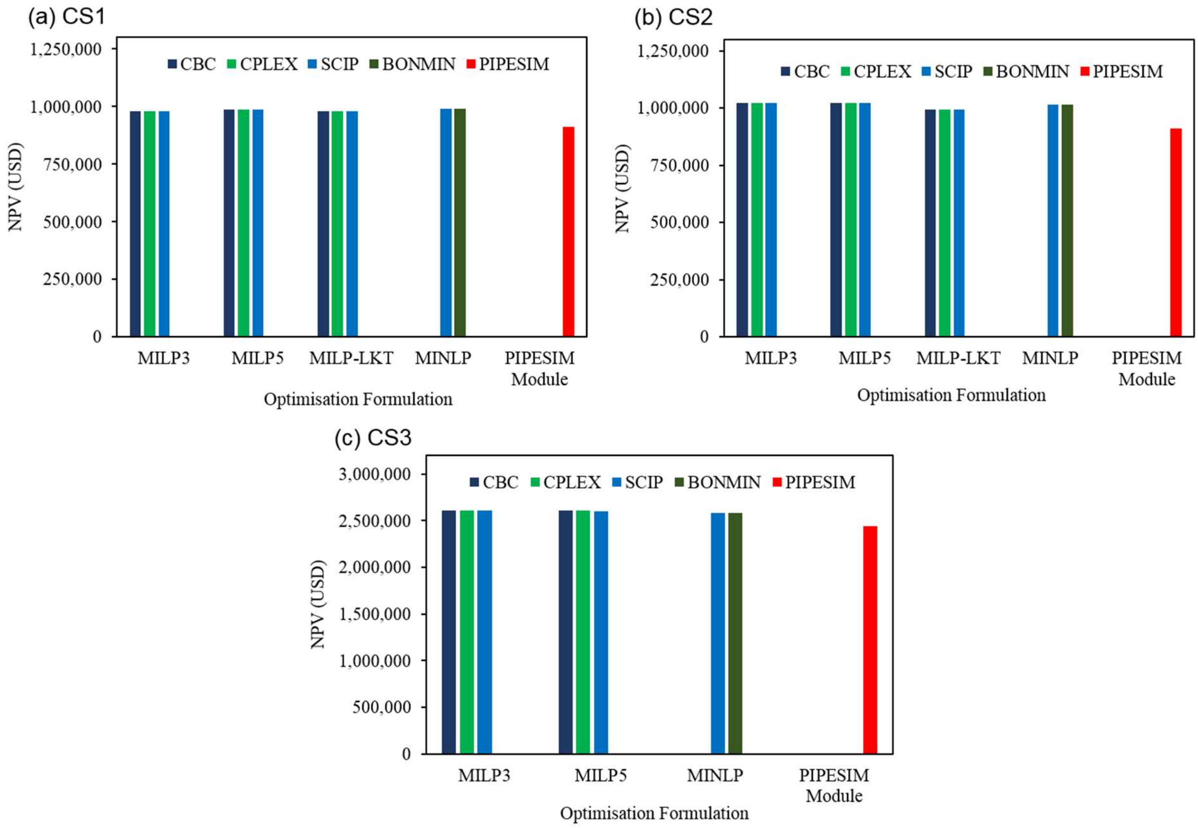

Figure 10: Comparison of the NPVs obtained for the different optimisation solvers and case studies.

5. Conclusion

This study proposed MINLP and MILP formulations for optimising production from a synthetic oil field consisting of 3 separators, 2 manifolds and 4-12 wells with complex downhole/ multiphase flow physics. The nonlinear models were developed using regression analysis that resulted in algebraic polynomial models; whereas piecewise linear models were developed from production points sampled from the look-up table and via linearization of the MINLP formulation. The following conclusions can be derived from the computational analyses performed herein.

- The resulting number of variables and the model development time for the MILP are significantly larger than that of the MINLP.
- Increased resolution of the MILP formulations from 3 to 5 breakpoints resulted in an improved NPV.
- CPLEX was the best performing solver with rapid computational speeds and low optimality gaps obtained for all MILP formulations.
- A computational analysis performed on the 4 formulations of CS1 showed superior performance of the MINLP formulation in terms of the NPV compared to the MILPs. However, for this case study (CS1), all formulations were solved in less than a second. Despite the similarities in the oil and water production rates of CS1, the optimal routing strategy for all formulations of CS1 were different.
- Increasing the optimisation search space (in terms of the wellhead pressure) as demonstrated in CS2 and CS3 favours the MILP-3 and MILP5 formulations. Higher NPVs are obtained in comparison to the MINLP because of the reformulation, even vs. a global MINLP solver.

- Compared to PIPESIM's® optimisation module, our optimisation formulation solves faster and yields higher NPVs.
- Future research could analyse in greater detail how the obtained solution times scale up with increased problem size (e.g. for fields containing hundreds of wells, manifolds and separators). This will further verify the adaptability of these formulations for real-time decision support. Incorporating the effect of temperature on the pressure drop functions, as well as other downhole phenomena like sand production is also worth investigating.

6. Acknowledgements

The authors gratefully acknowledge the financial support of the University of Edinburgh via a PhD scholarship awarded to Mr. E. I. Epelle. Moreover, Dr D. I. Gerogiorgis acknowledges a Royal Academy of Engineering (RAEng) Industrial Fellowship. Tabulated and cited literature data suffice for the reproduction of all original simulation results and no other supporting data are required to ensure reproducibility.

7. Nomenclature

Variables

f_{ESP}	ESP operating frequency (Hz)
GOR	Gas Oil Ratio (SCF/STB)
NPV	Net Present Value (USD)
ΔP	Pressure drop in pipeline (psi)
P	Pressure (psia)
P^m	Manifold pressure (psia)
P_{wf}	Bottomhole flowing pressure (psia)
P^{wh}	Wellhead pressure (psia)
ΔP_p	Pipeline pressure drop (psia)
Q,q	Flowrates (STB/day or MMSCF/day)
WC	Water cut (%)
y	Binary routing variable (-)
z	Binary routing variable (-)

Parameters

CWP	Cost of Water Production (USD)
LC^s	Separator liquid capacity (STB/day)
N_{prod}	Number of production wells (-)
$N_{g,inj}$	Number of gas lift wells (-)
$N_{l,ESP}$	Number of ESP-assisted wells (-)
PI	Productivity Index (STB/day.psi)
$P\text{-}ID$	Pipeline Internal Diameter (m)
$P\text{-}IR$	Pipeline Internal Roughness (m)
r_{op}	Unit oil sales price (USD/STB)
r_{wp}	Unit water production cost (USD/STB)
ROP	Revenue from Oil Production (USD)
RGP	Revenue from Gas Production (USD)
P_r	Reservoir pressure (psia)
P^s	Separator pressure (psia)
TVD	True Vertical Depth (ft)

Sets and other formulation properties

B	Binary variables
C_1, C_2	Continuous variables
$c_{\boldsymbol{P}}$	Intercept
D	Compact set
f	Continuous function
$m_{\boldsymbol{P}}$	Slope
L, L_1, L_2	Lower Bound
$\boldsymbol{P}$	Polytope
$\wp$	Set of polytopes
$\mathbb{R}^n$	Real coordinate space of n dimensions
U, U_1, U_2	Upper Bound (-)

v Vertex

V Set of vertices

Acronyms

CBC Coin-or Branch and Cut

B·B Branch and Bound

B·C Product of a binary and a continuous variable

BONMIN Basic Open-source Nonlinear Mixed INteger Optimiser

C·C Product of 2 continuous variables

ESP Electrical Submersible Pump

HP High-Pressure Separator

IP Intermediate Pressure Separator

IPR Inflow Performance Relationship

LKT Lookup Table

LP Low-Pressure Separator

MILP Mixed Integer Linear Program

MINLP Mixed-Integer Nonlinear Program

MPWLP Multicapacitated Platforms and Wells Location Problem

NF Natural Flowing

PCP Progressive Cavity Pump

RTPO Real-Time Production Optimisation

SCIP Solving Constraint Integer Programs

SOS2 Special Ordered Sets of type 2

STB Stock Tank Barrel

VFP Vertical Flow Performance

Greek symbols (parameters and variables)

$\alpha_0 - \alpha_5$ Proxy model coefficients

λ SOS weights

ξ Linearisation variable

τ Linearisation variable

Ω_{PCP} PCP impeller rotation speed (RPM)

δ SOS2 variables – sum of SOS weights

Subscripts and superscripts

i Fluid phase index

j Linearisation breakpoint

k Linearisation breakpoints

m Manifold index

o Oil phase index

p Pipeline index

s Separator index

w Production well index

wh wellhead

wt water phase index

8. Appendix

Table A1: Definitions of key variables and parameters of the optimisation formulation. Most of these definitions are adapted from the Schlumberger oilfield glossary (Schlumberger, 2020).

Variable	Symbol	Definition
Wellhead pressure	P^{wh}	The pressure at the surface component over a well that contains the flow from the well to other processing equipment.
Manifold pressure	P^{m}	The manifold is an arrangement of piping or valves designed to gather, control, distribute and monitor fluid flow. The pressure of this system is the manifold pressure.
Bottomhole pressure	P_{wf}	The pressure at the bottom of the well, near the producing formation.
Gas Oil Ratio	GOR	The ratio of produced gas to produced oil.
Water cut	WC	The ratio of produced water to the volume of the total liquids produced.
ESP frequency	f_{ESP}	The frequency of the impeller used to provide sustaining power to the Electrical Submersible Pump. The pump power can be obtained from this variable using the manufacturer's pump curves.
Pipeline pressure drop	ΔP	The decrease in pressure as fluid flows through a pipeline due to the pipe's roughness, friction, flowrate and geometrical properties of the pipe or conduit.
Routing variables	y, z	Binary (0,1) variables which are employed to select one of many manifolds or separators to channel the produced fluids.
Parameter	**Symbol**	**Definition**
Productivity index	PI	The ability of a reservoir to deliver fluids to the wellbore.
Reservoir pressure	P_r	The pressure of fluids within the pores of a reservoir.
Separator pressure	P^{s}	The operating pressure necessary for the efficient separation of gas and liquid phases or two liquid phases.
True Vertical Depth	TVD	The vertical distance from a point in the well (usually the current or final depth) to a point at the surface.

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