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γ-P_S-GENERALIZED CLOSED SETS AND γ-P_S-$T_{\frac{1}{2}}$ SPACES

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Abstract: This paper defines new class of sets called γ-P_S-generalized closed using γ-P_S-open set and τ_γ-P_S-closure of a set in a topological space. By using this new set, we introduce a new space called γ-P_S-$T_{\frac{1}{2}}$ and define three functions namely γ-P_S-g-continuous, γ-P_S-g-closed and γ-P_S-g-open. Some theoretical results and properties for this space and these functions are obtained. Several examples are given to illustrate some of the results.

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1. Introduction

Kasahara [8] defined the concept of α-closed graphs of an operation on τ. Later, Ogata [11] renamed the operation α as γ operation on τ. He defined γ-open sets and introduced the notion of τ_γ which is the class of all γ-open sets in a topological space (X,τ). Further study by Krishnan and Balachandran ([9], [10]) defined two types of sets called γ-preopen and γ-semiopen sets. The notion of α-γ-open sets have been defined by Kalaivani and Krishnan [7]. Meanwhile, Basu, Afsan and Ghosh [4] defined γ-β-open sets by using the operation γ on

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τ. Carpintero, Rajesh and Rosas [5] introduced another notion of γ-open set called γ-b-open sets of a topological space (X, τ). Recently, Asaad, Ahmad and Omar [1] defined the notion of γ-regular-open sets which lies strictly between the classes of γ-open set and γ-clopen set. They also introduced a new class of sets called γ-P_S-open sets, and they also defined γ-P_S-operations and their properties [2].

In the present paper, we define a new class of sets called γ-P_S-generalized closed using γ-P_S-open set and τ_γ-P_S-closure of a set and then invistigate some of its properties. A new space called γ-P_S-$T_{\frac{1}{2}}$ and functions called γ-P_S-g-continuous, γ-P_S-g-closed and γ-P_S-g-open are defined. Some theorems and results for this space and these functions are obtained.

2. Preliminaries and Basic Definitions

Throughout this paper, spaces (X, τ) and (Y, σ) (or simply X and Y) always mean topological spaces on which no separation axioms assumed unless explicitly stated. An operation γ on the topology τ on X is a mapping $\gamma\colon \tau \to P(X)$ such that $U \subseteq \gamma(U)$ for each $U \in \tau$, where $P(X)$ is the power set of X and $\gamma(U)$ denotes the value of γ at U [11]. A nonempty subset A of a topological space (X, τ) with an operation γ on τ is said to be γ-open [11] if for each $x \in A$, there exists an open set U containing x such that $\gamma(U) \subseteq A$. The complement of a γ-open set is called a γ-closed. The τ_γ-closure of a subset A of X with an operation γ on τ is defined as the intersection of all γ-closed sets containing A and it is denoted by τ_γ-$Cl(A)$ [11], and the τ_γ-interior of a subset A of X with an operation γ on τ is defined as the union of all γ-open sets containing A [10]. Now we begin to recall some known notions which are useful in the sequel.

Definition 2.1. Let (X, τ) be a topological space and γ be an operation on τ. A subset A of X is said to be:

1. γ-regular-open if $A = \tau_\gamma$-$Int(\tau_\gamma$-$Cl(A))$ [1].
2. γ-preopen if $A \subseteq \tau_\gamma$-$Int(\tau_\gamma$-$Cl(A))$ [9].
3. γ-semiopen if $A \subseteq \tau_\gamma$-$Cl(\tau_\gamma$-$Int(A))$ [10].
4. α-γ-open if $A \subseteq \tau_\gamma$-$Int(\tau_\gamma$-$Cl(\tau_\gamma$-$Int(A)))$ [7].
5. γ-b-open if $A \subseteq \tau_\gamma$-$Cl(\tau_\gamma$-$Int(A)) \cup \tau_\gamma$-$Int(\tau_\gamma$-$Cl(A))$ [5].
6. γ-β-open if $A \subseteq \tau_\gamma$-$Cl(\tau_\gamma$-$Int(\tau_\gamma$-$Cl(A)))$ [4].

7. γ-clopen if it is both γ-open and γ-closed.

8. γ-dense if τ_γ-$Cl(A) = X$ [6].

Definition 2.2. The complement of γ-regular-open, γ-preopen, γ-semiopen, α-γ-open, γ-b-open and γ-β-open set is said to be γ-regular-closed [4], γ-preclosed [9], γ-semiclosed [10], α-γ-closed [7], γ-b-closed [5] and γ-β-closed [4], respectively.

Definition 2.3. [2] A γ-preopen subset A of a topological space (X, τ) is called γ-P_S-open if for each $x \in A$, there exists a γ-semiclosed set F such that $x \in F \subseteq A$. The complement of a γ-P_S-open set is called a γ-P_S-closed.

The class of all γ-P_S-open and γ-P_S-closed subsets of a topological space (X, τ) are denoted by τ_γ-$P_SO(X)$ and τ_γ-$P_SC(X)$, respectively.

Definition 2.4. Let A be any subset of a topological space (X, τ) and γ be an operation on τ. Then:

1. the τ_γ-P_S-interior of A is defined as the union of all γ-P_S-open sets of X contained in A and it is denoted by τ_γ-$P_SInt(A)$ [2].

2. the τ_γ-P_S-closure, τ_γ-preclosure and $\tau_{\alpha-\gamma}$-closure of A is defined as the intersection of all γ-P_S-closed, γ-preclosed and α-γ-closed sets of X containing A and it is denoted by τ_γ-$P_SCl(A)$ [2], τ_γ-$pCl(A)$ [9] and $\tau_{\alpha-\gamma}$-$Cl(A)$ [7], respectively.

Remark 2.5. [2] Let (X, τ) be a topological space and γ be an operation on τ. For any subset A of a space X. The following statements are true.

1. A is γ-P_S-closed if and only if τ_γ-$P_SCl(A) = A$.

2. A is γ-P_S-open if and only if τ_γ-$P_SInt(A) = A$.

3. τ_γ-$P_SCl(X \backslash A) = X \backslash \tau_\gamma$-$P_SInt(A)$ and τ_γ-$P_SInt(X \backslash A) = X \backslash \tau_\gamma$-$P_SCl(A)$.

Remark 2.6. [2] Let (X, τ) be a topological space and γ be an operation on τ. For each element $x \in X$, the set $\{x\}$ is γ-P_S-open if and only if $\{x\}$ is γ-regular-open.

Theorem 2.7. *Let A be a subset of a topological space (X, τ) and γ be an operation on τ. Then:*

1. *$x \in \tau_\gamma$-$P_SCl(A)$ if and only if $A \cap U \neq \phi$ for every γ-P_S-open set U of X containing x [2].*

2. *$x \in \tau_\gamma$-$pCl(A)$ if and only if $A \cap U \neq \phi$ for every γ-preopen set U of X containing x [9].*

Definition 2.8. Let (X, τ) be a topological space and γ be an operation on τ. A subset A of X is called:

1. γ-pre-generalized closed (γ-pre$g$-closed) if τ_γ-$pCl(A) \subseteq G$ whenever $A \subseteq G$ and G is a γ-preopen set in X [9].

2. α-γ-generalized closed (α-γ-g-closed) if $\tau_{\alpha-\gamma}$-$Cl(A) \subseteq G$ whenever $A \subseteq G$ and G is a α-γ-open set in X [7].

Definition 2.9. [9] A topological space (X, τ) with an operation γ on τ is said to be γ-pre$T_{\frac{1}{2}}$ if every γ-pre$g$-closed set in X is γ-preclosed.

Theorem 2.10. *[9] For any topological space (X, τ) with an operation γ on τ. Then X is γ-pre$T_{\frac{1}{2}}$ if and only if for each element $x \in X$, the set $\{x\}$ is γ-preclosed or γ-preopen.*

Definition 2.11. [3] Let (X, τ) and (Y, σ) be two topological spaces and γ be an operation on τ. A function $f\colon (X, \tau) \to (Y, \sigma)$ is called γ-P_S-continuous if the inverse image of every closed set in Y is γ-P_S-closed set in X.

Definition 2.12. [3] Let (X, τ) and (Y, σ) be two topological spaces and γ be an operation on τ. A function $f\colon (X, \tau) \to (Y, \sigma)$ is called γ-P_S-closed if the image of every closed set in Y is γ-P_S-closed set in X.

3. γ-P_S-Generalized Closed Sets

In this section, we define a new class of sets called γ-P_S-generalized closed using γ-P_S-open set and τ_γ-P_S-closure of set. Also study some of its basic properties.

Definition 3.1. Let A be any subset of a topological space (X, τ) with an operation γ on τ is called γ-P_S-generalized closed (γ-P_S-g-closed) if τ_γ-$P_SCl(A) \subseteq G$ whenever $A \subseteq G$ and G is a γ-P_S-open set in X.

The class of all γ-P_S-g-closed sets of X is denoted by τ_γ-$P_SGC(X)$.

A set A is said to be γ-P_S-generalized open (γ-P_S-g-open) if its complement is γ-P_S-g-closed. Or equivalently, a set A is γ-P_S-g-open if $F \subseteq \tau_\gamma$-$P_SInt(A)$ whenever $F \subseteq A$ and F is a γ-P_S-closed set in X.

Lemma 3.2. *Every γ-P_S-closed set is γ-P_S-g-closed.*

Proof. Let A be any γ-P_S-closed set in a space X and $A \subseteq G$ where G is a γ-P_S-open set in X. Then τ_γ-$P_SCl(A) \subseteq G$ since A is γ-P_S-closed set. Therefore, A is γ-P_S-g-closed set. □

The following example shows that the converse of the Lemma 3.2 is not true.

Example 3.3. Let $X = \{a, b, c\}$ with the topology $\tau = \{\phi, \{a\}, \{b, c\}, X\}$. Define an operation $\gamma\colon \tau \to P(X)$ by $\gamma(A) = A$ for all $A \in \tau$. Then τ_γ-$P_SC(X) = \{\phi, \{a\}, \{b, c\}, X\}$ and τ_γ-$P_SGC(X) =$ all subsets of X. So $\{b\}$ is γ-P_S-g-closed, but it is not γ-P_S-closed.

Lemma 3.4. *The union of any two γ-P_S-g-closed sets may not be γ-P_S-g-closed.*

Lemma 3.5. *The intersection of any two γ-P_S-g-closed sets may not be γ-P_S-g-closed.*

Reverse implication of the above theorem does not hold as seen from the following example.

Example 3.6. Let $X = (0, 1)$ and τ be the usual topology on X. Define an operation γ on τ by $\gamma(U) = U$ for all $U \in \tau$. Let A be the set of rational numbers in X except the singleton set $\{\frac{1}{2}\}$ and B be the set of irrational numbers in X. Then A and B are both γ-P_S-g-closed sets, but $A \cup B$ is not γ-P_S-g-closed.

Theorem 3.7. *Let A be a subset of topological space (X, τ) and γ be an operation on τ. Then A is γ-P_S-g-closed if and only if τ_γ-$P_SCl(A)\backslash A$ does not contain any non-empty γ-P_S-closed set.*

Proof. Let F be a non-empty γ-P_S-closed set in X such that $F \subseteq \tau_\gamma$-$P_SCl(A)\backslash A$. Then $F \subseteq X\backslash A$ implies $A \subseteq X\backslash F$. Since $X\backslash F$ is γ-P_S-open set and A is γ-P_S-g-closed set, then τ_γ-$P_SCl(A) \subseteq X\backslash F$. That is $F \subseteq X\backslash\tau_\gamma$-$P_SCl(A)$. Hence $F \subseteq X\backslash\tau_\gamma$-$P_SCl(A) \cap \tau_\gamma$-$P_SCl(A)\backslash A \subseteq X\backslash\tau_\gamma$-$P_SCl(A) \cap \tau_\gamma$-$P_SCl(A) = \phi$. This shows that $F = \phi$. This is contradiction. Therefore, $F \nsubseteq \tau_\gamma$-$P_SCl(A)\backslash A$.

Conversely, let $A \subseteq G$ and G is γ-P_S-open set in X. So $X\backslash G$ is γ-P_S-closed set in X. Suppose that τ_γ-$P_SCl(A) \nsubseteq G$, then τ_γ-$P_SCl(A) \cap X\backslash G$ is a non-empty γ-P_S-closed set such that τ_γ-$P_SCl(A) \cap X\backslash G \subseteq \tau_\gamma$-$P_SCl(A)\backslash A$. Contradiction of hypothesis. Hence τ_γ-$P_SCl(A) \subseteq G$ and so A is γ-P_S-g-closed set. □

Corollary 3.8. *Let A be a γ-P_S-g-closed subset of topological space (X, τ) with an operation γ on τ. Then A is γ-P_S-closed if and only if τ_γ-$P_SCl(A)\backslash A$*

is γ-P_S-closed set.

Proof. Let A be a γ-P_S-closed set. Then τ_γ-$P_SCl(A) = A$ and hence τ_γ-$P_SCl(A)\backslash A = \phi$ which is γ-P_S-closed set.

Conversely, suppose τ_γ-$P_SCl(A)\backslash A$ is γ-P_S-closed and A is γ-P_S-g-closed. Then by Theorem 3.7, τ_γ-$P_SCl(A)\backslash A$ does not contain any non-empty γ-P_S-closed set and since τ_γ-$P_SCl(A)\backslash A$ is γ-P_S-closed subset of itself, then τ_γ-$P_SCl(A)\backslash A = \phi$ implies τ_γ-$P_SCl(A) \cap X\backslash A = \phi$. This implies that τ_γ-$P_SCl(A) = A$. Therefore, A is γ-P_S-closed in X. □

Remark 3.9. For any set $A \subseteq (X, \tau)$, τ_γ-$P_SInt(\tau_\gamma$-$P_SCl(A)\backslash A) = \phi$.

Proof. Obvious. □

Theorem 3.10. *In any topological space (X, τ), a set $A \subseteq (X, \tau)$ is γ-P_S-g-closed if and only if τ_γ-$P_SCl(A)\backslash A$ is γ-P_S-g-open set.*

Proof. Let F be a γ-P_S-closed set in X such that $F \subseteq \tau_\gamma$-$P_SCl(A)\backslash A$. Since A is γ-P_S-g-closed. Then by Theorem 3.7, $F = \phi$. Hence $F \subseteq \tau_\gamma$-$P_SInt(\tau_\gamma$-$P_SCl(A)\backslash A)$. This shows that τ_γ-$P_SCl(A)\backslash A$ is γ-P_S-g-open set.

Conversely, suppose that $A \subseteq G$, where G is a γ-P_S-open set in X. So τ_γ-$P_SCl(A) \cap X\backslash G \subseteq \tau_\gamma$-$P_SCl(A) \cap X\backslash A = \tau_\gamma$-$P_SCl(A)\backslash A$. Since $X\backslash G$ is a γ-P_S-closed and hence τ_γ-$P_SCl(A) \cap X\backslash G$ is γ-P_S-closed set in X and τ_γ-$P_SCl(A)\backslash A$ is γ-P_S-g-open set. Then τ_γ-$P_SCl(A) \cap X\backslash G \subseteq \tau_\gamma$-$P_SInt(\tau_\gamma$-$P_SCl(A)\backslash A) = \phi$. By Remark 3.9, τ_γ-$P_SInt(\tau_\gamma$-$P_SCl(A)\backslash A) = \phi$ implies that τ_γ-$P_SCl(A) \cap X\backslash G = \phi$ and hence τ_γ-$P_SCl(A) \subseteq G$. This means that A is γ-P_S-g-closed. □

Theorem 3.11. *Let (X, τ) be a topological space and γ be an operation on τ. If a subset A of X is γ-P_S-g-closed and γ-P_S-open, then A is γ-P_S-closed.*

Proof. Since A is γ-P_S-g-closed and γ-P_S-open set in X, then τ_γ-$P_SCl(A) \subseteq A$ and so A is γ-P_S-closed. □

Theorem 3.12. *Let (X, τ) be a topological space and γ be an operation on τ. If a subset A of X is γ-P_S-g-closed and γ-P_S-open and F is γ-P_S-closed, then $A \cap F$ is γ-P_S-closed.*

Proof. Since A is both γ-P_S-g-closed and γ-P_S-open set. Then by Theorem 3.11, A is γ-P_S-closed and since F is γ-P_S-closed, then $A \cap F$ is γ-P_S-closed. □

Corollary 3.13. *If $A \subseteq X$ is both γ-P_S-g-closed and γ-P_S-open and F is γ-P_S-closed, then $A \cap F$ is γ-P_S-g-closed.*

Proof. Follows from Theorem 3.12 and the fact that every γ-P_S-closed set is γ-P_S-g-closed. □

Corollary 3.14. *For any topological space (X, τ). If a subset A of X is γ-P_S-g-closed and γ-P_S-open, then A is γ-preg-closed.*

Proof. The proof follows directly from Theorem 3.11 and the fact that every γ-P_S-closed set is γ-preclosed and every γ-preclosed set is γ-pre$g$-closed [9]. □

Theorem 3.15. *If $A \subseteq (X, \tau)$ is both γ-regular-open and γ-P_S-g-closed, then A is γ-regular-closed and hence it is γ-clopen.*

Proof. Let A be both γ-regular-open and γ-P_S-g-closed. Since A is γ-regular-open set. Then A is γ-P_S-open and by Theorem 3.11, A is γ-P_S-closed and so it is γ-preclosed. Again since A is γ-regular-open set, then A is γ-semiopen. Therefore, A is γ-regular-closed in X. Thus A is both γ-open and γ-closed and hence it is γ-clopen. □

Lemma 3.16. *For any subset A in (X, τ). If A is γ-semiopen, then τ_γ-$P_SCl(A) = \tau_\gamma$-$pCl(A)$.*

Proof. Let $x \notin \tau_\gamma$-$pCl(A)$, then there exists a γ-preopen set U containing x such that $A \cap U = \phi$ implies that τ_γ-$Cl(\tau_\gamma$-$Int(A)) \cap \tau_\gamma$-$Int(\tau_\gamma$-$Cl(U)) = \phi$. Since A is γ-semiopen, then $A \cap \tau_\gamma$-$Int(\tau_\gamma$-$Cl(U)) = \phi$. Since U is γ-preopen set containing x, then $x \in \tau_\gamma$-$Int(\tau_\gamma$-$Cl(U))$ and τ_γ-$Int(\tau_\gamma$-$Cl(U))$ is γ-P_S-open set. So by Theorem 2.7 (1), $x \notin \tau_\gamma$-$P_SCl(A)$. Hence τ_γ-$P_SCl(A) \subseteq \tau_\gamma$-$pCl(A)$. But τ_γ-$pCl(A) \subseteq \tau_\gamma$-$P_SCl(A)$ in general. Then τ_γ-$P_SCl(A) = \tau_\gamma$-$pCl(A)$. □

Similar to Lemma 3.16, we can show that τ_γ-$pCl(A) = \tau_\gamma$-$Cl(A) = \tau_{\alpha-\gamma}$-$Cl(A)$ for every γ-semiopen set A in (X, τ). So we have the next corollary.

Corollary 3.17. *For each γ-semiopen A in (X, τ), we have*

$$\tau_\gamma\text{-}P_SCl(A) = \tau_\gamma\text{-}pCl(A) = \tau_\gamma\text{-}Cl(A) = \tau_{\alpha-\gamma}\text{-}Cl(A).$$

Lemma 3.18. *For any subset A in (X, τ). If A is γ-β-open, then τ_γ-$Cl(A) = \tau_{\alpha-\gamma}$-$Cl(A)$.*

Proof. The proof is similar to Lemma 3.16. □

Theorem 3.19. *If a subset A of (X, τ) is both α-γ-open and γ-preg-closed, then A is γ-P_S-g-closed.*

Proof. Suppose that A is both α-γ-open and γ-pre$g$-closed set in X. Let $A \subseteq G$ and G be a γ-P_S-open set in X. Since A is α-γ-open. Then A is γ-preopen. Now $A \subseteq A$. By hypothesis, τ_γ-$pCl(A) \subseteq A$. Again since A is α-γ-open, then A is γ-semiopen. By Corollary 3.17, we get τ_γ-$P_SCl(A) \subseteq A \subseteq G$. Thus, A is γ-P_S-g-closed. □

Theorem 3.20. *If a set A in X is both α-γ-open and α-γ-g-closed, then A is γ-P_S-g-closed.*

Proof. The proof is similar to Theorem 3.19 and using Corollary 3.17 to obtain $\tau_{\alpha-\gamma}$-$Cl(A) = \tau_\gamma$-$P_SCl(A)$ for every γ-semiopen set in X. □

The converse of Theorem 3.19 and Theorem 3.20 are true when A is γ-regular-open as it can be seen from the following corollary.

Corollary 3.21. *Let A be a γ-regular-open subset of a topological space (X, τ) with an operation γ on τ. Then the following conditions are equivalent:*

1. *A is γ-P_S-g-closed.*
2. *A is γ-preg-closed.*
3. *A is α-γ-g-closed.*

Theorem 3.22. *In a topological space (X, τ) with an operation γ on τ. Then every subset of X is γ-P_S-g-closed if and only if τ_γ-$P_SO(X) = \tau_\gamma$-$P_SC(X)$.*

Proof. Assume that every subset of X is γ-P_S-g-closed. Let $U \in \tau_\gamma$-$P_SO(X)$. Since U is γ-P_S-g-closed. Then by Theorem 3.11, we have U is γ-P_S-closed. Hence τ_γ-$P_SO(X) \subseteq \tau_\gamma$-$P_SC(X)$. If $F \in \tau_\gamma$-$P_SC(X)$, then $X \backslash F \in \tau_\gamma$-$P_SO(X)$ and $X \backslash F$ is γ-P_S-g-closed. Then by Theorem 3.11, $X \backslash F$ is γ-P_S-closed and hence F is γ-P_S-open set. Thus, τ_γ-$P_SC(X) \subseteq \tau_\gamma$-$P_SO(X)$. This means that τ_γ-$P_SO(X) = \tau_\gamma$-$P_SC(X)$.

Conversely, suppose that τ_γ-$P_SO(X) = \tau_\gamma$-$P_SC(X)$ and that $A \subseteq G$ and $G \in \tau_\gamma$-$P_SO(X)$. Then τ_γ-$P_SCl(A) \subseteq \tau_\gamma$-$P_SCl(G) = G$. So A is γ-P_S-g-closed. □

Theorem 3.23. *Let A, B be subsets of a topological space (X, τ) and γ be an operation on τ. If A is γ-P_S-g-closed and $A \subseteq B \subseteq \tau_\gamma$-$P_SCl(A)$, then B is γ-P_S-g-closed set.*

Proof. Let A be any γ-P_S-g-closed set in (X,τ) and $B \subseteq G$ where G is γ-P_S-open. Since $A \subseteq B$, then $A \subseteq G$ and hence τ_γ-$P_SCl(A) \subseteq G$. Since $B \subseteq \tau_\gamma$-$P_SCl(A)$ implies τ_γ-$P_SCl(B) \subseteq \tau_\gamma$-$P_SCl(A)$. Thus τ_γ-$P_SCl(B) \subseteq G$ and this shows that B is γ-P_S-g-closed set. □

From Theorems 3.7 and 3.23, we obtain the following proposition.

Proposition 3.24. *Let A, B be subsets of a topological space (X,τ) and γ be an operation on τ. If A is γ-P_S-g-closed and $A \subseteq B \subseteq \tau_\gamma$-$P_SCl(A)$, then τ_γ-$P_SCl(B)\backslash B$ contains no non-empty γ-P_S-closed set.*

Theorem 3.25. *Let A and B be subsets of (X,τ). If A is γ-P_S-g-open and τ_γ-$P_SInt(A) \subseteq B \subseteq A$, then B is γ-P_S-g-open set.*

Proof. Since τ_γ-$P_SInt(A) \subseteq B \subseteq A$ implies that $X\backslash A \subseteq X\backslash B \subseteq X\backslash\tau_\gamma$-$P_SInt(A)$. By Remark 2.5 (3), we get $X\backslash A \subseteq X\backslash B \subseteq \tau_\gamma$-$P_SCl(X\backslash A)$. Since A is γ-P_S-g-open and then $X\backslash A$ is γ-P_S-g-closed. So by Theorem 3.23, $X\backslash B$ is γ-P_S-g-closed and hence B is γ-P_S-g-open. □

Theorem 3.26. *A subset A in (X,τ) is γ-P_S-g-open if and only if $G = X$ whenever G is γ-P_S-open set in X and τ_γ-$P_SInt(A) \cup X\backslash A \subseteq G$.*

Proof. Let G be a γ-P_S-open set in X and τ_γ-$P_SInt(A) \cup X\backslash A \subseteq G$. This implies $X\backslash G \subseteq \tau_\gamma$-$P_SCl(X\backslash A) \cap A = \tau_\gamma$-$P_SCl(X\backslash A)\backslash(X\backslash A)$. Since G is γ-P_S-open and A is γ-P_S-g-open, then $X\backslash G$ is γ-P_S-closed and $X\backslash A$ is γ-P_S-g-closed. So by Theorem 3.7, $X\backslash G = \phi$ implies $G = X$.

Conversely, suppose F is a γ-P_S-closed set in X and $F \subseteq A$. Then $X\backslash A \subseteq X\backslash F$ and hence τ_γ-$P_SInt(A) \cup X\backslash A \subseteq \tau_\gamma$-$P_SInt(A) \cup X\backslash F$. Since τ_γ-$P_SInt(A) \cup X\backslash F$ is γ-P_S-open set in X, then by hypothesis τ_γ-$P_SInt(A) \cup X\backslash F = X$. It follows that $F \subseteq \tau_\gamma$-$P_SInt(A)$. Therefore, A is γ-P_S-g-open set in X. □

Recall that a topological space (X,τ) with an operation γ on τ is γ-semi-T_1 if for each pair of distinct points x, y in X, there exist two γ-semiopen sets U and V such that $x \in U$ but $y \notin U$ and $y \in V$ but $x \notin V$ [10].

Theorem 3.27. *[2] Let (X,τ) be a topological space and γ be an operation on τ. If X is γ-semi-T_1, then the notion of γ-P_S-open set and γ-preopen set coincide, or this means that the notion of γ-P_S-closed set and γ-preclosed set coincide.*

Remark 3.28. In the above theorem, we can conclude that τ_γ-$P_SCl(A) = \tau_\gamma$-$pCl(A)$ for any subset A of a γ-semi-T_1 space X.

Theorem 3.29. *Let (X, τ) be γ-semi-T_1 space and γ be an operation on τ. A set A is γ-P_S-g-closed if and only if A is γ-preg-closed.*

Proof. Follows from Theorem 3.27 and Remark 3.28. □

Recall that a topological space (X, τ) with an operation γ on τ is γ-locally indiscrete if every γ-open subset of X is γ-closed, or every γ-closed subset of X is γ-open [1].

Theorem 3.30. *[2] Let (X, τ) be a topological space and γ be an operation on τ. If X is γ-locally indiscrete, then τ_γ-$P_SO(X) = \tau_\gamma$.*

Lemma 3.31. *If a space (X, τ) is γ-locally indiscrete, then every γ-P_S-open subset of X is γ-P_S-closed.*

Proof. Follows directly from Theorem 3.30. □

The following theorem shows that if a space X is γ-locally indiscrete, then τ_γ-$P_SGC(X)$ is discrete topology.

Theorem 3.32. *If a topological space (X, τ) is γ-locally indiscrete, then every subset of X is γ-P_S-g-closed.*

Proof. Suppose that (X, τ) is γ-locally indiscrete space and $A \subseteq U$ where $U \in \tau_\gamma$-$P_SO(X)$. Then τ_γ-$P_SCl(A) \subseteq \tau_\gamma$-$P_SCl(U)$ and by Lemma 3.31, we have τ_γ-$P_SCl(A) \subseteq U$ and so A is γ-P_S-g-closed set. □

Recall that a topological space (X, τ) with an operation γ on τ is γ-hyperconnected if every γ-open subset of X is γ-dense [1].

Theorem 3.33. *[2] If a topological space (X, τ) is γ-hyperconnected if and only if τ_γ-$P_SO(X) = \{\phi, X\}$.*

Theorem 3.34. *In a topological space (X, τ) with an operation γ on τ, if τ_γ-$P_SO(X) = \{\phi, X\}$, then every subset of X is a γ-P_S-g-closed.*

Proof. Let A be any subset of a topological space (X, τ) and τ_γ-$P_SO(X) = \{\phi, X\}$. Suppose that $A = \phi$, then A is a γ-P_S-g-closed set in X. If $A \neq \phi$, then X is the only γ-P_S-open set containing A and hence τ_γ-$P_SCl(A) \subseteq X$. So A is a γ-P_S-g-closed set in X. □

Corollary 3.35. *If a topological space (X, τ) is γ-hyperconnected, then every subset of X is γ-P_S-g-closed.*

Proof. Follows from Theorem 3.33 and Theorem 3.34. □

Theorem 3.36. *In any topological space (X, τ) with an operation γ on τ. For an element $x \in X$, the set $X \backslash \{x\}$ is γ-P_S-g-closed or γ-P_S-open.*

Proof. Suppose that $X \backslash \{x\}$ is not γ-P_S-open. Then X is the only γ-P_S-open set containing $X \backslash \{x\}$. This implies that τ_γ-$P_S Cl(X \backslash \{x\}) \subseteq X$. Thus $X \backslash \{x\}$ is a γ-P_S-g-closed set in X. □

Corollary 3.37. *In any topological space (X, τ) with an operation γ on τ. For an element $x \in X$, either the set $\{x\}$ is γ-P_S-closed or the set $X \backslash \{x\}$ is γ-P_S-g-closed.*

Proof. Suppose $\{x\}$ is not γ-P_S-closed, then $X \backslash \{x\}$ is not γ-P_S-open. Hence by Theorem 3.36, $X \backslash \{x\}$ is γ-P_S-g-closed set in X. □

Lemma 3.38. *Let (X, τ) be a topological space and γ be an operation on τ. A set A in (X, τ) is γ-P_S-g-closed if and only if $A \cap \tau_\gamma$-$P_S Cl(\{x\}) \neq \phi$ for every $x \in \tau_\gamma$-$P_S Cl(A)$.*

Proof. Suppose A is γ-P_S-g-closed set in X and suppose (if possible) that there exists an element $x \in \tau_\gamma$-$P_S Cl(A)$ such that $A \cap \tau_\gamma$-$P_S Cl(\{x\}) = \phi$. This follows that $A \subseteq X \backslash \tau_\gamma$-$P_S Cl(\{x\})$. Since τ_γ-$P_S Cl(\{x\})$ is γ-P_S-closed implies $X \backslash \tau_\gamma$-$P_S Cl(\{x\})$ is γ-P_S-open and A is γ-P_S-g-closed set in X. Then τ_γ-$P_S Cl(A) \subseteq X \backslash \tau_\gamma$-$P_S Cl(\{x\})$. This means that $x \notin \tau_\gamma$-$P_S Cl(A)$. This is a contradiction. Hence $A \cap \tau_\gamma$-$P_S Cl(\{x\}) \neq \phi$.

Conversely, let G be any γ-P_S-open set in X containing A. To show that τ_γ-$P_S Cl(A) \subseteq G$. Let $x \in \tau_\gamma$-$P_S Cl(A)$. Then by hypothesis, $A \cap \tau_\gamma$-$P_S Cl(\{x\}) \neq \phi$. So there exists an element $y \in A \cap \tau_\gamma$-$P_S Cl(\{x\})$. Thus $y \in A \subseteq G$ and $y \in \tau_\gamma$-$P_S Cl(\{x\})$. By Theorem 2.7 (1), $\{x\} \cap G \neq \phi$. Hence $x \in G$ and so τ_γ-$P_S Cl(A) \subseteq G$. Therefore, A is γ-P_S-g-closed set in (X, τ). □

Theorem 3.39. *For any subset A of a topological space (X, τ). Then $A \cap \tau_\gamma$-$P_S Cl(\{x\}) \neq \phi$ for every $x \in \tau_\gamma$-$P_S Cl(A)$ if and only if τ_γ-$P_S Cl(A) \backslash A$ does not contain any non-empty γ-P_S-closed set.*

Proof. The proof is directly from Theorem 3.7 and Lemma 3.38. □

Corollary 3.40. *Let A be a subset of topological space (X, τ) and γ be an operation on τ. Then A is γ-P_S-g-closed if and only if $A = E \backslash F$, where E is γ-P_S-closed set and F contains no non-empty γ-P_S-closed set.*

Proof. Let A be any γ-P_S-g-closed set in (X, τ). Then by Theorem 3.7, τ_γ-$P_SCl(A)\backslash A = F$ contains no non-empty γ-P_S-closed set. Let $E = \tau_\gamma$-$P_SCl(A)$ is γ-P_S-closed set such that $A = E\backslash F$.

Conversely, let $A = E\backslash F$, where E is γ-P_S-closed set and F contains no non-empty γ-P_S-closed set. Let $A \subseteq G$ and G is γ-P_S-open set in X. Then $E \cap X\backslash G$ is a γ-P_S-closed subset of F and hence it is empty. Therefore, τ_γ-$P_SCl(A) \subseteq E \subseteq G$. Thus A is γ-P_S-g-closed set. □

4. γ-P_S-$T_{\frac{1}{2}}$ Spaces

This section introduces a new space called γ-P_S-$T_{\frac{1}{2}}$ by using γ-P_S-g-closed set.

Definition 4.1. A topological space (X, τ) with an operation γ on τ is said to be γ-P_S-$T_{\frac{1}{2}}$ if every γ-P_S-g-closed set in X is γ-P_S-closed set.

Lemma 4.2. *A topological space (X, τ) is γ-P_S-$T_{\frac{1}{2}}$ if and only if τ_γ-$P_SGC(X) = \tau_\gamma$-$P_SC(X)$.*

Proof. Follows from Definition 4.1 and Lemma 3.2. □

Theorem 4.3. *For any topological space (X, τ) with an operation γ on τ. Then X is γ-P_S-$T_{\frac{1}{2}}$ if and only if for each element $x \in X$, the set $\{x\}$ is γ-P_S-closed or γ-P_S-open.*

Proof. Let X be a γ-P_S-$T_{\frac{1}{2}}$ space and let $\{x\}$ is not γ-P_S-closed set in X. By Corollary 3.37, $X\backslash\{x\}$ is γ-P_S-g-closed. Since X is γ-P_S-$T_{\frac{1}{2}}$, then $X\backslash\{x\}$ is γ-P_S-closed set which means that $\{x\}$ is γ-P_S-open set in X.

Conversely, let F be any γ-P_S-g-closed set in the space (X, τ). We have to show that F is γ-P_S-closed (that is τ_γ-$P_SCl(F) = F$). Let $x \in \tau_\gamma$-$P_SCl(F)$. By hypothesis $\{x\}$ is γ-P_S-closed or γ-P_S-open for each $x \in X$. So we have two cases:

Case (1): If $\{x\}$ is γ-P_S-closed set. Suppose $x \notin F$, then $x \in \tau_\gamma$-$P_SCl(F)\backslash F$ contains a non-empty γ-P_S-closed set $\{x\}$. A contradiction since F is γ-P_S-g-closed set and according to the Theorem 3.7. Hence $x \in F$. This follows that τ_γ-$P_SCl(F) \subseteq F$ and so τ_γ-$P_SCl(F) = F$. This means that F is γ-P_S-closed set in X. Thus a space X is γ-P_S-$T_{\frac{1}{2}}$.

Case (2): If $\{x\}$ is γ-P_S-open set. Then by Theorem 2.7 (1), $F \cap \{x\} \neq \phi$ which implies that $x \in F$. So τ_γ-$P_SCl(F) \subseteq F$. Thus F is γ-P_S-closed. Therefore, X is γ-P_S-$T_{\frac{1}{2}}$ space. □

Proposition 4.4. *If a space (X, τ) is γ-P_S-$T_{\frac{1}{2}}$, then the set $\{x\}$ is γ-P_S-closed or γ-regular-open for each $x \in X$.*

Proof. The proof is directly from Theorem 4.3 and Remark 2.6. □

Corollary 4.5. *If a space (X, τ) is γ-P_S-$T_{\frac{1}{2}}$, then for each point $x \in X$ the set $\{x\}$ is γ-b-closed.*

Proof. Let X be a γ-P_S-$T_{\frac{1}{2}}$ space. Then by Theorem 4.3, $\{x\}$ of X is either γ-P_S-closed set or γ-P_S-open set. If $\{x\}$ is γ-P_S-closed set, then $\{x\}$ is γ-b-closed set. If $\{x\}$ is γ-P_S-open, then by Remark 2.6, $\{x\}$ is γ-regular-open set and hence it is γ-b-closed set. In both cases, we have $\{x\}$ is γ-b-closed set. □

Theorem 4.6. *Every γ-P_S-$T_{\frac{1}{2}}$ space is γ-pre$T_{\frac{1}{2}}$.*

Proof. Let (X, τ) be a γ-P_S-$T_{\frac{1}{2}}$ space. Then by Theorem 4.3, every singleton set is γ-P_S-closed or γ-P_S-open. This implies that every singleton set is γ-preclosed or γ-preopen. Therefore, by Theorem 2.10, (X, τ) is γ-pre$T_{\frac{1}{2}}$ space. □

The converse of the above theorem does not hold as seen from the following example.

Example 4.7. Let $X = \{a, b, c\}$ with the topology

$$\tau = \{\phi, X, \{b\}, \{c\}, \{b, c\}, \{a, b\}\}.$$

Define an operation $\gamma \colon \tau \to P(X)$ as follows: for every $A \in \tau$

$$\gamma(A) = \begin{cases} Cl(A) & \text{if } c \notin A \\ A & \text{if } c \in A \end{cases}$$

Then $\tau_\gamma = \{\phi, X, \{c\}, \{b, c\}, \{a, b\}\}$, τ_γ-$P_S C(X) = \{\phi, X, \{b\}, \{c\}, \{a, b\}\}$ and τ_γ-$P_S GC(X) = \{\phi, X, \{b\}, \{c\}, \{b, c\}, \{a, b\}\}$. Then (X, τ) is γ-pre$T_{\frac{1}{2}}$ but it is not γ-P_S-$T_{\frac{1}{2}}$ since the set {b, c} is γ-P_S-g-closed, but it is not γ-P_S-closed.

Theorem 4.8. *Let (X, τ) be a γ-semi-T_1 space. Then (X, τ) is γ-P_S-$T_{\frac{1}{2}}$ if and only if (X, τ) is γ-pre$T_{\frac{1}{2}}$.*

Proof. The proof follows from Theorem 3.27 and Theroem 3.29. □

5. γ-P_S-g-Continuous Functions

In this section, we introduce a new class of functions called γ-P_S-g-continuous by using γ-P_S-g-closed set. Some theorems and properties for this function are studied.

Definition 5.1. Let (X, τ) and (Y, σ) be two topological spaces and γ be an operation on τ. A function $f\colon (X, \tau) \to (Y, \sigma)$ is called γ-P_S-g-continuous if the inverse image of every closed set in Y is γ-P_S-g-closed set in X.

Theorem 5.2. *For a function $f\colon (X, \tau) \to (Y, \sigma)$ with an operation γ on τ, the following statements are equivalent:*

1. *f is γ-P_S-g-continuous.*
2. *The inverse image of every open set in Y is γ-P_S-g-open set in X.*
3. *For each point $x \in X$ and each open set V of Y containing $f(x)$, there exists a γ-P_S-g-open set U of X containing x such that $f(U) \subseteq V$.*

Proof. Straightforward. □

Remark 5.3. Every γ-P_S-continuous function is γ-P_S-g-continuous.

Proof. Obvious since every γ-P_S-closed set is γ-P_S-g-closed set. □

The converse of the above remark does not true as seen from the following example.

Example 5.4. Let (X, τ) be a topological space and γ be an operation on τ as in Example 4.7. Suppose that $Y = \{1, 2, 3\}$ and $\sigma = \{\phi, Y, \{1\}, \{1, 3\}\}$ be a topology on Y. Let $f\colon (X, \tau) \to (Y, \sigma)$ be a function defined as follows: $f(a) = 1$, $f(b) = 2$ and $f(c) = 3$. Then f is γ-P_S-g-continuous, but f is not γ-P_S-continuous since $\{2, 3\}$ is closed in (Y, σ), but $f^{-1}(\{2, 3\}) = \{b, c\}$ is not γ-P_S-closed set in (X, τ).

Theorem 5.5. *Let (X, τ) be γ-P_S-$T_{\frac{1}{2}}$ space and γ be an operation on τ. A function $f\colon (X, \tau) \to (Y, \sigma)$ is γ-P_S-continuous if and only if f is γ-P_S-g-continuous.*

Proof. Follows from Remark 4.2. □

Theorem 5.6. *Let $f\colon (X, \tau) \to (Y, \sigma)$ be a function and γ be an operation on τ. If (X, τ) is γ-locally indiscrete space, then f is γ-P_S-g-continuous.*

Proof. This is an immediate consequence of Theorem 3.32. □

Theorem 5.7. *Let γ be an operation on the topological space (X, τ). If the functions $f\colon (X, \tau) \to (Y, \sigma)$ is γ-P_S-g-continuous and $g\colon (Y, \sigma) \to (Z, \rho)$ is continuous. Then the composition function $g \circ f\colon (X, \tau) \to (Z, \rho)$ is γ-P_S-g-continuous.*

Proof. It is clear. □

Proposition 5.8. *Let γ be an operation on the topological space (X, τ). If $f\colon (X, \tau) \to (Y, \sigma)$ is a function, $g\colon (Y, \sigma) \to (Z, \rho)$ is closed and injective, and $g \circ f\colon (X, \tau) \to (Z, \rho)$ is γ-P_S-g-continuous. Then f is γ-P_S-g-continuous.*

Proof. Let F be a closed subset of Y. Since g is closed, $g(F)$ is closed subset of Z. Since $g \circ f$ is γ-P_S-g-continuous and g is injective, then $f^{-1}(F) = f^{-1}(g^{-1}(g(F)) = (g \circ f)(g(F))$ is γ-P_S-g-closed in X, which proves that f is γ-P_S-g-continuous. □

Definition 5.9. Let (X, τ) and (Y, σ) be two topological spaces and β be an operation on σ. A function $f\colon (X, \tau) \to (Y, \sigma)$ is called β-P_S-g-closed if for every closed set F of X, $f(F)$ is β-P_S-g-closed set in Y.

Remark 5.10. Every β-P_S-closed function is β-P_S-g-closed.

The converse of the above remark does not true as seen from the following example.

Example 5.11. Let $X = \{a, b, c\}$ with the topology $\tau = \{\phi, \{c\}, \{b, c\}, X\}$ and $\sigma = \{\phi, X, \{b\}, \{a, c\}\}$. Define an operation β on σ by $\beta(A) = A$ for all $A \in \sigma$. Define $f\colon (X, \tau) \to (X, \sigma)$ by $f(a) = a$, $f(b) = c$ and $f(c) = b$. Then f is β-P_S-g-closed, but f is not β-P_S-closed function since $\{a\}$ is closed set in (X, τ), but $f(\{a\}) = \{a\}$ is not β-P_S-closed set in (X, σ).

Theorem 5.12. *Let (Y, σ) be a topological space and β be an operation on σ. A function $f\colon (X, \tau) \to (Y, \sigma)$ is β-P_S-g-closed if and only if for each subset S of Y and each open set O in X containing $f^{-1}(S)$, there exists a β-P_S-g-open set R in Y such that $S \subseteq R$ and $f^{-1}(R) \subseteq O$.*

Proof. Suppose that f is β-P_S-g-closed function and let O be an open set in X containing $f^{-1}(S)$, where S is any subset in Y. Then $f(X \backslash O)$ is β-P_S-g-open set in Y. If we put $R = Y \backslash f(X \backslash O)$. Then R is β-P_S-g-closed set in Y containing S such that $f^{-1}(R) \subseteq O$.

Conversely, let F be closed set in X. Let $S = Y \backslash f(F) \subseteq Y$. Then $f^{-1}(S) \subseteq X \backslash F$ and $X \backslash F$ is open set in X. By hypothesis, there exists a β-P_S-g-open set

R in Y such that $S = Y \backslash f(F) \subseteq R$ and $f^{-1}(R) \subseteq X \backslash F$. For $f^{-1}(R) \subseteq X \backslash F$ implies $R \subseteq f(X \backslash F) \subseteq Y \backslash f(F)$. Hence $R = Y \backslash f(F)$. Since R is β-P_S-g-open set in Y. Then $f(F)$ is β-P_S-g-closed set in Y. Therefore, f is β-P_S-g-closed function. □

Theorem 5.13. *Let β be an operation on σ and $f: (X, \tau) \to (Y, \sigma)$ be β-P_S-$T_{\frac{1}{2}}$ space. Then f is β-P_S-g-closed if and only if f is β-P_S-closed.*

Proof. Follows from Remark 4.2. □

Theorem 5.14. *Let $f: (X, \tau) \to (Y, \sigma)$ be a function and β be an operation on σ. If (Y, σ) is β-locally indiscrete space, then f is β-P_S-g-closed.*

Proof. This is an immediate consequence of Theorem 3.32. □

Definition 5.15. Let (X, τ) and (Y, σ) be two topological spaces and β be an operation on σ. A function $f: (X, \tau) \to (Y, \sigma)$ is called β-P_S-g-open if for every open set V of X, $f(V)$ is β-P_S-g-open set in Y.

Definition 5.16. A function $f: (X, \tau) \to (Y, \sigma)$ is said to be γ-P_S-g-homeomorphism, if f is bijective, γ-P_S-g-continuous and f^{-1} is γ-P_S-g-continuous.

Theorem 5.17. *The following statements are equivalent for a bijective function $f: (X, \tau) \to (Y, \sigma)$ with an operation β on σ.*

1. *f is β-P_S-g-closed.*
2. *f is β-P_S-g-open.*
3. *f^{-1} is β-P_S-g-continuous.*

Proof. It is clear. □

Proposition 5.18. *Let α be an operation on the topological space (Z, ρ). If the function $f: (X, \tau) \to (Y, \sigma)$ is closed (resp., open) and $g: (Y, \sigma) \to (Z, \rho)$ is α-P_S-g-closed (resp., α-P_S-g-open). Then the composition function $g \circ f: (X, \tau) \to (Z, \rho)$ is α-P_S-g-closed (resp., α-P_S-g-open).*

Proof. Obvious. □

Proposition 5.19. *Let β be an operation on the topological space (Y, σ). If $g: (Y, \sigma) \to (Z, \rho)$ is a function, $f: (X, \tau) \to (Y, \sigma)$ is β-P_S-g-open and surjective, and $g \circ f: (X, \tau) \to (Z, \rho)$ is continuous. Then g is γ-P_S-g-continuous.*

Proof. Similar to Proposition 5.8. □

Proposition 5.20. *Let β be an operation on the topological space (Y,σ). If $g\colon (Y,\sigma)\to(Z,\rho)$ is a function, $f\colon (X,\tau)\to(Y,\sigma)$ is continuous and surjective, and $g\circ f\colon (X,\tau)\to(Z,\rho)$ is β-P_S-g-closed. Then g is β-P_S-g-closed.*

Proof. Similar to Proposition 5.8. □

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