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Analysing the effectiveness of vendor-managed inventory in a single-warehouse, multiple-retailer system

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This paper considers a two-stage supply chain, consisting of a single warehouse and multiple retailers facing deterministic demands, under a vendor-managed inventory (VMI) policy. It presents a two-phase optimisation approach for coordinating the shipments in this VMI system. The first phase uses direct shipping from the supplier to all retailers to minimise the overall inventory costs. Then, in the second phase, the retailers are clustered using a construction heuristic in order to optimise the transportation costs while satisfying some additional restrictions. The improvement of the system's performance through coordinated VMI replenishments against the system with direct shipping only is shown and discussed in the comparative analysis section.

Keywords: inventory control; VMI; supply chain optimisation; deterministic demand; inventory routing

1. Introduction

Vendor-managed inventory (VMI) is an integrated inventory management policy in which the supplier assumes, in addition to its own inbound inventory, the responsibility of maintaining inventory at the retailers, and ensuring that they will not run out of stock at any moment. The replenishment of the retailers is thus no longer triggered by retailers placing orders, but instead it is the supplier who determines when each of the retailers is replenished, and what the replenishment quantities are. The supply is thus proactive, as it is based on the available inventory information, instead of being reactive, in response to retailers' orders. This proactive policy has many advantages for both the supplier and the retailers. On the one hand, the supplier has the possibility to combine multiple deliveries to optimise truck loading and to minimise transportation costs. Moreover, since the supplier has direct information about retailers' demand, deliveries will become more uniform and predictable. As a consequence, the amount of inventory that must be held at the supplier can be drastically reduced. On the other hand, the retailers do not need to dedicate resources to the management of their inventories any longer. Furthermore, the service levels towards the retailers (i.e., product availability) can increase, as the supplier can track inventory levels and subsequently take into account the replenishment urgency.

VMI has gained popularity, thanks to the availability of many technologies that enable to monitor retailer inventories in an online and cost-effective manner. Inventory data can be made accessible much easier. However, this does not guarantee that implementing VMI always leads to improved results. Failure can happen, for example due to the unavailability of sharing the right pieces of information, or due to the inability of the supplier to make the right decisions. These two problems have to be solved in an integrated manner when implementing VMI, which only adds to the complexity, whereas managing inventory in a supply chain and optimising transportation between stages are already particularly challenging problems.

In this paper, we consider a two-stage supply system with deterministic demand, operating under VMI (see Figure 1). In particular, we focus on the single-warehouse, multiple-retailer (SWMR) case in which a supplier serves a set of retailers from a single warehouse. We assume that all retailers face a deterministic, constant demand rate. Deliveries to these retailers are made from the warehouse with a fleet of vehicles having a limited capacity. The warehouse in turn is replenished from an outside source. Incoming shipments into the warehouse have to be coordinated with outgoing shipments to the retailers in order to minimise the total cost. This total cost consists of inventory holding costs at the central warehouse and all retailers, costs for incoming shipment into the warehouse, and outbound shipment costs for the retailer replenishments. The optimisation problem of minimising the total of inventory and transportation costs encountered in this VMI system is known in the literature as the inventory routing problem (IRP).

This SWMR case has been studied before in particular by Roundy (1985) and Chu and Leon (2008), amongst

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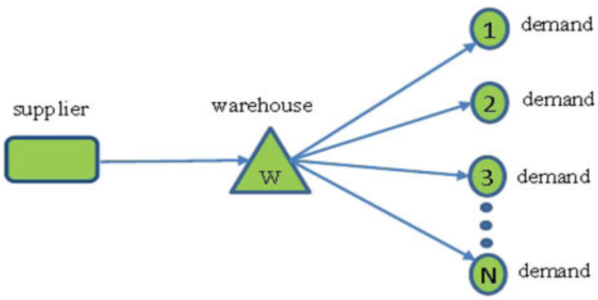

Figure 1. A two-echelon inventory system with SWMR.

others. However, they assumed that only direct shipping is used to replenish the retailers, i.e., each vehicle visits a single retailer and returns to the warehouse. Even under this assumption, it is shown that the problem cannot be solved in polynomial time.

We propose a two-phase heuristic solution approach to minimise the overall inventory and transportation costs of the SWMR system, under a VMI policy. In the first phase, retailers are partitioned into subsets in order to minimise the overall inventory costs of the system. Then, in the second phase, a vehicle routing problem (VRP) procedure is used to solve the routing in each of the retailer subsets with the objective of minimising the travelled distance and hence the transportation costs. As such, we drop the assumption of direct shipments from warehouse to retailers, but also include the option of combining multiple outbound shipments in so-called milk-runs. To evaluate the impact of VMI and milk-runs on the SWMR system, a comparative analysis of the SWMR system before and after the adoption of VMI and milk-runs is carried out. In particular, the inventory management practices in the different scenarios are examined and their related cost are compared.

The remainder of this paper is organised as follows. Section 2 reviews the relevant literature. Section 3 presents the formal description of the integrated SWMR–VMI deterministic model. Section 4 reviews existing direct shipping solutions and Section 5 describes the proposed two-phase approach for milk-run solutions. A detailed analysis of an illustrative supply chain example is given in Section 6. Section 7 provides conclusions and directions for future research.

2. A brief review of models and solution approaches

An important stream of research related to the SWMR is the one that takes transportation costs explicitly into account. Federgruen and Zipkin (1984) were probably among the first to integrate between inventory allocation and routing problems. They have considered a plant with a limited amount of available inventory serving a set of retailers with random periodic demand rates. The objective of their model is to allocate available inventory in the warehouse to the retailers in a way that minimises total transportation costs at the end of the period. They modelled the problem as a nonlinear mixed integer program, and proposed an approximation method for its solution. Federgruen, Prastacos, and Zipkin (1986) extended the work by Federgruen and Zipkin (1984) to the case in which the product considered is perishable. Chien, Balakrishnan, and Wong (1989) simulated a multiple period planning model based on a single period approach and formulated it as a mixed integer programming problem. Campbell, Clarke, Kleywegt, and Savelsbergh (1998) studied a two-phase heuristic based on a linear programming model. In the first phase, they calculated the exact visiting period and quantity to be delivered to each retailer. Then, in the second phase, retailers are sequenced into vehicle routes. Bertazzi, Paletta, and Speranza (2002) proposed a fast local search algorithm for the single vehicle case in which an order-up-to level (OU) inventory policy is applied. Aghezzaf, Raa, and Landeghem (2006) formulated a model for the long-term IRP when demand rates are stable and economic order quantity-like policies are used to manage the inventory of the retailers. The authors then proposed a column generation based heuristic. Other examples of recent contributions in the SWMR system were carried out by Aghezzaf (2008), Solyali and Sural (2011, 2012), Aghezzaf, Zhong, Raa, and Mateo (2012), Rahim and Aghezzaf (2012), Archetti, Bertazzi, Hertz, and Speranza (2012), Coelho, Cordeau, and Laporte (2012a, 2012b), and Coelho and Laporte (2013a, 2013b).

In the context of replenishment strategies, Gallego and Simchi-Levi (1990, 1994) considered a direct shipping approach in which each vehicle visits only a single retailer in each one of its trips. Retailers have deterministic demand rates and no shortages or backlogs are allowed. They assumed that a sufficient number of vehicles, each with a limited capacity, are available and that the storage capacities at the retailers are sufficiently large. Kim and Kim (2000) also examined a direct shipping method, but allow for more than one trip per vehicle and time period. They formulated the problem as a mixed integer linear program and proposed a Lagrangian heuristic to solve it. More recent works in the direction of direct shipping strategies with deterministic demand can be found in Zhao, Wang, and Lai (2007), Bertazzi (2008), Li, Wu, Lai, and Liu (2008), and Li, Chen, and Chu (2010). Barnes-Schuster and Bassok (1997) extended the direct shipping strategy to the case of independent stochastic stationary demand rates. Through simulation studies, they demonstrated that when the truck capacities are close to the mean of the demand, the direct shipping strategy performs well. Kleywegt, Nori, and Savelsbergh (2002) developed an approach that is designed for a different setting in which vehicle routes are limited and only allow for direct shipping. They introduced and modelled it as a Markov decision process and developed an approximate dynamic programming method in order to find good quality solutions with a reasonable computational effort. Direct shipping strategies are shown to be effective alternative to more complex strategies when the economic lot sizes for all retailers are close to the capacities of the vehicles. However, it may not be the ideal policy when many retailers require significantly less than a vehicle load.

When a direct shipping strategy is proven ineffective, a milk-run approach should be considered, where each vehicle serves multiple retailers in one delivery (one route). For this reason, most studies concentrated on a special type of distribution policies called fixed partition (FP) policies, as they are easily implemented and effective in many situations. Anily and Federgruen (1990, 1993) are among the first to adopt the ideas of FP policy. They analysed the replenishment strategy, where the set of retailers is partitioned into regions and each region is served independently. If a retailer in some region is visited, all retailers in that region are visited. Viswanathan and Mathur (1997) extended the work of Anily and Federgruen (1990) for the multi-period, multi-product problem. They presented a new heuristic that generates a stationary nested joint replenishment policy for the problem with deterministic demands. Then, they adopted a power-of-two policy and the results showed that when the replenishment periods are power-of-two multiples of a common base planning period, good performance can be achieved. Chan, Federgruen, and Simchi-Levi (1998) investigated the effectiveness of the class of FP policies and zero inventory ordering policies, and constructed an effective algorithm resulting in an FP policy that is asymptotically optimal. Jung and Mathur (2007) extended the replenishment strategy discussed in Anily and Federgruen (1993) by allowing different reorder intervals for each retailer in a cluster. They developed a three-step heuristic and the solution is rounded to fit the power-of-two policy constraints. Interesting studies in this research stream are found in Chan, Muriel, Shen, Simchi-Levi, and Teo (2002), Anily and Bramel (2004), Gaur and Fisher (2004), Zhao, Chen, and Zang (2008), Raa and Aghezzaf (2008, 2009), Bertazzi, Chan, and Speranza (2013), and Chu and Shen (2010).

An extension of this research line is concerned with the models that involve location–inventory network design, integrating the location and inventory decisions. Barahona and Jensen (1998) studied a practical distribution network design problem for computer spare parts. Their model takes into account the inventory costs at the various warehouses. Erlebacher and Meller (2000) developed an analytical model to minimise the total fixed operating costs and inventory holding costs incurred by warehouses, together with the transportation costs. Shen, Coullard, and Daskin (2003) and Daskin, Coullard, and Shen (2002) considered the case where retailers are facing uncertain demands following a Poisson distribution. Shen et al. (2003) studied a facility location problem in which the facilities manage their inventory through an (r,Q) policy, while Daskin et al. (2002) presented an efficient solution based on Lagrangian relaxation approach. Shu, Teo, and Shen (2005) solved the problem for general demand distributions. Sharma (2007) examined the increase/decrease in the demand level and the flexibility of the production rate in order to reduce the inventory cost at the warehouse. Shen and Qi (2007) defined a model for the stochastic supply chain design problem. Ozsen, Coullard, and Daskin (2008) introduced the capacitated warehouse location model with risk pooling, which captures the interdependence between the capacity issues and the inventory management at the warehouses. Chen, Li, and Ouyang (2011) studied a reliable inventory–location model to optimise facility location decisions, allocation of retailers and management of inventory in case warehouses are at a risk of disorder. More recent contributions in this research area are found in Tancrez, Lange, and Semal (2012), Berman, Krass, and Tajbakhshm (2012), and Hamedani, Jabalameli, and Bozorgi-Amiri (2013). In all models, the inventory holding costs at the retailers are ignored. The model examined here does not consider the design issue. However, it takes all inventories at the warehouse as well as at the retailers into account.

In this paper, we extend the SWMR model proposed by Roundy (1985) and Chu and Leon (2008) to allow for travel cost optimisation. Roundy (1985) introduced two new types of policies, namely, integer-ratio policies and power-of-two policies. Power-of-two policies are a subset of the class of integer-ratio policies in which each facility orders at a power-of-two multiple of a base planning period. Roundy has shown that for the SWMR inventory model, the cost rate of the optimal power-of-two policy is within 6% of the cost rate of any feasible policy. This result has made power-of-two policies very attractive. The complexity of each of the two policies developed by Roundy (1985) is $O(n\log(n))$, where n is the total number of retailers.

Chu and Leon (2008) consider the same problem as Roundy and proposed a solution method which only considers the feasible power-of-two policies. Instead of successively checking whether the optimal reorder period of the warehouse falls within a certain interval (Roundy, 1985), Chu and Leon (2008) proposed a method that takes advantage of the property that the total average cost of the system is convex.

The approaches proposed by Roundy (1985) and Chu and Leon (2008) consider the transportation costs as fixed costs. This means that there is no coordination between retailers to minimise transportation and fleet costs. Therefore, in order to integrate inventory management and routing cost optimisations, we extended these approaches to include routing optimisation. Some effective routing optimisation procedures for VRP are used to design an efficient heuristic for SWMR. Laporte, Gendreau, Potvin, and Semet (2000) classified the constructive techniques for solving VRP in two main groups. The first group consists of methods that combine existing routes using a savings method, and the second group consists of techniques assigning vertices to vehicle routes using an insertion cost. In this paper, we adopt the savings heuristic developed by Clarke and Wright (1964) and the improvement heuristic developed by Lin (1965) for the routing part of the problem.

3. The integrated SWMR–VMI deterministic model

In this section, the two-echelon SWMR–VMI system is formally described in a mathematical model. The model allows to find the optimal system order policy, i.e., minimising the sum of all operational costs. For the model development, let R be the set of retailers, indexed by j, and $R^+ = R \cup \{0\}$ the set of facilities, where 0 indicates the warehouse. We also define the following parameters:

- t_{ij}: trip duration from facility $i \in R^+$ to facility $j \in R^+$,
- τ_{ij}: transportation cost from facility $i \in R^+$ to facility $j \in R^+$,
- φ_0: fixed ordering cost incurred by the warehouse each time it places an order; the ordering cost is assumed independent of the order quantity,
- φ_j: fixed cost per delivery to retailer $j \in R$; the delivery cost is assumed independent of the replenishment quantity,
- h_0: inventory holding cost rate per unit per period in warehouse 0,
- h_j: inventory holding cost rate per unit per period at retailer j,
- d_j: constant demand rate per period faced by retailer j.

A solution to the problem is an order policy, which is described by the time between consecutive replenishments, or the replenishment interval, for all facilities in R^+. All of these replenishment intervals will be a power-of-two multiple of a base planning period, denoted by T_B.

Furthermore, for any retailer j, either of the two following cases must hold: (1) the retailer's replenishment interval, denoted by T_j, is a power-of-two multiple of the warehouse's replenishment interval, denoted by T_0, or (2) vice versa, that is T_0 is a power-of-two multiple of T_j.

Case 1: *T_j is a power-of-two multiple of T_0*

In the first situation, replenishments of retailer j (with a replenishment quantity of d_jT_j) can always be initiated at the moment an inbound shipment in the warehouse arrives. As a result, the warehouse serves as a cross-dock and never holds any inventory destined for that retailer. The resulting inventory cost rate for retailer j in this first case is denoted IC_j^1, and is given by

$$\mathrm{IC}_j^1 = \frac{\varphi_j}{T_j} + \frac{1}{2} h_j d_j T_j. \tag{1}$$

Case 2: *T_0 is a power-of-two multiple of T_j*

In the second situation, a replenishment of retailer j (with a replenishment quantity of d_jT_j) can only be initiated at the moment an inbound shipment in the warehouse arrives every T_0/T_j times. The other times, replenishments are made from inventory in the warehouse. As a result, the warehouse does hold inventory for that retailer. The resulting inventory cost rate for retailer j in this second case, denoted by IC_j^2, is then given by

$$\mathrm{IC}_j^2 = \frac{\varphi_j}{T_j} + \frac{1}{2} h_j d_j T_j + \frac{1}{2} h_0 d_j (T_0 - T_j). \tag{2}$$

Thus, given all the replenishment intervals, the total inventory cost rate IC is

$$\mathrm{IC}_{\mathrm{MR}} = \frac{\varphi_0}{T_0} + \sum_{j \in R} \left(\frac{\varphi_j}{T_j} + \frac{1}{2} h_j d_j T_j + \frac{1}{2} h_0 d_j \left[\max(T_0, T_j) - T_j\right] \right). \tag{3}$$

The second element in the total cost rate is the transportation cost rate. When milk-runs are used, decisions have to be made about clustering retailers and designing a trip per cluster, i.e., a VRP has to be solved. We assume that retailers in the same cluster all have the same replenishment interval. The notation used for the milk-runs is the following: V is the set of available vehicles, indexed by v; R^v is the cluster of retailers served by vehicle $v \in V$; trip^v is the (shortest possible) milk-run trip that visits all customers in R^v; $\tau^v = \sum_{(i,j) \in \mathrm{trip}^v} \tau_{ij}$ is the transportation cost of making trip^v; and T^v is the replenishment interval of all retailers in R^v. The transportation cost rate when using milk-runs, $\mathrm{TC}_{\mathrm{MR}}$, is then given by

$$\mathrm{TC}_{\mathrm{MR}} = \sum_{v \in V} \frac{\tau^v}{T^v}. \tag{4}$$

The total cost rate $\mathrm{TCR}_{\mathrm{MR}}$, which is the sum of the inventory cost rate $\mathrm{IC}_{\mathrm{MR}}$ and the transportation cost rate $\mathrm{TC}_{\mathrm{MR}}$, can then be written as follows:

$$\mathrm{TCR}_{\mathrm{MR}} = \frac{\varphi_0}{T_0} + \sum_{v \in V} \left(\frac{\tau^v + \sum_{j \in R^v} \varphi_j}{T^v} + \sum_{j \in R^v} \frac{h_j d_j}{2} T^v + \sum_{j \in R^v} \frac{h_0 d_j}{2} \left[\max(T_0, T^v) - T^v\right] \right). \tag{5}$$

For any trip^v visiting cluster R^v with interval T^v to be feasible; however, two conditions have to be met. First, the time between consecutive iterations, i.e., the interval T^v, must be longer than the duration of the trip, which results in a lower bound for the interval T^v, denoted by $T^v_{\min}$:

$$T^v \geq T^v_{\min} = \sum_{(i,j)\in \text{trip}^v} t_{ij}. \tag{6}$$

Second, the total quantity delivered to all retailers in the trip cannot exceed the vehicle capacity κ, which results in an upper bound for the interval T^v, denoted by $T^v_{\max}$:

$$T^v \leq T^v_{\max} = \frac{\kappa}{\sum_{j\in R^v} d_j}. \tag{7}$$

Apart from the first term, TCR_{MR} (5) is separable per cluster/vehicle, and therefore, the intervals T^v can be optimised individually. The two possible cases identified above reappear here.

Case 1: $T^v \geq T_0$

In this case, the warehouse never holds inventory for retailers in R^v, and the last term of the cost rate function is zero. The interval value T^{v*} that minimises the cost rate for vehicle v is as follows:

$$T^{v*} = \sqrt{\frac{2\left(\tau^v + \sum_{j\in R^v} \varphi_j\right)}{\sum_{j\in R^v} h_j d_j}} \quad (\geq T_0). \tag{8}$$

Case 2: $T^v < T_0$

In this case, the warehouse does hold inventory for retailers in R^v, and the last term of the cost rate function is nonzero. The interval value T^{v*} that minimises the cost rate for vehicle v is then

$$T^{v*} = \sqrt{\frac{2\left(\tau^v + \sum_{j\in R^v} \varphi_j\right)}{\sum_{j\in R^v} (h_j - h_0) d_j}} \quad (< T_0). \tag{9}$$

Since there is also a minimum and maximum value for the interval T^v, the optimal interval T^v_{opt} is as follows:

$$T^v_{\text{opt}} = \begin{cases} T^{v*}, & \text{if} \quad T^v_{\min} \leq T^{v*} \leq T^v_{\max} \\ T^v_{\min}, & \text{if} \quad T^v_{\min} > T^{v*} \\ T^v_{\max}, & \text{if} \quad T^{v*} > T^v_{\max} \end{cases}. \tag{10}$$

The problem to be solved is then to partition the set of retailers R into feasible clusters R^v, design a minimum cost trip per cluster, determine integer values for all T^v and T_0, such that the total cost rate TCR_{MR} is minimised. To solve this SWMR–VMI problem efficiently, we propose an algorithm that combines a solution method for the direct shipping with an effective heuristic for VRP as explained below.

4. Review of classical direct shipping solutions

To solve the SWMR–VMI problem, first, we describe the modelling algorithms developed in Roundy (1985) and the extension developed by Chu and Leon (2008).

4.1. *Roundy's algorithm*

For the case of direct shipping, all retailers are in separate clusters, and the total cost rate TCR_{DS} is

$$\text{TCR}_{\text{DS}} = \frac{\varphi_0}{T_0} + \sum_{j\in R}\left(\frac{\tau_{0j} + \tau_{j0} + \varphi_j}{T_j} + \frac{h_j d_j}{2} T_j + \frac{h_0 d_j}{2}\left[\max(T_0, T_j) - T_j\right]\right). \tag{11}$$

Apart from the first term, TCR_{DS} (11) is separable per retailer, and therefore, the intervals T_j can be optimised individually. Again, there are the same two possible cases.

Case 1: $T_j \geq T_0$

The interval value τ'_j that minimises the cost rate for retailer j is as follows:

$$\tau'_j = \sqrt{\frac{2\left(\tau_{0j} + \varphi_j + \tau_{j0}\right)}{h_j d_j}}. \tag{12}$$

Case 2: $T_j < T_0$

The interval value τ_j that minimises the cost rate for retailer j is then

$$\tau_j = \sqrt{\frac{2\left(\tau_{0j} + \varphi_j + \tau_{j0}\right)}{(h_j - h_0) d_j}}. \tag{13}$$

It is easy to verify that $\tau'_j \leq \tau_j$.

Since TCR_{DS} is convex in T_0, the optimal solution to the relaxed problem (without integer-ratio or power-of-two restrictions) given T_0, is the following:

$$T_j = \begin{cases} T_0 & \text{if} \quad \tau'_j \leq T_0 \leq \tau_j \\ \tau'_j & \text{if} \quad T_0 < \tau'_j \\ \tau_j & \text{if} \quad \tau_j < T_0 \end{cases}. \tag{14}$$

The algorithm introduced by Roundy (1985) assumes that no shortage or backlogging is permitted. Without loss of generality, replenishment is assumed to be instantaneous. Moreover, the base planning period T_B is assumed fixed, and only power-of-two policies are employed, i.e., the order intervals are all power-of-two multiples of T_B:

$$T_0 = 2^{k_0} T_B \qquad k_0 \geq 0 \text{ and integer}, \tag{15}$$

$$T_j = 2^{k_j} T_B \qquad k_j \geq 0 \text{ and integer}, \forall j \in R. \tag{16}$$

4.2. *Chu and Leon's solution algorithm*

The algorithm proposed by Chu and Leon (2008), which extends the method developed in Roundy (1985), is the algorithm we will adopt in the direct shipping phase of our solution procedure. The method starts by letting T_0 be a power of two of T_B. The proposed method then finds the corresponding optimal power-of-two multiples T_j for each retailer j, and calculates the corresponding total cost rate of the system. Then, T_0 is iteratively increased to the next power-of-two period until the total cost rate of the system increases. At this point, the optimal power-of-two policy is found.

The optimal power-of-two solutions, denoted by t'_j and t_j, are obtained by rounding the optimal solutions τ'_j and τ_j to the nearest power-of-two multiples of T_B.

Thus, this algorithm has proven that for a given T_0, the optimal power-of-two policy is given by (Chu & Leon, 2008)

$$T_j = \begin{cases} T_0 & \text{if} \quad t'_j \le T_0 \le t_j \\ t'_j & \text{if} \quad T_0 < t'_j \\ t_j & \text{if} \quad t_j < T_0 \end{cases} . \tag{17}$$

Based on (17), and the fact that $\mathrm{TCR}_{\mathrm{DS}}$ is convex in T_0, Chu and Leon proposed an iterative heuristic that monitors the changes in total cost rate, if interval T_i is used instead of interval T_j. The heuristic for the SWMR system is summarised as follows (see Chu & Leon, 2008).

Step 0: Calculate $\tau'_j = \sqrt{2(\tau_{0j}+\varphi_j+\tau_{j0})/h_j d_j}$ and $\tau_j = \sqrt{2(\tau_{0j}+\varphi_j+\tau_{j0})/(h_j-h_0)d_j}$, $\forall j \in R$ and round these to the nearest power of two to obtain t'_j and t_j. Find $t_{\min} = \min\{t'_j : j \in R\}$ and $t_{\max} = \max\{t_j : j \in R\}$. Let $i = 0$, $T_0 = t_{\min}$, $T^0 = \{\text{Ø}\}$, and $\mathrm{TCR}_{\mathrm{DS}}(T^0) = \infty$.

Step 1: Choose T_j according to condition (17). Set $i = i + 1$. Let $T^i = \{T_0, T_1, \ldots, T_n\}$ and calculate $\mathrm{TCR}_{\mathrm{DS}}(T^i)$ using (11). If $\mathrm{TCR}_{\mathrm{DS}}(T^i) < \mathrm{TCR}_{\mathrm{DS}}(T^{i-1})$, go to Step 2. Otherwise, the best power-of-two policy has been found and is given by $T^* = T^{i-1}$. Stop.

Step 2: If $T_0 < t_{\max}$, set $T_0 = 2T_0$ and go back to Step 1. Otherwise, the optimal T_0 is in the range $[t_{\max}, \infty]$. For any $T_0 > t_{\max}$, the optimal T_j remain the same as the values last calculated (in Step 1). Therefore, given these optimal T_j, $\forall j$, T_0 can be found by first minimising (11) with respect to T_0 and then rounding the solution such that $2^{k-1}\sqrt{2}T_B \le T_0 = 2^k T_B \le 2^k \sqrt{2} T_B$ with k integer. Stop.

5. Solution approach for SWMR–VMI with milk-runs

This section presents a solution approach for the problem presented in Section 3. Our method uses the work of Roundy (1985) and Chu and Leon (2008) for the case of direct shipping as a starting point, and then builds upon it to be able to tackle the case of milk-runs. The solution framework is illustrated in Figure 2 and consists of the following steps.

We start by initialising the set of clusters, with each retailer in a separate cluster, i.e., the direct shipping case. We then use the algorithm of Chu and Leon (2008) presented above to find the replenishment interval for each retailer as well as the warehouse. These power-of-two order intervals are then used in the next phase, the VRP phase.

For the VRP phase, the retailers are clustered per replenishment interval. We then use the savings heuristic of Clarke and Wright (1964) for each of the clusters. The main purpose of this algorithm is to optimise the transportation costs and to select retailers who can be replenished in a milk-run rather than with separate direct shipments.

For every cluster of retailers (that have the same replenishment interval after the previous step), we first perform the savings procedure as follows.

Step 1: Compute the savings, $S_{ij} = \tau_{i0} + \tau_{0j} - \tau_{ij}$, of combining every possible pair of retailers i and j in the cluster. Order the savings S_{ij} in a decreasing order.

Step 2: Find the first feasible link in the list which can be used to extend one of the two ends of the currently constructed route.

Step 3: If the route cannot be expanded further, terminate the route. Choose the first feasible link in the list to start a new route.

Step 4: Steps 2 and 3 are repeated until no further feasible links can be chosen.

Then, a 2-opt improvement heuristic is applied to further reduce transportation costs. This consists of deleting and re-inserting sub-routes. The possible sub-routes are inserted into the existing solution, and the cheapest alternative is kept. If no cheaper alternative is found, the solution is restored and no improvement is realised. However, if a profitable reconnection is identified, it means that the solution can be improved.

After the VRP phase, every route determines a new retailer cluster. If these retailer clusters are the same as before, we stop. Else, we return to the initial step to recalculate cycle times for each of the clusters and the central warehouse. Then, we can calculate the total cost for each of the clusters in the SWMR–VMI system.

To examine the impact of introducing milk-runs, we calculate the change in total inventory and transportation costs for each of the retailers and the warehouse.

6. A detailed analysis of an illustrative supply chain example

In this particular case, we consider 15 retailers as illustrated in Figure 3. These retailers are scattered around the

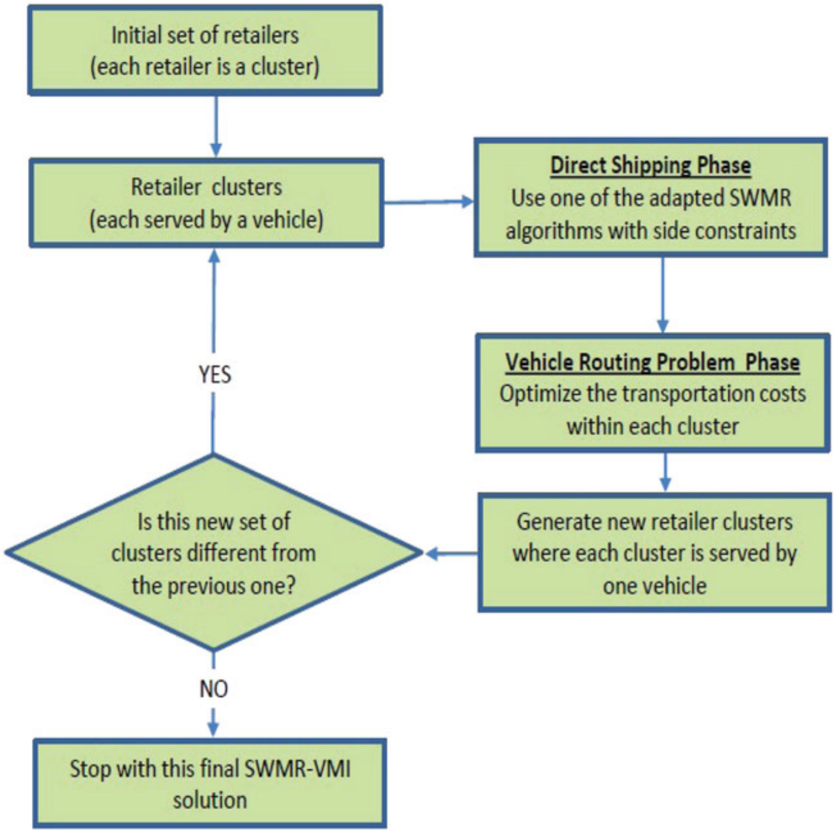

Figure 2. Solution framework for the SWMR–VMI system with milk-runs.

warehouse and have demand rates that are assumed to be stable, adding up to 6.341 ton/hour for all 15 retailers. We assume that a fleet of vehicles is available for product replenishment from the warehouse. The data of this case is obtained by Aghezzaf et al. (2006).

Table 1 shows the distances (in km) between the different retailers. Travel times can be obtained from Table 1 by considering an average speed of 50 km/hour for each vehicle. We assume that all vehicles in the fleet have a capacity of 60 ton and a transportation cost of €0.10 per km. We also assume that the fixed ordering cost of the warehouse is €75 and all retailers have the same fixed cost per delivery of €50. Finally, we assume that there is a difference in inventory holding cost rates at the warehouse versus at the retailers, with $(h_j - h_0) > 0$.

To solve this illustrative example, we use our solution method presented above. In the first step, we start from the direct shipping solution and use the method of Chu and

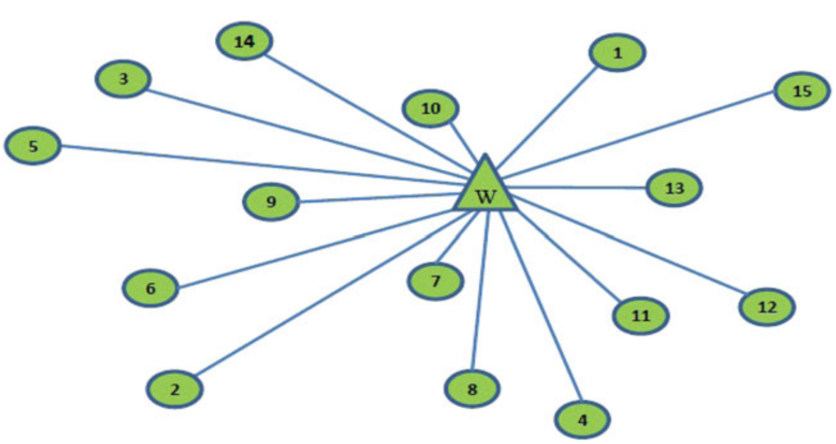

Figure 3. An example case with 15 retailers.

Table 1 Distance matrix (in km) for the example case.

	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
0	–	270	480	490	330	550	430	140	260	240	150	240	360	200	430	320
1		–	740	560	580	680	650	420	530	440	210	410	450	260	400	170
2			–	500	490	410	190	350	370	320	540	590	750	660	600	800
3				–	630	160	310	480	630	290	390	720	850	670	180	710
4					–	770	590	290	160	470	490	190	300	350	750	520
5						–	220	540	630	310	490	760	910	740	340	810
6							–	340	430	210	440	610	770	630	430	740
7								–	160	230	250	270	420	310	470	450
8									–	340	400	230	380	350	630	510
9										–	240	450	600	440	320	540
10											–	370	480	280	290	320
11												–	160	170	650	330
12													–	210	770	310
13														–	570	170
14															–	570
15																–

Leon (2008) to find the power-of-two order intervals for each retailer as well as the warehouse. We use $T_B = 1$ hour.

Initialisation:

Step 0: $\tau'_j = \sqrt{2(\tau_{0j}+\varphi_j+\tau_{j0})/h_j d_j}$ and $\tau_j = \sqrt{2(\tau_{0j}+\varphi_j+\tau_{j0})/(h_j-h_0)d_j}$, $\forall j \in R$ and round these to the nearest power of two. This results in t'_j={64, 32, 64, 32, 64, 64, 64, 32, 64, 64, 32, 32, 64, 64, 64} and t_j={64, 32, 64, 32, 128, 64, 64, 64, 64, 64, 32, 32, 64, 64, 128}. We find $t_{\min} = \min\{t'_j : j \in R\} = 32$ hours and $t_{\max} = \max\{t_j : j \in R\} = 128$ hours. $i = 0$; $T_0 = t_{\min} = 32$ hours; and $\mathrm{TCR}_{\mathrm{DS}}(T^0) = \infty$.

Iteration 1:

Step 1: We set $i = 1$ and choose T_j according to condition (17): T^1={32, 64, 32, 64, 32, 64, 64, 64, 32, 64, 64, 32, 32, 64, 64, 64} and find $\mathrm{TCR}_{\mathrm{DS}}(T^1)$ = €70.00 using (11). Since $\mathrm{TCR}_{\mathrm{DS}}(T^1) < \mathrm{TCR}_{\mathrm{DS}}(T^0)$, we go to Step 2.

Step 2: Set $T_0 = 64$ hours and return to Step 1.

Iteration 2:

Step 1: $i = 2$ and T^2={64, 64, 32, 64, 32, 64, 64, 64, 64, 64, 64, 32, 32, 64, 64, 64}. We find $\mathrm{TCR}_{\mathrm{DS}}(T^2)$ = €68.44, which is less than $\mathrm{TCR}_{\mathrm{DS}}(T^1)$, so we go to Step 2.

Step 2: Step 2: Set $T_0 = 128$ hours and return to Step 1.

Iteration 3:

Step 1: $i = 3$ and T^3={128, 64, 32, 64, 32, 128, 64, 64, 64, 64, 64, 32, 32, 64, 64, 128}. We find $\mathrm{TCR}_{\mathrm{DS}}(T^3)$ = €77.17, which is more than $\mathrm{TCR}_{\mathrm{DS}}(T^2)$. Therefore, the optimal power-of-two policy is T^2.

In the next step, we solve the VRP problem in order to reduce transportation costs. The problem is to define the allocation of retailers to routes, determine the sequence in which the retailers shall be visited on a route, and decide which vehicle shall cover which route.

To solve the constrained VRP sub-problems, first, we calculate the transportation costs between all pairs of points. The transportation cost is given by $\tau_{ij} = \delta.v.t_{ij}$ euro per tour, where t_{ij} represents the travel time between the pairs of retailers at a speed of v km per hour, and δ is the travel cost per km.

The replenishment quantity for each retailer is obtained by multiplying its cycle time with its demand rate. The resulting replenishment quantities are given in Table 2. The inventory holding cost rates are various at the retailers (see Table 2) and the inventory holding cost rate at the warehouse is €0.05.

A VRP is solved for two sets of customers: those with a cycle time of 64 hours {1, 3, 5, 6, 7, 8, 9, 10, 13, 14, 15}, and those with a cycle time of 32 hours {2, 4, 11, 12}. The result of the savings heuristic is shown in Figure 4. The

Table 2 Quantities delivered to each of the retailers.

Retailers	Demand (ton/hour)	Inventory holding cost (€)	Cycle time (hours)	Delivery (ton)
1	0.209	0.25	64	13.38
2	0.622	0.30	32	19.90
3	0.322	0.17	64	20.61
4	0.798	0.25	32	25.24
5	0.134	0.29	64	8.58
6	0.429	0.20	64	27.46
7	0.381	0.18	64	24.38
8	0.503	0.20	64	32.19
9	0.217	0.18	64	13.89
10	0.269	0.15	64	17.22
11	0.823	0.21	32	26.34
12	0.598	0.25	32	19.14
13	0.247	0.14	64	15.81
14	0.348	0.15	64	22.27
15	0.441	0.07	64	28.22

Table 3 Distribution results for the different tours.

Tours	Vehicle Load (ton)	Capacity utilisation (%)	Total cost, TCR_{MR} (€/hour)
$V_1 = (6, 5, 3)$	56.64	94.4	61.29
$V_2 = (13, 15, 1)$	57.41	95.7	58.55
$V_3 = (14, 9, 10)$	53.38	89.3	59.29
$V_4 = (8, 7)$	56.58	94.3	59.02
$V_5 = (12, 11)$	45.47	75.9	62.97
$V_6 = (2, 4)$	45.44	75.7	65.67

retailers of the first set are assigned to four routes: route $V_1 = (6, 5, 3)$ with a total demand of 56.64 ton, route $V_2 = (13, 15, 1)$ with a total demand of 57.41 ton, route $V_3 = (14, 9, 10)$ with a total demand of 53.38 ton, route $V_4 = (8, 7)$ with a total demand of 56.58 ton. The retailers in the other set are assigned to two routes: route $V_5 = (12, 11)$ which delivers 45.47 ton, and route $V_6 = (2, 4)$ which delivers 45.44 ton.

In Table 3, the vehicle load and the total cost rate TCR_{MR} for each of the sub-tours are clearly shown. From the results above, it shows that the truck loading is optimised efficiently with the average capacity utilisation for all tours being 87.48%.

As can be seen in Figure 4, sub-tour $V_3 = (14, 9, 10)$ can be improved. This improvement is found in the 2-opt heuristic that we apply next (see Figure 5). The existing route (0–14–9–10–0) is changed to a new route (0–10–14–9–0). This decreases total transportation costs from €114 to €100.

Figure 6 shows the new solution for SWMR–VMI after the savings and improvement heuristics. We now have six customer clusters (one per route), which are different from the initial clustering (where we had one cluster for each retailer). Therefore, the solution procedure starts a new iteration and will evaluate the reorder intervals for these new clusters.

Table 4 shows the total of the inventory costs, IC_{DS}, and round trip transportation costs, TC_{DS}, for every retailer with direct shipping. Table 5 shows the total costs of the inventory, IC_{MR}, and transportation, TC_{MR}, for every retailers cluster after the adoption of VMI and milk-runs. As we know, before implementing VMI and milk-runs, each retailer is exclusively served by a vehicle in trip visiting that retailer only. Then, once VMI and milk-runs are implemented, some retailers are clustered and served in a sub-tour of a the trip made by the vehicle.

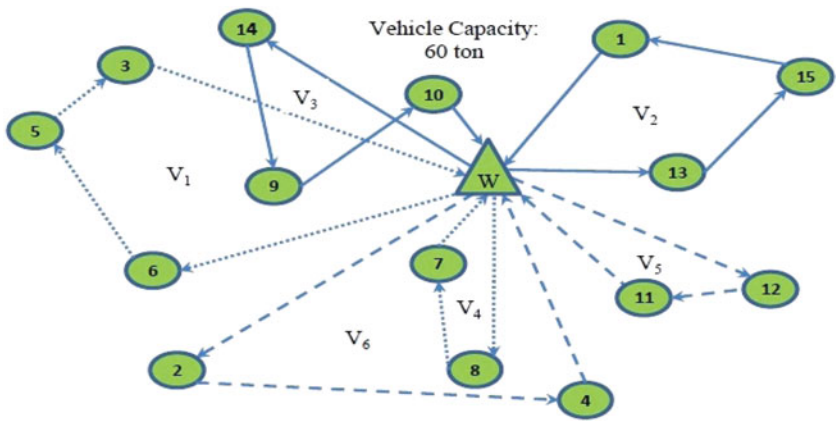

Figure 4. A VRP tour solution.

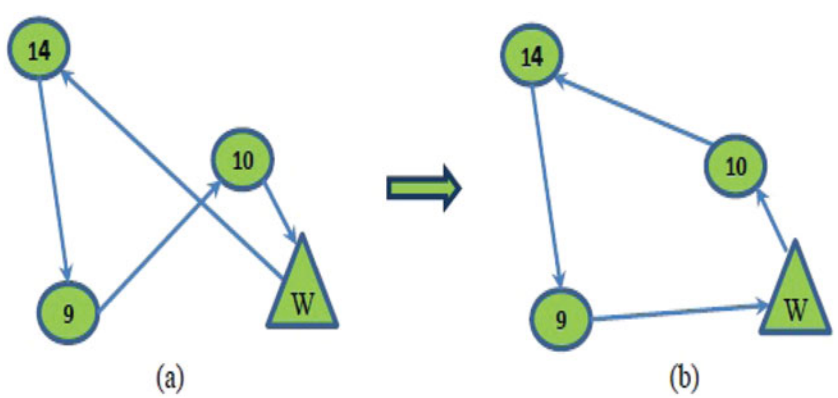

Figure 5. Solution of the sub-tour problem (improvement heuristic).

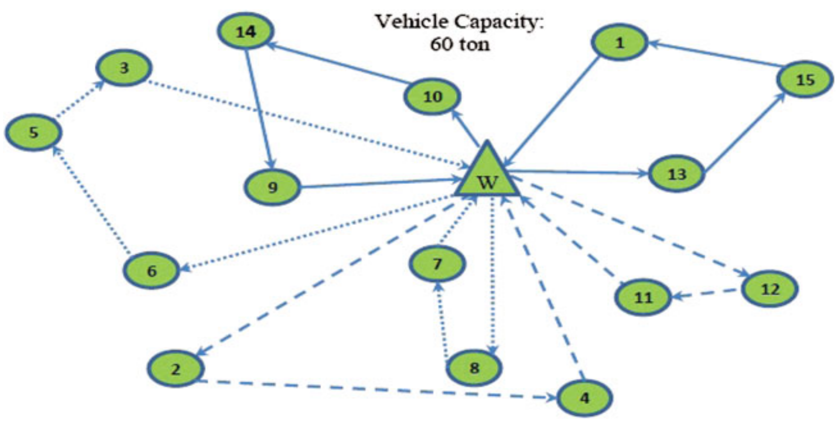

Figure 6. Solution of the example problem (savings + improvement heuristic).

Table 4 Inventory and transportation costs with direct shipping.

Retailers	IC_{DS} (€/hour)	TC_{DS} (€/hour)
1	3.63	0.84
2	6.22	3.00
3	3.70	1.53
4	6.56	2.06
5	3.20	1.72
6	4.70	1.34
7	4.15	0.44
8	5.17	0.81
9	3.20	0.75
10	3.24	0.47
11	6.16	1.50
12	5.60	2.25
13	3.06	0.63
14	3.62	1.34
15	2.94	1.00
Total cost	65.15	19.68

Table 6 gives the comparisons between the results obtained by the inventory management policy before and then after the adoption of VMI and milk-runs. From the table above, we can see that the inventory cost is reduced by 16.16% and the transportation cost is decreased by 38.10%, when implementing VMI and milk-runs in the system. Therefore, the total cost of the inventory and transportation costs in the system after implementing VMI and milk-runs is reduced by 21.25%.

Table 5 Inventory and transportation costs with milk-runs.

Retailers clusters	IC_{MR} (€/hour)	TC_{MR} (€/hour)
$V_1 = (6, 5, 3)$	9.26	2.03
$V_2 = (13, 15, 1)$	7.28	1.27
$V_3 = (10, 14, 9)$	7.73	1.56
$V_4 = (8, 7)$	8.15	0.88
$V_5 = (12, 11)$	10.59	2.38
$V_6 = (2, 4)$	11.61	4.06
Total cost	54.62	12.18

Table 6 Summary results for inventory and transportation costs.

	Milk-runs (€/hour)	Direct shipping (€/hour)	Gap (%)
Inventory cost	54.62	65.15	16.16
Transportation cost	12.18	19.68	38.10
Total cost	66.80	84.83	21.25

In addition, Table 7 gives the summary results for the main characteristics of the distribution pattern. For example, the vehicle 1 with a 60 ton capacity makes the tour $V_1 = (6, 5, 3)$. The tour has $T_{\min} = 26$ hours, $T_{\max} = 67.8$ hours, and $T^v = 64$ hours. The maximal cycle time is higher than the theoretical optimal cycle time. The actual cycle time is therefore, 64 hours, giving a total cost rate for this tour equal of €61.29/hour and the total demand is 56.64 ton.

From the table above, we also can evaluate the effect of the vehicle storage capacity restrictions. In this case, capacities of 60 ton, 80 ton, and 100 ton are used for delivering the product to each of retailers clusters. The vehicle capacity factor is used to show that our solution approach not only helps to decide on the fleet size, but can also be used to select the most appropriate vehicle type for a particular problem instance. For the size of 15 retailers, which are clustered by the same set partitions, the result shows that the average total cost rate is €366.79 when using a small vehicle of 60 ton, €315.08 when using a vehicle of 80 ton, and €261.93 when using a larger vehicle of 100 ton. Therefore, we can see that the smaller the delivery quantities to each of the clusters are, the less retailers are replenished per tour and more tours are made. Moreover, it also increased the number of vehicles and transportation costs. In this case,

Table 7 Summary results for characteristics of the distribution pattern.

Vehicle capacity	Tour	$T^v_{\min}$	$T^v_{\max}$	T^v	T^v_{opt}	Vehicle load	Total cost (€/hour)
60 ton	$V_1 = (6, 5, 3)$	26.00	67.80	64.00	64.00	56.64	61.29
	$V_2 = (12, 11)$	15.20	42.22	32.00	32.00	57.41	62.97
	$V_3 = (13, 15, 1)$	16.20	66.89	64.00	64.00	53.38	58.55
	$V_4 = (10, 14, 9)$	20.00	71.94	64.00	64.00	56.58	59.29
	$V_5 = (2, 4)$	26.00	42.25	32.00	32.00	45.47	65.67
	$V_6 = (8, 7)$	11.20	67.87	64.00	64.00	45.44	59.02
						314.92	366.79
80 ton	$V_1 = (6, 5, 3, 14)$	28.40	64.88	64.00	64.00	78.91	64.32
	$V_2 = (4, 11, 12)$	18.40	36.05	32.00	32.00	71.01	69.23
	$V_3 = (13, 15, 1, 10)$	18.00	68.61	64.00	64.00	74.62	60.76
	$V_4 = (9, 8, 7)$	17.60	72.66	64.00	64.00	70.46	61.55
	$V_5 = (2)$	19.20	128.62	32.00	32.00	19.90	59.22
						314.90	315.08
100 ton	$V_1 = (9, 6, 5, 3, 14)$	28.80	68.97	64.00	64.00	92.80	65.99
	$V_2 = (2, 4, 12, 11)$	33.40	35.20	32.00	33.40	94.89	76.16
	$V_3 = (13, 15, 1, 10)$	18.00	85.76	64.00	64.00	74.62	60.76
	$V_4 = (8, 7)$	11.20	113.12	64.00	64.00	56.58	59.02
						318.89	261.93

however, a smaller vehicle capacity is utilised efficiently instead of a larger one.

7. Conclusions

Managing inventory and routing in a supply chain is a very challenging optimisation problem. In this paper, we propose a global solution approach for a two-stage supply chain implementing VMI. We focused on the problem, denoted by SWMR–VMI, where a single warehouse delivers a single product to a set of independent retailers. These retailers draw the required material from the warehouse to satisfy their given individual demands. The warehouse, in turn, places orders to an outside supplier to fill the accumulated demands of the retailers.

An approach is proposed to minimise the overall inventory and transportation costs of the SWMR–VMI system, while satisfying the retailers demands. The approach integrates two effective algorithms, one for inventory management and the second for routing optimisation. In particular, the algorithms proposed by Roundy (1985) and improved by Chu and Leon (2008) are used to solve the SWMR direct shipping problem, and the heuristic of Clarke and Wright (1964) is used to solve the VRP sub-problem. The results of the proposed approach allowed us to investigate the effectiveness of an inventory management policy before and after the implementation of VMI and milk-runs in a two-stage supply chain. We found out that the transportation cost is relevant; the effect of VMI and milk-runs can result in significant inventory and transportation cost savings.

Further research will focus on adapting this solution approach to enriched IRP problems, including larger sets of retailers, driving-time restrictions on the vehicles and their drivers, delivery time windows at the retailers, heterogeneous vehicle fleets, multiple warehouses, multiple products, etc. Numerical experiments on the large-scale problems are currently under investigation. Finally, the basic assumption of constant demand rates is not always valid. So, it is worthwhile to investigate how the approach can be extended to explicitly take some demand variability into account. We will be extending this research to the stochastic case in the future research.

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