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A Hybrid Markov Chain Model of Manpower Data

Rahela Rahim

*Associate Professor, School of Quantitative Sciences
Universiti Utara Malaysia, Sintok, Kedah, Malaysia
e-mail: rahela@uum.edu.my*

Haslinda Ibrahim

*Associate Professor, School of Quantitative Sciences
Universiti Utara Malaysia, Sintok, Kedah, Malaysia
E-mail : linda@uum.edu.my*

Sahubar Ali Nadhar Khan

*Lecturer, School of Quantitative Sciences
Universiti Utara Malaysia, Sintok, Kedah, Malaysia
E-mail : sahubar@uum.edu.my*

Shafawati Saad

*Lecturer, Mathematical Engineering Institut
University Malaysia Perlis, Campus Pauh Putra, Arau, Perlis, Malaysia
E-mail : shafawatisaad@gmail.com*

Abstract

A hybrid model based on Markov chain and data interpolation is proposed for evaluating the manpower recruitment policy in higher learning institution. The model is developed and analysed on Excel spreadsheet. Based on the model, the new estimation of the states transition matrix for each category of manpower driven by interpolation technique is devised. The recruitment policy changes require some data needs modification while other data remains. This dataset are numerically intractable. But the revised transition matrix of Markov chain can be substituted by an interpolated data for which a revised transition probability matrix can be used as an equation solver to calculate mean time estimation for each category of manpower. The hybrid model results are then compared to the classical Markov chain result for both old and new policies by means of mean time estimation. Two scenarios were considered in the study; scenario 1 was based on historical data pattern between year 1999 – 2014 and scenario 2 was based on RMK 9 policies. The results showed the possibility average length of stay by position and

probability of loss for both scenarios. The greater impact is expected for average length of stay of senior lecturers compared to other faculty position considering the new policy.

Keywords: Markov chains, Excel spreadsheet, matrix transition diagram, interpolation.

Introduction

Year 2015 providing an apt background for higher learning institution to reinvent their academic staff or faculty appointment policy. The first mission stated by MOHE (Ministry of Higher Education) is to create a strategic and systematic plan for higher education. With 900,000 students pursuing higher education in 20 public universities, 33 private universities and university colleges, 4 foreign university branch campuses, 22 polytechnics, 37 community colleges and about 500 private colleges (Source: MOHE), it is the responsibility of each institution to remain its competitiveness. The management of the learning institution should be able to manage all the important elements that serve as the catalyst for the institutional development.

Obviously manpower is one of the crucial factor in achieving the mission and also an aid to the institutional planning process.. Earliest manpower analysis models in higher learning institutions were developed in Berkeley campus of University California [2]. The analysis contributes issues of staff recruitment, retirement, and retention towards improving recruitment success, faculty retention, and better planning for future campus faculty hiring needs. By understanding the manpower behaviour, management of the university can make a plan for the future. Central aspect of manpower planning problems lays in lecturers' shortages ([3],[2]). These shortages are reasoned by no commensurate of supply and demand of lecturers as it comes from poor recruitment by management in flow of lecturers in faculty.

Manpower planning in Higher learning institution in Malaysia has been an important issue especially in management. Basically the manpower planning analysis has been done using management approach. Other study on manpower planning in Malaysia is health manpower forecasting in Malaysia [4]. To meet the demand for higher education and to stem the outflow of foreign exchange, the government during the Sixth Malaysian Plan, 1991-1995, adopted a policy of expanding the role of the private sector as a provider of higher education. This was within the wider policy framework of promoting the private sector as a provider of higher education [5]. Because of this policy, more private universities appear and has become competitors to the public universities. Interesting offers from private universities and business organization attract more lecturers to resign from public universities. If careful attention is not given to forecast the impact of this scenario, public higher learning institution will face problem in recruiting and maintaining their academic staff of good quality. Indirectly this will influence their student's academic performance.

Based on the existing scenario, especially in the Malaysian context, some of the academic flow related questions decision makers must think of in long term planning of the institution are;

- a. What will the academic staff demographics such as rank mix, look like in the long term.
- b. What will the average length of stay of academic status look like in the long term if the current appointment policy is maintained? If it is changed?

Academic flow modeling can help answer these questions quickly. Once the questions have been answered, they will be used for faculty planning. Government has also considered an effort to improve the faculty employment in the public universities by introducing a new policy in RMK-9 (Malaysia 9th Planning) policies. The new policy stated that in the RMK-9, the status rank of lecturer should be decreased by 10% while senior lecturers and associate profesors should be increased by 70%. The question arise here was whether the new policy will have an impact on the distribution number of faculty with diverse status rank as compared to the present policy.

Any investigation on the faculty flow of an institution must take account for institutional policies and plans [23]. A model begins in analysing with the presence of historical data and also parameters for alternative personnel plans and the policies which is also known as variables [23]. The variables are distinguished by three planning variables which are the activity, stock and price variables. In terms of supply and demand, planning variables are categorized as activity variables. Activity variables are defined as almost anything that goes on at a university during a particular period of time. Since activity variables can be measured in terms of quantity per unit of time, they should be viewed as “flows” [27]. Thus, planning variables are important in the faculty flow. [26] stated that, the purpose of using planning variables is to investigate the influence of variables to the faculty flow. In this research, several variables will need to be considered in developing a faculty flow model. From previous studies, there are many variables used in faculty flow model. In Berkeley Campus of the University of California, variables used in the faculty flow model are tenured, untenured and retirement [35]. [19] developed a faculty cost and tenure model which involves the variables which are rank status, percentage at each rank, years of service required for tenure at each rank, present salary by rank, percentage of salary allocated for fringe benefits and average salary increases by rank. Meanwhile, several researches that have developed faculty flow models for Stanford University, Oregon State University, Capitol Campus (Pennsylvania State University), Michigan State University as well as Auburn University found that these universities have similar variables which are tenured, untenured, rank status, age, resignation, retirement, quit and death ([16],[27],[34],[23]). In this study, we focus on rank status and age as these two variables are related to the change of policy that is currently being the issue. Many studies model manpower flow using Markov chain in variety of field such as personnel planning by [36], military [38], management [25], healthcare [32] and education ([34],[23],[14])

Other method to model manpower flow can be found in studies such as equilibrium model ([35],[16]), Bowen and Sosa model [37], Cohort model ([33],[29]), simulation model ([38],[32],[30],[24]), System dynamic ([21],[22],[17]). Most studies focus on the application of Markov chain model in flow projection and mean time analysis.

Recently, studies in Markov chain model have been on improving the forecasting ability by hybridising it with other potential method such as in ([28],[18],[20]). The

hybrid approaches are all meant to supplement the Markov chain capabilities in data flow forecasting. Therefore in this study, we integrate the interpolation technique in the Markov chain model for modeling manpower flow in order to identify the recruitment and promotion behavior for academic staff in higher learning institutions. To increase the model flexibility a matrix form or high order Markov structure is used, which takes into account a few number of faculty categories in order to determine the next faculty flow. The underlying rationale behind this approach is that although the promotion of staff is random, it generally follows a probabilistic pattern which might be captured by a rich Markov model. Therefore high order model, in which the next flow depends on the recent history, say for the last few years seems more appropriate, as a modeling tool. The problem is that when changes required for some categories other categories are assumed to remain the same. Practically, this may seem impossible in real situation. Our basic approach to overcoming the constant states where some states are modified according to policy change has two key components (a) assign the respective state value to a new value as required by a new policy, (b) use interpolation technique to interpolate other states value. The steps identify new states value and form a matrix transition diagram of interpolated states value that can be used for forecasting.

Motivation of the Study

A Markov chain model is considered for modeling manpower flow in order to identify the recruitment and promotion behavior for academic staff in higher learning institutions. To increase the model flexibility a matrix form or high order Markov structure is used, which takes into account a few number of faculty categories in order to determine the next faculty flow. The underlying rationale behind this approach is that although the promotion of staff is random, it generally follows a probabilistic pattern which might be captured by a rich Markov model. Therefore high order model, in which the next flow depends on the recent history, say for the last few years seems more appropriate, as a modeling tool. The mixture transition distribution (MTD) was introduced by Raftery for high order Markov chain model. It is flexible and can represent a wide range of dependence pattern [6,7,8], however MTD model assumed homogeneous transient states. Later Ching, et. al extended Raftery model with non-homogeneous transient states. We consider the non-homogeneous transient states for generating the possible states transition diagram when certain states required a fixed change. The problem with the current application of Markov chain is when changes required for some categories other categories are assumed to remain the same. Practically, this may seem impossible in real situation. Our basic approach to overcoming the constant states where some states are modified according to policy change has two key components (a) assign the respective state value to a new value as required by a new policy, (b) use interpolation technique to interpolate other states value. The steps identify new states value and form a matrix transition diagram of interpolated states value that can be used for forecasting. Early studies of this approach can be found in [10, 11, 12].

Data and Model Design

The data are collected from the registrar office at the university, the detail of academic staff such age and status ranks are used in the model design. The data contains about 1033 individuals and for each of the user there are three categories of rank and five sub-categories of age interval. Considering these categories as 3 blocks of 5 sub-blocks. The block of staff categories are treated as training data. Suppose that training data consist of 5 sub-blocks including 5 A (Lecturer), 5 B (Senior Lecturer), and 5 C (Associate Professor). Then $K = 3$ and $M = \{s_1 = 22 - 27yrs, s_2 = 28 - 33yrs, s_3 = 34 - 39yrs, s_4 = 40 - 45yrs, s_5 = 46 - 51yrs\}$. Let $\{C_t; t = 1, 2, \dots\}$ be data arranged in sequence taking values in the state space $M = \{s_1, s_2, s_3, s_4, s_k\}$, the transition probabilities of an l -th order Markov chain as proposed by Raftery (1985) and Raftery and Tavare (1994) can be written as

$$P(C_t = s_{i_0} \setminus C_{t-1} = s_{i_1}, C_{t-2} = s_{i_2} \dots, C_{t-l} = s_{i_l}) = \sum_{j=1}^l \alpha_j q(s_{i_0} \setminus s_{i_j}) \quad (1)$$

$= l + 1, l + 2, \dots$

Where $Q = \{q(s_i \setminus s_j; i, j = 1, 2, \dots, K)$ and $\varphi = \{\alpha_i, i = 1, 2, \dots, l\}$ satisfy $q(s_i \setminus s_j) \geq 0, i, j = 1, \dots, K$ and $\sum_{i=1}^K q(s_i \setminus s_j) = 1, \forall j = 1, \dots, K$.

$$\alpha_i \geq 0, i = 1, 2, \dots, l \text{ and } \sum_{i=1}^l \alpha_i = 1$$

The model is known as a Mixture Transition Distribution (MTD) model. The advantage of MTD model is the reduction of the number of parameters, from $K^l(K - 1)$ to $K(K - 1) + l - 1$. It should be noted that (1) can be re-written as

$$X^{(n+k+1)} = \sum_{i=1}^k \alpha_i Q_i X^{(n+k+1-i)}$$

Raftery model assumed $Q_1 = Q_2 = \dots Q_k$. Later, Ching *et. al* (2013) have extended the MTD model by varying the Q_i , that is $Q_i \neq Q_j$ and the total number of independent parameters in the new model is $(k + km^2)$. However in this study we present a different method to estimate the parameter Q_i . Our method used Lagrange interpolation to estimate Q_i . To estimate Q_i , that is the i -th step transition matrix of the categorical data sequence $\{X^n\}$. Given $\{X^n\}$, we can calculate the transition frequency $f_{jl}^{(i)}$ in the sequence from state l to state j in the i -step. Hence the i -step transition matrix for the sequence $\{X^n\}$ can be constructed as follows.

$$F^{(i)} = \begin{pmatrix} f_{11}^{(i)} & \dots & \dots & f_{1m}^{(i)} \\ f_{21}^{(i)} & \dots & \dots & \dots \\ \vdots & \vdots & \vdots & \vdots \\ f_{m1}^{(i)} & \dots & \dots & f_{mm}^{(i)} \end{pmatrix} \quad (i)$$

Where $f_{ij}^{(i)} = \sum_{j=1}^m \sum_{i=1}^m L_{IJ}(x) f_{ij}$ and $L_{ij}(x) = \prod_{j=0}^n \frac{x-x_j}{x_i-x_j}, n = mxm$ (ii)

From $F^{(i)}$, $Q_i = [q_{ij}^{(i)}]$ can be estimated as follows:

$$\hat{Q}_i = \begin{pmatrix} \hat{q}_{11}^{(i)} & \cdots & \cdots & \hat{q}_{1m}^{(i)} \\ \hat{q}_{21}^{(i)} & \cdots & \cdots & \cdots \\ \vdots & \vdots & \vdots & \vdots \\ \hat{q}_{m1}^{(i)} & \cdots & \cdots & \hat{q}_{mm}^{(i)} \end{pmatrix} \text{ where } q_{ij}^{(i)} = \begin{cases} \frac{f_{ij}^{(i)}}{\sum_{l=1}^m f_{lj}^{(i)}} & \text{if } \sum_{l=1}^m f_{lj}^{(i)} \neq 0 \\ 0 & \text{otherwise} \end{cases} \quad (iii)$$

The following proposition extends the proposition of estimators by Ching et. al and shows that these estimators are unbiased.

Proposition: The estimators in (iii) satisfy

$$E(f_{ij}^{(i)}) = q_{ij}^{(i)} E \sum_{j=1}^m f_{lj}^{(i)}$$

Proof. Let T be the length of sequence, $[q_{ij}^{(i)}]$ be the i -step transition probability matrix and \hat{X}_l be the steady state probability that the process is in state l . Then we have

$$\begin{aligned} E(f_{lj}) &= T \cdot \hat{X}_l q_{lj} & (iv) \\ E \left(\sum_{j=1}^m f_{lj}^{(i)} \right) &= T \cdot \hat{X}_l \sum_{j=1}^m q_{lj}^{(i)} \\ E \left(\sum_{i=1}^n \sum_{j=1}^m f_{lj}^{(i)} \right) &= T \cdot \hat{X}_l \sum_{i=1}^n \sum_{j=1}^m q_{lj}^{(i)} \end{aligned}$$

Let

$$\sum_{j=1}^m f_{lj}^{(i)} = F_l$$

Therefore

$$\begin{aligned} E \left(\sum_{i=1}^n F_l \right) &= T \cdot \hat{X}_l \left(\sum_{i=1}^n 1 \right) \\ n \cdot E(F_l) &= T \cdot \hat{X}_l \cdot n \end{aligned}$$

$$E(F_l) = T \cdot \hat{X}_l$$

Then from (iv)

$$E(f_{lj}) = q_{lj} E(F_l)$$

Hence satisfy that the proposed interpolated estimators are unbiased.

Data Collection and States Transition Diagram

We have prepared the worksheet displays the states transition matrix of 5 years transition of staff categorized by rank and age as shown in figure 1. Two alternative scenarios were proposed in this study. The first scenario considers the policy of promoting the academic staff remain the same as in year 1999-2014. Therefore the transition probabilities developed from the previous five years of data would hold. The second scenario considered the new policy suggested by the government that the appointment of lecturer should be decreased by 10% while senior lecturers and associate professors should be increased by 70%. In this model the transition probabilities would change according to the respective change in the number of academic staff appointed.

| Status | A | A | A | A | A | B | B | B | B | B | C | C | C | C | TOTAL |
|----------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|
| rank | 22-27 | 28-33 | 34-39 | 40-45 | 46-51 | 28-33 | 34-39 | 40-45 | 46-51 | 52-57 | 34-39 | 40-45 | 46-51 | 52-57 | |
| age | | | | | | | | | | | | | | | |
| Interval | | | | | | | | | | | | | | | |
| A:22-27 | 346 | 123 | | | | 6 | | | | | | | | | 475 |
| A:28-33 | | 72 | 23 | | | 1 | 49 | | | | 3 | | | | 148 |
| A:34-39 | | | 33 | 3 | | | 10 | 21 | | | 39 | | | | 106 |
| A:40-45 | | | | 1 | | | | 3 | 8 | | | | | | 12 |
| A:46-51 | | | | | | | | | 2 | 2 | | | | | 4 |
| B:22-27 | | | | | | | | | | | | | | | 0 |
| B:28-33 | | | | | | | | | | | | | | | 0 |
| B:34-39 | | | | | | | | | | | | | | | 0 |
| B:40-45 | | | | | | | | | | | | | | | 0 |
| B:46-51 | | | | | | | | | | | | | | | 0 |
| C:34-39 | | | | | | | 13 | | | | | | | | 13 |
| C:40-45 | | | | | | | | 17 | | | | | | | 17 |
| C:46-51 | | | | | | | | | | | | 1 | | | 1 |
| C:52-57 | | | | | | | | | | | | | 11 | 6 | 17 |

Figure 1. States Transition Diagram of Scenario 1

The data frequency in the transition probability matrix of scenario 1 is arranged ascendingly and the minimum, median and maximum data are selected. The data is then referred to which category of staff they are in and later assigned to a new frequency as stated by the rule in scenario 2. For example, the median is 17 and falls on state B(40-45). The new policy indicated that the category should be raised by 70%. Therefore the median is assigned to new data frequency, 29. Based on the observed frequency and the three assigned frequency in Table 1, a new set of interpolated data are generated using quadratic interpolation formula as in Figure 2. Later, the obtained interpolated data are arranged accordingly to their states and a new states transition matrix is developed as shown in Figure 3.

Table 1. Interpolated state of scenario 2

| x | f(x) | Interpolated x |
|-----|-------|----------------|
| 1 | 2.00 | 2.00 |
| 2 | | 3.72 |
| 3 | | 5.44 |
| 6 | | 10.57 |
| 8 | | 13.96 |
| 10 | | 17.34 |
| 11 | | 19.02 |
| 13 | | 22.37 |
| 17 | 29.00 | 29.00 |
| 21 | | 35.56 |
| 23 | | 38.81 |
| 33 | | 54.77 |
| 39 | | 64.11 |
| 49 | | 79.30 |
| 72 | | 112.41 |
| 123 | | 176.75 |
| 346 | 311 | 310.98 |

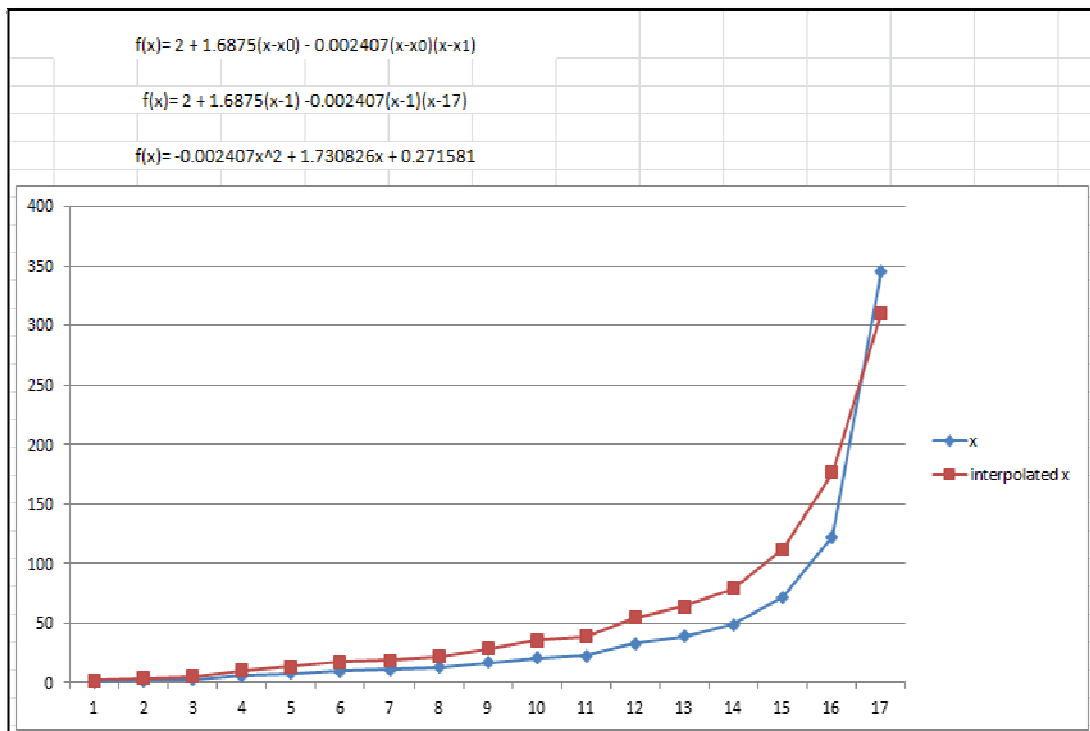


Figure 2. Interpolated transition data for each state based on Scenario 2 policy

| Status rank age Interval | A 22-27 | A 28-33 | A 34-39 | A 40-45 | A 46-51 | B 28-33 | B 34-39 | B 40-45 | B 46-51 | B 52-57 | C 34-39 | C 40-45 | C 46-51 | C 52-57 | TOTAL |
|--------------------------|---------|---------|---------|---------|---------|---------|---------|---------|---------|---------|---------|---------|---------|---------|-------|
| A:22-27 | 311 | 177 | | | | 11 | | | | | | | | | 499 |
| A:28-33 | | 113 | 39 | | | 2 | 80 | | | | 6 | | | | 240 |
| A:34-39 | | | 55 | 6 | | | 18 | 36 | | | 64 | | | | 179 |
| A:40-45 | | | | 2 | | | | 6 | 14 | | | | | | 22 |
| A:46-51 | | | | | | | | | 4 | 4 | | | | | 8 |
| B:22-27 | | | | | | | | | | | | | | | 0 |
| B:28-33 | | | | | | | | | | | | | | | 0 |
| B:34-39 | | | | | | | | | | | | | | | 0 |
| B:40-45 | | | | | | | | | | | | | | | 0 |
| B:46-51 | | | | | | | | | | | | | | | 0 |
| C:34-39 | | | | | | | 23 | | | | | | | | 23 |
| C:40-45 | | | | | | | | 79 | | | | | | | 79 |
| C:46-51 | | | | | | | | | | | | 2 | | | 2 |
| C:52-57 | | | | | | | | | | | | | 20 | 11 | 31 |

Figure 3. Interpolated States Transition matrix categorized by state such as age interval and status.

| Status rank age Interval | A 22-27 | A 28-33 | A 34-39 | A 40-45 | A 46-51 | B 28-33 | B 34-39 | B 40-45 | B 46-51 | B 52-57 | C 34-39 | C 40-45 | C 46-51 | C 52-57 |
|--------------------------|-----------|----------|----------|----------|---------|-------------|-------------|-------------|-------------|---------|----------|---------|----------|----------|
| A:22-27 | 0.6233246 | 0.354709 | 0 | 0 | 0 | 0.022044088 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| A:28-33 | 0 | 0.470833 | 0.1625 | 0 | 0 | 0.008333333 | 0.333333333 | 0 | 0 | 0.025 | 0 | 0 | 0 | 0 |
| A:34-39 | 0 | 0 | 0.307263 | 0.03352 | 0 | 0 | 0.100558659 | 0.201117318 | 0 | 0 | 0.357542 | 0 | 0 | 0 |
| A:40-45 | 0 | 0 | 0 | 0.090909 | 0 | 0 | 0 | 0.272727273 | 0.536363636 | 0 | 0 | 0 | 0 | 0 |
| A:46-51 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0.5 | 0.5 | 0 | 0 | 0 | 0 |
| B:22-27 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| B:28-33 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| B:34-39 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| B:40-45 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| B:46-51 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| C:34-39 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| C:40-45 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 |
| C:46-51 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 2 | 0 | 0 |
| C:52-57 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0.645161 | 0.354839 |

Figure 4. Transition Probability matrix of Interpolated data categorized by state such as age interval and status.

The new developed interpolated data transition matrix is transformed into transition probability matrix as shown in figure 4 using equation (iii).

Result and Analysis

Matrix transition probability for data distribution of year 1999-2014 was used to project faculty distribution. The assumption was the historical patterns for 1999-2014 will continue if policy of faculty appointment remains the same. The transition probability matrix indicates the probability that a faculty will move from one state to another within one period (5 years). Figure 5 provides the average length of stay for each level of position. The important use of Markov Chain is to predict future manpower distributions if there is policy changes in the current policy. As stated earlier, regarding the RMK-9 policies, which is the recruitments of tutor and lecturer will be reduced by 10% and the appointment of senior lecturer, associate professor and professor will be increased by 70%, a new formation of matrix transition diagram is realized. We consider this policy change as scenario 2 and the old policy as scenario 1. Additionally, we proposed quadratic interpolation to predict the expected

probability for each transition value so that it can be used as a guideline to monitor the transition of each state as given in Figure 4. Estimation of average length of stay, N for each category of staff is calculated using inverse matrix operation given by

| | | | | | | | | | | | | | | | |
|------------------|--|----------|----------|----------|----------|---|-------------|-------------|-------------|-------------|-----|----------|---|---|------|
| | | 2.654255 | 1.779192 | 0.417357 | 0.015389 | 0 | 0.073337242 | 0.826735429 | 0.088134596 | 0.009792733 | 0 | 0.193702 | 0 | 0 | 0 |
| | | 0 | 1.889764 | 0.443294 | 0.016345 | 0 | 0.015748031 | 0.88023876 | 0.093611887 | 0.010401321 | 0 | 0.20574 | 0 | 0 | 0 |
| | | 0 | 0 | 1.443548 | 0.053226 | 0 | 0 | 0.661290323 | 0.30483871 | 0.033870968 | 0 | 0.516129 | 0 | 0 | 0 |
| | | 0 | 0 | 0 | 1.1 | 0 | 0 | 0 | 0.3 | 0.7 | 0 | 0 | 0 | 0 | 0 |
| | | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0.5 | 0.5 | 0 | 0 | 0 | 0 |
| | | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| | | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| | | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 |
| | | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 |
| | | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 |
| | | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 |
| | | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 1 | 0 | 0 |
| | | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 2 | 0 | 0 | 0 | 2 | 1 |
| | | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 2 | 0 | 0 | 0 | 2 | 1 | 1.55 |
| $(I-Q)^{-1} = N$ | | | | | | | | | | | | | | | |

Figure 5. Average Length of Stay of Interpolated Data States for each Level of Position.

Finally, the total estimation of average length of stay, for each state category is obtained by matrix multiplication $N \cdot B$, where B is the total frequency of each state in the state transition matrix. The average length of stay for each category of staff is calculated for the three scenario, which is scenario 1: the distribution of staff faculties remain as year 1999-2014, scenario 2: the percentage of certain staff faculties has been changed with respect to the new policy while others remain, scenario 3: the percentage of certain staff faculties has been changed with respect to the new policy while others are interpolated with respect to minimum, median and maximum data of scenario 2. The analysis is done by comparing the average length of stay between the three category of states.

| | Old Policy | New Policy | Interpolate Data With New Policy |
|---------|-------------|-------------|----------------------------------|
| A:22-27 | 7.162696447 | 6.059895752 | 6.400788126 |
| A:28-33 | 3.601527168 | 3.555143510 | 2.973587047 |
| A:34-39 | 3.03113325 | 3.012903226 | 2.812769629 |
| A:40-45 | 2.090909091 | 2.1 | 2.052631579 |
| A:46-51 | 2 | 2 | 2 |
| B:22-27 | 1 | 1 | 1 |
| B:28-33 | 1 | 1 | 1 |
| B:34-39 | 1 | 1 | 1 |
| B:40-45 | 1 | 1 | 1 |
| B:46-51 | 1 | 1 | 1 |
| C:34-39 | 2 | 2 | 2 |
| C:40-45 | 2 | 2 | 2 |
| C:46-51 | 3 | 5 | 3 |
| C:52-57 | 4.545454545 | 6.55 | 4.526315789 |

Figure 6. Comparative Average Length of Stay Between Scenario 1: Old Policy, Scenario 2: New Policy and Scenario 3: Interpolated Data with New Policy

Figure 6 shows the comparative results between the average length of stay value for each category using classical TPM of Markov chain and the proposed

interpolated TPM of Markov chain for the old and new policy. The results for interpolated TPM has shown a better estimates of average mean queue length of stays by status for states towards a higher range of age and status.

Conclusion

Based on the results obtained, the Markov chain model developed in this study is an appropriate evaluation tool for policy change concerning the appointment of faculty. This paper demonstrates that if new policy is implemented, there will be a high impact on the number of academic staff by diverse rank especially towards more senior faculty members. Mean time for the faculty remain in current state does not show much difference between old and new policy. Based on the results, it is predicted the proposed policy will not have much changes if it were to be implemented. Otherwise a modified predicted approach is required such as interpolated data estimators approach as proposed in this study.

This paper presents the potential use of interpolation technique for predicting better estimate for projecting transition data in Markov chains. The states transition of staff faculties categories can easily be constructed using spreadsheet and the calculation of matrix performance can be done simply using excel built- in function.

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List of Symbols

| | |
|----------------|--|
| K | Category of rank |
| C_t | Data arrange in sequence |
| t | time |
| s_k | Sequence |
| l -th | order of transition probability |
| s_{il} | |
| q | |
| φ | |
| α_i | |
| $\forall j$ | all j |
| $K^l(K - 1)$ | Reduction of the number of parameter |
| X^n | Data sequence |
| Q_k | Estimate the parameter |
| $f_{jl}^{(i)}$ | Transition frequency in the sequence from state l to state j |
| \hat{Q}_i | Estimator |
| \hat{X}_l | Steady state probability that the process is in state l |
| T | |
| N | Average length of stay |

List of symbols and Abbreviations

| | |
|--------|-----------------------------------|
| MOHE | Ministry of Higher Education |
| RMK-9 | Malaysia 9 th Planning |
| et al. | (et alia): and others |
| yrs | years |
| MTD | Mixture Transition Distribution |
| UUM | Universiti Utara Malaysia |