

# Investigation of Channel Spacing for Hermite-Gaussian Mode Division Multiplexing in Multimode Fiber

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**Abstract**—Mode division multiplexing (MDM) may be used to increase the capacity of multimode fiber interconnects for data centers. This paper demonstrates the importance of controlling the channel spacing of a MDM system capitalizing on Hermite-Gaussian (HG) modes in order to mitigate modal dispersion and minimize the average system bit-error rate. The effect of channel spacing of a 25-channel hybrid MDM-wavelength division multiplexing (WDM) system was examined through the  $x$ -index and  $y$ -index separations of Hermite polynomials of HG modes for different MMF lengths. Simulations prove that by controlling the index separations of the Hermite polynomials, acceptable BER was achieved for 25Gb/s data transmission for a distance of 800 meters for a 25-channel HG-based MDM-WDM system at a center wavelength of 1550.12nm. The optimal  $x$ -index and  $y$ -index Hermite polynomial separations for the HG modes are 2, 3 and 4.

**Keywords**—mode division multiplexing (MDM); Hermite-Gaussian (HG) modes; control systems; channel spacing; multimode fiber;  $x$ -index separation; optical communications

## I. INTRODUCTION

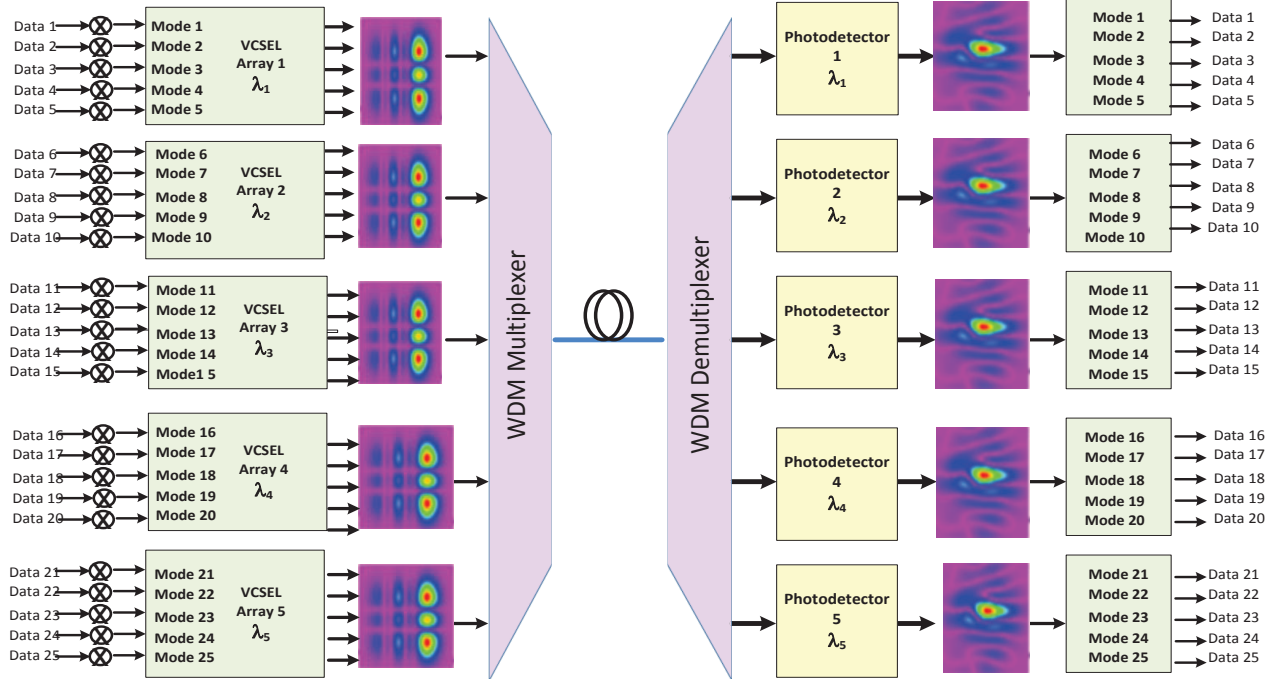
A data center consists of thousands of servers that form a parallel supercomputing infrastructure [1, 2]. The proliferation of triple-play services, big data analytics and the advent of supercomputers have led to the increasing significance of optical interconnects in data centers, predominantly multimode fiber (MMF) [3], to relieve bandwidth bottlenecks [4]. Today's 10Gb/s optical interconnects will be insufficient for handling future data bandwidth demands [5, 6]. Several possible methods for increasing the transmission capacity in data centers have been proposed, capitalizing on different amplitude [7, 8], polarization [9, 10], wavelength [11, 12] and time [13, 14] multiplexing schemes. A recent approach garnering remarkable attention is mode division multiplexing (MDM) in MMF, in which several channels are transmitted in mutually orthogonal fields through modes and then demultiplexed into separate channels in an attempt to break through current capacity barriers in data centers [15-18].

MMF has a modal dimension that may be leveraged in a multiplexing system [19, 20] known as mode division multiplexing, MDM, whereby single or groups of modes are used to transmit diverse data signals in a MMF [21], resulting in the ability to optimize propagation differences between modes [22]. Recent MDM demonstrations employ spatial light modulators [23-27], fiber gratings [28, 29], digital signal processing algorithms [30, 31], modal decomposition algorithms [32, 33], adaptive optics [33-37] and photonic crystal fiber [34]. The excitation of modes for each channel is realized by matching the incident field of MMF to the inherent modal field of MMF [24-26]. At the receiver side, signals will be demultiplexed based on modal characteristics [32].

This paper is organized as follows: Section II elucidates the recent literature review on MDM of HG launching conditions. Section III presents the research methodology for the simulation of the MDM model based on HG modes. Section IV reports on the investigation of the effect of channel spacing on the power coupling coefficients and bit-error-rate (BER) for different MMF lengths. The conclusion of the paper is presented in Section V.

## II. BACKGROUND AND LITERATURE SURVEY

HG modes have been recently used to control the impulse response of a MMF in a number of experiments. In [38, 39], a passive beam shaper formed on a fused silica substrate is used to generate a one-dimensional HG mode profile at the MMF input core from a collimated Gaussian beam. The one-dimensional HG launch excited predominantly mid-ordered modes, enhancing the bandwidth by more than five times and achieving 10Gb/s through a 220m MMF. In [40, 41], the one-dimensional HG mode profile varying only a single axis was extended to a two-dimensional square profile symmetrical about the  $x$ -axis and  $y$ -axis but with opposite phase distributions on both axes, achieving 10Gb/s data transmission and extending the link yield to 250m.


 Fig. 1: Hermite-Gaussian mode division multiplexing model for evaluating the effect of spacing of  $x$ -index on transmission performance

Although appreciable progress has been forged in experimental demonstration of MDM adopting HG modes [38-41], the effects of channel spacing of the excited modes as independent paths have not been analyzed. In this paper, we investigate for the first time the effect of channel spacing of a HG-based MDM system by investigating the effect of  $x$ -index and  $y$ -index separations of the Hermite polynomials in HG modes on the channel performance for different MMF lengths.

TABLE 1: LAUNCH OF HG MODES AT MMF INPUT

Simulation Run	HG $x$ -index generated	$x$ -index separation of Hermite-Gaussian modes, $\Delta x$	$y$ -index for all HG modes
1	$x=1,2,3,4,5$	1	$y=2$
2	$x=1,3,5,7,9$	2	$y=2$
3	$x=1,4,7,10,13$	3	$y=2$
4	$x=1,5,9,13,17$	4	$y=2$
5	$x=1,6,12,16,21$	5	$y=2$

### III. SIMULATION OF MDM OF HERMITE-GAUSSIAN MODES

An MDM-WDM model employing 25 HG modes as 25 independent channels carrying distinct data streams was designed in Optisim 5.2 simulator [42], as shown in Fig. 1.

In [38-41], an etched silica mask is used for the generation of HG modes. In contrast, in our work, vertical-cavity surface-emitting laser (VCSEL) arrays are used instead of an etched silica mask as in [38-41]. Also, in [38-41], the HG launches primarily target a single MMF mode group.

In our work, in order to investigate the effects of channel spacing, the launch comprises several HG modes as independent data streams with predetermined  $x$ -index and  $y$ -index separations of the Hermite polynomial in HG modes at the MMF input. The index separations are varied at each run in order to analyze the effects of channel spacing between independent HG channels on the system performance. The channel spacing for each simulation run is given in Table 1.

When the  $x$ -index separation of the Hermite polynomials of the HG modes,  $\Delta x$  is varied, the  $y$ -index is preserved for different simulation runs whereas when the  $y$ -index separation of the Hermite polynomials of the HG modes,  $\Delta y$  is varied, the  $x$ -index is preserved for different simulation runs.

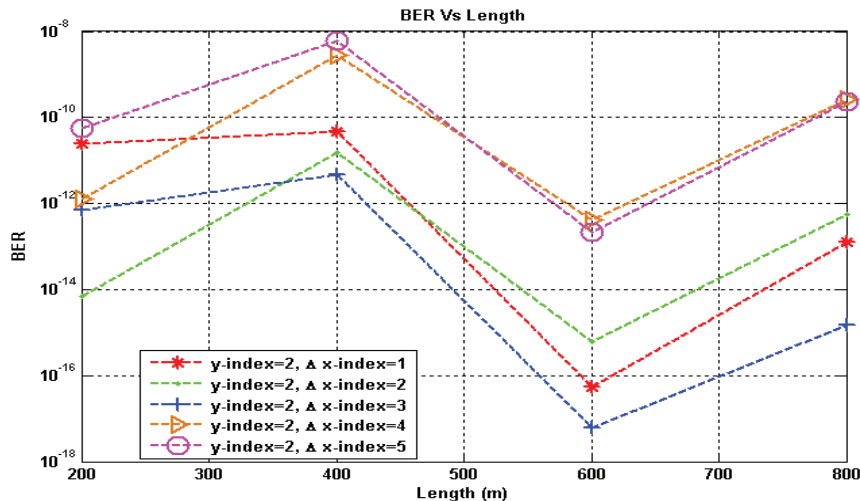


Fig. 2 Effect of x index spacing and MMF length on average system BER

The developed HG-MDM model transmitter consists of five VCSEL arrays at wavelengths between 1546.92nm and 1553.33nm separated by 1.6nm, driven by 1Gbps pseudo-random binary sequence (PRBS) to generate electrical sequences, which are independent from one another. The binary sequences are optical modulated using non-return-to-zero signals. The power from each of the five VCSEL arrays is assumed to be emitted uniformly into five 5 $\mu$ m HG modes as independent channels, thus realizing a 1Gbp/s per wavelength per mode  $\times$  5 wavelengths  $\times$  5 modes MDM-WDM system with an aggregate of 25Gbps on 25 channels. The generated electric field of the HG mode is defined as [42]:

$$\psi_{lm}(x,y) = \alpha H_l \left( \frac{\sqrt{2}x}{w_{0x}} \right) \cdot \exp \left( \frac{-x^2}{w_{0x}^2} \right) \cdot \exp \left( \frac{j\pi x^2}{\lambda R_{0x}} \right) \cdot H_m \left( \frac{\sqrt{2}y}{w_{0y}} \right) \cdot \exp \left( \frac{-y^2}{w_{0y}^2} \right) \cdot \exp \left( \frac{j\pi y^2}{\lambda R_{0y}} \right) \quad (1)$$

where  $w_{0x}$  and  $w_{0y}$  are the spot sizes in the  $x$ -axis and  $y$ -axis respectively;  $R_{0x}$  and  $R_{0y}$  are the  $x$  and  $y$  radii of curvature in the , respectively;  $H_l$  and  $H_m$  are Hermite polynomials in the  $x$ -axis and  $y$ -axis directions respectively;

The five HG modes on each VCSEL array are then mode division multiplexed and then wavelength division multiplexed with aggregate signals from other VCSEL arrays. The mode division multiplexed and wavelength division multiplexed signals are then propagated through a MMF. The refractive index profile of the MMF  $\alpha=1.81$  to reflect a typical manufactured MMF [24]. The assumed values for attenuation=1.5dB/km and power modal coupling have been

taken into consideration. To analyze the effect of the channel spacing, different MMF lengths are used, i.e. 200m, 400m, 600m and 800m. After the multiplexed signals are propagated through the MMF, at the receiver, the signal is demultiplexed into five separate wavelengths using a WDM demultiplexer and retrieved by five photodetectors of different wavelengths. Each photodetector signal will then be mode division demultiplexed into five HG modes based on the aforementioned  $x$ -index separation,  $\Delta x$  using noninterferometric modal decomposition [32].

To evaluate the channel spacing of the HG modes on the system performance, the  $x$ -index separation of the HG mode,  $\Delta x$ , is varied ranging from  $\Delta x=1$ ,  $\Delta x=2$ ,  $\Delta x=3$ ,  $\Delta x=4$  to  $\Delta x=5$  while preserving the  $y$ -index of the HG modes. Following this, the reverse is analyzed by varying the  $x$ -index separation from  $\Delta x=1$  to  $\Delta x=2$ ,  $\Delta x=3$ ,  $\Delta x=4$  to  $\Delta x=5$  while preserving the  $y$ -index of the HG modes. The system performance of the MDM of HG modes with different  $x$ -index and  $y$ -index channel spacing will be examined in terms power coupling coefficient versus modal delay and the BER for various MMF lengths up to 800m as in a typical data center, MMF lengths are mostly shorter than 500m [43].

#### IV. RESULT AND DISCUSSION

To examine the effect of  $\Delta x$  index separations of the HG modes on system performance, the average system BER for various  $\Delta x$  mode separations for different MMF lengths are plotted in Fig. 2. From 200m to 400m, the power between modes start to couple but the differential mode delay is large, thus resulting in pulse broadening and inter-symbol interference (ISI). From 400m to 600m, power coupling between modes increases and the differential mode delay is reduced. Thus, the pulse width is reduced and ISI is alleviated. From 400m to 600m, power modal

coupling between modes is still prevalent but the propagation time delay between modes is increased with distance, resulting in large pulse broadening and ISI in comparison to the cases of 400m to 600m. Interestingly, when the spacing between modes in the  $x$ -index is too narrow, the propagation time delay difference between modes is large, resulting in high ISI. Similarly, when the spacing between modes in the  $x$ -index is large, namely for  $\Delta x=4$  and  $\Delta x=5$ , the propagation delay between modes is large, resulting in high ISI. On the other hand, good BER is achieved when the mode spacing in the  $x$ -index is moderate, namely for  $\Delta x=2$  and  $\Delta x=3$ , hence, the propagation time between modes is minimized and ISI is relieved. It is worth noting that, the same BER distribution is achieved for equivalent  $\Delta y$  spacing given the symmetrical modal profile of the HG mode in the  $y$ -direction and the symmetry of MMF waveguide cross-section.

For further quantitative evaluation, the modal decomposition at the MMF output is analyzed for different  $\Delta x$  separations. The modal decomposition for Channel 12 is shown in Fig. 3. In Fig. 3(a) for  $\Delta x=1$ , the power is by largely coupled into higher-

order modes and medium order modes, therefore incurring high differential mode delay and a wide pulse. In Fig. 3(b) for  $\Delta x=2$ , the power is coupled more significantly into higher-order modes, but with a spattering of power in low-ranged and medium-ranged modes, resulting in a non-bell pulse. In Fig. 3(c) for  $\Delta x=3$ , the power distribution is similar to the case of  $\Delta x=2$  in Fig. 3(b) except for lower power coupling into lower-order modes. In Fig. 3(d) for  $\Delta x=4$  and Fig. 3(e) for  $\Delta x=5$ , it is observed that as  $\Delta x$  increases, the modes are more scrambled, producing a more non-uniform pulse. The differential modal delay for  $\Delta x=4$  is slightly better than in the case of  $\Delta x=5$  due to less power coupling into lower-order modes. Similar relative power coupling distributions are obtained for other channels.

Fig. 4 shows the comparison of the eye diagrams illustrating the effect of  $x$ -index separations for a 400m-long MMF when the  $y$ -index is fixed to 2. As the mode separation increases in the  $x$ -index, namely for  $\Delta x=4$  and  $\Delta x=5$ , the time deviation between the propagating modes decreases and the eye opening widens. The same eye diagrams are obtained for equivalent  $\Delta y$  separations.

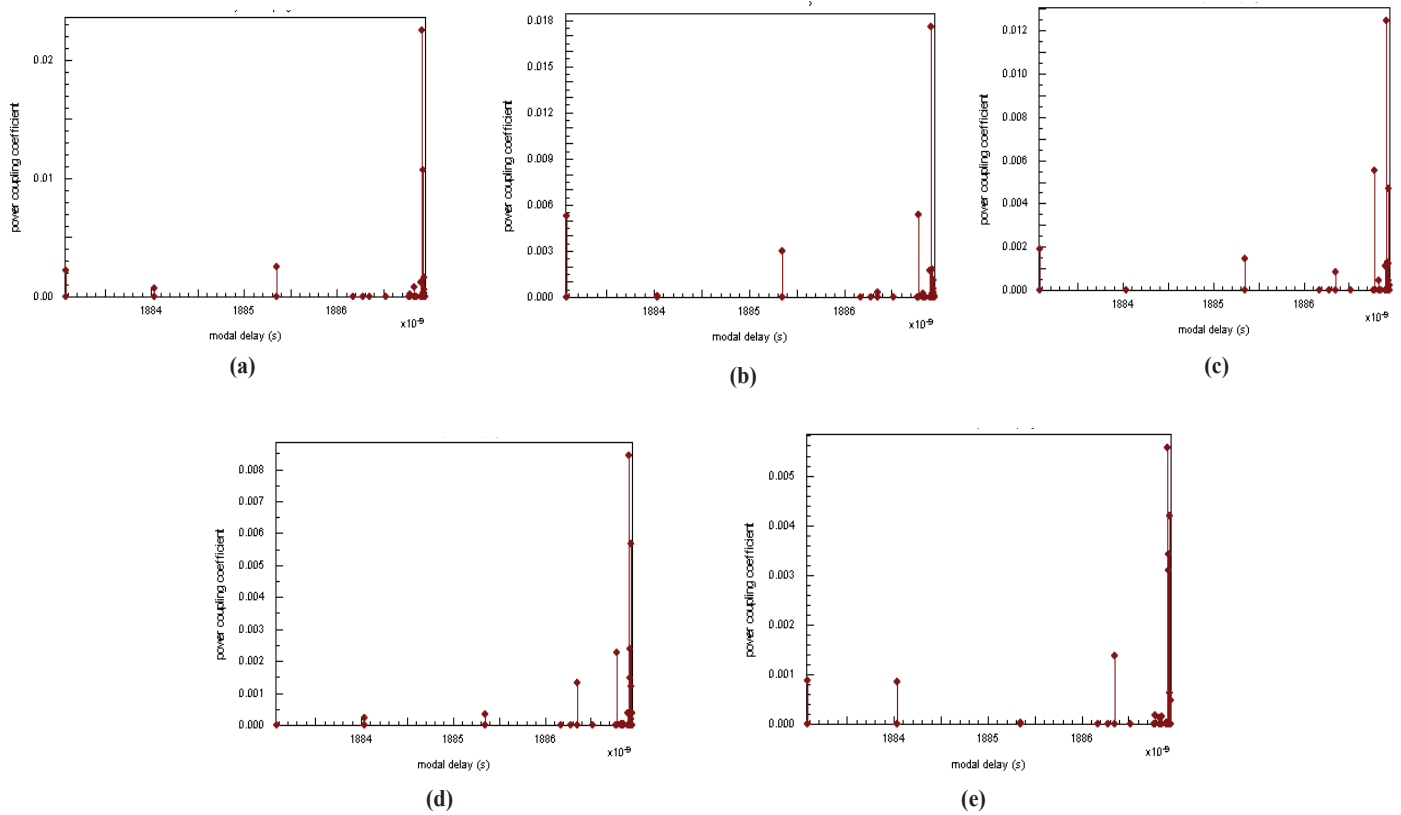


Fig. 3 Modal decomposition versus modal delay at the MMF output for different  $\Delta x$  mode separations at 400 m: (a)  $\Delta x = 1$  (b)  $\Delta x = 2$  (c)  $\Delta x = 3$  (d)  $\Delta x = 4$  and (e)  $\Delta x = 5$  for Channel 12

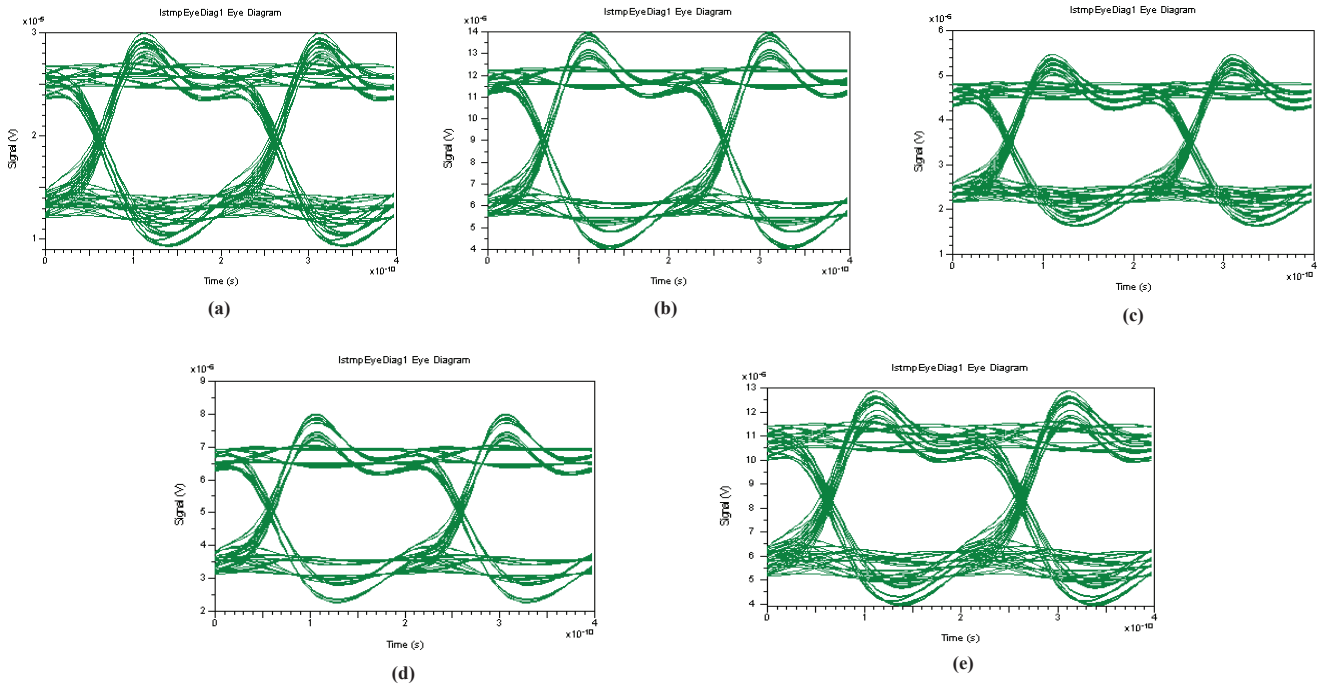


Fig. 4 Eye diagrams at the MMF output for different  $\Delta x$  mode separations at 400 m for Channel 12: (a)  $\Delta x = 1$  (b)  $\Delta x = 2$  (c)  $\Delta x = 3$  (d)  $\Delta x = 4$  and (e)  $\Delta x = 5$

It is important to note that differential modal delay and ISI still exist despite the selective HG mode excitation from the MDM launch but both values are significantly reduced compared to an overfilled launch. Also, more accurate results could be obtained by using a measured refractive index profile from a manufactured MMF with a dip in the center.

## V. CONCLUSION

Simulations on a HG-MDM-WDM system demonstrate the significance of controlling the separation of  $x$ -indices and  $y$ -indices of Hermite polynomials of HG modes,  $\Delta x$  and  $\Delta y$  respectively. By controlling the separation of Hermite polynomial indices in the VCSEL arrays, a data rate of 25Gb/s is achieved with acceptable BER for a distance of 800 meters for  $\Delta x = 2, 3, 4$  and  $\Delta y = 2, 3, 4$  at a central wavelength of 1550.12nm. The HG-MDM-WDM model has potential applications for multiplexing channels in optical interconnects for data centers.

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