Using Data Envelopment Analysis to Defuzzify a Group of Dependent Fuzzy Numbers

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ABSTRACT

The defuzzification process converts fuzzy numbers to crisp ones and is an important stage in the implementation of fuzzy systems. In many actual applications, relationships among data indicate their mathematical dependence on one another. Hence, this study proposes a new method based on the Data Envelopment Analysis (DEA) model to defuzzify a group of dependent fuzzy numbers. It also aims to obtain the crisp points that satisfy the characteristics of these data as a group by approximating the optimal solutions within the production possibility set of the DEA model. The proposed method partitions the fuzzy numbers, and the relationships among these numbers are observed as constraints. Finally, the usefulness of this new method is illustrated in a real-world problem.

Keywords: dependent fuzzy numbers, group defuzzification, Data Envelopment Analysis

1. Introduction

The modeling of complex systems is limited by incomplete knowledge and lack of information (Lai & Hwang, 1992). Hence, the fuzzy set theory developed by Zadeh (1965), along with its techniques, is an interesting and promising approach to address complex, real-world issues. In general, a fuzzy representation provides more information regarding a set than a crisp representation. However, this crisp representation remains necessary because it simplifies conception and clarification. Thus, the objective determination of the fuzzy structures of problematic systems is difficult. Thus, a crisp representation is typically easy to interpret and understand although it displays less information. To replace a fuzzy representation of sets with a crisp representation in fuzzy system applications, the process of defuzzification is applied (Leekwijck & Kerre, 1999; Mahdiani, Banaiyan, Haji Seyed Javadi, Fakhraie, & Lucas, 2013).

This definition enables the defuzzification of a set into a crisp subset of the original. Previous literature presents many defuzzification methods, but most of these methods generate fuzzy set results with the best information and composition. Furthermore, some of these methods lose their properties during actual observations of groups of related data. Meanwhile, defuzzification methods can generate similar results given data with various relationships.

This study mainly presents a new method to defuzzify a group of dependent fuzzy numbers with the tool Data Envelopment Analysis (DEA). The resultant defuzzify points can be considered crisp approximation of fuzzy numbers that maintain the relationships among a group of fuzzy numbers. Furthermore, the new method can overcome the shortcomings of previous methods, which could not address dependent data.

The rest of the paper is organized as follows. Section 2 presents the backgrounds of defuzzification, DEA, and fuzzy DEA (FDEA). Section 3 details the defuzzification of a group through examples and discusses the methodology in subsection 3.2. Section 4 presents the results of a case study. The final section sums up the main findings of the study.

2. Background

2.1 Defuzzification

Defuzzification is an important fuzzy system stage that replaces fuzzy numbers with a representative crisp number (Esogbue, Song, & II, 2000; Mahdiani et al., 2013). Some common defuzzification techniques are center of area (COA), weighted average method, and height method (Lee, 1990; Gunadi, Nurcahyo, Shamsuddin, & Alias, 2003 ; Nurcahyo, 2014) .

Related literature also describes various defuzzification methods with different levels of complexity. For instance, Ma, Kandel, and Friedman (2000) proposed a novel method to defuzzify fuzzy sets according to the metric distance between two symmetric and triangular fuzzy numbers. Similarly, Sladoje, Lindblad, and Nyström (2011) presented a novel defuzzification method for image processing. Their method determines the crisp set that is at a minimal distance from the fuzzy set by generating a family of distance functions. The distance between two fuzzy sets is expressed as a Minkowski distance.

Meanwhile, Naaz, Alam, and Biswas (2011) presented a simple model of the fuzzy load balancing algorithm in a distributed system and compared the effects of five defuzzification methods, namely, COA, bisector of area, mean of maximum (MOM), smallest of maximum, and largest of maximum. Asady and Zendehnam (2007) also Saneifard and Ezatti (2010) proposed defuzzification methods to rank fuzzy numbers. In the present study, we compare the proposed method with the center of gravity (COG) method and with that proposed by Asady and Zendehnam (2007).

2.1.1 Center of Gravity (COG)

The COG method was developed by Sugeno (1985) and is the most commonly used defuzzification method. This method calculates the position at which the left and the right areas are equal. COG refers to the centroid of the area, and the defuzzification method can be expressed as:

$$x_{COG} = \frac{\int \mu_{\tilde{F}}(x).\, x\partial x}{\int \mu_{\tilde{F}}(x).\, \partial x}$$

2.1.2 The method of Asady and Zendehnam

Asady and Zendehnam (2007) presented a defuzzification method based on the nearest point of a fuzzy number. The nearest point to the triangular fuzzy number $u = (x_0, \delta, \beta)$ to be:

$\mathcal{M}(u) = x_0 + \frac{\beta-\delta}{4}$, where δ and β are the left and the right fuzziness values, respectively.

2.2 Data Envelopment Analysis (DEA)

DEA is a recognized modern approach that stems from a linear programming (LP) model to evaluate the relative efficiencies of decision making units (*DMUs)* with multiple inputs and outputs. DEA is a non-parametric technique and was initially proposed by Charnes, Cooper, and Rhodes (1978) as a (*CCR*) model. This model was improved by other scholars, particularly in the form of the Banker, Charnes, and Cooper (1984) *(BCC)* model .

Assuming the inputs x_{ij} $(i = 1,2,..,m)$ and outputs y_{rj} $(r = 1,2,..,s)$ for DMU_j *(j=1,2,..,n)*. The programming statement for the CCR model is formulated as follows:

$$
\begin{aligned}
&\theta_p^* = min\, \theta_p \\
&s.t. \\
&\quad \sum_{j=1}^{n} \lambda_j x_{ij} \leq \theta_p x_{ip} \\
&\quad \sum_{j=1}^{n} \lambda_j y_{rj} \geq y_{rp} \\
&\quad \lambda_j \geq 0 \quad j = 1, \dots, n \\
&\quad \theta\ free
\end{aligned}
\tag{1}
$$

Where λ_j is a non-negative value related to the j^{th} *DMU*. The vector $\lambda = (\lambda_1, \lambda_2, \dots, \lambda_n)^t$ constructs a hull that covers all of the data points.

Model (1) is divided into three parts, namely, the left- and right-hand sides of the constraints and the objective function. The left-hand side generates the production possibility set (PPS), and retouching this set changes the space. The right-hand side and the objective function lead *DMUs* to the frontier. Thus, the *DMUs* located on the efficiency frontier are considered the relative ideal points in DEA evaluation. That is, each inefficient *DMU* probes its own ideal *DMU* on the frontier. However, the question is whether the ideal points always lie on the efficiency frontier. In this research, we indicate that the ideal point can be probed within PPS.

2.3 Fuzzy Data Envelopment Analysis (FDEA)

In real-world problems, the required input and output data are often not known precisely. Imprecise evaluations may be the result of unquantifiable, incomplete and non-obtainable information. Model (1) can only be used for cases where the data are precisely measured. As for the application of fuzzy sets theory in DEA, Fuzzy DEA that can be traced to Sengupta (1992) as a powerful tool for evaluating the performance of *DMUs* with imprecise data (or interval data). Fuzzy input–output variables can be introduced to *CCR* model as in the following;

$$
\begin{aligned}
&\theta_p^* = min\,\theta_p \\
&s.t. \\
&\quad \sum_{j=1}^{n} \lambda_j \tilde{x}_{ij} \le \theta_p \tilde{x}_{ip} \qquad (2)\\
&\quad \sum_{j=1}^{n} \lambda_j \tilde{y}_{rj} \ge \tilde{y}_{rp} \\
&\quad \lambda_j \ge 0 \quad j = 1, \dots, n \\
&\quad \theta\ free \\
&\quad \tilde{x}: fuzzy\ inputs \\
&\quad \tilde{y}: fuzzy\ outputs
\end{aligned}
$$

Since its inception, the FDEA method and its applications have generated increased interest. Wang, Greatbanks, and Yang (2005) used the α-level set approach to convert the fuzzy data into intervals. Hatami-Marbini, Saati, and Tavana (2010) developed a four-phase FDEA framework based on displaced ideal theory. Moreover, they (2011) also established an additive FDEA model. Meanwhile, Saati, Hatami-Marbini, and Tavana (2011) incorporated fuzzy discretionary and non-discretionary factors into a DEA model. Also Zerafat Angiz, Emrouznejad, and Mustafa (2012) proposed an alternative FDEA model to measure the efficiency of DMUs in a fuzzy environment according to the local α-level concept and an LP model. The approaches established by Luban (2009) and Zerafat Angiz, Emrouznejad, Mustafa, and Al-Eraqi (2010) have contributed to FDEA development as well.

3. Defuzzification of a Group of Dependent Fuzzy Numbers

3.1 Dependent fuzzy numbers

This section stresses on the origin of the premise underlying dependent fuzzy numbers. First, Asady and Zendehnam proposed the new defuzzification method that able to rank a group of independent fuzzy numbers in 2007. Although their method determined to the nearest point, it primarily concentrated on independent fuzzy numbers. Moreover, it is quite notable that all the proposed methods in literature deal with independent fuzzy numbers, which produce similar defuzzification data points under various conditions. Literature is also rife with independent fuzzy numbers in terms of various relationships emphasizing the transformation of individual fuzzy numbers into crisp numbers (e.g. COG, MOM) and the method brought forward by Saneifard (2009).

However, because real-world application data is noted in groups that display some relationships and properties that emphasize their dependence, dependent fuzzy numbers takes significance. Therefore, in this study, a new defuzzification method that stresses on dependent groups of fuzzy numbers is proposed. In other words, the present study is unique in that it addresses dependent fuzzy numbers rather than what has been extensively examined in literature namely independent fuzzy numbers.

To explain further, we refer to an example presented by Zerafat Angiz, Emrouznejad, Mustafa, and Rashidi Komijan (2009), where *(G_1, G_2,..,G_n)* indicates the supposed ranking places. Given a group of *p* experts *(E_1, E_2,..,E_p)* commenting on the weights of these places, the weights $w_{ik}(i = 1,2,..,n)(k = 1,2,..,p)$ can be aggregated into a group of fuzzy numbers. Thus, we first generate n fuzzy numbers. We then select the triangular fuzzy numbers (TFN) from among the various shapes of fuzzy numbers because it is the most popular one. Therefore, the triangular fuzzy numbers are denoted by three points as follows:

$$\widetilde{w}_i = (w_i^m, w_i^l, w_i^u)$$

In this case, we determine a representative of the fuzzy numbers given above $(w_1^*, w_2^*, .., w_n^*)$.This representative is established as the final weight of eachG_i $(i = 1,2, \ldots, n)$, and the sum of the representative weights must be one. However, this restriction may not be adhered to if a defuzzification method such as COG or that developed by Asady and Zendehnam is employed because these methods do not have a condition that maintains these relations among the representative weights. The following diagram illustrates this matter.

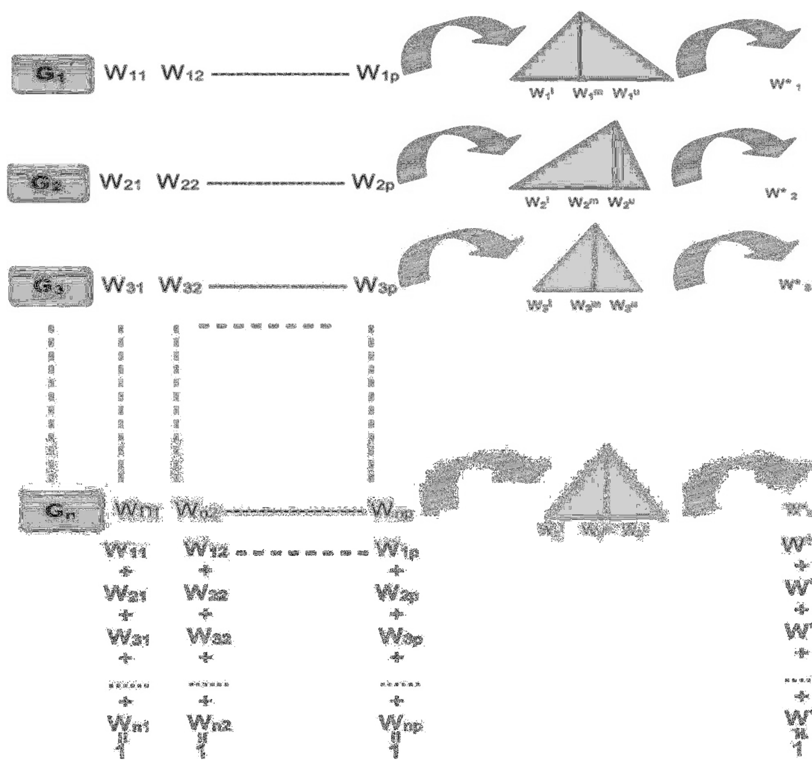

Fig1- Example to illustrate the shortcoming of existing defuzzification methods.

3.2 Defuzzification of a Group of Dependent Fuzzy Numbers

The method proposed to defuzzify a group of dependent fuzzy numbers operates in five stages:

Stage 1: *n* triangular fuzzy numbers are generated based on the method proposed by Yeh and Chang (2009) as follows:

$$\tilde{x}_i = (x_i^m, x_i^l, x_i^u)$$

where $x_i^m = \left(\prod_{k=1}^{p} x_{ik}\right)^{1/p}$, $x_i^l = min\{x_{i1}, x_{i2}, .., x_{ip}\}$ *and* $x_i^u = max\{x_{i1}, x_{i2}, .., x_{ip}\}$

This method displays the following membership functions:

$$\mu_{\tilde{x}_i}(\bar{x}_i) = \begin{cases} L\left(\dfrac{\bar{x}_i - x_i^l}{x_i^m - x_i^l}\right) & x_i^l \le \bar{x}_i \le x_i^m \\ R\left(\dfrac{x_i^u - \bar{x}_i}{x_i^u - x_i^m}\right) & x_i^m \le \bar{x}_i \le x_i^u \end{cases} \qquad i = 1,2,..,n$$

Stage 2: The interval $[x_i^l, x_i^u]$ of the fuzzy number *i* is divided into *m* subintervals$\{[x_i^l = x_{i1}, x_{i2}], [x_{i2}, x_{i3}].., [x_{i(m-1)}, x_{im} = x_i^u]\}$.

Stage 3: With these subintervals, *m DMUs* are created. The PPS of these *DMUs* generate all of the possible solutions in the fuzzy interval. In other words, $x_1^l = x_{1j}, x_{2j}, x_{3j}, .., x_{(n-1)j}, x_{nj} = x_n^u$ represents the input values of DMU_j *(j=1, 2,…, m)* that are used to produce PPS. The single output corresponding to DMU_j is assumed to be one. Figure 1 illustrates the interval partitioning of the fuzzy number *i*.

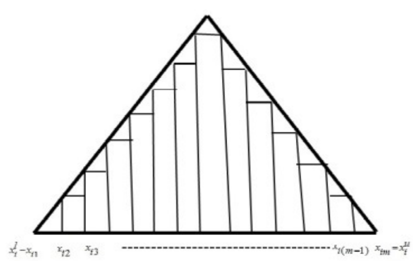

Fig 2- The interval partitioning of the fuzzy number *i*.

Stage4: The following non-linear programming model is proposed to the points nearest to fuzzy number *i* using the optimal solution. The objective functions approximate the COG of the distances from each partition.

$$
\begin{aligned}
& min \frac{\sum_{k=1}^{m} \mu_{\tilde{x}_i}(\grave{x}_{ik}) \left|\bar{x}\grave{}_i - \grave{x}_{ik}\right|}{\sum_{k=1}^{m} \mu_{\tilde{x}_i}(\grave{x}_{ik})} && i = 1,2, \dots, n \\
& s.t. \\
& \quad \sum_{k=1}^{m} \lambda_k \grave{x}_{ik} \geq \bar{x}\grave{}_i && i = 1,2,..,n \qquad (3) \\
& \quad \sum_{k=1}^{m} \lambda_k \leq 1 \\
& \quad \sum_{i=1}^{n} f(\bar{x}\grave{}_i) = t \\
& \quad \frac{x_{ik}^{l}}{x_{ik}^{u}} \leq \bar{x}\grave{}_i \leq 1 && i = 1,2, \dots, n \quad k = 1,2, \dots, m \\
& \quad \lambda_k \geq 0 && k = 1,2,..,m
\end{aligned}
$$

In the proposed method, the relationships among these groups of fuzzy numbers are expressed as constraints $\sum_{i=1}^{n} f(\bar{x}\grave{}_i) = t$. The fourth constraint on the right-hand side includes all of the intervals of the fuzzy numbers.

As shown in the model above, constraints including λ produce PPS that correspond to the CCR model. This model is solved only once unlike DEA evaluation, in which *DMU* evaluation, requires the calculation of many models.

By ignoring the third constraint $\sum_{i=1}^{n} f(\bar{x}\grave{}_i) = t$ in the aforementioned model, we can easily prove that the optimal solution (defuzzification points of each fuzzy number *i* of the above model is related to triangular fuzzy numbers of the mean value of each fuzzy number.

Stage5: We assume that $z_{ik} = \bar{x}\grave{}_i - \grave{x}_{ik}$ and $|z_{ik}| = z_{ik}^{+} + z_{ik}^{-}$ $\forall(i,k)$. The multi-objective nonlinear programming model (3) is then proposed as follows:

$$
\begin{aligned}
& min \frac{\sum_{k=1}^{m} \mu_{\tilde{x}_i}(\grave{x}_{ik}) \left(z_{ik}^{+} + z_{ik}^{-}\right)}{\sum_{k=1}^{m} \mu_{\tilde{x}_i}(\grave{x}_{ik})} && i = 1,2, \dots, n \\
& s.t. \\
& \quad \sum_{k=1}^{m} \lambda_k \grave{x}_{ik} \geq \bar{x}\grave{}_i && i = 1,2,..,n \qquad (4) \\
& \quad \sum_{k=1}^{m} \lambda_k \leq 1 \\
& \quad \sum_{i=1}^{n} f(\bar{x}\grave{}_i) = t \\
& \quad \frac{x_{ik}^{l}}{x_{ik}^{u}} \leq \bar{x}\grave{}_i \leq 1 && i = 1,2, \dots, n \quad k = 1,2, \dots, m \\
& \quad \bar{x}\grave{}_i - \grave{x}_{ik} - \left(z_{ik}^{+} - z_{ik}^{-}\right) = 0 && i = 1,2, \dots, n \quad k = 1,2, \dots, m \\
& \quad \lambda_k \geq 0 && k = 1,2, \dots, m
\end{aligned}
$$

If $\sum_{i=1}^{n} f(\bar{x}\grave{}_i) = t$ is linear, this model above is a multi-objective linear programming model (MOLP); hence, it can be solved using Archimedean goal programming model.

$$
\begin{aligned}
& min\ \textstyle\sum_{i=1}^{n} w_i d_i \\
& \quad s.t. \\
& \quad \frac{\sum_{k=1}^{m} \mu_{\tilde{x}_i}(\grave{x}_{ik})\,(z_{ik}^{+} + z_{ik}^{-})}{\sum_{k=1}^{m} \mu_{\tilde{x}_i}(\grave{x}_{ik})} - d_i \le t`_i && i = 1,2,\dots,n \qquad (5) \\
& \quad \sum_{k=1}^{m} \lambda_k \grave{x}_{ik} \ge \bar{x}`_i && i = 1,2,..,n \\
& \quad \sum_{k=1}^{m} \lambda_k \le 1 \\
& \quad \sum_{i=1}^{n} f(\bar{x}`_i) = t \\
& \quad \frac{x_{ik}^{l}}{x_{ik}^{u}} \le \bar{x}`_i \le 1 && i = 1,2,\dots,n \quad k = 1,2,\dots,m \\
& \quad \bar{x}`_i - \grave{x}_{ik} - (z_{ik}^{+} - z_{ik}^{-}) = 0 && i = 1,2,\dots,n \quad k = 1,2,\dots,m \\
& \quad \lambda_k \ge 0 && k = 1,2,\dots,m
\end{aligned}
$$

In model (5) ,w_i $(i = 1,2,\dots,n)$ denotes positive penalty weights. These weights can be determined through multi-criteria decision making techniques such as the analytic hierarchical process (AHP) developed by Saaty (1980). However, we assume that each objective is equally important and allocate equal weight without losing generality. That is $(w_1 = w_2 = \cdots = w_n = 1/n)$ allocated to each weight for this model d_i $(i = 1,2,\dots,n)$ measures the over-achievement from the target point $t`_i$ $(i = 1,2,\dots,n)$ which is obtained by computing the MOLP model as a single objective *n* times (i.e. by considering each objective individually).

4. An Application and Comparison with Other Methods

In this section, we apply the proposed methodology in real life by estimating the required number of hospital beds for the different wards of a Malaysian hospital. We collected the data on the number of beds used by patients who were hospitalized over a period of 150 days from the hospital database of the Malaysian Ministry of Health. The hospital patients are divided into five categories based on age [toddler (T), schoolchildren (S), adult (A), old (O), and elderly (E)]. This case study aims to aid managers in determining the optimal number of beds to be allocated to each group because the number of available beds at this hospital is limited.

Thus, the data are first compiled into a group of dependent fuzzy numbers. This group is represented as a triangular fuzzy number instead of individual fuzzy numbers, which is common in traditional defuzzification methods. The fuzzy number of each group I_{ji} $(j = 1,2,..,m)$ $(i = 1,2,\dots,n)$ is then generated using the method proposed by Yeh and Chang (2009). In DEA, the interval of each fuzzy number *i* is partitioned into *m* subintervals. The starting point of each interval of fuzzy number *i* is an input $(I_{11}, I_{12}, I_{13}, I_{14}, I_{15})$ for DMU_1. The second point is an input $(I_{21}, I_{22}, I_{23}, I_{24}, I_{25})$ for DMU_2, and so on until DMU_9.

These nine partitions therefore imply that nine *DMUs* exist for all fuzzy numbers, and the categorized elements are considered *DMUs* in this case study. Five groups of fuzzy numbers represent the inputs$(I_T\ , I_S, I_A, I_O, I_E)$. PPSs generated by these *DMUs* help generate the interval of fuzzy numbers in association with the five groups of inputs.

Therefore, the efficiency frontier produced by the starting points of the interval of fuzzy numbers is insignificant. In this case, optimal solution should occasionally be obtained from within the PPS rather than on the frontier, as demonstrated in the following example. The figure below indicates two groups of fuzzy numbers (T and S). These groups correspond to DMU_j where $j = 1,2, \ldots, m$ and $m = 9$.

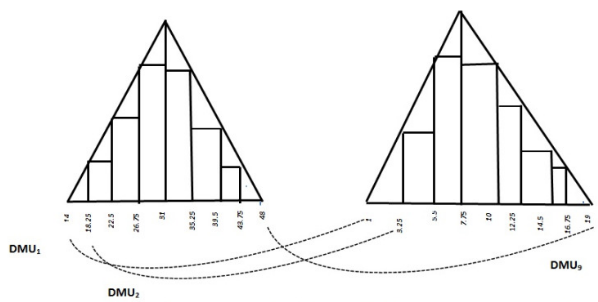

Fig 3- The first and second inputs of DMU_1, DMU_2,.., DMU_9

The flexibility of this method enables the increase in different numbers of *DMUs* by increasing the number of partitions to obtain the best solution. Hence, different numbers of partitions are introduced until we obtain a stable result.

Table 1

Results of the defuzzified values by using proposed method when the total of available beds is 130

Group fuzzy numbers	Defuzzified values				
(x_i^l, x_i^m, x_i^u)	*m=9*	*m=17*	*m=33*	*m=65*	*m=129*
(14,29.2397,48)	27	29	28	28	28
(1,6.7661,19)	8	7	7	7	7
(4,15.5450,34)	15	15	16	16	16
(15,29.4486,51)	29	29	29	29	29
(30,51.8925,76)	51	50	50	50	50
Sum of defuzzified values	130	130	130	130	130

Table 1 shows the group fuzzy numbers generated for each group of inputs (five groups of patients) over 150 days. For instance, when *i*=1, the first group of fuzzy numbers is described as follows as shown in column 1:

$$x_1^l = min\{x_{11}, x_{12}, .., x_{1\,150}\} = 14\ , x_1^m = \left(\prod_{k=1}^{150} x_{ik}\right)^{1/150}\ ,\ x_1^u = max\{x_{11}, x_{12}, .., x_{1\,150}\} = 48$$

The results obtained using *m* different numbers of partitions ranged from 9, 17, 33, and 65 to 129 are summarized in column 2 to 5. These values indicate the optimal number of beds for each group of patients under different numbers of partitions. Moreover, the observed results were stabilized by increasing the number of partitions to *m = 33, 65,129*. The optimal numbers of beds determined for these five groups of patients under various partitions satisfies the relationship($\sum_{i=1}^{5} \bar{x}`_i = 130$), which represents the total number of available beds. Table 2 depicts the results if the number of available beds is increased from 130 to 200.

Table2

Results of the defuzzified values by using proposed method when the total of available beds is 200

Group fuzzy numbers	Defuzzified values				
(x_i^l, x_i^m, x_i^u)	*m=9*	*m=17*	*m=33*	*m=65*	*m=129*
(14,29.2397,48)	43	42	42	42	42
(1,6.7661,19)	15	16	16	16	16
(4,15.5450,34)	27	29	29	29	29
(15,29.4486,51)	45	45	45	45	45
(30,51.8925,76)	70	68	68	68	68
Sum of defuzzified values	200	200	200	200	200

Based on this table, the optimal number of beds for each group of patients under different partitions remains similar given m =17, 33, 65,129, and the results vary only when the number of partitions is nine. Furthermore, the findings satisfied the constraint of the available beds($\sum_{i=1}^{5} \bar{x}`_i = 200$) in all of the considered numbers of partitions.

Subsequently, we compare the results of the proposed method with the results obtained using COG and using the method developed by Asady and Zendehnam. In this comparison, we ignore the third constraint in model (4), which represents the relationships among the groups of fuzzy numbers. In other words, we apply this method to a group of independent and unrelated fuzzy numbers.

Table 3

Comparison results of the proposed method with COG and the method of Asady and Zendehnam.

Group fuzzy numbers	Defuzzified values					COG	Asady & Zendehnam
(x_i^l, x_i^m, x_i^u)	*m*=9	*m*=17	*m*=33	*m*=65	*m*=129		
(14,29.2397,48)	30.00	31.00	29.99	29.99	30.28	30.41	30.12
(1,6.7661,19)	7.99	9.00	8.05	9.00	9.01	8.92	8.38
(4,15.5450,34)	19.00	17.34	17.07	17.73	17.73	17.85	17.27
(15,29.4486,51)	33.00	31.00	31.00	31.61	31.62	31.82	31.22
(30,51.8925,76)	53.00	53.00	53.00	53.00	51.92	52.63	52.45
Sum of defuzzified values	142.99	141.34	139.11	141.33	140.56	141.63	139.44

As, shown in the Table 3, the proposed defuzzification method presents results nearly similar results to the methods of COG and Asady and Zendehnam. The findings of different defuzzification methods vary in implementation (Saneifard & Saneifard, 2011; Chang, Yeh, & Chang, 2013).

The findings suggest that the proposed method can address dependent and independent fuzzy numbers individually and can generate results that are approximated to those determined using COG and the method of Asady and Zendehnam. Moreover, the proposed method has the ability dealing with fuzzy numbers whether as a group or as individuals.

The proposed methodology can also efficiently address nonlinear fuzzy numbers. In this case, many nonlinear membership functions can represent real-world problems to some extent, including the (hyperbolic and exponential) membership functions. This method can be followed by the matching of real-world problems to these functions using actual data and curve fitting or statistical techniques. Thus, we can obtain improved results in a real-world environment.

5. Conclusion

In this study, a new defuzzification method was developed to defuzzify a group of dependent fuzzy numbers using the DEA model. In fact, defuzzification was developed to group of dependent fuzzy numbers. Significantly, the context of the proposed method with respect to dependent fuzzy numbers reveals the crisp point that maintains the relationships and properties among these groups of fuzzy numbers. The proposed method is unique because no other methods in previous literature defuzzify dependent fuzzy numbers. The example and case study confirm that the proposed method is applicable to both dependent and independent fuzzy numbers even without conditions and even if these numbers are unrelated.

To demonstrate the influence of the new approach on application, an allocation problem was presented. In this case study, the proposed method was utilized to estimate the optimal number of available beds in a hospital by categorizing patients according to age.

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