The Construction of Distinct Circuits of Length Six for Complete Graph K_6

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ABSTRACT

The emergent applications of complete graph in diverse domains have invited numerous works in this subject matter. Studies related to the decomposition of complete graph such as one-factor, several n-gons and Cartesian product have been solved. Yet, the decomposition of complete graph into distinct circuits still has not been done. Thus, this paper aims to investigate the structure of complete graph, and in particular, the decomposing of complete graph of length six. The decomposition algorithm will be presented to enumerate distinct circuits of length six. Along this process, the adjacency matrices will be used to clarify distinct structures of circuits in a complete graph of length six.

Keywords: Graph decomposition, complete graph, circuit

Mathematics Subject Classification: 05C20, 05C45

1. INTRODUCTION

We shall follow the standard notations and definitions of graph. A complete graph with n vertices (K_n) is a simple graph whose vertices are pairwise adjacent. A circuit with n vertices is a closed walk which starts and ends at the same vertex. We define a complete graph decomposition as partitioning the edge set of the complete graph into other graphs or subgraphs (Rao, 2006).

The complete graph decomposition into cycles (Alspach and Gavlas, 2001; Sajna, 2002), one-factors (Kaski and Ostergard, 2009), and Cartesian product (Fu *et al.*, 2004) have been carried out. However, to our knowledge, the decomposition of complete graph with n vertices into distinct circuits with n vertices has remained unsolved.

Thus, in this paper, we aim to decompose a complete graph K_6 into distinct circuits of length six. By the graph K_6 we mean a finite, connected and undirected graph with six vertices without loops or multiples edges.

2. PRELIMINARY DEFINITIONS AND RESULTS

In this section, we provide some definitions which will be used in our algorithm development.

Definition 1. The adjacency matrix for a graph with n vertices is a binary $n \times n$ matrix whose (i, j) entry is 1 if the i^{th} vertex and j^{th} vertex are connected, and 0 if they are not.

Definition 2. Let M is a $m \times n$ matrix, then a transpose matrix of M denoted by M^T is an $n \times m$ matrix.

Definition 3. Let C_1^* and C_2^* be two circuits of length n. If the direction of C_1^* is opposite to the direction of C_2^*, then C_1^* is the mirror image of C_2^*.

Definition 4. Let $\begin{pmatrix} 1 & 2 & \cdots & 6 \\ 1f & 2f & \cdots & 6f \end{pmatrix}$ denotes the transposition of vertices set $\{1,2,\ldots,6\}$ of K_6, which maps $1 \longmapsto 1f$, $2 \longmapsto 2f$, …, $6 \longmapsto 6f$, for $n \in \mathbb{Z}^+$ and $1f$, $2f$, …, $6f$ be the images.

Definition 5. An *initial set* is a set of vertices of K_n that is used to develop the distinct circuits.

Definition 6. A *first wing* shows the direction from vertex 1 to other chosen vertices in an increasing order.

Definition 7. A *second wing* shows the direction for the remaining vertices from first wing in an increasing order.

Definition 8. Suppose $x_i, x_{i+1}, \ldots, x_n$ are the vertices of K_n where $i = 1$. Then, the set of vertices $\left\{x_i, x_{i+1}, \ldots, x_{\frac{n}{2}}\right\} \cup \left\{x_{\frac{n+2}{2}}, x_{\frac{n+4}{2}}, \ldots, x_n\right\} = \left\{x_i, x_{i+1}, \ldots, x_{\frac{n}{2}}, x_{\frac{n+2}{2}}, x_{\frac{n+4}{2}}, \ldots, x_n\right\}$ such that $\left\{x_i, x_{i+1}, \ldots, x_{\frac{n}{2}}\right\} \in$ first wing, and $\left\{x_{\frac{n+2}{2}}, x_{\frac{n+4}{2}}, \ldots, x_n\right\} \in$ second wing.

Definition 9. The *first endpoints* include all vertices that follow the first wing.

Definition 10. The *second endpoints* include all vertices that follow the second wing.

Definition 11. An endpoint (x_i, x_j) is a set of ordered pairs of vertices x_i and x_j such that $x_i \in$ *first endpoint* and $x_j \in$ *second endpoint*, where $i, j \in \mathbb{Z}^+$.

Definition 12. The union of any endpoints $(x_i, x_j) \cup (x_m, x_n) \cup \ldots (x_p, x_q) = (x_i, x_j, x_m, x_n, \ldots, x_p, x_q)$ where $i, j, m, n, p, q \in \mathbb{Z}^+$.

Now, we would like to investigate the structure of K_6 and present K_6 decomposition algorithm of length six.

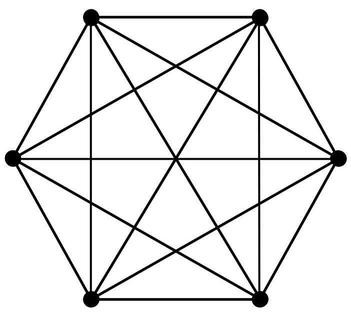


Figure 1. A complete graph K_6.

Suppose we have K_6 as shown in Figure 1. We can see several circuits of length six that can be decomposed from K_6 as presented in Figure 2. At this stage, one question may arise: how many distinct circuits of length six exist for K_6 decomposition? Thus, Section 3 will focus on algorithm development for K_6 decomposition into circuits of length six.

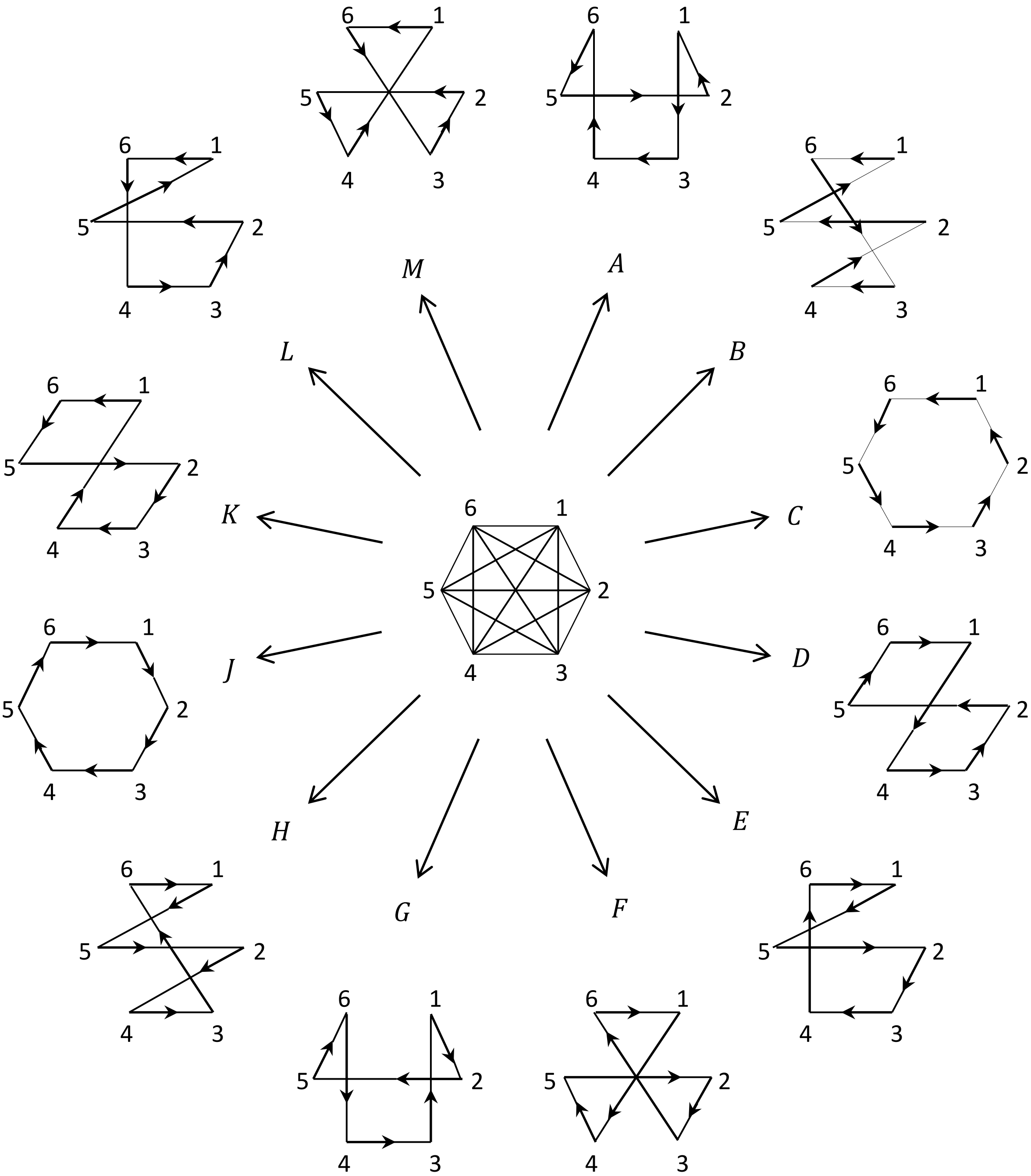


Figure 2. Several circuits from K_6

3. K_6 DECOMPOSITION INTO CIRCUITS

In this section, we provide the algorithm to decompose K_6 into distinct circuits of length six. The steps involved are discussed below.

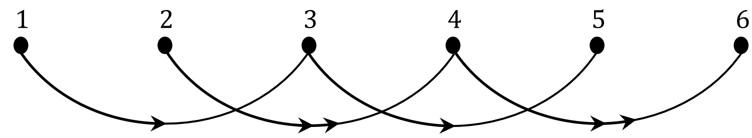
Step 1:

We first find the initial sets. For K_6, six initial sets are needed. Let $\{1,2,3,4,5,6\}$ be the set of vertices of K_6. We denote T_1 as the first initial set of K_6. We get the initial set by taking the first three vertices in the set which are 1, 2, and 3, for the first wing. Next, we take the remaining vertices which are 4, 5, and 6 for the second wing. The single arrow and the double arrow represent the first and second wing respectively.

T_1:

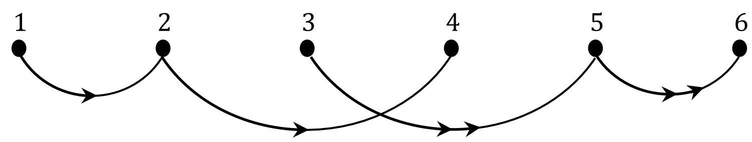
To generate T_2, we take the odd vertices in the set which are 1, 3, and 5 for the first wing. Next, we take the even vertices in the set which are 2, 4, and 6 for the second wing.

T_2:

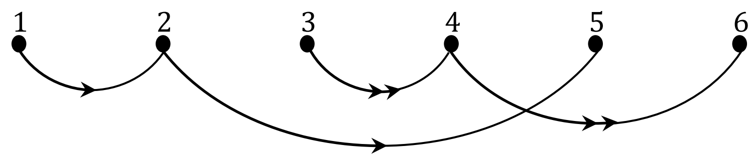


To generate the remaining initial sets T_3, T_4, T_5 and T_6, we take any arbitrary vertices for the first wing and the second wing. Both wings must be in increasing order which cannot turn back, and must have distinct vertices (both wings are not allowed to share the same vertices). The wing that contains vertex "1" will be the first wing. Thus, we obtain the remaining initial set as shown below.

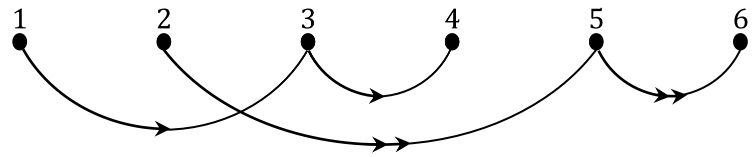
T_3:



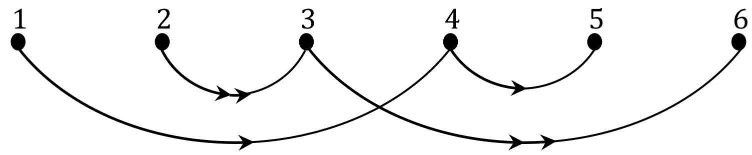
T_4:



T_5:



T_6:



We obtain the initial sets by following the 'Definition 8' as shown below. We use these initial sets to obtain the sixty distinct circuits as discussed in Step 2.

$$T_1 = \{1,2,3\} \cup \{4,5,6\} = \{1,2,3,4,5,6\}$$
$$T_2 = \{1,3,5\} \cup \{2,4,6\} = \{1,3,5,2,4,6\}$$
$$T_3 = \{1,2,4\} \cup \{3,5,6\} = \{1,2,4,3,5,6\}$$
$$T_4 = \{1,2,5\} \cup \{3,4,6\} = \{1,2,5,3,4,6\}$$
$$T_5 = \{1,3,4\} \cup \{2,5,6\} = \{1,3,4,2,5,6\}$$
$$T_6 = \{1,4,5\} \cup \{2,3,6\} = \{1,4,5,2,3,6\}$$

Step 2:

We start with the first initial set, $T_1 = \{1,2,3,4,5,6\}$. In this step, we put all vertices from first wing into first endpoints consecutively. Likewise, we put all vertices consecutively from second wing into second endpoints to obtain C_1^*. We call this step as Endpoint Strategy or Endp-S.

We describe in more detail here. C_1^* is obtained in this way: the vertices from first wing which are 1, 2, and 3 are put into first endpoints like this: $(1\ \ )(2\ \ )(3\ \ )$. Next, the vertices from second wing which are 4, 5, and 6 are filled into second endpoints like this: $(\ \ 4)(\ \ 5)(\ \ 6)$.
Thus, we obtain $C_1^* = (1,4)(2,5)(3,6)$.

Step 3:

In this step, we fix all the first endpoints, and shift all the second endpoints to the left. We call this step as Fix-And-Shift. When we say fix-and-shift C_1^* to obtain C_2^* and C_3^*, it means that we fix the first endpoints of C_1^*, and shift the second endpoints of C_1^* to the left. Therefore, C_2^* and C_3^* are obtained.

We describe in more detail here. We fix the first endpoints like this:

$$C_1^* = (1\quad)(2\quad)(3\quad)$$
$$C_2^* = (1\quad)(2\quad)(3\quad)$$
$$C_3^* = (1\quad)(2\quad)(3\quad)$$

Next, we shift the second endpoints of C_1^* to get the second endpoints of C_2^* and C_3^* as follows:

$$C_1^* = (\quad 4)(\quad 5)(\quad 6)$$
$$C_2^* = (\quad 5)(\quad 6)(\quad 4)$$
$$C_3^* = (\quad 6)(\quad 4)(\quad 5)$$

Thus, we obtain C_1^*, C_2^*, and C_3^* as follows:

$$C_1^* = (1{,}4)(2{,}5)(3{,}6)$$
$$C_2^* = (1{,}5)(2{,}6)(3{,}4)$$
$$C_3^* = (1{,}6)(2{,}4)(3{,}5)$$

Step 4:

In this step, we fix the first endpoints, and reverse the order of second endpoints to the left. We called this step as Fix-And-Reverse. When we say fix-and-reverse C_1^* to obtain C_4^*, it means that we fix the first endpoints of C_1^*, and reverse the order of second endpoints of C_1^*, to obtain C_4^*.

We explain further here. C_4^* is obtained by fixing the first endpoint of C_1^* like this:

$$C_4^* = (1\quad)(2\quad)(3\quad)$$

Then, the order of second endpoints of C_1^* is reversed to get this:

$$C_4^* = (\quad 6)(\quad 5)(\quad 4)$$

Thus, we obtain $C_4^* = (1\ 6)(2\ 5)(3\ 4)$.

Step 5:

In this stage, we do again the fix-and-shift step. But now we fix-and-shift C_4^* to obtain C_5^* and C_6^*.

$$C_4^* = (1\ 6)(2\ 5)(3\ 4)$$
$$C_5^* = (1\ 5)(2\ 4)(3\ 6)$$
$$C_6^* = (1\ 4)(2\ 6)(3\ 5)$$

At this stage, we have achieved complete rotation for C_1^* to obtain C_2^*, C_3^*, C_4^*, C_5^*, and C_6^* as listed above. These steps from Step 1 until Step 4 are important and they are the basis in developing the C_k^*. The development of the remaining circuits will use these four steps repetitively.

Step 6:

In this step, we still consider the initial set $T_1 = \{1,2,3,4,5,6\}$. We apply Endp-S for T_1 but now we start with the second element, 2, to produce C_7^*.

We explain detail in here. "Starts with the second element, 2," means that we apply Endp-S for T_1, but we start at the second element, "2". The elements 2, 3, and 4 are filled into first endpoints like this: $(2\ \ )(3\ \ )(4\ \ )$. Next, the elements 5, 6, and 1 are filled into second endpoints like this: $(\ \ 5)(\ \ 6)(\ \ 1)$. Thus, we obtain C_7^* as written below. Next, we fix-and-shift C_7^* to obtain C_8^* and C_9^*.

$$C_7^* = (2\ 5)(3\ 6)(4\ 1)$$
$$C_8^* = (2\ 6)(3\ 1)(4\ 5)$$
$$C_9^* = (2\ 1)(3\ 5)(4\ 6)$$

After that, we fix-and-reverse C_7^* to obtain C_{10}^*. Then, we fix-and-shift C_{10}^* to obtain C_{11}^* and C_{12}^*.

$$C_{10}^* = (2\ 1)(3\ 6)(4\ 5)$$
$$C_{11}^* = (2\ 6)(3\ 5)(4\ 1)$$
$$C_{12}^* = (2\ 5)(3\ 1)(4\ 6)$$

Step 7:

In this step, we apply Endp-S for T_1 but now we start with the third element, "3", to produce C_{13}^*. We discussed further here. "Starts with the third element, 3," means that we apply Endp-S for T_1, but we start at third element, "3". The elements 3,, 4, and 5, are filled into first endpoints like this: $(3\ \ )(4\ \ )(5\ \ )$. Next, the elements 6, 1,, and 2 are filled into second endpoints like this: $(\ \ 6)(\ \ 1)(\ \ 2)$. Thus, we obtain C_{13}^* as listed below. After that, we fix-and-shift C_{13}^* to obtain C_{14}^* and C_{15}^*.

$$C_{13}^* = (3\ 6)(4\ 1)(5\ 2)$$
$$C_{14}^* = (3\ 1)(4\ 2)(5\ 6)$$
$$C_{15}^* = (3\ 2)(4\ 6)(5\ 1)$$

Next, we fix-and-reverse C_{13}^* to obtain C_{16}^*. Then, we fix-and-shift C_{16}^* to obtain C_{17}^* and C_{18}^*.

$$C_{16}^* = (3\ 2)(4\ 1)(5\ 6)$$
$$C_{17}^* = (3\ 1)(4\ 6)(5\ 2)$$
$$C_{18}^* = (3\ 6)(4\ 2)(5\ 1)$$

For T_1, we stop until third element. In the following step, we apply the same procedure as applied for T_1 to enumerate the remaining circuits. We can conclude that, if steps from 1 to 6 are well understood, then the following steps are easy to follow.

At this stage, we obtain eighteen results for K_6 decomposition. The following section presents the algorithm to develop the remaining results.

3.1 Developing the Remaining Circuits

This section provided the algorithm to develop the remaining circuits. We follow the similar steps as discussed in the previous section.

Now, we consider the initial set $T_2 = \{1,3,5,2,4,6\}$. We apply Endp-S for T_2 and we start with the first element which is "1". Thus, elements 1, 3, and 5 are put into first endpoints, and elements 2, 4, and 6 are put into second endpoints, to obtain C^*_{19} as written below. Then, we fix-and-shift C^*_{19} to obtain C^*_{20} and C^*_{21}; fix-and-reverse C^*_{19} to obtain C^*_{22}; and fix-and-shift C^*_{22} to obtain C^*_{23} and C^*_{24}.

$$C^*_{19} = (1\ 2)(3\ 4)(5\ 6)$$
$$C^*_{20} = (1\ 4)(3\ 6)(5\ 2)$$
$$C^*_{21} = (1\ 6)(3\ 2)(5\ 4)$$
$$C^*_{22} = (1\ 6)(3\ 4)(5\ 2)$$
$$C^*_{23} = (1\ 4)(3\ 2)(5\ 6)$$
$$C^*_{24} = (1\ 2)(3\ 6)(5\ 4)$$

Next, we apply Endp-S at the second element in T_2 which is "3". The elements 3, 5, and 2 are put into first endpoints, and elements 4, 6, and 1 are put into second endpoints to obtain C^*_{25} as listed below. Then, we fix-and-shift C^*_{25} to obtain C^*_{26} and C^*_{27}, fix-and-reverse C^*_{25} to obtain C^*_{28}, and fix-and-shift C^*_{28} to obtain C^*_{29} and C^*_{30}.

$$C^*_{25} = (3\ 4)(5\ 6)(2\ 1)$$
$$C^*_{26} = (3\ 6)(5\ 1)(2\ 4)$$
$$C^*_{27} = (3\ 1)(5\ 4)(2\ 6)$$
$$C^*_{28} = (3\ 1)(5\ 6)(2\ 4)$$
$$C^*_{29} = (3\ 6)(5\ 4)(2\ 1)$$
$$C^*_{30} = (3\ 4)(5\ 1)(2\ 6)$$

We carry out the same procedure for the third element in T_2 which is "5". The elements 5, 2, and 4 are put into first endpoints, and elements 6, 1, and 3 are put into second endpoints to obtain C^*_{31}. Next, we fix-and-shift C^*_{31} to obtain C^*_{32} and C^*_{33}, fix-and-reverse C^*_{31} to obtain C^*_{34}, and fix-and-shift C^*_{34} to obtain C^*_{35} and C^*_{36}.

$$C^*_{31} = (5\ 6)(2\ 1)(4\ 3)$$
$$C^*_{32} = (5\ 1)(2\ 3)(4\ 6)$$
$$C^*_{33} = (5\ 3)(2\ 6)(4\ 1)$$
$$C^*_{34} = (5\ 3)(2\ 1)(4\ 6)$$
$$C^*_{35} = (5\ 1)(2\ 6)(4\ 3)$$
$$C^*_{36} = (5\ 6)(2\ 3)(4\ 1)$$

For T_2, we stop until the third element as T_1. In the following steps, we apply the same procedure as T_1 and T_2 to design the remaining circuits. We use the initial sets T_3, T_4, T_5, and T_6, but we just apply steps starting from Step 1 until Step 4.

We consider $T_3 = \{1,2,4,3,5,6\}$. Using Endp-S, we put elements 1, 2, and 4 into first endpoints and elements 3, 5, and 6 into second endpoints to obtain C_{37}^*. After that, we fix-and-shift C_{37}^* to obtain C_{38}^* and C_{39}^*, fix-and-reverse C_{37}^* to obtain C_{40}^*, and fix-and-shift C_{40}^* to obtain C_{41}^* and C_{42}^*.

$$C_{37}^* = (1\ 3)(2\ 5)(4\ 6)$$
$$C_{38}^* = (1\ 5)(2\ 6)(4\ 3)$$
$$C_{39}^* = (1\ 6)(2\ 3)(4\ 5)$$
$$C_{40}^* = (1\ 6)(2\ 5)(4\ 3)$$
$$C_{41}^* = (1\ 5)(2\ 3)(4\ 6)$$
$$C_{42}^* = (1\ 3)(2\ 6)(4\ 5)$$

Next, we consider $T_4 = \{1,2,5,3,4,6\}$. For Endp-S, we put elements 1, 2, and 5 into first endpoints, and elements 3, 4, and 6 into second endpoints to obtain C_{43}^*. Next, we fix-and-shift C_{43}^* to obtain C_{44}^* and C_{45}^*, fix-and-reverse C_{43}^* to obtain C_{46}^*, and fix-and-reverse C_{46}^* to obtain C_{47}^* and C_{48}^*.

$$C_{43}^* = (1\ 3)(2\ 4)(5\ 6)$$
$$C_{44}^* = (1\ 4)(2\ 6)(5\ 3)$$
$$C_{45}^* = (1\ 6)(2\ 3)(5\ 4)$$
$$C_{46}^* = (1\ 6)(2\ 4)(5\ 3)$$
$$C_{47}^* = (1\ 4)(2\ 3)(5\ 6)$$
$$C_{48}^* = (1\ 3)(2\ 6)(5\ 4)$$

Furthermore, we consider $T_5 = \{1,3,4,2,5,6\}$. Using Endp-S, we put elements 1, 3, and 4 into first endpoints and elements 2, 5, and 6 into second endpoints to obtain C_{49}^*. Then, we fix-and-shift C_{49}^* to obtain C_{50}^* and C_{51}^*, fix-and-reverse C_{49}^* to obtain C_{52}^*, and fix-and-shift C_{52}^* to obtain C_{53}^* and C_{54}^*.

$$C_{49}^* = (1\ 2)(3\ 5)(4\ 6)$$
$$C_{50}^* = (1\ 5)(3\ 6)(4\ 2)$$
$$C_{51}^* = (1\ 6)(3\ 2)(4\ 5)$$
$$C_{52}^* = (1\ 6)(3\ 5)(4\ 2)$$
$$C_{53}^* = (1\ 5)(3\ 2)(4\ 6)$$
$$C_{54}^* = (1\ 2)(3\ 6)(4\ 5)$$

Finally, we reach at the final level in developing all the sixty distinct circuits for K_6. We consider $T_6 = \{1,4,5,2,3,6\}$ to apply Endp-S. The elements 1, 4, and 5 are put into first endpoints and elements 2, 3, and 6 are put into second endpoints to obtain C_{55}^*. After that, we fix-and-shift C_{55}^* to obtain C_{56}^* and C_{57}^*, fix-and-reverse C_{55}^* to obtain C_{58}^*, and fix-and-shift C_{58}^* to obtain C_{59}^* and C_{60}^*.

$$C_{55}^* = (1\ 2)(4\ 3)(5\ 6)$$
$$C_{56}^* = (1\ 3)(4\ 6)(5\ 2)$$

$$C_{57}^{*} = (1\ 6)(4\ 2)(5\ 3)$$
$$C_{58}^{*} = (1\ 6)(4\ 3)(5\ 2)$$
$$C_{59}^{*} = (1\ 3)(4\ 2)(5\ 6)$$
$$C_{60}^{*} = (1\ 2)(4\ 6)(5\ 3)$$

It shows that the above sixty C_k^*, $1 \le k \le 60$, are not the complete circuits since they are in endpoint forms.

Step 8:

In this step, we follow 'Definition 12' to get the results as shown below.

$$C_1^* = (1\ 4) \cup (2\ 5) \cup (3\ 6) = (1\ 4\ 2\ 5\ 3\ 6)$$
$$C_2^* = (1\ 5) \cup (2\ 6) \cup (3\ 4) = (1\ 5\ 2\ 6\ 3\ 4)$$

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$$C_{60}^* = (1\ 2) \cup (4\ 6) \cup (5\ 3) = (1\ 2\ 4\ 6\ 5\ 3)$$

3.2 Drawing the Circuits

In this section, we provide steps to draw the circuits based on the results in Step 8 in Section 3.1.

Step 1:

We follow 'Definition 6' to find the mapping as shown below.

From Definition 6, $C_1^* = (1\ 4\ 2\ 5\ 3\ 6)$ have $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 4 & 5 & 6 & 2 & 3 & 1 \end{pmatrix}$ which maps $1 \longmapsto 4$, $4 \longmapsto 2$, $2 \longmapsto 5$, $5 \longmapsto 3$, $3 \longmapsto 6$, and $6 \longmapsto 1$. Likewise, $C_2^* = (1\ 5\ 2\ 6\ 3\ 4)$ have $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 5 & 6 & 4 & 1 & 2 & 3 \end{pmatrix}$ that maps $1 \longmapsto 5$, $5 \longmapsto 2$, $2 \longmapsto 6$, $6 \longmapsto 3$, $3 \longmapsto 4$, and $4 \longmapsto 1$. Similarly, $C_{60}^* = (1\ 2\ 4\ 6\ 5\ 3)$ have $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 2 & 4 & 1 & 6 & 3 & 5 \end{pmatrix}$ which maps $1 \longmapsto 2$, $2 \longmapsto 4$, $4 \longmapsto 6$, $6 \longmapsto 5$, $5 \longmapsto 3$, and $3 \longmapsto 1$.

Then, we have the mapping as follows:

$$C_1^* = (1\ 4)(4\ 2)(2\ 5)(5\ 3)(3\ 6)(6\ 1)$$
$$C_2^* = (1\ 5)(5\ 2)(2\ 6)(6\ 3)(3\ 4)(4\ 1)$$

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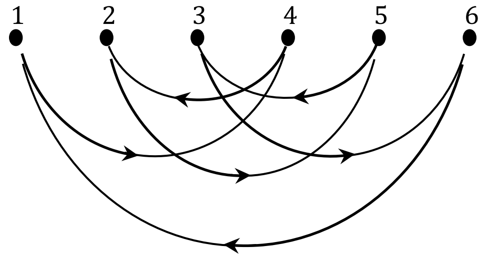
$$C_{60}^* = (1\ 2)(2\ 4)(4\ 6)(6\ 5)(5\ 3)(3\ 1)$$

We use this mapping in the following step to create the directions for each circuit.

Step 2:

We use the mapping $C_1^* = (1\ 4)(4\ 2)(2\ 5)(5\ 3)(3\ 6)(6\ 1)$ to get the direction as follows:

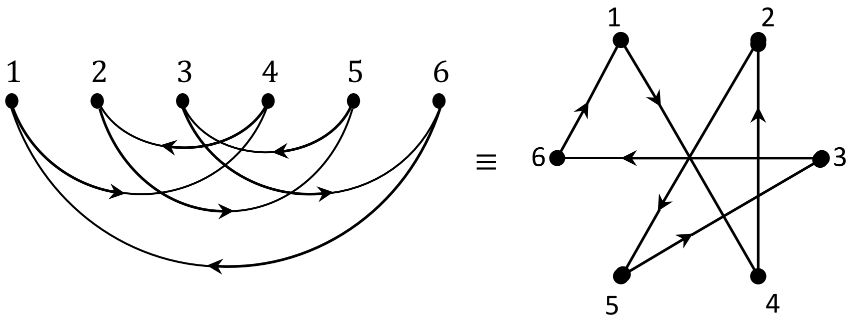


The remaining directions can be obtained by following the same steps as shown above.

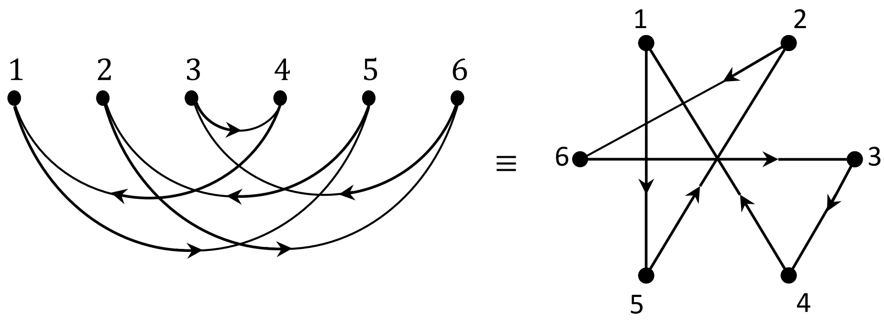
Step 3:

In this step, we draw the circuits using the same directions as discussed in Step 2.

C_1^*



C_2^*



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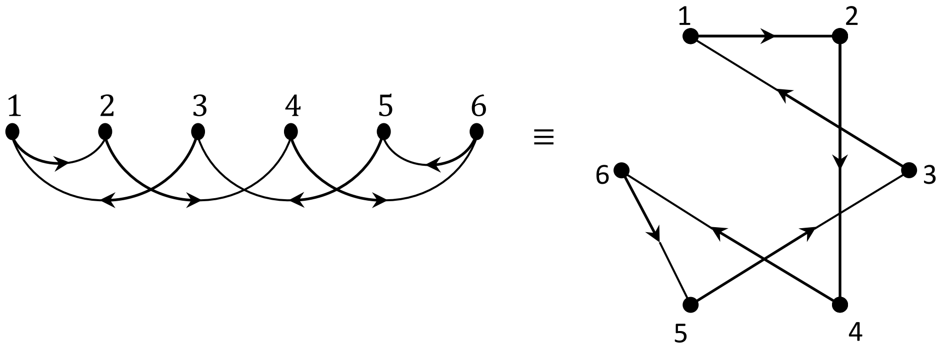
$C_{60}^* =$



By following the same argument for all the circuits $(C_1^*,\ C_2^*, \ldots,\ C_{60}^*)$, we can visualize all the circuits. This algorithm has led us to the following theorems:

Theorem 1: K_6 has 120 distinct circuits of order six.

Proof. K_6 has six vertices and one vertex is fixed to be the starter point of each circuit. Then K_6 can be decomposed into $(6-1)! = 120$ circuits.

Theorem 2: If circuit A has an opposite direction of circuit B, then A is the mirror image of B or vice versa such that A and B are the circuits of order six.

Proof. We consider the adjacency matrices and its transpose of circuits A, B, G and H in Figure 2 to investigate the mirror image as shown below.

Table 1: Adjacency Matrices and the Transpose of K_6

Adjacency Matrices

$A =$

	1	2	3	4	5	6
1	0	0	1	0	0	0
2	1	0	0	0	0	0
3	0	0	0	1	0	0
4	0	0	0	0	0	1
5	0	1	0	0	0	0
6	0	0	0	0	1	0

Transpose

$A^T =$

	1	2	3	4	5	6
1	0	1	0	0	0	0
2	0	0	0	0	1	0
3	1	0	0	0	0	0
4	0	0	1	0	0	0
5	0	0	0	0	0	1
6	0	0	0	1	0	0

$B =$

	1	2	3	4	5	6
1	0	0	0	0	0	1
2	0	0	0	0	1	0
3	0	0	0	1	0	0
4	0	1	0	0	0	0
5	1	0	0	0	0	0
6	0	0	1	0	0	0

$B^T =$

	1	2	3	4	5	6
1	0	0	0	0	1	0
2	0	0	0	1	0	0
3	0	0	0	0	0	1
4	0	0	1	0	0	0
5	0	1	0	0	0	0
6	1	0	0	0	0	0

$G =$

	1	2	3	4	5	6
1	0	1	0	0	0	0
2	0	0	0	0	1	0
3	1	0	0	0	0	0
4	0	0	1	0	0	0
5	0	0	0	0	0	1
6	0	0	0	1	0	0

$G^T =$

	1	2	3	4	5	6
1	0	0	1	0	0	0
2	1	0	0	0	0	0
3	0	0	0	1	0	0
4	0	0	0	0	0	1
5	0	1	0	0	0	0
6	0	0	0	0	1	0

$H =$

	1	2	3	4	5	6
1	0	0	0	0	1	0
2	0	0	0	1	0	0
3	0	0	0	0	0	1
4	0	0	1	0	0	0
5	0	1	0	0	0	0
6	1	0	0	0	0	0

$H^T =$

	1	2	3	4	5	6
1	0	0	0	0	0	1
2	0	0	0	0	1	0
3	0	0	0	1	0	0
4	0	1	0	0	0	0
5	1	0	0	0	0	0
6	0	0	1	0	0	0

At this stage, we have matrices $A \equiv G^T$, $B \equiv H^T$, $G \equiv A^T$, and $H \equiv B^T$. We conclude that circuit A is the mirror image of G, circuit B is the mirror image of H, and vice versa since they have similar structure. The remaining circuits in Figure 2 have the same impact.

Theorem 3: There exist sixty distinct circuits with different structure of order six from K_6.

Proof. By considering the mirror image, from Theorem 1, we have $(6-1)!/2 = 60$ distinct circuits with different structures of order six from K_6.

4. DISCUSSION AND CONCLUSION

We prove that K_6 has 120 circuits of length six. Hence, we prove that each circuits have their mirror image. We also prove that K_6 can be decomposed into sixty distinct circuits of length six. We hope that this proposed algorithm can be a basis to design K_n decomposition into distinct circuits of length n.

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