Linear Active Control Algorithm to Synchronize a Nonlinear HIV/AIDS Dynamical System

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ABSTRACT

Chaos synchronization between two chaotic systems happens when the trajectory of one of the system asymptotically follows the trajectory of another system due to forcing or due to coupling. This research paper addresses the synchronization problem of an In-host Model for HIV/AIDS dynamics using the Linear Active Control Technique. In this study, using the Linear Active Control Algorithm based on the Lyapunov stability theory, the synchronization between two identical HIV/AIDS chaotic systems and the switching synchronization between two different HIV/AIDS and Qi 4-D chaotic systems has been observed. Further, it has been shown that the proposed schemes have excellent transient performance and analytically as well as graphically found that the synchronization is globally exponential stable. Numerical simulations are carried out to demonstrate the efficiency of the proposed approach that support the analytical results and illustrated the possible scenarios for synchronization. All simulations have been done using *Mathematica* 9.

Key words: Synchronization, Linear Active Control, Lyapunov Stability Theory, HIV Model

INTRODUCTION

Today, mathematical theory of chaos is a fundamental base of natural sciences [1]. It proves that the complexity of the behavior of a chaotic system stems from the exponentially unstable dynamics rather than from the fluctuations or large degree of freedom. Mathematically, a chaotic system is a nonlinear deterministic system that plays unpredictable and exceedingly complex behavior [1]. After the pioneering work of Edwards Lorenz on Chaos [2], there has been tremendous interest worldwide in the possibility of using chaos in secure communication, physical and other basic sciences [3-5]. Chaos theory plays a vital role in the study of biological Sciences [6-7] and various chaotic dynamical models for human immunodeficiency virus (HIV) have been described and extensively studied in the literature in a large extent to understand the HIV dynamics, the mechanism and spread of disease, to predict its future conduct and some conclusion for better treatment and drug therapies [8-10]. These models provide a quantitative

understanding of the level of virus production during the long asymptotic stage of HIV infection [9]. HIV infects different body cells, but mainly target the CD4+T lymphocytes, which are the most copious white blood cells of the immune system. HIV inflects the most damage on the CD4+ T cells by causing their decline and destruction, decreasing the resistance of the immune system [10] and this causes a certain deaths.
Many mathematical models of HIV have been proposed and analyze to study the dynamics of HIV/AIDS. The utilization of mathematical models support in understanding the features of HIV/AIDS and virus infection dynamics have been substantial in the past two decades. The effect of saturation and delays have also been discussed [9, 10].
The chaos control and synchronization have been an attractive field of research in the last two decades [11]. Research efforts have explored the chaos control and synchronization and have been one of the critical issues in nonlinear sciences due to its potential applications in different fields including chemical, physical, biological and many engineering systems [12, 13]. In this line, a wide range of different effective control techniques and strategies have been proposed and applied successfully to achieve chaos control and synchronization of chaotic systems [14]. Noteworthy among those, chaos synchronization using linear active control techniques has recently been accepted and considered as one of the most efficient techniques for synchronizing both identical as well as nonidentical chaotic systems because of its implementation to practical systems such as, Bose-Einstein Condensate, Nonlinear Gyros, Ellipsoidal Satellite and Bonhoffer-van der Pol Oscillators [14] etc.
Chaos synchronization using active control technique was proposed by Bai and Longren using the Lorenz system and thereafter has been utilized to synchronize other chaotic and hyperchaotic systems [15]. If the nonlinearity of the system is known, linear active control techniques can be easily designed according to the given conditions of the chaotic system to achieve chaos control and synchronization globally. There are no derivatives in the controller or the Lyapunov exponents are not required for their execution and these characteristics gives an edge to the Linear Active Control Techniques on other conventional control approaches [16]. HIV/AIDS in its various forms have attracted the attention of mathematicians for the last two decades. At this stage, it is now significant to synchronize the HIV/AIDS chaotic system [17] for further research purposes in order to reduce the causes of mortality due to HIV in the future.
Motivated by the above, the main goal of this paper is to employ the Linear Active Control Technique [14] to study and examine the chaos synchronization problem of HIV/AIDS chaotic system [17] and to extend the applications of synchronization in treatment of HIV/AIDS related epidemics on theoretical ground.
Based on the Lyapunov Stability Theory [18] and using the Linear Active Control Technique, a class of feedback control strategies will be designed to achieve the synchronization globally. The controllers will be designed in a way that the nonlinearity of the system should not be neglected, and the error signals converges to the equilibrium point (origin) asymptotically global with less control effort and enough synchronization speed. Numerical simulations and graphs will be furnished to show the efficiency and the performance of the proposed approach.
The remainder of the paper is organized as follows: In unit 2, the Linear Active Control Methodology is given. In unit 3, description of the HIV/AIDS model is given, and chaos

synchronization of identical and nonidentical HIV/AIDS chaotic systems will be examined using the Linear Active Control Technique. In unit 4, numerical simulations will be provided to show the effectiveness of the proposed methods and finally the concluding remarks are then given in unit 5

DESIGN OF A LINEAR ACTIVE CONTROLLER

A particular chaotic system is called the drive or master system and the second system is called response or slave system. Many of the synchronization approaches belong to drive-response (master-slave) system configurations in which the two chaotic oscillators are coupled in such a way that the performance of the second (response/slave) system is controlled by the first (drive/master) system and the first system is not affected by the exertion of the second system. Consider a master system described by the following differential equation:

$$\dot{x} = Px + f(x) \tag{2.1}$$

and the slave system is described as:

$$\dot{y} = Qy + g(y) + \eta(t) \tag{2.2}$$

Where **x, y** $\in R^n$ are the state vectors, $f(x), g(y): R^n \to R^n$ are the nonlinear continuous sequential functions and $P, Q \in R^{n\times n}$ are constant system matrices of the corresponding master and slave systems respectively and $\eta(t) \in \mathrm{R}^{n\times 1}$ as a control input injected into the slave system. The synchronization error of the systems (2.1) and (2.2) is described as:

$$\begin{aligned} \dot{e} &= \dot{y} - \dot{x} \\ \Rightarrow \dot{e} &= Qy + g(y) - Px - f(x) + \eta(t) \\ \Rightarrow \quad \dot{e} &= Ae + F(x, y, e) + \eta(t) \end{aligned} \tag{2.3}$$

Where, $\dot{e}_i = y_i - x_i$, $\mathrm{i} = 1, 2, ..., n$, $A = \bar{Q} - \bar{P}$ is the common part of the system matrices in the master and slave systems and $F(x, y) = g(y) - f(x) + Qy - Px$ that contains the nonlinear functions and non-common terms and $\eta(t) = [\eta_1(t), \eta_2(t), ..., \eta_n(t)]^T \in R^{n\times 1}$ as the Linear Active Control input.

If $f(\square) = g(\square)$ *and / or* $P = Q$ then **x** and **y** are the states of the two unified chaotic systems and if, $f(\square) \neq g(\square)$ *and / or* $P \neq Q$, then **x** and **y** are the states of two nonidentical chaotic systems.

An appropriate active feedback controller ' $\eta(t)$ ' that satisfies the error system converges to the equilibrium point (zero), *i.e,*

$$\lim_{t\to\infty} e_i = \lim_{t\to\infty} \left| y_i(t) - x_i(t) \right| = 0, \ \forall\, x, y, e \in R^n.$$

Then the two chaotic systems (2.1) and (2.2) are said to be synchronized [14].

Thus the main issue to synchronize two identical/nonidentical chaotic systems is to design an appropriate linear active feedback controller that is injected into the slave system to follow the master system asymptotically in course of time. The active controller should be designed in a way that it vanishes the nonlinear terms and non-common parts and to sustain other linear part to attain asymptotically global stability [14].To achieve asymptotically global synchronization using Active Control Technique, let us assumes the following theorem.

Theorem 1. The trajectories of the two (identical or nonidentical) chaotic systems (2.1) and (2.2) for all initial conditions, $\left(x_{1m}(0), x_{2m}(0), \ldots., x_{nm}(0) \neq y_{1s}(0), y_{2s}(0), \ldots., y_{ns}(0)\right)$ will be asymptotically global synchronized with a suitable active controller ' $\eta(t)$ ' as:

$$\eta(t) = [\eta_1(t), \eta_2(t), \ldots., \eta_n(t)]^T \in R^{n\times 1}.$$

Proof: let us assume that the states of both systems (2.1) and (2.1) are measurable and parameters of the master and slave systems are known. A proper refinement of the Linear Active Controller locates the unstable eigenvalue(s) to a stable position. The control signal $\eta(t) \in \mathrm{R}^{n\times 1}$ is constructed in two parts. The first part eradicates the nonlinear terms from (2.3) and the second part $v(t)$ acts as an external impute to stabilize the error dynamics (2.3), *i.e,* $\eta(t) = -G(x, y) + v(t)$

Where $v(t) = -Be = -B(\mathrm{y}_i - x_i)$ is a linear feedback controller and $B \in R^{n\times n}$ as the feedback control matrix [14] that determines the strength of the feedback into the slave system. Thus the error dynamics (2.3) becomes:

$$\dot{e} = Ae + v(t) = Ae - Be = e(A - B) = Ce \qquad (2.4)$$

Where, $C = A - B$.

From equation (2.4), if the error system (2.4) is a linear system of the form, $\dot{e} = Ce$ and if the system matrix C is Hurwitz [18], i.e., the real parts of all eigenvalues of the system symmetric matrix C are negative, then by the Linear Control Theory [19] the error dynamics will be asymptotically stable.

To achieve globally exponential stability of the errors system (2.4), let us if we select a Lyapunov errors function candidate as:

$$V(t) = e^T Me$$

Where, $M = dig(m_1, m_2, \ldots m_n) \in \mathrm{R}^{n\times n}$ is a positive definite matrix and $V : R^n \to R^n$ is a positive definite function by construction [3].

If an active feedback controller $\eta(t) = [\eta_1(t), \eta_2(t), \ldots., \eta_n(t)]^T \in R^{n\times 1}$ is designed such that:

$$\dot{V}(e) = -e^T Ne,$$

Then, $\dot{V} : R^n \to R^n$ is a negative definite function [16] with N as a positive definite matrix, then the two systems (2.1) and (2.2) are globally exponential synchronized by the Lyapunov Stability Theory [18]..

DESCRIPTION OF AN HIV/AIDS CHAOTIC SYSTEM

HIV is a lentivirus that causes acquired immunodeficiency syndrome (AIDS). AIDS is an advanced phase of HIV infection. Recently, P. Das, et.al. [17] presented and studied an In-host chaotic model for HIV/AIDS dynamics with saturation effect and discrete time delays. In [17], the switching phenomena for the stable equilibria is observed when a discrete time delays is incorporated. Further, it has been analyzed the stability analysis with and without time delays and discussed the effect of various parameters that may control the disease transmission.

. The differential equations for the HIV/AIDS model [17] is given as:

$$\left.\begin{aligned} \dot{x} &= az - bx \\ \dot{y} &= \frac{c}{\kappa + x} - dy - \alpha xy \\ \dot{z} &= q_1 \alpha xy - s_1 z + \beta w(t-\tau) \\ \dot{w} &= q_2 \alpha xy - s_2 w - \beta w \end{aligned}\right\} \qquad (3.1.1)$$

Where $x, y, z, w \in R^{n\times n}$ are the state variables which represent the number of virions (virus particles), number of uninfected targets cells, number of productive infected cells and number of latent infected cells respectively at any time in an host cells [17]. $a, b, c, d, \kappa, \alpha, \beta, q_1, q_2, s_1$ *and* s_2 as the system parameters. The virus is reproduced by the infected cells at a rate of 'a' that is assumed to be proportional to number of latent infected cells 'z'. The uninfected cells are produced by the host cells at a particular rate of $\frac{c}{\kappa + x}$ which depends on the number of virions in the host cells. It is assumed that not all newborn cells are uninfected. These uninfected cells die at a rate of 'd' and become infected by the virus at a specific rate 'αx' (αx is the functional response of the viruses in the uninfected cells) entering 'z' class and 'w' class respectively in proportion. A proportion 'q_1' of the infected cells become productively infected while the remaining proportion, 'q_2' become latently infected where, $q_2 = (1 - q_1)$. Productive infected cells and latent infected cells die at particular rates '$s_1 = r_1 + d$' and $s_2 = r_2 + d$ respectively, where 'd' is a natural deaths rate, r_1 and r_2 are the additional death rates due to infection. Only the 'z' cells produce virions, and 'w' cells move to the 'z' class at a per capita rate β. Moreover, τ (0 < τ < ∞) is the delay due to the formation of productive infected class from the latent infected class. The parameter c is a constant and 'κ' is the half saturation constant.

IDENTICAL SYNCHRONIZATION OF AN HIV/AIDS CHAOTIC SYSTEM

To study and analyzed the identical synchronization problem for the HIV/AIDS chaotic system [17] using linear active control technique, the master-slave system configuration is described as:

$$\left.\begin{aligned}\dot{x}_1 &= az_1 - bx_1\\ \dot{y}_1 &= \frac{c}{\kappa + x_1} - dy_1 - \alpha x_1 y_1\\ \dot{z}_1 &= q_1\alpha x_1 y_1 - s_1 z_1 + \beta w_1(t-\tau)\\ \dot{w}_1 &= q_2\alpha x_1 y_1 - s_2 w_1 - \beta w_1\end{aligned}\right\} \quad \text{(Master system)}$$

(3.2.1)

and

$$\left.\begin{aligned}\dot{x}_2 &= az_2 - bx_2 + \eta_1\\ \dot{y}_2 &= \frac{c}{\kappa + x_2} - dy_2 - \alpha x_2 y_2 + \eta_2\\ \dot{z}_2 &= q_1\alpha x_2 y_2 - s_1 z_2 + \beta w_2(t-\tau) + \eta_3\\ \dot{w}_2 &= q_2\alpha x_2 y_2 - s_2 w_2 - \beta w_2 + \eta_4\end{aligned}\right\} \quad \text{(Slave system)}$$

(3.2.2)

Where $x_1, y_1, z_1, w_1 \in R^{n\times n}$ and $x_2, y_2, z_2, w_2 \in R^{n\times n}$ are the corresponding state vectors of master and slave systems respectively, $a, b, c, d, \kappa, \alpha, \beta, q_1, q_2, s_1 \text{ and } s_2$ are the parameters of the master and slave systems and $\eta(t) = [\eta_1(t), \eta_2(t), \eta_3(t), \eta_4(t)]^T \in R^{n\times 1}$ is the Linear Active Controller that is yet to be designed. The HIV/AIDS system describes chaotic behavior with the parameters:
$a = 5, b = 1, c = 10, d = 1, \kappa = 1, \alpha = 200, \beta = 2, q_1 = 0.3, q_2 = 0.7, \tau = 18, s_1 = 1 \text{and } s_2 = 0.1$.
From system of equations (3.2.1) and (3.2.2), the errors dynamics can be described as:

$$\left.\begin{aligned}\dot{e}_1 &= -be_1 + ae_3 + \eta_1\\ \dot{e}_2 &= -de_2 - \alpha(x_2 y_2 - x_1 y_1) + \frac{c}{\kappa + x_2} - \frac{c}{\kappa + x_1} + \eta_2\\ \dot{e}_3 &= -s_1 e_3 - bz_1 + s_1 z_1 + q_1\alpha(x_2 y_2 - x_1 y_1) + \beta(t-\tau)e_4 + \eta_3\\ \dot{e}_4 &= -(s_2 + \beta)e_4 + q_2\alpha(x_2 y_2 - x_1 y_1) + \eta_4\end{aligned}\right\}$$

(3.2.3)

The main goal of this section is to synchronize two identical chaotic systems (3.2.1) and (3.2.2) using linear active control technique by defining a feedback controller that the slave system force to track the master system and the states of two chaotic systems (3.2.1) and (3.2.2) show similar deportment for all future states. To achieve this goal, let us assume the following theorem.

Theorem 2. The trajectories of the two chaotic systems (3.2.1) and (3.2.2) will achieve synchronization asymptotically global for initial conditions, $(x_m(0), y_m(0), z_m(0), w_m(0)) \neq (x_s(0), y_s(0), z_s(0), w_s(0))$ with the following control law:

$$\left.\begin{aligned}
\eta_1(t) &= -ae_3 + v_1(t) \\
\eta_2(t) &= \alpha(x_2y_2 - x_1y_1) + \frac{c}{\kappa + x_1} - \frac{c}{\kappa + x_2} + v_2(t) \\
\eta_3(t) &= bz_1 - s_1z_1 - q_1\alpha(x_2y_2 - x_1y_1) - \beta(t-\tau)e_4 + v_3(t) \\
\eta_4(t) &= q_2\alpha(x_1y_1 - x_2y_2) + v_4(t)
\end{aligned}\right\}$$

(3.2.4)

Proof: Let us assume that the parameters of the master and slave systems are known and states of both systems (3.2.1) and (3.2.2) are measurable. Substituting equation (3.2.3) in equation (3.2.4), we get:

$$\begin{aligned}
\dot{e}_1 &= -be_1 + v_1(t) \\
\dot{e}_2 &= -de_2 + v_2(t) \\
\dot{e}_3 &= -s_1e_3 + v_3(t) \\
\dot{e}_4 &= -(s_2 + \beta)e_4 + v_4(t)
\end{aligned}$$

(3.2.5)

Where, $$\begin{pmatrix} v_1 \\ v_2 \\ v_3 \\ v_4 \end{pmatrix} = -\begin{pmatrix} b_{11} & b_{12} & b_{13} & b_{14} \\ b_{21} & b_{22} & b_{23} & b_{24} \\ b_{31} & b_{32} & b_{33} & b_{34} \\ b_{41} & b_{42} & b_{43} & b_{44} \end{pmatrix}\begin{pmatrix} e_1 \\ e_2 \\ e_3 \\ e_4 \end{pmatrix}$$

(3.2.6)

The error system (3.2.5) to be controlled is a linear system with control input v_1, v_2, v_3 *and* v_4 as function of e_1, e_2, e_3 *and* e_4 respectively where the constants $b_{ij's}$ are the feedback gains. As long as these feedbacks stabilize the error system then e_1, e_2, e_3 *and* e_4 converge to zero as time 't' goes to infinity [14]. This implies that the two identical chaotic systems (3.2.1) and (3.2.2) are synchronized asymptotically.

Replacing equation (3.2.6) in (3.2.5), we have:

$$\begin{pmatrix} \dot{e}_1 \\ \dot{e}_2 \\ \dot{e}_3 \\ \dot{e}_4 \end{pmatrix} = \begin{pmatrix} -b & 0 & 0 & 0 \\ 0 & -d & 0 & 0 \\ 0 & 0 & -s_1 & 0 \\ 0 & 0 & 0 & -(s_2+\beta) \end{pmatrix}\begin{pmatrix} e_1 \\ e_2 \\ e_3 \\ e_4 \end{pmatrix} - \begin{pmatrix} b_{11} & b_{12} & b_{13} & b_{14} \\ b_{21} & b_{22} & b_{23} & b_{24} \\ b_{31} & b_{32} & b_{33} & b_{34} \\ b_{41} & b_{42} & b_{43} & b_{44} \end{pmatrix}\begin{pmatrix} e_1 \\ e_2 \\ e_3 \\ e_4 \end{pmatrix}$$

i.e.,
$$\begin{pmatrix} \dot{e}_1 \\ \dot{e}_2 \\ \dot{e}_3 \\ \dot{e}_4 \end{pmatrix} = \begin{pmatrix} -b-b_{11} & -b_{12} & -b_{13} & -b_{14} \\ -b_{21} & -d-b_{22} & -b_{23} & -b_{24} \\ -b_{31} & -b_{32} & -s_1-b_{33} & -b_{34} \\ -b_{41} & -b_{42} & -b_{43} & -(s_2+\beta)-b_{44} \end{pmatrix} \begin{pmatrix} e_1 \\ e_2 \\ e_3 \\ e_4 \end{pmatrix}$$
(3.2.7)

Thus the aim of this paper is to choose a suitable coupling matrix B be in way that the closed loop system (3.2.7) must have all the eigenvalues with negative real parts so that the errors dynamics converge to zero as time t tends to infinity. For the particular choice of feedback gains:

$$B = \begin{pmatrix} b_{11} & b_{12} & b_{13} & b_{14} \\ b_{21} & b_{22} & b_{23} & b_{24} \\ b_{31} & b_{32} & b_{33} & b_{34} \\ b_{41} & b_{42} & b_{43} & b_{44} \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -0.1 \end{pmatrix}$$

With this particular choice of the feedback gain matrix and considering,
$b=1, d=1, \kappa=1, \beta=2, s_1=1 \ and \ s_2=0.1$, the errors system (3.2.7) becomes:
$$\begin{pmatrix} \dot{e}_1 \\ \dot{e}_2 \\ \dot{e}_3 \\ \dot{e}_4 \end{pmatrix} = \begin{pmatrix} -2 & 0 & 0 & 0 \\ 0 & -2 & 0 & 0 \\ 0 & 0 & -2 & 0 \\ 0 & 0 & 0 & -2 \end{pmatrix} \begin{pmatrix} e_1 \\ e_2 \\ e_3 \\ e_4 \end{pmatrix}$$
(3.2.8)

From equation (3.2.8), It can be seen that the error system (3.2.8) is a linear system of the form, $\dot{e} = Ce$. Thus the system matrix $C = \begin{pmatrix} -2 & 0 & 0 & 0 \\ 0 & -2 & 0 & 0 \\ 0 & 0 & -2 & 0 \\ 0 & 0 & 0 & -2 \end{pmatrix}$ is Hurwitz [19] and all the eigenvalues of the system matrix C are negative (-2, -2, -2,-2). Hence the above system (3.2.8) is asymptotically stable. To achieve globally exponential stability, let us assume a quadratic Lyapunov errors function of the form:
$$V(t) = e^T Me$$
(3.2.9)

where $M = \begin{pmatrix} 0.5 & 0 & 0 & 0 \\ 0 & 0.5 & 0 & 0 \\ 0 & 0 & 0.5 & 0 \\ 0 & 0 & 0 & 0.5 \end{pmatrix}$ which is a positive definite function.

It is clear that the Lyapunov errors function, $V(t) > 0$.

Now the time derivative of the Lyapunov function along the trajectory of the error system (3.2.3) is given as:

$$\dot{V}(t) = \dot{e}^T M e + e^T M \dot{e}$$

$$\dot{V}(t) = -(b+b_{11})e_1^2 - (d+b_{22})e_2^2 - (s_1+b_{33})e_3^2 - (s_2+\beta+b_{44})e_4^2$$

$$\dot{V}(t) = -e^T \begin{pmatrix} 2 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 2 \end{pmatrix} e < 0$$

Therefore, $-\dot{V}(t) = e^T N e$ and $N = \begin{pmatrix} 2 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 2 \end{pmatrix}$ which is also a positive definite matrix.

Hence based on the Lyapunov stability theory [18], the errors dynamics converge to the origin asymptotically global which implies that the two identical chaotic systems (3.2.1) and (3.2.2) are globally exponential synchronized.

NONIDENTICAL SYNCHRONIZATION OF AN HIV/AIDS CHAOTIC SYSTEM

In this section, two different chaotic systems are described. Both systems are 4-D chaotic systems. To achieve switching synchronization between two different chaotic systems, let us assume that the HIV/AIDS chaotic system drives the Qi 4-D chaotic system [20]. Thus the master-slave system arrangement is described as:

$$\left.\begin{aligned} \dot{x}_1 &= a z_1 - b x_1 \\ \dot{y}_1 &= \frac{c}{\kappa + x_1} - d y_1 - \alpha x_1 y_1 \\ \dot{z}_1 &= q_1 \alpha x_1 y_1 - s_1 z_1 + \beta w_1(t-\tau) \\ \dot{w}_1 &= q_2 \alpha x_1 y_1 - s_2 w_1 - \beta w_1 \end{aligned}\right\} \quad \text{(Master system)}$$

(3.3.1)

and

$$\left.\begin{aligned} \dot{x}_2 &= a_1(y_2 - x_2) + y_2 z_2 w_2 + \eta_1 \\ \dot{y}_2 &= a_2(x_2 + y_2) - x_2 z_2 w_2 + \eta_2 \\ \dot{z}_2 &= -a_3 z_2 + x_2 y_2 w_2 + \eta_3 \\ \dot{w}_2 &= -a_4 w_2 + x_2 y_2 z_2 + \eta_4 \end{aligned}\right\} \quad \text{(Slave system)}$$

(3.3.2)

Where $x_1, y_1, z_1, w_1 \in R^{n\times n}$ and $x_2, y_2, z_2, w_2 \in R^{n\times n}$ are the corresponding state vectors of master and slave systems respectively. $a, b, c, d, \kappa, \alpha, \beta, q_1, q_2, s_1$ *and* s_2 are the parameters of the master system where a_1, a_2, a_3 *and* a_4 are the parameters of the slave systems respectively and $\eta(t) = [\eta_1(t), \eta_2(t), \eta_3(t), \eta_4(t)]^T \in R^{n\times 1}$ are the Linear Active Controller.

From systems of equations (3.2.1) and (3.2.2), the error dynamics can be described as:

$$\left.\begin{aligned}\dot{e}_1 &= -a_1 e_1 + (b-a)x_1 + a_1 y_2 - a z_1 + y_2 z_2 w_2 + \eta_1\\ \dot{e}_2 &= -d e_2 + (d+a_2)y_2 + a_2 x_2 - x_2 z_2 w_2 + \alpha x_1 y_1 - \frac{c}{\kappa + x_1} + \eta_2\\ \dot{e}_3 &= -e_3 + x_2 y_2 w_2 - \beta(t-\tau)w_1 - q_1 \alpha x_1 y_1 + \eta_3\\ \dot{e}_4 &= -a_4 e_4 - a_4 w_1 + s_2 w_1 + \beta w_1 + x_2 y_2 z_2 - q_2 \alpha x_1 y_1 + \eta_4\end{aligned}\right\} \quad (3.3.3)$$

To achieve synchronization asymptotically globally using Linear Active Control Algorithm, re-defining the controller, $\eta(t) = [\eta_1(t), \eta_2(t), \eta_3(t), \eta_4(t)]^T \in R^{n\times 1}$ as:

$$\left.\begin{aligned}\eta_1(t) &= (a-b)x_1 - a_1 y_2 + a z_1 - y_2 z_2 w_2 + v_1(t)\\ \eta_2(t) &= -(d+a_2)y_2 - a_2 x_2 + x_2 z_2 w_2 - \alpha x_1 y_1 + \frac{c}{\kappa + x_1} + v_2(t)\\ \eta_3(t) &= -x_2 y_2 w_2 + \beta(t-\tau)w_1 + q_1 \alpha x_1 y_1 + v_3(t)\\ \eta_4(t) &= a_4 w_1 - s_2 w_1 - \beta w_1 - x_2 y_2 z_2 + q_2 \alpha x_1 y_1 + v_4(t)\end{aligned}\right\} \quad (3.3.4)$$

Substituting equation (3.2.4) in equation (3.2.3), we get:

$$\left.\begin{aligned}\dot{e}_1 &= -a e_1 + v_1(t)\\ \dot{e}_2 &= -d e_2 + v_2(t)\\ \dot{e}_3 &= -e_3 + v_3(t)\\ \dot{e}_4 &= -a_4 e_4 + v_4(t)\end{aligned}\right\} \quad (3.3.5)$$

Where $v_i(t) = -Be_i = -B(\mathrm{y}_i - x_i),\ i = 1, 2, 3, 4$

(3.3.6)

Replacing system of equations (3.2.5) in equation (3.2.6), we have:

$$\begin{pmatrix}\dot{e}_1\\ \dot{e}_2\\ \dot{e}_3\\ \dot{e}_4\end{pmatrix} = \begin{pmatrix}-a-b_{11} & -b_{12} & -b_{13} & -b_{14}\\ -b_{21} & -d-b_{22} & -b_{23} & -b_{24}\\ -b_{31} & -b_{32} & -1-b_{33} & -b_{34}\\ -b_{41} & -b_{42} & -b_{43} & -a_4-b_{44}\end{pmatrix}\begin{pmatrix}e_1\\ e_2\\ e_3\\ e_4\end{pmatrix}$$

(3.3.7)

For the following particular choice of feedback gain matrix *B*,

$$B=\begin{pmatrix} b_{11} & b_{12} & b_{13} & b_{14} \\ b_{21} & b_{22} & b_{23} & b_{24} \\ b_{31} & b_{32} & b_{33} & b_{34} \\ b_{41} & b_{42} & b_{43} & b_{44} \end{pmatrix}=\begin{pmatrix} -27 & 0 & 0 & 0 \\ 0 & -2 & 0 & 0 \\ 0 & 0 & -2 & 0 \\ 0 & 0 & 0 & -7 \end{pmatrix}$$

and considering, $b=1, d=1, \kappa=1, \beta=2, s_1=1 and\ s_2=0.1$, the error system (3.3.7) becomes:

$$\begin{pmatrix} \dot{e}_1 \\ \dot{e}_2 \\ \dot{e}_3 \\ \dot{e}_4 \end{pmatrix}=\begin{pmatrix} -3 & 0 & 0 & 0 \\ 0 & -3 & 0 & 0 \\ 0 & 0 & -3 & 0 \\ 0 & 0 & 0 & -3 \end{pmatrix}\begin{pmatrix} e_1 \\ e_2 \\ e_3 \\ e_4 \end{pmatrix} \tag{3.3.8}$$

It is now clear that the error system (3.3.8) is a linear system of the form, $\dot{e}=Ce$ with the system matrix, $C=\begin{pmatrix} -3 & 0 & 0 & 0 \\ 0 & -3 & 0 & 0 \\ 0 & 0 & -3 & 0 \\ 0 & 0 & 0 & -3 \end{pmatrix}$ is Hurwitz [19] and all the eigenvalues of the system matrix ***C*** are negative

(-3, -3, -3,-3). Hence the above error system (3.3.8) is asymptotically stable.

Let us construct the same Lyapunov errors function candidate with the same positive definite matrix as in (3.2.9). The time derivative of the Lyapunov errors function is given as: $\dot{V}(t)=-(a+b_{11})e_1{}^2-(d+b_{22})e_2{}^2-(1+b_{33})e_3{}^2-(a_4+b_{44})e_4{}^2$

$$\dot{V}(t)=-e^T\begin{pmatrix} 3 & 0 & 0 & 0 \\ 0 & 3 & 0 & 0 \\ 0 & 0 & 3 & 0 \\ 0 & 0 & 0 & 3 \end{pmatrix}e<0$$

Therefore, $-\dot{V}(t)=e^T Ne$.

Hence based on Lyapunov stability theory [18], the errors dynamics approaches to the origin asymptotically globally which implies that the two nonidentical chaotic systems (3.3.1) and (3.3.2) are globally exponential synchronized.

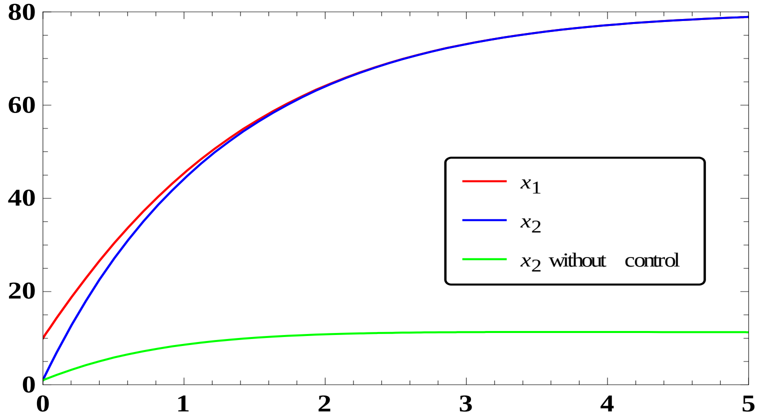

Fig 1: Time Series of x_1 & x_2 For Identical HIV systems.

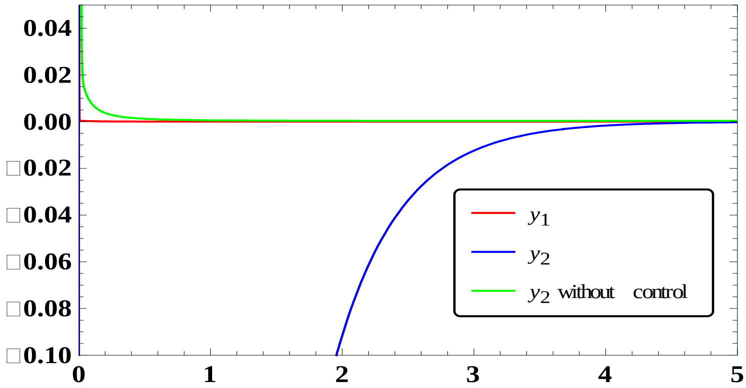

Fig 2: Time Series of y_1 & y_2 For identical HIV systems.

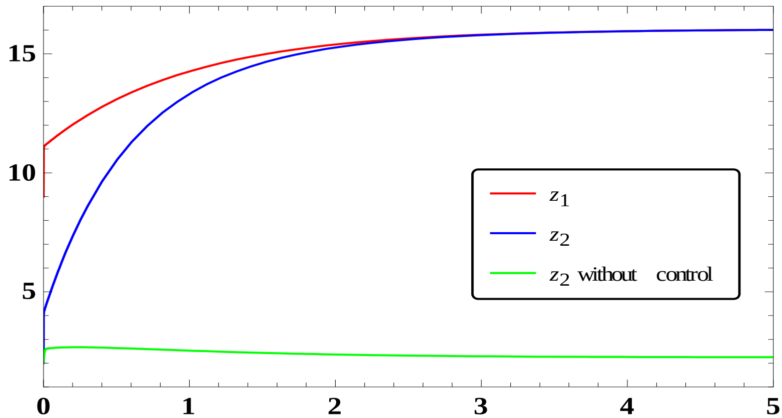

Fig 3: Time Series of z_1 & z_2 For identical HIV systems.

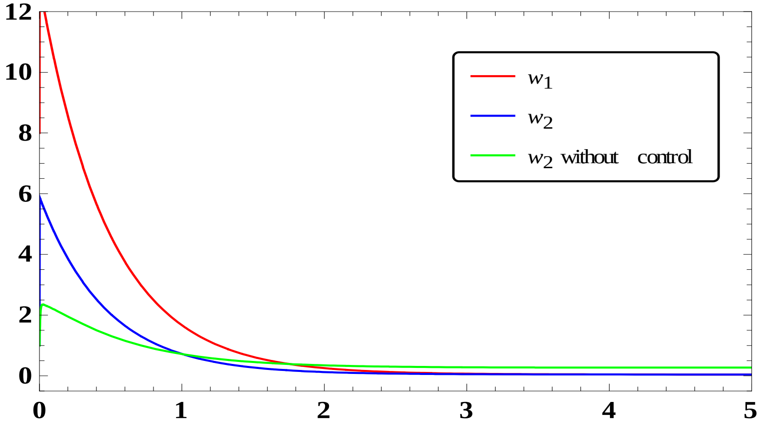

Fig 4: Time Series of w_1 & w_2 (For identical HIV systems)

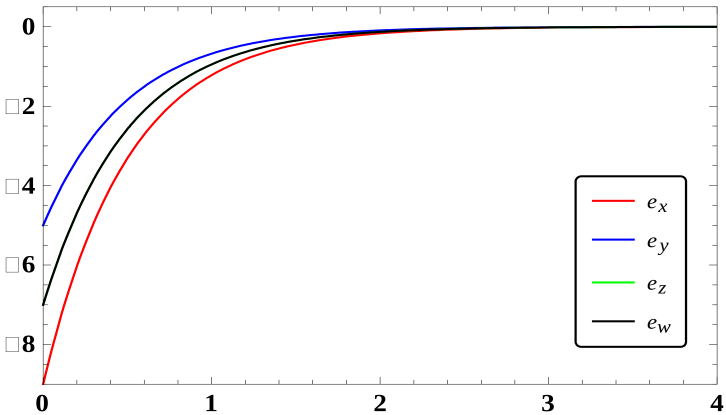

Fig 5: Time Series of errors (For identical HIV systems)

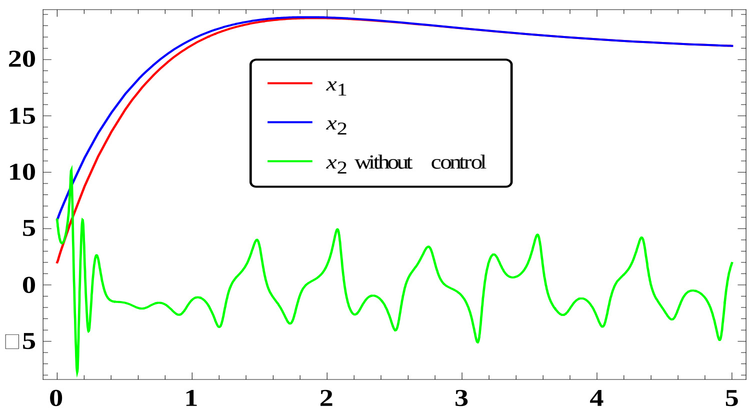

Fig 6: Time Series of x_1 & x_2 (HIV and Qi systems)

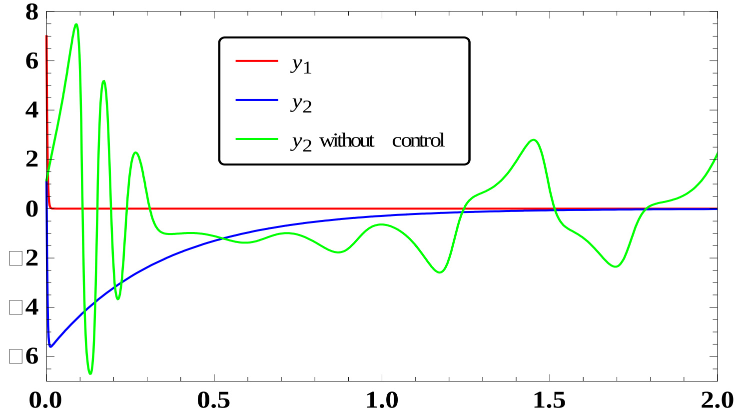

Fig 7: Time Series of y_1 & y_2 HIV and Qi systems

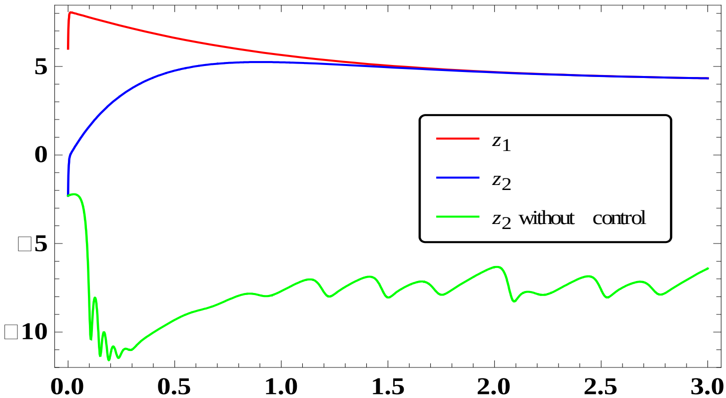

Fig 8: Time Series of z_1 & z_2 HIV and Qi systems

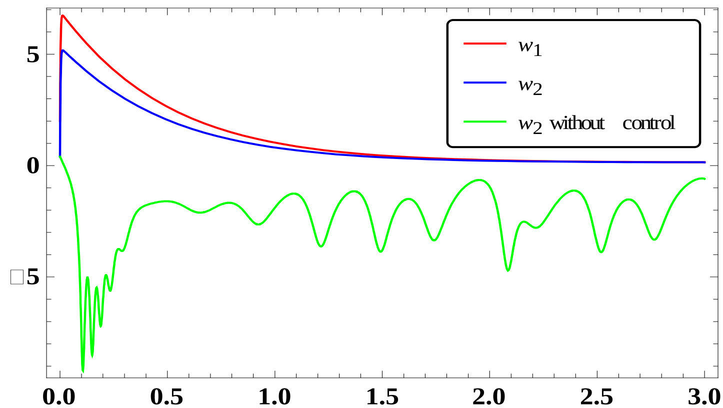

Fig 9: Time Series of w_1 & w_2 HIV and Qi systems

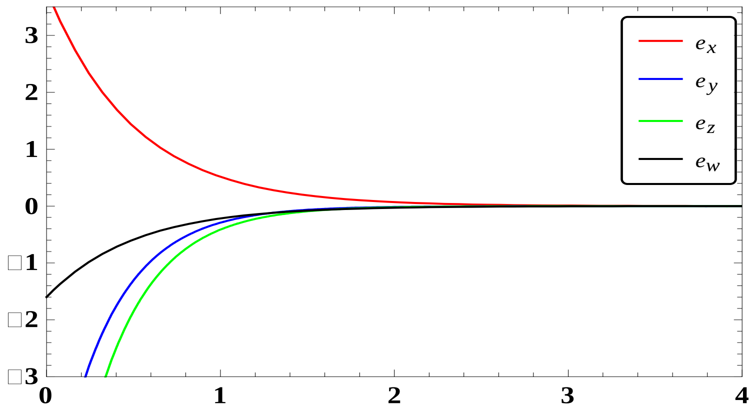

Fig 10: Time Series of errors (HIV and Qi systems)

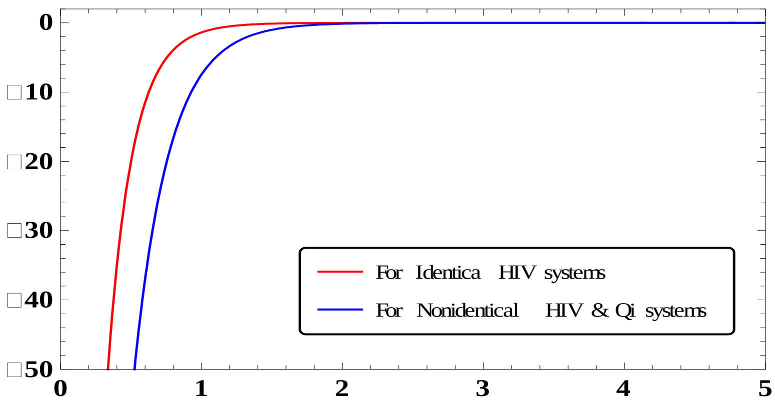

Fig 11: Time Series of $V(t)$

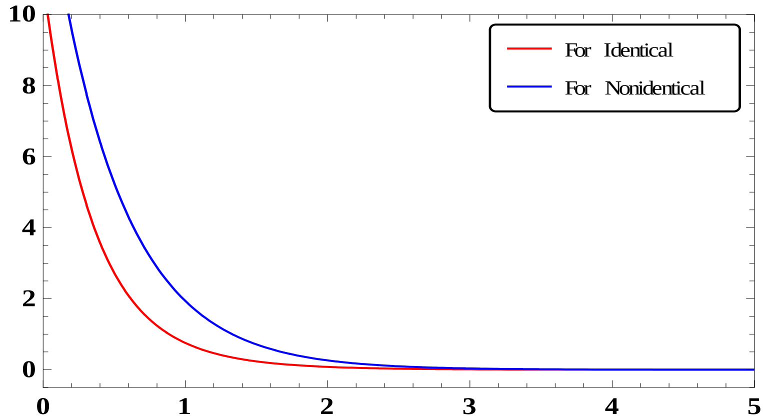

Fig 12: Convergence of errors

NUMERICAL SIMULATIONS

Numerical simulations are furnished to validate not only the advantages and potency of the proposed method as well as to overcome the chaotic nature of the HIV model. The parameters for the new chaotic system [17] are taken as:

$a=5, b=1, c=10, d=1, \kappa=1, \alpha=200, \beta=2, q_1=0.3, q_2=0.7, s_1=1 and\ s_2=0.1$,

with initial conditions are taken as: $(10,7,9,8)$ & $(2,7,6,2)$.

The parameters for Qi 4-D chaotic system [20] are selected as: $a_1=30, a_2=10, a_3=1 and\ a_4=10$, and initial values are taken as: $(1.1,-2.2,0.3,5.8)$ & $(5.8,1.15,-2.3,0.4)$.

For the above chosen values, we have plotted the time series of state variables for identical HIV systems (Figures 1- 6) and for non-identical HIV and Lu systems (Figures 7-10) and it is clear that the states grow chaotically in the absence of an acceptable controller.

The figure 5 illustrates the synchronization errors of two identical HIV/AIDS chaotic systems and figures 10 shows the synchronization errors of the two nonidentical (HIV/AIDS) and Qi 4-D chaotic systems respectively. For the two different chaotic systems (HIV/AIDS and Qi), that contain parameters mismatches and different structures, the controllers ware utilized to synchronize the states of master and slave systems asymptotically globally when the controls were switched on at *t* = 0 s. It has been shown that the HIV/AIDS chaotic system is forced to track the Qi 4-D chaotic system and the states of two chaotic systems show common conduct after a transient time of 2.8 s while for identical HIV/AIDS systems, the two systems show similar behavior after 3.5 s which illustrates that the errors signal (figure 10) for two different HIV/AIDS and Qi chaotic systems have fast response as compared to identical HIV/AIDS chaotic systems. It has been shown that the error signals converges to the origin very smoothly with a minimum rate of decay and enough synchronization speed showing that the investigated controllers are more robust to accidental mismatch in the transmitter and receiver.

The figure-11 depicts the derivative of Lyapunov errors functions of identical chaotic systems (HIV/AIDS) and nonidentical HIV/AIDS and Qi 4-D chaotic systems.

Moreover, the figure-12 illustrates the analysis of the synchronization between master and slave systems which has been also confirmed by the convergence of the synchronization quality defined by the breeding of the error signals:

$$e=\sqrt{(e_1)^2+(e_2)^2+(e_3)^2+(e_4)^2}$$

SUMMARY AND CONCLUSION

In this research article, global chaos synchronization of identical and nonidentical of an In-host model for HIV/AIDS chaotic system has been investigated. Based on Lyapunov Stability Theory and using the Linear Active Controller, a class of proper feedback controllers was designed to achieve exponentially global stability of the error signals. Since the Lyapunov exponents are not required for their execution, Linear Active Control Technique is a powerful algorithm for synchronizing two identical as well as nonidentical chaotic systems. Results are furnished in graphical forms with time history (Figures 1- 12).

In this study, using the Linear Active Control Technique, all graphical as well as analytical results shown that the proposed strategies have excellent synchronizing performance and that the synchronization is globally exponential stable.

In addition, the synchronization with negative derivative of the Lyapunov errors functions allows large synchronizable interval which shows that the non-progression of the HIV/AIDS virions could be maintained to a specific value for a long-term and would be especially significant for HIV infection treatment and thus biologically it would be more effective to react for treatments such as highly active antiretroviral therapy (HAART), etc. that provide a useful option for HIV/AIDS infection treatment.

This research can be significant for supplementary research in HIV/AIDS for the long-term immunological control of HIV/AIDS and crafting of such therapy that changes a progression patient in a long-term non-progression by slowing down the reproduction of HIV in the body and can prevent people from becoming ill for many years.

Since the synchronization of two identical as well as nonidentical chaotic systems presumes potential applications in the field of nonlinear dynamics, the result of this research work should be helpful and could be employed in the field of epidemiology and may be considered a good tools in analyzing the spread and control of infectious diseases in the field of HIV/AIDS.

In numerical simulations, the evolution of the synchronization can be modified by choosing different control gain. We have noticed that by using active control techniques, if the convergence time is reduced, the magnitude of the control signals is increased. This may lead to a large controller gain and a signal saturation which creates self-excitation and noise in the system and the synchronization might be disregarded completely.

This research work is totally based on theoretical ground, but in practice, it may also be disturbed by some additive noise. Further research can be done to overcome on these limitations by designing proper gain for the system in studies that can robustly against the internal or environmental noise.

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