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SVD-Based Signal Detector for Cognitive Radio Networks

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***Abstract*—This paper examines the implementation of the Singular Value Decomposition (SVD) method to detect the presence of wireless signal. The method is used to find the maximum and minimum eigenvalues. We simulated the algorithm using common digital signal in wireless communication namely rectangular pulse shape, raised cosine and root-raised cosine to test the performance of the signal detector. The SVD-based signal detector was found to be more efficient in sensing signal without knowing the properties of the transmitted signal. The execution time is acceptable compared to the favorable energy detection. The computational complexity of SVD-based detector is medium compared to the energy detector. The algorithm is suitable for blind spectrum sensing where the properties of the signal to be detected are unknown. This is also the advantage of the algorithm since any signal would interfere and subsequently affect the quality of service (QoS) of the IEEE 802.22 connection. Furthermore, the algorithm performed better in the low signal-to-noise ratio (SNR) environment.**

***Index Terms*—Cognitive radio, singular value decomposition (SVD), signal detector.**

I. Introduction

Spectrum sensing in cognitive radio (CR) has been a very important function to enable the state of the art technology in revolutionizing spectrum efficient utilization. In responding to the idea of CR coined by Joseph Mitola in [1], IEEE 802.22 Working Group (WG) was formed in 2004. The WG is expected to develop and incorporate CR functionality in a standard known as Wireless Regional Area Networks (WRAN). The new standard is going to operate in TV bands between 54-862MHz [2]. The standard is expected to deliver broadband access to data networks on vacant TV channels at the same time avoiding harmful interference to the licensed users in rural areas within a typical radius of 17km to 30km [3].

In order to rationalize the use of CR, a very efficient spectrum management needs to be implemented in cognitive radio networks. As stated in [4], the spectrum management process consists of four major steps: 1) spectrum sensing, 2) decision making, 3) spectrum sharing and 4) spectrum mobility. The first and second steps are very crucial in enabling the CR technology. CR users are expected to be able to detect primary user (PU) networks and find the spectrum holes or the unused spectrum in order to utilize them.

Several spectrums sensing algorithms such as classical energy detection (ED), the eigenvalue-based detection, the covariance-based detection and feature-based detection are reported in the literature to detect primary signal. Discussions about the these techniques and algorithms as well as their pros and cons can be found in [5], [6], [7], [8], [9]. According to Kortun et al. [10], the most accurate techniques that can simultaneously achieve both high probability of detection and low probability of false alarm with very minimal knowledge about the primary user signals and noise spectrum are the eigenvalue-based detection techniques introduced by Liang and Zhang in [11]. It is mentioned in the literature that maximum-maximum eigenvalue (MME) method has many advantages over the rest of sensing methods listed above. This is due to the fact that the decision of signal presence can be done without prior knowledge of the primary signal and noise.

In the eigenvalue-based detection methods, the decision threshold is derived from random matrix theory (RMT) to determine the hypothesis testing for signal detection. The methods are using the eigenvalue decomposition technique to find the eigenvalues in order to compare with the threshold. The SVD is quite similar to the eigenvalue decomposition method. However, the SVD is very general in the sense that it can be applied to any $m \times n$ matrix, whereas the eigenvalue decomposition method can only be applied to certain classes of square matrices. Nevertheless, the two decompositions are related. Furthermore, the SVD has got several advantages compared to other decomposition methods as listed below [12]:

i. more robust to numerical error;
ii. exposes the geometric structure of a matrix an important aspect of many matrix calculations; and
iii. quantify the resulting change between the underlying geometry of those vector spaces.

The rest of the paper is organized as follows. Next section gives the overview of related works done using SVD. Section III introduces the common signal detection model for spectrum sensing. In section IV, the SVD-based signal detection is highlighted in brief. Section V outlined the algorithm used for signal detection. In section VI, we discussed about the MME threshold used in this paper. Section VII, we describe the simulation parameters and results of implementing SVD-based signal detector. Finally, the conclusion is given in section VIII.

II. Related Works

Before the introduction of SVD-based detection, researchers discover that by analyzing the eigenvalues from a signal received matrix, the threshold for detecting of primary signal can be calculated. Most of the researchers used the eigenvalue decomposition technique as mentioned above. This method is known as eigenvalue-based detection. Discussion on the method can be found in [11], [13], [10].

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SVD-based detection was introduced by Xu et al. in [3] and [14]. Both papers used SVD technique to detect wireless microphone signal in a wideband cognitive radio network. In WRAN, the system needs to detect both digital TV and wireless microphone signals since both services are incumbent in the frequency bands. However, in some countries, wireless microphone is not protected by the law as in US where FCC Part 74 protects the service. For example, Malaysia only licenses the equipment through type approval, which is not protected by the law.

This paper used the SVD method to detect common signals rather than restrict it to the only digital TV and wireless microphone since any other signal would effect the WRAN quality of service (QoS).

III. System Model

In detecting a signal, two hypotheses are involved: H_0, signal does not presence; and H_1, signal presence. The received signal samples under two hypotheses are given respectively as follows [15], [16], [17], [18]:

$$\begin{array}{lcccl} H_0 & : & y_i & = & \eta_i(n) \\ H_1 & : & y_i & = & x_i(n)+\eta_i(n) \end{array} \tag{1}$$

where $x_i(n)$ is the transmitted signal samples, through a wireless channel consisting of path loss, multipath fading and time dispersion effects, and $\eta_i(n)$ is the white noise which is independent and identically distributed (iid) with zero mean and σ_η^2 variance. Note that $x_i(n)$ can be the superposition of the received signals from multiple primary users, hence, no synchronization is needed here.

There are two probabilities involved for signal detector: probability of detection, P_d, which defines, the hypothesis H_1, the probability of the detecting algorithm having detected the presence of the primary signal; and probability of false alarm, P_{fa}, which defines, at hypothesis H_0, the probability of the detecting algorithm claiming the presence of the primary signal. Test statistic for an energy detector is given by

$$T_i = \frac{1}{N_s}\sum_{n=i}^{N_s} |y_i(n)|^2 \tag{2}$$

Under the hypothesis H_0, it shows a Gaussian random distribution when number of signal sample (N_s) is large with mean σ_η^2 and variance $\frac{2}{N_s}\sigma_\eta^2$. Hence, for a given probability of false alarm P_{fa}, the threshold γ of an energy detector can be derived as

$$\gamma = \sigma_\eta^2\left(1+\frac{\sqrt{2}Q^{-1}\left(P_{fa}\right)}{\sqrt{N_s}}\right) \tag{3}$$

where $Q(x) = \left(1/\sqrt{2\pi}\right)\int_x^\infty e^{-t^2/2}dt$ is the normal Q-function.

In this paper we consider three types of signals at the receiver: rectangular pulse, raised cosine and root-raised cosine since these are common signals in today digital communication system.

IV. SVD Based Signal Detector

SVD plays an important role in signal processing and statistics, particularly in the area of a linear system. For a time series $y(n)$ with $n = 1,2,\ldots,N$, commonly, we can construct a Henkel matrix with $M = N-L+1$ rows and L columns as follows:

$$\mathbf{R} = \begin{bmatrix} y(1) & y(2) & \cdots & y(L) \\ y(2) & y(3) & \cdots & y(L+1) \\ \vdots & \vdots & \ddots & \vdots \\ y(N-L+1) & y(N-L+2) & \cdots & y(N) \end{bmatrix} \tag{4}$$

then $\mathbf{R}$ is an $M\times L$ matrix. Its elements can be found by substituting of $y(n)$

$$\mathbf{R}_{ml} = y(m+l-1), \quad m = 1,2,\ldots M \text{ and } l = 1,2,\ldots L. \tag{5}$$

Using SVD, $\mathbf{R}$ can be factorized as

$$\mathbf{R} = \mathbf{U}\Sigma\mathbf{V}^H \tag{6}$$

where $\mathbf{U}$ and $\mathbf{V}$ are an $M\times M$ and $L\times L$ unitary matrix, respectively. The columns of $\mathbf{U}$ and $\mathbf{V}$ are called left and right singular vectors, respectively. The $\Sigma = diag(\lambda_1,\lambda_2,\ldots,\lambda_m)$ is a diagonal matris whose nonnegative entries are the square roots of the positive eigenvalues of $\mathbf{R}^H\mathbf{R}$ or $\mathbf{R}\mathbf{R}^H$. These nonnegative entries are called the singular values of $\mathbf{R}$ and they are arranged in a decreasing manner with the largest number in the upper left-hand corner of the matrix. The $[\,]^H$ denotes the Hermitian transpose of a matrix.

Whenever no primary signal or other signal is present, the received signal $y(n)$ includes only AWGN contribution such that its singular values are similar and close to zero. When other signals are active whose power is higher than a threshold, there will exist several dominant singular values to represent these signals. As a result, the signal can be detected by examining the presence of dominant singular values.

V. Algorithm for Signal Detection

In implementing the SVD-based signal detector, we adopt method by Zeng and Liang (2007) in [11]. The algorithm to detect the presence of a signal is as follows:

Step 1: Select number of column of a covariance matrix, L such that $k < L < N-k$ [19], where N is the number of sampling points and k is the number of dominant singular values. In this paper, $k = 2$ and $L = 16$.

Step 2: Factorized the covariance matrix to form the equation as in (6).

Step 3: Obtain the maximum and minimum eigenvalues of the covariance matrix which are λ_{max} and λ_{min}.

Step 4: Compute threshold value, γ. The threshold value determination will be highlighted in the next section.

Step 5: Compare the ratio with the threshold. If $\frac{\lambda_{max}}{\lambda_{min}} > \gamma$, the signal is present, otherwise, the signal is not present.

VI. Threshold Determination

Decision threshold and probability of false alarm are derived based on limiting distribution of eigenvalues based on random matrix theory. The decision statistic for the maximum-minimum eigenvalue (MME) detection is defined as the ratio of maximum to minimum eigenvalues of received signal covariance matrix as follows:

$$T_y = \frac{\lambda_{max}}{\lambda_{min}} \tag{7}$$

Based on the decision statistic in (7), the detection threshold, γ, must be estimated for a required probability of false alarm. To define the threshold in terms of P_{fa} or vice versa, the density of the test statistic, T_y, is required. The density can be found asymptotically i.e. both the threshold values and the probabilities of detection and false alarm are derived based on asymptotical (limiting) distributions of eigenvalues that is mathematically tractable and less complicated [10].

An asymptotic formula of signal detection threshold in term of desired probability of false alarm for MME has been proposed in [11]. The detection threshold in terms of desired probability of false alarm is calculated by using the results of the theorem in [20] and [11], as follows (in our case, $M = 1$):

$$\begin{aligned} \gamma_{mme} &= \frac{\left(\sqrt{N_s}+\sqrt{L}\right)^2}{\left(\sqrt{N_s}-\sqrt{L}\right)^2} \\ &\times \left(1 + \frac{\left(\sqrt{N_s}+\sqrt{L}\right)^{-\frac{2}{3}}}{(N_s L)^{\frac{1}{6}}} \cdot F_1^{-1}\left(1-P_{fa}\right)\right) \end{aligned} \tag{8}$$

where F_1^{-1} denotes the inverse of cumulative distribution function (CDF) of the Tracy-Widom distribution of order 1 [21].

The threshold definition in (8) is formulated based on deterministic asymptotic values of the minimum and maximum eigenvalues of the covariance matrix, $\mathbf{R}$, when the number of samples, N_s is very large. As shown in the equation, it is defined only in terms of number of samples, N_s, level of covariance matrix, L and the desired probability of false alarm, P_{fa}.

VII. Simulations

A. Simulation parameters

It is assumed that the channel is not changing during the period of samples. The level of the covariance matrix, i.e. the column of the matrix is $L = 16$. The results are averaged over 10^3 tests using Monte-Carlo Simulations written in Matlab. Simulation results are taken using QPSK modulated random primary signal and independent and identically distributed (i.i.d.) noise samples with Gaussian distribution are used. Three types of signal namely rectangular pulse, raised cosine and root-raised cosine were tested and compared. To find the threshold, we require the probability of false alarm is $P_{fa} \leq 0.1$ and probability of detection is $P_d > 0.9$ as required by IEEE 802.22 standard.

B. Simulation results

Figure 1 shows simulation results of the P_d when the SVD-based method and a classical energy detector (ED) are used for comparison when SNR is from -16dB to -4dB. From these figures, it can be concluded that the SVD-based detection can overcome the flaws in ED when dealing with low SNRs. It can be noticed from the graphs that the performance of the ED drops dramatically below -8dB of the SNR.

Although ED at certain points better than the SVD based detection, but the overall performance of the detector is better than the ED. It is also shown in the graphs that the SVD-based method works better in detecting the rectangular pulse signal, raised cosine the second and root-raised cosine the third. The ED's performance for all three signals is quite similar.

In terms of performance of the detector, the receiver operating characteristics (ROC) curves are shown in figure 2. Both methods were simulated at -8dB SNR and tested for three types of signal. We plot the P_d under H_1 against P_{fa} under H_0 when P_{fa} changes from 0.01 to the desired value of 0.1. It is clearly shown that the ROC curves of the SVD-based detection are much higher than the ED's which proves the good performance of the detector.

Although the SVD-based detection is the best compared to the ED's on the overall, it is also noticing that performances of SVD are dropping but the ED's are rising. The dropping in SVD-based detection is consistent with previous results but the rising of ED's might be due to the type of the signals used where both raise cosine and root-raised cosine are higher in signal energy.

VIII. Conclusions

In this paper, we implemented a SVD-based approach to detect common signals in today's digital communication system. The rationale of detecting common signals is that, in order for a CR system to operate with an exceptable quality of service (QoS), the CR need to avoid interference not only to/from primary users but any other signals, which could affect the delivery of information to the system.

The brief simulation results show that SVD of the data matrix is very useful in finding the dominant singular values in which the presence of other signals can be detected. The method is more robust to numerical errors and very fast. These qualities are desirable in IEEE 802.22 standard since it is easily suited the need to shorten the period of sensing and hence making the system reliable.

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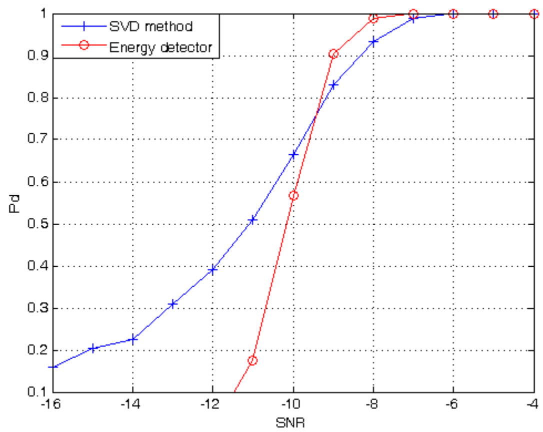
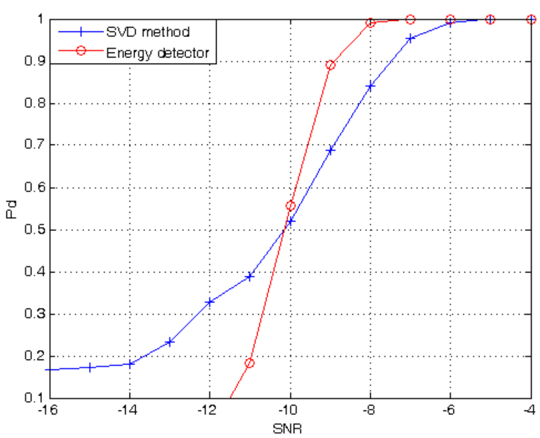
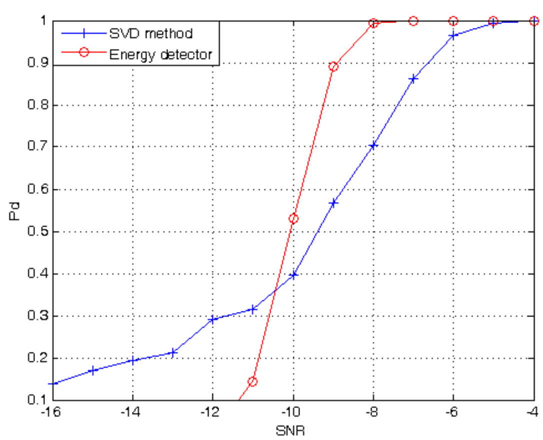


(a) Rectangular pulse signal

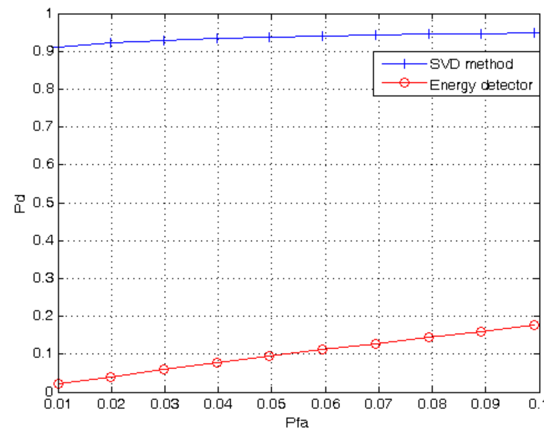


(b) Raised cosine

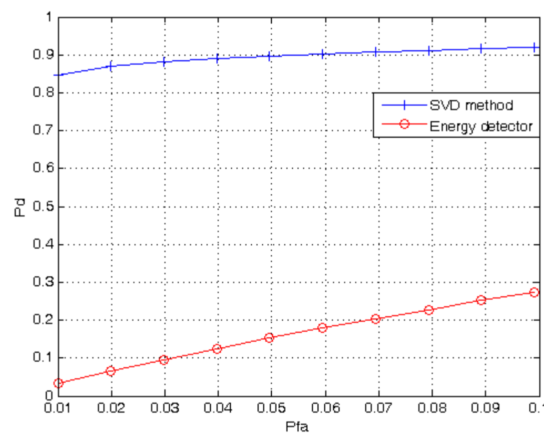


(c) Root-raised cosine

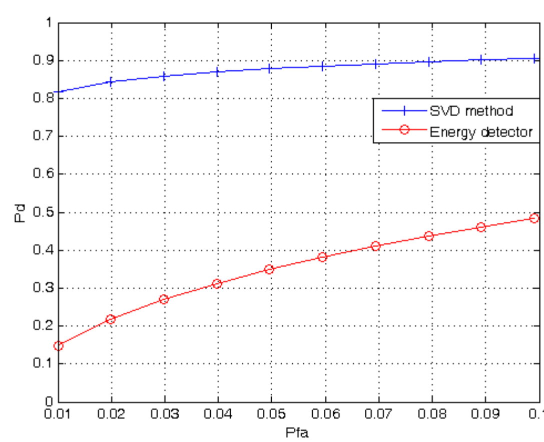
Figure 1: Comparison of P_d between the SVD method and energy detector for a) Rectangular pulse, b) Raised cosine and c) Root-raised cosine



(a) Rectangular pulse signal



(b) Raised cosine signal



(c) Root raised cosine signal

Figure 2: Comparison of ROC curves between SVD method and energy detector for a) Rectangular pulse, b) Raised cosine and c) Root-raised cosine

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