
Robust Transmit Beamforming Design Using Outage Probability Specification

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Abstract

Transmit beamforming (precoding) is a powerful technique for enhancing the channel capacity and reliability of multiple-input and multiple-output (MIMO) wireless systems. The optimum exploitation of the benefits provided by MIMO systems can be achieved when a perfect channel state information at transmitter (CSIT) is available. In practices, however, the channel knowledge is generally imperfect at transmitter because of the inevitable errors induced by finite feedback channel capacity, quantization and other physical constraints. Such errors degrade the system performance severely. Hence, robustness has become a crucial issue.

Current robust designs address the channel imperfections with the worst-case and stochastic approaches. In worst-case analysis, the channel uncertainties are considered as deterministic and norm-bounded, and the resulting design is a conservative optimization that guarantees a certain quality of service (QoS) for every allowable perturbation. The latter approach focuses on the average performance under the assumption of channel statistics, such as mean and covariance. The system performance could break down when persistent extreme errors occur. Thus, an outage probability-based approach is developed by keeping a low probability that channel condition falls below an acceptable level. Compared to the aforementioned methods, this approach can optimize the average performance as well as consider the extreme scenarios proportionally.

This thesis implements the outage-probability specification into transmit beamforming design for three scenarios: the single-user MIMO system and the corresponding adaptive modulation scheme as well as the multi-user MIMO system. In a single-user MIMO system, the transmit beamformer provides the maximum average received SNR and ensures the robustness to the CSIT errors by introducing probabilistic constraint on the instantaneous SNR. Beside the robustness against channel imperfections, the outage probability-based approach also provides a tight BER bound for adaptive modulation scheme, so that the maximum transmission rate can be achieved by taking advantage of transmit beamforming. Moreover, in multi-user MIMO (MU-MIMO) systems, the leakage power is accounted by probability measurement. The resulting transmit beamformer is designed based on signal-to-leakage-plus-noise ratio (SLNR) criteria, which maximizes the average received SNR and guarantees the least leakage energy from the desired user. In such a setting, an outstanding BER performance can be achieved as well as high reliability of signal-to-interference-plus-noise ratio (SINR).

Given the superior overall performances and significantly improved robustness, the probabilistic approach provides an attractive alternative to existing robust techniques under imperfect channel information at transmitter.

Declaration of originality

I hereby declare that the research recorded in this thesis and the thesis itself was composed and originated entirely by myself in the School of Engineering at The University of Edinburgh as a candidate for the Doctorate of Philosophy. I has not been submitted for any other degree or award in any other university or educational institution.

List your exceptions here and sign before your printed name.

Huiqin Du

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Acronyms and abbreviations

AWGN	Additive White Gaussian Noise.
BER	Bit error rate.
CCI	Co-channel Interference.
CSI	Channel state information.
CSIT	Channel state information at transmitter.
CSIR	Channel state information at receiver.
FDD	Frequency diversion duplex.
i.i.d	Independent and identically distributed.
KKT	Karush-Kuhn-Tucker.
MIMO	Multiple-input multiple-output.
MISO	Multiple-input single-output.
MMSE	Minimum mean square error.
MU-MIMO	Multi-user MIMO.
QAM	Quadrature amplitude modulation.
QoS	Quality of service.
SDMA	Spatial division multiple access.
SER	Symbol error rate.
SINR	Signal to interference plus noise ratio.
SISO	Single-input single-output.
SLNR	Signal to (leakage) plus noise ratio.
SNR	Signal-to-noise ratio.
STBC	Space-time block code.
TDD	Time division duplex.

Nomenclature

N	Number of transmit antennas.
M	Number of receive antennas in single-user MIMO system.
U	Number of active user in multi-user MIMO system.
M_k	Number of antenna for k -th user in multi-user MIMO system.
$\mathbf{c}, \mathbf{C}, \mathbf{W}_i, \overline{\mathbf{W}}_i$	Transmit beamforming vector or matrix.
\mathbf{E}	Estimation error.
\mathbf{H}	Wireless channel.
$\bar{\mathbf{H}}$	Channel mean.
h_{ij}, \mathbf{H}_{ij}	Path from transmit antenna j to receive antenna i .
$d_{Tx}(i, j), d_{Ty}(i, j)$	Locations of transmit antennas.
$d_{Rx}(m, n), d_{Ry}(m, n)$	Locations of receive antennas.
\mathbf{H}_{mn}	Path from transmit antenna m to receive antenna n .
$\hat{\mathbf{H}}, \hat{\mathbf{h}}$	Channel estimate.
Σ_c	Singular value of beamforming matrix \mathbf{C} .
\mathbf{V}_c	Right singular vector of beamforming matrix \mathbf{C} .
\mathbf{U}_c	Left singular vector of beamforming matrix \mathbf{C} or eigenvector of matrix $\mathbf{C}\mathbf{C}^H$.
\mathbf{D}_c	Eigenvalue of matrix $\mathbf{C}\mathbf{C}^H$.
d_i	i -th element of diagonal matrix \mathbf{D}_c .
u_i	i -th column of matrix \mathbf{U}_c .
Σ_h	Singular value of channel matrix \mathbf{H} .
\mathbf{V}_h	Right singular vector of channel matrix \mathbf{H} .
\mathbf{U}_h	Left singular vector of channel \mathbf{H} or eigenvector of matrix $\mathbf{H}\mathbf{H}^H$.

\mathbf{D}_h	Eigenvalue of matrix $\mathbf{H}\mathbf{H}^H$.
D_i	The i -th element of diagonal matrix \mathbf{D}_h .
$\hat{\mathbf{U}}_h$	Eigenvector of $\hat{\mathbf{H}}\hat{\mathbf{H}}^H$.
$\hat{\mathbf{D}}_h$	Eigenvalue of matrix $\hat{\mathbf{H}}\hat{\mathbf{H}}^H$.
\hat{D}_i	The i -th element of diagonal matrix $\hat{\mathbf{D}}_h$.
σ_n^2	Variance of noise.
σ_e^2	Variance of errors in CSIT.
δ	Noncentrality parameter of noncentral χ^2 distribution.
n	Degree of freedom.
α, μ	Scale for simplicity.
Δ_b	Difference induced by taking the lower bound of average BER.
ϵ	Outage probability.
γ_0	Threshold of probabilistic constraint.
$\tilde{\gamma}_0$	Scaled threshold of probabilistic constraint.
$\bar{\gamma}$	Average SNR.
ξ	Error bound of uncertainty region.
Δ_e	Mismatch in error variance, σ_e^2 .
\mathcal{L}	Lagrange function.
ν, λ	Lagrange multiplier.
δ_θ	Spread angle.
e_b	Target average BER.
$f(\mathbf{A})$	Function of matrix \mathbf{A} .
K	Constellation size.
\hat{K}	Constellation size based on lower bound of average BER.
L_k	Length of symbols transmitted to single user.

\mathbf{X}	Matrix \mathbf{X} (Bold-face capital letter).
\mathbf{x}	Vector \mathbf{x} (Bold-face lower-case letter).
\mathbf{X}^T	Transpose of \mathbf{X} .
\mathbf{X}^*	Complex conjugation without transposition of \mathbf{X} .
\mathbf{X}^H	Complex conjugate transposition of \mathbf{X} .
$\det(\mathbf{X})$	Determinant of \mathbf{X} .
$\mathbf{X} \geq 0$	\mathbf{X} is positive semi-definite.
\mathbf{I}	Identity matrix.
$\mathbb{E}[\cdot]$	Expected value.
$\Pr\{\cdot\}$	Probability.
$\exp(\cdot)$	Exponential function.
Re	Real part.
Im	Imaginary part.
$\ \mathbf{X}\ _F$	Frobenius norm of \mathbf{X} .
$\text{vec}(\mathbf{X})$	Vectorize \mathbf{X} by concatenating the column of \mathbf{X} .

Chapter 1

Introduction

1.1 Motivation

Multi-antenna diversity has been well established as an effective fading counter-measure for wireless communications, which can further strengthen benefits of multi-input and multi-output (MIMO) systems by taking advantage of transmit beamforming. The optimum exploitation of its strengths requires perfect channel state information at transmitter (CSIT) which is typically not available at transmitter because of the inevitable error induced by limited feedback channel capacity, time-varying channels or other physical constraints. Such channel imperfections degrade the system performance significantly. It motivates the effort to develop robust algorithms against channel imperfections.

Current robust transmit beamforming designs can be classified into the deterministic (or worst-case) and the stochastic approaches. In the worst-case analysis, channel uncertainty is modeled as deterministic and norm-bounded. Under such assumptions, conservative results are achieved as the worst operational condition is rare. On the other hand, the stochastic approach describes the channel state and channel uncertainty as random processes with mean or covariance known at transmitter, and focuses on average system performance without paying attention to the extreme error level. The system performance may break down when persistent extreme errors occur. It prompts the development of an outage probability-based approach that accounts the channel uncertainty by using probability measurement. This thesis develops the outage probability-based approach which provides robustness against the channel uncertainties for both single-user and multi-user MIMO systems, and offers a tight upper bound of average BER in robust adaptive modulation scheme.

1.2 Thesis Contribution

This section summarizes the contribution of this thesis, which focuses on robust transmit beamforming design against the channel imperfections using outage probability specification. The contribution can be divided into three parts: (1) designing a robust transmit beamformer for single-user MIMO system and maximizing the received average SNR performance; (2) building a robust adaptive modulation scheme and optimizing the system throughput; (3) proposing a robust SLNR-based downlink beamforming design for MU-MIMO system and improving the SINR reliability.

- (1) An outage probability-based beamforming is proposed in a single-user MIMO system under imperfect CSIT. This design maximizes the average SNR and takes the extreme conditions into account using the probability measurement. By keeping a low probability of the instantaneous SNR being below an acceptable level, the outage probability-based approach is more reasonable than the statistic-based beamformer and is less conservative than the worse-case beamformer. A deterministic form for probabilistic constraint is obtained which overcomes the main challenge in the probabilistic-constrained optimization problem. This approach achieves good average SNR performance with well-controlled outage probability, as well as a much broader error-tolerance range and more robustness against error variance misspecification. More importantly, the computational complexity of the probabilistic constraint is similar to the worst-case approach.
- (2) A robust adaptive modulation scheme based on a lower bound of average BER is established, which maximizes transmission rate and maintains an acceptable average BER performance. Without involving extra Monte-Carlo calculations or conservative channel conditions, an outage probability specification is introduced to provide a tight BER bound. The resulting final transmission is guaranteed to meet the target BER performance. The proposed robust adaptive scheme provides considerable improvement of the normalized system throughput and strong robustness against errors in CSIT, while guaranteeing the target BER under different scenarios.

- (3) A robust SLNR-based downlink beamformer is designed for both single-stream-per-user and multiple-stream-per-user MU-MIMO systems. The proposed transmit beamformer efficiently suppresses the inter-user-interference without perfect channel knowledge at transmitter. More specifically, a probabilistic constraint is introduced to keep a low outage probability that the leakage power of desired user is higher than an acceptable level. Under outage probability specifications, the extreme power leakage scenario is well controlled, which implicitly leads to the improvement of SINR reliability. For both cases, the proposed robust design reduces the leakage power as low as possible, and achieves the desirable BER performance as well as the good SINR reliability performance.

1.3 Thesis Outline

This thesis consists of three main chapters with background at the beginning, and conclusion followed with further work at the end. A brief outline of each chapter is as follows:

Chapter 2 discusses the wireless characteristics and MIMO modeling. It then focuses the transmit beamforming technique including the beamforming structure and optimal design as well as the consideration of the impact of channel uncertainties on transmit beamformer.

Chapter 3 designs a robust transmit beamformer for downlink single-user MIMO systems. The chapter first establishes the optimization problem that maximizes received SNR under imperfect CSIT. To ensure the robustness, an outage probability-based constraint is introduced by keeping a low probability of received SNR being below a pre-specified threshold. The chapter then obtains a deterministic form for this specification and converts the underlying probabilistic-constrained optimization problem into a convex problem, achieving the maximum average SNR. Finally, this chapter investigates the proposed robust design numerically, including average SNR performance, and robustness against to error and mismatched error variance.

Chapter 4 establishes a robust adaptive modulation scheme in the context of single-user MIMO

system. This chapter first formulates a robust scheme based on lower bound of average BER that maximizes the transmission rate while maintaining a target BER performance. It is proved that the probabilistic constraint works as an alternative BER constraint by keeping a low outage probability of instantaneous SNR below a pre-specified threshold. The resulting optimization problem is transferred into convex problem that can be solved by a standard toolbox. At the end of chapter, the proposed transmit beamforming is discussed about its benefits, including achieving the maximum transmission rate as well as the target average BER performance, and providing the strongest robustness against the channel imperfection.

Chapter 5 proposes a robust SLNR-based transmit beamforming design for both single-stream-per-user and multiple-stream-per-user MU-MIMO systems. The chapter first builds the leakage-based transmission scheme for the single-stream-per-user MU-MIMO system under imperfect channel information. The optimization problem is formulated as the maximization of average received SNR with a low outage probability of high leakage power. Using Markov's inequality, the probabilistic constraint is replaced by a deterministic form, and the resulting problem is solved through convex optimization tools. The chapter then extends the single-stream-per-user scenario into the multiple-stream-per-user case. Combining Alamouti code with SLNR-based solution, the hybrid scheme eliminates the inter-stream-interference with the help of Alamouti code, and suppresses the rest of interferences (inter-user-interference) by using the outage probability-based transmit beamformer. In the end, the chapter investigates its SINR reliability and average BER performance, and discusses the impact of parameters used in probabilistic constraint on SINR performance.

Chapter 6 summarizes the main results of this thesis, discusses the development of transmit beamforming design with scheduling algorithm for multi-user MIMO system, and outlines further work in cooperative transmission system.

Chapter 2

Background

The rapid growth of wireless communication services has brought several challenges in the design of reliable and efficient communication system. However, the physical susceptibility of the wireless channel limits the development of high speed and quality services transmission. Thus, channel characterization becomes a primary investigation before taking advantage of wireless channel. In response to reliable data transmission, the MIMO channel is investigated, which can provide diversity gain in the spatial domain without extra bandwidth expansion or transmit power. In order to exploit the benefits of MIMO system, transmit beamforming (precoding) is widely implemented for enhancing the performance and increasing the system throughput. A major drawback of most existing transmit beamforming techniques is that they require nearly perfect knowledge of the channel at the transmitter, which is typically not available in practice. The channel imperfections could lead to severe performance degradation. Hence, robust transmit beamforming design is required to provide robustness against the imperfect channel.

In this chapter, Section 2.1 characterizes the wireless channel, including a brief introduction of the wireless propagation, the small-scale fading channel and the frequent-nonselective narrow-band MIMO channel. As a powerful approach to exploit the benefits of MIMO channel, transmit beamforming technique is introduced in Section 2.2. This section first introduces the system model and transmit channel information acquisition in Subsection 2.2.1 and 2.2.2, respectively. In the context of its structure, depicted in Subsection 2.2.3, the transmit beamforming design is discussed in both perfect CSIT and imperfect CSIT cases.

2.1 Channel Characterizations

In wireless communication channel, the transmission path between transmitter and receiver varies from simple line-of-sight to multiple paths induced by the reflection from multiple ran-

dom scatters in the propagation environment. Since the combination of these paths creates a multi-tap impulse response, the wireless channel is extremely random and therefore is often characterized statistically. Under the time-varying wireless channel, it is critical to acquire channel state information. Different from channel acquisition at receiver that could produce accurate information by aiding pilots, transmit channel acquisition suffers information inaccuracy because of the errors from reciprocity/feedback channel [1].

2.1.1 Wireless Propagation

In wireless propagation, a signal is transmitted through wireless channel and arrives at the destination along multipath, arising from scattering, reflection, refraction, or dielectrics. The signal power drops off due to two effects: *large-scale* propagation and *small-scale* propagation [2]. In small-scale fading, the signal fades rapidly as the receiver moves, while the local average signal changes much more gradually with distance in large-scale fading. Fig. 2.1 illustrates large-scale and small-scale fading variations graphically.

Large-Scale Propagation

Large-Scale propagation captures the path loss and shadowing, where the average received signal power decreases logarithmically with distance [2]. The path loss is the difference between the effective transmitted power and receive signal power, and represents the signal attenuation as a positive quantity. On a log-log scale, the average large-scale path loss is a straight line shown in Fig. 2.1. The shadowing is caused by large terrain features, such as small hills and tall buildings over a long distance. It makes the main signal path from transmitter to receiver obscured by reflection, scattering and diffraction, representing as a dash line in Fig. 2.1.

Small-Scale Propagation

Small-Scale propagation, or simply *fading*, captures the variation of amplitudes, phases, or multipath delays of the signal over a short period of time or travel distance, so that large-scale propagation or path loss effects may be ignored [2]. A number of signals (two or more) arrive at receiver through different paths, known as *multipath* waves. Depending on their phase

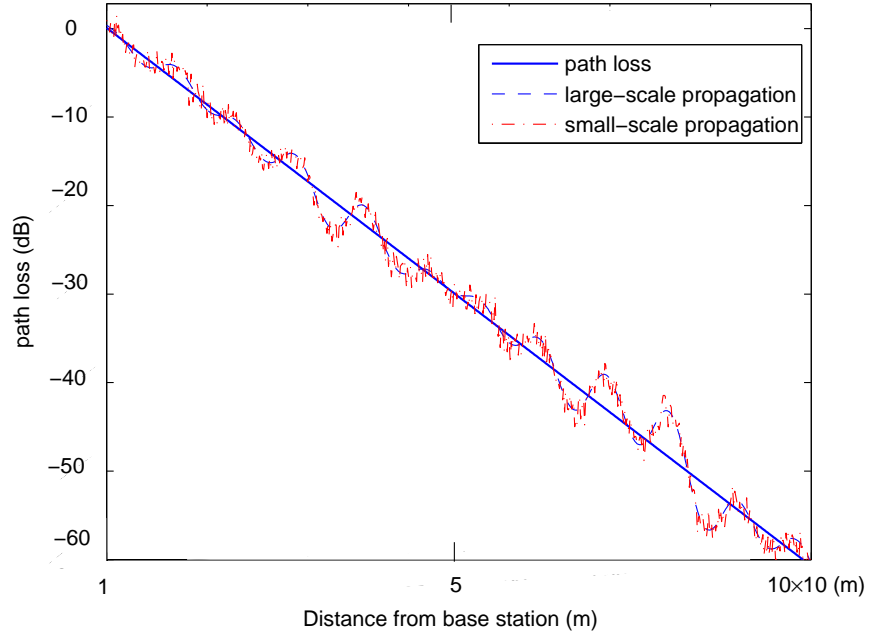


Figure 2.1: *Illustration of path loss, large-scale and small-scale*

and amplitude values, the received signals are combined constructively or destructively, resulting in the rapidly-changed signal strength, random frequency modulation and time dispersion, graphically displayed as dot line in Fig. 2.1.

This thesis will focus on the small-scale channel characteristics, leaving the large-scale characteristics to the references such as [3].

2.1.2 Small-Scale Fading Channel

As mentioned in previous subsection, multipath in the radio channel creates small-scale fading which could be influenced by following factors [2]:

- **Multipath propagation** : The signal energy is dissipated in amplitude, phase, and time because of the presence of reflecting objects and scatters which creates a constantly changing environment. More specifically, the random phase and amplitude of the different multipath components caused fluctuations in signal strength, thereby inducing small scale fading, signal distortion, or both. Moreover, multipath propagation can lengthen

the transmit time, resulting in inter symbol interference.

- Speed of the mobile : The relative motion between the base station and the mobile causes that the path lengths traveling from source to mobile are different. It changes the phase of the received signal with apparent change in frequency of a wave. This phenomenon is known as the *Doppler shift* which could be positive or negative depending on whether the mobile receiver is moving toward or away from the base station. Moreover, different signal components with multiple Doppler shifts contribute to a single fading channel tap, known as *Doppler spread*. In general, it is inversely proportional to *coherence time* that characterizes the time varying nature of the frequency dispersiveness of channel in the time domain. High mobility commonly results in large Doppler spread and fast channel time variation, consequently with high temporal channel selectivity.
- Speed of surrounding objects : If the surrounding objects in the transmission channel are in motion, a time varying Doppler shift will be induced on multipath components. This effect dominates the small scale fading as long as the surrounding objects have a higher rate than the mobile. Otherwise, it may be ignored and only the speed of the mobile is taken into account.
- The transmission bandwidth of the signal : In wireless propagation, the transmitted signal arrives via multiple paths. The identical signal received at the destination thus arrives at different time with different angles of arrival. The difference between the arrival moment of the first multipath component and the last one is called *delay spread*. The reciprocal of the delay spread is an approximative measure of the *coherence bandwidth* of the channel. Both delay spread and coherence bandwidth describe the time-varying dispersive nature of wireless propagation. If the transmission bandwidth exceeds the coherence bandwidth, an equalization is needed.

Depending on the relationship between signal parameters and channel parameters, the transmitted signals will undergo different types of fading.

Flat or frequency selective fading

If the channel coherence bandwidth is greater than the bandwidth of the transmitted signal, the received signal undergoes flat fading, otherwise, frequency selective fading. The flat fading channel is an amplitude varying channel, and related to *narrowband* channel, where the bandwidth of signal is narrow compared to the flat fading channel bandwidth. Under such conditions, the spectral characteristics of the transmitted signal are preserved, but the power of the received signal varies from time to time. On the other hand, the frequency-selective fading channel is known as *wideband* channel, when the bandwidth of the signal is wider than the channel coherence bandwidth. In this case, different frequency components of the signal are affected independently, and all parts of the signal could not be simultaneously affected by a deep fade. However, due to the dispersion of frequency selective fading, the signal energy associated with each symbol will be spread out in time. The resulting transmitted symbols are adjacent in time to interference with each other, known as *inter symbol interference*.

Fast or slow fading

Depending on the relative changes between the transmitted signal and the channel, a channel may be classified either as a *fast fading* or *slow fading* channel. In a fast fading channel, the channel response changes rapidly within the symbol duration, where the change rate is measured by Doppler spread. Frequency dispersion is caused by high Doppler spread, leading to a considerable variation in amplitude and phase of the transmitted signal. When the channel response changes much slower than the transmitted signal, the channel is known as a slow fading channel which implies that the Doppler spread of channel is much less than the bandwidth of signal. The amplitude and phase change imposed by the channel therefore can be considered roughly constant.

In this thesis, the slow fading narrowband channel is taken into account, which is a single-tap, frequency nonselective fading channel. Each tap contains multiple elements between all pairs of transmit-receive antenna, and the channel has the same response over the entire system bandwidth. In the following subsection, a model of the frequency-flat narrowband MIMO channel will be introduced.

2.1.3 Narrowband MIMO Channel Model

The MIMO wireless channel is created by using multiple antennas at both the transmitter and the receiver. It can be degenerated into the multiple-input and single-output (MISO) that equips only one antenna at receiver and the single-input and multiple-output (SIMO) with a single antenna at receiver. The channel contains multiple paths between the transmit and receive antennas. In this thesis, each path is assumed as frequency-flat narrowband channel.

Consider a complex narrowband MIMO channel with N transmit antennas and M receive antennas in Fig. 2.2, it can be presented as a matrix \mathbf{H} of size $M \times N$

$$\mathbf{H} = \begin{bmatrix} h_{11} & h_{12} & \dots & h_{1N} \\ h_{21} & h_{22} & \dots & h_{2N} \\ \vdots & & & \\ h_{M1} & h_{M2} & \dots & h_{MN} \end{bmatrix}, \quad (2.1)$$

in which h_{ij} indicates the channel from transmit antenna j to receive antenna i . The elements of narrowband MIMO channel matrix are assumed to be independent and identically distributed (i.i.d) [2]. Take $\mathbf{H} \in \mathbb{C}^{M \times N}$ for example, at least $M \times N$ scatters are required to get the i.i.d channel. In practice, however, because of insufficient spacing between antenna elements and limited scattering in the environment, the fading is not always independent, causing the correlation between each path [4]. In general, different assumptions about the channel matrix \mathbf{H} lead to different approaches to improve the system performance.

Consider the single-user narrowband MIMO wireless system, the base station equipped with N transmit antennas communicates with single user which has M antennas at receiver. In each time slot, the data vector \mathbf{x} is transmitted into fading channel. The received signal can be expressed as

$$\mathbf{y} = \mathbf{H}\mathbf{x} + \mathbf{n}, \quad (2.2)$$

where

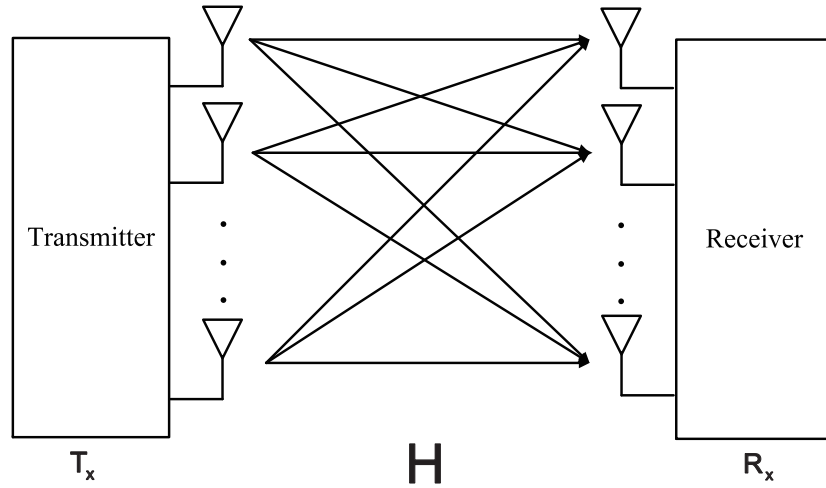


Figure 2.2: *Wireless communication channel*

- $\mathbf{H} \in \mathbb{C}^{M \times N}$ presents the wireless channel, and is assumed as flat fading channel.
- $\mathbf{x} \in \mathbb{C}^{N \times 1}$ contains the signal transmitted to wireless channel.
- $\mathbf{y} \in \mathbb{C}^{N \times 1}$ contains the received signals at input of MRC receiver.
- $\mathbf{n} \in \mathbb{C}^{N \times 1}$ is additive white Gaussian noise (AWGN) at receiver. Each element of \mathbf{n} is complex normally distributed with zero mean and covariance matrix $\sigma^2 \mathbf{I}_M$, i.e. $\mathbf{n} \sim \mathcal{CN}(\mathbf{0}, \sigma^2 \mathbf{I}_M)$.

The channel model in (2.2) indicates that the transmitted signal vector \mathbf{x} is projected onto the channel matrix \mathbf{H} and therefore, the number of independent data streams that can be supported must be at most equal to the rank of the channel matrix.

2.2 Transmit Beamforming Technique

Transmit antenna array system has the potential to promise higher data rates and improve link reliability without consuming extra bandwidth and transmit power. Including MISO to MIMO architectures, the following benefits are provided compared to SISO systems.

- **Array gain:** Array gain is available through processing at transmitter, resulting in an

Gain over SISO	Array/Diversity gain	Multiplexing gain	Interference reduction
$\text{MISO } (\mathbf{H}_{M \times 1})$ Transmit diversity Beamforming/Precoding SDMA	✓ ✓		✓
$\text{MIMO } (\mathbf{H}_{M \times N})$ Transmit/Receive diversity CCI nulling SDMA Spatial Multiplexing	✓	✓	✓ ✓

Table 2.1: Key Benefits of Different Smart Antenna Architectures

increase of average receive SNR. Transmit array gain depends on the number of transmit antennas, and can achieve N with perfect channel state information at transmitter.

- **Diversity gain:** Diversity gain relies on the independence of fading paths in time/ frequency/ space, used to mitigate fading in wireless links. Spatial (antenna) diversity is preferred over time/frequency diversity due to no expenditure in transmission time or bandwidth. Different from array gain, the spatial diversity gain is possible by using suitably-designed transmit signal, such as space-time coded signal [5, 6], precoding/beamforming signal [7, 8], and spatial division multiple access (SDMA) [9]. Given a MIMO channel with N transmit antennas and M receive antennas, the spatial diversity can achieve $(N \times M)$ if each link of MIMO channel fades independently and transmit signal is suitably constructed.
- **Spatial multiplexing gain:** Spatial multiplexing gain is realized by transmitting independent signals to individual antennas. Under rich scattering channel environment, the receiver can separate the different streams to yield a linear increase in capacity. Given a MIMO channel with N transmit antennas and M receive antennas, MIMO channel can offer a linear increase, $\min(N, M)$, in capacity without additional power and bandwidth.
- **Interference reduction:** With the help of multiple antennas, the interference can be reduced at both transmitter and receiver sides by using the co-channel interference nulling techniques [9]. Exact knowledge of the channel is required to reduce interference at receive side. For transmitter, the interference energy can be minimized under the signal

preprocessing scheme, such as SDMA [9].

Moreover, Table 2.1 summarizes the basic architectures with different algorithms. Note that the array gain, diversity gain and interference reduction are provided by both MISO and MIMO system, but multiplexing gain that increases the point-to-point throughput is only offered by MIMO systems.

There are two main techniques used to exploit the benefits of MIMO system under transmit antenna arrays:

- Space-time coding provides diversity gain in fading environment without any knowledge of spatial channel at transmitter [6, 9–13].
- Transmit beamforming/precoding provides spatially matched transmission or mitigates interference under perfect CSIT [7, 8, 14–20].

This thesis will only focus on the transmit beamforming technique, leaving the space-time coding as reference [5, 21].

In this section, the configuration of MIMO system with precoding will be illustrated, followed with the transmit channel acquisition and the beamforming structure. Finally, the design of transmit beamforming will be discussed based on both perfect and imperfect CSIT scenarios.

2.2.1 System Model

In the context of the narrowband MIMO channel, a single data is exploited by Gaussian distributed codeword into a vector \mathbf{s} and then is multiplied with a linear precoder \mathbf{C} before being transmitted through the flat fading MIMO channel, illustrated in Fig. 2.3. Based on (2.2), the received signal now can be presented as

$$\mathbf{y} = \mathbf{H}\mathbf{x} + \mathbf{n} = \mathbf{H}\mathbf{C}\mathbf{s} + \mathbf{n} , \quad (2.3)$$

where

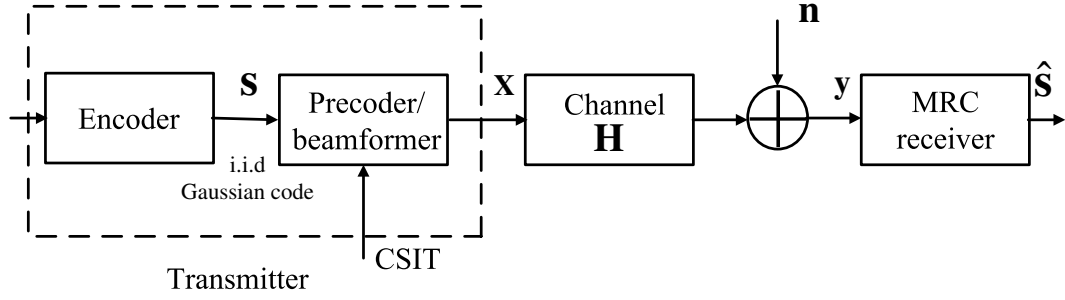


Figure 2.3: Configuration of a system with linear precoding

- $\mathbf{C} \in \mathbb{C}^{N \times L}$ denotes transmit beamforming matrix.
- $\mathbf{s} \in \mathbb{C}^{L \times 1}$ represents the transmit data exploited by Gaussian distributed codeword.

The receiver is assumed to have perfect knowledge of beamforming matrix \mathbf{C} and channel information \mathbf{H} (reasons referred to Subsection 2.2.2). Since it can maximize the signal to noise ratio (SNR) at output of thermal, the maximal ratio combining (MRC) technique is used at receiver. The average SNR at the output of MRC receiver can be expressed as

$$\text{SNR} = \frac{\mathbb{E} [(\mathbf{H}\mathbf{C}\mathbf{s})^H (\mathbf{H}\mathbf{C}\mathbf{s})]}{\mathbb{E} [\mathbf{n}^H \mathbf{n}]} = \frac{E_s}{N_0} \text{tr} \{ \mathbf{C}^H \mathbf{H}^H \mathbf{H} \mathbf{C} \} , \quad (2.4)$$

where $E_s = \mathbb{E} [||\mathbf{s}||^2]$ denotes the average signal power.

Regarding the importance of CSIT to transmit beamforming design, the next subsection will discuss the methods of channel acquisition at transmitter side.

2.2.2 Transmit Channel Acquisition

When a signal enters the wireless channel after leaving the transmitter, the receiver tries to detect the channel-modified signal correctly based on channel estimates, meanwhile the transmitter attempts to transmit data dependably according to channel conditions. In this case, the

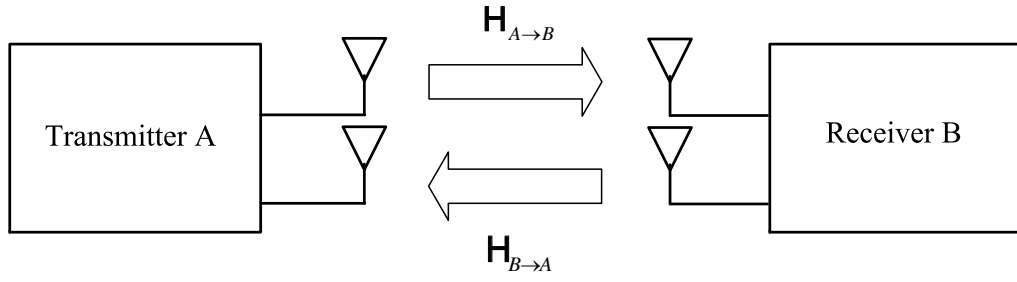


Figure 2.4: *Reciprocity-based methods*

system performance considerably depends on the channel information at receiver and transmitter.

In general, the receiver can estimate the channel accurately. The common method for obtaining channel estimates depends on training signals completely known at receiver, and estimates the channel using a least square approach [5]. Alternatively, blind techniques without explicit transmit signals estimate channel information based on second-order statistics or finite alphabet modulus [22, 23]. A more promising method is the semi-blind method which couples training-based and blind techniques for the unknown symbols [24].

Compared with the channel information obtained at receiver, it is difficult to guarantee the channel accuracy at transmitter, because of the errors in time-varying forward channel and limited-capacity feedback channel. Two general techniques are used in channel estimation at transmitter: reciprocity and feedback.

Reciprocity-based method : The reciprocity principle suggests that forward channel from antenna A to another B is identical to the reverse channel, which requires the same frequency in both forward and reverse channels at the same time and the same antenna locations. In a full-duplex system, this principle suggests that the transmitter at A can obtain the forward channel from the reverse channel, which the receiver at A can measure, as illustrated in Fig. 2.4 that the transmitter A can obtain the forward channel ($A \rightarrow B$) from the reverse channel ($B \rightarrow A$).

In practice, channel acquisition based on reciprocity can be implemented in time-division-

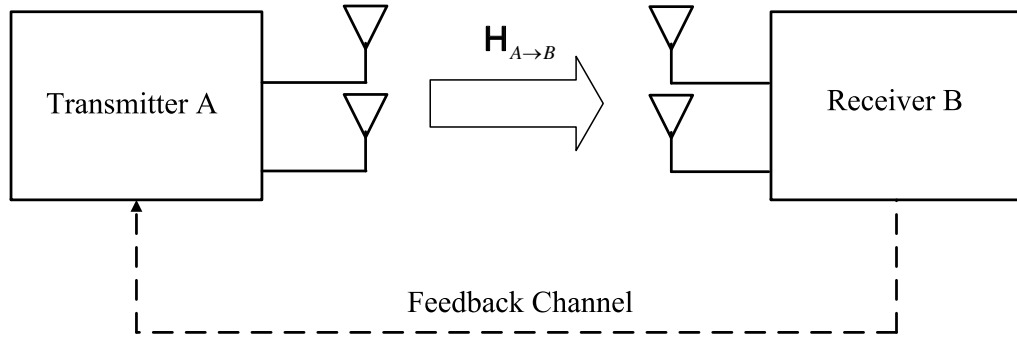


Figure 2.5: *Feedback-based methods*

duplex (TDD) systems. In TDD systems, the forward and reverse channel is generally assumed identically, with a negligible turn-around delay comparing with channel coherence time. For frequency-division-duplex (FDD) systems, the frequency offset between the forward and reverse links is much larger than the channel coherence bandwidth, which makes the reciprocity principle not applicable in FDD systems [5].

Feedback-based method : The channel information can be obtained using feedback channel, where channel information is measured at receiver B and resent to the transmitter A over the reverse link, depicted in Fig. 2.5. Through feedback channel, the outdated error occurs with large feedback delay between channel measurement at receive B and transmitter A , as well as quantization error because of limited feedback channel capacity. Feedback-based method can be applied in both TDD and FDD systems, but is more commonly used in FDD system. Although methods of reducing feedback overhead are of practical importance [25], it is not a focus of this thesis.

This thesis follows the assumption that imperfect CSIT is obtained while accurate CSI at receiver estimated. The models of channel uncertainties will be built up and the impact on transmit beamforming design will be discussed in the following section.

2.2.3 Transmit Beamforming Structure

Transmitter contains an encoder and a linear precoder, as shown in Fig. 2.6. The encoder assumes no channel knowledge, and could include either a channel code, space-time code or both [5]. On the other hand, the precoder that exploits the CSIT can be viewed a processing block to enhance system performance based on the available CSIT. The details of these processing blocks are discussed next.

Encoder Structure

The encoder intakes data bits and performs necessary coding for error correction, and then maps the coded bits into vector symbols. There are two methods in the symbol-mapping block: spatial multiplexing and space-time coding [5]:

- Spatial multiplexing: The output bits of channel coding are generated as independent bit streams which are mapped into vector symbols and fed directly into the precoder.
- Space-time coding: The output bits of channel coding are mapped into symbols first. These symbols are then processed in space-time fashion.

Note that these two approaches have the difference in the temporal dimension of the symbol-level code. Using spatial multiplexing, the symbols are spread over the spatial dimension alone, so that there is just one symbol fed into precoder block. Space-time coding, on the other hand, spread symbols over both the spatial and the temporal dimensions. Therefore, the spatial multiplexing can be considered as a special case of space-time coding with the block length of one [5].

In this thesis, the encoder is predetermined and is not the design target. It is assumed that the spatial multiplexing block spreads the output bits of channel coding by using a Gaussian-distributed codeword with zero mean and unit covariance, referred as *white multiplexing*. In the context of the system model (2.5), the covariance of the mapped symbol vector \mathbf{s} can be

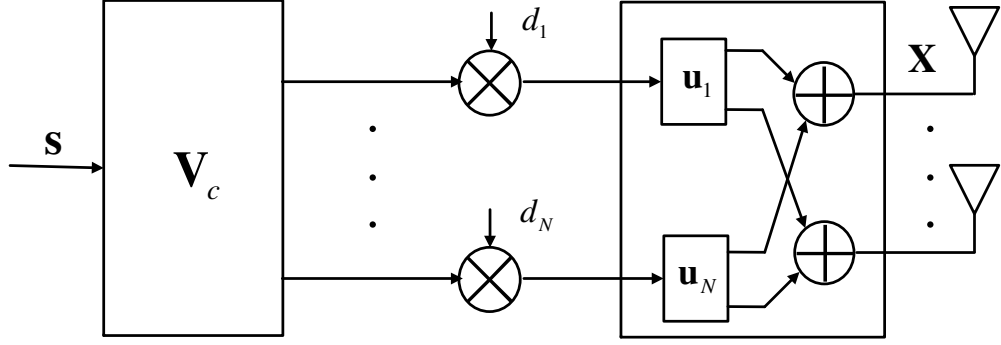


Figure 2.6: A linear beamforming structure

expressed as

$$\mathbb{E}[\mathbf{s}\mathbf{s}^H] = E_s \mathbf{I}_N, \quad (2.5)$$

where the covariance matrix of codeword is diagonal matrix.

Linear Precoder Structure

The precoder is a separate transmit processing block from the output symbols of the encoder block. A linear precoder combines input shaper and multimode beamformer with power allocation. The SVD of beamforming matrix \mathbf{C} can be written as

$$\mathbf{C} = \mathbf{U}_c \mathbf{\Sigma}_c \mathbf{V}_c^H, \quad (2.6)$$

where the right singular vector \mathbf{V}_c and the left \mathbf{U}_c are orthogonal and unitary matrices. Note that each column of \mathbf{U}_c represents a beam direction (pattern), and the matrix \mathbf{U}_c is also the eigenvectors of $\mathbf{C}\mathbf{C}^H$, which is often referred to eigen-beamforming with the corresponding beam power $\mathbf{\Sigma}_c^2$. Independent from CSIT, the right singular vector \mathbf{V}_c works as an input shaping matrix and combines the output of encoder to feed into each beam at each time instant. The optimal matrix of \mathbf{V}_c is achieved when it is equal to the left singular vector of code covariance [21]. The incoming vector-fed symbols then are allocated with the square singular values $\mathbf{\Sigma}_c$ as beam power, and finally transmitted through left singular vector \mathbf{U}_c , shown in Fig. 2.6.

To conserve the total transmit power, the precoder must satisfy

$$\text{tr}\{\mathbf{C}\mathbf{C}^H\} = 1. \quad (2.7)$$

It indicates that the sum of power over all beams is a constant. For the individual beam power allocation, it could be different according to the SNR, the CSIT, and the design criterion.

Under the assumption that the covariance of encoder is identity, this thesis focuses on the linear precoding design based on available CSIT. The next subsection designs the downlink beamformer with perfect CSIT.

2.2.4 Transmit Beamforming With Perfect CSIT

Perfect CSIT not only increases channel capacity, enhances system reliability but also reduces receiver complexity. According to the discussion in Subsection 2.2.3, the input shaping matrix is determined by the input code alone without channel information, while the precoding matrix including power allocation is determined by the CSIT. Moreover, in the special case of isotropic input (2.5), the optimal \mathbf{V}_c is an arbitrary unitary matrix and usually omitted [5]. Since the input-shaping matrix does not involve in the power constraint (2.7), this subsection only focuses on the designs of optimal precoding based on perfect CSIT.

Transmit Beamforming Matrix

Different from the input-shaping matrix, the beamforming matrix is a function of the CSIT. In the following, the optimal beamforming solution are presented based on perfect CSIT.

Consider a MIMO channel with $M \times N$ channel gain matrix, shown in (2.1). Each channel realization is perfectly known at both transmitter and receiver. Taking the singular value decomposition (SVD), it has

$$\mathbf{H} = \mathbf{V}_h \mathbf{\Sigma}_h \mathbf{U}_h^H, \quad (2.8)$$

where $\mathbf{V}_h \in \mathbb{C}^{M \times M}$ and $\mathbf{U}_h \in \mathbb{C}^{N \times N}$ are unitary matrix with $\mathbf{V}_h \mathbf{V}_h^H = \mathbf{I}_M$ and $\mathbf{U}_h \mathbf{U}_h^H = \mathbf{I}_N$.

$\mathbf{I}_N, \Sigma_h \in \mathbb{C}^{M \times N}$ is a diagonal matrix with singular values of \mathbf{H} , which is the square root of its corresponding eigenvalue of $\mathbf{H}^H \mathbf{H}$.

The optimal beam direction for all criteria are matched to the right singular vector of channel matrix [8, 16, 26–31]

$$\mathbf{U}_c = \mathbf{U}_h. \quad (2.9)$$

In this case, the input symbol vector is decoupled into orthogonal spatial modes of MIMO channel. On the other hand, the optimal power allocation has multiple solution. For example, to maximize the average SNR, the optimal power solution is to allocate all power on the strongest eigen-mode of the channel [16, 26, 27]. It has the same power solution when minimizing the mean square error (MMSE) between transmit signals and receive signals [28]. Under the system ergodic capacity criteria, the transmit power is allocated in water-filling fashion, that is, higher power is loaded on the direction that has larger eigenvalue, and reduced or no power in the weak directions [8, 29–31].

2.2.5 Transmit Beamforming With Imperfect CSIT

In real scenarios, perfect channel information is not available at transmitter side because of the error induced by limited feedback resources, delay or quantization, which leads to a severe system performance degradation. It motivates the effort to develop a robust precoding scheme against the channel imperfection. Before the development of robust transmit beamformer, the imperfect channel model will be outlined first. The solution of precoding with imperfect CSIT will be discussed with respect to the methods of addressing the channel uncertainty.

Imperfect Channel Model

Since the error in channel estimate at transmitter is inevitable, it is necessary to modify these uncertainty before exploiting the benefits of MIMO systems. Different sources of error can be identified depending on the CSI acquisition methods. In case of exploiting the channel reciprocity, the error from the uplink estimates is usually considered as Gaussian-distributed

random variable. When a feedback channel is used, additional errors induced by quantization are assumed as uniformly-distributed random variables. More details are outlined as follows:

- *Error in TDD systems:* The TDD system allows transmitter to estimate the users' channels on the uplink channel. As the channel is time varying, outdated channel knowledge could be obtained, introducing channel uncertainties. When the channel of each user is uncorrelated, each element of the error matrix can be considered as independent and Gaussian distributed random variable with zero mean and a given variance. Over correlated uplink channel, the error can be identified as jointly Gaussian with zero mean and a given covariance matrix.
- *Error in FDD systems:* For FDD systems, the transmitter could estimate CSIT through feedback channel. However, the feedback channel with finite capacity has to quantize the channel response, resulting in the imperfection. The quantization methods include scalar quantized and vector quantized. In scalar quantization, the real and imaginary parts of all the components in channel matrix are quantized uniformly with a given quantization step. Accordingly, the errors can be determined as independent and uniformly distributed random variables on a symmetric bounded interval. On the other hand, in the vector quantization¹, each estimated channel with its own uncertainty is determined based on the quantized index, corresponding to a given index region. And the error could be uniformly distributed over the ellipsoid volume.

Over the flat-fading narrowband wireless MIMO channel (illustrated in Subsection 2.1.3), one only has access to imperfect channel estimate, $\hat{\mathbf{H}} \in \mathbb{C}^{M \times N}$, that can be modeled as follows,

$$\mathbf{H} = \hat{\mathbf{H}} + \mathbf{E}, \quad (2.10)$$

where $\mathbf{E} \in \mathbb{C}^{M \times N}$ is the error in channel estimates. To address the channel uncertainty, current

¹Consider a space with N points $\{\mathbf{H}_i\}$, each one representing the region given by $\mathbf{H}_i + \mathcal{R}_i$, the i th index corresponding to \mathbf{H}_i is sent (the number of bits for the feedback is equal to $\log_2(N)$). Each region \mathcal{R}_i is polyhedron defined by the intersection of a finite number of half-spaces. In this case, the uncertainty regions depend on the channel estimates

robust precoding strategies can be classified into the deterministic, statistical and probabilistic approaches. The corresponding solutions will be discussed as follows.

Deterministic Approach

The deterministic approach considers the error belongs to a predefined uncertainty region and tries to provide the best performance in the worst-case CSI mismatch scenario. Consequently, it is also called as worst-case approach. A common assumption is that the error is bounded in a spherical region [12, 32–44],

$$\|\mathbf{E}\|_F \leq \xi, \quad (2.11)$$

where $\|\cdot\|_F$ represents the Frobenius norm operation, and ξ is a pre-specified bound of uncertainty region. According to the distribution of errors, the size of uncertainty region is determined by the inverse cumulative density function of the probability that provides the required QoS to the user [16]. Regarding to imperfect channel model in (2.10), the estimated channel $\hat{\mathbf{H}}$ perturbs within an ellipsoid centered at a nominal channel \mathbf{H} .

Based on the assumption of deterministic error (2.11), the worst-case technique is used to optimize the worst system performance. The optimal transmit directions are just the right singular vector of the nominal channel \mathbf{H} [12, 32–44]. This means that the eigenmode transmission is still optimal for the worst-case design [27]. Consequently, the power allocation problem is simplified to the scalar power problem, and its optimal solution is designed based on multiple criteria. For instance, [35] proposes a robust transmit beamforming design that embraces channel uncertainties both in the array response and the covariance. As for MIMO channel, [36] minimizes the worst-case MSE with a linear equalization. Moreover, in multi-user MIMO systems, the worst-case approach is implemented to minimize the total transmit power [37], and to optimize the QoS requirements, including minimizing MSE [34], maximizing SINR [38–41], maximizing SLNR [12, 42–44]. Note that although the above solutions of optimal power allocation could be different, they are consistent with the water-filling fashion.

In the worst-case approach, since the error are norm-bounded in region, the system performance is optimized according to the worst scenarios, no statistics of the error in CSIT is required to

be known. However, in wireless communication, it is not practical to use deterministic upper bounds on the norm of the channel errors. Furthermore, the worst-case approach maximizes the system performance based on the extreme cases. In this case, only conservative results could be achieved since the worst-case scenarios occurs with a very low probability.

Statistical Approach

The statistical approach addresses the error by using statistical models for the channel or mismatch between the presumed and actual transmitter CSI. In this approach, the channel uncertainty is assumed as Gaussian distributed, and its mean or covariance is known at transmitter.

The optimal power allocation still follows the water-filling fashion, that is, the weakest eigenmode of $\mathbf{C}\mathbf{C}^H$ may be dropped to ensure its positive semi-definiteness, and the total transmit power is re-allocated among the remaining modes [8, 14, 15, 28–30, 45–52]. The optimal beam directions, on the other hand, depend on either mean, covariance or both under different criteria. Taking BER criteria for example, the optimal beam directions depend on both the mean and covariance, but as the SNR increase, they asymptotically depend on the covariance alone [15, 21, 45, 48, 49]. For other criteria, the optimal beamforming matrix is approximately matched to the eigenvectors of the average channel gain, including optimizing the ergodic capacity [8, 29, 45], the received SNR [14, 30, 46, 47], the MMSE [28] and the average SINR for the multi-user case [14, 28, 50–52].

However, this approach is model based, and therefore, can suffer from mismodeling of the CSIT or channel statistics. For example, in real scenario, the accurate channel statistics is hard to be worked out, and mismatch could exist. Based on over-predicted or under-predicted channel statistics, the statistical approach could cause the performance degradation [16]. Moreover, this approach could break down if a persisting serious error occurs, because it only focuses the long-term performance without paying attention to extreme scenarios.

Probabilistic Approach

The probabilistic approach assumes that the imperfect channel estimate is deterministic with

Gaussian-distributed error. The impact of CSIT error on system performance is considered proportionally. More specifically, this approach measures the channel uncertainties by using the outage probability, i.e., the probability that the performance degradation caused by the error falls below a certain threshold. This approach is related to the statistical approach, which assumes that the covariance of the mismatched error matrix is known at transmitter.

Under the above assumption, the eigen-mode transmission is still the optimal solution for all criteria over single-user MIMO systems [41, 53–61]. However, the optimal power allocation is quite different from that obtained by the aforementioned approaches, which depends on the statistics of error matrix and the design criteria as well as the system configuration. Under the assumption of the identically and independently Gaussian-distributed CSIT errors, the outage probability specification for MISO system contains a single/mixture Gaussian distribution, and the solution is obtained by solving a convex optimization problem as long as that the outage probability specification is replaced by a deterministic form [41, 53–57, 61]. Note that such a transformation is easily achieved by using Markov's inequality or Q-function [1]. For MIMO system, the probabilistic approach involves a mixture of noncentral χ^2 distribution in the single user case [59, 60], and a mixture of noncentral Wishart distribution in the multi-user case [62], which causes the difficulty in obtaining the optimal solution of power allocation.

In the following chapters, the robust precoding for the single-user MIMO system will be considered first, followed by the devolvement of robust adaptive modulation scheme. Finally, the robust downlink beamforming matrix is designed for both the single-stream-per-user and multiple-stream-per-user multi-user MIMO systems.

Chapter 3

Probabilistic-Constrained Beamforming Design for Downlink Single-User MIMO Systems

Transmit beamforming (precoding) is a powerful technique for enhancing performance of wireless multiantenna communication systems. Standard transmit beamformers require perfect channel information at transmitter and are sensitive to errors in channel estimation. In practice, such channel imperfections are inevitable at transmitter due to the error induced by finite feedback resource, quantization and outdated information, leading to a significant performance degradation. Hence, it motivates robust design against channel imperfections. This chapter proposes an outage probability-based approach to maximize the average SNR and take the extreme conditions into account using the probability with which they may occur. Simulation results show that the proposed beamformer offers higher robustness against channel uncertainty than several popular transmit beamformers.

3.1 Introduction

Chapter 2 suggested that the transmit beamformer can further strengthen advantages of MIMO systems, such as higher data rates, better quality of service (QoS) and larger coverage areas, by exploiting CSIT. In the absence of accurate CSIT, the system performance degrades severely when non-robust beamformer is implemented. Hence, robustness is a crucial issue for transmit beamforming design.

Current robust transmit beamforming designs can be categorized into two classes with respect to imperfect CSI characterization: the deterministic (or worst-case) and stochastic approaches. The worst-case approach describes channel uncertainty in deterministic model with

norm-bounded error. In this case, the robust design aims at optimizing the worst-case performance, but leads to overly conservative results as the worst operational condition is rare. This philosophy has been proposed in [63, 64], and applied to robust receive beamformer design against mismatches in the array response and the covariance matrix [33, 34]. Based on imperfect CSIT, robust transmit beamforming can provide the optimum uniform power allocation for point-to-point MIMO channel [16, 26, 27, 65–67], MISO broadcasting channel [37, 68]. In the stochastic analysis, the channel is usually modeled as complex random matrix with normally distributed elements, with perfect knowledge of channel statistics at transmitter side. Here, the transmit beamforming design focuses on the optimization of average system performance without paying attention to the extreme error level. With the help of channel mean or channel covariance, examples include maximizing the ergodic capacity of MIMO/MISO channel [8, 29, 30], minimizing symbol error rate (SER) [14, 15, 45, 48, 49], and minimizing mean square error (MMSE) over linear transceiver scheme [28]. Recently, a related direction is the outage probability-based approach which considers the extreme scenarios proportionally by introducing probabilistic constraints on quality of service (QoS), including transceiver design of MISO system [56–58], receive adaptive beamforming [55, 69, 70]. In the MISO case, the probabilistic constraint only involves a single/mixture Gaussian distribution, because of single antenna equipped at receiver. The underlying design problem is easily solved by the standard convex tools as long as it is converted into a deterministic form [56–58]. However, the design becomes much complicated in MIMO systems, since the probability could be a mixture of noncentral χ^2 -distribution. There is no analytical solution for the probability of outage to a pre-specified threshold for MIMO case in the previous works.

This chapter introduces a probabilistic constraint into robust transmit beamforming design for downlink single-user MIMO systems, with the following contributions:

- A robust transmit beamformer is developed to maximize the average received SNR performance and ensure the robustness against channel imperfections by introducing the outage probability-based approach. This probabilistic constraint considers the extreme scenario proportionally by keeping a low probability of the received SNR being below

an acceptable level.

- A deterministic form is obtained for the probabilistic constraint, which overcomes the main challenge in the proposed beamforming design. In the context of MIMO system, the outage probability involves the weighted noncentral χ^2 -distributed random variables, which is more complicated compared to the MISO case where only Gaussian distribution is regarded. The resulting robust design is solved through convex optimization methods.
- An outstanding average received SNR performance is achieved under the proposed robust scheme. Furthermore, it also demonstrates a much broader tolerance range as well as higher robustness against mismatched error variance

The chapter is organized as follows. Section 3.2 proposes an outage probability-based approach which is formulated as a probabilistic-constrained optimization problem. Section 3.3 is devoted to reformulation of the outage probability specification into a convex deterministic form. Section 3.4 gives numerical results. Finally, Section 3.5 concludes this chapter.

3.2 Robust Design Based on Probabilistic Constrained Optimization

This chapter considers the MIMO wireless communication system. Due to limited feedback resources, delay or quantization errors, one has only access to imperfect channel information, which could degrade the system performance. To tackle the performance degradation, a probabilistic constraint approach is introduced to provide robustness against channel imperfections. The proposed algorithm maximizes the average SNR while keeping the probability for SNR being below a pre-specified threshold low. It has the advantage of achieving optimal overall performance while providing quality control for the extreme case. In contrast to the worst-case approach that focuses on the worst-case performance [16,26,27], the probability constraint takes the errors into account proportionally. On the other hand, the worst case scenario that is ignored by the stochastic approach [14, 15, 71] is also considered in the proposed approach.

Considering the MIMO channel imperfections model in (2.10)

$$\mathbf{H} = \hat{\mathbf{H}} + \mathbf{E} .$$

It is assumed that the error matrix $\mathbf{E} \in \mathbb{C}^{M \times N}$ consists of i.i.d complex normally distributed entries with zero mean and variance σ_e^2 , that is, $e_{ij} \sim \mathcal{CN}(0, \sigma_e^2)$, $(i = 1, \dots, M, j = 1, \dots, N)$. Hence, the received SNR in (2.4) becomes a function of the channel estimate $\hat{\mathbf{H}}$ and the random error \mathbf{E} ,

$$f(\hat{\mathbf{H}}, \mathbf{E}) = \frac{E_s}{N_0} \text{tr} \left\{ \mathbf{C}^H (\hat{\mathbf{H}} + \mathbf{E})^H (\hat{\mathbf{H}} + \mathbf{E}) \mathbf{C} \right\} . \quad (3.1)$$

The proposed design is to derive a precoding matrix \mathbf{C} that maximize the average SNR and keeps a low outage probability of the instantaneous SNR below an acceptable level. More specifically, the robust beamformer can be achieved by solving the following probabilistic-constrained quadratical optimization problem,

$$\text{maximize}_{\mathbf{C}} \quad \mathbb{E} [f(\hat{\mathbf{H}}, \mathbf{E})] , \quad (3.2)$$

$$\text{subject to} \quad \Pr \left\{ f(\hat{\mathbf{H}}, \mathbf{E}) \leq \gamma_0 \right\} \leq \epsilon , \quad (3.3)$$

$$\text{tr} \{ \mathbf{C} \mathbf{C}^H \} \leq 1 , \quad (3.4)$$

where $\Pr\{A\}$ denotes the probability of the event A , and γ_0 and ϵ are the pre-specified threshold and outage probability, respectively. The reformulation of problem (3.2)-(3.4) will be discussed as follows.

Consider the eigen decomposition of $\hat{\mathbf{H}}^H \hat{\mathbf{H}}$ and $\mathbf{C}^H \mathbf{C}$, we have

$$\hat{\mathbf{H}}^H \hat{\mathbf{H}} = \hat{\mathbf{U}}_h \hat{\mathbf{D}}_h \hat{\mathbf{U}}_h^H , \quad (3.5)$$

$$\mathbf{C} \mathbf{C}^H = \mathbf{U}_c \mathbf{D}_c \mathbf{U}_c^H , \quad (3.6)$$

where the diagonal matrix $\mathbf{D}_c = \text{diag}(d_1, d_2, \dots, d_N)$ where $d_1 \geq d_2 \geq d_N \geq 0$ consists of eigenvalues of $\mathbf{C}^H \mathbf{C}$ in descending order. The corresponding eigenvectors are columns of the

unitary matrix \mathbf{U}_c . The matrices $\hat{\mathbf{D}}_h = \text{diag}(\hat{D}_1, \dots, \hat{D}_N)$ and $\hat{\mathbf{U}}_h$ are similarly defined.

3.2.1 Objective Function

Given the channel estimate $\hat{\mathbf{H}}$, the objective function is obtained by taking expectation of $f(\hat{\mathbf{H}}, \mathbf{E})$ with respect to the random error \mathbf{E} , such as

$$\mathbb{E} [f(\hat{\mathbf{H}}, \mathbf{E})] = \frac{E_s}{N_0} \text{tr} \left\{ \mathbf{U}_c \mathbf{D}_c \mathbf{U}_c^H \left(\hat{\mathbf{U}}_h \hat{\mathbf{D}}_h \hat{\mathbf{U}}_h^H + M \sigma_e^2 \mathbf{I}_N \right) \right\}. \quad (3.7)$$

It is well established in the literature [26, 31] that a function with a structure similar to (3.7) can be maximized over the eigen-modes, \mathbf{U}_c , and the power allocated in each mode, \mathbf{D}_c , separately. In [21, 27], it is suggested that the eigen-mode transmission is optimal for SNR criteria. More specifically, given the matrix $\hat{\mathbf{U}}_h$, the optimal solution \mathbf{U}_c^* satisfies the relation $\hat{\mathbf{U}}_h \mathbf{U}_c^* = \mathbf{I}_N$. Inserting the optimal solution in (3.7), the objective function can be expressed as

$$\mathbb{E} [f(\hat{\mathbf{H}}, \mathbf{E})] = \mathbb{E} [f(\mathbf{D}_c)] = \frac{E_s}{N_0} \text{tr} \left\{ \mathbf{D}_c (\hat{\mathbf{D}}_h + M \sigma_e^2 \mathbf{I}_N) \right\}. \quad (3.8)$$

Note that the objective function depends on $\mathbf{C}\mathbf{C}^H$ only through its eigenvalues. Hence, the design of the beamforming matrix becomes a power allocation problem.

3.2.2 Probabilistic Constraint

To mitigate the impact of large errors, the system performance is guaranteed by keeping the a low probability that SNR falls below an acceptable level. Applying the eigen decomposition (3.5) and (3.6), the randomly varied SNR in (3.3) can be simplified as

$$f(\hat{\mathbf{H}}, \mathbf{E}) = \frac{E_s}{N_0} \sum_{i=1}^N d_i Z_i, \quad (3.9)$$

which is a weighted sum of *independent* noncentral $\chi_{n_i}^2(\delta_i)$ -distributed random variables Z_i , $i = 1, \dots, N$. For each random variable $Z_i \sim \chi_{n_i}^2(\delta_i)$, the noncentrality parameter is $\delta_i = \tilde{\mathbf{h}}_i^H \tilde{\mathbf{h}}_i$ and the degree of freedom is $n_i = 2M$. The vector $\tilde{\mathbf{h}}_i \in \mathbb{C}^{M \times 1}$ represents the i -th

column of the matrix $\tilde{\mathbf{H}} = \hat{\mathbf{H}}\mathbf{U}_c$. The derivation of (3.9) is provided in Appendix A.

3.2.3 Probabilistic Constrained Optimization

Based on the average SNR (3.8) and the compact expression (3.9), the proposed beamforming design (3.2)-(3.4) can be reformulated as the following probabilistic constrained optimization problem

$$\underset{\mathbf{D}_c}{\text{maximize}} \quad \text{tr} \left\{ \mathbf{D}_c \left(\hat{\mathbf{D}}_h + M\sigma_e^2 \mathbf{I} \right) \right\}, \quad (3.10)$$

$$\text{subject to} \quad \Pr \left\{ \sum_{i=1}^N d_i Z_i \leq \tilde{\gamma}_0 \right\} \leq \epsilon, \quad (3.11)$$

$$\text{tr} \{ \mathbf{D}_c \} \leq 1, \quad (3.12)$$

$$d_i \geq 0, \quad i = 1, \dots, N, \quad (3.13)$$

where $\tilde{\gamma}_0 = \gamma_0 \left(\frac{E_s}{N_0} \sigma_e^2 \right)^{-1}$, and (3.12) is a convex constraint derived from the power constraint $\text{tr} \{ \mathbf{C}\mathbf{C}^H \} \leq 1$.

3.3 Reformulation of Probabilistic Constraint

The major challenge in (3.10)-(3.13) is to covert the probabilistic constraint (3.11) into a deterministic term, so that the optimum solution can be efficiently computed by standard tools of mathematical programming. When the probabilistic constraint involves linear combination of normally distributed random variables, it can be reformulated as convex constraint [72]. However, (3.11) involves a mixture of weighted noncentral χ^2 distributed random variables. In the following, the probabilistic constraint (3.11) will be replaced by a deterministic convex constraint. Consequently, the original problem (3.10)-(3.13) can be reformulated as convex optimization problem.

Proposition: The outage probability constraint (3.11) can be replaced by the following convex constraint

$$\prod_{i=1}^N \left(\frac{1}{d_i} \left[\frac{\tilde{\gamma}_0/2}{1 + \delta_i/n_i} \right] \right) \leq \epsilon , \quad (3.14)$$

where $\tilde{\gamma}_0 = \gamma_0 (\frac{E_s}{N_0} \sigma_e^2)^{-1}$. If (3.14) holds, then (3.11) holds.

Proof: To decouple the design parameter d_i , the independence of Z_i is exploited. Define the event

$$\mathcal{A}_i = \{d_i Z_i \leq \tilde{\gamma}_0\} , \quad (3.15)$$

and

$$\mathcal{A} = \left\{ \sum_{i=1}^N d_i Z_i \leq \tilde{\gamma}_0 \right\} . \quad (3.16)$$

By definition, \mathcal{A} is a subset of the intersection of \mathcal{A}_i ($i = 1, \dots, N$), that is

$$\mathcal{A} \subset \{\mathcal{A}_1 \cap \mathcal{A}_2 \cap \dots \cap \mathcal{A}_N\} , \quad (3.17)$$

leading to the following inequality

$$\Pr \{\mathcal{A}\} \leq \prod_{i=1}^N \Pr\{\mathcal{A}_i\} . \quad (3.18)$$

The event \mathcal{B} decouples the random variables Z_i s so that the probability of event \mathcal{A}_i depends only on the noncentral $\chi_{n_i}^2(\delta_i)$ -distribution.

According to [73], the noncentral $\chi_{n_i}^2(\delta_i)$ distribution can be approximated by a central χ^2 distribution as

$$\Pr \left\{ \chi_{n_i}^2(\delta_i) \leq \frac{\tilde{\gamma}_0}{d_i} \right\} \approx \Pr \left\{ \chi_{n_i}^2 \leq \frac{\tilde{\gamma}_0/d_i}{1 + \delta_i/n_i} \right\} . \quad (3.19)$$

Let

$$x' = \frac{\tilde{\gamma}_0/d_i}{1 + \delta_i/n_i} , \quad (3.20)$$

which is determined by the SNR threshold $\tilde{\gamma}_0$, estimated channel and the number of receive

antenna M . Recall (3.19), we have

$$\begin{aligned}\Pr\{\chi_{n_i}^2 \leq x'\} &= 1 - \Pr\{\chi_{n_i}^2 \geq x'\} \\ &= 1 - \int_{x'}^{\infty} \frac{2^{-n_i/2}}{\Gamma(n_i/2)} t'^{\frac{n_i}{2}-1} e^{-\frac{t'}{2}} dt'\end{aligned}\quad (3.21)$$

Replacing the variable $t' = 2t$, (3.21) can be further expressed as

$$\begin{aligned}\Pr\{\chi_{n_i}^2 \leq x'\} &= 1 - \frac{1}{\Gamma(n_i/2)} \int_{\frac{x'}{2}}^{\infty} t^{\frac{n_i}{2}-1} e^{-t} dt \\ &= 1 - \frac{1}{\Gamma(n_i/2)} \Gamma\left(\frac{n_i}{2}, \frac{x'}{2}\right),\end{aligned}\quad (3.22)$$

where $\Gamma(a, x)$ denotes the incomplete gamma function, that is,

$$\Gamma(a, x) = \int_x^{\infty} t^{a-1} e^{-t} dt,$$

namely [74],

$$\int_x^{\infty} e^{-t^p} dt = \frac{1}{p} \Gamma\left(\frac{1}{p}, x^p\right). \quad (3.23)$$

Consequently, changing the variables

$$1/p = n_i/2 = M, \quad x = (x'/2)^{1/p}, \quad (3.24)$$

(3.22) can be reformulated as

$$\begin{aligned}\Pr\{\chi_{n_i}^2 \leq x'\} &= 1 - \frac{p}{\Gamma(1/p)} \int_x^{\infty} e^{-t^p} dt \\ &= 1 - \frac{1}{\Gamma(1 + 1/p)} \int_x^{\infty} e^{-t^p} dt.\end{aligned}\quad (3.25)$$

Note that the property of gamma function $y\Gamma(y) = \Gamma(y+1)$ ($y > 0$) leads to replacing $p/\Gamma(1/p)$ with $1/\Gamma(1 + 1/p)$ in (3.25).

Based on Appendix C, the upper bound of (3.19) can be expressed as

$$\Pr \left\{ \chi_{n_i}^2 \leq \frac{\tilde{\gamma}_0/d_i}{1 + \delta_i/n_i} \right\} < \left(1 - \exp \left\{ -\frac{\tilde{\gamma}_0/d_i}{2(1 + \delta_i/n_i)} \right\} \right)^{\frac{n_i}{2}}. \quad (3.26)$$

Let

$$f(x) = x - (1 - e^{-x}), \quad x \geq 0. \quad (3.27)$$

Taking derivative with respect to x , it has

$$\frac{d}{dx} f(x) = 1 - e^{-x} \geq 0, \quad x \geq 0. \quad (3.28)$$

Therefore, $f(x)$ is a monotonic nondecreasing function for $x \geq 0$. Furthermore, at $x = 0$, $f(0) = 0 - (1 - 1) = 0$. Thus, the inequality is obtained as follows

$$(1 - e^{-x}) \leq x, \quad x > 0. \quad (3.29)$$

The inequality with positive exponent $a > 0$ on both sides is also valid, such as

$$(1 - e^{-x})^a \leq x^a, \quad (3.30)$$

it immediately leads to

$$\left(1 - \exp \left\{ -\frac{\tilde{\gamma}/d_i}{2(1 + \delta_i/n_i)} \right\} \right)^{\frac{n_i}{2}} \leq \left(\frac{\tilde{\gamma}/d_i}{2(1 + \delta_i/n_i)} \right)^{\frac{n_i}{2}} \quad (3.31)$$

by replacing x in (3.30) with $\frac{\tilde{\gamma}_0/d_i}{2(1 + \delta_i/n_i)}$, and replacing a with $\frac{n_i}{2} = M$.

Combing the inequalities (3.19) (3.26) and (3.31), the following inequality is obtained

$$\Pr \{ \chi_{n_i}^2 \leq \tilde{\gamma}_0 \} \leq \left(\frac{1}{d_i} \left[\frac{\tilde{\gamma}_0/2}{1 + \delta_i/n_i} \right] \right). \quad (3.32)$$

According to (3.18) and (3.32), it is concluded that the outage probability constraint (3.11) is satisfied if the deterministic constraint (3.14) is satisfied.

Moreover, the constraint (3.14) can be represented as

$$\left(\left[\frac{\tilde{\gamma}_0/2}{1 + \delta_i/n_i} \right] \right) \leq \det(\mathbf{D}_c)^{1/M} \epsilon. \quad (3.33)$$

Since the n -th root determinant of semidefinite matrix is concave, it can be easily concluded that (3.33) is convex.

Remark 1 The upper bound on the specification (3.11) is obtained by implementing the sharp upper bound of incompletely Gamma distribution. Hence, the deterministic constraint (3.14) is a conservative approximation to (3.11).

Remark 2 The approximation of the noncentral $\chi_{n_i}^2(\delta_i)$ -distribution through the central $\chi_{n_i}^2$ -distribution is most accurate when the ratio between the noncentrality parameter and degrees of freedom is less than 0.2 [73]. For the proposed beamformer where $\delta_i = \tilde{\mathbf{h}}_i^H \tilde{\mathbf{h}} / \sigma_e^2$ and $n_i = 2M$, it corresponds to large estimation error for a fixed number of receive antennas. Therefore, in critical situations, the QoS is well controlled by the approximate deterministic convex constraint (3.14).

Remark 3 The inequality (3.14) is not convex, when $d_i = 0$ in (3.14). In practice, $d_i = 0$ can be a extreme small power and ignored as zero. Thus, the convex form (3.33) can be implemented in (3.10)-(3.13) instead of (3.14).

Replacing the outage probability specification (3.11) with the deterministic constraint (3.14), the original problem is transformed to the following convex optimization problem with respect to the design parameter \mathbf{D}_c ,

$$\underset{\mathbf{D}_c}{\text{maximize}} \quad \text{tr} \left\{ \mathbf{D}_c \left(\hat{\mathbf{D}}_h + M\sigma_e^2 \mathbf{I}_N \right) \right\}, \quad (3.34)$$

$$\text{subject to} \quad \prod_{i=1}^N \left(\frac{1}{d_i} \left[\frac{\tilde{\gamma}_0}{2(1 + \delta_i/n_i)} \right] \right)^{n_i/2} \leq \epsilon, \quad (3.35)$$

$$\text{tr} \{ \mathbf{D}_c \} \leq 1, \quad (3.36)$$

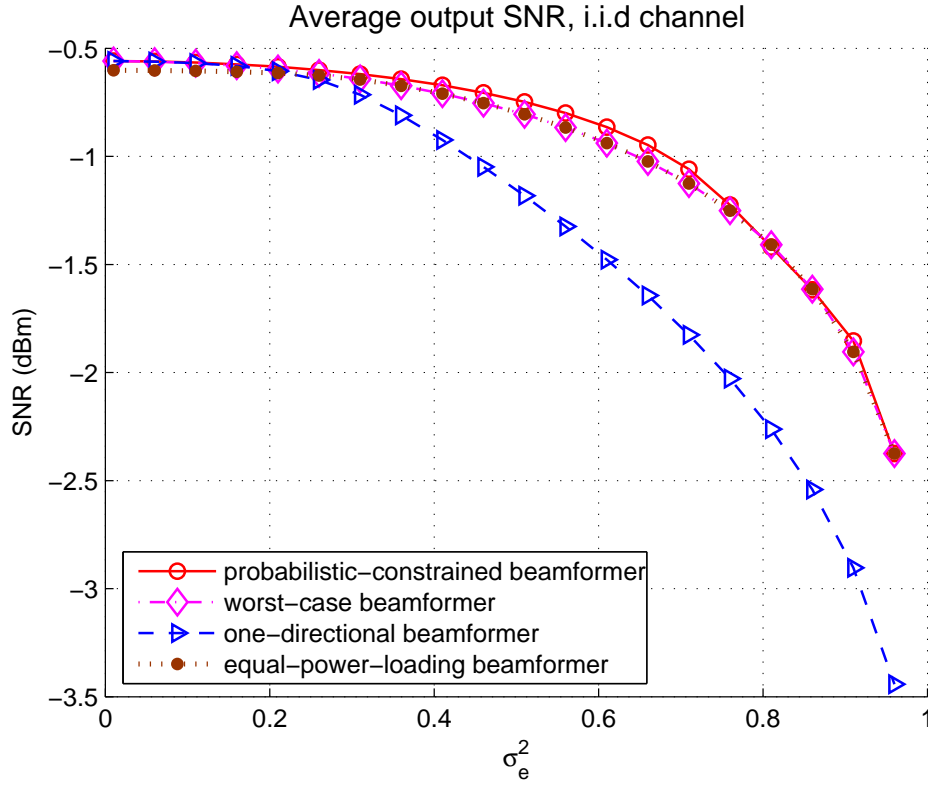


Figure 3.1: Average SNR vs. error variance σ_e^2 over i.i.d channel

$$d_i \geq 0, \quad i = 1, \dots, N, \quad (3.37)$$

which can be efficiently solved by standard tools of mathematical programming.

3.4 Simulation

In this section, numerical investigation of the proposed beamformer is presented under various scenarios. A single-user MIMO system with $N = 4$ transmit antennas and $M = 3$ receive antennas is considered. For comparison, standard designs for perfect channel information at transmitter including the conventional one-directional beamformer, equal-power-loading beamformer [5] and the robust minimax beamformer [26] are applied to the same batch of data. Note that the worst-case approach [26] is chosen because it uses the same type of channel information.

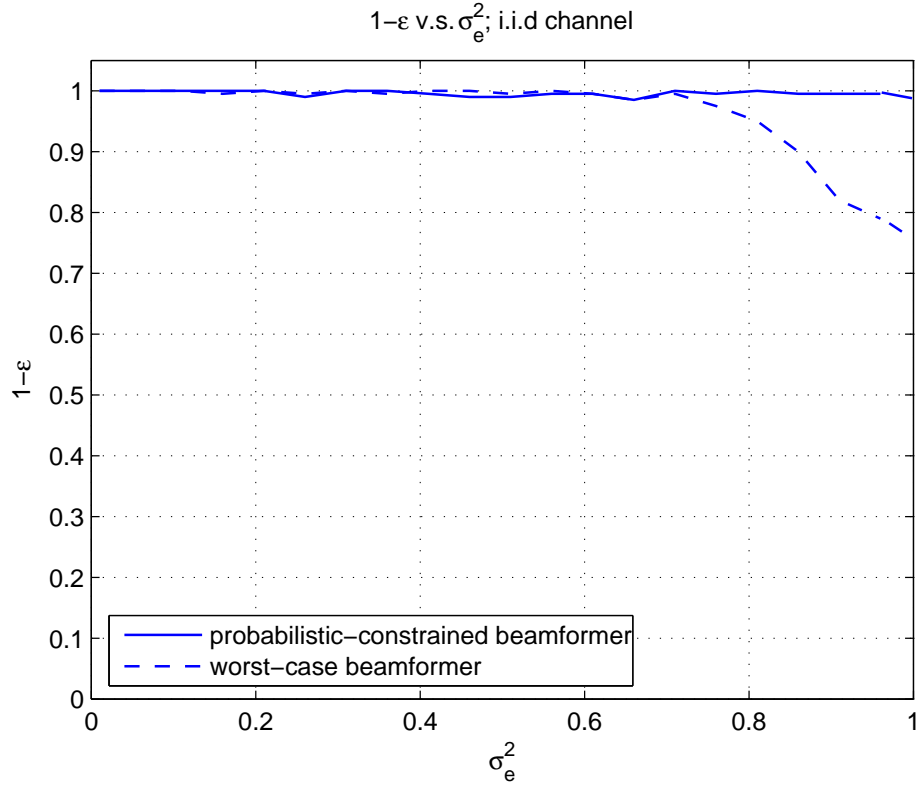


Figure 3.2: $(1 - \epsilon)$ vs. error variance over i.i.d channel

The simulation will be carried out for classical i.i.d channel and correlated fading channel. In addition, the impact of mismatch error variance σ_e^2 on the proposed algorithm is numerically investigated. Each realization performs 1000 Monte Carlo trials. Without any loss of generality, the assumption for simulation are follows:

- Parameters in outage probability specification: The probabilistic constraint is introduced to keep a low outage probability of SNR being below an acceptable level. In this case, the outage probability ϵ should be set at low level, such as $\epsilon = 10\%$. Since the received SNR is normalized by the variance of channel, the normalized SNR threshold $\tilde{\gamma}_0$ should set as high as possible, such as $\tilde{\gamma}_0 = 0.9$ in all experiments.
- For simplicity, the variance of error is normalized by the variance of channel. The entries in the error matrix \mathbf{E} are i.i.d zero mean complex Gaussian distribution with variance σ_e^2 . In the simulation, the normalized variance of error is varied from 0 to 1 with scale 0.05.

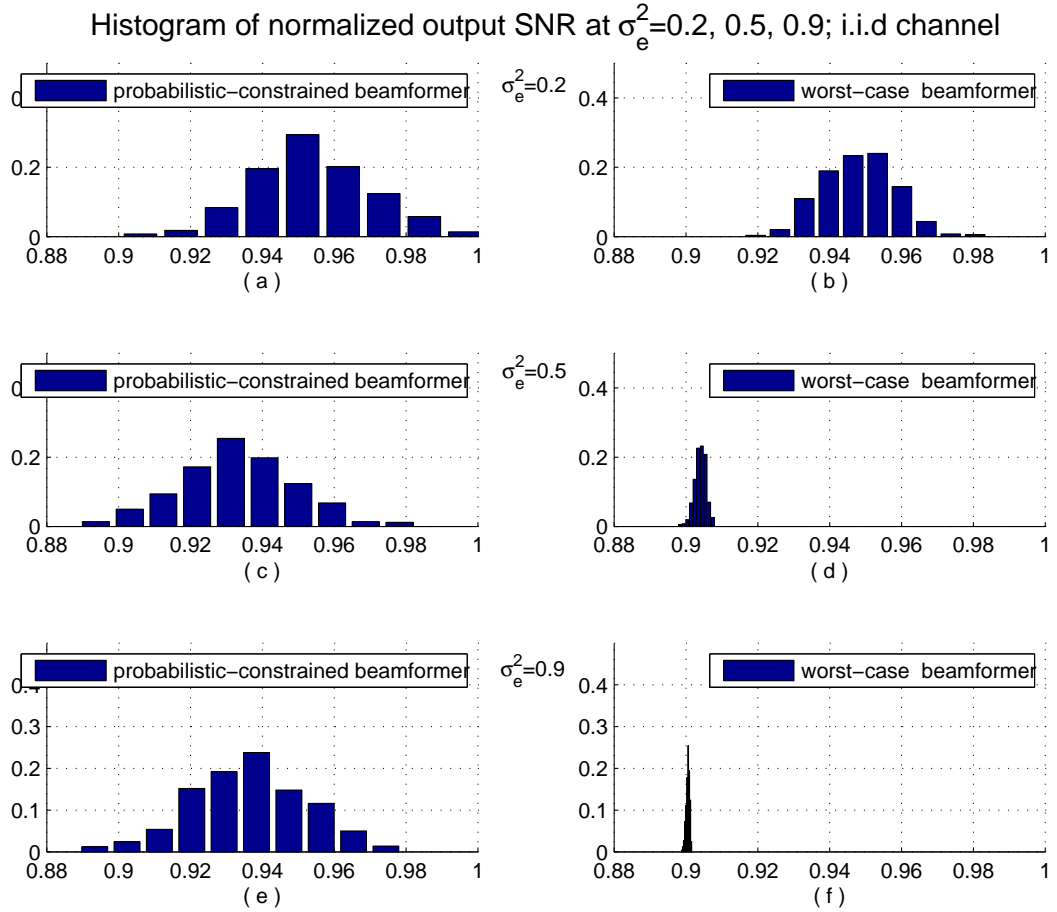


Figure 3.3: Histogram of normalized SNR for $\sigma_e^2 = 0.2, 0.5, 0.9$ over i.i.d channel

Classical i.i.d Channel

The classical i.i.d channel is modeled as independent circularly symmetric complex Gaussian random variables with zero mean and unit variance.

Fig. 3.1 shows the performance of average SNR versus noise over i.i.d channel. With increasing noise level, the SNR performances of all the beamformers have degradation. The proposed beamformer with the worst-case design and equal-power-loading beamformers perform similarly since the theoretical eigenvalues of channel covariance are equally distributed over the i.i.d channel. While, the one-directional beamformer exploits only one channel eigenmode and degrades rapidly with increasing channel errors.

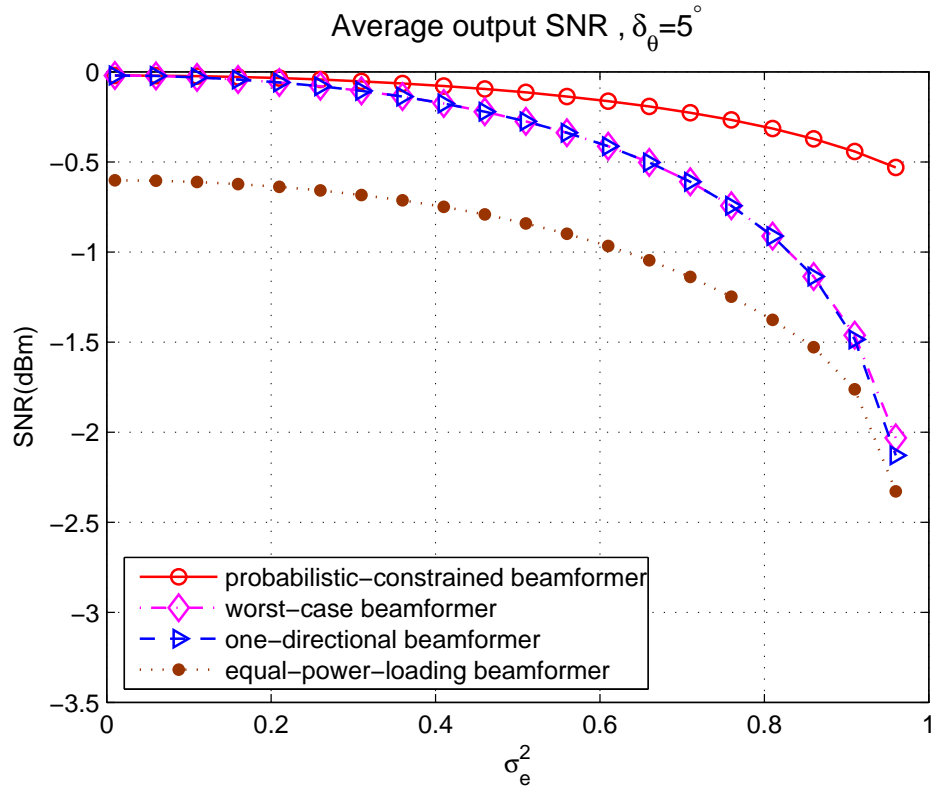


Figure 3.4: Average SNR vs. error variance σ_e^2 over correlated fading channel with spread angle $\delta_\theta = 5^\circ$

The empirical outage probability that the output SNR exceeds the pre-specified threshold is shown in Fig. 3.2. The proposed algorithm keeps the target SNR at a probability larger than 98% over the entire error ranges. On the other hand, the worst-case approach provides robustness over $0 \leq \sigma_e^2 \leq 0.7$, however, breaks down for the large noise variance region.

To investigate the behavior of the proposed and worst-case beamformers, the histograms of the normalized SNRs are shown in Fig. 3.3 with three noise variance, that is, $\sigma_e^2 = 0.2, 0.5, 0.9$. At small σ_e^2 , both approaches have similar distribution entirely above the threshold 0.9. In medium and high σ_e^2 cases, the worst-case beamformer trends to concentrate towards the threshold, while the proposed beamformer has similar distribution as the small σ_e^2 case. In practice, the channel uncertainty is not deterministic but randomly distributed, it is reasonable to measure the imperfection by using the proposed approach.

Correlated Fading Channel

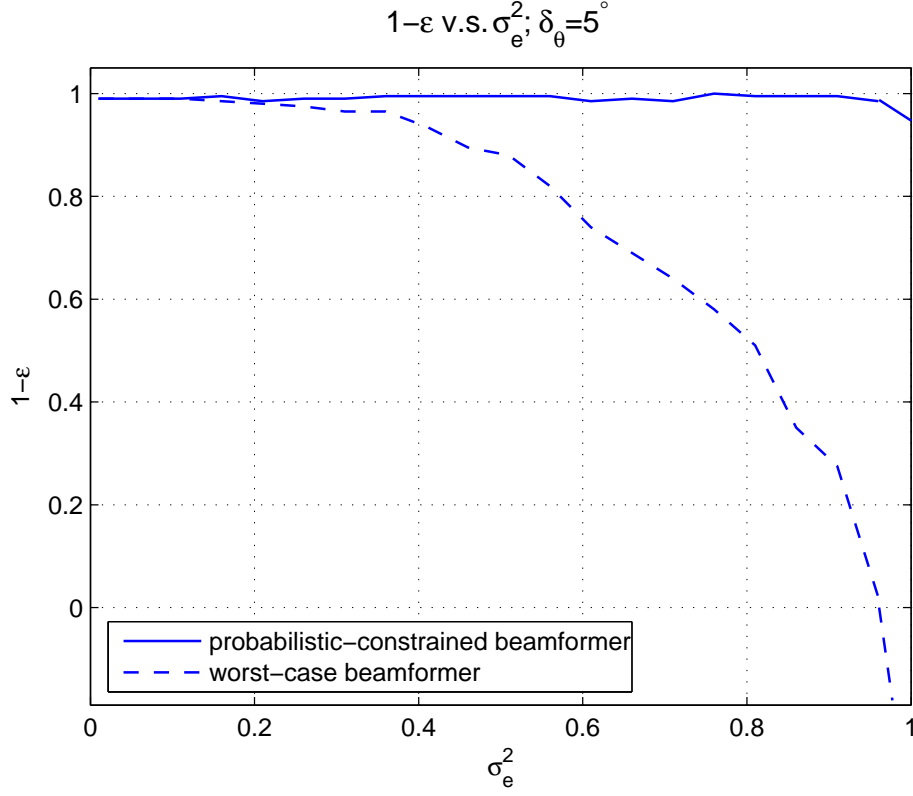


Figure 3.5: $(1-\epsilon)$ vs. error variance over correlated fading channel with spread angle $\delta_\theta = 5^\circ$

Under the condition of rich scatter environment at receiver side or inequality spaced transmit antennas, the correlated fading channel will occur. The channel is generated according to "One-Ring" model. Let λ be the wavelength of a narrow-band signal, d the antenna spacing, and δ_θ the angle spread (details in Appendix D). We assumed that the angle of arrival is perpendicular to the transmitter antenna array, for small angle spread, the transmit correlation is calculated based on (D.2) with the condition that $d = 0.5\lambda$ and $\delta_\theta = 5^\circ, 25^\circ$. Note that larger angle of spread leads to less correlation and better channels.

Fig. 3.4, 3.5 and 3.6 present the $\delta_\theta = 5^\circ$ case with the true eigenvalues $\{0.9546, 0.0452, 0.0002\}$. In Fig. 3.4, the probabilistic constraint approach provides the best performance of output SNR, especially in high error level. For large error variance, such as $\sigma_e^2 > 0.8$, the gap between the proposed approach and worst-case approach can be as large as 1.5 dB. With small spread angle where the channel energy concentrates on one eigenmode, the performance of worst-case approach and one-directional approach are similar, while the equal-power-loading scheme

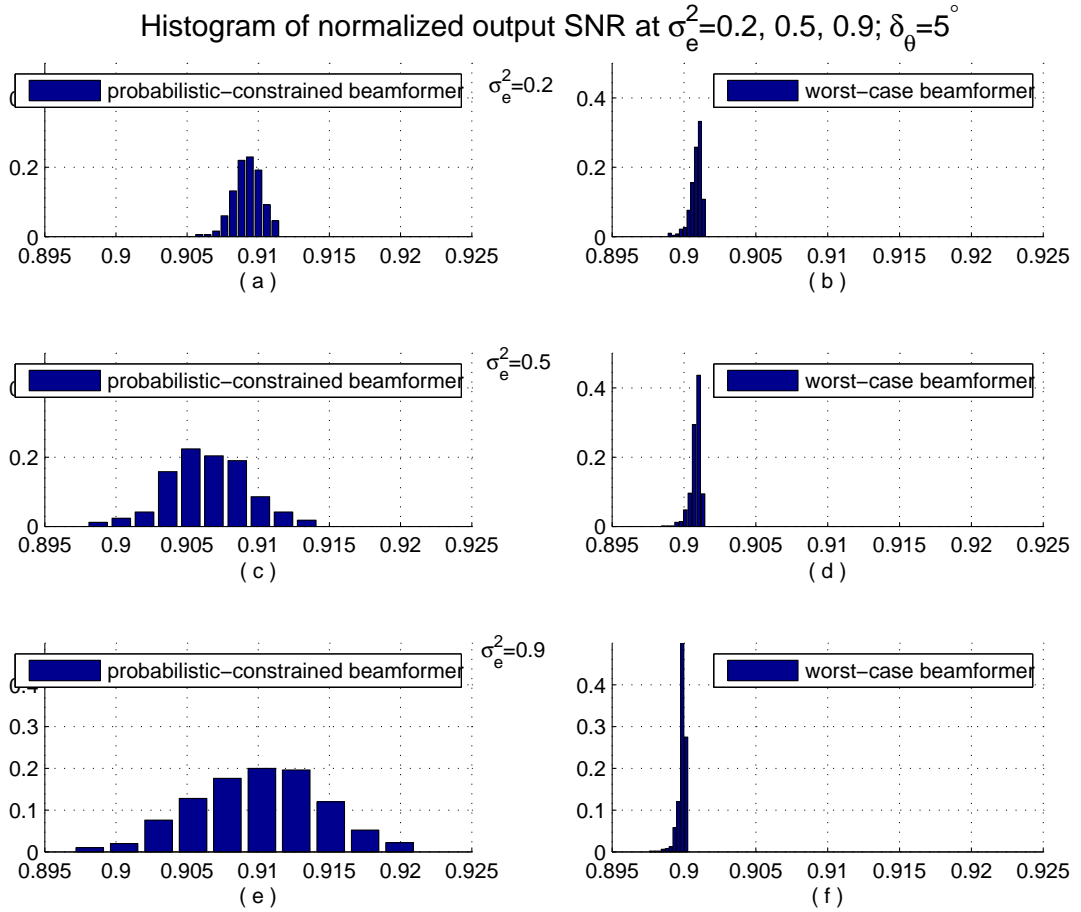


Figure 3.6: Histogram of normalized SNR for $\sigma_e^2 = 0.2, 0.5, 0.9$ over correlated fading channel with spread angle $\delta_\theta = 5^\circ$

performs worst.

Furthermore, Fig. 3.5 shows that the proposed algorithm always satisfies the probabilistic constraint and guarantees QoS at more than 98%. In contrast, the worst-case design significantly degrades with increased error level. The largest gap between them can be achieved 70% at $\sigma_e^2 = 0.9$. It can be consequently observed in Fig. 3.6 that the normalized SNR of the proposed approach is steadily distributed above $\tilde{\gamma}_0 = 0.9$ while the histogram of worst-case approach rapidly shift to left with increasing σ_e^2 .

Simulation results obtained from $\delta_\theta = 25^\circ$ are presented in Fig. 3.7, 3.8 and 3.9. The corre-

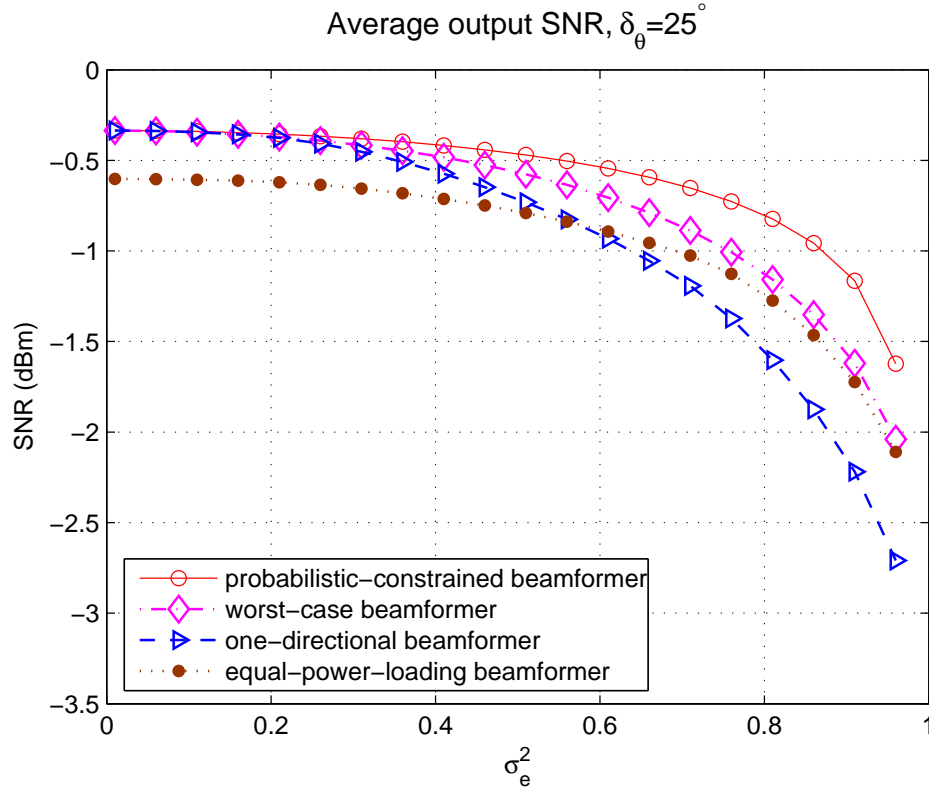


Figure 3.7: Average SNR vs. error variance σ_e^2 over over correlated fading channel with spread angle $\delta_\theta = 25^\circ$

sponding eigenvalues are $\{0.4474, 0.4351, 0.1137\}$ with more equally spread channel energy. More precisely, in the small σ_e^2 region, the proposed beamformer has similar performance as the worst-case and one-directional. With the increased σ_e^2 , the curve of one-directional degrades while the worst-case beamformer trends to be the same as equal-power-loading. Although the performance proposed beamformer degrades with increasing error level, but still outperforms the worst-case and other two designs. Similarly to the pervious two scenarios, the probabilistic constraint approach provides satisfying performance over the entire uncertainty region, while the worst-case approach is much more sensitive to the random-distributed error with large variance.

Performance for Mismatch Error Variance

In the proposed approach, the variance of channel estimates error is assumed to be known. In practice, the channel uncertainty is predicted according to the channel estimates and the

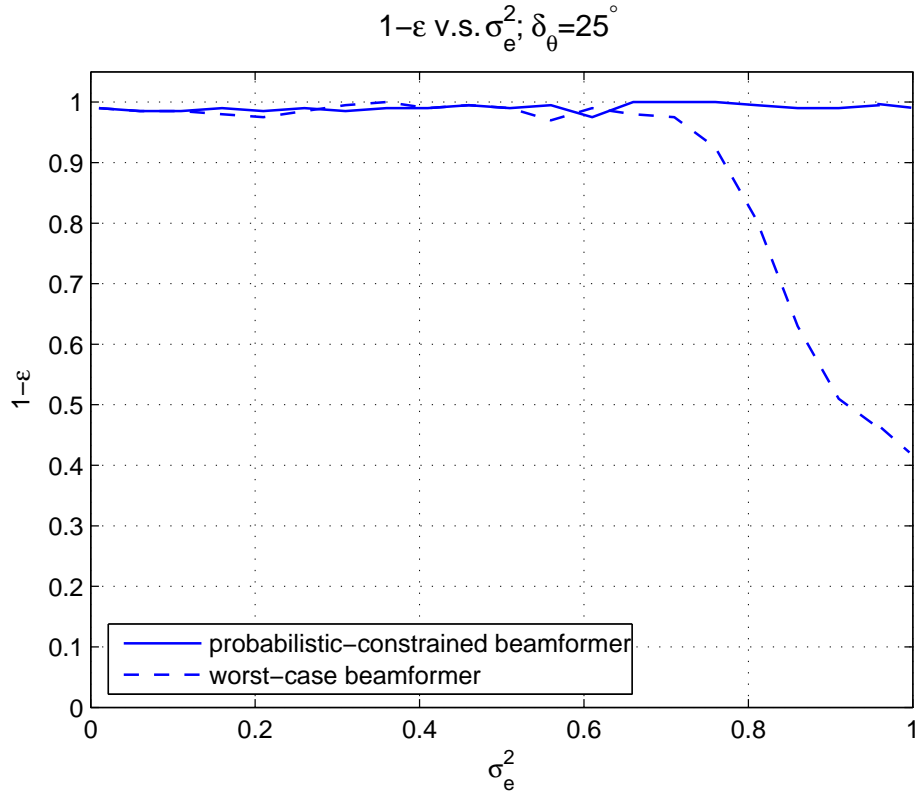


Figure 3.8: $(1 - \epsilon)$ vs. error variance over correlated fading channel with spread angle $\delta_\theta = 25^\circ$

nominal channel. However, this predicted error variance may have uncertainty as well. To get further sight of the robustness in proposed beamformer, an investigation is processed under a misspecified noise variance in estimated σ_e^2 . The correlated fading is also generated by using (2.6) with $\sigma_\theta = 25^\circ$. The robustness in proposed beamformer is shown in three scenarios σ_e^2 :

1. the true noise variance σ_e^2
2. mismatched noise variance $\sigma_{e,mis1}^2 = \sigma_e^2 + \Delta_e$,
3. mismatched noise variance $\sigma_{e,mis2}^2 = \sigma_e^2 - \Delta_e$,

Note that the deterministic mismatch $\Delta_e = 0.2$ is large relative to the considered range $\sigma_e^2 \in [0.25, 0.75]$.

As observed from Fig. 3.10, the proposed beamformer has the overall best performance under

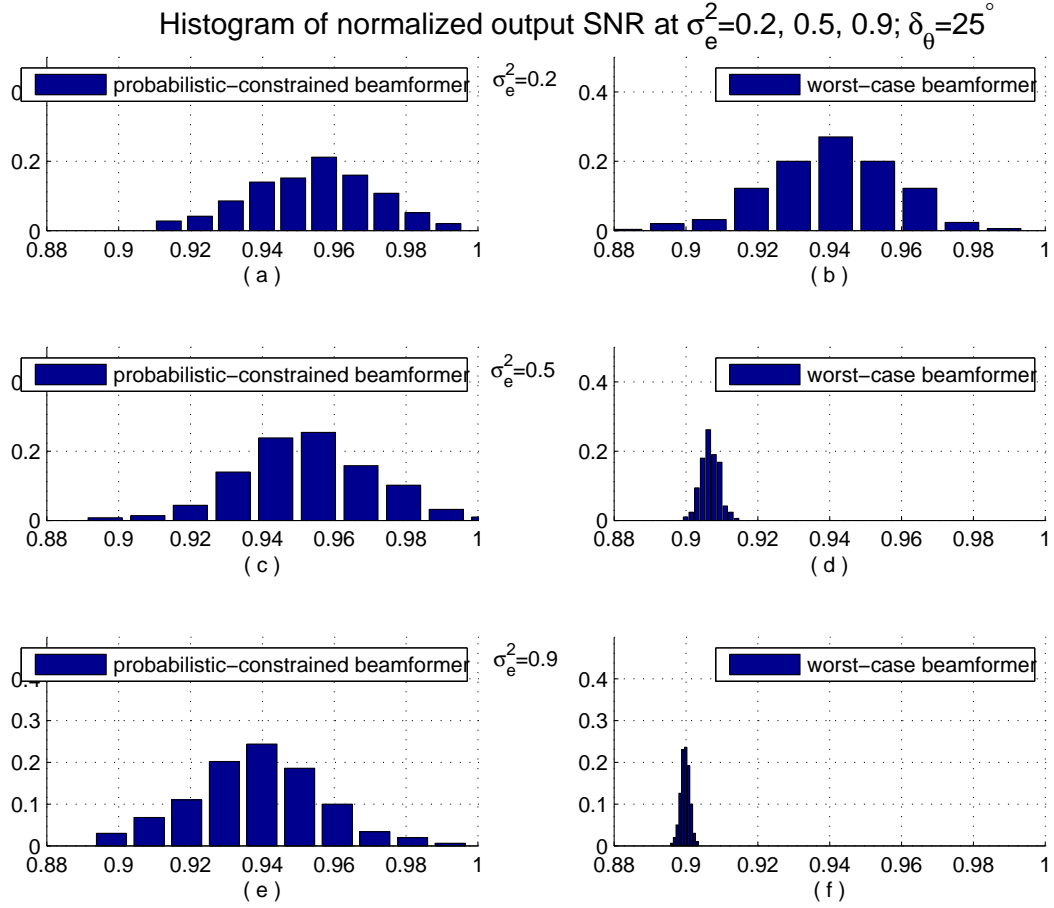


Figure 3.9: Histogram of normalized SNR for $\sigma_e^2 = 0.2, 0.5, 0.9$ over correlated fading channel with spread angle $\delta_\theta = 25^\circ$

perfect knowledge of error variance σ_e^2 , while followed by the curve associated with $\sigma_{e,mis2}^2$ and then with $\sigma_{e,mis1}^2$. The difference between these three cases becomes remarkable for $\sigma_e^2 > 0.65$, with twice degradation caused by $\sigma_{e,mis1}^2$ than that by $\sigma_{e,mis2}^2$. It is because that the normalized threshold in (3.35) $\tilde{\gamma}_0 = \gamma_0 \left(\frac{E_s}{N_0} \sigma_e^2 \right)^{-1}$ only depends on the *assumed* error variance, which leads to the following relationship of the implemented threshold among three cases

$$\tilde{\gamma}_{mis2} = \gamma_0 \left(\frac{E_s}{N_0} (\sigma_e^2 + \Delta_e) \right)^{-1} < \tilde{\gamma}_0 < \gamma_0 \left(\frac{E_s}{N_0} (\sigma_e^2 - \Delta_e) \right)^{-1}. \quad (3.38)$$

With the mismatch $\Delta_e = 0.2$ case, the *assumed* threshold $\tilde{\gamma}$ is smaller than the *true* threshold $\tilde{\gamma}_{mis1}$, and leads to a tighter constraint and a better QoS control. But it also reduces the

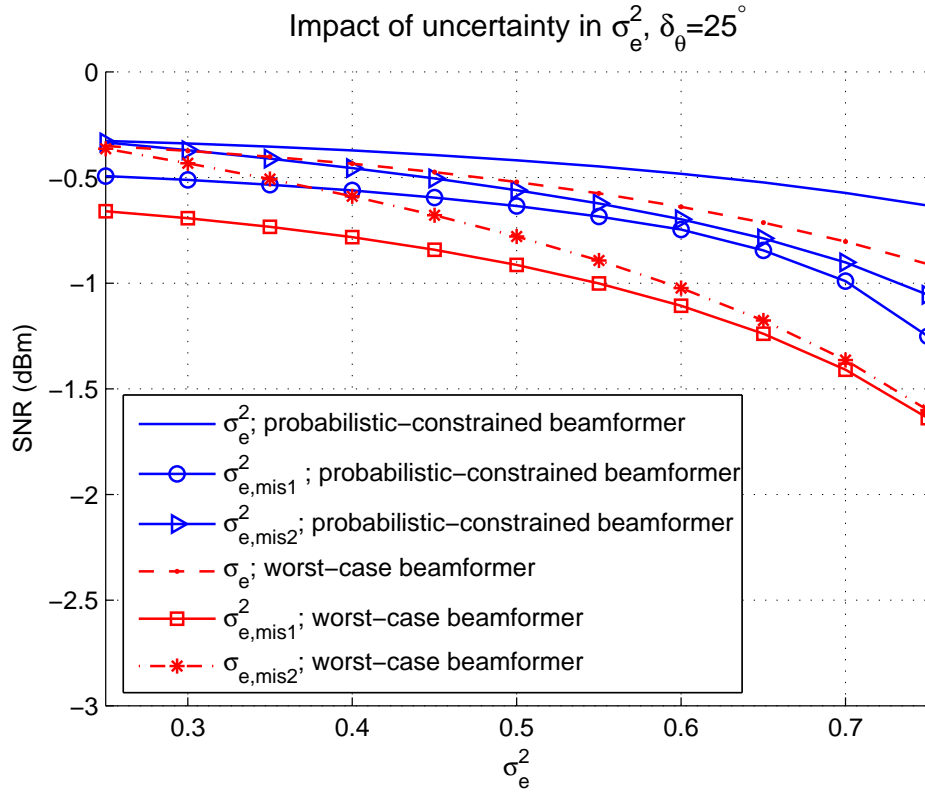


Figure 3.10: Average SNR vs. true error variance σ_e^2 , assumed error variance $\sigma_{e, mis1}^2 = \sigma_e^2 + \Delta_e$, $\sigma_{e, mis2}^2 = \sigma_e^2 - \Delta_e$, where $\Delta_e = 0.2$.

feasibility set of the optimization problem (3.34)-(3.37). In this case, the objective function becomes worse with the reduced feasibility set, and consequently the optimal solution may not be achieved. On the other hand, replacing $\tilde{\gamma}_0$ with a small threshold $\tilde{\gamma}_{mis2}$ relaxes the constraint (3.37) and QoS control, which has larger impact on the algorithm shown in Fig. 3.10.

The performance of the worst-case design against mismatched error variance is also illustrated in Fig. 3.10. The behavior of worst-case approach are similar as the proposed, that is, $\sigma_{e, mis2}^2$ lies between the best performance from perfect case σ_e^2 and the worst performance from $\sigma_{e, mis1}^2$. At $\sigma_e^2 = 0.7$, the SNR gap between the perfect and $\sigma_{e, mis2}^2$ is $\Delta \text{SNR}_{prob} \approx 0.42$ dB for the proposed approach, and $\Delta \text{SNR}_{worst} \approx 0.6$ dB for the worst-case approach, respectively. Over the entire observed region, since the performance of the latter one degrades 50% more than the former, the proposed beamformer is more robust to mismatched noise level.

3.5 Conclusion

In this chapter, a novel transmit beamforming design is proposed for single-user MIMO communication systems, which maximizes the average received SNR and guarantees the robustness against channel imperfections. The proposed beamformer optimizes average performance as well as proportionally considers worst-case scenarios, which is more reasonable than the statistic-based beamformer and is less conservative than the worse-case beamformer. The underlying design is formulated as a probabilistic-constrained optimization problem by introducing an outage probability specification for received SNR. The main challenge of the optimization problem is to find a deterministic form for probabilistic constraint. Under the assumption that the channel estimate error is complex Gaussian distributed, the probabilistic constraint was transformed into a convex one. The resulting convex optimization was efficiently solved by modern software package, such as cvx [75].

Simulation results show that the proposed beamformer provides the best performance compared to the popular maximin beamformer and outage probability are always well controlled. Compared to the worst-case beamformer, the proposed beamformer has a much broader error-tolerance range and more robustness against error variance misspecification. The proposed algorithm obtains the largest gain when the channel is highly correlated. More importantly, the computational complexity of the probabilistic constraint is similar to the minimax approach. Given its superior overall performance and significantly improved robustness, the probabilistic constraint beamformer provides an attractive alternative to existing transmit beamforming design under imperfect channel information.

Chapter 4

Robust Adaptive Modulation for Downlink Single-User MIMO Systems

Adaptive modulation could enable a spectrally-efficient transmission by adapting transmission parameters to a time-varying MIMO channel. Perfect channel information is crucial to adaptive modulation scheme, which is typically not available. To enhance the robustness against channel imperfections, an outage probability specification is introduced as a tight BER constraint by keeping the probability that SNR becomes smaller than a pre-specified threshold at a low level. Under such a constraint, the proposed scheme maximizes the transmission rate by taking advantage of transmit beamforming. Simulation results demonstrate that the proposed scheme offers higher robustness and transmission rate than several popular modulation schemes.

4.1 Introduction

Adaptive modulation has the potential to increase system throughput considerably over time-varying MIMO channels by adapting transmitter parameters to maintain acceptable BER performance [76]. On the other hand, transmit beamforming as an effective fading counter provides antenna diversity gain to further enhance the performance of wireless communication as well as relax the size and cost limitation of mobile units [5].

CSIT is crucial to an adaptive modulation scheme, where the transmission rate can be adapted to achieve an acceptable average BER performance. However, the performance of adaptive modulation could degrade significantly because only imperfect channel information is available at transmitter side [15]. This has motivated many efforts to develop robust adaptive modulation schemes that are robust against channel imperfections.

Current robust schemes adapt the transmission rate while maintaining the target average BER performance over MIMO channels. The average BER constraint consists of constellation size and instantaneous SNR, which is difficult to formulate analytically because of Gaussian-distributed errors in the CSIT. Thus, the existing robust schemes approximate the average BER by the lower bound of average BER or worst-case BER. Constrained by the lower bound of average BER, the system throughput is maximized by introducing an artificial modifying factor which holds the BER requirement at a lower level, so that the final transmission scheme can meet the target BER performance [47, 71, 77, 78]. However, the modifying factor depends on various system parameters. Without an analytical expression, this factor is determined empirically by extensive Monte Carlo simulations. Beside extra computation, Monte Carlo simulations could cause the factor underdetermined or overdetermined, inducing performance degradation. Alternatively, [26] represents the BER constraint deterministically based on worst-case SNR instead of taking the expectation of the BER with respect to random-varied errors, which is equivalent to setting the worst BER to satisfy BER requirement. In such a setting, a conservative solution of system throughput is achieved as the worst operational condition is rare. Thus, it is necessary to exploit an alternative constraint that can provide a tight BER bound to maintain target BER performance with respect to channel imperfection.

This chapter develops robust adaptive modulation scheme for single-user MIMO systems with imperfect channel information with the following contributions.

- The proposed approach maximizes the throughput based on the lower bound of the average BER while maintaining an acceptable BER performance by implementing an outage probability-based approach. This approach provides a tight BER bound by keeping a low probability that the received SNR falls below the pre-specified threshold.
- A deterministic form is given for the threshold in the outage probability specification. The implementation of probabilistic constraint is much efficient without involving extra Monte Carlo simulation.
- The proposed scheme offers a significant increase of the throughput by taking advantage

of transmit beamforming. It provides the strongest robustness against the channel uncertainties compared to the state-of-the-art robust adaptive modulation schemes, while guaranteeing the target BER.

The chapter is organized as follows. Section 4.2 gives a brief description of adaptive modulation scheme in the single-user MIMO system. Section 4.3 develops the proposed robust adaptive modulation scheme. Simulation results are presented and discussed in Section 4.4. Finally, a conclusion of this chapter is given in Section 4.5.

4.2 Adaptive Modulation Scheme

In this chapter, adaptive modulation scheme is considered in the context of a single-user MIMO system, shown in Fig. 4.1. By taking advantage of favorable channel conditions, adaptive modulation scheme balances the link budget through adaptive variation of transmitted power level, transmission rate and BER, which can provide a higher average link spectral efficiency as well as reliable data transmission.

The variation over time of the wireless channel makes the adaptation in wireless environment difficult. The transmitter must obtain the knowledge of current channel state via feedback channel in FDD systems, or from the reciprocity of the channel in TDD systems. However, these estimates are not only perturbed by noise, but also becomes the outdated estimates over the time-varying channel, which may degrades the system throughput significantly. Hence, these errors should be taken into account.

Based on the error model (2.10),

$$\mathbf{H} = \hat{\mathbf{H}} + \mathbf{E} ,$$

the resulting receiver SNR has the same expression in (3.1), that is,

$$f(\hat{\mathbf{H}}, \mathbf{E}) = \frac{E_s}{N_0} \text{tr} \left\{ \mathbf{C}^H (\hat{\mathbf{H}} + \mathbf{E})^H (\hat{\mathbf{H}} + \mathbf{E}) \mathbf{C} \right\} ,$$

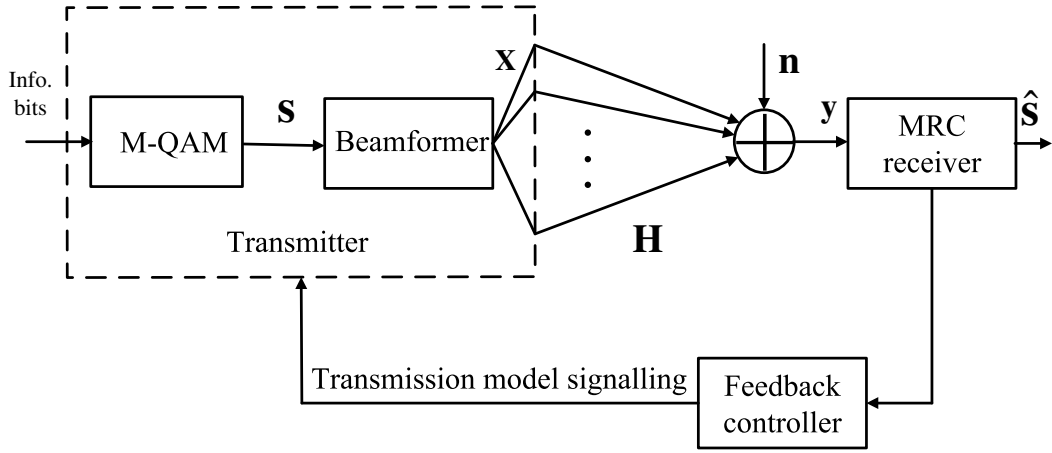


Figure 4.1: System model of adaptive modulation scheme

which is a function of channel estimate $\hat{\mathbf{H}}$ and random error \mathbf{E} . The function $f(\hat{\mathbf{H}}, \mathbf{E})$ is a mixture of noncentral χ^2 -distributed random variables with $2M$ degree of freedom [59].

Since the BER for an AWGN channel with MQAM modulation and ideal coherent phase detection can be expressed as [76, 79]

$$\text{BER} \left(K, f(\hat{\mathbf{H}}, \mathbf{E}) \right) \approx 0.2 \exp \left(-\frac{1.5 f(\hat{\mathbf{H}}, \mathbf{E})}{2^K - 1} \right), \quad (4.1)$$

which is a function of receive SNR $f(\hat{\mathbf{H}}, \mathbf{E})$ and constellation size K . In the fading channel with nonadaptive transmission (constant transmit power and rate), the received SNR varies with time. In this case, the BER is obtained by integrating the BER in AWGN over the fading distribution $p(f)$, which can be bounded as follows

$$\overline{\text{BER}}(K) \leq \int_0^\infty \text{BER} \left(K, f(\hat{\mathbf{H}}, \mathbf{E}) \right) p(f) df, \quad (4.2)$$

where $p(f)$ represents the probability density function of $f(\hat{\mathbf{H}}, \mathbf{E})$. Equation (4.2) indicates that given a constellation size, the average BER performance is determined by the instantaneous channel condition, namely, the instantaneous SNR.

The goal of robust adaptive modulation scheme is to maximize the system transmission rate while maintaining an acceptable average BER performance with imperfect channel informa-

tion. Under an average BER constraint, the transmission system can send more data by taking advantage of favorable channel conditions, otherwise set the data rate to be small or zero in poor channel condition. More specifically, the optimum transmission rate K can be achieved by solving the following problem

$$\text{maximize} \quad K, \quad (4.3)$$

$$\text{subject to} \quad \overline{\text{BER}}(K) \leq e_b, \quad (4.4)$$

where the transmission rate K is parameterized by average BER and target BER, and (4.4) represents BER constraint with a pre-specified target BER e_b , usually set as 10^{-3} .

However, the average BER constraint can not be calculated in closed form due to the mixture of noncentral χ^2 -distributed random variables $f(\hat{\mathbf{H}}, \mathbf{E})$. A common alternative is to take the lower bound of average BER [47],

$$\overline{\text{BER}}(K) \geq \text{BER} \left(K, \mathbb{E} \left[f(\hat{\mathbf{H}}, \mathbf{E}) \right] \right) = 0.2 \exp \left(-\frac{1.5 \bar{\gamma}}{2^K - 1} \right), \quad (4.5)$$

where $\bar{\gamma}$ presents the average SNR, $\bar{\gamma} = \mathbb{E} \left[f(\hat{\mathbf{H}}, \mathbf{E}) \right]$.

To satisfy the BER constraint, $\text{BER}(K, \bar{\gamma}) \leq e_b$, a candidate approximation \hat{K} for transmission rate can be found as

$$\hat{K} = \log_2 \left(1 - \frac{1.5 \bar{\gamma}}{\ln(5e_b)} \right), \quad (4.6)$$

which is determined by the average SNR. However, the transmission system based on this approximated rate (4.6) could lead to (4.4) being violated, because of the convexity of the average BER function. According to Jensen's inequality (shown in Appendix E), the maximum constellation size (4.6) is larger than the one obtained through the average BER constraint (4.4), that is

$$K \leq \hat{K} \Rightarrow \overline{\text{BER}}(K) \leq \overline{\text{BER}}(\hat{K}),$$

In this case, the target BER performance can not be maintained, since $\overline{\text{BER}}(\hat{K})$ could be larger than the target BER in some scenarios, that is, $\overline{\text{BER}}(\hat{K}) \geq e_b$. In [47], a modify factor is intro-

duced to set a low BER target but involves extra Monte Carlo computations. Alternate method is to maximize the system throughput based on the worst-case BER constraint which leads to pessimistic result. Unlike existing methods [26, 47], a probabilistic constraint is introduced in this chapter that works as an upper bound on $\overline{\text{BER}}(\hat{K})$, and consequently maintains the target BER performance.

4.3 Robust Adaptive Modulation Design

To efficiently guarantee that the system performance meets the target BER performance, a probabilistic constraint is presented instead of the average BER constraint (4.4). The proposed scheme adopts a lower bound of average BER without involving integral calculations. The corresponding system throughput \hat{K} is maximized subject to an outage probability specification that keeps the probability for SNR below a pre-specified threshold at a low level. It has the advantage of being an upper bound of $\overline{\text{BER}}(\hat{K})$, while maintaining the target BER performance.

Proposition: In the proposed adaptive modulation scheme design, the maximum transmission rate can be achieved by solving the following probabilistic-constrained optimization problem

$$\text{maximize} \quad \hat{K} = \log_2 \left(1 - \frac{1.5 \bar{\gamma}}{\ln(5e_b)} \right), \quad (4.7)$$

$$\text{subject to} \quad \Pr\{f(\hat{\mathbf{H}}, \mathbf{E}) \leq \gamma_0\} \leq \epsilon, \quad (4.8)$$

where $\Pr\{A\}$ denotes the probability of the event A , and ϵ is the outage probability. The probabilistic constraint (4.8) with pre-specified threshold γ_0 and low outage probability ϵ efficiently bounds the average BER based on approximated transmission rate $\overline{\text{BER}}(\hat{K})$, if the threshold γ_0 is satisfied

$$\gamma_0 \geq -\frac{\bar{\gamma}}{\ln 5e_b} \ln \left(\frac{1 - \epsilon}{5e_b - \epsilon} \right). \quad (4.9)$$

Proof: Defining $\alpha = 1.5/(2^{\hat{K}} - 1)$, the average BER based on approximated transmission rate,

$\overline{\text{BER}}(\hat{K})$, can be expressed as

$$\overline{\text{BER}}(\hat{K}) = \int_0^\infty 0.2 \exp\left(-\alpha f(\hat{\mathbf{H}}, \mathbf{E})\right) p(f) df, \quad (4.10)$$

with its upper bound

$$\begin{aligned} \overline{\text{BER}}(\hat{K}) &= 0.2 \left[\int_0^{\gamma_0} \exp\left(-\alpha f(\hat{\mathbf{H}}, \mathbf{E})\right) p(f) df + \int_{\gamma_0}^\infty \exp\left(-\alpha f(\hat{\mathbf{H}}, \mathbf{E})\right) p(f) df \right], \\ &\leq 0.2 \left[\int_0^{\gamma_0} p(f) df + \int_{\gamma_0}^\infty \exp\left(-f(\hat{\mathbf{H}}, \mathbf{E})\gamma_0\right) p(f) df \right], \\ &= 0.2 \left[\int_0^{\gamma_0} p(f) df + e^{-\alpha\gamma_0} \left(1 - \int_0^{\gamma_0} p(f) df\right) \right], \\ &= 0.2 \left[(1 - \exp(-\alpha\gamma_0)) \int_0^{\gamma_0} p(f) df + \exp(-\alpha\gamma_0) \right]. \end{aligned} \quad (4.11)$$

Note that the second step is possible because of the monotonically decreasing exponent function with global maximum at $\gamma = 0$ in the region $[0, \gamma_0]$ and local maximum at $f(\hat{\mathbf{H}}, \mathbf{E}) = \gamma_0$ in the region $[\gamma_0, \infty)$. The inequality (4.11) indicates that $\overline{\text{BER}}(\hat{K})$ is efficiently bounded by the probabilistic constraint (4.8). In order to maintain the target BER performance, $\overline{\text{BER}}(\hat{K}) \leq e_b$, we have

$$\overline{\text{BER}}(\hat{K}) \leq 0.2 [(1 - \exp(-\alpha\gamma_0)) \epsilon + \exp(-\alpha\gamma_0)] \leq e_b, \quad (4.12)$$

which is equivalent to,

$$\gamma_0 \geq \frac{1}{\alpha} \ln \left(\frac{1 - \epsilon}{5e_b - \epsilon} \right). \quad (4.13)$$

Substituting (4.6) into (4.13), the inequality (4.9) can be obtained immediately. In this case, the target BER performance is guaranteed, where $\overline{\text{BER}}(\hat{K})$ is efficiently bounded by the probabilistic constraint (4.8) if the threshold satisfies (4.9). \square

Remark : To guarantee the validity of (4.13) valid in practice, the threshold γ_0 should be non-negative,

$$\frac{1 - \epsilon}{5e_b - \epsilon} \geq 1, \quad \text{that is } e_b \leq 0.2.$$

In practice, the target BER is usually set lower than 10^{-3} [76]. Moreover, since the outage

1. Set up the maximum allowed BER target e_b , such as $e_b = 10^{-3}$,
2. Calculate the maximum average received SNR by taking advantage of the transmit beamforming:
2.1 Given the channel estimates $\hat{\mathbf{H}}$ and the statistics of the error matrix \mathbf{E} , apply robust transmit beamformer \mathbf{C} into the proposed scheme,
2.2 Under power constraint and outage probability specification (4.8), the maximum average received SNR, defined as $\bar{\gamma}_{max}$, is achieved.
The underlying optimization problem is now equivalent to (3.34)-(3.37), and consequently the optimal beamforming matrix can be achieved through the same processing [80].
3. Calculate the maximum achievable constellation size \hat{K} which fulfills $\overline{\text{BER}}(\bar{\gamma}_{max}, \hat{K}) \leq e_b$.

Table 4.1: Steps of Robust Adaptive Modulation Scheme

probability $\epsilon \ll 1$, we have to guarantee that

$$5e_b - \epsilon > 0, \quad \text{that is} \quad \epsilon < 5e_b,$$

so that the threshold γ_0 is nonnegative, that is, $\gamma_0 \geq 0$. It is reasonable in practice. Take $e_b = 10^{-3}$ for example, the outage probability could be any positive value such that $\epsilon < 0.005$, which is in accordance with the assumption of low outage probability.

Having the probabilistic constraint (4.8) as an upper bound of $\overline{\text{BER}}(\hat{K})$, the system throughput is maximized by the optimization problem (4.7)-(4.8). It is equivalent to maximizing the average SNR performance while keeping a low outage probability of the received SNR below a pre-specified threshold, which is similar to that in (3.34)-(3.37). More specifically, the maximum system throughput can be obtained by taking the three steps shown in Table 4.1.

Compared to the state-of-art robust schemes, simulation results show that the proposed adaptive modulation scheme provides the highest transmission rate and enables the strongest robustness against the CSIT errors while maintaining the target BER performance.

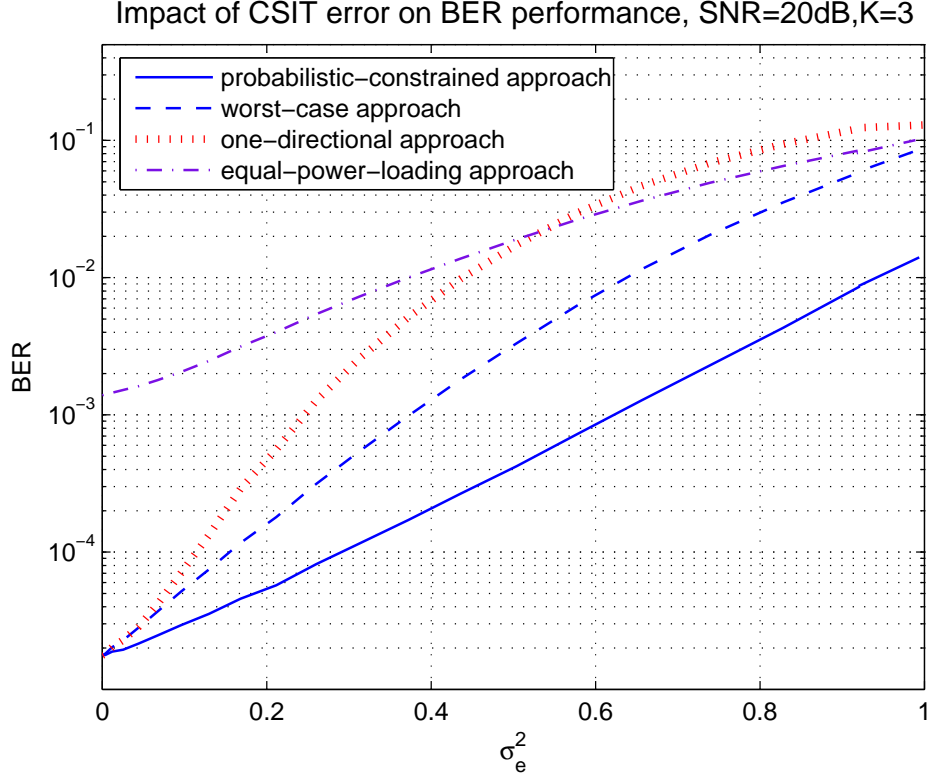


Figure 4.2: Impact of CSIT error on BER performance ($K = 3$ and SNR = 20 dB)

4.4 Simulation

In our simulation, we consider a single-user MIMO system with multiantenna at both transmitter and receiver sides ($N \geq M$). 10^3 Monte-Carlo runs are used to obtain each point. The proposed framework of adaptive modulation scheme is compared to other adaptive schemes based on different approaches, including the worst-case approach [26], one-directional approach and equal-power-loading approach [5]. Without any loss of generality, the assumptions are suggested as follows:

- *Imperfect channel estimates* : A correlated channel is based on (2.6) with fixed antenna spacing $d = 0.5\lambda$ and angle spread $\delta_\theta = 25^\circ$. The CSIT error (2.10) is assumed to be Gaussian distributed with zero mean and covariance matrix $\sigma_e^2 \mathbf{I}$, $\mathbf{E} \sim \mathcal{CN}(\mathbf{0}, \sigma_e^2 \mathbf{I})$, where the variance is varied from 0.01 to 1.

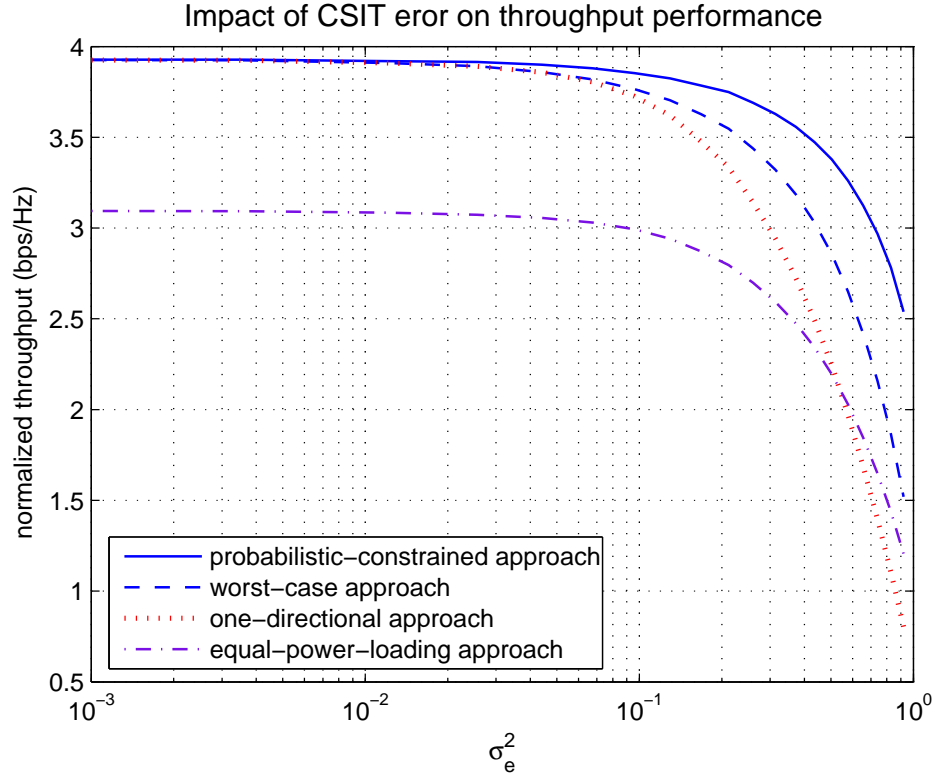


Figure 4.3: Impact of CSIT error on system throughput performance (SNR = 20 dB)

- *Other parameters* : We set the target BER as 10^{-3} . The outage probability is $\epsilon = 0.001$, and the corresponding normalized received SNR threshold $\gamma_0 = 1.0419$ based on (4.9).

Fig. 4.2 shows that the impact of CSIT error on BER performance with fixed modulation size and SNR, i.e., $K = 3$, and SNR = 20 dB. With imperfect CSIT, the performances of all the aforementioned beamforming techniques significantly degrade. Adaptive modulation based on the one-directional beamformer suffers from the worst degradation, while the proposed scheme suffers least. Note that, in the large error variance region, the difference becomes less, since little information can be obtained at transmitter and BER tends to be same.

The impact of CSIT error on the normalized throughput performance is illustrated in Fig. 4.3, where the normalized throughput is constellation size. It shows that the normalized throughput degradation is relatively sensitive to error variances. The error variance increases up to $\sigma_e^2 = 0.1$ can be tolerated without a noticeable performance degradation of the system through-

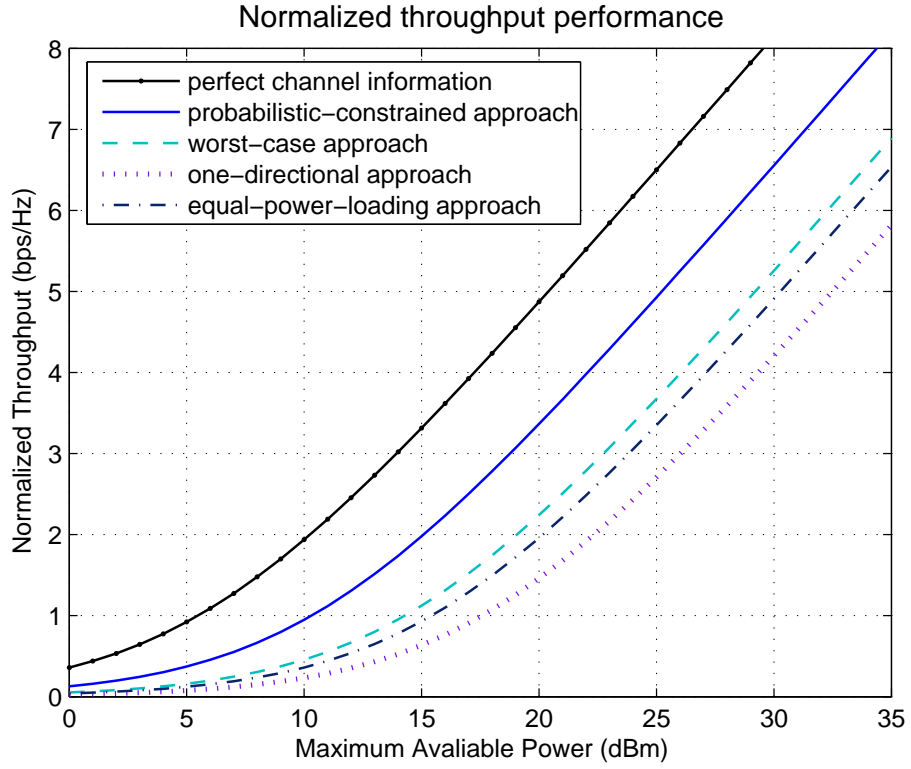


Figure 4.4: Average normalized throughput comparison, $\gamma_0 = 1.0419$ and $\epsilon = 0.1\%$

put performances. However, the system throughput drops sharply as long as $\sigma_e^2 > 0.1$, with the most degradation in the one-directional approach. In both Fig. 4.2 and Fig. 4.3, the one-directional approach has the worst ability to tolerate errors, followed by equal-power-loading approach and then worst-case approach. Since the proposed scheme could achieve higher average SNR [59], the corresponding adaptive modulation scheme outperforms other three.

In Fig. 4.4, the average throughput has been normalized with respect to the code rate, so that the gains provided by the robust technique itself for different number of transmit antennas can be compared directly. Here, we consider $N = 4$, $M = 3$ and $\sigma_e^2 = 0.5$. With the same channel conditions, the proposed scheme requires less transmit power than other schemes to fulfill the BER constraint, thus larger constellation size is allowed to modulate the transmit symbols, consequently, leading to the maximum normalized system throughput.

Fig. 4.5 shows that the average BER performance is well controlled below 10^{-3} under the proposed scheme. Here, we consider the BER performance with three different constellation

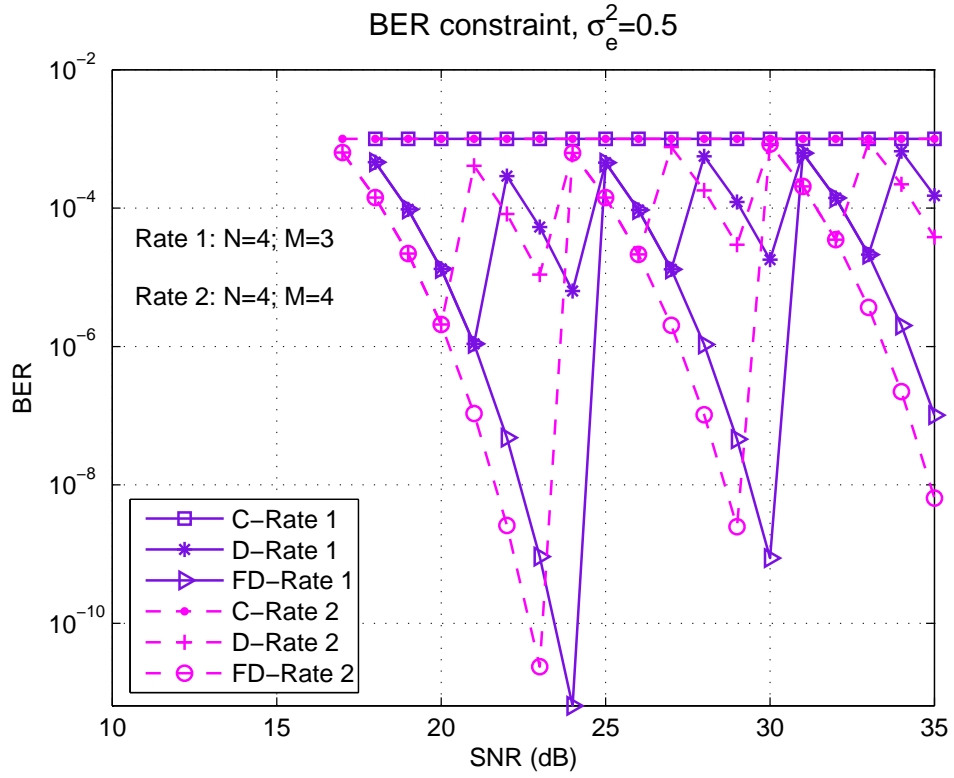


Figure 4.5: BER for the proposed robust adaptive modulation scheme, $\gamma_0 = 1.0419$ and $\epsilon = 0.1\%$

rates

$$M_i = 2^K \begin{cases} K \geq 2, K \in \mathcal{R}^+: \text{Continuous Rate (C-Rate)}, \\ K \in \{2, 3, 4, \dots\}: \text{Discrete Rate (D-Rate)}, \\ K \in \{2, 4, 6, 8\} : \text{Finite Discrete Rate (FD-Rate)}, \end{cases}$$

and two different numbers of receive antennas : $M = 3$ and $M = 4$. It indicates that no matter what the number of receive antennas is, the BER target can be achieved under probabilistic constraint. Note that the BER bound of 10^{-3} breaks down at low SNR, since (4.1) is not applicable to BPSK. Furthermore, because the BER increases monotonically with decreasing constellation size, the exact average BER is much lower than 10^{-3} with discrete rate and finite discrete rate, respectively.

4.5 Conclusion

This chapter proposes a novel robust adaptive modulation scheme that significantly improves the system throughput while satisfying the BER constraint. In the proposed scheme, the system throughput is obtained based on the lower bound of average BER. In order to maintain the BER target, a probabilistic constraint is introduced as a tight BER bound by keeping a low outage probability that the SNR falls below a pre-specified threshold. Under such a specification, the system throughput is maximized by utilizing transmit beamforming techniques under the assumption of Gaussian-distributed CSIT errors.

Simulation results demonstrate the proposed robust adaptive scheme not only provides the most significant improvement of normalized system throughput but also has the strongest error-tolerated ability among the state-of-art robust adaptive schemes. Moreover, the proposed scheme guarantees the target BER in different scenarios without involving extra Monte Carlo simulations.

Chapter 5

Probabilistic-Constrained Beamforming Design for Downlink MU-MIMO Systems

This chapter extends the single-user MIMO system to the MU-MIMO case. It has the potential to increase system capacity significantly by separating multiple users in the space domain through appropriate signal processing. Unfortunately, these techniques require accurate CSIT for their proper operations. With the inevitable channel imperfections, the main challenge for transmit beamforming is to efficiently suppress the multiple interference from other users. A robust transmit beamforming design based on SLNR criteria is proposed by introducing an outage probability specification. Under such a constraint, the corresponding design for the single-stream-per-user MU-MIMO system improves the average SINR performance implicitly by maximizing average SNR performance while keeping a low outage probability due to leakage power. Moreover, this chapter also considers the multiple-stream-per-user case and introduces a hybrid scheme that combines the proposed scheme with Alamouti code. Simulation results show that under the help of outage probability specification, both proposed beamformers achieve good BER performances, reliability of SINR levels as well as robustness against channel uncertainties.

5.1 Introduction

MU-MIMO wireless system has gained considerable amount of interest since it can increase data throughput and achieve higher diversity gain significantly. In a single-cell communication system, a base station communicates with several users in the same frequency and time slots, which leads to the multi-user interference at the end users. Thus, suppression of the multiple

interference is crucial to either transmit beamformer (precoder) or receiver decoder. In attempt to keep a low receiver complexity, it is reasonable to focus on transmit beamforming design.

To completely cancel the interference, accurate channel information is required at transmitter side. However, it is usually not available due to errors induced by imperfect channel feedback, estimation/quantization, leading to significant performance degradation. Hence, it motivates the design of robust transmit beamforming techniques which not only suppresses the interference but also ensures the robustness against the imperfect channel information.

With imperfect channel information, robust transmit beamformer is designed under a set of QoS measurements. For MU-MIMO system, one of the metrics is the minimization of the trace of the (weighted) MSE matrix [37, 41, 56]. Different from MMSE method that minimizes the system error, SINR maximization optimizes the system performance directly, which can be achieved through zero-forcing solution or iterative algorithm. In zero-forcing algorithm, the multiple interference among different users can be driven to zero under the condition that the number of antennas at base station has to be larger than the combined sum of all receive antennas by all users [54, 81]. When this configuration can not be met, an alternative method is under investigation, which iteratively finds the optimum solution by maximizing the SINR [18], or minimizing the MSE [39–41]. Since it couples optimization and feasibility simultaneously, the algorithm can not obtain a closed-form solution and easily arrives to infeasible region. Recently, a *leakage*-based approach is introduced in [12, 42, 52, 82] and further developed in [44, 62]. Although it is a suboptimal solution in terms of SINR metric, the SLNR criteria decouples the optimization and feasible problems and admits an analytical closed-form solution. In contrast to the zero-forcing solution, the leakage-based scheme does not require any dimension condition on the number of transmit/receive antennas. Moreover, the SLNR criteria also provides *fair* power allocation on the desired user, since larger power allocation on the desired user leads to more power leakage to other users. Hence, given its simplicity and fairness, the SLNR criteria is pursued as beamforming measurement in this chapter.

This chapter applies the outage probability-based approach to robust SLNR-based transmit beamforming design for both single-stream-per-user and multiple-stream-per-user MU-MIMO

systems, with the following contributions:

- The transmit beamforming maximizes the SLNR performance with the help of outage probability-based approach over single-stream-per-user MIMO systems. In this scheme, the average SNR performance is maximized with a low probability of the power leakage above an acceptable level.
- A deterministic expression of the probabilistic constraint is obtained by using the multivariate Markov's inequality for the single-stream-per-user case. Lagrangian relaxation is introduced to drop the non-convex rank constraint on beamforming matrix. The resulting optimization problem can be efficiently solved by modern convex optimization algorithms and a lower bound solution is obtained.
- The robust SLNR-based transmit design for single-stream-per-user systems provides a desirable BER performance and robustness against channel imperfections. Moreover, the SINR reliability is improved implicitly by achieving the maximum SLNR performance.
- The single-stream-per-user transmission is extended to the multiple-stream-per-user case. The resulting downlink beamforming design introduces a hybrid scheme that combines Alamouti code with the leakage-based scheme. The inter-user-interference can be eliminated by Alamouti code and the inter-symbol-interference is suppressed via probabilistic-constrained leakage-based approach.

The remaining of this chapter is organized as follows. Section 5.2 describes the system model of single-stream-per-user MU-MIMO transmission. The proposed design for the single-stream-per-user case is formulated in Section 5.3. Section 5.4 extends the single-stream-per-user case to the multiple-stream-per-user one. Numerical examples are presented and discussed in Section 5.5. Concluding remarks are given in Section 5.6.

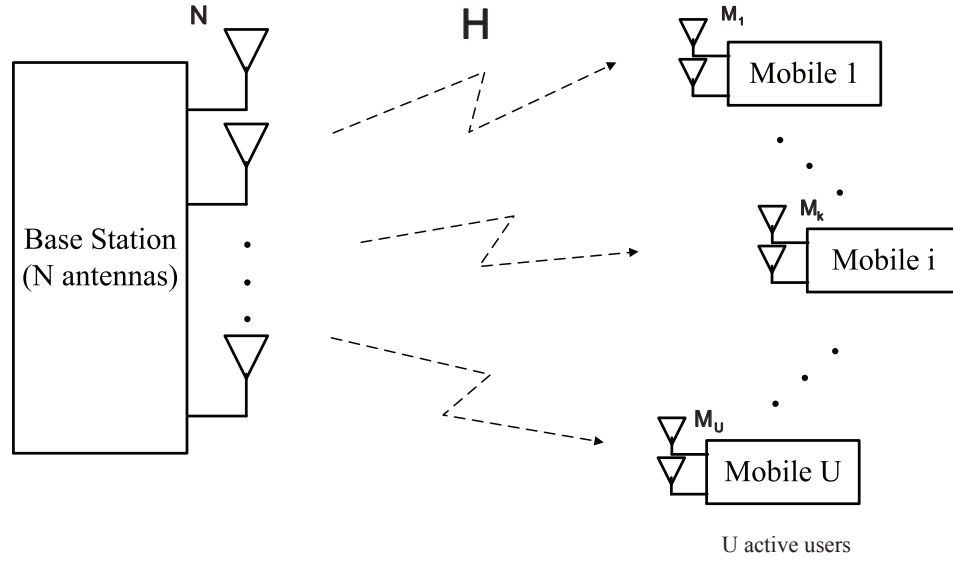


Figure 5.1: Block diagram of the MU-MIMO system

5.2 System Model of Single-Stream MU-MIMO System

From the traditional view of single-user MIMO systems, the capacity of MU-MIMO systems can be enhanced because of the spatial degrees of freedom provided by multiple antennas, if multiple users are properly scheduled to simultaneously share the spatial channel. This entails a fundamental paradigm shift from single user communications to multiple, resulting in substantial benefit experienced by MU-MIMO system. Several key advantages are included, such as providing a direct gain in multiple access capacity from multi-user diversity gain, holding the multiplexing gain without multiple antennas at mobile and immunizing to the ill-behavior of the propagation channel [82–84]. This section gives a basic model for MU-MIMO system, and discusses the criteria selection for transmit beamforming design.

5.2.1 Shift from Single-User MIMO to Multi-User MIMO Systems

In contrast to the single-user MIMO system, the basic model of MU-MIMO system is illustrated in Fig. 5.1. The base station equipped with N antennas communicates with U active users simultaneously, each of which is equipped with M_k antennas. This subsection focuses on the single-stream-per-user case firstly and the multiple-stream case will be discussed in Section

5.4. Let s_k denotes the transmitted data intended for user k . For each user the scale signal s_k is multiplied by a beamformer vector \mathbf{c}_k , thus the transmitted signal vector $\mathbf{x} \in \mathbb{C}^{N \times 1}$ can be presented as

$$\mathbf{x} = \sum_{k=1}^U \mathbf{c}_k s_k = \mathbf{C} \mathbf{s}, \quad (5.1)$$

where $\mathbf{C} \in \mathbb{C}^{N \times U}$ is beamforming matrix as $\mathbf{C} = [\mathbf{c}_1, \dots, \mathbf{c}_U]$. Assuming that the channel is a slow fading and i -th user is the desired user, the received signal vector \mathbf{y}_i for the i -th user can be written as

$$\mathbf{y}_i = \mathbf{H}_i \mathbf{C} \mathbf{s} + \mathbf{n}_i = \mathbf{H}_i \mathbf{c}_i s_i + \sum_{k=1, k \neq i}^U \mathbf{H}_i \mathbf{c}_k s_k + \mathbf{n}_i, \quad (5.2)$$

where the additive white noise $\mathbf{n}_i \in \mathbb{C}^{M_i \times 1}$ is independent complex Gaussian distributed, i.e $\mathbf{n}_i \sim \mathcal{CN}(\mathbf{0}, \sigma_i^2 \mathbf{I}_{M_i})$, and the MIMO channel for the desired user is $\mathbf{H}_i \in \mathbb{C}^{M_i \times N}$. Without loss of generalization, we assume that the desired signal power has unit power, that is, $\mathbb{E} [\|s_i\|^2] = 1$ ($i = 1, \dots, U$). At the output of maximum ratio combining (MRC) receiver, the estimate of the desired user can be expressed as

$$\tilde{s}_i = \frac{\mathbf{c}_i^H \mathbf{H}_i^H \mathbf{y}_i}{\|\mathbf{H}_i \mathbf{c}_i\|^2} = s_i + \frac{\mathbf{c}_i^H \mathbf{H}_i^H \sum_{k=1, k \neq i}^U \mathbf{H}_i \mathbf{c}_k s_k}{\|\mathbf{H}_i \mathbf{c}_i\|^2} + \frac{\mathbf{c}_i^H \mathbf{H}_i^H \mathbf{n}_i}{\|\mathbf{H}_i \mathbf{c}_i\|^2}. \quad (5.3)$$

In (5.3), the first term is the desired signal, and the second term quantifies the inter-user-interference. Note that all three parts (the desired signal, interference and sensor noise) are statistically independent of each other.

Next subsection will discuss several popular criteria for transmit beamforming design and introduce the SLNR criteria into the proposed designs.

5.2.2 Criteria Selection

Several works of transmit beamforming design have proposed to reduce the inter-user-interference for the multiuser case. According to the system configuration, standard schemes can be classified into zero-forcing and SINR designs.

Zero-Forcing Measure

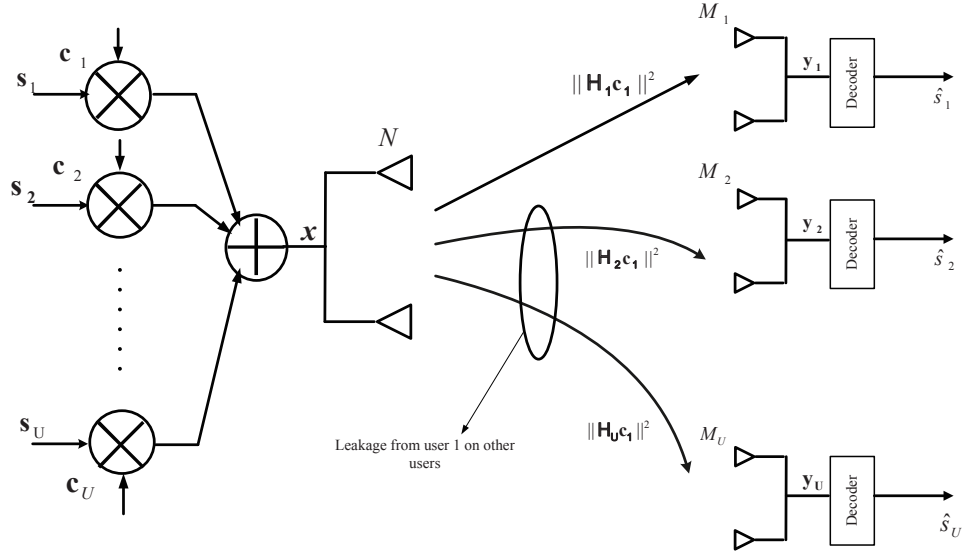


Figure 5.2: Block diagram depicting the leakage from user 1 on other users

The zero-forcing design can perfectly cancel the inter-user-interference by choosing the beam-forming matrix that enforces

$$\mathbf{H}_i \mathbf{c}_k = \mathbf{0}, \quad \forall i, k = \{1, \dots, U\}, i \neq k. \quad (5.4)$$

This criteria requires the number of transmit antennas at the base station to be larger than the combined sum of all receive antennas by all users [13, 81, 85],

$$N > \max \left\{ \sum_{k=1}^U M_k \right\}.$$

This configuration is a necessary condition for zero-forcing algorithm, since it provides at most $N - 1$ degree of freedom for the precoding to null out the interfering signals.

Two major disadvantages should be taken into account. First, the decoder at the receiver side will suffer when the noise level increases [13], since the additive noise component at the receiver has been ignored when designing the downlink beamformers. In addition, zero-forcing design imposes a restriction on the number of antennas: the number of transmit antennas at the base station should be larger than the combined sum of all receive antennas by all users.

However, it could be impractical when the number of active users is extremely large. In [85], it is proposed that the time scheduling to as an alternative, so that a subset of the users is allowed to communicate at each time slot, remaining the rest of them shut down. Under such a scheme, some of the users in the network may can not communicate with the base station because of unfair scheduling.

SINR Measure

In SINR-base scheme, the transmit beamforming design guarantees all the SINR performances achieving the target thresholds. According to (5.3), the SINR for user i at the *output* of maximum ratio combining (MRC) receiver is taken into account, that is

$$\text{SINR}_i = \frac{||\mathbf{H}_i \mathbf{c}_i||^2}{\sigma_i^2 + \frac{\sum_{k=1, k \neq i}^U ||\mathbf{c}_i^H \mathbf{H}_i^H \mathbf{H}_i \mathbf{c}_k||^2}{||\mathbf{H}_i \mathbf{c}_i||^2}} . \quad (5.5)$$

Regarding the limited transmit power, the transmit beamformer maximizes the SINR performance [18]

$$\begin{aligned} & \text{maximize} && \text{SINR}_i , \\ & \text{subject to} && \text{tr}\{\mathbf{C}\mathbf{C}^H\} \leq 1 , \quad i = 1, \dots, K . \end{aligned} \quad (5.6)$$

Two-stage approach is suggested in [18]: first checking the feasibility, then minimizing the transmission power. If the constraints are infeasible, the system should reduce the number of users by proper resource management.

Without requiring any dimension condition on the number of transmit/receive antennas, the transmission is nonorthogonal, leading to the invertible crosstalk between the desired user and other users. In this case, the SINR-based scheme is a complicated task, which couples the optimization and feasibility together. The resulting solution only can be obtained iteratively without a closed form.

SLNR-based Design

The concept of *leakage* is introduced in [42, 52], which considers the power leaked from the

desired user instead of the interference induced by other users. Regarding this criteria, the beamformer is designed to maximize SLNR. More specifically, the power allocated on the i -th user is given by $\|\mathbf{H}_i \mathbf{c}_i\|^2$, and the total power leaked from i -th user to all other user is $\sum_{k=1, k \neq i}^U \|\mathbf{H}_k \mathbf{c}_i\|^2$, depicting in Fig. 5.2. With perfect channel knowledge at the receiver, the average SLNR for the i -th user is obtained

$$\text{SLNR}_i = \frac{\|\mathbf{H}_i \mathbf{c}_i\|^2}{M_i \sigma_i^2 + \sum_{k=1, k \neq i}^U \|\mathbf{H}_k \mathbf{c}_i\|^2} = \frac{\|\mathbf{H}_i \mathbf{c}_i\|^2}{M_i \sigma_i^2 + \|\tilde{\mathbf{H}}_i \mathbf{c}_i\|^2}, \quad (5.7)$$

where $\tilde{\mathbf{H}}_i \in \mathbb{C}^{(\sum_{k=1, k \neq i}^U M_k) \times N}$ denotes an extended channel matrix that excludes \mathbf{H}_i , i.e.

$$\tilde{\mathbf{H}}_i = [\mathbf{H}_1^T, \dots, \mathbf{H}_{i-1}^T, \mathbf{H}_{i+1}^T, \dots, \mathbf{H}_U^T]^T.$$

The maximization of the average SLNR is achieved with the one directional beamformer [52]

$$\mathbf{c}_i^{\text{opt}} = \mathcal{P} \left\{ \left(M_i \sigma_i^2 \mathbf{I} + \tilde{\mathbf{H}}_i^H \tilde{\mathbf{H}}_i \right)^{-1} (\mathbf{H}_i^H \mathbf{H}_i) \right\}, \quad (5.8)$$

where $\mathcal{P}\{\cdot\}$ is the principal eigenvector of the matrix.

Although it is suboptimal in terms of the output SINR metric, the SLNR-based solution can be expressed in a closed form. More importantly, the maximum SLNR performance guarantees the least leakage power from the desired user to the rest. It implicitly reduces the interference to the desired user, consequently leading to a reliable SINR performance. In addition, there is no system configuration requirement for SLNR criteria. Subject to the above advantages, the SLNR scheme will be considered as a measurement in this chapter.

5.2.3 Channel Imperfections

The advantages of MU-MIMO system mentioned at the beginning of Section 5.2 bring challenges. The most critical to MU-MIMO system is the availability of channel knowledge at transmitter in order to properly serve the spatially multiplexed users. Perfect CSIT is crucial to achieving high QoS at the desired user. However, in real scenarios, only imperfect channel

information is available at transmitter which could degrade the system performance severely. It motivates the efforts of robust design in the presence of channel imperfections.

For i -th user, the presumed channel $\mathbf{H}_{i_p} \in \mathbb{C}^{M_i \times N}$ can be expressed as

$$\mathbf{H}_i = \mathbf{H}_{i_p} + \mathbf{E}_i, \quad (5.9)$$

where the error matrix $\mathbf{E}_i \in \mathbb{C}^{M_i \times N}$ consists of i.i.d. complex normally distributed entries with variance σ_e^2 . The subscript p is used to denote the *presumed* channel information. And the corresponding interference channel $\tilde{\mathbf{H}}_{i_p} \in \mathbb{C}^{(\sum_{k=1, k \neq i}^U M_k) \times N}$ can be written as

$$\tilde{\mathbf{H}}_i = \tilde{\mathbf{H}}_{i_p} + \tilde{\mathbf{E}}_i, \quad (5.10)$$

where the error matrix $\tilde{\mathbf{E}}_i$ is composed of $(U - 1)$ transport error matrix \mathbf{E}_i , that is,

$$\tilde{\mathbf{E}}_i = [\mathbf{E}_1^T, \dots, \mathbf{E}_{i-1}^T, \mathbf{E}_{i+1}^T, \dots, \mathbf{E}_U^T]^T. \quad (5.11)$$

Since the CSIT of each user is independent, the constructed matrix $\tilde{\mathbf{E}}_i$ has the same distribution of each component \mathbf{E}_i , that is, i.i.d complex normally distributed entries with variance σ_e^2 . Note that we assume that $\tilde{\mathbf{H}}_i$ and $\tilde{\mathbf{H}}_{i_p}$ have the same rank, that is,

$$\text{rank}(\tilde{\mathbf{H}}_i) = \text{rank}(\tilde{\mathbf{H}}_{i_p}) = \min \left(N, \sum_{k=1, k \neq i}^U M_k \right) = N.$$

The next section will design a transmit beamformer which maximizes the corresponding SLNR under inaccurate channel presumption.

5.3 Robust SLNR-based Beamformer Design for Single-Stream Case

To tackle performance degradation caused by the residual interference signals, the outage probability-based approach is introduced, which is favorable to an achievable SLNR performance of the desired user, and prevents a pessimistic result by considering the leakage power

proportionally.

According to the error model (5.9) and (5.10), SLNR of the desired user becomes a function of the presumed channel \mathbf{H}_{i_p} and $\tilde{\mathbf{H}}_{i_p}$ and the random errors \mathbf{E}_i and $\tilde{\mathbf{E}}_i$

$$f_i(\mathbf{E}_i, \tilde{\mathbf{E}}_i) = \frac{\mathbf{c}_i^H (\mathbf{H}_{i_p} + \mathbf{E}_i)^H (\mathbf{H}_{i_p} + \mathbf{E}_i) \mathbf{c}_i}{M_i \sigma_i^2 + \mathbf{c}_i^H (\tilde{\mathbf{H}}_{i_p} + \tilde{\mathbf{E}}_i)^H (\tilde{\mathbf{H}}_{i_p} + \tilde{\mathbf{E}}_i) \mathbf{c}_i} . \quad (5.12)$$

Instead of maximizing the SLNR directly, the average power allocation on the desired user is maximized while keeping a low outage probability of the leakage power being higher than an acceptable level. The proposed beamforming matrix can be obtained by solving the following problem

$$\text{maximize} \quad \mathbb{E} [\mathbf{c}_i^H (\mathbf{H}_{i_p} + \mathbf{E}_i)^H (\mathbf{H}_{i_p} + \mathbf{E}_i) \mathbf{c}_i] , \quad (5.13)$$

$$\text{subject to} \quad \Pr \left\{ M_i \sigma_i^2 + \mathbf{c}_i^H (\tilde{\mathbf{H}}_{i_p} + \tilde{\mathbf{E}}_i)^H (\tilde{\mathbf{H}}_{i_p} + \tilde{\mathbf{E}}_i) \mathbf{c}_i \geq \gamma_{0_i} \right\} \leq \epsilon_i , \quad (5.14)$$

where γ_{0_i} and ϵ_i denote the pre-specified threshold and outage probability for the desired user, respectively. In the following, the above problem will be discussed and reformulated one by one.

To simplify the expression (5.12), we define a new parameter \mathbf{W}_i as follow

$$\mathbf{W}_i \triangleq \mathbf{c}_i \mathbf{c}_i^H , \quad \mathbf{W}_i \geq 0 \text{ and } \text{rank}(\mathbf{W}_i) = 1, \quad (5.15)$$

where $\mathbf{W}_i \geq 0$ denotes the matrix \mathbf{W}_i is semi-positive definite.

5.3.1 Objective Function

Given the presumed channel \mathbf{H}_{i_p} at transmitter, the objective function (5.13) is obtained by taking the expectation of power allocated on i -th user with respect to the random error \mathbf{E}_i

$$\mathbb{E} [\mathbf{c}_i^H (\mathbf{H}_{i_p} + \mathbf{E}_i)^H (\mathbf{H}_{i_p} + \mathbf{E}_i) \mathbf{c}_i] = \text{tr} \left\{ \left(\mathbf{H}_{i_p}^H \mathbf{H}_{i_p} + \sigma_e^2 M_i \mathbf{I} \right) \mathbf{W}_i \right\} . \quad (5.16)$$

This chapter considers the system configuration that the number of transmit antennas is smaller than the number of all receive antennas combined. With N transmit antennas, only $N - 1$ degree of freedom is provided, which is less than the subspace of all users. That means, we can not guarantee the subspaces of all users are orthogonal to each other. Therefore, eigen-decomposition approach can not be easily implemented into objective function.

5.3.2 Outage Probability Specification

Besides the desirable average SNR performance, the power leakage should be well controlled. The outage probability-based approach is introduced, which guarantees a low probability of the power leakage being higher than a pre-specified threshold, formulated in (5.14). In order to efficiently achieve the optimum solution, the major problem is to convert the probabilistic constraint (5.14) into a deterministic form.

Proposition The probabilistic constraint (5.14) can be replaced by into the following convex form

$$\text{tr} \left\{ \mathbf{W}_i \left(n_i \sigma_e^2 \mathbf{I} + \tilde{\mathbf{H}}_{i_p}^H \tilde{\mathbf{H}}_{i_p} \right) \right\} \leq \epsilon_i \tilde{\gamma}_{0_i}, \quad (5.17)$$

where $\Pr\{A\}$ denotes the probability of the event A , and $\tilde{\gamma}_{0_i} = \gamma_{0_i} - M_i \sigma_i^2 > 0$.

Proof: Define

$$\mathbf{z}_i = \frac{(\tilde{\mathbf{H}}_{i_p} + \tilde{\mathbf{E}}_i) \mathbf{c}_i}{\sqrt{\text{tr}\{\sigma_e^2 \mathbf{W}_i\}}},$$

the probabilistic constraint (5.14) can be rewritten in terms of \mathbf{z}_i and \mathbf{W}_i ,

$$\Pr \left\{ \text{tr}\{\sigma_e^2 \mathbf{W}_i^H\} \|\mathbf{z}_i\|^2 \geq \bar{\gamma}_{th_i} \right\} \leq p_i, \quad (5.18)$$

where $\tilde{\gamma}_{th_i} = \gamma_{th_i} - M_i \sigma_i^2$. In order to guarantee the validity of the probabilistic constraint (5.18), $\tilde{\gamma}_{th_i}$ should be positive, namely $\tilde{\gamma} > 0$, because of nonnegative-definite random variable $\|\mathbf{z}_i\|^2$.

Based on Markov's inequality (Appendix F), an upper bound of the probability in (5.18) is

given by

$$\Pr \left\{ \|\mathbf{z}_i\|^2 \geq \frac{\bar{\gamma}_{th_i}}{\text{tr}\{\sigma_e^2 \mathbf{W}_i\}} \right\} \leq \frac{\mathbb{E} [\|\mathbf{z}_i\|^2]}{\bar{\gamma}_{th_i} / \text{tr}\{\sigma_e^2 \mathbf{W}_i\}} . \quad (5.19)$$

Under the assumption of Gaussian-distributed error $\tilde{\mathbf{E}}_i$, the random variable $\|\mathbf{z}_i\|^2$ is noncentral $\chi_{n_i}^2(\lambda_i)$ -distributed, with degree of freedom $n_i = 2 \sum_{k \neq i} M_k$ and noncentrality parameter $\lambda_i = \text{tr}\{\tilde{\mathbf{H}}_{i_p}^H \tilde{\mathbf{H}}_{i_p} \mathbf{W}_i\} / \text{tr}\{\sigma_e^2 \mathbf{W}_i\}$. The resulting expectation for the random variables $\|\mathbf{z}_i\|^2$ is [86]

$$\mathbb{E} [\|\mathbf{z}_i\|^2] = n_i + \lambda_i .$$

In order to efficiently guarantee a low probability of serious power leakage, the upper bound (5.19) is set less than p_i . that is,

$$\frac{n_i + \lambda_i}{\bar{\gamma}_{th_i} / \text{tr}\{\sigma_e^2 \mathbf{W}_i\}} \leq p_i , \quad \text{where} \quad \lambda_i = \text{tr}\{\tilde{\mathbf{H}}_{i_p}^H \tilde{\mathbf{H}}_{i_p} \mathbf{W}_i\} / \text{tr}\{\sigma_e^2 \mathbf{W}_i\} .$$

In such a setting, the inequality (5.17) is immediately obtained. To maintain the leakage power at acceptable level, the outage probability p_i is set as a small value.

Since \mathbf{W}_i and $(\tilde{\mathbf{H}}_{i_p}^H \tilde{\mathbf{H}}_{i_p})$ are semi-positive definite, the product of these two matrices is again semi-positive definite. Thus, the resulting inequality (5.17) is convex since the sum of semidefinite elements less than a positive value is convex. \square

Recall the beamformer matrix (5.15), the rank constraint on \mathbf{W}_i is non-convex. In order to convert the underlying design into convex one, Lagrangian relaxation is introduced to drop the rank constraint, only positive semi-definite matrix constraint left. In this case, a lower bound solution \mathbf{W}_i is obtained with a lower cost [87]. Moreover, regarding the limited transmit power, a constraint is set on the beamforming matrix \mathbf{W}_i ,

$$\text{tr}\{\mathbf{W}_i\} \leq 1 , \quad (5.20)$$

so that each user is allocated with unit power.

Regarding transmit power constraint (5.20), dropping the rank-one constraint (5.15), reformulated objective function (5.16) and probabilistic constraint (5.17), the proposed beamforming design (5.13)-(5.14) can be formulated as

$$\underset{\mathbf{W}_i}{\text{maximize}} \quad \text{tr} \left\{ \left(\mathbf{H}_{i_p}^H \mathbf{H}_{i_p} + \sigma_e^2 M_i \mathbf{I} \right) \mathbf{W}_i \right\}, \quad (5.21)$$

$$\text{subject to} \quad \text{tr} \left\{ \mathbf{W}_i \left(n_i \sigma_e^2 \mathbf{I} + \tilde{\mathbf{H}}_{i_p}^H \tilde{\mathbf{H}}_{i_p} \right) \right\} \leq \epsilon_i \tilde{\gamma}_{0_i}, \quad (5.22)$$

$$\text{tr} \{ \mathbf{W}_i \} \leq 1, \quad (5.23)$$

$$\mathbf{W}_i \geq 0, \quad i = 1, \dots, U \quad (5.24)$$

which can be efficiently solved by standard tools of mathematical programming [75]. Note that the rank of the solution \mathbf{W}_i is usually higher than one and, therefore, the optimal weight vector cannot be directly recovered from \mathbf{W}_i . As suggested in [20], a common approach is to use *randomization techniques*. First, a set of matrices is generated with the distribution of $\mathcal{CN}(\mathbf{0}, \mathbf{W}_i)$, and then the *best* solution is selected among such randomly generated candidates. Due to the randomization, the constraint (5.22) may be violated by some of the weight matrix candidates. The feasible weight vector can be found by simply scaling the vector. Finally, the best candidate that satisfies the constraint (5.22) and maximizes the objective function (5.21) is selected as the solution.

5.4 Robust SLNR-based Beamformer Design for Multi-Stream Case

The single-stream-per-user MU-MIMO system is now extended into the multiple-stream-per-user case where the base station simultaneously transmits multiple stream to single user in the selected users. Excluding interference coming from other users, the inter-stream-interference can lead to performance degradation without perfect channel information. This section designs a robust transmit beamforming combined with Alamouti codes with SLNR-based scheme, shown in Fig. 5.3.

In the multi-stream-per-user case, the base station is equipped with N transmit antennas and

each user has M_k receive antennas, where a multiple stream is transmitted from base station to each user with the length of multiple data equal to L_k . To prevent the inter-stream-interference caused by non-orthogonal beamforming matrix, the multiple stream $\mathbf{s}_k \in \mathbb{C}^{L_k \times 1}$ is first exploited by Alamouti scheme [6]. Note that this chapter only considers the simplest case $L_k = 2$. The transmitted coded block is given as follows

$$\mathbf{s}_k = \begin{bmatrix} s_{k,1} \\ s_{k,2} \end{bmatrix} \Rightarrow \mathbf{S}_k = \begin{bmatrix} s_{k,1} & -s_{k,2}^* \\ s_{k,2} & s_{k,1}^* \end{bmatrix}, \quad (5.25)$$

where the superscript $*$ denotes complex conjugation without transposition, and the power of data vector \mathbf{s}_k is assumed as $\mathbb{E}[\mathbf{s}_k \mathbf{s}_k^H] = \mathbf{I}/2$. The transmit coded block is multiplied by $\mathbf{C}_k \in \mathbb{C}^{N \times 2}$ before being transmitted. The transmit signal matrix $\mathbf{X} \in \mathbb{C}^{N \times 2}$ can be presented as

$$\mathbf{X} = \sum_{k=1}^U \mathbf{C}_k \mathbf{S}_k, \quad (5.26)$$

where beamforming matrix \mathbf{C}_k are assumed to be normalized as $\text{tr}\{\mathbf{C}_k^H \mathbf{C}_k\} \leq 2$. The received block for the desired user can be written as

$$\mathbf{Y}_i = \mathbf{H}_i \sum_{i=1}^U \mathbf{C}_i \mathbf{S}_i + \mathbf{N}_i = \mathbf{H}_i \mathbf{C}_i \mathbf{S}_i + \mathbf{H}_i \sum_{k=1, k \neq i}^U \mathbf{C}_k \mathbf{S}_k + \mathbf{N}_i, \quad (5.27)$$

where \mathbf{N}_i denotes the AWGN noise matrix, and each elements of \mathbf{N}_i is i.i.d complex normally distributed with zero mean and variance σ_i^2 .

Denote $\mathbf{F}_i = \mathbf{H}_i \mathbf{C}_i \in \mathbb{C}^{M_i \times 2}$, a reconstructed new matrix $\bar{\mathbf{H}}_i \in \mathbb{C}^{2M_i \times 2}$ for the desired user can be expressed as

$$\bar{\mathbf{H}}_i = \begin{bmatrix} \mathbf{F}_i^{(1,1)} & \mathbf{F}_i^{(1,2)*} & \dots & \mathbf{F}_i^{(M_i,1)} & \mathbf{F}_i^{(M_i,2)*} \\ \mathbf{F}_i^{(1,2)} & -\mathbf{F}_i^{(1,1)*} & \dots & \mathbf{F}_i^{(M_i,2)} & -\mathbf{F}_i^{(M_i,1)*} \end{bmatrix}^T, \quad (5.28)$$

where $\mathbf{F}_i^{k,l}$ denotes the (k, l) -th element in matrix \mathbf{F}_i . The rearranged receive block (5.27) can

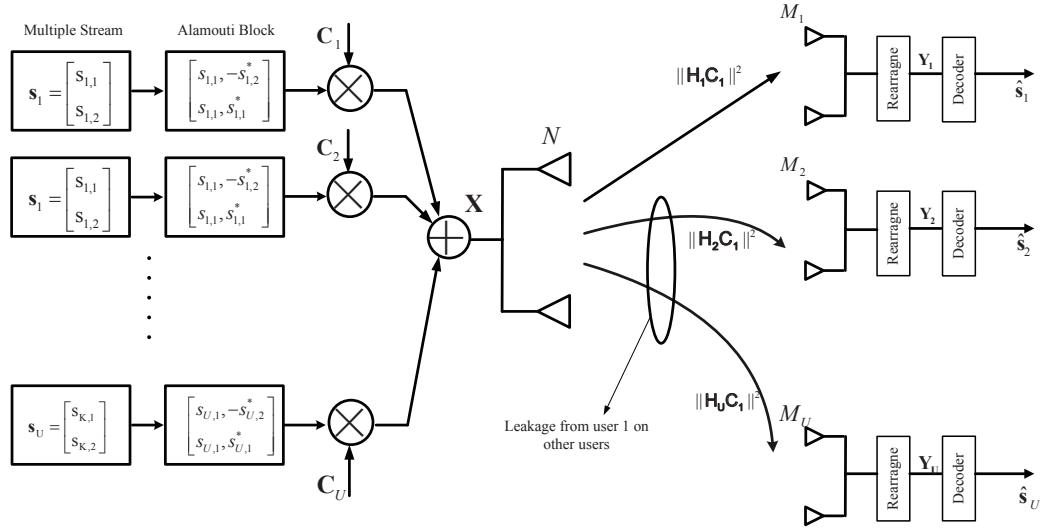


Figure 5.3: Block diagram of multi-stream MU-MIMO system depicting the leakage from user 1 on other users

be represented in terms of vector, that is

$$\mathbf{z}_i = \bar{\mathbf{H}}_i \mathbf{s}_i + \sum_{k=1, k \neq i}^U \bar{\mathbf{H}}_k \mathbf{s}_k + \mathbf{n}_i, \quad (5.29)$$

where $\mathbf{z}_i = [\mathbf{Y}_i^{(1,1)}, \mathbf{Y}_i^{(1,2)*}, \dots, \mathbf{Y}_i^{(M_i,1)}, \mathbf{Y}_i^{(M_i,2)*}]^T$, and therefore the vector \mathbf{n}_i is arranged correspondingly. According to (5.28), we have

$$\|\bar{\mathbf{H}}_i\|_F^2 = 2\|\mathbf{F}_i\|_F^2 = 2\|\mathbf{H}_i \mathbf{C}_i\|_F^2, \quad (5.30)$$

Based on the channel error model (5.10), (5.12) and (5.15), the SLNR at i -th user can be expressed as

$$\text{SLNR}_i(\mathbf{E}_i, \tilde{\mathbf{E}}_i) = \frac{\text{tr} \{ \mathbf{C}_i^H (\mathbf{H}_{i_p} + \mathbf{E}_i)^H (\mathbf{H}_{i_p} + \mathbf{E}_i) \mathbf{C}_i \}}{M_i \sigma_i^2 + \text{tr} \{ \mathbf{C}_i^H (\tilde{\mathbf{H}}_{i_p} + \tilde{\mathbf{E}}_i)^H (\tilde{\mathbf{H}}_{i_p} + \tilde{\mathbf{E}}_i) \mathbf{C}_i \}}, \quad (5.31)$$

where \mathbf{H}_{i_p} , $\tilde{\mathbf{H}}_{i_p}$, \mathbf{E}_i and $\tilde{\mathbf{E}}_i$ are defined in (5.10) and (5.12). Similar as the design problem in

(5.13)-(5.14), the proposed beamforming design can be formulated as follows

$$\underset{\mathbf{C}_i}{\text{maximize}} \quad \mathbb{E} \left[\text{tr} \left\{ \mathbf{C}_i^H (\mathbf{H}_{i_p} + \mathbf{E}_i)^H (\mathbf{H}_{i_p} + \mathbf{E}_i) \mathbf{C}_i \right\} \right], \quad (5.32)$$

$$\text{subject to} \quad \Pr \left\{ M_i \sigma_i^2 + \text{tr} \left\{ \mathbf{C}_i^H (\tilde{\mathbf{H}}_{i_p} + \tilde{\mathbf{E}}_i)^H (\tilde{\mathbf{H}}_{i_p} + \tilde{\mathbf{E}}_i) \mathbf{C}_i \right\} \geq \gamma_{0_i} \right\} \leq \epsilon_i, \quad (5.33)$$

$$\text{tr} \{ \mathbf{C}_i \mathbf{C}_i^H \} \leq 2, \quad \text{rank}(\mathbf{C}_i) = 2, \quad i = 1, \dots, U. \quad (5.34)$$

However, the underlying problem can not be solved unless it is converted into convex. Revising the above optimization problem (5.32)-(5.34), the major challenge still lies in obtaining deterministic form of the probabilistic constraint (5.33), which will be discussed as follows.

Proposition Under the assumption of Gaussian-distributed error, the probabilistic constraint (5.33) can be replaced by the following deterministic form, that is

$$\text{tr} \left\{ \left(\tilde{\mathbf{H}}_{i_p}^H \tilde{\mathbf{H}}_{i_p} + \sigma_e^2 n_i \mathbf{I} \right) \overline{\mathbf{W}}_i \right\} \leq \epsilon_i \tilde{\gamma}_{0_i}, \quad (5.35)$$

where $\tilde{\gamma}_{0_i} = \gamma_{0_i} - M_k \sigma_i^2 > 0$, and the matrix \mathbf{I} is an identity matrix.

Proof: Define

$$\mathbf{T} = (\tilde{\mathbf{H}}_{i_p} + \tilde{\mathbf{E}}_i)^H (\tilde{\mathbf{H}}_{i_p} + \tilde{\mathbf{E}}_i),$$

the probabilistic constraint (5.33) can be simplified as

$$\Pr \left\{ \text{tr} \{ \mathbf{T} \overline{\mathbf{W}}_i \} \geq \tilde{\gamma}_{0_i} \right\} \leq \epsilon_i, \quad (5.36)$$

where $\tilde{\gamma}_{0_i} = \gamma_{0_i} - M_k \sigma_i^2$. Applying the Markov's inequality (Appendix F), the upper bound for the probability in (5.36) could be obtained

$$\Pr \left\{ \text{tr} \{ \mathbf{T} \overline{\mathbf{W}}_i \} \geq \tilde{\gamma}_{th_i} \right\} \leq \frac{\mathbb{E} [\text{tr} \{ \mathbf{T} \overline{\mathbf{W}}_i \}]}{\tilde{\gamma}_{th_i}}. \quad (5.37)$$

In order to keep the power leakage below the pre-specified threshold, the upper bound (5.37) is

set less than p_i ,

$$\frac{\mathbb{E} [\text{tr}\{\mathbf{T}\overline{\mathbf{W}}_i\}]}{\tilde{\gamma}_{th_i}} \leq p_i. \quad (5.38)$$

Moreover, under the assumption that the error matrix $\tilde{\mathbf{E}}_i$ is complex Gaussian-distributed, we have

$$\mathbf{T} \sim \mathcal{CW}_N(n_i, (\sigma_e^2 \mathbf{I})^{-1} \tilde{\mathbf{H}}_{i_p}^H \tilde{\mathbf{H}}_{i_p}, \sigma_e^2 \mathbf{I}),$$

where $\mathcal{CW}_N(n_i, (\sigma_e^2 \mathbf{I})^{-1} \tilde{\mathbf{H}}_{i_p}^H \tilde{\mathbf{H}}_{i_p}, \sigma_e^2 \mathbf{I})$ denotes that the matrix $\mathbf{T}^{N \times N}$ is complex Wishart distributed with degree of freedom $n_i = 2 \sum_{k=1, k \neq i}^K M_k$, non-centrality parameter $(\sigma_e^2 \mathbf{I})^{-1} \tilde{\mathbf{H}}_{i_p}^H \tilde{\mathbf{H}}_{i_p}$ and covariance matrix $\sigma_e^2 \mathbf{I}$. The definition of noncentral Wishart distribution can be referred to Appendix G. Based on the result given by [88], the mean of complex Wishart-distributed matrix \mathbf{T}_i can be expressed as

$$\mathbb{E} [\mathbf{T}_i] = n_i \sigma_e^2 \mathbf{I} + \tilde{\mathbf{H}}_{i_p}^H \tilde{\mathbf{H}}_{i_p}. \quad (5.39)$$

Since the expectation and trace are both linear operators, $\mathbb{E} [\text{tr}\{\mathbf{T}\overline{\mathbf{W}}_i\}]$ can be expressed in the following form

$$\mathbb{E} [\text{tr}\{\mathbf{T}\overline{\mathbf{W}}_i\}] = \text{tr}\{\mathbb{E} [\mathbf{T}\overline{\mathbf{W}}_i]\} = \text{tr}\{\mathbb{E} [\mathbf{T}] \overline{\mathbf{W}}_i\}. \quad (5.40)$$

Substituting (5.39) and (5.40) into (5.36), it immediately leads to deterministic inequality (5.35). Note that since the Wishart-distributed random variables are nonnegative definite, the threshold $\tilde{\gamma}_{th_i}$ in the probabilistic constraint (5.36) should be positive, namely $\tilde{\gamma}_{th_i} > 0$.

Since both $\overline{\mathbf{W}}_i$ and $(\tilde{\mathbf{H}}_{i_p}^H \tilde{\mathbf{H}}_{i_p})$ are semi-positive definite, the constraint (5.35) is convex. \square

Taking the expectation of (5.32), and dropping the rank constraint in (5.34), the underlying optimization problem can be reformulated by replacing the probabilistic constraint to a deterministic one (5.35), such as

$$\underset{\mathbf{W}_i}{\text{maximize}} \quad \text{tr} \left\{ (\mathbf{H}_{i_p}^H \mathbf{H}_{i_p} + M_i \sigma_e^2 \mathbf{I}) \overline{\mathbf{W}}_i \right\}, \quad (5.41)$$

$$\text{subject to} \quad \text{tr} \left\{ \left(\tilde{\mathbf{H}}_{i_p}^H \tilde{\mathbf{H}}_{i_p} + \sigma_e^2 n_i \mathbf{I} \right) \overline{\mathbf{W}}_i \right\} \leq \epsilon_i \tilde{\gamma}_{0_i}, \quad (5.42)$$

$$\text{tr}\{\overline{\mathbf{W}}_i\} \leq 2, \quad (5.43)$$

$$\overline{\mathbf{W}}_i \geq 0, \quad i = 1, \dots, U, \quad (5.44)$$

where $\text{tr}\{\overline{\mathbf{W}}_i\} \leq 2$ is the power constraint on beamforming matrix, so that each symbol has unit power allocated. It can be solved similarly as the single-stream-per-user case in Section (5.3), that is, selecting the *best* solution from the randomly generated matrix candidates which are drawn from $\mathcal{CN}(\mathbf{0}, \overline{\mathbf{W}}_i)$.

5.5 Simulation

Consider MU-MIMO system with one base station (BS) equipped with 4 antennas and 3 users each equipped with 2 antennas. The data symbols are generated using QPSK modulation, and the results are averaged over 2000 channel realization. The probabilistic SLNR-based beamformer (abbr. Proposed LBeam) (5.22)-(5.25) in comparison to worst-case SLNR-based beamformer (abbr. as Worst-case LBeam) [44], uncertainty-modified SLNR-based beamformer (abbr. as Uncertainty-M LBeam) [52], non-robust SLNR-based beamformer (abbr. as Non-robust LBeam) [12], conventional single-user beamformer (abbr. as SU Beam) [5] and zero-forcing scheme (abbr. as ZF Beam) [81] with no-interference beamformer as a comparison benchmark. Without any loss of generality, we assume the following:

- The channel is zero-mean and unit-variance independent and identically distributed complex Gaussian random variables. The variance of AWGN noise per receive antenna is assumed to be the same for all user, $\sigma_1^2 = \dots = \sigma_k^2 = \sigma^2$. According to error model (5.10), the variance of uncertainty is set as $\sigma_e^2 = 0.9$.
- Parameters in outage probability specification: The normalized threshold is set as $\tilde{\gamma}_{0_i} = 0.9$ and $\epsilon_i = 5\%$.

The single-stream-per-user case is first examined. To understand the behavior of the proposed algorithm, the SINR outage performances at SNR = 0 dB and SNR = 10 dB are plotted in Fig. 5.4 and 5.5 respectively. In low SNR region (SNR = 0 dB), Fig. 5.4 shows that the proposed

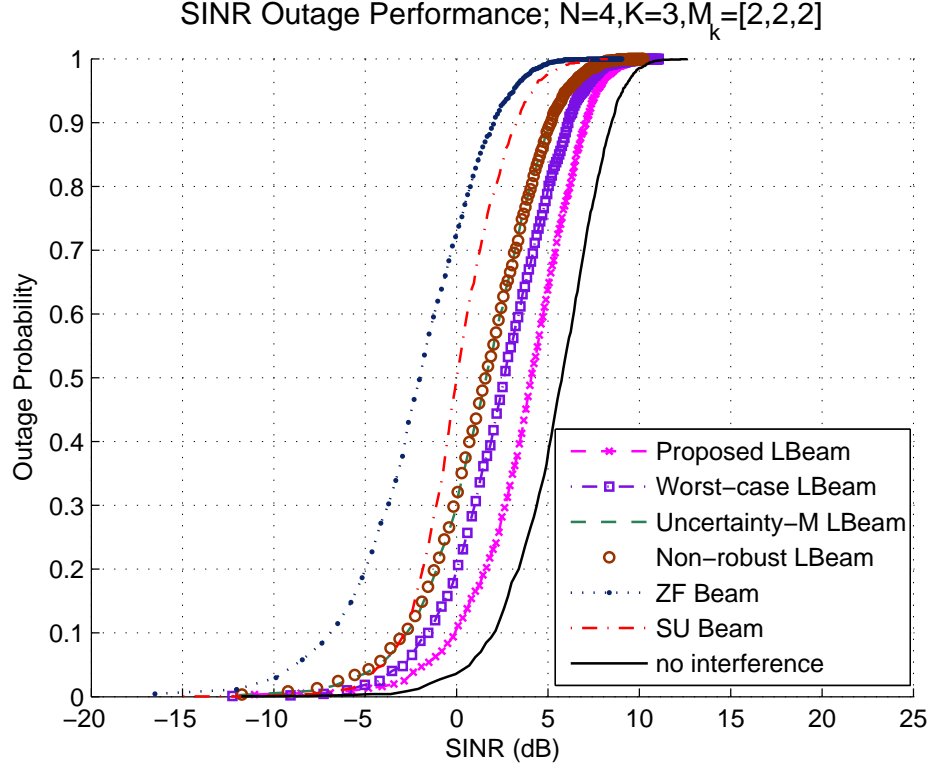


Figure 5.4: SINR outage probability performance at SNR = 0 dB ($\epsilon_i = 5\%$, $\tilde{\gamma}_{0_i} = 0.9$)

beamformer has the lowest outage probability, around 10% at SINR = 2 dB. It means for 90% of the channel realizations, the achieved SINR is larger than 2 dB. As shown in the figure, using the proposed beamformer results in an 1 dB improvement in 10% outage value compared to the worst-case SLNR-based beamformer, and an 2.5 dB improvement compared to the rest of SLNR-based beamformers and single-user one. Note that the zero-forcing beamformer provides worst performance because of the antenna configuration that the number of transmit antennas is smaller than the number of all receive antenna combined. When SNR increases to 10 dB (illustrated in Fig. 5.5), the proposed beamformer still provides the best performance of SINR reliability among all compared beamformers, where 90% SINR is higher than 6 dB. It has an 5 dB improvement compared to the worst-case approach. Meanwhile, the difference between uncertainty-modified SLNR-based beamformer and non-robust SLNR-based become larger, as the noise variance does not dominate in the SINR expression and the impact of uncertainty becomes obvious. As shown in both Fig. 5.4 and Fig. 5.5, although it is a suboptimal

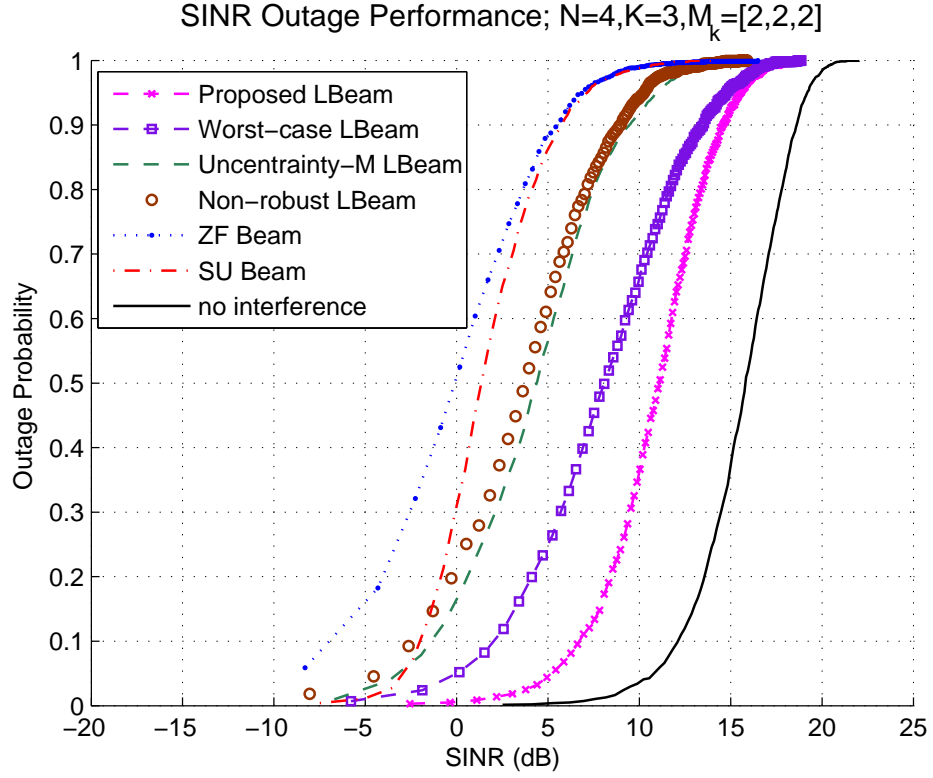


Figure 5.5: SINR outage probability performance at SNR = 10 dB ($\epsilon_i = 5\%$, $\tilde{\gamma}_{0_i} = 0.9$)

solution with respect to SINR criterion, the proposed scheme still outperforms than all other beamformers. It is because that the leakage power from the desired user is suppressed at low threshold, which consequently tends to reduce the interference from all other users. Moreover, in Fig. 5.5 the curves of the no interference, proposed and single-user beamformers present in a shifting from right to left, while the similar case for worst-case, uncertainty-modified, and non-robust leakage-based beamforming approaches and zero-forcing techniques. It indicates that the proposed beamforming more efficiently suppresses the interference from other users.

Fig. 5.6 shows the average BER performance to further depict the difference among the proposed beamformer and all other compared beamformers. The BER curves are plotted based on SINR [1], such as

$$P_e = \left[\frac{1}{2}(1 - \mu) \right]^{2M_k} \sum_{k=0}^{2M_k-1} \binom{2M_k-1+k}{k} \left[\frac{1}{2}(1 + \mu) \right]^k,$$

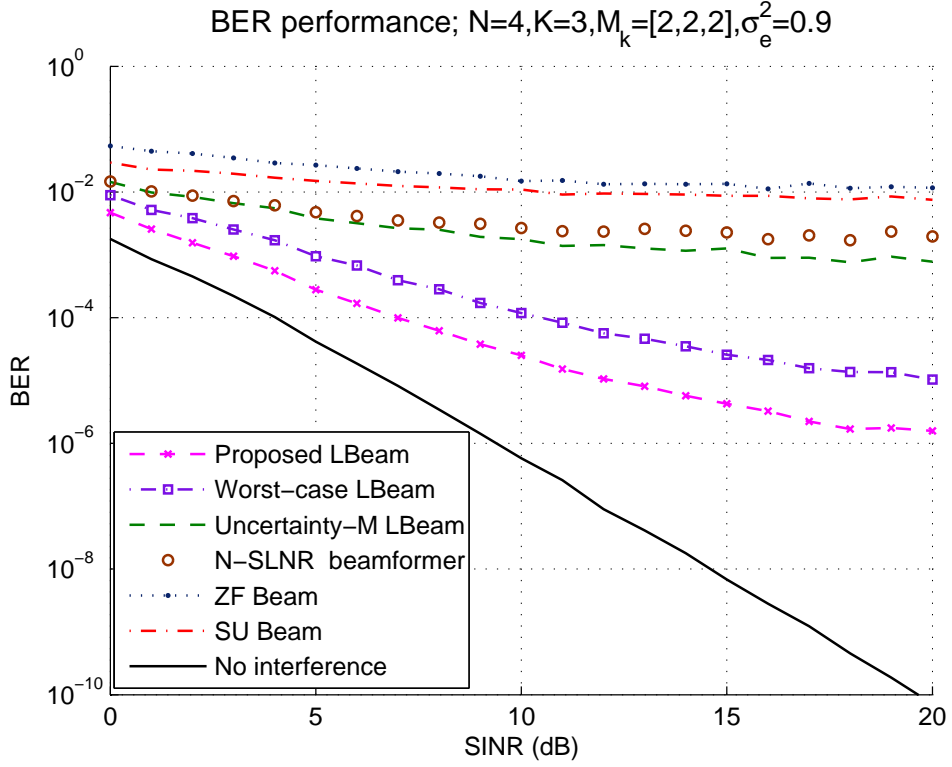


Figure 5.6: BER performance with $\sigma_e^2 = 0.9$ ($\epsilon_i = 5\%$, $\tilde{\gamma}_{0_i} = 0.9$)

where $\mu = \sqrt{\frac{\text{SINR}}{1+\text{SINR}}}$, and SINR is defined in (5.5). At low SNR, the proposed beamformer maintains an acceptable 10^{-3} BER at SNR = 3 dB for three simultaneously active users. To achieve the same BER performance, SNR required to proposed beamformer is 2 dB less than the worst-case SLNR-based beamformer. In medium and high SNR region, the proposed beamformer also outperforms in term of error floor performance among all other compared beamformers. Note that error floor occurs as long as that the interference is higher than noise level. Significant error floor suggests higher interference involved, consequently with low SINR output and poor BER performance. Thus, Fig. 5.6 indicates the SLNR criteria that reduces leakage power from desired user is a smart and simple method to improve SINR performance.

Fig. 5.7 illustrates the error-tolerance ability of beamformers. It shows that the proposed beamformer provides the strongest tolerance to error uncertainty, with absolute BER increased 1.15×10^{-5} , i.e. from 3.5×10^{-6} to 2.5×10^{-5} , when the variance of error is varied from 0 to 0.9. In the same scenario, the BER performance has the most degradation using uncertainty-

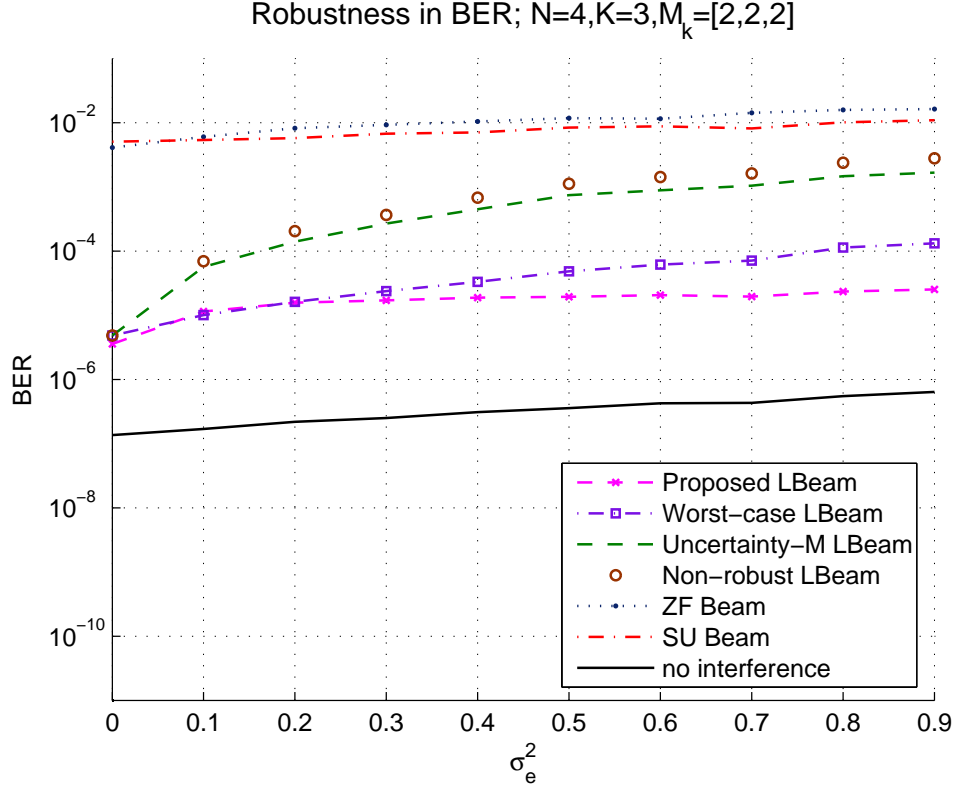


Figure 5.7: Robustness in BER at SNR = 10 dB ($\epsilon_i = 5\%$, $\tilde{\gamma}_{0_i} = 0.9$)

modified SLNR-based and the non-robust SLNR-based beamformers, having around 1.5×10^{-3} absolute BER increase. Note that compared to the zero-forcing and single-user beamformers, three leakage-based beamformers are sensitive to error uncertainty.

The parameters in probabilistic constraint (5.22) are crucial to the behavior of the probabilistic SLNR-based beamformer. In order to investigate these parameters, Fig. 5.8 displays the SINR outage probability performance under difference parameter selections, with two benchmarks (provided by no-interference and zero-forcing beamformers). It is suggested that a better SINR outage performance can be obtained with low outage probability and low leakage threshold, such as $\epsilon_i = 5\%$ and $\tilde{\gamma}_{th_i} = 0.3$. In addition, it demonstrates that the outage-probability selection has more significant impact on SINR performance than leakage-threshold selection, which gives the reason that in the previous simulation results, the proposed SLNR-based beamformer can achieve good performance with $\epsilon_i = 5\%$ and $\tilde{\gamma}_{th_i} = 0.9$.

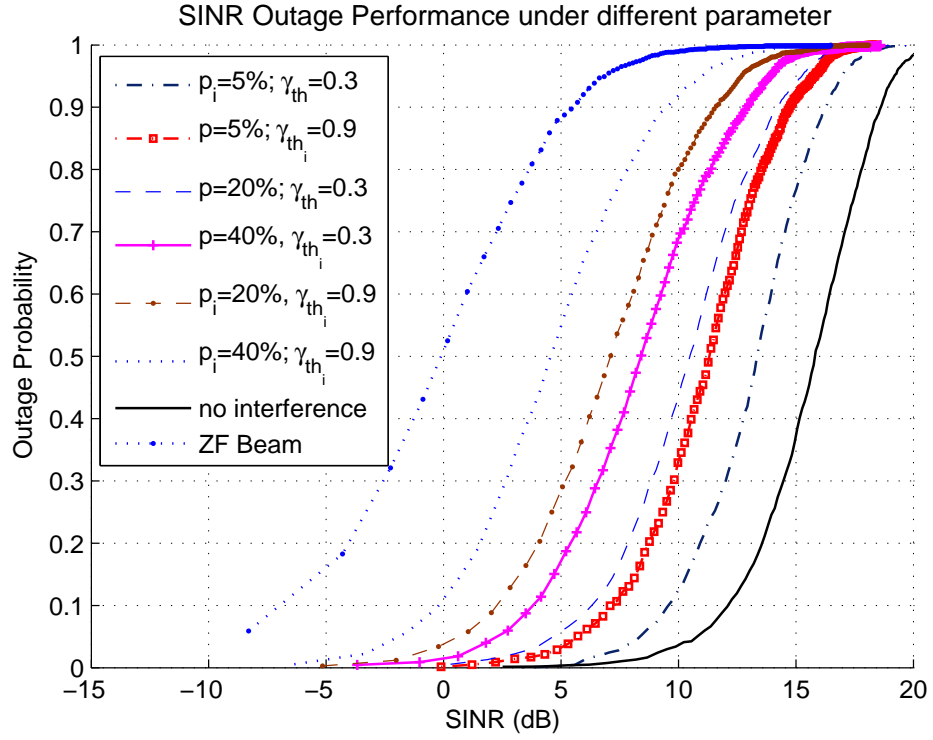


Figure 5.8: Impact of parameter choosing on SINR outage performance at SNR = 10 dB

Finally, the SINR reliability performance for multiple-stream-per-user case shows in Fig. 5.9, where the proposed beamformer with multiple stream is compared to the worst-case SLNR-based and uncertainty-modified SLNR-based beamformers. In this case, besides the interference induced by other users, the interference also comes from the symbols transmitted to the same user. Thanks to Alamouti code, the inter-symbol-interference has been eliminated. In the proposed hybrid scheme, the SINR reliability performance is only affected by the inter-user-interference. The resulting beamforming technique provides the lowest SINR outage probability, around 10% at SINR = 6 dB. As shown in the figure, using the proposed beamformer results in an 5 dB improvement in 10% outage value compared to the worst-case SLNR-based beamformer, and an 7 dB improvement compared to other SLNR-based beamformers. Note that under the same system configurations, the worst-case SLNR-based and uncertainty-modified SLNR-based beamformers perform worse than those in the single-stream-per-user case, because of the inter-symbol-interference.

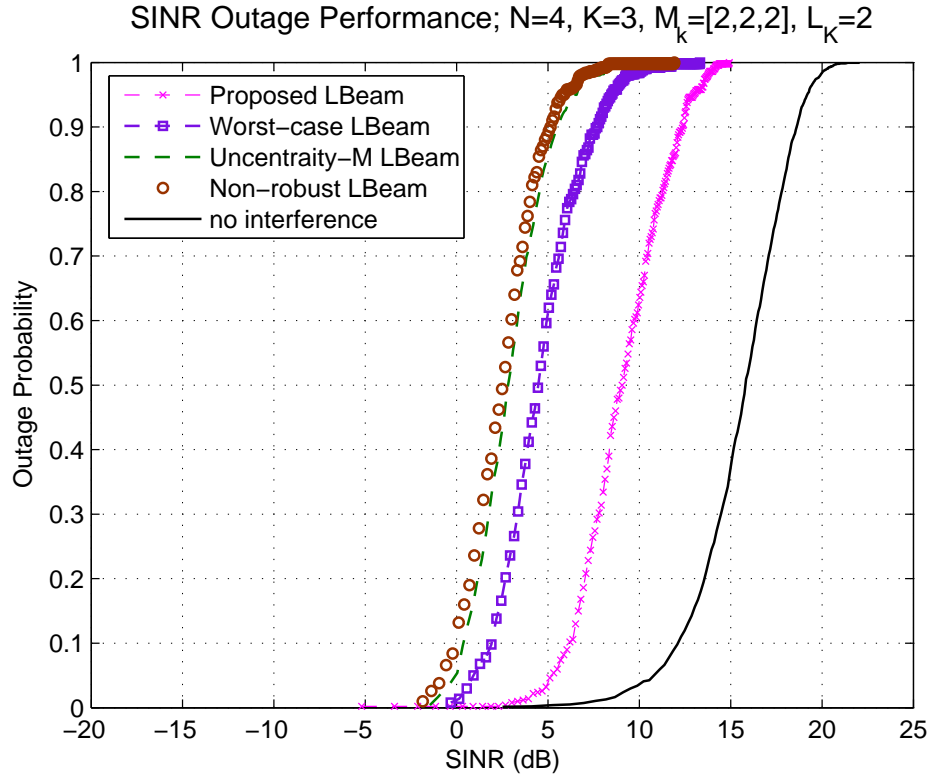


Figure 5.9: SINR outage probability performance over multiple stream MU-MIMO system at SNR = 10 dB ($\epsilon_i = 5\%$, $\tilde{\gamma}_{0_i} = 0.9$)

5.6 Conclusion

A leakage-based transmit beamforming design for MU-MIMO communications is proposed to maximize the average desired signal power and guarantee the leakage power under an acceptable level. This approach is formulated as a probabilistic-constrained optimization problem so that the probability of the leakage power higher than a pre-specified threshold is less than the target percentage. Under the assumption of complex Gaussian-distributed estimate errors, the probabilistic constraint is replaced by a deterministic convex one. By introducing Lagrange relaxation, the resulting convex optimization problem is efficiently solved by modern software packages. Furthermore, the proposed beamformer is further implemented into the multiple-stream-per-user case with the combination of Alamouti code. In such a hybrid scheme, the Alamouti scheme eliminates the inter-data-interference, while the probabilistic-constrained approach suppresses the inter-user-interference proportionally.

Simulation results show that the proposed beamformer provides the best BER performance and SINR reliability for both single-stream-per-user and multi-stream-per-user cases. Furthermore, it also demonstrates the highest robustness against imperfect channel information.

Chapter 6

Conclusion and Future Work

6.1 Conclusion

The MIMO system represents a promising technology in wireless communication which offers a significant performance improvement over SISO systems, such as higher data rates, better QoS and enhanced transmission reliability [5]. Transmit beamforming is one of the popular techniques to exploit the benefits of MIMO system with the requirement of perfect CSIT. However, only imperfect CSIT is available in real scenarios, which leads to significant performance degradation, and consequently posing challenges in system analysis and signal design. It motivates to exploit a robust transmit beamforming against errors in CSIT.

Existing robust techniques can enhance the system reliability and channel capacity by optimizing either the average system performance or the worst-case system performance, but followed with two major drawbacks. One of the disadvantages is that the statistic-based beamformers only optimize the average performance based on channel mean or covariance. Without considering the extreme scenario, it could break down when persistent error occurs. On the other hand, although the worst-case-based beamforming design provides robustness with the knowledge of the extreme channel conditions, the system performance only achieve conservative results, because the extreme case is rare in practice.

This thesis focuses on the exploiting of a flexible and reasonable transmit beamforming technique which provides a reliable and robustness transmission against the MIMO channel imperfections. The key in the proposed designs is the outage probability specification that measures the impact of channel imperfection on system performance proportionally.

Chapter 3 introduces an outage probability specification to the robust transmit beamforming design for single-user MIMO system. Regarding the average received SNR criteria, the prob-

abilistic constraint keeps a low probability of the SNR being below an acceptable level. In such a setting, the proposed transmit beamformer maximizes the average received SNR performance with the consideration of unacceptable scenarios by probability measurement. The probabilistic-constrained optimization problem is converted into a convex problem by transforming the probabilistic constraint into a convex form, so that the underlying problem is efficiently solved by modern software package. The proposed beamformer provides the best average received SNR performance compared to other popular transmit beamformers with well-controlled low outage probability. Moreover, it offers much broader error-tolerance range and more robustness against error variance misspecification than the worst-case beamformer.

Chapter 4 discusses the implementation of the probabilistic constraint in adaptive modulation design for single-user MIMO system. The proposed adaptive modulation scheme achieves the maximum transmission rate while maintaining an acceptable average BER performance. Under the assumption of Gaussian-distributed errors in CSIT, the expectation of the average BER is difficult to obtain, and replaced by its lower bound. To maintain the target average BER performance, the outage probability-based approach is introduced to provide a tight BER bound. Under such a constraint, the proposed scheme maximizes the transmission rate by taking advantage of transmit beamforming. Given the same channel conditions, the proposed scheme requires less transmit power than other popular schemes to fulfill the BER constraint, allows larger constellation size to modulate the transmit symbols, and consequently leads to a higher system throughput. Besides providing strong robustness against the CSIT errors, the proposed scheme also maintains the target average BER performance under different scenarios.

Chapter 5 extends the single-user MIMO system into the multi-user case, and designs a SLNR-based transmitter beamformer with outage probability specification when only imperfect CSIT is available. Two scenarios are taken into account, that is, single-stream-per-user and multiple-stream-per-user MU-MIMO systems. For single-stream-per-user MU-MIMO systems, the proposed beamformer maximizes the average SNR performance while keeping a low outage probability of a pre-defined power leakage level. In a multiple-stream-per-user scenario, a hybrid design combines Alamouti code with SLNR-based transmit beamforming technique. With the

assistance of Alamouti code, the inter stream interference is exterminated, so that the leakage power only comes from other users. The resulting optimized problem turns to the same problem as the single-stream case. Under outage probability specification, the proposed scheme achieves a reliable SINR level by maximizing the average SNR performance with well-controlled leakage power. Moreover, the proposed scheme not only achieves outstanding BER performance, but also withstands the impact of channel imperfection on system performance.

Given the superior overall performances and significantly improved robustness, the outage probability-based approach provides an attractive alternative to existing robust techniques under imperfect channel information at transmitter.

6.2 Limitation of Work

In Chapter 3, the proposed beamforming is designed based on the assumption of perfect channel information known at receiver. In practice, the channel information can not be perfectly estimated at receiver. In this case, the receive beamforming could be jointly designed according to the channel conditions.

In Chapter 5, the leakage-based robust transmit beamforming can achieve a sound QoS performance by using probabilistic constraint with an implicit condition that the number of total user is less than the number of transmit antenna. Under this assumption, the full multiplexing gain can be achieved [89]. However, in a large user regime $U \gg M$, the channel spatial information cannot simultaneously benefit from multiuser diversity, the transmitter performs user selection and the corresponding beamforming can only support up to M out of K users at a time. Moreover, Chapter 5 did not consider the total system throughput which can be improved using a proper scheduler. In order to improve the throughput performance and fair allocation of the power, scheduling strategy should be considered jointly with beamforming design.

6.3 Future Work

Considering the channel imperfections at both transmitter and receiver, a joint transmitter-receiver beamforming framework is required. In [90], MMSE V-BLAST structure provides an attractive approach to address the channel imperfection at receiver with an improved BER performance in comparison to the linear MMSE detector. Moreover, a worst-case MMSE V-BLAST scheme is also considered under the assumption of norm-bounded errors [91]. This algorithm outperforms in terms of BER and achieves the similar computationally efficient level as the V-BLAST algorithm with perfect CSI. Note that since the channel uncertainty is modeled as unknown but norm-bounded errors, the resulting solution could be conservative. Regarding the performance improvement provided by MMSE V-BLAST scheme, it could be an attractive research topic to jointly design the probabilistic-constrained beamforming and MMSE V-BLAST receiver against to random-varied channel uncertainty.

Moreover, one of the fundamental lessons learned from information theory is that resource allocation techniques help to exploit the gains of multiuser MIMO systems. Fairness in resource allocation among the users is a key parameter and should be taken into consideration. As a full-fair scheduling scheme, round-robin scheduler is a simple and efficient scheme, where all users have the same priority for accessing the channel, but it does not exploit the multiuser diversity. On the other hand, an exhaustive search that selects users that exhibit a compromise between a high level of instantaneous SINR and a good separability of their spatial signatures to facilitate user multiplexing [84]. However, its computation is high, roughly $\mathcal{O}(U^K)$ for $U \times K$ user set. A practical and low complexity algorithm has been developed [92–96]. In this algorithm, the transmitter chooses the single user with the highest channel capacity, then finds the next user that provides the maximum sum rate from the remaining unselected users, and repeats until K users are selected. In this case, the greedy scheduler achieves a higher throughput than round-robin scheduler does, and has a low complexity, roughly $U \times K$ which is much less than the full search method. But the price is the unfairness in resource allocation among the users. A tradeoff between the total throughput and the fairness among the user is proposed, known as the proportional fair scheduler which chooses the user with the highest normalized-throughput or

normalized-SNR [97]. Regarding the low complexity and fairness provided by the proportional fair scheduler, further work can focus on the joint beamforming design with this suboptimal scheduling in the context of leakage-based scheme.

To further improve spectral and power efficiency of wireless networks without the additional complexity of multiple antennas, the conventional MU-MIMO systems have been extended into the cooperative transmission that shares the antenna source and relays the signals in order to create a virtual antenna array. Different from the traditional antenna array technology, synchronization becomes a critical problem. It is because that the signals from the relay nodes tend to arrive at the destination node at different time, resulting in frequency-selective fading channel. Recently, the channel imperfection and asynchronization have been considered separately in transmit beamforming design. Based on perfect channel state information, imperfect synchronization in time and frequency has been addressed successfully, such as combining beamforming with OFDM schemes [98–101], and designing distributed STBC [102–104]. On the other hand, the QoS performance has been optimized under the worse-case channel condition by using convex optimization [105–107] without consideration of asynchronization. To survive in the real scenarios, both asynchronization and imperfect channel estimates should be taken into account simultaneously, which motivates the development of more robust beamforming. Regarding outage probability specification that provides robustness to channel imperfections and the role of OFDM scheme in frequency selective channels, it could be an attractive framework that combines the outage probability-based transmit beamforming with OFDM scheme against channel imperfection and asynchronization.

Appendix A

SNR Approximation in (3.9)

According to (3.5) and (3.6), the random varied SNR in (3.3) can be rewritten as

$$\begin{aligned}
 f(\hat{\mathbf{H}}, \mathbf{E}) &= \frac{E_s}{N_0} \text{tr} \left\{ (\hat{\mathbf{H}} + \mathbf{E})^H (\hat{\mathbf{H}} + \mathbf{E}) \mathbf{U}_c \mathbf{D}_c \mathbf{U}_c^H \right\} \\
 &= \frac{E_s}{N_0} \text{tr} \left\{ [(\hat{\mathbf{H}} + \mathbf{E}) \mathbf{U}_c]^H [(\hat{\mathbf{H}} + \mathbf{E}) \mathbf{U}_c] \mathbf{D}_c \right\} \\
 &= \frac{E_s}{N_0} \text{tr} \left\{ (\tilde{\mathbf{H}} + \tilde{\mathbf{E}})^H (\tilde{\mathbf{H}} + \tilde{\mathbf{E}}) \mathbf{D}_c \right\}, \tag{A.1}
 \end{aligned}$$

where $\tilde{\mathbf{H}} = \hat{\mathbf{H}} \mathbf{U}_c$, and $\tilde{\mathbf{E}} = \mathbf{E} \mathbf{U}_c$.

Since \mathbf{D}_c is a diagonal matrix, the trace operation output of (A.1) can be expressed as follows

$$\begin{aligned}
 &f(\hat{\mathbf{H}}, \mathbf{E}) \\
 &= \frac{E_s}{N_0} \begin{bmatrix} d_1 \sum_{j=1}^M (\tilde{h}_{j1} + \tilde{e}_{j1})^H (\tilde{h}_{j1} + \tilde{e}_{j1}) & \dots & \dots \\ \vdots & \ddots & \vdots \\ \vdots & \dots & d_N \sum_{j=1}^M (\tilde{h}_{jN} + \tilde{e}_{jN})^H (\tilde{h}_{jN} + \tilde{e}_{jN}) \end{bmatrix} \\
 &= \frac{E_s}{N_0} \sum_{i=1}^N d_i \sum_{j=1}^M (\tilde{h}_{ji} + \tilde{e}_{ji})^H (\tilde{h}_{ji} + \tilde{e}_{ji}). \tag{A.2}
 \end{aligned}$$

Eq. (3.9) is easily obtained by defining $Z_i = \sum_{j=1}^M (\tilde{h}_{ji} + \tilde{e}_{ji})^H (\tilde{h}_{ji} + \tilde{e}_{ji})$.

Moreover, under the assumption that each element of error matrix \mathbf{E} is zero-mean Gaussian distributed with variance σ_e^2 , each element in matrix $\tilde{\mathbf{E}}$ still follows the Gaussian distribution, such as $\tilde{e}_{ij} \sim \mathcal{CN}(0, \sigma_e^2)$, consequently, $(\tilde{h}_{ji} + \tilde{e}_{ji}) \sim \mathcal{CN}(\tilde{h}_{ji}, \sigma_e^2)$. According to Appendix B, the random variable Z_i , ($i = 1, \dots, N$) is noncentral $\chi_{n_i}^2(\delta_i)$ distributed with noncentrality parameter $\delta_i = \tilde{h}_i^H \tilde{h}_i$ and the degree of freedom $n_i = 2M$.

Appendix B

Noncentral χ^2 Distribution

Definition [86] : If X_i are k independent, normally distributed random variables with mean μ_i and variance σ_i^2 , then the random variable

$$\sum_{i=1}^k \left(\frac{X_i}{\sigma_i} \right)^2 \quad (\text{B.1})$$

is distributed according to the noncentral $\chi_k^2(\lambda)$ distribution with degrees of freedom k and noncentrality parameter $\lambda = \sum_{i=1}^k (\mu_i^2 / \sigma_i^2)$.

The probability density function is given by [86]

$$f_X(x; k, \lambda) = \frac{1}{2} e^{-\frac{x+\lambda}{2}} \left(\frac{x}{\lambda} \right)^{k/4-1/2} I_{k/2-1}(\sqrt{\lambda x}) , \quad (\text{B.2})$$

where $I_a(y)$ is a modified Bessel function of the first kind given by

$$I_a(y) = \left(\frac{y}{2} \right)^a \sum_{j=0}^{\infty} \frac{(y^2/4)^j}{j! \Gamma(a+j+1)} .$$

The moment generating function is given by [86]

$$M(t; k, \lambda) = \frac{\exp\left(\frac{\lambda t}{1-2t}\right)}{(1-2t)^{k/2}} . \quad (\text{B.3})$$

The mean, variance, skewness and kurtosis are [86]

$$\mu = \lambda + k , \quad \gamma_1 = \frac{2\sqrt{2}(3\lambda + k)}{(2\lambda + k)^{3/2}} , \quad (\text{B.4})$$

$$\sigma^2 = 2(2\lambda + k) , \quad \gamma_2 = \frac{12(4\lambda + k)}{(2\lambda + k)^2} . \quad (\text{B.5})$$

Appendix C

Proof of Inequality (3.26)

Corollary [74] Let $p \neq 1$ be a positive real number. The inequalities

$$[1 - e^{-\alpha x^p}]^{1/p} < \frac{1}{\Gamma(1 + 1/p)} \int_x^\infty e^{-t^p} dt < 1 - [1 - e^{-\beta x^p}]^{1/p} \quad (\text{C.1})$$

are valid for all positive x if and only if

$$\alpha \geq \max \{1, [\Gamma(1 + 1/p)]^{-p}\} , \quad \text{and} \quad 0 \leq \beta \leq \min \{1, [\Gamma(1 + 1/p)]^{-p}\} .$$

Taking the left side inequality in C.1, the inequality

$$\Pr \{ \chi_{n_i}^2 \leq x' \} = 1 - \frac{1}{\Gamma(1 + 1/p)} \int_x^\infty e^{-t^p} dt < (1 - e^{-x^p})^{\frac{1}{p}} \quad (\text{C.2})$$

is valid as $\max \left\{ 1, \Gamma(1 + M)^{-\frac{1}{M}} \right\} = 1$. Consequently, the upper bound on (3.26) is immediately obtained.

Appendix D

One-Ring Channel Model

The correlated-fading channel, also called spatial fading correlated channel, may occur at either one end of transmission link (i.e single-side correlated), or both ends (i.e. double-sided correlated), because of insufficiently spaced antennas or limited number of scatterers. In this thesis, the single-side correlated fading channel is considered and modeled by extending the "one-ring" model [4], which is appropriate in the fixed wireless communication system with a seldom-obstructed base station, shown in Fig. D.1.

The spatial fading correlation of the narrowband flat fading channel is determined from the physical parameters, including antenna spacing, antenna arrangement, angle spread, and angle of arrival. The covariance between \mathbf{H}_{ij} and \mathbf{H}_{mn} can be represented as [4]

$$\begin{aligned} \mathbb{E} [\mathbf{H}_{ij} \mathbf{H}_{mn}^H] &= \frac{1}{2\pi} \int_0^{2\pi} \exp \left\{ -j \frac{2\pi}{\lambda} \left[D_{TA_i \rightarrow S(\theta)} - D_{TA_j \rightarrow S(\theta)} \right. \right. \\ &\quad \left. \left. + D_{S(\theta) \rightarrow RA_m} - D_{S(\theta) \rightarrow RA_n} \right] d\theta \right\} \\ &\approx \frac{1}{2\pi} \int_0^{2\pi} \exp \left\{ -j \frac{2\pi}{\lambda} \left[d_{Tx}(i, j) \left(1 - \frac{\delta_\theta^2}{4} + \frac{\delta_\theta^2 \cos 2\theta}{4} \right) \right. \right. \\ &\quad \left. \left. + \delta_\theta d_{Ty}(i, j) \sin \theta + d_{Rx}(m, n) \sin \theta + d_{Ry}(m, n) \cos \theta \right] \right\} d\theta \quad (\text{D.1}) \end{aligned}$$

where

- Spread angle δ_θ can be approximated as $\delta_\theta \approx \arcsin(R/D)$, since the radius R and the distance D between base station and subscriber unit are typically large compared to antenna spacing.
- $D_{TA_i \rightarrow S(\theta)}$, $D_{TA_j \rightarrow S(\theta)}$, $D_{S(\theta) \rightarrow RA_m}$ and $D_{S(\theta) \rightarrow RA_n}$ present the distances between based station and subscriber unit, illustrated separately in Fig. D.1. More specially, $d_{Tx}(i, j)$ and $d_{Ty}(i, j)$ present the horizontal and vertical distances between antenna i and antenna j , respectively. $d_{Rx}(m, n)$ and $d_{Ry}(m, n)$ are similarly defined.

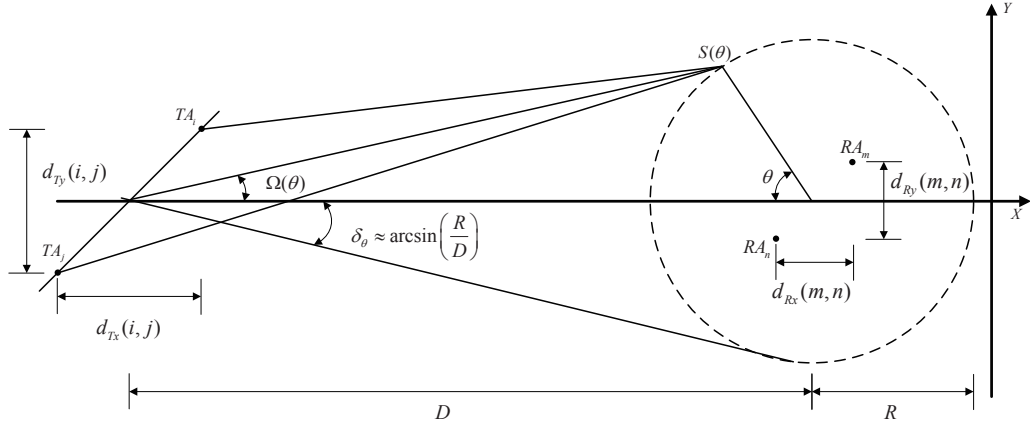


Figure D.1: One ring model for the single-side correlated fading channel

- The approximation is taken place when δ_θ is small, resulting in $D_{TA_i \rightarrow S(\theta)} - D_{TA_j \rightarrow S(\theta)} \approx d_{Tx}(j, n)(1 - 1/4(R/D)^2) + d_{Ty}(j, n)\delta_\theta \sin \theta$.

In this thesis, the correlated channel information is referred to the single-side correlation at transmitter where a uniform linear array is equipped, accordingly $d_{Rx}(m, n) = d_{Ry}(m, n) = d_{Tx}(i, j) = 0$. The resulting correlation between two paths can be simplified as

$$\mathbb{E} [\mathbf{H}_{ij} \mathbf{H}_{mn}^H] = \frac{1}{2\pi} \int_0^{2\pi} \exp \left\{ -j \frac{2\pi}{\lambda} \left[d_{Tx}(i, j) \left(1 - \frac{\delta_\theta^2}{4} + \frac{\delta_\theta^2 \cos 2\theta}{4} \right) \right] \right\}, \quad (\text{D.2})$$

Appendix E

Jensen Inequality

In mathematics, Jensen's inequality relates the value of a convex function of an integral to the integral of the convex function. The simplest form of the inequality states that the convex transformation of a mean is less than or equal to the mean after convex transformation.

Theorem: Let X be some random variable with a probability density function $p(x)$, and $f(x)$ be a convex function, the expected value of $f(X)$ is at least the value of f at the mean of X [108]

$$\mathbb{E}[f(X)] \geq f(\mathbb{E}[X]) . \quad (\text{E.1})$$

The opposite is true of concave transformations.

Appendix F

Markov Inequality

In probability theory, Markov's inequality gives an upper bound for the probability that a non-negative function of a random variable is greater than or equal to some positive constant. It relate probabilities to expectations, and provide loose but still useful bounds for the cumulative distribution function of a random variable.

Theorem [108] For any random variables, $X \geq 0$

$$\Pr\{X \geq a\} \leq \frac{\mathbb{E}[X]}{a} . \quad (\text{F.1})$$

It is also available for any positive function $f : X \rightarrow \mathbb{R}^+$ for X , that is

$$\Pr\{f(X) \geq f(a)\} \leq \frac{\mathbb{E}[f(X)]}{f(a)} . \quad (\text{F.2})$$

When f is a non-decreasing function, we have

$$\Pr\{X \geq a\} = \Pr\{f(X) \geq f(a)\} \leq \frac{\mathbb{E}[f(X)]}{f(a)} . \quad (\text{F.3})$$

Appendix G

Noncentral Wishart Distribution

Definition [86] : Consider \mathbf{X} is a random $n \times p$ ($n \geq p$) matrix, each row X_i of which is independently and identically distributed

$$\mathbf{X}_i \sim \mathcal{N}_p(\mu_i, \Sigma), \quad i = 1, \dots, n.$$

Then the matrix $\mathbf{S} = \mathbf{X}^T \mathbf{X}$ is Wishart distributed, such as

$$\mathbf{S} \sim \mathcal{W}_p(n, \mathbf{\Omega}, \Sigma), \tag{G.1}$$

with degree of freedom n , noncentrality $\mathbf{\Omega} = \Sigma^{-1} \sum_{i=1}^n \mu_i^H \mu_i$. The probability density function is given by

$$f(X) = (2\pi)^{-\frac{1}{2}np} |\Sigma|^{-\frac{1}{2}n} \exp \left(-\frac{1}{2} \text{tr} \{ \Sigma^{-1} \} (\mathbf{X} - \mathbf{M})^H (\mathbf{X} - \mathbf{M}) \right), \tag{G.2}$$

where the matrix \mathbf{M} is mean matrix of \mathbf{X} , that is, $\mathbf{M} = [\mu_1; \dots, \mu_n]$.

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Appendix H

Publications

H.1 Conference Papers

- [1] H.-Q Du, and P.-J. Chung, "Robust Leakage-Based Transmit Beamforming with Probabilistic Constraint for Downlink Multi-User MIMO System", in *IEEE Workshop on Statistical Signal Processing 2009 (SSP'09)*.
- [2] H.-Q Du, and P.-J. Chung, and B. Mulgrew, "Robust Adaptive Modulation With Imperfect Channel Information", in *European Signal Processing Conference 2009 (EUSIPCO2009)*.
- [3] P.-J. Chung, and H.-Q Du, and J. Gondzio, "A Probabilistic Constraint Approach for Robust Transmit Beamforming with Imperfect Channel Information", in *European Signal Processing Conference 2009 (EUSIPCO2009)*.
- [4] H.-Q, Du, P.-J. Chung, J. Gondzio and B. Mulgrew, "Robust Transmit Beamforming Based on Probabilistic Constraint", in *European Signal Processing Conference (EUSIPCO2008)*, August 25-29, Lausanne, Switzerland.

H.2 Submitted Journal Papers

- [1] P.-J. Chung, and H.-Q Du, and J. Gondzio, "A Probabilistic Constraint Approach for Robust Transmit Beamforming with Imperfect Channel Information", submitted to *IEEE Transactions on Vehicular Technology*, November, 2009.
- [2] H.-Q Du, and P.-J. Chung, and B. Mulgrew, "Robust Adaptive Modulation With Beamforming under Imperfect Channel Information", submitted to *IEEE Communication Letter*, November, 2009.

- [3] H.-Q Du, and P.-J. Chung "Robust Leakage-based MU-MIMO Downlink Beamforming with Outage Probability Specification", submitted to IEEE Transactions on Wireless Communication, January, 2010.
- [4] H.-Q Du, and P.-J. Chung "A Probabilistic-Constrained Leakage-Based Beamforming for Downlink Multiple Stream MU-MIMO System", submitted to ChinaCom2010, February, 2010.

ROBUST LEAKAGE-BASED TRANSMIT BEAMFORMING WITH PROBABILISTIC CONSTRAINT FOR DOWNLINK MULTI-USER MIMO SYSTEM

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ABSTRACT

Multi-user multiple-input and multiple-output (MU-MIMO) wireless systems have the potential to provide a substantial gain by using transmit beamforming to allow multi-user communication in the same frequency and time slots. The main challenge for transmit beamforming design is to suppress the co-channel interference (CCI) from other users. In order to completely cancel the CCI at each user, perfect channel state information (CSI) is required at base station, which is generally not available in practice. To overcome the performance degradation caused by the imperfections, the most common approach is the worst-case method, which leads to conservative result as the extreme (but rare) conditions may occur at a very low probability. In this work, we propose a probabilistic-constrained beamforming based on signal-to-leakage ratio (SLR) criterion under consideration of inaccurate channel information. The simulation results show that the proposed beamformer achieves the lowest bit error rate (BER) and leaks the least transmit power from the desired user to all other users among the state-of-art transmit beamformers.

Index Terms— Signal-to-leakage ratio, probabilistic constraint, robust transmit beamforming

1. INTRODUCTION

MU-MIMO wireless system has gained considerable amount of interest since it can significantly increase data throughput and achieve higher diversity gain [1]. In MU-MIMO systems, a base station (BS) communicates with several co-channel users by using the transmit beamforming in the same frequency and time slots, which leads to the CCI at the end users. Thus, it is crucial to design transmit beamformer which can suppress the CCI at the end users.

In the attempt to completely cancel CCI, accurate channel information is required, which is usually not available due to errors induced by imperfect channel feedback, estimation/quantization. It leads to significant performance degradation. Hence, it motivates to design robust transmit beamforming techniques which can not only suppress MU interference but also ensure robustness against the imperfections. Recent advances in robust MU-MIMO transmit beamforming techniques model the uncertainty as an arbitrary but Frobenius-norm bounded matrix, namely worst-case scenario [2] [3]. However, worst-case approach leads to excessively conservative performance as the worst operational condition is rare.

In this work, we adopt a recently developed transmit beamforming technique based on probabilistic constraint for single-user MIMO system [4] [5]. Note that the probabilistic constraint strategies have been applied in robust receive beamformer designs [6] [7] [8].

Moreover, two criteria work as performance measurement of robust transmit beamformer, that is, signal-to-noise ratio (SINR) [2] [9], and signal-to-leakage ratio (SLR) [1] [3] [10]. Due to coupling

between optimization and feasibility simultaneously, the SINR-based transmit beamformer can only be obtained iteratively, without a closed form solution. On the other hand, the leakage-based criterion leads to a decoupled optimization problem and admits an analytical closed form solution [1]. Hence, we pursue the SLR criterion for designing transmit beamforming.

Our approach maximizes the average signal power at the desired user and ensures the robustness against the CSI errors by keeping a low probability of the worst-case power leakage performance. According to multivariate Chebyshev inequality, we derive a deterministic expression for the probabilistic constraint. Moreover, we introduce Lagrangian relaxation to drop the non-convex rank constraint and formulate the beamformer design as probabilistic-constrained optimization problem. Under the assumption of Gaussian-distributed error, the underlying problem can be efficiently solved by modern convex optimization algorithms and a lower bound solution is obtained. Simulation results show the proposed approach provides the best BER performance, and also leaks the least power from desired user to all other users, compared with several popular transmit beamforming techniques.

In the next section, we give a brief description of the system model. Dropping rank constraint and transforming probabilistic constraint into deterministic form, the proposed approach is formulated as a stochastic optimization problem in Section 3. Simulation results are presented in Section 4. Finally, Section 5 concludes this work.

2. SYSTEM MODEL

Consider a downlink MU-MIMO system consisting of one base station communicating with K users. The base station employs N_t transmit antennas and each user is equipped with $N_{r,k}$ ($N_{r,k} \geq 1$) receive antennas. Let s_k denotes the transmitted data intended for user k . For each user the scale signal s_k is multiplied by a beamformer vector \mathbf{c}_k , thus the transmitted signal vector $\mathbf{x} \in \mathbb{C}^{N_t \times 1}$ can be presented as

$$\mathbf{x} = \sum_{k=1}^K \mathbf{c}_k s_k = \mathbf{C} \mathbf{s}, \quad (1)$$

where $\mathbf{C} \in \mathbb{C}^{N_t \times K}$ is beamforming matrix as $\mathbf{C} = [\mathbf{c}_1, \dots, \mathbf{c}_K]$. Assuming that the channel is slowly varied fading and i -th user is the desired user, the received signal vector \mathbf{y}_i for the i -th user can be written as

$$\begin{aligned} \mathbf{y}_i &= \mathbf{H}_i \mathbf{C} \mathbf{s} + \mathbf{n}_i \\ &= \mathbf{H}_i \mathbf{c}_i s_i + \sum_{k=1, k \neq i}^K \mathbf{H}_i \mathbf{c}_k s_k + \mathbf{n}_i, \end{aligned} \quad (2)$$

where the noise $\mathbf{n}_i \in \mathbb{C}^{N_{r,i} \times 1}$ is independent complex Gaussian distributed, i.e. $\mathbf{n}_i \sim \mathcal{CN}(\mathbf{0}, \sigma_e^2 \mathbf{I}_{N_{r,i}})$, and the MIMO channel for i -th user is $\mathbf{H}_i \in \mathbb{C}^{N_{r,i} \times N_t}$. In (2), the first term is the desired signal, and the second term quantifies the CCI caused to i -th user from all other users. Note that all three parts (the desired signal, interference and sensor noise) are statistically independent components. We consider the case that the number of transmit antennas is smaller than the number of all receive antennas combined, such as

$$N_t \leq \max_i \left\{ \sum_{k=1, k \neq i}^K N_{r_k} \right\},$$

in which the interference can not be exterminated by zero-forcing scheme [1].

Since the transmit beamformer based on SINR criterion easily arrives in infeasible region [9], we design the transmit beamformer based on SLR criterion instead. Assuming $\mathbb{E}[\|s_i\|^2] = 1$, ($i = 1, \dots, K$), the power allocated on the i -th user is given by $\|\mathbf{H}_i \mathbf{c}_i\|^2$, and the total power leaked from i -th user to all other user is $\sum_{k=1, k \neq i}^K \|\mathbf{H}_k \mathbf{c}_i\|^2$. With perfect channel knowledge at the receiver, the average SLR obtained from maximum ratio combining for the i -th user is given by

$$\text{SLR}_i = \frac{\|\mathbf{H}_i \mathbf{c}_i\|^2}{\sum_{k=1, k \neq i}^K \|\mathbf{H}_k \mathbf{c}_i\|^2} = \frac{\|\mathbf{H}_i \mathbf{c}_i\|^2}{\|\tilde{\mathbf{H}}_i \mathbf{c}_i\|^2}, \quad (3)$$

where $\tilde{\mathbf{H}}_i \in \mathbb{C}^{\sum_{k=1, k \neq i}^K N_{r,k} \times N_t}$ denotes an extended channel matrix that excludes \mathbf{H}_i , i.e. $\tilde{\mathbf{H}}_i = [\mathbf{H}_1^T, \dots, \mathbf{H}_{i-1}^T, \mathbf{H}_{i+1}^T, \dots, \mathbf{H}_K^T]^T$. When perfect CSI is available at transmitter, maximization of the average SLR is achieved with the one directional beamformer [3]

$$\mathbf{c}_i^{\text{opt}} = \mathcal{P} \left\{ (\tilde{\mathbf{H}}_i^H \tilde{\mathbf{H}}_i)^{-1} (\tilde{\mathbf{H}}_i^H \mathbf{H}_i) \right\},$$

where $\mathcal{P}\{\cdot\}$ is the principal eigenvector of the matrix.

However, in real scenario, only imperfect channel information can be accessed at transmitter. For i -th user, the presumed channel $\mathbf{H}_{i_p} \in \mathbb{C}^{N_{r,i} \times N_t}$ can be expressed as

$$\mathbf{H}_i = \mathbf{H}_{i_p} + \mathbf{E}_i, \quad (4)$$

where the error matrix $\mathbf{E}_i \in \mathbb{C}^{N_{r,i} \times N_t}$ consists of i.i.d. complex normally distributed entries with variance σ_e^2 . The subscript p is used to denote the *presumed* channel information. And the corresponding interference channel $\tilde{\mathbf{H}}_{i_p} \in \mathbb{C}^{\sum_{k=1, k \neq i}^K N_{r,k} \times N_t}$ can be written as

$$\tilde{\mathbf{H}}_i = \tilde{\mathbf{H}}_{i_p} + \tilde{\mathbf{E}}_i, \quad (5)$$

where the error matrix $\tilde{\mathbf{E}}_i$ is composed of $(K-1)$ transport error matrix \mathbf{E}_i , that is, $\tilde{\mathbf{E}}_i = [\mathbf{E}_1^T, \dots, \mathbf{E}_{i-1}^T, \mathbf{E}_{i+1}^T, \dots, \mathbf{E}_K^T]^T$. Since the CSITs for each user are independent, the constructed matrix $\tilde{\mathbf{E}}_i$ has the same distribution of each component \mathbf{E}_i , that is, i.i.d complex normally distributed entries with variance σ_e^2 . Note that we assume that $\tilde{\mathbf{H}}_i$ and $\tilde{\mathbf{H}}_{i_p}$ have the same rank, that is,

$$\text{rank}(\tilde{\mathbf{H}}_i) = \text{rank}(\tilde{\mathbf{H}}_{i_p}) = \min(N_t, \sum_{k=1, k \neq i}^K N_{r_k}) = N_t.$$

In this paper, we design a transmit beamformer \mathbf{C} which maximizes SLR under inaccurate channel presumption.

3. BEAMFORMER DESIGN BASED ON PROBABILISTIC-CONSTRAINED OPTIMIZATION

To tackle performance degradation caused by imperfect channel estimates, we consider a probabilistic constraint approach. In contrast to the minimax approach [1] that focuses on the worst-case performance, the probabilistic constraint takes into account the degradation performance proportionally. This approach is favorable to an achievable optimal power allocation on the desired user, and prevents a pessimistic result due to worst-case performance of the power leakage proportionally considered.

According to the error model (4) and (5), SLR becomes a function of the presumed channel \mathbf{H}_{i_p} and $\tilde{\mathbf{H}}_{i_p}$ and the random errors \mathbf{E}_i and $\tilde{\mathbf{E}}_i$

$$\text{SLR}_i(\mathbf{E}_i, \tilde{\mathbf{E}}_i) = \frac{\mathbf{c}_i^H (\mathbf{H}_{i_p} + \mathbf{E}_i)^H (\mathbf{H}_{i_p} + \mathbf{E}_i) \mathbf{c}_i}{\mathbf{c}_i^H (\tilde{\mathbf{H}}_{i_p} + \tilde{\mathbf{E}}_i)^H (\tilde{\mathbf{H}}_{i_p} + \tilde{\mathbf{E}}_i) \mathbf{c}_i}. \quad (6)$$

To simplify the expression (6), we define a new parameter \mathbf{C}_i as follow

$$\mathbf{C}_i \triangleq \mathbf{c}_i \mathbf{c}_i^H, \quad \mathbf{C}_i \geq 0 \text{ and } \text{rank}(\mathbf{C}_i) = 1, \quad (7)$$

where \mathbf{C}_i is positive semidefinite.

In this work, instead of maximizing the SLR directly, we separately maximize the average power allocated on the desired signal while keeping a low probability that the leakage power from the desired signal is larger than a pre-specified threshold.

3.1. Objective Function

Given the presumed channel \mathbf{H}_{i_p} at transmitter, the objective function is obtained by taking the expectation of the power allocated on i -th user with respect to the random error \mathbf{E}_i

$$\begin{aligned} & \mathbb{E} \left[\text{tr} \{ (\mathbf{H}_{i_p} + \mathbf{E}_i)^H (\mathbf{H}_{i_p} + \mathbf{E}_i) \mathbf{C}_i \} \right] \\ &= \text{tr} \left\{ \left(\mathbf{H}_{i_p}^H \mathbf{H}_{i_p} + \sigma_e^2 N_{r_k} \mathbf{I} \right) \mathbf{C}_i \right\}. \end{aligned} \quad (8)$$

Note that as we can not guarantee the subspaces of all users are orthogonal to each other, eigen-decomposition approach can not be easily implemented into objective function.

3.2. Probabilistic Constraint

To maximize the SLR performance, we also keep the probability of the worst-case power leakage at i -th user larger than a threshold low. That is, for a given pre-specified leakage power level γ_{th} and an outage probability p_i , the varied leakage power has to satisfy the following probabilistic constraint,

$$\Pr \left\{ \text{tr} \{ (\tilde{\mathbf{H}}_{i_p} + \tilde{\mathbf{E}}_i)^H (\tilde{\mathbf{H}}_{i_p} + \tilde{\mathbf{E}}_i) \mathbf{C}_i \} \geq \gamma_{th_i} \right\} \leq p_i, \quad (9)$$

where $\Pr\{A\}$ denotes the probability of the event A .

Define

$$\mathbf{Z}_i = \frac{(\tilde{\mathbf{H}}_{i_p} + \tilde{\mathbf{E}}_i) \mathbf{c}_i}{\sqrt{\text{tr} \{ \sigma_e^2 \mathbf{C}_i \}}},$$

it is easy to show that the random vector \mathbf{Z}_i has the following distribution

$$\mathbf{Z}_i \sim \mathcal{CN} \left(\frac{\tilde{\mathbf{H}}_{i_p} \mathbf{c}_i}{\sqrt{\text{tr} \{ \sigma_e^2 \mathbf{C}_i \}}}, \mathbf{I} \right).$$

Since the rank of $\tilde{\mathbf{H}}_{i_p}$ is equal to N_t and $\text{rank}(\tilde{\mathbf{H}}_{i_p}^H \tilde{\mathbf{H}}_{i_p}) = N_t$, the random variables $\|\mathbf{Z}_i\|^2$ is non-central $\chi_n^2(\lambda)$ -distributed, with

degree of freedom $n_i = 2N_t$ and noncentrality parameter $\lambda = \text{tr}\{\tilde{\mathbf{H}}_{i_p}^H \tilde{\mathbf{H}}_{i_p} \mathbf{C}_i\} / \text{tr}\{\sigma_e^2 \mathbf{C}_i\}$.

According to the Chebyshev inequality [11], we reformulate the probabilistic constraint (9) as follow

$$\Pr\{\text{tr}\{\sigma_e^2 \mathbf{C}_i\} \|\mathbf{Z}_i\|^2 \geq \gamma_{th_i}\} \leq \frac{\mathbb{E}[\|\mathbf{Z}_i\|^2]}{\gamma_{th_i} / \text{tr}\{\sigma_e^2 \mathbf{C}_i\}}, \quad (10)$$

where the mean of the non-central χ^2 -distributed variable $\|\mathbf{Z}_i\|^2$ can be expressed as

$$\mathbb{E}[\|\mathbf{Z}_i\|^2] = n_i + \lambda.$$

Then, for $p_i \in (0, 1)$, the constraint (10) can be rewritten as

$$\frac{n_i + \lambda}{\gamma_{th_i} / \text{tr}\{\sigma_e^2 \mathbf{C}_i\}} \leq p_i,$$

Since $\lambda = \text{tr}\{\tilde{\mathbf{H}}_{i_p}^H \tilde{\mathbf{H}}_{i_p} \mathbf{C}_i\} / \text{tr}\{\sigma_e^2 \mathbf{C}_i\}$, we have

$$\begin{aligned} \frac{1}{\text{tr}\{\sigma_e^2 \mathbf{C}_i\}} \left(\text{tr}\{\sigma_e^2 \mathbf{C}_i\} n_i + \text{tr}\{\tilde{\mathbf{H}}_{i_p}^H \tilde{\mathbf{H}}_{i_p} \mathbf{C}_i\} \right) &\leq \frac{p_i \gamma_{th_i}}{\text{tr}\{\sigma_e^2 \mathbf{C}_i\}}, \\ \text{tr}\left\{\mathbf{C}_i \left(n_i \sigma_e^2 \mathbf{I} + \tilde{\mathbf{H}}_{i_p}^H \tilde{\mathbf{H}}_{i_p} \right)\right\} &\leq p_i \gamma_{th_i}. \end{aligned} \quad (11)$$

In order to guarantee a low probability of worst-case performance, the outage probability p_i is set as a small value, and so does the threshold.

3.3. Probabilistic Constrained Optimization

Recall the beamformer matrix (7), the rank constraint on \mathbf{C}_i is non-convex. In order to convert the optimization problem into convex form, we introduce Lagrangian relaxation to drop the rank constraint, such as

$$\mathbf{C}_i \triangleq \mathbf{c}_i \mathbf{c}_i^H, \text{ and } \mathbf{C}_i \geq 0. \quad (12)$$

It means that we expect to find a lower bound solution \mathbf{C}_i with a lower cost than (7) but with high rank [12].

Based on average transmit power allocated on i -th user (8), the reformulated probabilistic constraint (11), and relaxed rank constraint (12), the proposed beamformer design can be formulated as

$$\max_{\mathbf{C}_i} \text{tr}\left\{\left(\mathbf{H}_{i_p}^H \mathbf{H}_{i_p} + \sigma_e^2 N_{r_k} \mathbf{I}\right) \mathbf{C}_i\right\}, \quad (13)$$

subject to

$$\text{tr}\left\{\mathbf{C}_i \left(n_i \sigma_e^2 \mathbf{I} + \tilde{\mathbf{H}}_{i_p}^H \tilde{\mathbf{H}}_{i_p} \right)\right\} \leq p_i \gamma_{th_i}, \quad (14)$$

$$\text{tr}\{\mathbf{C}_i\} \leq 1, \quad (15)$$

$$\mathbf{C}_i \geq 0, \quad i = 1, \dots, K \quad (16)$$

which can be efficiently solved by standard tools of mathematical programming [13]. Note that the rank of the solution \mathbf{C}_i is usually higher than one and, therefore, the optimal weight vector cannot be directly recovered from \mathbf{C}_i . As suggested in [14], a common approach is to use *randomization techniques* whose essence is to draw multiple Gaussian random vectors from $\mathcal{CN}(\mathbf{0}, \mathbf{C}_i)$ and the *best* solution is selected among such randomly generated candidates.

4. SIMULATION RESULTS

In our simulation, we consider multi-use MIMO system with one base station (BS) equipped with 6 antennas and 3 users each equipped

with 3 antennas. The data symbols are generated using QPSK modulation. The proposed probabilistic-constrained beamformer is compared with non-robust SLR-based beamformer [15], and worst-case beamformer [3]. Without any loss of generality, we assume the following:

- Channel Mean Feedback: The channel coefficients are slowly time-varying according to Jake's model with Doppler frequency f_d . For i -th user, assume that the accurate channel and the presumed channel are distributed as follows

$$\mathbf{H}_i \sim \mathcal{CN}(\mathbf{0}, \sigma_{\mathbf{H}_i}^2), \quad \tilde{\mathbf{H}}_{i_p} \sim \mathcal{CN}(\mathbf{0}, \sigma_{\tilde{\mathbf{H}}_{i_p}}^2).$$

And $\mathbb{E}[\mathbf{H}_i^H \tilde{\mathbf{H}}_{i_p}] = \rho \sigma_{\mathbf{H}_i}^2 \mathbf{I}$ where the correlation coefficient ρ determines the feedback quality. According to error model (4), we have

$$\mathbf{E}_i \sim \mathcal{CN}(\mathbf{0}, \sigma_e^2 \mathbf{I}), \quad \text{where } \sigma_e^2 = (1 - \rho^2) \sigma_{\mathbf{H}_i}^2$$

Here we set $\sigma_{\mathbf{H}_i}^2 = 1$, and the error variance $\sigma_e^2 = 0.01$.

- Parameters in Probabilistic Constraint: According to [3], the errors in covariance matrix of the desired user are bounded by $\epsilon_i = 2.1852$, while errors in leakage part bounded as $\epsilon_k = 11.5992$. In order to make a fair comparison, we set threshold $\gamma_{th_i} = \mathbb{E}[\tilde{\mathbf{H}}_{i_p}^H \tilde{\mathbf{H}}_{i_p}]_{i_p} + \epsilon_k$ and outage probability to $p = 0.1$ for the proposed beamformer.
- Other Parameters: The white noise variance per receive antennas is assumed the same for all users, $\sigma_e^2 = \dots = \sigma_K^2 = 1$. And BER is based on SNR at receiver side.

The proposed beamformer outperforms the state-of-art beamformers in MU-MIMO system in Fig. 1. More specifically, the proposed beamformer maintains an acceptable 10^{-3} uncoded BER at SNR -2 dB for three simultaneously active users. To achieve the same BER, the SNR required to the proposed beamformer is 4 dB less than the worst-case SLR-based beamformer, and 6 dB less than the non-robust SLR-based beamformer, where the conventional one-directional beamformer has the worst performance. Moreover, the BER performance of the proposed beamformer trends to the same performance as all other popular beamformers since the errors in CSIT is not dominant in high SNR region.

To understand the behavior of the proposed algorithm, its SINR outage at SNR $= 0$ dB is plotted in Fig. 2. It shows that an outage value of 10% at SINR $= 6.5$ dB for the proposed scheme, which means the achieved SINR is larger than 2dB for 90% channel realizations. Moreover, using the propose scheme, there is around 3.5 dB improvement in 10% outage value compared to the non-robust SLR-based beamformer. It is because that the proposed beamformer is designed to maximize SLR which reduces the power leakage from i -th desired user to all other users, and consequently tends to reduce the interference from all other users.

5. CONCLUSION

We proposed a novel SLR-based transmit beamforming design that maximizes average desired signal power and guarantees a low probability of worst-case power leakage by using probabilistic constraint. By introducing Lagrangian relaxation approach to relax rank constraint and transferring the probabilistic constraint into deterministic form, the underlying problem was transformed into a convex optimization problem, and a lower bound solution is efficiently obtained

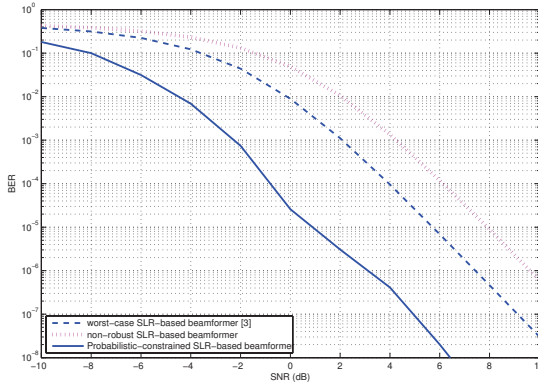


Fig. 1. The average BER performance over MU-MIMO system, where one base station with $N_t = 6$ transmit antennas and $K = 3$ users equipped with $N_{r_k} = 3$ receive antennas

by modern tools under the assumption of the complex Gaussian-distributed errors. Simulation results show that the proposed beamformer achieves the lowest bit error rate at the same SNR stage, compared with the worst-case design. Moreover, it effectively reduces power leakage from the desired user and provides the highest SINR reliability among the popular beamforming techniques.

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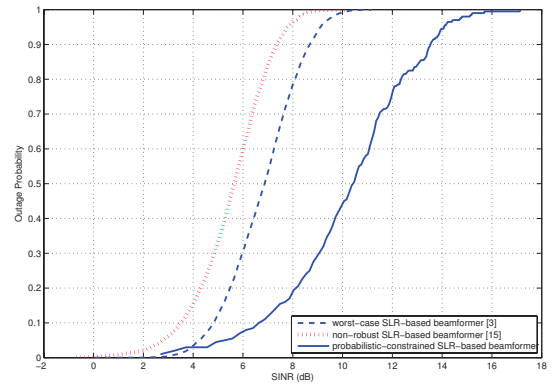


Fig. 2. The outage probability of SINR at the output of the receiver, where one base station with $N_t = 6$ transmit antennas and $K = 3$ users equipped with $N_{r_k} = 3$ receive antennas for SNR = 0 dB

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ROBUST ADAPTIVE MODULATION WITH IMPERFECT CHANNEL INFORMATION

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ABSTRACT

Adaptive modulation is a promising technique to increase system throughput considerably. However, it relies on perfect channel state information (CSI), and is sensitive to errors in CSI. In this work, we maximize the system transmission rate based on a lower bound of average bit error rate (BER) while satisfying the transmit power and BER constraint. In order to further enhance the system throughput, adaptive modulation scheme is combined with a robust transmit beamformer to obtain extra diversity gain. Moreover, to pay the penalty for the lower bound of the average BER, we introduce a probabilistic constraint by keeping a low outage probability of signal-to-noise ratio (SNR). Simulation results show that the proposed scheme provides the maximum system throughput compared with several state-of-the-art robust adaptive schemes, and always guarantees the target BER.

1. INTRODUCTION

Adaptive modulation has the potential to increase the transmit rate by taking the advantage of favorable channel conditions [1] [2] [3] [4] [5] [6] [7]. Perfect channel state information is crucial to adaptive modulation, but is typically not available due to errors induced by the imperfect (quantized, erroneous, or outdated) feedback channel [8]. Thus, a robust adaptive modulation scheme is required based on imperfect CSI.

Although existing robust adaptive modulation schemes at transmitter [5] [6] [7] take errors in CSI into account, the system throughput does not achieve the maximum rate, due to the improperly-paid compensation on average BER. More specifically, the system throughput is determined by the target BER and the average BER that the system achieves. However, the latter is difficult to evaluate, and is usually replaced by its lower bound, which carries a performance penalty. To ensure that adaptive modulation still meets the BER target, the compensation can be employed in two ways. One approach is to artificially introduce a modifying factor which can only be empirically determined through extensive Monte Carlo simulations [5]. In another approach [6], the BER constraint is satisfied under the consideration of worst-case SNR scenario. Due to the excessive compensation, only a conservative throughput can be achieved. Therefore, it is necessary to investigate an efficient approach that employs appropriate compensation on the average BER.

Recently, transmit diversity has been well developed to enhance the performance of wireless communication when perfect CSI is not known [6] [9] [10]. In order to reduce its performance degradation caused by imperfect CSI, adaptive

modulation scheme incorporates transmit beamforming technique and leads to further improvement of system throughput. For instance, in partial channel information scenarios, the transmit beamformers based Alamouti scheme provide extra two-dimensional diversity gain to adaptive modulation scheme, which increase the system throughput [3] [5]. By applying the transmit beamformer based on worst-case CSI scenario, the robust adaptive modulation scheme achieves the maximum transmission rate for any possible error in the uncertainty region [6]. In this work, the recently proposed transmit beamforming techniques [11] [12] are incorporated into adaptive modulation scheme.

We design robust adaptive modulation scheme for multi-antenna transmissions with imperfect channel information. Under transmit power constraint, the transmitter here optimally adjusts the power allocation and the signal constellation to maximize the system throughput while maintaining a prescribed BER constraint. In order to obtain an extra diversity gain, the proposed adaptive modulation scheme is combined with the transmit beamformer. Thus, a necessary compensation is required. Here, we introduce a probabilistic constraint to efficiently pay for the penalty to keep the outage probability of SNR as low as possible. The proposed robust adaptive problem is transformed into maximization of SNR while satisfying a probabilistic constraint and transmit power constraint, which can be solved by standard mathematical tools. Simulation results show that the proposed adaptive scheme significantly increases the system throughput compared with other state-of-the-art robust adaptive modulation schemes, while guaranteeing the target BER.

This paper is organized as follows. The system model is described in Section 2. After a brief introduction of the standard adaptive modulation schemes in Section 3, the proposed robust adaptive modulation schemes is developed in Section 4. Simulation results are presented and discussed in Section 5. Concluding remarks are given in Section 6.

2. SYSTEM MODEL

Consider a single-user wireless communication system with N_t transmit antennas and N_r receive antennas ($N_t \geq N_r$). The channels are assumed as slow time-varying, and the transmitter can track the channel variations via feedback channel. However, perfect channel realization can not be accessed, leading the imperfection taken into account in real scenario. In this work, we assume that the transmitter can obtain the imperfect channel information and the error statistics over slow-fading channel.

Defining the perfect channel as $\mathbf{H} \in \mathbb{C}^{N_r \times N_t}$ and the esti-

mate as $\hat{\mathbf{H}} \in \mathbb{C}^{N_r \times N_t}$, we have

$$\mathbf{H} := \hat{\mathbf{H}} + \mathbf{E}, \quad (1)$$

where the channel matrix is $\mathbf{H} = [\mathbf{h}_1, \dots, \mathbf{h}_{N_t}]$, and the error matrix $\mathbf{E} \in \mathbb{C}^{N_r \times N_t}$ consists of i.i.d complex normally distributed entries with variance σ_e^2 . The information-bearing symbol $\mathbf{s} \in \mathbb{C}^{P \times 1}$ is drawn from an appropriate signal constellation of size M with average energy E_s , spread by a precoding matrix $\mathbf{C} \in \mathbb{C}^{N_t \times P}$ and transmitted through multiple channels.

According to the error model (1), the SNR is a function of the channel estimate $\hat{\mathbf{H}}$ and the random error \mathbf{E} ,

$$\gamma = \frac{E_s}{N_0} \text{tr} \{ \mathbf{C}^H (\hat{\mathbf{H}} + \mathbf{E})^H (\hat{\mathbf{H}} + \mathbf{E}) \mathbf{C} \}, \quad (2)$$

where N_0 is the energy of the additive white Gaussian noise (AWGN) with zero mean and variance $N_0/2$ per real and imaginary dimension.

3. STANDARD ADAPTIVE MODULATION

The goal of adaptive modulation is to maximize the system transmission rate, subject to BER constraint and power constraints. To simplify the design, we rely on the approximation of the instantaneous BER, which is a function of received SNR γ and constellation size 2^k [2]

$$\text{BER}(k, \gamma) \approx 0.2 \exp \left(-\frac{1.6\gamma}{2^k - 1} \right), \quad (3)$$

where k is the transmission rate. The average BER can be calculated by taking the expectation of the instantaneous BER with respect to γ , as follows

$$\overline{\text{BER}}(k) = \int_0^\infty \text{BER}(k, \gamma) p(\gamma) d\gamma. \quad (4)$$

Here, we define the BER constraint as

$$\overline{\text{BER}}(k) \leq \text{BER}_0, \quad (5)$$

where BER_0 is pre-specified value, usually defined as 10^{-3} .

According to (3), (4) and (5), the optimization problem can be formulated as

$$\max k, \quad (6)$$

$$\text{subject to } \overline{\text{BER}}(k) \leq \text{BER}_0, \quad (7)$$

where the transmission rate k is parameterized by the average BER and BER constraint.

4. ROBUST DESIGN WITH IMPERFECT CHANNEL INFORMATION

In practice, the CSI can not be perfectly known, leading a significant degradation performance of system throughput. Thus, for robust adaptive modulation scheme, it is crucial to take the errors in CSI into account. In this section, we combine robust adaptive modulation scheme with recently developed robust transmit beamforming technique [11] [12], which can significantly enhance the system throughput.

Since the integral in (4) can not be calculated in closed form, a common method is to take the lower bound of average BER [5],

$$\overline{\text{BER}}_L(k) = 0.2 \exp \left(-\frac{1.6 \bar{\gamma}}{2^k - 1} \right), \quad (8)$$

where $\bar{\gamma}$ is the average SNR. By considering BER constraint (5), a suboptimal transmission rate can be expressed as

$$k' = \log_2 \left(1 - \frac{1.6 \bar{\gamma}}{\ln(5 \text{BER}_0)} \right), \quad (9)$$

where k' denotes as the suboptimal transmission rate. Given a pre-specified BER constraint, the maximum achievable transmission rate increases with the average SNR [5].

However, according to Jensen's inequality, the average BER (4) may be larger than the target BER, leading the constraint (5) violated [5]. Two approaches are used to prevent this. One introduces a modifying factor to set a smaller BER target [5], which only can be empirically determined by extensive Monte Carlo simulation. Another approach [6] considers the worst-case SNR, which leads conservative solution due to extreme rare worst operational condition. In order to efficiently maximize the system throughput, we propose a novel approach which can intelligently and efficiently prevent the constraint violation.

In order to avoid the suboptimal transmission rate violating the average BER constraint, we introduce a probabilistic constraint that keeps a low outage probability of SNR.

To illustrate the novelty in the proposed scheme, we investigate the relationship between average BER and its lower bound. According to [13], the instantaneous BER, $\text{BER}(k, \gamma)$, can be approximated by a Taylor series about the mean SNR $\bar{\gamma}$ that is truncated after the quadratic term, such as

$$\begin{aligned} \text{BER}(k, \gamma) &\approx \overline{\text{BER}}_L(k) + (\gamma - \bar{\gamma}) \overline{\text{BER}}'_L(k) \\ &+ \frac{(\gamma - \bar{\gamma})^2}{2} \overline{\text{BER}}''_L(k) + o(\gamma), \end{aligned}$$

where $\overline{\text{BER}}'_L(k)$ and $\overline{\text{BER}}''_L(k)$ are defined as first derivative and second derivative of $\overline{\text{BER}}_L(k)$ with respect to k . Ignoring higher order terms and taking the expectation of BER, we have

$$\begin{aligned} \overline{\text{BER}}(k) &\approx \int_0^\infty [\overline{\text{BER}}_L(k) + (\gamma - \bar{\gamma}) \overline{\text{BER}}'_L(k) \\ &+ \frac{(\gamma - \bar{\gamma})^2}{2} \overline{\text{BER}}''_L(k)] p(\gamma) d\gamma \\ &= \overline{\text{BER}}_L(k) + 0.2 \frac{\text{var}\{\gamma\}}{2} \overline{\text{BER}}''_L(k) \\ &= \overline{\text{BER}}_L(k) \left(1 + \frac{\text{var}\{\gamma\}}{10} \left(\frac{-1.6}{2^k - 1} \right)^2 \right). \end{aligned} \quad (10)$$

Note that the first-order term vanishes as a result of the expectation operation, and the approximation will be accurate if the instantaneous SNR is well concentrated about its mean, namely the variance of SNR $\text{var}\{\gamma\}$ is small. It also clearly indicates that the penalty of lower bound BER comes from the ignored higher order terms.

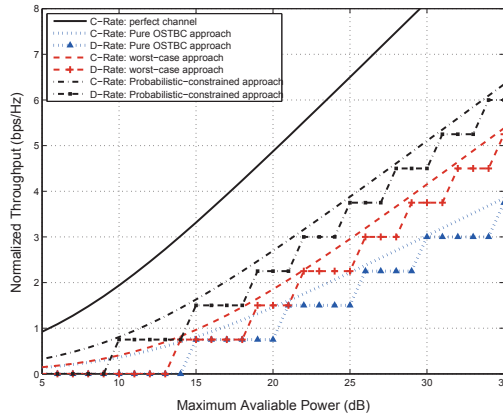


Figure 1: Average normalized throughput comparison, $\gamma_{th} = 0.95$ and $p_{out} = 10\%$

In recently proposed probabilistic-constrained transmit beamforming techniques [11] [12], we find that the high-order terms in (10) can be reasonably taken into account with a properly defined outage probability constraint. We define the probabilistic constraint that the SNR γ falls below a threshold,

$$Pr\{\gamma \leq \gamma_{th}\} \leq p_{out}, \quad (11)$$

where the SNR threshold is defined as γ_{th} , and p_{out} is a pre-specified probability value that satisfies QoS requirements, and $Pr\{A\}$ stands for the probability of event A . Note that by setting the threshold equal to or larger than the average SNR, $\bar{\gamma}$, and the outage probability at a low level, leading well-concentrated random variables γ , correspondingly, the difference between average BER and its lower bound is reduced without any extra compensation.

By taking the lower bound of average BER (8) and introducing the probabilistic constraint (11), our adaptive modulation scheme (6)-(7) can be formulated as follows

$$\max \log_2 \left(1 - \frac{1.6 \bar{\gamma}}{\ln(5\overline{BER}_L(k))} \right), \quad (12)$$

subject to

$$\overline{BER}_L(k) \leq BER_0, \quad (13)$$

$$Pr\{\gamma \leq \gamma_{th}\} \leq p_{out}. \quad (14)$$

It indicates that the system throughput is determined by the achievable average SNR and outage probability of varied SNR. Applying the robust transmit beamforming [11] [12], the average SNR is maximized while the robustness is achieved by taking errors in CSI proportionally. Compared to other popular robust designs [10] [6], simulation results show that the probabilistic constraint approach has the best performance. Consequently, it leads to the highest transmit rate in the proposed adaptive modulation scheme.

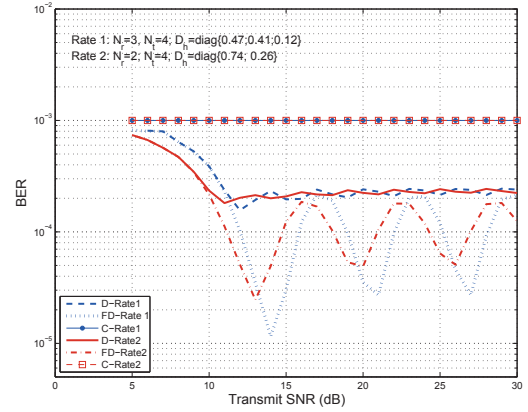


Figure 2: BER for the proposed robust adaptive modulation scheme, $\gamma_{th} = 0.95$ and $p_{out} = 10\%$

5. SIMULATION RESULTS

In our simulation, we consider a single-user MIMO system with multi-antenna at both transmitter and receiver sides ($N_t \geq N_r$). 10^5 Monte-Carlo runs are used to obtain each point. The proposed adaptive modulation scheme is compared with other adaptive schemes based on different approaches, such as the worst-case approach [6] and the orthogonal space-time block code (OSTBC) approach [14]. Without any loss of generality, we assume the following:

- Channel parameters : The channel between p th and q th transmit antennas can be presented as [15]

$$[H^H H]_{p,q} \approx \frac{1}{2\pi} \int_0^\pi \exp \left[-j2\pi(p-q)\Delta \frac{d_t}{\lambda} \sin \theta \right] d\theta,$$

where angle of spread Δ is related to the channel state information, λ is the wavelength of a narrow-band signal, and d_t the antenna spacing and Δ the angle of spread. We set $d_t = 0.5\lambda$ and $\Delta = 30^\circ$.

- Error in CSI : We assume that the error is Gaussian distributed with zero mean and covariance matrix $\sigma_e^2 \mathbf{I}$, i.e.

$$\mathbf{E}_{N_t \times N_t} \sim \mathcal{CN}(0, \sigma_e^2 \mathbf{I}).$$

In our simulation, the variance of the error is set as 0.6.

- Other parameters : We set the target BER as 10^{-3} . The SNR threshold is $\gamma_{th} = 0.95$, and $E_s/N_0 = 1$. The outage probability is $p_{out} = 10\%$.

In Fig. 1, the average throughput [6] has been normalized with respect to the code rate, so that the gains provided by the robust technique itself for different number of transmit antennas can be compared directly. In this case, 4 transmit antennas and 3 receive antennas are considered. The eigenvalues of $\hat{\mathbf{H}}^H \hat{\mathbf{H}}$ are (0.4676, 0.4104, 0.1220, 0). With the same channel condition, the proposed scheme requires less transmit power among other schemes to fulfill the BER constraint, thus larger constellation size is allowed to modulate

the transmit symbols, consequently, leading to the maximum normalized system throughput.

In Fig. 2, it shows that the average BER performance has been well controlled below 10^{-3} under the proposed scheme. Here, we consider the BER performance with three different constellation rates

$$M_i = 2^k \begin{cases} k \geq 2, k \in \mathbb{R}^+ : \text{Continuous Rate (C-Rate)}, \\ k \in \{2, 3, 4, \dots\} : \text{Discrete Rate (D-Rate)}, \\ k \in \{2, 4, 6, 8\} : \text{Finite Discrete Rate (FD-Rate)}, \end{cases}$$

and two different numbers of receive antennas : $N_r = 2$ and $N_r = 3$. It indicates that no matter the number of receive antennas, the BER achieves the target by using the continuous rate. Note that the BER bound of 10^{-3} breaks down at low SNR, since (4) is not applicable to BPSK. Furthermore, because the BER increases monotonically with decreasing constellation size, the exact average BER is much lower than 10^{-3} with both discrete rates, such as D-Rate and FD-Rate.

6. CONCLUSION

We propose a novel robust adaptive modulation scheme that significantly improves the system throughput while satisfying the BER constraint. In contrary to the conventional schemes, the proposed scheme introduces the probabilistic constraint to control the varied SNR, which efficiently minimizes the penalty for the lower bound on average BER by keeping the probability that the SNR falls below a threshold low. Under imperfect channel conditions, the robust adaptive modulation scheme gains an extra transmit diversity gain by combining with transmit beamforming, and a high average SNR. With the robustness provided by probabilistic constraint, the resulting system throughput achieves the maximum rate. Simulation results demonstrate the proposed robust adaptive scheme provides the most significant improvement of the normalized system throughput among the state-of-art robust adaptive schemes, and guarantees the target BER in different scenarios, such as different numbers of receive antennas and different constellation rates.

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A PROBABILISTIC CONSTRAINT APPROACH FOR ROBUST TRANSMIT BEAMFORMING WITH IMPERFECT CHANNEL INFORMATION

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ABSTRACT

Transmit beamforming is a powerful technique for enhancing performance of wireless communication systems. Most existing transmit beamforming techniques require perfect channel state information at the transmitter (CSIT), which is typically not available in practice. In such situations, the design should take errors in CSIT into account to avoid performance degradation. Among two popular robust designs, the stochastic approach exploits channel statistics and optimizes the average system performance. The maximin approach considers errors as deterministic and optimizes the worst-case performance. The latter usually leads to conservative results as the extreme (but rare) conditions may occur at a very low probability. In this work, we propose a more flexible approach that maximizes the average signal-to-noise ratio (SNR) and takes the extreme conditions into account proportionally. Simulation results show that the proposed beamformer offers higher robustness against channel estimation errors than several popular transmit beamformers.

1. INTRODUCTION

Multi-antenna diversity is well motivated in wireless communication systems because it offers significant advantages over single antenna [1]. Perfect or partial knowledge of the channel state information at transmitter (CSIT) can provide further performance improvement.

However, in practical wireless systems, accurate channel estimates are not available due to errors induced by imperfect channel feedback, estimation/quantization errors or outdated channels. It is well known that the performance of several nonrobust designs for multi-antenna diversity degrades rapidly with increasing error levels. This has motivated many works that take imperfect channel information into account.

Existing robust transmit beamforming (or precoder) designs can be categorized into the stochastic and the maximin approaches. The stochastic approach [2] [3] exploits channel statistics such as mean or covariance and optimizes the average system performance. On the other hand, the maximin approach considers channel estimation errors as deterministic and optimizes the worst-case performance [4] [5]. While the stochastic approach focuses on the average performance without paying attention to the extreme error level, the worst-case approach is overall too conservative as the worst operational condition is rare.

To overcome this problem, we proposed a more flexible design based on probabilistic constraint using channel covariance in [6]. In this work, we apply this approach to transmit beamforming design under consideration of

imperfect channel estimates. Note that a similar strategy was introduced into the design of adaptive beamformer at the receiver side in [7].

Our approach maximizes the average Signal-to-Noise Ratio (SNR) and ensures robustness against the CSIT error by keeping the probability of the worst-case performance at a very low level. Under the assumption that the CSIT error is complex Gaussian distributed, this stochastic optimization problem is further simplified to an equivalent deterministic form which can be efficiently solved by modern convex optimization algorithms [8]. Simulation results show that the proposed approach provides the best performance and highest robustness among several popular transmit beamformers.

In the following section, we give a brief description of the system model. The proposed approach is formulated as a stochastic optimization problem in Section 3. Section 4 is devoted to transformation of the probabilistic constraint to a deterministic, convex constraint. Simulation results are presented in Section 5. Finally, Section 6 concludes this paper.

2. SYSTEM MODEL

Consider a single-user wireless communication system with N_t transmit antennas and N_r receive antennas. The encoded signal $\mathbf{s} \in \mathbb{C}^{P \times 1}$ is spread by the precoding matrix $\mathbf{C} \in \mathbb{C}^{N_t \times P}$ and then transmitted through a flat fading channel. The received signal \mathbf{y} in the presence of additive white Gaussian noise \mathbf{w} is given by

$$\mathbf{y} = \mathbf{H}\mathbf{C}\mathbf{s} + \mathbf{w}. \quad (1)$$

The (i, j) element of the channel matrix $\mathbf{H} = [\mathbf{h}_1, \dots, \mathbf{h}_{N_t}] \in \mathbb{C}^{N_r \times N_t}$ represents the response between the i th receive antenna and the j th transmit antenna. Assuming perfect channel knowledge at the receiver, the average signal-to-noise-ratio (SNR) obtained from maximum ratio combining (MRC) is given by

$$\text{SNR} = \frac{E_s}{N_0} \text{tr}\{\mathbf{C}^H \mathbf{H}^H \mathbf{H} \mathbf{C}\}, \quad (2)$$

where $E_s = \mathbb{E}[\|\mathbf{s}\|^2]$ is the average energy of the signal and N_0 is the noise power.

When perfect channel knowledge is available at transmitter, maximization of the average SNR leads to the conventional one directional beamforming which allocates all power on the strongest eigen-mode of the channel correlation matrix $\mathbf{H}^H \mathbf{H}$. In practice, one has only access to an imperfect estimate for the channel matrix $\hat{\mathbf{H}} \in \mathbb{C}^{N_r \times N_t}$, which is related to \mathbf{H} as follows:

$$\mathbf{H} = \hat{\mathbf{H}} + \mathbf{E}, \quad (3)$$

where the error matrix $\mathbf{E} \in \mathbb{C}^{N_r \times N_t}$ consists of i.i.d. complex normally distributed entries with variance σ_e^2 . The goal of this work is to design a transmit beamformer \mathbf{C} that maximizes SNR under consideration of inaccuracy in channel estimates.

3. ROBUST DESIGN BASED ON PROBABILISTIC CONSTRAINED OPTIMIZATION

To tackle performance degradation caused by imperfect channel estimates, we consider a probabilistic constraint approach. The proposed algorithm maximizes the average SNR while keeping the probability for SNR being below a pre-specified threshold γ_{th} low. It has the advantage of achieving optimal overall performance while providing quality control for the worst case. In contrast to the minimax approach that focuses on the worst-case performance, the probability constraint takes the errors into account proportionally. On the other hand, the worst case scenario ignored by the stochastic approach is considered in our approach.

Assuming the error model (3), the average SNR (2) becomes a function of the channel estimate $\hat{\mathbf{H}}$ and the random error \mathbf{E}

$$f(\hat{\mathbf{H}}, \mathbf{E}) = \frac{E_s}{N_0} \text{tr}\{\mathbf{C}^H (\hat{\mathbf{H}} + \mathbf{E})^H (\hat{\mathbf{H}} + \mathbf{E}) \mathbf{C}\}. \quad (4)$$

To simplify the expression (4), we consider the eigen-decomposition of $\mathbf{C}\mathbf{C}^H = \mathbf{U}_c \mathbf{D}_c \mathbf{U}_c^H$ and $\hat{\mathbf{H}}^H \hat{\mathbf{H}} = \mathbf{U}_h \mathbf{D}_h \mathbf{U}_h^H$. The diagonal matrix $\mathbf{D}_c = \text{diag}(d_1, d_2, \dots, d_{N_t})$ where $d_1 \geq \dots \geq d_{N_t} \geq 0$ are eigenvalues of $\mathbf{C}\mathbf{C}^H$. The corresponding eigenvectors are summarized in the unitary matrix \mathbf{U}_c . The matrices $\mathbf{D}_h = \text{diag}(D_1, \dots, D_{N_t})$ and \mathbf{U}_h are similarly defined.

3.1 Objective function

Given the channel estimate $\hat{\mathbf{H}}$, we obtain the objective function by taking the expectation of $f(\hat{\mathbf{H}}, \mathbf{E})$ with respect to the random error \mathbf{E}

$$\mathbb{E}[f(\hat{\mathbf{H}}, \mathbf{E})] = \frac{E_s}{N_0} \text{tr}\{\mathbf{U}_c \mathbf{D}_c \mathbf{U}_c^H (\mathbf{U}_h \mathbf{D}_h \mathbf{U}_h^H + \sigma_e^2 N_r \mathbf{I}_{N_t})\}. \quad (5)$$

It is well established in the literature [4] that a function with a structure similar to (5) can be maximized over \mathbf{U}_c and \mathbf{D}_c separately. Inserting the optimal solution for \mathbf{U}_c so that $\mathbf{U}_c^H \mathbf{U}_c = \mathbf{I}$, we obtain the following objective function

$$\bar{f}(\mathbf{D}_c) = \frac{E_s}{N_0} \text{tr}\{\mathbf{D}_c (\mathbf{D}_h + \sigma_e^2 N_r \mathbf{I}_{N_t})\}. \quad (6)$$

Note that $\bar{f}(\mathbf{D}_c)$ depends on $\mathbf{C}\mathbf{C}^H$ only through its eigenvalues. Hence, the design of the beamforming matrix becomes a power allocation problem.

3.2 Probabilistic constraint

To mitigate the impact of large errors, we guarantee the system performance by keeping the probability that SNR becomes smaller than an acceptable level γ_{th} to be low. More precisely, given an acceptable SNR level γ_{th} and the outage probability p_{out} , $f(\hat{\mathbf{H}}, \mathbf{E})$ satisfies the following probabilistic constraint

$$\Pr\{f(\hat{\mathbf{H}}, \mathbf{E}) \leq \gamma_{th}\} \leq p_{out}, \quad (7)$$

where $\Pr\{A\}$ denotes the probability of the event A .

As shown in (7), the distribution of $f(\hat{\mathbf{H}}, \mathbf{E})$ is crucial to the implementation of our algorithm. Applying the eigen-decomposition of $\mathbf{C}\mathbf{C}^H = \mathbf{U}_c \mathbf{D}_c \mathbf{U}_c^H$ and permutation property of the trace operation, (7) can be simplified to a mixture of independent noncentral $\chi^2_{n_i}(\delta_i)$ -distributed random variables Z_i , $i = 1, \dots, N_t$

$$f(\hat{\mathbf{H}}, \mathbf{E}) = \frac{E_s}{N_0} \sum_{i=1}^{N_t} d_i \sigma_e^2 Z_i. \quad (8)$$

The noncentrality parameter is $\delta_i = \frac{1}{\sigma_e^2} \tilde{\mathbf{h}}_i^H \tilde{\mathbf{h}}_i$ and the degree of freedom is $n_i = 2N_r$. The vector $\tilde{\mathbf{h}}_i \in \mathbb{C}^{N_r \times 1}$ represents the i th column of the matrix $\tilde{\mathbf{H}} = \hat{\mathbf{H}} \mathbf{U}_c$.

3.3 Probabilistic constrained optimization

Having derived the average SNR (5) and the compact expression (8) for $f(\hat{\mathbf{H}}, \mathbf{E})$, our design can be formulated as the following constrained optimization problem:

$$\begin{aligned} & \max_{\mathbf{D}_c} \text{tr}\{\mathbf{D}_c (\mathbf{D}_h + \sigma_e^2 N_r \mathbf{I}_{N_t})\}, \\ & \text{subject to} \\ & \Pr\left\{\sum_{i=1}^{N_t} d_i Z_i \leq \bar{\gamma}\right\} \leq p_{out}, \\ & \text{tr}\{\mathbf{D}_c\} \leq 1, \\ & d_i \geq 0, \quad i = 1, \dots, N_t \end{aligned} \quad (9)$$

where $\bar{\gamma} = \gamma_{th} (\frac{E_s}{N_0} \sigma_e^2)^{-1}$ and (10) is a convex constraint derived from the power constraint $\text{tr}\{\mathbf{C}\mathbf{C}^H\} \leq 1$.

4. REFORMULATION OF PROBABILISTIC CONSTRAINT

The major challenge in our approach is to convert the probabilistic constraint (9) into a deterministic one so that the solution can be efficiently computed by standard tools of mathematical programming. When the chance constraint involves linear combination of normally distributed random variables, it can be reformulated as a convex constraint [9]. However, (9) involves a mixture of noncentral χ^2 -distributions. The following result shows that the probabilistic constraint (9) can be replaced by a deterministic convex constraint.

Proposition The probabilistic constraint (9) can be replaced by the following convex constraint

$$\prod_{i=1}^{N_t} \left(\frac{1}{d_i} \left[\frac{\bar{\gamma}/2}{1 + \delta_i/n_i} \right] \right)^{n_i/2} \leq p_{out}, \quad (12)$$

where $\bar{\gamma} = \gamma_{th} (\frac{E_s}{N_0} \sigma_e^2)^{-1}$, $\delta_i = \frac{1}{\sigma_e^2} \tilde{\mathbf{h}}_i^H \tilde{\mathbf{h}}_i$ and $n_i = 2N_r$. If (12) holds, then (9) holds.

Proof: To decouple the design parameter d_i , we exploit the independence of Z_i , $i = 1, \dots, N_t$. Define the event

$$\mathcal{A}_i = \{d_i Z_i \leq \bar{\gamma}\} \quad (13)$$

and

$$\mathcal{A} = \left\{ \sum_{i=1}^{N_t} d_i Z_i \leq \bar{\gamma} \right\}. \quad (14)$$

By definition, \mathcal{A} is a subset of the intersection of \mathcal{A}_i , $i = 1, \dots, N_t$,

$$\mathcal{A} \subset \mathcal{B} = \mathcal{A}_1 \cap \mathcal{A}_2 \cap \dots \cap \mathcal{A}_{N_t} \quad (15)$$

which leads to the following inequality

$$\Pr\{\mathcal{A}\} \leq \Pr\{\mathcal{B}\} = \prod_{i=1}^{N_t} \Pr\{\mathcal{A}_i\}. \quad (16)$$

The above expression has the advantage that the event \mathcal{A}_i depends only on the noncentral $\chi_{n_i}^2(\delta_i)$ -distribution.

According to [10], the distribution of noncentral χ^2 -distribution can be approximated by a central χ^2 -distribution. Application of this result leads to the following approximation

$$\Pr\{\chi_{n_i}^2(\delta_i) \leq \frac{\bar{\gamma}}{d_i}\} \approx \Pr\{\chi_{n_i}^2 \leq \frac{\bar{\gamma}/d_i}{1 + \delta_i/n_i}\}. \quad (17)$$

To transform (17) to a deterministic form, we apply the sharp upper bound on the integral $\frac{1}{\Gamma(1+1/u)} \int_x^\infty e^{-t} dt$ derived in [11] to obtain the following inequality

$$\Pr\{\chi_{n_i}^2 \leq \frac{\bar{\gamma}/d_i}{1 + \delta_i/n_i}\} < \left(1 - \exp\left(-\frac{\bar{\gamma}/d_i}{2(1 + \delta_i/n_i)}\right) \right)^{\frac{n_i}{2}}. \quad (18)$$

Due to limited space, details about the derivation of (18) will be given in a future publication.

To achieve a convex constraint, we apply the following inequality to the right hand side of (18)

$$\left(1 - \exp\left(-\frac{\bar{\gamma}/d_i}{2(1 + \delta_i/n_i)}\right) \right)^{\frac{n_i}{2}} \leq \left(\frac{\bar{\gamma}/d_i}{2(1 + \delta_i/n_i)} \right)^{\frac{n_i}{2}}. \quad (19)$$

Eq (19) follows immediately from the inequality $(1 - e^{-u})^x \leq u^x$ for $u = \frac{\bar{\gamma}/d_i}{2(1 + \delta_i/n_i)} \geq 0$ and $x = \frac{n_i}{2} > 0$. Moreover, it is easy to verify that for positive $\frac{n_i}{2}$ and $\frac{\bar{\gamma}/d_i}{2(1 + \delta_i/n_i)}$, (12) is a convex set in d_i s.

Combing the inequalities (16),(17),(18) and (19), we conclude that the probabilistic constraint (9) is satisfied by the convex constraint (12). \square

Replacing the probabilistic constraint (9) with the deterministic constraint (12), the original problem is transformed to the following convex optimization problem

$$\max_{\mathbf{D}_c} \text{tr}\{\mathbf{D}_c(\mathbf{D}_h + \sigma_e^2 N_r \mathbf{I}_{N_t})\},$$

subject to

$$\begin{aligned} \prod_{i=1}^{N_t} \left(\frac{1}{d_i} \left[\frac{\bar{\gamma}/2}{1 + \delta_i/n_i} \right] \right)^{n_i/2} &\leq p_{\text{out}}, \\ \text{tr}\{\mathbf{D}_c\} &\leq 1, \\ d_i &\geq 0, \quad i = 1, \dots, N_t, \end{aligned}$$

that can be efficiently solved by standard tools of mathematical programming.

5. SIMULATION

In this section, we present simulation results to demonstrate robustness of the proposed beamformer in various scenarios. Here a single-user MIMO system with $N_t = 4$ transmit antennas and $N_r = 3$ receive antennas is considered. We also compare the proposed beamformer with existing techniques, such as the conventional one-directional beamformer, two-directional, equal-power loading beamformer [1] and the robust minimax beamformer [4]. We choose [4] for comparison because it uses the same type of channel information. The outage probability p_{out} is 10% and the normalized SNR threshold $\bar{\gamma}$ is 0.9 in all experiments.

In the first experiment, the proposed approach is applied to a well conditioned channel $\mathbf{D}_h = \text{diag}(0.8064, 0.1901, 0.0035, 0)$ with the first eigenvalue much larger than the remaining eigenvalues. The error variance σ_e^2 varies from 0 to 1. The SNR averaged over 10^4 Monte Carlo trials is plotted in Fig 1. With increasing error levels, the performance of all beamforming techniques degrade. For σ_e^2 between 0 and 0.4, the proposed approach, the maximin approach [4] and the one directional beamformer perform similarly. For $\sigma_e^2 > 0.4$, our approach has a much slower decline in SNR than other beamformers. The equal power loading beamformer has a significantly lower SNR than other three beamformers because channel information is not fully incorporated in its design.

In the second experiment, we consider the channel condition $\mathbf{D}_h = \text{diag}(0.4676, 0.4104, 0.1220, 0)$ with two closely spread eigenvalues. This indicates a larger correlation between antennas. Fig 2 shows that the probabilistic constraint beamformer still outperforms other three beamformers. For large error region $0.5 < \sigma_e^2 < 1$, the performance of all beamformers degrade more rapidly than in the previous experiment. However, our approach shows least sensitivity to channel errors.

In Fig 2, we also observe that when σ_e^2 increases from 0 to 0.8, the decrease in SNR associated with our approach is $\Delta \text{SNR} = -0.38$ dB and other beamformers lead to a decrease of more than $\Delta \text{SNR} = -1.83$ dB. This is 4.8 times as high as that caused by the probabilistic constraint approach. On the other hand, given a target SNR level, for example, -0.8 dB, the probabilistic constraint beamformer has the largest error tolerance range, $\sigma_e^2 \in [0, 0.78]$, while the worst case design achieves the desired performance only for $\sigma_e^2 \in [0, 0.63]$.

To summarize, the probabilistic constraint approach outperforms the worst case design and other classical beamformers over the entire error range. The gain in SNR is most significant at high error levels. In other words, our approach has the broadest tolerance range for channel

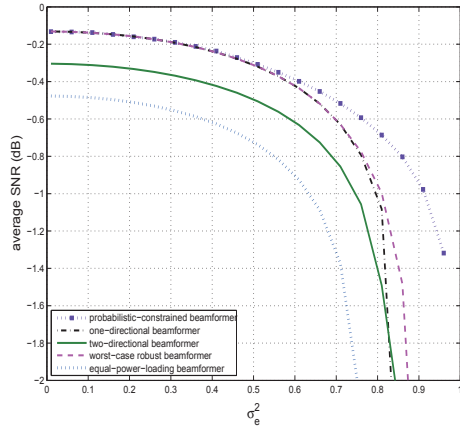


Figure 1: Average SNR versus σ_e^2 . $p_{\text{out}} = 10\%$, $\bar{\gamma} = 0.9$, $\mathbf{D}_h = \text{diag}(0.8064, 0.1901, 0.0035, 0)$.

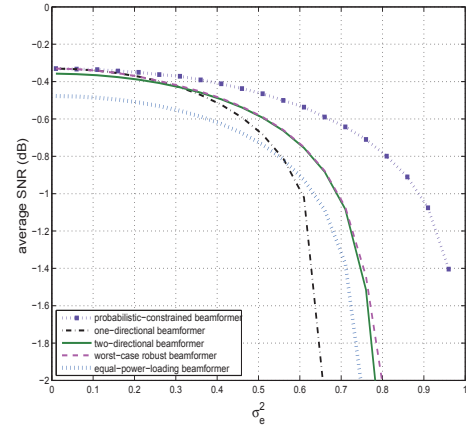


Figure 2: Average SNR versus σ_e^2 . $p_{\text{out}} = 10\%$, $\bar{\gamma} = 0.9$, $\mathbf{D}_h = \text{diag}(0.4676, 0.4104, 0.122, 0)$.

errors. Since the original stochastic optimization problem has been transformed to a convex optimization problem, the computational complexity is similar to the worst-case approach such as [4].

6. CONCLUSION

We proposed a novel transmit beamforming design that maximizes the average SNR performance and also guarantees robustness against channel estimation errors. Our approach was formulated as a probabilistic constrained optimization problem. Under the assumption that the channel estimation error is complex Gaussian distributed, the underlying problem was transformed into a convex optimization problem which can be efficiently solved by modern software packages. The resulting computational cost is similar to many state-of-the-art robust transmit beamformers. Simulation results show that the proposed beamformer achieves higher robustness than the maximin approach and leads to a much broader tolerance range for channel estimation errors. It provides a promising alternative to existing robust transmit beamforming techniques.

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ROBUST TRANSMIT BEAMFORMING BASED ON PROBABILISTIC CONSTRAINT

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ABSTRACT

Transmit beamforming is a powerful technique for enhancing performance of wireless communication systems. Most existing transmit beamforming techniques require perfect channel state information at the transmitter (CSIT), which is typically not available in practice. In such situations, the design should take into account errors in the channel estimates, so that the beamformers are less sensitive to these errors. Two robust approaches are widely used. The stochastic approach optimizes the average performance of the system and assumes that the statistics, such as mean and covariance, of the errors are known. The maximin approach assumes that the errors belong to a worst-case uncertainty region and optimizes the worst-case system performance. This type of design usually leads to conservative results as the worst-case conditions may occur at a very low probability. In this paper, we propose a more flexible approach that optimizes the average beamforming performance and takes the extreme (but rare) conditions into account proportionally. Simulation results show that the proposed beamformer offers higher robustness against errors in CSIT than several state-of-the-art beamformers.

1. INTRODUCTION

Multi-antenna diversity is well motivated in wireless communication systems because it offers significant advantages over single antenna [7]. Perfect or partial knowledge of the channel state information (CSIT) can provide further performance enhancement [10][11].

However, in practical wireless systems, the accuracy of the CSIT is impossible to know due to errors induced by imperfect (quantized, erroneous, or outdated) channel feedback. In such situations, the transmit beamforming design should take into account errors in channel estimates. Existing robust transmit beamforming designs can be categorized into stochastic and maximin approaches. The stochastic approach [6] [10] [11] assumes that statistics of errors in CSIT, such as mean and covariance, are known and optimizes the average performance of the system. On the other hand, the maximin approach considers channel estimation errors as deterministic and optimizes the worst-case system performance [1] [2]. This approach provides robustness against any error in the worst-case region. However, it is overly conservative as the worst operational condition is rare. To overcome this problem, a more flexible probabilistic constraint is introduced in [9] into the design of adaptive beamformer at the receiver side.

In this work, we propose a robust transmit beamforming technique that maximizes the average SNR performance and use probabilistic constraints to keep the worst-case performance at a very low probability. The aforementioned stochastic approach only optimizes the average performance without considering the worst-case scenario. On the other hand, although the maximin approach provides the best performance in the worst case, it is overall too conservative. To keep balance between the average and the worst-case performance, we take a more flexible approach in which the extreme (but rare) conditions are taken into account proportionally. Our approach maximizes the average SNR performance and ensures robustness against the CSIT error by keeping the probability of the worst-case performance at a very low level. Under the assumption that the CSIT error is Gaussian distributed, this stochastic optimization problem can be further simplified to equivalent deterministic forms which can be efficiently solved by modern convex optimization algorithms [3]. Simulation results show the proposed approach provides the best performance among several state-of-the-art beamforming techniques.

The paper is organized as follows. The system model is described in Section 2. We formulate the proposed method as a stochastic optimization problem in Section 3 and simplify it to an equivalent convex optimization problem in Subsection 3.1 and 3.2. Simulation results are presented and discussed in Section 4. Concluding remarks are given in Section 5.

Notation: $(\cdot)^H$ denotes Hermitian transpose; $E[\cdot]$ stands for expectation; $\text{tr}\{\cdot\}$ is the trace of a matrix; \mathbf{I}_K denotes the identity matrix of size K ; $\mathbf{0}_{K \times P}$ denotes an all-zero matrix of size $K \times P$; $\text{diag}\{\mathbf{x}\}$ stands for a diagonal matrix with \mathbf{x} on its diagonal; $\{\cdot\}_j$ denotes the j th entry of a vector, \mathbf{h}_j denotes the j th column of matrix \mathbf{H} .

2. SYSTEM MODEL

We consider a single-user wireless communication system with M transmit antennas and a single receive antenna. The information-bearing signal s is spread by the precoding matrix \mathbf{C} and then transmitted through the flat fading channel. As we focus on symbol-by-symbol detection, the received signal \mathbf{y} in the presence of additive white Gaussian noise \mathbf{w} is given by

$$\mathbf{y} = \mathbf{C}\mathbf{h}s + \mathbf{w}. \quad (1)$$

In the perfect CSIT case, the estimated channel at the transmitter is error free and the output \hat{s} of maximum ratio combining (MRC) at the receiver is given by

$$\hat{s} = (\mathbf{C}\mathbf{h})^H \mathbf{y} = \mathbf{h}^H \mathbf{C}^H \mathbf{C} \mathbf{h} \mathbf{s} + \mathbf{h}^H \mathbf{C}^H \mathbf{w}. \quad (2)$$

The average signal-to-noise ratio (SNR) at MRC receiver output is

$$\text{SNR} = \mathbb{E} \left[\frac{(\mathbf{h}^H \mathbf{C}^H \mathbf{C} \mathbf{h} \mathbf{s}) (\mathbf{h}^H \mathbf{C}^H \mathbf{C} \mathbf{h} \mathbf{s})^H}{\mathbf{h}^H \mathbf{C}^H \mathbf{w} \mathbf{w}^H \mathbf{C} \mathbf{h}} \right] = \frac{E_s}{N_0} \mathbf{h}^H \mathbf{C}^H \mathbf{C} \mathbf{h}, \quad (3)$$

where $E_s = \mathbb{E}[|s|^2]$ is the average energy of the signal and $N_0/2$ is the noise variance.

To extend the model to a system with N receive antennas, we assume that the channel vectors observed on different receive antennas are mutually uncorrelated. The channel vector denotes as \mathbf{h}_j for j th receive antenna, and is arranged into a $M \times N$ matrix $\mathbf{H} = [\mathbf{h}_1, \dots, \mathbf{h}_N]$. Similar to the single-receive-antenna case, the received signal at the j th antenna is $\mathbf{y}_j = \mathbf{C} \mathbf{h}_j s + \mathbf{w}_j$. The total receiver SNR at the output of the MRC is

$$\text{SNR} = \frac{E_s}{N_0} \sum_{j=1}^N \mathbf{h}_j^H \mathbf{C}^H \mathbf{C} \mathbf{h}_j = \frac{E_s}{N_0} \text{tr}\{\mathbf{H}^H \mathbf{C}^H \mathbf{C} \mathbf{H}\} \quad (4)$$

which includes (3) as a special case corresponding to $N = 1$.

3. ROBUST BEAMFORMING BASED ON PROBABILISTIC-CONSTRAINED OPTIMIZATION

We consider the case in which the transmitter does not have exact channel state information (CSI) but has an estimate $\hat{\mathbf{H}}$ of the channel matrix \mathbf{H} . The CSIT error matrix is given by

$$\mathbf{E} = [\mathbf{e}_1, \dots, \mathbf{e}_N] = \mathbf{H} - \hat{\mathbf{H}}. \quad (5)$$

We assume that \mathbf{e}_j is complex normally distributed and independent from the estimate channel $\hat{\mathbf{h}}_j$, i.e. $\mathbb{E}[\hat{\mathbf{h}}_j^H \mathbf{e}_j] = 0$. In the proposed approach, we optimize the average SNR at the output of MRC receiver and achieve the robustness by keeping the outage probability of the instantaneous SNR below a pre-specified level. For simplicity, assuming E_s/N_0 is constant in one symbol interval, we will drop the constant factor E_s/N_0 from the SNR expression.

Our objective is to derive the precoding matrix \mathbf{C} that maximizes the average SNR and has a low outage probability. More specifically, the design of robust beamforming matrix can be achieved by solving the following optimization problem

$$\max_{\mathbf{C}} \mathbb{E} \left[\text{tr}\{(\hat{\mathbf{H}} + \mathbf{E})^H \mathbf{C}^H \mathbf{C} (\hat{\mathbf{H}} + \mathbf{E})\} \right],$$

subject to

$$\begin{aligned} P \{ \text{tr}\{(\hat{\mathbf{H}} + \mathbf{E})^H \mathbf{C}^H \mathbf{C} (\hat{\mathbf{H}} + \mathbf{E})\} \leq \gamma \} &\leq p, \\ \text{tr}\{\mathbf{C}^H \mathbf{C}\} &= 1, \end{aligned} \quad (6)$$

where γ denotes the SNR threshold, p is a pre-specified probability value that satisfies quality of service (QoS) requirements, and $P\{A\}$ stands for the probability of event A . Typically we select a low probability value p and high threshold value γ . The deterministic constraint $\text{tr}\{\mathbf{C}^H \mathbf{C}\} = 1$ reflects the fact that the total transmitted power is limited by the system.

To simplify the above problem, we consider the eigen-decomposition of $\mathbf{C}^H \mathbf{C}$

$$\mathbf{C}^H \mathbf{C} = \mathbf{U}_c \mathbf{D}_c \mathbf{U}_c^H, \quad (7)$$

where $\mathbf{U}_c = [\mathbf{u}_{c1}, \dots, \mathbf{u}_{cM}]$ consists of eigenvectors of $\mathbf{C}^H \mathbf{C}$ and $\mathbf{D}_c = \text{diag}\{d_{c1}, \dots, d_{cM}\}$ is a diagonal matrix with corresponding eigenvalues $d_{c1} \geq \dots \geq d_{cM} \geq 0$. The precoding matrix \mathbf{C} can be viewed as a weight matrix. The error covariance matrix \mathbf{R}_e is positive definite and can be factorized as

$$\mathbf{R}_e = \mathbf{V}_e \mathbf{V}_e^H, \quad (8)$$

where \mathbf{V}_e is a nonsingular matrix. Then the product $\mathbf{V}_e^H \mathbf{C}^H \mathbf{C} \mathbf{V}_e$ can be simplified as follows

$$\mathbf{V}_e^H \mathbf{C}^H \mathbf{C} \mathbf{V}_e = (\mathbf{U}_c^H \mathbf{V}_e)^H \mathbf{D}_c (\mathbf{U}_c^H \mathbf{V}_e) = \mathbf{P}^H \mathbf{D}_c \mathbf{P}, \quad (9)$$

where $\mathbf{P} = \mathbf{U}_c^H \mathbf{V}_e$.

Since the average SNR depends on the beamforming matrix \mathbf{C} through $\mathbf{C}^H \mathbf{C}$, it suffices to optimize the objective function with respect to \mathbf{U}_c and \mathbf{D}_c . Define $\check{\mathbf{H}} = \mathbf{U}_c^H \hat{\mathbf{H}}$ and $\check{\mathbf{E}} = \mathbf{U}_c^H \mathbf{E}$. The objective function in (6) can be rewritten as

$$\begin{aligned} &\mathbb{E} \left[\text{tr}\{(\hat{\mathbf{H}} + \mathbf{E})^H \mathbf{C}^H \mathbf{C} (\hat{\mathbf{H}} + \mathbf{E})\} \right] \\ &= \mathbb{E} \left[\text{tr}\{(\check{\mathbf{H}} + \check{\mathbf{E}})^H \mathbf{D}_c (\check{\mathbf{H}} + \check{\mathbf{E}})\} \right] \\ &= \text{tr} \left\{ \mathbf{D}_c [\check{\mathbf{H}} \check{\mathbf{H}}^H + \mathbb{E}[\check{\mathbf{H}} \check{\mathbf{E}}^H] + \mathbb{E}[\check{\mathbf{E}} \check{\mathbf{H}}^H] + \mathbb{E}[\check{\mathbf{E}} \check{\mathbf{E}}^H]] \right\} \\ &= \text{tr} \left\{ \mathbf{D}_c (\check{\mathbf{R}} + \check{\mathbf{R}}_e) \right\}, \end{aligned} \quad (10)$$

where $\check{\mathbf{R}} = \mathbb{E}[\check{\mathbf{H}} \check{\mathbf{H}}^H]$ and $\check{\mathbf{R}}_e = \mathbb{E}[\check{\mathbf{E}} \check{\mathbf{E}}^H]$.

The probabilistic constraint in (6) becomes mathematically tractable if we can find a closed expression for the distribution of the random variable $\text{tr}\{(\hat{\mathbf{H}} + \mathbf{E})^H \mathbf{C}^H \mathbf{C} (\hat{\mathbf{H}} + \mathbf{E})\}$. Applying a non-singular linear transformation [4], this random variable can be written as

$$\begin{aligned} \text{SNR} &= \text{tr} \left\{ (\hat{\mathbf{H}} + \mathbf{E})^H \mathbf{C}^H \mathbf{C} (\hat{\mathbf{H}} + \mathbf{E}) \right\} \\ &= \text{tr} \left\{ (\hat{\mathbf{H}} + \mathbf{E})^H (\mathbf{V}_e^H)^{-1} \mathbf{V}_e^H \mathbf{C}^H \mathbf{C} \mathbf{V}_e \mathbf{V}_e^{-1} (\hat{\mathbf{H}} + \mathbf{E}) \right\} \\ &= \text{tr} \left\{ \left[\mathbf{P} \mathbf{V}_e^{-1} (\hat{\mathbf{H}} + \mathbf{E}) \right]^H \mathbf{D}_c \left[\mathbf{P} \mathbf{V}_e^{-1} (\hat{\mathbf{H}} + \mathbf{E}) \right] \right\} \\ &= \text{tr} \left\{ (\hat{\mathbf{H}} + \check{\mathbf{E}})^H \mathbf{D}_c (\hat{\mathbf{H}} + \check{\mathbf{E}}) \right\} \\ &= \sum_{i=1}^M d_{ci} \sum_{j=1}^N (\tilde{h}_{ij} + \tilde{e}_{ij})^2, \end{aligned} \quad (11)$$

where $\tilde{\mathbf{H}} = \mathbf{P} \mathbf{V}_e^{-1} \hat{\mathbf{H}}$ and $\tilde{\mathbf{E}} = \mathbf{P} \mathbf{V}_e^{-1} \mathbf{E}$. The random matrix $\tilde{\mathbf{E}}$ has normal distribution with zero mean and covariance matrix $\mathbf{I}_{M \times M}$.

Using (10) and (11), the proposed approach can be reformulated as follows:

$$\max_{\mathbf{D}_c} \text{tr}\{\mathbf{D}_c (\check{\mathbf{R}} + \check{\mathbf{R}}_e)\}, \quad (12)$$

subject to

$$P \{ \text{tr}\{(\tilde{\mathbf{H}} + \tilde{\mathbf{E}})^H \mathbf{D}_c (\tilde{\mathbf{H}} + \tilde{\mathbf{E}})\} \leq \gamma \} \leq p, \quad (13)$$

$$\text{tr}\{\mathbf{D}_c\} = 1. \quad (14)$$

The robust beamformer design is now in the form of a probabilistic-constrained stochastic optimization problem.

Under the assumption that the error in CSIT is Gaussian, the stochastic optimization can be converted into a convex optimization problem which can be efficiently solved using modern convex optimization methods.

3.1 Relaxation of Convex Constraint

In convex programming, both the objective function and the constraints are required to be convex. We replace $\text{tr}\{\mathbf{D}_c\} = 1$ with an inequality constraint which is easier to satisfy, that is

$$\text{tr}\{\mathbf{D}_c\} \leq 1. \quad (15)$$

This is equivalent to relaxing the constraint (6) to $\text{tr}\{\mathbf{C}^H \mathbf{C}\} \leq 1$.

Theorem The optimization problem defined in (12)-(14) is equivalent to that with the strict constraint (14) being replaced by the relaxed constraint (15)

Proof: Suppose the optimal solution $\bar{\mathbf{D}}_c$ lies in the region $\text{tr}\{\bar{\mathbf{D}}_c\} < 1$. This implies that the maximum of (14) is given by

$$\text{tr}\{\bar{\mathbf{D}}_c (\check{\mathbf{R}} + \check{\mathbf{R}}_e)\}.$$

However, we can always construct another matrix \mathbf{D}_c^* by multiplying $\bar{\mathbf{D}}_c$ with a positive constant $c = 1/\text{tr}\{\bar{\mathbf{D}}_c\} > 1$, so that the constraint $\text{tr}\{\mathbf{D}_c\} = 1$ is satisfied. This leads to the following inequality:

$$\text{tr}\{\mathbf{D}_c^* (\check{\mathbf{R}} + \check{\mathbf{R}}_e)\} > \text{tr}\{\bar{\mathbf{D}}_c (\check{\mathbf{R}} + \check{\mathbf{R}}_e)\}. \quad (16)$$

This inequality (16) contradicts our assumption that $\bar{\mathbf{D}}_c$ maximizes (12). Thus, a matrix $\bar{\mathbf{D}}_c$ satisfying the constraint $\text{tr}\{\bar{\mathbf{D}}_c\} < 1$ can not be the optimal solution. In other words, the optimal solution always satisfies the original constraint $\text{tr}\{\mathbf{D}_c\} = 1$. Hence, the objective function (12)-(14) can be equivalently transformed into a convex optimization problem by relaxing the constraint $\text{tr}\{\mathbf{D}_c\} = 1$ to $\text{tr}\{\mathbf{D}_c\} \leq 1$. \square

3.2 Reformulation of Probabilistic Constraint

To make the proposed approach tractable, we apply Imhof's results [5] to approximate the distribution of the quadratic form $\text{tr}\{(\check{\mathbf{H}} + \check{\mathbf{E}})^H \mathbf{D}_c (\check{\mathbf{H}} + \check{\mathbf{E}})\}$ and transform the probabilistic constraint into a deterministic constraint.

We consider the quadratic form (11) as a linear combination of noncentral χ^2 -distributed random variables

$$\sum_{i=1}^M d_{c_i} \sum_{j=1}^N (\tilde{h}_{i,j} + \tilde{e}_{i,j})^2 = \sum_{i=1}^M d_{c_i} \chi_{n_i, \delta_i^2}^2, \quad (17)$$

where $\chi_{n_i, \delta_i^2}^2$, $i = 1, \dots, M$ are independent noncentral χ^2 -distributed random variables with degree of freedom $n_i = N$ and non-centrality parameter $\delta_i^2 = \sum_{j=1}^N \tilde{h}_{i,j}^2$. Imhof has derived an integral form of the cumulative distribution function for random variables in the form of (17). Based on the results of [5], the probabilistic constraint can be rewritten as

$$\begin{aligned} P \left\{ \sum_{i=1}^M d_{c_i} (n_i + \delta_i^2) \leq \gamma \right\} \\ = 1 - \left(\frac{1}{2} + \frac{1}{\pi} \int_0^\infty \frac{\sin \theta(u)}{u \rho(u)} du \right) \\ = \frac{1}{2} - \frac{1}{2\pi} \gamma + \frac{1}{2\pi} \sum_{i=1}^M d_{c_i} (n_i + \delta_i^2) \end{aligned} \quad (18)$$

where

$$\begin{aligned} \lim_{u \rightarrow 0} \frac{\sin \theta(u)}{u \rho(u)} &= \frac{1}{2} \sum_{i=1}^M d_{c_i} (n_i + \delta_i^2) - \frac{1}{2} \gamma \\ \lim_{u \rightarrow \infty} u \rho(u) &= +\infty \\ \lim_{u \rightarrow \infty} \theta(u) &= \begin{cases} -\infty & : \text{ if } \gamma > 0 \\ +\infty & : \text{ if } \gamma < 0 \\ \frac{\pi}{4} \sum_{i=1}^M n_i d_{c_i} |d_{c_i}|^{-1} & : \text{ if } \gamma = 0 \end{cases} \end{aligned}$$

With (18), the probabilistic constraint (11) can be transformed into a convex constraint as follows

$$\frac{1}{2} - \frac{1}{2\pi} \gamma + \frac{1}{2\pi} \sum_{i=1}^M d_{c_i} (n_i + \delta_i^2) \leq p. \quad (19)$$

With the relaxation (15) and the expression (19), the original stochastic optimization problem (6) is now converted into the convex optimization problem defined as follows.

$$\max_{\mathbf{D}_c} \text{tr}\{\mathbf{D}_c (\check{\mathbf{R}} + \check{\mathbf{R}}_e)\}, \quad (20)$$

subject to

$$\begin{aligned} \frac{1}{2} - \frac{1}{2\pi} \gamma + \frac{1}{2\pi} \sum_{i=1}^M d_{c_i} (n_i + \delta_i^2) &\leq p, \\ \text{tr}\{\mathbf{D}_c\} &\leq 1 \end{aligned}$$

The optimal solution can be efficiently found by modern convex optimization algorithms, such as CVX [3]. CVX software package is a Matlab-based modeling system for convex optimization that allows constraints and objective functions to be specified using standard Matlab expression syntax.

4. SIMULATION RESULTS

The proposed beamformer is tested by simulation. We consider a single-user MIMO system with $M = 4$ transmit antennas and $N = 3$ receive antennas. A hundred Monte Carlo trials were performed in each experiment. The proposed beamformer is compared with existing techniques, such as the worst-case one-directional, equal-power loading beamformer and robust beamformer [1]. Without any loss of generality, we assume the following:

- Channel paraments: Angle of spread Δ is related to the channel state information. The angle of spread determines the spatial correlations of the channel. For the small angle spread, the correlation coefficient between the p th and q th transmit antenna can be presented as [8]

$$[\mathbf{R}]_{p,q} \approx \frac{1}{2\pi} \int_0^\pi \exp \left[-j2\pi(p-q)\Delta \frac{d_t}{\lambda} \sin \theta \right] d\theta,$$

where λ is the wavelength of a narrow-band signal, d_t the antenna spacing and Δ the angle of spread.

- Sample covariance matrix: The channel covariance matrix $\hat{\mathbf{R}}$ is estimated by sampling the instantaneous channels

$$\hat{\mathbf{R}} = \frac{1}{N} \sum_{i=1}^N \hat{\mathbf{H}} \hat{\mathbf{H}}^H.$$

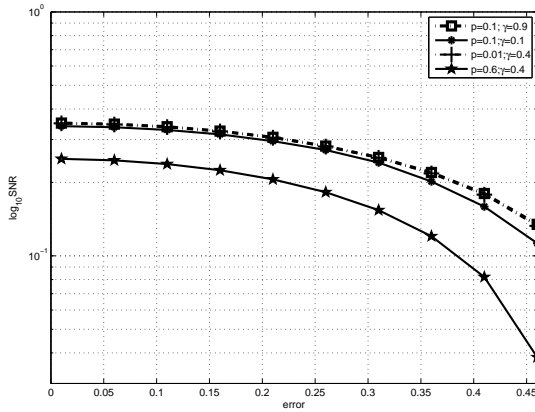


Figure 1: SNR performance under the different parameter selection, where $\Delta = 45^\circ$

- Estimated error at the transmitter: We assume that the error is Gaussian distributed with zero mean and covariance matrix $\sigma^2 \mathbf{I}$, that is,

$$\mathbf{E}_{M \times M} \sim \mathcal{CN}(0, \sigma^2 \mathbf{I}).$$

In our simulation, the error is varied from 0.01 to 0.9.

Firstly, we compare performances under various choices of parameters γ and p , shown in Fig. 1. We set the spread angle $\Delta = 45^\circ$. With the same probability $p = 0.1$, the high-threshold beamformer ($\gamma = 0.9$) outperforms the low-threshold one ($\gamma = 0.4$). On the other hand, under the same SNR threshold $\gamma = 0.4$, the beamformer with $p = 0.01$ achieves an overall higher SNR than $p = 0.6$. This implies that a low outage probability ensures robustness against errors. In Fig. 1, we can also observe that the proposed transmit beamformer is sensitive to the selection of the outage probability p .

Then we compare the average SNR performance of the proposed transmit beamformer and four other existing methods. According to the quality of service (QoS) requirements, we select a low probability value $p = 0.1$ and a high SNR threshold $\gamma = 0.9$.

In Fig. 2, the angle of spread is 5° and the correlation between p th and q th channel is high. That means less knowledge of CSIT can be obtained and the MRC output of SNR is more sensitive to the error. In this case, worst-case robust beamformers [1] [6] and one-directional beamformer [7] prefer to focus all available power on the channel's strongest direction. And the equal-power-loading beamformer equally loads the transmit power without considering CSIT. However, in the proposed beamformer, the instantaneous SNR is controlled by the probabilistic constraint and the proposed robust design offers the best performance over other beamformers.

In Fig. 3, the spread angle is $\Delta = 25^\circ$ and the channel environment is better than the channel in the previous experiment. In this case, for the maximum MRC output of SNR,

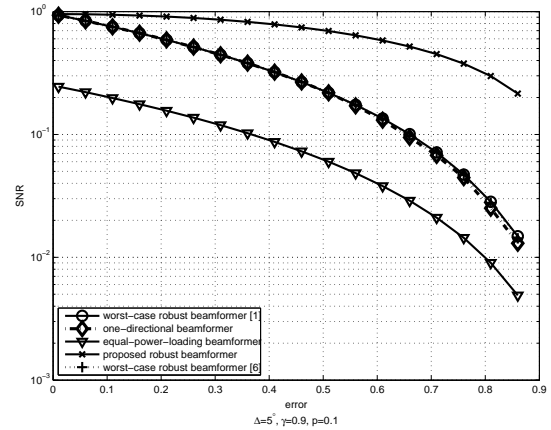


Figure 2: SNR performance of one-directional beamformer, equal-power-loading beamformer, worst-case robust beamformers [1] and [6], proposed beamformer versus error: $\gamma = 0.9$, $p = 0.1$, $\Delta = 5^\circ$

the transmit power tends to be loaded equally. The performances of both worst-case robust beamformers tend to that of the equal-power-loading robust beamformer. Meanwhile, the one-directional beamformer offers the worst performance as the error increases. On the other hand, the proposed beamformer still offers the highest average SNR in the entire error range.

5. CONCLUSION

In this work, we propose a novel transmit beamformer design that maximizes average SNR performance and also guarantees robustness against the CSIT errors. The robust transmit beamformer design is formulated as a stochastic optimization problem. Under the assumption that the CSIT error is Gaussian distributed, the underlying stochastic optimization problem is transformed into a convex optimization problem which can be efficiently solved by modern software packages. Simulation results show that the proposed robust transmit beamformer is less sensitive to the errors in CSIT and outperforms several state-of-the-art robust beamforming algorithms.

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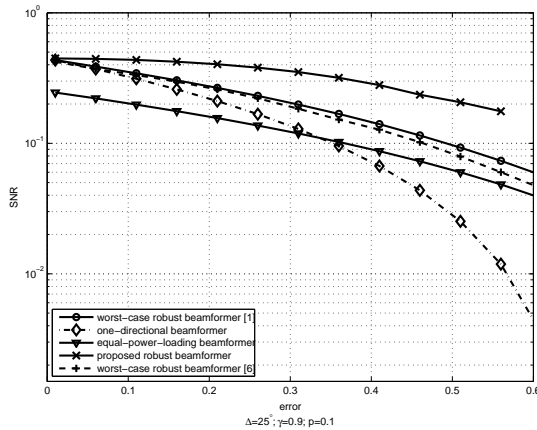


Figure 3: SNR performance of one-directional beamformer, equal-power-loading beamformer, worst-case robust beamformers [1] and [6], proposed beamformer versus error: $\gamma = 0.9$; $p = 0.1$; $\Delta = 25^\circ$

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