DYNAMIC WIND LOADS IN RELATION TO THE DESIGN OF STRUCTURES

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CONTENTS PAGE

SYNOPSIS.

Wind loads constitute one of the major forms of structural loading. The calculation of the dynamic response of structures to wind loads requires the use of specialised techniques due to the complexity of wind excited oscillating systems. Current techniques are at an early stage of development and the need for more research in this field is recognised.
An analytical investigation of the dynamic behaviour of structures subjected to simulated wind loads is carried out using a conventional spectrum analysis technique. The relative importance of parameters relating to the wind and the structures are discussed and the insensitivity of the spectrum analysis to variations in structural data is demonstrated. The fact that the non-linearity of the systems and the inertia of the structures are neglected is suggested as reason for this.
A further analytical investigation, using an energy technique developed by the author, is described and the importance of making an allowance for the non-linearity of the systems and of structural inertia is demonstrated. The energy method is reduced to a dimensionless form so as to be suitable for general application in design and its use as a substitute for the spectrum analysis is suggested.
Appendix 1 contains the analytical development of power spectrum analysis theory while Appendix 2 deals with detailed aspects of the energy analysis.
Appendices 3, 4 and 5 are concerned with the development of a method for calculating the natural frequencies and mode shapes of multi-storey shear wall structures. This was carried out concurrently with the main topics./

topics. Appendix 3 contains the analysis of a shear wall building using a continuum theory adapted by the author for use in dynamic analysis. The results predicted by the continuum theory are compared with those obtained from more approximate methods and a model test carried out to confirm the theoretical results. The continuum theory is reduced to a set of dimensionless design curves from which the natural frequencies of shear wall structures may be determined after the evaluation of simple structural parameters. Appendices 4 and 5 describe the solution of equations and the evaluation of certain constants used in the shear wall analysis.

CHAPTER 1

SHORT HISTORY OF WIND LOADING RESEARCH

The effect which wind loading has on a structure is an enigma which has troubled engineers for centuries and which assumed importance in the latter half of the eighteenth and beginning of the nineteenth centuries, with the advent of structural analysis and a more rational approach to design. The need for new types of structure brought about by the industrial revolution, together with the development of the techniques of structural mechanics and stress analysis, led to a change in emphasis of design criteria from those of aesthetics to those of economy and efficiency. The calculation of stresses became a standard part of design procedures for engineering structures and the dimensions of their components were arrived at on the basis that the maximum stresses should not exceed certain specified limits. The maximum loading condition had therefore to be known, and it was the problem of determining this which constituted a major difficulty in the case of wind loads.

Although it was recognised that the wind was highly turbulent and that it gave rise to forces which were far from steady, it was considered that the worst loading condition to which a structure would be subjected would result from the highest velocity gust which struck it in its lifetime, and that this could be regarded as a static phenomenon. Early investigators were concerned with the problem of predicting the velocity of the worst gust which would blow over a particular structure, and the resulting pressure which this would produce on its surface.

The controversy as to what design wind pressure ought to be adopted for general use went on throughout the nineteenth century and consisted of arguments which were based more on inspired guesswork than on the result of scientific investigation. The matter was brought to a head after the collapse of the Tay Railway Bridge in a gale in 1879. This bridge had been designed to resist a maximum wind pressure of 10 lbs./sq. ft., a figure which was recommended by the Astronomer Royal of the day, but which appears to have been based on recommendations made by Smeaton in a letter to the Royal Society in 1759. Smeaton advocated pressures of 6 lb./sq. ft. for high winds, 8-9 lb./sq. ft. for very high winds and 12 lb./sq. ft. for storms or tempests.
As a result of the Tay Bridge disaster and the investigation which followed, the Board of Trade issued a regulation that a pressure of 56 lb./sq. ft. be used in future for the design of engineering structures. This brought Britain into line with other countries, where typical accepted values at the time were 55 lb./sq. ft. in France and 50 lb./sq. ft. in the U.S.A.
One of the first scientific studies of wind pressure was made by Baker in connection with the design and construction of the Forth Railway Bridge in the 1880's.[1] Baker erected four wind pressure gauges on an island in the River Forth near the intended site of the bridge. These consisted of a square board of area 300 sq. ft. and three circular boards approximately 1.5 sq.ft. in area. Readings were taken continuously between 1883 and 1890, and maximum pressures recorded were 31 lb./sq.ft. for the small boards and 19 lb./sq.ft. for the large board. Baker also conducted wind tunnel experiments to obtain the drag properties of the proposed bridge by comparing the force on a model of the bridge with that on plane laminae of different areas.

Perhaps the most important result from Baker's experiment was the discovery that the average pressure over a large area is much less than the maximum load pressure due to a gust, and that high intensity gusts appeared to have a small spatial extent. This question of the difference in the average pressure between surfaces of different area now became one of the main problems attracting the interest of investigators.

By the beginning of the twentieth century, a large number of meteorological stations had been set up all over Great Britain and these were keeping continuous records of the wind velocity. It was therefore possible for the designer to get some idea of the maximum wind velocity which was likely to occur in the area of a proposed structure, and the problem of estimating the maximum gust pressure on a structure was on the way to being solved. As civil engineering structures usually presented a large area to the wind, however, the new problem of estimating the extent to which this maximum pressure ought to be reduced to allow for the small spatial extent of gusts now assumed some importance. The extent of the interest in this issue can be gauged from the discussion which followed a paper on wind pressure which was presented to the Institution of Civil Engineers in 1924 by Stanton. Stanton's paper described an experiment to measure average wind pressures across different areas, at a site near the National Physical Laboratory at Teddington and another on Tower Bridge in London. A comparison was also made between anemometer readings taken on these sites and readings taken at the nearest meteorological station at Kew. Stanton found that while average pressures were lower than local pressures for the Teddington experiment, this was

not the case on the Tower Bridge site. On the basis of his results Stanton made the following recommendations for design:

1. That for the purposes of assessing the maximum design velocity for a structure, an anemometer should be erected on the site of a proposed structure to facilitate a comparison between the wind at the site and the wind at the nearest meteorological station. The design wind should then be found by extrapolating the maximum velocity recorded at the meteorological station to the site.
2. That no reduction should be made from the maximum gust pressure to allow for the fact that the gust may not totally envelope the structure.

The second of these recommendations was criticised during the discussion and it was suggested that the result obtained at Tower Bridge was not typical of most structures and could be attributed to peculiarities in the topography surrounding the Tower Bridge site and to the positioning of the pressure gauges in the structure of the bridge. The general practice at the time seems to have been that reductions were made for average pressures over large areas, and typical values are:-

300 sq.ft. 0.67p

40 sq.ft. 0.79p

10 sq.ft. 0.89p

where p is the local pressure due to the highest gust likely to occur at the site.

The general state of knowledge at this time was not very far advanced, however, especially so far as application to design was concerned. One of the main shortcomings was the lack of accurate

data on the wind itself. The Dine's pressure tube anemometer, for instance, which was used at most meteorological stations, was incapable of recording gusts of short duration due to its long response time, and it was realised that the maximum gust velocity recorded by this instrument was probably considerably less than the maximum gust velocity which would actually occur at a given site. It was also realised that the extent to which the average pressure over a large area became reduced from the maximum local pressure depended on the characteristics of the turbulence at the site in question and that this depended on local topography. Sites in open country where there were few obstructions and where the air stream was comparatively smooth were distinguished from those in urban areas where the intensity of turbulence was much greater and gusting more prevalent. No quantitative data was available, however, to help a designer to decide what allowance ought to be made for the variation in the characteristics of turbulence at different sites. The situation was, therefore, that while the general principles of the effect of wind on buildings were understood, the crudity of the available data was such that large safety factors were required for design.

As the use of steel and concrete-framed buildings became more widespread during the first half of this century, the tendency in the building industry was to use ever lighter material for the purpose of cladding. Concern became concentrated on wind-induced failure of building components and not, as previously, on failure of the whole structure due to wind loading. The emphasis in research therefore shifted away from a consideration of the overall stability of a structure, and investigations were begun

on the pressure distribution over a building due to the wind. In 1942, Bailey and Vincent published a paper entitled "Wind Pressure on Buildings including the effect of Adjacent Buildings".[2] This paper described a series of wind tunnel tests carried out on model buildings in which pressure distributions were measured. Graphs of pressure distribution for many building shapes were given. Also, as previous investigators had done, they recommended that the maximum gust velocity recorded at meteorological stations should be the criterion for design. Perhaps the most significant result of their work, however, was their appreciation of the effect which the internal pressure in a building has on panel loads. Bailey and Vincent realised that the load on a cladding panel depended on the difference in pressure across the panel, and that the pressure inside the building was as important as that outside. They also realised that the internal pressure depended on the location of the dominant openings in the building and their position with respect to the direction of the wind. A large opening on the leeward side of a building, for instance, would give rise to a negative pressure inside which would greatly increase the load on the windward wall as it would then be subjected to a positive pressure on one side and a negative one on the other. Bailey and Vincent, therefore, illustrated that care was needed in the design of walls and roofs of buildings and that the location and sizes of windows and doors, as well as the wind velocity, had to be taken into account during design.

One aspect of the wind loading problem which has not yet been touched on so far is the variation in wind speed with height.

This has been found to be dependent on the temperature gradient and ground roughness. Attempts were made in the 1930's to find empirically, a formula which would define the variation in mean velocity with height and early work in the field was mainly based on the formula,

$$V_Z = a \log Z + b$$

where V_Z = mean velocity at height Z

a, b = constants

More recent work has concentrated on the simpler "Power Law" profile given by,

$$V_Z = V_o\left(\frac{Z}{Z_o}\right)^{\alpha'}$$

where V_o is a reference velocity at height Z_o and α' a constant dependent on ground roughness. The "Power Law" formula is not altogether satisfactory, however, especially for areas near changes in ground roughness such as occur at city boundaries, but its simplicity has led to its adoption in the wind loading field and values of α' have been tabulated for a range of ground roughnesses.

The Code of Practice CP 3 (Ch. 5), which was published in 1952, formulated the procedure for design to resist wind loads, and followed more or less the lines of the Bailey and Vincent paper. One possible shortcoming, however, was that although the variation in wind speed with height was allowed for in the Code and higher loadings were recommended for high buildings, the pressure coefficients given were based on wind tunnel tests carried out in a uniform air stream. Later work has suggested that the neglect of the velocity profile leads to incorrect modelling of the flow

around buildings and to an inaccurate assessment of the pressure coefficients. A further criticism of the model tests is that they were carried out on solid models. Recent work by Newberry on full scale buildings has suggested that buildings are much more permeable than was previously assumed and that this has a considerable effect on the pressure distribution across their surfaces. Another aspect of the 1952 Code, which was perhaps rather surprising, was the adoption of the maximum one minute mean wind speed for design purposes rather than the maximum gust speed. This gave lower loadings than had previously been used and may be criticised in retrospect in the light of the many cladding failures which have occurred since the introduction of this standard. The new Code, which was published in draft form in 1968, reverts to the older usage of maximum gust velocities and generally brings the 1952 Code up to date by the use of more recent data on pressure coefficients and the introduction of a more detailed procedure for assessing the internal pressure of buildings. The method suggested for checking the overall stability of a structure is to sum the cladding loads vectorially to obtain an overturning moment.

Throughout the development of methods for assessing wind loads, it has always been assumed that the consideration of the wind as a static form of loading would give a good enough approximation for design purposes, although the turbulent nature of the wind had always been appreciated. In recent years, however, it has been realised that slender structures, such as lattice towers or tall buildings, are capable of responding dynamically to the wind, and vibrations of such structures have been recorded, both

the along wind and across wind directions. Vibrations in the lateral direction are usually due to the phenomenon of vortex shedding. This is a problem which is particularly prevalent with tall chimneys and it has been the subject of a considerable research effort at the National Physical Laboratory by Scruton and others. Vibration due to vortex shedding may be controlled either by designing the structure so that its natural frequency does not correspond to the frequency of vortex shedding, or by preventing the vortices from forming by the attachment of spoilers to the surface of the structure which break up the air stream. Vibration of slender structures in the along wind direction as a result of buffeting by high frequency gusts presents a more difficult problem from the point of view of design. It is unlikely that such vibrations could be eliminated by suitable design, although the possibility of limiting the amplitude by the introduction of damping devices does exist. The feasibility of this remedy is discussed in a later chapter, but it may be noted here that there are many structures for which it would probably not be an economic solution to the problem.

For the calculation of stresses in a vibrating structure, there is needed an assessment of the amplitude of vibration, and to obtain this a dynamic analysis must be performed. This, in addition to being a rather complicated operation, requires a detailed knowledge of the loading conditions and fairly precise data on the nature of wind turbulence are therefore necessary. Wind turbulence is a highly complex phenomenon which has to be dealt with on a statistical basis. Investigations during the past two decades into the spectral density of wind velocity,

notably by Van der Hoven[37], Panofsky, McCormick[28], Davenport[9] and Harris[19] have yielded promising results, however, and Davenport and Harris in particular have evolved empirical formulae from which the spectrum of wind turbulence at any site may be approximated if certain elementary parameters associated with ground roughness are known. On the basis of these formulae, Davenport has developed a method for predicting the statistical properties of the response of cantilever-like structures to wind turbulence, and has presented it in a form which is suitable for general application in design. This is the only method available at present, in a simplified form, for assessing the stability of a slender structure, which is likely to respond dynamically to the wind.

The present state of knowledge on the subject of wind loading of structures, after approximately a century of research is therefore as follows. Fairly comprehensive data is available on the static effect of wind and on the distribution of pressure over buildings. It is possible, as a result of this, to make fairly accurate predictions of the loads on building components such as cladding panels, and of the general overall stability of stiff structures which are unlikely to respond dynamically. The study of the dynamic effects of wind, however, is still in its infancy. The Davenport method, by which the stresses due to wind induced vibration may be estimated, uses rather a crude mathematical model for what is a highly complicated dynamic system. This was inevitable when one considers the gaps in the existing knowledge on wind turbulence and its interaction with vibrating structures, and also the necessity to keep the method simple enough for general use. The answers given by it probably constitute no more than a

very rough approximation to the real situation. There is, therefore, a need for further research on this topic, and the investigation presented here consists of a theoretical study of the parameters which affect a vibrating system and the relative importance of these so far as the wind loading problem is concerned.

CHAPTER 2

DESCRIPTION OF RELEVANT BACKGROUND MATERIAL

2.1 STRUCTURE OF THE WIND

Wind is the result of differential heating of the earth's surface by the sun which causes pressure gradients to be set up in the atmosphere. These give rise to air movement, and the velocity of the wind, at heights above approximately 1500 ft., where it is unaffected by the ground, is dependent on the magnitude of these pressure gradients. This air movement at heights large enough to be unaffected by the ground is called the gradient wind. Movement of the layers of atmosphere immediately adjacent to the ground is retarded by the friction associated with ground roughness. The mean velocity at ground level is, therefore, lower than the gradient wind velocity and it increases with height, up to the gradient velocity, which is reached at what is called the gradient height. The rate of increase in mean velocity with height, and the gradient height itself, are functions of the ground roughness. The mean velocity profile is generally taken to follow a simple power law given by,

$$Vz = \left(\frac{Z}{Zg}\right)^{\alpha'} Vg$$

where, V = mean velocity at height z
Vg = gradient velocity
Zg = gradient height
α' = coefficient dependent on ground roughness

The shearing effect between the ground and the air in contact with its surface, and between adjacent layers above the surface, gives rise to turbulence. The intensity of turbulence is defined as the ratio of the r.m.s. of the time varying component of velocity to the mean velocity and this too depends

on the ground roughness. In urban areas, where buildings generate large eddies, the intensity of turbulence is high, while over a smooth surface (open grassland for instance) where the surface drag is much less, the air stream near the ground is much more uniform and the intensity of turbulence less. The nature of the wind at a particular time and place, therefore, depends on the pressure differences in the atmosphere, which determine the gradient wind speed, and the topography and ground roughness surrounding the site in question, which determine the gradient height, mean velocity profile and turbulence characteristics. The pressure differences in the atmosphere are associated with the passage of weather systems and give rise to variations in wind speed which occur slowly over comparatively long periods of time, in the region of days. The fluctuations caused by ground roughness, however, are high frequency variations with periods of from five minutes to fractions of a second. The different effects caused by these two mechanisms is clearly illustrated if a spectrum* of wind velocity for a particular site is examined. The wind velocity spectrum is the breakdown of the time varying wind velocity function into frequency components as in a Fourier analysis. It may be found by converting the velocity function into its electrical analogue (e.g. by hot wire anemometer), passing this through a range of filters with different frequency characteristics and measuring the root mean square output at each frequency. The graph of root mean square output against frequency gives the spectrum of the signal.

* For explanation of spectrum see Appendix 1.

Fig. (1) shows a wind velocity spectrum obtained by Van der Hoven and it shows the maxima corresponding to the fluctuations caused by the basic mechanisms mentioned above. It is significant that no large fluctuation occurs with a period of between five hours and five mintues. This is thought to be due to the fact that no mechanism exists for generating turbulence in this frequency range. As a result of this gap in the spectrum it is possible to regard the wind as consisting of a mean velocity with turbulence superimposed on it. Long term variations are seen as movements of the mean with high frequency variations regarded as fluctuations about this mean.

In the design of structures to resist wind loads, the worst wind conditions which will occur in the vicinity of the structure in its lifetime are those which are of interest. These occur as a result of a high mean velocity and the superimposed turbulence caused by ground roughness. The estimation of the worst load on the structure is essentially a prediction of the future so the problem becomes a statistical one requiring the examination of past records.

In Britain we are fortunate in having a large number of meteorological stations throughout the country which keep a continuous record of the wind. The continuous records are split up into hour long portions and hourly means taken. The worst hourly mean on each day is recorded together with the highest gust speed. This is taken to be a three second gust as three seconds is the minimum response time of the equipment used. The length of record varies from sixty years to ten years approximately.

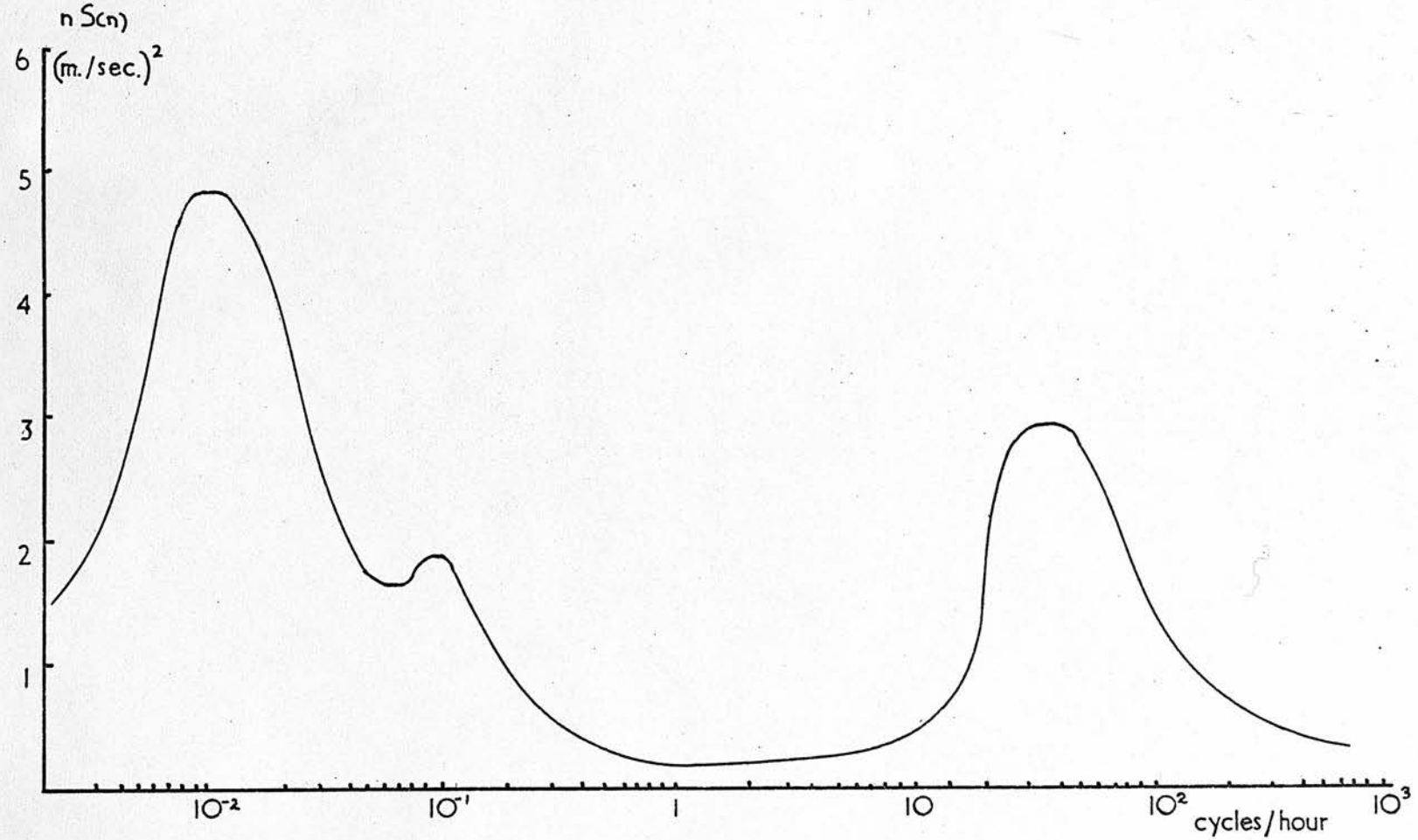


Fig. 1 Spectrum of horizontal wind speed, (Van der Hoven).

With such records at their disposal, the meteorological office are in a position to make fairly reliable predictions of the worst hourly mean wind speed and the worst three second gust speed which are likely to occur in a particular area for any return period. Predictions of velocities with averaging periods other than one hour or three seconds have to be interpolated from these results. Work on the relationship between the magnitudes of mean wind velocities for different averaging periods has been carried out by the meteorological office, notably by Shellard, and values for one minute, fifteen second and ten second periods, together with the likely spatial extent of such gusts are available. The British Code of Practice on wind loading is based on this 'worst gust' approach. Structures are designed to resist the worst single gust which is likely to blow over them in their lifetime, and this is considered as a static load. The fact that the wind is a dynamic form of load and may cause a structure to respond in a dynamic way is not taken into account. For the majority of stiff, low rise structures, this is probably a good enough approach to design but in the case of slender structures which respond dynamically to the wind a check on possible amplitude of vibration should be made.

In a dynamic analysis, the concern is not so much with the worst single gust to which a structure is likely to be subjected as with the worst sequence of gusts and the time varying properties of the wind load on a structure must be known if the amplitude of resulting vibrations are to be calculated. If a vibration analysis were to become part of a standard design procedure suitable for incorporation into a code of practice, a mathematical/

model of turbulence, which could be used to predict the characteristics of turbulence at different sites, would have to be available. Attempts have been made to provide this and work on the topic has been concentrated mainly on trying to provide a means of predicting the spectrum of turbulence at any site. Notable contributions in this field have been made by Davenport and Harris who have both produced empirical formulae for the spectrum of horizontal wind speed.

Davenport splits the wind velocity function at a site into its mean and turbulent components. The turbulence is considered as a fluctuation about zero mean and is regarded as a steady state phenomenon dependant on the mean velocity and the ground roughness. In a given mean wind speed, the roughness is considered to generate a particular size of eddy which is dispersed to smaller eddies, which are in turn dispersed, the energy finally being dissipated as heat. Davenport considers that the rate at which eddies are broken down and dispersed is the same as the rate at which new eddies are created. The proportion of large to small to smaller eddies in a particular batch of wind is therefore always the same. The size of an eddy, combined with the mean wind speed, determines the frequency of a particular component of turbulence. If the proportion of different sizes of eddy in the wind remains the same, the distribution of components of turbulence with respect to frequency will be constant and will not be a function of time. The spectral density of wind velocity will therefore also be constant. A site of particular roughness, should, therefore, always yield the same wind spectrum in the same mean wind speed and this should be similar to spectra from other sites with the/

same roughness characteristics.

To develop this theory, Davenport examined data from a large number of sites with different roughness and for different mean wind speeds. By operating on the spectra of turbulence at these various sites, with parameters pertaining to ground roughness and mean wind speed, he was able to fit all the spectra onto one curve called the reduced spectrum, and he also derived an empirical formula for this curve.

It is possible, with the use of this formula and appropriate roughness parameters, to predict the turbulence spectrum at any site for any mean wind speed, The availability of a reduced spectrum, therefore, enables a picture of the worst wind which is likely to occur at a particular site to be built up. The mean velocity of the worst hour of wind can be found from meteorological office data, and an idea of the nature of the turbulence during this hour, obtained from the reduced spectrum. This is of course a limited amount of information. The spectrum gives only the distribution with frequency of the various components of turbulence in the wind and it applies only to one point in space. It gives no indication of the spatial extent of any of the components of turbulence or of the way in which different components are related to one another in the time domain.

The size of a component of turbulence in relation to the size of a structure with which it is interacting is an important consideration so far as calculation of structural response is concerned. Some idea of the area over which a gust sequence is likely to be effective can be obtained if the cross-correlation properties of the wind with respect to frequency are/

known. The cross-correlation between two signals is a measure of the extent to which they are similar. Two identical signals would have a cross-correlation coefficient of one, while completely dissimilar signals would have a cross-correlation coefficient of zero. The extent to which the wind at two points in space is similar depends on the distance between the points and on the frequency of the component of turbulence which is being examined. High frequency turbulence, resulting from small eddies, has high correlation only over small areas while low frequency turbulence, which results from much larger eddies, is correlated highly over larger areas.

The cross-correlation coefficient between two points in space for a particular component of turbulence, therefore, depends on the eddy size of the turbulence. This may be related to the more easily measured parameters of frequency and mean wind velocity. By analysing data so as to determine the extent to which wind signals, recorded at points with different separation distances were correlated, Davenport evolved an empirical formula for the cross-correlation coefficient of wind turbulence as a function of separation distance, frequency, and mean wind speed. This may be used in conjunction with the turbulence spectrum to obtain the frontal area over which particular components of turbulence are likely to be effective.

The time varying properties of the wind function cannot be obtained from the spectrum, which gives only the statistical properties of the turbulence. By dealing with the basic parameters of turbulence, however, it does have the great advantage of enabling the the statistical properties of turbulence at any site at which/

these parameters are known, to be predicted. This is an essential requirement of any design method. Wind velocity spectra are the only data on wind loading, available at present, which give enough detail on the loading conditions for a dynamic analysis to be carried out. The information which can be obtained from such an analysis is, of course, restricted to the statistical properties of the response of a structure to turbulence. This is a limited amount of information but it is sufficient for the purposes of checking the suitability of a design. By dealing with turbulence on a statistical basis, along the lines shown by Davenport, it is possible to reduce the computations involved in a dynamic analysis to a manageable level and still obtain a useful result.

2.2 RESPONSE OF STRUCTURES TO DYNAMIC LOADING

When a structure is loaded it deflects. Work is done on it and it is therefore given energy in the form of strain energy. This is potential energy which is returned to the loading mechanism if the load is subsequently released. The speed with which the structure can be returned to its original position depends on the acceleration which the force due to its stiffness can impose on the mass of the structure. If the load is withdrawn faster than the recovery speed of the structure, the strain energy will not be returned to the forcing mechanism and will remain in the structure in the form of kinetic energy. In the absence of a dissipative mechanism this energy is not destroyed and through inertia is reconverted into strain energy by the deflection of the structure in the reverse direction to the original dis-

placement. An oscillatory system is therefore set up in which energy is continuously being converted and re-converted from the potential to the kinetic form. The frequency of the resulting motion depends on the mass and the stiffness of the structure and is therefore one of its fundamental properties. It is called the natural frequency.

If a structure is forced by a load varying sinusoidally at its natural frequency, energy is added at each cycle and the amplitude of vibration increases with each cycle. In the absence of any dissipative forces, such a system is theoretically capable of reaching an infinite amplitude. In practice, dissipative forces are always present and a steady state equilibrium is reached when the amplitude is such that the energy being supplied to the structure per cycle is exactly balanced by the energy being dissipated by damping forces. The amplitude reached by a structure which is being forced to vibrate at its natural frequency is therefore dependent on the amount of damping present. If a structure is subjected to a sinusoidally varying load with a frequency which is lower than its natural frequency the rate of unloading in each cycle is not so great that the strain energy cannot be returned to the forcing mechanism. Energy may therefore be continuously added to and subtracted from the system and no accumulation of energy occurs. The amplitude of the deflection is dependent on the relationship between the stiffness of the structure and the load, as in a static system. If a structure is forced at a frequency above its natural frequency, the inertia forces become so great that the amplitude of vibration is minimal.

It is possible to distinguish between two types of system which undergo time varying loading conditions therefore. One is the truly dynamic system where the structure is excited at its natural frequency, inertia forces play a significant role and the ultimate deflection depends on the damping. The other is a quasi-static system where, although the load is a function of time, the frequencies are low enough to make inertia forces insignificant resulting in the deflection being controlled by the stiffness of the structure. The deflection at any instant is directly related to the load at that instant.

All vibrating systems may be described in roughly the same way mathematically. The basic equation may be derived by considering the forced vibration of a simple spring-mass-dashpot system.

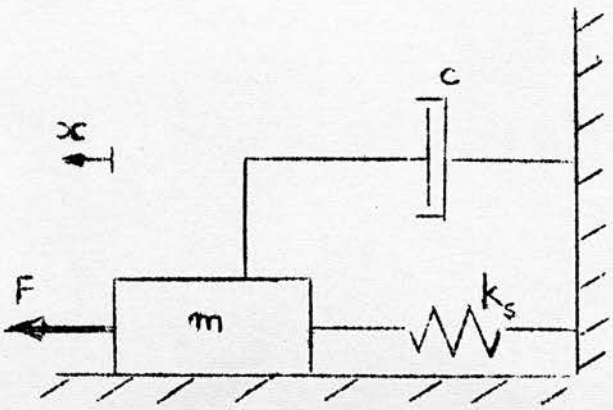


Fig. 2

Single degree of freedom system

The force exerted by the spring is proportional to the displacement, and by the dashpot to the velocity of the motion. The equation of motion is, therefore,

$$m\frac{d^2x}{dt^2} + c\frac{dx}{dt} + k_s x = P\,\mathrm{Cos}\,wt \qquad (2.1)$$

The complete solution to this equation consists of a complementary function, which describes the free vibration of the starting transient, and the particular integral which relates to the steady state forced vibration. In most mechanical vibration problems it is the steady state vibration which is of interest and the complementary function is usually neglected. The particular integral solution to equation (2.1) is,

$$x = \frac{P\,\mathrm{Cos}(wt - \phi)}{\left[(k_s - mw^2)^2 + c^2w^2\right]^{\frac{1}{2}}} \qquad (2.2)$$

$$\tan\phi = \frac{cw}{(k_s - mw^2)}$$

The substitution $w_o = \frac{k_s}{m}$ can be made where w_o is the natural frequency of the system in which case,

$$x = \frac{P\,\mathrm{Cos}(wt - \phi)}{k_s\left[\left(1 - \frac{w^2}{w_o^2}\right)^2 + \left(\frac{cw}{k}\right)^2\right]^{\frac{1}{2}}}$$

If $x_{st} = \frac{P}{k_s}$ is the displacement for a static force P, and X is the amplitude of the steady forced vibration ($x = X\,\mathrm{Cos}(wt - \phi)$) the magnification due to the load being applied dynamically is given by,

$$M_g = \frac{x}{x_{st}} = \frac{1}{\left[\left(1 - \frac{w^2}{w_o^2}\right)^2 + \left(\frac{cw}{k_s}\right)^2\right]^{\frac{1}{2}}}$$

M_g = magnification factor or frequency response function

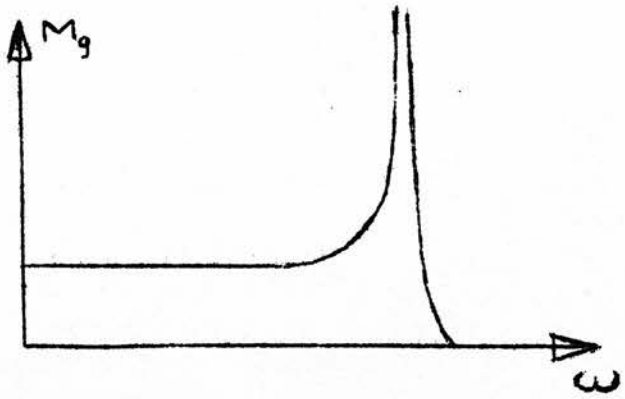


Fig. 3 Frequency response function for single degree of freedom system.

The graph shows M_g plotted against frequency and it illustrates the fundamental properties of a single degree of freedom, forced vibration system which are:-

1) That a high magnification occurs at a particular frequency ω_o which depends on the ratio of the mass to the stiffness of the structure.

2) That at frequencies below ω_o the deflection of the system is quasi-static, i.e., there is a distinct relationship between deflection and force at any instant. $x = \frac{P \cos(\omega t - \phi)}{k_s}$

3) That the amplitude of the vibration at the natural frequency ω_o depends on the damping value. $M_g(\omega_o) = \frac{k_s}{c\omega}$

Thus, the qualitative assessment of the behaviour of a vibrating structure, stated previously, can be completely/

described mathematically by the magnification factor.

Equation (2.1) describes the motion of a single degree of freedom vibration system and one displacement variable is sufficient for this purpose. A simple approximation to a tall building is a cantilever which is a multi-degree of freedom system. If vibration in one plane only is considered the motion of any part may be defined by the co-ordinate y (see fig. 4) which is a function of x and t. Both the natural frequency of the structure and the shape into which it deflects must be determined before the motion may be completely described.

Fig. 4

The strain energy at the point of maximum deflection depends on the shape into which the structure deflects, and the kinetic energy at the mean position on the natural frequency, so that a unique relationship between deflection shape and natural frequency, would be expected.

The following equation may be derived to describe the free vibration of a cantilever.

$$EI \frac{\partial^4 y}{\partial x^4} - M \frac{\partial^2 y}{\partial t^2} = 0 \qquad (2.3)$$

where:- E = Young's Modulus, I = Second Moment of Area of section, M = generalised mass $y(x, t)$ may be expressed as a product of a shape vector $f(x)$ and a time dependant function $g(t)$. If a vibrational solution is assumed as in the single degree of freedom case, $g(t)$ will be a trigonometric function and the substitution $g(t) = G_r \sin(w_r t)$ may be made where w_r is the natural frequency of the system. The equation then becomes,

$$EI \frac{d^4 f(x)}{dx^4} - M\omega_r^2 f(x) = 0 \qquad (2.4)$$

The solution to this equation contains five unknowns, four constants of integration and ω_r. As there are only four boundary conditions a complete solution is impossible and $y(x, t)$ can only be found in terms of an arbitrary constant. The general solution to equation (2.3) is,

$$y(x, t) = g(t) \quad f(x)$$

where,

$$f(x) = A \operatorname{Sin}\lambda x + B \operatorname{Cos} \lambda x + C \operatorname{Sinh} \lambda x + D \operatorname{Cosh} \lambda x$$

$$\lambda = \sqrt[4]{\frac{\rho A w_r^2}{EI}}$$

The eigenvalues, $(w_1, w_2, w_3, \ldots w_r)$ may be found by applying the boundary conditions to equation (2.4). Then, for a cantilever,

$$f_r(x) = F_r\left[\operatorname{Cosh} \lambda_r x - \operatorname{Cos}\lambda_r x - k_r(\operatorname{Sinh} \lambda_r x - \operatorname{Sin}\lambda_r x) \right]$$

where, $k_r = \dfrac{\operatorname{Cos}\lambda_r l + \operatorname{Cosh} \lambda_r l}{\operatorname{Sin}\lambda_r l + \operatorname{Sinh} \lambda_r l}$

$r = 1, 2, 3, \ldots n.$

F_r is an arbitrary constant. The modes are usually 'normalised' such that $\int_0^1 f_r^2(x)dx = 1$. A value for F_r is found from this equation.

The multi degree of freedom system, therefore, has many natural frequencies and for each natural frequency there is a specific shape of deflection called the mode shape. This unique relationship between ω_r and $f_r(x)$ results from the necessity for each deflected shape to be related to a specific frequency so that the maximum kinetic and strain energies of vibration are equal. It is possible to regard the vibration of the structure in each mode as a single degree of freedom system and the total motion of the structure as a superposition of these uncoupled single degree of freedom motions.

The forced vibration of a cantilever under the action of a distributed harmonic load P(x, t) may be dealt with by this 'normal mode' approach. If $P(x, t) = \sum_r p_r(t) f_r(x)$

and $y(x, t) = \sum_r g_r(t) f_r(x)$

the equation of motion (2.3) becomes,

$$EI\sum_r g_r(t) \frac{d^4 f_r}{dx^4} + M\sum_r \frac{d^2 g_r}{dt^2} f_r(x) = \sum_r p_r(t) f_r(x) \qquad (2.5)$$

This may be written as an infinite set of uncoupled equations,

$$EI\, g_r(t) \frac{d^4 f_r}{dx^4} + M \frac{d^2 g_r}{dt^2} = P_r(t) f_r(x)$$

$$EI\, g_r(t) \lambda^4 f_r(x) + M \frac{d^2 g}{dt^2} f_r(x) = p_r(t) f_r(x)$$

$$\frac{d^2 g_r}{dt^2} + \frac{EI}{M} \lambda^4 g_r(t) = \frac{1}{M} p_r(t)$$

Assuming viscous damping of magnitude $c\, g_r'(t)$ this becomes,

$$\frac{d^2 g_r}{dt^2} + \frac{c}{M}\frac{dg_r}{dt} + \frac{EI}{M}\lambda^4 g_r(t) = \frac{1}{M} p_r(t) \qquad (2.6)$$

Equation (2.6) has the same solution as equation (2.1). The motion of a multi-degree of freedom system such as a cantilever is therefore given by,

$$y(x, t) = \sum_r f_r(x)\, g_r(t) \qquad (2.7)$$

where $$g_r(t) = \frac{p_r(t)\cos(\omega t)}{M\left[\left(I - \frac{\omega^2}{\omega_r^2}\right)^2 + \left(\frac{c\omega}{EI}\right)^2\right]^{\frac{1}{2}}}$$

The response characteristics of a multi degree of freedom system are evident if $\sum g_r(t)$ is plotted against frequency as in Fig(5).

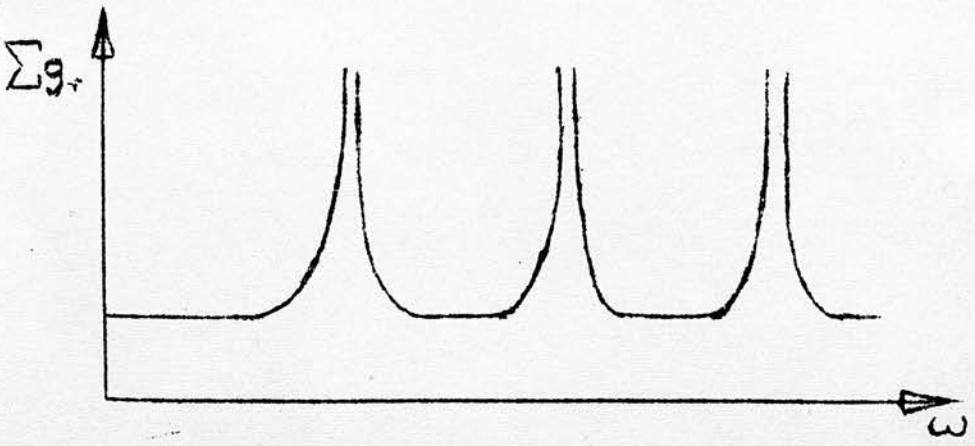


Fig. 5 Frequency response function for multi degree of freedom system

As with the single degree of freedom case the height of the resonance peaks depends on the value of c. Excitation at

frequencies above about 5 Hz is very small in the case of wind loading and if the second mode natural frequencies for cantilever structures are quite high ($>$10 Hz), it is usual to consider the first mode only in wind load calculations.

Summary of response characteristics of slender structures

For the purposes of assessing the effect of wind loading it is possible to split the response of a slender structure into three parts:-

1) The static response which results from the static effect of the mean wind. The magnitude of this is proportional to the stiffness of the structure.

2) The quasi-static response. This results from all the time varying loads including harmonic loads, which do not have the same frequency as the natural frequency of the structure. The deflection at any instant is proportional to the load at that instant and depends on the stiffness of the structure.

3) The dynamic response. This is periodic and occurs at the natural frequency of the structure. It is due to the component of the load which occurs at the natural frequency and its magnitude, once steady state conditions have been reached, depends on the amount of damping in the structure.

Fig. (6) shows a record of the deflection of a 30 ft. high lattice tower (natural frequency 3.3 Hz) with a simultaneous wind recording. The trace is one of many results of an experiment to measure the response of a slender structure to the wind, which was carried out at Edinburgh University by/

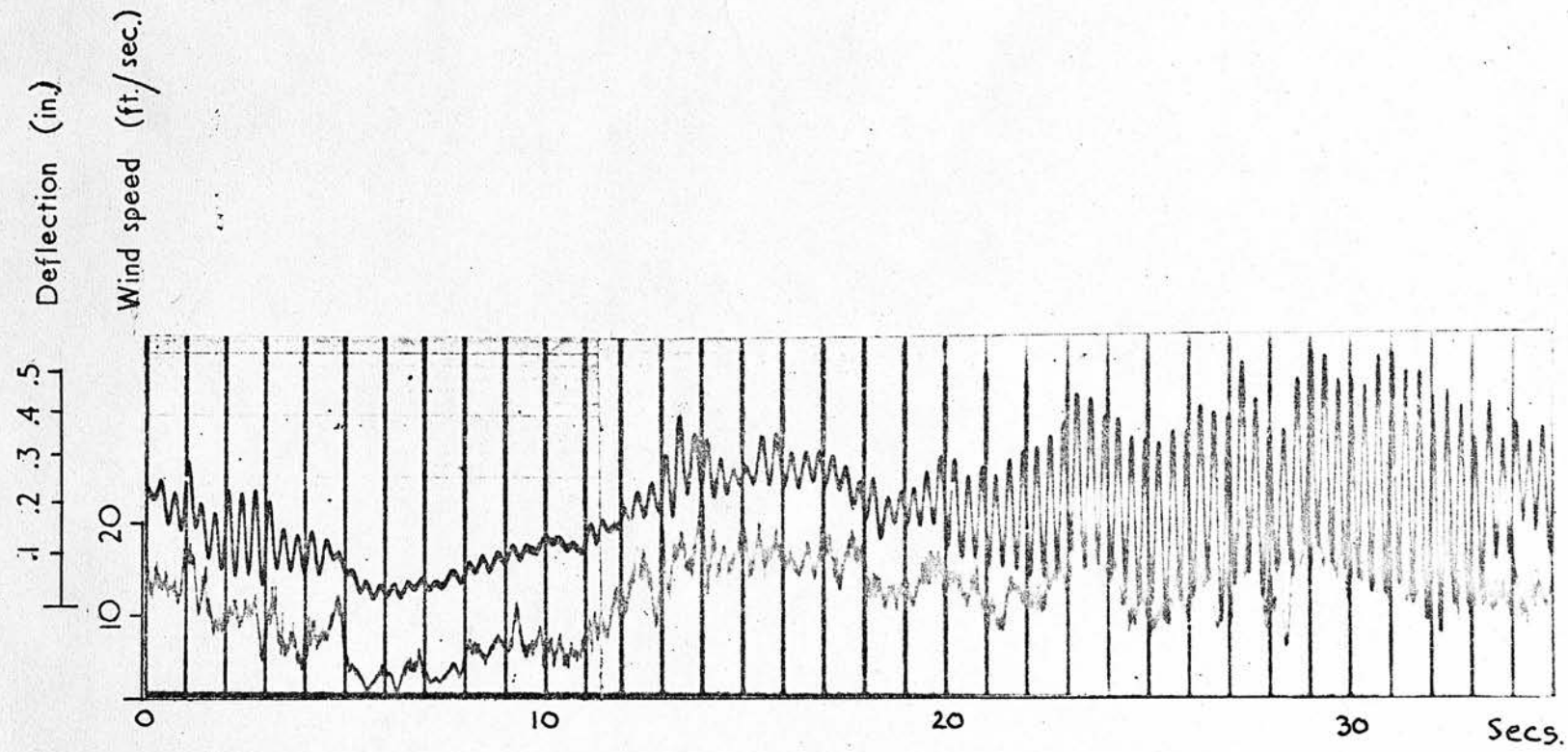


Fig. 6. Deflection of 30 ft. high lattice tower with simultaneous wind record.

M.J. Dareau(see ref. 6) and it illustrates the three types of response mentioned above. The graph shows the mean wind, mean deflection, the good correlation between the load variations and the quasi-static response, and the dynamic response superimposed on this.

2.3 DESIGN METHODS

The object of a design method is to predict the worst loading condition which will occur during the lifetime of a structure and determine the stresses which will result from this. As wind loading is a natural phenomenon statistical techniques have to be applied to existing records in order to achieve this.

There are two approaches to the problem. One is to apply extreme value statistics to existing wind records in order to define the worst single gust to which a structure is likely to be subjected in its lifetime. This is then used as a static load and an accurate conventional analysis performed to evaluate deflections and stresses. The advantage of this method, which has been adopted for the new Code of Practice, is that it uses existing techniques of static analysis which are known to be reliable. It suffers from the disadvantage, however, that it deals solely with the static and quasi-static components of the response of a structure and makes no allowance for any additional deflection which might occur due to resonance. Its validity, therefore, depends on whether or not a particular structure undergoes its state of maximum distress as a result of the action of one single gust or the combined effect of a sequence of gusts.

The second approach to the wind loading problem is to postpone the use of extreme value statistics until after the analysis of the structural response has been carried out. The overall behaviour of a structure in response to all components of turbulence in the worst average conditions (i.e. the worst hour of wind) is first assessed. Once the characteristics of the worst average deflection are known, extreme value statistics are then used to predict the most likely maximum deflection. The use of this method eliminates the need to specify exactly the worst gust or sequence of gusts to which a structure is likely to be subjected.

A general assessment of the response of a structure to turbulence however, inevitably involves some sort of dynamic analysis. The second of the two methods, therefore, which must be able to deal with static, quasi-static and dynamic components of deflection, uses techniques of analysis which are comparatively new so far as wind loading problems are concerned and their reliability is still not proven.

The spectral method of Davenport is based on the second of these two approaches. It is the only method available at present which takes account of resonant vibration, and which is presented in a form which could be used by a design engineer. A brief outline of the method, which is fully described in references (9), (10) and (14) will be given here.

Davenport realised that a rigorous dynamic analysis could not be contemplated as part of a standard design procedure and has attempted to formulate a simple method for determining the/

ratio of maximum dynamic to static deflection due to wind. This is then used in conjunction with a conventional static analysis to get the maximum stresses due to the combined effects of static and dynamic loading. The method is presented in the form of a set of design curves from which the required ratios of dynamic to static loading (called 'gust factors') may be derived.

Davenport works from the worst mean wind condition which is likely to occur in the lifetime of a structure. In Britain, the available data limits this to the worst hourly mean. He therefore deals with the worst hour of wind which a structure will be called upon to withstand. The problem is to assess the additional deflection which will occur due to turbulence in this hour. Davenport split the wind velocity function into its mean and turbulent components. The turbulence is regarded as a fluctuation about zero mean.

$$V(t) = \bar{V} + v(t)$$

where $\bar{V}$ = mean wind speed

$v(t)$ = turbulence comp.

The deflection of the structure is also regarded as the sum of mean and dynamic components.

$$Y(t) = \bar{Y} + y(t)$$

where $\bar{Y}$ = mean deflection

$y(t)$ = dynamic component of deflection

It is the most probable maximum deflection rather than the time varying properties of the deflection which are required and Davenport derives this from the equation,

$$Y_{max} = \bar{Y} + k\sigma_y$$

where Y_{max} = most probable maximum deflection

$\bar{Y}$ = mean deflection (worst hour)

where σ_y = r.m.s. deflection during worst hour.

k_c = a statistical coefficient

$$Y_{max} = \bar{Y}\left(1 + \frac{k\sigma_y}{\bar{Y}}\right) = \bar{Y}.G \qquad (2.8)$$

$$G = 1 + \frac{k_c\sigma_y}{\bar{Y}}$$

where G is the gust factor.

To evaluate the gust factor it is necessary to find values for k_c and $\frac{\sigma_y}{\bar{Y}}$. k_c is called the 'peak factor' by Davenport and is given by the equation,

$$k_c = \sqrt{2 \ln \nu T} + \frac{0.57}{\sqrt{2 \ln \nu T}} \quad *$$

where, $\nu = \frac{\omega_o}{2\pi}$ T = averaging period of $\bar{V}$

The information required about the time varying part of the response (i.e. the combined quasi-static and dynamic components) is its root mean square or variance. This may be obtained from the spectrum of response, which, as is explained in appendix (1), is a measure of the frequency distribution of the various components of the response.

$$\sigma_y^2 = \int_0^\infty S_y(n)\, dn \qquad (2.9)$$

The distribution of the load components with respect to frequency is given by the wind velocity spectrum which may be obtained for any site and any mean wind speed from Davenport's/

* Derivation of k_c given in reference (11).

empirical formula. The response characteristics of a structure are given as a function of frequency by the frequency response function. The response spectrum may therefore be evaluated from these functions as in equation (2.10).

$$\frac{S_y(n)}{\bar{V}^2} = 4 X_a^2 \, X_m^2 \, \frac{S_v(n)}{\bar{V}^2} \qquad (2.10)$$

where,

$S_y(n)$ = deflection spectrum

X_a^2 = 'aerodynamic admittance' - this function is a measure of the frequency distribution of the drag properties of the structure. The following formula for it is suggested by Davenport.

$$X_a^2 = \frac{C_D^2(\xi) + \frac{\pi^4}{4} \xi^2 C_m^2(\xi)}{C_D^2(0)}$$

where,

C_D = drag coefficient

C_m = virtual mass coefficient

$\xi = \frac{nD}{\bar{V}}$

D = diameter of object

X_m^2 = 'mechanical admittance' - this is the square of the frequency response function.

$S_v(n)$ = The wind velocity spectrum, given by,

$$S(n) = \frac{4k\bar{V}x^2}{n(1 + x^2)^{4/3}}$$

$$x = \frac{4000\, n_o}{\bar{V}} \qquad k = \text{surface drag coef.}$$

Equation (2.10) is applicable to a single degree of freedom structure which occupies a point in space. It is therefore not a very practical equation but may be used as an approx-

imation for such structures as floodlighting towers, where most of the drag is concentrated in one place, and only the motion in the first mode is of importance. For more complex structures such as tall buildings a more sophisticated equation is required. As only the first mode of vibration is considered significant for most wind loading problems the single degree of freedom 'mechanical admittance' is considered adequate for use with multi degree of freedom structures. A more complicated 'aerodynamic admittance' is used however to allow for the variation in the spatial extent of gusts with frequency and the size of the various components of turbulence in relation to the size of the structure under consideration. The derivation of this function is given in Chapter 3 and it will only be quoted here. For multi degree of freedom structures,

$$\frac{S_y(y,n)}{Y} = X_m^2 \cdot J^2 \cdot \frac{\dot{S}_v(n)}{V} \qquad \ldots (2.11)$$

$$J^2 = \frac{1}{N_r^2} \int_0^{\ell}\int_0^{\ell} \exp\left(-\frac{C_1 |x-x'| n}{V}\right) \cdot f_r(x)\, f_r(x')\, dx\, dx' \qquad \ldots (2.12)$$

$$N_r = \int_0^{\ell} f_r^2(x)\, dx$$

$f_r(x)$ = mode shape

C_1 = constant dependant on ground roughness

The Davenport method, therefore, provides a means of assessing the behaviour of a structure in the wind, which takes all components of deflection into account. The use of spectral analysis enables this to be done from the very limited data which are available on the characteristics of wind tur-

bulence. The mathematical model used to describe a vibrating system, however, is highly simplified so as to be applicable to a wide range of structures and the method is probably only capable of giving a rough indication of the possible extent of the dynamic response of any particular structure. It cannot be considered as a rigorous design method.

2.4 APPRAISAL OF THE PRESENT SITUATION

Of the two approaches to the wind loading problem outlined in the preceding section, the one which takes account of all mechanisms of deflection must be considered fundamentally the better. The ultimate solution is a reliable design method which takes dynamic response into account and which could be applied to all structures as a standard procedure. This is not feasible at present due to lack of data on wind turbulence and to the difficulty of reducing the dynamic part of the analysis to a simple procedure capable of general application. The current Code of Practice on wind loading therefore makes allowance for only the static and quasi-static components of deflection. There is no doubt, however, that some structures do respond dynamically to wind loads and that in certain cases the amplitude of resonant vibration of a structure constitutes a significant part of its total deflection. Such structures should be subjected to some form of dynamic analysis at the design stage if their subsequent performance is not to be unsatisfactory or even dangerous. The rigorous dynamic analysis of structures is without the scope of general civil engineering practice and there is,/

in the absence of a general design method, a need for a simplified procedure for predicting the response of slender structures to dynamic wind loads. It is felt that the provision of the following might fulfil this requirement:-

1. A simple test which would help a designer to decide whether a structure was likely to respond dynamically to wind turbulence or not. Structures which were found to be safe against a vibratory response could then be analysed for static and quasi-static loads only, as prescribed by the Code of Practice.
2. A procedure for assessing the additional deflection which might occur in a structure due to resonant vibration. This would be carried out in conjunction with the static and quasi-static analysis and would only be applied to structures which were thought to be liable to have a dynamic response.

Before either of these facilities can be provided much more will have to be known about the parameters upon which the behaviour of wind excited vibrating systems depend. Much of this information will ultimately have to be obtained experimentally but the present state of knowledge is such that it is difficult to know along which lines an experimental investigation should proceed. The scope of an experiment is bound to be limited and once the general pattern has been set it is often difficult to alter.

It was felt that at this stage much useful information could be obtained from a theoretical study, provided its limitations were recognised, and it was decided to try and simulate the/

process of wind excited oscillation of structures on a digital computer. It was thought that if a suitable mathematical model were chosen it would be possible to vary important parameters over wide ranges and also to maintain great flexibility. The object was to gain an insight into the relative importance of the various parameters in the wind excited system in the hope that this would provide an indication of the lines along which any further experimental investigation should proceed.

CHAPTER 3

INVESTIGATION OF THE MAIN PARAMETERS IN WIND EXCITED OSCILLATION SYSTEMS USING SPECTRAL ANALYSIS

3.1 INTRODUCTION.

The preliminary investigation consisted of an attempt to simulate the behaviour of a wind driven vibrating system on a digital computer. The object was to study the effects of variations in the system parameters on the predicted overall behaviour of a slender structure vibrating in response to wind turbulence. Of particular interest was the ratio of the dynamic to the quasi-static components of response. This was considered to be a good indication of the response characteristics of a structure to wind loads and it was also regarded as a measure of the difficulty which the structure is likely to present so far as wind loading calculations are concerned because it is the calculation of the dynamic part of the response which presents the greatest difficulty to the designer of a structure.
Most of the data currently available on the characteristics of wind turbulence are in the form of wind spectra. Any mathematical simulation of a wind driven vibrating system must therefore be based on a mathematical model of the wind which can be built up from the information obtainable from a spectrum. The investigator is therefore forced to assume that the wind velocity function is a stochastic variable with constant statistical properties. If the deflection function of the structure is also assumed/

also assumed to be stochastic a deflection spectrum may be found from which statistical information concerning the deflection of the structure may be obtained. The area under the deflection spectrum gives the root mean square deflection and the contribution of any component of deflection with a specific bandwidth may also be obtained. It is therefore possible to calculate the ratio of the resonant component of response to the total broad band response and hence find the extent to which the dynamic part of the response of the structure contributes to the total time varying response.

Spectrum analysis was therefore adopted as the best technique for conducting the preliminary investigation. The Davenport reduced spectrum was used as the mathematical model of the wind and transfer functions similar to those formulated by Davenport were derived and used to convert from the wind velocity spectrum to the pressure and deflection spectra of the various structures analysed. The quasi-static component of the deflection of each structure was found by direct area measurement of the spectrum, as suggested by Davenport. The area of the resonance peak was found from the formula,

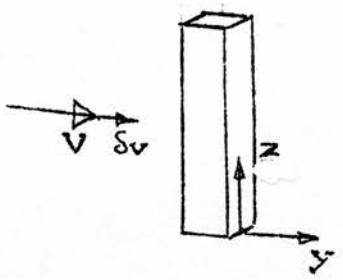
$$\text{Area of peak} = \frac{\pi^2}{2\delta} \times \text{ordinate of the force spectrum at the resonant frequency}$$

where δ = the logarithmic damping decrement

3.2 DEVELOPMENT OF TRANSFER FUNCTIONS

Aerodynamic admittance.

A rectangular structure was assumed



$V_{(z)}$ = mean wind velocity
$\delta v_{(z)}$ = fluctuation in velocity
ρ = air density
C = coefficient of drag
$P_{(z)}$ = mean pressure
$\delta p_{(z)}$ = fluctuation in pressure

Fig. 7

$$P(z) = 1/2 \rho C V^2_{(z)}$$

If the fluctuation in pressure is considered to occur about $V_{(z)}$, then,

$$\delta p(z,t) = \rho C V(z) \delta v(z,t)$$

$$\delta p(z,t) = \tfrac{1}{2} \rho V^2(z) \left[2C \frac{\delta v(z,t)}{V(z)} \right] \quad \ldots(3.1)$$

A 'normal mode' analysis was used to evaluate the structural response. This necessitated the splitting of the pressure function into modal components. The pressure over the whole structure at any time is a function of z and t. It may be subdivided into modal components and represented as a series:-

$$P(z,t) = p_1(t)f_1(z) + p_2(t)f_2(z) + \ldots p_r(t)f_r(z) \quad \ldots\ldots(3.2)$$

where $f_r(z)$ are the normal modes of the structure.

The components $p_r(t)$ may be evaluated as follows. If (3.2) is multiplied by $f_r(z)$ and integrated with respect to z from 0 to ℓ (where ℓ is the height of the structure), it becomes:-

$$\int_0^\ell P(z,t) f_r(z) dx = \int_0^\ell p_1(t) f_1(z) f_r(z) dz + \ldots \int_0^\ell p_r(t) f_r^2(z) dz \quad \ldots\ldots(3.3)$$

Due/

Due to the orthogonal properties of the mode functions,

$$\int_0^{\ell} f_l(z) f_m(z)\,dz = 0 \text{ where } l \neq m$$

$$= N_r \text{ " } l = m$$

$$\int_0^{\ell} P(z,t) f_r(z)\,dz = p_r(t) \int_0^{\ell} f_r^{\,2}(z)\,dz = p(t) N_r$$

$$p_r(t) = \frac{1}{N_r} \int_0^{\ell} P(z,t) f_r(z)\,dz \qquad \ldots(3.4)$$

The load component causing excitation in each mode is given by (3.4). The mean square of each load component with respect to time is given by,

$$\overline{\delta p_r^{\,2}(t)} = \frac{1}{N_r^{\,2}} \int_0^{\ell}\int_0^{\ell} \overline{\delta P(z,t)\,\delta P(z',t)}\, f_r(z)\, f_r(z')\,dz\,dz' \qquad \ldots(3.5)$$

where the bar denotes a time average. The mean square of the fluctuating part of the load only is given by,

$$\overline{\delta p_r^{\,2}} = \frac{1}{N_r^{\,2}} \int_0^{\ell}\int_0^{\ell} \overline{\delta p(z,t)\,\delta p(z',t)}\, f_r(z) f_r(z')\,dz\,dz' \qquad \ldots(3.6)$$

$$\overline{\delta P_r^{\,2}} = \frac{1}{N_r} \int_0^{\ell}\int_0^{\ell} \left[\tfrac{1}{2}\rho V(z)\left\{2C\frac{\delta v_z}{V(z)}\right\}\right]\left[\tfrac{1}{2}\rho V(z')\left\{2C\frac{\delta v_{z'}}{V(z')}\right\}\right] f_r(z) f_r(z')\,dz\,dz'$$

$$= \frac{4\left[\frac{1}{2}\rho C V_0^{\,2}\right]^2}{N_r^2} \int_0^{\ell}\int_0^{\ell} \frac{1}{V_0^{\,2}} \frac{V(z)\,V(z')}{V_0^{\,2}}\, \delta v_z\, \delta v_{z'}\, f_r(z) f_r(z')\,dz\,dz'$$

where V_0 is the mean velocity at a reference height.

$$= \frac{4\,P_0^{\,2}}{N_r^{\,2} V_0^{\,2}} \int_0^{\ell}\int_0^{\ell} \frac{V(z)}{V_0} \frac{V(z')}{V_0}\, \delta\bar{v}_{(z)}\, \delta\bar{v}_{(z')}\, f_r(z) f_r(z')\,dz\,dz' \qquad \ldots(3.7)$$

If the force $\delta p_r(t)$ is assumed to be a stationary random

function of time the above equation may be written in terms of spectra as follows:-

$$nS_{Pr}(n) = \frac{4 P_o^2}{N_r^2 V_o^2} \int_0^l \int_0^l \frac{V(z)}{V_o} \frac{V(z')}{V_o} \, nS_v(z,z';n) f_r(z) f_r(z') dz dz' \quad ..(3.8)$$

where $S_v(z,z';n)$ is the cross spectrum of the velocity function at z and z'.

$$nS_v(z,z';n) = nS_{v_o}(n).R(z,z';n) \quad ...(3.9)$$

where $S_{v_o}(n)$ is the spectrum of velocity fluctuation at the reference point on the structure and $R(z,z';n)$ the normalised cross spectral density function.

$$n S_{P_r}(n) = \left[\frac{2P_o}{V_o}\right]^2 \frac{1}{N_r^2} n S_{v_o}(n) \int_0^l \int_0^l \frac{V(z)}{V_o} \frac{V(z')}{V_o} R(z,z',n) f(z) f(z') dz dz' \quad (3.10)$$

$$n S_{P_r}(n) = \left[\frac{2P_o}{V_o}\right]^2 J_r(n) \, n S_{v_o}(n). \quad (3.11)$$

where $J_r(n) = \frac{1}{N_r^2} \int_0^l \int_0^l \frac{V(z)}{V_o} \frac{V(z')}{V} R(z,z',n) f(z) f(z') dz dz' \quad (3.12)$

and is called the 'joint acceptance function'.

$nS_{Pr}(n)$ = the spectrum of the r th modal load component.

$nS_{v_o}(n)$ = the spectrum of wind velocity at a reference point on the structure

$$= \frac{4k V_o x^2}{(1 + x^2)^{4/3}}$$

$$x = \frac{4000n}{V_o}$$

k = ground roughness coefficient.

the shape of the wind velocity spectrum can be seen in Fig.10.

The 'joint acceptance' is a frequency dependent function which converts the velocity spectrum into any of the modal pressure spectra. The variation with frequency is due to different degrees of correlation between different frequency components of the load. The function also makes allowance for variations in the characteristics of turbulence at different sites. This property may be seen if the constituents of the function are examined. The function $R(z,z';n)$ has been studied by Davenport who has suggested the following formula.

$$R(z,z';n) = e^{-k_c \frac{\Delta z n}{V_o}} \qquad \ldots(3.14)$$

where k_c = a coefficient dependent on ground roughness

n = frequency

$\Delta z = |z - z'|$

The velocity ratios $\frac{V(z)}{V_o}$ and $\frac{V(z')}{V_o}$ may be found from the power law equation $\frac{V(z)}{V_o} = \left(\frac{z'}{z_o}\right)^{\alpha'}$ where α' is a coefficient dependent on ground roughness. Thus, both the $R(z,z';n)$ function and the velocity ratio constituents of the 'joint acceptance' function are dependent on ground roughness coefficients.

Mechanical/

Mechanical admittance.

It was assumed that there would be no interaction between the modes of vibration and that each mode could be considered as a single degree of freedom system. The motion of the structure in each mode is therefore described by the equation:-

$$m_r \frac{d^2y}{dt^2} + c_r \frac{dy}{dt} + k_r y = p_r(t) \qquad \ldots(3.15)$$

The 'particular integral' solution to this equation gives the response characteristics of the mode as a function of frequency.

$$y_r = \frac{p_r(t)}{k_r} \cdot \frac{f_r(z)}{\left[\left\{1 - \left(\frac{n}{n_r}\right)^2\right\}^2 + \left(\frac{2\pi n c_r}{k_r}\right)^2\right]^{1/2}} \qquad \ldots(3.16)$$

This gives the well known frequency response function,

$$\frac{1}{k_r}\left[\left\{1 - \left(\frac{n}{n_r}\right)^2\right\}^2 + \left(\frac{2\,nc_r}{k_r}\right)^2\right]^{-1/2}$$

where, k_r = generalised stiffness = $EI\int_0^{\ell} f_r^{\,2}\,(dz)$

c_r = generalised damping coefficient

m_r = generalised mass = $\int_0^{\ell} \rho A f_r^{\,2}\,(dz)$

The mechanical admittance is the square of the frequency response function.

$$\left|X_r\right|^2 = \left[\left\{1 - \left(\frac{n}{n_r}\right)^2\right\}^2 + \left(\frac{2\pi n c_r}{k_r}\right)^2\right]^{-1} \qquad \ldots(3.17)$$

$$S_{y_r}(n) = \frac{\left|X_r\right|^2}{k_r^2} S_{p_r}(n) \qquad \ldots(3.18)$$

3.3 COMPUTER ANALYSIS.*

Table (1) shows the properties of the structures used in the spectral analysis. They were selected to give a range of natural frequency and mass per unit length. As the object of the analysis was to investigate the relationship between the resonant and quasi-static components of deflection for each structure it was felt that a range of drag properties was not required. The flow diagram for the computer program is shown in fig.(9) along with the relevant functions for each step in the calculation.

Except where specifically mentioned the input spectrum was kept the same for all structures. The input spectrum parameters were chosen to be relevant to a high wind condition in a city centre. They were,

V_o = 100 ft/sec

k = 0.05

α' = 0.41

L = 4000 ft

P_o = 12 lb/ft^2

3.4 RESULTS

The deflection spectra obtained from the analysis are shown in graphical form. As the mode functions were normalised with respect to an arbitrary constant it was not possible to obtain the actual magnitudes of the deflections/

* See over

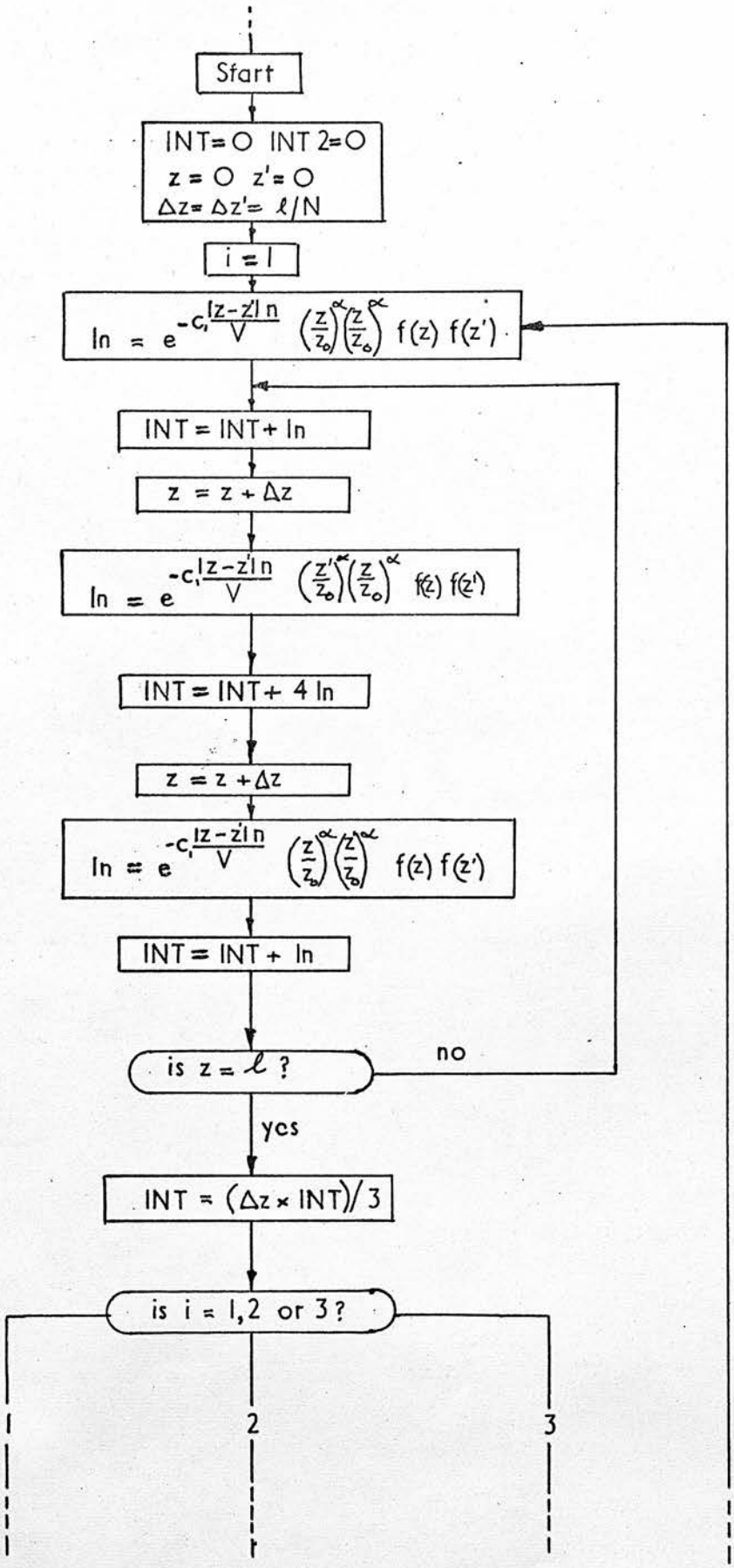
*

The computer simulation was carried out in order that a comparison could be made between the relative magnitudes of the resonant to quasi-static ratios of response of the structures concerned. As this could be obtained from the ratio of the areas under the relevant parts of the response spectra, which depended on the shapes of the spectra only, it was considered unnecessary to ensure that the magnitudes of the spectra were correct. To simplify the analysis, therefore, the constant k_y, which does not affect the shape of a response spectrum, was omitted from equation (3.18), and the response spectra found from the equation,

$$S_y(n) = |X_r|^2 S_{p_r}(n).$$

Type of Structure	B in	I in^4	m lb/in	l ft	E lbf/in^2	n_0 Hz
Lattice Tower T1	120	30·2	1·58	30	30×10^6	0·106
" " T2	120	2455 ·0	2·36	30	30×10^6	0·76
" " T3	120	12142·0	2·36	30	30×10^6	1·71
" " T4	12	12142·0	0·72	30	30×10^6	9·58
" " T5	120	$0·7 \times 10^6$	6·30	100	30×10^6	0·72
" " T6	120	$0·2 \times 10^6$	4·00	200	30×10^6	0·12
" " T7	120	$0·5 \times 10^6$	4·90	200	30×10^6	0·18
" " T8	120	$7·0 \times 10^6$	3·51	200	30×10^6	1·76
" " T9	120	$8·2 \times 10^6$	3·51	200	30×10^6	2·54
Shear wall building B1	120	155×10^6	220	100	4×10^6	0·65
" B2	120	155×10^6	445	200	4×10^6	0·11
Concrete Chimeny B3	120	155×10^6	10	200	4×10^6	1·76

TABLE 1 Properties of structures used in spectrum analysis.


Start
INT=0 INT 2=0
z = 0 z'= 0
Δz= Δz'= ℓ/N
i = 1
$In = e^{-c_1 \frac{|z-z'|n}{V}} \left(\frac{z}{z_0}\right)^{\alpha}\left(\frac{z}{z_0}\right)^{\alpha} f(z)\, f(z')$
INT = INT+ In
z = z + Δz
$In = e^{-c_1 \frac{|z-z'|n}{V}} \left(\frac{z'}{z_0}\right)^{\alpha}\left(\frac{z}{z_0}\right)^{\alpha} f(z)\, f(z')$
INT = INT+ 4 In
z = z + Δz
$In = e^{-c_1 \frac{|z-z'|n}{V}} \left(\frac{z}{z_0}\right)^{\alpha}\left(\frac{z}{z_0}\right)^{\alpha} f(z)\, f(z')$
INT = INT + In
is z = ℓ ?
no
yes
INT = (Δz × INT)/3
is i = 1, 2 or 3 ?
1
2
3

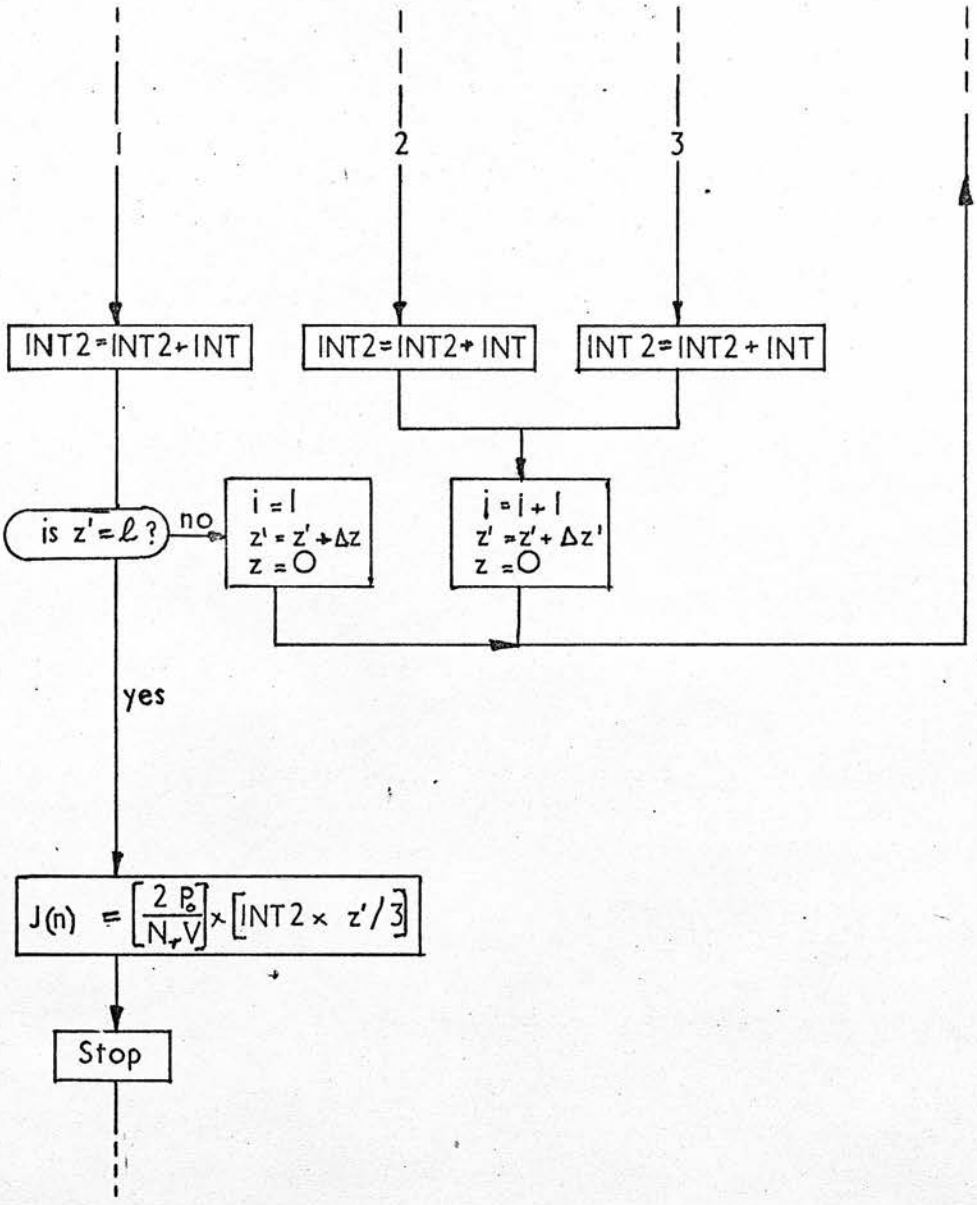


Fig 8. Flow chart for routine, used in spectrum analysis programme, to calculate J(n).

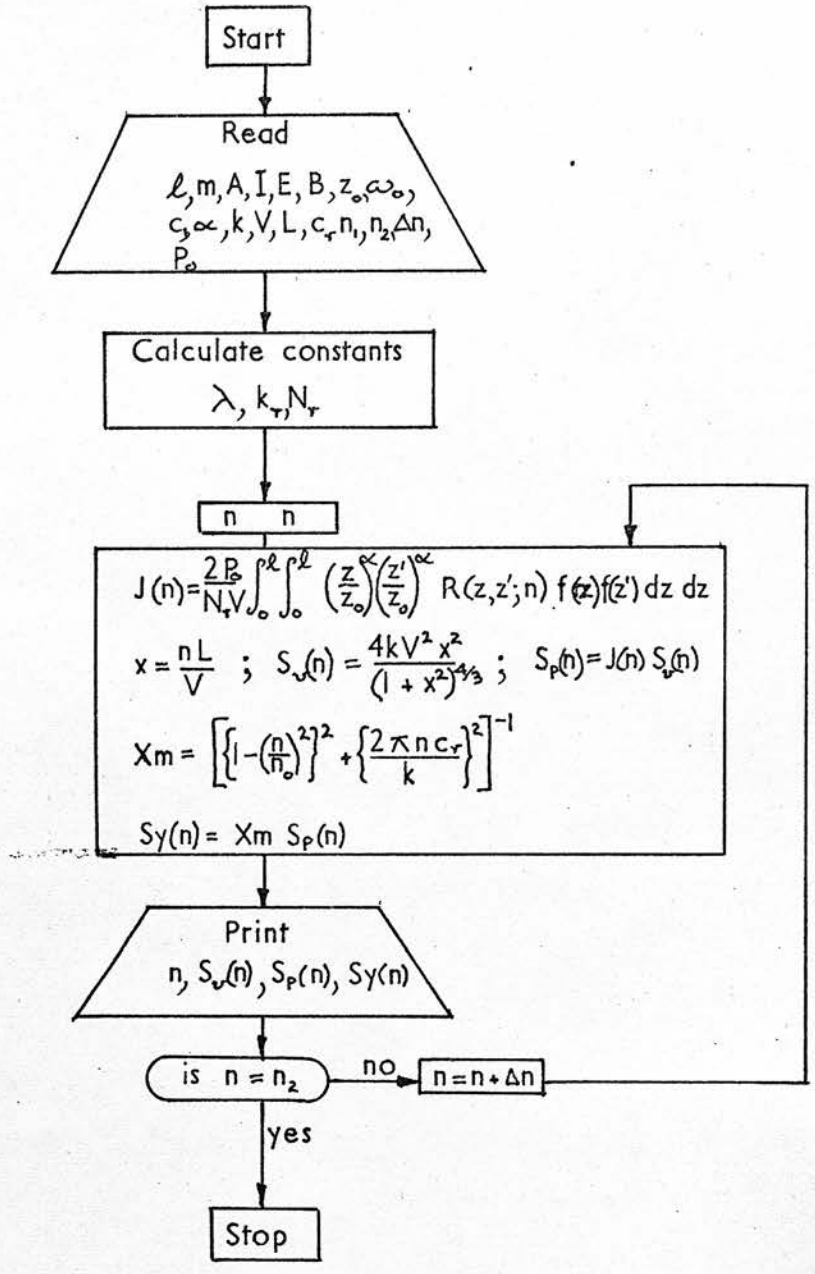


Fig 9. Flow chart for spectrum analysis programme.

deflections of the structures from the response spectra shown in graphs 1 to 12. Neither was it possible to make direct comparisons between the spectra. The ratio of the resonant to the quasi-static deflection for each structure was, however, given by the ratio of the area under the resonance peak to the area under the background turbulence part of each spectrum. Graph 13 is a chart which shows the relationship between this ratio and the natural frequency for each structure. The effects of the various system parameters on the dynamic behaviour of the structures in the wind may be deduced from this chart.

Structural Parameters

1. Natural Frequency

Graph 13 shows that the ratio of resonant to quasi-static deflection is highly sensitive to variations in the natural frequency parameter and that the predicted dynamic component of response increases as the natural frequency decreases. This effect may be attributed almost entirely to the shape of the excitation spectrum which has a peak at around 0.05Hz and diminishes rapidly with rising frequency.

The predicted ratio for structures with a natural frequency around 1Hz (typical of most slender structures) suggests that for such structures the resonant component of deflection may be of the same order or larger than the quasi-static component.

2. Damping

As would be expected, the predicted dynamic response of the structures/

structures was very much dependant on the degree of damping. For values of 1% critical damping or lower, which is typical of most slender structures, the analysis predicted that the dynamic component of response constitutes a significant proportion of the overall time varying response for all the structures analysed.

3. Height of Structure.

For structures with similar natural frequencies a slight variation in the dynamic/quasi-static ratio of response was observed with variation in the height of structure. Smaller structures tended to have a higher component of response at the resonant frequency. The effect was due to the cross-correlation coefficient function which allows for the fact that the higher frequency gusts have greater influence on smaller structures. The effect of this function was very small, however, compared to that of the previous parameters.

4. Mass and Stiffness of Structure.

The mass and stiffness of the structures only affected the results in so far as their ratio determined the natural frequency of each structure. This was due to the fact that the frequency response function was taken to represent fully the dynamic characteristics of each structure. The predicted response of structures as different as lattice towers and tall buildings were therefore almost identical provided their natural frequencies and damping coefficients were the same.

Wind Parameters.

Two wind parameters appear in the input spectrum. These are V,/

V, the mean velocity, and k, the ground roughness coefficient. Both of these parameters affect the overall size of the spectrum without affecting its shape. This has the effect of increasing the size of the time varying part of the reponse in relation to the mean response but has no effect on the dynamic to quasi-static ratio of response.†

The V parameter is one of the constituants of the x variable in the input spectrum, however, and so affects the position of the spectral peak in the frequency domain. Variations in V therefore have an effect on the dynamic/quasi-static ratio of response of each structure similar to that of a variation in the natural frequency parameter. V is consequently one of the most important parameters in the system.

3.5 DISCUSSION OF RESULTS

Although the preliminary analysis cannot be considered comprehensive it is possible to draw a number of tentative conclusions from the results. These may be divided into two categories. The first concerns the pin-pointing of important parameters which significantly effect the wind driven vibrating system and which will have to be known accurately if an accurate prediction of its behaviour is to be made. The second concerns the ability of the transfer functions in the spectrum to represent faithfully the behaviour of the system and of the spectrum method as a whole to predict its response.

Important Parameters

It is evident from the large variation in the results with variation in the natural frequency parameter that the position of the natural frequency of the structure with respect to the/

the high frequency tail in the excitation spectrum is of great importance. It follows from this that it is also essential for the profile of this steeply sloping part of the spectrum to be correct. A glance at Davenport's own graph showing the experimental curves superimposed on the theoretical spectrum curve (reproduced in fig 10) illustrates the magnitude of the variation which is possible between theory and practice in this part of the spectrum. Some experimental curves are well above the theoretical curve in this region while others are below it.

At the 0.01 wave number*, for instance, the value of the reduced spectrum according to the theoretical curve is 0.8 whereas the lowest and highest measured values, corresponding to data from Brookhaven and Sale, are 0.4 and 0.95 respectively. The use of the theoretical value for the Brookhaven site could therefore lead to an estimation of the dynamic component of response of a structure which was in error by a factor of two. If a mean wind speed of 100 ft/sec. were assumed the wave number used in this illustration would correspond to a frequency of 0.28 H_z. This is probably lower than the natural frequency of most structures and, as can be seen from fig 10, the discrepancy between theoretical and true values of the reduced spectra is likely to be larger at higher frequencies.

This result suggest, that the reduced spectrum formula in its present form may not be sophisticated enough to be capable of predicting reliably the spectrum of turbulence at any site. The result calls into question the validity of dynamic wind loading/

* $\frac{V}{n} = 358$ ft.

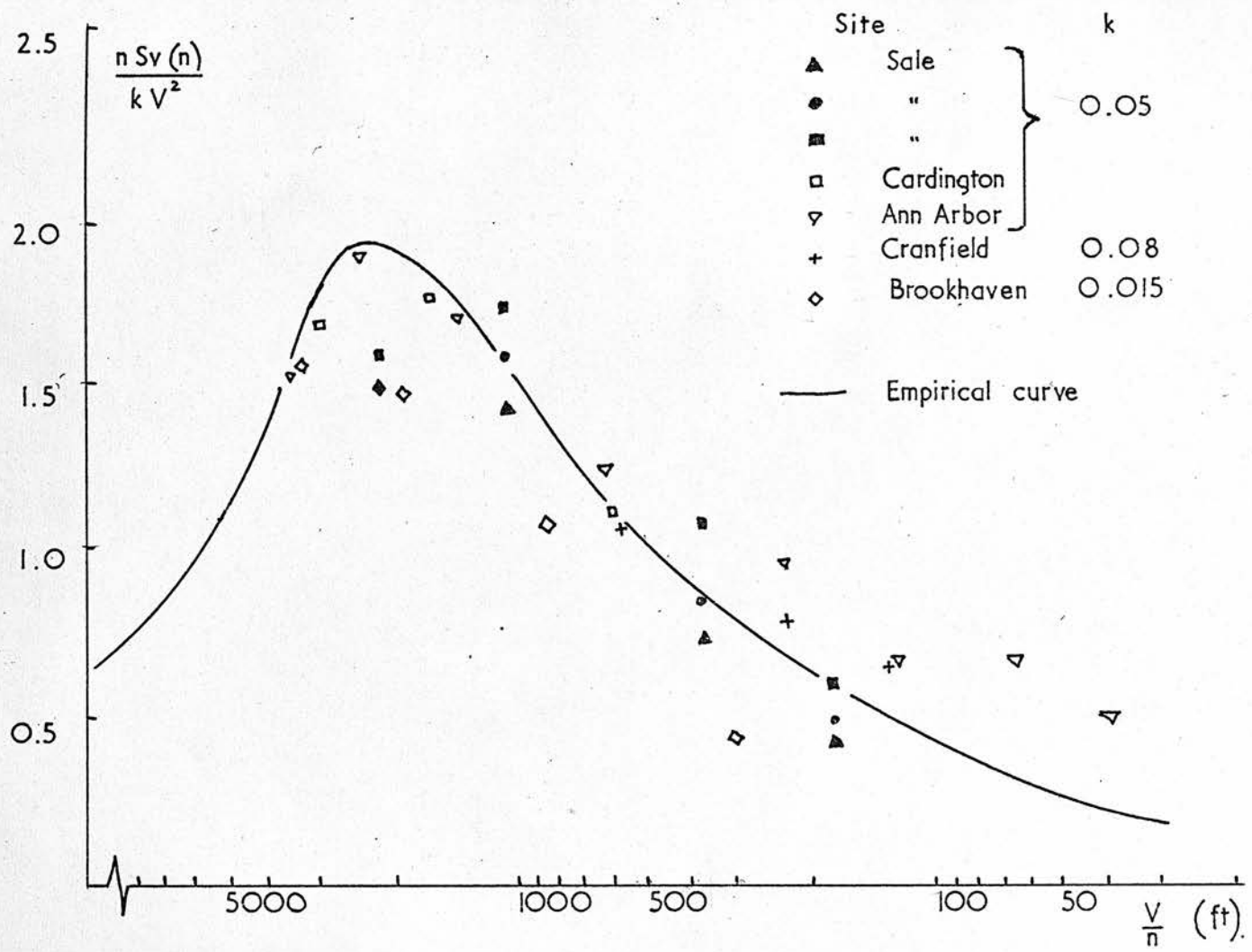


Fig. 10 Spectrum of horizontal wind speed (Davenport)

loading calculations based on the limited data which is currently available and suggests that before a prediction of the response of a structure will be able to be carried out with any confidence, much more data will have to be acquired on wind turbulence. In particular, the acquisition of data covering a range of ground roughness parameters is required so that the validity of existing empirically derived reduced spectrum formulae may be checked.

It is also evident that before a spectrum analysis may be attempted, correct values must be obtained for the natural frequency and damping parameters of the structure concerned as these have also been demonstrated to be critical factors.

Transfer Functions.

It is possible from the results to make a critical appraisal of the ability of the transfer functions to represent the behaviour of the various structures analysed. Two points are worth commenting on.

The first concerns the fact that for all the structures analysed the dynamic component of deflection was either of the same order of magnitude or larger than the quasi-static component. The predicted dynamic components were, in the opinion of the author, higher than would be expected in practice, especially for the heavier structures. The second point is that although the group of structures investigated was chosen so as to provide a range of structural properties the results obtained were all fairly similar. The transfer function used to convert from the pressure to the deflection spectra (mechanical/

(mechanical admittance) was insensitive to variations in structural parameters other than natural frequency and damping. In practice some variation in the response characteristics would be expected for different types of structure. Large vibration amplitudes can be envisaged to occur with light slender structures such as lamp posts and lattice towers but this type of behaviour is not generally associated with large buildings. The effect is probably partly due to the fact that buildings are likely to be more heavily damped than lighter structures but the author believes that other factors may be important.

One of the main structural differences between buildings and lattice towers is that the mass per unit length and stiffness of buildings are much greater than those of lattice towers. Because the natural frequency of a cantilever type structure depends on the ratio of mass to stiffness, however, these two types of structure may have natural frequencies very close to one another and their frequency response functions may be almost identical. If the frequency response function is taken as the sole criterion of dynamic response, the spectrum method of analysis may yield the same gust factor for structures as dissimilar as lattice towers and tall buildings.

The frequency response function is the particular integral solution of the vibration equation and it is a mathematical treatment which has been widely applied in mechanical vibration problems. The following conditions must be satisfied for it/

it to be valid.

1. The load function must be presented in the form of harmonic components of constant phase and amplitude.
2. The system must have reached a steady state condition.
3. The system must be linear.

These conditions are satisfied by most mechanical systems but the author believes that they may not be satisfied by all wind driven systems. The first is satisfied if a wind spectrum is available. The second is satisfied mathematically if a stochastic approach is made but is only valid if the structure concerned is capable of instantaneous response to high frequency gusts.

Physically the wind velocity is a random function of time and the assumption that its statistical properties are constant with time can be justified. If the velocity function is split into components which have a fixed bandwidth, the root mean square amplitude of each component is theoretically independent of its averaging period. A graph of these average amplitudes against the frequency on which each bandwidth is centred gives the wind velocity spectrum. In a spectrum analysis the average amplitude of any component of the response function is found by multiplying the corresponding ordinate in the velocity spectrum, first by a coefficient which converts it to a pressure and then by the appropriate value of the frequency response function.

The assumption that the structure responds instantaneously to any variation in the input function is implicit in this step.

The/

The argument that the average response is related to the average input by a simple transfer function such as the frequency response function cannot be justified unless the response of the structure is assumed to follow faithfully the input function.

Such a situation can be envisaged for components of turbulence and response whose bandwidths are centred on frequencies below the natural frequency of the structure. This is the quasi-static case and the response and excitation functions are related by the stiffness of the structure. The stiffness acts as a simple transfer function from which the response may be obtained from the input.

At the natural frequency, however, the situation is more complicated. A single impulse of duration $\frac{\pi}{2\omega_o}$, for instance, would cause a structure to deflect to an amplitude which would be defined by the stiffness of the structure in the same way as for a quasi-static impulse. The energy imparted to the structure would be stored, however, and such an impulse would cause the structure to vibrate for a short period until the energy had been dissipated by damping. A chain of such impulses (i.e. a sinusoidal input of constant amplitude) would cause an accumulation of energy in the structure and lead to an amplitude of vibration dictated by the ratio of input to damping energies per cycle and not by the stiffness of the structure alone. The relationship between input and response/

response in this case is given by the frequency response function. A finite number of cycles is required, however, for a steady state amplitude, as defined by this function, to be reached.

The forcing function, in the case of wind loading, is a component of turbulence, of narrow bandwidth, which is centred on the natural frequency of the structure. The amplitude of this forcing function is constantly varying, as observation of a spectrum analyser operating on wind data will show. It is probable that its phase is varying also. A structure excited by wind turbulence to vibrate at its natural frequency therefore, does so subject to a forcing function whose amplitude and phase are constantly changing. In such a situation the steady state condition as defined above is probably never achieved and the system is in a state of continuous transition. Under such conditions, the inertia of a structure must influence its behaviour.

A light structure is likely to respond very quickly to any high intensity batch of turbulence and may reach a high amplitude in a few cycles. Similarly, if the intensity of turbulence drops suddently, or the phase changes, the structural response will be quickly damped by the aerodynamic forces which now act in reverse. The response therefore follows the forcing function fairly closely and the averages of the forcing and response functions are probably linked by a simple relationship such as the frequency response function. A heavy

structure/

structure, however, may need so much energy for vibration as to require a batch of turbulence to maintain a constant phase and amplitude for a large number of cycles before any appreciable amplitude can be built up. The average response of such a structure is unlikely to be greatly affected by any large fluctuation or series of large fluctuations in the input function unless they last for sufficient time for the full structural response to become established. Short duration batches of turbulence, even if they are of high intensity, may have very little effect on the average response.

It is possible, therefore, that the inertia of a structure has a significant influence on its response to a random forcing function such as wind turbulence and that the average of the resonant component of response is not related to the average input in the simple way suggested by the frequency response function. Whether or not the inertia of a structure is important depends on the relationship between the energy which can be imparted to the structure from the wind and the energy of vibration of the structure itself. If the ratio of input energy to energy of vibration is high then the inertia is probably unimportant. If it is low, however, it is possible that the frequency response function does not constitute a sufficiently accurate mathematical model of the system to provide a reliable prediction of the response. It was thought that further investigation of this question was required and

Chapter 4 describes an attempt to evaluate the energy levels concerned in order to assess the importance of this factor. The third condition upon which the spectrum analysis, in its present form, is based, is that the system concerned should be linear. In order that the wind can exert a force on a structure there must be a relative velocity between the air and the structure. To cause resonance this relative velocity must be maintained throughout the cycle of deflection. It must, therefore, be periodic and in phase with the structure. If wind turbulence is regarded as a fluctuation about zero mean, as in the spectrum analysis, resonance would be expected to result from the action of the component of turbulence which has the same frequency as the natural frequency of the strucure. Once a structure begins to vibrate in response to a wind load, however, it has a periodic velocity of its own, so that the relative velocity and consequently the force on it decrease. If the amplitude builds up to the extent that the velocity of the structure approaches the velocity of the wind component causing the vibration, the energy input to the structure will tend to zero and further increase in amplitude will be impossible

The use of a conventional frequency response function presumes that the amplitude of the periodic force is independent of the amplitude of vibration; a condition which is not satisfied in the case of a periodic wind load. It is possible, therefore, that even if high vibration amplitudes do develop in response to/

to wind loads, the maximum deflection is controlled, not by the degree of damping in the structure, as would be expected from a frequency response analysis, but by the onset of the zero relative velocity condition described above. The use of the frequency response function could, therefore, lead to the prediction of higher amplitudes at resonance than would occur in practice.

Returning to the two points which were made at the beginning of this section which were firstly that the predicted dynamic components of response given by the spectrum method seemed to be too high and secondly that the method was insensitive to the differences between different types of structure, it may be said in conclusion that the first of these could be due to the fact that, in its present form, the spectrum analysis neglects the non-linearity of the wind driven system and the second to the fact that the inertia of a structure being analysed is neglected despite the fact that the randomness of the load may necessitate its being taken into account.

3.6 CONCLUSIONS

Wind turbulence is a highly complex phenomena and in the present state of knowledge a stochastic approach to the problem of predicting structural response to wind loads is probably the only one feasible. An analysis to obtain the dynamic response of a structure must therefore be based on the wind velocity spectrum. In the light of the foregoing investigation/

investigation four observations can be made about this type of analysis.

1. The reduced wind velocity spectrum, in its present form, is not sophisticated enough to be capable of providing a sufficiently reliable prediction of the wind conditions at any site. Much more data are required on the characteristics of turbulence at sites of different roughness and for different mean wind speeds so that the reduced spectrum may be improved.

2. The most important structural parameter is the natural frequency. A correct value for this parameter is an essential prerequisite to the successful prediction of the response of a structure to wind turbulence. The problem of obtaining the natural frequency is particularly difficult for tall buildings which usually have a highly complicated structural form and mass distribution. Several empirical formulae based on simple parameters such as the overall dimensions of buildings are currently in use but these are approximate and are not likely to be capable of producing results within the accuracy required for wind loading calculations.

More work is therefore required in this field so as to provide a method for predicting the natural frequency of a building which is both accurate and simple enough for general use.

An attempt to do this for a particular type of multi-storey structure had been carried out by the author. A description of this is given in Appendix 3.

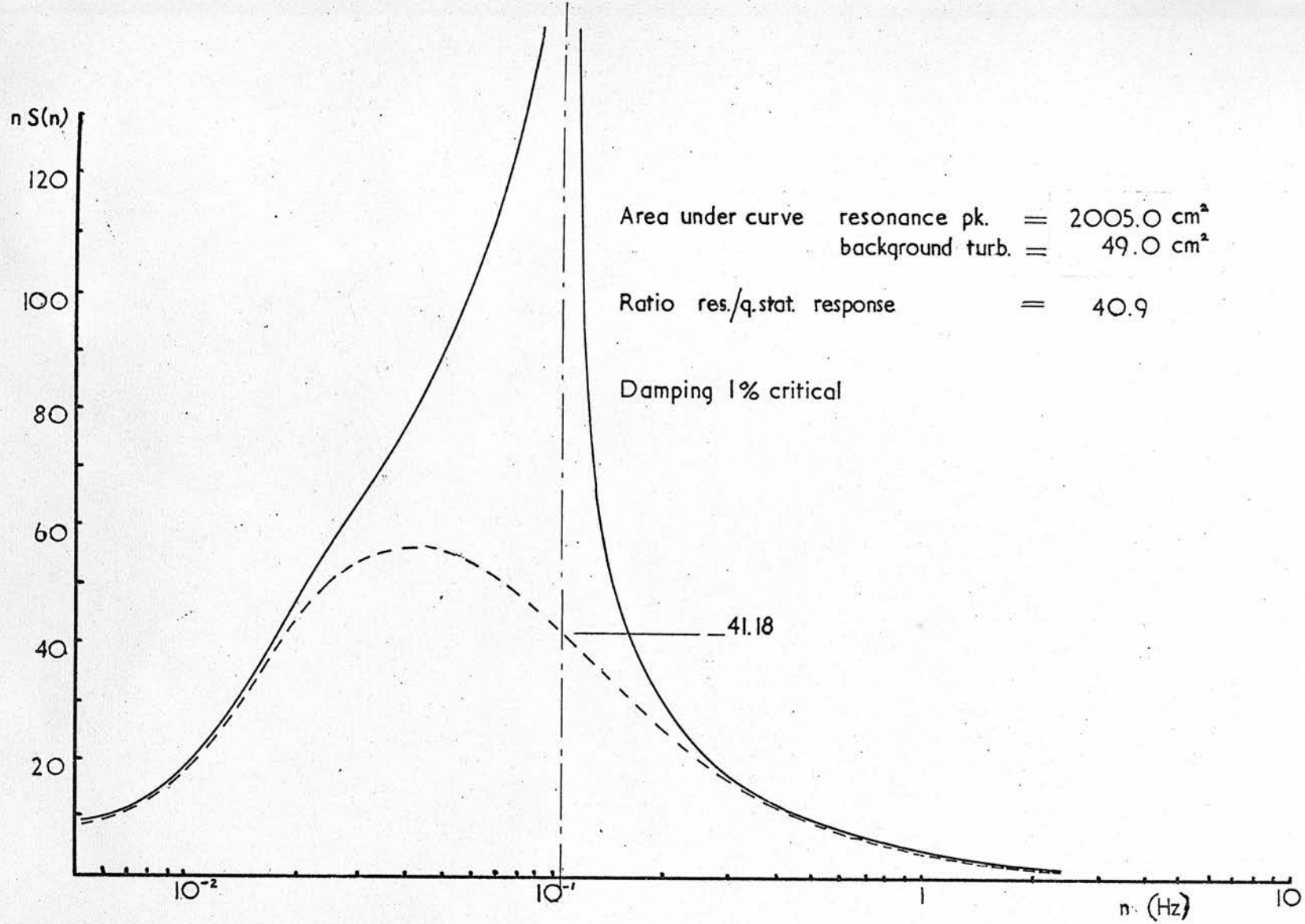
3. The/

3. The spectrum analysis technique, in its present form, is insensitive to variations in structural parameters other than the natural frequency and damping. Although there is a slight distinction between structures of different height, all structures with the same natural frequency and damping are regarded as being identical. It is felt that the behaviour of structures with different stiffness and inertia properties may not necessarily be the same, especially if the excitation is of a random nature, and that an investigation into the effect of these parameters should be carried out to determine whether or not the transfer function used in the present spectrum analyses is capable of producing reliable results.

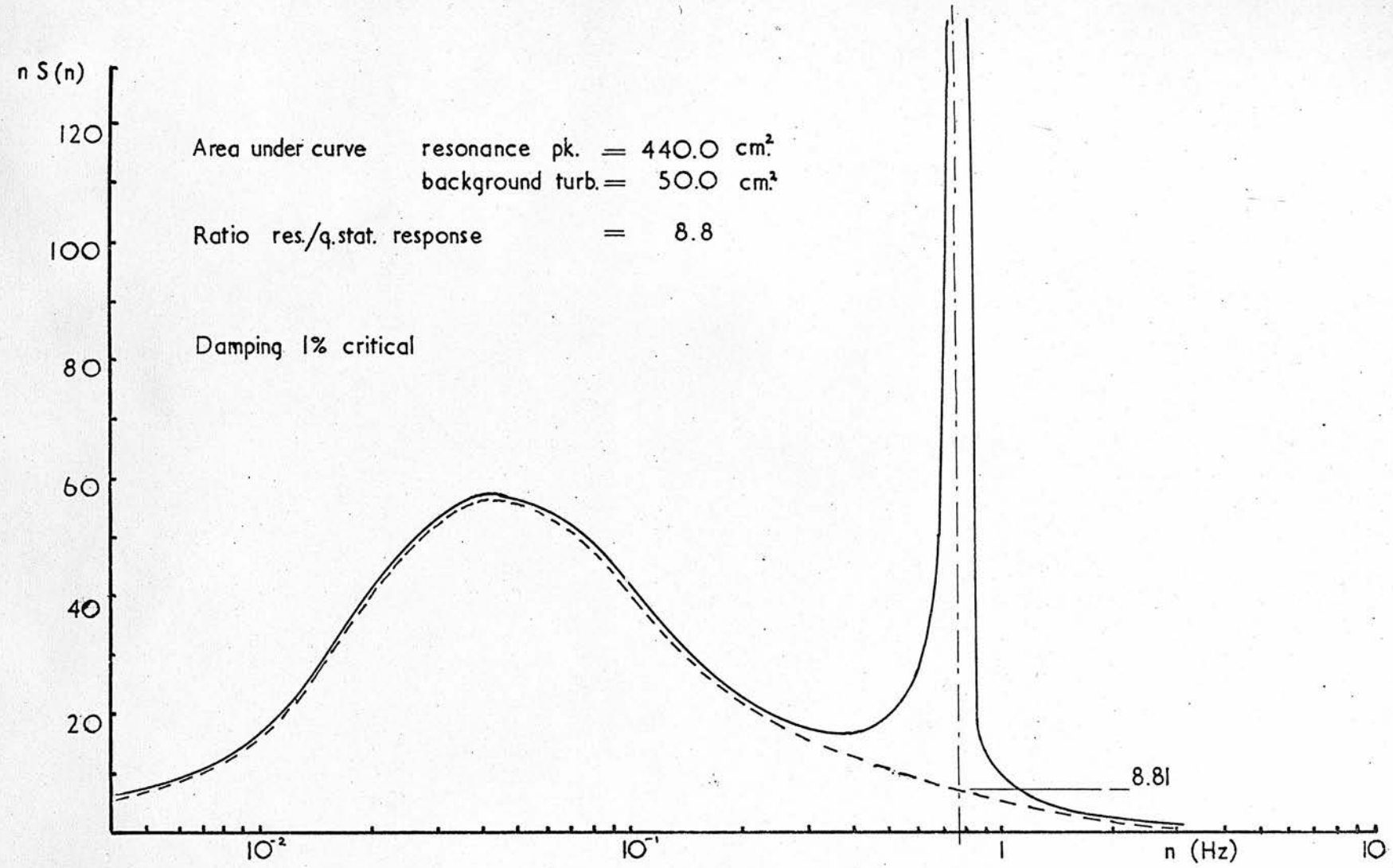
4. A wind driven vibrating system is non-linear due to the fact that the forcing function is influenced by the amplitude of the vibration. This fact is neglected by the spectrum analysis in its present form and it is felt that the possible effect of this on the reliability of the method should be investigated.

Chapter 4 describes further computer simulation of a wind driven vibrating system using a mathematical model designed to highlight the effects of the two points raised in 3 and 4 above.

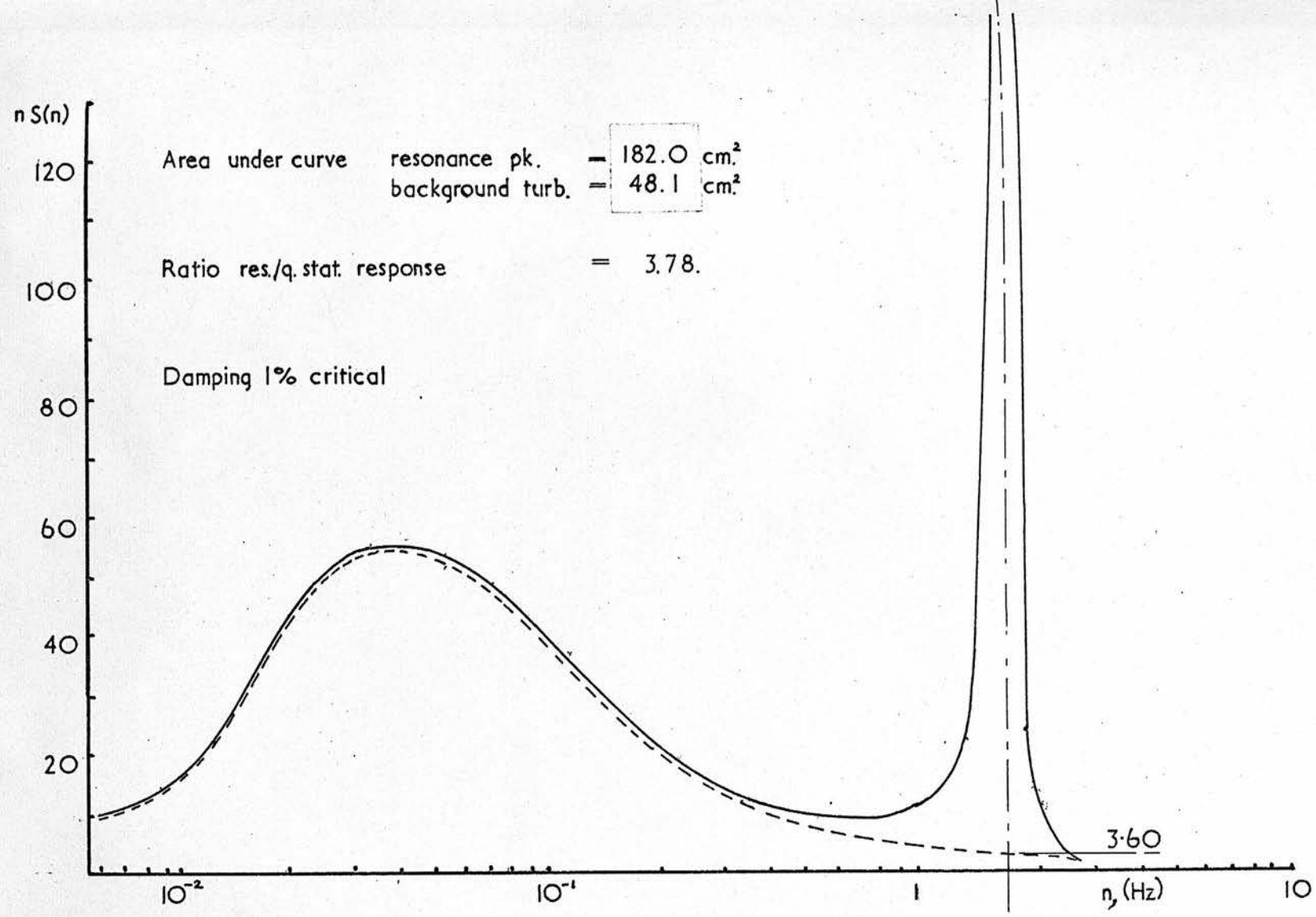
† The dynamic to quasi-static ratio of response is affected by the α' wind parameter, which appears in the joint acceptance function. The extent of the influence of this parameter may be judged from graph 5.



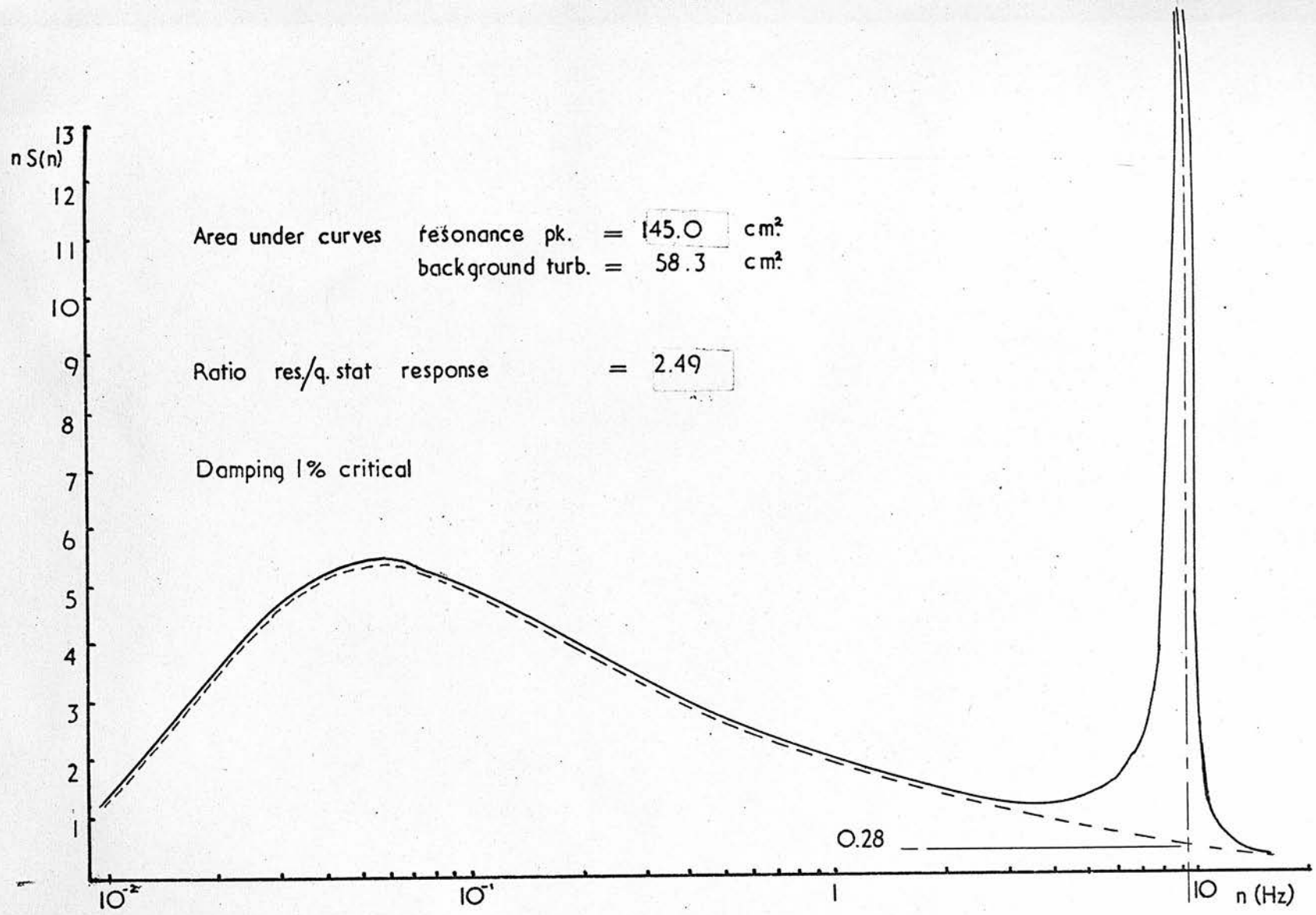
Graph I. Deflection and pressure spectra Structure T I.



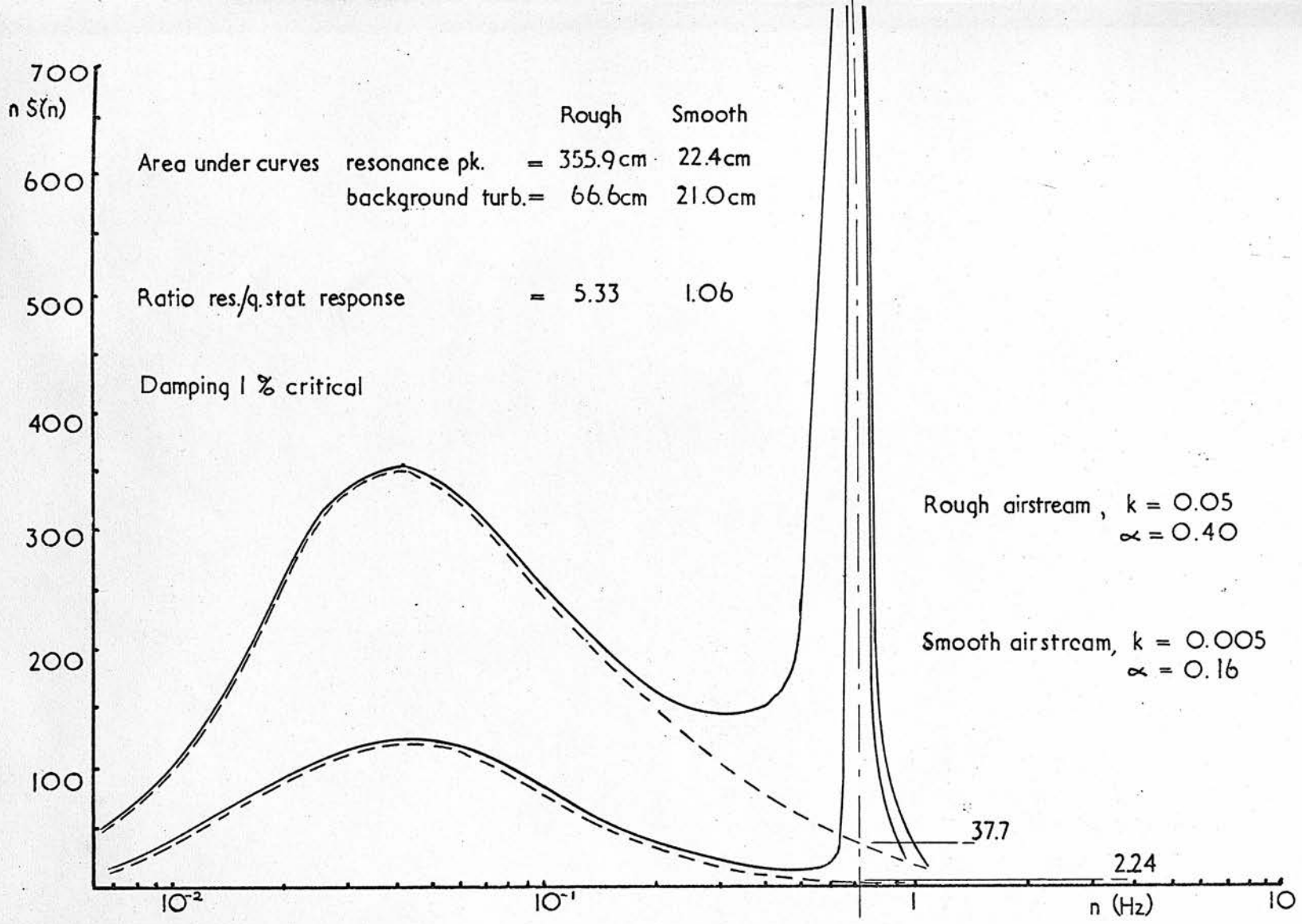
Graph 2. Deflection and pressure spectra Structure T 2.



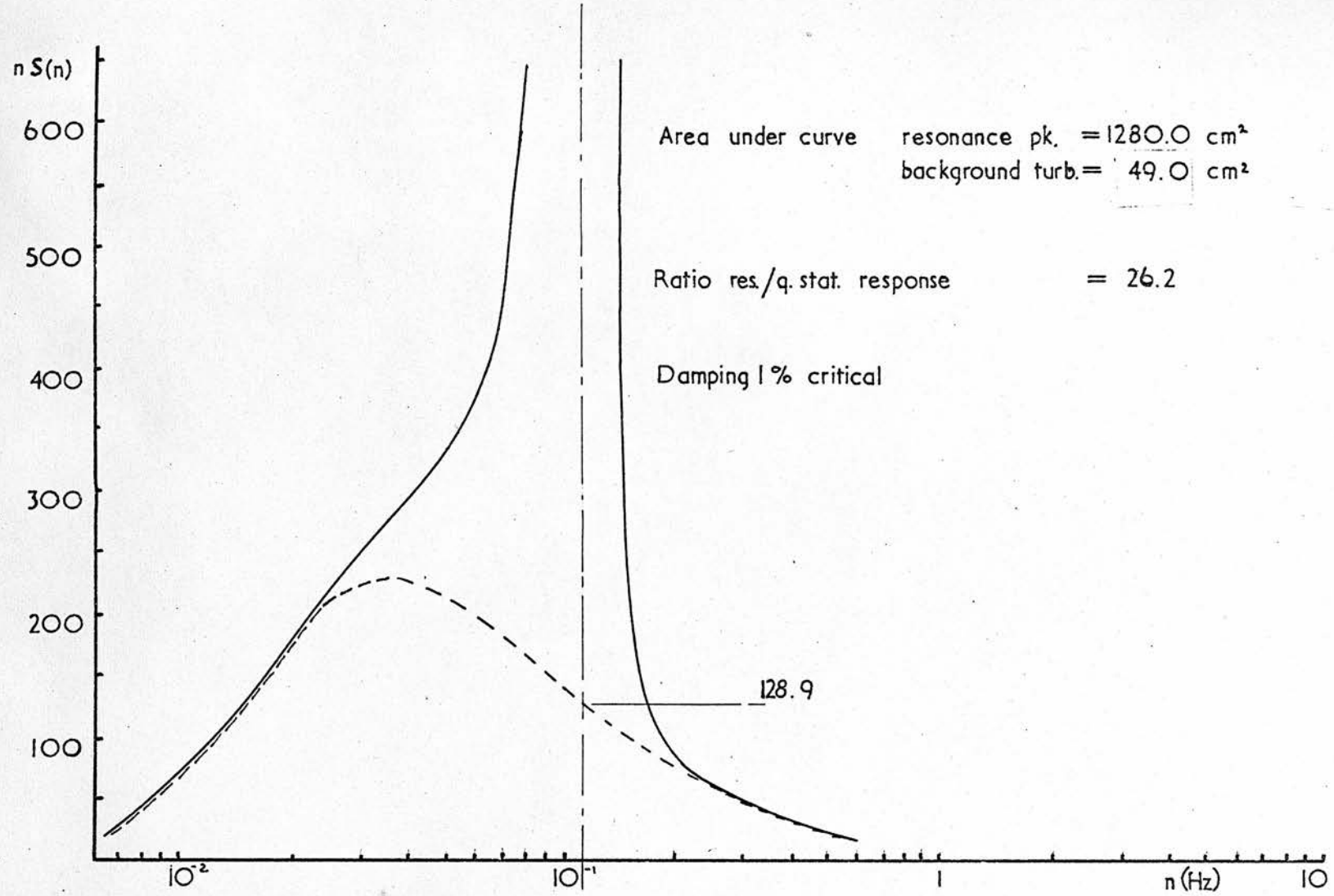
Graph 3. Deflection and pressure spectra Structure T3



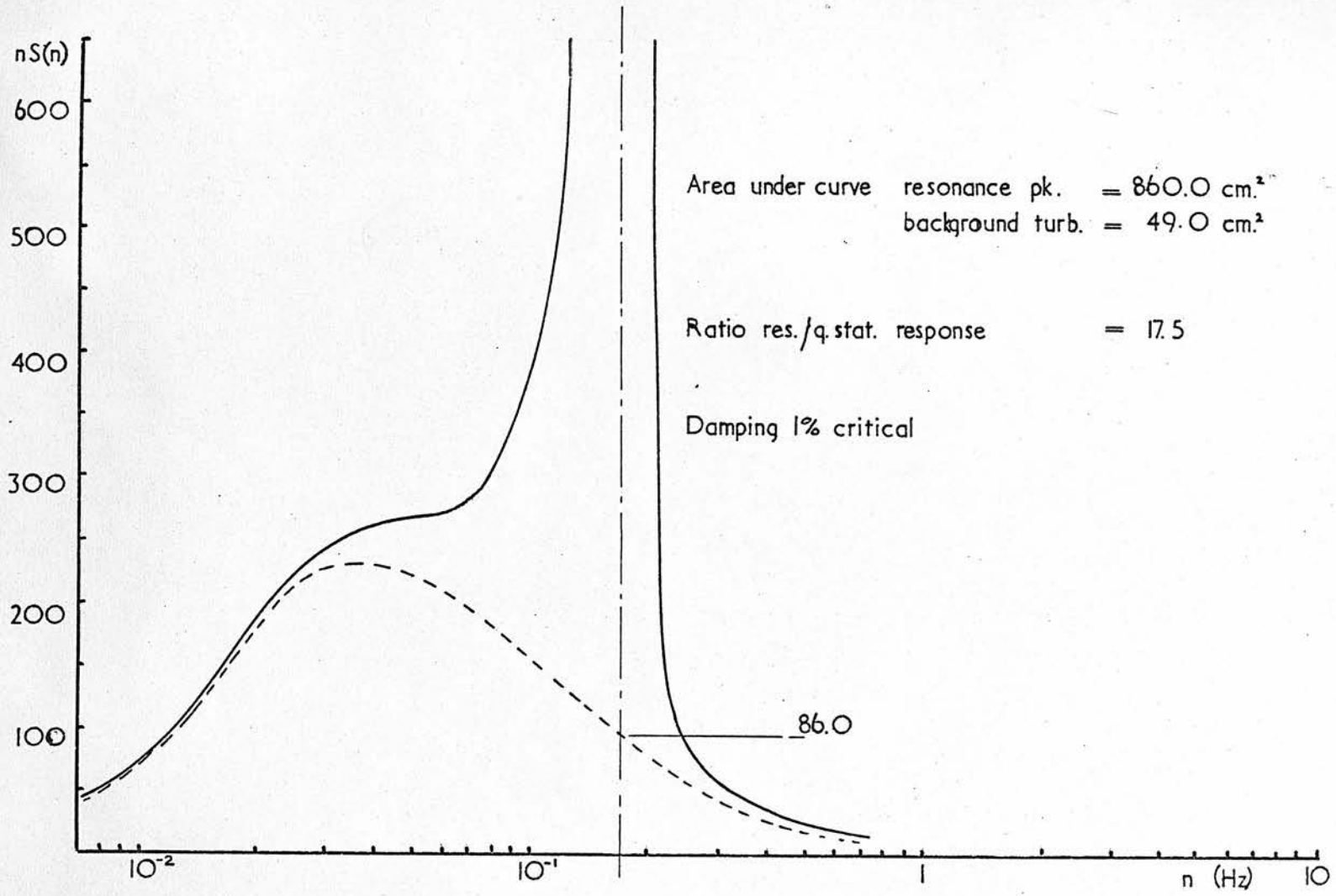
Graph 4 Deflection and pressure spectra Structure T4



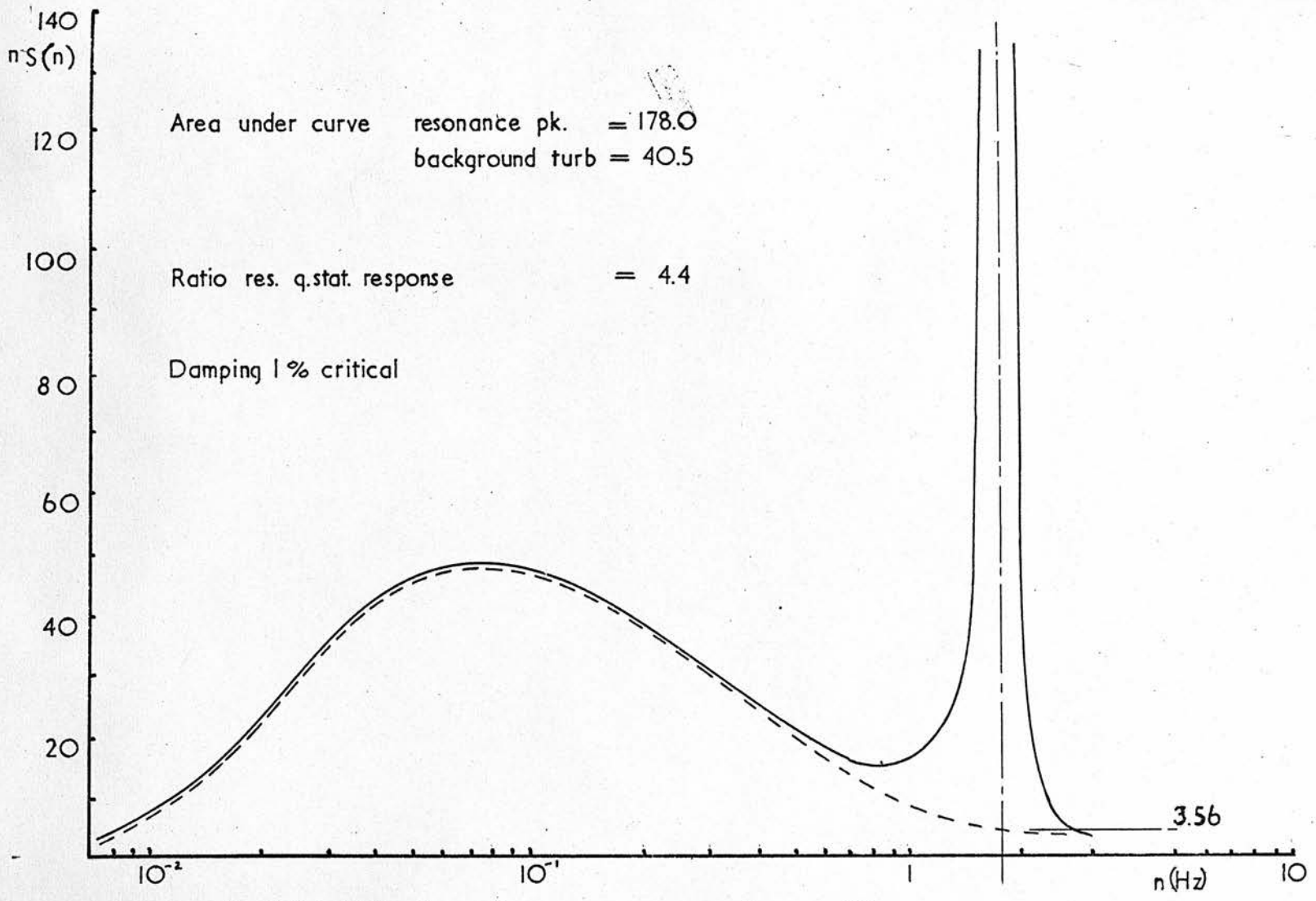
Graph 5 Deflection and pressure spectra (rough and smooth airstreams) Structure T5.



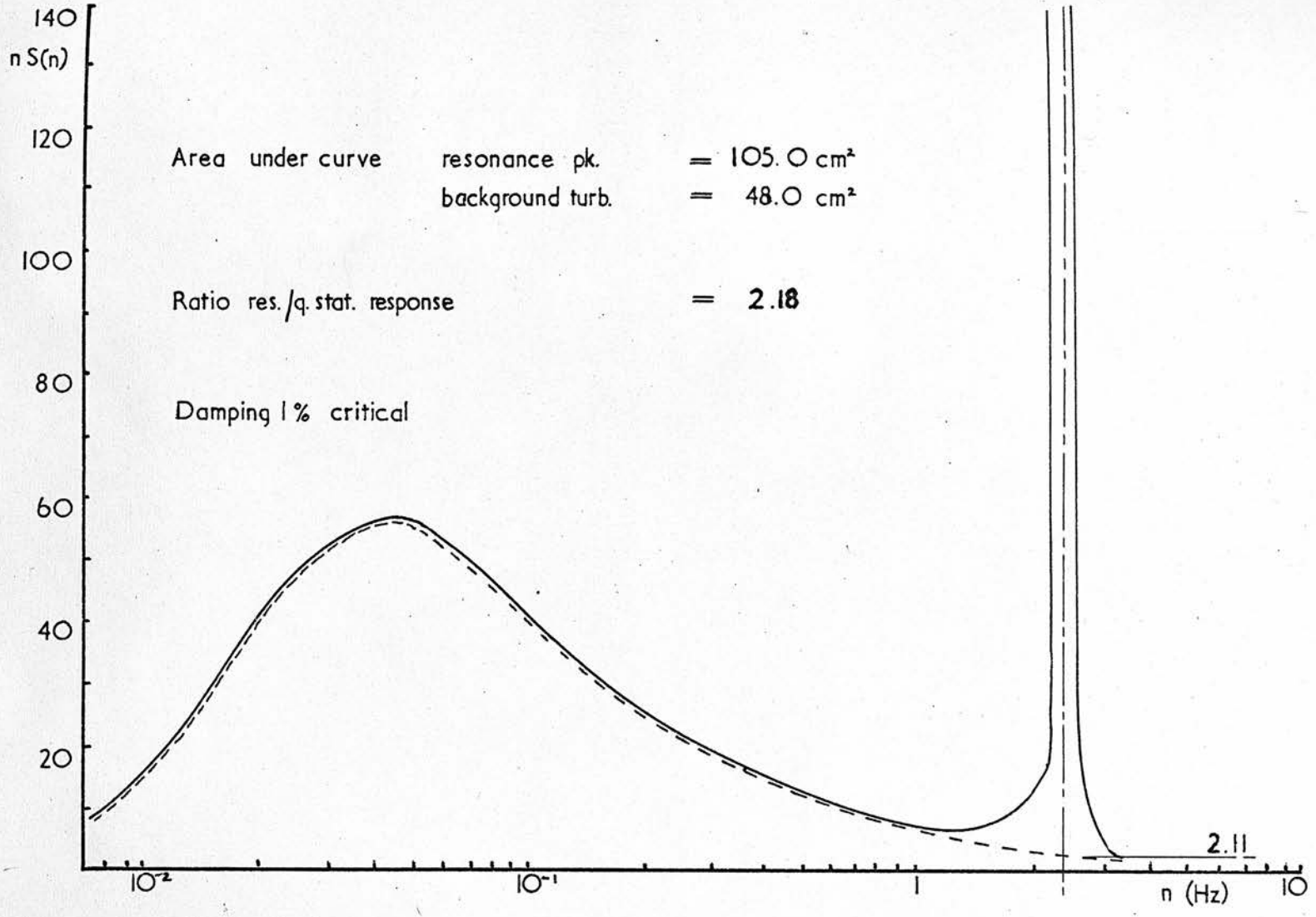
Graph 6 Deflection and pressure spectra Structure T 6



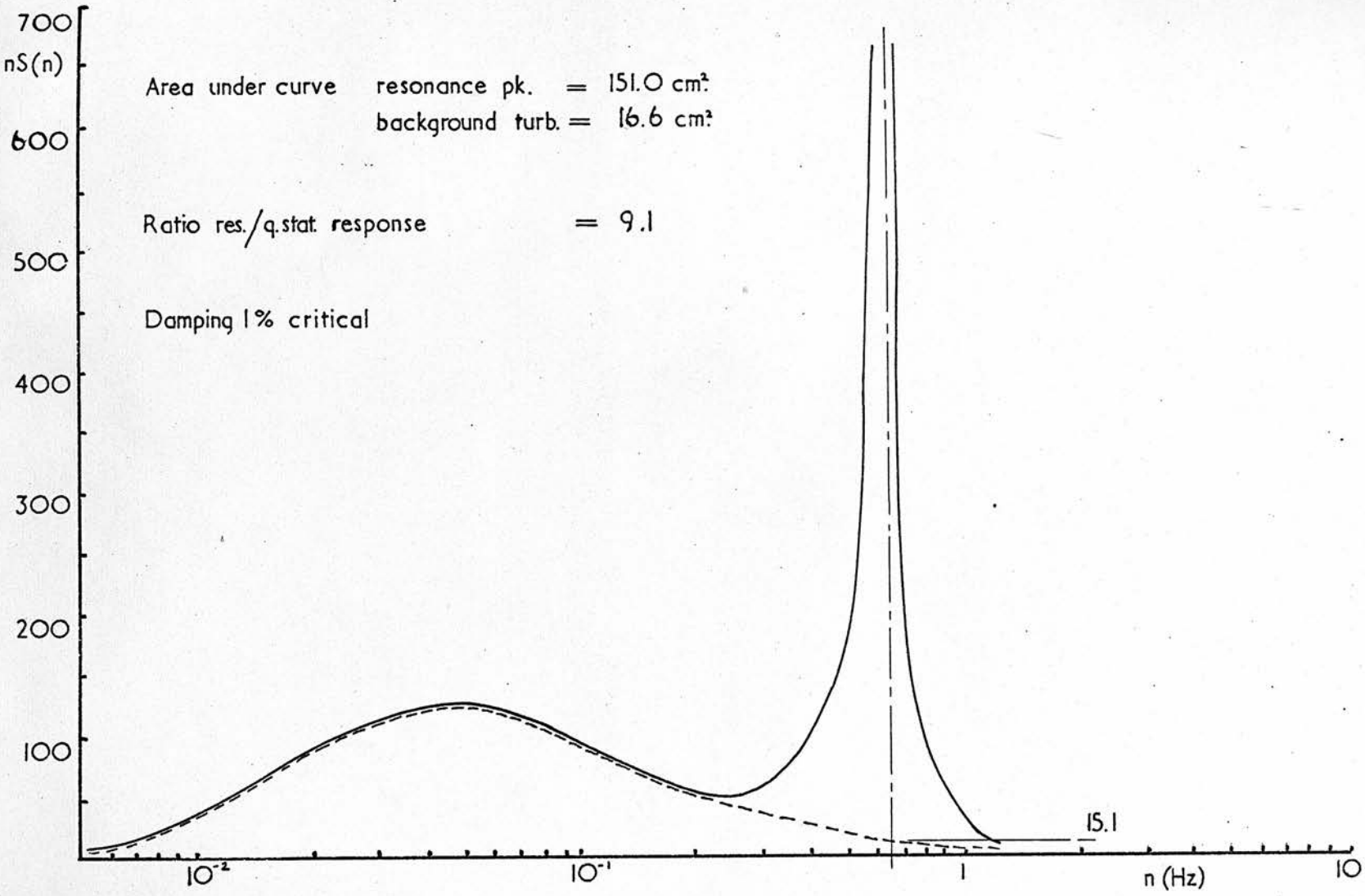
Graph 7 Deflection and pressure spectra Structure T7



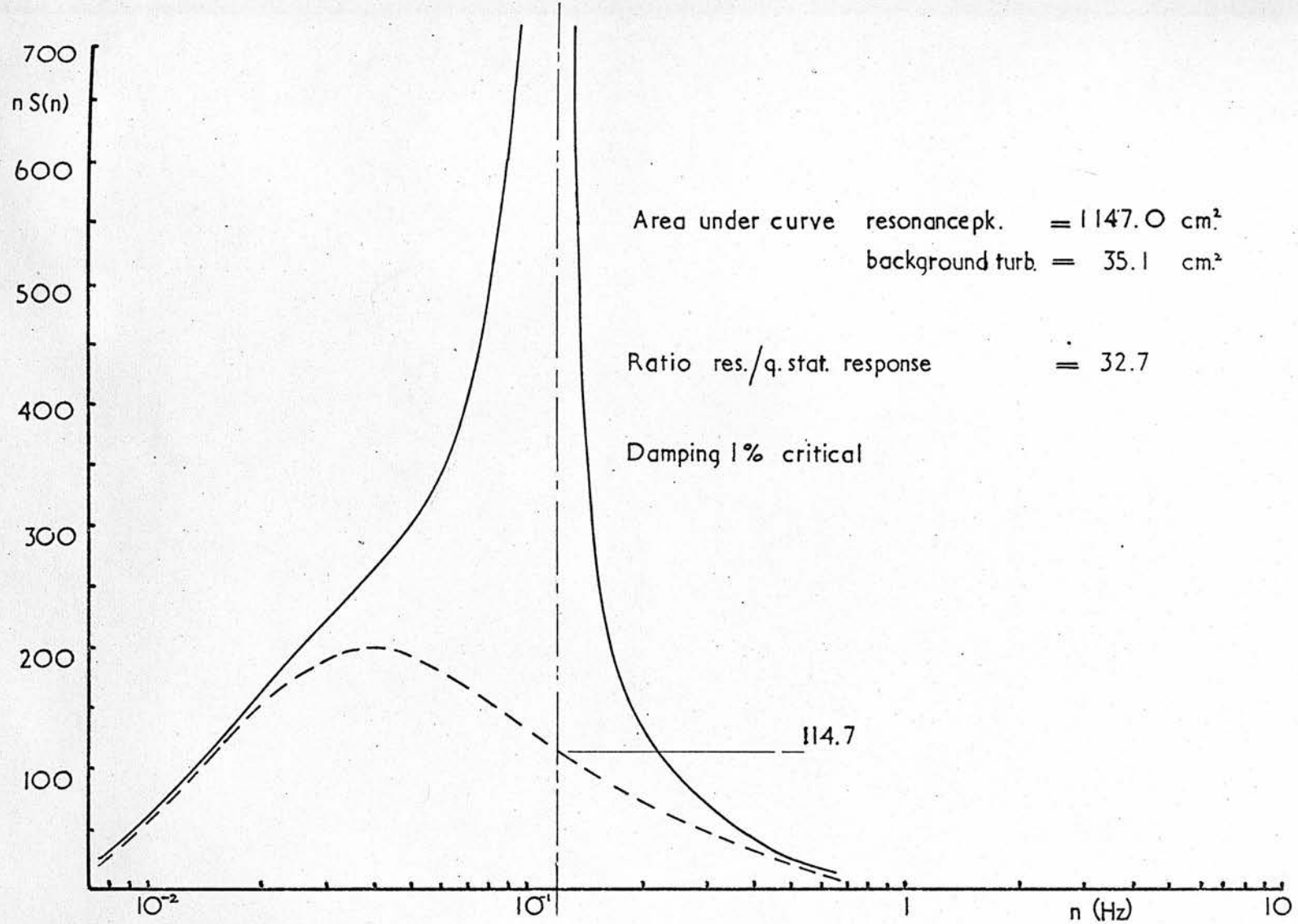
Graph 8 Deflection and pressure spectra Structure T8



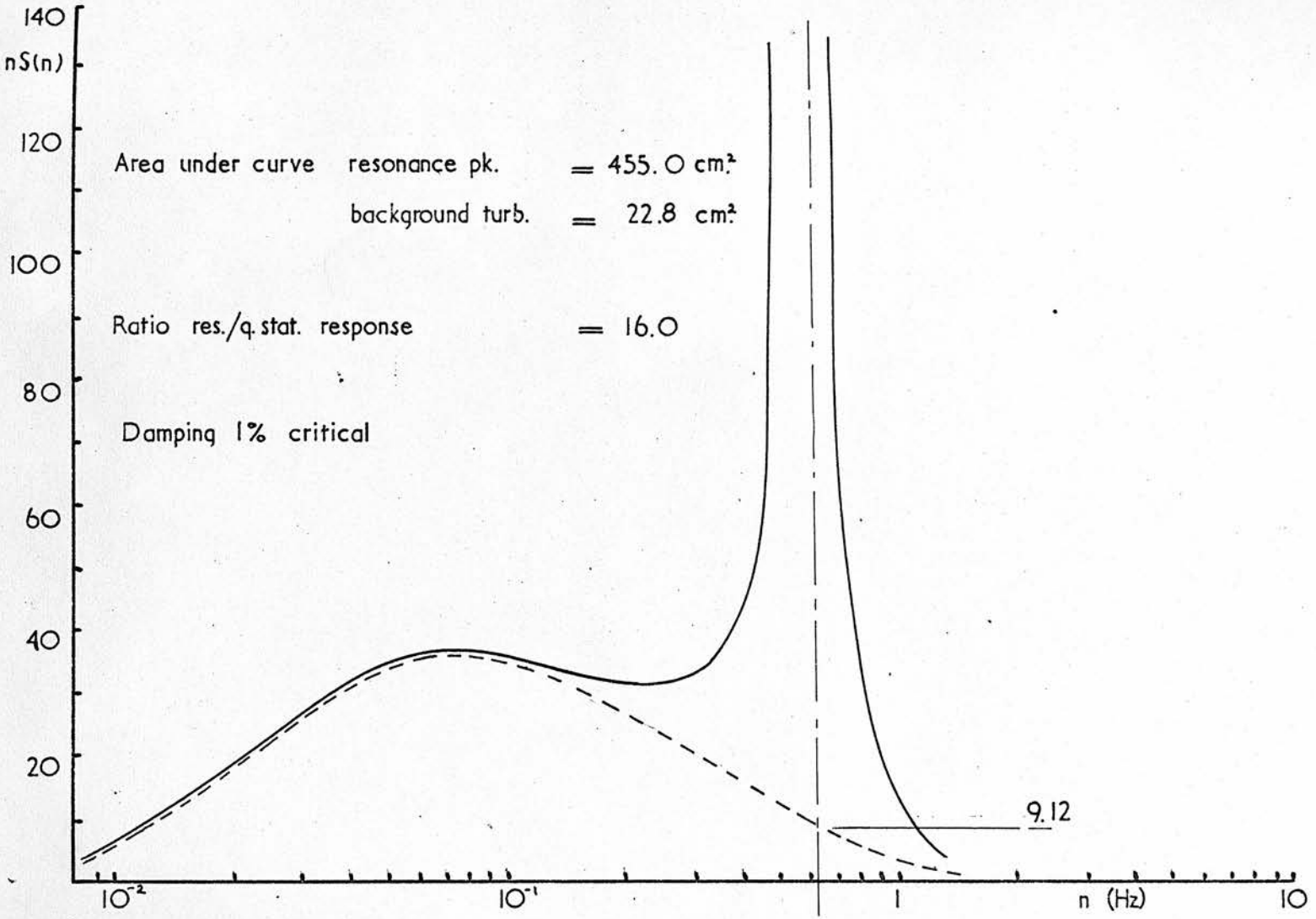
Graph 9 Deflection and pressure spectra Structure T 9



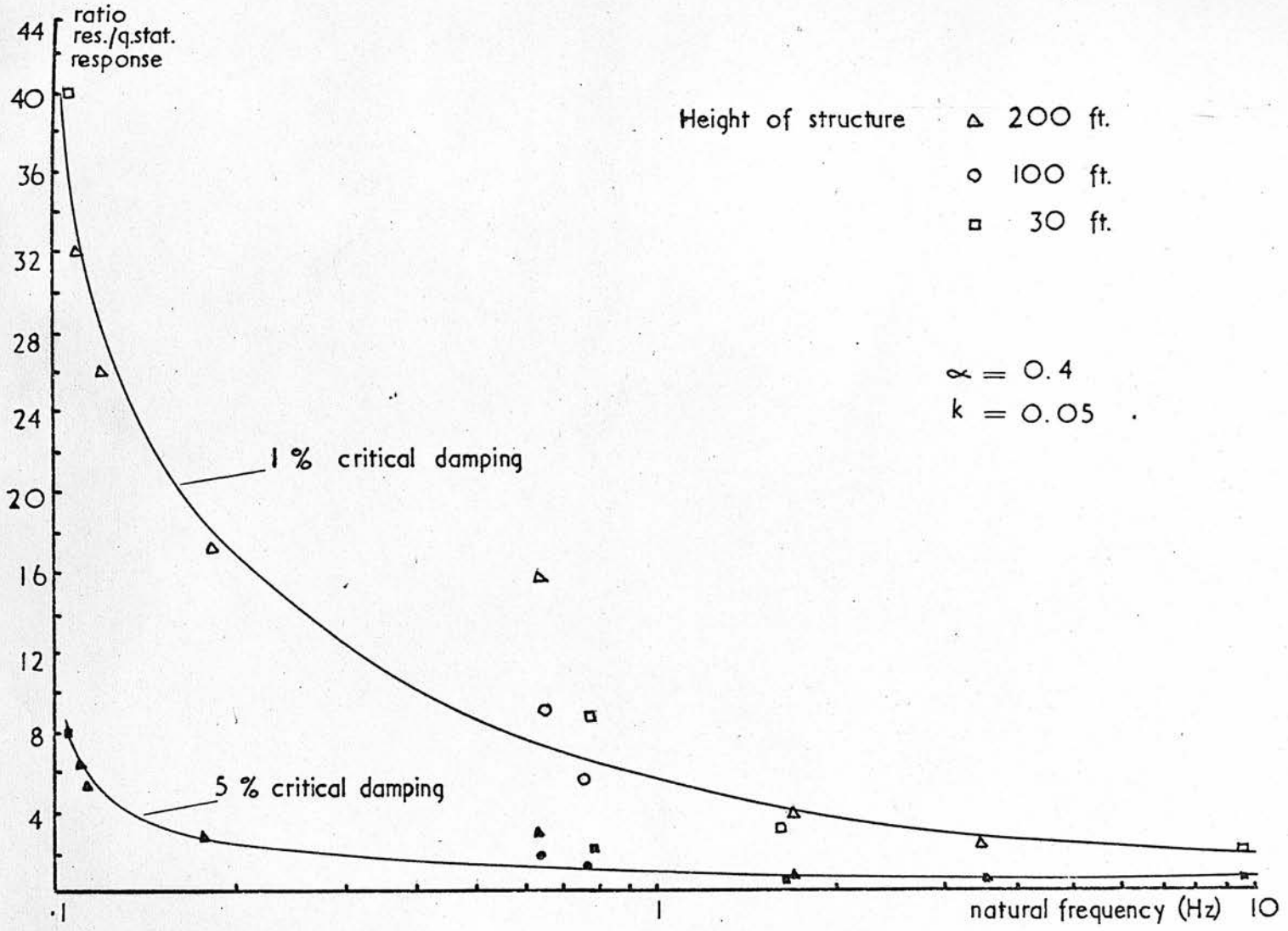
Graph 10 Deflection and pressure spectra Structure B1



Graph 11 Deflection and pressure spectra Structure **B2**



Graph 12 Deflection and pressure spectra Structure B 3



Graph 13 Relationship between ratio of res. q.stat. response and natural frequency.

CHAPTER 4

ENERGY ANALYSIS

4.1 INTRODUCTION

The energy analysis was carried out with a view to investigating the effects on a vibrating system of the two points raised at the end of Chapter 3. The first of these was the suggestion that the relative levels of energy input and energy of vibration in a wind driven vibrating structure may be such that the transient part of the response will be of sufficient duration to influence the long term behaviour of the system. It was considered that this factor might be of importance when the forcing function was random as is the case with turbulent wind loads. The second point raised in Chapter 3 was that a wind driven vibrating structure is not a linear system. It was thought that this fact might lead to an overestimation of the predicted vibration amplitude of a structure if an analytical technique was used in which no allowance for the non-linearity was made.

The object of the analysis which follows was to try and estimate the extent of the error which neglect of these two factors might cause so as to determine whether an allowance for them ought to be made as a standard part of a dynamic wind loading analysis.

4.2 DEVELOPMENT OF ENERGY FUNCTIONS.

There are two main energy transfer mechanisms in a wind driven vibrating system. One is the energy exchange which takes place between the wind and the structure and which provides the excitation energy for the system. The other is the energy dissipation which occurs due to mechanical damping. In a given wind situation the relationship between these two energy mechanisms, which is a function/

function of the amplitude of vibration, defines the net energy input to the structure. The zero net energy input condition defines the maximum amplitude of vibration which can occur in that wind.

A vibrating structure stores energy in the form of strain energy (potential energy) and kinetic energy. The proportion of strain energy to kinetic energy at any instant depends on the displacement and velocity of the structure i.e. on the position of the structure in the vibration cycle. The total energy stored is a function of the amplitude of vibration. The relationship between the total energy and the input energy at any amplitude governs the rate of build up of the vibration.

In order that the behaviour of a structure in the wind could be simulated mathematically, expressions were derived for excitation energy, damping energy and energy stored in vibrating structures. These expressions deal solely with the resonant component of structural response.

Excitation Energy

In the derivation of the expression for the excitation energy a very simple mechanism was assumed as an approximation to the interaction between a structure and the wind. The initial assumptions were:-

1. The structure is a cantilever
2. The component of turbulence which has the same frequency as the natural frequency of the structure is the only one which contributes to the resonant component of response.

3. This/

3. This component varies sinusoidally with time, has constant amplitude and envelops the whole structure.

4. The maximum energy transfer occurs when the wind velocity is in phase with the velocity of the structure.

No allowance was made for the variation in wind speed with height. If the y and z axes are defined as in Fig (11) the deflection of the structure is given by the expression,

$$y = A_{st}\, f(z) \sin\omega t$$

$$\therefore\ V_{st} = \omega A_{st}\, f(z) \cos\omega t$$

where, A_{st} = amplitude of vibration at top of structure

V_{st} = velocity of structure

Pressure per unit width and length on the structure, $P(z,t) = \frac{1}{2} \rho\, Cd\ V_r^2$

where ρ = density of air

Cd = drag coefficient

V_r = velocity of wind relative to structure

$$P = \bar{P} + p = \tfrac{1}{2}\rho\, Cd\, (\bar{V} + v_r)^2$$

$$= \tfrac{1}{2}\rho\, Cd\, \bar{V}^2 + \rho\, Cd\bar{V}v_r + \tfrac{1}{2}\rho\, Cd\, v_r^2$$

where P = mean pressure

p = fluctuating component of pressure

$\bar{V}$ = mean velocity of wind

v_r/

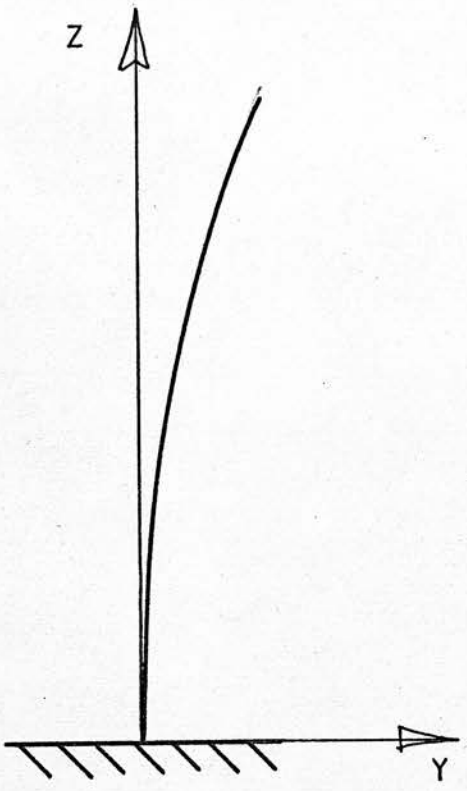


Fig 11 Reference axes in energy analysis.

v_r = fluctuating component of velocity of wind relative to structure.

The last two terms in this equation give the time varying component of pressure and neglecting the term in v_r^2,

$$p = \rho C_d \bar{V} v_r$$

but $v_r = V_w - V_{st}$

where V_w = fluctuating component of wind velocity.

$$\therefore \quad v_r = A_w \sin\omega t - f(z)\omega A_{st} \sin\omega t$$

$$= (A_w - f(z)\omega A_{st}) \sin\omega t$$

$$\therefore \quad p = \rho C_d \bar{V} (A_w - f(x)\omega A_{st}) \sin\omega t$$

where A_w = amplitude of wind turbulence component.

The force per unit width on an element δz of the structure is given by:-

$$\delta F = p\,\delta z = \rho C_d \bar{V} (A_w - f(z)\omega A_{st}) \operatorname{Sin}\omega t\, \delta z$$

The work done on the element in moving a small distance δy,

$$\delta E_e = p\,\delta z\,\delta y$$

$$= p\,V_{st}\,\delta z\,\delta t$$

$$= \rho C_d \bar{V} (A_w - f(z)\omega A_{st})\, f(z)\omega A_{st} \operatorname{Sin}^2\omega t\, \delta z\, \delta t$$

Energy input to the whole structure in one cycle,

$$E_e = \int_0^{\frac{2\pi}{\omega}} \int_0^{\ell} B\rho C_d \bar{V} (A_w - f(z)\omega A_{st})\, f(z)\omega A_{st} \operatorname{Sin}^2\omega t\, dz\, dt$$

$$= B/$$

$$= B\rho C d\bar{V} \omega \pi \int_0^{\ell} \left(\frac{A_W}{\omega} - f(z)A_{st}\right) f(z)A_{st}\, dz$$

where B = width of structure

This equation applies to a structure subjected to a wind component which totally envelops it. To allow for the fact that gusts may be of small spatial extent, A_W was multiplied by a vertical cross-correlation coefficient given by,

$$C_s = e^{\frac{-7.7(\ell-z)n}{V}}$$

This expression is based on the coherence or co-spectrum of wind velocity and it was derived empirically by Davenport. A discussion concerning its applicability in this situation is given in appendix 2. Full correlation was assumed across the face of the structure in the horizontal direction. The energy input to the structure in one cycle is therefore given by the expression

$$E_e = B\rho C d\bar{V} \omega \pi \int_0^{\ell} \left(\frac{C_s A_W}{\omega} - f(z)A_{st}\right) f(z)\, A_{st}\, dz \quad \ldots(4.1)$$

Equation (4.1) gives the energy transfer per cycle between a structure and the wind and as would be expected it is a function of the amplitude of vibration. The non-linearity in the system can be seen from an examination of the parameter $\left(\frac{A_W}{\omega} - f(z)A_{st}\right)$. This is positive when the periodic wind velocity is greater than the structural velocity and the structure is being excited and negative when the structural velocity is greater than the wind velocity. In the latter case the energy transfer is from structure to wind and a condition of aero-

dynamic/

dynamic damping exists. The maximum possible amplitude of vibration for a given A_w occurs when the net energy transfer given by equation (4.1) is zero. This value cannot be exceeded no matter how light the mechanical damping.

If the system is assumed to be linear, the corresponding excitation energy equation is,

$$E_e = \rho\, Cd\, \bar{V}\pi \int_0^{\ell} C_s\, A_w\, A_{st}\, f(z)dz \qquad \ldots(4.2)$$

The maximum possible amplitude in this case cannot be deduced from examination of the relationship between A_w and A_{st} but depends solely on the amount of mechanical damping in the system.

Damping Energy

The deflection of the structure may be represented by,

$$y\,(z,t) = f(z)\, g(t)$$

where $g(t)$ is a generalised co-ordinate.

The time varying properties of the system are described by the equation,

$$K\ddot{g} + c\dot{g} + Mg = p$$

The damping force is therefore given by $c\dot{g}$.

Work done by damping force $= c\dot{g}\, dg$

$= c\dot{g}^2\, dt$

Work done / cycle by damping force,

$$Ed = \int_0^{\frac{2\pi}{\omega}} c\dot{g}^2\, dt$$

but/

but $g = A_{st} \sin \omega t$

$$E_d = A_{st}^2 \quad c \int_0^{\frac{2\pi}{\omega}} \sin^2 \omega t \, dt$$

$$= \frac{A_{st}^2 \; c \pi}{\omega}$$

The damping coefficient c can be expressed as,

$$c = \nu \; c_r$$

where ν = critical damping ratio

c_r = critical damping coefficient

and $c_r = 2M\omega$

$$= 2 \omega m \int_0^{\ell} f^2(z) dz$$

where m = mass/unit length of structure

$$E_d = 2A_{st}^2 \, \pi \, \nu \, \omega^2 m \int_0^{\ell} f^2 (z) dz \qquad \ldots(4.3)$$

Equation (4.3) gives the energy dissipated per cycle by mechanical damping.

Energy of Vibration

The energy stored in the structure exists in the form of strain energy and kinetic energy.

Total Stored Energy = strain energy + kinetic energy

$$E_s = E_u + E_k$$

If a cantilever type structure is assumed the following expressions, which relate the strain and kinetic energies to the deflection at the top/

top of the structure may be derived.

Kinetic energy of an element δz of the structure,

$$\delta E_k = \tfrac{1}{2} m v_{st}^2 \, dz$$

$$v = y_1 \, f(z) \, \omega \cos\omega t$$

where y_1 = deflection at the top of the structure

$$\delta E_k = \tfrac{1}{2} m y_1^2 f^2(z) \, \omega^2 \cos^2\omega t \, dz$$

Total kinetic energy,

$$E_k = \tfrac{1}{2} m y_1^2 \omega^2 \cos^2\omega t \int_0^{\ell} f^2(z) \, dz$$

Strain energy $E_u = \int_0^{\ell} \frac{M^2(z)}{2EI} \, dz$

where $M(z)$ = bending moment at z

$= EIy_1 f''(z)$

E = Young's Modulus

I = 2nd Moment of area of cross-section

$$E_u = \tfrac{1}{2} EI \, y_1 \int_0^{\ell} f''(z) \, dz$$

$$\therefore \quad E_s = \tfrac{1}{2} m y_1^2 \omega^2 \cos^2\omega t \int_0^{\ell} f(z) \, dz + \tfrac{1}{2} EIy_1 \int_0^{\ell} f''(z) \, dz$$

As $E_k = 0$ when $y_1 = A_{st}$ and $E_u = 0$ when v_{st} is a maximum the total energy stored is also given by the equations,

E_s /

$$E_s = \tfrac{1}{2} m\omega^2 A_{st}^2 \int_0^{\ell} f^2(z)\, dz \qquad \ldots(4.4)$$

$$E_s = \tfrac{1}{2} EI A_{st}^2 \int_0^{\ell} f''^2(z)\, dz \qquad \ldots(4.5)$$

4.3 COMPUTER SIMULATION

A digital compu.er was used to evaluate the vibration energy levels for a number of structures. The analysis was carried out in the form of a simulation of the build up of vibration in a structure from a small initial amplitude to the r.m.s. value. The net energy input in each cycle was calculated from equations (4.1) and (4.3) and this was added to the total energy of vibration (given by equation 4.5) at the end of each cycle. The amplitudes of successive cycles were computed from equation (4.5)

This procedure allowed energy levels to be obtained over a range of amplitudes and the format of a vibration build up allowed an estimation to be made of the probable quickness of response of the respective structures to batches of high intensity turbulence. This gave an indication of the ability of the frequency response function of each structure to predict its r.m.s. response from the r.m.s. excitation. The simulation was also carried out using equation (4.2) to calculate the excitation energy so that the linear case could be compared with the non-linear case.

The computer used for the analysis was an I.B.M. 360:50 which was programmed in Atlas Autocode. A flow chart for the program is shown in Fig (12).

The wind input parameters used were relevant to a 70 m.p.h. mean wind speed in city centre conditions of roughness. The root mean square of/

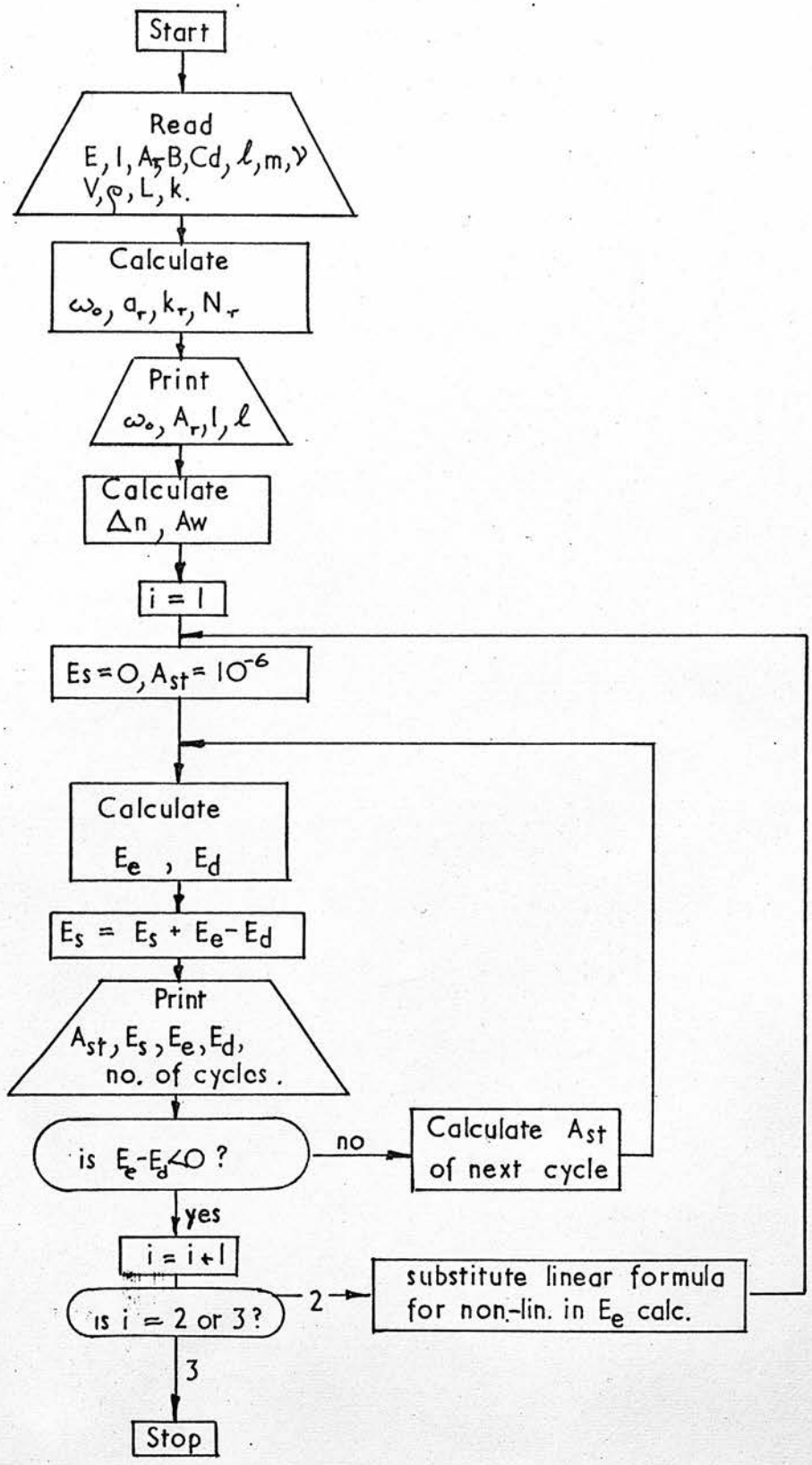


Fig 12a Flow chart for energy analysis programme.

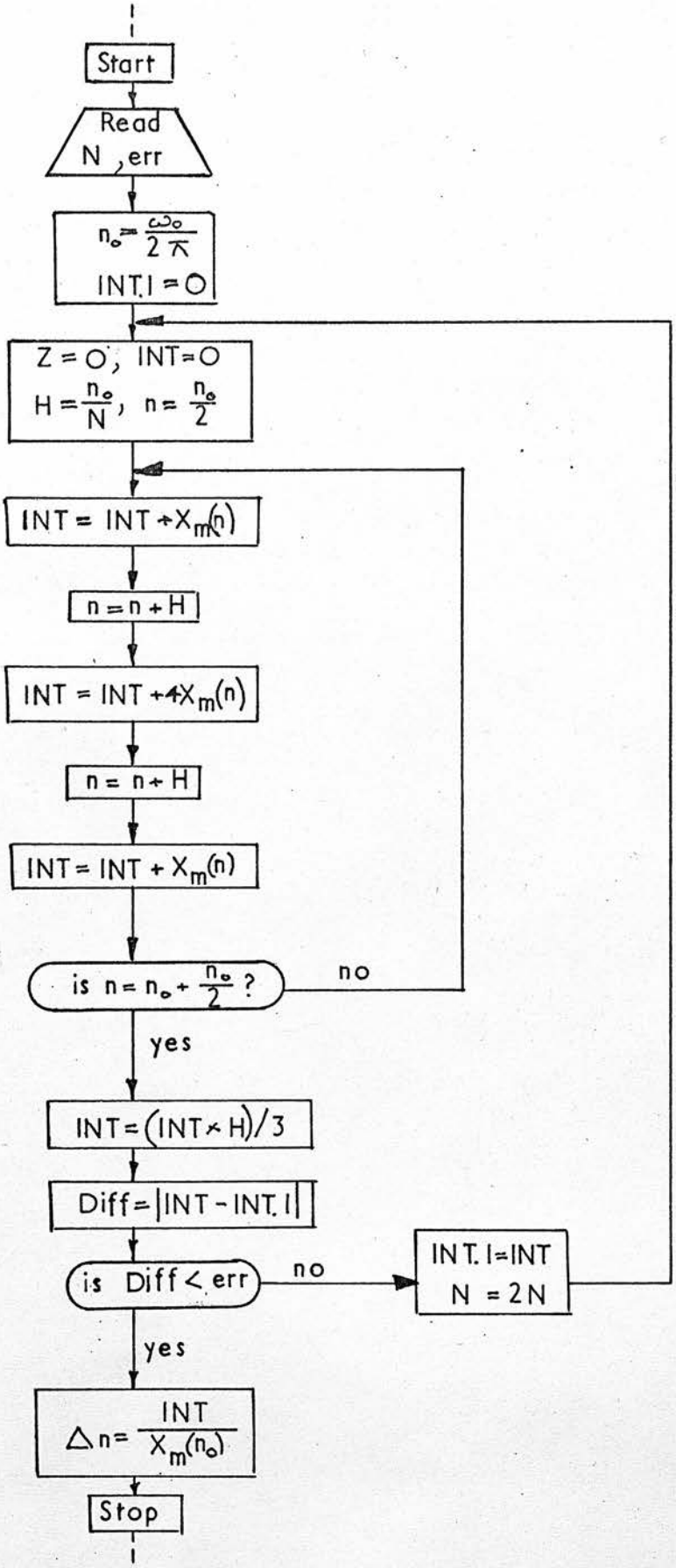


Fig 12 b Flow chart for routine to calculate Δn in energy analysis programme.

Type of Structure	A_r in^2	B in	I in^4	E lbf/in^2	m lb/in	l ft	n_b Hz
Lattice Tower T'1	8·36	10	2,455	30×10^6	·005	30	16·10
" " T'2	8·36	10	2,455	30×10^6	·005	60	4·02
" " T'3	8·36	10	2,455	30×10^6	·005	100	1·45
" " T'4	8·36	10	2,455	30×10^6	·005	140	0·74
" " T'5	72·00	10	20,000	30×10^6	·046	60	4·46
" " T'6	72·00	10	4×10^6	30×10^6	·046	200	2·53
" " T'7	72·00	10	32,400	30×10^6	·046	200	0·45
" " T'8	72·00	10	32,400	30×10^6	·046	100	1·93
" " T'9	72·00	10	1×10^6	30×10^6	·046	100	10·00
" " T'10	72·00	10	21,500	30×10^6	·046	180	1·16
" " T'11	72·00	10	2×10^6	30×10^6	·046	200	3·52
" " T'12	6220·00	10	5×10^6	30×10^6	1·40	150	9·64
Shear Wall Building B'1	6220·00	10	5×10^6	4×10^6	1·40	100	1·45
" B'2	6220·00	10	5×10^6	4×10^6	1·40	200	0·35
" B'3	6220·00	10	155×10^6	4×10^6	1·40	200	2·04

Table 2 Properties of Structures used in energy analysis

of the wind velocity component at the natural frequency of each structure was computed from the Davenport reduced wind spectrum. The bandwidth used was obtained from the following expression,

$$\Delta\omega = \frac{\text{Area under resonance peak in frequency response function}}{\text{height of resonance peak}}$$

It was, therefore, the width of a rectangular resonance peak with the same area and height as the resonance peak for the structure as given by the frequency response function. The reasons for the adoption of a bandwidth calculated in this manner are given in appendix 2. The properties of the structures for which the vibration simulation was carried out are given in table (2).

The static response of the structures to the mean wind was also estimated. This was done by calculating the mean wind pressure from the equation,

$$P = \tfrac{1}{2} \rho V^2$$

and assuming this to be uniformly distributed over the face of the structure. The deflection at the top of the structure was found from the equation,

$$\text{deflection} = \frac{wl^3}{8EI}$$

where w = total load

$= PBl$

l = height of structure

4.4 RESULTS/

4.4 RESULTS

A summary of the results is given in table (3). The results are shown in detail in graphs (14) to (28) in which the levels of excitation energy and damping energy are plotted against amplitude of vibration. Both the linear and non-linear cases are shown. The ordinates between the excitation and damping energy curves denote the net energy input to the various structures at each amplitude. Graphs (29) to (43) show the net energy input curves for the non-linear case added to the curves of total vibration energy. These illustrate the relative levels of input energy to energy of vibration for the structures and show their variation with amplitude.

The number of cycles required for a given build up of vibration may be deduced from these curves. For example, at amplitude A in graph (31) the energy of vibration is given by AB and the net input energy by BC. The total energy in the next cycle is therefore AC and the amplitude of this cycle is found by moving horizontally from C until the total energy curve is cut at D. The operation may be repeated and a stepping procedure developed to obtain the number of cycles required for the amplitude to build up to the root mean square value.

4.5 DISCUSSION OF RESULTS

The object of the study was to evaluate the levels of energy input and energy of vibration in structures vibrating in response to wind loads. in order that their relative magnitudes could be compared. Of particular interest were the relative levels of input energy calculated using linear and non-linear theories and the relationship between the levels of input energy and energy of vibration. These two topics are/

are discussed separately.

Before discussing the results in detail, however, it is appropriate to mention the question of the accuracy with which the analytical simulation is capable of representing a full scale system and the effect which any errors may have on the validity of the conclusions. As the investigation was a comparative study it was felt that it was necessary to ensure only that the calculated energy levels were of the correct order of magnitude. The use of standard equations of strain energy and damping energy in conjunction with highly simplified structural models(cantilevers with uniform distribution of mass and stiffness) was therefore considered justified and the calculated levels of damping energy and energy vibration are considered to be of the correct order of magnitude.

The calculated levels of input energy from the wind to the structures are of more doubtful validity. As with the structural models a highly simplified system was assumed. It is felt that the initial assumptions of constant phase input, non random cross-correlation etc. are likely to cause overestimation of the input energy levels. The extent of the errors is difficult to determine, however, without more detailed data on the interaction of the wind with vibrating structures. It is only possible at this stage to note that such errors are likely to occur and to bear this in mind when considering the conclusions.

Comparison of Input Energies calculated from linear and non-linear theories

It can be seen from table (3) and graphs (14) to (28) that the error in the calculated root mean square deflection, which is incurred if linearity is assumed, can be large, especially in the case of light structures/

structures such as lattice towers. The discrepancy between the energy input curves increases with amplitude of vibration because the aerodynamic damping energy, which is allowed for in the non-linear case, is a function of the velocity of the structure and this increases with amplitude. As the root mean square deflection is determined by the relationship between the excitation energy and mechanical damping energy curves, the error in the calculated r.m.s. deflection depends on the amount of mechanical damping in the system. The lighter the damping, the larger the error is likely to be.

The relationship between the mechanical damping energy and the total energy of vibration is also important because, for a particular value of the critical damping ratio, the mechanical damping energy is related to the mass of the structure. For large, heavy structures, such as the buildings simulated in this study, the energy dissipated per cycle by mechanical damping is relatively high, even at low structural velocities. As a result of this, the mechanical damping energy curve crosses the excitation energy curve at a low amplitude, even when the critical damping ratio is small. The error due to the assumption of linearity is therefore small.

With light structures, however, large amplitudes, and consequently large structural velocities, are possible before the damping energy approaches the excitation energy level. The errors for these structures are therefore considerable.

The discrepancy between the linear and non-linear cases depends on the velocity of the structures in question. This is a function of the amplitude and frequency of the vibration. The calculated r.m.s. deflection in/

in both the cases depends on the value of the mechanical damping ratio, is a function of the total energy of vibration. A measure of the total energy is the stiffness of the structure EI. The discrepancy between the linear and non-linear theories could therefore be expected to be a function of the parameter $\frac{EI}{\omega}$, for a range of structures for which the critical damping ratio is constant. The ratios of linear to non-linear r.m.s. deflections calculated for the structures in this investigation were plotted against $\frac{EI}{\omega}$. The resulting curve is shown in graph (44).

It can be seen that there is a relationship between the ratio of calculated r.m.s. deflections and the parameter $\frac{EI}{\omega}$. A considerable discrepancy between the linear and non-linear cases is evident for values of $\frac{EI}{\omega}$ below 1×10^{11}. The error decreases with rising $\frac{EI}{\omega}$ and tends to zero at values of $\frac{EI}{\omega}$ greater than 1×10^{13} where the ratio of r.m.s. deflections approaches unity.

The significance of this curve is that it suggests that slender structures which are sufficiently light to have a high natural frequency are likely to suffer quite high degrees of aerodynamic damping and that an allowance for the non-linearity of the system should be made when calculating their response to turbulent winds. The curve shown in graph (44) was compiled from data relevant to structures with a critical damping ratio of 0.01. In practice, light lattice structures are likely to have lower damping ratios and the error involved in assuming linearity could be expected to be greater than those indicated here.

The results also suggest that in the case of structures which have a low natural frequency or in which the vibration energy level is high (i.e. structures/

structures for which $\frac{EI}{\omega} > 1 \times 10^{13}$), the error involved in assuming linearity is likely to be small.

It is thought that a series of curves such as graph (44), plotted for a range of critical damping ratios, could be used in design to distinguish between those structures for which an allowance for the non-linearity is necessary and those for which it is not.

Comparison of Input Energy with Energy of Vibration

The energy input from the wind to a structure is determined by the drag forces which the wind can induce to act on its surface. In the foregoing analysis the drag per unit length of the structures was calculated from a drag coefficient (assumed unity for all the structures analysed) and the breadth of the structures B (in addition to the wind velocity and air density). From table (2) it can be seen that B was not increased in proportion to EI over the range of structures analysed. It was thought that this was representative of the full scale situation. It resulted in the ratio of energy input to energy of vibration decreasing as the mass and stiffness of the structures were increased. The effect was a large variation in the energy input to energy of vibration ratio with mass of the structures.

The results show that for light structures, such as lattice towers, the ratio of energy input to vibration energy was high causing the number of cycles required for the amplitude of vibration to build up from a small initial value to the root mean square deflection to be small. With the heavier structures, however, this ratio was low, and a large number of cycles was required before the vibration build up was complete.

In/

In practice a structure in a turbulent wind is in a state of continuous vibration in response to a periodic forcing function whose amplitude is continually changing. Light structures, with high input energy to vibration energy ratios, are likely to follow variations in the wind closely. When the turbulence intensity increases the vibration amplitude will rise fairly rapidly and when the wind decreases the aerodynamic damping forces, which are high relative to the total vibration energy are capable of quickly reducing the vibration amplitude. The result is that a light structure can be expected to undergo a response which follows wind deviations closely and the excitation and response are related by the frequency response function at almost all times. In this situation, the root mean square response may be calculated from the root mean square excitation using the frequency response function.

The heavier structures, however, require time to respond to increases in the intensity of turbulence and unless a batch of high intensity turbulence lasts for sufficient time for the transient part of the response to be completed, the vibration amplitude will not reach the level which would be predicted by multiplying the amplitude of the excitation function by the frequency response function. Similarly, when the turbulence intensity decreases the structure will tend to carry on vibrating through the period of lull because the aerodynamic forces acting on it are small compared to the total energy of vibration. Heavy structures are therefore likely to have a much smoother response to wind turbulence than light structures and their amplitudes of vibration are likely to remain more steady.

The analysis presented in this investigation represents a physical system/

system which is much simpler than the one which occurs in practice. It does show, however, that there is likely to be a considerable variation, in the ratio of input to vibration energy for different structures and it demonstrates that this ratio is likely to be small for heavy structures such as tall buildings. The ability of the frequency response function to represent the mechanical properties of a structure in a dynamic wind loading calculation depends on whether or not the transient parts of the response are of significant duration. This depends on the mass of the structure and on the characteristics of the turbulence. If the turbulence is fairly steady the effect of neglecting the transients may be slight but if the turbulence is of high intensity with large randomly spaced fluctuations in wind speed the effect of neglecting the transients may be considerable for certain types of structure. The important parameters are the duration of constant phase sequences in the component of turbulence causing the vibration and the relationship between these and the lengths of the transients of the structure concerned. Little data are available at present on the detailed characteristics of turbulence or on the possible duration of constant phase batches and it is therefore difficult to estimate the importance of the transient components of response. It seems likely, however, that in the case of tall buildings, subjected to high intensity random turbulence, a more sophisticated technique than that of simply multiplying the excitation function by the frequency response function may be required for the accurate prediction of their dynamic response.

4.6 CONCLUSIONS/

4.6 CONCLUSIONS

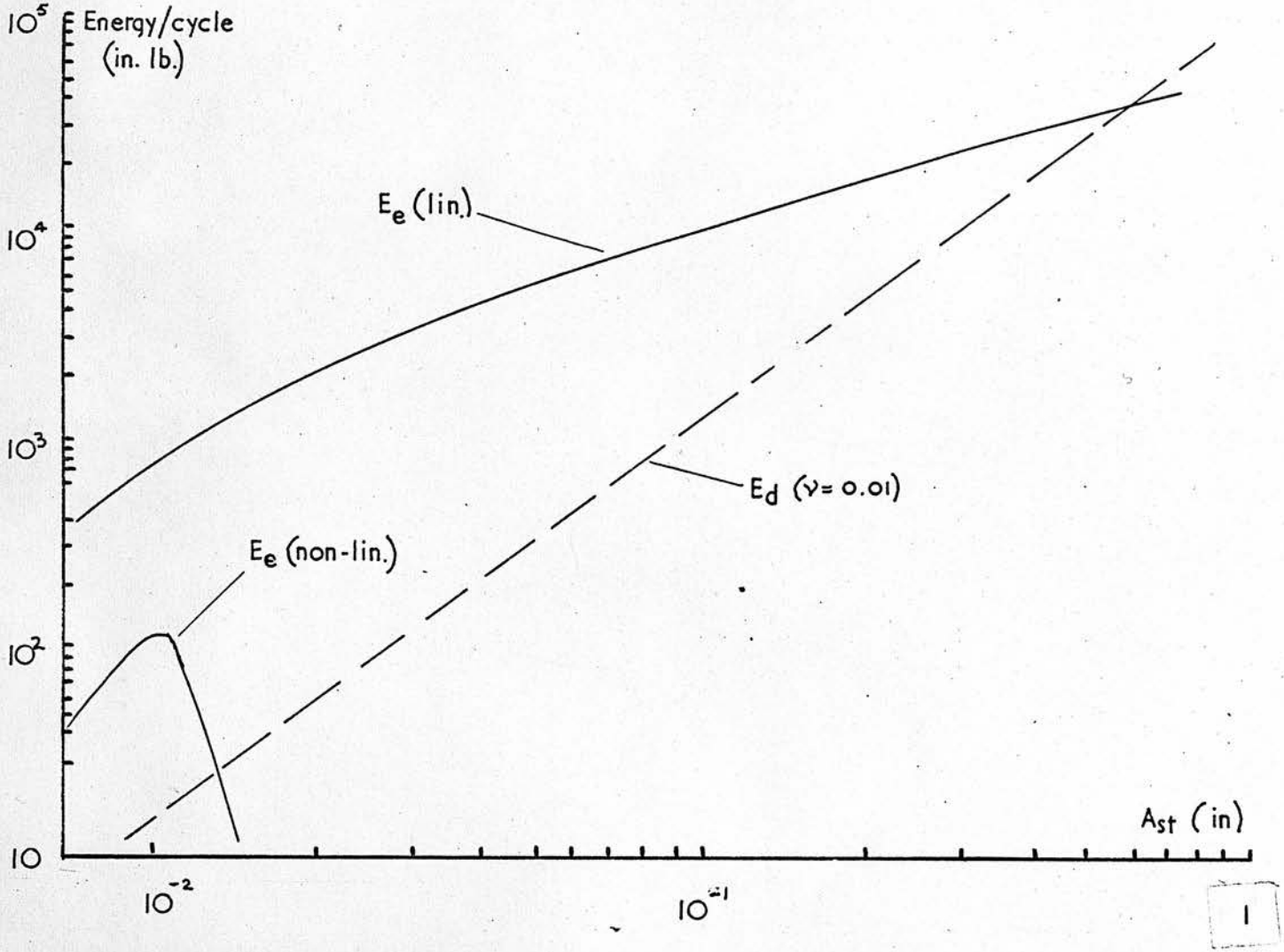
The investigation presented here was a theoretical study in which a number of simplifying assumptions were made. It is felt that the principal effect of these was to cause an overestimation of the predicted levels of input energy. It is likely that this caused the predicted errors due to the assumption of linearity to be exadurated but diminished the predicted lengths of the transients in the response and thus reduced the apparent importance of these. The author believes that the effects described here are nevertheless of the correct order of magnitude and that the following conclusions may be drawn.

1. The non-linearity of the wind driven vibrating system must be taken into account when predicting the dynamic response of slender structures. This is especially true for light structures with high natural frequencies.

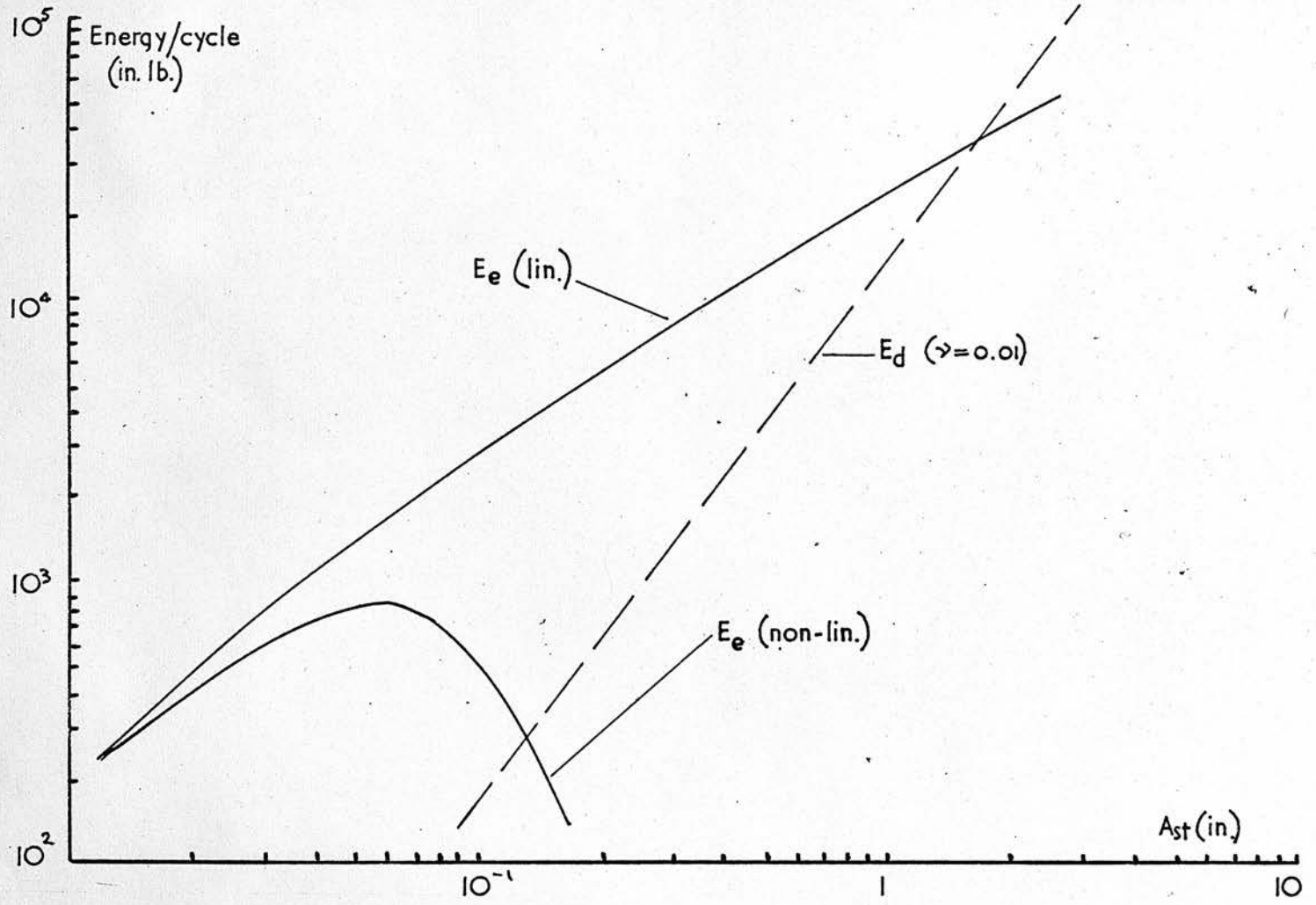
2. The energy stored in a vibrating structure is usually considerably greater than the energy input from the wind/cycle. In this situation the frequency response function is not capable of giving a reliable prediction of the dynamic response of a structure to the wind and a more sophisticated mathematical model, which takes account of the inertia of the structure, is required to represent its mechanical properties.

Struct.	no Hz	m lb/in	No. of cycles to reach r.m.s. deflection non-lin	lin	r.m.s. deflection in non-lin	lin	lin / non-lin	static deflection in
T'1	16·10	·005	5	48	0·013	0·579	44·5	0·28
T'2	4·02	·0051	7	48	0·13	1·60	12·3	4·45
T'3	1·45	·005	15	54	0·72	3·50	4 ·86	35·0
T'4	0·74	·005	21	45	3·20	9·51	2·98	136·0
T'5	4·46	·046	24	58	0·043	0·10	2·33	0·56
T'6	2·53	·046	29	59	0·061	0·11	1·8	1·39
T'7	0·45	·046	41	56	0·77	0·87	1·13	42·50
T'8	1·93	·046	32	60	0·096	0·15	1·57	2·65
T'9	10·00	·046	15	53	0·011	0·045	4·0	0·086
T'10	1·16	·046	36	44	0·35	0·47	1·34	4·18
T'11	3·52	·046	26	59	0·036	0·076	2·11	0·69
T'12	0·64	1·40	42	44	0·023	0·024	1·04	0·087
B'1	1·45	1·40	41	44	0·013	0·013	1·00	0·0023
B'2	0·35	1·40	43	44	0·035	0·035	1·00	0·036
B'3	2·04	1·40	38	43	0·0045	0·0046	1·01	0·0089

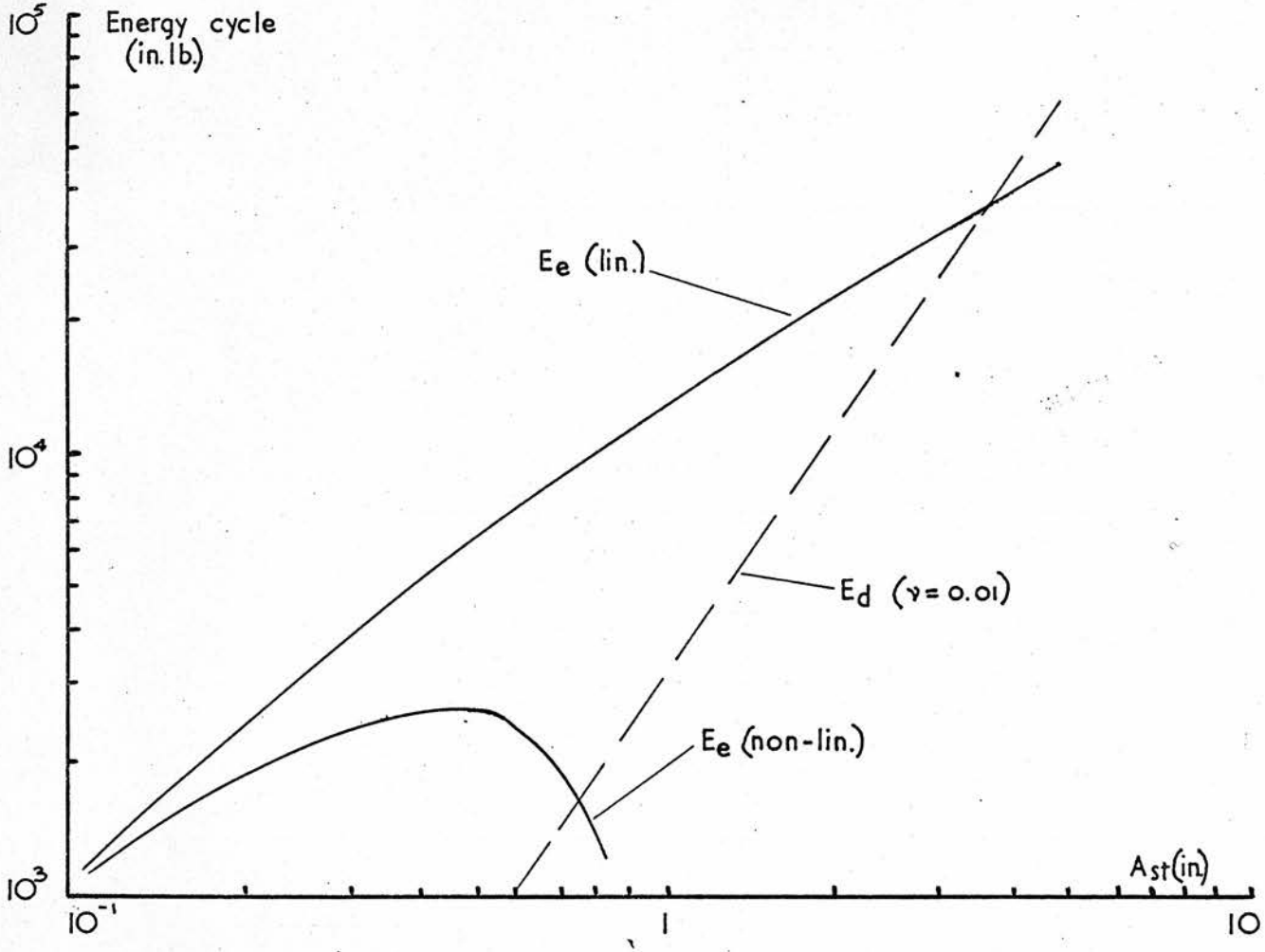
Table 3 Summary of results of energy analysis.



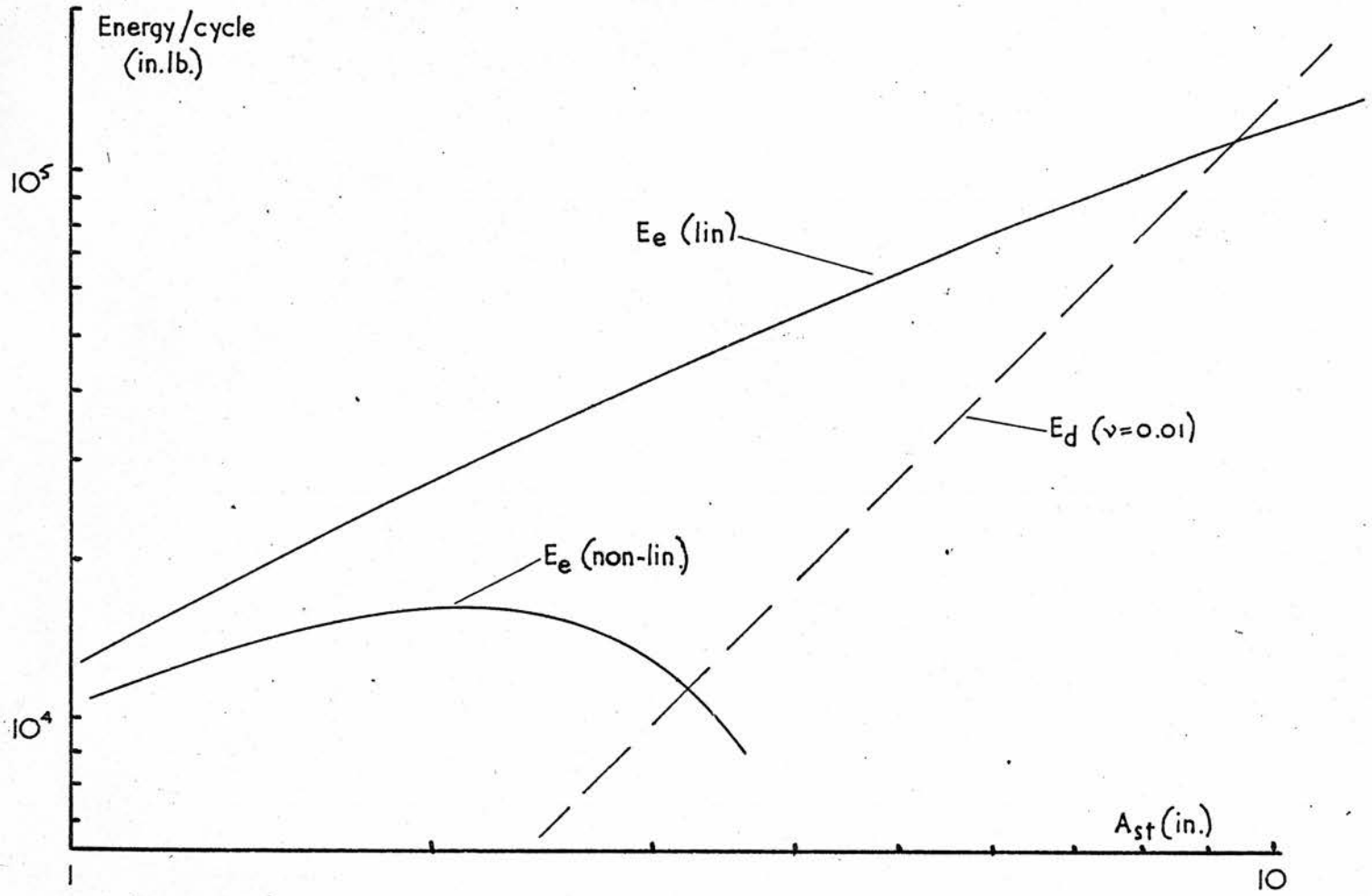
Graph 14 Excitation and damping energy curves Structure T'1



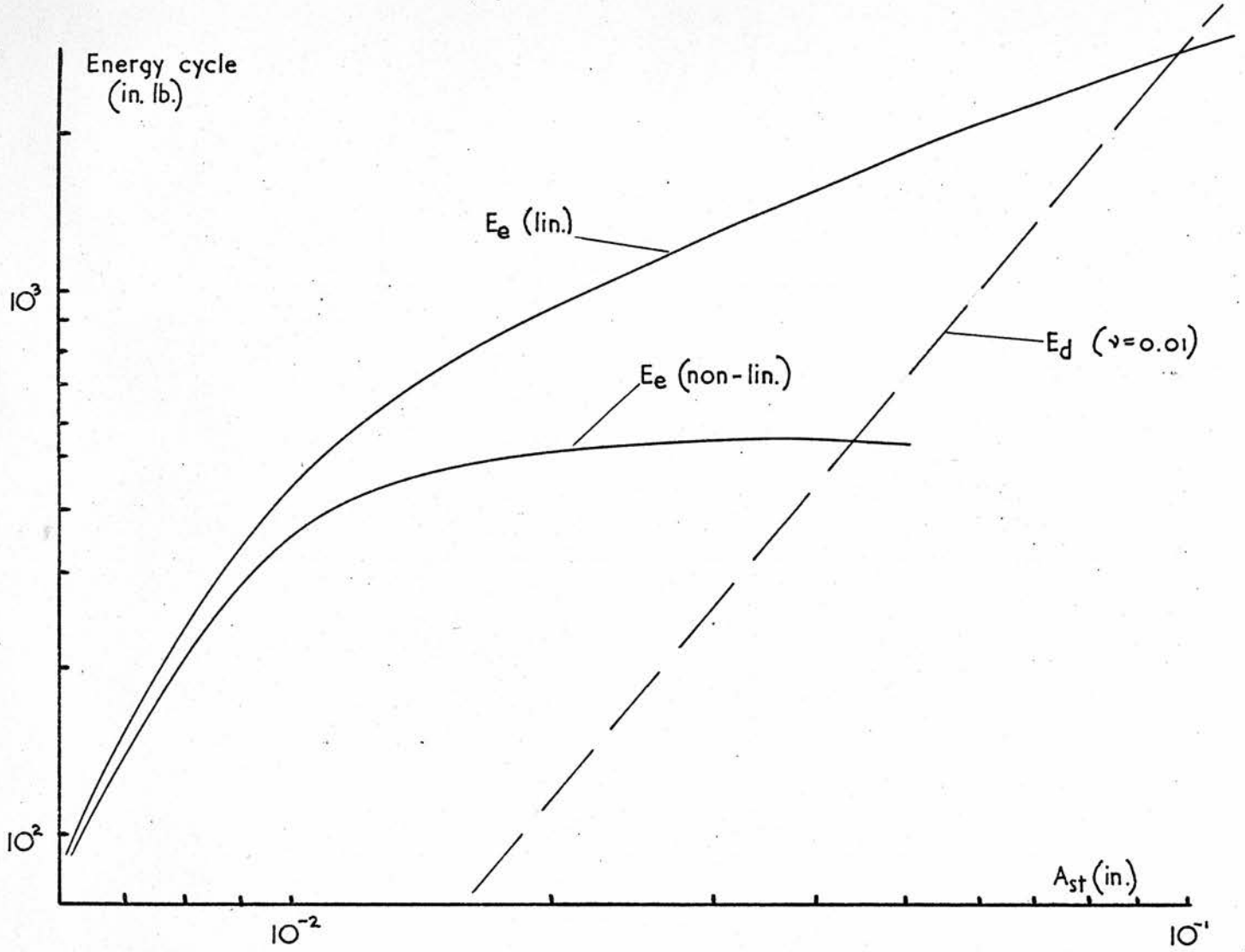
Graph 15 Excitation and damping energy curves Structure T'2



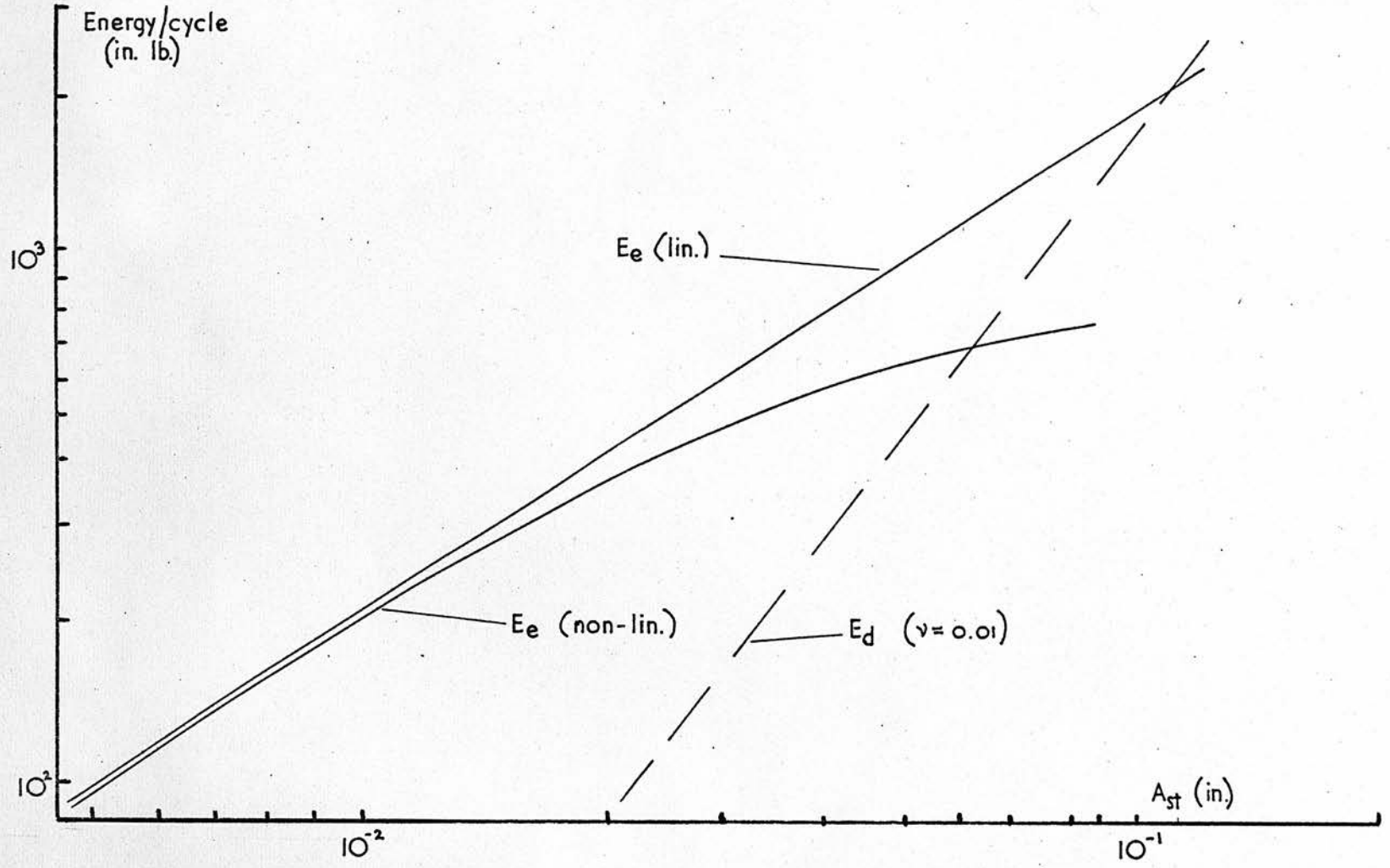
Graph 16 Excitation and damping energy curves Structure T'3



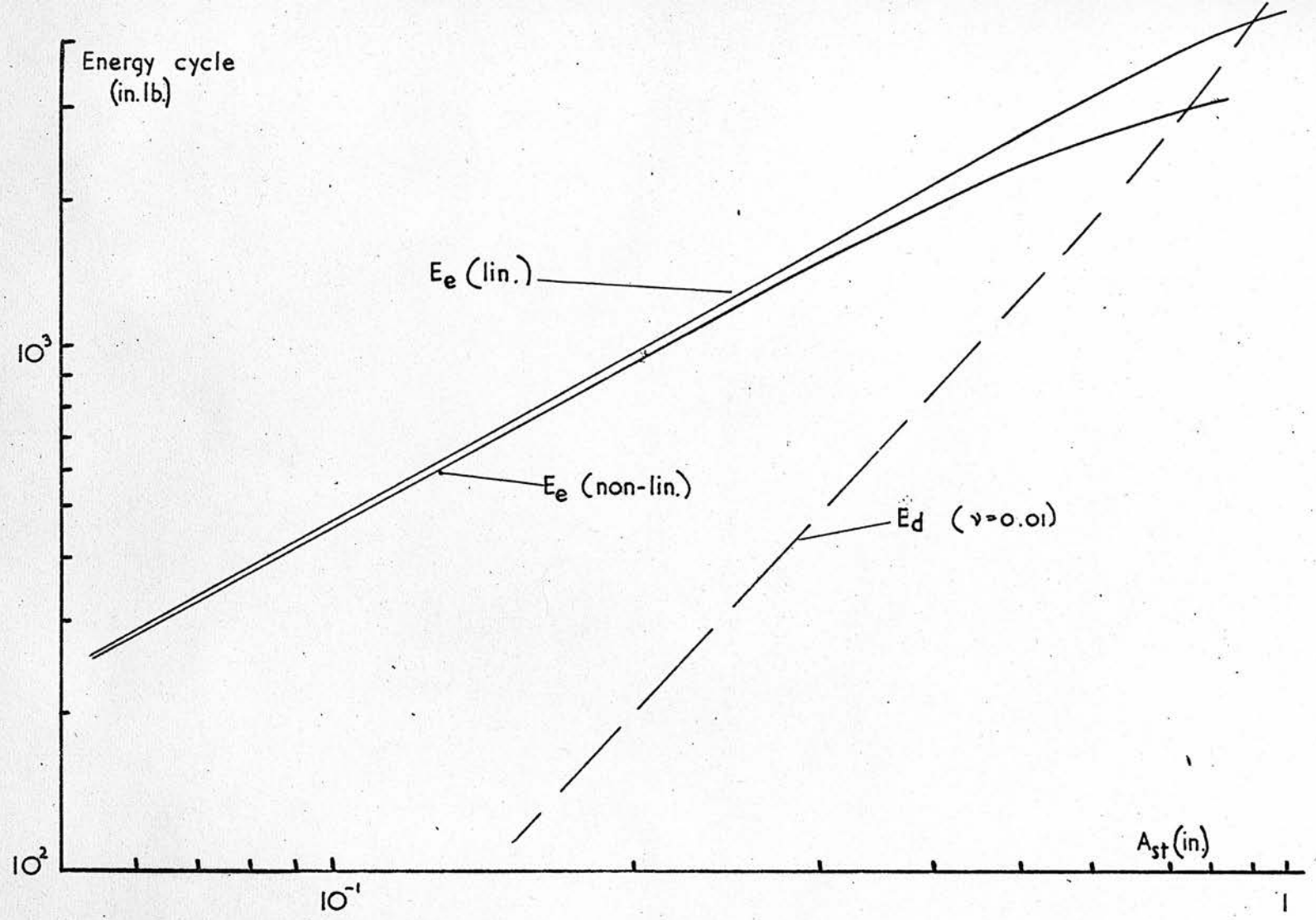
Graph 17 Excitation and damping energy curves Structure T'4



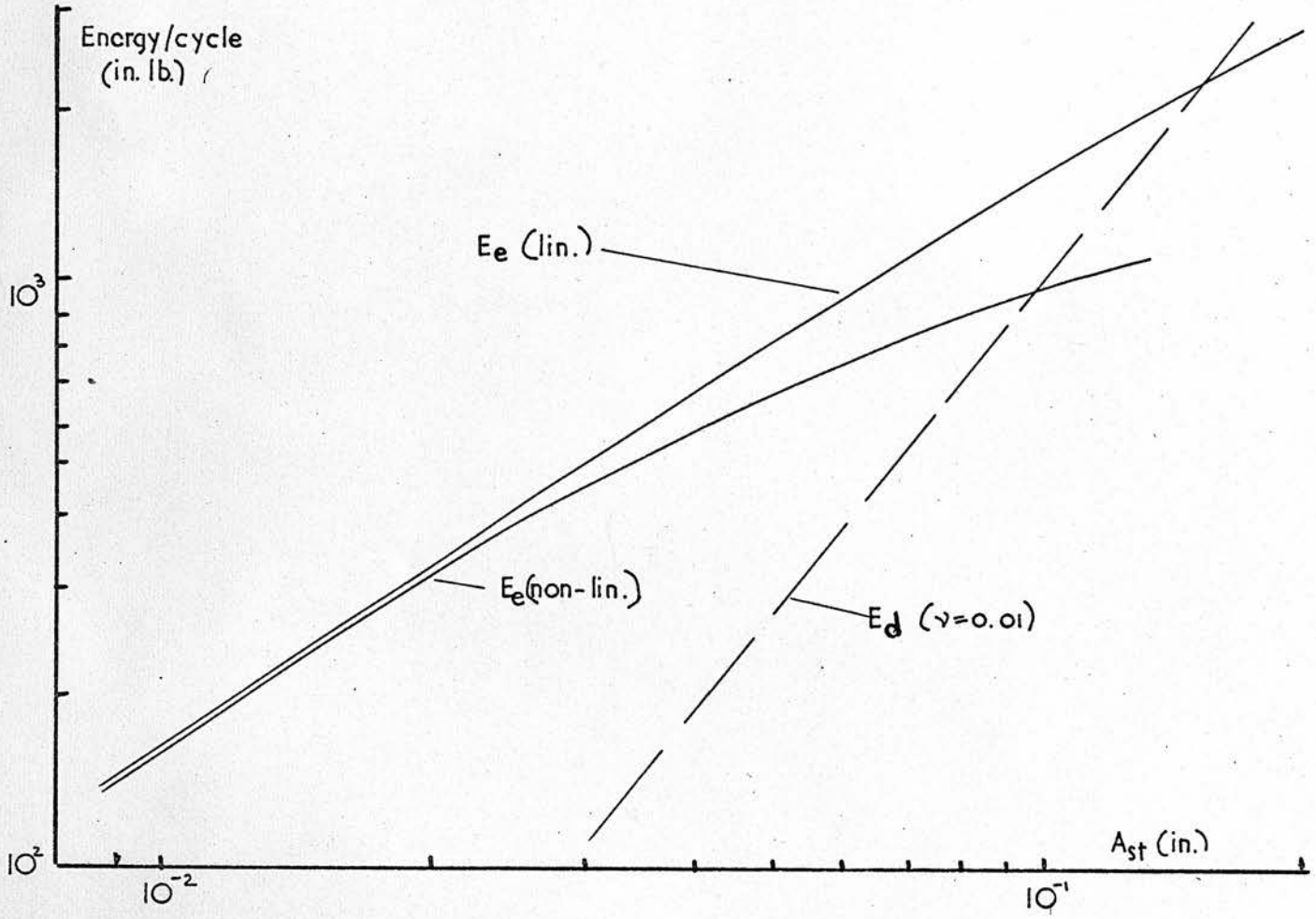
Graph 18 Excitation and damping energy curves Structure T'5



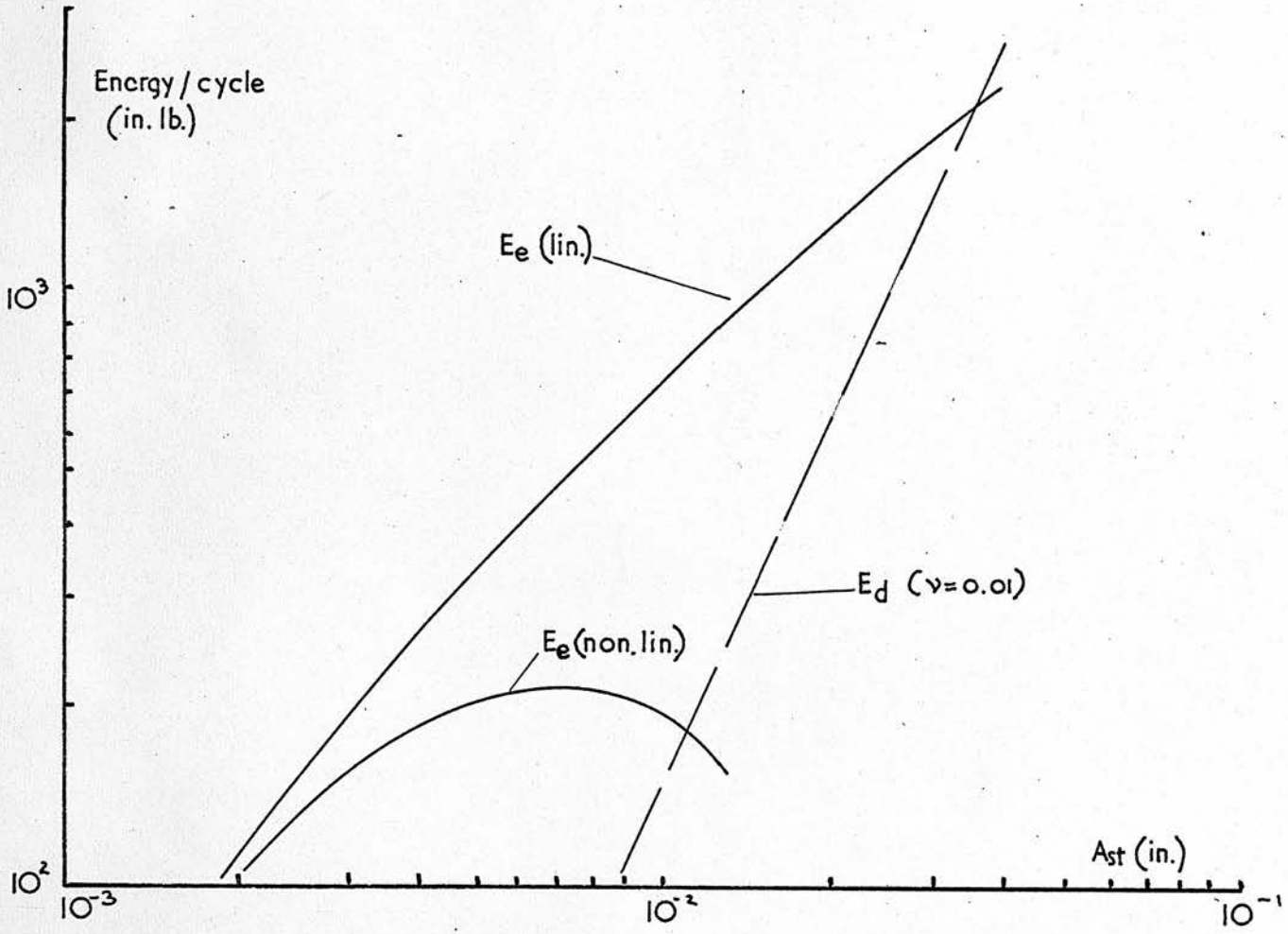
Graph 19 Excitation and damping energy curves Structure T'6



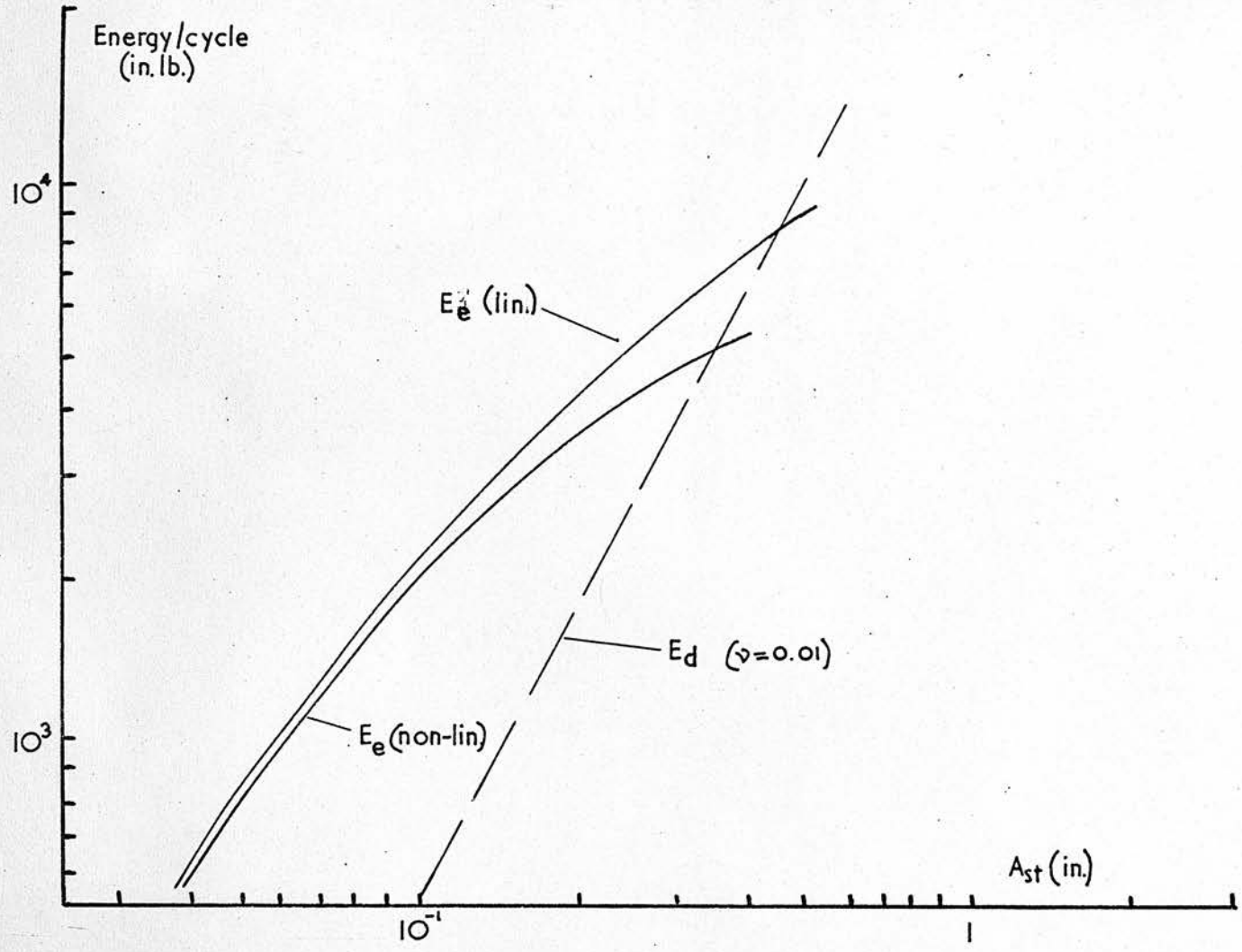
Graph 20 Excitation and damping energy curves Structure T7



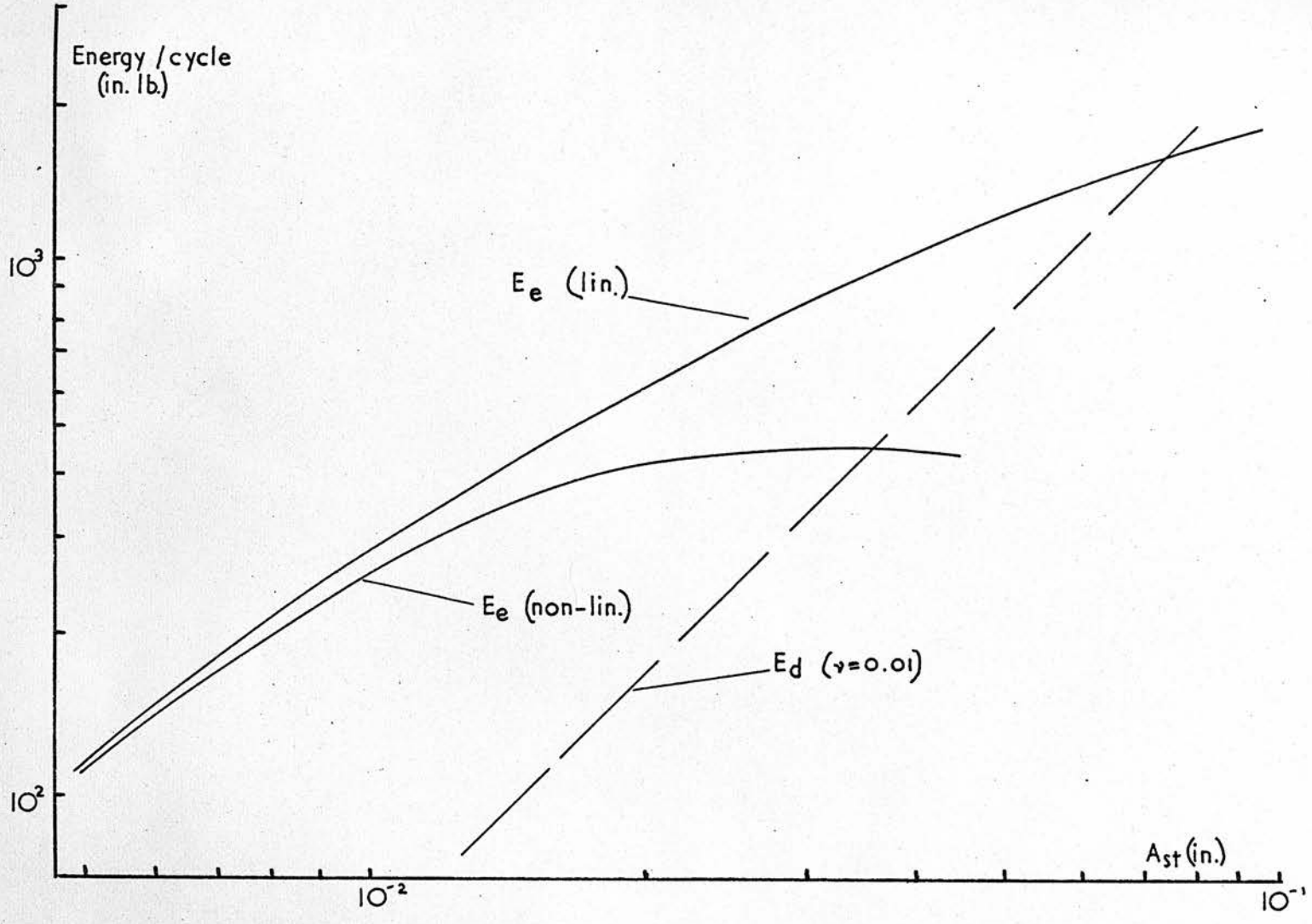
Graph 21 Excitation and damping energy curves Structure T'8



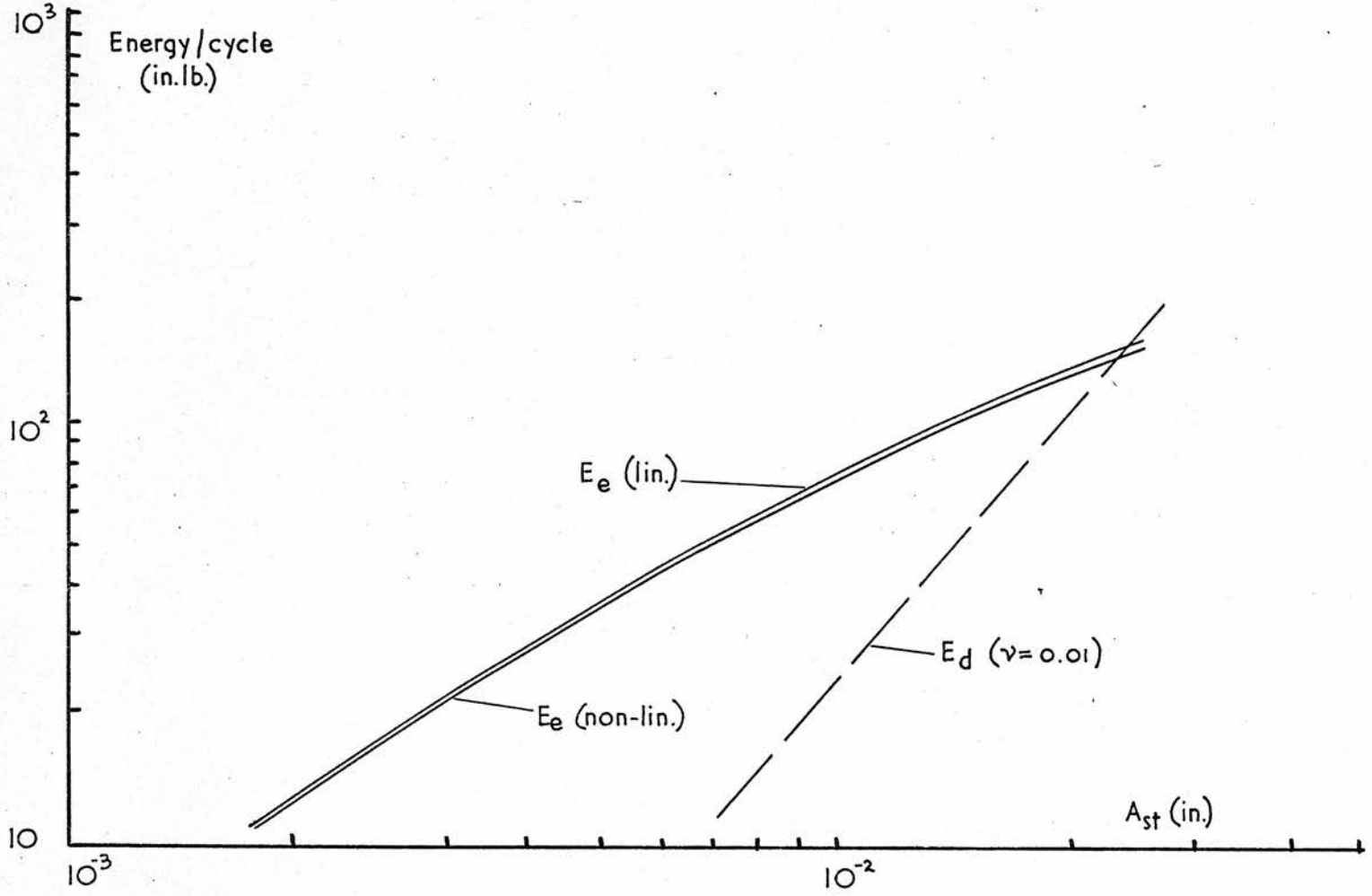
Graph 22 Excitation and damping energy curves Structure T'9



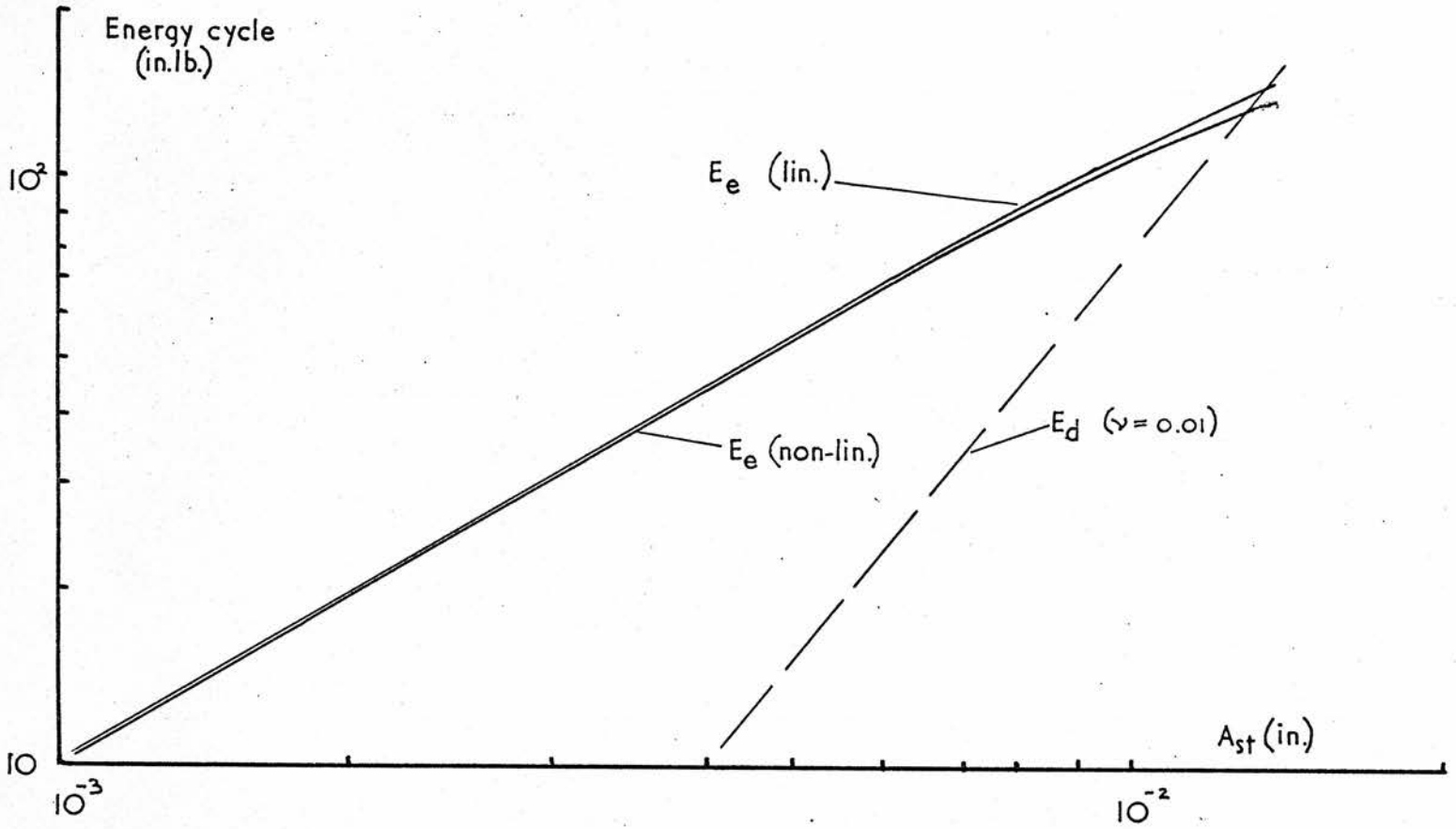
Graph 23 Excitation and damping energy curves Structure T'10



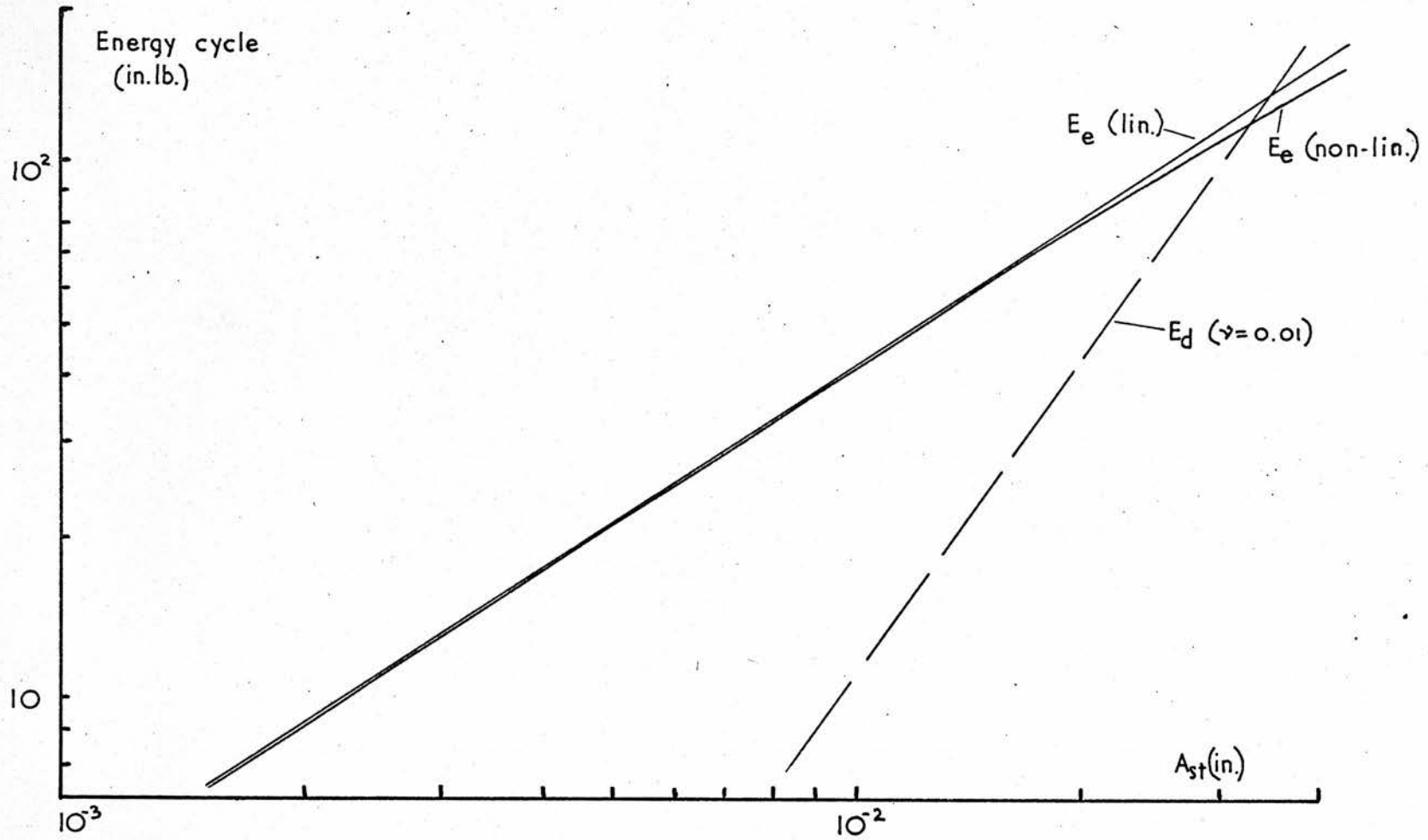
Graph 24 Excitation and damping energy curves Structure T'11



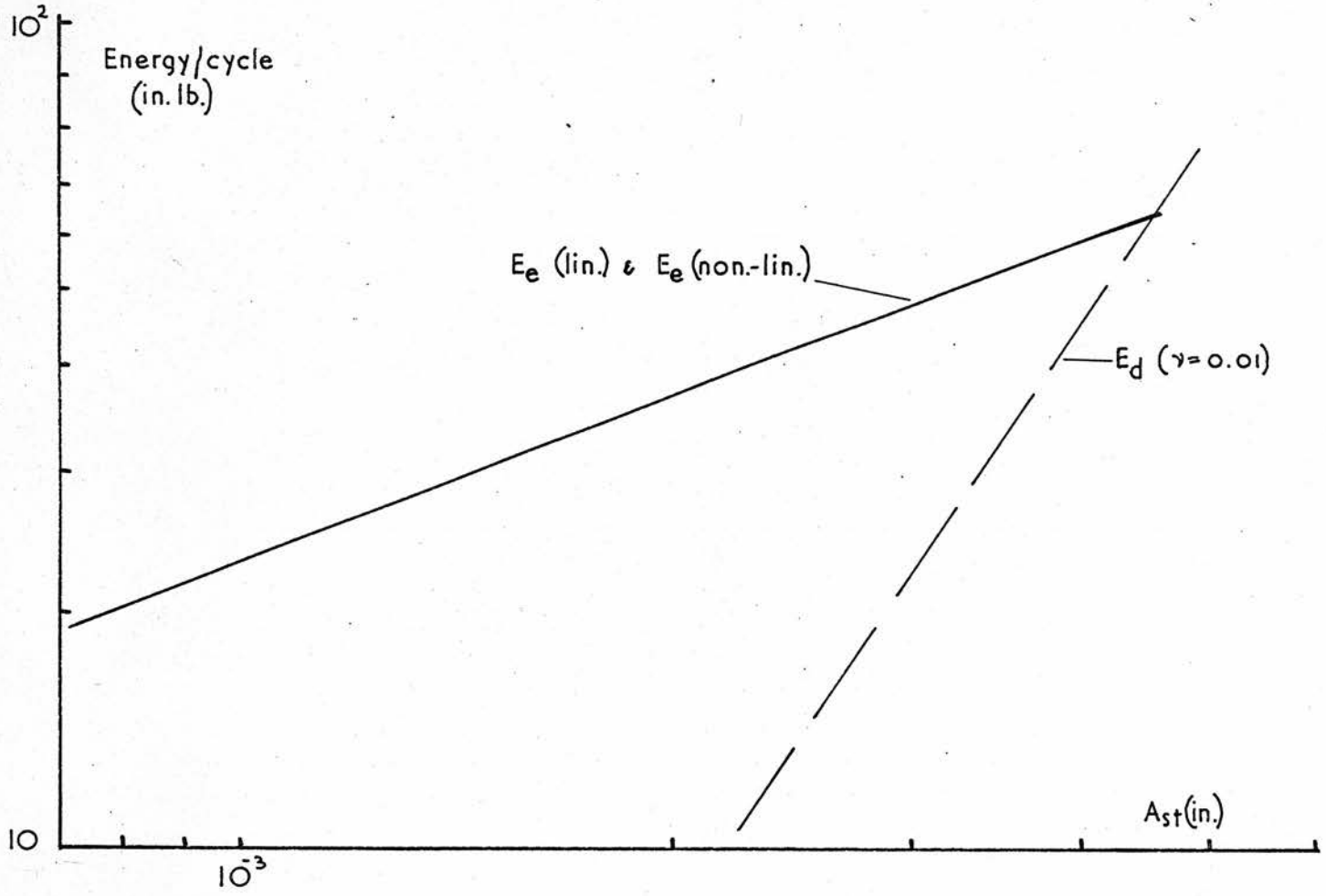
Graph 25 Excitation and damping energy curves Structure T' 12



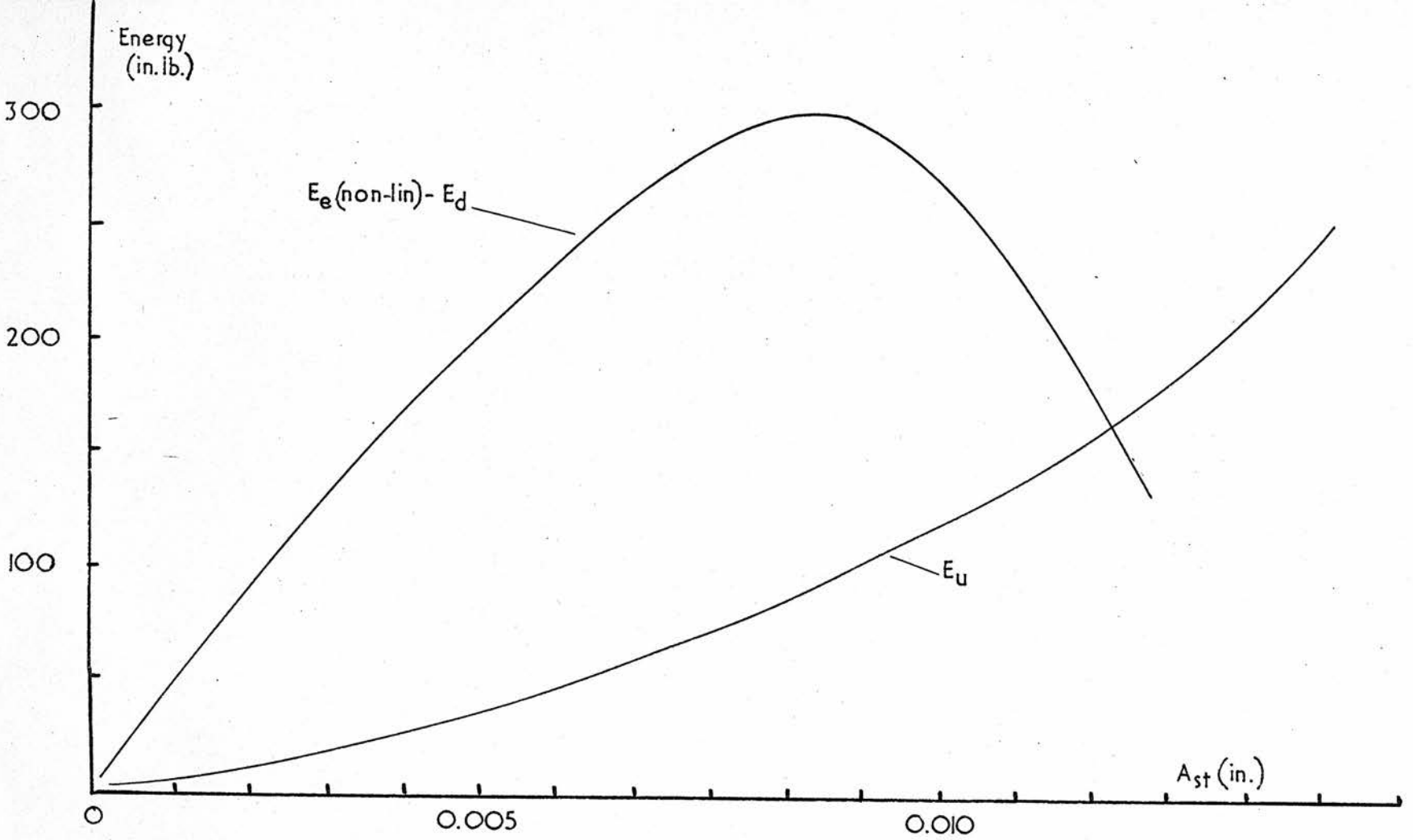
Graph 26 Excitation and damping energy curves Structure B1



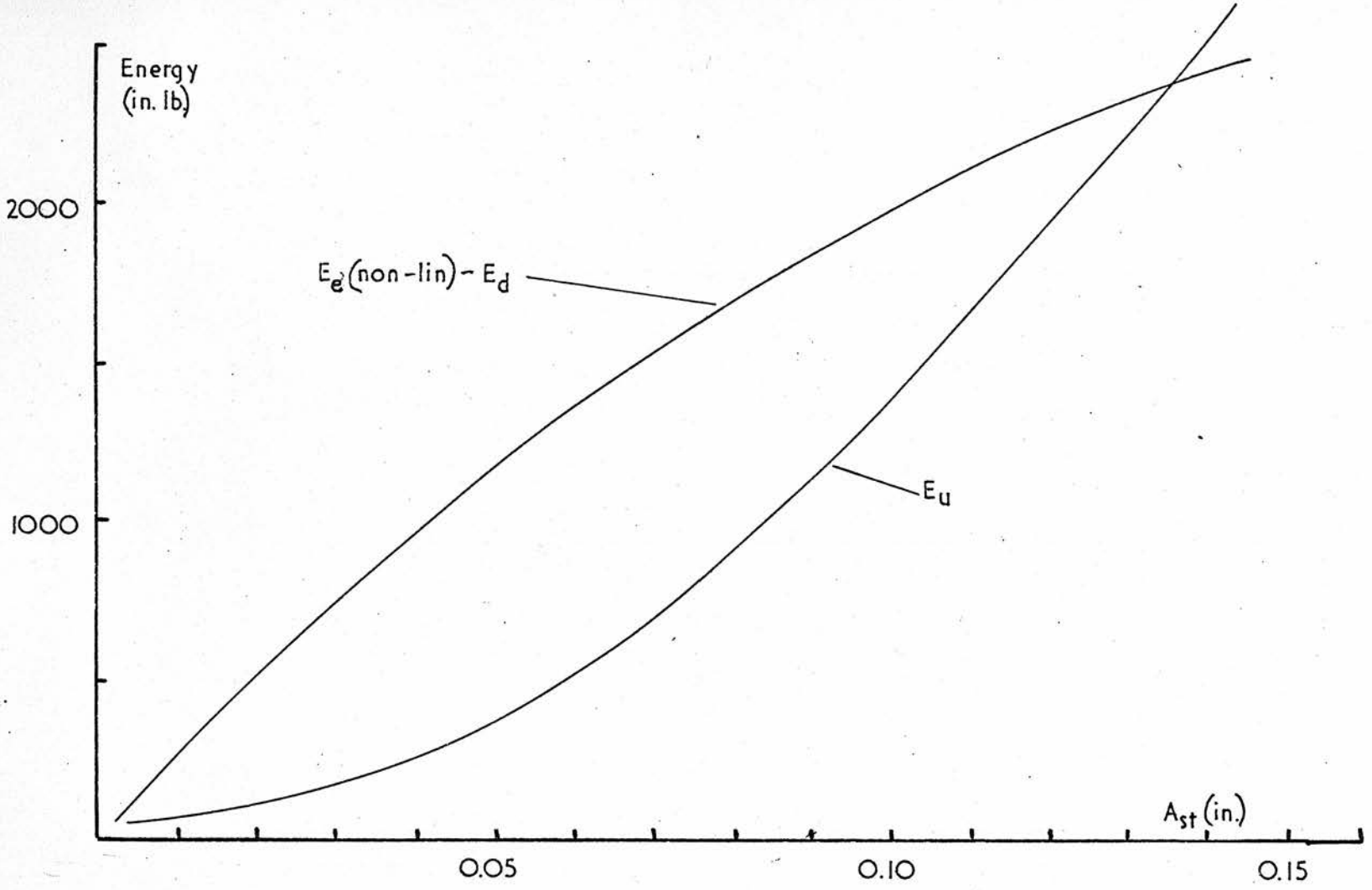
Graph 27 Excitation and damping energy curves Structure B'2



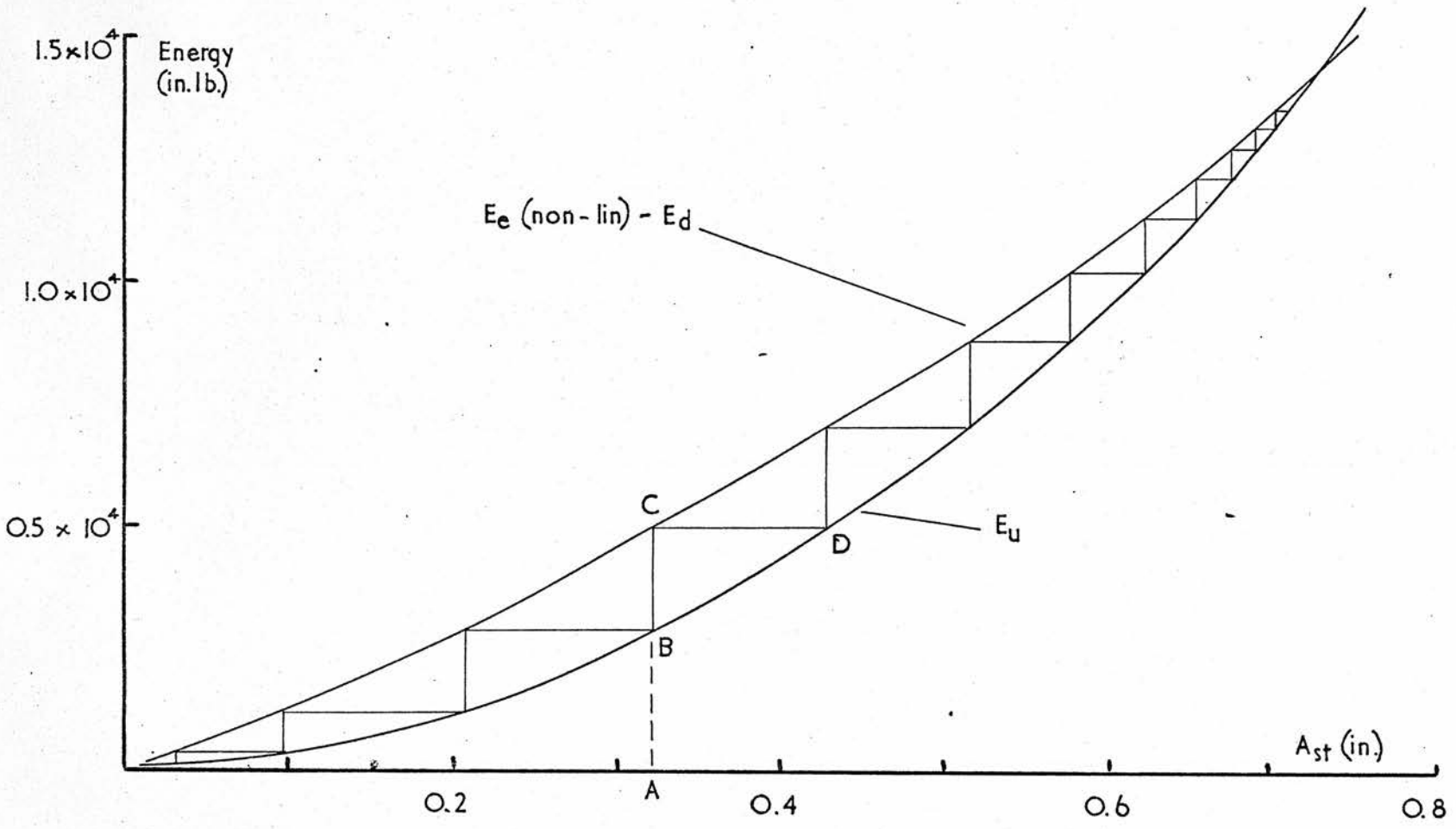
Graph 28 Excitation and damping energy curves Structure B'3



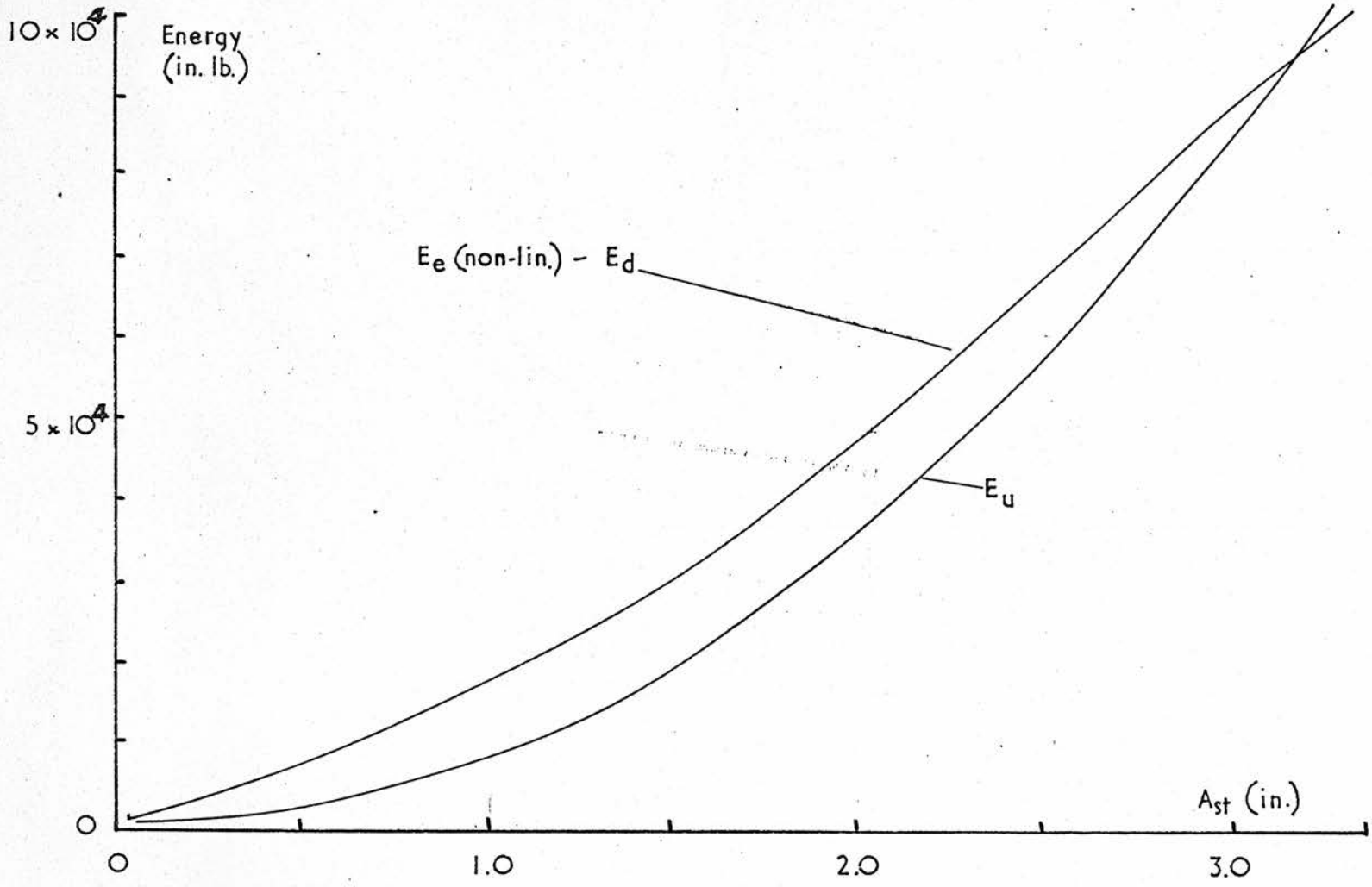
Graph 29 Net input energy added to energy of vibration Structure T'1



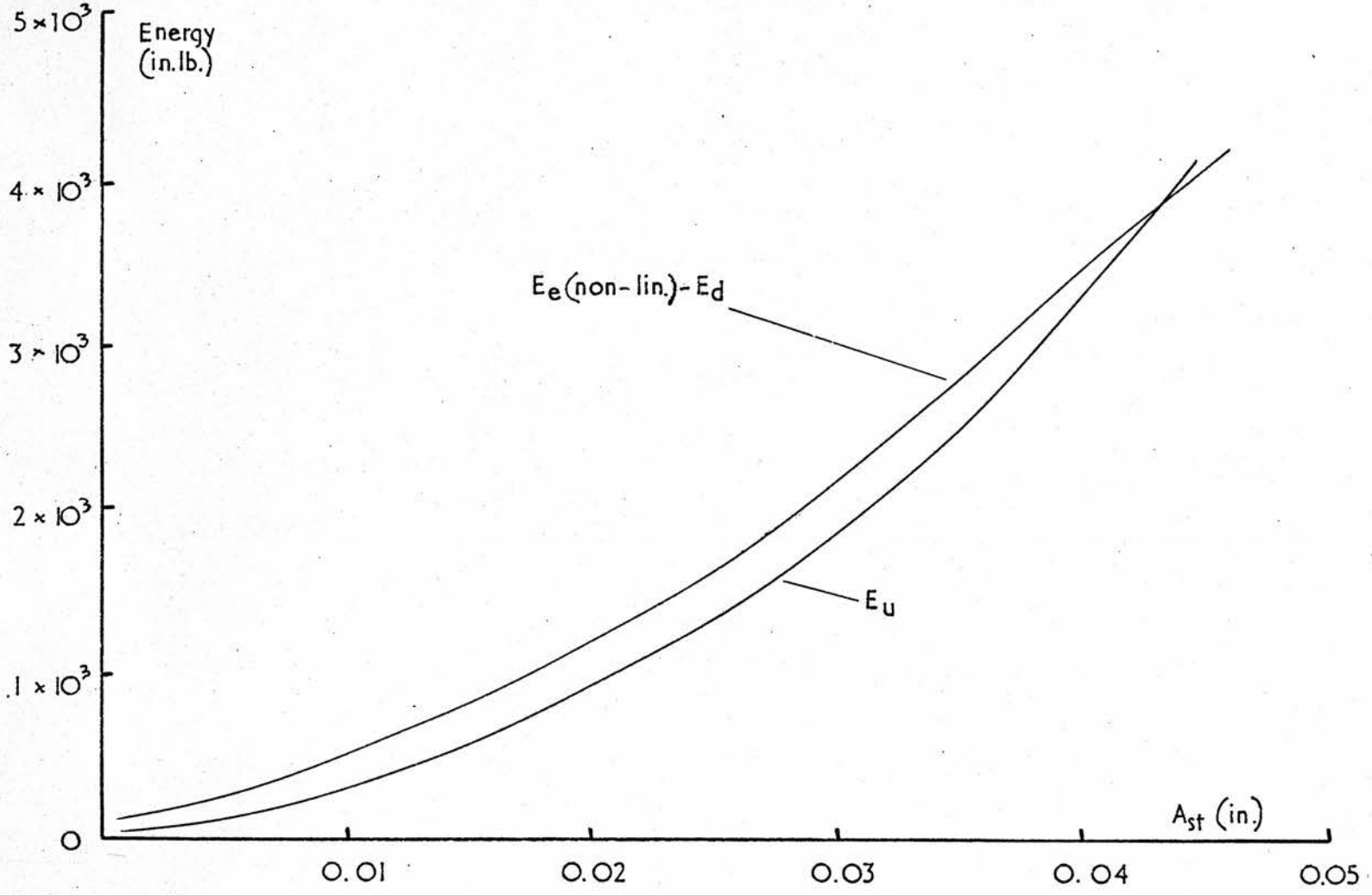
Graph 30 Net input energy added to energy of vibration Structure T'2



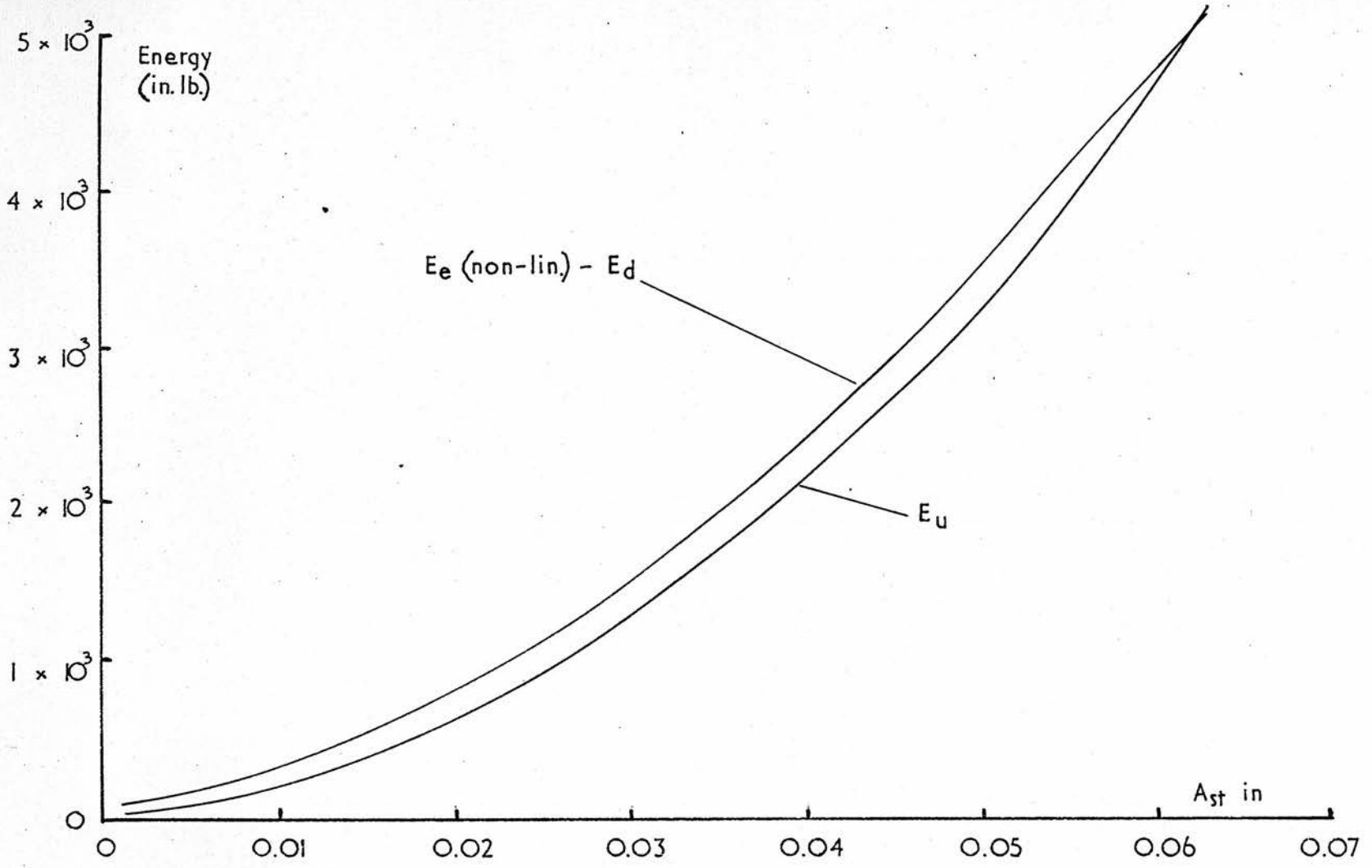
Graph 31 Net input energy added to energy of vibration Structure T' 3



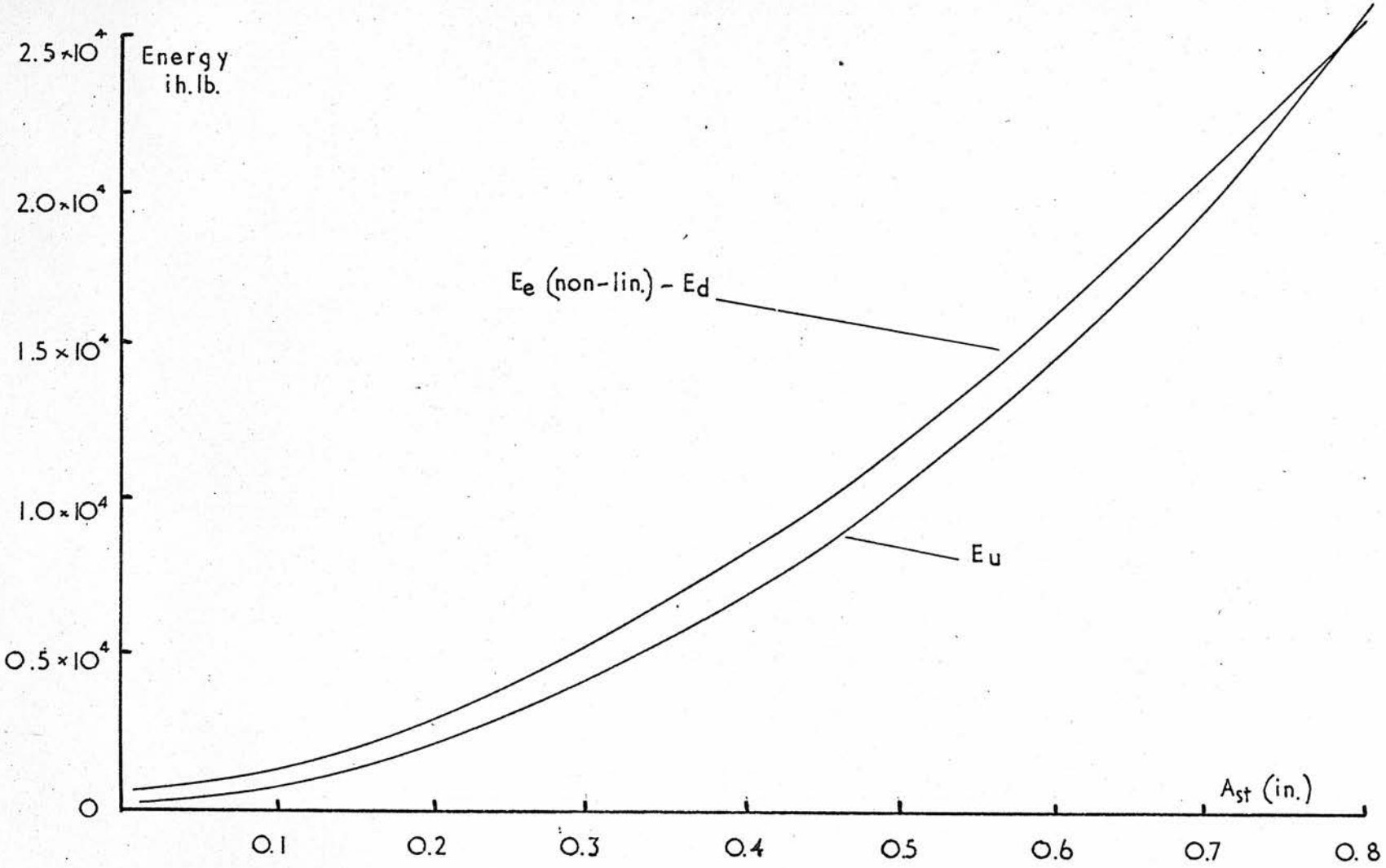
Graph 32 Net input energy added to energy of vibration Structure T'4



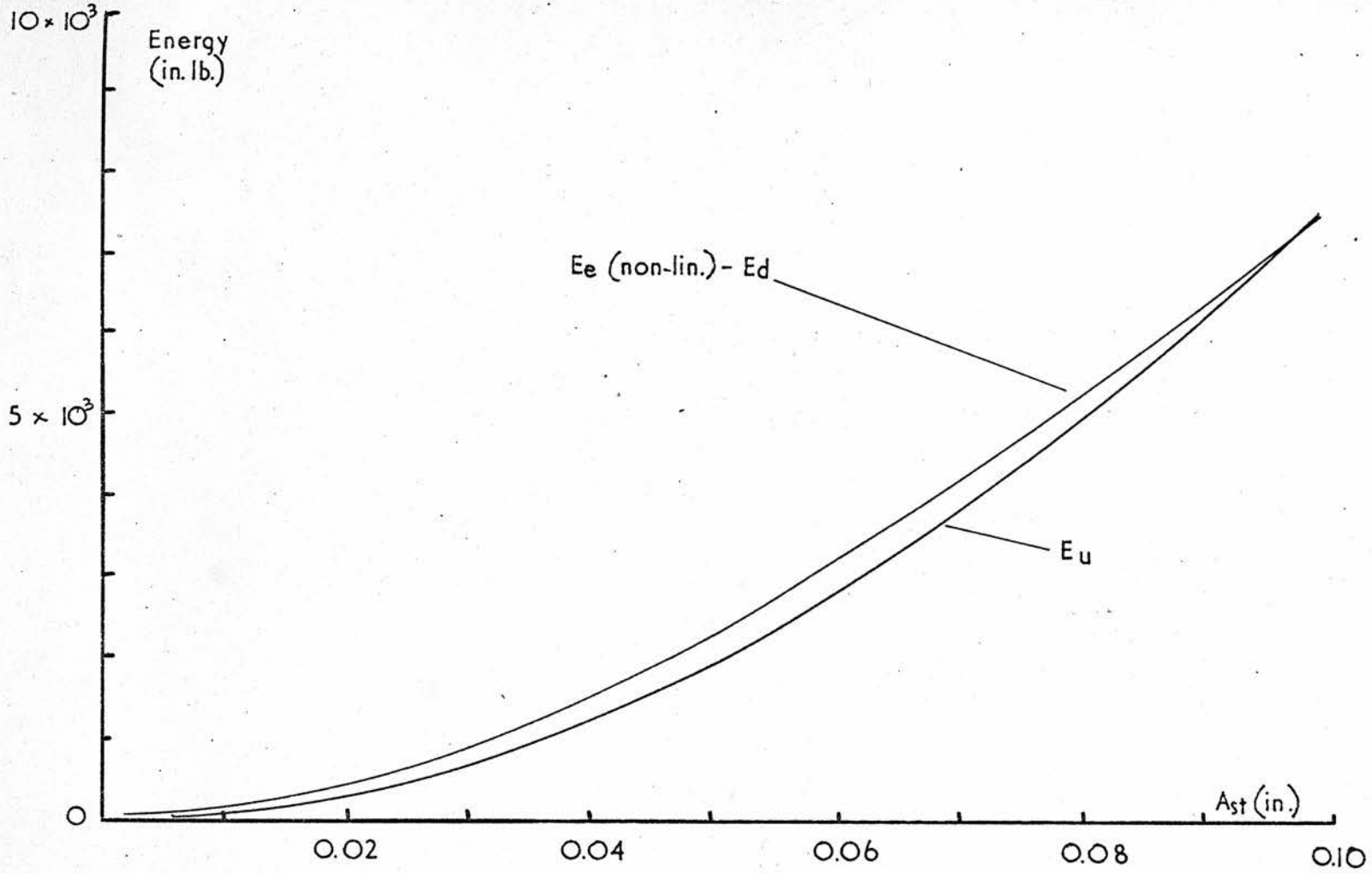
Graph 33 Net input energy added to energy of vibration Structure T' 5



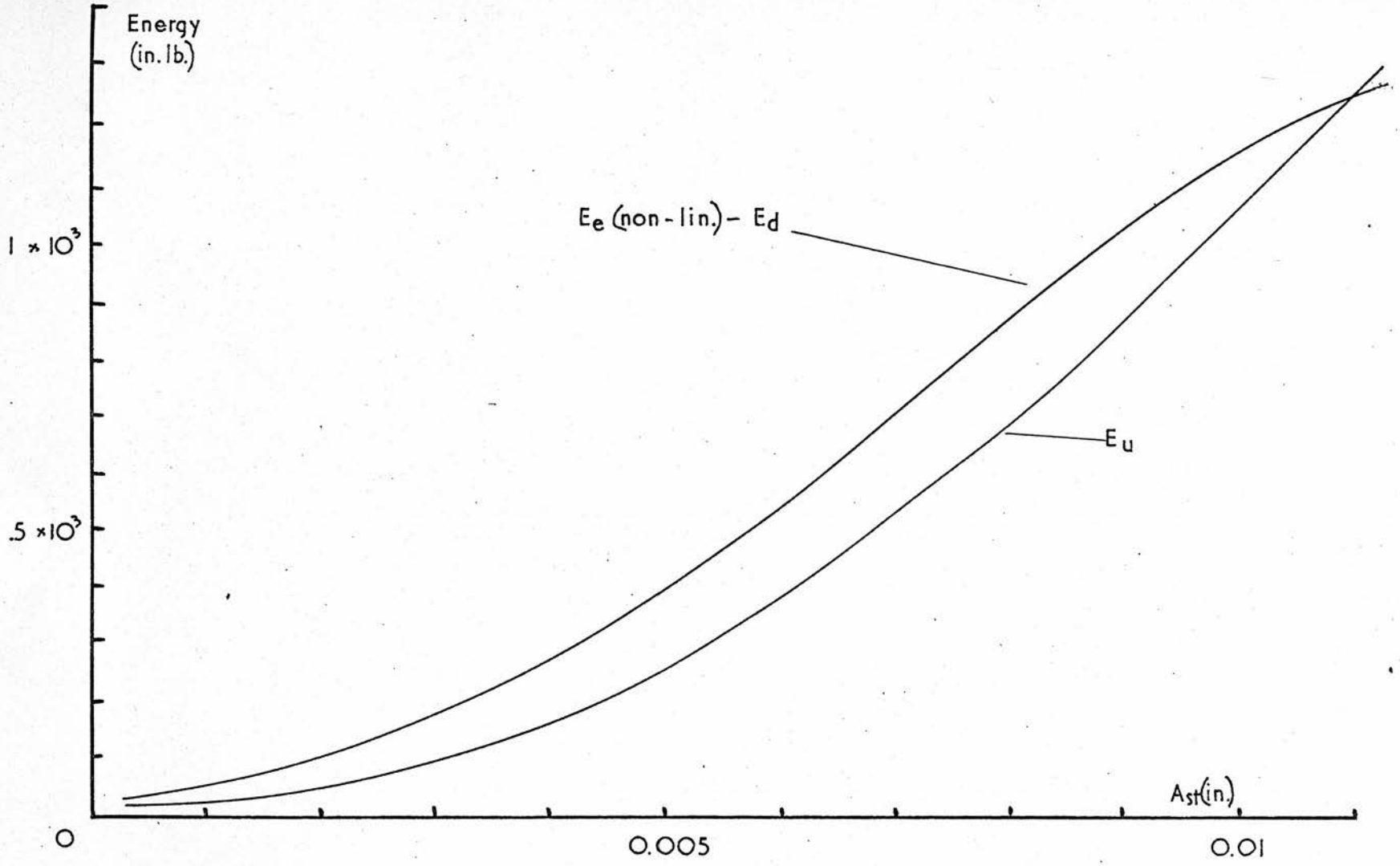
Graph 34 Net input energy added to energy of vibration Structure T'6



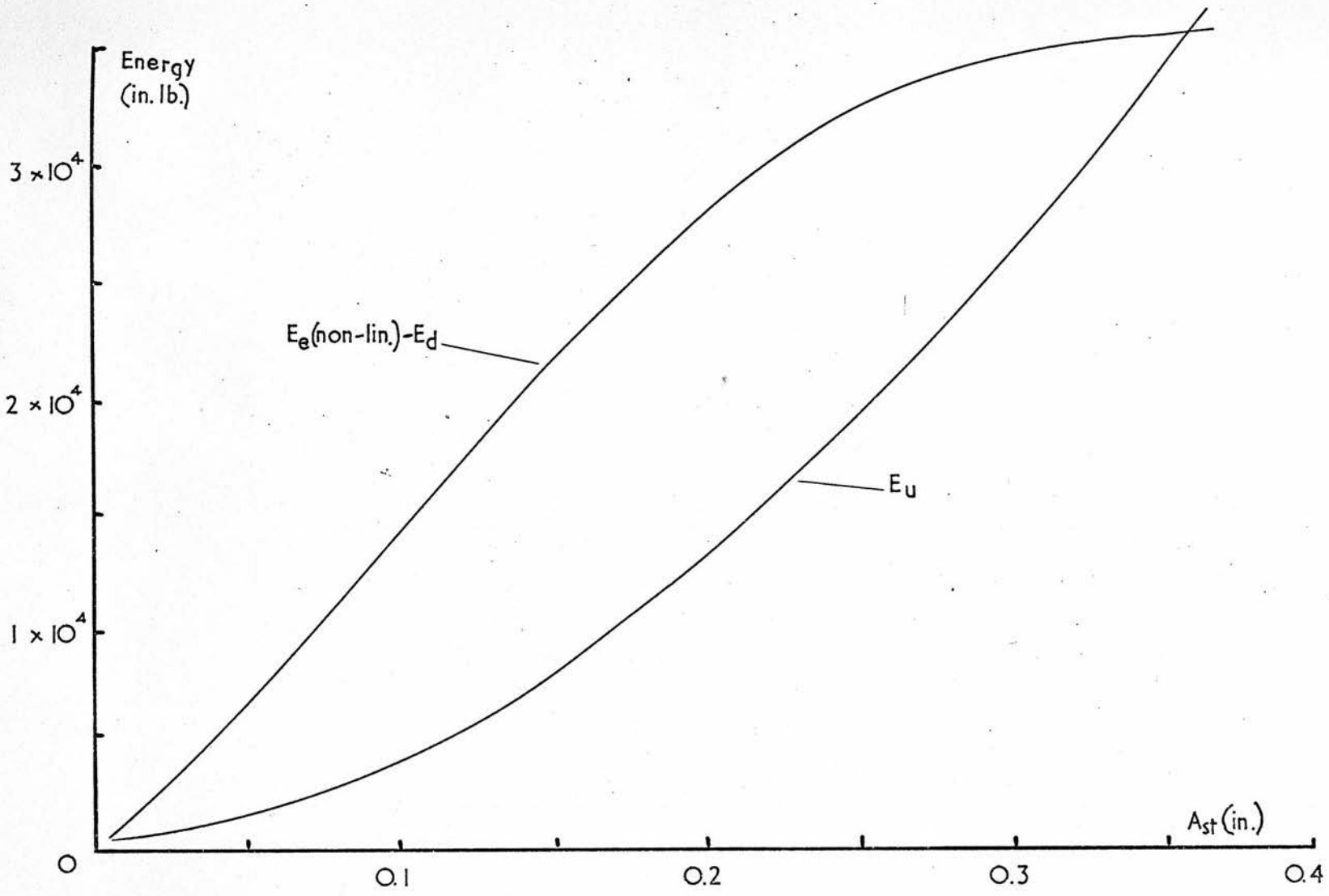
Graph 35 Net input energy added to energy of vibration Structure T'7



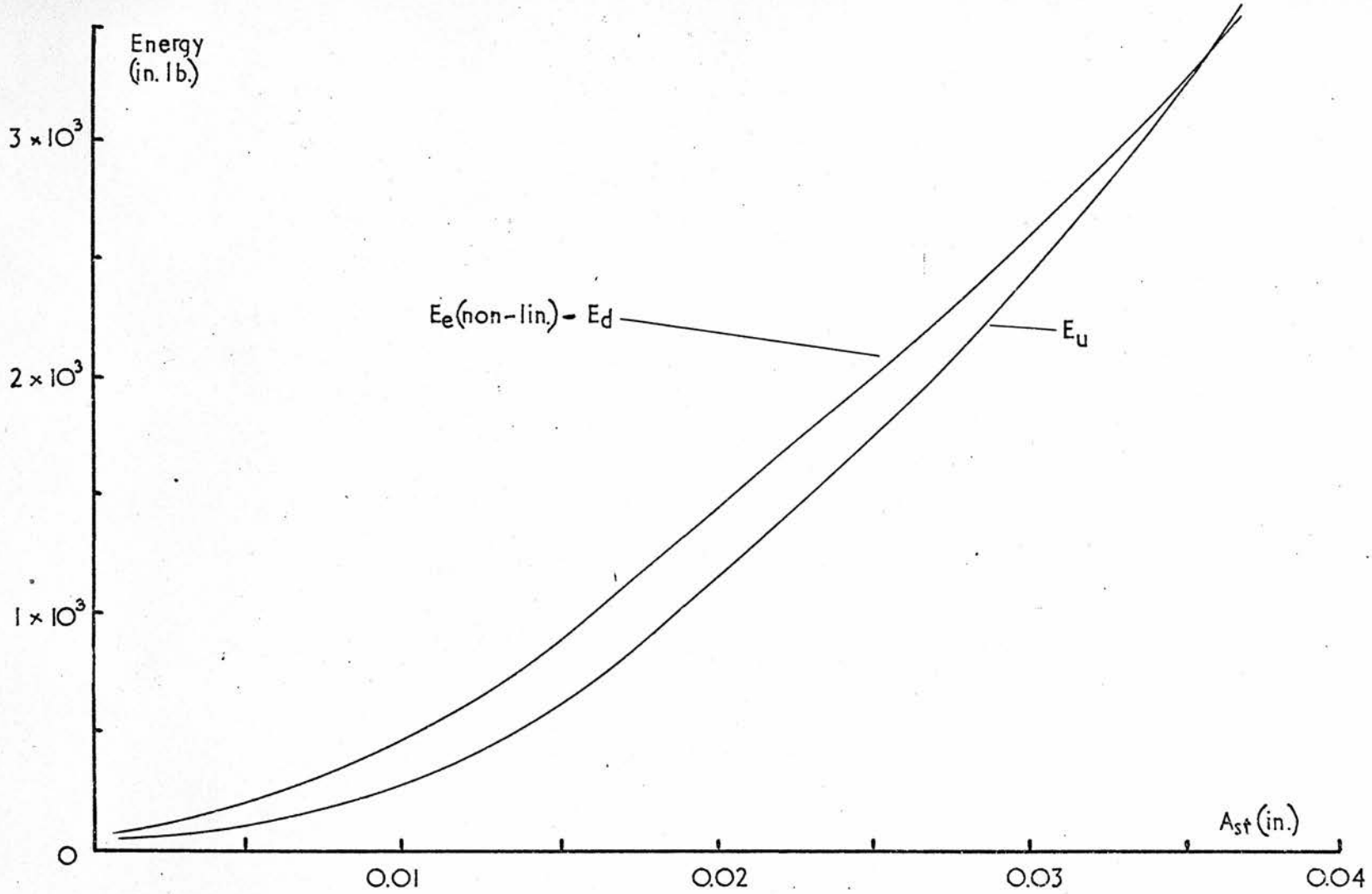
Graph 36 Net input energy added to energy of vibration Structure T'8



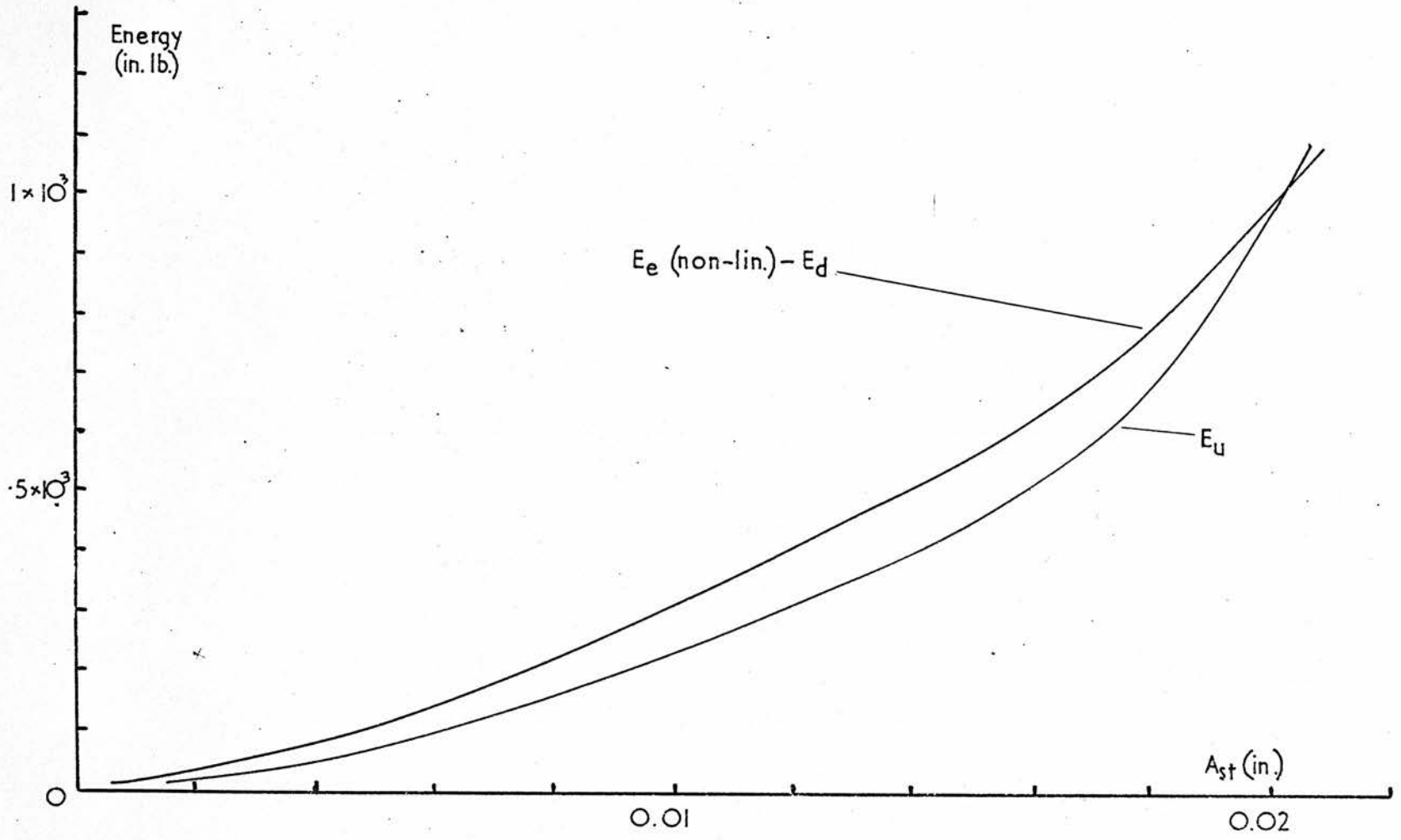
Graph 37 Net energy input added to energy of vibration Structure T'9



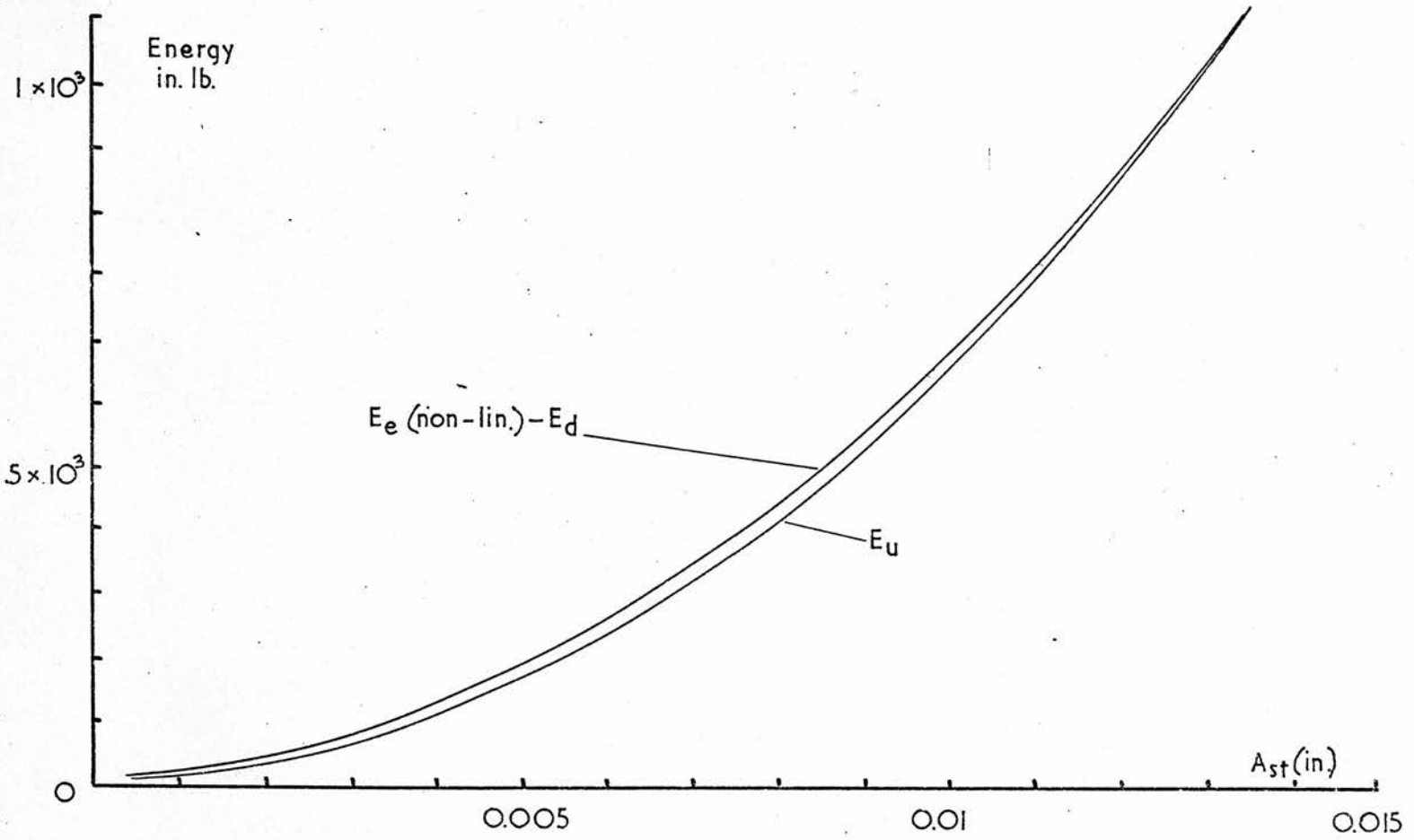
Graph 38 Net input energy added to energy of vibration Structure T'10



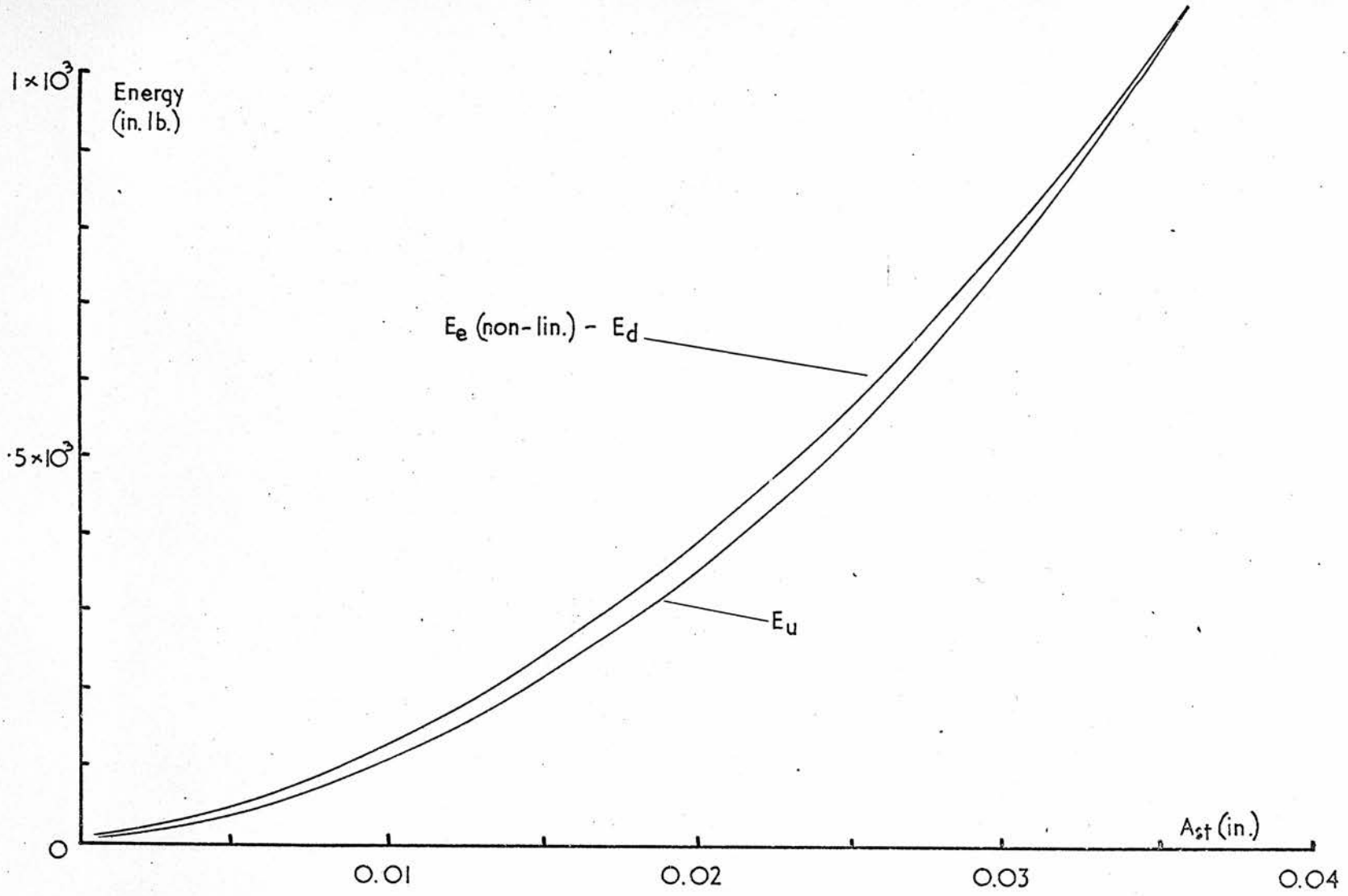
Graph 39 Net energy input added to energy of vibration Structure T'11



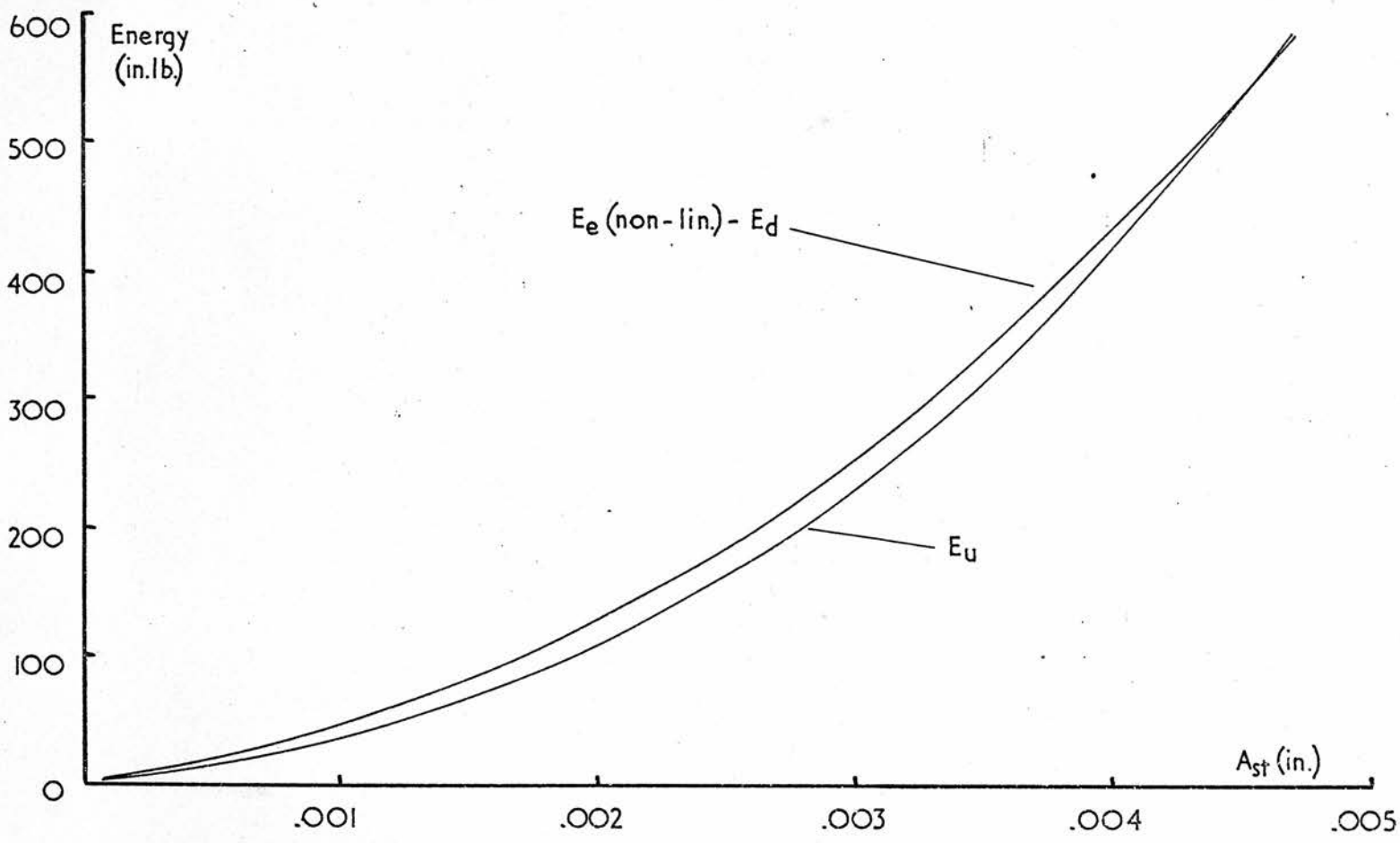
Graph 40 Net input energy added to energy of vibration Structure T' 12



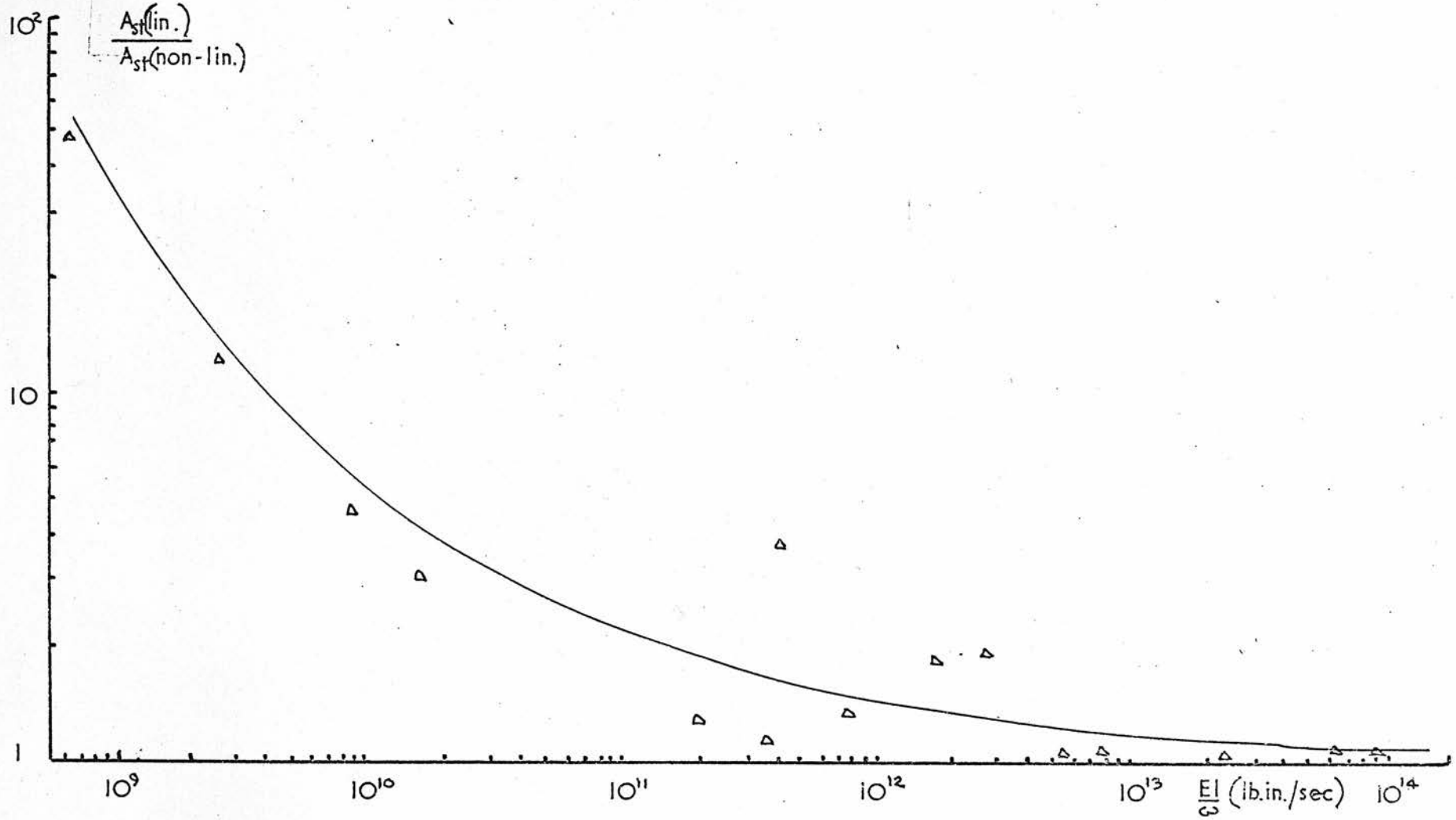
Graph 41 Net input energy added to energy of vibration Structure B'1



Graph 42 Net input energy added to energy of vibration Structure B'2



Graph 43 Net input energy added to energy of vibration Structure B'3



Graph 44 Relationship between $\frac{EI}{g}$ and ratio of response predictions based on linear and non-linear theories.

CHAPTER 5

USE OF ENERGY METHOD IN DESIGN

5.1 INTRODUCTION

It was seen in Chapter 4 that an energy analysis provided a comparatively simple means of calculating the R.M.S. deflection of a structure vibrating in response to the buffeting effect of wind turbulence. The only existing method for predicting the dynamic response of structures to buffeting by gusts, which is suitable for general application in design, is the spectrum analysis in the form proposed by Davenport. Two sources of error which might lead to inaccurate results with this method were discussed in Chapter 4. These are the neglect of the non-linearity in the system and of the inertia of the structures concerned. It is felt that the energy analysis may be capable of giving a more accurate prediction than the spectrum technique in cases where these effects may be large. Two techniques whereby the energy analysis can be used as part of a design procedure for tall structures are now outlined. The first is a rigorous analysis, performed on a computer, and intended for use only if preliminary checks indicate that large errors might be incurred if the spectrum analysis were employed. The second is a simplified method using energy equations which are reduced to a dimensionless form. It is thought that this technique could be used, as a substitute for spectral analysis, to obtain quickly, gust factors for use in equivalent static analysis.

It/

It may be said at the outset that while the energy analysis, in both forms, is capable of incorporating an allowance for the effect of the transient part of the response, where this may be considered to affect the overall performance of a structure, the data currently available on the structure of wind turbulence are not sufficiently detailed for this type of analysis to be carried out. The proposed analyses are therefore seen, at present, only as means of overcoming the problem of allowing for the non-linearity in a wind driven vibrating system although the simplified technique may be considered as a direct substitute for the spectrum analysis in the form suggested for code of practice use.

5.2 USE OF COMPUTER TO DETERMINE R.M.S. OF RESONANT COMPONENT OF RESPONSE.

The most satisfactory means of calculating the dynamic component of response of a structure from the energy equations is to solve these equations on a computer. This was carried out in Chapter 4 and the programme used for the energy investigation has been modified so as to be suitable for more general application in design. A flow chart for the altered programme is given in Fig (13). The principal modifications are discussed below.

The computer programme used in Chapter 4 was designed to simulate the build up in vibration of a structure from a small initial amplitude to the r.m.s. value. It was necessary to specify a starting value for A_{st} and the programme was in effect/

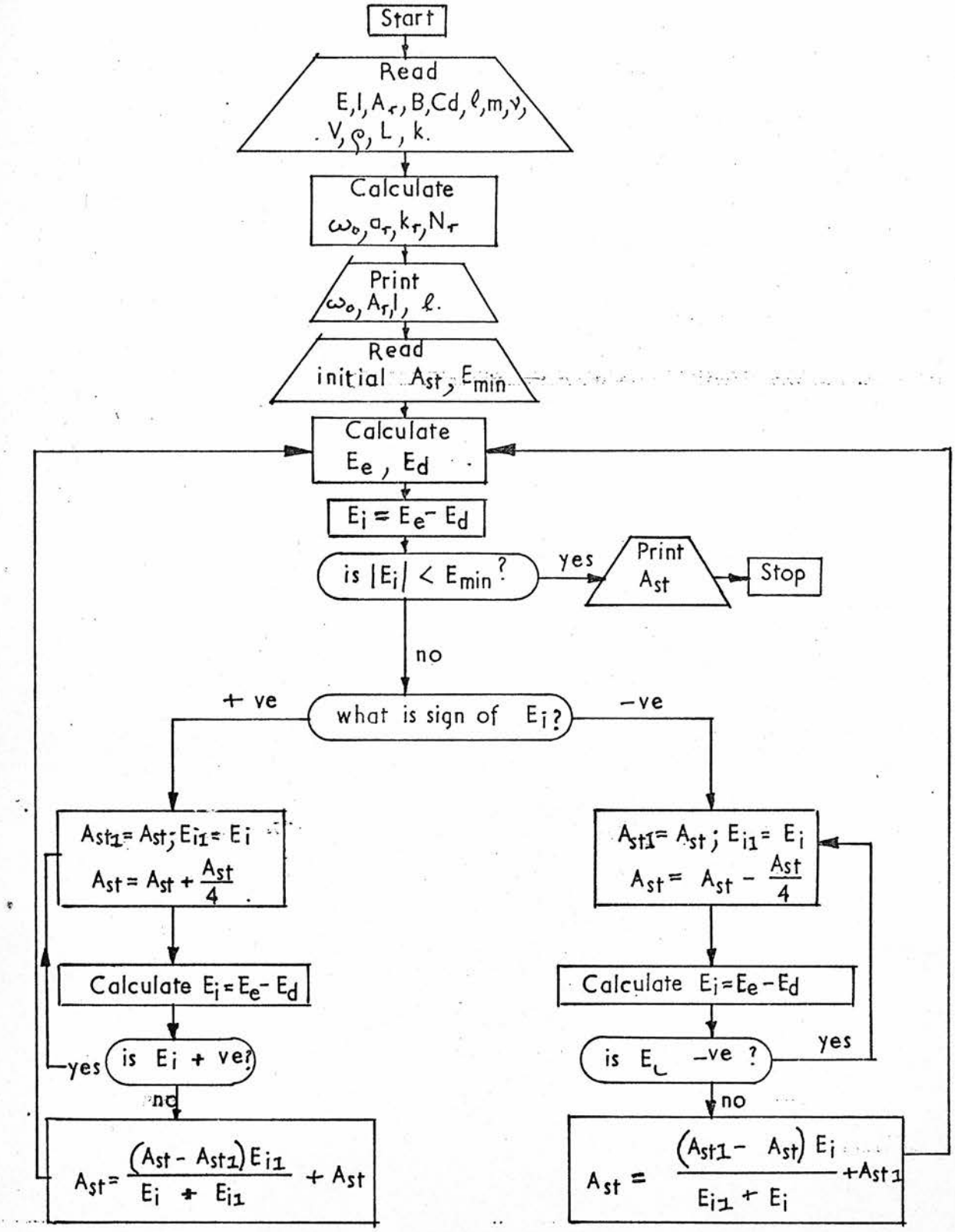


Fig 13 Flow chart for ammended energy programme, suitable for use in design.

effect an itterative process for solving the energy equations. For design purposes, it is the r.m.s. value only which is required. In the design programme an itterative technique is again used and it is still necessary to specify an initial value of A_{st}. The program has been written, however, so that rapid convergence on the r.m.s. deflection is obtained and the accuracy of the initial guess is not important.

In Chapter 4, it was assumed that a cantilever was a good approximation to a tall structure and that the first mode of vibration was the only one of importance. It is felt that, in practice, these approximations will provide a satisfactory solution for a wide range of structures. As will be seen from Appendix 3, however, the assumption that a building acts like a cantilever can lead to errors in the calculated natural frequency. The programme has therefore been written in such a way that substitution of expressions for mode shape and natural frequency other than the cantilever formulae can easily be made. Provision is also made for determining the r.m.s. value of the deflection in the higher modes as well as the first. The formulae used in the programme as it stands are,

mode shape $f_r(z) = \cosh\ a_r z - \cos a_r z - k_r(\sinh . a_r z - \sin a_r z)$

$$a_r = \frac{A_r \omega_r^2}{EI}$$

$k_r/$

$$k_r = \frac{\cos a_r l + \cosh . a_r l}{\sin a_r l + \sinh . a_r l}$$

$$\omega_1 = \frac{3{\cdot}516}{l^2} \sqrt{\frac{EI}{\rho A_r}}$$

$$\omega_2 = \frac{22{\cdot}03}{l^2} \sqrt{\frac{EI}{\rho A_r}}$$

$$\omega_3 = \frac{61{\cdot}70}{l^2} \sqrt{\frac{EI}{\rho A_r}}$$

As in the research programme the wind parameters are based on the Davenport reduced spectrum and cross-correlation coefficient. The programme was written in Atlas Autocode and its compiling time on the I.B.M. 360/50 is 3·90 seconds. The running time for an average job is 7·7 seconds.

5.3 USE OF REDUCED ENERGY EQUATIONS TO DETERMINE THE R.M.S. OF THE RESONANT COMPONENT OF DEFLECTION

It was considered that, in addition to a computer analysis, there was a need for a simple method for determining approximately the r.m.s. deflection of the resonant component of response. In an attempt to satisfy this the energy equations have been simplified, reduced to a non-dimensional form and solved directly for A_{st}. The resulting expression may be used to determine A_{st} from two dimensionless coefficients. The analysis was carried out for the first mode of vibration only, but extension to include higher modes is possible although more complicated/

complicated.

The excitation energy per cycle is given by the equation

$$E_e = B\rho Cd\bar{V}\omega\pi \int_0^l \left[\frac{Aw}{\omega} e^{\frac{-7\cdot7(1-z)n}{\bar{V}}} - f(z)A_{st} \right] f(z)\, A_{st}\, dz \qquad \ldots\ldots (5.1)$$

The part under the integral sign may be made non-dimensional by means of the substitution $u = \frac{z}{l}$

Then $z = l\,u$

$dz = l\,du$

$u = 0$ when $z = 0$

$u = 1$ when $z = l$

$\therefore$ the limits of integration become 0 to 1.

$$e^{-\frac{7\cdot7(1-z)n}{\bar{V}}} = e^{\frac{-7\cdot7(1-ul)n}{\bar{V}}} = e^{-7\cdot7(1-u)\frac{ln}{\bar{V}}}$$

Equation (5.1) becomes,

$$E_e = B\rho Cd\bar{V}\omega\pi \int_0^1 \left[\frac{A_w}{\omega} e^{-7\cdot7(1-u)\frac{ln}{\bar{V}}} - f(u)\, A_{st} \right] f(u)A_{st}\, du$$

$$= \frac{B\rho Cd\bar{V}\pi l A_w^2}{\omega} \int_0^1 \left[e^{-7\cdot7(1-u)\frac{ln}{\bar{V}}} - f(u)\frac{A_{st}\omega}{A_w} \right] f(u)\frac{A_{st}\omega}{A_w}\, du$$

$$\therefore E_e = \frac{B\rho Cd\bar{V}\pi l A_w^2}{\omega} \int_0^1 \left[e^{-7\cdot7(1-u)\alpha} - f(u)\beta \right] f(u)\beta\, du \qquad \ldots\ldots (5.2)$$

where/

where $\alpha = \frac{nl}{\bar{v}}$; $\beta = \frac{A_{st}\,\omega}{A_w}$

To make equation (5.2) applicable to a range of structures the reduced excitation energy E_{re} is defined by,

$$E_{re} = \frac{\omega}{B\,\rho\,Cd\,\bar{v}\,\pi\,l A_w^2}\; E_e$$

$$E_{re} = \int_0^1 \left[e^{-7\cdot7(1-u)\alpha} - f(u)\beta\right] f(u)\,\beta \; du$$

Assuming that modes higher than the first may be neglected and making a straight line approximation,

$$E_{re} = \int_0^1 \left[e^{-7\cdot7(1-u)\alpha} - u\,\beta\right] u\,\beta \; du \qquad \ldots\ldots(5.3)$$

Equation 5·3 may be integrated without difficulty and becomes,

$$E_{re} = \beta\left[\frac{1}{7\cdot7\alpha} - \left(\frac{1}{7\cdot7\alpha}\right)^2\left(1 - e^{-7\cdot7\alpha}\right)\right] - \tfrac{1}{3}\beta^2 \qquad \ldots\ldots(5.4)$$

The damping energy per cycle is given by the equation,

$$E_d = 2\,A_{st}^2\,\pi\,\nu\,\omega^2\,m \int_0^l f^2(z)\,dz$$

If the substitution $u = \frac{z}{l}$ is made and a linear mode shape assumed this equation becomes,

E_d /

$$E_d = 2A_{st}^2 \nu \omega^2 m l \int_0^1 u^2 \, du$$

$$\therefore E_d = \tfrac{2}{3} A_{st}^2 \nu \omega^2 m l \qquad \text{.....(5.5)}$$

The condition for A_{st} = r.m.s. amplitude is

$$E_e - E_d = 0 .$$

$$\text{i.e. } \frac{B\rho C d\bar{V} \pi l A_w^2}{\omega} \cdot E_{re} - \tfrac{2}{3} A_{st}^2 \nu \omega^2 m l = 0$$

$$\therefore E_{re} - \tfrac{2}{3} \frac{m\nu\omega}{B\rho C d\bar{V}} \cdot \frac{A_{st}^2 \omega^2}{A_w^2} = 0$$

$$\therefore E_{re} - \tfrac{2}{3} \frac{m\nu\omega}{B\rho C d\bar{V}} \cdot \beta^2 = 0 \qquad \text{.....(5.6)}$$

Substituting (5.4) for E_{re} in (5.6) gives

$$\left[\frac{1}{7{\cdot}7\alpha} - \left(\frac{1}{7{\cdot}7\alpha}\right)^2 (1 - e^{-7{\cdot}7\alpha})\right]\beta - \tfrac{1}{3}\beta^2 - \tfrac{2}{3}\frac{m\nu\omega}{B\rho C d\bar{V}} \beta^2 = 0$$

$$\beta^2 \left(\tfrac{2}{3}\frac{m\nu\omega}{B\rho C d\bar{V}} + \tfrac{1}{3}\right) - \beta\left[\frac{1}{7{\cdot}7\alpha} - \left(\frac{1}{7{\cdot}7\alpha}\right)^2 (1 - e^{-7{\cdot}7\alpha})\right] = 0 \qquad \text{.....(5.7)}$$

The solutions to equation 5·7 are,

$$\beta = 0 \quad \text{and} \quad \beta = \frac{\left[\frac{1}{7{\cdot}7\alpha} - \left(\frac{1}{7{\cdot}7\alpha}\right)^2 (1 - e^{-7{\cdot}7\alpha})\right]}{\tfrac{2}{3}\frac{m\nu\omega}{B\rho C d\bar{V}} + \tfrac{1}{3}}$$

thus,

$$A_{st} = \frac{A_w}{\omega} \left[\frac{\frac{1}{7{\cdot}7\alpha} - \left(\frac{1}{7{\cdot}7\alpha}\right)^2 (1 - e^{-7{\cdot}7\alpha})}{\gamma + \tfrac{1}{3}}\right] \qquad \text{.....(5.8)}$$

where/

where $\alpha = \frac{nl}{\bar{V}}$

$$\gamma = \frac{2}{3} \frac{m \vartheta \omega}{B \rho C d \bar{V}}$$

The r.m.s. of the resonant component may be obtained directly from equation (5.8) after evaluation of the dimensionless parameters α and γ. The equation may also be written,

$$A_{st} = \frac{A_w}{\omega} \gamma' \qquad \ldots\ldots(5.9)$$

where $$\gamma' = \left[\frac{\frac{1}{7{\cdot}7\alpha} + \left(\frac{1}{7{\cdot}7\alpha}\right)^2 \left(1 - e^{-7{\cdot}7\alpha}\right)}{\gamma + \frac{1}{3}} \right]$$

Graph (46) gives values of γ' for a range of α and γ.

A_w may be evaluated from the Davenport reduced spectrum if a bandwidth is specified. An approximation to the bandwidth which excites a structures to resonate is given by the frequency separation of the "half-power points" in the resonance peak. This is given by the formula,

$$\Delta n = 2\nu n_o$$

A_w may be obtained from the equation,

$$A_w^2 = \frac{S_v k \bar{V}^2 \Delta n}{n_o}$$

where/

where $S_y = \dfrac{4{\cdot}0 \left(\dfrac{n_o L}{\bar{V}}\right)^2}{\left[1 + \left(\dfrac{n_o L}{\bar{V}}\right)^2\right]^{\frac{4}{3}}}$

A curve of S_y against $\frac{Y}{h}$ is shown in graph (45).

A worked example illustrating the use of the reduced energy formula to calculate A_{st} is now given. Structure T6 from the energy analysis of Chapter 4 is used.

The appropriate input parameters are:

$\bar{V}$ = 100 ft/sec

k = 0·05

ρ = 1·39 x 10^{-6} slug/in^3

l = 200 ft = 2400 ins

m = 0·045 slug/in

ν = 0·01

B = 10 in

I = 1·03 x 10^6 in ^{4}

E = 30 x 10^6 lb/in^2

Cd = 1

$$\omega_o = \frac{3{\cdot}516}{l^2}\sqrt{\frac{EI}{m}}$$

$$= \frac{3{\cdot}516}{2400^2}\sqrt{\frac{30 \times 1{\cdot}03 \times 10^{12}}{0{\cdot}045}}$$

$$= 15{\cdot}85 \text{ rad/sec}$$

n_o = 2·53 Hz.

$\alpha = \dfrac{n_o l}{\bar{V}}$ /

$$\alpha = \frac{n_o l}{\bar{V}} \qquad \gamma = \frac{2}{3} \frac{m \nu \omega_o}{B \rho C d \bar{V}}$$

$$= \frac{2 \cdot 53 \times 200}{100} \qquad = \frac{2 \times \cdot 045 \times \cdot 01 \times 15 \cdot 85}{3 \times 10 \times 1 \cdot 39 \times 10^{-6} \times 1 \times 100 \times 12}$$

$$= 5 \cdot 06 \qquad = 0 \cdot 285$$

$$\gamma' = \frac{\frac{1}{7 \cdot 7 \alpha} - (\frac{1}{7 \cdot 7 \alpha})^2 (1 - e^{-7 \cdot 7 \alpha})}{\gamma + \frac{1}{3}}$$

$$= \frac{\frac{1}{7 \cdot 7 \times 5 \cdot 06} - (\frac{1}{7 \cdot 7 \times 5 \cdot 06})^2 (1 - e^{-7 \cdot 7 \times 5 \cdot 06})}{0 \cdot 285 + 0 \cdot 333}$$

$$= 0 \cdot 041$$

$$\Delta n = 2 n_o \nu$$

$$= 2 \times 2 \cdot 53 \times 0 \cdot 01$$

$$= 0 \cdot 506$$

From graph (45). $S_v = 0 \cdot 25$

$$\therefore A_w = \sqrt{\frac{S_v k \bar{V}^2 n}{n_o}}$$

$$= \sqrt{\frac{0 \cdot 25 \times \cdot 05 \times 100^2 \times \cdot 506}{2 \cdot 53}}$$

$$= 1 \cdot 58 \text{ ft/sec}$$

$$= 18 \cdot 95 \text{ in/sec}$$

$$A_{st} = \frac{A_w}{\omega} \gamma' = \frac{18 \cdot 95 \times 0 \cdot 041}{15 \cdot 85}$$

$$= \underline{\underline{0 \cdot 049 \text{ in}}}$$

The/

The value of A_{st} obtained from the computer analysis was 0.061 in.

Table (4) shows a comparison between the values of A_{st} calculated using the reduced energy equation and the computer analysis for the other structures in Chapter 4. It may be seen that the agreement is fairly good.

5.4 FINAL COMMENT

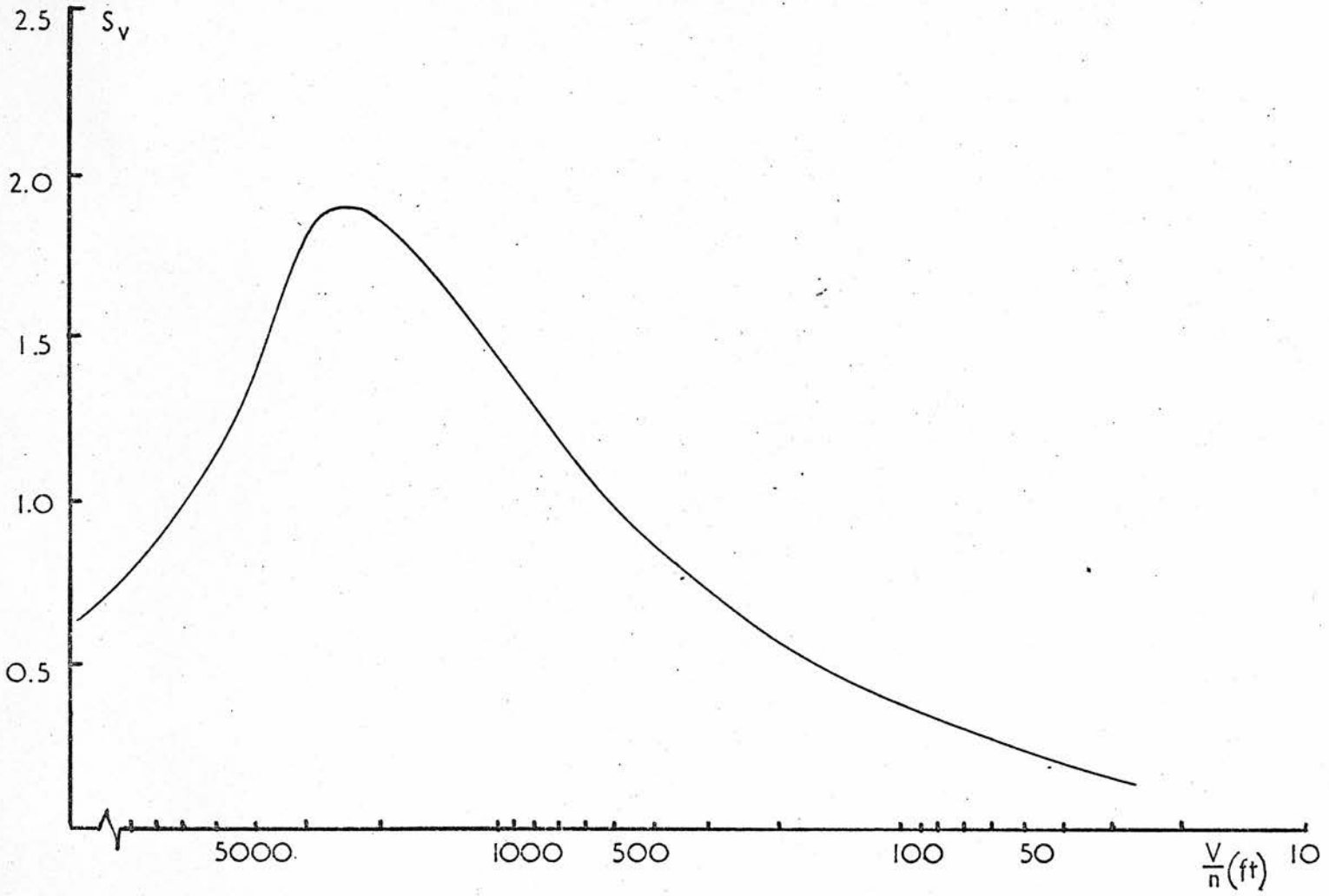
Both the techniques outlined here are designed to calculate the r.m.s. of the resonant component of deflection. If the extreme value is required a final calculation must also be made using extreme value statistics.

Neither of the techniques proposed here is capable of giving more than a rough approximation to the vibration amplitudes which would occur in practice. It is felt, however, that they constitute a realistic approach to the problem. As with the conventional spectral approach the weakest feature of the methods is their reliance on inadequate wind data.

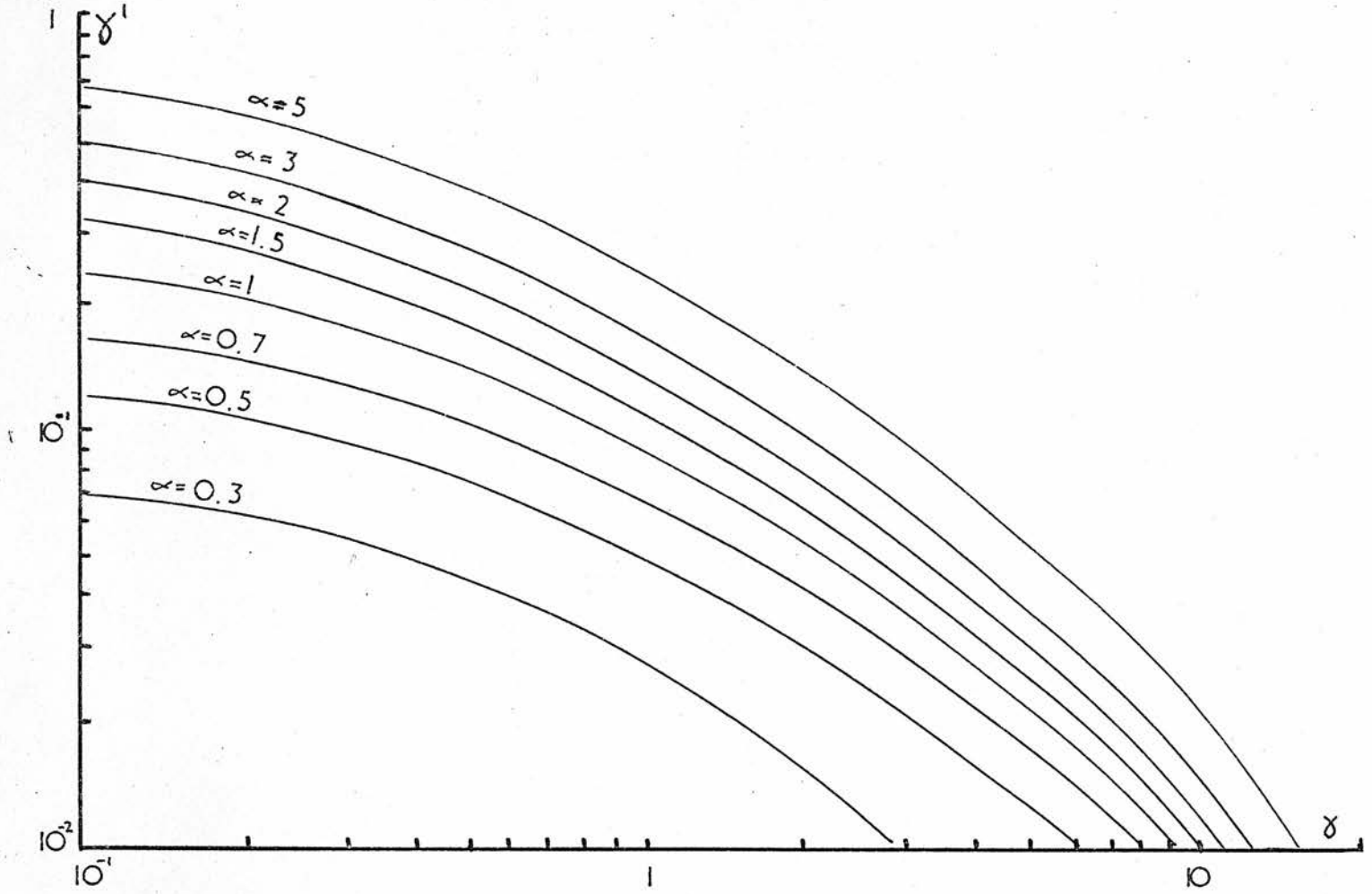
The techniques presented here are the result of a theoretical analysis and the examples are intended to be illustrative. It is not suggested that they be adopted for design without further study. It is felt, however, the the approach is capable of development into a practical method for assessing the behaviour of slender structures in the wind.

Structure	α	γ	A in/sec	A_{st}	
				(reduced equation) in.	(computer) in.
T'1	4·82	0·05	14·0	0·039	0·013
T'2	2·41	0·03	18·0	0·180	0·130
T'3	1·45	0·02	23·0	0·650	0·720
T'4	1·04	0·01	40·0	3·19	3·20
T'5	2·68	0·51	10·0	0·020	0·043
T'6	5·06	0·29	19·0	0·049	0·061
T'7	0·90	0·01	50·0	7·80	0·77
T'8	1·93	0·02	20·0	0·320	0·096
T'9	10·00	0·12	13·0	0·006	0·011
T'10	2·10	0·01	30·0	0·75	0·35
T'11	7·00	0·44	17·0	0·039	0·036
T'12	0·96	2·24	28·0	0·370	0·023
B'1	1·45	5·10	23·0	0·042	0·013
B'2	0·70	1·20	35·0	1·900	0·035
B'3	4·08	7·10	15·0	0·005	0·005

Table 4 Comparison of results from computer and reduced energy equation.



Graph 45 Reduced spectrum of horizontal wind speed.



Graph 46 Relationship between constants in reduced energy solution.

CHAPTER 6

CONCLUSIONS

6.1 SUMMARY OF CONCLUSIONS

The investigation presented in this thesis consisted of an analytical simulation of the response of slender structures to turbulent wind loads. It was carried out in two parts, the first being a conventional spectrum analysis and the second an evaluation of the levels of transfer and vibration energies in wind driven vibrating systems using a technique developed by the author. The main conclusions may be summarised as follows:-

Spectrum Analysis.

The spectrum analysis suggested that slender structures, exposed in turbulent winds were likely to be severely buffetted and that the dynamic component of response was likely to be a major constuent of the total response. The magnitude of the dynamic component of response was found to be largely dependent on the natural frequency of the structure due to the fact that the intensity of excitation was dependent on frequency. The spectrum analysis was found to be insensitive to variations in input data, especially those concerning the mechanical properties of the structures, and it was concluded that the mechanical admittance function might not provide a sufficiently accurate mathematical model of a structure for this type of analysis. It was thought that this factor, combined with the fact that the spectrum analysis, in the form usually applied to wind loading calculations, makes no allowance for the non-linearity in the system, might lead to an overestimation of the dynamic component of response./

response.

Energy Analysis.

In the energy analysis, the levels of input energy and energy of vibration per cycle were calculated for a number of structures considered to be vibrating in response to simulated wind loads. The analysis was carried out in such a way that the effect of neglecting the non-linearity in the system could be evaluated. It was found that for most types of structure the level of input energy was very low compared to the energy of vibration. It was felt that in this situation the inertia of the structures would affect the extent of their dynamic response, especially to a random form of load such as wind turbulence, and that some allowance for this should be made when attempting to predict the magnitude of the dynamic response.

It was also found that certain types of structure were likely to be subjected to large amounts of aerodynamic damping when buffeted by turbulence and that the non-linearity in such systems could not be ignored when predicting dynamic response.

6.2 DESIGN FOR DYNAMIC WIND LOADS.

In recent years considerable advances have been made in the field of assessing the effects of wind loads on buildings and the current CP3 Chapter 5 enables reliable predictions of the static and quasi-static effects of wind to be made. The problem of determining the dynamic response has still not been satisfactorily resolved, however/

however, and further work is required in this field. Of the several wind induced excitation mechanisms which are capable of causing a dynamic response in a structure, the buffeting of structures by gusts is the one which has been considered in this investigation. The following comments refer only to this form of excitation.

It is likely that there are many structures for which a large dynamic response to buffeting by gusts will not occur and in such cases the inclusion of a dynamic analysis as part of the design procedure is not justified. It is felt that there is a need for a quick means of testing, at the beginning of a design, whether or not a structure is in this category. From the results of this study two criteria are suggested. The first is the natural frequency of the structure concerned. A chart such as graph (13) could be used to determine whether or not the natural frequency of the structure was low enough to make large dynamic response possible. If the natural frequency were found to be greater than a specified value a dynamic analysis could be considered unnecessary.

The second criterion suggested is the mass or inertia of the structure. Due to the fact that the ratio of energy input to energy of vibration is very low for tall buildings and that wind turbulence is a random form of loading, the author believes that many slender structures, especially tall buildings, will have a negligible dynamic component of response even though their natural frequencies may be low. The important parameter is the ratio of the/

the maximum duration of a constant phase batch of turbulence to the length of the transient part of the response of a structure. The present state of knowledge is such that a meaningful assessment of the duration of a constant phase batch of turbulence is not possible. The author believes that research into this property of wind turbulence would be valuable and a suggested procedure is given in section 6.3.

If preliminary checks indicate that a structure is likely to be adversely affected by dynamic loading a designer may be able to reduce its effect by altering the design, either to increase the natural frequency or the damping. It seems likely, however, that for some structures the prevention of dynamic response will not be possible and there is a need for a method for calculating the extent of this component of the overall response.

The spectrum analysis, in the form suggested by Davenport, provides such a technique but it suffers from a number of drawbacks. In addition to those already discussed a further inconvenience with this method is that the static, quasi-static and dynamic components of response must be dealt with together, in one all-embracing calculating to produce a gust factor. The designer who uses this method is, therfore, forced to calculate the non-dynamic part of the response from wind spectrum data. Better data are now available, however, for calculating the static and quasi-static components of response, than the spectrun, which is still in an early stage of development. The author believes that a more logical approach is to evaluate the static and quasi-static response/

response by the method suggested in CP3 Chapter 5 and to perform a separate analysis to determine the dynamic response.
The energy analysis proposed in Chapter 5 of this thesis lends itself to this approach and the simplified form enables the R.M.S. deflection of the resonant component of response of a structure to be calculated directly from two dimensionless coefficients. The dynamic part of the analysis is therefore kept separate from the static and quasi-static parts and in no way interferes with their accuracy.

6.3 FURTHER RESEARCH

One of the objects of the investigation presented in this thesis was to determine which were the parameters in a wind excited vibrating system, which required further experimental investigation. From the results of the study a number of suggestions may be made concerning the direction in which further research should proceed.

1. The splitting of the wind velocity function into mean and time varying components and the reduction of the latter to the format of a reduced spectrum and roughness coefficients is a feasible approach to the problem of devising a mathematical model of wind turbulence. The spectrum undergoes large variations with frequency, however, in the range of frequencies likely to excite tall structures. A study is now required to obtain accurate data so that the reliability of the reduced spectrum may be improved. This will require the collection of data on a large scale over the whole range of roughness conditions/

conditions but it is an essential prerequisite to any code of practice which attempts to deal with the buffeting effect of wind turbulence.

2. One of the conclusions reached from the energy analysis was that some structures may not achieve high amplitudes of vibration in response to wind loads because the relatively constant conditions required for a large build up in energy may never occur with this random form of loading. Data on the average lengths of constant phase batches of turbulence would be useful so that the possible extent of energy build up in large structures could be assessed. An insight into orders of magnitude of the duration of constant phase sequences could be gained from examination of records such as that shown in Fig. 6. The structure in that case was 30 ft high and had a natural frequency of 3.3 Hz. From the records, the average length of constant phase sequences, for that component of turbulence, could be deduced. If many such records, for different structures, were examined, an assessment of the lengths of turbulence components at their frequencies, could be made. If data from a variety of conditions were analysed the parameters on which the lengths of constant phase sequences depend, could possibly be evaluated

3. It is possible that the cost of collecting the data necessary for accurate dynamic wind load analysis would be such that other solutions to the wind load problem should be examined. One such possibility is the elimination of resonant vibration in/

in structures by the introduction of additional damping. This could be done, either by building in damping (e.g. friction joints) or by attaching damping devices to structures. The structures which are worst affected by dynamic loads are those which are light as well as slender, such as lattice towers. The energy dissipative capacity of damping devices attached to such structures would not have to be high to bring about a significant reduction in vibration amplitude. A study of the feasibility of such an approach to the problem, including an evaluation of required damping energy levels, would be useful at this stage.

4. The energy method proposed in Chapter 5, for predicting the dynamic response of structures to wind loads, has not been checked experimentally. Wind tunnel studies on aeroelastic models in a turbulent airstream are now being planned so that the ability of this technique to model the behaviour of a wind driven vibrating system may be determined. It is hoped that the results will also enable an assessment of the accuracy of the conventional spectrum analysis to be made.

APPENDIX 1

POWER SPECTRUM ANALYSIS THEORY

If a function x(t) is periodic, it may be expanded as a series of harmonically varying quantities in the form,

$$x(t) = a_o + \sum_{1}^{\infty} (a_r \operatorname{Cos} r\omega_1 t + b_r \operatorname{Sin} r\omega_1 t)$$

where,

$$a_o = \frac{1}{T} \int_{-T/2}^{T/2} x(t) \, dt$$

$$a_r = \frac{2}{T} \int_{-T/2}^{T/2} x(t) \operatorname{Cos} r\omega_1 t \, dt$$

$$b_r = \frac{2}{T} \int_{-T/2}^{T/2} x(t) \operatorname{Sin} r\omega_1 t \, dt$$

T = period of $x(t)$

$$\omega_1 = \frac{2\pi}{T}$$

The components of the series are sinusoids whose frequencies are multiples of the fundamental frequency, ω_1. The amplitudes of the components can be plotted against frequency to give a discrete spectrum in which the spectral lines have spacing ω_1.

The series may also be expressed in the complex form,

$$x(t) = \sum_{-\infty}^{\infty} c_r e^{ir\omega_1 t}$$

where, $c_r = \frac{1}{T} \int_{-\frac{T}{2}}^{\frac{T}{2}} x(t) \, e^{-ir\omega_1 t} \, dt$

It is possible to express the mean square value of x(t) in terms of the coefficient a_o, a_r, and b_r,

x^2 /

$$\overline{x^2(t)} = \frac{1}{T}\int_{-\frac{T}{2}}^{\frac{T}{2}} x^2(t)\,dt = a_o^2 + \tfrac{1}{2}\sum_{1}^{\infty}(a_r^2 + b_r^2).$$

A non-periodic function may be expressed as a Fourier Series if it is considered to be periodic with infinite period. The fundamental frequency is then infinitely small and the discrete spectrum becomes continuous. Assuming that ω_1 is very small $(\Delta\omega)$ the equation may be written,

$$x(t) = \sum_{-\infty}^{\infty} \frac{\Delta\omega}{2\pi}\left[\int_{-\frac{T}{2}}^{\frac{T}{2}} x(t)e^{-ir\Delta\omega t}\,dt\right] e^{ir\Delta\omega t}$$

As $\Delta\omega \to d\omega$ this becomes,

$$x(t) = \int_{-\infty}^{\infty} \frac{d\omega}{2\pi}\left[\int_{-\infty}^{\infty} x(t)e^{-i\omega t}\,dt\right] e^{i\omega t}$$

which may be written,

$$x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} A(i\omega)e^{-i\omega t}\,d\omega$$

where, $A(i\omega) = \int_{-\infty}^{\infty} x(t)e^{-i\omega t}\,dt$

These equations give the Fourier Integral expression for x(t). $A(i\omega)$ is called the Fourier Transform of x(t). The equations may also be written,

$$x(t) = \int_{-\infty}^{\infty} A(if)\,e^{i2\pi f t}\,df$$

$$A(if) = \int_{-\infty}^{\infty} x(t)e^{-i2\pi f t}\,dt$$

Also/

Also,

$$\int_{-\infty}^{\infty} x^2(t)dt = \int_{-\infty}^{\infty} x(t)x(t)dt$$

$$= \int_{-\infty}^{\infty} x(t) \left[\int_{-\infty}^{\infty} A(if)e^{i2\pi ft} df \right] dt$$

$$= \int_{-\infty}^{\infty} A(if) \left[\int_{-\infty}^{\infty} x(t)e^{i2\pi ft} dt \right] df$$

$$\int_{-\infty}^{\infty} A(if)A*(if)df$$

where A(if) and A*(if) are complex conjugates.

$$\therefore \int_{-\infty}^{\infty} x^2(t)\, dt = \int_{-\infty}^{\infty} \left| A(if) \right|^2 df$$

As $\left| A(if) \right|^2$ is an even function of f this may be written,

$$\int_{-\infty}^{\infty} x^2(t)dt = 2 \int_{-\infty}^{\infty} \left| A(if) \right|^2 df$$

A random signal is not periodic and cannot be expressed as a Fourier Series. It cannot be expressed as a Fourier Integral either because to have stationary properties a random signal must be assumed to extend over an infinite time. The Fourier Transform A(if) of a signal which begins at $t = -\infty$ and continues until $t = \infty$ cannot be defined. It is possible, however, to obtain the Fourier Transform of a signal $x_T(t)$ which is defined as equal to $x(t)$ over the interval $-\frac{T}{2} < t < \frac{T}{2}$ and zero at other times.

$$\overline{x_T^2(t)}/$$

$$\overline{x_r^2(t)} = \frac{1}{T}\int_{-T/2}^{T/2} x_r^2(t)\,dt$$

$$= \frac{2}{T}\int_0^{\infty} |(A_r(if)|^2\,df$$

The limit of this as $T \to \infty$ gives the mean-square value of $x(t)$,

$$\overline{x^2(t)} = \int_0^{\infty} \lim_{T\to\infty}\left[\frac{2}{T}\,|A_r(if)|^2\right] df$$

$$= \int_0^{\infty} S(f)\,df$$

The power spectrum of the signal is defined by,

$$S(f) = \lim_{T\to\infty}\left[\frac{2}{T}\,|A_r(if)|^2\right]$$

$$\sigma^2 = \overline{x^2(t)} = \int_0^{\infty} S(f)\,df$$

APPENDIX 2

DETERMINATION OF A_{ω} AND THE USE OF A CROSS-CORRELATION COEFFICIENT IN THE ENERGY ANALYSES

A 2.1 DETERMINATION OF A_{ww}

The equation for excitation energy which is derived in Chapter 4 is,

$$E_e = B \rho C d \bar{V} \omega \pi \int_0^l \left[\frac{A_{ww}}{\omega} - f(z) A_{st} \right] f(z) A_{st} \, dz \quad .....(4.1)$$

This equation is based on the assumption that the exciting force is one of constant phase and amplitude and in response to this type of exciting force a structure would achieve a steady state condition of vibration. In practice, however, the amplitudes of narrow bandwidth components of wind turbulence fluctuate erratically, even over short periods of time, and in such a situation the concept of a steady state response has little meaning. Equations 4.1 and 4.2 therefore constitute a much simplified model of the real system and a realistic value for the input parameter A_{ww} is impossible to obtain because it has no equivalent in the real system.

The object of the analysis was to compare the relative levels of the three main energy forms, however, and this may be done using the root mean square value of the turbulent velocity components. This can be considered to remain constant with time. The energy levels calculated from the r.m.s. velocities are not true energy levels but a comparison of them is still a valid exercise for the purposes of assessing the effects of non-linearity and of possible rate of build up of vibration.

The/

The r.m.s. value of a narrow band component of turbulence may be obtained from a wind velocity spectrum. To do this it is necessary to specify the bandwidth. All components of turbulence within the bandwidth in which the frequency response function of a structure is greater than one are capable of inducing a dynamic response in that structure. All such components should therefore be included in the bandwidth which is specified to obtain A_w from the spectrum.

The response of the structure within this bandwidth varies with frequency in proportion to the variation in the frequency response function. Equation 4.1, however, applies only to that component of response which occurs at the natural frequency of the structure. Use of a value for A_w obtained from the spectrum using a bandwidth for which the frequency response function is greater than one would be equivalent to assuming that all the energy in the wind in this bandwidth is concentrated at a discrete frequency. This would lead to an overestimation of the dynamic response of the structure.

An approximation to the real situation is obtained if the bandwidth is selected such that,

$$\Delta n = \frac{\text{Area under resonance peak in frequency response function}}{\text{height of resonance peak}}$$

where Δn = bandwidth

This is the width of the rectangular resonance peak with the same area and height as the resonance peak from the frequency response/

response function. By reducing Δn in this way the energy input to the system is reduced proportionately so as to allow for the fact that different components of turbulence, within the bandwidth for which the frequency response function is greater than one, excite the structure by different amounts.

A_w is therefore given by,

$$A_w = \sqrt{Sp_v(n)}$$

$$Sp_v(n) = \frac{4 k \bar{V}^2 x^2 \Delta n}{n(1 + x^2)^{4/3}}$$

$$x = \frac{4000n}{\bar{V}}$$

k = roughness coefficient

A2.2 USE OF CROSS-CORRELATION COEFFICIENT

The value for A_w obtained from a spectrum of horizontal wind velocity applies to one point in space only. One of the initial assumptions of the preceeding analysis is that the resonant frequency turbulence component is effective over the whole structure. This assumes that all gusts are large enough to fully envelop the structure and that full correlation occurs in both the horizontal and vertical directions.

The constituant gusts of high frequency turbulence are usually of small spatial extent, however. This is due to the fact/

fact that the vertical and longitudinal dimensions of eddies in a turbulent airstream are usually of the same order of magnitude. The extent to which the wind velocity at two points in space across an airstream are correlated depends, therefore, on the frequency of the turbulence being examined. It has been shown that the extent to which high frequency turbulence ($n \geq 1{\cdot}0$ Hz) is correlated is small compared to the dimensions of most engineering structures. In practice even slender structures have relatively high natural frequencies (around 1 Hz) and it is unlikely that the turbulence components which excite such structures to vibrate at their natural frequencies will be fully correlated over their surfaces. Input energy calculations based on the assumption that full correlation exists are therefore likely to err on the high side.

It was considered that an allowance for lack of full correlation in the vertical direction was essential in the energy analysis but it was felt that, as the hypothetical structures used in the analysis were considered to be of small width, full correlation in the horizontal direction could be assumed.

In order to comply with the other assumptions in the analysis the vertical cross-correlation properties of wind turbulence components for zero time lag are required. These are given by the co-spectrum of wind velocity, which is a measure of the contribution made by different frequency components to the co-varience between the velocity functions at two points in an airstream for zero time lag.

Most/

Most of the work on the cross-correlation properties of wind turbulence has been centred on a complex quantity known as the Cross-Correlation Spectrum. This is given by the expression,

$$\text{Cross-Correlation Spectrum} = \frac{Co_{12}(n) + i\ Q_{12}(n)}{S_1(n)\ S_2(n)}$$

where Co_{12} = co-spectrum of velocity fluctuations at points 1 and 2

Q_{12} = quadrature spectrum of velocity fluctuations at points 1 and 2.

$S_1(n)$ and $S_2(n)$ = spectra at points 1 and 2 respectively

The quadrature spectrum is similar to the co-spectrum. The difference is that in the quadrature spectrum, the velocities at points 1 and 2 are compared for a time lag of $\frac{1}{4}$ period, instead of zero time lag.

Davenport has derived an expression for the modulus of the cross-correlation spectrum, based on data from a number of sites. The Davenport formula is,

$$\text{Modulus of Cross-Correlation Spectrum,} \quad C_s = e^{-c' \frac{\Delta zn}{\bar{V}}}$$

where c' = a constant dependent on ground roughness.

In the spectrum analysis, Davenport uses this function to represent the co-spectrum of wind velocity and computes an amended excitation spectrum by multiplying the ordinates in the/

the velocity spectrum by c_s. Davenport justifies this by demonstrating that the modulus of the cross-correlation spectrum and the co-spectrum are almost identical for most frequencies. Fig. A2.1, which is taken from Davenport, shows the co-and quadrature spectra plotted against wave number. It can be seen that the quadrature spectrum rises to one maximum, at a wave number of approximately 0·002, then dies away to almost zero. This means that the cross-correlation spectrum is dominated by the co-spectrum at almost all wave numbers and is only slightly affected by the quadrature spectrum. The c_c function, used by Davenport, is therefore quite a good approximation to the co-spectrum. Davenport, in his 1962 paper, justifies its use in spectrum analysis by saying "In spite of the non-zero quadrature component it is nevertheless small, and it seems adequate for practical purposes to use the square root of the coherence* as a measure of the cross correlation".

The existance of the maximum in the quadrature spectrum does however indicate that there is a slight correlation between wind turbulence at two points across an airstream for a $\frac{1}{4}$ phase time lag.

In the energy analysis, the cross-correlation properties of the wind are allowed for, in the same way as in the Davenport analysis, by multiplying the A_{ww} term by c_s. The use of c_s in this application is less easily justified. There are two main sources of error. Firstly, by appearing under the integral/

* coherence is square of cross-correlation spectrum

integral sign in equation 4.1 it has the effect of making the first term in this equation very small at the base of the structure, where z = 0, and causes it to rise to a maximum at the top. This situation is the same for all the cycles of vibration and the correlation is therefore assumed to remain fixed spatially for every cycle. This is equivalent to assuming that all the constituant gusts of the resonant component of turbulence hit the top of the structure and while it represents the worst case it is not a true representation of the full scale situation in which the turbulence is random in space. The second source of error is the neglect of the quadrature spectrum. The fact that some correlation is possible between gusts hitting different parts of the structure $\frac{1}{4}$ period out of phase, as is suggested by Fig A2.1 could increase the aerodynamic damping. No allowance for this is made in the energy analysis.

The c_s function cannot, therefore, be considered an ideal expression for the cross-correlation properties of the wind but it does give an indication of the orders of magnitude involved. The errors due to its use in this application are likely to be on the conservative side and the calculated levels of excitation energy are likely to be larger than would occur in practice. This should be kept in mind when interpreting the results.

APPENDIX 3

ANALYSIS OF MULTI-STOREY SHEAR WALL STRUCTURES

A3.1 SUMMARY

As was stated in Chapter 3, the dominant structural parameter so far as dynamic wind loading is concerned is the natural frequency. For a complicated structure such as a building a rigorous analysis to find this tends to be beyond the scope of a design engineer and it has become common practice to regard such structures as simple cantilevers. It was decided to examine a particular type of building in some detail so as to establish whether such an approximation was likely to lead to large errors. This appendix is devoted to the analysis of a multi-storey shear wall type structure to find its natural frequencies. The purpose was firstly to try and check the accuracy of simpler and more approximate methods which have previously been used and secondly to provide a better means of determining such an important structural parameter should the previously used methods prove inaccurate. A mathematical model was chosen such that any resulting design method would be applicable to all buildings of the shear wall type.

The analysis was carried out using the continuum theory which has been widely applied in the case of static loadings on such structures. A solution was obtained in terms of constants which are simple to evaluate and a number of hypothetical buildings were analysed by the continuum theory and more approximate methods so that comparisons could be made. A set of simple experiments was carried out on model structures to confirm the conclusions reached from the theoretical results.

A3-2 THEORY

The type of building analysed was of the multi-storey shear wall type, a typical example of which is shown in Fig. A3.1. The main structural elements of these buildings are load bearing walls which act both as the main vertical supporting members and as wind bracing. They are connected to one another at each storey level, either by continuous floor slabs or by floor beams. Each building usually consists of a number of similar bays, each of which contains elements to provide stiffness along the two principal axes of the building. The walls therefore tend to be T, L, H or E shaped in plan. The buildings vibrate about different axes with different natural frequencies and it is convenient to split the analysis into two parts and deal with each axis separately. Soane has shown that for the static case, a wall with a complicated plan shape can be replaced in the analysis, without loss of accuracy, by one of rectangular cross-section, which has an equivalent second moment of area about the axis concerned. This simplified approach is used here. Fig A3.2 illustrates how the building in Fig. A3.1 would be simplified for further analysis.

To simplify the problem further it is assumed that all bays in the building are of identical mass and stiffness and that individually they will have the same natural frequency as the whole building. i.e. the natural frequency of bay abcd in Fig. 3.2, vibrating about the x-x axis, will be the same as that of the whole building, vibrating about the same axis. It was decided that/

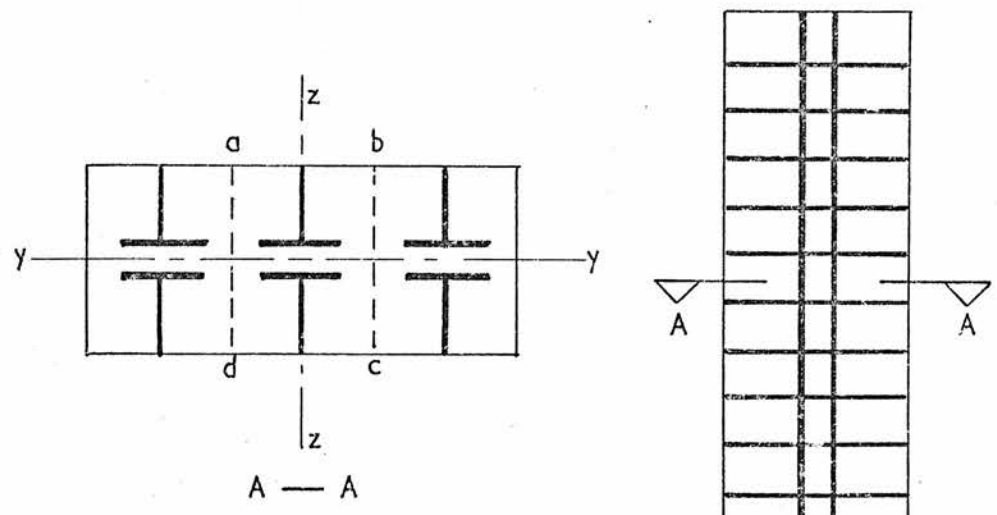


Fig A3.1 Typical shear wall building.

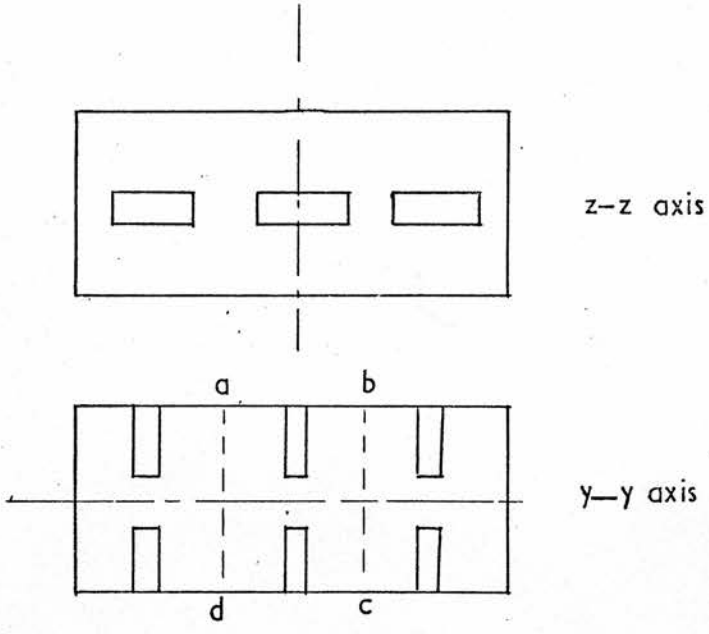


Fig A3.2 Building simplified for analysis.

that in order to achieve versatility of application, any resulting design method would have to be capable of dealing with buildings with different numbers of walls. The mathematical model of one bay, therefore, is a plane structure with n walls and (n-1) interconnecting sets of beams or slabs, (Fig. A3.3).

The analysis has been carried out in accordance with the following assumptions:-

(a) That the mass of the structure is uniformly distributed along its length.

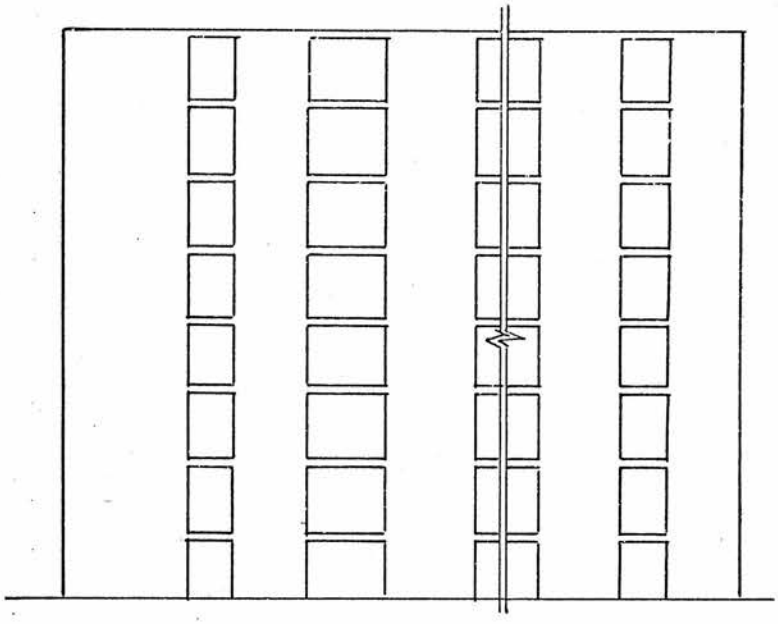
(b) That all behaviour is elastic. Shear wall structures are normally constructed of reinforced concrete or brickwork and this is approximately true in the range of design stresses used.

(c) That a condition of complete fixity exists at foundation level.

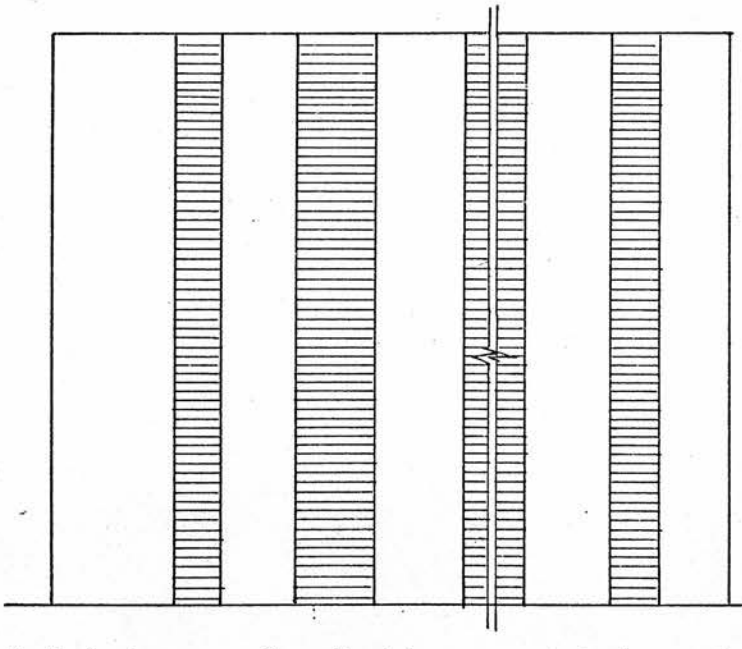
(d) That the deformation of the connecting elements due to normal forces in the connecting elements themselves are negligible. i.e., the lateral deformations of the walls are the same at any given level. This has been verified experimentally for the static case.

(e) That the points of inflection of the connecting elements are at their centres.

The discrete connecting elements were replaced by continuous media having appropriate stiffness properties and the model split up into individual free standing cantilevers. The effect of the media on these was represented by equivalent external forces/



Plane array of n walls interconnected by n-1 sets of beams or slabs.



Equivalent array of walls interconnected by continuous media.

Fig A 3.3.

forces and moments, distributed along the edges of the walls. The i th wall is shown in Fig. (A3.4) and a section of this in Fig. A3.5. The notation used in these figures is:-

m_i = the bending moment per unit length transmitted from medium i.

t_i = the normal force per unit length transmitted from medium i.

r_i = the shear force per unit length transmitted from medium i.

M_i = the bending moment in wall i.

N_i = the normal force in wall i.

w_i = the width of wall i.

A_i = the cross-sectional area of wall i.

ρ = the density of the material.

Considering the element of wall i shown in Fig A3.5 and taking moments:-

$$M_i + Q_i \frac{dx}{2} + Q_i \frac{dx}{2} + r_i \frac{w_i}{2} + r_j \frac{w_i}{2} + m_i dx + m_j dx - M_i - \frac{\partial M_i}{\partial x} dx = 0 \quad \text{.....(A3.1)}$$

If higher powers of increments are neglected this simplifed to:-

$$Q_i + (m_i + m_j) + (r_i + r_j)\frac{w_i}{2} - \frac{\partial M_i}{\partial x} = 0 \quad \text{.....(A3.2)}$$

Resolving/

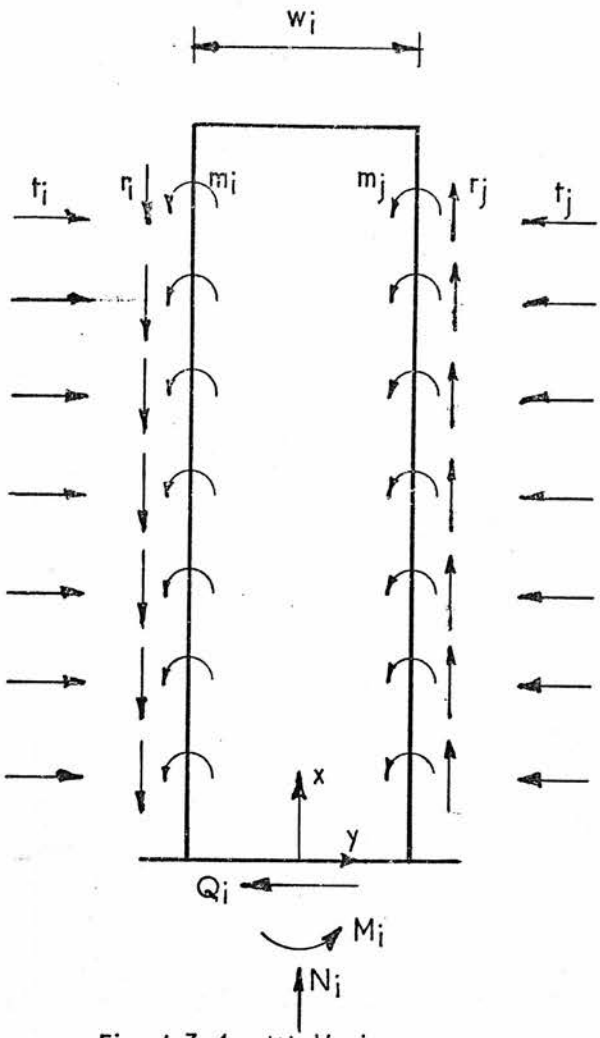


Fig A 3.4. Wall i.

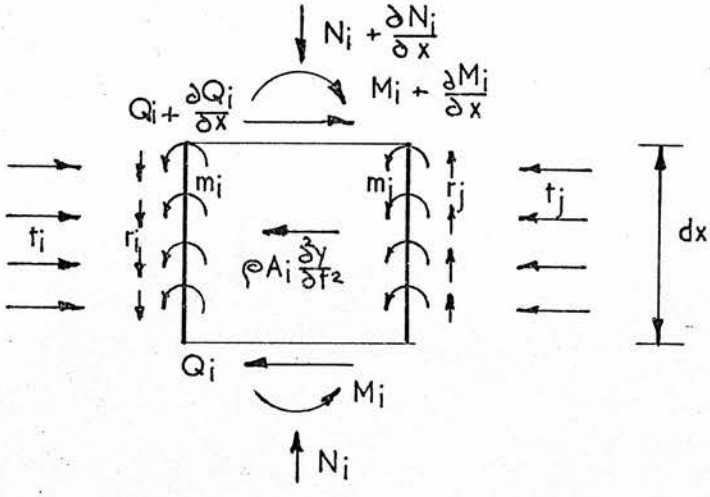


Fig A3.5. An element of wall i

Resolving horizontally and vertically leads to:-

$$\frac{\partial Q_i}{\partial x} + t_i - t_j + \rho A_i \frac{\partial^2 y_i}{\partial x^2} = 0 \qquad \text{.....(A3.3)}$$

$$\frac{\partial N_i}{\partial x} + r_i - r_j = 0 \qquad \text{.....(A3.4)}$$

It has been shown by Chitty that the effect of the normal forces in the walls is negligible, and so equation A3.4 was considered redundant. Differentiating equation (A3.2) and substituting in A3.3 gives:

$$\frac{\partial^2 M_i}{\partial x^2} - \left(\frac{\partial m_i}{\partial x} + \frac{\partial m_j}{\partial x}\right) - \left(\frac{\partial r_i}{\partial x} + \frac{\partial r_j}{\partial x}\right) \frac{w_i}{2} + t_i - t_j + \rho A_i \frac{\partial^2 y_i}{\partial t^2} = 0 \qquad \text{.....(A3.5)}$$

An equation similar to (A3.5) can be derived for each wall in the structure. Assuming there are n walls of equal width, and adding all such equations, the terms in t cancel out and the equation simplified to:-

$$\sum_n \frac{\partial^2 M_i}{\partial x^2} - 2 \sum_n \frac{\partial m_i}{\partial x} - w \sum_n \frac{\partial r_i}{\partial x} + \rho \sum_n A_i \frac{\partial^2 y_i}{\partial t^2} = 0 \qquad \text{.....(A.36)}$$

The assumption of equal wall widths involves a loss of generality but this can easily be restored once the differential equation has been/

been solved, as is demonstrated later.

To simplify (A3.6) it is necessary to relate $\frac{\partial m_i}{\partial x}$ and $\frac{\partial r_i}{\partial x}$ to derivatives of y. This is done by examining the moments and reactions in one of the connecting elements. A diagram of the forces involved is shown in Fig. A3.6. In accordance with assumption (d), the walls deflect equally but AB does not remain perpendicular to them. The connecting element is constrained to remain at right angles to the walls at its ends, and this imposes resultant moments and forces on the walls. The end displacement of the connecting element is the sum of the longitudinal strains in the walls and the relative displacement of the walls due to bending. As was previously stated, the former of these has been proved negligible by Chitty. The bending moment in the connecting elements can therefore be directly related to the end displacement, as in equation (A3.7).

$$R_{ci} = \frac{2M_i}{e_i} \qquad \text{.....(A3.7)}$$

Thus at a distance q from A:-

$$EI_{ci} \frac{\partial^2 p}{\partial q^2} = -M_{ci} + \frac{2M_{ci}\, q}{e_i}$$

$$\therefore\ EI_{ci} \frac{\partial p}{\partial q} = -M_{ci}\, q + \frac{M_{ci}\, q^2}{e_i} + V$$

$$\therefore\ EI_{ci}\, p = \frac{-M_{ci}\, q^2}{2} + \frac{M_{ci}\, q^3}{3e_i} + Vq + W \qquad \text{.....(A3.8)}$$

$p = 0$ at A where $q = 0$, $\therefore\ W = 0$

From/

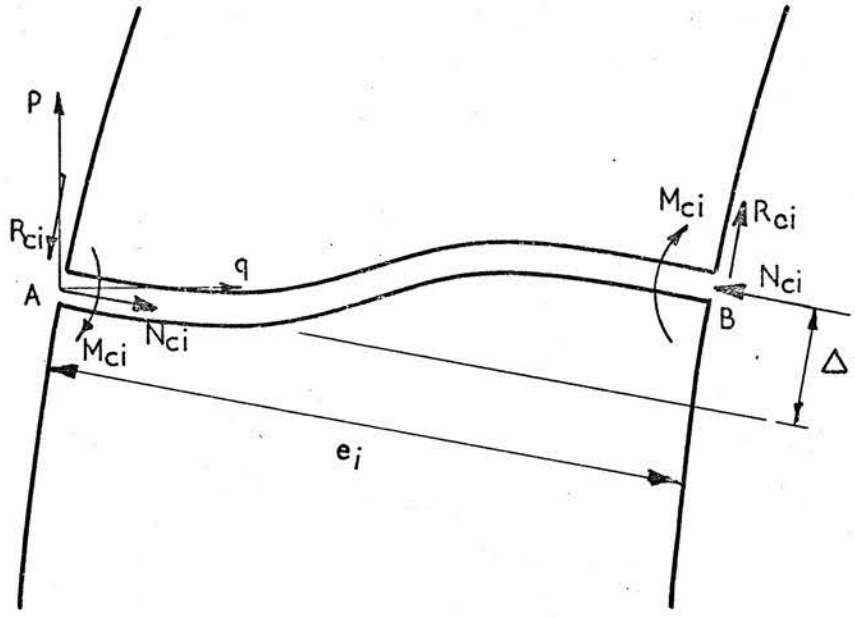


Fig A3.6. A connecting element in space i.

From assumption (e), $P = 0$ when $q = \frac{e_{ci}}{2}$

$\therefore \quad V = \frac{M_{ci} e_i}{6}$

Thus when $q = 0$, the slope at the end, $\frac{\partial P}{\partial q} = \frac{M_{ci} e_i}{6 EI_{ci}}$

$$\therefore \quad M_{ci} = \frac{6 EI_{ci}}{e_i} \frac{\partial P}{\partial q} \qquad \text{.....(A3.9)}$$

$$\text{and} \quad R_i = \frac{12EI_{ci}}{e_i} \frac{\partial p}{\partial q} \qquad \text{.....(A3.10)}$$

According to the sign convention adopted, a positive end moment results from a positive wall slope, thus

$$M_{ci} = \frac{6EI_{ci}}{e_i} \frac{\partial y_i}{\partial x} \qquad \text{.....(A3.11)}$$

$$R_{ci} = \frac{12EI_{ci}}{e_i^2} \frac{\partial y_i}{\partial x} \qquad \text{.....(A3.12)}$$

If the height between connecting elements is h_i,

$$m_i = \frac{6EI_{ci}}{h_i e_i} \frac{\partial y_i}{\partial x} \qquad \text{.....(A3.13)}$$

$$r_i = \frac{12EI_{ci}}{h_i e_i} \frac{\partial y_i}{\partial x} \qquad \text{.....(A3.14)}$$

Thus,

$$\frac{\partial m_i}{\partial x} = \frac{6EI_{ci}}{h_i e_i} \frac{\partial^2 y_i}{\partial x^2} \qquad \text{.....(A3.15)}$$

$\frac{\partial r_i}{\partial x}$ /

$$\frac{\partial r_i}{\partial x} = \frac{12EI_{ci}}{h_i e_i^2} \frac{\partial^2 y_i}{\partial x^2} \qquad \text{.....(A3.16)}$$

The following relationship may also be derived,

$$M_i = - EI_i \frac{\partial^2 y_i}{\partial x^2}$$

$$\therefore \frac{\partial^2 M_i}{\partial x^2} = - EI_i \frac{\partial^4 y_i}{\partial x^4} \qquad \text{.....(A3.17)}$$

Substituting (A3.15), (A3.16) and (A3.17) in (A3.6) and simplifying

$$\frac{\partial^4 y_i}{\partial x^4} + \frac{12 \sum \frac{I_{ci}}{h_{ci} e_{ci}} \left(1 + \frac{w}{e_i}\right)}{I_i} \frac{\partial^2 y_i}{\partial x^2} -$$

$$\frac{\rho \sum A_i}{E \sum I_i} \frac{\partial^2 y_i}{\partial t^2} = 0 \qquad \text{.....(A3.18)}$$

Assuming a vibrational solution of the form $y(x,y) = f(x)g(t)$ where $g(t) = \text{Sin}(\omega t + \alpha)$ and ω is an eigenvalue, (A.318) can be rewritten,

$$\frac{\partial^4 f}{\partial x^4} + \frac{12 \sum \frac{I_{ci}}{h_i e_i} \left(1 + \frac{w}{e_i}\right)}{I_i} \frac{\partial^2 f}{\partial x^2} + \omega^2 \frac{\rho}{E} \frac{\sum A_i}{\sum I_i} f$$

$$= 0 \qquad \text{.....(A3.19)}$$

which/

which simplifies to,

$$\frac{d^4 f}{dx^4} + a_1 \frac{d^2 f}{dx^2} + b_1 \omega^2 f = 0 \qquad \text{.....(A3.20)}$$

with a_1 and b_1 constant

$$a_1 = \frac{12 \sum \frac{I_i}{h_i e_i} \left(1 + \frac{w_i}{} \right)}{\sum I_i}, \quad b_1 = \frac{\rho}{E} \frac{\sum A_i}{\sum I_i}$$

The boundary conditions of equation (A3.20) are found by examining the end conditions of the walls. According to assumption (c), the deflection and slope at the base of each wall will be zero.

$$f = 0 \quad \text{when} \quad x = 0 \qquad \text{.....(i)}$$

$$\frac{df}{dx} = 0 \quad \text{when} \quad x = 0 \qquad \text{.....(ii)}$$

At the free ends of the walls it may be assumed that the bending moments and shear forces will be zero. Thus,

$$\frac{d^2 f}{dx^2} = 0 \quad \text{when} \quad x = H \qquad \text{.....(iii)}$$

$$\frac{d^3 f}{dx^3} = 0 \quad \text{when} \quad x = H \qquad \text{.....(iv)}$$

where H is the total height of the walls.

The calculation is simplified if (A3.20) is made non-dimensional. This may be done with a substitution of the form,

$$f = Hf' \qquad a = H^{-2} a_1$$

$$x = Hx' \qquad b = H^{-4} b_1$$

(A3.20)/

(A3.20) becomes,

$$\frac{d^4 f'}{dx'^4} + a\frac{d^2 f'}{dx^2} + \omega^2 b' f' = 0 \qquad \text{.....(A3.21)}$$

with boundary conditions,

$f' = 0$ when $x' = 0$

$\frac{df}{dx} = 0$ when $x' = 0$

$\frac{d^2 f}{dx^2} = 0$ when $x = 1$

$\frac{d^3 f}{dx^3} = 0$ when $x = 1$

A power series solution to (A3.20) may be obtained using the method suggested by Frobenius. A detailed description of this is given in Appendix (4) and it is only dealt with briefly here. If the substitution,

$$f' = \sum_n P_n x^{n+c}$$

is made in (A3.21) the equation becomes,

$$\sum_n P_n (n + c)(n + c - 1)(n + c - 2)(n + c - 3) x^{n + c - 4} +$$

$$a \sum_n P_n (n + c)(n + c - 1) x^{n + c - 2} + b\omega^2 \sum P_n x^{n + c} = 0 \qquad \text{.....(A3.22)}$$

By equating coefficients, expressions can be found from which the coefficients of the terms in each power series may be determined./

determined. This leads to the four series below.

$$c = 0: \quad f_1 = 1 + a\frac{x^2}{2!} + (b\omega^2 + a^2)\frac{x^4}{4!} + (2b\omega^2 a + a^3)\frac{x^6}{6!} + (b^2\omega^4 + 3b\omega^2 a^2 + a^3)\frac{x^8}{8!} + \cdots \quad \text{.....(A3.23)}$$

$$c = 1: \quad f_2 = x + a\frac{x^3}{3!} + (b\omega^2 + a^2)\frac{x^5}{5!} + (2b\omega^2 a + a^3)\frac{x^7}{7!} + (b^2\omega^4 + 3b^2\omega^2 a^2 + a^3)\frac{x^9}{9!} + \cdots \quad \text{.....(A3,24)}$$

$$c = 2: \quad f_3 = x^2 + a/12x^4 + \frac{1}{300}(b\omega^2 + a^2)x^6 + \frac{1}{2{\cdot}01 \times 10^4} \times (2b\omega^2 a + a^3)x^8 + \cdots \quad \text{.....(A3.25)}$$

$$c = 3: \quad f_4 = x^3 + a/20\,x^5 + \frac{1}{840}(b\omega^2 + a^2)x^7 + \frac{1}{6{\cdot}0^4 \times 10^4}(2b\omega^2 a + a^3)x^9 + \cdots \quad \text{.....(A3.26)}$$

The mode shape,

$$f = Af_1 + Bf_2 + Cf_3 + Df_4 \quad \text{.....(A3.27)}$$

where A, B, C and D are arbitrary constants. Application of the boundary conditions gives,

$$f_3''(1) \times f_4'''(1) - f_3''(1) \times f_4'''(1) = 0 \quad \text{.....(A3.28)}$$

If/

If the derivatives of (A3.25) and (A3.26) are substituted in (A3.28), an equation is obtained which is a polynomial in ω. The roots of this give the eigenvalues of equation (A3.21), which are the natural frequencies of the system. By substituting these in turn in equation (A3.27) and again applying the boundary conditions, f(x) may be found in terms of one arbitrary constant. The resulting expressions give the mode shapes of the system, corresponding to the various natural frequencies.

By this method, therefore, it is possible to calculate the natural frequencies and mode shapes of shear wall buildings from two constants, a and b. As can be seen from the formulae, these are easily calculable functions of the buildings' dimensions and of the properties of the construction material. a, is dependent on the stiffness of the connecting elements, on there separation distance and on the ratio of wall width to wall spacing. It therefore represents the extent to which the behaviour of each wall is influenced by the action of adjacent walls and ultimately the extent to which the connecting elements influence the overall behaviour of the building. The value of b depends on the mass and stiffness of the walls themselves. b would therefore be expected to be the dominant of the two constants, a decrease in the value of which should lead to an increase in natural frequency. a will have a lesser effect; a decrease in its value should bring about a decrease in natural frequency. That the two constants do behave in this way can be seen from graph (A3.7).

It/

It was found that for values of a less than 10, which covers a wide range of buildings, the fifth and larger terms in each series became negligible and only the first four terms in each series were required for an accurate solution. This results in the polynomial in ω being a quadratic which is simple to solve. For some types of building, a is greater than 10, in which case more terms in each series are required and the polynomial becomes more involved. More work is envisaged to provide an itterative method by which these polynomials may be solved with the aid of a digital computer.

The expressions given for a and b are applicable to buildings with any number of walls of equal width. The formulation of expressions for buildings with different wall widths is easily carried out and is demonstrated in appendix (5).

A.3.3 COMPARISON OF CONTINUUM THEORY WITH MORE APPROXIMATE METHODS

As was stated previously, it has become common practice to use simple cantilever approximations to buildings for the purposes of calculating their natural frequencies. For a building such as that shown in Fig (A3.1), there are two possible approximations. One is to assume complete interaction between the walls, and to consider the combined section to act as a cantilever. This is the most favoured approach, as the overall dimensions of the building are usually considered to have the major influence on its natural frequency. Another approximation is to assume no interaction between the walls, and to consider the stiffness of

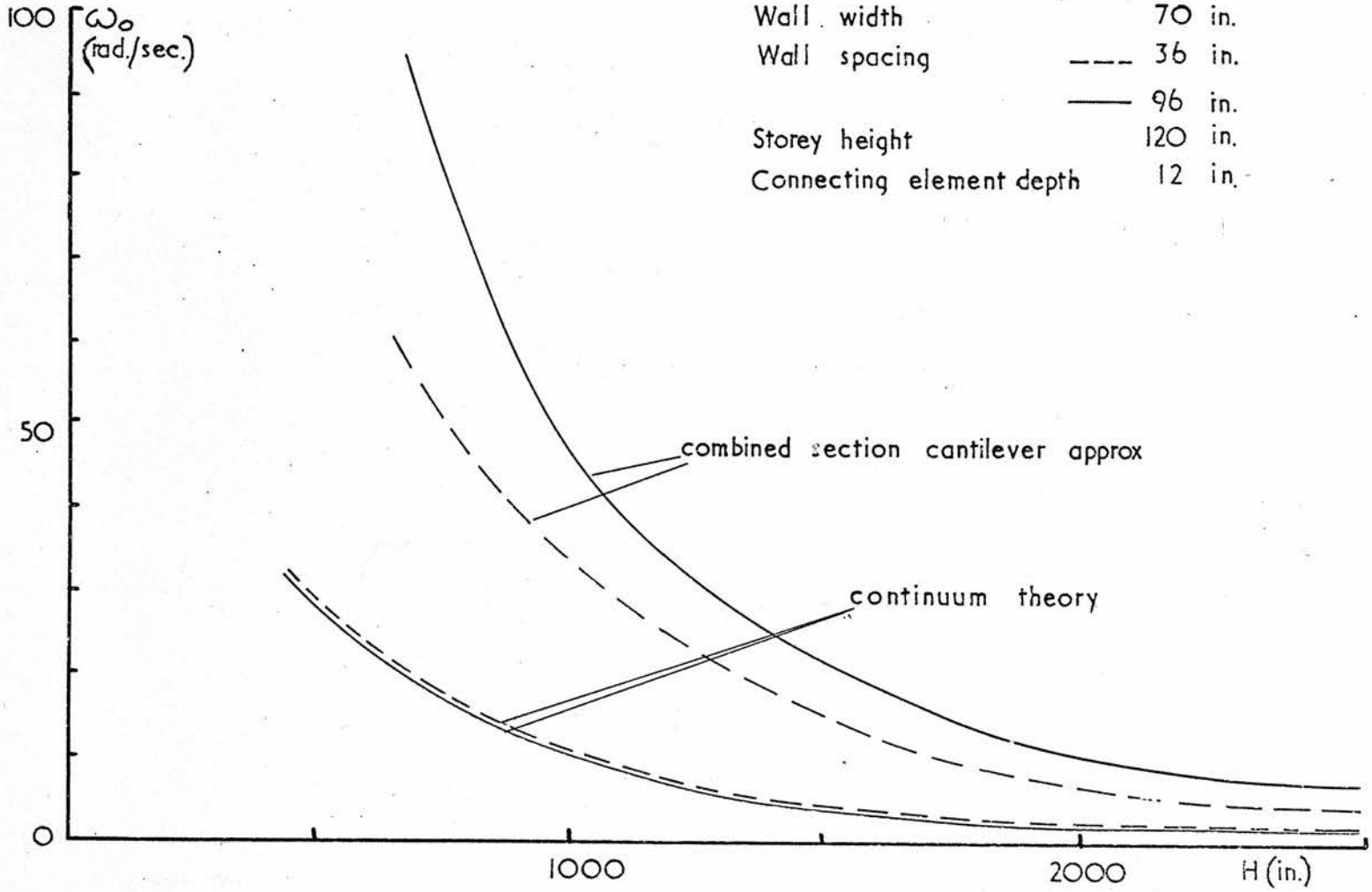
one/

one wall to be a good indication of the stiffness of the whole building. Clearly, these are limiting cases and the true situation lies somewhere between them.

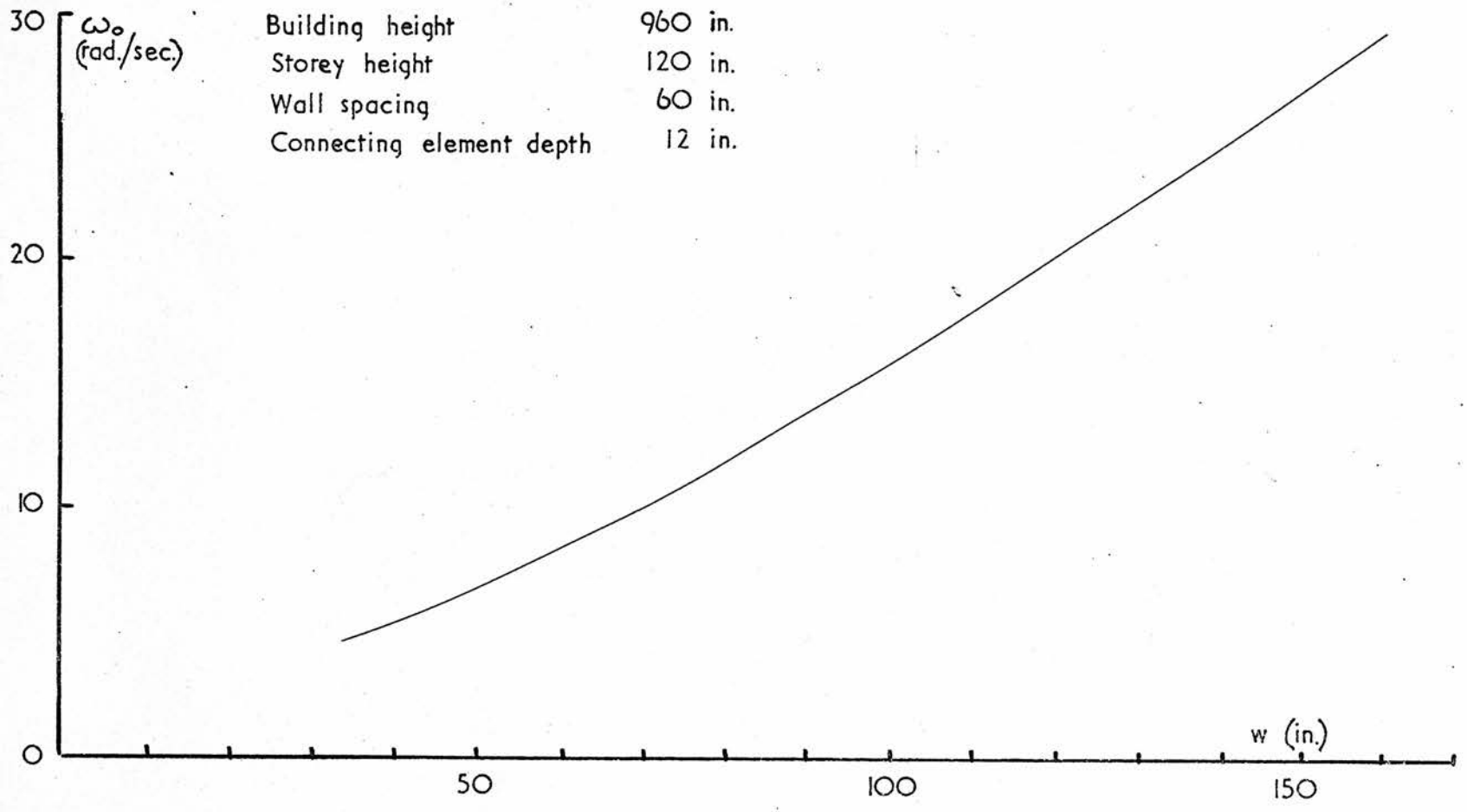
In order to compare the results obtained using the continuum theory outlined previously with those from cantilever approximations, a number of two wall buildings were analysed. Graphs (A3.1) and (A3.2) show typical results.

It can be seen that the dominant parameters in the case of the continuum theory are the actual wall widths, while in the "combined section" cantilever approximation it is the overall width of the building which is important. Also, in every case the continuum theory predicts a much lower natural frequency than the"combined section" cantilever approximation. In fact, the values given by the continuum theory are nearer those appropriate to one wall in the section, given by the "no interaction" approximation rather than the "combined section". This suggests that the connecting effect of the beams is relatively small and that the walls, while constrained to vibrate together, exhibit more or less the same dynamic characteristics as they would do if acting separately. It would appear, therefore, that the "no interaction" case gives a better indication of what is likely to happen in practice than the "combined section" approximation. It tends to underestimate the total stiffness of the building however, because it fails to take account of the stiffness of the cross beams.

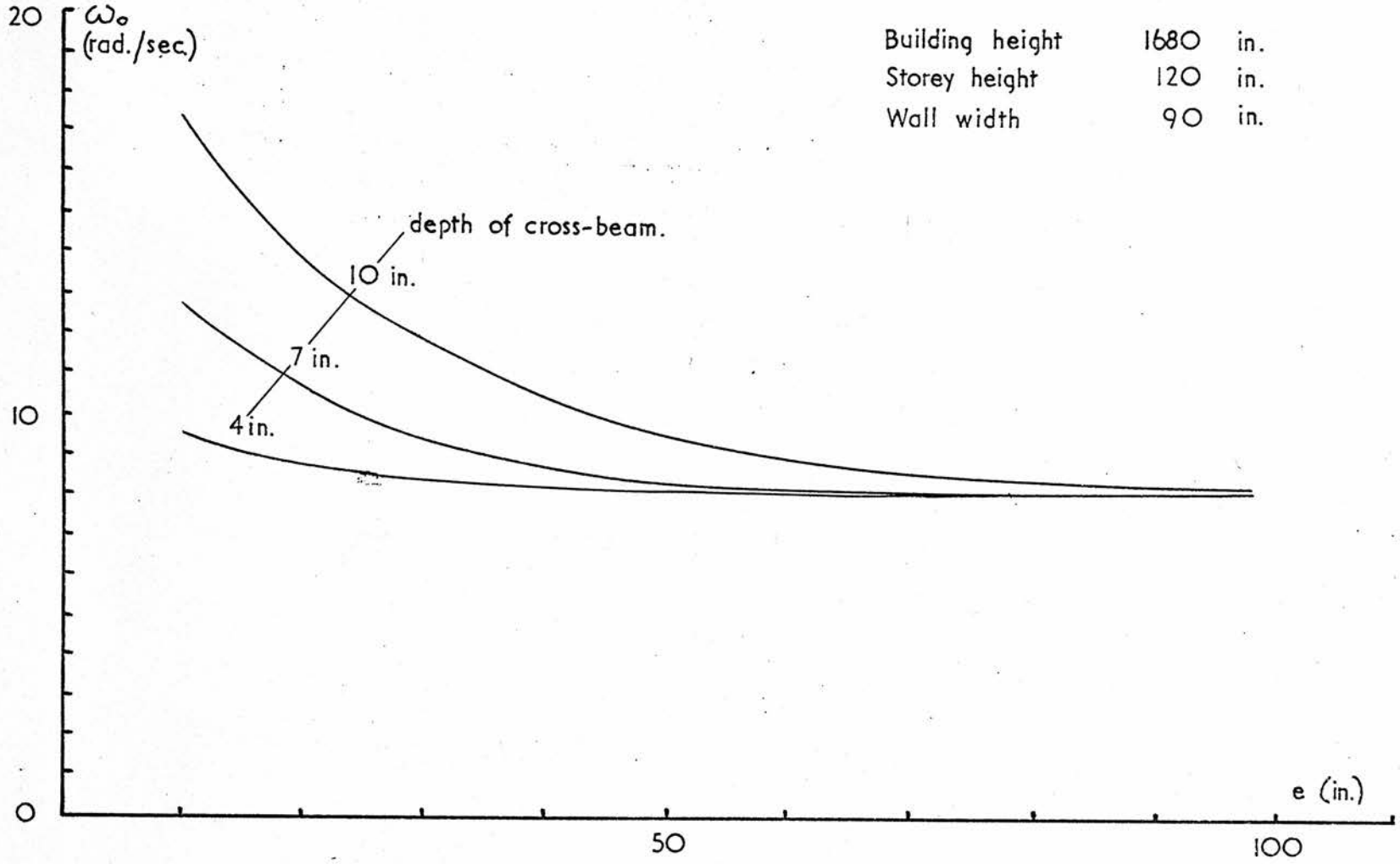
The effect of the cross beams is shown in Graph (A3.3), which demonstrates/



Graph A 3.1 Variation in natural frequency with building height and wall spacing.



Graph A3.2 Variation in natural frequency with wall width.



Graph A 3.3 Variation in natural frequency with stiffness of cross-beams.

demonstrates that the natural frequency decreases slightly with decreasing stiffness of the cross beams and tends to a limit equivalent to the "no interaction" case. This means that the natural frequency falls slightly as the distance between the walls is increased.

A3.4 MODEL INVESTIGATION

According to the continuum theory the natural frequency of a building tends to be affected only slightly by a variation in its overall dimensions if the dimensions of the walls remain the same. A simple experiment was carried out to try and verify this. Three perspex models of two-wall structures were constructed. The wall widths in each of them were kept the same and the wall spacing varied. A diagram showing the dimensions is given in Fig. (A3.7), and the test set up is shown in Fig. (A3.8). Fixity at the base of the models, was attempted by bolting them through 2" x 2" steel angles to a 2' 9" square, $\frac{3}{4}$" thick steel base plate. The models were excited by an electrical vibrator driven by a power oscillator. The amplitude of the vibration was measured by an accelerometer fixed to the top of each model, the output of which was fed through an oscilloscope.

The procedure adopted to find the natural frequency of each model was to sweep through a range of frequencies with the oscillator and note at which frequency resonance occurred. To verify that the resonance peak found was that of the first mode, the/

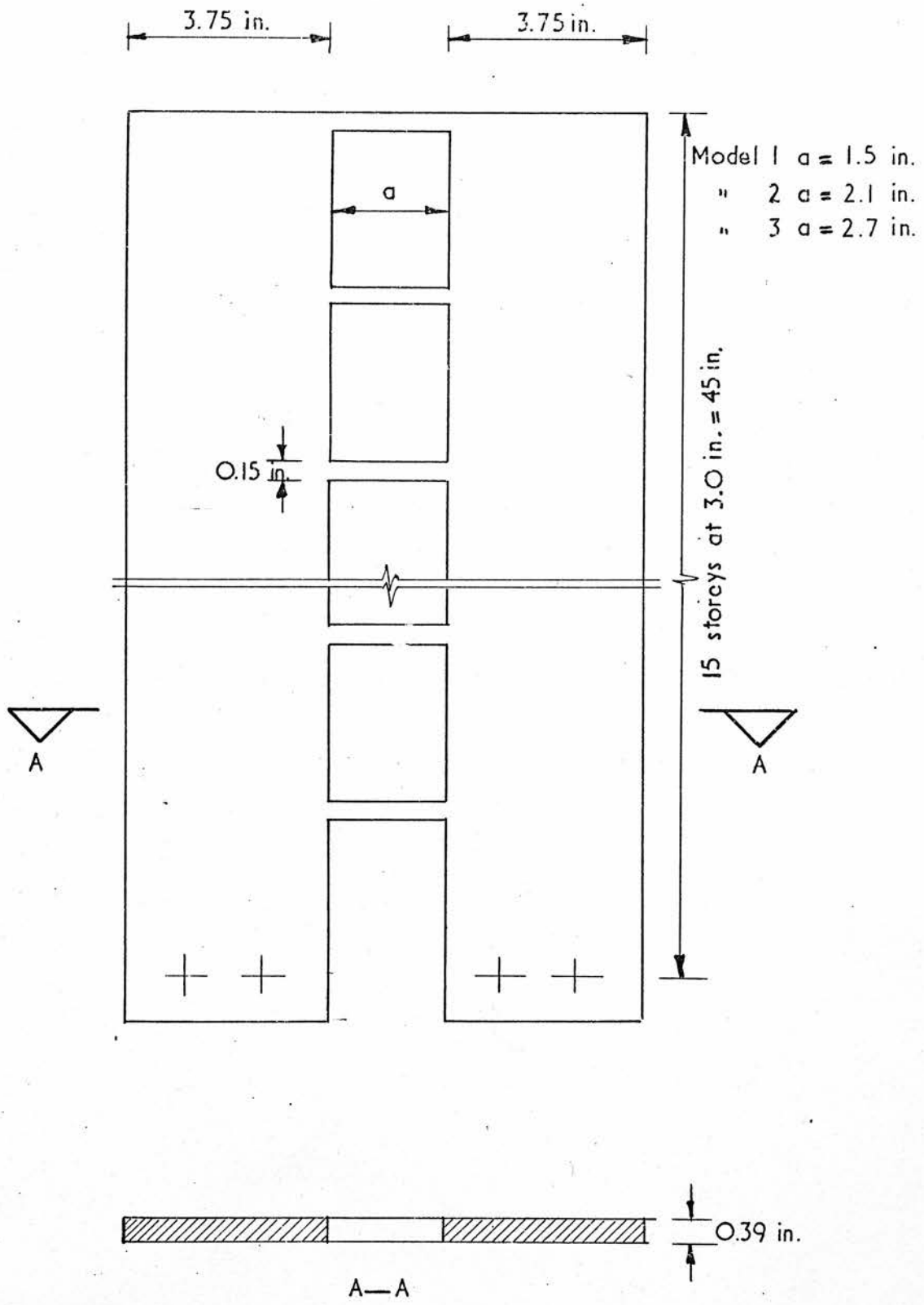


Fig A 3.7 Dimensions of perspex models.

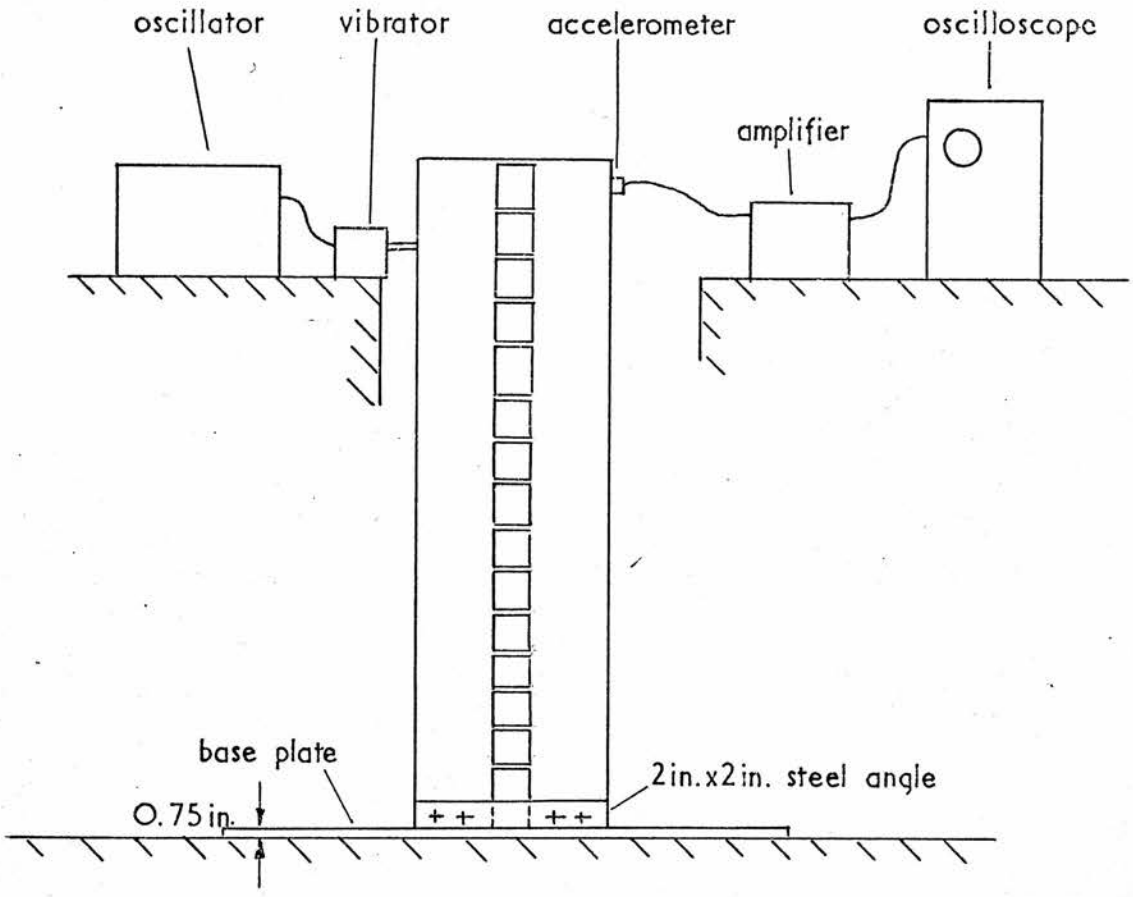


Fig A3.8. Test set up.

the accelerometer was then moved gradually down the model so that the mode shape could be checked. In practice the resonance peaks were very sharp and the resonant frequency could be determined to within $\frac{1}{2}$ Hz. Each model was constructed 15 storeys high and was subsequently shortened, one storey at a time, so that readings appropriate to a number of building heights could be obtained. A plane cantilever sheet of perspex with the same outside dimensions as the medium sized model was also tested. The results of these tests are shown in tables (A3.1) and (A3.2) and Graphs (A3.4) to (A3.6).

The dynamic modulus of elasticity of perspex was found by testing 3/8" thick cantilever strips of different length in a similar manner to the shear wall models. The standard formula for a vibrating cantilever, $\omega = \frac{3.516}{l^2}\sqrt{\frac{EI}{\rho A}}$ was used to relate E to ω. The results of these tests are shown in table (A3.2) and Graph (A3.5)

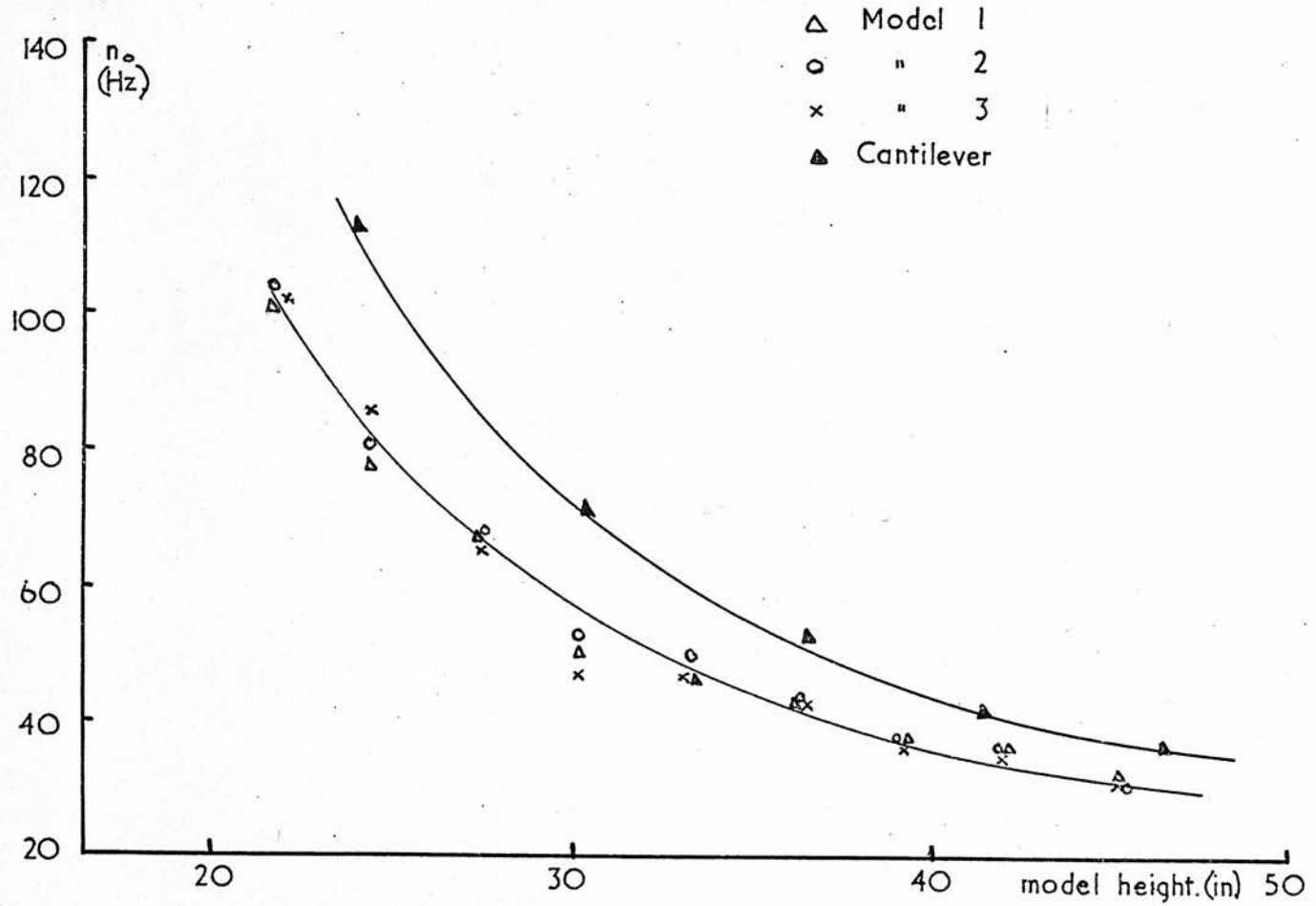
A3.5 RESULTS AND CONCLUSIONS.

As can be seen from Graph (A3.4), altering the wall spacing appeared to have little effect on the natural frequencies of the models. If anything the tendency was for wider spacing to lead to lower natural frequencies as predicted by the continuum theory. In this respect, therefore, the continuum theory gives a better representation of what happens in practice than the cantilever approximation. Also, the plane cantilever of perspex was much stiffer than the shear wall models. In fact, when allowance is made for the change in E value, evident from Graph/

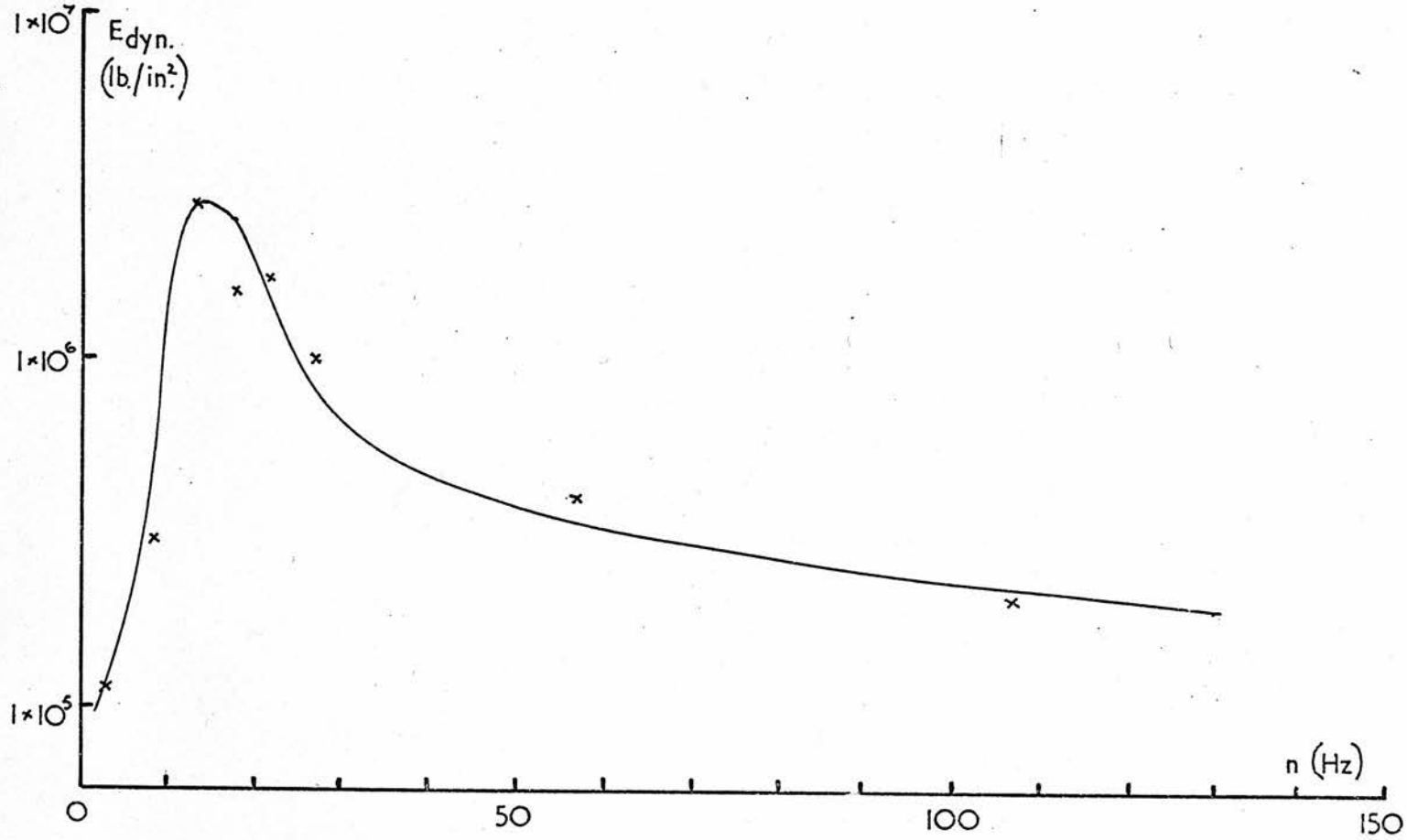
Building Height in	No of storeys	NATURAL FREQUENCY (Hz)			
		Model 1 e = 1·5	Model 2 e = 2·1	Model 3 e = 2·7	Cantilever
47	16				38·5
45	15	31·0	29·0	29·5	
42	14	33·5	33·0	31·5	41·0
39	13	37·0	37·0	35·0	
36	12	41·0	42·0	39·0	53·0
33	11	44·0	50·0	44·0	
30	10	50·0	53·0	48·0	76·0
17	9	65·0	65·0	66·0	
24	8	78·0	80·0	86·0	110·0
21	7	98·0	100·0	98·0	

TABLE A3.1

Length of Strip (in)	Natural Frequency		Dynamic Young's Modulus $\times 10^5$ lb/in^2
	Hz.	Rad/s	
6	110·0	691	2·29
9	57·0	358	6·46
12	33·0	207	6·80
14	29·0	182	9·70
20	21·0	131	21·0
21	19·5	122	18·0
29	15·0	96	49·0
42	1·5	80	1·47
45	2·0	110	4·0



Graph A3.4. Results of tests on shear wall and cantilever models.

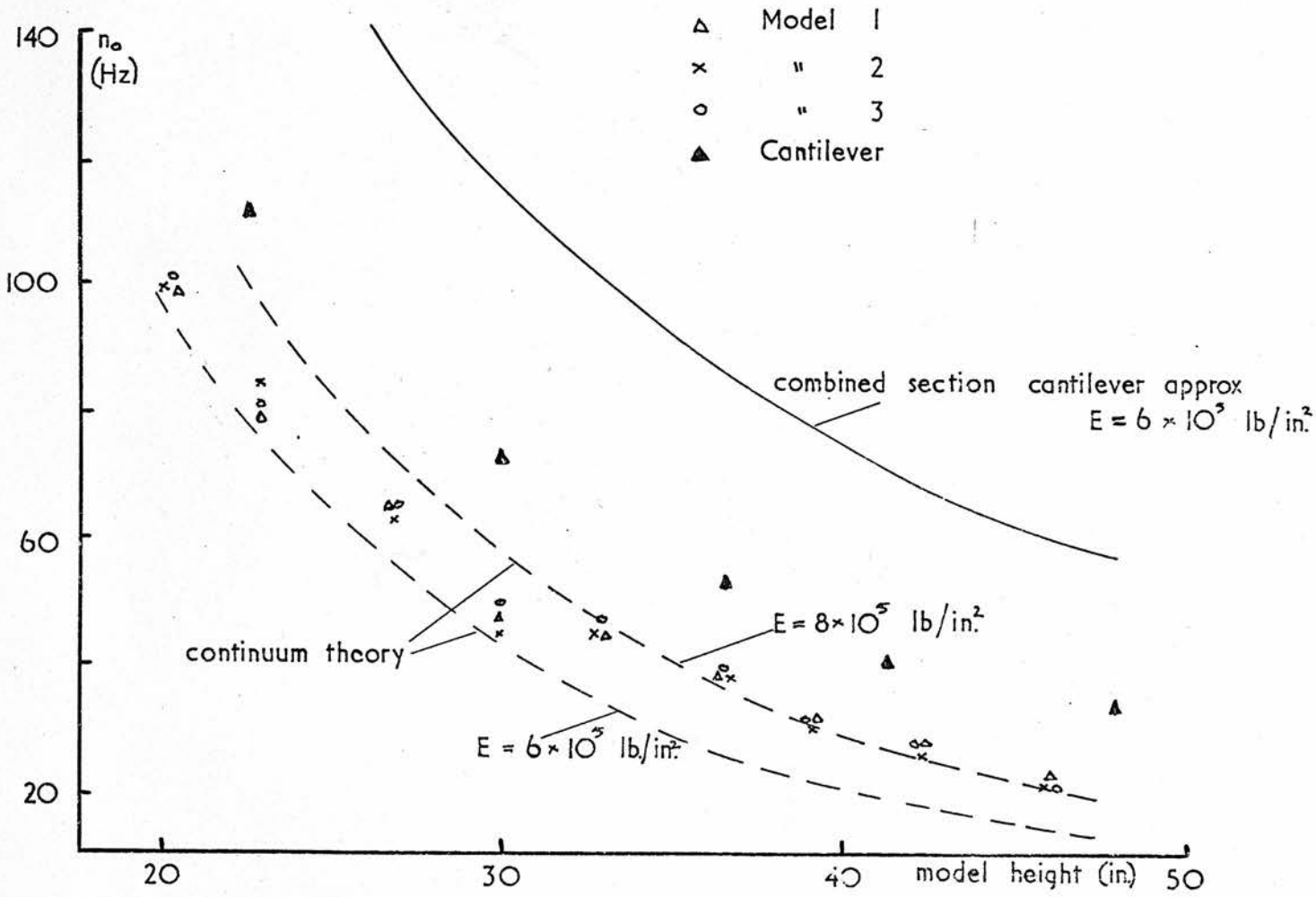


Graph A 3.5. Variation in dynamic modulus of elasticity of perspex with frequency.

Graph (A3.5), the experimental curves for cantilever and shear walls bear much the same relationship to one another as do the theoretical curves for cantilever approximation and continuum theory.

Graph (A3.5) illustrates the large variation in dynamic modulus of elasticity of perspex measured, in the range of frequencies used in the tests. This large variation made direct comparison of theoretical with experimental results difficult. Two sets of theoretical results appropriate to E values of 6×10^5 lb/sq in and 8×10^5 lb/sq in were calculated as being representative of conditions at the ends of the experimental curves. These are shown in conjunction with the experimental points in Graph (A3.6). From this graph it can be seen that the values predicted by the continuum theory match the experimental values well. Considering that the range of E values is from approximately 9×10^5 lb/sq in at 25 Hz to $5{\cdot}5 \times 10^5$ lb/sq in at 100 Hz and that most of the variation occurs from 25 Hz to 45 Hz, if each point on the theoretical curve was worked out for its correct E value, the theoretical and experimental curves would coincide almost exactly.

One disturbing aspect, however, is the lack of agreement of the cantilever results with those predicted by the cantilever approximation, although the discrepancy appears worse than it is, due to the fact that the theoretical curves were worked out for constant E values. The explanation of this is thought to be that the end condition of complete fixity at the base was not in fact achieved with the models and that as a result the experimental curves, were displaced downwards. If this were the case, then the agreement between the experimental/



Graph A3.6 Comparison of experimental with theoretical results.

curves in the case of the continuum theory is not as good as it seems and the conclusion is that the continuum theory predicts values which are too low. Whether or not this is the case could only be established by further investigation.

The results of the experiments, therefore, were rather inconclusive. Good qualitative agreement was achieved, however, between the theoretical and the experimental results in the case of the shear walls and the tests do show that the natural frequencies of the models were not dependent on their overall dimensions. Further tests are envisaged to try and verify the relationship between natural frequency and wall width.

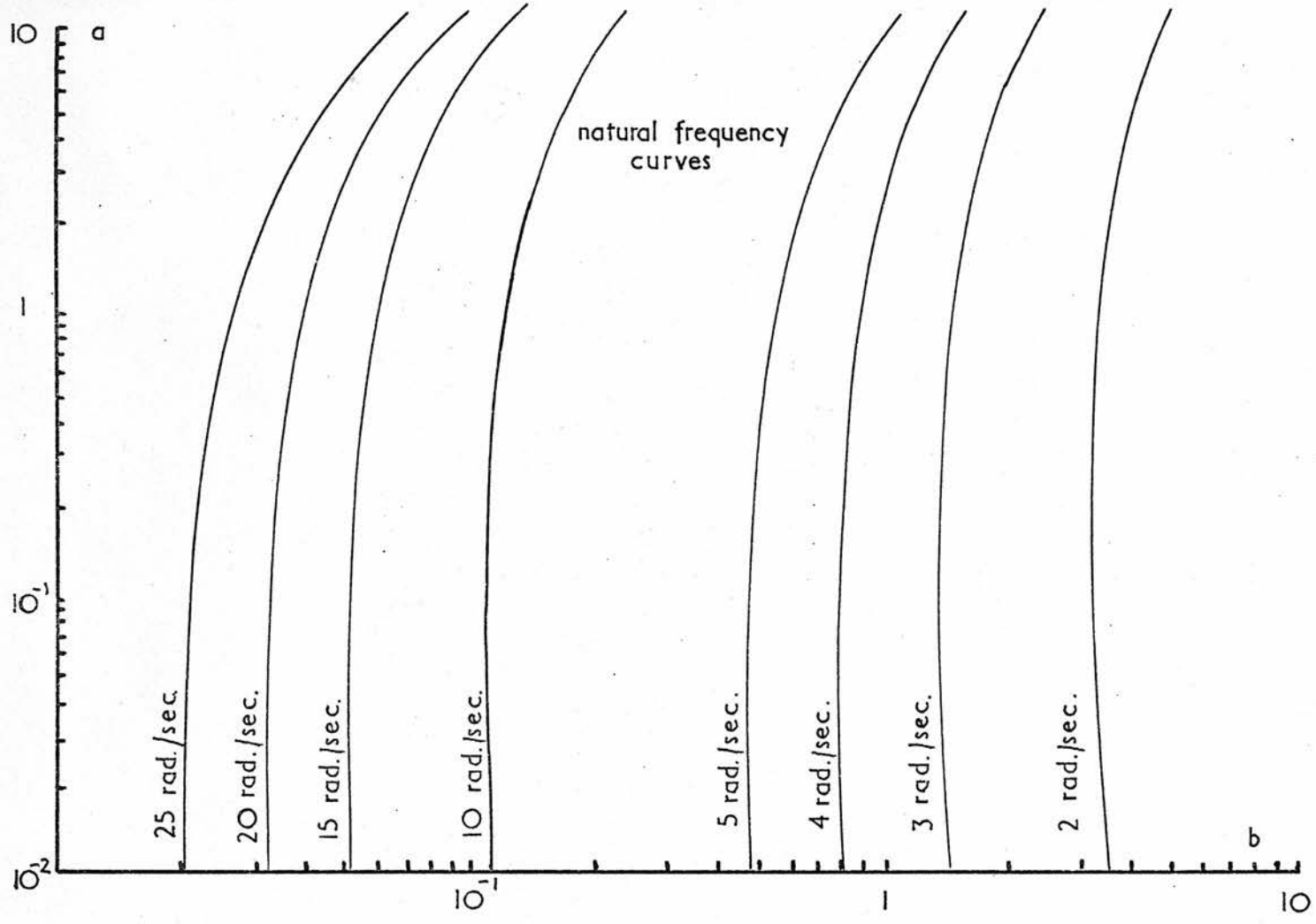
A3.6 FINAL CONCLUSIONS

By analysing shear wall type buildings with the continuum theory, the natural frequency can be obtained in terms of two constants a and b. These are simple expressions involving only the elementary properties of the building components. A chart similar to Graph (A3.7) may be constructed for design purposes. To find the natural frequency of a building, a designer has only to evaluate a and b. The theory therefore provides a method which is simple enough for design office use. It is not considered that Graph (A3.7) itself should be used for design purposes. Before the method could be used, tests on full scale buildings would be required in conjunction with more model tests to establish properly the validity of the theory.

The most significant conclusion is that a simple approximation

to/

to a building, such as a cantilever, can lead to misleading results when used in dynamic calculations. The natural frequency of a building is not necessarily a function of its overall dimensions as has often been suggested. The investigation demonstrates that while the height is an important parameter, for multi storey shear wall structures, the natural frequency is dependent on the size of individual structural components. Thus, for dynamic wind loading calculations, which are highly sensitive to the value obtained for natural frequency, such rough approximations as have previously been made could lead to large errors.



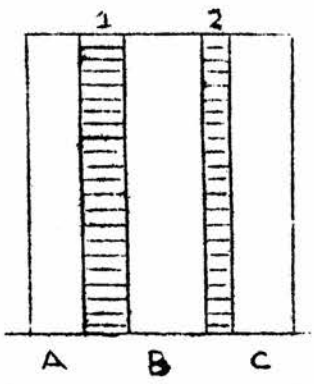
Graph A3.7 Natural frequency curves plotted from equation A3.28.

APPENDIX 4

FORMULATION OF 'a' CONSTANT FOR A BUILDING WITH WALLS OF UNEQUAL WIDTH.

A three-wall building was chosen to illustrate the analysis.

Let the walls be A, B and C and the spaces 1 and 2.



Applying equations (A3.1) and (A3.2) to each wall,

$$Q_A + m_1 + r_1 \frac{w_A}{2} - \frac{\partial M_A}{\partial x} = 0 \qquad \text{.....(i)}$$

$$Q_B + m_1 + m_2 + (r_1 + r_2)\frac{w_B}{2} - \frac{\partial M_B}{\partial x} = 0 \qquad \text{.....(ii)}$$

$$Q_C + m_2 + r_2 \frac{w_C}{2} - \frac{\partial M_C}{\partial x} = 0 \qquad \text{.....(iii)}$$

Differentiating these gives,

$$\frac{\partial Q_A}{\partial x} + \frac{\partial m_1}{\partial x} + \frac{\partial r_1}{\partial x} \cdot \frac{w_A}{2} - \frac{\partial^2 M_A}{\partial x^2} = 0 \qquad \text{.....(iv)}$$

$$\frac{\partial Q_B}{\partial x} + \frac{\partial m_1}{\partial x} + \frac{\partial m_2}{\partial x} + \frac{\partial r_1}{\partial x} \cdot \frac{w_B}{2} + \frac{\partial r_2}{\partial x} \cdot \frac{w_B}{2} - \frac{\partial^2 M_B}{\partial x^2} = 0 \qquad \text{.....(v)}$$

$$\frac{\partial Q_C}{\partial x} + \frac{\partial m_2}{\partial x} + \frac{\partial r_2}{\partial x} \cdot \frac{w_C}{2} - \frac{\partial^2 M_C}{\partial x^2} = 0 \qquad \text{.....(vi)}$$

From/

From equation (A3.2).

$$\frac{\partial Q_A}{\partial x} - t_1 + \rho A_A \frac{\partial^2 y}{\partial t^2} = 0 \qquad \text{.....(vii)}$$

$$\frac{\partial Q_B}{\partial x} - t_1 - t_2 + \rho A_B \frac{\partial^2 y}{\partial t^2} = 0 \qquad \text{.....(viii)}$$

$$\frac{\partial Q_C}{\partial x} + t_2 + \rho A_C \frac{\partial^2 y}{\partial t^2} = 0 \qquad \text{.....(ix)}$$

Substituting in (vii), (viii) and (ix) from (iv), v) and (vi) and adding gives,

$$\frac{\partial^2 M_A}{\partial x^2} + \frac{\partial^2 M_B}{\partial x^2} + \frac{\partial^2 M_C}{\partial x} - 2\frac{\partial m_1}{\partial x} - 2\frac{\partial m_2}{\partial x} - \frac{\partial r_1}{\partial x}\left(\frac{w_A}{2} + \frac{w_B}{2}\right)$$

$$-\frac{\partial r_2}{\partial x}\left(\frac{w_B}{2} + \frac{w_C}{2}\right) + \rho(A_A + A_B + A_C)\frac{\partial^2 y}{\partial t^2} = 0$$

Substituting for $\frac{\partial m}{\partial x}$, $\frac{\partial r}{\partial x}$ and $\frac{\partial^2 M}{\partial x}$ gives,

$$E(I_A + I_B + I_C)\frac{\partial^4 y}{\partial x^4} - 12E\left[\frac{I_u}{h_1 e_1} + \frac{I_{e2}}{h_2 e_2}\right]\frac{\partial^2 y}{\partial x^2} -$$

$$12E\left[\frac{I_{c1}}{h_1 e_1} + \frac{I_{c2}}{h_2 e_2}\right]\frac{\partial^2 y}{\partial x^2} + \rho(A_A + A_B + A_C)\frac{\partial^2 y}{\partial t^2} = 0$$

$$\therefore \frac{\partial^4 y}{\partial x^4} - \frac{12}{\Sigma I}\left[\frac{I_{c1}}{h_1 e_1}\left(1 + \frac{1}{e_1}\right) + \frac{I_{c2}}{h_2 e_2}\left(1 + \frac{1}{e_2}\right)\right] \times$$

$$\frac{\partial^2 y}{\partial x^2} + \frac{\rho}{E}\frac{\Sigma A}{\Sigma I}\frac{\partial^2 y}{\partial t^2} = 0$$

Thus, $a = \frac{12}{\Sigma I}\left[\frac{I_{c1}}{h_1 e_1}\left(1 + \frac{1}{e_1}\right) + \frac{I_{c2}}{h_2 e_2}\left(1, + \frac{1}{e_2}\right)\right]$

APPENDIX 5

SOLUTION OF EQUATION (A3.20) BY THE METHOD OF FROBENIUS

Equation (A3.20) is of the form,

$$\frac{d^4f}{dx^4} + a\frac{d^2f}{dx^2} + \omega^2 bf = 0$$

where a and b are constants, and ω an eigenvalue. The boundary conditions are,

$$f = 0, \quad \text{when} \quad x = 0$$

$$\frac{df}{dx} = 0, \quad \text{when} \quad x = 0$$

$$\frac{d^2f}{dx^2} = 0, \quad \text{when} \quad x = 1$$

$$\frac{d^3f}{dx^3} = 0, \quad \text{when} \quad x = 1$$

A power series solution may be obtained by means of the substitution,

$$f = \sum_n p_n x^{n+c}$$

Equation (6.21) becomes

$$\sum_n p_n(n+c)(n+c-2)(n+c-3)x^{n+c-4} + a\sum_n p_n(n+c)(n+c-1)x^{n+c-2} + b\omega^2 \sum_n p_n x^{n+c} = 0$$

Writing out the first few terms in each series, (6.21) becomes,

$$p_o(c)(c-1)(c-2)(c-3)x^{c-4} + p_1(c+1)(c)(c-1)(c-2)x^{c-3}$$

$$+ \ /$$

$$+ p_2(c+2)(c+1)(c)(c-1)x^{c-2} + p_3(c+3)(c+2)(c+1)(c)x^{c-3} + \ldots$$

$$+ ap_o(c)(c-1)x^{c-2} + ap_1(c+1)(c)x^{c-1} + ap_2(c+2)(c+1)x^{3} + \ldots$$

$$+ b\omega^2 p_o x^c + b\omega^2 p_1 x^{c+1} + b\omega^2 p_2 x^{c+2} + \ldots\ldots = 0$$

Equating coefficients,

$$x^{c-4} :- p_o(c)(c-1)(c-2)(c-3) = 0 \quad \ldots\ldots (A)$$

$$x^{c-3} :- p_1(c+1)(c)(c-1)(c-2) = 0 \quad \ldots\ldots (B)$$

Four independent solutions may therefore be obtained by letting $c = 0, 1, 2$ and 3 with $p_1 = 0$ and $p_o = 0$.

$$x^{c-2} :- p_2 = \frac{a'}{(c+2)(c+1)} p_o \quad \ldots\ldots (C)$$

$$p_{n+2} = \frac{a'}{(c+n+2)(c+n+1)} p_n + \frac{b'}{(c+n+2)(c+n+1)(c+n)(c+n-1)} p_{n-2} \quad (D)$$

where $a' = -a$ and $b' = -b$

The coefficients for each series are found by substituting the appropriate value for c in equations (C) and (D).

The four series are,

$c = 0 = /$

$$c = 0 = f_1 = 1 + a' \frac{x^2}{2!} + (b'\omega^2 + a'^2)\frac{x^4}{4!} + (2b'\omega^2 a' + a'^3)\frac{x^6}{6!}$$

$$+ (b'^2\omega^4 + 3b'\omega^2 a^2 + a'^4)\frac{x^8}{8!} + \cdots$$

$$c + 1 = f_2 = x + a'\frac{x^3}{3!} + (b'\omega^2 + a'^2)\frac{x^5}{5!} + (2b'\omega^2 a' + a'^3)\frac{x^7}{7!} \cdots$$

$$+ (b'^2\omega^4 + 3b'\omega^2 a^2 + a'^4)\frac{x^9}{9!} + \cdots$$

$$c = 2 = f_3 = x^2 + \frac{a'}{12}x^4 + \frac{1}{360}(b'\omega^2 + a'^2)x^6 + \frac{1}{2.01 + 10^4}x$$

$$(2b'\omega^2 a' + a'^3)x^6 + \cdots$$

$$c = 3 = f_4 = x^3 + \frac{a'}{20}x^5 + \frac{1}{840}(a^2 + b\omega^2)x^7 + \frac{1}{6.04 + 10^4}x$$

$$(a'^3 + 2a'b\omega^2)x^9 \cdots$$

The solution to (A3.20) is given by,

$$f(x) = Af_1 + Bf_2 + Cf_3 + Df_4$$

where A, B, C and D are constants.

Applying the boundary conditions,

$$Af_1(0) + Bf_2(0) + Cf_3(0) + Cf_4(0) = 0$$

$$Af_1'(0) + Bf_2'(0) + Cf_3'(0) + Df_4'(0) = 0$$

$$Af_1''(1) + Bf_2''(1) + Cf_3''(1) + Df_4''(1) = 0$$

$$Af_1'''(1) + Bf_2'''(1) + Cf_3'''(1) + Df_4'''(1) = 0$$

The/

The eigenvalues are found by setting the determinant to zero.

$$\begin{vmatrix} f_1 & (0) & f_2 & (0) & f_3 & (0) & f_4 & (0) \\ f_1' & (0) & f_2' & (0) & f_3' & (0) & f_4' & (0) \\ f_1'' & (1) & f_2'' & (1) & f_3'' & (1) & f_4'' & (1) \\ f_1''' & (1) & f_2''' & (1) & f_3''' & (1) & f_4''' & (1) \end{vmatrix} = 0$$

the determinant becomes,

$$\begin{vmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ f_1''(1) & f_2''(1) & f_3''(1) & f_4''(1) \\ f_1'''(1) & f_2'''(1) & f_3'''(1) & f_4'''(1) \end{vmatrix} = 0$$

Thus $f_3''(1) \times f_4'''(1) - f_4''(1) \times f_3'''(1) = 0$

The solution to this equation gives the eigenvalues of the system

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PRINCIPAL NOTATION

α	dimensionless coefficient used in energy analysis
α'	index of power law profile
A_r	area of cross-section
A_{st}	amplitude of structural vibration
A_w	amplitude of resonant frequency component of wind turbulence
B	breadth of structure
β	dimensionless coefficient used in energy analysis
C, Cd	drag coefficient
C_m	virtual mass coefficient
c_1	coefficient dependent on ground roughness
c_r	critical damping coefficient
c_s	cross-correlation coefficient
D	diameter of structure
δ	logarithmic damping decrement
E	modulus of elasticity
E_d	damping energy per cycle
E_{rd}	reduced damping energy per cycle
E_e	excitation energy per cycle
E_{re}	reduced excitation energy per cycle
E_k	kinetic energy
E_u	strain energy
F	force
f(z)	mode shape
G	gust factor
g(f)	generalised/

$G(f)$	generalised displacement co-ordinate
I	second moment of area
$J(n)$	joint acceptance function
k	ground roughness coefficient
k_c	peak factor
k_r	generalised stiffness
L	scale of turbulence
l	height of structure
M, m_r	generalised mass
m	mass per unit length
N_r	normalised mode function
n	frequency
ω	circular frequency
P_o	mean pressure at reference height
p_r	modal component of forcing function
$P(z)$	mean pressure at height z
ρ	density
$R(z,z';n)$	(cross-correlation coefficient (co-spectrum of wind velocities at z and z'
$S_v(n)$	spectrum of wind velocity
$S(z,z';n)$	cross-spectrum of wind velocity at z and z'
$S_p(n)$	spectrum of wind pressure
$S_y(n)$	spectrum of structural deflection
σ_y	root mean square deflection
$\bar{V}_o$, V_o	mean wind velocity at reference height
$\bar{V}_z$, V_z	mean wind velocity at height z
Vg	gradient wind velocity
$V(t)$	/

$V(t)$	wind velocity
$V \quad v(t)$	fluctuating component of wind velocity
V_r	relative velocity between wind and a structure
v_r	fluctuating component of relative velocity
V_{st}	velocity of a structure
X_a	aerodynamic admittance
X_m	mechanical admittance
$Y(t)$	deflection
$\bar{Y}$	mean deflection
$y(t)$	fluctuating component of deflection
Y_{max}	maximum deflection