A NEW PHOTOMECHANICAL METHOD

OF

HARMONIC ANALYSIS AND SYNTHESIS.

being Part I

of a thesis for the degree of

Doctor of Philosophy

submitted by

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April 1944.

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SECTION A:

CHAPTER I.

INTRODUCTION.

The original purpose of this research was to produce an instrument which would be of assistance to the X-ray crystallographer in the determination of crystal structure. A wider field of interesting and fruitful applications in electrical engineering and especially in acoustics is now also envisaged, together with the provision of a means of demonstrating in a remarkably vivid manner the result of summing a Fourier Series term by term.

The present apparatus permits us to work as far as the ninth harmonic in a given Fourier Series, which means that we can determine the coefficients of nine sine and nine cosine terms (n = 1, 2 ----- 9), i.e. eighteen coefficients in all. This number suffices for many purposes but there would be no real difficulty in extending the range.

There are, of course, many existing harmonic analysers and synthesisers of all degrees of accuracy, and while it is certainly not suggested that a degree of precision has as yet been attained with this instrument/

instrument to equal the best of these, unique features both in its principle of operation and in its performance can be claimed. These will be discussed fully in the text.

In order to emphasise how the method could be applied to expansions in terms of any set of orthogonal functions a chapter has been written on some theoretical aspects of the subject. This will also serve to introduce particular examples of Fourier Series to be dealt with later and a standard nomenclature.

Section B is devoted to a discussion of the principle of operation and the performance of the Synthesiser (first suggested by Dr. R. Fürth) and includes a detailed description of its constructional features. The necessary adjustments for Analysis and the use of an electronic mixer valve (which was proposed by Professor Max Born) are reserved for Section C. The latter subject will be discussed at some length for, as far as the author is aware, it is only now that the possibility has been recognised of a method of Harmonic Analysis by means of electronic multiplication.

It may be of interest to note here that, since this work has been undertaken, plans have been made for a photomechanical method of evaluating the Fourier Integral - a method based to some extent on the optical multiplication/

multiplication method of Montgomery[1] for Harmonic Analysis. This method if successful may prove to be of use to the mathematical physicists but details will be reserved for a later paper.

SECTION A:

CHAPTER 2.

THE REPRESENTATION OF PERIODIC FUNCTIONS BY MEANS OF SERIES OF ORTHOGONAL FUNCTIONS.

A rigorous treatment will be found in the first 45 pages of "The Fourier Integral" (Wiener) where the theoretical foundations of the subject are discussed very completely. An attempt will be made here to present only such material as seems necessary before a discussion of the construction and performance of the present instrument.

Continuity and Boundary Conditions.

We will deal with real and complex functions u (x) of the real variable x which satisfy the conditions,

(1). u (x) and $du(x)/dx$ are both continuous for all values of x .

(2). $|u|$ remains finite when $|x| \to \infty$

Orthogonal Sets.

Two functions $u_1(x)$ and $u_2(x)$ are said to be orthogonal in the interval (ab) if

$$\int_a^b u_1(x)\,\overline{u_2}(x)\,dx = 0$$

where $\overline{u_2}(x)$ is the complex conjugate of $u_2(x)$.

If in the set of complex functions $\left\{ u_n(x) \right\}$ every/

every two functions are orthogonal in the interval (a, b) i.e. if

$$\int_a^b u_m(x)\,\overline{u_n}(x)\,dx = 0 \qquad \text{---}(1)$$

then the set is said to be <u>orthogonal</u> in the interval (ab) of x.

Suppose we consider the infinite set of functions,

$$1, \cos x, \cos 2x, \ldots\ldots$$
$$\sin x, \sin 2x, \ldots\ldots \qquad \text{---}(2)$$

We notice that the integral within limits $(-\pi, \pi)$ of the product of any member of the set and the complex conjugate of any other member is zero. (As all members of the set are real in this case the complex conjugate is just the function itself). Thus the set is orthogonal in the interval $(-\pi, \pi)$ of x. It can easily be seen that the set is orthogonal for any interval x of length 2π.

<u>Fourier Series.</u>

The importance to mathematical physics of the orthogonal set of functions quoted above cannot be overemphasised. This is due to the discovery by Fourier in the course of his researches on heat conduction that almost all functions (in fact those obeying what we now know as the Direchlet conditions though simplification takes place if we relax to those of mean convergence), and not just analytic functions as had been proved almost half a century earlier by Euler/

Euler, could be expressed in an interval 2π in the form

$$f = \alpha_1 u_1 + \alpha_2 u_2 + \dots + \alpha_k u_k + \dots \qquad \text{--(3)}$$

where the u's are members of set (2). Thus <u>periodic</u> functions of period 2π can be expressed in this way over the range $(-\infty, \infty)$.

We refer to (2) as a <u>Fourier Series</u>, and the expansion coefficients in this series as <u>Fourier coefficients</u>. Generalised Fourier series and generalised Fourier Coefficients are the names used when this expansion is in terms of orthogonal sets other than (2).

<u>Evaluation of the Fourier Coefficients.</u>

Let us now assume that the series (3) - considering now any orthogonal set - can be integrated within the limits (a, b).

Multiply (2) by $\bar{u}_k$ giving us

$$f\bar{u}_k = \alpha_1 u_1 \bar{u}_k + \alpha_2 u_2 \bar{u}_k + \dots + \alpha_k u_k \bar{u}_k + \dots \qquad \text{-(4)}$$

and integrate over the interval (a, b) of x. With the exception of the term involving $u_k\, u_k$ all terms on the right hand side of (4) will vanish because of the orthogonality of the u's expressed by relation (1). Hence we obtain

$$\int_a^b f\bar{u}_k\,dx = \alpha_k \int_a^b u_k \bar{u}_k\,dx$$

$$\text{i.e. } \alpha_k = \frac{\int_a^b f\bar{u}_k\,dx}{\int_a^b u_k \bar{u}_k\,dx} \qquad (k=1,2,3\dots) \qquad \text{--(5)}$$

Our harmonic analyser which is a machine for the evaluation of the Fourier coefficients corresponding to a given function is based on the evaluation of the right hand side of (5).

As a result of (5) it is evident that <u>if</u> an expansion of f (x) in terms of a given orthogonal set exists, as in (3) then it must be <u>unique</u>.

All functions cannot be expressed in terms of <u>any</u> orthogonal set but it can be shown that every periodic function of period 2π satisfying the Dirichlet conditions can be expanded in terms of set (2). We say this set is <u>complete</u> with respect to continuous periodic functions of period 2π.

Combining (2) and (3) we can write

$$f(x) = \frac{a_0}{2} + a_1 \cos x + a_2 \cos 2x + \ldots + a_n \cos nx + \ldots + b_1 \sin x + b_2 \sin 2x + \ldots + b_n \sin nx + \ldots \quad (6)$$

where the factor $\frac{1}{2}$ is introduced into the first term in order to give us a general expression for the coefficients.

From (5) we see that

$$a_n = \frac{1}{\pi}\int_{-\pi}^{+\pi} f(x) \cos nx \, dx \qquad b_n = \frac{1}{\pi}\int_{-\pi}^{+\pi} f(x) \sin nx \, dx \quad (7)$$

An alternative form of (6) which we will make use of later is

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} c_n \cos(nx - \theta_n) \quad (8)$$

$$\text{where} \quad c_n = \sqrt{a_n^2 + b_n^2}$$

$$\theta = \tan^{-1}(b_n/a_n)$$

The form

$$f(x) = \sum_{n=-\infty}^{n=\infty} k_n \, e^{inx} \tag{9}$$

where $k_n = \frac{1}{2}(a_n - i\, b_n)$ for n +ve,
conjugate for n -ve.

is sometimes very convenient especially in the development of the Fourier Integral.

It should now be evident that the presence of both sine and cosine terms in a given waveform implies that the harmonic components have not only different amplitudes but different relative initial phase angles as well.

<u>Examples.</u>

Two simple examples of curves made up of straight line segments, and of special interest to us in connection with the harmonic synthesiser will be discussed now.

(1). Suppose the function f (x) to be given by

$$f(x) = \begin{cases} -1 & -\pi \leq x < 0 \\ 0 & x = 0 \\ 1 & 0 < x \leq \pi \end{cases} \tag{10}$$

$$f(x+2\pi) = f(x) \quad \text{for} \quad -\infty < x < \infty$$

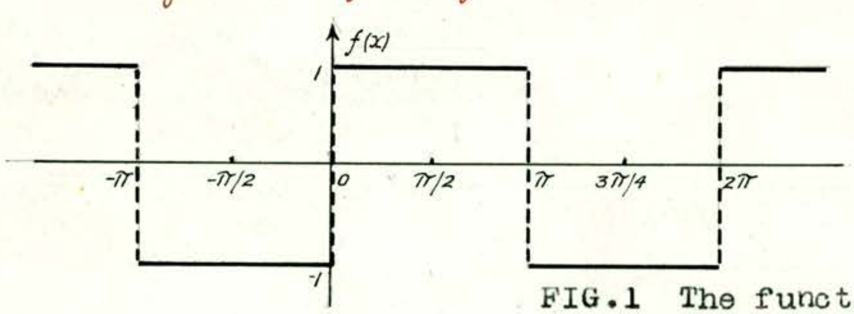


FIG.1 <u>The function defined by (10).</u>

Equations (7) give us

$$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x) \cos nx \, dx = 0$$

as f (x) is odd in x , cos n x even in x and the interval of integration symmetric about x = o ; and

$$b_n = \frac{1}{\pi}\int_{-\pi}^{+\pi} f(x)\sin nx\,dx$$
$$= \frac{1}{\pi}\int_{-\pi}^{0}(-1)\sin nx\,dx + \frac{1}{\pi}\int_{0}^{+\pi}\sin nx\,dx$$
$$= \frac{2}{\pi}\int_{0}^{+\pi}\sin nx\,dx \quad = \begin{cases} 0 & \text{if } n \text{ is even} \\ 4/2\pi & \text{if } n \text{ is odd} \end{cases}$$

$$\therefore f(x) = \frac{4}{\pi}\left(\frac{\sin x}{1} + \frac{\sin 3x}{3} + \frac{\sin 5x}{5} + \dots\right)$$
$$= \frac{4}{\pi}\sum_{0}^{\infty}\frac{\sin(2n+1)x}{2n+1} \qquad (11)$$

The formation of a connected curve by the addition to (10) of vertical lines through 0, π, 2 π gives the function which is referred to by electrical engineers as a 'square wave'

(II). Suppose the function f (x) to be given by

$$f(x) = \begin{cases} -1 - \frac{x}{\pi} & -\pi \leqslant x < 0 \\ 0 & x = 0 \\ +1 - \frac{x}{\pi} & 0 < x \leqslant \pi \end{cases} \qquad (12)$$
$$f(x+2\pi) = f(x) \quad \text{for} \quad -\infty < x < \infty$$

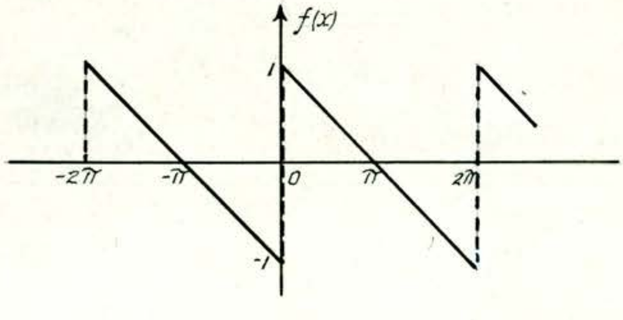


FIG.2 The function defined by (12)

(i.e straight line segments through the points π, 3 π, ---- with discontinuities at 0, 2 π, ---).

Equations (7) give us

$$a_n = \frac{1}{\pi}\int_{-\pi}^{+\pi} f(x) \cos nx \, dx = 0$$

as in the last example,

$$\text{and } b_n = \frac{1}{\pi}\int_{-\pi}^{+\pi} f(x) \sin nx \, dx$$

$$= \frac{1}{\pi}\int_{-\pi}^{0}\left(-1-\frac{x}{\pi}\right) \sin nx \, dx + \frac{1}{\pi}\int_{0}^{+\pi}\left(+1-\frac{x}{\pi}\right) \sin nx \, dx$$

$$= -\frac{1}{\pi^2}\left[-2\pi\int_0^{\pi} \sin nx \, dx + 2\int_0^{\pi} x \sin nx \, dx\right]$$

$$= -\frac{1}{\pi^2}\left[\frac{2\pi}{n}(\cos n\pi - 1) - \frac{2\pi}{n}\cos n\pi\right] = \frac{2}{n\pi}.$$

$$\text{Thus } f(x) = \frac{2}{\pi}\left(\frac{\sin x}{1} + \frac{\sin 2x}{2} + \frac{\sin 3x}{3} + \dots\right)$$

$$= \frac{2}{\pi}\sum_{1}^{\infty}\frac{1}{n}\sin nx. \qquad (13)$$

This waveform with vertical lines inserted at the discontinuities would be referred to in television practice as a 'saw-tooth' with zero fly-back time.

We shall later use the harmonic synthesiser to investigate how, for the simple curves whose analyses are given by (11) and (13), the successive sums Sn (x) i.e. of the first n terms of the appropriate series tend to f (x) with increase of n. The limit curve will now clearly be connected, as for any one of the successively approximating curves <u>x</u> is variable and <u>n</u> is fixed.

It will also be found possible to illustrate in

a/

a vivid manner a very interesting phenomenon discussed by Gibbs in 1899. Suppose we consider series (13). Gibbs showed that in fact the limit of Sn (x) when n becomes infinite is such that the vertical part of the limit curve projects beyond f (x + o) and f (x - o) by a finite amount so that the limit curve has the remarkable form sketched below

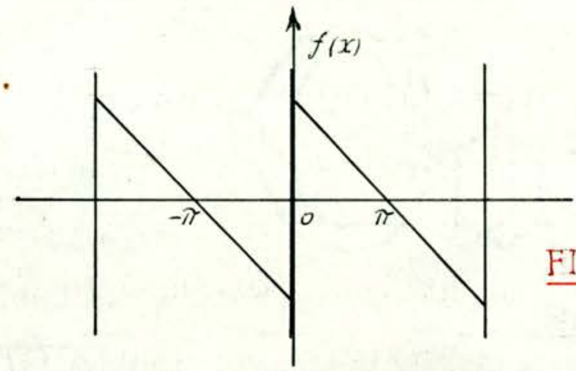


FIG. 3 The Gibbs Phenomenon

The length of the extension according to Gibbs is $0.09 \times |f(x+o) - f(x-o)|$.

SECTION B:

CHAPTER 1.

THE HARMONIC SYNTHESISER.

Harmonic synthesis is the composition of a number of given periodic functions to form a resultant or synthesised waveform. In practice these given waveforms are almost always trigonometrical so although the method to be described is equally applicable to any given type of function and thus can be used for general Fourier synthesis, we shall confine our attention to the composition of sine and cosine functions of given amplitude, phase and frequency - the latter in the present instrument not going beyond the ninth harmonic of the fundamental.

Many types of synthesisers are already in existence. In the study of sound vibrations, of X-ray analysis, of crystal structure and of the response of certain electrical networks to voltages of complex waveform, the necessity of a means of rapid and accurate harmonic synthesis is soon recognised.

However, in addition to simplicity and speed of operation, the present instrument possesses the unique features that

(1) it is possible to see immediately the effect of adding a given component harmonic or of altering its phase/

phase or amplitude by arbitary amounts.

(2) We produce not merely a set of numbers or a graph but an electrical oscillation as our synthesised waveform. Thus electrical oscillations of any form but of very low frequency can be obtained (20 ~ /sec, which could in a later instrument be considerably increased.)

In principle the method is no more than the photo-electric summation of the sinoidally varying intensities of eighteen light beams (sine and cosine functions up to the ninth harmonic), where the amplitudes of the periodic disturbances are capable of independant control and where the sinoidal variations are caused by a variable area modulation method.

The output from the photocell is fed to a pre-amplifier and then to the amplifiers of a commercial cathode ray oscilloscope. Observations of the synthesised waveform may be sufficient for some purposes but if necessary oscillograms may be made quite rapidly by photographic means.

The whole apparatus is sketched in Fig.4. A large part of this section will be taken up with a description of its construction - full details will be given, for as we shall see, with certain modifications the same apparatus is capable of performing the inverse operation of Harmonic Analysis.

Fig. 4. THE HARMONIC SYNTHESISER.

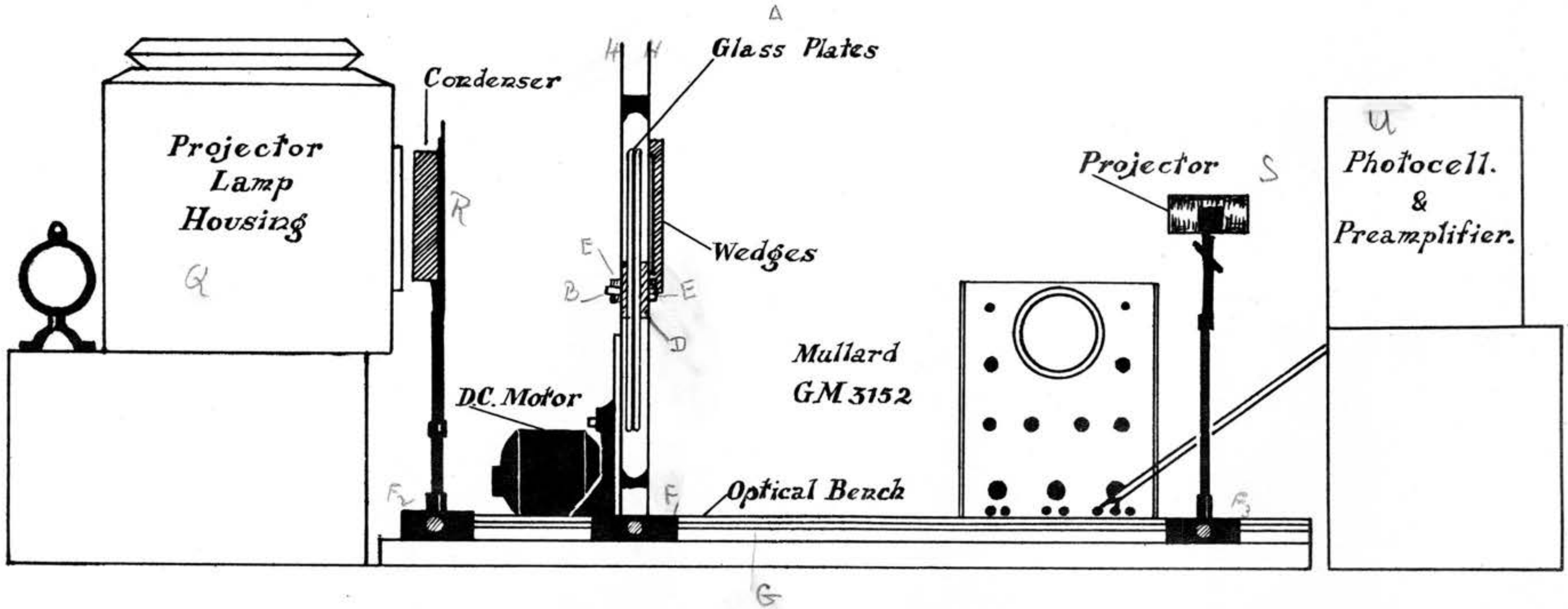


Fig 1

SECTION B:

CHAPTER 2.

THE CONSTRUCTION AND MOUNTING OF THE VARIABLE AREA SINE AND COSINE FUNCTIONS.

It is evident that a light beam the intensity of which varies sinoidally in time, can be produced by rotating a circular strip containing a variable area sine function in front of a uniformly illuminated slit as shown below.

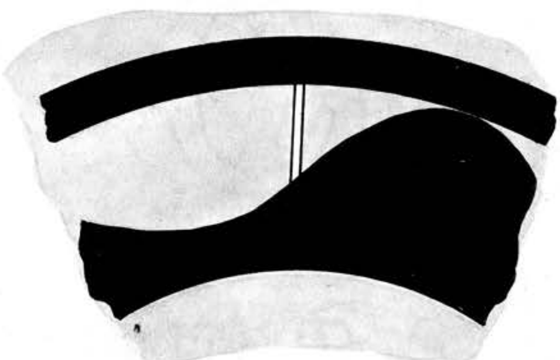


FIG.5 Production of a light beam of sinoidally varying intensity.

The light passing through this slit varies as (1 + sin x) for, of course, we can never produce a 'negative' light intensity: that is to say it is represented by sin x plus a constant term. We will find later (page 57) that this constant term must be considered carefully.

To/

To produce the required eighteen light beams it was therefore proposed to mount a film bearing this number of variable area sine and cosine functions, drawn about a common centre in a system of polar co-ordinates, between two glass plates and to rotate these in a vertical plane in front of a uniformly illuminated vertical radial slit. Figure 6 shows the appearance of the pattern required.

The possibility of selecting any number of these light beams and of varying their intensities independently has then to be provided for, but this problem will be left for the next chapter. We now proceed to discuss how the practical realisation of the above proposal has come about.

The first task was the drawing to a very large scale of the sine and cosine functions - these would eventually be reduced to the correct size photographically. The drawing was a long and laborious operation but well worth the time devoted to it as the success of the whole instrument depended on the accuracy with which it could be performed. The original is of 112 cm. diameter, each of the functions having an amplitude of 2 cm and a spacing of .4 cms. It was decided to have the fundamental cosine function nearest the centre with one complete period in each quadrant - which means, for example, that 36 complete periods of the 9th harmonic/

FIG. 6
The variable area functions.

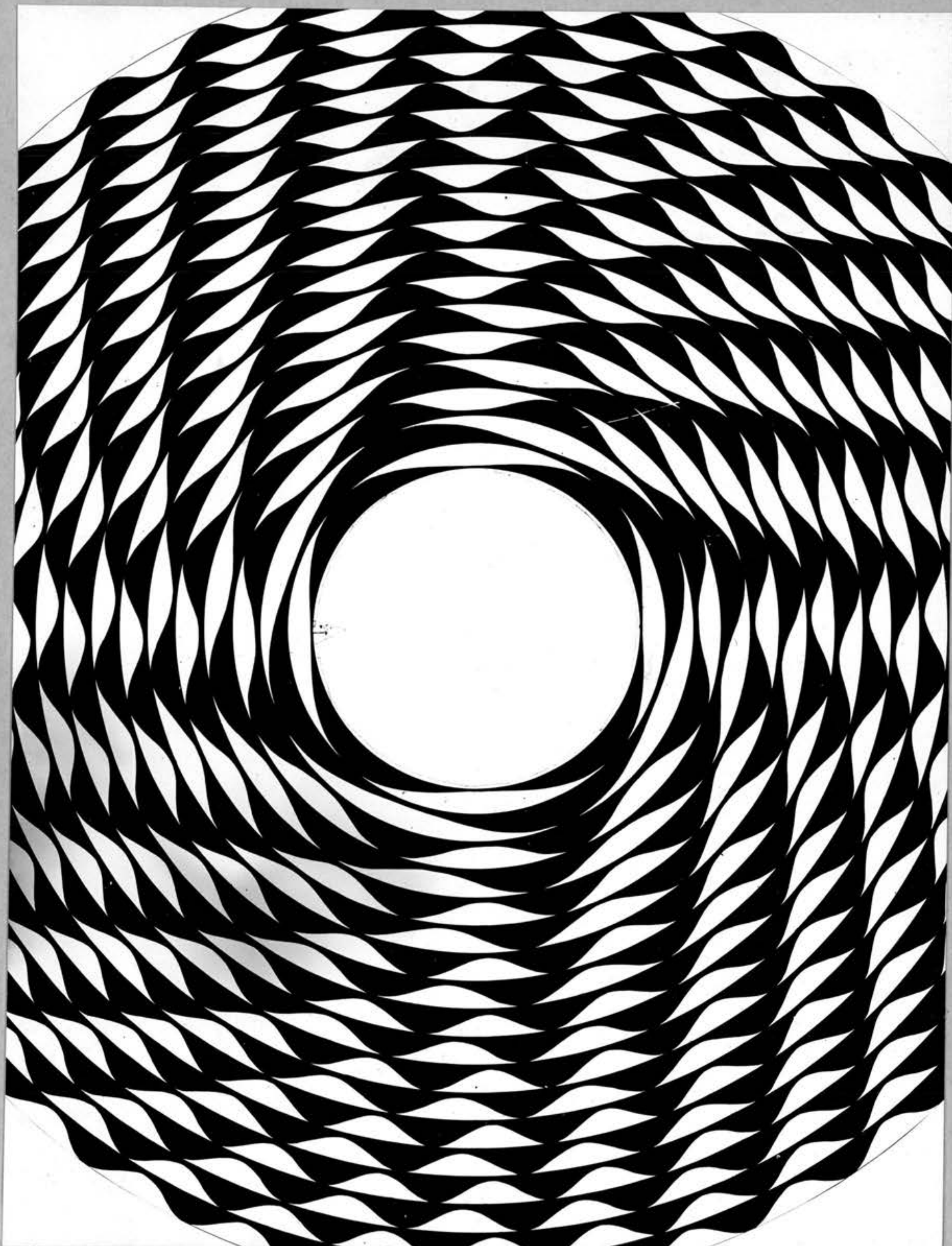


fig 2

harmonic are required round the outer circle.

A large beam compass was used for the drawing of the circles in their exact positions, the outer circle being divided into 360 parts. The beginning and the end of each period in the diagram could then be marked (to avoid cumulative errors), and one period of each function accurately plotted. A series of 18 templates, one for each function, was prepared with great care from the plotted figures. These templates were constructed of stiff paper mounted on cardboard in order that tracings could easily be made in ink. In this way a considerable decrease was obtained in the labour required to complete the whole drawing, which was filled in with Indian ink until it had the appearance of Fig.6 on a much larger scale.

The photographic reduction of this drawing was then necessary. It was essential that it should be achieved with a negligible amount of aberration, so the photography was undertaken by Messrs. Bartholemew, the well-known firm of map makers who of course have at their disposal cameras admirably suited to this sort of work. The reduction took place in two stages - the first was to a wet plate 38 cm. in diameter and the second provided a copy 21 cm in diameter with functions 3.5 mm. in amplitude and .75 mm spacings. This was on film ('Kodaline') which has a fine grain emulsion of extreme contrast. Care was taken to avoid dust spots and/

and the photographic reduction was regarded as having been entirely successful.

We now come to the question of how best to mount the film which, as we have seen, is to be rotated in a vertical plane. The decisions were made to enclose the film between two glass discs which would then be supported on a horizontal axle, and that the bearings for this should be provided with a rigid mount on an optical bench.

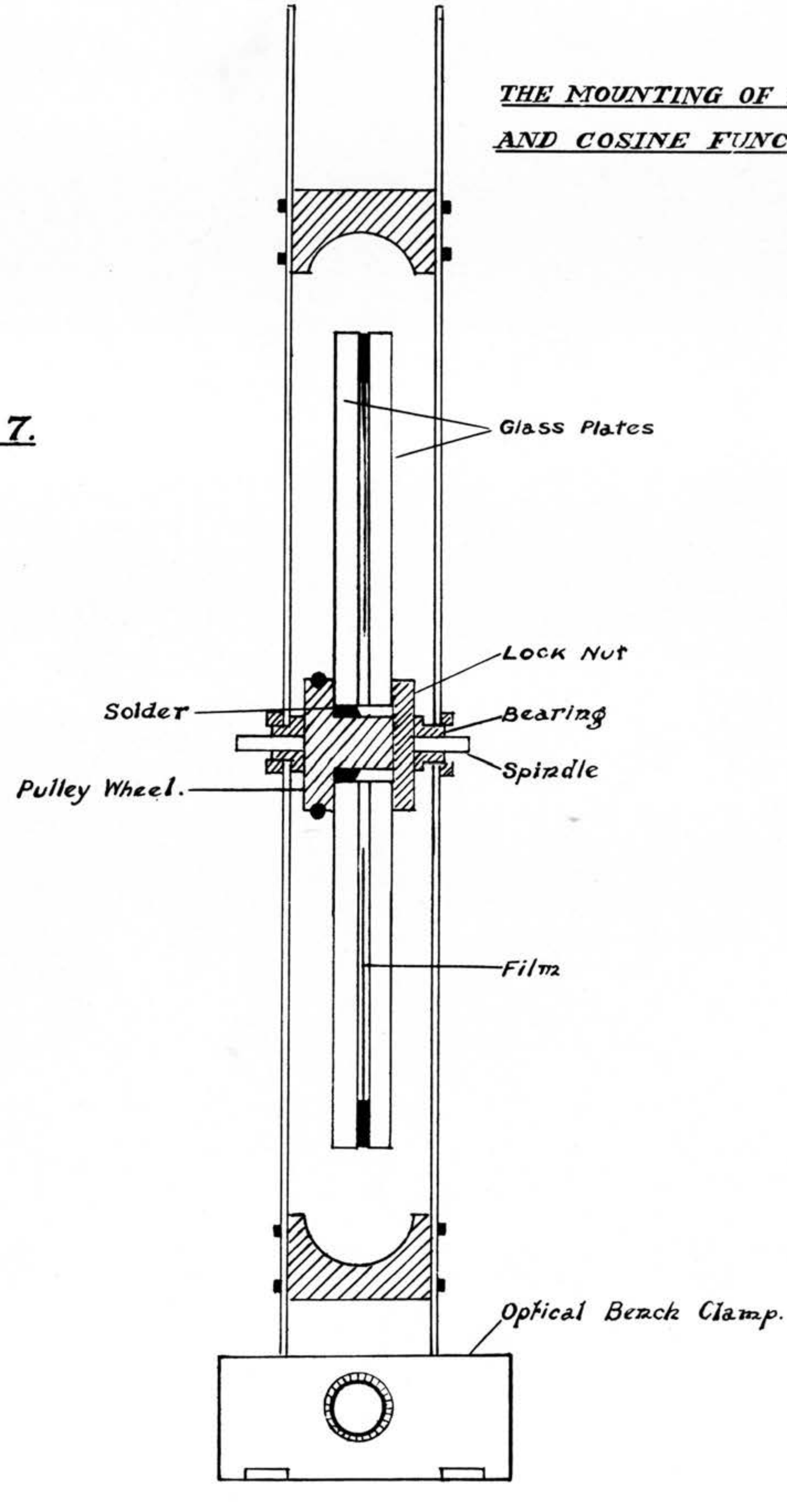
Two circular glass plates each $\frac{1}{8}$th inch thick and 10 inches in diameter were cut and circular holes $\frac{1}{2}$ inch in diameter were bored at the centres. The axle complete with pulley wheel and lock nut, and also the optical bench mount with suitable bearings had then to be designed (see the diagrammatic cross-section in Fig.7). The axle was carefully made in order that no wobble of the glass plates should be possible. It was later found that the washers had to be of thin celluloid before this was finally achieved.

Problems of centering arose. It was evidently a most important matter that the functions should be free (to within a small fraction of a millimetre) of any radial motion along the slit direction when in rotation, and also that the glass plates be accurately centered for mechanical reasons. The following technique was therefore devised.

First/

THE MOUNTING OF THE SINE AND COSINE FUNCTIONS

Fig. 7.



First one glass plate was centered accurately, using a material known as liquid solder to fill in the space between the glass and the axle (this was necessary as the holes, having been bored after the cutting of the plates, are not themselves exactly centered).

A circular strip of cardboard inscribed with a scale and extending inwards about 1 inch from the outer circumference was stuck on one face of the plate with seccotine. The system being used as a lathe, the cardboard was then cut so as to leave a ring (approx. .8 inch broad) the inner circumference of which, exactly concentric with the centre of revolution of the plate, was just of such a diameter to contain exactly the film bearing the sine and cosine functions.

The second plate is used to hold the film in position and is itself secured with a lock nut (see Fig 7). This arrangement has the obvious advantage that the film is not damaged in any way. Also, as we shall see, it is a necessary one if the same apparatus is to be used for Harmonic Analysis, in which case it must be possible to dismantle the plates with ease.

It is as well to mention here that the motor used in the process described above - and subsequently during the operation of the complete instrument - is of $\frac{1}{16}$ H.P. and operates from the D.C. mains (230 v) with a variable series resistance of maximum value 2000 ohms/

ohms. A suitable system of reduction gearing is used and the glass plates are belt driven. In normal operation the plates are turned at about 300 r.p.m.

SECTION B:

CHAPTER 3.

THE CONSTRUCTION OF THE NEUTRAL OPTICAL WEDGES.

Some time was devoted to a consideration of the means by which it might be possible to effect the required control of the intensities of the eighteen sinoidally varying light beams. A system of a similar number of neutral optical wedges each of which could move independantly in a horizontal direction across the vertical slit seemed to be the only practical solution.

The first task then was the preparation of a broad optical wedge (10 cm x 10 cm) on Kodaline film from which subsequently could be prepared eighteen identical wedges of the required dimensions (that is to say 10 cm. long and 4.3 mm. broad - the breadth of any of the sine and cosine functions plus the spacing.)

There is no real necessity for the wedge to have a linear variation in optical transmission along its length and so the following method was adopted and carried out in the dark room. The sheet of Kodaline film, covered with an opaque screen, was placed in the uniformly illuminated field of a photographic enlarger. A small electric motor suitably geared was then used to/

to remove this screen at a uniform rate from its position covering the film and in this way a linear variation in exposure time along the film was obtained (the density variation depending of course on the film characteristics). The light was shut off before the screen reached the edge of the film. A series of test exposures was required until the correct aperture to use in the enlarger was discovered. This corresponds to a variation in optical transmission along the wedge from complete transparency to less than a tenth of 1% of this. A simple photometric circuit is required for this measurement (- and a calibration graph is shown in figure 8). Metolhydroquinone developer is suitable for this film and the importance of avoiding all dust spots (which cannot in this case be touched up) must again be emphasised.

The wedge was now cut up into a series of 18 strips with a photograph cutter - strips which would allow of a final trimming to the exact size later. The mounting of these wedges and their accurate positioning was recognised to be no easy task. It seemed that we could not do better than utilise for this purpose the glass-like thermoplastic material called 'Perspex' manufactured by I.C.I. (Plastics) Ltd. in the form of precast sheets.

This substance is a polymerised methyl methacrylate which/

which has most remarkable optical properties. It possesses a refractive index higher than that of dense flint glass and a transparency equal to that of the finest optical glass made. It is admirably suited to maching and drilling but as is found this otherwise perfect material suffers from one grave defect. It is much less scratch - resistant than glass. It is interesting to note that commercially this has been overcome by giving the Perspex a very thin deposit of quartz, but the technique required is beyond our means. The difficulty, however, is partly overcome by the use of special polishing materials. These are No.1 and No.2 Perspex polishing liquids which should be rubbed in lightly with cotton wool. One contains an oil having the same refractive index as the Perspex and can be used to fill up scratches.

From a 1 foot by 1 foot sheet of $\frac{1}{16}$ inch Perspex, eighteen strips each 30 cm. long and .5 cm. broad were cut with a circular saw. It was found necessary to go fairly slowly with the cutting. The strips were then milled, filed and sandpapered at the edges until they had the shape shown in the upper half of figure 9. The aid of a milling machine, once it had been properly set, ensured that each strip was accurately cut to the size and shape required. Holes suitable for use with number/

number 10 B.A. screws were drilled in the positions indicated.

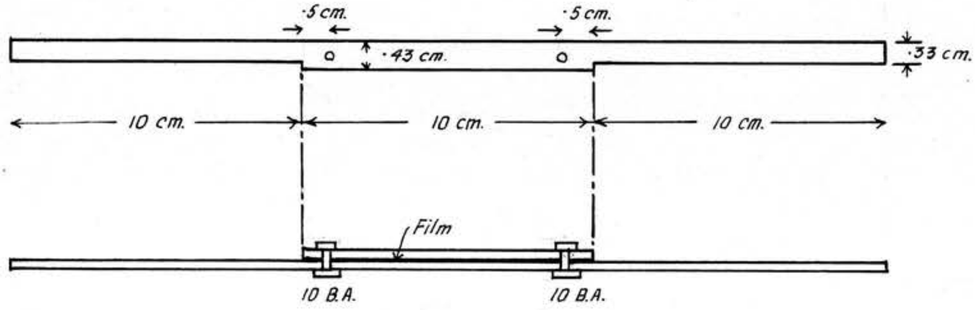


FIG.9 <u>The Perspex strips.</u>

Eighteen cover pieces 10 cm long and 4.3 mm broad and also of Perspex were prepared by cutting and milling. These were also drilled for 10 B.A. screws and the arrangement shown in the lower half of the figure could then be assembled with out optical wedge mounted between the long Perspex strip and the cover piece. The film is held securely in position by the screws which pass through small holes in it at either end. All that was then required was the trimming of the film to a breadth of exactly 4.3 mm.

So much for the runners on which the wedges are to move. Provision has now to be made for eighteen channels along which these can pass - the channels being so designed that the movement of one wedge will not interfere with any of the others.

Two pieces of angle brass of the dimensions shown in/

in Fig 10 (a) were cut and one slit $\frac{1}{16}$ inch broad was milled out of each.

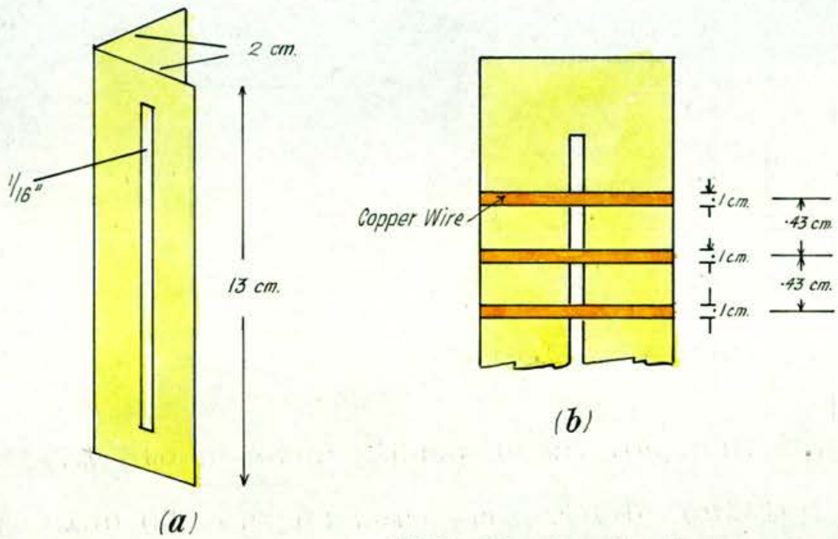


FIG.10 Angle Brass Supports.

In the next operation use was made of a dividing machine to produce a series of nineteen grooves perpendicular to the slit and accurately spaced according to what we might call the radial periodicity (4.3 mm) of the sine and cosine functions on the film. Deepish groves were cut using a sharp tool in the machine and short lengths of copper wire 1 mm. in diameter soldered on to the brass using the grooves as guides. Reference to Fig. 9 shows that the spacing of these wires and their diameter is such that the ends of the Perspex strips will fit exactly between them.

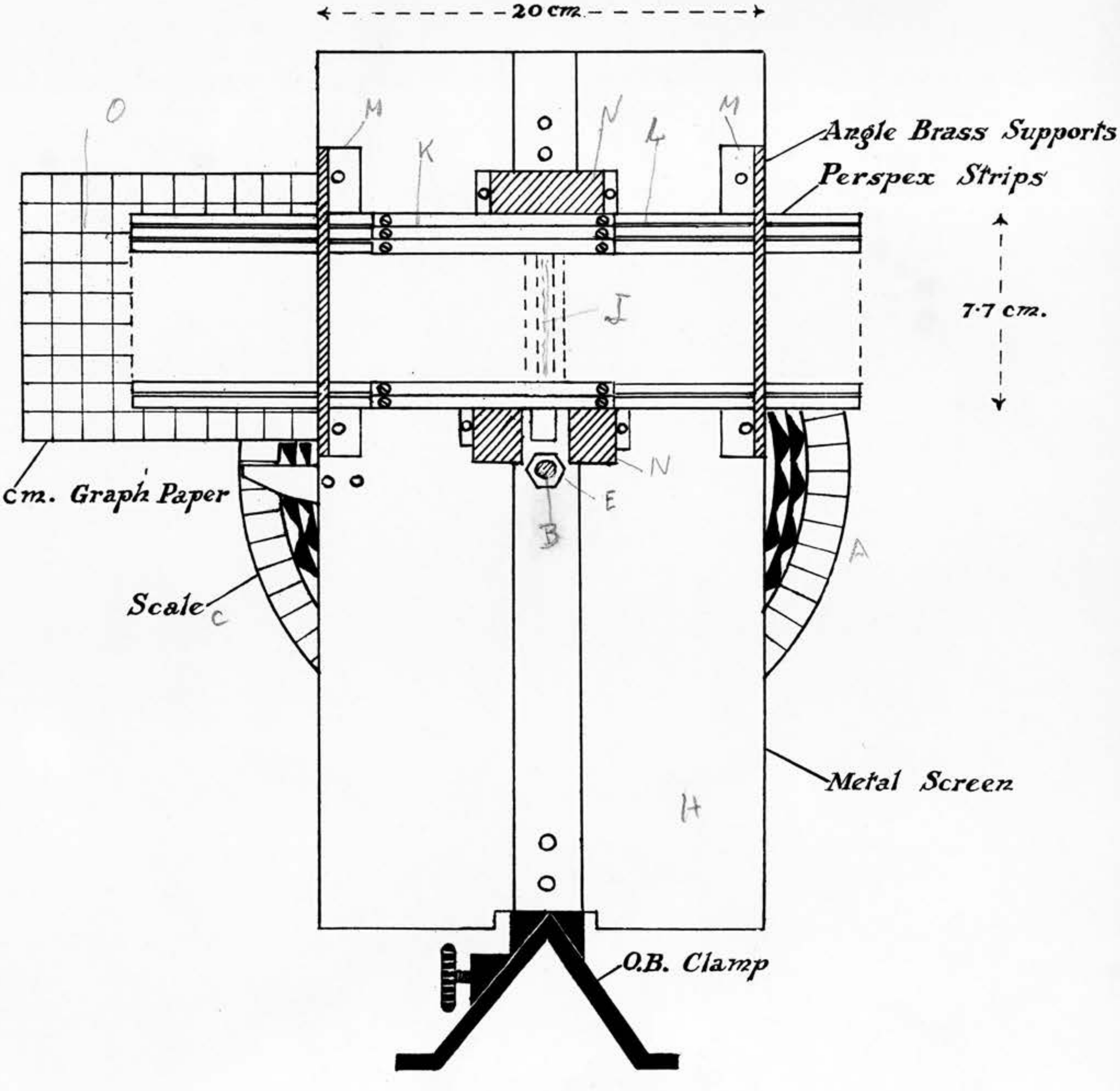
The two pieces of angle brass, one the mirror image of the other, and the strips could now be assembled/

assembled as in figure 11. The brass supports are shown screwed into a stout piece of metal sheeting, which also forms a part of the mount for the discs and acts as a light shield. The figure is drawn as if we were looking from the projector lens system against the direction of the light (see Fig 4, page 13). The dotted lines represent two wooden runners over which the Perspex strips slide, the optical slit being on the far side of the glass plates. The erection on the left is a paxoline square covered with centimetre graph paper and serves to locate the position of each strip (measurements are taken from the ends of the strips which are well squared off). The strips can be manipulated from the right hand side.

The whole system of wedges could be moved to the exact vertical position required relative to the sine and cosine functions as the screw supports for the angle brass pieces passed through short vertical slits in metal sheeting.

Eventually then we had what we set out to produce, - eighteen optical wedges properly positioned and moving in a system which was sound both mechanically and optically.

Fig. 11. OPTICAL WEDGE ASSEMBLY.



SECTION B:

CHAPTER 4.

THE OPTICAL ARRANGEMENT.

Mention has already been made of the use of an optical bench (see Fig 4) on which the trigonometrical functions and optical wedges, as well as a condenser and a projector lens system, are mounted. This bench has a heavy metal base of triangular cross section which serves to retain the various components securely in their correct positions once these have been located. The projector lamp housing and the photocell unit have their own permanent supports of such dimensions that the lamp and cell are correctly aligned with respect to the optical axis of the rest of the system.

A Mazda 115 volts 200 watts projector lamp with tungsten filament is used and in series with a variable resistance of maximum value 90 ohms (current carrying capacity 3 amps) can be worked from the D.C. mains (230 volts). It is interesting to notice that this is quite a suitable source to use because in spite of the presence of a small mains ripple voltage the light output of the lamp does not fluctuate within a range which can be detected by the photocell circuit, due to the comparitively large heat capacity of the filament system. This was found however not to be the case (see/

(see page 33) for the ordinary bulbs used for lighting purposes which have a smaller thermal inertia. Small changes in mains voltage are unimportant as we are interested eventually in a record of the synthesised waveform which is produced practically instantaneously.

That the slit immediately preceding the rotating system be uniformly illuminated is a necessity and hence a large condenser lens system (11 cm diameter) placed 20 cm from the lamp, with a fine ground glass screen as diffuser is made use of to obtain this result. To begin with it was considered that filtering of the infra-red rays from the lamp should take place in order to prevent damage to the film bearing the sine and cosine functions. A water tank with plane glass walls was used for this purpose but was later dispensed with.

The condenser is 20 cm from the slit, which is of length 10 cm. During the series of experiments to be described a slit of width .5 mm. was used though provision had been made for this to be adjustable. Nothing appeared to be gained by using a smaller width which would of course require an increase in the amplification available in the photocell circuit.

The slit and system of rotating functions form eighteen light sources, the intensities of which vary sinoidally. Reduced images of these sources are focussed/

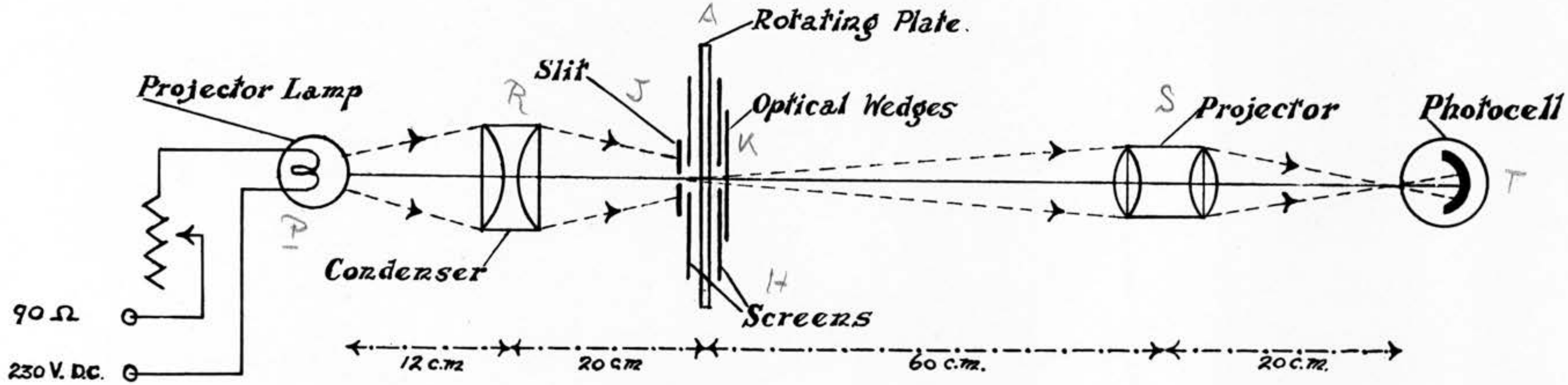
focussed on the photocell by a projector lens system (of aperture 5 cm), placed at a distance of 60 cm. from the slit. The photocell-lens distance being 20 cm. we see that the reduction factor is $\frac{1}{3}$. It was thought desirable that the images on the light sensitive cathode surface should be slightly out of focus so as to spread the light over a bigger area and thus decrease the possibility that small but sudden fluctuations in sensitivity over this surface might produce distortive effects.

The complete optical system is given in figure 12. It will be noted that blackened screens are provided to prevent light from the projector lamp falling directly on to the photocell.

A series of calibration curves for the identical optical wedges show in some cases small displacements (especially between those wedges in a central position and those in extreme positions) between one graph and another, which can be attributed to a slight lack of uniformity in the illumination of the slit. This is only to be expected and, unfortunately, not very much can be done about it with the present arrangement. However the matter is not as important as it seems to be at first sight for all that is required is that the illumination be sensibly constant over the range not/

Fig. 12.

THE OPTICAL SYSTEM.



not of all (7.6 cm) but over the range of each filter (.43 cm) in order that the light falling on the photo-cell should be a system with pure sine variations in intensity. This has been achieved in the present instrument.

SECTION B:

CHAPTER 5.

THE ELECTRICAL ARRANGEMENT.

An Osram caesium type gasfilled photoelectric cell with a maximum 'anode' voltage of 110 volts is used. This cell has anode and cathode connections at either end - which is preferable as it eliminates base leakage. It is quite suitable for use with the projector lamp as can be seen by considering its spectral response curve. This shows a maximum sensitivity in the region of 5000 Å, and is not too unlike that of the human eye.

A high vacuum cell should be superior from the standpoint of the linearity of the light input - signal output characteristic but none was available. However, unless a secondary emission type such as the Osram C.W.S.24 were used, such a cell would necessitate further stages of amplification - to be avoided if possible - and in any case the departure from linearity is probably of no practical importance since the cell is operated only at a voltage of 70 volts (a region in which the characteristic is tolerably straight and the amount of light involved is small. This means the cell operates over only a small portion of its characteristic and there is no evidence to show that the use/

use of the gas filled cell in fact impairs the accuracy of the instrument.

The cell and pre-amplifier are contained in an earthed metal box which serves both to shield the cell from strong light and screen the amplifier (and cell) from electric and magnetic disturbances. The light from the projector lens enters through a small window.

The circuit diagram of the preamplifier circuit is shown in figure 13.

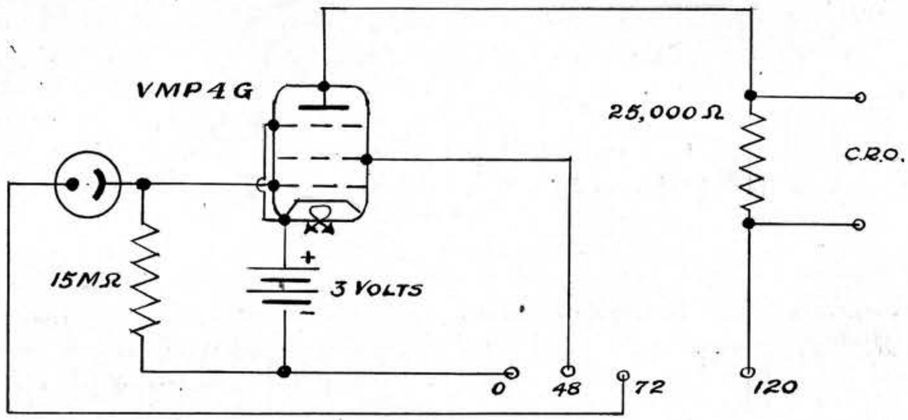


FIG.13 Photoelectric cell and Preamplifier.

One of the first problems in the design of this circuit was to consider the most suitable value of load resistance. The thermal agitation hiss in the high load resistance of the photocell varies as the square root of the resistance but there is a direct proportionality between the resistance and the signal output/

output of the cell. Thus the higher we make the resistance the less the signal to hiss ratio. Loss in high frequency response would be the result of using an excessively high load resistance and so 15 megohms was chosen as a suitable compromise.

A V M P 4 G Osram tube (a 4 volt variable - mu high frequency pentode) is used as the preamplifying valve. Other types would of course give satisfactory performances but it may be worth noting that the Miller effect which increases the apparent input capacity of triodes, renders the use of such valves unsatisfactory for this purpose. The operating potentials were chosen so that the photocell and preamplifier could be worked from one H.T. battery, which is enclosed in the same box. An anode resistance of 25,000 ohms and a grid biassing potential of 3 volts then gave us suitable operating conditions. The stage gain is approximately 30.

Hum problems (due to stray electric and magnetic fields inducing voltages on the various circuit components) at first seemed formidable but were overcome by the use of a carefully screened filament battery of two accumulators and the enclosure as already mentioned of cell and preamplifier in an earthed metal case. The leads from the accumulator to the preamplifier case also were shielded and kept as short as possible.

A/

A certain amount of microphonic noise, caused by the mechanical vibration of the elements of the tubes was encountered at first but suitable rebuilding of the unit eliminated this troublesome factor.

The presence of a very small reverse grid current of the order of $10^{-9} - 10^{-10}$ ampere can be detected with a grid bias of - 3 volts (the photocell being disconnected) by including a current meter in the anode circuit of the valve and by then short circuiting the grid resistance. This current is due to the collection by the grid of +ve ions formed in the grid-anode space but the magnitude of the effect did not seem to be such that it could seriously influence the operation of the unit.

In spite of the screening provided work should be carried out in a dimly lit room, illumination coming only from out-of-doors. This is necessary even though the lighting is D.C. as the presence of a slight ripple (probably communator hum) means that any appreciable stray light from a D.C. lamp will completely upset the functioning of the photoelectric cell. As already explained, this is due to the small heat capacity of the lamp filament.

We now come to the last section of the 'receiving system' the cathode ray oscillograph, a Mullard Type G M 3152/

G M 3152 instrument, which is located as shown in Fig 4. A screened lead connects the preamplifier output direct to the grid condenser and leak of the first valve of the oscillograph amplifier. This is a two-stage resistance capacity coupled system with a push-pull output and when operated with negative feedback - which almost eliminates the frequency, phase and amplitude distortions present in all amplifiers - is ideal for our purpose. The frequency response curve is sensibly flat down to 10 ~/sec. The degree of amplification provided in this way is sufficient for the light from one sine function to produce a vertical line on the oscillograph screen 2 cm. in length.

The question now arises of a horizontal or X deflection to delineate the synthesised waveform in a suitable manner. There are at least two possibilities - to use the internal 'linear time base' of the oscillograph or to use an external time base operated mechanically by the rotation of the glass plates.

The internal time base of this oscillograph is of the 'hard' valve type and in the lowest frequency range can operate from 2 ~/sec to 18 ~/sec. This range is used because, as will be later explained, it is desirable to have four complete cycles on the oscillograph screen, and the glass plates (with one complete period of/

of the fundamental in each quadrant) rotates approximately 5 times per second i.e. a time base with repetition frequency 5~/ sec is required.

In order to synchronise this frequency with the frequency of the synthesised waveform the initiation of each cycle from the time base is controlled by a signal derived from the Y deflecting voltage. The amount of this signal can be varied by a synchronising adjustment control. It is unfortunately found necessary to apply heavy synchronisation in order to obtain a stationary picture of the phenomenon under investigation. At the low repetition frequency of 5~/ sec the effect of this is to cause non-linearity in the time base, as can be seen by examining the photographs exhibited in the next chapter. Furthermore in spite of this synchronisation any appreciable change in the speed of the motor will cause the picture to start moving across the screen.

It becomes obvious then that a mechanically operated time base is essential for the satisfactory operation of the instrument and a suitable arrangement is shown in figure 14. R is a continuously rotating potentiometer, resistance 100,000 ohms, which is motor driven on the same axle as the glass plates and V is a 240 volt battery.

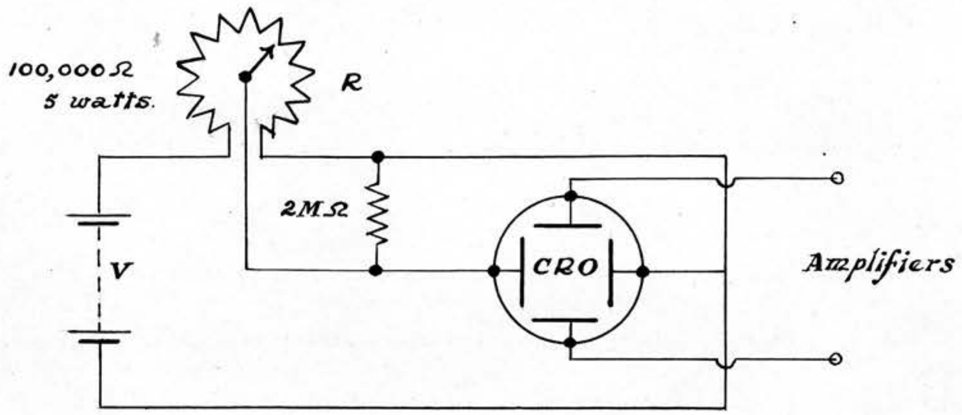


FIG.14 Mechanically-Operated Time-Base.

The x-coordinate of the oscillograph spot will correspond to some definite angular position of the plate and synchronisation will be automatic. The ends of the wire-wound potentiometer R must of course be sufficiently far apart to avoid short circuiting of the battery by the rotating contact and because of this a high resistance path must be provided between the two X plates.

The potentiometer is required to be of sufficiently high resistance to prevent any appreciable drain on the battery V and to prevent interference with the optical system should not be more than 2 inches in diameter. At first a wire-wound potentiometer of high resistance could not be obtained and a number of attempts were made to adapt different types of the common carbon element potentiometer. While these are of/

of suitable value and no difficulty is experienced in making them capable of revolving continuously, the carbon elements wear out after a few minutes of use as is to be expected. Most of the work described in this thesis had therefore to be performed using the internal time base of the oscillograph.

SECTION B.

CHAPTER 6.

PERFORMANCE.

Preliminary Galvanometer Tests.

Before the oscillograph and preamplifier part of the circuit was set up, a preliminary investigation of the rest of the instrument was carried out using a Tinsley reflecting type moving coil galvanometer to measure the photocell output. The motor was of course disconnected during this series of tests and galvanometer readings of photoelectric current observed for given angular positions of the plates - which could be determined accurately using the scale mentioned on page 18 .

The sensitivity of the galvanometer is such that a current of 1 μA would give a deflection of 185 cm. with the galvanometer scale at a distance of 1 metre. If the galvanometer is used unshunted and if the photocell is operated at a potential of about 107 volts this is quite satisfactory as then a single sine function will give a deflection of about 6 cm. on the galvanometer scale, corresponding to a photoelectric current of 3.2×10^{-8} amperes.

It was this arrangement which was used to obtain the series of calibration curves for the optical wedges - a representative curve was given in figure 8.

Then/

Then it was decided that a fairly stiff test of efficiency should be applied by finding how well the instrument could synthesise the 'saw-tooth' waveform which was discussed on page 10 . We wish to demonstrate graphically the functions

$$S_n(x) = \frac{2}{\pi}\sum_{1}^{n}\frac{1}{n}\sin nx$$

where n may have any value between 1 and 9. Six of these, corresponding to n = 1, 2, 3, 4, 6 and 8 are given in figure 16. The scales are quite arbitrary but the factor $\frac{2}{\pi}$ allows $S_{\infty}(x)$ to be plotted. One period of each of these functions is given. The graphs are of galvanometer current against angular position of the plate and are obtained by setting the filters to positions determined by the appropriate Fourier coefficients and calibration graphs. A slight horizontal displacement of each curve can be attributed to lack of coincidence between the zero of the scale determining the plate position and the actual zero of the fundamental on the film. Slight defects can of course be observed but the curves give good enough evidence of the reasonable operation of the sine functions, optical wedges and photocell.

Though the curves are drawn in about horizontal axes it must be remembered that as the galvanometer deflections/

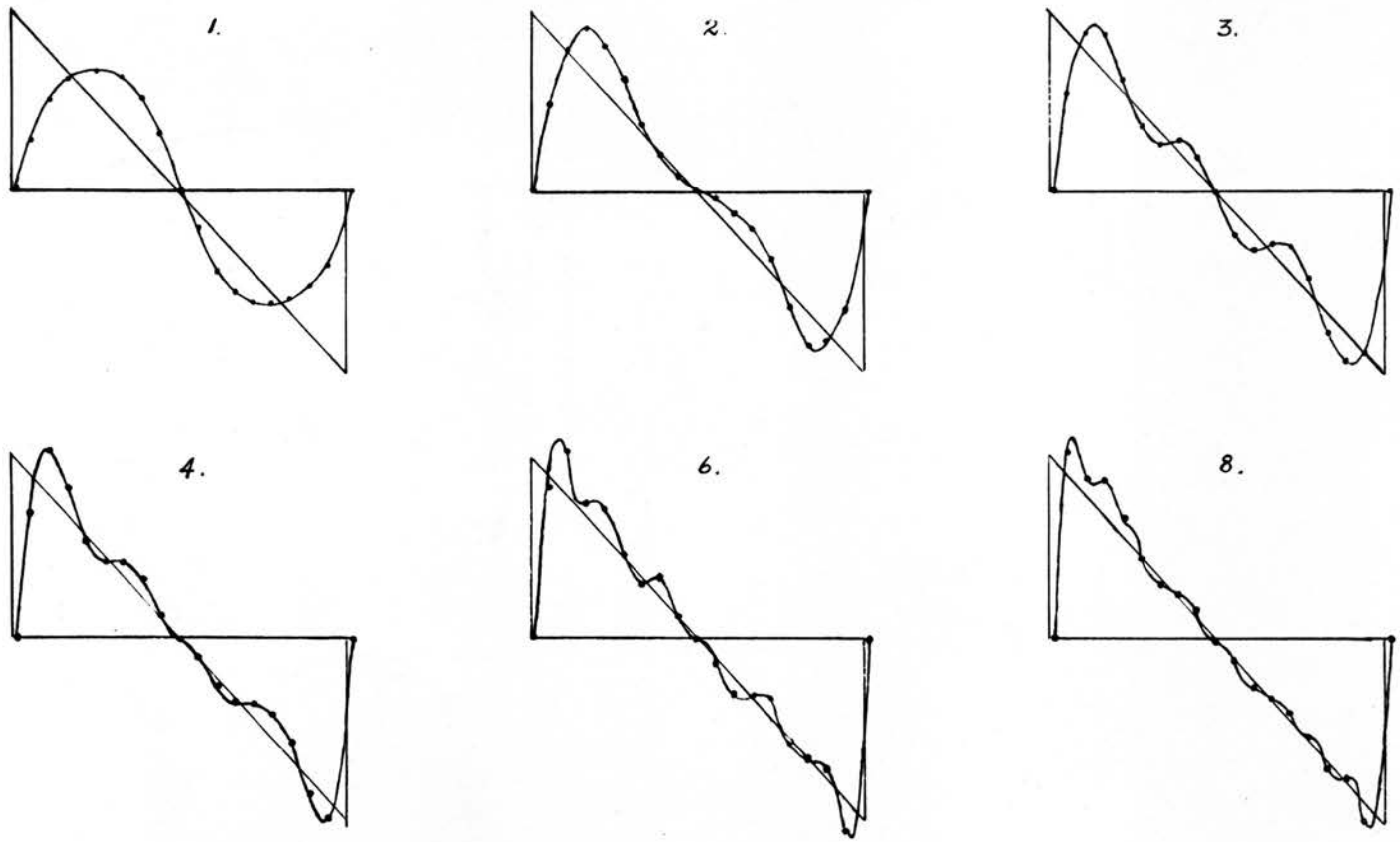


Fig. 16. *Graphs obtained of $\frac{2}{\pi} \Sigma \frac{1}{n} \sin nx$ using a galvanometer to measure photocell current.*

deflections are all unidirectional these axes can only be inferred from the symmetry of the results.

This method is a reliable one as it eliminates the possible distortions of a valve amplifying circuit or of time base circuits. Moreover these graphs are themselves of interest - for example they show how the successively approximating curves try to imitate the discontinuities of Sn (x) by going through x = o with ever increasing steepness and how with only 6 or 8 terms of the series the Gibb's phenomenon is becoming apparent. They are however obtained only after laborious plotting and hence in this form the instrument exhibits none of the unique features which have already been claimed for the complete assembly.

<u>Operation with the C.R.O.</u>

With the projector lamp and motor rheostats set to their correct values, A. C. and D. C. mains supplies are switched on, the photoelectric cell and amplifier battery circuits closed, and the necessary adjustments made to the focussing and brilliancy controls of the oscillograph (+ frequency and synchronising controls if the internal time base is to be used). It is assumed that the electrical screening difficulties already described have been overcome and that work is taking place in a room free from artificial light.

With/

With all the filters set to zero the oscillograph trace is a straight line.

Let us suppose we wish to synthesise a given Fourier series. The 'sin x" wedge, let us say, is adjusted until we have on the oscillograph screen a sine wave of a suitable amplitude - say 3 cm. This reading can be quite accurately made with the use of a transparent millimetre square raster. Then, leaving the 'sin x' wedge at this setting but covering it up, we adjust the 'sin 2 x' wedge, say, until we have on the screen a sine wave double the frequency of the first and with amplitude in a proportion to 3 cm which is determined by the coefficients of the Fourier Series.

In this way accurate settings can be made of amplitudes (and phases by combining sin n x and cos n x in the correct proportion) of harmonics as far as the 9th. The synthesised waveform is then obtained by uncovering all the wedges. This method has the advantage, over using the wedge calibrations obtained with the galvanometer, that difficulties relating to the frequency response of the amplifiers are eliminated and in any case it only requires, for example, about 2 minutes to complete the operations leading to the synthesis of a 'saw-tooth' waveform. If required, a still more accurate setting of the higher harmonics is possible using instead of the millimetre raster a cathetometer

to/

to measure the vertical deflections which are produced with the time base inoperative.

For recording purposes four cycles are always used on the screen. These are obtained by suitably adjusting the frequency control of the internal time base. This is desirable for it must be remembered that the synthesiser produces four quite independent cycles of the synthesised waveform. If small differences exist between the cycles due to the imperfections of the apparatus as constructed, then a slight blurring of the oscillograph trace will occur if, say, only two periods are present. This has, unfortunately, so far proved to be unavoidable but it is suspected that slight errors in the drawing of the fundamental may be the cause. A perfectly steady trace can be photographed when four periods are used.

Photography of the Cathode Ray tube trace.

For many purposes observation of the oscillograph trace is sufficient but permanent records of synthesised waveforms can easily be obtained photographically if required.

The use of a Zeiss Contax camera (Sonnar lens, f = 5 cm) and 35 mm. film was found to be very convenient, but it was first necessary to provide a lens extension piece of length 2.2 cm. It was then possible to take quite large pictures (reduction factor only

3½/

3½) with an oscillograph screen - lens distance of 22.5 cm. As the range finding system of this camera is of course put out of operation by the extension, a small ground glass screen must be used for focussing. The extension is fixed and so it is an easy matter to obtain the correct screen - lens distance once and for all.

With Kodak high-speed orthochromatic film (35 m.m.) it is possible to take 36 exposures on strips 1.6 metres in length. The use of a fast orthochromatic emulsion is preferable to the use of a panchromatic emulsion, with the attendant disadvantage of having to work in the dark. It is of course essential with a screen giving a green fluorescence, as in the present case, to use one of these two emulsions.

A few trial exposures at a suitable brilliance give the correct exposure time of $\frac{1}{5}$ sec. A good metolhydroquinone developer will give satisfactory contrast with this film and subsequent intensification is unnecessary. Tank developer was used and Kodak 'Elan' hydroquinone gave good results with a development time of 8 mins. at 18°C. It is necessary when printing for measurement or for reproduction to enlarge the negative. It is said that greater contrast is obtained by printing through a condenser enlarger with projector lamp illumination and no diffuser but the/

the author has not found this to have much effect. A high contrast printing paper is required such as Ilford Ultra-Contrasty Glossy No.6 which was used for the photographs in Figs. 17-19.

The question of the errors introduced by the photography of the trace due to screen curvature will not be discussed as these are smaller than the errors due to other factors. An interesting article on this subject by Moss and Cattanes[2] will be found in a recent copy of 'Electronic Engineering.'

Fourier Synthesis.

A number of representative photographs have been taken both as evidence of the correct functioning of the instrument and to illustrate some of its special features. The first group of these (Fig. 17) deals with our now familiar 'saw-tooth' waveforms

$$S_n(x) = \frac{2}{\pi}\sum_1^n \frac{1}{n}\sin nx$$

A, B and C give this function for values of n = 3, 4 and 8 and the remarks which were made about the curves obtained by using a galvanometer apply equally to these. It is interesting how nearly $S_8(x)$ approaches to the form $S_\infty(x)$. The discontinuities at $x = 0, 2\pi$, ---- are taking place so rapidly that the 'fly-back' is almost lost in the photograph.

The important thing now of course is, that to pass/

FIG.17A

$$S_3(x) = \frac{2}{\pi} \sum_{1}^{3} \frac{1}{n} \sin nx$$

FIG.17B

$$S_4(x) = \frac{2}{\pi} \sum_{1}^{4} \frac{1}{n} \sin nx$$

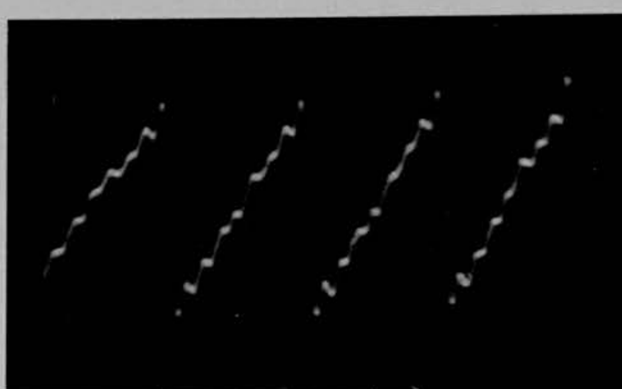


FIG.17C

$$S_8(x) = \frac{2}{\pi} \sum_{1}^{8} \frac{1}{n} \sin nx$$

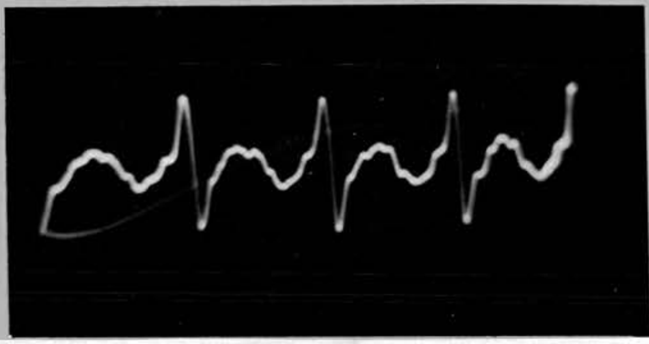


FIG. 17D

$S_8(x) - S_1(x)$ where $S_n(x) = \frac{2}{\Pi} \sum_{1}^{n} \frac{1}{n} \sin nx$

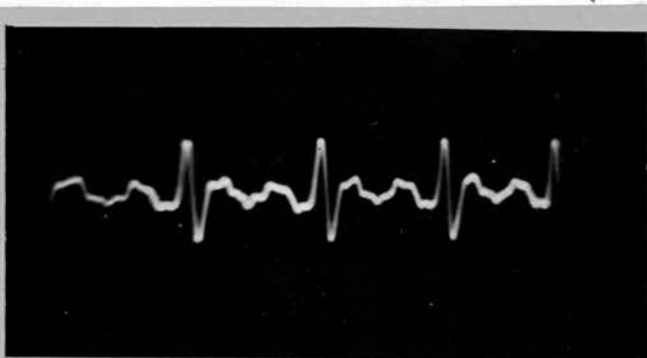


FIG. 17E

$S_8(x) - S_2(x)$

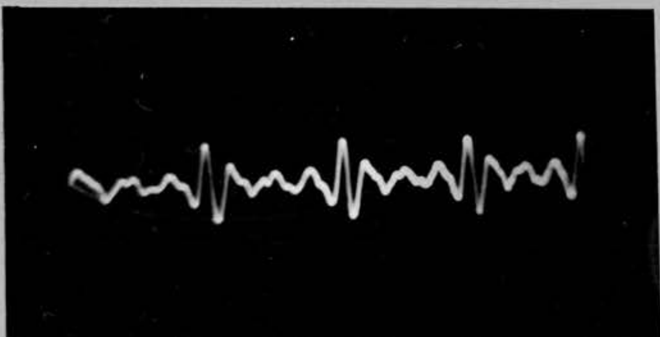


FIG. 17F

$S_8(x) - S_3(x)$

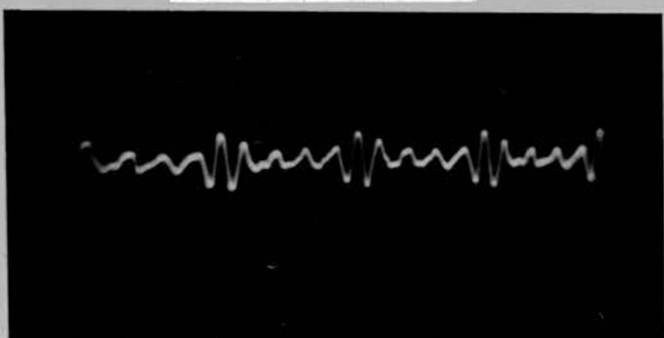


FIG 17G

$S_8(x) - S_4(x)$

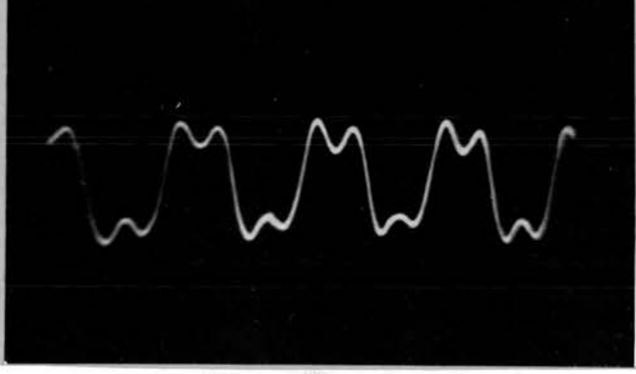


FIG. 18A

$$S_1(x) = \frac{4}{\pi} \sum_{0}^{1} \left(\frac{1}{2n+1}\right) \sin(2n+1)x$$

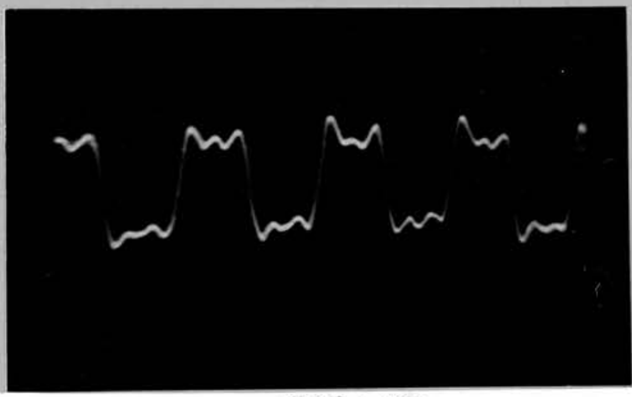


FIG. 18B

$$S_2(x) = \frac{4}{\pi} \sum_{0}^{2} \left(\frac{1}{2n+1}\right) \sin(2n+1)x$$

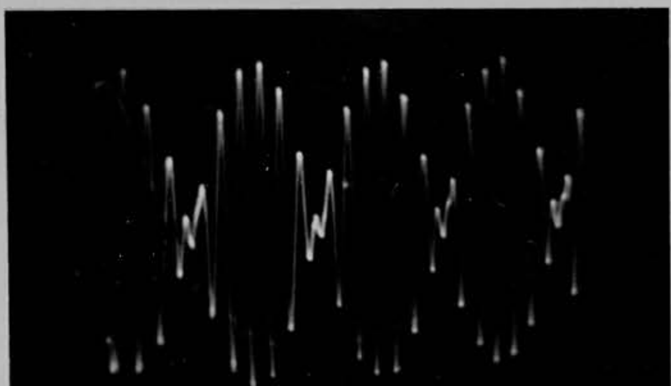


FIG. 18C

$\sin 6x + \cos 7x$

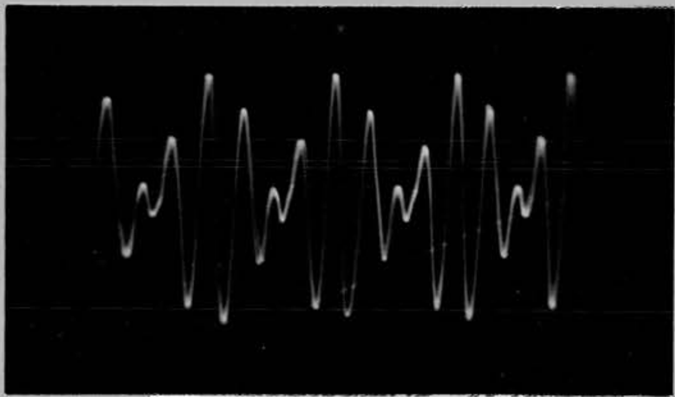


FIG. 18D

sin 3 x + sin 4 x

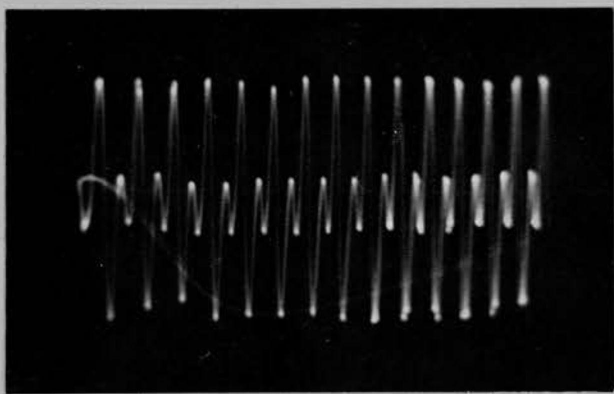


FIG. 18E

sin 4 x + sin 8 x

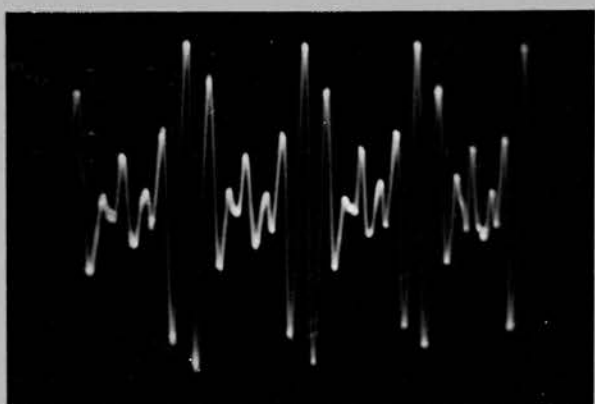


FIG. 18F

sin 4 x + sin 5 x + sin 6 x

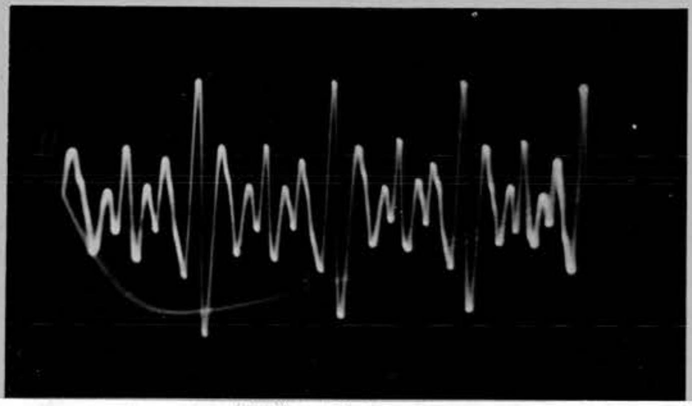


FIG. 19A

sin 4 x + sin 5 x + sin 6 x + sin 8 x

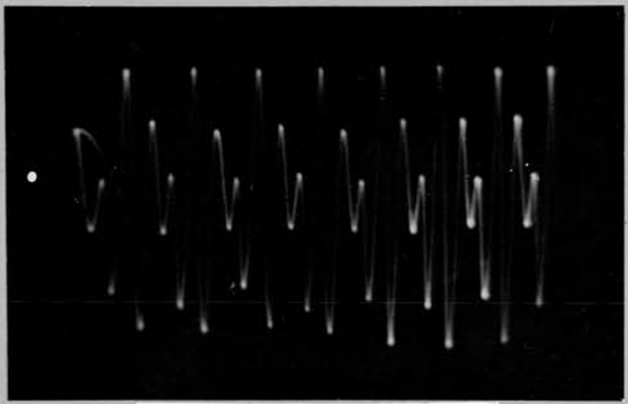


FIG. 19B

sin 4 x + sin 6 x

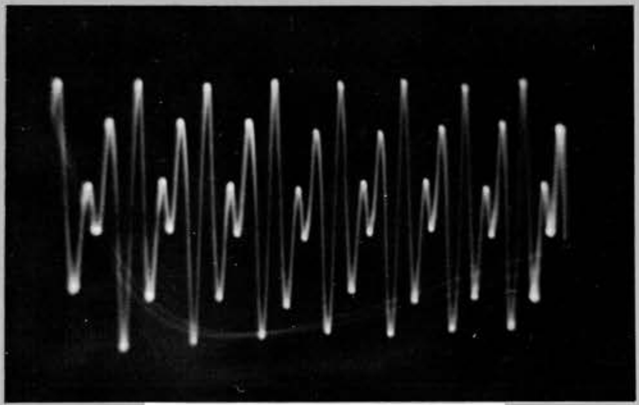


FIG. 19C

sin 4 x + cos 6 x

pass from one of these forms to another, we need only uncover the appropriate set of optical wedges (assuming these have been already set in the manner described). Thus the mere movement of an opaque screen across the series of wedges exposing the fundamental first shows in a remarkable fashion how the series builds up term by term.

A further interesting demonstration is obtained by moving this screen in the <u>opposite</u> direction when traces are obtained which correspond to the synthesis of the higher harmonics only. D shows the series to 8 terms minus the fundamental, E the series minus the fundamental and second harmonic, and so on. D is of special interest as it demonstrates so clearly the well-known acoustical fact that the extraction of the fundamental from a complex sound will leave us with a note which has the pitch of the original - for a frequency must be exhibited which is the highest common factor of the various harmonics. D, E, F and G all possess the periodicity of the fundamental but the more terms removed, the greater the blurring of this. These curves give quite a good idea of what would be the result of performing the same operation with an infinite series. It only takes a few seconds to demonstrate the effect so far described.

Two photographs, Figs. 18A and 18B, have also been/

been taken to illustrate the building up of a 'square wave' from its components.

$$\frac{4}{\pi}\sum_{0}^{n}\frac{1}{(2n+1)}\sin(2n+1)x.$$

is shown for n = 1 and 2.

In the same figure C shows the beating effect of waves whose frequencies are in the ratio of 6:7, the higher frequency component having an initial phase angle of 90^{o}. Another example of a beating effect is obtained in D with two ~~co~~sine waves, frequency ratio 3 : 4.

A few intervals of interest in music are also displayed. E shows a note with its octave sin 4x + sin 8 x, and F the major chord sin 4 x + sin 5 x + sin 6 x, and A (Fig. 19) the result of adding the octave to this. In figure 19, B and C are intervals of a major fifth (3 : 2) but in C an initial phase difference of 90^{o} has been introduced. Translated into sound these would be indistinguishable to the ear (see later for a description of how Ohm's Law has been verified). D is an interval of a major third composed of cos 4 x + cos 5 x

E is of interest for it demonstrates how the series .3 sin 5 x + sin 6 x + .3 sin 7 x i.e. 'carrier' plus side bands, gives an amplitude modulated wave of frequency 6. It is easy to show how/

how the depth of modulation depends on the amplitude of the sidebands. Photograph F will be discussed shortly.

This seems a suitable place to point out one unfortunate limitation of the instrument. There is at present no provision by which positive and negative terms of a Fourier series can be summed simultaneously. This would only be possible if four functions sin n x, - sin n x, cos n x, - cos n x were drawn for each harmonic; or if another uniformly illuminated slit and series of wedges were provided in the correct position, together with an extra photocell and projector lens. From the sin n x and cos n x functions we could then apply to the preamplifier signals 180° out of phase with those from the first photocell. The second of these possibilities would probably be the more difficult to carry out in practice.

A large number of series can still be dealt with however, but if a series with positive and negative terms must be summed it is possible to treat these as separate series and obtain the final result by graphical subtraction.

Calculation of the response of electrical networks to voltages of complex waveform.

It is proposed to show by a simple example how the synthesiser is well suited to be of assistance in calculations of this kind.

We/

We know from elementary alternating current theory that if a sinoidal voltage wave $v = V \sin wt$ is applied across a resistance - capacity series circuit (see Fig. 20) the voltage across the resistance is given by

$$v_R = V \frac{\omega CR}{\sqrt{1+\omega^2 C^2 R^2}} \sin(\omega t + \alpha) \qquad (14a)$$

where $\alpha = \tan^{-1} \frac{1}{WCR}$

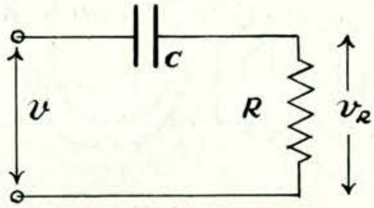


FIG.20 Resistance - Capacity Coupling.

Therefore if the applied voltage (a 'square wave) has the form given by

$$\frac{4}{\pi} \sum_{n=0}^{\infty} \frac{1}{(2n+1)} \sin(2n+1)\omega t$$

the voltage across R will be obtained by operating upon each component as in (14a) giving

$$\frac{4}{\pi} \sum_{n=0}^{\infty} \frac{\omega CR}{\sqrt{1+(2n+1)^2 \omega^2 C^2 R^2}} \sin\left\{(2n+1)\omega t + \tan^{-1}\frac{1}{(2n+1)\omega CR}\right\} \qquad (14b)$$

Suppose we wish to calculate the result of applying a wave form, composed of the first two terms of (14b) (Fig. 18A) to a resistance capacity coupling of time constant RC which is .1 of a complete period of the fundamental. The first two terms of (14b) give

$$.68 \sin(\omega t + 58^\circ) + .38 \sin(3\omega t + 11^\circ)$$

$$= .36 \sin \omega t + .58 \cos \omega t + .37 \sin 3\omega t + .07 \cos 3\omega t$$

and a photograph of the form of this function is shown in Fig 19F. In this simple case it would have been possible to obtain the result graphically, but with a larger number of harmonics the process is extremely tedious. The waveform which is shown in Fig 19F no longer possesses the symmetry of Fig. 18A and is tending towards the exponential form of the complete series (14b).

If the series of the original voltage wave is known it is only necessary to operate upon it term by term with a factor which expresses the characteristic of the circuit (as in 14a). The resulting waveform can then be obtained by the present instrument with sufficient accuracy for most purposes.

Verification of the Acoustical Law of Ohm.

The last application of the instrument which will be given here is an interesting one that makes use of the fact that the synthesised wave is not a mere series of numbers but has a definite physical existence. It is possible to apply the deflecting voltage across the/

the plates of the oscillograph to an output valve with loud speaker, and actually listen to the synthesised wave.

It would therefore seem that we have a system ideally suited to a study of the well known acoustical law of Ohm, which states that from a complex sound the amplitudes only of the various components can be detected by the human ear. That is to say, if the amplitudes are left unchanged, alternatives in the phases of the components do not cause the slightest audible difference in the sound.

Van der Pol has described a method of testing this out in which the phases of the various components of sounds spoken into a microphone can be affected and yet their amplitudes left unchanged. Special circuits had to be designed before this was possible but here we can synthesise any type of wave, and, while listening to it, control the phases and amplitudes as we please.

The oscillogram can be observed simultaneously of course and it is curious to watch different waveforms producing identical sound effects - an example is that of a major fifth interval as shown in Figs 19B and 19C.

If the speed of rotation of the glass plates could be considerably increased - and the building of another/

another instrument in which this would be possible is at present thought desirable - so that the frequency of the fundamental would be, say, 200$\sim$/sec, then the effects of organ pipes, flutes, violins etc. could be imitated from the known analysis of these notes. Even at present however interesting demonstrations are possible with beating effects, and some of the common consonant intervals.

SECTION C.

CHAPTER 1.

THE HARMONIC ANALYSER.

The process of generalised Fourier analysis means the separation of a function into periodic components without other restriction on the form which these components may take. Our discussion will be limited to the analysis of periodic functions into components which satisfy the conditions of orthogonality as expressed on page 5.

By a harmonic analyser we of course imply a device for calculating the coefficients in a series of the form

$$f(x) = \frac{a_0}{2} + a_1 \cos x + a_2 \cos 2x + \ldots$$
$$+ b_1 \sin x + b_2 \sin 2x + \ldots .$$

The diverse nature of the methods which have been used is a reflection of the great practical importance of the problem in many fields of Physics. A rough classification of these methods would be

(1) Graphical methods using a planimeter (e.g. the Henrici[3] Analyser)

(2) Electrical (Resonance and Bridge methods)

(3) Optical (Montgomery[1])

(4) Diffractive (Meyer[4])

The method to be described falls somewhere between

(2)/

(2) and (3) - for although the use of an electrionic mixing device is the essential and novel feature, the function to be analysed must first be translated into the modulations of a light beam. This chapter will be taken up with a description of the fundamental principle of operation.

Product terms with multigrid tubes.

Consider the case of a thermionic valve with two grids and let the instantaneous grid potentials be eg_1 and eg_2.

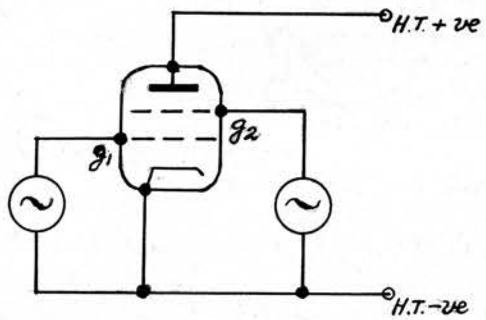


FIG.21 Two grid tube.

The corresponding value of anode current, ia, is a function of both eg_1 and eg_2 and so the characteristic is a surface drawn in three dimensions, the axes being those of anode current ia, grid potential eg_1, and grid potential eg_2. In general a cross-section of this surface is a curve but the plot of anode current against the voltage on a particular grid may be substantially linear over a considerable range.

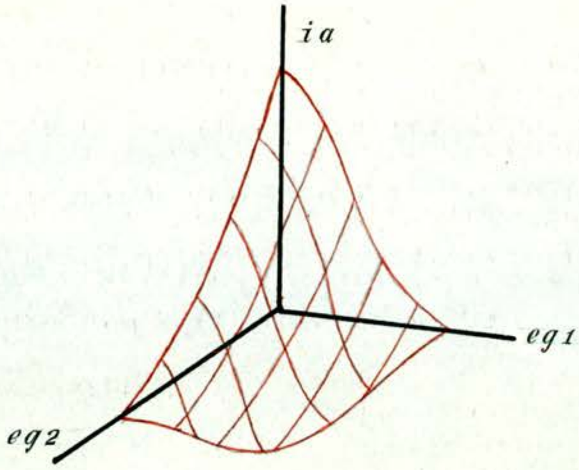


FIG.22. Characteristic Surface.

The following treatment applies only to small signals within this range.

The anode current can obviously be written in the form

$$i = (a_1 + b_1 g_1)(a_2 + b_2 g_2)$$

where a_1, a_2, b_1, b_2 are constants and g_1 and g_2 are small changes of potential from the operating point.

We have

$$i = a_1 a_2 + a_2 b_1 g_1 + a_1 b_2 g_2 + b_1 b_2 g_1 g_2$$

$$= i_0 + c_1 g_1 + c_2 g_2 + \frac{c_1 c_2}{i_0} g_1 g_2 \quad \text{---(15)}$$

where the c's are constants.

Suppose now we write

$$\begin{cases} g_1 = A_1 \cos n\omega t \\ g_2 = A_2 f(\omega t) \end{cases}, \quad \text{---(16)}$$

f (wt) being some function of period 2 π.

Then/

Then

$$i = i_0 + c_1 A_1 \cos n\omega t + c_2 A_2 f(\omega t) + \frac{c_1 c_2}{i_0} A_1 A_2 f(\omega t) \cos n\omega t$$

$$= i_0 + C_1 \cos n\omega t + C_2 f(\omega t) + \frac{C_1 C_2}{i_0} f(\omega t) \cos n\omega t$$

$$\text{where} \quad C_1 = c_1 A_1$$
$$C_2 = c_2 A_2$$

The average of i over a complete cycle will be

$$\bar{i} = \frac{1}{2\pi}\int_0^{2\pi} i \, d(\omega t) = i_0 + \frac{C_1 C_2}{i_0} \frac{1}{2\pi}\int_0^{2\pi} f(\omega t) \cos n\omega t \, d(\omega t)$$

assuming $\overline{f(wt)}$ is zero.

Now we have seen in Section A, Chapter 2 that the n^{th} Fourier cosine coefficient in the expansion of f (x) in terms of the orthogonal set (2) can be written

$$a_n = \frac{1}{\pi}\int_0^{2\pi} f(x) \cos nx \, dx = \frac{1}{\pi}\int_0^{2\pi} f(\omega t) \cos n\omega t \, d(\omega t)$$

$$\therefore \bar{i} - i_0 = \left(\frac{C_1 C_2}{2 i_0}\right) a_n \qquad (17)$$

i.e. a_n is proportional to $(\bar{i} - i_0)$. This can be measured of course if we have a direct current meter in the anode circuit. Hence to obtain the Fourier analysis of the function f (wt) we have only to find some method by which it is possible to apply potentials to the two grids which satisfy (16) for n = 1, 2, ... and for sine functions as well as cosine functions. The readings on a current meter in the anode circuit, balanced against the standing current i_0, will then give a measure of the appropriate coefficient.

That the above method is immediately extendible to expansions in terms of orthogonal sets can be seen/

seen from the form of the general Fourier coefficient (5) page 6. For simplicity we will continue to deal with set (2) only, always remembering the wider application which is possible.

In practice, then, a method has to be found by which it will be possible to satisfy (16) as to both the frequency and phase relations of the sinoidal voltages to one another and to the voltage representing the unknown function. No direct method of achieving this is possible. However in the Harmonic Synthesiser we already have a device which can produce sine and cosine voltage waves with the required frequency and phase relationships. All we need do therefore is to rotate a suitable variable area drawing of the function f (x) together with the film bearing the sine and cosine functions, and allow the resultant modulated light beam to fall on another photoelectric cell. The two photoelectric cell outputs are then applied to the two grids of the tube.

The theory of the method already given does not apply exactly to this practical case. In fact (16) must now be written

$$\begin{cases} g_1 = A_1(\cos n\omega t + \varphi_1) \\ g_2 = A_2(\cos n\omega t + \varphi_2) \end{cases} \qquad (18)$$

[φ_1 and φ_2 constants]

to allow for the introduction of the constant term as /

as described at the beginning of Chapter I, Section B. We must therefore have

$$i = i_0 + c_1 A_1(\cos n\omega t + \varphi_1) + c_2 A_2(f(\omega t) + \varphi_2)$$
$$+ \frac{c_1 c_2}{i_0} A_1 A_2 \left\{(\cos n\omega t + \varphi_1)(f(\omega t) + \varphi_2)\right\}$$

The average of i over a complete cycle will now be

$$\bar{i}_1 = \frac{1}{2\pi}\int_0^{2\pi} i\, d(\omega t) = i_0 + c_1 A_1 \varphi_1 + c_2 A_2 \varphi_2$$
$$+ \frac{c_1 c_2}{i_0} A_1 A_2 \left\{\frac{1}{2\pi}\int_0^{2\pi} f(\omega t)\cos n\omega t\, d(\omega t) + \varphi_1\varphi_2\right\}$$
$$= i_0 + C_1\varphi_1 + C_2\varphi_2 + \frac{C_1 C_2}{i_0}\left\{\frac{1}{2\pi}\int_0^{2\pi} f(\omega t)\cos n\omega t\, d(\omega t) + \varphi_1\varphi_2\right\}$$
$$= i_0 + C_1\varphi_1 + C_2\varphi_2 + \frac{C_1 C_2}{i_0}\left\{\frac{a_n}{2} + \varphi_1\varphi_2\right\}$$

where a_n has the same meaning as before and

$$\begin{cases} C_1 = c_1 A_1 \\ C_2 = c_2 A_2 \end{cases}$$

If we can now cut out the function f (wt) but retain as before cos n wt, φ_1 , and φ_2 then the new average value of the current is

$$\bar{i}_2 = i_0 + C_1\varphi_1 + C_2\varphi_2 + \frac{C_1 C_2}{i_0}\varphi_1\varphi_2 ,$$

the value of the integral now being zero.

$$\therefore \bar{i}_1 - \bar{i}_2 = \left(\frac{C_1 C_2}{2 i_0}\right) \times a_n \qquad (19)$$

Equations (17) and (19) should be compared. It can be seen that to obtain a measure of a_n we must again take the difference between two readings of a direct current meter in the anode circuit of the valve, but one of these readings is <u>not</u> as before of the anode current at the operating point.

We/

We must now take the difference between a reading which corresponds to the obtaining of conditions (18) and that in which

$$\begin{cases} g_1 = A_1 (\cos n\omega t + \varphi_1) \\ g_2 = A_2 \varphi_2 \end{cases} \tag{20}$$

A general idea has been given of how grid voltages as in (18) can be produced. The experimental details of this and of how the transition from (18) to (20)can be effected, are reserved for the next chapter.

SECTION C.

CHAPTER 2.

ADJUSTMENTS REQUIRED IN THE SYNTHESISER LAY-OUT AND THE RESULTS OBTAINED.

The function to be analysed must first be drawn in variable area form using a system of polar co-ordinates; this can be done quite accurately on thin celluloid. Fig. 23, for the special case of a square wave illustrates the sort of thing required. Four complete cycles are necessary in order to correspond with the arrangement of the sine and cosine functions. The dimensions are chosen so that the celluloid can be mounted between the two glass plates and yet not interfere with the Kodaline film. This gives us 19 variable area functions - from the centre outwards these are

$f(x), \cos x, \sin x, \cos 2x, \ldots\ldots \sin 9x$

Care must be taken in the mounting of the celluloid that f (x) has the correct phase relationship to sin x. After this operation is complete the whole system is locked securely in position as described on page 2. For the analysis of f (x) in terms of any <u>other</u> orthogonal set then the film must of course be replaced by another bearing the appropriate set of functions. It seems worth while also pointing out here that the original/

original drawings need only be made once and would provide copies for a number of instruments.

The optical wedges are set so that the trigonometrical functions all produce periodic light fluctuations of the same amplitude. Otherwise they are not concerned with the operation of analysis.

Two projector lens systems are necessary. The first as before focusses an image of the trigonometrical functions on one photocell and the second, placed below, focusses an image of the function f (x) on another. The use of black paper screens facilitates this adjustment.

For the rest of this chapter we will deal with the design and operation of the new 'receiving' system. Two Osram gas-filled emission-type photoelectric cells are used, and the remarks previously made on this subject (page 30) apply again here. Load resistances of 10 MΩ are provided and the outputs across these go to the two grids of a mixer valve. The photocells, mixer valve and batteries are adequately screened from electric and magnetic disturbances.

In the basic mixing circuit of Fig. 21 a two grid tube was shown. The modern 'frequency changers' used for the supersonic heterodyne reception of wireless signals have however additional grids for the suppression of secondary electrons and to diminish interelectrode capacities, and also a 'local oscillator' section/

section. These lead to the construction of the eight electrode valve or octode.

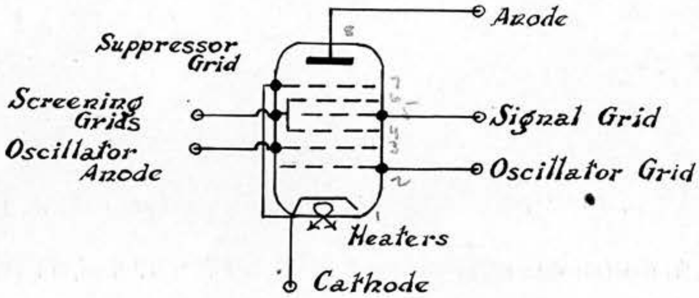


FIG. 24. The Octode.

In spite of this more complex arrangement, the potentials on the other grids being constant, direct multiplication of the signal grid and oscillator grid voltages takes place when these modulate the common electron stream. It was therefore decided that a Mullard FC 4 Octode would be a suitable valve to use and a set of characteristic curves was taken for the anode and screen potentials indicated in Figure 25.

Curves are plotted to show the dependence of the anode current on the potential of one grid for various constant potentials of the other. These curves are useful as they allow us both to choose suitable operating conditions and to calculate roughly the magnitudes to be expected in the phenomenon under investigation.

In addition a simple examination was carried out of the multiplicative action of the octode:

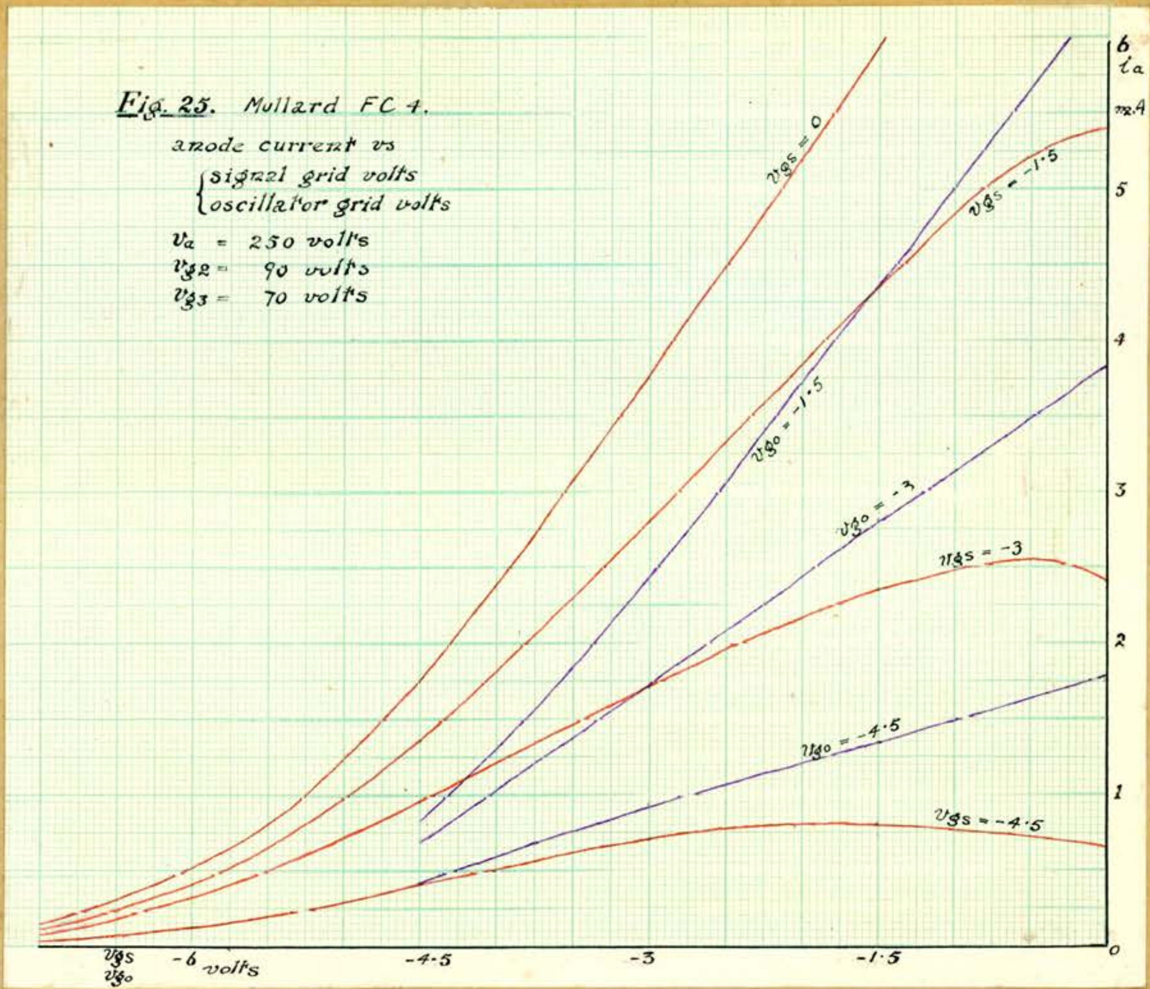
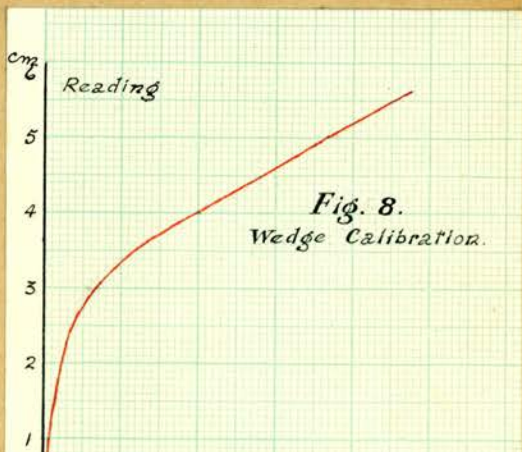


FIG. 23
Square Wave in a system
of polar coordinates.



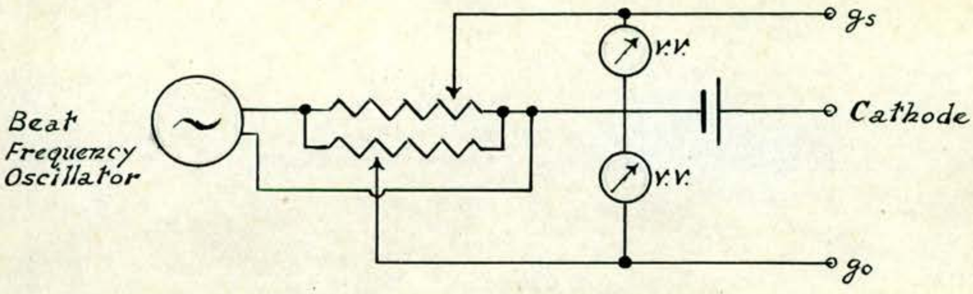


FIG.26. Test of Multiplication.

From an inspection of this Figure it will be seen that we can apply to the two grids voltages which are in the same phase but the amplitudes of which can be varied independently. A d.c. meter reads the anode current and observations of this and of valve voltmeter readings are recorded. In the following table Si_a is the change in anode current from the steady value to that under the conditions which are stipulated.

g_o (volts RMS)	g_s (volts RMS)	$g_o g_s$	Si_a (μA)	$Si_a/g_o g_s$
.2	.49	.098	21	213
.3	.64	.192	40	208
.4	.85	.340	71	209
.5	1.0	.500	106	212
.6	1.2	.720	150	208

The values of ($Si_a/g_o g_s$) are constant to within 2% and show that a change of anode current of 210 μA /(volt RMS)^2 is to be expected.

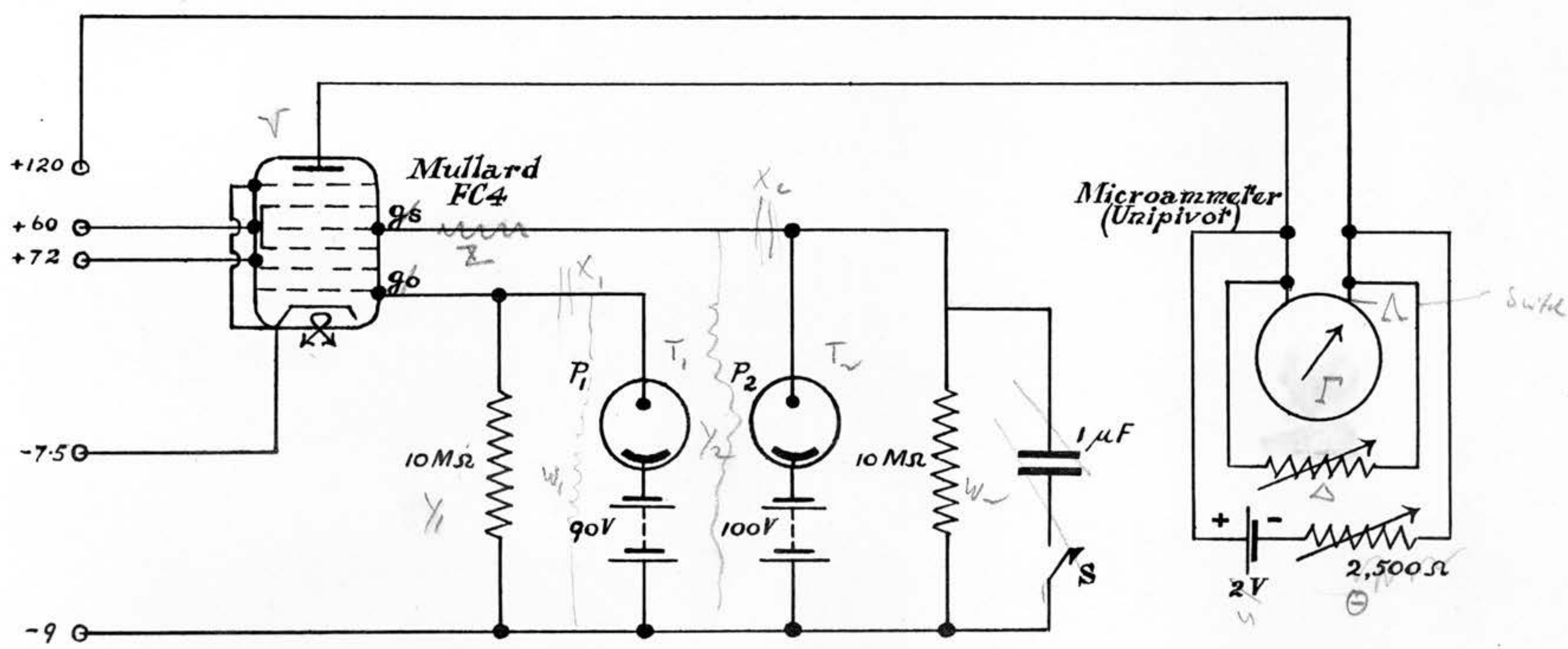
A/

A good deal of experimenting was required before the complete receiver unit was suitably designed as shown in Figure 27. The shunted microammeter in the anode circuit of the mixer valve is a Cambridge Unipivot type instrument with a maximum scale deflection of 240 μA, and is connected in parallel with a two-volt accumulator cell and a variable resistance of maximum value 2,500 ohms. This arrangement permits the balancing of steady currents down to about .8 m A and is therefore a suitable one.

The operating potentials of the mixer valve are shown. A common negative grid bias of - 1.5 volts is used. The screen grid has roughly one half the anode potential but is somewhat below the potential of the oscillator anode. The mixer valve is operated from one high tension battery but separate photoelectric cell batteries are required because of different cell sensitivities. The image of the periodically varying light source, corresponding to the function to be analysed is concentrated on cell P_1, and P_2 receives the images, <u>one at a time</u>, of the sine and cosine functions.

Excepting the presence of the condenser and switch, the working of the unit should be sufficiently clear from what has gone before. Leads to this switch and to the microammeter circuit pass to a separate table for ease of observation. Otherwise all the components and/

Fig. 27. THE HARMONIC ANALYSER. (Receiver Unit)



and batteries are enclosed.

All supplies are switched on and the glass plates set into rotation. We therefore have potentials on the oscillator and signal grids which, relative to the operating point, are given respectively by

$$g_o = A_1 (\cos n\omega t + \varphi_1)$$
$$g_s = A_2 (f(\omega t) + \varphi_2)$$

(see 18)

If now the switch S is closed the effect of the 1 μF condenser (which has, even at 20$\sim$/sec, a negligible impedence in comparison with 10 MΩ) is to present to the alternating component of g_s a total load resistance of only a few thousand ohms. Thus the voltage output of the cell corresponds to the mean intensity of the light falling on it and we can therefore regard the grid potentials as given by

$$g_o = A_1 (\cos n\omega t + \varphi_1)$$
$$g_s = A_2 (\varphi_2)$$

(see 20)

The microammeter reading is now observed (the total anode current is $\bar{i}_2$). The switch S is opened and the new meter reading taken (total anode current now $\bar{i}_1$). Subtraction gives ($\bar{i}_1 - \bar{i}_2$) a quantity which as we have seen is proportional to the n th cosine Fourier coefficient in the expansion of f (x). Repetitions of this procedure for the uncovering one by one of each of the sine and cosine functions therefore give the Fourier Analysis of f (x).

There/

There are certain difficulties relating to the use of the condenser. The insulation resistance must not be low enough to shunt the 10 MΩ resistance appreciably. A good mica condenser must therefore be used or spurious results will be obtained. Further when the switch S is closed the condenser is subjected to a voltage change of amount $A_2 \varphi_2$. Thus a charging current will flow for some time subsequent to the operation of closing the switch. The effect of dielectric hysteresis therefore means that about a minute must be allowed prior to any observations of the anode current $\bar{i}_2$. Then if the switch is open only for a few seconds during the observation of $\bar{i}_1$ the condenser voltage will not have time to change appreciably and no serious time lag need take place during the subsequent recording of the different harmonics.

A preliminary examination of the practicability of the method was made by analysing a 'square wave'. For example the following, a typical series of values of ($\bar{i}_1 - \bar{i}_2$) as far as the 5th harmonic, was deduced:

sin x	21 microamperes
sin 3x	8 microamperes
sin 5x	4 microamperes,

other values of $(\bar{i}_1 - \bar{i}_2)$ in this range being of the order of 1 microampere. These figures are in reasonable agreement with theory. It was not found possible to increase the magnitudes of the current changes because, to produce even these, the photocells had to be worked so near to the maximum permissible anode voltages that a certain degree of instability in operation was present.

It would therefore seem that the use of electron multiplier photocells or the provision of two preamplifier circuits is required before inaccuracies (as much as 8% in some cases) can be reduced.

To within this limit then the work which has been carried out has shown the method to be valid, but cannot be said to have produced a precision instrument. It is considered however that the further development suggested is warranted, not so as to compete with existing harmonic analysers, but because of the unique applications to general Fourier analysis which are possible.

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ACKNOWLEDGMENTS.

I am deeply indebted to Professor Born for proposing the construction of the Analyser and for much helpful discussion and interest in my work; and to Dr Fürth for the suggestion of the Synthesiser and for his valuable cooperation and encouragement.

I also wish to express my sincere thanks to Professor Barkla for his kind permission to undertake the work herein described; to Dr Childs for the loan of various pieces of apparatus; and to Messrs Bartholomew for undertaking so successfully the work described on page 16.

A NEW PHOTOMECHANICAL METHOD

OF

MICROPHOTOMETRY.

being Part II

of a thesis for the degree of

Doctor of Philosophy

submitted by

ROBERT W. PRINGLE, B.Sc.

University of Edinburgh

April 1944.

The second part of this thesis consists of a paper to be published in the May (1944) issue of 'Electronic Engineering' and is reproduced exactly as it will appear there.

THE FURTH MICROPHOTOMETER.

A description of an improved form of an instrument which is entirely based on electronic principles.

by R.W. PRINGLE, B.Sc., A. Inst. P.,
Physics Department,
University of Edinburgh.

INTRODUCTION.

It is a matter of considerable importance in physical research to have an instrument which is capable of recording accurately the variations in transmissibility along a photographic plate. We have therefore seen the development of more than one highly successful microphotometer - for example, the Moll or Koch types. It may be remarked in passing that only recently there has been published an excellent bibliography of the somewhat extensive literature on the subject.[1]

A new type of microphotometer has been recently designed by R. Fürth[2,3] which might be of some interest to the readers of this journal as it is entirely based on electronic principles. It has three unique features which distinguish it from all other types of existing Microphotometers:-

(1) The required curve is obtained on the screen of a cathode ray oscillograph and thus a quick visual survey/

survey of the whole plate under examination is possible.

(2) Magnifications (200 or more) can be obtained which are much greater than with any other existing type.

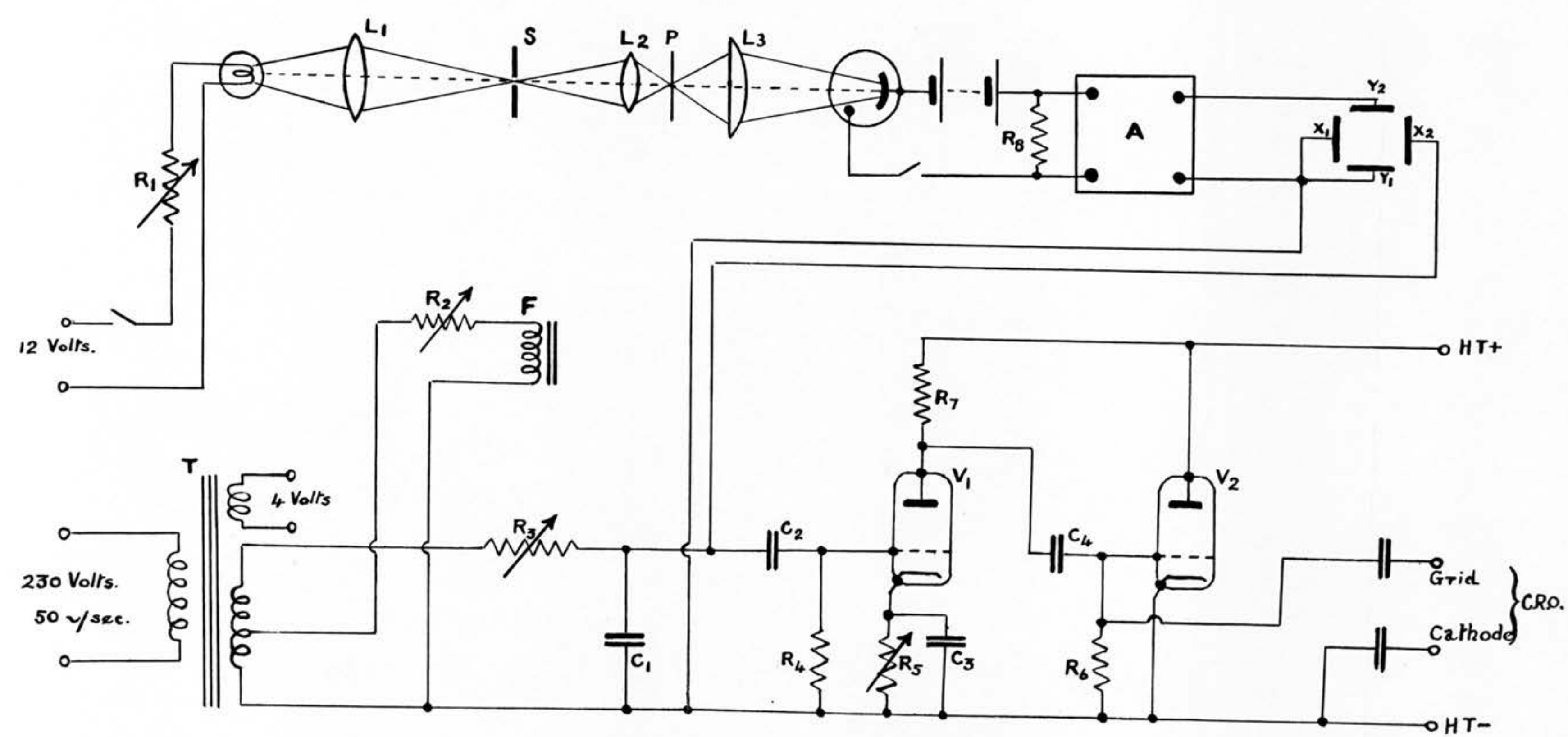
(3) No elaborate gearing mechanism is incorporated.

One disadvantage of the new device was that the photometric curves obtained were rather thick and therefore lacking in detail. The present author has been able to improve the instrument in this respect[4] so that its resolving power is now comparable with that of other existing instruments. In the following a short account of the principles of the Fürth Microphotometer is given and a description of the recent improvements.

DESCRIPTION OF THE INSTRUMENT.

A. The Y-Deflection.

A schematic diagram of the complete instrument is given in figure (1). Lens L concentrates the light from a 12 volt 6 watt lamp on to a slit S, and a reduced image ($\frac{1}{12}$) of the slit is formed by the microscopic objective L_2 at the photographic plate P. The transmitted light is then condensed by the lens system L_3 on to the cathode of a gas-filled caesium-type photoelectric cell which requires a potential of about 100 volts for operation. Voltages developed across the load resistance R_8 are passed, after suitable amplification, to the Y deflector plates of a cathode ray/



C_1	·01 μF	R_3	Variable 25 x 10⁴ ohms
C_2	·02 μF	R_4	5 x 10³ ohms
C_3	25 μF	R_5	Variable 1 x 10³ ohms.
C_4	·1 μF	R_6	25 x 10⁴ ohms
R_1	Variable 20 ohms	R_7	5 x 10⁴ ohms
R_2	Varible 20 ohms	R_8	1 megohm

FIG. 1

ray oscillograph (Mitcham Type GM 3152 in which the amplifier is incorporated). The photocell, battery, and the amplifier input leads must all be carefully shielded against external disturbances.

The small plate P which is to be examined is fixed in a holder carried by one end of a tuning fork (natural frequency about 50$\sim$/sec.) which is supported in a horizontal position and the arrangement is such that oscillations of the fork cause the plate to move in a vertical direction with its plane normal to the axis of the optical system. The tuning fork is maintained magnetically and the current for the fork coil F is supplied by a 12 volt winding on the mains transformer T. The amplitude of the forced vibrations of the fork can be controlled by the rheostat R_2 but it should be noticed that owing to the finite inertia of the moving parts there will be a phase difference between this vibration and the controlling voltage.

B. The X-Deflection.

The 50 volt terminals of the transformer T are connected across a phase-shifting device which consists of a condenser C (.1μF) and a variable resistance R_3 (maximum value .25 MΩ) in series. The X plates of the oscillograph, being placed in parallel with this condenser, can now be supplied with a sinoidal time base/

base which has the same frequency and phase as the mechanical oscillation of the plate P.

Suppose now the plate to contain a photograph of a spectrum where the lines are parallel to the slit S. The vibration of the plate across the light beam being equivalent to a vibration of the slit image along the spectrum over a range equal to twice the amplitude of vibration of the fork, it will be seen that the X deflection of the oscillograph is an enlarged copy of this motion of the slit image.

Now the Y deflection is proportional to the intensity of illumination of the photocell and, since this is proportional to the transparency on the plate of the particular line just passing the slit image, the spot of the oscillograph traces the transparency curve automatically. Different parts of this curve can be brought successively into the field of observation by moving the vibrating mechanism vertically with the aid of a micrometer adjustment.

THE IMPROVED RECORD.

The curve obtained in this way is really the superposition of two traces writing in opposite directions on the screen of the cathode ray oscillograph, and corresponding to the two directions of the vibrational motion of the plate through the slit image, and hence fluctuations/

fluctuations in the mains frequency or voltage can cause slight relative displacements of the two traces which result either in a thick curve or a splitting up in two curves.

A periodic 'square-wave' voltage is now applied between grid and cathode of the oscillograph and this controls the brughtness of the fluorescent spot in such a way as to make only one trace visible. To do this the time base voltage is applied across resistance R_4 (5000 Ω) and condenser C_2 (.02μF) in series, and the result is to produce across R_4 an alternating voltage of about 1 volt and removed more than 88° in phase from the time base voltage. Triode valve V_1 functions as an amplifier for this voltage and V_2 which is unbiassed and has a high grid resistance clips off the positive peaks of the output from V_2 due to grid current flow (see Fig.2). The dotted line indicates that potential corresponding to the limit of visibility, and the thick line therefore shows approximately the fluctuations in brightness of the fluorescent spot. Comparison of curves (1) and (2) shows that in the interval AB we have a bright spot and in the interval BC nothing is visible. So a single trace has been obtained.

<u>EXAMPLES</u>./

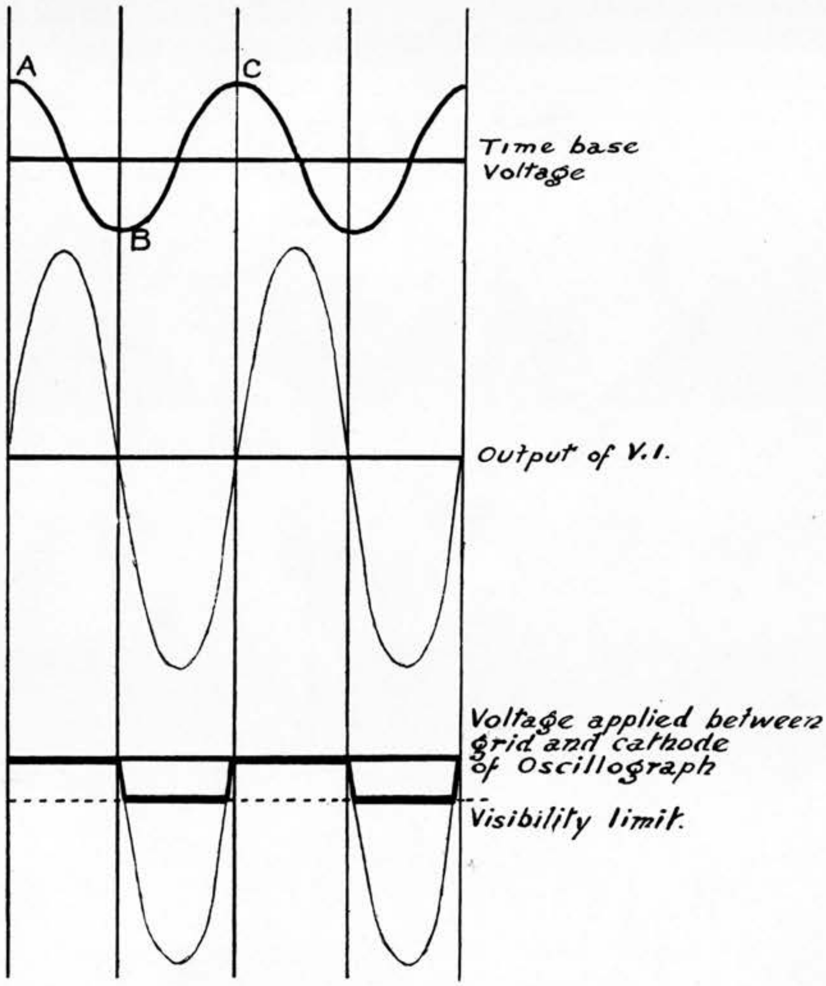

A
B
C
Time base
Voltage
Output of V.1.
Voltage applied between
grid and cathode
of Oscillograph
Visibility limit.

Fig. 2.

EXAMPLES.

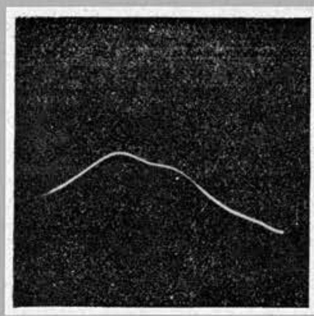
In Figure 3 are shown two photographic records taken with a Ross 9 in. lens using Ilford Rapid Process Panchromatic Plates with an exposure time of 3 seconds.

Both records relate to the density distribution in a photograph of the arc spectrum of copper in the neighbourhood of 3440 A^{o}, taken with an ordinary glass spectrograph. Record B (with original magnification of 30) covers a range of 15 A^{o} approximately. Record A is of the central hump in record B; it corresponds to an original magnification of 200 and covers a range of 2.2 A^{o}, and a distance of 0.24 mm. on the spectrum photograph. On the oscillograph screen a distance of 1 mm. corresponded to 0.3 A^{o} in the case B and to 0.045A^{o} in case A.

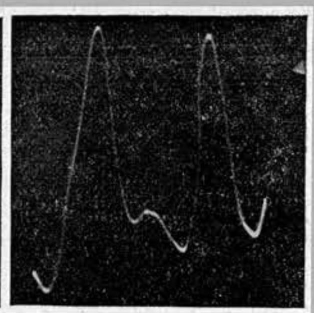
The two records not only give some indication of the improved performance of the instrument, but serve to illustrate how the magnification can be altered at will to study more closely any desired portion of the photometric curve.

This article would be incomplete without my thanks to Prof. Born and Dr. Fürth of the Mathematical Physics Department of this University for their encouragement in this work.

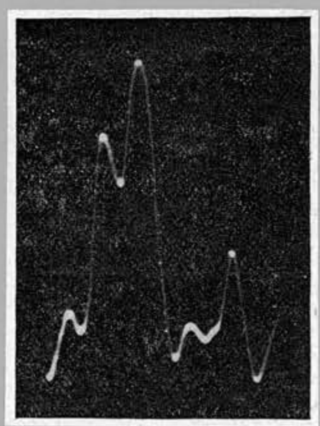
(1) Ronnebeck, H.R. J. Sci. Instr. 20 (1943), p.159.
(2) Fürth, R. Nature 149, (1942) p.730.
(3) Fürth, R. Proc. Phys. Soc. London, 55, (1943), p.34.
(4) Pringle, R.W. Nature 153 (1944) p.81.



3 A.



3 B.



From the same plate.
(Magnification 30.)