KANT'S FIRST ANTINOMY:

AN ESSAY IN PHILOSOPHICAL COSMOLOGY

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I declare that this thesis has been composed by myself and is the result of my own work.

A B S T R A C T

This thesis attempts to provide a reinterpretation and de-
fense of the arguments of the Thesis and the Antithesis of
the First Antinomy, taking account of contemporary interpre-
tations and objections. The objective is to provide not on-
ly illuminating interpretations of Kant's text but also to
provide arguments which are valid granted the assumption
that Kant believed to underlie the Antinomy, namely the trans-
cendental reality of the sensible world. The discussion is
at no point a wholly historical one, but rather touches on
various connected issues of contemporary or recent concern:
this is particularly true of the chapter on the Thesis.

Having defended the arguments of the Thesis and the Anti-
thesis the immediate problem is to solve the resulting Anti-
nomy. Kant's solution, in terms of transcendental idealism,
is rejected, and an alternative sought. This is initially
introduced as the suggestion that the universe exists inter-
nally but not externally, and some explication and defence
of this suggestion is provided. A view of space and time is
developed, largely drawing on the material in the Metaphysi-
cal Expositions, which supports this suggestion. This view
of space and time is called *mundocentrism*. It is not an i-
dealist theory.

A separate chapter seeks to relate the predominantly philo-
sophical discussion in the main part of the thesis to con-
temporary scientific thinking on the same area, in particu-
lar the Big Bang theory and General Relativity.

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PREFACE

It is necessary to explain the method by which quotations in the text are ascribed to their sources. All the sources of quotations are to be found in the Bibliography. Where an author has only one work included in the Bibliography, the quotation is assigned to that work by the author's name. If more than one work by the author appears in the Bibliography then the quotation is assigned by the initials of the title of the work (e.g., "TLP" stands for Wittgenstein's Tractatus Logico-Philosophicus). Generally the page number of the appropriate edition is then given, with the following exceptions. In the case of the Critique of Pure Reason the page number of the second edition is given, as in "B125". These numbers are given in the margins of the Kemp Smith translation. In the case of quotations from The Metaphysical Foundations of Natural Science the page number of the Academy edition is given (also provided in the margin of the cited text). In quotations from Leibniz (all of which are from the letters to Clarke), the number and paragraph of the letter are given. Finally, in the case of quotations from Wittgenstein's Tractatus, the number of the remark is given. In recognition of its pre-eminent importance in this work the Critique of Pure Reason is indicated solely by the "B" number, without accompanying initials.

The Bibliography is quite a short one. It would be impossible to list all the material in any way relevant to

this project since I find that any philosophy I read helps me to understand other philosophical problems, and ideas encountered long ago in other contexts can prove useful with current problems. To mention all the relevant works in this sense would be to list all my philosophical reading over the last ten years. Therefore the works cited in the Bibliography are those which either are quoted in the text or from which I have derived some specific benefit in my understanding of the problems this thesis covers.

INTRODUCTION

As the title suggests this thesis has much to do with Kant, however it is not primarily *on* Kant. That is to say, I am not a Kant scholar in the conventional sense of the term. My main concern is not with the minutiae of Kant's text, or the historical relations of that text to other texts of the period. Rather, the focal point of this work is a problem, and arguments connected with that problem. The problem is the problem of the First Antinomy. Both the problem and many of the arguments can be found in Kant's text and it is largely in terms of Kant's text that I shall discuss them. Nonetheless it is the problem, not the text, that shall be at the centre of attention. In particular when it comes to solving the problem I shall argue that Kant's solution is inadequate and offer a different one of my own.

However I would not wish to claim that the two main approaches to a philosophical classic, which we might call the *scholarly* and the *argumentative*, are necessarily and completely incompatible. The careful study of past masters can throw light on contemporary problems (the Renaissance is only the most extreme example of this common occurrence); and in any case if we do not properly understand what the text is telling us then we cannot argue with it, but only with a disputant of our own imagination, or a distortion created by the popular image of what the dead master is supposed

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to have said. Accordingly I am far from cavalier in my approach to Kant's text, and I am on several occasions critical of interpretations that I find inadequate. Moreover the argumentative approach, if it is tempered by humility, can have scholarly benefits, especially with a philosopher such as Kant. Where a philosopher's treatment of a problem is obscure or brief or convoluted -- and Kant's, at times, is all of these -- then a desire on the part of the interpreter to solve the problem and a knowledge of its intricacies and its pitfalls can help him to illuminate a text that someone who had only the text could find impossibly difficult.

In fact the importance and worth of Kant's thinking in this area has been somewhat under-rated in recent years partly because his interpreters have not done him justice. One of the secondary aims of this thesis is to re-establish Kant's thought on this subject in a prominent position. Kant's relative neglect to-day is particularly surprising if we consider the historical background to the Antinomy. Kant belonged to the second generation of German intellectuals after Leibniz (there is about a sixty year gap between Leibniz's death and Kant's writing the Critique of Pure Reason), and important features of his intellectual landscape were "the philosophers of the Leibnizian school", as Kant calls them in the Observations on the Antithesis. At the same time Newtonian mechanics was establishing the control over European thought that it was to maintain throughout the following century. In this context Al-Azm is quite right to stress, as a matter of Kant's historical situation, the importance of the debate between Leibniz and the English

Newtonian Samuel Clarke although, as I shall argue, he is mistaken in the particular points of interpretation he makes on that basis. The difference of opinion between Leibniz and Newton, left as a legacy to their successors (the debate was cut short by Leibniz's death) meant that cosmology was probably a more live issue in Kant's time than in any subsequent time until our own.

However in our own time the issues have been dramatically re-opened. The era of unchallenged supremacy of the Newtonian world-view came to an abrupt end in the first two decades of this century with the publication of the Special and General Theories of Relativity. There has been, throughout the century, a development in our techniques for exploring the universe, from radio telescopes to space probes. The "Big Bang" theory has come to be generally (though not universally) accepted among astrophysicists. There is a good deal of interest, not only among academics in the relevant disciplines, but also among the educated general public, in cosmological questions. The historical parallel with Kant's time is stronger still if one remembers that Einstein supposedly had strong leanings towards Leibniz, derived via Mach.

Given the situation it might be thought to be an opportune moment to revisit the Antinomy. This thought appears not to be shared by the majority of Kant's commentators. The First Antinomy tends to be quickly passed over in books that aim to provide a general introduction to the Critique and is often subjected to brief and ill-considered criticisms. Rarely does one find writings wholly concerned with

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the First Antinomy, and even more rarely do these concern themselves with the validity or otherwise of the arguments of the Thesis or Antithesis. I have suggested that part of this neglect is due to an inadequate grasp of Kant's arguments. However I believe there is also a wider issue involved, namely a fear among contemporary philosophers, at least in the Anglo-American world, to stray into an area which has been claimed by physical science. There are perhaps historical justifications for caution in this regard, but taken to extremes it serves only to isolate Western philosophy from some of the most pressing intellectual problems of the culture in which it is imbedded, and, at the very least, removes from the scientific arena a great store of human thinking on the nature of the world.

Part of the problem lies in a misunderstanding of the nature of scientific work. The popular prejudice is that philosophers "just think" and that by "just thinking" we can never find out about the world. Certainly thought alone is not enough, as Kant would be only too keen to acknowledge, but we should not ignore the amount of "just thinking" that goes on within scientific citadels. For example, it is not impossible for a theoretical physicist, after his initial training, never to enter a laboratory in his life. Someone must do the empirical work; but without a good deal of thinking first and afterwards, the empirical work is liable to be barren. Kant's famous phrase, "thoughts without content are empty, intuitions without concepts are blind," (B75) is not inappropriate here.

There is therefore no necessary incompatibility between

philosophy and, at any rate, the more theoretical parts of physical science. The main obstacle to a rapprochement is institutional. Theoretical science, particularly in the physical sciences and increasingly so elsewhere, is highly mathematical; and most philosophers come from Arts backgrounds, and know little mathematics. This creates particular difficulties in discussing a physical theory which is highly complex mathematically, such as the General Theory of Relativity, and which uses as ethereal a branch of mathematics as tensor calculus (which, as far as I can tell, is rarely studied or used except in the context of Einstein's theory). Nonetheless I have tried, despite my own mathematical shortcomings, to relate the philosophical discussion of the Antinomy to the main theories in contemporary science in the same area, one of which is the General Theory of Relativity. In the case of the General Theory I in fact do little more than describe points of comparison and identify points of disagreement and suggest why, on the points on which Kant (and myself) and Einstein disagree, Kant would not accept Einstein's view. However even doing this little is better than ignoring Einstein altogether or suggesting that the disagreement cannot be pursued to a total resolution because of the nature of philosophy and of science rather than simply because of the limits to my knowledge of Einstein's theory. I also seek by this discussion to show that one cannot simply assume that Einstein is right and Kant is wrong, although that may be one's considered judgement.

I said at the beginning of this introduction that my central concern was with the problem of the First Antinomy

and its attendant arguments, both those required to estab-
lish the problem and those required to solve it. To some ex-
tent this can be regarded as one long sustained argument.
This has consequences for the treatment given to particular
issues that arise in the course of this thesis, namely that
I discuss particular problems as they are presented by the
dictates of the argument, rather than in general terms.
Thus, for example, I discuss at various places issues that
belong to the epistemology of mathematics, but at no point
is there a sustained general treatment of this area for its
own sake. There are stylistic reasons for this: such a dis-
cussion would interrupt the continuity of the essay. How-
ever there is a philosophical reason as well, which is that
I prefer to do philosophy as a problem-centred rather than
a topic-centred subject. That is to say, I prefer to dis-
cuss the epistemology of mathematics as that discussion is
forced on me by the needs of a particular problem (not ne-
cessarily, and in this case not ultimately, one in that
area itself), than to sit down with an otherwise blank sheet
of paper headed "The Epistemology of Mathematics". Of
course, that is to put crudely a distinction that is large-
ly one of emphasis, and I would not want to say that either
approach is superior to the other; simply it is the problem-
centred approach that I prefer.

One aspect of this is that I do not attempt a general
discussion of transcendental idealism, relating it to other
forms of idealism perhaps or to problems in the philosophy
of perception. I discuss it only as it intrudes upon the
First Antinomy, which is why I do not even characterise it
until Chapter Three, although in Chapter Five I do discuss

its roots in the epistemology of geometry. I do not pretend to have provided anything near an exhaustive discussion of these entities, but only those aspects that most closely concern the First Antinomy.

There are many topics that naturally arise in the general context of this thesis that receive an even more cursory treatment, often being dismissed with the excuse that to discuss them would be too great a diversion from the main thesis. On other occasions, most notably the discussion of mathematics and reality to be found in Appendix Two, shortcuts are taken and assumptions made which would be inexcusable in a work solely concerned with those topics. In all these cases where a topic has been too relevant to be ignored but too large to be fully encompassed within the space available for it, I have had to make fine judgements about the amount of coverage it would be justified to give each such subject in the light of the requirements and general ambitions of this thesis. I imagine that the extent to which a reader agrees with the correctness of those judgements will depend greatly on the extent to which he shares the assumptions that are made; and so one can expect a good deal of disagreement among readers. Unless one were, like the early Wittgenstein, to claim to have solved all philosophical problems I do not see how this difficulty is to be avoided. I trust that at any rate no mere assumption has been made whose opposite would have been fatal for the main thrust of this thesis.

A point must be made about ontology. Sometimes in this work I use the word "object" and sometimes I use the word

It has been suggested to me that there is unclarity in my use of the distinction between "actual" and "potential" infinite: accordingly I should like to clarify this. By the actual infinite I understand a model of the spatio-temporal world in which some two objects are separated by an infinite number of objects. By a potential infinite I understand a model of the universe -- such as the one provided by Bell in the paper cited -- in which although the universe is held to be infinite there is always only a finite distance between any two objects. I believe this to be a natural use of the distinction in this context and would suggest that Lane Craig in the work cited understands it in the same way. If the reader disagrees then I hope he will take me to be capriciously re-defining these terms to refer to these models: it is the criticisms that I make of the models that matter, not the labels I use to refer to them. It *is* important that these two possibilities should comprise the exclusive range of possible models of an infinite world: and this they do.

"event". No ontological bifurcation is intended by this difference. I use each term to refer to the individuals, whatever they are, that make up the universe. As this is a work in cosmology, not ontology, a certain haziness about ontology is unavoidable. I would hope that it does not affect the validity of any of the arguments. In general, though I have not been totally consistent about this, I have tended to use the word "object" when considering space and "event" when considering time.

Finally, as the logical structure of the Antinomy is quite complicated it is best to become clear about this now. Kant does not, of course, accept unreservedly the arguments of the Antinomy. However he does believe that the Antinomy necessarily results, and thus that its arguments are valid, if we make one assumption. That assumption is that the sensible world is transcendentally real. Kant goes on to deny the assumption, and claims that the fact that an antinomy follows upon our adopting it is an indirect argument in favour of his own view of transcendental idealism.

Is is clear from this that Kant's purposes fail if the Antinomy does *not* result from our adopting that assumption. Thus it is an argument against Kant, though not directly against Kant's own views, to argue that even given transcendental realism no antinomy results because one or both of the arguments given is invalid. This is the usual approach among commentators. Thus it is necessary to defend the arguments against these criticisms in order to protect Kant's aims.

The situation is further complicated by my own belief that transcendental idealism is false. My reasons for this belief are given primarily in Chapter Three, but also in Chapter Five and Appendices One and Two. From this it follows that I endorse the assumption on which the arguments are based, and so accept the arguments as straightforwardly valid. However, since each argument proceeds as a *reductio ad absurdum* of the opposing view, I take them to show not that the universe is both finite and infinite, but that it is neither finite nor infinite. How this can be is the main topic of the second half of this thesis. It means that my defences of the arguments of the Antinomy have a somewhat duplicitous character since I shall also defend them against Kant himself.

CONTENTS

T H E S I S

As I have intimated in my introduction, this chapter is an argument against the possibility of an infinite world. Partly it is concerned with defending the argument put forward by Kant in the Thesis of the First Antinomy. However before that I want to consider some arguments on the same subject that are straightforwardly advanced by Kant in an earlier work, his Inaugural Dissertation. Consequently we must be especially clear about the structure of this chapter. Put simply it is this. I believe that the arguments of the Inaugural Dissertation are valid (with one exception which we will come to) and that the arguments of the First Antinomy are valid. Kant, when he wrote the Inaugural Dissertation, believed that its arguments were valid. Kant, when he wrote the First Antinomy, believed that its arguments were valid granted the (to him) mistaken assumption of transcendental realism. Thus from the point of view of Kant scholarship there is an important distinction between the two cases, which is why I have sign-posted it clearly here. As my dominant concern is with the development of a valid argument that distinction is likely to be glossed over hereafter. As long as this warning is borne in mind no damage should accrue from this.

Similarly, there must certainly have been an actual

historical development of Kant's thinking in this area
from the time of the Inaugural Dissertation to the time
of the first Critique. I am simply ignorant of these his-
torical facts; and the development of the arguments in
this chapter is not an attempt to recreate them.

To turn to the Inaugural Dissertation itself, it is
clearly deserving of the title of "a pre-critical work".
Firstly, it is easy to understand that the man who wrote
the Inaugural Dissertation later went on to write the Cri-
tique of Pure Reason. Yet secondly, if we bear in mind
Kant's comparison of himself to Copernicus, the Inaugural
Dissertation is definitely a pre-revolutionary work. Its
tone as one reads it is more scholastic and less modern
than that of the Critique. Much of the Transcendental
Aesthetic is present in an embryonic form and the Disser-
tation does make the distinction between the sensible and
the intelligible and asserts that much bad metaphysics re-
sults from confusing the two. However not only is a
transcendental analytic not attempted, or even hinted at,
but also no sharp distinction is drawn between a transcen-
dental analytic and a transcendental dialectic. The idea
of synthetic *a priori* categories required by and limited
to possible experience is altogether lacking. Indeed,
far from arguing that any attempt to describe the intelli-
gible world can only result in paralogisms, antinomies
and illusions, Kant tries to outline some of its features
himself.

Most of this enterprise does not directly concern us.
However, to properly appreciate the following discussion

it is necessary to be clear about how I intend to treat
Kant's distinction here between a sensible and an intelli-
gible world. It is not necessary for our purposes here
to become entangled in what could only be a long and tor-
tuous discussion of the psychological and epistemological
aspects of the distinction. I shall simply treat the sen-
sible world as a spatio-temporal world and the intelli-
gible world as a non-spatio-temporal world. As long as
we are aware that we are doing this it will not lead to a
misrepresentation because for Kant the sensible world is
a spatio-temporal world and the intelligible world is
not -- though for him this is a matter of discovery, not
of definition.

Early in the _Dissertation_ Kant sets out to describe
"the notion of a world in general". He claims that there
are three factors involved in the definition of a world:
matter, form and wholeness. We are interested in the
third, for it is under this heading that we find the ar-
guments with which we are concerned.

The discussion is in two parts, the first part deal-
ing with the world in time and the second dealing with
the world in space. Taking the world in time first, we
are asked to view it as spread out before us in time as a
whole "comprehending absolutely all changes". (p. 42)
Kant thinks that there is something problematic about
this infinitely temporally extended universe. He offers
two reasons for this belief. Firstly he says:

> On account of its very infinitude it is
> necessary that it be without limit, and
> therefore that no series of successive

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events be given save as part of some fur-
ther series. Consequently, for the same
reason, an all round completeness, an ab-
solute totality, seems to be ruled out al-
together. (p. 42)

Then he goes on to say:

> Though the notion of a part can be taken
> universally and though all the things
> that are contained under this notion, if
> regarded as placed in the same series,
> may constitute a unity, yet the concept
> of a *whole* seems to demand that all those
> things should be taken together; and this,
> in the given case, is impossible. For
> since to the series as a whole there is
> nothing subsequent, and in the case of a
> given series of successives the only mem-
> ber to which there is nothing subsequent
> is the last, there will be to eternity a
> last member in (the succession); and that
> is absurd. (p. 42)

Kant then goes on to argue that the conception of the
world as infinite in space is equally problematic:

> (I)f a simultaneous infinite be admitted,
> the totality of the successive infinite
> must also be conceded while if the lat-
> ter is denied, the former must also be
> given up. (p. 42 - 43)

He describes the problem for the simultaneous infinite as
follows:

> For the simultaneous infinite supplies
> eternity with inexhaustable material
> to be successively traversed, through
> its innumerable parts *in infinitum*
> and yet this series, freed from all
> (the bonds of) number, would be sup-
> posed to be given in a simultaneous
> infinite, and so a series which
> could never be completed by succes-
> sive addition would be supposed to
> be capable of being given as a
> whole. (p. 43)

Allow me to summarise and label these arguments. The
first argues that if the world were infinite in time then

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there could be no single series of states of the world --
rather every series would be part of some greater series.
I will label this the *one series argument*.

The second argues that if the world in time were to
exist as a whole there cannot be anything after the world.
Therefore the temporal world as a whole must have a last
term. However if it is infinite it exists to eternity.
Therefore it must have a last term to eternity -- which
is absurd. I will label this the *last term argument*.

Thirdly there is the argument concerning space. This
argues that as the world can be seen as a collection of
parts distributed in space contemporaneously just as it
can be seen as a collection of parts distributed succes-
sively, then it would seem that if either distribution is
impossible, so *prima facie*, must be the other. To use an
analogy, if I cannot write down all the natural numbers
then they cannot all be written down. I will label this
the *infinite sum argument*.[1]

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[1]
  It may seem that I have ignored Kant's use of the phrase
"successively traversing" in my summary of this argument.
The point Kant is seeking to make there is this. If the
spatial world is infinite then we can imagine a journey in
a straight line through the spatial world as a model of an
infinite temporal world, since in both cases there would
be an infinite successive passing of events. So if such
an infinite successive passing of events is impossible as
a temporal sequence then *a fortiori* it is impossible as a
journey; and if it is possible as a journey, which it would
appear to be if the spatial world is indeed infinite, then
it must be possible as a temporal sequence.
  However it is important to bear in mind that this refers
to the possibility of such a journey, not to anybody's ac-
tually making it. Kant is not suggesting here that an in-
finite spatial world could only be created by someone's in-
finite traversing of it (as might appear to be the case in
transcendental idealism). In fact all the reference

How valid are these arguments? The one series argument seems valid as far as it goes, because of the nature of infinity. An infinite series does not have two termini[2]: accordingly there is always another member to the series: this means that there is always another series that any given series is a sub-series of: which is what the argument claims.

It may be objected that there is still one series to which all the members belong, namely the infinite series. This may be intended to mean that there is an "actual infinite" series to which all the members belong. The difficulty with this (a difficulty we shall return to) is that an "actual infinite" is not a possible model for the temporal world as it would require that there be some two events in the series such that there is an infinite number of events between them, which would mean that it would be impossible for

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to successive traversing establishes is that if either distribution is impossible then so, *prima facie*, must be the other; since by thinking in terms of a journey we can, as it were, convert a spatial distribution into a temporal one and vice versa: so their possibility or impossibility must stand or fall together.

[2] Of course if we think in terms of the real number line then we have a line with an infinite number of points but that can have two termini. This is due to the infinite divisibility of the line. Yet if we write the series of denominators of the fractions into which the line is divided ("halves", "quarters", "eighths", and so on), and thus produce a series of the divisions, then that series cannot have two termini. In any case infinite divisibility is the subject of the Second Antinomy and cannot be superimposed onto our discussion here without creating insuperable confusions. There is a difference, at least in terms of the traditional approaches to them, between the question of an infinitely extended universe of objects, and the question of an object's being infinitely divisible, and it is only the first of these questions that this thesis is able to consider.

both the earlier event and the later event to occur: since an infinite series does not have two termini.

I therefore tentatively declare that the one series argument is valid.

I cannot be as generous to the last term argument because it rests on a mistake. Of course, to the world as a whole there is nothing subsequent. However this does not entail that there is a last term: for to an infinite world there is nothing subsequent yet there is no last term. Kant may have been misled into believing that "nothing subsequent" required that there be a time subsequent at which there is nothing, whereas in fact it requires that there be no time subsequent at which there is something; and a temporally infinite universe satisfies this requirement by there being no time subsequent.

The case for the infinite sum argument also appears strong. Just as we can think of time as extended from "now" into the past and the future, so we can think of space as extended in three directions from "here". So there seems no reason why if the one series argument applies to time it should not also apply to space.

I imagine that an opponent of these views might accept that these two arguments are valid, but deny that they prove anything. This would be an alternative reading of the objection that there is one series, namely an infinite one, to the "actual infinite" reading of it I dismissed above. On this view there is no difficulty with accepting the one series argument that there is always another member to any given series if we simply remind ourselves that that is what an infinite series is like. Perhaps this has the

consequence that the world cannot be considered "as a whole"
but then so much the worse for considering the world as a
whole. If considering the world as a whole is equal to the
demand that there be only one world, then that demand is
met by there being only one series.

Essentially this is a demand for an explanation and justi-
fication of the claim that wholeness is a necessary fea-
ture of the world. One possible attempt to meet this de-
mand is provided by Al-Azm, and so, while his remarks are
mainly addressed to the First Antinomy, it is more conve-
nient to discuss them here. The criticisms I shall make of
them are valid in either context.

The answer that Al-Azm provides to our question: "Why
is wholeness a necessary feature of the world?" is: because
Newton said so. His interpretation of wholeness is that
it is equal to determinate position. He discusses the case
of space. In particular he says:

> The claim I am referring to is Newton's
> inclusion of position in absolute space
> in the definition of a physical (material)
> entity. Accordingly, such an entity has
> to be finite or else it will fill all
> spaces i.e. it will have no determinate
> position relative to absolute space which
> amounts to a violation of its own defi-
> nition. (Al-Azm, p. 14)

However, even if we allow this Newtonian claim (including
this strange talk of an object violating a definition) it
does not rule out the possibility of an infinite world.
Consider the following model, for simplicity dealing with
only one dimension. At any point in the absolute space of
this one-dimensional universe we either encounter an object
or we do not. To map this universe we write down a "1"

when we encounter an object and an "O" when we do not. Thus we describe it by a binary sequence, part of which may look like this:

$$- - - - 1 \ 0 \ 1 \ 0 \ 1 - - - -$$

This satisfies the condition that every object have a determinate position in absolute space. However there is no guarantee from this sequence that it forms part of a finite rather than an infinite sequence. Thus the finitude of the world does not follow from this Newtonian definition of a material object.

However this may not be what Al-Azm intends, as he goes on to say:

> From the point of view of the thesis the physical entity called the universe requires, then, by definition, a determinate position in absolute space i.e. it cannot fill all spaces. It has to be finite. (Al-Azm, p. 14)

Here Al-Azm treats the universe itself as a material object in the sense in which its parts are material objects. But what reason have we for treating the universe as anything more than the sum of its parts? Occam's Razor speaks against it, as it is introducing a new object unnecessarily. If the universe is just the sum of its parts then its "position" is nothing more than the set of positions of its parts: and this is the same as the case considered above, and does not preclude an infinite universe.

At any rate the universe is a discontinuous entity in the sense that it consists of discrete parts with determinate positions (on the Newtonian view). Therefore it is *prima facie* implausible to suppose that it has a position

in the same sense as each discrete part does.

Further problems await Al-Azm. He discusses a claim
by Kemp Smith that if a whole is finite then space cannot
form an infinite whole despite claims by Kant (at B39 of
the Critique of Pure Reason) that it does. Al-Azm says that
there is no inconsistency here as space is an analytic to-
tality whereas the world in space is a synthetic totality.
It is not clear what Al-Azm intends by this difference of
nomenclature, but in any case it does not avoid an inverse
form of Kemp Smith's objection. For if space is an infi-
nite whole (of whatever kind) why may not the world in
space also be an infinite whole? In an infinite space
there are an infinite number of possible positions of ob-
jects. There are therefore several proper subsets of these
possible positions which are also infinite (just as the set
of even numbers is an infinite proper subset of the set of
natural numbers). Thus even if not all the points of an
infinite space are occupied it is still possible for there
to be an infinite world.

This is a problem not only for Al-Azm, but for anyone
who wishes to defend Kant's rejection of an infinite world
and so, *a fortiori*, for me. My response can be found in
Chapter Five.

Even though I have shown there to be serious faults in
Al-Azm's application of Newtonian science to Kantian meta-
physics in this instance, is there anything wrong with this
practice in general? Certainly, if one wishes to describe
the intellectual climate in which Kant worked or to describe
the psychological genesis of some of his views, this method

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may have value; but this should not be confused with des-
cribing the structure or validity of the finished arguments.
It is presumably confusion of this sort that leads Al-Azm
to claim:

> The kind of objection that one can bring
> against Kant in this context is to ques-
> tion the accuracy of his statement of
> the implications of the Newtonian con-
> cept of a *world*. (Al-Azm, p. 16)

This would be true if Kant were offering us a classroom
primer in Newtonian mechanics. In the Dissertation, how-
ever, he is offering arguments he straightforwardly believes
to be valid. Even if in the more complicated setting of the
Antinomy he requires the Antinomy to result from the single
assumption of Newtonian mechanics then it would be a very
dated and uninteresting piece of philosophy. Thus if Straw-
son suggests that no antinomy arises from the assumption of
transcendental reality because the arguments Kant puts for-
ward are bad ones, then it is beside the point for Al-Azm
to chide Strawson for "historically irrelevant criticisms".
(Al-Azm, p. 16) Strawson was not doing history, he was do-
ing philosophy -- and so was Kant.

Therefore Al-Azm's interpretation is ultimately unsa-
tisfying. However I think he is correct in introducing
the notion of "determinacy" as a central idea, although
wrong in his use of it. As we have just seen, he uses it
to mean a determinate position of an object or of the en-
tire universe. I think determinacy comes into the argument
in a different way, namely this: if the universe is infinite
then it is not determinate which possible objects are actual.

Firstly we have to consider what this means, and secondly whether it is true. Consider some event which we will name e_n. If this event actually occurs in the temporal extent of the universe then the statement "e_n occurs" is true; if not, then the statement is false. The claim that it is determinate which possible objects are actual is the claim that all statements of the form "e_n occurs" are either true or false. Thus this is a particular case of what can be called the "principle of bivalence" (c/ Dummett: Introduction). For reasons which will become clearer later in this thesis I shall call it *internal determinacy*. My major thesis of this entire chapter can now be simply stated: an infinite world is not internally determinate.

The question of whether this is true brings us back to the one series argument. The one series argument is an argument about the series of events in time. The statement "e_n occurs" means that e_n has a temporal position (and a spatial one as well of course, but we are only considering time at the moment). Thus the core issue here is the relationship between a temporal event (an event with a temporal position) and the temporal series. In fact, it is the unity of the temporal series which is paramount.

This needs justification. We can consider it this way. A temporal position is a position in the temporal series. That means that if we take some arbitrary member of the temporal series, call it e_o, then for any e_n, if e_n occurs then e_n must be temporally related to e_o. This is intuitively clear if we remember that any event that occurs must occur before or after e_o. (We are considering the tem-

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poral series as unidimensional and so ignoring questions of
simultaneity. The complications of this concept pointed
out by Einstein do not affect the major point at issue, sin-
ce there is still a temporal order in Special Relativity al-
though temporal distances are relative to the observer).
This point about temporal relatedness is brought out by
Kant later in the <u>Dissertation</u> in his general discussion of
time. There he says:

> Thus time is an absolutely primary, for-
> mal principle of the sensible world. For
> all things that are in any way sensibles
> can be apprehended only as at the same
> time or in successive times, and so as in-
> cluded and definitely related to each o-
> ther within the course of the one single
> time. Thus through this concept, primary
> in the domain of sense, there necessarily
> arises a formal whole which is not a part
> of any other i.e., the phenomenal world.
> (p. 59)

This passage means this: if an event occurs (is "in any
way sensible") then it must occur at a time, and this means
that it must be temporally related to all other events.
It is through this temporal relatedness that time causes
the world to be a "whole".

Temporal relatedness (excluding the possibility of cir-
cular time, which I discuss in Chapter Four) is a transi-
tive concept. If a occurs later than b and b occurs later
than c, then a occurs later than c. I propose that we now
think of the temporal series as a series of events e dis-
tinguished by numerical subscripts, so:

$$\dots\ e_{-3},\ e_{-2},\ e_{-1},\ e_{1},\ e_{2},\ e_{3}\ \dots$$

We can, if we like, regard e_1 as the present and e_n where
$n > 1$ as the future, and e_n where $n < 1$ as the past. No-

thing of importance depends on that: I have used both positive and negative integers solely to show that the series is open at both ends. I shall call this the *e-series*. Given the transitive nature of temporal relations, it follows that all members of the e-series must be related to all other members. The argument is then that the one series argument shows that if the universe is infinite then there is no one series to which all events belong (effectively there is no e-series). However if there is no one series to which all events belong then they are not all related to each other. Since they must all be related to each other, the universe cannot be infinite.

This is clearer if we consider the temporal relations themselves. We can represent these by a set Q of all the temporal relations. Q is a set, not a series, since the temporal relations are not themselves serial, only the events ordered by them are. If one were to argue the converse one would have to argue that temporal relations hold between temporal relations and so on *ad infinitum*. Thus if a is later than b, and b is later than c, then the temporal relation "b is later than c" is not later than the temporal relation "a is later than b", unless one simply takes this as a misleading way of expressing the transitive rule, namely that it then follows that a is later than c.

This means that Q cannot be a series, but can only be a set. As a set is defined by its members it cannot be "open": in other words it cannot be indeterminate which members belong to the set. Of course we can refer to the set without referring to all its members, or knowing what all its mem-

bers are. We can also describe the set in terms of some membership qualification, which appears to blur the distinction between a set and a series. For example, if H is the set of horses then the membership qualification could be: $a \in H$ *iff* "*a* is a horse" is true. However this is only another way of saying that H is the set of horses, and as it is more long-winded and involves the notoriously difficult concept of truth I do not see that this particular form of expression is to be recommended. (To cite just one difficulty, are there true statements if there are no humans to make them and no languages in which they could be expressed? Yet undoubtedly there could be horses without there being humans).

The fact that the set Q is defined by its members has the following consequence. Suppose the series of temporal events is infinite. Then there must be an infinite number of temporal relations (since each pair of events has a temporal relation): the set Q must also be infinite. However as Q is a set, and not an open series, then if it is infinite it must be an actual infinite. In other words, a possible infinite of temporal events requires an actual infinite of temporal relations.

However, the set of temporal relations cannot be an actual infinite for, as we have seen, the actual infinite is not a possible model of the temporal world. An actual infinity of temporal relations would require an actual infinity of temporal events standing in those relations (excluding circular time) and so two events would have an infinite number of events between them; the situation discussed be-

fore.

When discussing a set such as the set of all horses it is common to think of it as an atemporal set, as the set of all horses past, present and future. One of the reasons we must be able to refer to the set without naming all its members is that we are necessarily ignorant of future horses. It is worth noting here the distinction between this case and that of the set Q. There can be future horses because a horse is a temporal *object*. There cannot however be future temporal relations since to be future is a temporal relation between an event occurring now and an event occurring in the future, or at any rate presupposes a temporal relation even if it does not amount to one[3], and, as noted above, a temporal relation cannot stand in a temporal relation. Thus we could not defend the idea of an infinite temporal series by saying that it requires that the set Q be "open" only in the sense that some of its members exist in the future (as the set of horses is "open" in this sense); since the members of the set Q are not the sorts of things to which the phrase "exists in the future" can be applied.

The only manoeuver left that I can imagine would be to

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[3] Of course there are great philosophical difficulties with the notions of past, present and future (McTaggart's paradox, for example) but as these do not affect the arguments of the First Antinomy to an extent that would justify the lengthy discussion that they would have to receive I am forced to ignore them altogether. The point I am seeking to make here is that even if the distinction between the "presentness" and "futureness" of two events does not *amount to* their standing in some temporal relation (such as "four hours later than") then it at least *presupposes* some such relation, and that is all that my argument requires.

deny the temporal unity thesis: to claim that it is possible for there to be temporally unrelated events. Perhaps the possibility considered by Bell is like this (Bell, p. 165). I shall deal with this possibility in Chapter Five.

It may help to recap this argument by showing its relation to the principle of internal determinacy. This principle states that any statement of the form "e_x occurs" must be either true or false. If we adopt some arbitrary standard event e_0 then the principle of internal determinacy requires that either e_x is temporally related to e_0 or it is not, for all e_x. This is a feature of temporal unity. It requires that the set of temporal relations be complete which, if it is to be infinite, requires that it be an actual infinite: which is absurd. Therefore it, and so the temporal world, cannot be infinite.

Perhaps any lingering uncertainties about this argument, or unclarities as to its meaning, can be dispelled by showing its application to the world in space. As I said when setting out the infinite sum argument the major assumption is that the spatial and the temporal worlds have a shared form. This can be brought out as follows. We shall have to avoid complicating issues of simultaneity (as I believe matters are complex enough already) and imagine that we can speak of the spatial distribution of objects at a fixed time (rather as we treat a variable as a constant in a partial differentiation). I repeat that this is merely an argumentative convenience: dealing with, say, Minkowski space-time would be much more complicated, and though it would no doubt be very rewarding, it would lead us too far from our central

purposes. I will also, to simplify the argument, consider
space as one-dimensional: a move to more than one dimension
would involve only a greater complexity in the subscripts
I shall be using. Suppose then we begin at some arbitrary
event in this one-dimensional space and label it e_1. We
can then label the event to the right of it e_2, the event
to the left of it, e_{-1}, the event to the right of e_2, e_3;
and so one. We then have the same series as that we noted
for time. This is the basic similarity of space and time:
each of them involves a distribution of events (I use the
neutral word "involves" advisedly: the precise nature of
this involvement will prove highly contentious).

There is however an important difference. In the case
of time the serial nature of the e-series is a feature of
the nature of time. In the case of space it is a feature
only of our method of constructing the series. It is a pri-
mitive feature of the spatial world that its parts co-exist.
Thus we cannot speak of the infinity of the spatial world
in terms of the possibility of there always being another
place, as all places co-exist together.

The effect of this is that I do not need to appeal to
a parallel set Q' for spatial relations to the set Q of
temporal relations. For spatial events themselves form a
closed set rather than an open series, through the basic
nature of space.

Perhaps this is more intuitively obvious if we think
in terms of possibility. An infinite series involves a re-
ference to possibility: the possibility of there always be-
ing another member of the series. That is why it has seem-

ed plausible to think of the temporal world as infinite because as it forms a series it always is possible for something to happen in the future. In space however there are no such possibilia: everything in space is actual.

This mention of "possibility" is at the moment an appeal to intuition (in the everyday, not the Kantian, sense) in an attempt to make the argument understood. I shall discuss it in more detail later in this chapter.

This then means that if the spatial world were to be infinite then it must be an "actual infinite". This however is not a possibility. Although there is no logical necessity for us to have gone through all the spatial regions of the universe to arrive here, in the way in which there is a logical necessity for us to have gone through all the past stages of the universe to reach the present, it would still mean that there are some two spatial objects such that it is conceptually impossible to leave one and arrive at the other, since there would be an infinite number of spatial objects between them. This is absurd whether one intends to make the journey or not. More formally, it would violate the requirement of spatial unity which I shall be defending in Chapter Five.[4]

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[4] An actual infinite spatial world cannot consist of a finite number of objects of finite size at finite distances from each other. Ignoring size, if there is such an actual infinite number of objects then there must be two objects such that there is an infinite number of objects between them, in which case it would be impossible, hopping, as it were, from object to object, to travel from one of these objects to the other. The other possibility is of a finite number of objects separated by infinite distances. But how are we to comprehend the notion of an infinite distance? Presumably it means two objects separated by such a distance are spatially isolated from each other: but then they could not both belong to the same space.

One possible objection that could be made at this point is that we have no difficulty with the notion of infinity in mathematics, so why should we have difficulty with it in regard to the world? I believe that the first part of this claim would find a good deal of agreement among the general educated public although of course many people whose business is the foundations of mathematics would not agree with it, most notably those with a Kantian or neo-Kantian background (Kaufmann, for instance, whose immediate intellectual inspiration was Husserl). However I am prepared to grant it for the sake of explaining why the same does not hold true of the spatio-temporal world.

At the same time I want to explain Kant's final remarks on these arguments in the <u>Dissertation</u> where he says:

> (E)ven if such totalities are not conceivable in sensitive terms, they do not thereby cease to be intellectual concepts. For these it suffices that co-ordinate things be in some way given, and be thought as all belonging to one (whole). (p. 43)

In this I take Kant to be saying that the problems we have been discussing for the spatio-temporal world do not apply to the intelligible world.

I intend to treat these two issues together by using an axiomatic logicist view of the natural numbers (one on which I take there to be no problem with infinity) as a model of Kant's notion of the intelligible world. I shall offer some textual grounds for the appropriateness of the model. I can then show why Kant thought the problem existed for the sensible but not the intelligible world, at the same time as showing why the assumedly unproblematic nature of the in-

finite in pure mathematics does not extend to the spatio-
temporal world.

Please notice that I am not committed to holding that
such a view of mathematics is true, though I believe it is
widely held. Nor do I want to suggest that it is Kant's
view, or that he thought that numbers belonged to the intel-
ligible world. This is of course not the case. Kant's
view of number is given in a section of the Critique of Pure
Reason called the Schematism where he says:

> Number is therefore simply the unity of
> the synthesis of the manifold of a homo-
> geneous intuition in general, a unity
> due to my generating time itself in the
> apprehension of the intuition. (B182)

Thus for Kant numbers are connected with time and so with
the sensible rather than the intelligible world. However,
the point of my model is not to give a Kantian account of
number, but to use an axiomatic logicist view of number as
a model of the Kantian notion of an intelligible world.

On such a view we can specify, say, the natural numbers
with a few axioms, e.g.:

(1) 0 is a natural number;
(2) The successor of a natural number is
    a natural number;
(3) Every natural number has a successor;
(4) 0 is not the successor of any natural
    number;
(5) No two natural numbers have the same
    successor.

It is intuitively obvious that any series which satis-
fies these axioms must be infinite. This satisfies the re-
marks of Kant's quoted above. The same way in which the
co-ordinate things are given is via the axioms, and they

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are thought of as all belonging to one whole as the series satisfies the one set of axioms. Similarly the parts of the intelligible world are connected by their all being effects of the same cause: "We can conclude from a given world to a sole cause of all its parts." (p. 69: here "world" means "intelligible world"). Kant cannot of course mean "cause" here in a spatio-temporal sense. In fact he means "cause" in the more general sense of dependence, in this case the dependence of a contingent being on a necessary being. That is why Kant goes on to argue:

> If we could similarly argue, vice versa,
> from a given cause common to all things
> to their interconnection, and so to the
> form of a world ... then owing to the
> substances all being maintained by a com-
> mon principle the fundamental connection
> of the substances would not be contin-
> gent but necessary. (p. 69)

Similarly in our model if the axioms are logical truths from which the numbers series can be deduced then the truths of arithmetic would be logical truths. Indeed, this was the main aim of the logicist movement. I mention this analogy to show the appropriateness of the chosen model.

It should be obvious how this case is different from that of the spatio-temporal world. The number series follows from the axioms and the parts of the intelligible world follow from their cause, but there is nothing equivalent in the spatio-temporal world. The distribution of objects in space and time does not follow from space and time in an analogous manner, if this idea can even be given any sense. The relationship between the natural numbers and the axioms, on the view considered here, or the

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intelligible world and its cause, is a conceptual relation (which is what one would expect from an intelligible world). But the relationship between the various parts of the spatio-temporal world is not conceptual. The axioms specify the number series while it is not possible to specify the spatio-temporal world. That is why one can learn mathematics without leaving one's room, while to find out about the world (at first hand) one has to go out and look at it.

Of course the view of mathematics I have been using is beset by severe theoretical difficulties, most notably Gödel's Theorem. But then on any view of mathematics which regards the natural numbers as an extension existing independently of some set of axioms it is not clear that regarding the numbers as infinite is unproblematic. However it is perhaps possible to describe the difference between numbers and spatio-temporal events without going into the theoretical reason for this difference. This is not as satisfactory as an illustration of Kant's distinction between the sensible and intelligible worlds, but it should fulfill the important function of showing why statements about the infinity of the natural numbers do not serve as counter-examples to statements about the infinity of the spatio-temporal world. The differences, which I do not seek here to explain, but merely note, are firstly that it is immediately obvious whether some number, even one that no one has considered before, is a member of the number series, yet if some event is described then it cannot be ascertained whether that event is a member of the spatio-temporal world without some form of empirical search (assuming the description of the event is not self-contradictory). Secondly

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it is possible to offer proofs on the number series yet there is nothing analogous in the case of the spatio-temporal world. In the spatio-temporal world I can *predict* but I cannot *prove*. Because of these disanalogies it would not be correct to say that because we are comfortable with the notion of an infinite series of numbers, if indeed we are, then there should be no difficulty with the idea of an infinite world.

We should find these considerations helpful as we now consider the Antinomy itself.

The First Antinomy

I have already noted the difference in intent between the Inaugural Dissertation and the First Antinomy. In the Inaugural Dissertation arguments are put forward which are claimed to be straightforwardly valid; in the Antinomy arguments are put forward which claim to show that an inevitable antinomy follows from the mistaken assumption of the transcendental reality of the spatio-temporal world. We would expect this to have two consequences for the form of argument offered in the Antinomy as opposed to that in the Dissertation. Firstly we would expect the Antinomy arguments to have fewer assumptions since they are not straightforwardly part of the general development of a theoretical position (it is the Antinomy as a whole that has a place in Kant's general scheme of things, not its component ar-

guments) and so additional assumptions would be merely additional hostages to fortune. This is reflected most clearly in the fact that the arguments use the *reductio* form. Secondly we would expect the assumption of reality to function clearly in the arguments. Perhaps clarity is too much to hope for from Kant; nevertheless I do hope to show how that assumption operates in these arguments.

I shall begin with the argument considering the temporal world. It can be set out thus:

(1) Assume the world has no beginning in time.
(2) Then up to any moment there has been an infinite series of successive states of things.
(3) The infinity of a series consists in the fact that it can never be completed by successive synthesis.
(4) Therefore an infinite world series cannot have passed away.
(5) Therefore the world must have a beginning.

Strawson is not impressed by this argument. He writes:

> We can indeed validly argue that as the series ... has a final member it cannot be both: (*a*) that it has an infinite number of members and (*b*) that it has a first member .... But since the argument is supposed to be based on the hypothesis that (*b*) is false, it is clearly (*b*) that must be rejected and not (*a*). (Strawson, p. 176)

This objection is too brief, for it ignores the possible objection that with an infinite past there must be some proper subset of the series of past events which has two termini (one being the present) and an infinite number of members, whether or not this other terminus is the first member of the series. In other words, to maintain his objec-

tion to Kant's argument Strawson must make the claim that not only is the past infinite, but also that all such sub-series are finite. This is not as obvious as the brevity of Strawson's objection suggests.

Nevertheless such a position is explicitly proposed and defended in the much more thorough review of this sort of argument by Bell (Bell's paper is not explicitly concerned with Kant). Bell says,

> (L)et us observe that such a (temporally ordered) sequence of past events would have the order type ω^* of the set of negative integers with its natural ordering. (Bell, p. 163)

Bell then considers the assertion that such a series of past events could never arrive at the present. His conclusion is:

> (T)he assertion is evidently *false* when the sequence has order type ω^*. For then each event in the sequence will be of the form $\mathcal{E} - n$, where n is a natural number, and starting from there we will duly arrive at the present in precisely n steps. (Bell, p. 164)

This then is the model we are being offered. The past is of infinite length but the subsequence of the past between any given past event and the present is finite. It is against this model that we should consider Kant's argument as directed, since it is the obvious model of an infinite past.

Firstly, I am not sure what Bell means when he speaks of the negative numbers "in its natural ordering". What is the force of the word "natural"? Presumably he intends us to think of them as having "-1" as the last member.

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This is surely mistaken, for -1 is not the last member, but the first, in terms of which the others are defined. Thus, just as 2 = 1 + 1, and so on, -2 = -1 + -1, and so on. After all, a small bank overdraft is not an infinite amount of negative money.

William Lane Craig, in a contribution to the same debate as Bell's paper, argues that if the past is to be infinite then it must be an "actual infinite": there must be an event an infinite distance away in the past from the present. The reason he gives is this: "To be a potential infinite, the series of past events would have to be finite, but always growing backwards." (Craig, p. 168) This of course is absurd, as the direction of the past series is forwards, towards the present, not backwards into the past. By a "potential infinite" I take Craig to mean an infinite series whose sub-series defined by two terminal events are all finite.

I think Craig is near the truth here, but at best he has not expressed himself well. An opponent could reply as follows: Craig is confusing the conceptual fact that in an infinite past there is always an earlier event to any event you care to take, which is guaranteed by there being no begining with the irrelevant temporal fact that new events can only occur in the future. In this he is misled by his metaphor of "growing" which suggests a temporal process, whereas the "growing" of the series into the past does not designate a temporal process but merely re-expresses the fact that there is no first term.

I think that this objection would be correct. I sus-

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pect moreover that Craig has taken Kant's term "synthesis", as used in this argument to mean a temporal process of successive addition of event to event. However as the above objection to Craig suggests, while this notion of "synthesis" is crucial to Kant's argument, it must not be taken in a temporal but a conceptual sense.

So I should now give my view of the argument. It rests on considering the relationship between these finite subsequences of the past and the infinitude of the past. How do we get from the one to the other? To use the Kantian word, how do we synthesise an infinite past out of its finite subsequences? I must repeat that I regard this as a conceptual and not a temporal question. The answer is not that the addition of a sufficient number of the finite subsequences would give us an infinite sequence: "infinity" is not a very large finite number (as Kant points out in the Observations). What more is required is freedom in the addition of the parts, the possibility of there always being another member of the series.

However this possibility is a conceptual possibility and cannot be expressed as a temporal one. This is clear if we think of the past as divided completely into discrete temporal sub-sequences. There is no temporal reason for not thinking of the past this way since, after all, everything in the past has, *ex hypothesi*, happened. Yet even if we divide the past into these discrete temporal sub-sequences then by adding them together we cannot arrive at an infinite past, since for the past to be infinite it would require that it is always possible to add another subse-

quence to the construction. Thus we need, to move from
the finite subsequences to the infinite past, not only the
sub-sequences and their addition but also the possibility
of sub-sequences as an irreducible factor of the synthesis
(by "irreducible" I mean that we can never at any point re-
place the possibility of adding further subsequences with
the actual adding of further subsequences since the actual
adding of further subsequences will always only produce a
finite sequence).

However this possibility of adding further subsequen-
ces conflicts with the *reality* of the past. Since the
past is over and done with there are no possibilities in
it -- everything in it is actual. To speak metaphorically,
the past must represent one picture of events. If it is
infinite it cannot do so.

Of course I am not suggesting that the possible adding
of actual sub-sequences, which is what is required for in-
finity, is the same as the actual adding of possible sub-
sequences. I am not claiming that the infinitist must
hold that the past contains possible events. What I am
claiming that he must hold is that·the past must contain
the possibility of (actual) events in a way that cannot be
reduced to the actuality of (actual) events. In the past
there is no possibility of events.

Thus Kant's argument ought not to be interpreted as the
position that an infinite past is inconsistent with our e-
ver arriving at the present, as it so often is. Rather,
as one would expect if one remembers what the mistaken as-
sumption of the Antinomy is meant to be, it is that an in-

finite past is inconsistent with the reality of the past, as expressed by the fact that there are no possibilities of events in the past.

It will be remembered that I defined the principle of internal determinacy as the requirement that it is determinate which possible events are actual. It is clear that it is this principle that expresses the reality of the past in the sense relevant to this argument, and which the infinite past violates. Its alternative formulation is that for any particular event description a then $(\exists x)(x = a)$ is either true or false. I shall now support my interpretation of Kant, and its validity as an argument, by showing that an infinite past conflicts with this formulation of the requirement.

This is quite simple. Of course, it is always possible for $(\exists x)(x = a)$ to be true. That would be the case if a referred to the battle of Waterloo for example, as that event is indeed part of the past of the world. However it is possible for $(\exists x)(x = a)$ to not be true and yet not be false, if the past is infinite. For in that case it is always possible for there to be another event in which case it is always possible, *a fortiori*, for that event to be a.

In other words, since in an infinite series there is always another member it is always possible that that member should be a. Thus $(\exists x)(x = a)$ is never false, while it is not the case that it is always true (since there is no requirement that a must occur). So in an infinite world the principle of internal determinacy as expressed by the claim that $(\exists x)(x = a)$ must be true or false for any a is

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violated.[5]

Notice that this is an ontological, not an epistemological, argument. It follows from a fact about infinite series, not a fact about our knowledge of infinite series. "Knowledge" is nowhere mentioned.[6] Moreover, it is important to distinguish truth from meaning. We can say what $-(\exists x)(x = a)$ *means* in an infinite world. It *means* that while there is always another event, that event is never a. We can specify this because meaning is intentional. Truth about events in the world is not and so we can never specify the *truth* of $-(\exists x)(x = a)$ in a similar fashion. To put

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[5] Of course we can assert that $(\exists x)(x = a)$ is false for an infinite domain if a is an inappropriate object to be a member of that domain. Thus if the domain is the natural numbers and a is the name of a horse then we can assert $\sim(\exists x)(x = a)$ since horses are not numbers. However if a is the name of a spatio-temporal event and the domain is the spatio-temporal world, then if the domain is infinite $(\exists x)(x = a)$ is never false. For it to be false it would have to be the case that for all events none of them is a, and there is no "all events" since the infinity of the domain consists in the fact that however many events there are, there are always more.

[6] The relation between epistemology and ontology in this case can perhaps be brought out in this way. Suppose that there is a book with an infinite number of pages. Consider the statement, "The letter y occurs within this book". It is easy to say what it is for this to be true: on a certain page the letter y occurs. It is also easy to say what it is for us know that this is true: we turn to that page and see that the letter y is there.
However let us consider the possibility of the falsehood of the statement. It is clear that we could not *know* that the statement is false. As there are an infinite number of pages there must always be another page for us to turn, and it may be that y is written on that page. However here the ontological case has an essential similarity to the epistemological one. For what does it mean to say that there is an infinite number of pages? It means that there are more pages than any given number of pages. The fact that there always are more pages is an ontological analogue of the epistemological case where we always have more pages to turn. So just as the fact that there is always another page to look at however many pages we have already looked at

it another way, we can say under what conditions $(\exists x)(x = a)$ is false -- the totality of events does not include a -- but if the world is infinite then those conditions can never be satisfied as there cannot be a totality of events.

I claim therefore to have shown why Kant believed that an infinite past is inconsistent with the assumption that the past is real.

Kant's argument in the case of space can be set out thus:

> (1) Suppose the world is an infinite given whole of co-existing things.
> (2) The magnitude of a quantum not given in the intuition can be thought only through the synthesis of its parts.
> (3) The successive synthesis of the parts of an infinite world would require an infinite time.
> (4) An infinite time cannot have elapsed.
> (5) Therefore the world cannot be infinite in space.

Strawson is no more impressed by this than he was in the case of the temporal world. He interprets it thus:

> Kant maintains that the very thought of the infinite spatial extent of the world involves the thought of its being possible to *complete* a temporally infinite process of spatial surveying of the world .... This, says Kant, is impossible. (Strawson, p. 177)

To this Strawson offers three objections. Firstly, he argues that the supposition that the universe is infinite in

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means that we could never know that it is false, so the fact that there always are more pages than any given number of pages means that it can never be false.

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space cannot be equivalent to the supposition that an infi-
nitely long process of surveying the universe has been com-
pleted, as the second implies that the universe has existed
forever and the first does not. Secondly, he argues that
an infinite process of surveying could be completed as long
as it did not begin. However, these really only set the
stage for the third criticism, which is: "(W)ith what re-
levance and by what right (is) the notion of surveying ...
introduced into the discussion at all." (Strawson, p. 177)

Thus essentially Strawson is setting before us a mani-
festly invalid argument as his interpretation of Kant's
text. An adequate response would be to show that this is
a misinterpretation, and this is the line I shall follow.
Strawson's version can be set out thus:

> (1) Suppose the world is infinite in space.
> (2) Then an infinite task of surveying
>   the world must have been completed.
> (3) An infinite task cannot be complet-
>   ed.
> (4) Therefore, the world is not infi-
>   nite in space.

The attraction of this interpretation is that it ac-
counts, quite cleverly, for the intrusion of time into the
argument. Surveying is a spatial concept that involves tem-
poral duration and thus supplies the link between space and
time that the text demands. However I think Strawson not
only mis-interprets the argument, but also fails to appre-
ciate its intention. It is not a theoretical piece of Kan-
tian philosophy, but a *reductio ad absurdum*. All Kant has
to do is show that the supposition that the universe is in-
finite in space leads to absurd consequence. So even Straw-

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son's interpretation would do if the impossibility of completing a survey of the world were an absurd enough consequence. Strawson does not argue that it is not.[7]

However it must be conceded that this argument of Kant's is the least well-expressed of the four arguments that make up the Antinomy. Not only is it extremely brief and obscure, it is also written in the vocabulary of transcendental idealism: "synthesis" and "intuition", for example. Given this it is not surprising that Strawson should be misled in his interpretation. I propose to deal with the argument as follows. Firstly I shall explain the first premise. I shall then translate premise (2) out of the transcendental idealist vocabulary in which it is written. As I maintain that Kant's use of this vocabulary is unfortunate rather than deliberate this translation should reveal the true sense of the premise. I shall then construct an argument against an infinite spatial world based on this premise. Finally I shall seek to show the relationship between this argument and the form of argument Kant actually offers, thus in this rather roundabout way establishing an interpretation of Kant's text.

The first premise is fairly simple. The phrase "a given whole of co-existing things" means only a collection of things related to each other contemporaneously, i.e., distributed in space. As spatial relatedness is transitive

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  Of course absurdity requires more than mere implausibility, and it may well be that the impossibility of completing a survey is not an absurd enough consequence. My main point here is that Strawson's rejection of Kant's argument is too brief and dismissive.

all these objects in space must be related to each other. For example, if a has some spatial relation to b and b has some relation to c, then a must have some spatial relation to c. The relation between a and c may not be the *same* relation as that between a and b or b and c. For example, if $a = c$ then a could be some distance from b and b could be some distance from c, yet a would not be any distance from c. Yet a would still have a spatial relation with c, namely it would be coincident with it. Thus all the spatially distributed objects must be related to each other. Hence the phrase "a given whole". We are asked to consider the possibility that this given whole be infinite.

Next we must consider premise (2). It is generally possible to translate remarks of Kant's out of their transcendental idealist idiom by substituting "space" and/or "time" for "intuition". In this case we have to deal with the phrase "given in intuition". The idea of a magnitude's being "given in intuition" is that it can be seen whole at a glance (subject to the normal constraints of human vision. I can see a pen whole at a glance but I cannot see the whole pen, since one side of it is hidden from me. The pen does not, however, extend beyond my visual field). This is possible for any finite object provided that we are far enough away from it (on a clear night in Edinburgh I cannot take in the whole city at a glance, because I am too near to it, but I can take in the Andromeda nebula at a glance). So if we express this same idea in a non-idealist fashion then the equivalent notion is "is spatially finite": in other words, an object whose magnitude can be given at a

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glance is an object which does not fill all space, is finite, for then there is some spatial position from which its magnitude can be given at a glance. Translating "magnitude" into the more usual word "size" and "quantum" into "object" the first half of premise (2) now reads: "The size of an object which is not finite...."

By the phrase "can be thought" Kant does not mean a particular human act of imagination or any other actual mental process. He means rather "can be considered" or "can be understood". Finally there is the phrase "through the synthesis of its parts". "Through" should not be taken here as indicating a process: rather it means "in terms of". "Synthesis" I take to mean "addition". So the phrase as a whole reads "in terms of the addition of its parts".

The natural development of this argument, which I shall now offer, is this. How is it that we understand the infinitude of an object in terms of the addition of its parts? It is not that the infinite object is formed by our adding together its parts. Rather it is that the infinitude of the object lies in the impossibility (*logical* impossibility) of such an addition's being completed.

In other words, what is being offered here is a point about infinity. What is an infinite size? It is not a very large finite size. Rather, if we think in terms of the parts of an infinitely large object then its infinity lies in the fact that its parts stretch out without end. To put it another way, if we think of the addition of its parts, then that addition is never-ending. This does not mean that the object must at some time have been *formed* from the

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coming together of its constituent parts, however it must *consist* of them.

So we can recap the argument so far. We are to consider the possibility of an infinite spatial world. To consider the infinity of the spatial world is to think of the adding together of its parts as unending. However now comes the dénouement. If we refer back to premise (1) we see that we are discussing the universe as a "given whole". All its parts co-exist: the world in space is complete. Therefore this logically impossible infinite addition actually exists in the world. As the whole is the sum of its parts, so the infinite spatial universe must be the sum of its parts: but its infinity means that the summing of its parts could never be completed. This is a contradiction: therefore the world cannot be infinite.

How does this argument relate to the development of the argument as given by Kant? I believe that the connection is this. It is intelligible to suppose that the adding together of the parts of an infinite object could be completed, and therefore it is intelligible to suppose that an infinite number of parts could co-exist in space, if an infinite time had elapsed during the addition. Thus the *intelligibility* of an infinite spatial world could be guaranteed by the intelligibility of an infinite process of addition. However that would require the intelligibility of an infinite time's beginning and ending -- and that is not intelligible.

In fact, I think it is highly misleading for Kant to introduce time into the argument. Strawson has clearly

been misled. By doing so Kant appears to suggest
that the impossibility of an infinite spatial world lies in
the impossibility of some actual temporal process of *form-
ing* such a world, to which the appropriate response is that
it could always have been infinite. Instead the impossi-
bility of an infinite spatial world lies in the conflict-
ing demands of its infinitude -- that it have an infinite
number of parts -- and its spatiality -- that all its parts
co-exist.

We can demonstrate this argument in another way, using
diction unavailable to Kant. The claim that the spatial
world is infinite is the claim that the objects in the uni-
verse have a one-to-one correspondence with the natural
numbers. Therefore let us suppose that this correspondence
has been carried out and that each object is uniquely num-
bered. Then the claim that the world in space is an infi-
nite given whole is modelled by the claim that all the na-
tural numbers can be (logical "can") written down together.
This second claim is surely absurd since the infinity of
the natural numbers lies in there always being another one.
Therefore, says Kant, the first is impossible also. He
does *not* claim that the infinitude of the world is equiva-
lent to our having counted an infinite number of stars.

Strawson suggests that Kant in these two arguments is
"reaching out to the doctrines which he is later to advan-
ce as a solution to the conflict he is ostensibly display-
ing." (Strawson, p. 176) (In fact he says this explicit-
ly only about the first argument, but as he thinks the se-
cond incorporates the mistakes of the first we can take

him to believe it of both). This can only imply that Strawson does not understand those solutions either. The assumption which those solutions deny is the transcendental reality of the world, and with it the demand that the past be over and done with and that the spatial world be given as a whole. Far from these arguments requiring such a view, on such a view these arguments are not possible. As I have shown, it is the assumption of the transcendental reality of the world that makes these arguments work, as manifested in the case of the temporal world by the requirement that the past should not contain possibilities, and in the case of the spatial world that it be regarded as a given whole. This will be clearer still, when, in Chapter Three, I discuss Kant's solution of the Antinomy.

In the meantime, I have established the first leg of the Antinomy: the spatio-temporal world, if it is real, is not infinite.

A N T I T H E S I S

The arguments Kant puts forward in the Antithesis are,
if anything, even briefer and are certainly much more ob-
scure than those of the Thesis. Accordingly any discussion
of the Antithesis, even one, like this, more concerned with
producing important arguments than with the fine details
of Kant scholarship, must devote a good deal of time to mat-
ters of interpretation. I therefore treat, as separate to-
pics in their own right, two interpretations of Kant's text
which are at variance with my own. In each case I discuss
the interpretations in terms of the temporal argument, as
that is the first in the text. I then offer my own inter-
pretation of the text and seek to show that it produces a
valid argument (subject, as ever, to the assumption of
transcendental realism). I then extend the same line of
interpretation to the spatial argument, and I seek to show
that it yields a valid argument there also.

It is a commonly held principle, not only in philosophy,
to give the dead the benefit of the doubt, since they have
no power of reply. Accordingly my criteria for acceptili-
ty of an interpretation should be not only that it be con-
sistent with the text but that it should also be valid.
Only if the most assiduous search for such an interpreta-
tion had failed would I be prepared to attribute to Kant
an argument that was not valid, or to put forward a valid

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argument that was not Kant's.

The two interpretations I shall consider first are the *First Analogy Interpretation* and the *Leibnizian Interpretation*. These interpretations capture the main views of most commentators on Kant, but without, as they are presented here, either corresponding exactly to a particular commentator's perception of the argument. For that reason I shall not associate either interpretation with a particular commentator's name.

The first task is to set out the argument we are considering. That can be done thus:

> (1) Assume the world has a beginning.
> (2) Then there was a time before the world: an empty time.
> (3) Nothing can begin in empty time because no part of empty time possesses, as compared with any other, a distinguishing condition of existence rather than non-existence.
> (4) Therefore the world cannot have a beginning.

The crucial premise here is obviously premise (3), and it is the meaning of this premise that any interpretation must endeavour to provide.

The First Analogy Interpretation

At the end of the presentation of the above argument, Kant remarks: "In the world many series of things can, indeed, begin; but the world itself cannot have a beginning."

(B455) To borrow a phrase from Ryle, it looks here as if Kant is trying to point out a category mistake in our use of the term "beginning". To express it prepositionally, a word that can only be used *in* the world is being wrongly used *of* the world.

We may amplify this suggestion thus: a beginning implies a change from not-being to being. This change presupposes a contrast with something that is unchanging. Within the world such a contrast is always possible but of the world as a whole it is not, as there is nothing else in terms of which the change from not-being to being is intelligible.

This notion of change requiring permanence can be found, used for a different purpose, in the Refutation of Idealism ("All determination of time presupposes something *permanent* in perception." (B275)), but for our purposes the most promising text is that of the First Analogy. In that context he says:

> These transitions can be empirically
> known only as changing determinations
> of that which is permanent. If we as-
> sume that something absolutely begins
> to be, we must have a time at which it
> is not. But to what are we to attach
> this point, if not to that which alrea-
> dy exists? For a preceding empty time
> is not an object of perception. (B231)

The main thrust of the argument appears to be this. For us to notice change we must notice the contrast bet-ween what is changing and something that is not changing. To notice the sun's movement, for example, we must see it change its position with respect to the fixed landscape. Kant says that moreover if there is to *be* change then it must be change with respect to something else. In the case of the beginning of the world this change can only be with

respect to empty time. However it is a dominant theme of the _Critique_ that nothing can happen that is not a possible experience. Empty time is not a possible experience. Therefore the world cannot begin.[1]

My position is this. Kant clearly did hold such an argument to be valid. It is not a very strong argument. It is unacceptable as an argument in the First Antinomy. It is clearly not -- and is stated by Kant not to be -- the argument of the Antithesis of the First Antinomy.

Firstly let me say why I do not find it a strong argument. This is due to the dubiousness of the premise that change must always be with respect to something. Of course if something is to change then it must be different from what it was before, but that is not what Kant means. He means that there must be something other which endures throughout the change. This seems plausible if we think of motion as a paradigm of change, but is less so if we think of other cases. Consider changes of colour for example. Suppose I observe a ball that changes colour from red to purple. It may be said that this change is with respect to my unchanging mental state (though according to the Refuta-

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[1]
  It may be argued that something can change with respect to something that is unchanging _relative to it_ though not necessarily itself permanent. Kant would presumably want to argue that that would only lead to an infinite regress unless there was something that was absolutely unchanging. Perhaps a regress could be avoided by assuming a pattern of objects interlocking in such a way that at any time something was changing there was something unchanging relative to it: I do not know if that would be possible or not. In any case Kant's argument here would appear unaffected as it would still require that empty time serve the rôle of the relatively unchanging which would, for Kant, still require it to be a possible object of experience.

tation of Idealism that is ultimately unsatisfactory) but the change in colour is not necessarily dependent on my observing it. Why should not the only existent in the universe be a patch of red colour that changes to a patch of purple colour? We may have doubts about this possibility because of the nature of colour; surely not because of the nature of change.

Another example can be drawn from sound. Suppose an otherwise sensationless and thoughtless consciousness can contain a note of constant pitch. Why should it be impossible, from the nature of change, not of sound, for that note to change pitch? Indeed as it has some pitch at different times why should the fact that it has the same pitch be unproblematic and the fact that it has a different pitch be impossible?

For these reasons I am highly dubious of the premise that all change must be change with respect to something else.

However that is not the reason that this argument would be unacceptable as an argument in the First Antinomy. That is because of the premise that anything that happens in the world must be a possible experience. This expresses a central tenet of transcendental idealism. That means that this argument presupposes transcendental idealism. However it is not allowed to do so if it is to be an argument in the First Antinomy, for the Antinomy is supposed to occur inexorably on the assumption of transcendental realism with transcendental idealism required for its resolution.

However I suggest that Kant was well aware of this.

The argument that a finite world is impossible because empty time and empty space are not possible experiences is found in a passage describing the solution of the Antinomy, once Kant has already appealed to transcendental idealism. He argues thus:

> (T)he world would be limited on the one hand by empty time, and on the other by empty space. Since, however, as appearance, it cannot in itself be limited in either manner -- appearance not being a thing in itself -- these limits of the world would have to be given in a possible experience, that is to say, we should require to have a perception of limitation by absolutely empty time or space. But such an experience, as completely empty of content, is impossible. (B549)

Kant then adds a footnote to make it clear that this argument is quite different from that of the Antithesis, which proceeds on the assumption of transcendental realism:

> It may be noted that this proof is presented in a very different manner from the dogmatic proof of the Antithesis of the First Antinomy. In that argument we regarded the sensible world, in accordance with the common and dogmatic view, as a thing given in itself, in its totality, prior to any regress; and we asserted that unless it occupies all time and all places, it cannot have any determinate position whatsoever in them. (B549n)

I shall have cause to return to that footnote later. In the meantime it is clear from these passages that Kant did support an argument based on transcendental idealism at least similar to the one I have outlined above, but that that is not the argument he is offering here. It goes without saying that because of the complex logical relation between the arguments of the Antinomy and the rest of the Cri-

<u>tique</u>, one should be very careful about using material from elsewhere in the work to interpret those arguments.

Finally, it might be worth noticing that a weaker version of this interpretation is offered by Bennett. On this view the problem is not that the universe cannot begin because that would involve a change with respect to empty time, but that it cannot begin because its beginning would have to be dated with respect to empty time. Its beginning could not be dated in relation to other events except internally. We could not say when it began. Bennett is not happy with his own interpretation:

> The inference is invalid, though, for
> it merely assumes without reason that
> if the world began then it must have
> done so at a "determinate time" in the
> non-internal sense. (Bennett, p. 160)

This is correct; it is only to be regretted that the weakness of this argument did not lead Bennett to question whether it may not be a misinterpretation. Although, as we shall see, Bennett is right to locate the heart of the argument in the word "determinate"; even though he misses its significance.

The Leibnizian Interpretation

The First Analogy interpretation was relatively easy to treat. It was possible to delimit the argument and to assign it to its proper place in the <u>Critique</u>, while showing that it was not a correct interpretation of the Anti-

nomy. The case of the Leibnizian interpretation is, unfortunately, more complicated. It is not as easy to discern, in Leibniz's text, the original argument that it is claimed will serve as the interpretation. Secondly, it is impossible that Leibniz's argument should serve as an interpretation of Kant's unless it is altered in a plausibly Kantian, but very unLeibnizian, manner. Thirdly, Kant himself makes some very unclear remarks about Leibniz's approach to this problem. Yet fourthly, there is certainly a similarity in the sorts of arguments used by Leibniz and those of the Antithesis.

Given these complications, some guide must be given to the reader as to the structure of this section. The Leibnizian Interpretation holds that the arguments of the Antithesis rely on the principle of sufficient reason. To put it simply, it reads the argument thus: if the world is finite then there must be empty time before the world. There could be no reason for the world to begin at one point of empty time rather than another. Nothing happens without a reason. Therefore the world could not begin.

The interpretation is put forward most clearly by Al-Azm; although the discussion here shall not follow Al-Azm's. He writes:

> The question is why should the first
> state of the world series occur at any
> one moment of time rather than another
> when all moments are wholly alike?
> It is the problem of the principle of
> sufficient reason all over again.
> (Al-Azm, p. 45)

The relevant text for this interpretation is the se-

ries of letters exchanged between Leibniz and the English Newtonian Samuel Clarke, shortly before Leibniz's death. Accordingly the first part of this section is a textual examination of those letters. As I shall show something like this argument can be found there, although it is not as clear as one might wish. However Leibniz takes it to show not that the world could not begin, but that there could not be empty time. On his relational view of time there could be a finite past with no empty time before it. In fact Leibniz believed that it was philosophically open whether the past was finite or infinite -- he was inclined to the belief that it *was* finite, though for scriptural reasons, namely the story of creation. Thus it appears that far from being a plausible interpretation of Kant's argument, Leibniz's argument is an argument against Kant since it denies Kant's very first inference, from a beginning of the world to the existence of an empty time before it.

Someone who favoured the Leibnizian Interpretation might respond as follows. While the Antithesis may not be a straightforward presentation of Leibniz's views, it can be taken as a presentation of Leibniz's argument modified by a rejection of Leibniz's relational view of time. If the first premise of Kant's argument could be defended then Leibniz's argument would give the required conclusion. In other words, Leibniz can be taken as showing that either there would not be empty time before the beginning of the world or the world could not have a beginning. Leibniz opts for the first of these options. If a Kantian could rule that option out, could show that a beginning of the

world would require empty time before the world, then there is only the second option left, that the world could not have a beginning, and that is the conclusion that the Kantian wants to establish.

Thus the pursuance of the Leibnizian Interpretation of the Antithesis involves an attack on Leibniz's view of space and time. Therefore the second part of this section is a criticism of Leibniz. Some form of criticism can be found in Kant's Observations on the Antithesis, though I suggest that his remarks there rest on a misunderstanding of Leibniz's views. I therefore offer other arguments that aim to show that Leibniz is mistaken and that the first inference of the argument of the Antithesis is valid. However these arguments, which occupy a large part of this section, have an importance beyond the needs of the Leibnizian Interpretation. As Leibniz's relational view of space and time is essentially an alternative solution to the Antinomy, or at least a way of avoiding the Antinomy, it is necessary in any case to show why that view is mistaken. Rather than repeat myself by attacking Leibniz again later I take the opportunity to criticise him here. Thus the criticisms of Leibniz have a dual function. One is the furtherance of the Leibnizian Interpretation. The other is the closure of an escape route from the Antinomy.

Finally, however, in the third part of this section, I argue that the Leibnizian Interpretation is ultimately mistaken as an interpretation of the Antithesis text. I argue that there are similarities between some of Leibniz's arguments against Clarke and the point of view of the Anti-

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thesis, but that no more can be said than that.

Regrettably any discussion of the relationship between Leibniz's letters to Clarke and the Antithesis of the Antinomy, if it is to be complete and accurate, must be complicated.

As I stated, the work of Leibniz's that we are concerned with is the letters to Clarke concerning Newtonian cosmology that Leibniz wrote just before his death. It is partly the nature of this correspondence that makes it difficult to extricate the relevant arguments. Leibniz's letters are vigorous *ad hominem* attacks. The correspondence begins with Leibniz's questioning Newton's religious orthodoxy (more out of mischief, I suspect, than as any serious attempt to damage Newton's position, though we must remember that Newton was, at the time, a senior government employee -- Master of the Mint); and the debate with Clarke about the meaning of the term "sensorium" -- at one point Leibniz suggests that Clarke should look it up in a dictionary -- is not conducted at the highest abstract levels of debate. It is understandable that in such circumstances the presentation of Leibniz's position is not as clear and unequivocal as we might have wished. It is necessary to bear the polemical nature of his writing in mind.

The aspect of the debate that concerns us is Leibniz's opposition to Newtonian absolute space and time. As a point of attack on the Newtonian conception Leibniz makes the assumption of a finite world and seeks to show the chimerical nature of the relationship between this finite world of ob-

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jects and infinite Newtonian space and time. Thus some of
Leibniz's arguments could be seen as arguments against a
finite world if one accepted the Newtonian view of space
and time. This, it must be emphasised, was not Leibniz's
intention. He discusses a finite world as an argumentative
device; it is not itself the object of his criticism.

Nonetheless we should now consider the form that criti-
cism takes. As we are, at the present, particularly inter-
ested in the temporal case, let us consider his argument
there. It is this:

> Supposing any one should ask, why God
> did not create everything *a year soon-
> er*; and the same person should infer
> from thence, that God has done some-
> thing, concerning which it is *not pos-
> sible* there should be a *reason*, why he
> did it *so*, and not *otherwise*: the an-
> swer is, that his inference would be
> right, if *time* were any thing distinct
> from things existing in time. For it
> would be *impossible* there should be
> any *reason*, why things should be ap-
> plied to such *particular instants*, ra-
> ther than to *others*, their succession
> continuing the same. (III6)

This argument appears to operate thus. If the temporal
world and time are totally distinct entities then it is pos-
sible that the temporal world could be transposed to a dif-
ferent position in time. No rational choice is possible be-
tween the two cases: thus it is contrary to the principle
of sufficient reason, by which God acts, that one case
should occur rather than the other.

The objection one might put to this is that if the two
cases are different then God could indeed rationally choose
between them -- although we may not be able to understand
his reasoning -- and if they are not different then the

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question of God's choice does not arise. Leibniz is not very clear about this. Take, for example, this passage:

> To say that God can cause the whole universe to *move forward* in a straight line, or in any other line, without making otherwise any alteration in it, is another *chimerical* supposition. For, *two states indiscernible* from each other, are the *same* state; and consequently, 'tis a change without any change. Besides, there is neither *rhyme* nor *reason* in it. But God does nothing without *reason*; and 'tis *impossible* there should be any here. Besides it would be acting without doing anything ... because of the indiscernibility. (IV13)

In this passage Leibniz seems unable to make up his mind whether a finite universe could move in space but there would be no reason for its doing so, or could not move in space because that would not constitute a change. It would seem that on the Newtonian view (on which this is an attempt at a *reductio*) the second argument is ineffective because there is a change in position in absolute space. Yet if we allow that, that there is a change, then why may it not be a change pleasing to God, albeit for some inscrutable reason. Leibniz is working with two principles here, the *principle of sufficient reason* -- that nothing happens unless there is a reason for it -- and the *identity of indiscernibles* -- that if two things are indiscernible then they are the same thing. It appears on this passage above that the principle of sufficient reason argument is being supported by the identity of indiscernibles. The two cases, where the universe moves with respect to absolute space and where it does not, are indiscernible: therefore they are identical. Therefore there cannot be a reason for

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one to happen rather than the other. This is unhappily put,
because if the two cases are really one, then it makes no
sense to speak of "one happening rather than the other".

In contrast to this, it appears later in the corres-
pondence that Leibniz is seeking to justify the identity of
indiscernibles by appealing to the principle of sufficient
reason:

> This supposition of two *indiscernibles*,
> such as two pieces of matter perfectly
> alike, seems indeed to be *possible* in
> abstract terms; but it is not consis-
> tent with the order of things, nor with
> the divine wisdom, by which nothing is
> admitted without reason. (V21)

This is an admission that it is theoretically possible that
two different objects should be indiscernible, but that God
would never create them so, because, presumably, he could
not have a reason to assign each its separate place in the
cosmos rather than each having the place occupied by the
other.

There is no reason to suppose Leibniz is entirely con-
sistent here. These are *ad hominem* attacks on Clarke, not
the development of a general metaphysical theory. (though,
as we shall see, Leibniz does offer his own view of space
and time). However it appears that the principle of suf-
ficient reason argument is the predominant one. It is this
principle that Leibniz stresses and it is the one that makes
most sense, if we remember that he is offering a *reductio
ad absurdum* of the Newtonian position. On that view we sup-
pose *ex hypothesi* that it is possible for the universe to
have different positions in space and time. It would not

be a satisfactory *reductio* to then assert that these differently positioned universes are identical. Instead Leibniz secures Clarke's agreement to a more general thesis -- the principle of sufficient reason -- and then defies Clarke to produce a reason for creating the universe in one of these positions rather than other.

Clarke's response is to say that the rationality of the choice rests in the mere will of God. This exposes the theological divide between Clarke and Leibniz, Clarke obviously believing that if x is some act of God then x is good because God does x, whereas Leibniz believes that God does x because x is good. However there is no need for us to follow the protagonists into the theological arena. We have now examined enough of Leibniz's text to be able to examine whether it provides the grounds for a plausible interpretation of the Antithesis arguments.

The first problem that must be met is this: if Leibniz's arguments are taken at face value then far from ruling out the possibility of a finite world, instead they rule out a crucial premise of the Antithesis argument, namely premise (2), that there must be an empty time before the world. In order to use them against a finite world, we must establish first of all that there must be empty time before a finite world.

It is possible to treat Kant's remarks in the Observations on the Antithesis as an attempt to do just that. He begins:

> I am aware that attempts have been made
> to evade this conclusion by arguing that

> a limit of the world in time ... is
> quite possible without our having to
> make the impossible assumption of an ab-
> solute time prior to the beginning of
> the world .... With the latter part of
> this doctrine, as held by the philoso-
> phers of the Leibnizian school, I am
> entirely satisfied. (B461)

He continues:

> But we are here treating only of the
> *mundus phaenomenon* and its magnitude,
> and cannot therefore abstract from the
> ... conditions of sensibility without
> destroying the very being of that
> world. (B461)

This is a clear misconception of Leibniz's position,
made either by Kant himself, or by the mediating "philoso-
phers of the Leibnizian school" that Kant refers to. Kant
takes Leibniz's affirmation that time is unreal to be equi-
valent to his own transcendental idealist view that time
does not apply to noumena. He then argues that the Antinomy
is not discussing noumena, but phenomena, and they are ne-
cessarily in time. However the unreality of space and time
for Leibniz lies not in their not applying to noumena but
in their being only relations between objects. Leibniz's
view would still hold of a Kantian phenomenal world, since
it does not deny that such a world is spatio-temporal, but
only that its spatio-temporality is merely a relation be-
tween phenomena and does not require space and time as in-
dependent existents.

So Kant's criticism of Leibniz on this point is very
unsatisfactory. However there are other arguments that can
be advanced. Kant scholars will recognise that the two arguments I am about to advance are very similar to the argu-

ments in paragraphs (1) and (5) of the Metaphysical Exposition of the Concept of Time, in the Transcendental Aesthetic. I discuss these paragraphs directly in Chapter Five, where I treat them as part of a general theory of space and time. In this chapter I have no general aim, but am developing variants of these arguments as *ad hominem* refutations of Leibniz. The first argument here in particular is somewhat different from the corresponding argument in the Exposition. Both arguments are valid independently of the question of the truth of transcendental idealism.

The first argument (which corresponds to paragraph (5) of the Exposition) is as follows: processes in time may have limits, but time cannot have limits.

The argument for this is reasonably simple. Any process takes a certain amount of time, but could take longer. To take a homely example, if I were an even worse typist than I am, then typing that last sentence would have taken even longer than it did. However if any process could take longer, then there must be a longer time that it could take. However if there is always a longer time that a process could take then any period of time as delimited by a process could be longer, and so time cannot have limits: it must be unlimited.

It may be said in Leibniz's defense that he is only committed to allowing that a process could take longer relative to some standard process (the revolution of the Earth for example), and that the standard process could not take less or more time than it does. This however would be a confusion. In one sense the standard process could never

take more or less time than it does since however long it takes is the time of the standard process. Yet in another sense it quite clearly could take more or less time. If the period of one revolution of the Earth is our standard process then clearly that process could be accelerated or decelerated (by gravitational forces for example). A day would still be a day, but days would not be the same length as they used to be. There is a difference between the adoption of a unit and the existence of a length (roads do not become longer when we express their length in kilometres rather than miles). Therefore there is a difference between being the same number of units (in the case of the standard process, one) and being the same length.

We can apply these considerations to Leibniz directly. In paragraph 56 of his fifth letter to Clarke, Leibniz says that there is a sense in which the universe could have begun earlier than it did, namely there could have been a stage before the first stage. Then there would have been a time prior to the first time. But for it to be possible for there to be an event before the first event then "before the first event" must have a reference. To refer to Leibniz's diagram, he could not draw the section $ARSB$ if he had no space to draw it in, and this space above the line AB corresponds to the time before AB. To use an analogy, we can consider the universe as a train and time as the track it moves along. Leibniz's view is that the track is built as the train goes along (rather as they did when they built the railroads across the United States). My objection is that the track must have somewhere to go -- there must be

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somewhere to put it.[2]

Perhaps essentially the same point can be made in another way: the length of time of the world is not defined, on Leibniz's view, in terms of some independent temporal scale. It is not possible even to compare it to some other process, since all processes are within the world. Therefore the length of time of the world is simply the length of time of the world. It is a primitive, a notion not definable in terms of some other notion.

Let us call the length of time of the world without the extra stage A. Let us call the length of time of the world with the extra stage B. Let us call the length of time of the actual world C. If the first of these worlds is the actual world then $A = C$. If the second of these worlds is the actual world then $B = C$. But C is a primitive -- it does not have a value in terms of some independent time scale -- so it has the same value whether $A = C$ or whether $B = C$. So it would appear A must equal B. Yet *ex hypothesi* $A \neq B$. This contradiction arises through considering the length of time to be equivalent to the length of the world.

The second argument (which corresponds to paragraph 1 of the Exposition) is that any attempt to derive the concept of time from relations between objects is bound to be circular. To apply this to Leibniz we need to examine his

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[2]
   It cannot be said that the train has somewhere to go because of the track and the track has somewhere to go because of the train, because then presumably the relative position of the train to the track is unchanged, and what is the difference between the train's moving and its standing still?

attempt to do so. Leibniz does not describe his theory of time in detail, but from the remarks he makes throughout the correspondence it is easy to construct it on analogy with the theory of space which he does articulate, in paragraph 47 of the fifth letter.

As space is the order of co-existences, time is the order of succession (third letter, paragraph 4). So presumably as Leibniz develops his theory of space on the basis of the notion of "same place" he would develop his theory of time on the basis of the notion "successive time". He may do it as follows: suppose that object A has a certain relation to objects BCD. Next suppose that A has a different relation to the same BCD. These different relations $ABCD$ and $ABCD'$ correspond to different times. Next, we have to suppose that for any $ABCD$ there is an $ABCD'$ such that for any $ABCD''$, $ABCD'$ is between $ABCD$ and $ABCD''$. Thus $ABCD$ and $ABCD'$ are successive. Time is then defined as the order of successive relations.

The contents of this theory are objects (A, B, C, D): relations ($ABCD$, $ABCD'$, $ABCD''$); and second order relations (between, different relation). In his letters to Clarke Leibniz did not call A, B, C, D monads, and so neither shall we.

The critical notion here is the notion "between". We could not derive the concept of time from the notion of "different relation" without the notion "between". It does not follow from the fact that $ABCD$ and $ABCD'$ are different relations that they represent different times, for the same objects can have more than one relation at the same

time: A could be heavier than B, C, and D, and longer than B, C, and D. Nor can we make the requirement that $ABCD$ and $ABCD'$ be logically incompatible, for then our theoretical conjunction of them would be unintelligible unless time were already assumed; so that would beg the question. For example, our conclusion would be "x and $-x$ are different times", where x and $-x$ stand for incompatible relations. But the theoretical conjunction "x and $-x$" has the form of a self contradiction. It is not one only because x and $-x$ are already taken to signify different times. Thus the conclusion is an empty tautology -- to be intelligible it has to be true -- and tells us nothing about the nature of . time.

Thus Leibniz needs the concept "between" or an equivalent concept. Unfortunately for him the use here of this concept involves our already having a concept of a temporal sequence. Consider the example of a traffic light. If we say that the amber is between the red and the green then that is ambiguous between the possibility that the lights shine in fixed positions, and the possibility that they shine in a fixed sequence. In other words it is ambiguous between a temporal order and a spatial order. But we cannot specify that we mean "between" to be taken temporally rather than spatially without being guilty of circularity.

We can be more general than this. Consider the series of natural numbers. The number 2 is between the number 1 and the number 3. Yet the series of natural numbers is neither a spatial nor a temporal series. So it must be specified that "between" is not intended in a numerical sense.

The general point is: to specify the appropriate sense of "between" we must specify the kind of sequence we intend; but to do that we must already have the appropriate concepts.

We should have suspected that this problem would arise when we read "I hold space to be an order of co-existence as time is an order of successions" (third letter, paragraph 4) -- for co-existence is a temporal concept. Thus he has used a temporal concept to distinguish space and time.

Notice that Leibniz is not free to say that the sense of "between" always stays the same but the sequences are spatial or temporal or numerical depending on whether the objects are spatial, temporal or numerical. Firstly, the spatial and temporal objects are the same, and secondly that would only put the circularity on a different part of the theory.

For these reasons Leibniz's attempt to reduce time to a relation between objects is doomed to failure. Combined with our first argument it gives us strong grounds to reject the Leibnizian option of saying that time and the universe began together. We have thereby defended premise (2) of the Antithesis argument. Where does this leave the Leibnizian Interpretation?

The natural thing to do at this point would be to conclude that as there must be empty time before the beginning of the world then, as God could not rationally choose between creating it at one time or another, which would contradict the principle of sufficient reason, then the world must not have a beginning.

However this is still not an accurate representation of the Antithesis argument. Kant clearly does not intend his argument to refer to God since he says explicitly: "and this applies whether the thing is supposed to arise of itself or through some other cause." (B455) However if we do not regard the principle of sufficient reason as founded in God's rationality then it becomes much less plausible. If the universe is going to "arise of itself", which seems an arbitrary thing to do anyway, then there is no greater arbitrariness in its arising of itself at one moment rather than another. For the principle of sufficient reason to work there must be some moment of choice to which it can apply, which means there must be a chooser, commonly God.

On the other hand, if it were to be suggested that it is the very arbitrariness of the world's arising of itself that the Antithesis argument is intended to exploit, then it is impossible to escape the other half of the quotation that the argument applies if we regard the beginning of the world as the effect of some other cause. Moreover, the question of the world's beginning as an "uncaused cause" is discussed in the Third Antinomy and I find it unlikely that Kant would discuss the same issue there and here, especially as he gives no sign of doing so.

The Leibnizian interpretation is mistaken, then, in as much as it seeks to represent the Antithesis argument in terms of the principle of sufficient reason. However the weaker thesis that there are parallels between Leibniz's letters to Clarke and the point of view of the Antithesis does have value. For example, I shall find a use, not only

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in this chapter, for the identity of indiscernibles, or for something closely resembling it.

In more general terms, Kant (as the proponent of the Antithesis) and Leibniz both seek to show that there is something paradoxical in the notion of a finite world and infinite time and space, and they have, to some extent, similar sorts of cases in mind, although the conclusions they draw are different. It would be imprudent to try to draw a closer parallel than that.

The Temporal Argument

I must now attempt to give a correct interpretation of the argument of the first half of the Antithesis. We have already shown in the previous section that time is not a relation between objects, and is unlimited, so, if the world's past were to be finite, then there must be empty time before the world began. It is important to be clear about the logical structure of this. The argument is an attempt at a *reductio ad absurdum* of the position that the past of the world is finite. The notion of empty time is introduced *via* the assumption of a finite past. Therefore, if the Antithesis were to argue that a pre-mundial empty time were impossible, that would not constitute a contradiction in the Antithesis argument (that there both must be and cannot be empty time), but would disprove the possibility of a finite past (a finite past requires a previous empty time, a

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previous empty time is impossible; so a finite past is impossible). However this is not quite the form the argument actually takes.

I stated in the previous section that Kant seeks to show that there is something strange about the relationship of the past to the prior empty time. The particular paradox he has in mind is this. If there is empty time before the world then there must be different points of empty time. Let us call two of these points t_1 and t_2. I can now express simply the meaning of premise (3) of the argument: there is no difference between the universe's beginning at t_1 and the universe's beginning at t_2.

The rest of the argument then proceeds as follows: as this holds for any two points of empty time it holds for all points of empty time. So if the universe began at *any* point of empty time then it began at *all* points of empty time, i.e., it did not begin at all. Or to put it another way: "unless it occupies all time ... it cannot have any determinate position whatsoever." (B549n)

This is a more radical argument than the previous two interpretations allow. It does not argue that we could not say when it began, except internally; nor does it argue that there is no reason for it to begin at t_1, rather than t_2 (Leibniz, on balance, appears to allow that it is *intelligible* that it should begin at t_1 rather than t_2, but that it would be contrary to God's wisdom). Rather it argues that there is no difference between its beginning at one time and its beginning at another, and that if two things are not different then they are the same.

The obvious objection to this is that since *ex hypothesi*
t_1 and t_2 are different points of empty time, the difference
between these points constitutes the difference between the
universe's beginning at one and its beginning at another.

I shall argue that this is, to use a Leibnizian phrase,
a chimerical difference. Firstly I shall reject the view
of time on which it might be taken to be a real difference.
Secondly I shall show that t_1 and t_2 would not be uni-
quely specified by the length of a supposedly finite past.

The general conception of time by which a finite past
would have a definite temporal position is to see time as a
ruler against which the universe is measured. A temporal
length and temporal position of the universe is determined
by where it lies against this infinite ruler of time. There
are various problems with this conception. There is first-
ly the disanalogy that a ruler is an object and time is
not. If time were an object then it would be in time, which
is absurd. Yet it is the object-hood of the ruler which is
doing the work in this conception of time.

Another problem for the notion of time as a ruler is
this: is it finite or infinite? We have already, in the
previous section, dismissed the possibility that it be finite.
But if it is infinite then we encounter all the problems of
the previous chapter. For in an infinite time it is always
possible that there should be an infinite past: and we have
already shown that there could not be an infinite past.

There is also the following implausibility inherent in
this conception. Suppose that time is marked out in some

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way, as if it were a ruler. Then, from the point of view
of an instant in pre-mundial time, what is the reason for
this marking? Why does time have this form if, up to this
point, there are not and never have been events? Indeed,
unless we take the existence of the universe as a necessary
fact, which would need a great deal of argument, it is not
even certain at that instant that there ever will be events.
This marking of time would be as strange and as arbitrary
an occurence as if houses in the eighteenth century had all
been equipped with electric sockets even though there was
no electricity and the possibilities of electric power had
not yet been conceived.

Any attempt to distinguish t_1 and t_2 by reference to
time must commit the mistake of giving time a temporal
structure and time is not a thing in time. Thus suppose
we were to say that t_1 was at a later time than t_2. But
t_1 and t_2 are not *at* points of time, they *are* points of
time. For two times to be earlier and later they must first
of all be different, so they cannot be differentiated by be-
ing earlier and later. In effect, this would be to treat
points of empty time as if they were temporal objects. My
second argument gives further reasons for not doing this.

Let us take some temporal object in the world's past,
say the Battle of Waterloo. Then if t_1 and t_2 are to be
different, there should be a different relation between the
Battle of Waterloo and t_1 and the Battle of Waterloo and t_2.
However consider the following possibility. Suppose that
the history of the universe should be moved forward in time
so that everything occurred later in empty time. How would

such a universe be different from our own? Nothing within
the universe would speak of this change, as all the tempo-
ral relations within the world would remain the same. No-
thing in the world would be any older by virtue of begin-
ning at a later moment of empty time. Neither, however,
from the point of view of infinite empty time would there
be any difference. Since an infinite amount cannot be di-
minished or enlarged by the subtraction or addition of a
finite amount, there would still be the same amount of emp-
ty time before and after the world in each of the two cases.

However on this supposition the supposed relation-
ship between Waterloo and t_1 would have changed and, for
the sake of argument, we will suppose that same relation-
ship now holds with t_2. However as this supposed change is
absolutely unnoticeable either from inside the universe or
from outside it, then t_1 must be indistinguishable from t_2.

Of course if t_1 and t_2 were objects, rather than points
of empty time, then there would be a real difference and the
move would be a real change, just as it would be if the
whole temporal universe were to move except for some event,
such as a particular human life, so that that human lived
earlier in history than he otherwise would have.

Another objection may perhaps take this form. It is
perfectly intelligible to debate whether the universe began
in 4004 B.C. or millions of years ago. Therefore there
must be a difference between its beginning in 4004 B.C.
and its beginning millions of years ago. Therefore it must
be intelligible to say it began at one point of empty time
rather than another.

The trouble with this is that different points in the history of the universe only correspond to different points of empty time if the universe itself has a fixed position in empty time. Thus any attempt to provide such a fixed point "internally" is bound to fail. After all, if one is travelling by train from Edinburgh to London, one does not assume oneself to be in Yorkshire from the fact of sitting in the middle carriage.

We can now understand the sense attached to "determinate" in this context. It is not epistemological determinacy that is required, not determinate knowledge of the universe's temporal position, but ontological determinacy, that the universe actually have one temporal position rather than another. I shall call this *external determinacy*. In the next chapter I will discuss the relation between external determinacy and internal determinacy. At the moment I want to see whether this interpretation of the temporal argument also holds for the spatial argument in the Antithesis.

The Finite World in Space

The argument of the second half of the Antithesis can be set out thus:

> (1) Assume that the world in space is
>   finite.
> (2) Then it exists in unlimited empty
>   space.

(3) Then things are related not only in
    space, but also to space.
(4) The relation of the world to empty
    space is a relation to no ob-
    ject.
(5) Such a relation is nothing at all.
(6) Therefore the world in space is not
    finite.

I had best say at once that Einstein will be discussed
at some length in Chapter Four, and so will not be mentioned
here. He has not been forgotten. To turn to the argument,
we must first make the now familiar reminder that it is a
reductio ad absurdum argument, attempting to show that there
is an absurdity in the very idea of a finite spatial world.
The second premise of the argument amounts to a denial of
the Leibnizian view of space. The arguments advanced ear-
lier in the case of time are directly transferrable to the
case of space so I shall not rehearse them again here.
Clearly any spatial object could be longer than it is, which
sets up the first argument, and clearly also the problem of
the ambiguity of "between" tells against space's being a re-
lation as much as against time's.

Premises (3) to (5) seek to show where the absurdity
is to be found. Premise (3) argues that if the world is
finite then not only are the various parts of the world re-
lated to each other in space, but also the world as a whole
is related to space: there is the world and there is the
space around it. Thus this premise is not pointing to a
misuse of language as Strawson appears to think (Strawson,
p. 182), but merely points out that a finite world has a
relation to empty space, namely it is surrounded by it. I
do not see how that *is* a misuse of language. Premises (4)

and (5) then argue that this relation is vacuous, because it is not a relation between objects. This is the difficult part of the argument. What does Kant mean by saying that this relation is nothing and that therefore the notion of a finite world is absurd?

To show this let us consider some point in empty space, call it *a*, and some point in the universe, say Edinburgh. If the universe is related to empty space then there ought to be a unique relationship between *a* and Edinburgh: they ought to be a unique distance apart (we can ensure that this relation is not straightforwardly ambiguous by treating distance as a vector). Certainly if *a* were an object this would be unobjectionable. We have no problem of conceiving the spatial relationship between Edinburgh and London. However, as we shall see, where *a* is a point of empty space, this supposed relation is chimerical.

Consider the following case. Suppose that there is some latter-day Gulliver who, wisely eschewing the sea, sets up a prosperous private practise, but nonetheless wakes up one morning to find that the entire universe except for him has doubled in size. This is an intelligible though unlikely event. Suppose now that Gulliver had also doubled in size. There would in that case be no difference between the doubling's happening and its not happening. All the relations between objects would be exactly as they were before the doubling. However the same relations would not hold between Edinburgh and *a*, since *a* is a point in space and not an object. So since in such a case there is no difference -- it is change without a change -- the relation be-

tween Edinburgh and a cannot be a real relation.

This can be brought out clearly if we consider oursel-
ves to inhabit the outermost planet of a finite universe
surrounded by infinite space. We think of a point in empty
space a mile away as a. Suppose then the entire universe
doubles in size during the night. We could not tell that
the point a mile away was not a. What difference would
there be? It would still be a mile away. We would still
be surrounded by infinite space. There is no difference at
all between its being a and its not being a.

A possible objection here is this: Assume that the
planet has a diameter of two miles and a is a mile away
from the surface of the planet. Then we can say that if
the planet doubles it now has a diameter of four miles and
a is now on the surface.

Firstly it would be impossible to determine that the
planet now had a diameter of four miles since, as all the
measuring equipment will also have doubled in size, it will
still show the diameter as two miles. It may be said that
the planet now occupies twice as much space as yesterday,
but how is that determined? For the amount of space not oc-
cupied by the planet is the same (infinite) amount.

Similarly, suppose one day somebody were to say: "That
point there on the surface is a point that yesterday was a
mile out into space." What could be made of this claim?
When we look around at the universe we find it unchanged
from yesterday. All other stars and planets are still as
far away from us as before and staring out into space there
appears to be (and in fact is) as much of it as there was

the previous day. It is not just that we *cannot notice* any difference, there *really is* no difference; except that we now have the claim that a point that was a mile away in space is now on the surface of the planet. That claim could be made with as much support and as much real content on any morning.

Of course this argument shows that it is impossible to suppose that the universe could in any intelligible sense double in size. That is not a contradiction in the argument, it is merely another way of expressing the same point, since the universe's doubling in size is another example of a vacuous relationship between the world and empty space. Of course if the universe were to double in size in relationship to some object (say Gulliver on our first supposition), then this would be intelligible (although even here the most we could say is that either the universe has doubled in size or Gulliver has shrunk by one-half).

Another variation on the same theme is provided by the idea of a scale model of the universe. Suppose this model to be identical in every detail except that it is on a different scale. Now suppose that this model is set in empty space instead of the real universe. There would be absolutely no difference. Yet we have not specified what scale the model is on. If we imagine a miniature of our universe the size of a grain of sand set in empty space instead of the universe there would not be, neither from a point of view inside nor outside it, any difference at all; unless we regard a "point of view" as embodied in an object.

This idea can perhaps be conveyed by trying the follow-

ing thought experiment: would you say that the universe is, in absolute terms, not only in relation to some subsection of itself, large or small?

Another paradox is mentioned by Kant. He says:

> For example, a determination of the relation of the motion (or rest) to the world to infinite empty space is a determination which can never be perceived, and is therefore the predicate of a mere thought-entity. (B547n)

Let us suppose that no actual movement is involved (and thus that there is no question of forces), but simply in one split second the entire universe is transposed from one point of empty space to another. This again would be a change without a change, to use a Leibnizian phrase to describe a Leibnizian example, since from no point of view would there be any difference. All the relations between objects within the universe would be the same, and there would be the same amount of empty space surrounding it on all sides.

I suspect that other paradoxes could be developed on the same theme. Kant subsumes all of them in premises (4) and (5) of the argument where he points out that the relation of the world to the empty space around it is nothing at all. Since these absurdities follow from the assumption of a finite spatial world, then we can conclude with Kant that the world in space cannot be finite.

I can now show how the assumption of the reality of the universe functions in the Antithesis. The paradoxes arise by trying to apply the real spatio-temporal relations that hold between the parts of the universe to the relationship

between the universe and space and time themselves. Moreover the reality of the universe needs to be assumed if we are to insist that it must be just one size, that it must be in just one place, that it must begin at just one time. That is to say, in this context also the demand that the universe be real is the demand that it be determinate; externally determinate.

We have therefore now established the Antithesis also: the universe cannot be finite, if it is real.

S Y N T H E S I S

In this chapter Kant's views and mine diverge as he takes the road into transcendental idealism. In Kant's work we are offered a logical solution, which involves regarding the world as sequential, and a doctrine of space and time to support it, namely transcendental idealism. I shall show that this does not work. While I seek to delineate the essential features of Kant's solution, since I shall shortly reject the whole, I do not try to expose and examine every single feature. That would be akin to a U-boat's doing a complete inventory of a ship before sinking it.

I then offer my own solution. It too is in two parts, a logical structure, which is offered in this chapter, and a view of space and time to support it, which is offered in Chapter Five. I regard space and time, and thus the spatio-temporal world, as transcendentally real. I do not see the spatio-temporal world as in any way sequential, except in the obvious temporal sense that one day follows another. However I do not regard space and time as independent of the world. While Kant can be said to treat space and time as "species-centric" my view treats them as "mundocentric". However I am pre-judging matters somewhat. The immediate task is a discussion of Kant's solution.

This is intended to work in the following way. According to transcendental idealism, there is a distinction be-

tween the sensible world -- the world of phenomena, and the intelligible world -- the world of noumena. The intelligible world does not exist in space and time so our considerations do not apply to it. The problem lies solely with the sensible world. However, also according to transcendental idealism, the sensible world does not exist independently of our experiences of it. So the solution of the First Antinomy lies in a correct description of our experience of the sensible world.

This Kant continues, immediately rules out the possibility of a finite world, since an absolute limit to the sensible world is not a possible experience -- and anything that is not a possible experience cannot belong to the sensible world. Kant argues: "(S)uch an experience must represent to us phenomena as limited by nothing or the mere void." (B545) The problem, I take it, is that space and time are not themselves perceivable (for example, "Now time cannot by itself be perceived." (B225)), so the limitation of the world of objects by space and time demands their limitation by something imperceivable, and that is not a possible experience. I have already discussed this line of thinking in the last chapter, in the First Analogy Interpretation.

Kant also argues that our experience is sequential. It regresses both from present stages to earlier stages (as, for example, when an astrophysicist asks what the universe must have been like in the past to be as it is today), and also from this place to other places, as when we build more powerful telescopes to see further out into the universe, or, conceivably, build spaceships to travel out into it. Kant

makes no distinction between the temporal and spatial cases, both of which he calls the regress from the conditioned to its conditions.[1] These sequences Kant calls the regress of experience. It is in terms of this regress that the problem of the Antinomy is to be solved.

It has already been claimed that the sensible world does not exist independently of this regress, and that the regress cannot be finite as a limit cannot be a member of it. Therefore it only remains to ask whether the regress is *in infinitum* or *in indefinitum*. Since we can have no grounds for knowing in advance that the regress will be infinite (Kant argues this in a section entitled "The Critical solution of the Antinomy", which I do not propose to discuss in detail), we are only entitled to say that the regress is indefinite. Thus the solution of the First Antinomy is to replace an independently existing world with an indefinitely extended regress of our experiences.

Transcendental idealism is not the most clearly stated and consistent doctrine in philosophical history. Therefore to discuss this solution of Kant's it is necessary to give a clear statement of what transcendental idealism is taken to involve. Firstly it is clear that Kant intended transcendental idealism to operate at the level of the species rather than the individual. The angels may see the world in a different way from me but another human being does not. The connection between this intersubjective doc-

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[1] It is in fact in *measuring* space that Kant considers it as conditioned by other parts of space since they form the limits around the area measured. (See B439 - 440)

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trine and its presumed roots in the psyches of individuals
is one of the great problems of transcendental idealism. I
propose to ignore it by treating the doctrine as a purely
ontological and epistemological doctrine, and dismissing the
individual psychological aspects. As I shall not be defend-
ing this view I do not need to solve all its problems. In
fact I am taking the term "experience", as it is used in
Kant's solution, in a rather Popperian manner, to mean
"scientific research". Moreover, because of my interest in
relating Kant's views to the formal sciences I shall fur-
ther restrict the sense of "experience" to those, although
the word could encompass any thinking and learning about
the world in an empirical fashion. The important point is
to distinguish the social epistemological sense of experien-
ce which Kant is invoking here from the personal psychologi-
cal one. The solution to the First Antinomy is not a doc-
trine of psychology.

Thus we can restate Kant's solution as follows. The
sensible world does not exist independently of our physi-
cal sciences. For example, it is necessary to explain the
present state of the universe in terms of its past states,
and so the past states exist in the regress of physical
science into the necessary previous states, necessary in ac-
cordance with physical law, of the present. Similarly our
scientific researches describe regions at greater and great-
er distances from the Earth. Space and time exist, as it
were, only as variables in scientific equations.

However, this view is ultimately unacceptable. Consi-
der the case of the moon-men that Kant mentions:

> That there may be inhabitants in the
> moon, although no-one has ever perceiv-
> ed them, must certainly be admitted.
> This however only means that in the pos-
> sible advance of experience we may en-
> counter them. For everything is real
> which stands in connection with a per-
> ception in accordance with the laws of
> empirical advance. They are therefore
> real if they stand in an empirical con-
> nection with my actual consciousness,
> although they are not for that reason
> real in themselves, that is, outside
> this advance of experience. (B521)

This is the most obvious objection to the view Kant is de-
veloping here -- objects can exist without our being aware
of them, or ever having been aware of them. Kant's answer
to this is that their existence is allowed in his scheme by
their standing "in connection with a perception in accordan-
ce with the laws of empirical advance." This can mean ei-
ther, or both, of two things. It could mean that the moon-
men are real if their existence has causal consequences for
us here and now, or it could mean that they are real if we
could come to perceive them as a result of some process in
scientific research -- going to the moon and looking for
example. So that we cannot be accused of showing Kant's
view to be inadequate only by omitting part of it, we will
allow that moon-men exist if either condition is satisfied.

However this is highly problematic, as neither option
protects the necessarily sequential nature of Kant's solu-
tion. If we allow that anything exists which operates cau-
sally on the here and now then the regress of experience be-
comes ontologically otiose and we can define the world as
this set of causal operators. Then the problem returns as
to whether this set is finite or infinite. Similarly, if

anything exists prior to our discovering it as long as it is discoverable, then we can define the sensible world as the set of discoverable things and then the question returns as to whether this set is finite or infinite. The non-sequential set of discoverable things is not defined by the sequential process of actual discovery but by the non-sequential set of methods of discovery.

This can be illustrated by a mathematical analogy. The regress of experience is rather like counting the positive integers. Once a number is counted it exists in the sense of belonging to the regress. However unless we make each member as we "come" to it -- in which case we can hardly be said to be counting -- then there must be numbers we have not yet counted. These numbers correspond to the moon-men we have not yet perceived. It is an intelligible question to ask: how many numbers are there? Thus it is an intelligible question to ask: how large is the sensible world?

One solution to this problem is to opt for a more thorough idealism: to deny that the moon-men exist until they are experienced. It is pertinent to wonder how they are ever to be experienced if they do not exist. Further this option is not open to someone like Kant, or myself, who wishes to preserve the objectivity of the physical sciences. For on such a view what they say is true by virtue of its being said. Indeed our experience could not be said to "advance" at all -- we would simply tell ourselves new stories.

Also of course such a view is no longer transcendental idealism as it gives up the notion of the sensible world as empirically real which, as Kant tells us, is the difference

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between his idealism and the sort of idealism which is a scandal to philosophy.

There is another problem for Kant's view, which I shall call the problem of continual discovery. Normally this is a problem for epistemology. All it asserts is that our present system of beliefs can be altered by new discoveries. An obvious example is the move from a geocentric to a heliocentric system. This effected an enormous shift in European values and beliefs, many far removed from astronomy. Not all new discoveries can be simply added on to our existing stock of knowledge. Even when the discovery is not inconsistent with any already known fact it can still cause great changes in the patterns of our beliefs, in the way we see things.

This is a fairly commonplace description of one of the main sources of excitement and interest in the development of human culture. The problem of continual discovery is not a crisis in epistemology because these epistemological changes are underwritten by an assumption of ontological stability. All I mean by that is that while our beliefs about the universe may change, the universe itself does not. Thus the epistemological problem of continual discovery is ultimately soluble, when our beliefs about what there is completely coincide with what there actually is. Even if we never expect in the lifetime of our species to be in that position, its possibility means that the problem is not a fundamental difficulty.

Of course, in contemporary epistemology this issue is not as clear as I have made it appear. I have sought a sim-

ple exposition of it to show more markedly the difficulty for Kant. On the solution he puts forward to the First Antinomy, the problem becomes an ontological one, and accordingly much more severe. If reality is indeed sequential then even that which has already been encompassed by the sequence has no clear or determinate nature, since that nature (not only our knowledge of it) is, as the Americans say "up for grabs" in the light of future terms of the sequence, and so on *in indefinitum*.

A powerful illustration of this (I am sure science fiction devotees could think of many others) is the conjecture that seems to be common among children that our universe may have roughly the same relation to some greater universe that an atom has to our universe. Such a universe would be very different to a universe of which that were not true, yet on Kant's view, not only do we not know whether this is such a universe or not, it is not yet the case that it is such a universe or not.

This can also be illustrated by a mathematical analogy. The nature of the sequence of numbers, "12312" is different depending on whether they belong to this sequence:

$$1, 2, 3, 1, 2, 3, 1, 2, 3 \ldots$$

or this sequence:

$$1, 2, 3, 1, 2, 1, 2, 3, 1, 2, 1 \ldots$$

Their character would again be different in different possible extrapolations of these series.

These difficulties suggest the need for a re-assessment of the situation. Since these objections are directed again-

st the idealist content of Kant's solution, I will make the opposite assumption, that the spatio-temporal world is transcendentally real. This may seem strange as I have argued that the Antinomy follows once we assume transcendental realism. The answer to this is that, as we shall see, the form of transcendental realism I shall adopt will not be identical with that of the Antinomy. I retain the demand expressed in the Thesis that the world be internally determinate and I retain also the demand that it not be externally indeterminate. However I reject the assumption that connects the Thesis and the Antithesis together into an Antinomy, namely that the world must have a determinate magnitude; must be either finite or infinite. This rejection I share with Kant; however our grounds for it are different. Kant thinks that the demand that the world have a determinate magnitude follows from the supposition of its (transcendental) reality; and so he denies that reality. I shall be claiming that it follows from thinking of the world as an object, and it is that that I shall be denying. It follows from this denial that from the *reductio ad absurdum* of the infinite option one can no longer infer the truth of the finite option, and vice versa. Thus what the arguments of the Antinomy now establish is that a transcendentally real world, but which is not an object, is *neither* finite *nor* infinite; and this indeed shall be my conclusion.

This solution involves a reassessment of Kant's doctrines of space and time -- although in fact a great deal can be retained -- and this I shall do in Chapter Five. It is also necessary to offer a different logical structure to

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that offered by Kant of a sequential progress of experience. That task I attempt here. No justification is offered here for the logical form my solution takes, except that it *is* a solution. However, once I have presented it I shall defuse several possible objections to it. The justification for my solution is found in Chapter Five where I offer a general theory of space and time. Moreover in Chapter Four I seek to show how my remarks here relate to contemporary thinking in this area in the physical sciences. Thus this chapter is largely a foil for what is to follow.

It will be noticed from the previous two chapters that the difficulties of the Antinomy can be traced to the idea of "determinacy". The impossibility of the infinite world lay in its not being determinate for some possible object whether it existed or not. This I call the lack of internal determinacy. Likewise in the case of the finite world we discovered that the universe would have no determinate position within time and space. I described that case as the lack of external determinacy.

Since, however, one of these cases is a case of internal determinacy, and one of external determinacy, it may be considered possible that some compromise be reached. It may be possible to protect the internal determinacy of the universe without committing it to external indeterminacy. That is the course I shall follow. However there are two approaches to this. One is to seek a position of which it could be said that the universe is *both* finite and infinite. The other is to treat it as *neither* finite nor infinite. The two are not equivalent. I shall opt for the second.

Let me begin by describing a model of the kind of logic I shall invoke. The model comes from mathematics. Consider the function $y = \dfrac{1}{x - 1}$ at the point $x = 1$. Normally we would say that the function is infinite at that point, but that is to speak loosely. Certainly it does not have a finite value, but neither does it have an infinite one. What is in fact the case is that the function is *undefined* at that point. This is clear if you draw a graph of the function. The graph is in two parts, separated by the point $x = 1$. This breaking down of a function at a point is called a singularity, and I shall have more to say about these in the next chapter. At the moment I want to say that the "infinity" of the universe is like this.

We are pressed towards admitting that the world is infinite because of the problem of external determinacy. It appears that unless we allow the world to occupy all space and time then it has no determinate spatio-temporal position. My response to this is that the universe is not infinite, but is externally undefined. This means that it does not exist externally, but only internally. There is no "outside the universe", either temporally or spatially.

This does not mean the universe is finite either, for its finitude would require that it existed externally as, using a Kantian phrase, "a limited whole". In fact, if the universe exists only internally then it has no size. Size is an external concept. Of course, we can guess the size of a house while standing inside it, but that is guessing a fact about its exterior from facts about its interior. If we were inside a house which had no exterior -- which is

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how I am suggesting we think of the universe -- then it would make no sense to speak of its size. Therefore, the universe, on my view, is neither finite nor infinite.

It also follows that phrases such as "the whole universe", or any statements which presuppose such expressions, are illegitimate since they require us to think of the universe as delineated. These expressions would have us treat "the whole universe" as an *object*, similar to "the United States" or "the moon", and the lesson of the First Antinomy is that the universe is not an object. The mistake Kant made was to read "the universe is not an object" as meaning "the universe is not transcendentally real". It is real, it is not a real object.

It might perhaps be said that what is tricky about the universe is its universality, the fact is that everything is, by definition, inside it, and nothing is outside it. My solution to the Antinomy is really just a formal acknowledgement of that fact.[2]

As can be seen this incorporates one of the central features of Kant's solution, namely that the universe does not exist as a whole. In each case this is supported by a

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2

An analogy which may illustrate this notion of something's existing internally but not externally, can perhaps be provided by a possible view of consciousness. An individual who was not a materialist might think of his field of experience or consciousness as something that his ego exists within but which has no outside from which it can be circumscribed and in terms of which it can be located. It would thus be something that existed internally but not externally. We would in this case note the way that this view does not regard consciousness as an object by calling it "subjective". Of course by drawing this analogy I do not mean to suggest that the universe is a form of consciousness, or vice versa.

view of space and time. Kant's is transcendental idealism.
My own, which is developed in Chapter Five, I call mundocen-
trism.[3]

It may help also to notice a similarity, on this point,
between my view and that of Wittgenstein in the _Tractatus_.
His first sentence: "The world is all that is the case,"
(_TLP_, 1) is as good a definition of what I mean by "the world"
as any. He certainly regarded the world as internally de-
terminate (for example, "(T)he totality of existing states
of affairs is the world." (_TLP_, 2.04)), but he regarded any
talk about "the whole world", or "all states of affairs" as
strictly meaningless, including of course such talk as he
might use himself. The reason is that to discuss the whole
of reality one would have to do so from a position outside
it, and this is impossible because: "(L)ogic pervades the
world: the limits of the world are also its limits." (_TLP_,
5.61) In fact, Wittgenstein treats the thought of the
world as a whole as "mystical":

> To view the world sub specie aeterni is
> to view it as a whole -- a limited whole.
> Feeling the world as a limited whole --
> it is this that is mystical. (_TLP_, 6.45)

There are similarities then between the _Tractatus_' view
and my own. Of course I do not seek to endorse the wider

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[3] Interestingly, Swinburne picks on this denial of wholeness
by Kant _as_ the solution, largely disregarding the view of
space and time which supports it: "Kant claimed that, given
that talk of the Universe as a whole was proper, both his con-
clusions were correct. Since, however, the conclusions con-
tradicted each other, this only showed that all talk about
the Universe as a whole was improper." (Swinburne, p. 282)
Presumably then, from Swinburne's point of view Kant's so-
lution and my own are very similar.

Tractatus viewpoint -- my position will be supported by a view of space and time whereas Wittgenstein's arises out of a view of logic and language -- nor do I share Wittgenstein's mystical feelings (and if I did I would consider it irrelevant to mention them here), yet I agree with the Tractatus that it is illegitimate to think of the world as a totality.

Incidentally it will be noticed that the quotation from 2.04 contains an expression, "the totality of existing states of affairs" that I later claim Wittgenstein would regard as meaningless. This is not unusual in the Tractatus. As Wittgenstein says: "My propositions serve as elucidations in the following way: anyone who understands me eventually recognises them as nonsensical." (TLP, 6.54) My own formulation of the internal determinacy principle in Chapter One refers to "any event" rather than "all events".

I ought now to formally serve notice, as it were, that my solution does solve the Antinomy in that it avoids the arguments in both the Thesis and the Antithesis. It clearly avoids the problems raised in the Antithesis since as the universe does not exist externally there is no empty space and time outside it, and thus no problematic relations between it and empty space and time.

It also meets the objections of the Thesis since there is no need on my view to suppose an infinite distribution of objects in space and time and thus there is no commitment to possibilia as irreducible elements in the ontology. As I do not admit a *total* distribution of objects then, *a fortiori*, I do not admit an *infinite total* distribution.

It is not surprising that my view should meet the ob-

jections that it was created to deal with. I shall now further articulate it by answering objections directed specifically against it. The general line of these objections will be that the universe must exist externally.

The first objection is this. It is impossible for there to be an infinite number of stars, for the reasons given in the Thesis. Therefore there must be a finite number of stars. Therefore the universe must be finite.

The answer to that is this. If we remove from the concept "star" all its spatio-temporal connotations and give it some empty symbol, say x, then the argument becomes: it is impossible that there should be an infinite number of x, so there must be a finite number of x. However since x is a wholly empty symbol this becomes simply the claim: there cannot be an infinite number, therefore there must be a finite number, and I do not accept that this is true. There *are* infinite numbers. Whether the uses they are put to are entirely legitimate is an abstruse mathematico-philosophical debate, and cannot provide an indubitable premise on which this objection could proceed. I myself, as I have suggested earlier in this work, see no reason to deny the possibility of infinite numbers.

Thus the infinite world is not precluded because there cannot be an infinite *number* of stars, it is precluded because there cannot be an infinite number of *stars*; that is, there cannot be an infinite spatio-temporal distribution of objects. But there cannot be a *finite* spatio-temporal distribution of objects either, for the reasons given in the Antithesis, so the finite option is no better off.

The next objection is that if the past is real then it must have a determinate length, so it must be finite. I shall discuss this issue more fully in the next chapter, but a general response to this objection can be offered here. The answer is in two parts. Firstly, although we are precluded from assigning a beginning to the universe *externally*, we can do so *internally* simply by assigning it the value $t = 0$, where t refers to some internal measure of time. Thus the beginning of the universe in this sense is a purely analytical result of our adoption of a particular time-scale. All that we must insist is that $t = 0$ is not taken to be equivalent to some value of extra-mundial time, upon pain of suffering the paradoxes of the Antithesis. Nor of course does the beginning of the universe on this view designate an event since $t = 0$ does not designate a time. Zero is a mathematical fiction. Just as a bag with five apples in it contains a number of apples, but an empty bag does not, so $t = 5$ denotes a time but $t = 0$ does not. "Nothing exists at $t = 0$" is a mathematical, not an empirical, truth.[4]

This leads naturally to the second part of the answer, which is that the past can have a determinate length measured internally. What this length is is partly determined by the choice of time-scale, and is partly an empirical matter. Thus we can say for scientific reasons that the time from $t = 0$ to the present is billions of years, which is to say

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[4]
There are of course mathematical reasons for regarding zero as a number, namely to ensure that arithmetical operations on number always yield numbers (for example, if we subtract five from five then we want whatever this yields, which happens to be zero, to be a number). Zero is a very useful mathematical fiction.

that if the Earth had been going around the sun throughout
it would have have done so billions of times by now. It is
a harmless internal comparison of the same sort as: while
you were looking out of the window we were eating all your
potatoes, or any other comparative remark. I shall argue
in the next chapter that it does not have any fixed meaning,
nor does it require that the universe has a beginning.

I can also deal with the claim that the universe ei-
ther has a beginning or it does not. This is beguiling be-
cause we think of there being some first event and then ar-
guing that this event must either have occured or not. It
is certainly true for any event described simply as an e-
vent that that event either occurs at some point in the
history of the universe or not. However the first event can-
not be described simply as an event but instead involves an
essential reference to the totality of events, i.e., to the
whole universe, and this is illegitimate. This is clear if
we examine the logical form of the statement "there is a
first event": $(\exists y)(x)(x \neq y \supset Byx)$, where B means "is before".
The illegitimacy of this expression lies in the term (x)
which as a reference to all events is a reference to the
whole universe.

Similar considerations apply to the next objection
which raises corresponding issues in the case of space. It
may be claimed that it must have a determinate size. First-
ly, the internal determinacy of the world does not guaran-
tee its external determinacy. It does not follow from "x
exists" that anything else exists. Nor does it follow from
"it is not the case that x exists" that anything else ex-

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ists. Thus from "either it is the case that x exists or it is not the case that x exists" (the principle of internal determinacy) we cannot infer anything about the rest, or all, of the universe. Secondly, we cannot define two end-points between which the universe exists, since to define such a point would involve a reference to the universe as a whole in the same way that "the first event" was shown to do.

Another possible objection is that my account is in fact idealist since it assures one that if cne travels out into space one will always encounter another object, i.e., one will never leave the universe. Thus it makes *a priori* ontological claims.

The answer here is that my view gives no such assurance. It is perfectly consistent with my view that you should travel *out from* the universe without encountering objects. What is not possible is to, as it were, stumble across the universe and travel *into it*. Many roads lead away from any point in the universe, but the only roads that lead to it come from other points in the universe.

However, while it is possible to travel *out from* the universe, it is not possible to *be outside* it. Any position in space only exists in relation to objects and to the fact of our occupying it at any given time. As we saw in the Antithesis, points of empty space have no absolute meaning on their own (remember the example of the universe doubling). Thus if we call our spaceship's position x, there is no point of space which can be called x, there is only a position of our spaceship relative to other objects which can be called x (clearly saying spatial positions can only be

defined by relations between objects is different from the Leibnizian claim that space is just a relation between objects).

Similar observations apply to the following perfectly possible scenario. It is possible to travel out in a spaceship and at some point to turn about and look back and see a mass of matter close together with no other matter around it. It is, as far as I am concerned, a totally empirical matter whether there is such a view anywhere or not. However I insist that this is only a view, like any other, and not a specifically privileged one. Firstly, as we have noted above, our position can be defined in terms of our ship, the mass of matter we have left, and our intervening journey. That is to say, our position can only be defined in an internal manner, in relation to objects in the universe, and so is itself in the universe. It is not possible for us to arrive at that point in some extra-universal manner and thus consider it is a point outside the universe. Moreover the statement "that mass of matter is all the matter that there is," is not significant, on my view, since it contains an illegitimate totality. It is always possible that as we continue our journey we shall encounter further matter, it is also possible that we shall not. Whether we do or do not is an empirical consequence of the vicissitudes of any particular journey.

It is perhaps worth inserting a reminder at this point that I am not, at the moment, trying to justify my conception; I am only trying to show that it can deal consistently with various sorts of objections. Of course the answers it gi-

ves to these objections are only true if the conception is it-self true; but the fact that it can give consistent answers means that the objections do not show that it is false. The positive grounds in favour of my solution offered so far are that it solves the Antinomy without running into the difficulties that encounter Kant's solution. I will be offering a justification for it in terms of a theory of space and time in Chapter Five.

The final objection I shall consider at this point is as follows. If we consider some object combined with all the nearest spatio-temporal objects to it, then this aggregate still has the properties of an object: in particular it has a size and a spatio-temporal position. Thus the Earth is an object; its combination with the other planets yields the solar system, which is an object; the combination of the stars and stellar systems yields our galaxy, which is an object. Why should not all the galaxies in combination yield the universe, which in turn would be an object?

Since the argument seeks a disanalogy I will reply by giving one. The combination of planets yields a solar system which is in contrast with other stars and stellar systems. It is in contrast with these that it has a size and spatio-temporal position. Similarly for our galaxy, and for clusters of galaxies -- they are objects because they contrast with objects outside them. But for the universe, an absolute totality of all objects, there is no object outside it, in relation to which it has a size or spatio-temporal position.[5]

[5]

It follows that, contrary to assertions made in the Thesis,

These then are my replies to various possible objections to my position. There may well be other objections I have not considered; it is asking too much of a philosopher to expect him to anticipate all the arguments that could be advanced against his conception. At least I hope, in my replies above, to have shown the general line of response I would adopt to other possible objections.

To conclude this chapter I wish to rescue from Kant's solution one aspect, which in an altered form, will prove useful in the next chapter. This is the notion of a "regulative idea". Kant's use for it in this context is this: if the universe is not actually infinite then what justification is there for our continuing with this progress of experience? Why should we not, as individuals and as a culture, live entirely in the here and now? Kant's answer is that while the idea that the universe is infinite cannot be "constitutive", cannot express a truth about the world, it can be "regulative", that is, it can operate as a rule enjoining us never to cease seeking to discover new spatio-temporal regions.

As it is used by Kant this notion shares all the drawbacks of transcendental idealism, since, while there is not reality independently of the progression of experience, this regulative idea tells us that if we do not embark and ceaselessly continue on this progress we are in some way missing something. In other words, it shares in the commitment of

the universe in space does not form a given whole. This is not an inconsistency, as in the Thesis we made the unspoken assumption that the universe is an object, which is here denied.

transcendental idealism, demonstrated earlier in this chapter to, to adapt Berkeley's phrase, the ghosts of undiscovered objects. Calling the principle "regulative" rather than "constitutive" does not resolve this paradox.

My own use of the notion of a regulative idea lies within the context of transcendental realism and my embargo on expressions which refer to the whole universe. Quite often physical theories make claims about the whole universe, yet maintain a high level of empirical accuracy in localised or restricted applications. I can think of three kinds of case:

> (1) The Big Bang Theory, which gives differential equations intended to describe the entire development of the universe.
> (2) Newton's theory of gravitation, which posits a universal force acting between all objects.
> (3) Thermodynamics, and in particular the law of entropy, which treats the universe as a closed system.

The roles that regulative ideas play in these cases are different, but if one may speak generally, the regulative ideas are used in the formation of the theory in order to give it a simple and convenient mathematical form. Thus I see the question of regulative ideas as part of the question of the relationship between mathematics and reality.

The case of the Big Bang theory can be dealt with briefly as it is discussed in detail in the next chapter. It is enough to say that for various empirical reasons a finite mathematical model gives the most favoured representation of the current behavior of the universe. Therefore mathematical equations are adopted which regard the past of the

universe as finite. There is then a problem in interpret-
ing the physical significance of these equations at the va-
lue of t = 0. I shall argue that this is because t = 0 has
no physical significance, but is a regulative idea.

In the case of gravitation it is simply the desire to
be able to give a uniform explanation for diverse phenomena
that leads to the demand that gravity be regarded as a uni-
versal force. This is a common regulative idea in modern
science, although much effort in recent years has gone into
trying to establish it as a constitutive principle with lit-
tle obvious success (this usually under the heading "the
problem of induction"). Kant would probably treat this case
as a synthetic *a priori* -- and so constitutive -- principle
himself -- the principle of universal causality. At least
it appears to be a plausible generalization of that princi-
ple. We might call it the principle of universal explana-
tion.

The case of thermodynamics is different again. We can
see two different regulative ideas here. One is the notion
of an ideal experiment. The laws of thermodynamics apply
only to closed systems, but no laboratory experiment is ever
perfectly closed. So the laws of thermodynamics apply only
approximately to laboratory systems. Thus we introduce (con-
sciously or not) the notion of an ideal experiment to which
the theory applies. We can see that Plato's ideal circle
to which the proofs of geometry apply is a regulative idea
in this sense. Another possibility, if we wished to avoid
ideal experiments, would be to say that the theory applies
exactly, not approximately, to the whole universe, since

that must be a closed system. This would still be a case of idealisation. A different way in which the expression "the whole universe" could enter thermodynamics would be if we wished to use the laws to explain, describe, or predict the large scale behaviour of the universe. Whether it is profitable to do this is an empirical matter. To apply these laws to the universe we would have to think of the universe as a closed system since it is to closed systems that the theory applies. Regarding the universe as a closed system involves regarding it as a whole. Here the regulative idea functions by sanctioning the application of theory to reality.

Thus as in Kant's original use, a regulative idea is an "as if" idea: for example "think of the universe as if it were a closed system." They can clearly be easily accomodated within an instrumentalist view of science, where all theoretical terms are treated more or less as regulative ideas. The difficulty on a realist view of science is how to demonstrate them and how to tell which uses are legitimate, and which are not. The main demarcation rule I am using is that regulative ideas refer to the whole universe. That there are other kinds is shown by the case of ideal experiments. As for the second question, that of legitimacy, I suspect that each case must be considered on its merits, and that these are disputes on which philosophers and scientists might fruitfully co-operate. One case is considered in detail in the next chapter. It would be too much of a diversion from our main purposes to consider others here.

Having now put together the general structure of my so-

lution I want next to show how it relates to current scientific thinking in this area.

P H Y S I C S

In this chapter I discuss the consequences for physics of our metaphysical discussions of the last three chapters. Contemporary physics has settled for the finite option, for both the temporal and the spatial world, encapsulated in the first case by the standard model in cosmology (the "Big Bang" theory) and in the second by Einstein's view that space is finite but unbounded. These two views are the subject of this chapter. My treatment of them differs. In the case of the Big Bang theory I shall seek to show how it can be accomodated within my general conceptual outlook, and that moreover this has advantages for the theory itself in that it accounts for the initial singularity.

In the case of Einstein I seek to explore the points of agreement and disagreement between us and, on the points on which we disagree, I give the reasons for my being unable to share Einstein's opinion. This is a less thorough and less ambitious treatment than that accorded to the Big Bang theory, but then the subject matter is more complicated.

I have decided that it is best to try to keep these discussions as free of mathematical formulae as possible, although some discussion of mathematics is inevitable. Although this will result in some parts of the discussion's being more long-winded than would otherwise be the case, I know that there are many people who could follow the argu-

ments but who are instantly dismayed by the sight of mathematical symbolism.

The Big Bang Theory

The Big Bang theory seeks to provide differential equations which represent, in a general way, the development of the universe through time. In the 1920's the astronomer Hubble postulated that the red shift which could be observed in the light from the galaxies could be accounted for in terms of the Doppler effect if we assume that the galaxies are moving away from us at great speed. By taking the inverse of the rate of expansion and assuming it to be roughly constant through time it is possible to calculate the time at which all the galaxies would have been in the same place, i.e., the age of the universe. This is known as the "Hubble Time". This can be seen to be an internal measure of time in the sense of the last chapter as it is calculated from the internal measurement of distance between here and the nearest galaxies. Further empirical evidence for the theory came with the discovery in the 1960's of the background radiation claimed to be left over from the large amounts of radiation created by the Big Bang. It was this discovery that led to the large amount of conformity in contemporary scientific cosmology.

Of course no empirical evidence is ever theoretically unequivocal and there is always the possibility of discover-

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ing more. It is quite possible that the finite models are simply mistaken. However, they are the models we currently have. By showing how they can be accomodated within my general position, I naturally do not mean to suggest that they are the only models possible, or are metaphysically necessary. Consistency is all I seek.

Nor shall I be concerned here with questions of causality, either in a scientific or a philosophical manner. Thus I do not ask why the universe began expanding, nor why the universe exists at all.

There are of course various finite models each of which gives different values to particular equations. Since all of them are finite models and all of them share the initial singularity, I shall not distinguish them, but shall continue to speak as if there were one theory.

These models set a temporal origin to the universe, $t = 0$, and construct a differential equation to describe the development of the universe from $t = 0$ onwards. The equation is generally in three parts. The first term has a positive sign and expresses the acceleration imparted to the universe by the initial "bang". The second term has a negative sign and it expresses the braking force of gravity. The third term is usually set to zero, and it represents Einstein's cosmological constant, an *ad hoc* device with no known physical meaning.

We may notice that even within the terms of reference of the theory there is a difficulty with regarding $t = 0$ as representing an absolute point in space and time. In the

construction of the theory in order to generalise from observations made from a particular place (i.e., the Earth) to statements about the universe in general an assumption known as the cosmological principle is made. This states that the universe is homogeneous and isotropic. But if the universe began with a big bang then there must be some point at which the bang occurred, a centre of the expansion -- which violates the principle. If the bang is supposed to have occurred everywhere simultaneously then how could the universe be expanding? However this is not the problem we shall be concerned with here.

Our problem is the singularity at $t = 0$. The first term of the equation represents an acceleration, that is, a second order differentiation of distance with respect to time. Since the differentiation is with respect to time, when the value of the time variable is zero the equation breaks down (as one cannot divide by zero). There is also a singularity in the second term of the equation at $t = 0$, but this is because the distances between objects are zero at $t = 0$ and so this is not a direct consequence of the temporal variable.

The mathematical reason for the singularity is clear but the problem is to understand its physical significance. Landsberg and Evans describe the universe at $t = 0$ as "unphysical". (p. 21) The other extreme would be to say that while there is a singularity in the equations, the universe must be in some state or another. An intermediate position, perhaps, would be to think of $t = 0$ as an absolute zero for time, just as we have come to accept the idea of an absolute

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zero temperature.

I shall, in my own way, adopt a form of the first of these three alternatives. I shall offer reasons for not accepting the second. The third is essentially a version of the Leibnizian view and is therefore refuted in the second chapter. We understand the notion of an absolute zero temperature because we think of temperature in terms of the motions of particles, and we think we understand the notion of rest: and so, if there is no motion of the particles, there is no temperature. For the case of time to be analogous we would presumably have to think of time as a process, or as process as such, or as the causal chain (the particular candidate does not matter), and then say that where there was no process happening, no cause and effect taking place, then that is an absolute zero time. This is clearly a restatement of Leibniz's position and subject to the objections to it provided earlier, as well as this further one. Suppose that $t = 0$ is a zero time in the sense outlined. It is quite possible at some later date for all processes and all causality to cease. In that case that too would be a zero time. It will also have to be $t = 0$. But *ex hypothesi* it is later than $t = 0$. So it cannot be a zero time. Therefore there is a contradiction in the idea that an absence of physical processes equals a zero time.

My own position is this. We cannot think of $t = 0$ as a limit of the world's past in time, for in that case there is either empty time before it or it constitutes a beginning of time, and neither of these options are acceptable. In fact we accept $t = 0$ as a regulative idea adopted in the

formulation of the theory in order to yield a finite model. I have already suggested that $t = 0$ is not a time just as no apples is not an actual amount of fruit. I intend now to show that it is not an actual limit to the world but only an ideal limit.

I have suggested that the problem of the singularity is part of the larger question of the physical interpretation of mathematical equations, a question that lies at the heart of much work in science and ought to lie nearer the heart of work in the philosophy of science. Of course no one would suggest that all the mathematical operations in a theory should correspond to real physical processes in the world. On the other hand it is a reasonable demand that they should preserve empirical significance. I have expressed that vaguely because I do not want to enter a general discussion of the question here. However I will discuss a related example before considering the case of the initial singularity of the Big Bang theory.

This case is the technique of integration over a singularity. This is a common practice in physics as it gets rid of the otherwise troublesome breakdowns in equations. There is even a joke in scientific circles (no funnier than such jokes usually are) that a professor, seeing one of his students looking especially frustrated and upset, walked up to him and murmured, "Integrate it, integrate it". I am not seeking to question integration, considered solely as a mathematical technique, but rather as a part of empirical physical science.

To take a simple example, consider the equation $y = x^{-\frac{1}{2}}$.

This has no value at the point of $x = 0$. Yet if we take the definite integral of the equation between $x = 0$ and $x = 1$ then this does have a value, namely 2. To put it another way, if you drew a graph of the function you would find that it does not touch the y axis. Yet the area bounded by the y axis, the x axis, a line corresponding to $x = 1$, and the line representing the function, does have a value, namely 2. How is it possible that while the function does not reach the y axis, its integral which has the y axis as a border, has a definite value?

Physicists that I have questioned on this problem have tended to say that as the curve nears the y axis the area under it becomes so small that it can be discounted, and that 2 is an approximation to the value of the total area. But this is simply not true, as any mathematician could tell them: 2 is not an approximation to the value of the area in the way in which 3.14 is an approximation to the value of π. Of course, the physicists I spoke to were not simply ignorant of mathematics. Rather, they gave their answer with one eye on integration's physical significance. To try to protect that they felt they had to treat the values obtained in this sort of case as approximation.

A better answer lies in what I shall call an "operationalist" approach. We cannot say that the integration is valid because it encompasses all the points of the curve (i.e., all the values of the function between $x = 0$ and $x = 1$) as the function tends to infinity as x approaches zero. Rather it is valid because it encompasses any value of x one cares to take, no matter how small. If one were to

take the definite integrals between $1/n$ and 1, where n is
an increasing positive integer, one would find that as n in-
creased the value of the integral approached 2. Of course,
$1/n$ could never equal zero and so in that sense the value
of 2 for the definite integral is an ideal, but it is an i-
deal whose validity lies in the series of operations which
produces a closer approximation to 2.[1]

This is not intended as a contribution to pure mathe-
matics. It is an attempt to understand a perfectly accept-
able piece of pure mathematics in a way which gives it em-
pirical significance.

We can apply a similar line of reasoning to the case
of the singularity in the Big Bang theory. Firstly I need
to give an account of what I take the past of the world to
consist of. As matter is not present at all stages of the
universe's past, I propose to use an ontology consisting of

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[1]
I would adopt a similar approach to the notion of a limit
as the difference in a variable approaches zero as it is us-
ed in differential calculus. Thus, for me, the value of
$f(x)$ as $\Delta x \rightarrow 0$ is an ideal derived from the series of values
for $f(x)$ for successively lower values of Δx. Thus I would
reply to Berkeley's famous phrase that we are not dealing
with "the ghost of departed quantities", but with "the ideal
of departing quantities".
   This has as a consequence that many scientific concepts
which have been defined, since Newton, in terms of the dif-
ferential and integral calculus, are themselves ideals. For
example, velocity is defined as:

$$\frac{ds}{dt}$$

where s is a distance (or a position) and t is a time. Thus
the scientific notion of "velocity" is an idealisation of our
everyday notion of "speed" and not a straightforward analo-
gue of it. That is a conclusion I am quite happy to accept.
It also implies that the idea of an "instant" of time -- as
in "instantaneous rate of change", is also an ideal. With
that consequence, too, I am quite content.

ranges of values of variables that astrophysicists in this area are most concerned with, namely temperature, density, velocity, and so on. A given band of values of these variables will be taken to constitute a stage of the universe.

In the early periods of the universe, as described by the models, the changes in these variables are much more rapid than in the later periods. This means that there are many more stages of much shorter duration. As t approaches zero, these variables tend to infinity, which means that the closer we go to zero the more shorter stages we encounter and we never actually reach $t = 0$ itself. For example in the first few seconds the drop in temperature is enormous, so there are many stages in those few seconds.

This is really only a re-expression of the original paradox of the singularity, but it puts it in a form in which it can be dealt with in an analogous manner to the treatment of integration above. It is important to bear in mind that I am discussing the possible physical significance of a model science has adopted. I am not saying what the universe must be like.

In the case of integration we regarded the value of the integral as an ideal whose validity lies in the series of operations I described. In this case I want to say that $t = 0$ is an ideal whose validity lies in the way that the stages of the universe contract temporally as we near $t = 0$. At each small fraction the stages become smaller, so we assume as an ideal an end-point at which there is not time at all and at which the stage is an extensionless point. Of course the series of stages never could result in an exten-

sionless point any more than in the case of integration $1/n$ could ever equal zero. That is why I call this an ideal. This ideal point in time is $t = 0$.

It is important to realise that by calling $t = 0$ an ideal I avoid having to say that it is an actual point a finite distance away from the present, at the same time as avoiding treating the corresponding extensionless stages as an actual stage at which the values of the variables are all actually infinite. If one wished to insist, against all reason, that $t = 0$ must denote an actual time in the past, then one must explain the corresponding phenomenon of, for example, an actually infinite temperature, or an actually infinite (not just maximal) velocity.

Of course it is quite possible for physicists to decide, on whatever grounds, that there was a stage before the present first stage of the universe. That would not contradict my treatment of $t = 0$, as an ideal, as it would simply mean that the point hitherto taken to be $t = 0$ was not in fact so, that $t = 0$ has to be, so to speak, moved backwards. My considerations would still apply to it.

My task here is a rather Wittgenstinean one, of removing a picture and substituting a different one. I think the image most people have is this. Empty time is flowing along, and at one point, which corresponds to $t = 0$, God says "Let there be light", and the universe begins. The difficulty for us human beings, which we believe we may solve or believe we shall never solve, depending on the extent of our hubris, is to understand what is going on in everyday physical terms at that actual point in the past at which God is saying "Let

there be light".

It is this picture which the argument of the Antithesis is intended to destroy (although it does not see God as a necessary feature of the picture). I have offered reasons, based on our everyday use of numbers, for not thinking that $t = 0$ is a time. To that it may be possible to reply that $t = 0$ is only a label which, in our present ignorance, we assign to this point, which is some non-zero point of empty time. However as soon as one begins speculating on the nature of this empty time, one is engulfed in difficulties.

Another part of the illusion is this. We encounter many finite intramundial processes with a beginning and an end which we can describe scientifically. We then come to think of the entire course of the universe as some gigantic intramundial process which also has a beginning and an end. The attractions of analogy here are very strong. We are then dismayed when we discover that the event which corresponds to the universe's beginning is such a very odd one. Still, some of our braver souls respond, we have accomodated some very odd things within science before, so we can probably just learn to live with this one.

My aim is to substitute a better picture. The general landscape of my picture has been sketched in the last chapter. I would draw the reader's attention to the absence of points of extramundial time, and the mistake of drawing an analogy between all things intramundial, including intramundial processes, and the universe itself. Next I stress that the adoption of a finite model is a human epistemological

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act. It is not strictly warranted by my picture of reality but it is a useful way of describing the current behavior of the universe. The idea of a time $t = 0$ is a necessary feature of the model, if it is to be a finite model, but it is not provided by reality. Thus I treat it as a regulative idea, that it is to say, an ostensibly empirical claim which is useful epistemologically (for Kant's use of the term read "or morally") but which cannot be given a straightforward physical significance (or, for Kant, "transcends the bounds of possible experience").

However in deference to the present empirical success of the theory and to its wide support among the scientific community I seek to show not only that the regulative idea is used, but also that its use can be justified. That is what I have tried to do in my operationalist view of integration and its application to the initial singularity.

I want to make two final points before I leave this subject. I became aware in the course of developing this view that my notion of mathematics, developed in accordance with requirements of physical significance, is very similar to the view developed by Greek mathematics. The Greeks also did not admit zero or infinity as numbers, as I do not admit them as numbers with physical significance, and my operationalist view of calculus is very similar to the Greek method of approximations. It appears that, at least in these respects, the *empirical significance* of modern mathematics does not exceed that of Greek mathematics, despite our improved symbolism and techniques.

Secondly, there may still be people who will insist

that either the world has a beginning, in which case its be-
ginning must be some kind of event or other, or it does not
have a beginning. The scientific evidence at present points
to its having a beginning, so there must be, unless one sim-
ply wishes to disregard science, some event which constitu-
tes its beginning.

For such people it may be sufficient to point out to
them that the heart of their argument is not a scientific
claim at all, nor, as they would probably want to say, a
belief in common-sense (although "common-sense" is mostly
out-of-date science anyway), but is a philosophical claim,
that the principle of bivalence applies not only within the
universe (which I insist it does) but also of the whole uni-
verse itself. For the refutation of this philosophical
claim I would refer them to Chapter Three.

Einstein's Cosmology

Einstein's view of space as "finite but unbounded" is
often put forward as a solution to the Antinomy. For exam-
ple, Popper has written:

> As far as space is concerned a fascinat-
> ing solution has been suggested since,
> by Einstein, in the form of a world which
> is both finite and without limits. This
> solution cuts right through the Kantian
> knot, but it uses more powerful means
> than those available to Kant and his con-
> temporaries. (CR, p. 178)

Some philosophers appear to regard Einstein's achievement

as relegating the Antinomy, at least in respect to space, to the status of an historical curiosity (although I would not wish to accuse Popper of this). This is somewhat extreme, and ignores, in any case, the fact that Einstein's view is not universally accepted. Indeed, the position would appear to be more like this: if Kant's Antinomy is a genuine one, and if the notion of finite but unbounded space, as proposed by Einstein, is the only available solution to it, then Einstein's conception would appear to have strong philosophical support. Thus instead of Einstein's views vitiating the Antinomy, the Antinomy gives grounds for adopting Einstein's view.

However I do not wish to suggest that I feel able to accept Einstein's view unreservedly. There are points on which it is in accord with my solution to the Antinomy offered in the last chapter. There are also points on which it is inconsistent with my solution, and these are the points which I find troublesome or dubious. It is not my intention here to give a thorough philosophical critique of General Relativity, a task for which I have neither the space, nor, I suspect, the mathematical competence. Instead I wish to point out the areas of agreement and disagreement between Einstein and myself, and to suggest, in the cases where we disagree, why I have difficulty with accepting his view.

Some points of agreement can be quickly sketched. We agree that the universe is not infinite. We further agree that there should not be relations between objects and empty space (I shall return to this point). Furthermore, on

both our views all routes leading to an object in the universe come from another object in the universe.

It is in terms of routes connecting objects that the main disagreement between Einstein's view and my own can be spelled out. On Einstein's view there are routes which run in a straight line from an object and ultimately back to the same object. This is normally put in terms of light rays. It is possible for a light ray to be sent from Earth, and, not being affected by extraneous forces, to eventually return to Earth again, having travelled all around the universe. In this way it is quite easy to give a sense to the size of the universe: we send the light ray out and count the years until it comes back, and that gives us the size of the universe in light-years. We can even speak in these terms of the universe's expanding or contracting. Suppose that we send out two light signals at different times. If the times they take to return are different then the universe has expanded or contracted, depending on whether the second took a longer or a shorter time than the first.

On these points Einstein's view conflicts with mine, for, as I stated in the last chapter, I do not allow that the universe has a size. Our disagreement can be expressed quite simply: on Einstein's view the universe exists as a whole while on mine it does not. This wholeness is illustrated by the notion that a light ray could travel right round the universe and return to the point it originally left. However this wholeness does not result in relations between the world and empty space because space itself is also held to form a whole, being a closed finite continuum.

My unease with this conception can be brought out in the following way. Suppose that our spatial world is finite in the manner Einstein describes. It appears possible that as our world is finite, there may be other finite worlds. There are three positions which one can adopt with regard to this possibility:

> (1) It is possible that there are other spatial worlds, but all these worlds, and our own, would be spatially related to each other.
>
> (2) It is possible that there are other spatial worlds, but all these worlds, and our own, would not be spatially related to each other.
>
> (3) It is not possible that there are other spatial worlds.

It may be wondered how the first of these can be a real possibility when we have described the space of our world as a closed continuum. The answer would be in these terms: the two-dimensional surfaces of two soccer balls form closed continua, yet they are spatially related in three-dimensional space. Similarly, it could be that our closed three-dimensional world is spatially related to some other closed three-dimensional world in four-dimensional space.

There are various difficulties with this conception. One is that this is not the view that Einstein intended. While he mathematically introduces the notion of a closed three-dimensional continuum by a reference to a four-dimensional space, this is "only for a convenient definition of our hyper-surface". (p. 185) We are not intended to take this mathematical technique as an ontological claim. A more substantial difficulty is that the analogy with the surface of a sphere in three-dimensional space tells, if

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anything, against the possibility of our three-dimensional space's actually physically existing in a four-dimensional space. For there are no two-dimensional objects in our three-dimensional space. The surface of a sphere is not a two-dimensional object. For it to be one it would need to have no thickness, and I defy anyone to produce a sphere whose surface has no thickness. The point is that we are being asked not to consider the actual surface of an actual sphere but some mathematical ideal of "surface" which we pretend, in discussions such as this, to be instantiated by real soccer balls (one has only to run one's fingers over the surface of a soccer ball to discover that it is not two-dimensional, as there are bumps and cracks). Similarly there is the April Fool's request: "Bring me one side of a sheet of paper".

However, the most serious difficulty for this option, from our point of view, is that it no longer solves the Antinomy. For the problems of the Antinomy then arise in terms of this four-dimensional space: is the number of three-dimensional worlds co-existing within this four-dimensional space finite or infinite? If we propose the same solution for the four-dimensional space, claiming that it forms a closed continuum within a five-dimensional space, then the problem can be pursued in terms of this five-dimensional space, and so on *ad infinitum*.

The second option to be considered is that it is possible for two (or more) spatial worlds to exist, but for them to be spatially unrelated. This option would, I think, be the most popular one among contemporary philosophers. I

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cannot adopt it, for adopting it contradicts a principle I
hold, which I call the strong unity thesis for space, which
denies the possibility of a plurality of spatially unrelat-
ed spaces. This principle has a general significance with-
in this thesis, and its arguments form part of the develop-
ment of my view of space, so I think it better for me to re-
serve those arguments for the next chapter. Nor would it do
the arguments justice to attempt to summarise them here.
However I can at least state the central difficulty, which
is, if there are two (or more) spaces then they must be in
some sense external to each other (since they cannot co-in-
cide), but in what way, if not a spatial one, is this exter-
nality to be expressed? I am aware that this brief state-
ment of the difficulty is hardly likely to convince anyone
who denies the strong unity thesis, but the matter is dis-
cussed in more detail in the next chapter.

The third option one may adopt is that it is not pos-
sible that there be more than one space. On this assumption
Einstein's view becomes much closer to my own. However I
have difficulty with regarding this assumption as compatible
with the assumption of a finite space. This may be a naïve
misconception, but I do not see how we can insist that some-
thing is finite if we do not allow the *possibility* that it
is not unique, even if it is in fact unique (though how it
could ever be known for certain I do not know). If it is
possible for there to be one finite, spatially self-contain-
ed world how can it be impossible for there to be two such
worlds? And if it is impossible for there to be two such
worlds, how is it possible for there to be one?

Having reviewed the three options it appears that the most promising course for a philosopher who wishes to endorse Einstein's conception is to adopt the second, that it is possible for there to be several spatially unrelated worlds. I have not yet provided a refutation of this option, but I have identified the principle which I (and Kant) hold with which it is in conflict, namely the strong unity thesis for space; and I shall defend that principle in the next chapter.

Of course it may be argued that Einstein's conception is not as close to mine as I have been suggesting, but is instead a form of Leibniz's. However I think too much can be made of Einstein's Leibnizian inheritance. Certainly, like Leibniz, Einstein was opposed to the idea of Newtonian absolute space, but we must be careful in deciding the form his opposition took, and how successful he was in avoiding the Newtonian concept. Einstein, of course, was a physicist, not a philosopher, and the philosophical aspects of his work are not as clear and well-defined as they might be if philosophy had been his main concern. Certainly his work contains philosophical elements, though these are secondary to, or expressed in terms of, his physics; and he himself, in his approach to science, displayed certain metaphysical attitudes (as came out clearly in his opposition to the Copenhagen interpretation of quantum mechanics). But we do not find a sustained philosophical treatment of some of the fundamental concepts of his theories, such as space and time.

The nearest we have in Einstein to a philosophical discussion of space and time is the celebrated discussion of simultaneity in the Special Theory. This passage suggests not an attempt at a Leibnizian reduction of space and time to non-spatial-temporal relations between objects, but rather a form of operationalism, where space and time are regarded in terms of (though not reduced to) the behaviour of the objects -- rigid rods and clocks -- used to measure them. For example, Einstein writes:

> It might appear possible to overcome all the difficulties attending the definition of "time" by substituting "the position of the small hand of my watch" for "time". And in fact such a definition is satisfactory when we are concerned with defining a time exclusively for the place where the watch is located. (Einstein, p. 39)

In the General Theory the situation is more complicated for this operationalism appears to be one of the features of the Special Theory which does not survive, although Einstein does continue to speak in terms of rigid rods and clocks. This rejection of operationalism appears to lie in the rejection of any system of co-ordinates' being privileged. For example:

> For the laws of geometry, even according to the special theory of relativity, are to be interpreted directly as laws relating to the possible relative positions of solid bodies at rest; and, in a more general way, the laws of kinematics are to be interpreted as laws which describe the relations of measuring bodies and clocks. To two selected points of a stationary rigid body there always corresponds a distance of quite definite length, which is independent of the locality and orientation of the body, and is also independent of the time. To two

selected positions of the hands of a
clock at rest relatively to the privile-
ged system of reference there always cor-
responds an interval of time of a defi-
nite length, which is independent of
place and time. We shall soon see that
the general theory of relativity cannot
adhere to this simple physical interpre-
tation of space and time. (Einstein, p.
112)

Einstein's move away from "this simple physical inter-

pretation" owes, in fact, more to Mach than to Leibniz.

Einstein holds, with Mach, that empty space and time should

not be endowed with physical properties, which is to say

that the properties of a physical system must be describable

without reference to empty space or time. This, or an al-

ternative formulation of it, is Mach's Principle. In Ein-

stein it takes the form:

The laws of physics must be of such a
nature that they apply to systems of
reference in any kind of motion. (Ein-
stein, p. 113)

Re-expressed in terms of spatio-temporal co-ordinate systems

this becomes the requirement of general co-variance:

The general laws of nature are to be
expressed by equations which hold good
for all systems of co-ordinates, that
is, are co-variant with respect to any
substitutions whatever (generally co-
variant). (Einstein, p. 117)

The demand for general co-variance is opposed to the Newton-

ian view of absolute space and time in the following way.

If there were a system of co-ordinates that was privileged

with respect to the laws of nature then that set of co-or-

dinates could be taken as representing the co-ordinates of

empty space and time. As the laws of nature are different

depending on whether these co-ordinates apply or not, then

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empty space and time have a physical reality, since they affect the laws of nature.

However there is nothing in Mach's Principle that commits Einstein to Leibniz's reductionism, nor is there anything here that I cannot accept. Indeed, I actively welcome it. I have already argued, in my defence of the arguments of the Antithesis, that there cannot be physical relations between objects and empty space and time. The difference is that the First Antinomy is not concerned with dynamics and so the Antithesis concentrates on paradoxes of *position* relative to empty space and time, rather than *acceleration* relative to empty space and time, which was Mach's main concern. In a sense the requirement of general co-variance can be seen as an extension of the requirement arising out of the Antithesis that the universe not be thought of as located in empty space and time (which is expressed in the original Antithesis text as the requirement that the universe be infinite).

This similarity between the Antithesis of the Antinomy and the sorts of considerations that lead to the requirement of general co-variance is further shown if we consider the question of how successful Einstein has been in eliminating the Newtonian concept of space. Grünbaum has suggested that Einstein has not been totally successful:

> It is now clear that the GTR (General Theory of Relativity) *cannot* be said to have resolved the controversy between the absolutistic and relativistic conceptions of space in favour of the latter on the issue of the implementation of Mach's Principle. (Grünbaum, p. 422)

Grünbaum's point of view has been developed by Sklar, who

offers five points on which the General Theory contravenes
Mach's Principle and appears to re-introduce empty space
and time as physical realities. For anyone who has studied
the Antithesis the sorts of cases Sklar mentions have a fa-
miliar ring, since they consist of occasions where, without
contravening the General Theory, it is possible to speak
of relations between the physical universe and empty space
and time. For example, Sklar cites a proof by Gödel that it
is possible, within the constraints of General Relativity,
to regard the entire universe as in a state of rotation.
Since the rotation must presumably be relative to something,
and since it is the entire universe that is rotating, it
can only be a rotation relative to empty space. I do not
have Sklar's competence to appraise the technical details
of the particular examples he produces. However the moral
I draw from these examples is that Mach's Principle, whether
Einstein's theory fully realises it or not, is very close
to a principle which we could express loosely as: do not
fall victim to the paradoxes of the Antithesis.

In this light it appears that the question:"How Leib-
nizian is Einstein?" is, from the point of view of this es-
say, very similar to the question, "How Leibnizian are the
arguments of the Antithesis?"; and to this question I have
already attempted to provide an answer in my section on the
Leibnizian Interpretation of the Antithesis. Certainly if
one traces an intellectual tradition of thinking on ques-
tions of space and time from the Leibniz-Clarke debate on-
wards, then Einstein falls on Leibniz's side rather than
Clarke's, but in the course of a tradition one can move a

great way from the starting-point; and in any case it is un-reasonably limiting to demand that a view be all Leibnizian or all Newtonian. In general, the most that one can safely say is that Einstein and Leibniz are united in what they oppose but not in what they propose. They are both opposed to the i-dea that the physical universe could stand in physical rela-tions to empty space and time, but in this they are conjoined not only with each other but also with Kant and with myself. I hope that these issues will become clearer in the next chap-ter where I discuss the relations between Newton, Leibniz, Kant and myself, and seek to show how Kant and I offer differ-ent compromises between the Newtonian and the Leibnizian positions.

There is some evidence to suggest that Einstein's ap-proach is ultimately not ontological but topological. This aspect is apparent in remarks Sklar makes on an attempt by Wheeler to remove some, at least, of the non-Machian ele-ments of the General Theory:

> Another approach, suggested by Wheeler, is to insist that the space-time of gener-al-relativistic solutions be spatially closed. Now this topological constraint on the allowable solutions to the field equations turns out to be a little more complicated than one might at first sus-pect ... but still, the motive for impo-sing this constraint upon solutions is fairly clear. If the spacetime is spati-ally closed at every time, there is no need to impose the boundary conditions of "Minkowski spacetime at infinity" since there is no boundary to the space at the time and no "points infinitely distant from a given point" or even "points as far from a given point as one likes".
> (Sklar, p. 221)

If General Relativity were thoroughly Leibnizian, and held that there was no space and time apart from an order between

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objects, then it would not need topological constraints to rule out the existence of space and time at an infinite distance from objects.

My interim conclusion is therefore that there is some scope for intellectual sympathy between Einstein's views and my own as it is developed in the last chapter and the next, while the issue that divides us is that on Einstein's view the universe can be regarded as a whole whereas on my view it cannot. It is worth stressing that it is the wholeness I object to, not the geometry. It may be believed that as Einstein's conception cannot be stated within Euclidean geometry, Euclidean geometry forms an unspoken assumption of the Antinomy. It may then be further explained that since Kant believed in the synthetic *a priori* truth of Euclidean geometry it is not surprising that it forms such an unspoken assumption. There are several points that need to be made here. Firstly there is the psychological point that undoubtedly Kant's faith in the certainty of Euclidean geometry would blind him to any requirement to discuss any geometrical aspects of the Antinomy. However there are two more philosophical points to be made. The first is that only a spherical space, not merely a curved space, could escape the dilemma of the Antinomy. There are many possibilities within non-Euclidean geometry on which the Antinomy could still be presented much as it is presented by Kant. It is the fact that space is closed, not merely curved, that Einstein's solution requires. Secondly it would not be possible for Kant (or a Kantian) to use the *a priori* nature of Euclidean

geometry as an additional assumption of the Antinomy in order to exclude the Einsteinian solution, for the *a priori* nature of geometry leads directly, for Kant, to the truth of transcendental idealism. But then the Antinomy would be making two incompatible assumptions, as it would be assuming the truth of both transcendental realism and transcendental idealism.

S P A C E   A N D   T I M E

Since I shall be attempting several distinct but inter-
locking tasks in this chapter it is important for me to be-
gin by stating clearly what my objectives shall be. The
central ambition of this chapter shall be to develop a view
of space and time, which I shall call mundocentrism, which
provides the ontological support for the solution to the An-
tinomy which I developed in the third chapter. I will ar-
rive at this position *via* a consideration of issues in the
theory of space and time which I passed over in the first
two chapters, promising to discuss them here, namely spatial
and temporal unity, and spatial and temporal infinity. The
arguments I shall advance are, at least, plausible interpre-
tations of the arguments advanced by Kant in his discussion
of the "metaphysical" nature of space and time. This leads
to my third objective, which is to discover how much of the
Transcendental Aesthetic follows naturally from the First
Antinomy. Kant himself regarded his meditations on the pro-
blems of the Antinomies as an important formative influence
on the Critique as a whole, so here I examine one obvious
possible connection. My conclusion is that the views advan-
ced in the "metaphysical" discussions of space and time are
a natural part of an attempt to deal with the difficulties
of the First Antinomy.

These objectives are interwoven into the form of a

single argument (just as I regard this thesis as in essence one sustained argument). In order to avoid distractions from the argument I shall not signpost each of these topics as it occurs: that is why I am stating them clearly here. On the assumption that clarity of expression is the most important principle of literary style I will now set them out in the otherwise unaesthetic form of a list:

> (1) To develop the mundocentric views of
> space and time as the ontological ba-
> sis for the views expressed in Chap-
> ter Three.
> (2) To deal with spatio-temporal issues
> remitted from earlier chapters, in
> particular unity and infinity.
> (3) To discern how much of the Transcen-
> dental Aesthetic is justified by or
> follows naturally from the First An-
> tinomy.

It will be noticed that I do not include among my ob-
jectives any attempt to provide a reductive account of space and time, to say what space and time "really are". I nei-
ther have such an account nor know whether such an account is a realisable philosophical or scientific aim. In this respect I am to some degree in agreement with Kant. He thought that space and time were "forms of our intuition" but that they are so is simply a brute fact. There is no-
thing especially privileged about spatio-temporal forms of intuition, but these are the forms that we have. The angels may perceive the world in a non-spatio-temporal fashion, but we cannot imagine what this is like. Similarly I shall ar-
gue the transcendentally realist equivalent of Kant's view, which is that space and time are the forms of the universe: in other words that the universe exists spatio-temporally.

This I see as simply a brute fact, just as I see the fact that things exist at all as a brute fact. I cannot answer the perhaps ill-defined philosophical worry of why there is something instead of nothing; nor can I say why that something is a spatio-temporal something. This does not prevent my describing some of the features of that brute fact.

The first feature to be discussed is the claim that space and time each have a necessary unity. I made this claim in Chapter One and it appeared again in the discussion of Einstein in Chapter Four. It is also to be found in the Aesthetic as the third point in the metaphysical exposition of space and the fifth point in the metaphysical exposition of time. As the two passages are similar I shall quote only from the one on space:

> We can represent to ourselves only one space; and if we speak of diverse spaces, we mean thereby only parts of one and the same unique space. (B39)

There are two different possible forms of this claim. The first is that there cannot be two distinct, but spatially related, spaces, or two distinct, but temporally related, times. This I shall call *the weak unity thesis*. The second interpretation is that there cannot be two spatially unrelated spaces or two temporally unrelated times. This I shall call *the strong unity thesis*.

The weak unity thesis is certainly true. If two spaces or times are spatially or temporally related then there must be an intervening stretch of space or time connecting them. There must therefore be a single spatial or temporal route from one extreme of one space or time to the opposite

extreme of the other (since each space or time has internal unity). This is equivalent to saying that they are part of one single space or time.

The defence of the strong unity thesis is necessarily more complicated. I shall defend it in the case of space, since it is in that case that philosophers have recently most strongly attacked it; there would be no difficulty in adapting the arguments to the case of time. Further, within my discussion of the case of space I shall ignore the temporal dimension. Thus if I say that two things must exist at different places I am ignoring, rather than forgetting, the possibility that they could exist in the same place at different times. This is merely to facilitate the expression of the argument and does not affect its validity -- as I shall show at the end of this discussion.

The reasons for adopting the strong unity thesis are along these lines. Consider the claim that there are two horses. This involves two claims:

> (a) The horses are different.
> (b) The horses co-exist.

The differences between the horses would lie in the horses themselves. Their co-existence depends on space: the two horses co-exist in space by existing at different places. Indeed there have been philosophers who hold that by saying that there are two horses we are in fact only committed to the second claim since the co-existing horses could be otherwise identical.[1]

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[1] These philosophers include Kant, of course: "Thus in the

In general, the plurality of spatial objects is guaranteed by the fact that the objects all co-exist within one space. If there were no way in which they co-existed then there would be no plurality.[2] However, since space is a necessary condition for the plurality of spatial objects it would appear to be a category mistake, to use Ryle's expression, to suppose that plurality can be applied to space itself. For the question then arises: in what way can two spaces co-exist? It cannot be that they co-exist in space, for then, by the weak unity thesis, there would not be three spaces (the two co-existing spaces and the space in which they co-exist), but only one all-embracing space. But then, if they do not co-exist in space, by reference to what medium is their co-existence to be understood? Two *places* can co-exist because they exist in the one space, but two *spaces* cannot co-exist.

The need for co-existence in order to have plurality can be illustrated if we consider how someone could prove that there was a plurality of objects. Suppose, for example, that I were foolish enough to insist to my wife that we possessed only one flower vase, while she insisted

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case of the two drops of water we can abstract altogether from all internal difference ... and the mere fact that they have been intuited simultaneously in different spatial positions is sufficient justification for holding them to be numerically different." (B319 - B320)

[2] Of course, the necessity of space for plurality only applies to physical objects, though the necessity of co-existence does not. For example, even if musical notes cannot be regarded as wholly physical entities they still co-exist either in the medium of the musical scale or the score of a particular composition, or in the consciousness of the listener. If there were no medium in which they co-existed there could not be a plurality of them.

that we owned two. She could win the argument at once, and establish that there were two vases, by holding them both up in front of me, i.e., by demonstrating their co-existence. If it were in some way logically impossible for her to do so (not just practically impossible: if for example, she could not find one), then there could be no justification for her claim that there were two vases.

I am aware that this argument is of a somewhat tentative character, as it does not rule out the possibility that in the advance of science some sense can be given to the notion of two spaces co-existing. If that were to happen my immediate reaction would be to enquire to what extent the weak unity thesis applied to this case (as I do above to the supposition that two spaces could co-exist in a third space). At the very least, I suspect that such an advance would commit us to the claim that there was a reality more fundamental than spatial reality,[3] and in such a case the entire spatial half of the Antinomy would have to be reconsidered.

I am also not certain that the correct conclusion of this argument ought to be that space is unitary. The reason for my doubts is that the conceptual conditions for there being a plurality of spaces -- namely a way in which those spaces co-exist -- is lacking, and it appears to me that to assert that there is one of something is to allow the intelligibility -- although not the actuality -- of there being more than one. In other words, to assign

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[3] I.e., the reality of the medium in which the two spaces co-exist.

the number "one" is to assert the intelligibility of assign-
ing numbers, and it is that intelligibility that the argu-
ment denies. However it is at any rate clear that the ar-
gument rules out a plurality of spaces.

The opposite conclusion has been most notably argued
for by Quinton. He seeks to describe a situation in which
he thinks it would be intelligible to speak of more than one
space. He asks us to imagine that a man in England falls a-
sleep each night and immediately finds himself awakening to
a day within a social community at a tropical lakeside.
When he falls asleep again at the lakeside, he re-awakens to
a normal day in England. A mid-day nap in England results
in a nocturnal awakening at the lakeside; when asked at the
lakeside to describe his dreams he tells of the previous day
in England. Thus Quinton argues that an individual's expe-
rience could exist in more than one space.

However, does Quinton's myth (as he himself calls it)
solve the co-existence problem? I would not seek to legis-
late about the experiences an individual may or may not
have, but for those experiences to have any ontological for-
ce there must be some way around the ontological problem:
otherwise we have only some very strange experiences. The
ontological problem is the co-existence problem. Quinton's
myth only appears to solve this problem if we ignore an im-
portant shift in the middle of the argument. We are intro-
duced to the lakeside space by means of the dreams of this
particular individual. In that way the lakeside space and
England space appear to co-exist: one exists "in the world"
and the other exists in the consciousness of the dreamer.

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However in that case the distinction is not between two spaces, but between reality and dreams. Therefore Quinton asserts that the lakeside's space is as ontologically sound as England's space. But then the individual's consciousness drops out of the ontological picture since the existence of the lakeside space can no longer be in terms of the individual's dreams (since the existence of England's space does not depend on the individual's dreams). Therefore we cannot say that the co-existence of the lakeside space and England's space is explained in terms of the consciousness of the dreamer, since the putative existence of each is independent of that consciousness. Therefore Quinton's argument fails to solve the co-existence problem.*

In the last chapter I claimed that the strong unity thesis spoke against the possibility of there being two or more spatially unrelated Einsteinian worlds, each within a closed spatial continuum. It is perhaps a coincidence that Quinton should challenge the strong unity thesis at a time when the General Theory is exerting a strong influence on philosophy, at least to the point of making philosophers wary of it. In any case Quinton's highly psychological approach does not allow his argument to be easily utilised within theoretical physics, where we would require a myth which involved a plurality of normal physical spaces. In such a myth both spaces would have to be introduced objectively, unlike Quinton's myth where one space is introduced subjectively, in terms of an

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* Thus whatever Quinton's dreamer reported to us of his experiences (and one cannot legislate for such reports, or even for such experiences), we would not take this as evidence of a plurality of spaces since we have prior reasons for rejecting that plurality -- namely the co-existence problem -- which these reports or experiences do not remove.

individual's dreams. As I have already admitted I do not rule out the possibility of such a myth's ever being produced, or even adopted within science, but I make this admission with the caveat that its adoption would force a revision not only of the strong unity thesis but also of all the rest of our thinking about space and the spatial world.

Finally, in this discussion, I must redeem my promise to show that the exclusion of time from this argument does not affect its validity. It is possible, of course, for plurality to involve co-existence in time, if we take "co-existence" timelessly to mean "existence at different points in the temporal series". Thus we can speak of there being two different buildings on the same site because one is demolished and replaced by the other: they exist at different times. It may perhaps be argued that there could be a plurality of spaces which co-exist in this way, that is, exist at different times. I doubt whether a philosopher who sought to deny the strong unity thesis for space would be satisfied with only this. He would want to argue that there can be a multiplicity of spaces existing *simultaneously*.[4] Moreover, it is not clear that a multiplicity of spaces can be established in this way as easily as a multiplicity of buildings. Suppose that space and the spatial world were instantly annihilated and then replaced. Would this create two spaces or one space existing at different times? Even if we suppose space and the spatial world to be instantly

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[4] How simultaneity is to be defined across spaces is a further problem, but one that I shall not go into.

annihilated and replaced with some space and a spatial world quite different in content, the issue would not be clear-cut. Would we then have two spaces and two worlds, or one space existing at different times and containing different worlds? I do not see that it matters which we say, so I am prepared to concede that in these circumstances the strong unity thesis has no force, but that in these circumstances it also has no importance.

The unity theses have the following important consequence: they rule out the possibility of there being distinct and separable spaces or times. However, as spatial and temporal concepts imply the existence of a space or time within which those concepts can apply, this means that spatial and temporal concepts cannot be applied to space or time. This is important when we think about the infinity of space and time.

I claimed in the first chapter that Kant's claim that space and time are infinite should not be taken to mean that space and time are infinitely extended. The previous paragraph shows that the infinity of space and time could not consist in their being ubiquitous and eternal -- which is what their infinite extension would require -- since "ubiquitous" and "eternal" are spatio-temporal concepts (as is clear if their negations are considered), and so cannot be applied to space and time. Thus my objections to an infinite world in Chapter One are not confounded by the existence of infinitely extended space and time.

But then what does Kant mean when he says in the Aesthetic that space and time are infinite? In the case of

space he says: "Space is represented as an infinite *given* magnitude," (B39) where it is the word "given" that is stressed. He develops this:

> (N)o concept, as such, can be thought as containing an infinite number of representations *within* itself. It is in this latter way however that space is thought. (B40)

This is clearly an argument against the possibility that space might be a concept. Kant is arguing, it appears, that a concept can have an infinite number of instances, but that these instances are all outside the concept. The metaphor implicit in the use of the word "outside" (which I take to be the opposite of "within") may not prove easy to interpret. Space on the other hand contains all its parts "within" itself, and this metaphor too is not easy to understand. However these matters become clearer when the corresponding passage on time is examined. There Kant makes clear what he takes infinity to involve in this context:

> The infinitude of time signifies nothing more than that every determinate magnitude of time is possible only through limitations of one single time that underlies it. The original representation, *time*, must therefore be given as unlimited. (B47-48)

This argument rests on a proper understanding of the relationship between a "determinate magnitude of time" and the "one single time that underlies it". A determinate magnitude of time I take to be a temporal distance, the interval between two temporal events. Suppose we consider two such events, e_1 and e_2. Consider now the temporal interval between e_1 and e_2. Call this temporal interval t_1, and call analogously introduced intervals t_2 - t_n. Then, Kant

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argues, it would be a mistake to consider time as a conjunction $t_1 + t_2 + ... + t_n$. Rather, Kant says, the temporal interval is a limitation of time -- it is arrived at by imposing limits, namely e_1 and e_2, on time. As limitations of time constitute temporal intervals, and as time is not a temporal interval, it follows that time has no limits. It is in this sense that it is infinite, not in the sense that it is an infinite conjunct of temporal intervals: for it is not a conjunct of temporal intervals.

Another consideration in Kant's favour here is the fact, already noted, that temporal concepts cannot be applied to time. When we say "a long time" it is a temporal interval we are referring to. As we can apply temporal concepts to temporal intervals, but cannot apply them to time, it follows that time is not a temporal interval.

The same arguments also apply to space. It is perhaps not as intuitively obvious that space is not extended, but that may be because we tend to think of space as equivalent to air. If we refrain from thinking of space in this material way then we can see that space cannot be equivalent to spatial distances. When we say that it is a long way from x to y it is a spatial distance we are referring to, and so by our principle that spatial concepts cannot be applied to space itself ("long" here being a spatial concept) it follows that space cannot be a spatial distance, so *a fortiori* it cannot be an infinite spatial distance.

This interpretation of the infinity of space and time is clearly an unusual one, but then the usual notion of infinite extension has been rejected. Thus it is perhaps bet-

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ter to use the word "unlimited" rather than "infinite".

This view of the unlimitedness of space and time can provide a theoretical rejection of the possibility of a finite world, to supplement the *reductio* rejection provided in the Antithesis. Consider the case of time. Suppose we represent time by the usual image of an extended line:

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Suppose then that we represent the finite temporal world thus:

(- - - -)

That is to say, it occupies a limited amount of time. Then the picture of a finite world in infinite time is this:

(- - - -)

That is, the temporal world can be given a place in time of which it occupies a finite amount. But this picture cannot be accurate because, as shown above, the infinity of time does not consist in its being infinitely extended. Time is not "stretched out" in the manner the picture suggests. Thus it does not provide an extension of which a finite world could occupy a part.

We can obtain independent confirmation of this using a Leibnizian principle, the identity of indiscernibles. Applied to objects this principle is usually held to be false as it is possible that there should be two identical objects with different spatio-temporal locations. That may or may not be the case. However, that defence cannot be used if the principle is applied to points of empty space and time,

since they do not have spatio-temporal positions. If they
did then there would be an infinite regress, since if a point
in space and time had a spatio-temporal position, and since
that spatio-temporal position would itself be a point in
space and time, then it too would have a spatio-temporal po-
sition, and so on *ad infinitum*. But then what would distin-
guish one point of time from another if there were no events,
if nothing happened? It cannot be said, "Their different
relations to other points," since that presupposes that the
points are distinguishable. Thus on this ground also there
cannot be an infinitely extended empty time. Thus the idea
of a finite world as a small island of events in a sea of
empty space and time is an inappropriate one.

This discussion is reminiscent of the Antithesis. As
in the case of the Antithesis it is tempting to draw a Leib-
nizian conclusion from it. For the inescapable conclusion
of these arguments is that spatio-temporal extension only
occurs within the world, among events and objects. From
this it is natural to draw the further conclusion that space
and time are solely properties of objects, and are not in
any way real. Kant was aware of this possible move and was
strongly opposed to it. Two of the metaphysical expositions
of each of space and time express this anti-reductionism.
As the corresponding passages for time are very similar, I
will deal only with those of space. The first passage reads
thus:

> For in order that certain sensations be
> referred to something outside me ...
> and similarly in order that I may be a-
> ble to represent them as outside and a-
> longside one another, and accordingly as

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not only different but as in different
places, the representation of space
must be presupposed. The representa-
tion of space cannot, therefore, be em-
pirically obtained from the relations
of outer appearance. (B38)

I interpret this argument as a generalised circularity char-
ge: in any attempt to reduce space to relations between ob-
jects, Kant is saying, there will be some point at which the
notion of space has to be smuggled in illegitimately. Thus
the full force of this claim can only be seen in the expo-
sure of the circularity in a particular case of reduction-
ism. This I have done in the Antithesis chapter. The ac-
tual form that the circularity took, as I saw it then, is
slightly different from the form Kant suggests that it is
likely to take, though the two are very close. I sought to
show that the relations between objects from which the con-
cept of space is to be derived must be assumed to be *spatial*
relations, not just relations *per se*. Here Kant seems to
suggest that the very idea of objects already commits us to the
existence of space since these objects must be external to
one another and their externality can be understood only
spatially. I think, once again, that this claim would need
to be considered in the context of a particular attempt at
reductionism.

The other anti-reductionist argument Kant offers is
more difficult to assimilate. He says: "We can never repre-
sent to ourselves the absence of space, though we can quite
well think it as empty of objects." (B38) This is a very
difficult claim to interpret. It appears to contradict the
claim throughout the Antinomy chapter that empty space and
time are not possible objects of experience. How then can

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we think space "as empty of objects" if empty space is not a possible experience? Furthermore, it is far from clear what the logic of this passage is intended to be. If the passage is intended as a psychological note -- a remark on Kant's own powers of imagination, or a challenge to the imagination of the reader -- then not only is it difficult to see what the philosophical force of it could be, it is also unclear just what it is we are supposed to be trying to do. If I allow my mind to go completely blank then have I succeeded in representing the absence of space? Or if I concentrate on logical truths does that involve representing the absence of space? (for logical truths have no spatial or temporal component for Kant). Or on the other hand, if I let my mind go completely blank am I then representing to myself space empty of all objects? If there are no objects then what is the difference in a mental image between the presence of space and the absence of space?

However no non-psychological interpretation of this passage lies immediately to hand. Under these circumstances the best that I can do is to offer an argument with a rather similar point, but which is not similar enough to be plausibly called an interpretation of Kant's text. Let us imagine a universe of objects that do not exist in space and time. Such a universe could not contain objects in any way similar to our own. It is not simply that such a universe would be rigidly determinate. In a rigidly determinate universe, for some object description e it is determined which of $e \text{ v} \sim e$ is the case. In a spaceless and timeless universe the disjunction of $e \text{ v} \sim e$ would not even make sense. For

its intelligibility requires that there be some place which either e or $\sim e$ could occupy: that is, if e is to be anything like an object. If e is merely a concept then this requirement may not be necessary. For example we may think the description "the lowest even prime greater than 2" can designate or fail to designate a number even if numbers are non-spatial and non-temporal, but then a number is not similar to tables, chairs and stars and the other objects that make up the world. A world of concepts, such as a world consisting of numbers, would not be anything like our world.

We might express it this way: space and time are necessary for the possibility of objects.[†] If we say, "That yellow cushion might have been a blue cushion," we mean, "Instead of there being a yellow cushion *there* there might have been a blue cushion *there*." If no location is allowed then the sentence makes no sense, for we should not forget that there is a trite but important sense in which a yellow cushion cannot be blue (as the Fool told Lear, the reason seven stars are seven is a pretty reason).

Thus in a world which is neither spatial nor temporal it is

[†] Strawson (in Chapter Two of his book Individuals) seeks to arrgue that there could be an objective world of objects that was non-spatial, offering the model of a purely auditory world. There is no space to discuss Strawson's argument in detail here, so I will remark only that even if we allow Strawson's unargued assumption that an auditory world would be non-spatial (which involves our forgetting all our actual knowledge of the physical nature of sound) then Strawson's assumption that it would be natural to re-identify qualitatively similar patches of sound encountered at identifiable points in the varying pitch of a "master sound" as independently existing particulars only holds because we have the analogy of re-identifying bodies in space before us: there is no reason to suppose that someone born into a purely auditory world, without the experience of bodies existing apart in different regions of a space he could move around in, would be able to interpret his auditory sensations in the Strawsonian manner.

not simply that things could not be other than they are because it is determined how things are, it is rather that the things could not be other than they are because it makes no sense to speak of things as being other than they are; but in that case is it possible to speak of things *as* they are? To that question I have no sharp answer. However I am prepared to commit myself to the claim that such a world would not be a world of objects, where objects are things like tables, chairs, and stars, or even protons, electrons, and quarks. So Kant's anti-reductionism is vindicated.

However this does not lead to the claim that space and time could exist entirely empty of objects, which has already been dismissed. Our considerations so far seem to lead to this conclusion: spatio-temporal relations exist only within the world, but are not ontologically reducible to other components of the world. The question then arises: so what are space and time? It is in the light of this question (rather than as a solution to the Antinomy or as a doctrine of the epistemology of geometry) that transcendental idealism looks most appealing. For it says: space and time are forms of our intutition. As forms of intuition they order the objects of the world into spatio-temporal magnitudes. Moreover, as a world presented by intuition is the only world we can know, it is not surprising that objects are (for us) irreducibly spatio-temporal. Thus we can understand how spatio-temporal extension is a *necessary* feature of the world, while extended space and time are not a *possible* feature of it, for space and time themselves are features of our sensibility, rather than the world. The answer lay within ourselves all the time.

However, while this looks an attractive answer to our

present problem, adopting it would commit us to all the shortcomings of transcendental idealism as a solution to the Antinomy, and so I must reject it. I shall instead adopt what I take to be the realist equivalent of this view, namely that space and time are not forms of our intuition, but are instead forms of the universe. This appears only to consist of the unhelpful claim that our world is a spatio-temporal world, but it is not unhelpful to understand this claim correctly. Firstly, by asserting that space and time are forms of the world I am denying that they are objects in the world, which amounts to denying that they are objects at all. This takes account of the requirement that empty space and time cannot be extended. Yet, secondly, any object that does exist in the world must fall under the form of the world: i.e., it must be spatio-temporal. That takes account of the irreducibly spatio-temporal character of objects. Thus the claim that space and time are forms of the world follows naturally from the features of space and time we have been considering.

Moreover the unity of space and time, defended earlier in this chapter, guarantees the unity of the world. We are thus justified in calling the world a *uni*verse.

Unfortunately, I do not think we can separate the sense of "form of the world" from these points about space and time. For not only are space and time forms of the world, they are the *only* forms of the world, so the nature of this ontological pigeon-hole cannot be explicated except by referring to space and time. Still, this ontological uniqueness is preferable to an ontological mis-classification.

On Kant's view space and time are forms of our intuition where "our" refers to the human race (though it is the human race in contrast to the angels, rather than in contrast to other corporeal terrestrial beings). So we might call his view "species-centric". On my view, space and time are forms of the universe, so I call this view *mundocentric.*

Thus my view is a modified form of Kant's. I largely agree with Kant on what we might call the logic of space and time (the views developed by the metaphysical expositions), but I disagree with him on the ontology (the view developed in the transcendental expositions).

As forms of the world, rather than objects in it, space and time cannot be said to be *within* the world. They are not perceivable (as Kant says, "Now time cannot by itself be perceived." (B225)). What are perceivable are spatio-temporal intervals which hold between objects or events. Space and time can no more be perceived without objects than objects can be perceived without space and time. Space and time on the one hand, and objects on the other, do not have a *separable reality.*

This is a point of the utmost importance in our present context, for it holds the key to unlocking the Antinomy. There are two dichotomies central to the Antinomy. One is the dichotomy between the world on the one side and space and time on the other, and the second is the dichotomy between a finite world and an infinite world. It is because the first of these dichotomies is misunderstood that the second arises and presents insoluble difficulties. Once we think of space and time as having extension and an existen-

tence apart from objects we are led to worry about how much space and time the world occupies, a finite amount or an infinite amount. We see the size of the universe as a relation between objects on the one hand and space and time on the other. Thus the finite-infinite dichotomy becomes a feature of the objects space and time dichotomy, and the choice between a finite and an infinite world appears unavoidable.

By showing that space and time do not have a reality independently of objects, and thus that the world as a whole cannot be said to be "in" space and time, in any ordinary sense of that preposition, and thus need not have any determinate magnitude "in" space and time, the mundocentric view of space and time vitiates the Antinomy.

In Chapter Three I offered a logical solution to the Antinomy which regarded the universe as existing internally but not externally. This was equivalent to regarding the universe as not being an *object*. We can now deduce this same consequence from mundocentrism. Space and time are the forms of the world. Therefore any object is a spatio-temporal object, for example it has a spatio-temporal position. Yet the universe cannot have a spatio-temporal position as that would require that it have a place in a space and time that extended beyond it, which would in turn require that space and time exist separately from objects. Therefore as all objects are spatio-temporal objects, the universe cannot be an object.

The relation between mundocentrism and the no external existence thesis can perhaps be expressed most crisply thus: if the world is to exist externally there must be something

other than the world which the world is external to. That
something other would be space and time: but mundocentrism
shows that space and time do not exist as something other
in the required sense.

How does this approach relate to the different views ex-
pressed in the debate between Leibniz and Clarke? This is
an important question since Leibniz and Clarke represent
the Scylla and the Charybdis of thinking on space and time,
and to be successful a new view must show that it can sail
between them.

Clarke's view, which is a statement of Newton's, that
space and time are independent extended entities, is as un-
satisfying to me as it was to Kant. In the Metaphysical
Foundations of Natural Science, where Kant is perhaps keen-
er than in the Critique to be indulgent towards the Newton-
ian view, he allows that absolute space may perhaps be ad-
mitted to science as a regulative idea, but may not be treat-
ed as a straightforward constituent of the universe:

> To make this absolute space an actual
> thing means to mistake the logical uni-
> versality of any space, with which I
> can compare each empirical space as be-
> ing included in it, for a physical uni-
> versality of actual compass, and to mis-
> understand reason in its idea. (MFNS,
> 482)

In the Critique he suggests that absolute space and time
would lead inevitably to Berkeleian idealism:

> (I)f we reflect on the absurdities in
> which we are then involved, in that two
> infinite things, which are not substan-
> ces, nor anything actually inhering in
> substances, must yet have existence,
> nay, must be the necessary condition of
> the existence of all things, and more-

> over must continue to exist, even
> though all existing things be removed--
> we cannot blame the good Berkeley for
> degrading bodies to mere illusion.
> (B70 - 71)

Thus both Kant and I agree with Leibniz in rejecting

the Newtonian conception defended by Clarke. However nei-

ther of us is in any closer agreement with Leibniz's posi-

tive doctrines than we were with Clarke. That is because

both Kant and I are anti-reductionists with regard to space

and time while Leibniz is in this regard the reductionist

par excellence. I will not repeat the passages from Kant,

quoted throughout this thesis, which express his anti-reduc-

tionism. It could be said that the word "absolute" in "ab-

solute space and time" contains an ambiguity. On the one

hand it expresses the view that space and time are extended

existents in their own right, and in this sense it is rejec-

ted by Leibniz, Kant and myself. However, it can also be

taken to express the irreducibility of space and time and

in this sense it is rejected by Leibniz and defended by

Kant and myself. Since Leibniz rejects absolute space and

time in both senses and Clarke maintains it in both senses,

the two senses become confused in their correspondence so

that it can come to appear that to oppose absolute space in

the sense of regarding empty space and time as extended ex-

istents one must believe that space and time can be reduced

to fundamentally non-spatio-temporal relations between fun-

damentally non-spatio-temporal objects. However if one does

conflate these two issues then Kant's doctrines of space and

time must appear hopelessly self-contradictory and incompre-

hensible. Perhaps it was frustration generated by others'

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making this mistaken synthesis that led Paton to write:

> It is sometimes assumed to-day, perhaps
> by a lack of transcendental reflexion,
> that if we accept a relational theory
> of space we must reject Kant and fol-
> low Leibniz; but ... Kant's theory of
> space is at least as relational as that
> of Leibniz. (Paton, p. 79)

It may be that the relational aspects of Kant's doc-
trines are unnoticed not only because "relational" is as-
sumed to mean "reductive", but also because attention, when
thinking about this aspect of Kant's work, tends to concen-
trate on transcendental idealism. Thus my own view stresses
the relational aspects of Kant's theory not only by avoid-
ing the distracting influence of transcendental idealism,
but also by making those relational aspects do all the work
of solving the Antinomy. For the doctrines of mundocent-
rism -- that there is no independently existing space and
time but that objects are necessarily spatio-temporal --
is, at least in contrast to Newtonian absolute space, a re-
lational view.

It may be thought that because my view has these re-
lational aspects and does not contain transcendental ideal-
ism it is really just a disguised form of Leibniz's. It
is instructive to show that this is not so. On Leibniz's
view a finite universe is possible since, as space and time
are not really present in the universe, such a world would
not be subject to the paradoxes of the Antithesis. However
on my view a finite universe is not permissible. I hold
that any object must be a spatio-temporal object. So if
the universe forms a finite totality -- which is the same
as regarding it as an object -- then it must be a spatio-

temporal object. Thus it must, for example, have a spatio-temporal position: and in this case we *are* in the position of the Antithesis, and subject to the objections produced there. So Leibniz's views and mine are quite different.

To bring the debate up to date would involve a thorough discussion of space and time in General Relativity. Some discussion of this is provided in Chapter Four. However, the time is not yet right for an authoritative assessment of Einstein's position *vis-a-vis* Leibniz and Clarke (and the subsequent development of the debate) partly because General Relativity is far from fully developed and tested as a physical theory in its primary area of the explanation and prediction of gravitational phenomena, and so it is unlikely that we have as yet the final version, and also because many different groups with different approaches are working on the conceptual development of the theory. Einstein's achievement is still too recent to be given an accurate historical placing. In Chapter Four I expressed a general intellectual sympathy with Einstein, but there are possible developments of General Relativity, for example along the road of spatio-temporal reductionism, from which I would feel intellectually more distant.

There are two final points I wish to raise before ending this chapter. When discussing the infinitude of space and time I said that the sense of "infinite" when applied to them was an unusual one and that a better word was "unlimited". It is now possible to give some greater sense to that word. Space and time are unlimited in that they are not bound by spatio-temporal structures: only objects

are. Space and time do not have their existence limited to particular spatio-temporal regions as tables, stars, empires and people do. This is because they do not exist in spatio-temporal regions at all, not because they exist in all of them.

An analogy from another area of philosophy may be useful. It is this: the unlimitedness of space and time is akin to the absoluteness of the Hobbesian sovereign. The Hobbseian sovereign is not bound by any legal strictures or obligations. This is not because he is an anarchist. The anarchist and the man of law live on the same legal level defined by the laws: but the anarchist lives outside the law and the man of law lives within it. The Hobbesian sovereign however transcends this legal level: he is the source of all law. He is not as much *outside* the law as *beyond* it. His absoluteness is a matter of logic, not political ideology.

Similarly the unlimitedness of space and time lies not in their existing at all places and moments but in their existing *beyond* places and moments as, in some sense, their source.

The second point I wish to finish with is this: how Kantian is my view of space and time? I have suggested throughout that it is a realist analogue of Kant's idealist view. I wish now to end by suggesting, provocatively, that it *is* Kant's view, but with the idealist content discarded. If this is true, then it is important for two reasons. Firstly it shows that transcendental idealism is not an essential feature of Kant's view of space and time, since it can

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be rooted out while the rest of the doctrines remain undisturbed. Secondly, as this thesis attempts to show, transcendental idealism is not needed to solve the First Antinomy. The material provided by the Metaphysical Expositions is adequate for that. Thus if we consider the importance of the Antinomies in the development of Kant's critical philosophy, then the evidence from the First Antinomy would appear to be that Kant need never have become a transcendental idealist. Whether in that case he would ever have written a work recognisably similar to the Critique of Pure Reason is one of those hypotheticals of history to which no sure answer can be given. Certainly any attempt to remove all traces of transcendental idealism from the work would require a substantial rewriting of it. However, advances since the eighteenth century in science and mathematics and, to a lesser extent, in psychology and philosophy, make such a rewriting desirable in any case. If the rewriting were to incorporate realism, instead of idealism of whatever form, it would make more readily available to our present intellectual pursuits the wealth of Kant's long labours.

A P P E N D I X   O N E :

The Structure Of The First Antinomy

I have concentrated, especially in the second half of this thesis, with developing my own views, which can be summarised as the inter-twined doctrines that the universe exists only internally and that space and time are mundo-centric. In this appendix I want to return to Kant and to offer an overview of the structure of the First Antinomy. This makes no claim to completeness. It is not a companion or guide to the First Antinomy but an attempt to display clearly the main points, to show their development, and to comment on some of these points which have not already been discussed in the main text. In any case, as Kant was prone to repeat himself and to offer the same argument in several different forms, a point by point account of his text is not always a service to someone whose aim is a basic understanding of Kant's position, and is finding it difficult to see the wood for the trees.[1]

There are two areas of Kant's discussion of the Antinomy that I intend to omit from my description of its structure. One is his concern with the "conditioned" and the "unconditioned (see especially sections 1 and 7). This phrase-

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[1] Kant sometimes seems to take seriously Lewis Carroll's dictum in "The Hunting of the Snark" that "what I tell you three times is true."

ology lends itself most obviously to questions of causality, but, while Kant does apply it to causality, he means it to have a wider sense than that. The meaning he intends for it is roughly this: if some member of a series exists then all the preceding members of the series must also exist. Kant speaks naturally in section 1 of the world as a whole as existing as a "composition". (B443) When he speaks of space he says:

> Nonetheless the synthesis of the manifold parts of space, by means of which we apprehend space, is successive, taking place in time and containing a series. (B439)

Thus at this point, before the Antinomies have been produced, Kant is already thinking of the world in a *serial* fashion. He then poses the problem of the First Antinomy in terms of this series: if the present state is a member of a series of states of the world then the totality of previous states must exist, and this totality must be either finite or infinite.

Or as Kant puts it:

> If the conditioned is given, the entire series of all its conditions is likewise given; objects of the senses are given conditioned; therefore, etc. (B525)

Kant's answer to this is that the conditions are not given in the world as a totality but are only set as a task:

> (T)hey are possible only through successive regress, which is given only in the process in which it is actually carried out. (B529)

Clearly this presentation of the issues of the Antinomies in terms of the conditioned and the unconditioned is

one that Kant thought important and worth discussing, but it seems to me to be one peculiar to Kant's own way of thinking and without wider interest. As it parallels the general development of the Antinomy without adding anything to it, and can only be a source of confusion, I propose to disregard it henceforth.

The other aspect of the Antinomies chapter that I do not intend to discuss is Kant's characterisation of the proponents of the Thesis and the Antithesis (to be found in Section 3). In a prolonged and difficult work such as the <u>Critique of Pure Reason</u> an author can be excused for relaxing occasionally and speaking in a general fashion, and that is what Kant does here. His remarks may be interesting, but they are not important, nor relevant to the development of the argument.

To turn to the Antinomy itself, the central assumption underlying it is the assumption that the sensible world is transcendentally real. This assumption operates at two levels. It operates *within* the arguments themselves, and it operates *between* the arguments. It operates between the arguments as the demand that the world have a determinate magnitude, i.e., that it be either finite or infinite. As the arguments are of a *reductio* form this means that from the *reductio ad absurdum* of one position the other can be inferred.

Transcendental realism works within the arguments by setting the conditions whose violation constitutes a *reductio ad absurdum*. In the Thesis this is the demand that possibilia do not exist in the universe of past time or of space:

all things in space or past time are actual. In the Antithesis there is the demand that a finite temporal or spatial world must have a determinate position in empty space and time. Kant then demonstrates that in each case the appropriate demand of transcendental realism cannot be satisfied. Then in each case he infers -*x* from "*x* is absurd" and so arrives at the Antinomy. Not only must the world be finite, because it is absurd to suppose that it is infinite, it must also be infinite because it is absurd to suppose that it is finite.

At this point one possible reaction might be to adopt an epistemologically pessimistic position (as some religious people do towards knowledge of God) and claim that the truth about the whole world is simply ineffable, beyond human comprehension. In sections 4 and 5 Kant argues against this, adopting, though without stating it explicitly a transcendentally idealist pose. The object, which corresponds to the whole world, is not given to us in experience. Therefore our conflicting claims about its finitude or its infinitude cannot arise from the object, but can only arise from within our own reason; therefore the conflict ought to be resoluble by our own reason. The way it is to be resolved, Kant suggests in section 5, is to cut our conceptual clothes to suit our experiential cloth. As he puts it himself:

> We have said that in all these cases the
> cosmical idea is either too large or too
> small for the empirical regress, and
> therefore for any possible concept of the
> understanding. We have thus been main-
> taining that the fault lies with the i-
> dea, in being too large or too small for
> that to which it is directed, namely, pos-
> sible experience. (B517)

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These ideas are developed in the next section in which Kant explicitly introduces transcendental idealism. However at this point it is natural to ask what the logical position of the Antinomy becomes once the transcendental realist assumption upon which it is based is denied. J. E. Llewelyn has suggested that there is some confusion in the text between the view that both sides of the Antinomy are now "mere nonsense", (B513) or whether "both statements might be false". (B532) Kant does appear uncertain about this. My own view of the situation is this. If we concentrate on how the assumption of transcendental realism operates between the arguments then we no longer have any reason, once we deny the assumption, to suppose that the world is finite or infinite, nor any requirement that it should be one or the other. Moreover given the importance of the rôle of transcendental realism within the arguments, then once it is denied neither of the arguments have any force. However this does not of itself prove that the world *is* neither finite nor infinite, though if it is either it is not for any reason given in the Antinomy.

Kant appears to have perceived the situation in a similar fashion for once he has introduced his solution in terms of transcendental idealism (which I criticise in Chapter Three), he then seeks to prove that on this view the world, or the regress of experience as it now is, is neither infinite nor finite.

Firstly Kant rules out the possibility that the regress is infinite. His argument is that as we have no means of determining the magnitude of the regress in advan-

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ce we cannot say that it is a regress *in infinitum*. The
most that can be said is that it is a regress *in indefinitum*.
This distinction suffers the same difficulties that I sought
to expose in Kant's sequential solution in general, in that
it depends on the ill-defined ontological status of objects
not yet countenanced within the regression. As Kant says:

> (W)e cannot therefore say that this re-
> gress proceeds to infinity. In doing
> so we should be anticipating members
> which the regress has not yet reached,
> representing their number as so great
> that no empirical synthesis could at-
> tain thereto, and so should be deter-
> mining the magnitude of the world (al-
> though only negatively) prior to the
> regress -- which is impossible. (B547)

Thus the difference appears to be that a regress *in infini-
tum* makes a claim about objects not yet encountered, while
a regress *in indefinitum* makes no such claim. Yet the re-
gress *in indefinitum* appears to claim at least that at any
point there is always another object yet to be encountered:

> This rule says no more than that, how-
> ever far we may have attained in the
> series of empirical conditions, we
> should never assume an absolute limit,
> but should subordinate every appearan-
> ce, as conditioned, to another as its
> condition, and that we must advance to
> this condition. (B547)

Thus the distinction appears to be between, "there are an
infinite number of *x*'s," and "However many *x*'s there are,
there are always more," a distinction I find highly dubious.
Equally dubious is the distinction Kant is employing between
regarding the regress as infinite and having a regulative
idea that there are always more members in the regress than
have already been encountered. The difference is not, as
Kant suggests at B542 - 3, between *finding* another term and

enquiring for another term; it is between *finding* another term and *enquiring successfully for* another term: and enquiring successfully seems very close to finding.

In part the guarantee that the enquiry will be successful is provided by the argument that the regress cannot be finite. This argument has already been encountered in Chapter Two, in the First Analogy Interpretation, and is assessed there. Kant expresses it as follows:

> (T)he world would be limited on the one hand by empty time and on the other by empty space. Since, however, as appearance, it cannot in itself be limited in either manner -- appearance not being a thing in itself -- these limits of the world would have to be given in a possible experience, that is to say, we should require to have a perception of limitation by absolutely empty time or space. But such an experience, as completely empty of content, is impossible. (B549)

This argument is in two stages. Firstly it is argued that, because of transcendental idealism, the finitude of the sensible world requires a possible experience of that finitude. It is then argued that no such experience is possible since absolutely empty space and time cannot be experienced. I have, in my main text, shown that there is an intellectual consonance between this passage and the First Analogy (where, for example, he says, "For a preceding empty time is not an object of perception." (B231)), and with the Refutation of Idealism (where he states, "All determination of time presupposes something *permanent* in perception." (B275)). This connection between time and the perception of objects, which is found in these three passages but nowhere receives

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a sustained treatment in its own right (as opposed to as a means to other ends) can perhaps be taken to be a minor theme of the Analytic and Dialectic.

Kant's treatment of the First Antinomy ends with a statement of what he claims to have proven:

> All beginning is in time and all limits of the extended are in space. But space and time belong only to the world of sense. Accordingly, while appearances *in the world* are conditionally limited, *the world itself* is neither conditionally nor unconditionally limited. (B550)

"Conditionally limited" and "unconditionally limited" can be understood as "finite" and "infinite" respectively. Notice that Kant says only that appearances are conditionally limited, since if any appearances in the world were to be unconditionally limited, then the world would have to be unconditionally limited. It is also worth noting that if one ignores the transcendental idealist features of this statement then it could almost be a statement of my own conclusions: while objects within the world are finite the world itself is neither finite nor infinite. Thus Kant's solution and my own share this feature, while differing on others.

APPENDIX TWO:

The Dimensionality Of Space

It would be a grave ommission in a work on Kant's doc-
trines of space and time, especially one critical of trans-
cendental idealism, to say nothing about the epistemology
of geometry. As is well known, Kant held that the truths
of Euclidean geometry were synthetic *a priori* truths about
the world, and that this fact could only be accounted for
by transcendental idealism. This view is generally dismiss-
sed by post-Einsteinian commentators on Kant, as not only
being wrong but, worse, being simply outdated. An example
of this view is provided by Russell:

> On the one hand, there is pure geometry,
> which deduces consequences from axioms,
> without inquiring whether the axioms are
> "true" .... On the other hand, there is
> geometry as a branch of physics as it ap-
> pears, for example, in the general theo-
> ry of relativity; this is an empirical
> science .... Thus of the two kinds of
> geometry one is *a priori* but not synthe-
> tic, while the other is synthetic but
> not *a priori*. This disposes of the
> transcendental argument. (HWP, p. 743)

Needless to say, matters are not as simple as Russell
makes them appear here, especially when one comes to study
how geometry functions "as a branch of physics". Nonethe-
less it is, I think, generally accepted that such an ap-
proach is at least adequate to remove the compulsion for re-
garding geometry as a body of synthetic *a priori* truths, and
thus fatally weakens Kant's transcendental argument to the

point where no-one would be convinced by it who was not already well-disposed to transcendental idealism. This is not to say that synthetic *a priorism* is not a possible position to take on the epistemology of geometry; it is, and will remain so until all the problems about the application of geometry to reality are solved. However it is no longer seen as a necessary, or even a leading, position: with a correspondingly debilitating effect on the transcendental argument.

I do not intend in this appendix to discuss this whole field. Instead I want to discuss the particular example Kant gives of a synthetic *a priori* truth. This is the claim that space has three dimensions. I shall not argue that this cannot be a synthetic *a priori* truth, but I shall argue that there is no reason to regard it as one, and Kant's argument only works if we are compelled to think of it as one.

I have made some remarks about the dimensionality of space in Chapter Four during my discussion of Einstein. Then I suggested that if space has *x* number of dimensions then there can be no objects in space with a different number of dimensions: the number of dimensions is all-or-nothing. I claimed, for example, that there are no two-dimensional objects in our three-dimensional world. However I have no philosophical argument for that position, nor do I believe there can be one. For I regard it as a wholly empirical (but nonetheless true) remark. That is also the position I shall take towards the claim that space has three dimensions.

If we ignore the possibility that this statement is a

synthetic *a priori* truth then there are two other possibilities:

> (1) It is a logical truth.
> (2) It is an empirical truth.

There is, of course, the further possibility that it is not the sort of statement that can be true or false at all, but I see no reason to suppose that that might be the case.

The objections to (1) are as effective as they are simple. Firstly, if the statement is a logical truth then how is it clearly about the world? Secondly, and more importantly, consistent mathematical descriptions exist of spaces of other numbers of dimensions. The Euclidean geometry of the plane is one example. Therefore it cannot be a logical truth that the space we live in is three-dimensional.

The objections to the empirical option are along these lines. We do not infer from the fact that this chair, for example, has three dimensions the fact that space has three dimensions. The dimensionality of space seems prior to the dimensionality of objects in space. One does not look to see how many dimensions space has. Besides, try to imagine that the chair has a different number of dimensions -- one cannot do so. Therefore it cannot be a merely empirical fact.

Nonetheless I shall defend the empirical option. It is perhaps best to begin by suggesting that philosophers in general take an overly simplistic view of the empirical. I think that this may be due to giving too much attention to the "foundations of knowledge" and to the accompanying be-

lief that all empirical statements are connected to these foundations by some simple procedure, usually induction although Popperians prefer falsification. This is less true than it used to be, perhaps because of Wittgenstein who realised that the empirical comes in many forms and in various degrees: consider his metaphor of a river in "On Certainty". I regard "space has three dimensions" as a highly theoretical part of our knowledge more on a par with the most abstruse statements of the theoretical sciences than with "the cat is on the mat", but as empirical nonetheless. In keeping with the practice of this thesis of treating particular problems as they arise, rather than offering general philosophical theories, I shall not offer a general demarcation of the empirical. It should be clear from my argument that this statement is empirical even if it is not clear what "being empirical" in general involves.

Let us then return to the problem. If "Space has three dimensions" functions as an empirical scientific assumption I ought to be able to show it so functioning and to show the similarity between it and other similar, but indubitably empirical, assumptions. I shall now attempt to do this. In particular I shall seek to show that the assumption of the three-dimensionality of space is on a par with the assumption of the orthogonality of space. Thus it is important to consider this argument in the context of the general approach to Kant's epistemology of geometry outlined at the beginning of this appendix. I am not seeking to show that the claim, "Space has three dimensions" is *more* certainly empirical than the other claims of Euclidean geometry, but I am seeking to show that it is *as* certainly empirical.

Since the relationship between geometry and dynamics is particularly close, I have chosen an example from this area. In Newtonian mechanics we have the equation $F = ma$, which means force = mass multiplied by acceleration. Although this is a highly theoretical statement it is not a logical one. I want to break it down into its component parts. Acceleration is a second order differentiation of distance with respect to time. So if we take s as a symbol for distance we can rewrite the equation as:

$$F = (d^2 s/dt^2) \times m$$

The letter s respresents the distance between two points, but how is that given? If we assume space is orthogonal then s is given by Pythagoras' Theorem. However to use Pythagoras' Theorem we must also make an assumption about the number of dimensions involved. If we assume that space is three-dimensional then we have to have three terms on the right-hand side of the equation. Thus if s is the distance between x and y, then s is given by:

$$s = \sqrt{(y_1 - x_1)^2 + (y_2 - x_2)^2 + (y_3 - x_3)^2}$$

Thus the law now becomes:

$$F = m \cdot \frac{d^2 \left(\sqrt{(y_1 - x_1)^2 + (y_2 - x_2)^2 + (y_3 - x_3)^2}\right)}{dt^2}$$

Here we can see clearly spelled out the geometrical assumptions the law makes. The form of the equation for s shows that space is assumed to be orthogonal, and the fact that it has three terms shows that space is assumed to have three dimensions.

Suppose then that for empirical reasons it was decided

that this law was unsatisfactory. We then have three op-
tions. We can alter our concept of mass, we can reject the
orthogonality of space, or we can reject the three-dimension-
ality of space. Historically each of the first two options
has been adopted. In Special Relativity Einstein introduced
a new concept of mass (in which "The mass of a body is a
measure of its energy-content." (Einstein, p. 71)), and in
General Relativity he adopted a geometry in which space is
not orthogonal. As the third option is epistemologically
equivalent to the second, at least, it too is not immune
from future revision.

There remains the objection that four-dimensional space
is inconceivable. I do not doubt that many people genuine-
ly found the idea that the Earth is a rock flying through
space inconceivable until that belief formed part of a sys-
tem that was conceivable as a whole. No doubt even if the
idea of four-dimensional space were to be accepted, many of
our commonsense beliefs would still treat space as three-
dimensional, just as many of our commonsense beliefs remain
geocentric (we all of us, even astronomers on nights off,
I suspect, still look "up" at the stars, do we not?). Sci-
ence is a formal epistemological enterprise and cannot al-
low itself to be chained to the imaginative powers of com-
monsense, since we have no guarantee that reality accords
with those powers.

A similar reply can be made to the point that the three-
dimensionality of space is a pervasive aspect of our exper-
ience. For the immobility of the Earth was, for many cen-
turies at least, an equally pervasive aspect of experience.

Nothing said here shows that synthetic *a priorism*, is not a *possible* position, but it does show that it is not a necessary one, and it is that that Kant's transcendental argument requires. Indeed, one might argue that its transcendental idealist consequences are grounds for not adopting that possibility.

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[1]
This view as I have expounded it is clearly close to the view of geometry developed by Poincaré and Reichenbach and generally known as conventionalism, although I would want to give more weight to the rôle of empirical evidence in determining the adoption of the geometry than conventionalism is traditionally supposed to allow, although I would not claim that a geometry could be adopted in total isolation from the rest of physics. An extreme form of conventionalism extending beyond merely the intimate connection between geometry and dynamics to encompass the whole of science, is encouraged by the holism inherent in Quine's attack on the analytic/synthetic distinction. This holism is to some extent foreshadowed in Duhem (e.g., "the statement of the result of an experiment implies, in general, an act of faith in a whole group of theories." (Duhem, p. 183)), though as an instrumentalist Duhem could not be expected to be concerned with the problem of the realist interpretation of geometrical theories.

A courageous attempt along different lines has been provided by Whitrow who has argued that the three-dimensionality of space has a teleological necessity for the development of higher, and in particular human, life on Earth. In a historical survey included in his paper he claims that the logical option (which he rejects) was adopted by Ptolemy in a lost work on distance in which he apparently argued that a distance must be defined along a perpindicular line and that at any point there can only be three lines which are mutually perpindicular. Whitrow also cites Galileo as having held a similar view (Whitrow, pp. 16 - 17; 1955).

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