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THESIS

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INTRODUCTION

In 1925, on the basis of their work on atomic spectra, Uhlenbeck and Goudsmit[1] postulated that the electron had a mechanical moment and a magnetic moment and to these properties they ascribed the name of spin. In 1928 Dirac[2], by means of a proper relativistic treatment of the wave equation, showed that the electron spin was a necessary consequence of the principle of relativity. In the following year Mott[3] showed that by means of a suitable scattering process an initially unpolarized electron beam should become partially polarized, and further that a partially polarized electron beam, when scattered, should produce an angular distribution of scattered electrons which would depend on the azimuthal angle (i.e. that angle measured around the direction of the incident beam).

A considerable number of unsuccessful attempts were made to detect the effects predicted by the Mott scattering theory[4 - 12] but in 1942 Skull, Chase and Myers[13] carried out an experiment which gave results in qualitative agreement with some of the predictions of the Mott theory. Since then several experiments have been carried out to investigate more closely the various aspects of the scattering process but quantitative agreement between theory and experiment is still lacking in some important details.

Interest in this field was considerably increased at the end of 1956 when Lee and Yang[14] advanced the hypothesis that parity was not conserved in weak interactions. One of the

consequences of this theory was that the electrons from β-decay should be longitudinally polarized. In a second paper Lee and Yang[15] postulated that the neutrino could be adequately described by a two-component theory. The work of Lee and Yang initiated a series of experiments which has considerably increased our knowledge of the nature of weak interactions.

The experiment described in this thesis was carried out to test, as accurately as possible, one of the predictions of the two-component theory, by measuring the degree of longitudinal polarization of the β-particles emitted by unaligned nuclei.

CHAPTER 1.

THE NATURE OF WEAK INTERACTIONS

1.1 The conservation of parity in weak interactions

Prior to the work of Lee and Yang[14] an apparent contradiction had arisen in the study of K particles. In particular the $K_{\pi 2}^{+}$ ($\equiv \Theta^{+}$) and the $K_{\pi 3}^{+}$ ($\equiv \tau^{+}$) particles had been found to have the same lifetimes and the same masses, within the limits of experimental error, and this, together with the fact that both particles had the same nuclear interactions, suggested that they were simply different decay modes of the same particle[16]. By consideration of the decay schemes of the two particles, together with the use of the conservation laws of angular momentum and spin momentum, it was shown that irrespective of the initial spin asigned to the K particle, the Θ^{+} and τ^{+} mesons were particles of different parity[16]. Various attempts were made to explain the apparent contradiction but without success[16, 17, 18].

The problem prompted Lee and Yang to investigate the status of the law of conservation of parity and they found that in strong interactions there was considerable experimental evidence for its acceptance but that in weak interactions there was no such evidence. The type of evidence required was that from experiments which determined whether weak interactions differentiated left from right, since the principle of parity conservation demands that Nature should give rise indifferently to left-handed and right-handed situations. If, in fact, parity was not conserved in weak interactions then the K particle problem was solved since the Θ^{+} and τ^{+} mesons could be said to be two different decay modes of the same particle which necessarily had a single mass and a single life-time.

1.2 The classic experiment of Wu

The first experiment to detect the non-conservation of parity was carried out by Wu et al[19]. To do this they measured the angular distribution of the electrons from the β - decay of polarized nuclei. If Θ be the angle between the spin of the parent nucleus and the direction of the emitted β -particle then an asymmetry of distribution between Θ and $\Pi - \Theta$ clearly constituted a break-down of parity conservation in β -decay. Co^{60} was chosen as the source of β -particles because of the relative ease with which Co^{60} nuclei could be polarized by the Rose-Gorter method[20]. The direction of polarization of the nuclei was reversed by reversing the direction of the applied magnetic field thus enabling the elimination of spurious effects. The numbers of electrons emitted in a fixed direction obtained with opposite settings of the magnetic field were compared and a large asymmetry was obtained.

1.3 The two-component theory of the neutrino

The results of Wu's experiment prompted Lee and Yang[15] (also independently Landau[21] and Salam[22]) to consider a hitherto rejected theory of the neutrino, namely the two-component theory. This particularly simple theory of the neutrino was originally put forward by Weyl[23] but had been rejected because it violated the conservation of parity. As a result of Wu's experiment this objection was no longer valid.

In the two-component theory the neutrino has only one spin state, that is the spin is always parallel or always anti-parallel to the momentum. The helicity K of a particle is defined by

the relationship

$$K = \frac{\hat{I} \cdot \hat{p}}{|\hat{p}|}$$

$\hat{I}$ is defined as a unit vector in the spin direction of the particle or photon and $\hat{p}$ as the momentum of the particle or photon in the laboratory space. According to the two-component theory the neutrino has a helicity of $\overset{+}{-}$ 1 and the anti-neutrino a helicity of $\overset{-}{+}$ 1, the upper signs applying for a S,T transition and the lower signs for a V,A transition[44]. Under the parity operator designated by P, K changes sign because under space inversion $\hat{p} \rightarrow - \hat{p}$, $\hat{I} \rightarrow \hat{I}$, and since the two-component theory of the neutrino stipulates that the sign of the helicity of the neutrino is fixed, then parity is not conserved.

1.4 β -decay

The fact that parity was not conserved in weak interactions required modifications to be made to the theory of β -decay. Prior to 1956 the generally accepted Hamiltonian describing β - decay was based on the original work of Fermi[24] and was characterized by five coupling constants which were measures of the relative strengths of the possible interactions. The most general Hamiltonian density which was invariant under proper Lorentz transformations, under time reversal and under space inversion, which conserved leptons and which did not include derivatives of the fermion field, was given by the following expression.

$$H_{int.} = C_s (\overline{\psi}_p \, \psi_n)(\overline{\psi}_e \, \psi_{\nu})$$

$$+ C_v (\overline{\psi}_p \, \gamma_\mu \, \psi_n)(\overline{\psi}_e \, \gamma_\mu \, \psi_{\nu})$$

$$+ \tfrac{1}{2} C_T (\overline{\psi}_p \, \sigma_{\lambda\mu} \, \psi_n)(\overline{\psi}_e \, \sigma_{\lambda\mu} \, \psi_\nu)$$

$$- C_A (\overline{\psi}_p \, \gamma_\mu \gamma_5 \, \psi_n)(\overline{\psi}_e \, \gamma_\mu \gamma_5 \, \psi_\nu)$$

$$+ C_P (\overline{\psi}_p \, \gamma_5 \, \psi_n)(\overline{\psi}_e \, \gamma_5 \, \psi_\nu)$$

$$+ \; h.c.$$

$$\sigma_{\mu\nu} = -\tfrac{i}{2} (\gamma_\mu \gamma_\nu - \gamma_\nu \gamma_\mu)$$

where all the C's, or at least their ratios, are real. C_S, C_V, C_T, C_A and C_P are the coupling constants for the scalar (S), vector (V), tensor (T), axial-vector (A) and pseudoscalar (P) interactions[25].

The most general Hamiltonian density which conserves leptons, which does not include derivatives of the fermion field, which is invariant under proper Lorentz transformations, but which is not invariant under space inversion nor time reversal is given by the following expression.

$$H_{int.} = (\overline{\psi}_p \, \psi_n)(C_S \, \overline{\psi}_e \, \psi_\nu + C_S' \, \overline{\psi}_e \, \gamma_5 \, \psi_\nu)$$

$$+ (\overline{\psi}_p \, \gamma_\mu \, \psi_n)(C_V \, \overline{\psi}_e \, \gamma_\mu \, \psi_\nu + C_V' \, \overline{\psi}_e \, \gamma_\mu \gamma_5 \, \psi_\nu)$$

$$+ \frac{1}{2} (\overline{\psi}_p \, \sigma_{\lambda\mu} \, \psi_n)(C_T \, \overline{\psi}_e \, \sigma_{\lambda\mu} \, \psi_{2'} + C_T' \, \overline{\psi}_e \, \sigma_{\lambda\mu} \, \gamma_5 \, \psi_{2'})$$

$$- (\overline{\psi}_p \, \gamma_\mu \, \gamma_5 \, \psi_n)(C_A \, \overline{\psi}_e \, \gamma_\mu \, \gamma_5 \, \psi_{2'} + C_A' \, \overline{\psi}_e \, \gamma_\mu \, \psi_{2'})$$

$$+ (\overline{\psi}_p \, \gamma_5 \, \psi_n)(C_P \, \overline{\psi}_e \, \gamma_5 \, \psi_{2'} + C_P' \, \overline{\psi}_e \, \psi_{2'})$$

$$+ \, h.c.$$

where parity conservation demands either all $C_i' = 0$ (even couplings) or all $C_i = 0$ (odd couplings). Time-reversal invariance requires that all the coupling constants be real with respect to one another. The two-component neutrino theory requires that the parity-conserving and the parity non-conserving coupling constants be equal in magnitude.

$$\text{i.e.} \quad C_i = \pm \, C_i'$$

It can be shown to a first approximation that the S and V nuclear matrix elements vanish unless there be no change in spin or parity in a transition (Fermi selection rule). Similarly, to a first approximation, the T and A nuclear matrix elements vanish unless there be no change in parity and a spin change of 0 or ± 1 (but not $0 \to 0$) in a transition (Gamow-Teller selection rule).

From evidence regarding the absence of Fierz interference terms together with the electron-neutrino angular correlation results available in 1956, though the position regarding the latter was by no means clear, it was thought at that time that

$$C_V^2 \ll C_S^2 \quad \text{and} \quad C_A^2 \ll C_T^2 \qquad (26).$$

1.5 The detection of interference terms in weak interactions

The reason why the numerous experiments carried out in the field of β-decay before 1956 could not provide an answer to the question of parity conservation in weak interactions was that the phenomena studied contained no interference terms between the coupling constants for parity-conserving and parity non-conserving interactions. In order to detect such interference a pseudoscalar, formed out of the experimentally measured quantities, had to be obtained, a pseudoscalar being defined as an observable which is invariant under rotation but which reverses sign under reflection.

It was recognised that the problem of the detection of such interference phenomena was essentially that of the observation and the determination of the helicity of particles and photons since it could be shown that if a non-zero helicity was observed then parity could not be conserved in the interaction from which the particles and photons resulted. Further by the measurement of the sense and the degree of the particle and the photon polarization, information could be obtained regarding the nature of the relevant interactions. Consequently it was necessary to develop a range of techniques for the determination of the polarization of electrons, positrons, γ-rays and neutrinos.

1.6 β-γ circular polarization technique

One of the predictions of the two-component theory of the neutrino is that in β-decay the electron spin direction is correlated with its momentum direction and by the application of

the principle of the conservation of angular momentum to the
nuclear disintegration it can be shown that the spin of the
residual nucleus is correlated with the direction of the β -emission.
Further if the residual nucleus emits a γ -ray then, except for
pure Fermi transitions and for transitions in which the γ -emitting
state has zero spin, the γ -ray is circularly polarized.   The
study of the circular polarization of the γ -ray, together with a
knowledge of the direction of the emitted β -particle, provides
essentially the same information as that obtained from an experi-
ment such as Wu's[19] but the former technique has two advantages
over the latter insofar as less expensive equipment is required and
a wider range of nuclei can be studied by its use.

The helicity of the γ -rays, emitted at an angle Θ to the
β -direction, is given by the expression

$$K_\gamma = \pm \, \frac{v}{c} A \cos \Theta \qquad\qquad 1.6.1$$

(+ for right-handed polarization, - for left-
handed polarization).

The parameter A depends on a number of factors, namely, the
interaction matrix elements and coupling constants, the relative
amount of Fermi to Gamow-Teller interactions and a factor dependent
on the spins of the initial and final states and the multipole
order of the γ -rays.   Theoretical values of A for different
types of transitions have been evaluated by several authors[27, 28, 29].

The spin dependence of the Compton scattering cross-section
has been utilized in the experimental study of the circular
polarization of γ -rays, by scattering the rays in magnetized

iron[30]. This technique can only be used for nuclei with suitable $\beta - \gamma$ decay characteristics and, moreover, the γ - ray energy must be such that the photo-electric effect and the pair production effect in the magnetized iron are small. Several experiments[31-35] have been carried out using this technique and the results obtained are in agreement with the following con-clusions.

(a) Parity is not conserved in pure Gamow-Teller transitions and the predictions of the two-component theory for these transitions are correct to within an accuracy of about 5%.

(b) The velocity and the cosine dependence of the expression for the helicity of the γ -ray (1.6.1) have been verified to within an accuracy of about 10%.

(c) The existence of interference terms in mixed Fermi and Gamow-Teller interactions has been established and since such interference can only exist if the neutrino emitted in the Fermi channel is of the same helicity as the neutrino emitted in the Gamow-Teller channel then the combination of interactions must be S and T or V and A, or possibly all four. The work of Burgy et al[36] on the decay of polarized neutrons indicates that the interference is maximal.

1.7   The longitudinal polarization of β -particles

It has been pointed out by several authors[15, 21, 22] that if parity is not conserved in weak interactions then the β - particles emitted from unaligned nuclei should be longitudinally polarized. The expected degree of polarization has been evaluated for different types of transitions by several groups[27, 29, 37].

In particular, Curtis and Lewis[37] have shown that for allowed
transitions the degree of polarization (P) should take the
following form:

$$P = \pm \frac{V_e}{C} \frac{d}{1 + \frac{b}{E_e}}$$

1.7.1

where E_e is the energy of the β-particle, in units of mc^2, and
d and b are quantities involving the Fermi and Gamow-Teller
matrix elements and the coupling constants. If the two-component
theory of the neutrino is valid and if either of the following
conditions is satisfied,

(a) $C_V = C_A = 0$ and $C_S = -C_S'$: $C_T = -C_T'$

(b) $C_S = C_T = C_P = 0$ and $C_A = C_A'$: $C_V = C_V'$,

then d = 1 and b = 0 and the degree of polarization is given by
$P = \pm \frac{v}{c}$ where the negative sign applies to the case of electrons
and the positive sign to the case of positrons.

Since the predicted degree of electron polarization depends
directly on the β-particle velocity it is most desirable that
investigations be made over as wide a range of electron energy as
possible. For this purpose three general techniques have been
developed, namely the investigation of the polarization of
bremsstrahlung produced by electrons of energy greater than 1 MeV,
the Møller scattering of electrons in the energy range 400 keV –
1 MeV and the Mott scattering of electrons in the range 50 keV –
750 keV.

1.8 The bremsstrahlung technique

The direct determination of the longitudinal polarization of

β-particles of energy greater than 1 MeV would be difficult but it has been shown[40] that, under suitable conditions, longitudinally polarized electrons produce circularly polarized external bremsstrahlung and essentially the same information can be obtained from the study of the polarization characteristics of the bremsstrahlung as from the examination of the electron polarization. Another process which has proved of interest in this field is that of the production of internal bremsstrahlung during β-emission and K-capture; in both cases the photon production is due to a displacement of charge density during the decay process.

Schopper and Galster[38] (also, independently, Boehm and Wapstra[39]) detected the circular polarization of both internal and external bremsstrahlung by means of Compton scattering with the oriented electrons available in magnetized iron; if the bremsstrahlung is circularly polarized then the number of quanta scattered in a particular direction changes when the direction of magnetization is reversed and from the study of such changes it is possible to determine the sense and magnitude of the polarization. Goldhaber et al[40] obtained essentially the same information by measuring the variation in transmission of bremsstrahlung through magnetized iron on the reversal of the direction of magnetization. Such work has shown that the general technique is suitable for the study of the polarization of high-energy electrons ($\frac{v}{c} \rightarrow 1$) and the results obtained in this energy region agree with the predictions of the two-component theory to within an accuracy of $(5 - 10)\%$[38-43]. This work has also shown that the helicity of the electron is negative. Similar results have been obtained

from the study of the bremsstrahlung accompanying K-capture[103].
These methods tend to lose their efficiency and accuracy at
energies lower than about 1 MeV since various effects which
may reasonably be neglected at higher energies become considerably
more important at lower energies.

1.9 Møller scattering

In the energy range 400 keV - 1 MeV the most direct method
of determining the longitudinal polarization of β -particles
is by the use of the Møller scattering technique, which makes use
of the fact that the electron-electron scattering cross-section
depends on the relative spin orientation of the incident and
scattering electron. Møller scattering leads to an asymmetry
because of the indistinguishability of elementary particles and is,
essentially, a low-energy effect which can be extended to higher
energies.

In a normal Møller scattering experiment a well-collimated
beam of β -particles is allowed to strike a thin, highly satu-
rated, magnetic foil which has a large component of electron spin
in the direction of the incident beam and the initial and the
scattered electrons are recorded in coincidence in counters which
preferably select energy spectra such that the sum of the energy
losses in the two counters is equal to the incident electron
energy. The variation in the coincidence rates between opposite
directions of the magnetic field in the foil is a measure of the
degree of the longitudinal polarization of the initial electron
beam. Experiments[44-47] based on this technique have yielded
results in reasonable agreement with the predictions of the

two-component theory but it would appear that the method is not capable of giving results of high accuracy, at least at present, due to uncertainties in the determination of the magnetic field in very thin foils and also in the amount of plural scattering taking place at the scattering foil. This latter restriction tends to be relatively more important for electrons of energy less than 500 keV.

1.10   Bhabha scattering

Theoretically the scattering of high-energy positrons by polarized electrons should lead to an asymmetry due to the dependence of the positron-electron annihilation rate on the relative spin directions of the two particles[48, 49]. No experiment has been reported, however, which makes use of this theory.

1.11   Mott scattering

The use of the Mott scattering theory for the detection of electron polarization depends on the presence of a spin-dependent term in the scattering of an electron by a nucleus. If a beam of transversely polarized electrons is scattered by a foil (of high Z value) then an azimuthal asymmetry results and the measurement of this asymmetry leads to a knowledge of the sense and the degree of the electron polarization[30]. In principle the Mott scattering technique has a greater sensitivity than the other methods in the energy range 40 keV - 200 keV and it is precisely in this region that the velocity dependence of the polarization can best be established. The results obtained by the use of Mott scattering are discussed in Chapter 3.

1.12   The determination of the polarization of positrons

In principle thebremsstrahlung technique and the Mott
scattering technique may be used for polarization measurements on
positrons as well as on negatrons but in practice it has been found
necessary to develop more efficient methods for the examination of
positron polarization.   The experimental techniques which have
been devised to measure the degree and sense of the longitudinal
polarization of positrons may be divided into two categories,
namely those which require the positrons to be slowed down to
near zero-energy and those which make use of an annihilation-in-
flight technique.

1.13   The formation of positronium

Positronium is formed in two states, namely in the singlet
state when the positron and electron spins are anti-parallel and
in the triplet state when the positron and electron spins are
parallel.   In the presence of a magnetic field it is found that
if the incident positrons have their spins parallel to the magnetic
field then the formation of the singlet state is preferred and
if anti-parallel to the field, the formation of the triplet
state is preferred.   The polarization of the incident positron
beam can therefore be examined by the determination of the relative
abundance of the two positronium states.   Due to magnetic quenching
both states decay in effectively the same way but the triplet state
has a longer lifetime than the singlet state and consequently has
a greater opportunity to "thermalize" with the result that the
two-photon annihilation of the triplet state has a narrower angular
correlation than that of the singlet state.   Page and Heinberg[50]

determined the longitudinal polarization of positrons from Na^{22} by studying the angular correlation of the two-photon annihilation yield. Their results were not accurate, due primarily to the difficulty of estimating the amount of depolarization which took place when the positrons were being slowed down, but their work was sufficiently conclusive to show that the positron helicity was of opposite sign to that of the electron.

If a beam of very slow positrons is directed into a piece of iron, magnetized either parallel or anti-parallel to the direction of the beam, then it can be shown that positronium can be formed between all the incident positrons and the slow conduction electrons in the iron, but that only the positrons whose spins are parallel to the magnetic field can form positronium with the relatively fast polarized d-electrons[51]. The relative abundance of the two types can be determined by the examination of the angular distribution of the two-photon annihilation yield. Hanna and Preston[51] have carried out experiments using this technique and although the accuracy they achieved was poor, their results did show that for all transitions studied, the emitted positrons had positive helicity.

1.14   The annihilation-in-flight technique

It can be shown[52] that when polarized positrons are annihilated in an unpolarized material the high energy photons in the two-quanta annihilation are almost completely circularly polarized in the direction of the positron beam. By the study of the polarization characteristics of the annihilation photons, using a method similar to that described in 1.8., Deutsch et al[53] were

thus able to investigate the longitudinal polarization of

positrons.   They obtained the result that positrons produced in

Fermi transitions had positive helicity and their results were not

inconsistent with the predictions of the two-component theory.

Boehm et al[54] also used an annihilation-in-flight technique to

investigate the polarization of positrons from N^{13}, which was of

particular interest since it was a mixed transition.   Their

results showed that positrons from Fermi and from Gamow-Teller

transitions had the same helicity.

1.15   Conclusions to be drawn from the parity experiments

From the considerable amount of experimental evidence obtained

by the methods outlined in the above paragraphs it was clear that

parity was not conserved in weak interactions and further that

the degree of longitudinal polarization of β -particles was

$\pm \frac{v}{c}$, for all types of transitions, at least to within an accuracy

of (5 – 10)%.   There was only one known exception to the latter

statement namely the decay of RaE[47, 55].

The above range of experiments did not, however, give infor-

mation as to the exact nature of the covariants which participate

in the fundamental β -interaction.

1.16   The helicity of the neutrino and the nature of the β-inter-
action

The nature of the β -interaction and the helicity of the

neutrino are very closely connected since an interaction which

yields the Fermi radiation with a neutrino of positive helicity

(i.e. a right-handed neutrino) is known as the scalar (S) coupling

while one yielding a neutrino of negative helicity (i.e. a left-

handed neutrino) is called the vector (V) coupling.   Similarly an

interaction which yields the Gamow-Teller radiation with a neutrino of positive helicity is called the tensor (T) coupling while an axial-vector (A) coupling produces neutrinos of negative helicity. Most of the information on the relative magnitudes of the various possible types of interactions has come from electron-neutrino correlation measurements. Although at one time the weight of the experimental evidence indicated the opposite conclusion[26] it now appears certain that the β -coupling has a VA form[56, 57].

1.17 The direct determination of the helicity of the neutrino

In 1958 Goldhaber et al[58] carried out an experiment to determine the helicity of the neutrino. Their experiment was based on the following facts, namely that in the case of K capture, the residual nucleus must recoil with a momentum equal and opposite to that of the neutrino and therefore a knowledge of the direction of the recoiling nucleus determines the neutrino momentum direction and also from the conservation of angular momentum, that a know-ledge of the helicity of the recoil nucleus implies a knowledge of the helicity of the neutrino. Further if the recoiling nucleus is polarized then any emitted γ -ray must be circularly polarized and consequently the problem of the measurement of the helicity of the neutrino is essentially reduced to the problem of the identi-fication, and the measurement of the degree and sense of the circular polarization, of the γ -ray emitted in a direction opposite to that of the neutrino momentum.

The circular polarization of the γ -rays was analysed by trans-mission through magnetized iron (1.8). The direction of the emitted neutrinos was selected by resonant scattering of the emitted

γ-rays since the conditions necessary for this type of scattering were best fulfilled by those γ-rays which were emitted in a direction opposite to that of the neutrinos, which had an energy comparable to that of the neutrinos and which were emitted before the recoil energies of the nuclei were lost. The results of Goldhaber et al indicated that the Gamow-Teller interaction was predominantly axial-vector (A), at least for positron emitters, in agreement with the work of Herrmannsfeldt et al[56].

1.18 Conclusion

The results on the longitudinal polarization of electrons and positrons together with the conclusions to be drawn from the $\beta - \gamma$ circular polarization measurements and from the experiments on the helicity of the neutrino may be explained in terms of the two-component theory of the neutrino with real coupling constants and also

$$C_V = C_V'$$

$$C_A = C_A'$$

$$\frac{C_V}{C_A} \sim -1$$

(There is little experimental evidence on the question of the reality of the coupling constants).

Moreover the results are in agreement with a Universal Fermi interaction of the form V-A. As shown by Sakurai[59] the principle of lepton conservation is also established provided the following three conditions are satisfied:

(a)   the β-interaction consists of a linear combination of V-A

(b)   the longitudinal polarization of $e^{\pm}$ is $\mp \frac{v}{c}$.

(c)   the amount of V-A interference is maximal.

CHAPTER 2.

A REVIEW OF MOTT SCATTERING

2.1   Mott scattering

Dirac's relativistic wave equation, which successfully
accounted for many of the phenomena interpreted as being due to
the spin of orbital electrons, also predicted that the free
electron should have a spin and, in consequence, that each electron
wave should be characterized by a definite direction other than
that of propagation.   On this basis an electron beam should
therefore be capable of exhibiting polarization.   In 1929 Mott[3]
showed that if an unpolarized electron beam be scattered by the
Coulomb field of a nucleus then, under certain conditions, the
scattered beam should be partially polarized and further that
this polarization should be capable of being observed experimen-
tally by the presence of an azimuthal asymmetry in a second
Coulomb scattering.   The polarization and the asymmetry effects
in Mott scattering are due to the interaction of the electron spin
with the non-uniform magnetic field, through which the electron
moves in the Coulomb field of the nucleus.

The scattering must be considered as a relativistic, quantum-
mechanical process since the effect of the non-uniform magnetic
field on the electron is negligible except when the electron
is travelling with a relativistic velocity.   The scattering must
be treated as a quantum-mechanical problem since a 100 keV
electron has a De Broglie wavelength of $\sim 3 \times 10^{-10}$ cms and for
an electron of such energy to be scattered through an angle of $90°$
by a gold nucleus its classical impact parameter must be $\sim 10^{-11}$ cms.
There is, however, an interesting classical model of the process

2.2   A classical model

For simplicity the quantum-mechanical definition of an un-
polarized electron beam is chosen, namely that half the electrons
in the beam have their spins parallel to a certain direction and
the other half have their spins anti-parallel to this direction;
for convenience we choose the direction to be at right-angles to
the paper so that half the electrons have their spins pointing
into the paper (spin-down) and half the electrons have their spins
pointing out of the paper (spin-up).   Consider such a beam
incident on a nucleus of charge Ze (figure 1(A)).   If there
is no interaction between the non-uniform magnetic field surround-
ing the nucleus (the magnitude of which may be obtained by a
Lorentz transformation of the nuclear electric field from the
laboratory system to a co-ordinate system in which the electron
is at rest) and the magnetic moment of the electron then the
electrons proceed along path (a).   For the case when the inter-
action is not zero then for electrons with spin-up the Coulomb
force and the spin-orbit force act in conjunction and the electrons
proceed along path (b).   For electrons with spin-down the Coulomb
force and the spin-orbit force are in opposition and the electrons
proceed along path (c).   To enter the detector spin-up electrons
must be incident along the impact parameter b_2 and spin-down
electrons must be incident along impact parameter b_0 (fig. 1(B)).
But the number of electrons incident along a certain impact
parameter is proportional to the magnitude of that impact parameter
and since $b_2 > b_0$ then more spin-up electrons than spin-down
electrons enter the detector and consequently a partially-polarized
beam is produced.   For the case of the incidence of this partially
polarized beam on a second scattering nucleus only the excess of

Figure 1.   Azimuthal Asymmetry on Double Scattering

spin-up electrons need be considered, since the remainder will produce a symmetrical distribution. For a spin-up electron incident at the bottom of the nucleus then the Coulomb force and the spin-orbit force act in conjunction and the electron is scattered through an angle Θ (figure 1(c)): for a spin-up electron incident at the top of the nucleus the Coulomb force and the spin-orbit force are in opposition and the electron is scattered through an angle which is less than Θ. For a spin-up electron, incident at the top of the nucleus, to be scattered through an angle Θ it must be incident along an impact parameter b_o ($b_o < b_1$) and since there are more electrons incident along impact parameter b_1 than along impact parameter b_o an asymmetry in scattering results.

By the use of a classical model it is possible therefore to illustrate the production of a transversely polarized electron beam by a scattering process and also the presence of an asymmetry in the scattering of a transversely polarized electron beam.

By considering the dependence of the scattering angle and of the relative magnitudes of the Coulomb force and the spin-orbit force on the classical impact parameter and on the atomic number of the scatterer, it is possible to obtain in a qualitative way the angular dependence and the Z dependence of the Mott asymmetry. Since the spin-orbit force depends on a Lorentz transformation and consequently tends to zero as the electron energy tends to zero and further, since the spin-orbit force is proportional to the magnetic moment of the electron and consequently is inversely proportional to the relativistic mass of the electron with the result that the spin-orbit force approaches zero as $\frac{v}{c} \rightarrow 1$, it is possible to

explain the energy dependence of the Mott asymmetry by the use of the classical model[60]. The full relativistic quantum-mechanical treatment of the problem must be used however in order to obtain the actual values of the expected asymmetry.

2.3 The Mott theory

By applying the Dirac relativistic wave equation to the scattering problem Mott[3] found that the wavefunction, in asymptotic form, describing the scattering process could be expressed in the following way:

$$\psi_3 \sim A e^a + [Af(\theta) - Bg(\theta) e^{-i\phi}] \frac{e}{r}^b \qquad 2.3.1$$

$$\psi_4 \sim B e^a + [Bf(\theta) + Ag(\theta) e^{i\phi}] \frac{e}{r}^b \qquad 2.3.2$$

$$a = iKz - i\gamma \ln K(r - z) \qquad 2.3.3$$

$$b = iKr + i\gamma \ln Kr \qquad 2.3.4$$

$$f(\theta) = \frac{1}{K} [-i\gamma' F(\theta) + G(\theta)] \qquad 2.3.5$$

$$g(\theta) = \frac{1}{K} [i\gamma' \cot(\tfrac{\theta}{2}) F(\theta) + \tan(\tfrac{\theta}{2}) G(\theta)] \qquad 2.3.6$$

$$F(\theta) = \tfrac{i}{2} \sum_{\ell=0}^{\infty} (-1)^{\ell} [\ell C_{\ell} + (\ell+1) C_{\ell+1}] P_{\ell} (\cos \theta) \qquad 2.3.7$$

$$G(\theta) = \tfrac{i}{2} \sum_{\ell=0}^{\infty} (-1)^{\ell} [\ell^2 C_{\ell} - (\ell+1)^2 C_{\ell+1}] P_{\ell} (\cos \theta) \qquad 2.3.8$$

$$C_{\ell} = -\frac{e^{-\pi \rho_{\ell}} \Gamma(\rho_{\ell} - i\gamma)}{\Gamma(\rho_{\ell} + 1 + i\gamma)} \qquad 2.3.9$$

$$\rho_{\ell} = + (\ell^2 - \alpha^2)^{\tfrac{1}{2}} \qquad 2.3.10$$

$$\alpha = \frac{Ze^2}{\hbar c} \qquad 2.3.11$$

$$\gamma = \frac{\alpha}{\beta} \qquad 2.3.12$$

$$\gamma' = \gamma(1-\beta^2)^{\tfrac{1}{2}} : \beta = \tfrac{v}{c} \qquad 2.3.13$$

$$K = \frac{m_0 c \beta}{\hbar(1-\beta^2)^{\tfrac{1}{2}}} \qquad 2.3.14$$

The double scattering cross-section is given by

$$\sigma(\theta_1, \theta_2, \phi_2) = \sigma(\theta_1)\sigma(\theta_2)[1 + P(\theta_1)P(\theta_2) \cos \phi_2] \qquad 2.3.15$$

with θ_1 the first scattering angle, θ_2 the second scattering

angle and ϕ_2 the angle between the plane of the second scattering and the plane containing the electron source, the first scattering nucleus and the second scattering nucleus (the azimuthal angle).

Also

$$\sigma(\theta_1) = |f(\theta_1)|^2 + |g(\theta_1)|^2 \qquad 2.3.16$$

$$\sigma(\theta_2) = |f(\theta_2)|^2 + |g(\theta_2)|^2 \qquad 2.3.17$$

$$P(\theta_1) = \left[i \left[\frac{f(\theta_1) g^*(\theta_1) - f^*(\theta_1) g(\theta_1)}{|f(\theta_1)|^2 + |g(\theta_1)|^2} \right] \right] \qquad 2.3.18$$

$$P(\theta_2) = i \left[\frac{f(\theta_2) g^*(\theta_2) - f^*(\theta_2) g(\theta_2)}{|f(\theta_2)|^2 + |g(\theta_2)|^2} \right] \qquad 2.3.19$$

From equation 2.3.16 it is clear that there is no asymmetry produced by the single scattering of an unpolarized beam and from equation 2.3.15 it follows that the double scattering cross-section has an azimuthal asymmetry. The Mott asymmetry factor is given by

$$\delta(\theta_1, \theta_2) = P(\theta_1) P(\theta_2) \qquad 2.3.20$$

Because of the slow convergence of the series for both $F(\theta)$ and $G(\theta)$, numerical calculations of the single scattering cross-section $\sigma(\theta)$ and the polarization $P(\theta)$ are difficult. Of the calculations published[61-67] the most accurate are those of Sherman[67] but these do not include the effects of the screening of the nuclear scattering field by the atomic electrons. In some earlier work by Mohr and Tassie[66] the screening effects were taken into account but their calculations were neither so accurate nor so extensive as those of Sherman.

2.4 Experiments on Mott scattering (1928-1942)

The experiments[4-12] carried out in the period 1928 - 1942 to observe the asymmetry in a double scattering experiment, as predicted by the Mott theory, were unsuccessful. In some of the early work done by Cox et al[4] and later by Chase[5], asymmetries were detected but they could not be explained on the basis of the Mott theory. It is interesting to note that it is possible that these early investigations were the first experiments to show evidence of the non-conservation of parity in weak interactions since a possible explanation for the obtained asymmetries is that they were due to the longitudinal polarization of electrons. Another of the early experiments in which an asymmetry was found was that of Kikuchi[12] but subsequent work[68] has shown that the asymmetry was most likely of instrumental origin.

The experiments carried out in the period 1928 - 1942 served to draw attention to the type of effects which can mask the true polarization asymmetry.

2.5   The elimination of instrumental asymmetries

In early work attempts were made to eliminate instrumental
asymmetries by careful attention to the geometry of the apparatus
but this was not particularly satisfactory and in most of the later
work the instrumental asymmetries were determined by replacing
the first or second gold scattering foils by aluminium scattering
foils.   Since the atomic number of aluminium is considerably
smaller than that of gold and since the polarization asymmetry is
proportional to the atomic number of the scatterer then, to a
reasonable degree of approximation, any asymmetry obtained with
an aluminium scattering foil can be ascribed to instrumental
causes and can be measured.

2.6   Elastic scattering

It is essential to have pure elastic scattering in order to
show up the polarization asymmetry.   Inelastic scattering can
reduce the asymmetry in two ways; firstly by a simple reduction
in the electron energy before the large angle scattering takes
place and secondly by depolarization of the beam due to a change
in the spin direction of the incident electron during the
inelastic collision.

The rate of energy loss in gold for electrons is two MeV
per gm/cm^2 and with gold foils of thickness 10^{-5} cms the energy
loss and the resultant effect on the asymmetry are negligible[68].

Rose and Bethe[68] have evaluated the effect of spin flip
in inelastic collisions between the incident electrons and the
atomic electrons consistent with the condition that the energy
loss be much less than the initial energy of the incident electron.

They found that for 100 keV electrons scattered at gold foils of thickness 10^{-5} cms the effect is a negligible source of depolarization.

2.7 Single scattering

The opposite of single scattering is multiple scattering where the scattering in a target consists of more than one large-angle scattering together with a number of small-angle scatterings. In 1922 Wentzel[69] gave as a criterion for single scattering of the Rutherford type that the angle Θ , at which scattering is observed, should be several times greater than 4 W where W is given by the following expressipn:

$$ W = 2 \cot^{-1} \frac{2V}{Ze} \left(\frac{2}{\pi nt} \right)^{\frac{1}{2}} \qquad 2.7.1 $$

where eV is the kinetic energy of the electron, Z is the atomic number of the scatterer, n is the number of atoms per unit volume of the scatterer and t is the thickness of the scattering foil. It has been customary to consider scattering as single at angles greater than 12 W.   Chase and Cox[70] applied a more empirical test to single scattering and found that provided t be taken as the mean length of path in the foil of the electrons scattered at an angle Θ and not as that of an undeflected electron then Wentzel's criterion was satisfactory.

Rose and Bethe[68] evaluated the degree of depolarization caused by multiple scattering in gold foils and found that for a foil-thickness of approximately 10^{-5} cms and for electrons of energy 100 keV incident on the foil at an angle of 45° the depolarization due to multiple scattering is less than 1%.

2.8   Exchange scattering

An electron beam can be depolarized by exchange scattering, that is a scattering process in which the outgoing electron has the opposite spin orientation to that of the incident electron. This effect is extremely small for gold foils of thickness 10^{-5} cms under normal conditions, since only the valence electrons of gold can participate in the exchange process[68].

2.9   Miscellaneous scattering problems

The difficulties associated with the creation of bremsstrahlung in the apparatus, with electron sources which also emitted γ-radiation, with electron guns which emitted beams, the characteristics of which varied with time, and with the scattering of electrons from parts of the apparatus other than the scattering foils, were encountered in early work in this field[4, 5, 9].

2.10   Plural scattering

The presence of the effects discussed in the above paragraphs was known to and allowed for by some of the early workers but nevertheless no positive detection of the Mott asymmetry was made until 1942.   There were two reasons for the failure of the early experiments.   Firstly when criteria for single scattering were evaluated it was assumed that the probability of scattering at a large angle by a combination of two deflections of the same order of magnitude could be ignored.   Secondly when electron beams were incident on scattering foils at angles other than 90° it was assumed that no more allowance for obliquity had to be made than to use the oblique thickness as the effective thickness of the foil. That such assumptions were wrong was shown theoretically by

Goertzel and Cox[71] and experimentally by Skull, Chase and Myers[13].

It was found by Chase and Cox[70] that when an unpolarized electron beam was incident on a target foil (fig. 2 (A)) then the scattered intensity for a given scattering angle Θ depended o n which side of the foil thedetector was located. They found that the detector on the so-called reflection side received more electrons than the detector placed on the transmission side. This transmission-reflection asymmetry was explained by Goertzel and Cox[71] as being due to plural scattering which is a combination of two deflections of the same order of magnitude. The detector on the reflection side of the foil received electrons scattered once through $90°$ together with electrons scattered twice through $45°$ whereas the detector on the transmission side of the foil received electrons scattered once through $90°$ together with electrons scattered once at an angle of $135°$ and then through an angle of $45°$ (fig. 2(B)). Because of the difference in scattering cross-section between scattering angles of $45°$ and $135°$ more electrons entered thedetector placed on the reflection side than that on the transmission side.

The failure of such experiments as those of Dymond and Richter [9, 11] was therefore probably due to the fact that they used both scattering foils in the reflection position (that is both the incident and scattered beams were on the same side of the foil) and consequently, because of the presence of plurally scattered electrons in the beam incident on the second scattering foil, their experiments did not satisfy the single-scattering criterion.

Figure 2.   Reflection-Transmission Effect.

Skull, Chase and Myers[13] carried out two sets of experiments, one with both scattering foils in the reflection position and the other with both foils in the transmission position (that is the incident and scattered beams were on opposite sides of the foil). In the first they obtained results in agreement with those of Dymond whereas in the second they obtained results in reasonable agreement with the predictions of the Mott theory. Their results were verified shortly afterwards by Trounson and Simpson[72].

2.11   Further investigations into the Mott scattering theory

Since the work of Trounson and Simpson the double scattering experiment has been repeated, with relatively minor modifications, by six groups[72-78] with a view to examining experimentally the various predictions of the Mott scattering theory. The four effects which have been studied are

(a)   the cosine dependence in azimuth of the asymmetry,

(b)   the dependence of the asymmetry on the atomic number

of the scatterers,

(c)   the angular dependence of the asymmetry,

and (d)   the energy dependence of the asymmetry.

2.12   The azimuthal dependence and the Z dependence of the

Mott asymmetry

The best agreement between theory and experiment has been obtained in the experiments which have measured the angular dependence in azimuth of the asymmetry[74, 75, 77, 78]. In the angular range $\Theta = 80° - 140°$ and the energy range $E = 60$ keV $- 130$ keV no significant discrepancy between

theory and experiment has been encountered. It should be noted however that such agreement between theory and experiment is proof only of the correctness of the concept that the first scattering produces a transversely polarized electron beam and that the second scattering acts as a detector of transversely polarized electrons, and not as a proof of the detailed theory of Mott scattering.

The only experimental evidence from double-scattering experiments on the dependence of the Mott asymmetry on the atomic number of the scatterer comes from an experiment by Louisell, Pidd and Crane[75] which was designed to measure the gyromagnetic ratio of the free electron. In this experiment use was made of Mott scattering and, as a check on the validity of the results obtained, one of the gold foils was replaced by a silver foil and the resultant reduction in asymmetry measured. The accuracy of this particular aspect of their work was not, however, high.

2.13 The energy dependence and the angular dependence of the
                                                           Mott asymmetry

Most of the work in double-scattering experiments has been concentrated on the angular dependence and on the energy dependence of the Mott asymmetry.

Shinohara and Ryu[73] studied the energy dependence of the Mott asymmetry in the energy range 45 keV - 92 keV and obtained some agreement with theory but the instrumental asymmetry of their apparatus was not measured and this reduced the value of their work. In later experiments Ryu[74] extended the range of scattering angles and energies studied and found that in some

cases the discrepancies between theory and experiment were as high as 50%. In Ryu's experiment the first foil was in the transmission position but the second foil appeared to be in the reflection position.

Pettus[76] carried out a fairly extensive study of the energy dependence of the Mott asymmetry in the electron energy range 80 keV - 200 keV and found large discrepancies between theory and experiment at low energies but at high energies the discrepancies were only of the order of 10%. Because of the large second scattering angle (120°) used it was necessary for each counter, in turn, to view the reflection side of the scattering foil. Schneider and Barnard[78] carried out an experiment similar to that of Pettus but, by the use of a smaller second scattering angle, were able to use the transmission side of both foils. They worked in the electron energy range 60 keV - 100 keV and in this range obtained asymmetries which were only about half the value of those predicted by theory.

One feature common to the above experiments was that no attempt was made to determine the energy of the electrons recorded at the counters, the only control over the electron energy being at the electron gun. In their experiment Pidd and Nelson[77] chose to investigate the angular dependence of the Mott asymmetry at one energy (121 keV). They first obtained a set of asymmetry values with no energy discrimination at the counters and these values were in rough agreement with those obtained in the experiments discussed above[74, 76, 78] and not with the predictions of the Mott scattering theory. They repeated their measurements with energy discrimin-

ation at the counters and the results obtained in this way agreed relatively well with the Mott theory except at large scattering angles (120° - 140°) where the discrepancies were sufficiently small to be explained by plural scattering. The authors were unable to explain why the presence of energy dis- crimination at the counters made such a large difference to their measurements since analysis of their results gave the conclusion that approximately one-third of the electrons recorded with no energy discrimination had suffered energy losses of 0 - 40 keV while travelling between the gun and the counters. Theoretically the most probable energy loss at scattering at gold foils of thickness 10^{-5} cms of 121 keV electrons is approximately 125 eV and the probability of a loss in excess of ten times the most probable loss is only a few percent[77].

2.14 The present status of the Mott scattering theory

In view of the difficulties encountered in obtaining a satisfactory explanation for the observed energy losses in the work of Pidd and Nelson, it is difficult to assess the importance of this effect in earlier double-scattering experiments. It is interesting to note that prior to the work of Pidd and Nelson, the best agreement between theory and experiment had probably been obtained by Skull, Chase and Myers[13] who carried out their measurements using 400 keV electrons while Pettus[76] obtained better agreement between theory and experiment for 200 keV electrons than for 80 keV electrons. These results could be explained by the fact that energy losses tend to be more serious for low energy electrons than for high energy electrons, particu- larly in view of the nature of the energy dependence of the Mott

asymmetry.

The above explanation for the observed discrepancies between theory and experiment cannot be accepted as conclusive, however, since effects due to multiple and plural scattering, and the corrections to the theory for the screening effects of the atomic electrons are all more important at low energies.

The theory of plural scattering put forward by Goertzel and Cox[71] is only capable of putting a lower limit on the magnitude of the effect. Ryu[74] evaluated the effect of plural scattering at a gold foil, oriented at 45° to both the incident and scattered beams, for a scattering angle of 90° but his theory was unsuccessful in explaining his experimental results.

Recently a theory for the multiple scattering of electrons in thin foils has been put forward by Mühlochlegel and Koppe[79] and Wegener[80] has made calculations on the effects of both multiple and plural scattering on the Mott asymmetry. Since plural scattering effects are much more important than multiple scattering effects in normal double-scattering experiments the work of the last named author appears to be more relevant to this field. However the application of Wegener's theory is restricted to the case of an electron beam incident normally on the scattering foil and is further limited by the condition that the combined effects of multiple and plural scattering on the Mott asymmetry must be considerably smaller than the asymmetry itself. With the normal scattering foil thicknesses, electron energies and scattering angles used, this latter condition is seldom fulfilled in double-scattering experiments.

Another explanation for the better agreement between theory and experiment obtained at high, as opposed to low, energies may lie in the fact that screening corrections are larger at low energies than at high energies. As already pointed out (2.3) accurate values of these corrections are not yet available but it is interesting to note that Pidd and Nelson[77] interpreted their results as indicating the existence of a screening effect at least at a scattering angle of 80°.

2.15   Modifications to the Mott theory

Due to the lack of agreement between theory and experiment, attempts have been made to modify the Mott scattering theory by the introduction of deviations from the Coulomb scattering field other than those due to screening effects[81, 82]. In view of the results of Pidd and Nelson together with the work described in 3.2 it is extremely doubtful whether such modifications are applicable.

2.16   Complete verification of the Mott theory

It has been pointed out by Tolhoek[30] and also by Schopper[83] that for a complete verification of the Mott scattering theory it would be necessary to determine the change in polarization of an initially polarized electron beam due to a scattering process. The latter author examined the effects which would be expected to appear in such a second-order experiment but no experimental evidence is available on this question at present.

CHAPTER 3.

THE LONGITUDINAL POLARIZATION OF β-PARTICLES

3.1 The use of Mott scattering

The use of Mott scattering for the measurement of the
longitudinal polarization of β-particles had an advantage
over the other methods employed for this purpose insofar as
considerable experience in its use in the measurement of electron
polarization had been acquired prior to 1956. Since Mott
scattering could only be used to detect transverse polarization
however, it was necessary to devise methods for the conversion
of the longitudinal polarization of β-particles into a trans-
verse polarization before the scattering took place. The three
methods which have been successfully developed for this purpose
are discussed in the following paragraphs. In each case the
scattering foil and counter system used in conjunction with the
"spin-rotator" have been very similar to those discussed in
Chapter 2, with the exception that only one scattering foil had
to be used.

3.2 The electrostatic deflection technique

If an electron enters a transverse electric field then its
momentum direction is changed due to the interaction of the
electric field with the electronic charge but its spin direction
remains almost unaltered and it is therefore possible to vary
the angle between the momentum direction and the spin direction[30].
With reference to figure 3, if a longitudinally polarized electron
beam enters the space between the cylindrically-shaped electric

Deflection of an electron beam in an
electric field. [non-relativistic
approximation].
Dotted lines : electric field lines.
Short arrows : spin orientation.

Figure 3   The Electrostatic Deflection Technique

field plates B and C at the point A then, as shown schematically in the diagram, the beam which emerges at the point D has a transverse component of polarization.

One of the main advantages of this method lies in the fact that the transverse electric field acts as an energy selector. This technique has the disadvantage that no method exists whereby a null measurement can be obtained with a gold scattering foil nor is it possible to reverse the direction of the asymmetry (cf. method 3). In consequence, errors due to incorrect positioning of the source relative to the electric field and to the non-uniform deposition of the source material are difficult to eliminate[85].

The general method of deflection in an electrostatic field has been used by several groups[85-89] and their work is summarized in Table 1. The low values of $P/\frac{v}{c}$ obtained by Fraunfelder et al[84] were due to depolarization effects in the scattering foils[80]. Depolarization effects in the scattering foils and also in the rather thick sources used by Langevin-Joliot et al[89(a) and 89(b)] were probably the reason for the low values of $P/\frac{v}{c}$ obtained in their experiments. The other results are consistent with the predictions of the two-component theory within the limits of the rather large experimental errors.

The presence of electrons in the beam emergent from the electric field which have been scattered at the electric field plates reduces the degree of polarization of the beam. This effect has been studied by Bienlien et al[88] by using the electric field plates, employed in their polarization experiments, as part of a β-ray spectrometer. By varying the magnitude of

Table 1.

Results on the longitudinal polarization of β-particles
obtained by the use of the method described in 3.2

Group	Source	Energy K keV.	Foil thickness	ψ	θ	P/$\frac{v}{c}$	X (%)
Fraunfelder et al [84]	Co^{60}	50 68 77 77	.15G .15G .05G .15G	108°	95°–140°	0.1 0.34 0.82 0.71	100 15 17 30
De Waard and Poppema (85)	Co^{60}(a) Sc^{46}(b) P^{32} (c) Tm^{170}(d) Au^{198}(e)	170	.05–.25G	90°	90°	Relative measurements P^a/P^b = 1 P^c/P^d = 1 P^c/P^e = 1	15 N.Q. N.Q.
Vishnevsky et al [86]	Cu^{64}	145	.12, .24G .12, .24G	90°	90°	1.03 0.79	14 19
Langevin–Joliot et al [89(a)]	Sr^{90}	204 204 128 128	0.4G 0.4G 0.2G 0.2G	127°20"	99°–129°	0.70 0.57 0.38 0.32	36 38 65 63
Langevin–Joliot and Marty (89(b))	S^{35}	128	0.26G 0.35G	127°20"	99°–129°	0.67 0.62	35 38
Bienlien et al [88]	Co^{60}	160	Various G	110°	120°	0.96	6
Malone et al [87]	Co^{60}	200	Various G	N.Q.	70°	1.03	5

ψ   is the angle turned through by the electron momentum
while traversing the electrostatic field.

θ   is the scattering angle: N.Q. $\equiv$ not quoted in text.
Foil thicknesses are quoted in units of mgm/cm^2.
G signifies gold foils.

X   is the quoted percentage error on the P/$\frac{v}{c}$ result.

the applied potential they studied the energy distribution of
the electron beam transmitted by the electric field; they
obtained a linear Kurie-plot down to an electron energy of 150 keV
and consequently they assumed that above this energy the effect
of inelastic scattering could be ignored. Further advantages of
working with relatively high energy electrons in this type of
experiment are that the depolarization effects at the source and
the effects of plural and multiple scattering at the scattering
foil are reduced and that the screening effects are likely to be
of less importance though Bienlien et al encountered difficulties
with the last named effect at an electron energy of 160 keV.

Bienlien et al[100] have used their apparatus to investigate
the energy dependence of the Mott asymmetry for electrons in the
energy range 120 keV - 210 keV at a scattering angle of 120°.
They measured the Mott asymmetry for Au, Ce, Ag and Cu scattering
foils. At high energies their results were in agreement with
the theoretical calculations of Sherman[67] and the discrepancies
which they observed at low energies between theory and experiment
were attributed to the effects of screening.

The same group have also investigated the angular dependence
of the Mott asymmetry for electrons of energy 155 keV over the
angular range 40° - 150°[101]. Within the limits of the accuracy
of their work ($\sim$ 10%) they obtained asymmetry values in agreement
with the calculations of Sherman[67]. The accuracy of their
results was not sufficiently good to indicate the magnitude of
the screening corrections. A possible conclusion to be drawn
from the consideration of their results together with those of

Pidd and Nelson[77] is that the Mott scattering theory is correct and that the suggested modifications to it are unnecessary (2.15).

3.3   The Multiple scattering technique

The second technique which has been used to convert the longitudinal polarization of an electron beam into a transverse one is that of multiple scattering at a foil of low atomic number. By virtue of the low atomic number of the scatterer the spin-orbit force is very small and consequently the Coulomb field of the nucleus is primarily responsible for the scattering; under such conditions the momentum vector of an incident electron is rotated through a much larger angle than that of the spin vector with the result that the polarization direction is altered.   The theory of such a scattering process has been worked out by Bernardini et al[102].

The main advantage of this method is its simplicity, since no electric or magnetic fields are required for its operation. In the other two methods, however, the electric and magnetic fields act as energy selectors and the absence of such discrimination in the multiple-scattering technique constitutes a serious difficulty since the degree of polarization is, in theory, directly proportional to the electron velocity.   It is not only necessary to know the range of electron energies recorded but it is also necessary to take into account the energy dependence of the multiple scattering at the first foil, of the multiple and plural scattering at the second foil and of the scattering cross-section at both foils (for a full discussion of these effects see reference 92).   The magnitude of these effects are difficult to evaluate accurately and it is for this reason that the stated accuracy

of the work of Alikhanov et al[90] has been questioned[44].

The multiple-scattering technique has been used quite extensively[90 - 96] and the results obtained by the use of this method are shown in Table 2. Due to the presence of the phenomena described in the previous paragraph the method is not suitable for the attainment of accurate absolute values of the degree of electron polarization and its main application has been to the relative measurement of the degree of polarization of electrons from different types of interactions, particularly for transitions with approximately the same shape of β -spectrum and approximately the same end-point energy.

Bühring and Heinzte[93] have used this technique to determine the ratio of the longitudinal polarization of β -particles from RaE to that of β -particles from Tl^{204} and Y^{91}. RaE is a particularly interesting decay since it is the only case that has been reported which gives rise to electrons which do not have a full $(\frac{v}{c})$ degree of polarization[47]. Bühring and Heintze have been able to account for this discrepancy by the use of the nuclear matrix elements suggested by the characteristics of the RaE β - spectrum.

Using the same apparatus as that employed for the RaE measurements, Bühring[94] investigated thedegree of longitudinal polarization of β -particles from Ho^{166} in order to obtain an estimate for the pseudoscalar contribution to the interaction. On the basis of the two-component theory of the neutrino, the results obtained placed an upper limit of 3×10^{-7} on the pseudoscalar contribution.

Table 2.

Results on the longitudinal polarization of β-particles
obtained by the use of the method described in 3.3

Reference	Source	Energy ke.V.	1st Foil	2nd Foil thickness	ψ	$\ominus$	P/$\frac{v}{c}$	X
(95)	P^{32}	> 900	Al	2.5G	90°	75°	C.f.P.	N.Q.
(96)	$P^{32}(a)$ $Au^{198}(b)$	~ 250	Al		90°	73°	$P^a/P^b = 1$	N.Q.
(91)	$Sr^{90} + Y^{90}$	200-400	Al	0.25G -20.8 Pb	30°	135°	0.82	18
(92)	$Tl^{204}(c)$ $Au^{198}(d)$ $Sr^{90}(e)$ + Y^{90}	> 200	Cu	0.73G 1.95 Pb	90°	135°	$P^d/P^c = 0.98$ $P^c/P^e = 0.99$ $P^d/P^e = 0.97$	5 3 4
(93)	RaE (f) $Tl^{204}(g)$ Y^{91} (h)	250-600	Cu	N.Q.	90°	135°	$P^h/P^g = 1.016$ $P^f/P^g = 0.83$	1.5 2
(94)	$Ho^{166}(i)$ P^{32} (j)	250 -600	Cu	N.Q.	90°	135°	$P^i/P^j = .992$	2
(90)*	$Sr^{90}+Y^{90}$ Tm^{170} Sm^{153} Lu^{177} Re^{177} Au^{198}	145-650	Al	0.12-1.97 G	90°	112.5°	0.99 0.98 0.98 1.00 1.06 0.97	5* 3 5 6 11 6

For explanation of symbols see next page

Table 2 (contd.)

G $\equiv$ gold: Al $\equiv$ aluminium: Pb $\equiv$ lead: Cu $\equiv$ copper

The thicknesses of the 2nd scattering foils are quoted in mg/cm^2.

ψ is the angle of scattering at the 1st scattering foil

Θ is the angle of scattering at the 2nd scattering foil

N.Q.   not quoted in text.

C.f.P.   comparable with full polarization

X   is the quoted accuracy of the P/$\frac{v}{c}$ results.

*   for a discussion of these errors see text.

3.4 The crossed fields technique

The third method which has been developed to change the longitudinal polarization of an electron beam into a transverse one utilizes crossed electric and magnetic fields. This technique is discussed in detail in Chapter 4 but essentially its operation depends on the fact that for one particular electron velocity the forces on the electron due to the electric and magnetic fields are equal in magnitude and opposite in direction with the result that electrons of this particular velocity pass undeflected through the fields. The electric field due to a parallel-plate condenser (which is normally used to provide the electric field) has no effect on the magnetic moment of the electron but the magnetic field exerts a couple on the magnetic moment which therefore precesses as the electron traverses the crossed fields. Hence it is possible to control the sense of polarization of an electron beam. This technique has not been used so extensively as the other two probably because of the technical difficulties associated with the production of electric fields of the required magnitude and with the attainment of magnetic fields which are sufficiently uniform over the required distances. The results obtained by the three groups who have used this method are summarised in Table 3.

The first two groups[97, 98] chose to keep the electron path length in the crossed fields constant and varied the sense of the transverse polarization and consequently the sign of the azimuthal asymmetry by reversing the direction of both the electric and magnetic fields. For the satisfactory operation of this technique the electrons, having left the crossed fields region, must be

Table 3.

Results on the longitudinal polarization of β-particles obtained by the use of the method described in 3.4

Group	Source	Energy keV	Spin Control	Foil Thickness	θ	$P/\frac{v}{c}$	X %
Cavanagh et al [97]	Co^{60} Au^{198}	128	$\pm 90°$	Various G	90°	0.65 0.98 0.95	20 11 21
Alikhanov et al [98]	$Sr^{90} + Y^{90}$	300 750 300	$\pm 90°$	0.537G 1.9G 0.17G	90° 90° 105°	0.80 1.15 1.10	25 35 17
Mikaélyan and Spivak (99)	P^{32} (a) Sm^{153}(b) Lu^{177}(c) Ho^{166}(d) In^{114}(e)	340	$+90°$	0.55 G	120°	Relative Measurements $P^a/P^b = 1.047*$ $P^c/P^b = 0.945$ $P^d/P^b = 0.930$ $P^e/P^b = 0.965$ Absolute Measurement	1.1 1.3 1.3 3.1
	Sm^{153}	340	$+90°$	.18, .36, .55 G	120°	0.90	4

The thicknesses of the scattering foils are quoted
in mg/cm^2.

θ is the scattering angle

X   is the quoted accuracy of the $P/\frac{v}{c}$ results

*   for a discussion of these values see text.

unaffected by the sense and the magnitude of either the electric
or the magnetic field.   For these conditions to be fulfilled the
electric and magnetic fringe fields must be as small as possible.
Further, since it would be likely that some of the effects
associated with the fringe fields would change sign on reversal
of the field directions, then any systematic errors introduced
by these effects would be unlikely to be apparent in the relevant
magnitude of the measured asymmetries and would therefore be
difficult to eliminate.   The technique used by Cavanagh et al[97]
and Alikhanov et al[98] has the advantage, however, that it can
be used over a range of electron energies without changing the
position of the scattering foil or the counters (of the method
described in Chapter 4).

It appears that the effects of multiple and plural scatter-
ing were not taken into account in the work of Alikhanov et al
though these effects would probably have been small for the
electron energies studied.   Cavanagh et al eliminated these
effects from their final results by measuring the asymmetries for
a range of thicknesses of scattering foil and extrapolating the
measurements to zero foil thickness.   (See Chapter 5).

Both groups investigated the azimuthal dependence of the
scattering asymmetry and obtained good agreement with theory.

The work of Mikaélyan and Spivak[99], us ing the crossed
fields technique, differed from that of the other two groups
insofar as no attempt was made to reverse the direction of the
polarization-asymmetry by reversing the directions of the
electric and magnetic fields.   Further, for the relative

measurements, the instrumental asymmetry was not measured and consequently it would appear possible that the non-uniform deposition of the source materials on the source-holders was the reason for the varying degrees of polarization obtained for the different sources. Since the sign of the polarization asymmetry was not reversed during the course of the experiment it would appear possible that inherent asymmetries in the gold foils used for the absolute measurement introduced an effective instrumental asymmetry, for which correction was not made. It must be concluded, therefore, that although the statistical accuracy of the work of Mikaélyan and Spivak was considerably better than that achieved by the other two groups, their experimental technique was more prone to give rise to systematic errors.

Cavanagh et al[97] studied the degree of polarization of electrons in the energy range 58 keV - 178 keV by applying potentials ranging from -70 KV to +50 KV to the source thus eliminating the troublesome effects associated with the direct investigation of low energy electrons[80, 84]. By such a technique it was hoped to place an upper limit on the magnitude of a Coulomb term in the expression for the degree of polarization but their work was not sufficiently accurate to do this.

CHAPTER 4.

THE THEORY OF THE EXPERIMENT

4.1 The proposed experiment

At the time when this experiment was begun (Autumn, 1957) the position in the field of weak interactions was very confused due to conflicting experimental evidence on the degree of longitudinal polarization of β-particles from different types of interactions, and on the nature of the coupling constants in β-decay[108,109,51,53]. In particular the electron polarization experiments gave results of relatively poor statistical accuracy and the methods used to obtain these results appeared likely to give rise to systematic errors.

In order to obtain good statistical accuracy in a measurement of the velocity dependence of the electron polarization the Mott scattering technique was chosen because the velocity dependence could best be investigated in the region $\frac{v}{c} = 0.4 - 0.7$ and it was precisely in this region that Mott scattering had a higher sensitivity than the other methods[97].

As discussed in the previous chapter, three methods have been used to transform the longitudinal polarization of β-particles into a transverse polarization namely by deflection in an electrostatic field, by multiple scattering and by using crossed electric and magnetic fields. Because of the difficulties associated with the first two methods, particularly in their application to the measurement of absolute values of the degree of polarization, the third technique was adopted. The idea was conceived that, by varying the position of a radioactive source placed in crossed

electric and magnetic fields, that is by varying the time spent by the emitted β -particles in the crossed fields, the polarization direction could be altered in a way unlikely to introduce appreciable systematic errors.

In order to convert the longitudinal into a transverse polarization, electrons from a radioactive source were allowed to pass through crossed electric and magnetic fields, the relative values of which were chosen to select electrons of a convenient energy (100 keV) while the absolute value of the magnetic field was selected so that the electron spin axis would be rotated through an angle of 90° in the time taken for an electron to traverse approximately one-third of the total length of the crossed fields.   By means of a movable radioactive source the electron path length could be varied and hence the electron spin axis could be rotated through any angle between 0° and 270°.   On emerging from the crossed fields the electrons were acted on by the magnetic field alone and consequently traversed a circular path before being incident, at an angle of 90°, on a gold foil. Electrons scattered through an angle in the range $110^{\circ}-165^{\circ}$ were detected by means of two electron-sensitive plates placed symmetrically with respect to the electron beam (Figure 4).   It was considered that the large degree of control over the direction of the spin of the electron incident on the scattering foil should lead to results of good statistical accuracy.

4.2   The basic theory of the experiment

It can readily be shown that for electrons of velocity V to pass undeflected through crossed electric and magnetic fields, the following relation must hold:

$$V = \frac{E}{B}$$

4.2.1

Figure 4.   Schematic Diagram of Apparatus.

where V is in metres/sec, E is the electric field strength in volts/metre and B is the magnetic flux density in webers/metre^2.

The influence of electric and magnetic fields on the spin orientation of electrons in a beam has been calculated according to the Pauli spin theory and the Dirac theory[30] and also by the use of a consistent set of covariant classical equations of motion[104]. From such work it follows that if an electron travels a distance l metres through crossed electric and magnetic fields then its spin axis is rotated through an angle Θ, where Θ is given by the following equation.

$$\Theta = \frac{e}{m_o} \quad \frac{B^2 l (1 - \beta^2)}{E} \quad \text{radians} \qquad 4.2.2$$

where e and m_o are the charge and the rest mass of the electron, respectively, E and B are defined as above and β is the ratio of the velocity of the electron to the velocity of light in vacuo.

Strictly speaking equation 4.2.2 is valid only when the gyro-magnetic ratio of the free electron is equal to 2.

4.3   The equations of motion for an electron in crossed fields

It is a general characteristic of electron polarization experiments that a certain amount of depolarization occurs due to the spread of electron energies and to the finite dimensions of the electron beams.

Figure 5

Consider an electron emitted in the x-y plane at an angle to the x-axis from the source at O (Figure 5). The electric field E is in the -y direction and the magnetic field B in the -z direction (i.e. directly into the paper). If the initial velocity of the electron be v_o, with x and y components $\dot{x}_o$ and $\dot{y}_o$ and if the velocity of the electron at a later time t be v, with x and y components $\dot{x}$ and $\dot{y}$, then the equations of motion are given by the following expressions:

$$m\ddot{y} = -Be\dot{x} + Ee \qquad 4.3.1$$

$$m\ddot{x} = Be\dot{y} \qquad 4.3.2$$

From the principle of the conservation of energy it follows that

$$\tfrac{1}{2} m (\dot{x}^2 + \dot{y}^2) = \tfrac{1}{2} mv_o^2 + Eey \qquad 4.3.3$$

If Θ be the angle between the electron momentum direction and the x-axis at time t then, provided both Θ_o (the value of Θ at time t = 0) and Θ are small, the following equations are valid

$$\Theta_o = \frac{\dot{y}_o}{\dot{x}_o} \qquad \text{and} \qquad \Theta = \frac{\dot{y}}{\dot{x}} \qquad 4.3.4$$

Integrating equation 4.3.2 once with respect to time and substituting the resultant expression (with the appropriate boundary conditions) into equation 4.3.1 we obtain

$$\ddot{y} = -\left(\frac{Be}{m}\right)^2 y - \left(\frac{Be}{m}\right) \dot{x}_o + \frac{Ee}{m} \qquad 4.3.5$$

By the use of this equation together with the relationship

$$v_o^2 = \dot{x}_o^2 + \dot{y}_o^2 \qquad 4.3.6$$

and, on making the assumption that $\dot{y}_o$ is small, we obtain

$$\ddot{y} = -\left(\frac{Be}{m}\right)^2 y + \left(\frac{Be}{2m}\right)v_o \times \left(\frac{\dot{y}_o}{v_o}\right)^2 \qquad 4.3.7$$

4.4 The transmitted energy range

For the special case when the electron is emitted in the x direction with velocity $v_o + \delta v_o$ then equation 4.3.5 takes the form

$$\ddot{y} = -\left(\frac{Be}{m}\right)^2 y - \left(\frac{Be}{m}\right) \times (v_o + \delta v_o) + \frac{eE}{m} \qquad 4.4.1$$

By the use of equation 4.2.1, equation 4.4.1 reduces to

$$\ddot{y} = -\left(\frac{Be}{m}\right)^2 y - \left(\frac{Be}{m}\right) . \delta v \qquad 4.4.2$$

i.e. $$y = -w^2 \left(y + \frac{\delta v}{w} \right) \qquad 4.4.3$$

where $$w = \frac{Be}{m} \qquad 4.4.4$$

The solution of equation 4.4.3, under the appropriate conditions, leads to an expression for the range of electron energies, δ E, transmitted by a defining slit of width t mms, of the form

$$\frac{\delta E}{E} \sim t \times 10^{-2} \qquad 4.4.5$$

where E is the mean electron energy in keV. Under the condition that E and B were set to transmit electrons of energy 100 keV and taking into account the height of the source, the width of the slit and the geometry of the apparatus, it was found by use of equation 4.4.5 that electrons in the energy range (98 - 103)keV emerged from the slit.

4.5 The angular range of the transmitted electron

In order to calculate the magnitude of the depolarization effects in the beam it was necessary to determine the magnitude of the solid angle subtended at the source by the slit.

From equation 4.3.7 it is clear that electrons of initial velocity v_o, emitted at an angle Θ_o to the x axis, oscillate about a line which is at a distance X above the x axis where X is given by the relationship

$$X = \frac{\left(\frac{Be}{m}\right) v_o \left(\frac{\dot{y}_o}{v_o}\right)^2}{\left(\frac{Be}{m}\right)^2} \qquad 4.5.1$$

The periodic time of the oscillation is given by the expression

$$\tau = \frac{2\pi m}{Be} \qquad 4.5.2$$

The solution of equation 4.3.7 is

$$y = \left(\frac{m}{2eBv_o}\right) \dot{y}_o^2 + A \sin(wt + \varepsilon) \qquad 4.5.3$$

On inserting the appropriate conditions into equation 4.5.3 the following expression is obtained

$$y = \frac{mv_o}{2Be} \Theta_o^2 + \frac{mv_o}{2Be} \Theta_o \times \left(1 + \frac{\Theta^2}{2}\right)^{\frac{1}{2}} \sin(wt + \varepsilon) \qquad 4.5.4$$

By inserting the appropriate values for m, v_o, B and e into equation 4.5.4 and, making the assumption that Θ_o is small, equation 4.5.4 gives the result

$$y \sim 27\,\Theta_o \qquad \text{(y is in mms and } \Theta_o \text{ in radians)}$$

For the height of slit used in this experiment

$$\Theta_o \sim \pm 2^o$$

The angle Θ_o has been obtained by assuming that the electron moves only in the x-y plane. From the consideration of the path of an electron emitted by the source at an angle ϕ

to the x-y plane then, from calculations similar to those carried out in the determination of the value of Θ_o , it was found that electrons emitted in the angular range $\phi = \pm 2\frac{1}{2}^o$ about the central position were incident on the scattering foil.

4.6   Depolarization effects in the crossed fields

The electron energy range gives rise to depolarization in two distinct ways.   Firstly, electrons of different velocities have their spin directions rotated by different amounts due to the variation in the time spent by the electrons in the crossed fields. Secondly, because of the different paths followed in the crossed fields, electrons of different velocity which are emitted from the source at the same angle, will not emerge from the defining slit at the same angle and therefore will not be incident on the scattering foil at the same angle.

The angular spread of the electron beam results in depolarization since, if electrons are emitted at different angles by the source, then the initial spin directions, and therefore the spin directions after traversal of the crossed fields, are different. Further, electrons of the same energy emitted at different angles by the source spend different times in the crossed fields and consequently their spin directions are rotated through different angles.

Alikhanov et al[98] have shown that if an electron of velocity v_o has its spin direction rotated through an angle ϕ_o when traversing crossed fields then an electron of velocity v has its spin rotated through an angle ϕ when traversing the same crossed fields, where ϕ is given by the following expression.

$$\sin \phi = \frac{\sin \phi_o}{\left[\frac{\left(\frac{v}{v_o} - 1 + \left[1 - \frac{v v_o}{c^2}\right] \cos \phi_o\right)^2}{\left(1 - \frac{v^2}{c^2}\right)\left(1 - \frac{v_o^2}{c^2}\right)} + \sin^2 \phi_o\right]^{\frac{1}{2}}} \qquad 4.6.1$$

where $\phi_o = \dfrac{300\, H\, \ell \left[1 - \left(\dfrac{v_o}{c}\right)^2\right]^{\frac{1}{2}}}{p_o\, c}$ \qquad 4.6.2

ℓ being the path length of the electron in the magnetic field (in cms), H the magnetic field strength in oersteds and p_o the momentum in units of $\frac{eV}{c}$ corresponding to the velocity v_o defined by the relationship $\frac{v_o}{c} = \frac{E}{H}$.

The application of equation 4.6.1 to electrons with the largest and smallest values of energy and emission angles, consistent with the condition that they be finally emergent from the defining slit, (i.e. E = 103 keV, E = 98 keV : $\Theta = +2^o$, $\Theta = -2^o$: $\phi = +2\frac{1}{2}^o$, $\phi = -2\frac{1}{2}^o$) together with the consideration of the work of Mendlowitz and Case[107] on the depolarization effects in a double-scattering experiment carried out in a magnetic field, led to the conclusion that the depolarization effects due to the energy range and to the finite dimensions of the electron beam were less than 1% in this experiment.

A particularly useful property of the crossed fields technique lies in the fact that a fairly large error (e.g. 10%) in the angle of rotation of the spin axis in the crossed fields leads to only a very small error ($\sim$1%) in the polarization asymmetry value. This fact has been experimentally verified by Mikaélyan and Spivak[99]. Because of the presence of this factor it is permissible to neglect

the effect of the acceleration of the electrons in the crossed
fields by the electric field.   For example, with reference to
equation 4.6.1, if $v = 0.6$ c (121 keV) and $V_o = 0.55$ c (100 keV)
and if the parameters of the crossed fields be chosen such that
$\phi_o = 90^o$, then it is found that $\phi = 82^o$, but since it is the
cosine of the angle of deviation which is of importance, the
resulting discrepancy is only about 1%.

Depolarization effects may also arise from the presence of
non-uniformities in the magnetic and in the electric fields.
The depolarization occurs partly because the spins of the electrons
in different parts of the beam precess through different angles,
due to the varying magnitude of the magnetic field, and partly
because electrons, in different parts of the beam follow paths
which are not geometrically similar, with results identical to
those discussed at the beginning of this section.

CHAPTER 5.

THE APPARATUS

5.1 The vacuum chamber

The apparatus was contained in a rectangular brass box which was securely clamped between the pole-faces of the permanent magnet (5.2) and which was continuously evacuated to a pressure of 10^{-4} - 10^{-5} mms Hg. The breakdown potential of the electric field depended rather critically on the quality of the vacuum and care was taken to maintain the pressure at as low a value as possible.

5.2 The magnetic field

A large permanent cobalt steel magnet, originally designed by Cockcroft et al[110], supplied the transverse magnetic field, the strength of which could be adjusted by passing a suitable current through six energising coils surrounding the laminated steel magnet arms. The current for adjusting the magnetic field was obtained from the 230 volts D.C. mains through a reversing switch and adjustable series resistances. The most important feature of the magnet, as far as this experiment was concerned, was its ability (as originally investigated by Ellis[111]) to provide a uniform magnetic field, to better than 1%, over a distance of 23 cms and it was in this region that the experiment was carried out. The fact that the air gap between the pole-pieces of the magnet was only 5.5 cms constituted a difficulty (5.17).

The magnetic field strength was measured in two ways, firstly by the use of a search-coil and a Grassot fluxmeter, which had previously been calibrated by means of a Hibbert standard and

secondly by measuring the radii of curvature of electrons from three conversion electron lines of known $H\rho$ values from the spectrum of $Th(B \to C)$, using a well-defined slit system and photographic plates. A field strength of approximately 350 gauss was used and its value was known to better than 1%. No variation was noted in the magnitude of the magnetic field over long periods.

5.3 The electric field

The power to supply the electric field was obtained from H.T. apparatus capable of providing 100 KV D.C. and consisting of an H.T. variac, a large transformer, a rectifier and an R-C smoothing device (figure 6(A)). Difficulty was experienced in getting ordinary resistances to operate satisfactorily under the experimental conditions and a liquid resistance was used (Figure 6(B)). The H.T. ripple was measured using a resistance chain and a double-beam C.R.O. Under the operating conditions the ripple was approximately 0.02%. The H.T. output voltage was calibrated against the input voltage of the transformer using a resistance chain together with an electrostatic voltmeter. The absolute value of the H.T. voltage was known to better than 1%. Slow fluctuations were noted in the input voltage, which was obtained from the mains, and manual adjustments were made to the H.T. variac during the course of the experiments to correct for this effect.

It was necessary to produce an electricfield of approximately 60 KV/cm between two rectangular plates, 18.8 cms in length and 0.5 inches in breadth (figure 7(A)). An attempt was made to

OUTPUT.

G

F .033μF

A and C are Variacs; B is a transformer [input voltage = 0 - 230v, output voltage = 0-10v output current = 0-8 amps]; D is a transformer [input voltage = 0-230v, maximum peak voltage = 100 k.v.; maximum current = 0.05 amps]; E is a rectifying valve [maximum peak voltage = 150 k.v.]; G is a liquid resistance [see below].

Figure 6(A)   The H.T. Apparatus.

B

D

A

C

230v

230v

E

← - - - - - - - 20 cms - - - - - →

Made of glass tubing, ¾" diameter; Contents = 50% distilled water + 50% tap water. Resistance value = 5 MΩ.

Figure 6(B) The Liquid Resistance.

E

D

X   Y

4.1 cms.

19 cms

A   B   C

4.1 cms.

A, B and C are made of duralumin

D and E are made of polystyrene.

The trough [see below] slides in between X and Y.

Figure 7(A).   The Electric Field Plates.

Made either of ebonite or polystyrene.
The trough contains a brass bar which is
connected to the H.T. supply [not shown]

19.5 cms.

1.5 cms.

4.2 cms.

Figure 7(B).   The Trough.

produce the field by applying a voltage of 60 KV across a plate
gap of 1 cm but, because of the limited space available for input
connections to the field plates, considerable trouble was exper-
ienced with corona discharge and consequently the effective gap
between the plates was reduced to 6 mms with a consequent decrease
in the necessary voltage.   The lower plate was made of duralumin
in order to reduce scattering and, to achieve the required field
strength without electrical breakdown, the upper plate was
enclosed in a trough of insulating material (figure 7(B)).
Troughs were made of two materials, ebonite and polystyrene, the
former being more durable and more easily machined while the
latter is a better insulator.   The insulating properties of
both types deteriorated with time and had to be replaced.

5.4   Errors in the electron velocity

As shown in 4.4, electrons in the energy range 98 keV –
103 keV emerged from the defining slit under ideal conditions.
There were two possible sources of error in the value of the
selected energy range, the one arising from the incorrect setting
of the magnitudes of the electric and magnetic fields and the
other from the imperfect control of the input voltage during the
course of the experiments.   It was considered that the maximum
error in the mean electron velocity due to both these effects
was about 2%.   By the use of equation 4.6.1 the change in the
electron spin precession due to such changes in the electron
velocity, was calculated and found to be negligible.   It was
also calculated that such a change in the electron velocity would
produce a change of 0.5% in the polarization asymmetry (5.17)
(neglecting the effect of the variation in velocity on the

angular distribution of the electrons, scattered by the foil, and incident on the emulsion (5.17)). The most serious effect of a 2% error in the mean electron velocity was the resulting 2% error in the theoretical degree of longitudinal polarization $(P = \frac{v}{c})$. However, since the final value of the longitudinal polarization was calculated from the results of six different experiments and since it would be expected that the above effects would vary in a random way over these experiments, then any errors due to the uncertainty in the electron velocity would be expected to appear in the statistical error of the final result (7.9).

5.5 The radioactive source

A considerable number of factors affected the choice of a suitable radioactive source for this experiment. It was essential that no γ-radiation came from the source as the presence of such radiation would have seriously affected the electron-sensitive plates. In order to obtain results of good statistical accuracy in a reasonable time, it was necessary to have as large an electron counting rate as possible. This was particularly true when using electron-sensitive plates as recorders since the emulsion tended to peel from the glass backing if placed in the vacuum for a period exceeding approximately twenty-five hours. It was therefore necessary for the β-spectrum of the selected source to have a sufficiently low end-point energy to provide a reasonable fraction of electrons with energy in the range (98 - 103) keV. The source had to be carrier free and had to have as small an amount of impurity in it as possible in order to reduce depolarization effects (5.8). It was also

necessary for the source to have a half-life at least of the order of months.

S^{35} appeared to satisfy these requirements and calculations were made on the required source strength. It was possible to evaluate the number of electrons incident on the scattering foil for different strengths and positions of the source by making suitable calculations on the known shape of the β -spectrum of S^{35} (106) and by the use of the resolving power of the crossed fields and of the relevant solid angles, both of which had been determined previously (4.4 and 4.5).

Using a corrected form of the Rutherford scattering cross-section[112] and the calculations of Doggett and Spencer[62] on the Mott cross-section, the elastic scattering cross-section of the system was evaluated. From these data, together with a knowledge of the angular distribution of the electrons incident on the emulsion (5.17) and of the solid angle subtended at the foil by the collimating windows (5.14), it was possible to obtain a simple relation connecting the thickness of the scattering foil, the source strength, the source position and the total number of electrons registered on the emulsions. Experiments were carried out to check this relationship and fair agreement between theoretical and experimental results was obtained. On the basis of this work it was decided that, under the proposed conditions of the experiment, a source strength of the order of (10 - 100) mC was required.

A carrier free 100 mC source of S^{35} (an allowed transition corresponding to $\Delta I = 0$ (no), end-point energy of 167.4 keV, half-life of 87.1 days[106])was obtained having a volume of 1.1 cc, the amount of solid present being approximately 5 μ g/cc.

5.6   The preparation of sources

Sources were prepared by evaporating the radioactive liquid onto thin aluminium foils (1 mg/cm^2). The effective parts of the source foils were approximately 8 mms in breadth and 3 mms in height. De Waard and Poppema[85] and possibly also Mikaélyan and Spivak[99] experienced trouble from the non-uniform deposition of the source material on their foils. Under normal conditions the additional depolarization effects due to non-uniform deposition would be negligible and the only result of such an effect would be to introduce an additional instrumental asymmetry. Since it would be expected that this additional instrumental asymmetry would be the same for all source positions it could be considered as part of the "normal" instrumental asymmetry and treated accordingly (5.16).

5.7   The source-holders

The source foils had to be earthed since, as is well-known from work in β-ray spectroscopy, an unearthed foil charges up and distorts the energy spectrum of the emitted particles. For this condition to be satisfied, source-holders had to be designed to withstand fields of 180 KV/cm, to be such as to produce as little electron back-scattering as possible and to have sufficient mechanical strength to stand up to considerable movement.

Of the source-holders designed and tested, two were reasonably successful (figure 8). Source-holder E stood up well to the electric field but the earthing foils tended to break down under the mechanical stresses involved in the movement of the source. No earthing difficulties were encountered with source-holder F but slow deterioration of the insulation was noted and the various

SOURCE-HOLDER E

Length of holder = 18 cmo.
D is a brass connection block [Figure 10].
E-E is a polystgrene strip.
F-F is a polystgrene cover for the source foil.
G-G is a strut: the source foil is connected
between the inside roof of the cover and
the strut.
The source is earthed by means of an
aluminium foil running under E-E.

0.5 cmo

SOURCE-HOLDER F

Length of holder = 18 cmo.
D is a brass connection block [Figure 10]
I-I is a polystyrene strip.
K-K is a polystyrene cover for the source foil.
L-L is a brass foil-support. and the source
foil is connected across it.
The source is earthed by means of the brass
strip J-J.

0.5 cmo

Figure 8   The Source-Holders

components had to be renewed regularly.

Source-holder E was used for the preliminary work and source-holder F was used in the first set of experiments (6.1). For the second set of experiments a new carrier-free source of S^{35} (volume 1.0 ml, strength 30 mC) was obtained and since a new source-holder had to be constructed, the opportunity was taken to make two improvements on source-holder F; the height was increased to 0.55 cms and the connecting strip J-J was enclosed on the top and the sides by a polystyrene trough. With these modifications a very successful source-holder was obtained.

It was found that small particles of radioactive material came off the sources and contaminated the apparatus. To minimize this, pieces of thin aluminium foil (0.2 mg/cm^2) were placed over the sources. In the first set of experiments (6.1) a thin piece of mica ($\sim$ 1 mg/cm^2) was put over the defining slit in order to prevent radioactive material getting into the plate-foil holder (5.14), an eventuality which would have had serious consequences since even a weak source outside the crossed fields would have contributed a proportionally large number of electrons to the electron-sensitive plates. During the second set of experiments no such piece of mica was put in position since it was found that electrical breakdown took place along its surface. The apparatus was regularly decontaminated using a strong caustic-soda solution. It is considered that the decontaminated equipment did not introduce a significant error into the final result (6.2).

5.8   Depolarization at the source

Depolarization in the region of the source may occur due to

backscattering in the foil on which the source is evaporated,
and also to multiple and large-angle scattering in the source
layer and in the foil covering the source.   Many workers have
carried out investigations on the magnitude of these depolari-
zation effects in conjunction with their experiments on electron
polarization (Chapter 3) but their results tend to be of signifi-
cance only for the particular experiments from which they were
derived.   This conclusion is drawn from the fact that the
depolarization effects are dependent on the energy of the electrons
studied, the end-point energy of the β -spectrum of the source
used, the thickness and the atomic number of both foils, the
thickness of the source layer, the atomic number and the atomic
weight of the source material and the geometry of the apparatus.

The degree of depolarization of β -particles due to multiple
and single scattering in the source layer has been evaluated
theoretically by Mühlochlegel and Koppe[79] and more fully by
Mühlochlegel[105].   As noted previously, one of the reasons for
choosing S^{35} as the source for this experiment was the fact that
it could be obtained in a carrier-free state and thus could be
used to make very thin sources[106].   It was estimated that the
sources used had a mean thickness of 20 μ g/cm^2.   Direct substi-
tution of this value and the appropriate parameters for the
experiment into the formulae of Mühlochlegel[105] gave the result
that the depolarization in the source layer was less than 0.1%.
It was rather doubtful if the theory applied to such a small
source thickness, however, but consideration of the theoretical
value of the depolarization together with the experimental work
of Heintze[92] and Bühring and Heintze[93] led to the conclusion

that the depolarization due to this factor was considerably
less than 1%.

From the work of De Waard and Poppema[85], together with
that of Heintze[92], it was clear that the depolarization due to
the presence of the 0.2 mg/cm^2 aluminium foil in front of the
source was negligible.

The most serious depolarization in the source region was
due to backscattering in the 1 mg/cm^2 aluminium foil on which
the sources used for both sets of experiments (6.1) were deposited.
From the work of Cavanagh et al[97], together with that of
Heintze[92], it appeared that the depolarization due to back-
scattering was about 1% in this experiment.

5.9 The slit system

In order to improve the resolution and decrease the number
of electrons getting to the electron-sensitive plates without
first being scattered by the foil, a defining slit was fitted to
the end of the electric field plates (figure 9). In order to
reduce scattering, the sides of the slit were bevelled. The
bevelling was done by hand and since it was conceivable that this
might have introduced an instrumental asymmetry, different slits
were used in the two sets of experiments (6.1).

To improve the resolution and decrease the background still
further, a second defining slit was attached to the side of the
plate-foil holder nearest the electric field plates (5.14) but
it was found that its presence gave rise to a sharp increase in
general background on the electron-sensitive plates (6.2) which
was attributed to the creation of low-energy bremsstrahlung
in the slit material. The second slit was therefore removed.

The end of the trough passes through aperture F
The slit is made of polystyrene.
G is the slit through which the electron
beam passes.

Figure 9   The Slit

5.10   The position of the source

The position of the radioactive source in the crossed fields could be altered by means of a control rod which entered the box through a vacuum seal (figure 10).   The screw C fitted into the block D on the source-holders (figure 8).   The position of the source relative to the electric field plates was known to 0.6% accuracy and the movements of the source were accurate to within 0.2%.   The errors in the changes of the spin precession angles due to errors in the positions of the source were negligible.

The lower electric field plate, on which the source moved, was securely attached to a large lead block which, in turn, was firmly held between the walls of the magnet box so that the movements of the source did not disturb any other parts of the apparatus.

5.11   The electron beam emergent from the crossed fields

Since it was essential that the electron beam, after emerging from the crossed fields, should be incident on the scattering foil at an angle of 90°, it was necessary to know the electron path accurately.   This was done by exposing photographic plates (Ilford H.P.3) at right-angles to the beam at various distances from the end of the electric field plates.   As well as the expected trace of the beam other images were found on the plates.   After some investigation it was decided that these were due to low-energy bremsstrahlung created at or near the source and, since it was unlikely that these would interfere with the experiment, they were neglected.

It was known that electric field-plates of the type used in this experiment had an end-effect insofar as the electric field at the ends of the plates was weaker than that at the centre.

Control Rod

27 cms.

10 cms.

A-A is the end-plate of the box.

B-B is an O-ring.

H-H is a hat packing

C is screwed into D [Figure 8]

The whole is made of brass

excepting the vacuum seals.

Figure 10.   The Source-Control.

The presence of such an effect might have affected the electron path and so various arrangements of earthed plates were put at the end of the electric field plates in order to reduce its magnitude but it was found that these were unnecessary as the end-effect was small and its influence on the electron path reproducible. The resulting small deviation in the electron path from that expected on theoretical grounds was taken into account in the calculations for the position of the scattering foil.

It was verified, using the photographic technique, that apart from changes in intensity, the characteristics of the beam emergent from the crossed fields were independent of the position of the source.   This was an important property since any variation in the beam position would have led to differences in the ranges of scattering angles of the electrons accepted by the windows in the plate-foil holder (5.17), and also to variations in the instrumental asymmetry;   both of these effects would have been very difficult to take into account.

5.12   The effects of non-uniformities in the electric field

As well as having an effect on the trajectory of the electron beam, the non-uniformity of the electric field at the source and at the end of the electric field plates could have had an effect on the degree and sense of the polarization of the electron beam. As discussed above, the end-effect of the field plates was small but it was unlikely that the same was true of the non-uniformity of the electric field at the source.

There was a volume in front of the source in which the characteristics of the electric field were unknown and in this region equation 4.2.2 was not valid.   The exact trajectories

of the electrons and the amount of precession of the electron polarization in this region could not be evaluated without a detailed knowledge of the electric field. It was postulated that the actual source at a distance ℓ from the end of the field plates could, for all practical purposes, be replaced by an imaginary, ideal source at a distance ℓ' from the end of the field plates ($\ell' < \ell$). Further, it was postulated that the electrons were acted on by the full value of the electric field immediately they left the imaginary source, that they had the same degree of polarization as the electrons leaving the actual source and that the angle between the electron polarization and the momentum of the electrons at the imaginary source was ψ. The main assumption contained in these postulations was that the electrons suffered no depolarization while passing through the region in which the electric field was non-uniform. No matter the degree of complexity of the trajectories followed by the electrons in this region, there would be no depolarization provided all the electrons travelled along paths which were geometrically similar. Since there were no obvious asymmetries in the geometry of the source-holders it was considered that this condition was satisfied for the electrons which finally emerged from the slit.

The effects due to the non-uniformity of the electric field at the end of the field plates would be much less than that at the source and could be corrected for by small additions to the values of ℓ' and ψ so that the electrons could be considered to have travelled a distance ℓ'' through crossed fields of the required magnitude and the electron polarization to have turned through an angle of ψ' with respect to the electron momentum during the time the electrons travelled in regions where the

crossed fields were not of the required magnitude.

Let Δ_0 be the polarization asymmetry produced by the scattering of an initially longitudinally polarized electron beam which has traversed a distance ℓ in crossed fields of the required magnitude, and in so doing has its polarization direction rotated through $90°$. Then, for an electron beam which has traversed a distance ℓ'' under similar conditions and which has associated with its initial polarization direction the angle ψ', as defined in the previous paragraph, the polarization asymmetry $\Delta_{1\ell}$ is given by the following relation:

$$\Delta_{1\ell} = \Delta_0 \, \sin(K\ell'' + \psi') \qquad 5.12.1$$

where K is defined by the expression

$$K\ell = \frac{\pi}{2} \qquad 5.12.2$$

Further, since the value of ψ' is independent of the position of the source and since the magnitude of $\ell - \ell''$ is independent of the value of ℓ, then the value of the polarization asymmetries $\Delta_{2\ell}$ and $\Delta_{3\ell}$ for the source positions $\ell'' + \ell$ and $\ell'' + 2\ell$ are given by the equations

$$\Delta_{2\ell} = \Delta_0 \, \sin\left[K(\ell'' + \ell) + \psi'\right] \qquad 5.12.3$$

$$\Delta_{3\ell} = \Delta_0 \, \sin\left[K(\ell'' + 2\ell) + \psi'\right] \qquad 5.12.4$$

By the use of equation 5.12.2 equations 5.12.3 and 5.12.4 reduce to

$$\Delta_{2\ell} = \Delta_0 \, \cos\left[K\ell'' + \psi'\right] \qquad 5.12.5$$

$$\Delta_{3\ell} = -\Delta_0 \, \sin\left[K\ell'' + \psi'\right] \qquad 5.12.6$$

It is to be noted that the polarization asymmetries $\Delta_{1\ell}$ and $\Delta_{3\ell}$ must be of opposite sign irrespective of the values of ℓ'' and ψ'. The asymmetry values $\Delta_{1\ell}$, $\Delta_{2\ell}$ and $\Delta_{3\ell}$, are of the form that were measured in this experiment and, clearly values of Δ_o could be obtained by using the above equations.

5.13   The determination of the electron energy

When the electrons emerged from the slit they were acted on by the magnetic field and consequently traversed a circular path of radius p, where p was given by the expression

$$p = \frac{mv}{Be} \qquad\qquad 5.13.1$$

where e and m are the charge and mass of the electron in m.k.s. units, respectively, B is the magnetic flux density in webers/metre^2 and v is the velocity of the electron in metres/sec. By measuring the radius of curvature of the electron path using photographic plates the electron velocity was determined from equation 5.13.1. This was done for various values of the electric and magnetic fields and the results compared with the values predicted by equation 4.2.1. The experimental and theoretical values for the electron velocity agreed to within 2%.

The energy of the emergent electrons was also determined using nuclear emulsions. An electron-sensitive plate was placed at right-angles to the electron beam and given a short exposure. After development the electron tracks were examined under a microscope and, by the technique of grain-counting, the electron energy was found to a fair degree of accuracy. Good agreement was obtained between theoretical and experimental results over a range of electric and magnetic field values.

5.14   The plate-foil holder

The plate-foil holder (figure 11) was constructed of aluminium in order to reduce electron scattering at the walls.   The electron-sensitive plates were contained in stirrups which were made to run between the inner and outer walls of the plate-foil holder, that is in spaces C and D.   The position of the stirrups could be altered by means of a control rod which entered the box through a vacuum seal (similar in design to the source-control, figure 10) and which enabled the position of the plates to be altered without breaking the vacuum or switching off the H.T.

The dotted lines indicate the position of the scattering foil

Figure 12   The Foil-Holder

The scattering foils were placed in small holders (figure 12) which were made to run in grooves A and B, cut in the inner walls of the plate-foil holder.   The foil holders were made of brass for rigidity.   Since it was known that the positions of the scattering foils were important[85, 108], particularly when comparisons were being made between aluminium and gold foils, the position of the plate-foil holder was controlled at both top and

The whole construction is of aluminium.

A and B are two grooves in which the foil holder slides. The nuclear emulsions move in spaces C and D.

E is one of the two windows through which the scattered electrons pass to reach the emulsion.

Figure 11.   The Plate-Foil Holder.

bottom by guide pieces. This, together with the fact that both the position of a foil on the foil-holder and the position of the foil-holder in the grooves were easily reproducible, led to the conclusion that the comparison of asymmetries was justified.

5.15 Mott scattering in a magnetic field

It is of interest to assess what effect an applied magnetic field might have on the Mott scattering process. The spin precession frequency in such an applied field is given by

$$\omega_s = \frac{eB}{2m_0 c} \cdot \left[1 - \beta^2\right]^{\frac{1}{2}} \cdot g \qquad 5.15.1$$

where g is the gyromagnetic ratio of the free electron and where the other quantities are defined as in 4.2.2.

Within a region of about 10^5 wavelengths from the scatterer, that is about 10^{-5} cms for an electron of energy 100 keV, the effect of the magnetic field upon the particle is negligible compared with that of the scattering potential, since for a magnetic field of 350 gauss the spin precession is of the order of 0.3×10^{-5} radians. Further, the change in orbit direction is of the same order of magnitude as the spin precession and consequently in the above region the particle can be considered as travelling in free space and the scattering is completely determined by the scattering potential[107].

5.16 The scattering foils

The azimuthal asymmetry in the Mott scattering of transversely polarized electrons depends on the following factors:

(a) the degree of electron polarization,

(b) the velocity of the electrons,

(c)　　the atomic number of the scatterer,

(d)　　the degree to which the scattering is of the pure

　　　　single elastic type,

(e)　　the scattering angle,

(f)　　the angle of incidence on the foil.

Factors (a) and (b) are not independent of each other since
Lee and Yang's theory predicts that the degree of electron
polarization is equal to $\frac{v}{c}$.　The asymmetry is greatest for
electrons of velocity $v = (0.6 - 0.7)c$ (depending on the
scattering angle) but, on consideration of the required values
for the electric and magnetic fields (4.2.1) together with a
survey of the possible radioactive sources (5.5), it was decided
to work with electrons of velocity $v = 0.55c$, that is with
electrons of energy 100 keV.

Gold and aluminium scattering foils were used for the
determination of the degree of electron polarization and of the
instrumental asymmetry respectively because of their suitable
atomic numbers.　Polarization-asymmetry values are available in
the literature for these atomic numbers[67] and these foils could
be obtained commercially.

From the theoretical and experimental work discussed in
Chapter 2 it was clear that it was essential that very thin gold
foils should be used in this experiment.　For gold foils of
thickness 10^{-5} cms the most important source of error was due to
plural scattering, the effect of multiple scattering being of
secondary importance.　For the ranges of energy and scattering
angle used in this experiment it could not be assumed that the
depolarization effects due to plural and multiple scattering

were small and consequently use could not be made of the theory of Wegener[80]. There was, however, a considerable amount of theoretical and experimental evidence[80, 85, 97, 98] for the concept that the depolarization effects in a foil due to plural and multiple scattering were proportional to the thickness of the foil. Therefore, by measuring the polarization asymmetries for various thicknesses of foil and by extrapolating these values to zero foil thickness, the effects of plural and multiple scattering could be eliminated from the final result.

One of the important features of this experiment was that, as a result of the ability to reverse the direction of the transverse polarization of the electron beam and consequently the sign of the polarization asymmetry, it was unnecessary to measure the absolute value of the instrumental asymmetry. In general it is difficult to obtain an accurate value of the instrumental asymmetry since three factors contribute to the asymmetry obtained with an aluminium foil. They are the small but finite polarization-asymmetry, the instrumental asymmetry (including the asymmetry due to the non-uniform deposition of the source (5.6)) and the asymmetry due to the non-uniformity in thickness of the aluminium foil, and it is difficult to assess the magnitude of these various contributions to the total asymmetry.

It was, however, necessary to verify that the instrumental asymmetry was independent of the position of the source in the crossed fields and this was done using a rather thick aluminium foil (10 mg/cm^2).

5.17 The theoretical value of the polarization asymmetry for
this experiment

The polarization asymmetry depends strongly on the scattering

angle, being very small at small angles and increasing sharply at angles greater than 90°. Double-scattering experiments, carried out to test the angular dependence of the Mott scattering asymmetry, have, in general, found better agreement between theory and experiment at relatively small scattering angles ($< 110^{o}$) than at large angles (2.13). These experiments did not take into account the effects of multiple and plural scattering, however, and the work of Bienlien et al[101] has shown that when such effects are eliminated from the results, the agreement between theory and experiment is as good for large-angle scattering as for small-angle scattering.

Because of the narrowness of the gap between the pole faces of the magnet (5.2) it was necessary to work at large scattering angles with the consequent disadvantage of the small scattering cross-section at these angles.

The windows through which the scattered electrons entered were cut in the inner walls of the plate-foil holder, care being taken to ensure that they were symmetrical with respect to the scattering foil in order to reduce the instrumental asymmetry. It was found necessary to put thin aluminium foils (0.4 mg/cm^2) over the windows in order to prevent light from corona discharges, produced by the electric field, reaching the electron-sensitive plates. The sides of the windows were bevelled to reduce background.

In order to determine the degree of electron polarization associated with the measured asymmetry it was necessary to have a detailed knowledge of the range of angles through which the electrons could be scattered in order to enter the windows in the

plate-foil holder.   Consideration was therefore given to the general trajectory of an electron, incident normally on the scattering foil and deflected through an angle Θ .   With reference to figure 13, if X, Y and Z be the distances travelled in the x, y and z directions respectively by an electron scattered through an angle Θ at the point (x_o, y_o, O) before striking the plate-foil holder or the emulsion, then the following equations hold:

$$X + x_o = p\cos\Theta - p\left[\cos^2\Theta + \sin^2\Theta\cos^2\phi\right]^{\frac{1}{2}}\sin[\psi - \gamma]$$

$$5.17.1$$

$$Z = p\left[\cos^2\Theta + \sin^2\Theta\cos^2\phi\right]^{\frac{1}{2}}\cos[\psi - \gamma] - p\sin\Theta\cos\phi$$

$$5.17.2$$

where $\qquad \psi = $ arctan $(\cot\Theta\sec\phi)$ $\qquad 5.17.3$

and $\qquad \gamma = \dfrac{Y + y_o}{p\sin\Theta\sin\phi}$

where p is identical to that defined by equation 5.13.1 and the angle ϕ is as shown in figure 13.

The above equations were solved graphically giving the range of values of Θ and ϕ for which an electron would enter the window, after being scattered from a certain point on the foil. By means of repeated numerical integrations the angular distribution of the electrons admitted by the window was determined (figure 14).

The angular variation of the Mott scattering cross-section was taken into account using the calculations of Doggett and Spencer[62].   Sherman[67] has published a very full set of calculations on the Mott asymmetry factor for electron energies in the range $\beta = 0.2$ to 0.9 and at scattering angles varying

PQRS is the scattering foil. The axes are
chosen as shown.
AOB is the direction of the incident electron
OC is the direction of the scattered
electron. [at an angle θ to OB].
OD [constructed as shown] is at an angle φ
to the axis.

Figure 13   The Scattering Foil

Figure 14. Graph of Scattering Angle against Contribution.

from 15° to 165°. These calculations were carried out for
scattering foils of atomic number Z = 13, 48 and 80. Sherman
and Nelson[114] have evaluated the Mott asymmetry factor for a
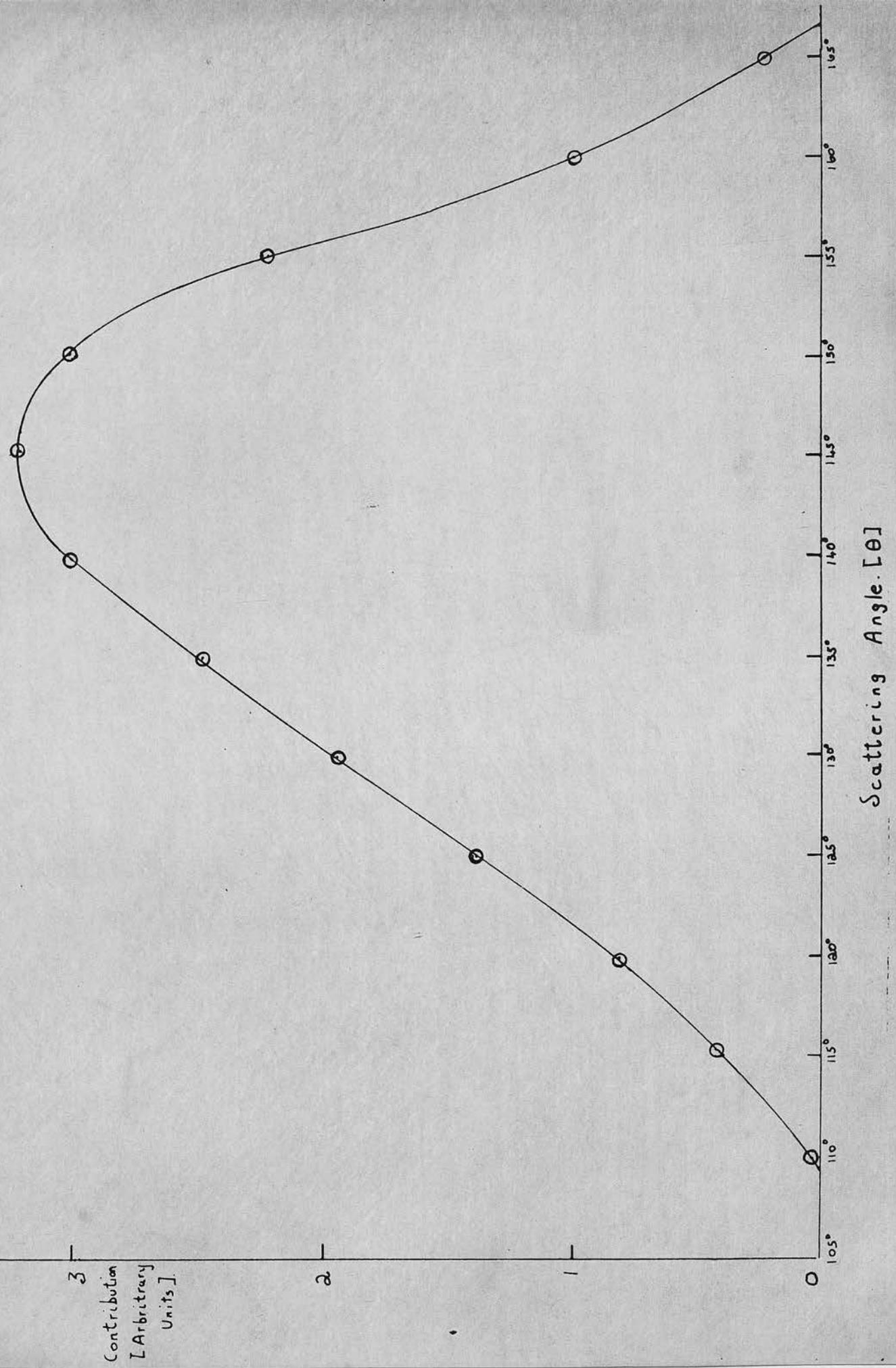gold scattering foil (Z = 79) in the angular range 15° to 165° and
for electron energies corresponding to $\beta = 0.49$ and $\beta = 0.59$.
From a comparison of the results of Sherman with those of Sherman
and Nelson (making interpolations where necessary) it was observed
that the percentage difference in the asymmetry factors for Z = 79
and Z = 80 was only slightly dependent on the energy and on the
scattering angle and for this reason it was possible to make
accurate corrections to Sherman's values for Z = 80 in order to
use them for a gold scattering foil. The average value of the
correction factor was 1.9% of the asymmetry value for Z = 80, in
agreement with the calculations of Alikhanov et al[90].

The effect of the azimuthal dependence of the Mott asymmetry
on the polarization asymmetry was taken into account by considering
the range of azimuthal angles, through which the electrons had to
be scattered in order to reach the area of emulsion examined.
The calculation on the magnitude of the correction took only partial
account of the effect of the magnetic field on the paths of the
scattered electrons; it ignored completely the finite angular
spread of the incident electron beam. The magnitude of the cor-
rection was 2.0% and reduced the value of the expected asymmetry.

As shown in Table 4, the theoretical value of the asymmetry
(δ_{theor}) produced by the scattering of a fully polarized beam of
electrons of energy 100 keV, under the conditions of this experiment,
was evaluated. The value obtained was

$$\delta_{theor} = \pm 35.22\%$$

Table 4.

θ (degrees)	C	$\operatorname{cosec}^4\frac{\theta}{2}$	D	$C.D.\operatorname{cosec}^4\frac{\theta}{2}$	B	δ_1	δ_2	$B\delta_2$
110	.020	2.221	1.815	.081	.16	38.4	37.7	.60
115	.384	1.977	1.842	1.400	2.82	40.1	39.4	111.11
120	.826	1.778	1.866	2.742	5.51	41.3	40.5	223.16
125	1.368	1.616	1.886	4.169	8.39	41.6	40.8	342.31
130	1.952	1.482	1.904	5.509	11.09	41.1	40.3	446.93
135	2.494	1.373	1.920	6.572	13.22	39.9	39.1	516.90
140	3.026	1.283	1.933	7.502	15.09	37.8	38.1	559.84
145	3.237	1.209	1.945	7.608	15.31	35.1	34.5	528.20
150	3.026	1.149	1.955	6.797	13.68	31.6	31.0	424.08
155	2.224	1.101	1.963	4.806	9.67	27.5	27.0	261.09
160	.991	1.063	1.969	2.073	4.17	22.6	22.2	92.57
165	.230	1.035	1.974	.454	.91	17.4	17.1	15.56

θ is the scattering angle

C   is a measure of the number of electrons, scattered through the relevant angle, at the foil, which enter the windows (see figure 14).

D   is a correction factor to the Rutherford scattering cross-section, taken from the work of Doggett and Spencer[62].

B   is defined by the following expression

$$B = \frac{C.D. \operatorname{cosec}^4\frac{\theta}{2} \times 100}{\sum\limits_{110}^{165} C.D. \operatorname{cosec}^4\frac{\theta}{2}}$$

δ_1 is the Mott asymmetry factor for z = 80 taken from the work of Sherman[67].

δ_2 is the Mott asymmetry factor for z = 79 calculated on the basis of the work of Sherman[67] and that of Sherman and Nelson[114].

$$\delta_{theor.} = \sum\limits_{110}^{165} \frac{B\delta_2}{100} = 35.22\%$$

the sign depending on the direction of the polarization.

5.18   Errors in the theoretical value of the polarization asymmetry

There was a considerable number of possible sources of error
in the value of δ_{theor}. Errors may have occurred in the
theoretical calculations of Doggett and Spencer but, since only
the relative magnitude of the values were of importance, it was
considered that such errors could reasonably be neglected. On
the other hand the theoretical values of Sherman were of considerable
importance in obtaining the theoretical value of the polarization
asymmetry and the presence of errors in these calculations is
discussed in 7.11. The value of δ_{theor} was calculated for
electrons of velocity $v = 0.55c$ whereas electrons in the energy
range (98 - 103) keV were incident on the foil. From an examin-
ation of the velocity dependence of the polarization-asymmetry
it was found that the error from this source was negligible. In
obtaining the value of δ_{theor} the assumption was made that the
electrons were incident normally on the scattering foil whereas
the calculations in 4.5 indicate that electrons were incident
on the foil in the approximate angular range $90° \pm 3°$. Since,
however, it was the cosine of the angle of deviation from the normal
that was of importance in deriving the value of δ_{theor} the error
due to the above angular range was less than 0.1%.

Apart from errors in the work of Sherman the most likely
source of error in the value of δ_{theor} was due to uncertainties
in the determination of the scattering angle distribution. It
was calculated that a 1% error in the scattering angle distribution
would lead to an approximate error of 1% in the value of δ_{theor}.

The values of the distribution were calculated independently of one-another for values of Θ at 5° intervals between $\Theta = 110^{\circ}$ and $\Theta = 165^{\circ}$ and since a smooth curve could be drawn through the points obtained in this way (figure 14) it was considered that the error from this source was of the order of 1%.

5.19   The electron detectors

Nuclear emulsions were used as detectors principally because of the technical difficulties associated with the satisfactory operation of two electron counters of another type in the small working gap between the pole faces of the magnet (5.2).   The nuclear emulsions had the advantage of being extremely reliable, of being unlikely to have inherent asymmetries and of enabling simultaneous energy determination and counting to be carried out. Their main disadvantage lay in the fact that they were manufactured weekly and that they had to be used as soon after processing as possible, otherwise heavy backgrounds tended to mask the desired effects.   This, together with the fact that two days had to elapse between exposure and microscope examination, tended to retard progress in the preliminary stages of the experiment.

Ilford G5 electron-sensitive plates, with an emulsion thickness of 100 microns, were used.   The development procedure adopted was essentially that of Dilworth, Occhialini and Payne[113]. Emulsions were examined using a microscope with an oil immersion lens and also by a microphotometer.   The latter did not appear capable of giving sufficiently accurate results for this experiment since it was difficult to translate microphotometer readings into electron numbers and also because spurious effects, such as small patches of surface stain, could give distorted galvanometer readings.   The counting of electron tracks using the microscope,

although probably considerably slower, appeared to be a more

satisfactory method of examination.

CHAPTER 6.

THE EXPERIMENTAL PROCEDURE

6.1 The general experimental procedure

As previously discussed (4.1), exposures were taken for
three positions of the source, namely the "1ℓ" position, in
which transversely polarized electrons were incident on the
scattering foil, the "2ℓ" position in which the electrons
incident on the scattering foil had a longitudinal polarization
in the direction opposite to that in which they were emitted,
and the "3ℓ" position in which the electrons incident on the
scattering foil had a transverse polarization opposite in sense
to that in the "1ℓ" position. These statements regarding the
polarization directions do not take into account the effects
discussed in 5.11. The three exposures, corresponding to the
three positions of the source, were recorded on different areas
of the same electron-sensitive plates.

Two sets of experiments were carried out, using different
source foils, source-holders, defining slits and scattering foils,
with the object of finding if there were any systematic errors
associated with the design or the characteristics of these com-
ponents. In the first set of experiments, exposures were taken
for each source position with no scattering foil, with an aluminium
foil and with 0.19 mg/cm^2 and 0.38 mg/cm^2 gold foils in position.
After polarization-asymmetry values had been obtained for these
foil thicknesses, a second set of exposures were taken for each
source position, with no scattering foil, with an aluminium foil
and with 0.19 mg/cm^2, 0.57 mg/cm^2, 0.76 mg/cm^2 and 0.965 mg/cm^2

gold foils in position.

The eight electron-sensitive plates in the first set and the twelve electron-sensitive plates in the second set were developed and examined in the way described in 5.18.

6.2   Background effects in the nuclear emulsions

The electron tracks observed throughout the emulsions could be divided into three categories.   Firstly, those which were due to the presence of very small quantities of radioactive material in the emulsion when manufactured.   Secondly, due to cosmic radiation and to the presence of γ-emitting sources in the laboratory the plates had a background of electron tracks.   The amount of this background depended on the time lapse between manufacture and development and for this reason exposure times should have been as short as possible.   It was found that a plate, placed in the plate-foil holder, received an additional background which depended on the length of the exposure time.   This effect was attributed to the production of bremsstrahlung at the inelastic scattering of electrons from S^{35} either inside or outside the electric field plates, because even the most energetic electrons from S^{35} could not have penetrated the walls of the plate-foil holder. These three effects were grouped together under the term general background, since they provided a nearly uniform density of electron tracks over the whole plate.

Electrons from the beam were scattered into the windows of the plate-foil holder from places other than the scattering foil. It was also possible that electrons emitted from places other than the source (e.g. from contaminated equipment) could have entered the windows directly.   These two effects were termed the specific

background since, in both cases, the electrons were recorded on
only one part of the emulsion, namely that part opposite the window
in the plate-foil holder. The magnitude of the specific background
was measured by taking exposures for each source position with no
scattering foil in the holder.

It was noted that the presence of a scattering foil could
have increased the specific background in two ways. Firstly, some
of the electrons in the beam, when traversing the scattering foil,
were scattered in such a direction as to be incident on the inner
walls of the plate-foil holder and there was a small but finite
probability that such electrons were scattered by the plate-foil
holder so as to enter the windows. Considerations based on the
geometry of the plate-foil holder and on the thickness of the
scattering foils led to the conclusion that this effect was very
small. Secondly, electrons emitted from places other than the
source (e.g. from contaminated equipment) could have been scattered
by the foil into the windows. The number of such electrons regis-
tered on the emulsions in a given time would not depend on the
position of the source and consequently would be of much greater
importance for the exposure corresponding to the "3ℓ" position
than for the exposure corresponding to the "1ℓ" position. The
effect of the presence of such electrons would be to reduce the
polarization asymmetry and consequently, if such an effect were
present, the polarization asymmetry obtained with the source in
the "3ℓ" position would have been invariably smaller than that
obtained in the "1ℓ" position, irrespective of the thickness of
the scattering foil. The results (Table 5) showed that this was
not the case and so it was concluded that this effect did not play
an important part in this experiment.

6.3   The exposure times

Since at least part of the general background did not depend on the exposure time but only on the time lapse between the manufacture and the development of the emulsion, it was desirable to keep the ratio of the former to the latter as high as possible. There were, however, other factors which influenced this ratio. If the exposure times were made too short the density of electron tracks was small, and the process of examining the emulsions became rather lengthy as large areas had to be scanned if the total number of tracks counted was to be sufficiently high. If, on the other hand, the exposure times were too long then the density of electron tracks was large, with the result that the tracks tended to overlap, and the rate of counting was slow due to the large time spent examining one field of view in the microscope.

The field of view in the microscope used was approximately 67 microns square.   A 100 keV electron has a mean range of 46.7 microns and a mean number of grains per track of 43.3 in the type of emulsion used[115] and under these conditions it was found that the fastest and easiest counting conditions existed when there were 2 - 6 electron tracks per field of view.

One other factor which indirectly affected the magnitude of the exposure times was the decision to use one set of exposures taken with an aluminium foil and one set of exposures taken with no scattering foil, with more than one set of gold foil exposures. It was considered that such a procedure was permissible provided all the sets were taken within a period of time small enough to be able to neglect changes in source intensity and in background, and

provided no changes were made in the apparatus which altered the instrumental asymmetry. Changes in the background could have occurred in two ways, firstly due to fluctuations in the cosmic radiation intensity, which would have affected the general, but not the specific background, and consequently would not have influenced the polarization asymmetry results, and secondly by a fall in the source activity. The effect on the polarization asymmetry of a reduction in the source intensity during the course of an experiment depended on several factors which included the time interval between the background exposures and the gold foil exposures, the ratio of the electron track density in the specific background to that obtained in the gold foil exposure, the magnitude of the polarization asymmetry and the half-life of the radioactive source. A first-order calculation was carried out on the magnitude of this effect, making the assumptions that the specific background was directly proportional to the beam intensity and that no instrumental or foil asymmetries were present. It was found that, under the most unfavourable conditions present in any of the experiments, the variation in the source intensity introduced an uncertainty of approximately 1% into the value of the relevant polarization asymmetry. Since, however, some of the gold foil exposures were taken before the background exposures and some after, and since the resultant effect on the polarization asymmetry was of opposite sign for the two cases then, to a large extent, the error from this source was included in the statistical error of the final asymmetry value because of the method used to calculate the latter (7.8).

6.4   The examination of the electron-sensitive plates

Criteria for the identification of tracks of 100 keV electrons in the emulsion had been developed during the grain counting work carried out previously (5.13) and these were used when counting the number of electron tracks.   If the selection criteria were constant but too strict, so that only a fraction of the 100 keV electron tracks present in the emulsion were counted, the polariz-ation asymmetry values would not be affected since only the ratios of the numbers of electrons in the positions on the various plates were of importance (6.6).   Alternatively, if the selection criteria were constant but not strict enough, that is electrons which had not been elastically scattered by the foil were also counted, then either these "additional" electrons would have appeared in the exposures taken with no scattering foil, in which case they would have been eliminated from the final results, or they would have been electrons which had undergone inelastic scattering at the foil. As discussed in 2.6 this latter effect was small for the foil thick-nesses used.   This argument does not take into account the small effects discussed in 6.2.   Considerable laxity was therefore permitted in the choice of the selection criteria but it was essential that once they had been established they should have remained constant throughout the work.   It was found, by repeated examination of the same section of the emulsion, that the selection criteria did vary initially, but after some practice consistency was achieved.

In order to reduce still further the possibility of variations in selection criteria influencing the final asymmetry values the

following scanning technique was adopted.

$$OF = y_1 : OE = y_2$$

Figure 15

The microscope was set to view a particular strip of emulsion, e.g. AB, at a distance y from the edge of the plate. Scanning started at the point A and proceeded in the x direction, ten fields of view in every forty being examined, until the point C was reached. The plate was then removed and another plate, chosen at random, put in its place. Scanning continued along the same y-line but over a different range of x values. The process was continued until all plates had been examined in this way. The complete cycle was repeated for the same y value but a different ten fields of view in every forty were examined.

The double cycle completed, a new value of y, within the limits $y_1 < y < y_2$, was chosen and the above procedure repeated. This was carried out for six values of y. It was considered that this technique reduced the effects of variations in selection criteria

and also any effects due to variations in the density of electron tracks in the exposed areas.

Consideration was given to the effect of inaccurate electron counting. There was a finite probability that a certain fraction of the 100 keV electron tracks present on the plates were not counted. This was a different effect from that due to too strict selection criteria since in that case the electron tracks were examined, then rejected, whereas in this case the tracks were not examined. The technique, previously described, of counting ten fields of view in every forty in the first examination and another ten in the second examination would be expected to bring to light any variations in the accuracy of counting but could not give any indication of the absolute degree of accuracy. Since the densities of electron tracks in the exposures used to obtain a single asymmetry value did not vary greatly, then as a first-order approximation it was considered that the number of tracks missed was proportional to the number of tracks counted. Under this assumption it can be shown that the degree of accuracy of electron counting did not influence the final asymmetry values.

6.5   The rate of counting and energy discrimination

After some practice it was found possible to count 1,750 electron tracks per day. Approximately 100,000 electron tracks were counted in the scanning of the twenty plates. With the above rate of counting, good energy discrimination could not be achieved. From the consideration of the work of Ross and Zajac[115] it was concluded that electron tracks which had a number of grains between 30 and 60 were counted, that is electrons of energy between 75 keV and 120 keV (approximately) were accepted as
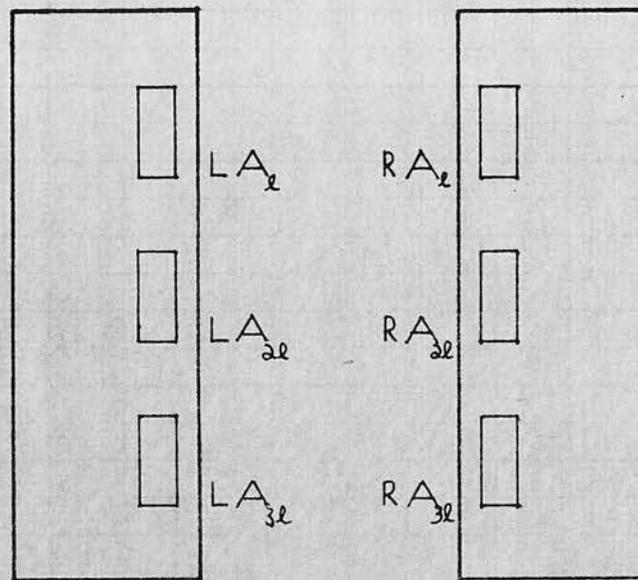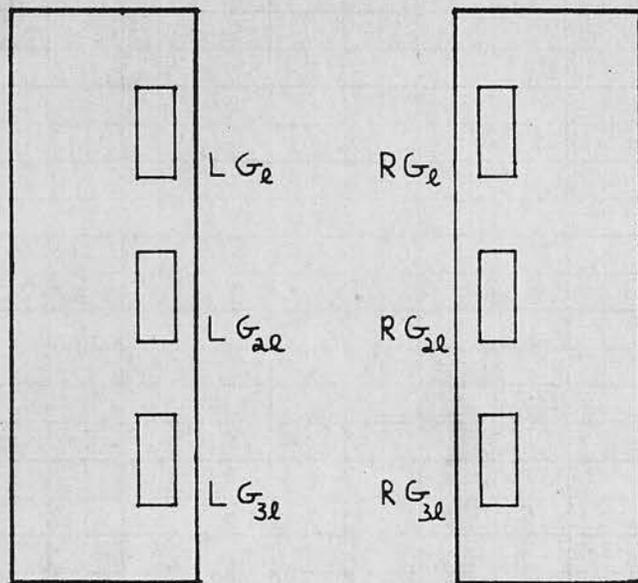
being genuine.

6.6 The mathematical analysis of the results

Each plate had three exposed areas on it corresponding to the three positions of the source. It was also necessary to examine the unexposed areas in order to determine the general background so that it could be subtracted from the counts recorded for the exposed areas. This correction having been made, correction was made for the specific $b_a$ckground by subtracting the number of electrons per field of view obtained from the exposure taken with no scattering foil from the number obtained from the exposure taken with a scattering foil. This was carried out for the exposures taken for each source position.

Due correction was made for differences in the exposure times and in the areas scanned by standardizing all measured quantities to an exposure time of 100 minutes and by expressing the results in terms of the average number of electron tracks per field of view.

Full corrections having been made for background effects, a set of values were obtained as shown diagramatically in figure 16. LG_ℓ and RG_ℓ represented the average number of electrons per field of view registered on the left-hand and right-hand emulsions (as viewed by the source), the electrons having been emitted by the source in the "1ℓ " position and having been scattered by a gold foil. Similar definitions applied to the other quantities, A representing an aluminium foil exposure reading and the 2ℓ and the 3ℓ subscripts denoting the fact that the electrons registered in the particular exposure had been emitted by the source in the 2ℓ and 3ℓ positions respectively. The values LG_ℓ, RG_ℓ, LA_ℓ etc.

LG_{ℓ} RG_{ℓ}

$LG_{2\ell}$ $RG_{2\ell}$

$LG_{3\ell}$ $RG_{3\ell}$

LA_{ℓ} RA_{ℓ}

$LA_{2\ell}$ $RA_{2\ell}$

$LA_{3\ell}$ $RA_{3\ell}$

For explanation see text

Figure 16

were each the average of the readings obtained from the examination of about 60 fields of view. For each foil, twelve sets of the type shown in figure 16 were obtained, representing the double cycles for each of the six values of y examined (6.4).

The values LG_ℓ ..... $RG_{3\ell}$, LA_ℓ ..... $RA_{3\ell}$, were expressed in the following manner.

$$LG_\ell = G_1(a_1 + \Delta_1) \qquad 6.6.1 \qquad RG_\ell = G_1(a_1^1 + \Delta_2) \qquad 6.6.2$$

$$LG_{2\ell} = G_2(a_1 + \Delta_3) \qquad 6.6.3 \qquad RG_{2\ell} = G_2(a_2^1 + \Delta_4) \qquad 6.6.4$$

$$LG_{3\ell} = G_3(a_3 + \Delta_5) \qquad 6.6.5 \qquad RG_{3\ell} = G_3(a_3^1 + \Delta_6) \qquad 6.6.6$$

$$LA_\ell = xG_1 a_1 \qquad\qquad 6.6.7 \qquad RA_\ell = xG_1 a_1^1 \qquad\qquad 6.6.8$$

$$LA_{2\ell} = yG_2 a_2 \qquad\qquad 6.6.9 \qquad RA_{2\ell} = yG_2 a_2^1 \qquad\qquad 6.6.10$$

$$LA_{3\ell} = zG_3 a_3 \qquad\qquad 6.6.11 \qquad RA_{3\ell} = zG_3 a_3^1 \qquad\qquad 6.6.12$$

The quantities G_1, G_2 and G_3, were dependent on the following three factors:

(a)   the intensity of the electron beam incident on the scattering foil,

(b)   the value of the expression

$$\frac{e^4(1 - \beta^2)\ \mathrm{cosec}^4 \frac{\theta}{2}}{4\ m_o^2\ c^4\ \beta^4} \qquad (\text{see } 5.5)$$

when the $\mathrm{cosec}^4 \frac{\theta}{2}$ term had been integrated over the angular range of the scattered electrons admitted by the windows (5.17),

(c)   the atomic number and thickness of the scattering foil.

The factor (b) was the same for both gold and aluminium foils and the terms x, y and z, were introduced in order to take into account variations in factor (c) and also possible variations in factor (a), for the two foils.

The quantities a_1, a_2, a_3, $a_1^{\;1}$, $a_2^{\;1}$, $a_3^{\;1}$, were expressions for the instrumental asymmetries for the three source positions and were defined in the following way:

$$a_1 = \frac{LA_\ell}{LA_\ell + RA_\ell} \qquad 6.6.13 \qquad\qquad a_1^{\;1} = \frac{RA_\ell}{LA_\ell + RA_\ell} \qquad 6.6.14$$

$$a_2 = \frac{LA_{2\ell}}{LA_{2\ell} + RA_{2\ell}} \qquad 6.6.15 \qquad\qquad a_2^{\;1} = \frac{RA_{2\ell}}{LA_{2\ell} + RA_{2\ell}} \qquad 6.6.16$$

$$a_3 = \frac{LA_{3\ell}}{LA_{3\ell} + RA_{3\ell}} \qquad 6.6.17 \qquad\qquad a_3^{\;1} = \frac{RA_{3\ell}}{LA_{3\ell} + RA_{3\ell}} \qquad 6.6.18$$

i.e. $\quad a_1 + a_1^{\;1} = a_2 + a_2^{\;1} = a_3 + a_3^{\;1} = 1 \qquad 6.6.19$

Using these equations it was possible to obtain values of the instrumental asymmetry.

The quantities Δ_1, Δ_2, Δ_3, Δ_4, Δ_5, Δ_6, each represented the sum of two asymmetries namely the polarization asymmetry and the asymmetry due to non-uniformities in the gold scattering foil (hereafter termed the foil asymmetry). One possible assumption regarding the magnitude of these composite asymmetries was that the following relationships were valid.

$$\Delta_1 = -\Delta_2 : \Delta_3 = -\Delta_4 : \Delta_5 = -\Delta_6$$

Under this assumption the values of x, y and z could be expressed thus:

$$x = \frac{LA_\ell + RA_\ell}{LG_\ell + RG_\ell} \quad 6.6.20 \qquad\qquad y = \frac{LA_{2\ell} + RA_{2\ell}}{LG_{2\ell} + RG_{2\ell}} \quad 6.6.21$$

$$z = \frac{LA_{3\ell} + RA_{3\ell}}{LG_{3\ell} + RG_{3\ell}} \quad 6.6.22$$

It was thus possible to determine the values of x, y and z from the available data.

The problem of finding the values of $\Delta_1 \ldots \Delta_6$ was essentially that of determining the values of G_1, G_2 and G_3. From equations (6.6.7 - 6.6.12) the following relationships were obtained:

$$\frac{G_2}{G_1} = \frac{x}{y} \cdot \frac{a_1}{a_2} \cdot \frac{LA_{2\ell}}{LA_\ell} \quad 6.6.23 \quad \text{or} \quad = \frac{x}{y} \cdot \frac{a_1^1}{a_2^1} \cdot \frac{RA_{2\ell}}{RA_\ell} \quad 6.6.24$$

$$\frac{G_3}{G_1} = \frac{x}{z} \cdot \frac{a_1}{a_3} \cdot \frac{LA_{3\ell}}{LA_\ell} \quad 6.6.25 \quad \text{or} \quad = \frac{x}{z} \cdot \frac{a_1^1}{a_3^1} \cdot \frac{RA_{3\ell}}{RA_\ell} \quad 6.6.26$$

$$\frac{G_3}{G_2} = \frac{y}{z} \cdot \frac{a_2}{a_3} \cdot \frac{LA_{3\ell}}{LA_{2\ell}} \quad 6.6.27 \quad \text{or} \quad = \frac{y}{z} \cdot \frac{a_2^1}{a_3^1} \cdot \frac{RA_{3\ell}}{RA_{2\ell}} \quad 6.6.28$$

It was noted that any one value of G_1, G_2 or G_3 could be obtained using any one of the following assumptions:

$$\Delta_1 = -\Delta_2 : \Delta_3 = -\Delta_4 : \Delta_5 = -\Delta_6 : \Delta_1 = -\Delta_5 : \Delta_2 = -\Delta_6$$

The last two assumptions follow from the work discussed in 5.12 It was therefore possible to obtain values of $\Delta_1 \ldots \Delta_6$ using any one of the above five assumptions.

Example

Assumption : $\Delta_1 = -\Delta_2$

$\therefore \quad G_1 = LG_\ell + RG_\ell \ldots 6.6.29 \qquad \text{(from 6.6.1)}$

$$G_2 = \frac{x}{y} \cdot \frac{a_1}{a_2} \cdot \frac{LA_{2\ell}}{LA_\ell} (LG_\ell + RG_\ell) \quad 6.6.30 \quad \text{(from 6.6.23 and 6.6.29)}$$

$$G_2 = \frac{x}{y} \cdot \frac{a_1^1}{a_2^1} \cdot \frac{RA_{2\ell}}{RA_\ell} (LG_\ell + RG_\ell) \quad 6.6.31 \quad \text{(from 6.6.24 and 6.6.29)}$$

$$G_3 = \frac{x}{z} \cdot \frac{a_1}{a_3} \cdot \frac{LA_{3\ell}}{LA_\ell} \left(LG_\ell + RG_\ell \right) \quad 6.6.32 \quad \text{(from 6.6.25 and 6.6.29)}$$

$$G_3 = \frac{x}{z} \cdot \frac{a_1^{\,1}}{a_3^{\,1}} \cdot \frac{RA_{3\ell}}{RA_\ell} \left(LG_\ell + RG_\ell \right) \quad 6.6.33 \quad \text{(from 6.6.26 and 6.6.29)}$$

By assuming that $\Delta_1 = -\Delta_2$ it was possible, therefore, to obtain one value of Δ_1 and Δ_2 and two values for each of Δ_3, Δ_4, Δ_5 and Δ_6. This was achieved by substituting the values of G_1, G_2 and G_3, obtained from equations 6.6.29 – 6.6.33, into equations 6.6.1 – 6.6.6 and by making use of equations 6.6.13 – 6.6.18. Similarly, results were obtained by the use of assumptions $\Delta_3 = -\Delta_4$ and $\Delta_5 = -\Delta_6$.

Example

Assumption : $\Delta_1 = -\Delta_5$

From the consideration of equations 6.6.1 – 6.6.6, 6.6.7 – 6.6.12, and 6.6.23 – 6.6.28, it was found that the following equations were valid.

$$G_1 = \frac{1}{a_1 + a_3} \left(LG_\ell + \frac{z}{x} \cdot \frac{a_3}{a_1} \cdot \frac{LA_\ell}{LA_{3\ell}} \cdot LG_{3\ell} \right) \quad 6.6.34$$

$$G_1 = \frac{1}{a_1 + a_3} \left(LG_\ell + \frac{z}{x} \cdot \frac{a_3^{\,1}}{a_1^{\,1}} \cdot \frac{RA_\ell}{RA_{3\ell}} \cdot LG_{3\ell} \right) \quad 6.6.35$$

$$G_2 = \frac{x}{y} \cdot \frac{a_1}{a_2} \cdot \frac{LA_{2\ell}}{LA_\ell} \cdot \frac{1}{a_1 + a_3} \left(LG_\ell + \frac{z}{x} \cdot \frac{a_3}{a_1} \cdot \frac{LA_\ell}{LA_{3\ell}} \cdot LG_{3\ell} \right) \quad 6.6.36$$

$$G_2 = \frac{x}{y} \cdot \frac{a_1^{\frac{1}{1}}}{a_2} \cdot \frac{RA_{2\ell}}{RA_{\ell}} \cdot \frac{1}{a_1 + a_3} \left(LG_{\ell} + \frac{z}{x} \cdot \frac{a_3^{\frac{1}{1}}}{a_1} \cdot \frac{RA_{\ell}}{RA_{3\ell}} \cdot LG_{3\ell} \right)$$

6.6.37

$$G_3 = \frac{x}{z} \cdot \frac{a_1}{a_3} \cdot \frac{LA_{3\ell}}{LA_{\ell}} \cdot \frac{1}{a_1 + a_3} \left(LG_{\ell} + \frac{z}{x} \cdot \frac{a_3}{a_1} \cdot \frac{LA_{\ell}}{LA_{3\ell}} \cdot LG_{3\ell} \right)$$

6.6.38

$$G_3 = \frac{x}{z} \cdot \frac{a_1^{\frac{1}{1}}}{a_3} \cdot \frac{RA_{3\ell}}{RA_{\ell}} \cdot \frac{1}{a_1 + a_3} \left(LG_{\ell} + \frac{z}{x} \cdot \frac{a_3^{\frac{1}{1}}}{a_1} \cdot \frac{RA_{\ell}}{RA_{3\ell}} \cdot LG_{3\ell} \right)$$

6.6.39

Using equations 6.6.34 – 6.6.39, together with equations 6.6.1 – 6.6.6 and 6.6.13 – 6.6.18, it was possible to obtain two values each of $\Delta_1, \Delta_2, \Delta_3, \Delta_4, \Delta_5$ and Δ_6. Results were obtained using the assumption $\Delta_2 = -\Delta_6$ in a similar way.

Consideration was given to the suitability of using the assumptions $\Delta_1 = \Delta_6$ and $\Delta_2 = \Delta_5$ but it was found that the values of the polarization asymmetry obtained in this way were less accurate by an order of magnitude than those obtained by using equations 6.6.29 – 6.6.33 and 6.6.34 – 6.6.39, due to the greater amount of data required; consequently these assumptions were not used.

By the method outlined above eleven values each for $\Delta_1, \Delta_2, \Delta_3, \Delta_4, \Delta_5$ and Δ_6 were obtained from one set of results of the type shown in figure 16. With twelve sets of results it was therefore possible to obtain 132 values for each of the asymmetry values and the average of these 132 values for $\Delta_1, \Delta_2, \Delta_3, \Delta_4, \Delta_5$ and Δ_6, for each gold foil are contained in Table 5.

Table 5

	Gold Foil Thickness (mg/cm^2)					
	First set of experiments		Second set of experiments			
	0.19	0.38	0.19	0.57	0.76	0.965
$\triangle_1^T$	-14.1	-16.6	-7.5	-7.3	-4.3	-3.8
$\triangle_2^T$	10.9	14.4	7.5	6.7	2.5	2.3
$\triangle_3^T$	15.3	10.9	20.5	15.8	16.7	16.0
$\triangle_4^T$	-17.2	-12.4	-22.9	-17.7	-20.0	-17.4
$\triangle_5^T$	14.3	13.0	18.7	9.5	20.4	9.0
$\triangle_6^T$	-14.9	-14.8	-20.5	-10.7	-22.9	-10.2

$\triangle_1^T \cdots\cdots \triangle_6^T$ are the asymmetry values obtained for the different

source positions; they are the algebraic sum of the polarization

asymmetry, the foil asymmetry and the instrumental asymmetry

(see text).

CHAPTER 7.

DISCUSSION OF RESULTS

7.1 The instrumental asymmetries

By the use of equations 6.6.13 - 6.6.18 the values of the instrumental asymmetry for the different source positions were calculated for the two sets of experiments (Table 6). The average value of the instrumental asymmetry for each set of experiments was calculated on the assumption that the instrumental asymmetry was the same for each source position. It appeared that this was certainly the case in the first set of experiments but the evidence in favour of this assumption was not so strong in the second set of experiments. The statistical accuracy of the individual values in Table 5 did not permit the determination of the polarization asymmetry for scattering at an aluminium foil nor the measurement of the aluminium foil asymmetry (5.16).

7.2 The elimination of foil and instrumental asymmetries

Examination of Table 5 gave the result that

$$\Delta_1^T \neq \Delta_5^T : \Delta_2^T \neq \Delta_6^T \qquad 7.2.1$$

nor was

$$\frac{\Delta_1^T - \Delta_2^T}{-\Delta_5^T + \Delta_6^T} \qquad 7.2.2$$

a constant for the different scattering foils used. This latter fact suggested that the inequalities in 7.2.1 were due to some property of the foils. The effect was ascribed to the presence of variations in the thickness of the gold foil used and this, although in the nature of an instrumental asymmetry, was not present in the expos ures taken using an aluminium foil. The ability to reverse the direction of the polarization asymmetry was very

Table 6

Instrumental asymmetry	First set of experiments	Second set of experiments
a_1	$0.51 \pm .03$	$0.48 \pm .01$
a_2	$0.49 \pm .03$	$0.53 \pm .01$
a_3	$0.52 \pm .01$	$0.46 \pm .02$
Average	$0.51 \pm .01$	$0.49 \pm .01$

For definitions of the instrumental asymmetries see text.

useful since, from the discussion in 5.12 it was clear that the effects due to the foil asymmetry and to the instrumental asymmetry could be accurately determined and hence eliminated from the results by using the following relation:

$$\Delta_1^T - \Delta_2^T = -\Delta_5^T + \Delta_6^T - 4C \qquad 7.2.3$$

where C is the algebraic sum of the foil asymmetry and the instrumental asymmetry.   Equation 7.2.3 was valid only when the instrumental asymmetry was the same for all positions of the source.   The foil asymmetry and the instrumental asymmetry occurred in the exposures for the "2ℓ" position, as well as in the "1ℓ" and "3ℓ" positions, and consequently the appropriate Δ_3^T and Δ_4^T values were adjusted by using the correction factor C.   The asymmetry values obtained after correction for the foil and instrumental asymmetries are shown in Table 7.

7.3 Second-order effects due to foil asymmetries

The presence of rather large foil asymmetries, as shown in Table 7, raised the question as to the type of errors introduced by the non-uniform thickness of the foils used.   The final value of the polarization asymmetry (δ) was obtained by measuring the values of the polarization asymmetry (Δ_o) obtained for the different foil thicknesses and extrapolating to zero foil thickness. Any errors in the mean thicknesses of the foils used would be included in the statistical error in δ , because the effect of such errors would be simply to increase the spread of the individual values, and it is from the magnitude of this spread that the statistical error in δ is calculated.

Consideration must also be given to the question as to whether the mean value of the thickness is the appropriate one to use when

Table 7

	Gold Foil Thickness (mg/cm^2)					
	First set of experiments		Second set of experiments			
	0.19	0.38	0.19	0.57	0.76	0.965
Δ_1^c	-15.1	-15.8	-13.5	-8.8	-13.4	-7.1
Δ_2^c	11.9	13.6	13.5	8.2	11.6	5.6
Δ_3^c	14.3	11.7	14.5	14.3	7.6	12.7
Δ_4^c	-16.2	-13.2	-16.9	-16.2	-10.9	-14.1
Δ_5^c	13.3	13.8	12.7	8.0	11.3	5.7
Δ_6^c	-13.9	-15.6	-14.5	-9.2	-13.8	-6.9
C	-1.0	+0.8	-6.0	-1.5	-9.1	-3.3

Δ_1^c ..... Δ_6^c are the polarization asymmetry values obtained for the different source positions. C is the algebraic sum of the foil asymmetry and the instrumental asymmetry.

taking into account the depolarization effects in the foil.

Basically the problem is whether or not the same amount of de-

polarization occurs in two foils of the same mean thickness, one

of which is uniform and the other is non-uniform. Provided the

linear relationship between polarization asymmetry and foil thickness

(7.8) still exists for the thickest part of the non-uniform foil

(i.e. other effects such as inelastic scattering are not of import-

ance) then it may be concluded that the amount of depolarization

is the same in both foils and consequently that the mean thickness

is the correct parameter to use when evaluating depolarization

effects.

7.4 Effects due to the non-uniformity of the electric field at the source (A)

From an examination of the results in Table 7 it was clear

that $\triangle_3{}^C$ and $\triangle_4{}^C$ were not zero as would be expected from simple

theory (6.1). The discrepancies were almost certainly due to the

presence of a volume in front of the source where the magnitude

and the characteristics of the electric field were unknown (5.12).

It was clear from the results that the electrons did not leave the

ideal source with their spin directions anti-parallel to their

momentum directions.

From the fact that $\triangle_1{}^C$ was opposite in sign to both $\triangle_3{}^C$ and

$\triangle_5{}^C$, it was concluded that the amount of spin precession which an

electron experienced while traversing the region between the actual

source and the ideal source was greater than that which it would have

experienced in traversing an equal distance in crossed fields of

the correct magnitude. This was in agreement with the theoretical

predictions, since if a Lorentz transformation is applied to a

system in which there is a magnetic field of the full value and an electric field of reduced value (the values being defined by 4.2.1) then the amount of spin precession in this case is greater than that for a system in which both fields are of full value. An accurate calculation of this effect was impossible owing to ignorance of the nature of the electric field in the source region. From an examination of the results in Table 7 it was clear that the effect was not small; indeed the magnitude of the effect suggested that the electrons, whilst traversing the distance between the real source and the ideal source, followed paths in which the momentum direction experienced changes and the spin direction remained constant (cf. the electrostatic field method 3.2) with the effective result of a spin precession. As shown in 5.12, the effects due to the non-uniformity of the electric field at the source could be eliminated from the final results provided they were not dependent on the position of the source in the crossed fields.

7.5 The polarization asymmetry values

From Table 7 it was clear that, to a fairly high degree of accuracy, $\Delta_1{}^C = -\Delta_2{}^C$, $\Delta_3{}^C = -\Delta_4{}^C$ and $\Delta_5{}^C = -\Delta_6{}^C$ for all gold foils examined. These facts were physical properties of the results themselves, rather than consequences of the mathematical analysis, since only one-fifth of the results were obtained on the assumption that $\Delta_1 = -\Delta_2$, one-fifth on the assumption that $\Delta_3 = -\Delta_4$ and a further one-fifth on the assumption that $\Delta_5 = -\Delta_6$. In this connection it was noted that although one-fifth of the results were obtained by assuming that $\Delta_1 = \Delta_5$, the results were not in accordance with this assumption until the correction factor C had been applied (Tables 5 and 7). The fact

that the assumptions $\triangle_1 = -\triangle_2, \triangle_3 = -\triangle_4, \triangle_5 = -\triangle_6$ were used to obtain the values of x, y and z (6.6.20 - 6.6.22) which were used throughout the calculations, modifies the above argument to a small extent for the following reason.   From an examination of equations 6.6.30 - 6.6.39, it was clear that the factors x, $\bar{y}$ and z could not introduce or remove discrepancies between the polarization asymmetry values but could only alter the magnitude of such discrepancies; from a scrutiny of the results obtained it was concluded that such alterations were small.

The near equality of $\triangle_1^C$ and $-\triangle_2^C$, $\triangle_3^C$ and $-\triangle_4^C$, $\triangle_5^C$ and $-\triangle_6^C$, notwithstanding the scanning technique used (6.4), was taken as an indication that the selection criteria and the accuracy of counting had remained constant during the period of examination.

Inspection of the asymmetry values in Table 7 revealed that, for each foil, $-\triangle_1^C > \triangle_2^C$, $\triangle_3^C < -\triangle_4^C$, $\triangle_5^C < {}^-\triangle_6^C$, the only exception being the equality of $\triangle_1^C$ and $\triangle_2^C$ for the 0.19 mg/cm^2 gold foil exposures in the second set of experiments.   Further, by the use of the results in Table 7, the following results were obtained

$$\left(\frac{-\triangle_1^C}{\triangle_2^C} \right)_{Av} = 1.15 \pm .04$$

$$\left(\frac{\triangle_3^C}{-\triangle_4^C} \right)_{Av} = 0.85 \pm 0.03$$

$$\left(\frac{\triangle_5^C}{-\triangle_6^C} \right)_{Av} = 0.87 \pm 0.02$$

the averages being taken over the results for all foils.

Within the limits of the statistical accuracy of the ratios it was clear that

$$\left(\frac{\Delta_3^C}{\Delta_4^C}\right)_{Av} \simeq \left(\frac{\Delta_5^C}{\Delta_6^C}\right)_{Av}$$

and that

$$\left(\frac{\Delta_3^C}{-\Delta_4^C}\right)_{Av} \neq \left(\frac{-\Delta_1^C}{\Delta_2^C}\right)_{Av}$$

$$\left(\frac{\Delta_5^C}{-\Delta_6^C}\right)_{Av} \neq \left(\frac{-\Delta_1^C}{\Delta_2^C}\right)_{Av}$$

These relationships implied that the discrepancies between Δ_1^C and Δ_2^C, Δ_3^C and Δ_4^C, Δ_5^C and Δ_6^C, were associated with the direction of the polarization asymmetry.

The following results were obtained by summation of the ratios from Table 7 over all foils:

$$\frac{\sum_1^6\left(\frac{-\Delta_1^C}{\Delta_2^C}\right) + \sum_1^6\frac{\Delta_5^C}{-\Delta_6^C}}{6} = 2.02 \pm 0.03$$

$$\frac{\sum_1^6\left(\frac{-\Delta_1^C}{\Delta_2^C}\right) \times \sum_1^6\left(\frac{\Delta_5^C}{-\Delta_6^C}\right)}{36} = 1.02 \pm 0.04$$

$$\frac{\sum_1^6\left(\frac{-\Delta_1^C}{\Delta_2^C} \times \frac{\Delta_5^C}{-\Delta_6^C}\right)}{6} = 1.01 \pm 0.05$$

In theory, when $-\Delta_1^C = \Delta_2^C$ and $\Delta_5^C = -\Delta_6^C$, the results for the above expressions would be 2, 1 and 1 respectively. From this it was concluded that the factor which was causing $-\Delta_1^C$ to be greater than Δ_2^C, and Δ_5^C to be less than $-\Delta_6^C$, was equal in magnitude and opposite in direction for the "1ℓ" and "3ℓ" source positions.

On the basis of these observations it appeared that the most likely cause of the discrepancies was the fact that the asymmetry values had been calculated before the foil and instrumental asymmetries had been eliminated from the results rather than the preferable but, unfortunately, impractical reverse procedure. From Table 7 it was clear that the effect was not large and from equations 6.6.29 - 6.6.39 it was recognised that due correction could be made for the effect by giving equal weight to all values of the polarization asymmetry in the final calculations.

The above theory to explain the discrepancies between the values of $-\Delta_1^C$ and Δ_2^C, Δ_3^C and $-\Delta_4^C$, Δ_5^C and $-\Delta_6^C$, could only be justified if all the foil and instrumental asymmetries were of the same sign. Five of the six foils used did satisfy this condition. It was noted that it was statistically improbable that the sum of the instrumental asymmetry and the foil asymmetry should be of the same sign for five of the foils, particularly in view of the smallness of the instrumental asymmetries (Table 6). However, the 0.57 mg/cm^2 and 0.76 mg/cm^2 gold foils were made up of three and four layers, respectively, of the 0.19 mg/cm^2 gold foil and since these were cut from the same sheet and mounted on the foil holders in a systematic way, it was not surprising that they should have asymmetries of the same sense. It was rather

difficult, using the above theory, to explain why the asymmetry values for the 0.38 mg/cm^2 gold foil followed the same pattern (i.e. $-\Delta_1{}^C > \Delta_2{}^C : \Delta_3{}^C < -\Delta_4{}^C : \Delta_5{}^C < -\Delta_6{}^C$) as those for the other foils when the sum of the instrumental and foil asymmetries for the 0.38 mg/cm^2 foil was of opposite sign to the others; the composite asymmetry for the 0.38 mg/cm^2 foil was small, however, and it was considered that the effect might have been the result of statistical fluctuations in the values of the asymmetry factors used in the calculation of the magnitude of C (7.2).

7.6 The effect of an instrumental asymmetry on a polarization asymmetry

There were two effects (other than the one discussed in 7.5) which could have caused discrepancies between the values of $\Delta_1{}^C$ and $\Delta_2{}^C$, $\Delta_3{}^C$ and $\Delta_4{}^C$, $\Delta_5{}^C$ and $\Delta_6{}^C$, both being due to the effect of an instrumental asymmetry on a polarization asymmetry.

Firstly, if the windows in the plate-foil holder had subtended different angular ranges at the scattering foil then the differential scattering cross-section, integrated over the appropriate angular ranges, would have been different for the two windows. Such an effect would have appeared in both the aluminium and gold foil exposures and hence could have been eliminated. The polarization asymmetry value, δ_{theor} (5.16), for the two windows would have been different due to the angular dependence of the Mott scattering asymmetry and, since such an effect would not have appeared in the aluminium foil results, it could not have been easily eliminated from the gold foil values. If such an effect were present, then its existence would have been demonstrated in the following manner:

either $\quad -\Delta_1^c > \Delta_2^c : \Delta_3^c > -\Delta_4^c : \Delta_5^c > -\Delta_6^c \qquad 7.6.1$

or $\quad -\Delta_1^c < \Delta_2^c : \Delta_3^c < -\Delta_4^c : \Delta_5^c < -\Delta_6^c \qquad 7.6.2$

Secondly, if the range of azimuthal angles through which electrons could be scattered in order to reach the emulsion were different for the two windows in the plate-foil holder, then the polarization asymmetry value, δ_{theor} (5.16), for the two windows would have been different due to the azimuthal dependence of the Mott asymmetry. As above, such an effect would not have appeared in the aluminium foil results and consequently would have been difficult to eliminate from the gold foil values. The presence of such an effect would have been demonstrated by polarization asymmetry values which were of the form shown in 7.6.1 or in 7.6.2.

Since the experimental results contained in Table 7 were not consistent with the conditions of 7.6.1 or 7.6.2 and since the instrumental asymmetries were small (Table 6) it appeared that the effects of the instrumental asymmetries on the polarization asymmetry were not of importance in this experiment.

7.7 Effects due to the non-uniformity of the electric field at the source (B)

By the use of equations 5.12.1 - 5.12.6 and the results contained in Table 7, the values of $K\ell'' + \psi'$ were calculated for each gold foil exposure (Table 8). For the case when the real source and the ideal source coincide (5.12) then $K\ell'' + \psi' = 90°$ and deviations from this value indicate the magnitude and the importance of the volume in front of the source in which the

Table 8

Gold Foil Thickness (mg/cm^2)					
First set of experiments		Second set of experiments			
0.19	0.38	0.19	0.57	0.76	0.965
C					
-1.0	$+0.8$	-6.0	-1.5	-9.1	-3.3
$K\ell'' + \psi'$					
$42° \pm 1°$	$52° \pm 1°$	$41° \pm 1°$	$30° \pm 1°$	$54 \pm 2°$	$25° \pm 1°$
Δ_0					
20.4 ± 0.4	19.3 ± 0.3	20.7 ± 0.4	17.5 ± 0.3	15.6 ± 0.5	14.8 ± 0.3

C is the algebraic sum of the foil asymmetry and the
instrumental asymmetry.   The terms $K\ell'' + \psi'$ are as defined
in 5.11.

electric field is not uniform. There did not appear to be any correlation between the values of $K\ell" + \psi'$ and the respective magnitudes of the foil and instrumental asymmetries, the foil thicknesses or the final polarization asymmetry values (7.7). The fact that the values of $K\ell" + \psi'$ for the two 0.19 mg/cm^2 gold foils were almost identical was considered to be a coincidence.

Possible explanations for the variation in value of $K\ell" + \psi'$ during the experiments are discussed later (7.10). Such variations did not introduce any uncertainties into the values of the polarization asymmetries but theirexistence did raise the question as to whether the value of $K\ell" + \psi'$ changed as the source was moved from the "1ℓ" to the "2ℓ" or to the "3ℓ" position. If such an effect did exist the basic principle of the experiment would be invalid, since the comparison of the polarization asymmetry values obtained for the different source positions would not be permissible. The presence or absence of such an effect could only be established by examination of the final asymmetry values (7.8).

7.8   The final polarization asymmetry values (Δ_o)

The value of the polarization asymmetry (Δ_o) for each foil was found by the insertion of the values in Table 7 into equations 5.12 (1-6) (Table 8). The errors in these values are purely statistical. The required degree of statistical accuracy has been achieved, the slightly poorer accuracy of the 0.76 mg/cm^2 value being ascribed to the large foil asymmetry present. By using the method of least squares, the polarization-asymmetry value for a gold foil of zero thickness was obtained from the results in Table 8, due account being given to the varying degree of statistical accuracy of the latter. The value obtained was

$$\delta = (22.05 \pm 0.16)\% \quad \text{(figure 17)}$$

i.e. a statistical accuracy of about $\frac{3}{4}\%$.

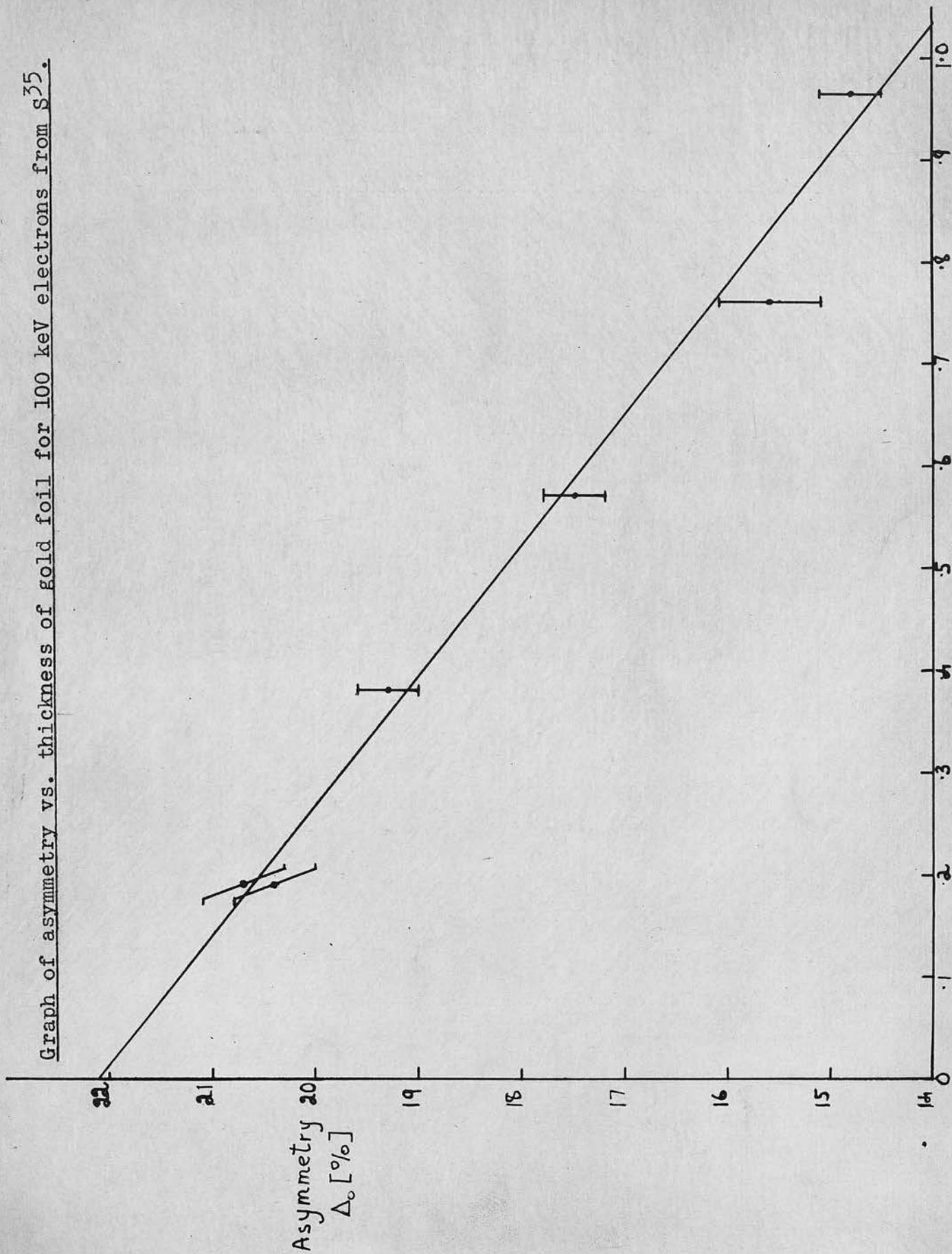### 7.9 The linear relationship between Δ_o and the foil thickness

A number of important conclusions could be drawn from the fact that the plot of polarization asymmetry (Δ_o) vs. foil thickness was linear and that the spread of points about the line was no more than would have been expected from the statistical accuracy of the individual values.

Since the exposures for one foil thickness were examined in a random order (6.4) and since the exposures for the different foil thicknesses in the second set of experiments were not examined in any particular order, then the linearity of the plot of the polarization asymmetry value (Δ_o) vs. foil thickness suggested that the selection criteria and the accuracy of electron track counting had remained constant throughout the work.

It was noted that the points obtained in the first set of experiments lay on the same straight line as those obtained in the second set (6.1) and, since the two sets were carried out under different conditions, it was considered that the experimental technique was such as to eliminate any systematic errors associated with the parameters which were different in the two experiments. In particular, the fact that the results obtained using 0.19 mg/cm^2 gold foil in the two sets were in agreement, within the statistical errors, notwithstanding the fact that they had different foil asymmetries, led to the conclusion that the final polarization asymmetry values were reproducible.

The linear relationship between the measured polarization asymmetry values and the foil thicknesses was a clear indication

Graph of asymmetry vs. thickness of gold foil for 100 keV electrons from S^{35}.

Figure 17.

Thickness of gold foil $[mg/cm^2]$.

Asymmetry Δ_o $[\%]$

that the value of $K\ell" + \psi'$ had remained constant during the exposures for one foil thickness (7.7). Since the value of $K\ell" + \psi'$ apparently varied in a random way from the exposures for one foil thickness to another then it would be reasonable to expect that if such variations occurred during the exposures for one foil thickness then they would result in a spread of points about the line greater than that expected from the statistical accuracy of the individual values.

7.10 Variations in the electric field at the source

It is clear from Table 8 that the value of $K\ell" + \psi'$ was different form the various experiments. From the discussion in the last paragraph of the previous section it would appear equally certain that the value of $K\ell" + \psi'$ remained constant during the exposures made for each experiment. The one significant factor which emerges from an examination of the conditions under which the experiments were carried out is that air was allowed into the apparatus when the foils were changed between experiments but not during the exposures made for one foil thickness. There are three possible ways in which the entry of air into the apparatus could have affected the electrical conductivity of the insulating material surrounding the source (and consequently the characteristics of the electric field near the source) namely by the deposition of dust particles onto the surface of the insulator, by the chemical interaction of the constituents of the air with the irradiated polystyrene and by the absorbtion of water vapour by the polystyrene. The presence of this last factor has been noted by workers carrying out measurements on the dissipation factor of polystyrene and its existence has led to discrepancies in the work published in this field(116).

The resultant electric field at the source is made up of two components namely that due to the potential applied to the field plates and that due to the accumulation of electrons from the source on the polystyrene hood (5.7); the characteristics of both components depend on the electrical conductivity of the walls of the source-holder. The position is further complicated by the fact that the form of the electric field due to the accumulation of electrons on the source-holder will depend, to some extent, on the characteristics of the electric field produced by the applied potential and also by the fact that there are two separate mechanisms by which the angle between the momentum direction and the spin direction can be altered in the volume in front of the source (7.4).

It would appear that a considerable amount of experimental work would have to be carried out before any definite conclusions could be reached on the precise nature of the effects which govern the variations in the value of $K \ell'' + \psi'$.

7.11   The main systematic errors in δ.

As previously discussed (5.16), the effects of plural and multiple scattering on the polarization asymmetry value δ were eliminated by extrapolating to zero the thickness of the scattering foil. On the assumption that errors due to fluctuations in the velocity of the electrons emergent from the crossed fields (5.4), due to the time lapse between background exposures and gold foil exposures (6.4), and due to uncertainties in the values of the thicknesses of gold foil used (7.3), are contained in the statistical error of the value of δ, then the main systematic errors in the latter value are due to backscattering in the source foil (5.8),

to depolarization in the crossed fields (4.6), and to uncertainties in the calculated angular distribution (5.17). From the calculations on the magnitudes of these effects it is to be concluded that they give rise to an uncertainty of approximately 3% in the value of S . It is worthy of note that of the three main sources of systematic error listed above and of the minor sources of systematic errors discussed in previous chapters, only one, namely the error in the calculation of the angular distribution of the electrons entering the windows (5.17), could have led to the value of S being greater than the "correct" value of the polarization asymmetry for this experiment. This factor is of particular importance in assessing the significance of the value of P in the following section.

7.12   The measured value of the degree of polarization (P)

In order to use the measured value of the polarization asymmetry to establish the degree of longitudinal polarization of the β - particles examined, it was necessary to use theoretical calculations on Mott scattering. As previously discussed (5.17), the most accurate values obtainable were those of Sherman[67] and according to these calculations, for the parameters of this experiment, a fully polarized electron beam would have produced an asymmetry of 35.22% (5.17). On this basis, the value of the asymmetry obtained in this experiment gave the result that the degree of polarization (P) of 100 keV electrons from S^{35} was

$$P = 0.626 \pm 0.005$$
$$= (1.14 \pm 0.01) \frac{v}{c}$$

From the work of Alikhanov et al[98] it was clear that the

observed asymmetry was such that the β-particle polarization was negative (i.e. the preferred spin direction of the emitted electron was anti-parallel to its momentum), in agreement with all results published in this field.

7.13 The effects of screening

The major part of the discrepancy between the measured value of P and the value predicted by Lee and Yang on the basis of the two component theory of the neutrino (i.e. $P = \frac{v}{c}$) is almost certainly due to the fact that the theoretical values of the polarization asymmetry computed by Sherman are for a pure Coulomb scattering field. In their calculations Mohr and Tassie[66] did take into account the screening effects of the atomic electrons but, because of the particular energies studied (1.95 keV, 5.4 keV, 12.2 keV, 33 keV, 121 keV), it does not appear justifiable to interpolate their results at an energy of 100 keV. It would also appear that the results of Mohr and Tassie are not so accurate as those of Sherman[114]. The values obtained by Mohr and Tassie, and by Sherman, of the Mott asymmetry produced by the scattering of a fully polarized beam of 121 keV electrons are shown in graphical form in figure 18[77]. As originally pointed out by Sherman and Nelson[114], there is a discrepancy of 50% between the two sets of values at a scattering angle of 165°. For two reasons, it is not permissible to use the differences between the screened and unscreened values for an electron energy of 121 keV to determine correction factors to Sherman's values for 100 keV. Firstly, it would be expected that the screening corrections at an energy of 100 keV would be larger than those at an energy of 121 keV. Secondly, the angle at which the Mott asymmetry is a maximum varies with energy and consequently the differences between the two curves would also be expected to be
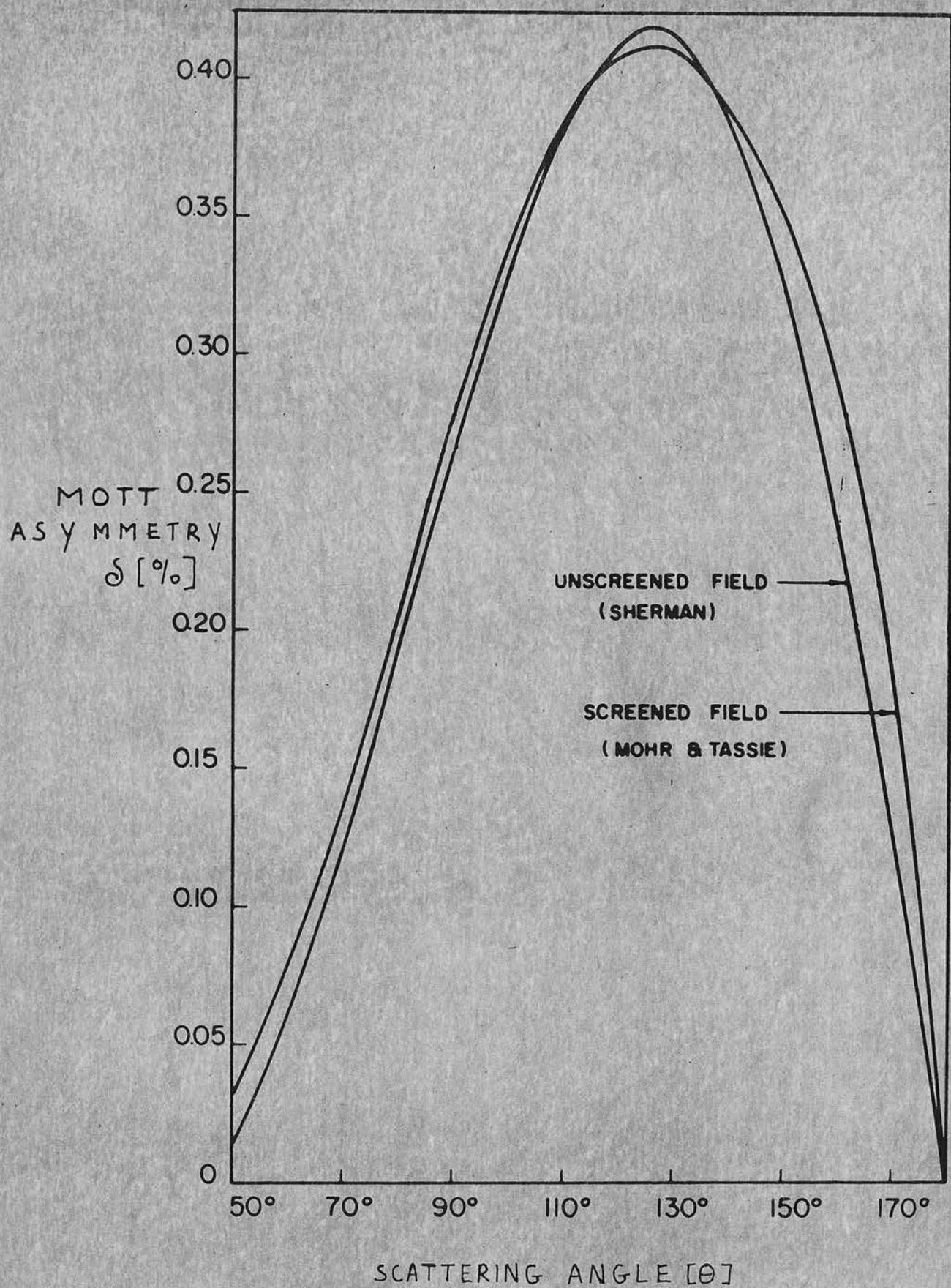
Figure 18.

energy-dependent; such an effect would be of considerable import-
ance for the scattering angle range used in this experiment.

From a consideration of the values of Mohr and Tassie, and
of Sherman, it would appear reasonable to conclude that if the
effects of screening were taken into account, the measured degree
of polarization would be in much better agreement with that
predicted by theory.

7.14 Coulomb effects and the value of P

Jackson, Treiman and Wyld[29] have obtained an expression
for the degree of longitudinal polarization of β -particles
emitted in allowed transitions. They found that

$$P = \frac{G \frac{v}{c}}{1 + b \cdot \frac{m}{E}} \qquad 7.14.1$$

where E is the energy of the electron, m is its mass and v its
velocity, and b and G may be obtained from the following
expressions:

$$G \, \xi = |M_F|^2 \left[\pm 2 \, Re \, (C_S C_S'^* - C_V C_V'^*) + \frac{\alpha Z m}{p_e} 2 \, Im. (C_S C_V'^* + C_S' C_V^*) \right]$$

$$+ |M_{GT}|^2 \left[\pm 2 \, Re \, (C_T C_T'^* - C_A C_A'^*) + \frac{\alpha Z m}{p_e}. 2 \, Im (C_T C_A'^* + C_T' C_A^*) \right]$$

$$7.14.2$$

$$b \, \xi = \pm 2 \, \gamma \, Re \left[|M_F|^2 (C_S C_V^* + C_S' C_V'^*) + |M_{GT}|^2 (C_T C_A^* + C_T' C_A'^*) \right]$$

$$7.14.3$$

$$\xi = |M_F|^2 (|C_S|^2 + |C_V^2| + |C_S'|^2 + |C_V'|^2)$$

$$+ |M_{GT}|^2 (|C_T|^2 + |C_A|^2 + |C_T'|^2 + |C_A'|^2)$$

$$7.14.4$$

$$\gamma = (1 - \alpha^2 Z^2)^{\frac{1}{2}} \qquad\qquad 7.14.5$$

    Z   is the atomic number of the final nucleus

    α is the fine structure constant.

These expressions, which include all Coulomb effects, are quite general in that no assumptions have been made as to invariance with respect to space inversion, charge conjugation or time-reversal.

For pure VA interactions it is clear that

$$\frac{P}{\frac{v}{c}} = 1 \qquad\qquad 7.14.6$$

If, however, the S and T type interactions contribute appreciably then the degree of polarization is given by

$$\frac{P}{\frac{v}{c}} = 1 \pm \frac{K m Z \alpha}{p_e} \qquad\qquad 7.14.7$$

where K is a measure of the contribution of the S and T type interactions.   Theoretically K can have any value between +1 and −1 though, in view of the experimental work on the relative magnitudes of the coupling constants discussed in chapter 1, it would be surprising if the value of K differed much from zero.

For K = +1 and for electrons of energy 100 keV emitted from S^{35} nuclei equation 7.12 gives the result

$$\frac{P}{\frac{v}{c}} \sim 1.058$$

Although no definite conclusions may be drawn from the measured degree of polarization until accurate polarization asymmetry values, which include the effects of screening, are available,

it might be concluded that the experimental result indicates a small Coulomb effect whose sign is positive.

A similar conclusion might be drawn from the work of Cavanagh et al[97] on the degree of longitudinal polarization of β -particles from Co^{60}, in the energy range 58 keV - 178 keV (figure 19).
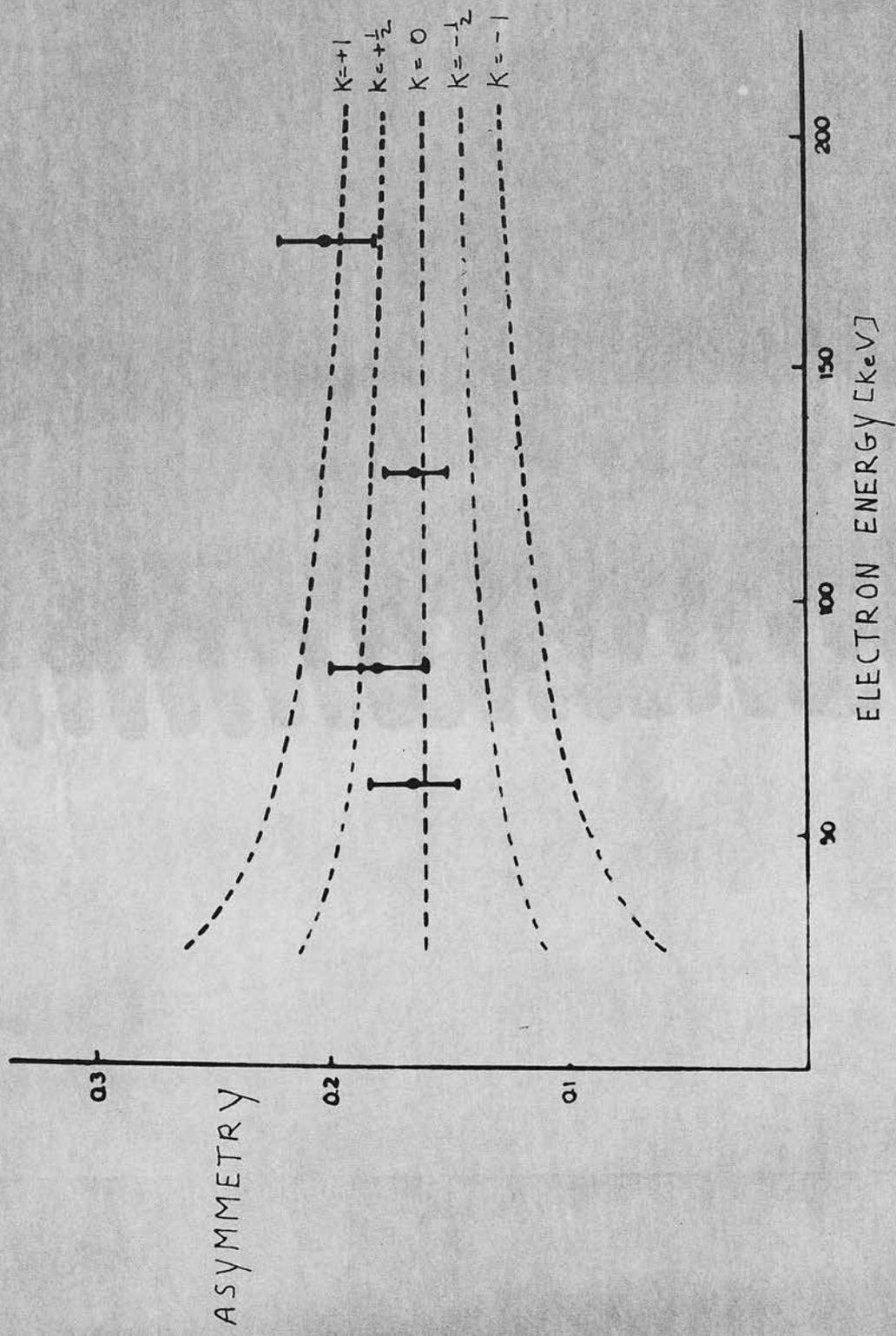
Figure 19.

CHAPTER 8.

THE CONCLUSION

A new method for the determination of the degree of longitudinal polarization of β -particles has been successfully developed. The results obtained by its use are of better statistical accuracy than any hitherto published. Further, experimental evidence has been obtained which suggests that the results are relatively free of systematic errors.

The degree of longitudinal polarization of 100 keV β - particles from S^{35} is

$$P = (1.14 \pm 0.01) \frac{v}{c}$$

It is estimated that the systematic error in the value of P is about 3%. This value is not in agreement with that predicted by Lee and Yang on the basis of the two component theory of the neutrino. It is considered that the major part of the discrepancy between theory and experiment is due to the use of theoretical values of the Mott asymmetry which do not include the effects of the screening of the nuclear scattering field by atomic electrons. Accurate theoretical values which include the effects of screening are not, at present, available. The experimental value of P does not exclude the possibility of the presence of a small Coulomb effect in the degree of longitudinal polarization. If such an effect exists then it would appear that it is positive in sign.

ACKNOWLEDGEMENTS

REFERENCES

1. Uhlenbeck and Goudsmit, 1926, Nature, Lond., 117, 264.

2. Dirac, 1928, Proc. Roy. Soc. A, 117, 610.

3. Mott, 1929, Proc. Roy. Soc. A, 124, 425.

4. Cox, McIlwraith and Kurrelmeyer, 1928, Proc. Nat. Acad. Sci. 14, 544.

5. Chase, 1929, Phys. Rev. 34, 1069.
   1930, Phys. Rev. 36, 1060.

6. Rupp, 1932, Phys. Z., 33, 158.
   1932, Phys. Z., 33, 937.

7. Rupp, 1932, Zeit. Phys., 79, 642.
   1934, Zeit. Phys., 88, 242.
   1934, Zeit. Phys., 90, 166.

8. Langstroth, 1932, Proc. Roy. Soc. A, 136, 558.

9. Dymond, 1932, Proc. Roy. Soc. A, 136, 638.
   1934, Proc. Roy. Soc. A, 145, 657.

10. Thomson, 1934, Phil. Mag. 17, 1058.

11. Richter, 1937, Ann. Physik 28, 533.

12. Kikuchi, 1939, Proc. Phys. Math. Soc. Japan, 21, 524.
    1940, Proc. Phys. Math. Soc. Japan, 22, 805.
13. Skull, Chase and Myers, 1943, Phys. Rev. 63, 29.

14. Lee and Yang, 1956, Phys. Rev. 104, 254.

15. Lee and Yang, 1957, Phys. Rev. 105, 1671.

16. Rochester Conference, 1955, 1956, 1957, Proceedings of
    the Fifth, Sixth, Seventh Annual Conference on High
    Energy Physics (Interscience Pub., New York)
    reported by Sakurai, 1959, Prog. Nucl. Phys. 7, 243.

17. Dalitz, 1957, Reports on Progr. in Phys. 20, 163.

18. Lee and Orear, 1955, Phys. Rev. 100, 932.
    Lee and Yang, 1956, Phys. Rev. 102, 290.

19. Wu, Ambler, Hayward, Hoppes and Hudson, 1957,
    Phys. Rev. 105, 1413.

20. Gorter, 1948, Physica 14, 504.
    Rose, 1949, Phys. Rev. 75, 213.

21. Landau, 1957, Nucl. Phys. 3, 127.

22. Salam, 1957, Nuovo Cimento 5, 299.

23. Weyl, 1929, Zeit. Phys. 56, 330.

24. Fermi, 1934, Zeit. Phys. 88, 161.

25. Sakurai, 1959, Progr. Nucl. Phys. 7, 244.

26. Allen, The Neutrino, (Princeton University Press, Princeton,
    1958) Chapters 4, 5, 7 and 8.
    Allen, 1959, Rev. Mod. Phys. 31, 791.

27. Alder, Stech and Winther, 1957, Phys. Rev. 107, 728.

28. Morita and Morita, 1957, Phys. Rev. 107, 1316.

29. Jackson, Treiman and Wyld, 1957, Phys. Rev. 106, 517.
                                1957, Nucl. Phys. 4, 206.

30. Tolhoek and De Groot, 1951, Physica 17, 1.
    Tolhoek and De Groot, 1951, Physica 17, 17.
    Tolhoek   1956, Rev. Mod. Phys. 28, 277.

31. Schopper, 1957, Phil. Mag. 2, 710.
    Schopper, Appel and Bloom, 1958, Phys. Rev. 109, 2211.

32. Boehm and Wapstra, 1957, Phys. Rev. 106, 1364.
                        1957, Phys. Rev. 107, 1202.
                        1957, Phys. Rev. 107, 1462.
                        1958, Phys. Rev. 109, 456.

33. Lundy, Orear, Sens, Swenson, Telegdi, Yovanovitch,
    1958, Bull. Am. Phys. Soc. Ser II, 3, 198.

34. De Brunner and Kundig, 1957, Helv. Phys. Acta, 30, 261.

35. Steffen, 1958, Proc. of the Rehovoth Confer. Edited by
    Lipkin, (North Holland Pub. Co., Amsterdam, 1958)
    pp 419 ⇔ 436.

36. Burgy, Krohn, Novey, Ringo and Telegdi,
    1958, Phys. Rev. 110, 1214.

37. Curtis and Lewis, 1957, Phys. Rev. 107, 543.

38. Schopper and Galster, 1958, Nucl. Phys. 6, 125.

39. Boehm and Wapstra, 1958, Phys. Rev. 109, 456.

40. Goldhaber, Grodzins and Sunyar, 1957, Phys. Rev. 106, 826.

41. Cohen, Wiener, Wald and Schmorak, Proc. of the Rehovoth
    Confer., Edited by Lipkin, (North Holland Pub. Co.,
    Amsterdam, 1958) pp 404 - 405.

42. Amaldi, Bernardini, Brovetto and Ferroni, 1959,
    Nuovo Cimento, 11, 415.

43. Bisi and Zappa, 1959, Nucl. Phys. 10, 331.
    Bisi, Fasana and Zappa, 1960, Nuovo Cimento 16, 350.

44. Grodzins, 1959, Progr. Nucl. Phys. 7, 163.

45. Fraunfelder, Hanson, Levine, Rossi and De Pasquali,
    1957, Phys. Rev. 106, 386.

46. Koller, Schwartzschild, Vise, and Wu, 1958,
    Phys. Rev. 109, 89.

47. Geiger, Ewan, Graham and Mackenzie, 1958,
    Phys. Rev. 112, 1684.

48. Bincer, 1957, Phys. Rev. 107, 1434.

49. Bhabha, 1936, Proc. Roy. Soc. A, 154, 19.

50. Page and Heinberg, 1957, Phys. Rev. 106, 1220.

51. Hanna and Preston, 1957, Phys. Rev. 106, 1363.
                         1957, Phys. Rev. 108, 160.
                         1958, Phys. Rev. 109, 716.

52. Page, 1957, Phys. Rev. 106, 394.

53. Deutsch, Gittelman, Bauer, Grodzins and Sunyar,
    1957, Phys. Rev. 107, 1733.

54. Boehm, Movey, Barnes and Stech, 1957, Phys. Rev. 108, 1497.

55. Bühring and Heintze, 1958, Zeit. Phys. 153, 237.

56. Herrmannsfeldt, Burman, Stahelin, Allen and Braid, 1958, Phys. Rev. Let. 1, 61.

57. Rustad and Ruby, 1958, Post-deadline paper, New York Meeting, Am. Phys. Soc. reported by Sakurai, 1959, Prog. Nucl. Phys. 7, 243.

58. Goldhaber, Grodzins and Sunyar, 1958, Phys. Rev. 109, 1015.

59. Sakurai, 1959, Prog. Nucl. Phys. 7, 243.

60. Nelson, 1958, Ph.D. Thesis, University of Michigan.

61. Bartlett and Watson, 1940, Proc. Am. Acad. Arts Sci. 74, 53.

62. Doggett and Spencer, 1956, Bull Am Phys. Soc. Ser. II, 1, 37. 1956, Phys. Rev. 103, 1597.

63. Yadav, 1955, Proc. Phys. Soc., Lond., A 68, 348.

64. Curr, 1955, Proc. Phys. Soc., Lond., A 68, 156.

65. Feshback, 1951, Phys. Rev. 88, 295.

66. Mohr and Tassie, 1954, Proc. Phys. Soc., Lond., A67, 711.

67. Sherman, 1956, Phys. Rev. 103, 1601.

68. Rose and Bethe, 1939, Phys. Rev. 55, 277. 1940, Phys. Rev. 57, 280.

69. Wentzel, 1922, Ann. Physik, 69, 335.

70. Chase and Cox, 1940, Phys. Rev. 58, 243.

71. Goertzel and Cox, 1943, Phys. Rev. 63, 37.

72. Trounson and Simpson, 1943, Phys. Rev. 63, 55.

73. Shinohara and Ryu, 1950, J. Phys. Soc. Japan, 5, 119.

74. Ryu, 1952, J. Phys. Soc. Japan, 7, 125. 1952, J. Phys. Soc. Japan, 7, 130. 1953, J. Phys. Soc. Japan, 8, 804.

75. Louisell, Pidd and Crane, 1954, Phys. Rev. 94, 7.

76. Pettus, 1958, Phys. Rev. 109, 1458.

77. Pidd and Nelson, 1959, Phys. Rev. 114, 728. Nelson, 1958, Ph.D. Thesis, University of Michigan.

78. Schneider and Barnard, 1958, Nuovo Cimento, <u>8</u>, 772.

79. Mühlochlegel and Koppe, 1958, Zeit. Phys. <u>150</u>, 474.

80. Wegener, 1958, Zeit. Phys. <u>151</u>, 252.

81. Massey and Mohr, 1941, Proc. Roy. Soc. A, <u>177</u>, 341.

82. Van DerSpuy, 1959, Nucl. Phys. <u>10</u>, 53.

83. Schopper, 1959, Nucl. Phys. <u>11</u>, 675.

*84. Frauenfelder, Bobone, Goeler, Levine, Lewis, Peacock, Rossi and De Pasquali, 1957, Phys. Rev. <u>107</u>, 386.

*85. De Waard and Poppema, 1957, Physica <u>23</u>, 597.

86. Vishnevsky, Grigoriev, Ergakov, Nikitin, Pushkin and Trebakhovsky, 1957, Nucl. Phys. <u>4</u>, 271.

87. Malone, Greenberg, Gluckstern and Hughes, 1958, Phys. Rev. to be published; reported by Grodzins, 1959, Prog. Nucl. Phys. <u>7</u>, 163.

*88. Bienlien, Fleischmann and Wegener, 1958, Zeit. Phys. <u>150</u>, 134.

*89. Langevin-Joliot, Marty and Sergent, 1957, Comptes rendus <u>244</u>, 3142.
Langevin-Joliot and Marty, 1958, Comptes rendus, <u>245</u>, 670.

90. Alikhanov, Eliseiev and Luibimov, 1958, Nucl. Phys. <u>7</u>, 655.

91. Heintze, 1957, Zeit. Phys. <u>148</u>, 560.

92. Heintze, 1958, Zeit. Phys. <u>150</u>, 134.

93. Bühring and Heintze, 1958, Zeit. Phys. <u>153</u>, 237.

94. Bühring, 1959, Zeit. Phys. <u>155</u>, 566.

*95. de-Shalit, Cuperman, Lipkin and Rothem, 1957, Phys. Rev. <u>107</u>, 1459.

*96. Lipkin, Cuperman, Rothem and de-Shalit, 1958, Phys . Rev. <u>109</u>, 223.

*97. Cavanagh, Turner, Coleman, Gard and Ridley, 1957, Phil. Mag. <u>2</u>, 1105.

98. Alikhanov, Eliseiv, Lubimov and Ershler, 1958, Nucl. Phys. <u>5</u>, 588.

99. Mikaélyan and Spivak, 1960, J.E.T.P. 37, 831.

100. Bienlien, Felsner, Guthner, Issendorf and Wegener, 1959, Zeit. Phys. 154, 376.

101. Bienlein, Felsner, Fleischmann, Guthner, Issendorf and Wegener, 1959, Zeit, Phys. 155, 566.

102. Bernardini, Brovetto and Ferroni, 1958, Nucl. Phys. 8, 294.

103. Bernardini, Brovetto, De Benedetti and Ferroni, 1958, Nuovo Cimento, 7, 419.

104. Bargmann, Michel and Telegdi, 1959, Phys. Rev. Let. 2, 435.

105. Mühlochlegel, 1959, Zeit. Phys. 155, 69.

106. Connor and Fairweather, 1957, Proc. Phys. Soc., Lond., A 70, 769.

107. Mendlowitz and Case, 1955, Phys. Rev. 97, 33.

108. Fraunfelder, Bobone, Goeler, Levine, Lewis, Peacock, Rossi and De Pasquali, 1957, Phys. Rev. 107, 909.

109. Fraunfelder, Hanson, Levine, Rossi and De Pasquali, 1957, Phys. Rev. 107, 910.

110. Cockcroft, Ellis and Kershaw, 1932, Proc. Roy. Soc. A, 138, 318.

111. Ellis, 1932, Proc. Roy. Soc. A. 138, 318.

112. Mott and Massey, The Theory of Atomic Collisions, (Clarendon Press, Oxford, 1949), second edition, p. 80.

113. Dilworth, Occhialini and Payne, 1948, Nature (London) 162, 102.

114. Sherman and Nelson, 1959, Phys. Rev. 114, 1541.

115. Ross and Zajac 1948, Nature (London), 162, 923.
1949, Nature (London), 164, 311.

116. Warner, Dielectric Materials and Applications, Edited by von Hippel (The Technology Press of M.I.T. and John Wiley and Sons, Inc., New York 1954) pp. 219 - 220.