Signalling Game Models of the Auditing Process

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DECLARATION

I hereby declare that this thesis is my own composition and that the work it contains is entirely my own

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ABSTRACT

Theoretical auditing models have recently changed from a single-person to a multi-person setting. This change has been prompted by the inability of decision theory to recognise that the manager has the potential to influence the outcome of the audit. The auditor's uncertainty about the manager's motivations can also influence the outcome. One of the motivating factors for audit work is the auditor's uncertainty about the rate of error or fraud occurrence. This is incorporated in the Audit Risk model in the "inherent risk" term, which has only been considered in a decision-theoretic setting. A game theoretic consideration leads to a signalling game.

Two models of the audit are developed to consider settings of both error detection and fraud prevention. In the model of error detection the players' actions include the effort put into maintaining the internal control system and investigation of these controls by the auditor followed by substantive testing and qualification. The effects of changes in the players' outcome costs on the number and type of equilibrium pairs is investigated. The model is shown to have the following properties; Costly information acquisition can form part of a pure strategy equilibrium, and the manager can send signals that are conditional upon the inherent chance of errors occurring. An example is given to illustrate the above properties. This also shows that raising an outcome cost to encourage hard work can be counter-productive.

Fraud and its detection do not occur in isolation. A model is therefore developed where fraud detection occurs against a background of unintentional errors. The auditor must divide his resources between error detection and fraud prevention. The manager is classified into two types by his difficulty in committing a fraudulent act. The manager has a choice over the level of effort to put into maintaining the internal control system and whether or not to commit a fraudulent act. The auditor chooses the level of substantive testing and subsequent in depth testing to carry out before issuing an audit report. It is shown that no equilibrium exist where the manager always reveals his type to the auditor.

The equilibrium set is shown to be dependent on the probability that the manager is the type who finds it easier to commit fraud. The effects of varying the costs of actions on the equilibrium behaviour of an example are considered. Whilst lowering the cost of in depth testing will reduce the equilibrium fraud rate, a decrease in the cost of substantive testing may have the opposite effect. The components of audit risk are assessed and it is shown that measures to reduce the risk of errors going undetected may increase the risk of fraud going undetected. In both models there are costs for which there is not a unique equilibrium. This suggests that there may not be a unique assessment of audit risk if srategic interaction is taken into account.

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Any remaining errors or omissions are entirely my responsibility,

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INTRODUCTION

For many years theoretical models of the auditing process were based on a decision theoretic approach, where it is assumed that the likelihood of events occurring is not influenced by the actions of the decision maker. This approach effectively regards the audit as a sampling problem for the auditor. Although such models have given a theoretical foundation to various areas of auditing, such as the assessment of sampling risk, they suffer from a crucial shortcoming; Decision theory cannot recognise that the managers of a company have an interest in, and the ability to influence, the outcome of the audit.

Within the last twenty five years game theory has been used to consider various aspects of the strategic interaction between the auditor, the management and the shareholders during a financial audit. It is suggested that one of the motivating factors for audit work is the auditor's uncertainty about aspects of his client's business. In particular the auditor will not know the rate of error or fraud occurrence in the accounting systems. This is recognised in the Audit risk model which has been traditionally assessed in a decision theoretic setting. This model includes an "Inherent Risk" term that recognises that the risk of errors occurring varies between companies and will have a crucial effect on the auditor's testing strategy. In a one-person decision problem the auditor's uncertainty about this inherent risk term is overcome by estimation. The auditor uses his knowledge of the company and experience with similar companies in the past to make an educated estimate of the level of this risk. In a strategic setting the auditor may be able to infer the level of this risk from the manager's behaviour. This leads to a formulation of the audit as a signalling game.

An increasingly difficult problem in auditing is the auditor's degree of responsibility to shareholders in those cases where a serious error goes undetected. At present the penalty associated with this outcome is determined through the courts, either directly through litigation or through an out of court settlement. Since the auditor's responsibility in this situation is determined through the courts, auditors can find themselves facing a large penalty for negligence despite having performed the audit to a level deemed responsible by the profession. An associated problem is the method used to allocate responsibility, and subsequent financial penalties, for such an error between the auditor and the manager of the company.

This problem can be analysed using a game theoretic model. If the costs of some of the outcomes are considered as variables, the effects of different levels of financial penalty on the auditor's and manager's behaviour can be observed. This can be used to consider whether the current high levels of litigation provide suitable motivation for both parties to work hard during an audit. The problem of the allocation of responsibility between the auditor and manager can also be considered by including additional constraints in the model.

This thesis develops two signalling game models of the auditing process. The first model regards the audit as a means of preventing accounting errors from going undetected and investigates the auditor's acquisition of costly information in such a setting. The second model views the audit as a means of both detecting random errors and preventing fraudulent activity and considers how these two potentially conflicting responsibilities influence both the auditor's and manager's optimal behaviour. The optimal behaviour of the participants in each model is analysed using game theory. The models are then used to consider how changes in factors such as cooperation, the degree of liability, the relative costs of hard work, and the responsibility for fraud detection will affect the players' optimal behaviour.

The following chapter discusses the audit in so far as is necessary to provide a setting for the theoretical models to be developed. It starts by sketching the development of the modern audit function and describing some of the defining characteristics of an audit. The auditor's responsibilities and potential actions during an audit are described by dividing the audit into five stages; Planning, investigating the internal controls, substantive testing, drawing conclusions from the evidence and qualification. The Audit Risk model mentioned above is introduced and the auditor's legal responsibilities are discussed.

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The second chapter describes the relevant concepts and definitions of game theory which will be used to both develop and analyse the models. The chapter starts by discussing the assumptions made about the players in a game theoretic setting and introduces some of the basic concepts and definitions of game theory. The concept of a pure strategy Nash equilibrium is defined and this is used to motivate the introduction of mixed strategy profiles and the idea of equilibrium set refinement. Games of incomplete information are discussed in broad terms before signalling games are defined in more detail.

Chapter three outlines previous work that has considered auditing in a game theoretic setting. These models are divided into five categories by their approach to modelling the audit. A brief outline of the models is given which highlights the important results of each. The relationship between the models to be developed here and the existing models is then discussed.

Chapter four develops a model of the audit as a means of error detection. The model is used to consider the implications of the present increase in the rise of auditor liability to third parties. This is achieved by regarding the cost of the unqualified material error outcome as a variable The model includes the three main stages of the audit described in section 1.3; Investigation of the internal controls, substantive testing and qualification. The model is analysed using game theory and the auditor's strategy set is reduced. This enables the auditor's optimal strategy set to be classified. A method for determining the equilibrium set of the model as two of the costs vary is developed and this is illustrated with a numerical example. The effects of increased litigation damages on the behaviour of both the auditor and the manager are discussed for the numerical example.

Chapter five extends the analysis of the optimal behaviour to include randomised strategies and pareto domination as an equilibrium refinement. The example of chapter four is investigated in two alternative settings. Firstly, the model is considered in a cooperative setting. This allows the auditor and manager to enter into pre-play commitments to minimise their joint costs. In such a setting the auditor's perceived independence may be called into question. In the error detection model the players behaviour can vary between implicitly cooperative (where both players prefer the same outcome) to non-cooperative where the players' interests are at odds. The player's behaviour is compared with the optimal behaviour in chapter four to see to

what extent the ability to explicitly cooperate influences the optimal behaviour. Secondly the optimal behaviour is considered in a setting of proportionate liability. This is achieved by an additional constraint modelling the interdependence between the penalties imposed on the auditor and manager. The optimal behaviour as the penalties increase in this setting is compared with the unconstrained case.

Chapter six develops a model of the audit where the manager has the ability to commit fraud. This fraudulent activity is considered against a background of random errors so that the auditor must divide his resources between error and fraud detection. The effects on the optimal behaviour of altering the costs of actions are considered. The equilibrium set is classified and the auditee's motives for concealing his private information are discussed. A numerical example is given to illustrate the types of optimal behaviour that occur. For this example the components of the audit risk model are considered and it is shown that measures to reduce the risk of errors going undetected may increase the risk of fraud going undetected.

Chapter seven starts by comparing and contrasting the analysis of the two models. The interesting equilibrium behaviour and the conclusions that can be drawn from each model are then discussed. The limitations of each model are considered and these limitations motivate a discussion of areas for future research. Finally, a summary is given which describes the development of the models, the contributions that these have made and the conclusions which can be drawn from them.

1 AN INTRODUCTION TO AUDITING

1.1 Background

The idea of an audit has been around since at least the fifteenth century. Originally an auditor's responsibility was to ensure the absence of fraud in the accounts kept by stewards of wealthy estates. Until the 19th century businesses were mainly small and owned either by individuals or partnerships. There was therefore little demand for either complex accounting or auditing. The Companies Act of 1844 made the first distinction between the providers of capital (shareholders) and the management of a business. This led to a need for an independent examination of accounts to ensure the safety of the shareholders' interests. Initially a company would appoint one of its shareholders as an auditor, with no requirements for either relevant qualifications or independence. As companies became more commonplace and increased in complexity there was a corresponding increase in the demands on the auditor. This led to the development of auditing as a profession. Flint [18] gives seven postulates that define an audit. These can be summarised to give the following three requirements:

The Need For an Investigation

The primary condition for an audit is that there is a relationship of accountability between two parties where one party is dependent in some way upon the actions or information issued by another whose aims may not coincide. Furthermore, the second party owes a duty of acceptable conduct to the first. There is therefore a need for reassurance that the second party is fulfilling his duty.

The Existence of a Profession

The subject matter must be suitably complex or remote to deter the interested party from reassuring themselves directly. Thus audits are restricted to matters where a large degree of formal training and experience are needed to form a reasonable opinion. There must also be certain standards of behaviour that both parties perceive as appropriate. Conformance to these standards must be verifiable (albeit by someone possessing the necessary skills and judgement) if the auditor is to express a considered opinion on that conformity. Standards of accountability can be set and actual performance can be measured against these by reference to known criteria. The process of this measurement and comparison requires both special skill and the exercise of judgement.

Credibility of The Report

For an auditor's report to carry any weight it must be possible to collect sufficient evidence to reach an informed opinion. Also, the meaning of the manager's statement to the interested party must be unambiguous if the auditor is to comment on the reasonableness of its content. For his client to gain any reassurance from the report, the auditor must be independent and free from investigative and reporting constraints. If the auditor is not given freedom of investigation then any opinion that he forms about the state of affairs as a whole will be fundamentally flawed.

These requirements can be combined to give a broad definition of an audit. Although a precise definition can only be given for a particular field, such as financial auditing, the Auditing Concepts Committee [4] give the following general definition:

> "A systematic process of objectively obtaining and evaluating evidence regarding assertions about economic actions and events to ascertain the degree of correspondence between those assertions and established criteria and communicating the results to interested users"

Although auditing developed as a practical discipline, much work has been done to develop a theoretical basis. This work falls into two categories, a philosophical approach which considers the essential characteristics of an audit and theoretical models which have been developed to reflect different aspects of the audit. The philosophical approach concerns itself with the ethical side of auditing. It discusses concepts such as auditor independence and considers how factors such as personal relationships or financial interest might compromise (or be seen to compromise) independence. An abstract discussion of the concepts involved in auditing can serve as a good introduction to a complex subject. This discussion can lead to a generalised description of an audit which is applicable to a wide range of circumstances and can be easily understood. Secondly, discussion may lead to a consensus as to what makes up an audit. This can be used, often in conjunction with a theoretical model, to establish criteria for "good practice".

The theoretical models of the auditing process ignore the ethical implications of the interaction between an auditor and the manager (and shareholders) and consider how the participants behave as economically rational agents. This approach can be used to consider how self-interested individuals will interact. For example principal/agent models have considered the circumstances under which the shareholders will benefit from the appointment of an auditor. These economic models have several uses. Focusing on one aspect of auditing can act as an aid to understanding the whole process. The simplified nature of any such model can highlight interactions that may be obscured in practice. The models can then be used in practice as a planning aid (such as the audit risk model described in section 1.4). The sensitivity of the models to changes in certain costs or factors can also be considered. This can suggest the possible effects of a similar change in a real-world setting. This thesis develops two theoretical models of the audit considering both error and fraud detection. This chapter gives a brief description of the auditing process and mentions some of the issues which are later considered using the models.

1.2 Financial Auditing

The need for reassurance on the credibility of a set of financial accounts led historically to the development of the auditing profession. The management of a company is responsible for the stewardship of the shareholders' investment. To demonstrate the effectiveness of their stewardship the management annually issues a financial statement. There is a clear need for an independent investigation of the truth and fairness of this report. The size and complexity of modern businesses mean that such an investigation will require a degree of formal training.

Other types of audit now exist, such as internal or operational audits, which use many of the same techniques and procedures. However, these types of audit are often used by management to improve and monitor the efficiency of the company, whereas financial audits are primarily for the benefit of the shareholders. Although an external auditor is appointed by the shareholders, in practice the shareholders usually agree with the appointment suggested by the management. A financial audit differs from other types of audit as the auditor's and manager's interests may be at odds. This thesis considers the interaction between the management and the auditor and will therefore focus on financial auditing. The purpose of a financial audit has been defined by the American Institute of Certified Public Accountants [1] as:

> "The objective of the ordinary examination of financial statements by the independent auditor is the expression of an opinion on the fairness with which they present financial position, results of operations and cash flows in conformity with generally accepted accounting principles. The auditor's report is the medium through which he expresses his opinion or, if circumstances require, disclaims an opinion"

Detailed information about a company's activities would be of use to their competitors. The shareholders access to detailed accounting information is restricted as otherwise competitors could also gain access by becoming shareholders. However, the auditor must have access to this private information if he is to make an informed judgement on the fairness of the financial statements. It is important therefore that the management regards the auditor as independent so that he can gain access to this information. Any concerns about this independence will at the very least put a strain on the working relationship between the auditor and management and may lead to restrictions upon the freedom of the investigation. If on the other hand the shareholders do not feel that the auditor is impartial they will gain no further confidence from the audit as they will now question the "truth and fairness" of the auditor's report as well as the manager's. An obvious lack of independence on behalf of the auditor can negate any benefits that his presence and investigation may have given.

Perhaps even more important than the auditor's independence is the commonly held views on the independence of the profession as a whole. It is the public's trust in the auditor's independence that gives the audit report credibility. Furthermore, the shareholders can tell little about an individual auditor's independence and integrity apart from that commonly associated with his profession.

It is therefore in every auditors' interest to avoid even the appearance of compromising their independence. The professional bodies issue ethical guidelines to help auditors on questions of independence. These guidelines help to maintain public confidence in the independence of the auditing profession.

If the auditor's fee is determined by management then again the auditor's independence is brought into question. The size of the fee being conditional upon a certain report being given is clearly unacceptable. However, the fee can influence the audit in other ways. For example, if the fee is agreed before hand (which is usually the case) then a low fee may influence the amount of work done by the auditor which will impair his freedom of investigation. A business can effectively limit the fee (and thus the investigation) by giving the job to the lowest bidder. If a disproportionate amount of an auditing firm's revenue comes from one company (through consultancy for example) then the auditors may feel reluctant to issue a qualified report for fear of losing revenue. A combination of the two issues can occur, when a firm bids a lower auditing fee than they think reasonable with the aim of making up this deficit with the more lucrative consultancy for the business.

1.3 The Stages of an Audit

Materiality is a key concept that is used throughout the auditing process. The three main points at which it is used are during planning, evidence gathering and drawing conclusions. Ideally, an auditor would be able to account for every misstatement in the accounts. The constraints of time and cost-effectiveness mean that this is not feasible. The auditor needs to decide which misstatements will affect the fairness of the financial report. Another problem is that if an auditor was to draw attention to every discrepancy then those reading his report may miss important points that are snowed under by irrelevancies. The decision of which items to consider important is a matter of judgement and is very case-dependent. It is thus very difficult to formally define materiality, yet it is a concept so central to the auditing function that many attempts have been made. DePaula's Auditing [3] offers the following definition:

> "A matter is material if its non-disclosure, misstatement or omission would be likely to distort the view given by the accounts or other statement under consideration."

Planning the Investigation

The auditor needs a thorough familiarity with the business. This should take into consideration any special tax laws or regulations peculiar to their business, the location and scope of their operations, the accounting methods for previous years and the financial performance of the industry as a whole. Such information can be gained from prior experience with the company, other clients in the same industry, or by talking to auditors that have the relevant experience. The management of the company will obviously be a good source of information but their views may be biased. Perhaps the most useful means of learning about the client is by reviewing the working papers from previous years. These factors are included in the "inherent risk" term of the audit risk model discussed in section 1.4. Having familiarised himself with the business the auditor can make an assessment of the likelihood that the accounts may contain a material error.

Investigating the Internal Controls

A business will take measures to minimise the risk of fraud and error occurring and going undetected. These measures will include both accounting information processing techniques and organisational policies. All these measures are referred to collectively as internal controls. These should be documented by the company, who should also ensure that these procedures are known and understood by the relevant employees. The standard of the internal controls will be of great interest to the auditor when planning an audit. Clearly, the less faith the auditor can place in the internal control system, the more evidence he will need to gather to reasonably assure himself that the accounts are in order.

The first step in examining the internal controls is to review the procedures, through review of the documents and discussions with staff. This gives the auditor an idea of how the internal controls should be working. However, before he can place too much faith in them he needs to see how they work in practice. One of the simplest ways to achieve this is to observe the personnel going about their duties to determine to what extent the procedures are followed. Of course for this to work it is important that the personnel do not realise that they are being observed. The very fact that there is an audit about to take place may also give workers a tendency to follow procedures closely.

Another method of investigation is compliance testing, where the auditor runs sample accounts through the system and observes their progress through each stage. If this reveals serious flaws in the system the auditor may make recommendations for changes. If these changes are made he will then be able to re-test the system. The degree to which the internal control procedures are followed, and the stringency of these procedures will determine how much substantive testing needs to be done.

Substantive Testing

After considering the effectiveness of the internal controls the auditor can resort to searching for errors himself. Probabilistic sampling techniques provide a theoretical basis on which to plan substantive testing, although sampling is seldom used in a formal sense in practice. These techniques arose in response to the threat of litigation and the competitiveness of audit fee which have forced auditors to become more cost effective. These methods are used to estimate a characteristic (the error rate) of a population (the accounts). As the items to be sampled are randomly selected, an assessment of the accuracy of the estimate can be made. There are four main benefits in principle to probabilistic sampling:

- The techniques are cost-effective as they are designed to give a good estimate for as small a sample size as possible.
- It forces the auditor to formally plan the sample of a population.
- The required sample size is determined objectively from the acceptable risk level.
- The sampling risk is quantified and can thus be used to show due care in the face of litigation.

The auditor conducts substantive testing to determine whether the level of error in the accounts is material. There are two ways that the sample may lead to the wrong conclusion:

Type I error The estimate of the error rate from the sample is too high. The auditor incorrectly concludes that the level of errors is material. [false positive]

Type II error The estimate of the error rate from the sample estimate is too low. The auditor incorrectly concludes that the level of errors is not material. [false negative]

Type I errors will lead to a waste of time and resources on behalf of the auditor as he gathers further unnecessary evidence. Type II errors on the other hand will mean issuing an unqualified audit report when there are in fact material errors. Clearly the auditor wishes to avoid this, but for a given level of materiality and sample size, reducing the risk of one type of error increases the risk of the other.

It has been argued that objective sampling can detract from the application of the auditor's intuition and experience as to where the problems may lie. Also, although the sampling is objective and formalised, the assessment of acceptable risk upon which the sampling is based is itself based upon the auditor's judgement of other risks. Thus the objectivity can be thought to be only skin-deep.

Drawing Conclusions from the Evidence

In the case where an error in the financial statements has been discovered during the evidence gathering phase, the auditor needs to decide whether this error should be disclosed to the interested parties. Ideally any error should lead the auditor to question the accuracy of the accounting system, or the honesty of those using it. The concept of materiality is used here to consider whether the errors found are significant. Another question to be considered is whether the errors discovered are indicative of other errors in the population that would, when taken together, be material. The auditor can use the evidence gathered and the assessment of acceptable risk to help him decide the likelihood of other such errors having gone undetected.

If the auditor has gathered sufficient evidence, to his mind, to show that the financial statement is a fair one he may decide that an unqualified report is in order even if errors are present. In this situation, he is using the concept of materiality and his judgement to decide that the level of error indicated is unlikely to affect his clients' views. A second factor based upon the auditor's judgement is the "going concern" consideration. The financial statement of a business is expected to conform to certain accepted accounting principles. Most of these deal with acceptable approaches to the recording of financial transactions and events. However, financial statements are usually prepared under the assumption that the enterprise will continue for the foreseeable future. The shareholders' biggest fear in any business is the collapse of operations which may irretrievably swallow their investment. An important assurance they will want from the statements is that the company will continue to be a going concern. An auditor will consider explicitly whether any uncertainty relating to the company's going concern status is adequately disclosed in the financial statements. If the uncertainty is fundamental it will be referred to in the audit report.

Qualification

Once the auditor has drawn his conclusions about the materiality of any errors and the compliance to accepted auditing standards he will be in a position to give an informed opinion on the state of the accounts. This takes the form of a short, standardised opinion. If the auditor has found nothing wrong, the report will positively state that the financial statements give a true and fair view of the state of affairs. If the auditor is unable to report affirmatively he will qualify his report by referring to all the matters about which he has reservations. The Auditing Practices Board [5] describe circumstances that, if deemed material, lead to a qualified opinion. Either there is a limitation on the scope of the auditors' examination or the auditors disagree with the treatment or disclosure of a matter in the financial statements. The APB give two forms of qualified opinion for a disagreement:

"An **Adverse opinion** is issued when the effect of a disagreement is so material or pervasive that the auditors conclude that the financial statements are seriously misleading. An adverse opinion is expressed by stating that the financial statements do not give a true and fair view. When the auditors conclude that the effect of a disagreement is not so significant as to require an adverse opinion, they express an opinion that is qualified by stating that the financial statements give a true and fair view except for the effects of the matter giving rise to the disagreement."

If the scope of the auditor's investigation is limited the APB give two other forms of qualification:

A **Disclaimer of opinion** is expressed when the possible effect of a limitation on scope is so material or pervasive that the auditors have not been able to obtain sufficient evidence to support, and accordingly are unable to express, an opinion on the financial statements. Where the auditors conclude that the possible effect of the limitation is not so significant as to require a disclaimer, they issue an opinion that is qualified by stating that the financial statements give a true and fair view except for the effects of any adjustments that might have been found necessary had the limitation not affected the evidence available to them."

1.4 The Audit Risk Model

A widely used theoretical model is the Audit Risk model. This considers the overall risk of an error going undetected to be the product of separate underlying risks. Because of its simplicity the audit risk model is frequently used as a conceptual tool during the planning process. After the auditor has investigated the internal control system and familiarised himself with the company background and area of operations this model can be used to help determine the amount of testing to be done. By assessing the risks that are not under his control, the auditor can determine a level of testing needed to limit the overall risk to an acceptable level. Audit risk and its components are set out by the Auditing Standards Board [6]:

"Audit Risk is the risk that the auditor may unknowingly fail to appropriately modify his (or her) opinion on financial statements that are materially misstated. It can be viewed as comprising the components of inherent risk, control risk and detection risk. Inherent risk relates to the susceptibility of an account balance or class of transactions to error that could be material ... assuming that there were no related internal controls."

Control risk is the risk that material errors are not prevented or detected by the internal control system. Detection risk is the risk that errors that are not prevented or detected by the control structure are not detected by the auditor. This gives the following relationship:

$$AR = IR \times CR \times DR$$

where
AR = Audit Risk (also known as universal risk)
IR = Inherent Risk
CR = Control Risk
DR = Detection Risk

One of the three factors (IR) is determined by the nature of the business, one factor (CR) can be influenced by the manager of the company, and the remaining factor (DR) can be determined by the auditor. To achieve a certain level of audit risk, the auditor can rearrange the audit risk equation and solve for DR. This gives the level of audit assurance required from the substantive testing as (1-DR)%. This value of audit assurance can be used to calculate the size of sample needed to obtain this level of assurance. However, despite the neatness of the calculation, all the variables have been assigned values by the auditor. Thus the method can be no better than the judgements upon which it rests.

1.5 Legal Responsibility

The basic legal responsibility of an auditor is that he can be brought to account if the standard of his work falls below a level deemed reasonable. The advantage of this, to the profession as a whole, is that it lends authority to the opinions of an auditor. As mentioned above, the auditing profession is worthless unless there is widely accepted trust in the standards of their work and their integrity. However, deciding what constitutes a reasonable standard of work will vary from case to case and is not always clear cut.

The biggest problem is that when an auditor's standard of work is called into question it will often be in a courtroom setting. In this case the reasonable standard will be determined by a Judge and jury. This can cause problems since the auditors are expected to conform to standards of work which others outside their profession deem reasonable. If the public expectation is higher than standards thought reasonable within the profession (the "expectation gap") then auditors can be found negligent despite having performed with due care as they understood it. The auditor's liability can arise in two ways; statute law and common law.

Statute law covers cases concerning criminal or wilfully dishonest acts. These might be the wrongful use of authority, wilfully making a materially false statement in a report, or dishonestly obtaining funds. These kinds of offences are not specific to the auditing profession but rather apply to anyone in a position to commit them. Common law covers cases where the auditor is accused of negligence in his duties to his clients. These are the more common liabilities that an auditor will face and largely define the legal requirements upon the auditing profession. Some of the auditors main legal responsibilities, as determined by previous legal cases, are outlined below:

- The auditor is liable for any damage sustained by his client by reason of his omission of verification of assets stated in the balance sheet.
- The auditor is liable for any damage sustained by his client as a result of falsifications that should have been uncovered by the exercise of reasonable care and skill.
- The auditor is responsible for the full disclosure of any material inadequacies.
- On the other hand, the auditor is not responsible for guaranteeing the accuracy of the accounts.

The auditor's responsibility for reporting and detecting fraud has been ambiguous for many years. Although this was at one time a major objective of the audit, as the modern audit function developed (and the accounting became more complex) the audit came to be regarded as a report on the quality of the financial statements. So whilst fraud was of concern as a potential source of error its detection was a secondary objective. However fraud, particularly by management, has always remained a source of concern for shareholders. In recent years the auditors responsibility for fraud prevention has increased in response to shareholders' concerns about a number of highly publicised cases of fraud. This may lead to another "expectation gap" because, as Tweedie [37] points out, "a properly designed and executed audit may not detect a material or other irregularity." since the auditor is unable to take into account the effects of collusion and concealment. At present the auditor's responsibility is limited to planning and performing his work so that he has a reasonable expectation of detecting material misstatements caused by either fraud or error. It is generally considered sufficient for the auditor to bring evidence of fraud to the attention of the management. It is normally only in extreme situations, where the auditor suspects the management of being involved for example, that the auditor reports directly to a regulatory body (the Department of Trade and Industry).

1.6 Limiting Auditor Liability

The auditor has inherited a legal responsibility to third parties. It used to be the case that no right of negligence existed to third parties (those outside the contract) except for cases of physical injury. However, it has been successfully argued in the courts that in some cases if an auditor's report is acted upon in good faith by a third party, and the information proves to be incorrect, then the auditor should bear some responsibility for this (although this depends on proximity). The third party can claim that, although the report was not intended for him, he would not have acted without the added assurance of the report. This raises alarming questions for the profession as to how far this liability will extend.

Both the auditor and the manager have a responsibility for preventing and detecting errors. In the UK this is reflected by a policy of joint and several liability. This basically means that both parties can be held responsible for losses as a result of negligence by either party. However the largest losses are suffered by shareholders when a company goes bankrupt. As the management's finances are often closely related to the company, bankruptcy can leave them unable to pay damages to the shareholders. This then results in very high costs to the auditor, no matter how small his share of the blame. The extent of this potential liability has led to calls for the liability to be limited in some way.

One such proposal would permit the auditors to form limited liability companies. Until 1989 audit firms existed as partnerships (or single practicioners). Under partnership law all partners within a firm are joint and severally liable. If damages are awarded that exceed the firms indemnity insurance the shortfall is made up from the personal assets of the negligent partner, with any excess made up from the personal assets of the other partners. Thus a partner may lose personal assets as a result of a poorly performed audit which he had no involvement in. The Companies Act 1989 permits audit firms to form limited liability companies. With such an arrangement the assets of the company are used to meet the excess of any claim greater than the insurance cover. The personal assets of the responsible partner can also be used to meet any deficiency, although the assets of the other partners cannot. However, even with incorporation, a large claim may still force the audit firm out of business. Furthermore, it seems that some firms question whether an incorporated entity provides the appropriate environment for carrying out professional services. Major audit firms in the US, for example, have instead formed limited liability partnerships registered in the state of Delaware. These combine limited liability with the working environment of a partnership.

A second proposal being considered is a statutory cap to limit the size of a claim, or to fix the maximum liability as a multiple of the audit fee. The latter method is already used in some European countries as it has the advantages of simplicity and reducing the chance of an audit firm being forced out of business by a single damages claim. Within such a system however audit firms have the potential to limit their liability by reducing the size of their audit fee, with a corresponding reduction in audit quality. Another potential drawback is that a client who suffered losses as a result of negligence may be prevented from fully recouping his losses.

A change in policy has recently occurred in the US to proportionate liability which awards damages against those responsible in proportion to their responsibility for that loss. This has the advantage of linking the damages awarded against the auditor to his degree of responsibility without limiting the size of the claim. Thus aggreived clients can fully recoup their losses whilst the audit firm shoulders no more than its share of the blame. The UK auditing profession is currently lobbying hard for such an arrangement to be adopted.

2 GAME THEORY

2.1 Introduction

Game theory considers the area of multi-person decision making processes. Its aim is to predict the optimal behaviour of the participants. To do this it is necessary to consider which outcomes each player prefers. The theory of utility was developed to express non-quantifiable preferences by assigning a numerical value to each outcome, so that preference could be reflected by an ordering of these numbers. Ordinal utility (best, second best, etc.) can be insufficient to describe a player's motivations in a game theoretic setting. If a compromise is to be reached between outcome A and outcome B for example, what matters is not just which outcome is preferred (ordinal utility) but by how much (cardinal utility). Shubik [35] points out that in situations where the benefit from an outcome is subjective, the assignment of cardinal utility may be inappropriate. However, if the outcomes can be assigned monetary values by the decision makers then cardinal utility can be used. Monetary values seem a fair approximation to the payoffs in an auditing setting where the costs involve the time spent working and potential legal costs. The model developed here will therefore assign cardinal utility to each outcome.

In game theory the players are considered to be rational and risk neutral, and each player's motivations and preferences are described entirely by the utility of an outcome. A rational player will aim to maximise his utility from the interaction. To derive an optimal strategy, a given player must consider what every other player is likely to do. Effectively this means that each player needs to consider the game from every players point of view. This leads to the assumption of "common knowledge", discussed in Binmore [9]. Not only is each player rational and aware of all the rules and options in the game, but all players know that all players are rational and aware of the rules, and all players know that all players know this (and so on....). The assumption of risk neutrality means that the player's preferences are not influenced by the occurrence of chance. This means that we do not need to distinguish between a guaranteed value and some chance events with the same expected value.

2.2 Basic Definitions

A game theoretic model of decision making must consider the actions available to each player, a timetable for when each decision can be made, a set of outcomes and a means of describing how the actions taken lead to an outcome. There are two ways of representing such a decision problem, the extensive form and the strategic form. The former of these is more useful for considering situations that involve timing.

Extensive Form: A graphical method of describing in chronological order the actions that are available to each of the players and the choices determined by chance for the game in question. These decision trees consist of nodes and branches. Each node either represents a decision by one of the players or a chance event. Each branch represents an action and the sub-tree below a branch determines the consequences of that decision. At the end of each final branch there is a payoff determined by the choices that led to that point.

The extensive form is a useful means of expressing situations in which the players move in sequence. If this is the case then the game also needs to specify whether each player will know the actions of the players that preceded him before making his choice. This leads to a consideration of the information available to each player.

Information set: A set of nodes within the decision tree where a certain player must make a decision but he does not know which of these nodes (with identical options for him) represents the actual state of play. This happens if the player has incomplete knowledge of the history of the game, caused either by a simultaneous move on behalf of the other player or by an element of chance. If a player does not have this uncertainty he is said to have a singleton information set. There are two ways in which the players can have non-singleton information sets. This leads to two classes of a lack of information:

Incomplete Information: A game has incomplete information if some or all of the players in it do not know all the structure of the game. Each player may not know the payoffs, utility functions or strategies available to the other players.

Imperfect Information: In contrast, a game has imperfect information if the players do not have all the information about the previous moves.

Each player will have a set of actions that can influence the outcome. He will want to choose amongst these actions to find a "best" action that maximises his payoff. In some games however the players choose actions more than once. In these cases the player's sequence of choices is equivalent to choosing a "best" plan of action for all eventualities before play starts. Such a plan of action is known as a strategy:

Strategy: A strategy outlines the action a player will take for any eventuality. It tells the player which action to take at each node of the tree where such a decision is called for.

In situations where a player observes the outcome of some event before choosing his action a strategy will prescribe an action for each possible outcome. Thus each player can choose a "best" strategy before play begins and then effectively plays no further part in the proceedings. Player i's choices are reduced to choosing a strategy σ_i from his set of possible strategies Σ_i. We can then describe the payoffs of the game as functions of these strategies:

Payoff function: Player i's payoff in an n-player game can be described as a function $U_i(\sigma_1,\sigma_2, ..., \sigma_n)$ of each player's strategy.

A comprehensive list of strategies and subsequent payoffs for each player is sufficient to describe a game. This leads to a second way of representing a multi-person decision problem.

Strategic form: The strategic form (normal form) of an n-player game G specifies the player's strategy sets and their payoff functions. $G = \{\Sigma_1,\Sigma_2,...., \Sigma_n ;U_1,U_2,...,U_n\}$

A game can be categorised by the nature of the payoff functions and the ability of the players to make binding pre-play commitments. If the players can make pre-play commitments the game is said to be cooperative. Luce and Raiffa [26] identify a further division that can be made between cooperative games where the payoffs have a monetary equivalent and those where payoffs are in terms of subjective utility. If payoffs are in monetary terms (and the player's utility is linear in money) then players can commit themselves to side payments in the pre-play agreements. In these situations the players should all agree upon the best outcome (the largest amount of money) and the only remaining problem is how to divide this amongst the players.

If the payoffs represent subjective utility (preferences over outcomes) then, even if comparisons of utility can be made between the players, side payments are meaningless. In these situations there will be a pre-play negotiation game, where the player's attempt to agree to a mutually beneficial course of action through arbitration. In this case the theory, discussed in Thomas [36], focuses on the likely results of this negotiation. Attention here will be focused upon the non-cooperative theory since the auditing process will be modelled as a non-cooperative game. In chapter 5 the model is considered in a cooperative framework as a comparison. However since the player's payoffs can be considered in monetary terms it will be assumed that they act to maximise their joint profit.

Non-cooperative games can be classified in terms of the player's payoffs. If the player's interests are diametrically opposed, so that what one player wins another loses, the game is said to be zero-sum. In these settings the players behaviour will always be competitive. However, in many situations the players' payoffs, although influenced by the others' actions, are not influenced by their payoffs. Every player's payoff function therefore needs to be considered separately. Since zero-sum games are a special case of non-zero sum games the discussion below will focus on the latter.

The important question for any game is "how are the players likely to behave?" Since we are considering rational utility maximising players this question can be rephrased to "how should the players behave to maximise their payoffs?". To answer these questions we will need to consider the strategic suitability of each potential combination of strategies. For an n-player game each way of playing the game will be an n-tuple with a strategy for each player.

Strategy profile A strategy profile $\sigma_N = (\sigma_1, \sigma_2, ..., \sigma_n)$ contains a strategy for each player and therefore describes one way in which the game could be played. The set of all such strategy profiles contains every possible play of the game G:

$$\Sigma_N = \{ (\sigma_1, \sigma_2, ..., \sigma_n) \mid \sigma_1 \in \Sigma_1, \sigma_2 \in \Sigma_2, ..., \sigma_n \in \Sigma_n \}.$$

Opposing strategy profile: Any approach to finding a "best play" solution to a game must consider how some player i can maximise his payoff for a given set of strategies for the other (n-1) players. Define the following:

$$\Sigma_{N-i} = \{(\sigma_1, \sigma_2, ..., \sigma_{i-1}, \sigma_{i+1}, ..., \sigma_n) \mid \sigma_1 \in \Sigma_1, \sigma_2 \in \Sigma_2, ..., \sigma_{i-1} \in \Sigma_{i-1}, \sigma_{i+1} \in \Sigma_{i+1}, ..., \sigma_n \in \Sigma_n \}$$

Each player i may be able to rule out some of his strategies by considering strategies that he will never want to use. Suppose that one of player i's strategies **x** is consistently better than **y**, for any strategies that the other n-1 players might use. Then a rational player i will never use strategy **y**. Formally we can say that strategy **x** dominates strategy **y**:

Domination: A strategy **x** dominates a strategy **y** for player i if

$$U_i(\mathbf{x}, \sigma_{N-i}) \geq U_i(\mathbf{y}, \sigma_{N-i}) \ \forall \sigma_{N-i} \in \Sigma_{N-i}$$

This can be used for the iterated elimination of strategies. After player i removes strategy y from his strategy set, player j may find that one of his strategies (which is consistently worse than another unless y is played) can also be eliminated. This in turn may mean that further strategies can be eliminated. This process can reduce the number of strategies that we need to consider to predict the optimal play. One concern with this might be that the solution we find after eliminating strategies is not a solution of the original game. However Owen [31] proves that for a two person game:

Lemma *If a dominated strategy is removed from a two player game then the solution of the reduced game is a solution of the original game.*

There is a problem however that solutions of the original game can be lost through domination unless a strict inequality is a used in the definition of domination (referred to as strict domination). Although domination can reduce the number of potential strategies it is usually insufficient to find a solution to a game.

2.3 Equilibrium Concepts

A basic requirement of any solution concept should be stability. An optimal play suggestion for the n players is worthless if any of the players can improve their payoff by ignoring the suggested strategy. This motivates the following definition:

Nash equilibrium If each player i has a set of strategies Σ_i then a strategy profile $\sigma_N^* = (\sigma_1^*, \sigma_2^*, ..., \sigma_n^*)$ is a Nash equilibrium if for each player i = 1, ..., n

$$U_i(\sigma_i^*, \sigma_{N-i}^*) \geq U_i(\sigma_i, \sigma_{N-i}^*) \qquad \forall\, \sigma_i \in \Sigma_i$$

Thus no player can benefit by unilaterally deviating from a Nash equilibrium. However, not all games have a such an equilibrium. It is easy to construct a game where the players interests are at odds in such a way that no combination of strategies satisfies the requirements for equilibrium. In these games the players best action can be to randomly choose between several of their strategies. This led to an expansion of the idea of a strategy to include chance. Playing strategies with a certain probability makes the choice irrational, however we can still choose the randomisation scheme rationally.

Mixed Strategy: A mixed strategy s_i for player i consists of a probability distribution over player i's strategy set Σ_i so that player i plays strategy σ_i^k with probability p^k where $\sum_{j=1}^{k} p^j = 1$ and $\sigma_i^1, \sigma_i^2, ..., \sigma_i^k \in \Sigma_i$

A mixed strategy may only put a positive probability on some subset of the available strategies. To distinguish between those strategies involved in the randomisation and those which are not we refer to the support of a mixed strategy s_i as those strategies σ_i^k that occur with positive probability. Clearly the set of pure strategy equilibrium is a subset of mixed strategies (i.e. mixed strategies with a support of one). The above definitions for pure strategies (such as the strategy profile) can easily be extended to the broader mixed-strategy case. It can be shown that with the inclusion of mixed strategies a Nash equilibrium will always exist:

Theorem (Nash 1950) *Every finite strategic-form game has a mixed strategy equilibrium*

Another potential difficulty with the Nash equilibrium set is its non-uniqueness. In zero-sum games each equilibrium will have the same value (gain or loss) to each player and so each equilibrium is equivalent. In non-zero sum games each equilibrium can have a different value for each player. The players may therefore disagree on which equilibrium is preferable. This leads to the idea of domination of strategy profiles

Domination: A strategy profile $(s_1, s_2, ..., s_n)$ dominates another, $(s_1', s_2', ..., s_n')$ if $U_i(s_1, s_2, ..., s_n) \geq U_i(s_1', s_2', ..., s_n')$ for each player i (with strict inequality in one case)

Pareto Optimality: A strategy profile is Pareto optimal if it is not dominated. A two person non-cooperative game is an antagonistic game if all outcomes are Pareto optimal.

As equilibrium pairs are not necessarily equivalent or interchangeable it is not clear which equilibrium point should be chosen as a solution. Therefore further solution criteria are needed to select a "best" solution to a game. One of the most compelling equilibrium refinements can be applied to dynamic games. In these games equilibrium points can exist which are "unreasonable" as they rely on an empty threat. This involves one of the player's threatening to play an action if a certain point off the equilibrium path is reached, even though it would not be optimal for the player to do this. Selten (1965) proposed the idea of sub-game perfect equilibrium which requires the players to act rationally at each decision point (sub-game). This eliminates equilibria which rely on a threat of irrational behaviour.

Subgame: A subgame G' of an extensive form game G consists of a single node and all of its successors in G, where the structure of information sets and payoffs of G' are inherited from G.

Subgame-perfect equilibrium: A strategy profile s_N of an extensive-form game G is a subgame-perfect equilibrium if the restriction of s_N to G' is a Nash equilibrium of G' for every subgame G'.

2.4 Games of Incomplete Information

Very little work had been done on games of incomplete information before Harsanyi's [23] three papers in 1967 - 1968. This was due to the difficulties in analysing how one player could predict the actions of his opponents (which is a necessary step in determining his best strategy) if he does not know what they can do or what they prefer. This situation leads to an infinite regression of expectations between the players. Harsanyi's work developed an alternate theory for the analysis of such games based on the construction of equivalent games which have complete but imperfect information. These equivalent games could then be analysed using existing techniques for games of imperfect information. There are two main assumptions that Harsanyi uses in constructing these equivalent games:

"Bayesian Hypothesis": Each player will assign a subjective joint probability to all unknown independent variables. Having done this they will try and maximise their expected payoff in terms of this probability distribution.

Consistency: Beliefs held by different players can be regarded as conditional probability distributions derived from a basic probability distribution over all variables unknown to the players.

Harsanyi's method for constructing an equivalent game of imperfect information is to introduce a chance move by nature at the beginning of the game that exogenously determines the variables unknown to all players. The consistency assumption ensures that the probability distribution for this chance move is common knowledge. Each player then learns the values of those variables he is entitled to know before the game begins in earnest. This leads to the concept of "types" of players, where for example different types may have different utility functions. The chance move by nature can then be thought of as a lottery choosing which types of player get to play. Each player will know his own type but not necessarily those of the other players. Games of incomplete information which are modelled in this way are referred to as Bayesian games. The strategic form representation of an n-player Bayesian game G^* is given by:

$G^*=\{S_1,S_2, ..., Sn ;U_1,U_2, ..., U_n ;T_1,T_2, ...,T_n; p_1, p_2, ..., p_n\}$

Where Si and Ui are strategy spaces and payoff functions (similar to a game G)
T_i is player i's type space
p_i is player i's belief about the types of the other players.

A strategy in a Bayesian game will be contingent upon type and will therefore be described as a function of type $s_i(t_i)$. Player i's payoff function can in general be a function of each player's type as well as action. Each player's belief is determined using Bayes' rule after learning any new information. In a static Bayesian Game G*, we can extend the definition of a Nash equilibrium to a Bayesian-Nash equilibrium. This essentially applies the Nash requirement that each player's strategy be a best response to the others' to a Bayesian game.

Bayesian Nash Equilibrium: The strategies $s_N^* = (s_1^*, ..., s_n^*)$ are a Bayesian Nash equilibrium if for each player i and for each of i's types $t_i \in T_i$ $s_i^*(t_i)$ satisfies:

$$s_i^*(t_i) = \underset{s_i \in S_i}{\mathrm{ARGMAX}} \sum_{t_{N-i} \in T_{N-i}} p(t_{N-i}|t_i)\times U_i(s_1^*(t_1), ..., s_i(t_i), ..., s_n^*(t_n); t_{N-i}, t_i)$$

In dynamic games of incomplete information the concept of Bayesian-Nash equilibrium can be extended to include subgame perfection. Since the player's optimal actions are influenced by their beliefs the concept of perfection must be extended to impose restrictions on the player's beliefs after a certain history. A slightly stronger equilibrium refinement is Kreps and Wilson's sequential equilibrium which restricts the beliefs at an information set which is off the equilibrium path. Both of these equilibrium concepts require lengthy definitions for a general case, which can be found in Fudenberg and Tirole [19]. Instead a formal definition of the requirements for a perfect Bayesian Nash equilibrium is given below for a particular class of dynamic games of incomplete information. It has been shown that for simple games the two equilibrium refinements are equivalent.

Theorem (Fudenberg and Tirole 1991): *If either each player has at most two types or there are two or less periods then the equilibrium sets for perfect Bayesian equilibrium and sequential equilibrium are the same.*

2.5 Signalling Games

Signalling games are amongst the most simple dynamic games of incomplete information. These are two player games of one sided incomplete information, where the informed player moves first. Since the informed player moves first, the other player may be able to infer the first's type by his choice of actions. The players can therefore be referred to as the Sender (the informed player that moves first) and the Receiver. This class of games is discussed in some detail since the audit is modelled as a signalling game in subsequent chapters. The timing of the basic game is described by Gibbons [20] as follows:

(1) Nature draws a type $t_i \in T_i$ for the sender, according to a probability distribution (known to both players) $p(t_i)$ where

$$p(t_i) > 0 \qquad \forall\, t_i \in T_i$$

$$\sum_{t_i} p(t_i) = 1$$

(2) The sender learns his type t_i and then chooses a message m_j

(3) The receiver observes m_j and then chooses an action a_k

(4) Payoffs are given by $U_S(t_i, m_j, a_k)$ and $U_R(t_i, m_j, a_k)$.

The requirements for Perfect Bayesian equilibrium (which is equivalent to sequential equilibrium in this class of games) are simplified for a signalling game:

Belief about uncertainty After observing any message m_j the Receiver must have a belief about which types would send m_j.

$$\sum_{t_i \in T_i} p(t_i | m_j) = 1$$

Receivers optimality: For each message the Receiver will act to maximise his expected utility given his belief about which types could have sent the message:

$$a^*(m_j) = \underset{a_k \in A}{MAX} \sum_{t_i \in T} p(t_i | m_j) \times U_R(t_i, m_j, a_k) \qquad \forall\, m_j \in M$$

Senders optimality: Since the Receiver will act to maximise his expected utility after receiving message m_j the Sender can effectively predict how the Receiver will behave after each message. The Sender will therefore choose a message so that the message and the Receivers subsequent response maximises the Sender's utility

$$m^*(t_i) = \max_{m_j \in M} U_S(t_i, m_j, a^*(m_j))$$

Bayesian updating: beliefs on the equilibrium path can be obtained from Bayes' rule and the Sender's strategy. Given $m^*(t_i)$ let T_j denote the set of types whose optimal signal is m_j. After observing m_j the Receiver's beliefs are given by:

$$p(t_i \mid m_j) \quad = \quad \frac{p(t_i)}{\sum_{t_i \in T_j} p(t_i)} \qquad \text{if } t_i \in T_j$$

$$= \quad 0 \qquad \text{if } t_i \notin T_j$$

There are three kinds of pure strategy equilibrium in a signalling game:

Pooling equilibria - each sender type sends the same message. The Receiver learns nothing about the Senders type from the message.

Separating equilibria - each sender type sends a different message. The Receiver is certain of the Sender's type after receiving the message.

Partially pooling - with more than two types of sender all types in a subset send the same message, but different subsets send different messages. The Receiver can update his beliefs about the Sender's type, as certain types are ruled out by each message. However, the Receiver will only know that the Sender is one of the types in that subset.

There are also mixed strategy equilibria which, since they can show aspects of both pooling and separating equilibria are referred to as hybrid equilibria. If only one auditee type has a randomised signalling strategy the equilibrium will be referred to as partially hybrid.

Hybrid equilibria - one or more Sender types has a randomised signalling strategy. The Receiver can update his beliefs about the Sender's type but he will not always know this type with certainty.

3 LITERATURE REVIEW

3.1 Introduction

For many years theoretical models of the auditing process were firmly grounded in decision theory. An important assumption in decision theory is that the likelihood of events occurring is not influenced by the decision-maker's actions. In other words the events are considered to be actions of "nature" determined by probability distributions known to the decision maker. Auditing can be modelled using this theory by considering the auditor as the decision maker. A theoretical basis for the interaction between, for example, sampling and audit risk can be developed. Whilst this approach has advanced the understanding of the audit process considerably it suffers from one crucial shortcoming: *It cannot recognise that the auditor's actions may influence the actions of the managers of the company.* The ability of the auditor to influence the behaviour of the managers has been recognised in practice for a long time. An example of this phenomenon is the anticipatory effect of an audit. The prospect of a particular department being audited can affect the performance of that department. To capture this strategic interaction between the managers and the auditor a theory permitting multi-person strategic decision choice must be used - game theory.

Game theory was first used in various principal-agent models to consider such topics as the value of information, the need for an auditor and the design of optimal contracts. In a principal-agent model the principal determines contracts which define the payoff for the agent. In auditing terms the shareholders are clearly the principal player (as they collectively own the means of producing wealth) whilst both the auditor and the manager of the business can be regarded as agents since they are employed by (or on behalf of) the shareholders.

Attention subsequently focused on the strategic interaction between the auditor and the manager. The models developed so far can be divided into two classes; models of major directional choice and hypothesis testing models. Models of directional choice such as Fellingham and Newman [16] consider a sequence of distinct but limited actions, such as High or Low effort by the manager or two levels of substantive testing by the auditor. These actions represent large changes in the players' conduct during the audit and clearly illustrate the strategic interaction between the two players.

In more operational models, such as Newman and Noel [30], the auditor uses a hypothesis test to accept or reject the reported account balance. The auditor's testing is regarded as an imperfect signal of the account balance. An important assumption in these models is that the distribution of the signal (or the errors) is known to the auditor. The auditor's strategic decision is the size of the region of rejection in the hypothesis test. An extension of this approach allows the auditor to choose the sample size, which determines the accuracy of the testing signal on which the hypothesis test is based. These hypothesis testing models have recently been used to consider the auditor's responsibility for fraud detection.

3.2 Principal / Agent Models

A principal / agent model is used by Froystein Gjesdal [21] to consider the value of an audit report. He identifies two reasons for financial statement demand; Decision making and stewardship demand. The latter of these represents the situation where the owners of a business have delegated responsibility to a manager and wish to check on his performance. Whereas the theory of the value of information in decision making can be analysed using decision theory, the value of information for stewardship requires a game theoretic analysis. To this end, Gjesdal develops a very general agency model where the results of some information system are used to motivate the players. Using a two player principal-agent model as a simplified version of the general case, Gjesdal shows that an information system (such as an auditor's report) is of value to the principal. This is an advancement over previous accounting literature which starts with the assumption that such information is of value.

In a similar vein, Baiman and Evans [7] consider six principal-agent models where the agent is hired to provide some sort of productive input to a project funded by the principal. Two information systems are considered; A private pre-decision signal to the agent and a public post-decision signal correlated to the state of the project and the agent's action. In two of the models the agent also has the option of sending a signal to the principal. The models are compared in terms of the Pareto optimal frontiers (neither player can increase their payoff without reducing the other's) of the payoff regions. These comparisons are used to determine conditions for the introduction of an information system or a communication system to benefit the players.

In contrast to Baiman and Evans' assumption that the agent learns his private information before his action, Dye [14] considers a similar model where the agent receives private post-decision information. This signal can then be (perhaps falsely) relayed to the principal. By making the agent's contract depend upon this communication, the principal is in effect offering the agent a choice between a family of contracts conditional on the signal. The communication is valuable if offering such contracts can give a Pareto improvement over a single payment scheme. Dye develops a scheme that improves upon the single optimal compensation scheme available if there is no communication. This new scheme makes both players better off and also enforces truth telling in the agent's communication. However, this result depends upon the original solution - if public knowledge of the agent's private information does not improve the contract then communication is useless.

Having considered the value of information systems and communication in Baiman and Evans; Baiman, Evans and Noel [8] develop a principal-agent model in which the agent has private information, which he agrees to communicate to the principal. To reduce the inefficiency caused by this information asymmetry the principal hires an auditor to attest to the validity of the agents message. The model used, in which the agent has a private information system which he reports to the principal and there is public ex-post information is similar to the most general model considered in Baiman and Evans. Here however they include a second agent (the auditor) to attest to the validity of the manager's report.

Restrictions are placed upon the punishment for the manager misreporting. Otherwise, since the ex-post public information system reveals the managers private information with positive probability, a sufficiently large penalty will ensure the best result without the need for an auditor. The auditor's inability to misappropriate assets directly is one reason why the principal may find it easier to motivate the auditor (who then subsequently influences the manager's behaviour) rather than trying to motivate the manager directly.

The model can be considered one of fraud prevention since the manager keeps any production not reported and handed over to the principal. The principal will only hire an auditor for those manager reports where the auditor can improve the situation (for example suspiciously low reports). The reports that lead to an audit can be divided into audit regions. The size of these regions influences the auditor's behaviour. For example, larger regions reduce the attractiveness of shirking since it becomes harder for the auditor to guess the right outcome. Having determined the optimal audit region (all reports below a certain level result in an audit) and an optimal contract pair, the only subgame perfect Nash equilibrium is shown to be consistent reporting where the manager reports honestly but not fully i.e. "the outcome lies in this subset ...." and effective auditing followed by honest reporting by the auditor.

Antle [2] uses an agency model to tackle another area of auditing. He attempts to form a plausible definition of auditor independence within an agency model. A general model is developed involving the principal (owner) and 2 agents (manager and auditor). Firstly the owner chooses an incentive scheme based on the manager's report, the auditor's report and some publicly observable ex-post information variable (such as the company's gross profit for the year). Secondly the manager observes some private information (such as net income), then chooses a productive act and a reported value of the privately observed variable (reported net income).Thirdly the auditor has a non-productive action that generates a privately observed variable correlated in some way to the action / information of the manager. The outcome is then determined, influenced by both the manager's action and a random variable (this outcome can be thought of as some amount of money received by the owner). The owner then pays both the agents according to the incentive scheme.

Having derived this model Antle turns his attentions to the question of independence. He points out that the model cannot be regarded as a cooperative one because if it were the owner and manager could enter into binding pre-play agreements and there would be no need for an auditor. Each of the players is considered to be utility-maximising. The problem is where to draw the line between self interest (which involves at least maximising utility) and collusion to the detriment of the owner. With utility maximising players the manager and auditor will play a Nash equilibrium in the subgame generated by the owners incentive plan. However, the Nash equilibrium may not be unique, indeed all equilibrium refinements are intended to reduce this set of feasible solutions. Antle suggests that auditor independence could be used as an equilibrium refinement. He defines a strongly independent auditor to be one who plays the Nash equilibrium most preferred by the owner.

The owners incentive problem with a strongly independent auditor can be simplified by considering only those incentive schemes that lead to truthful reporting (by the revelation principle). A second weaker definition of independence is given in which the auditor will choose, amongst those equilibria that maximise his own utility, the one preferred by the owner. With this definition of independence the auditor's optimal contract depends only upon the ex-post financial indicator. This would seem to be at odds with rules prohibiting the auditor from owning shares in the company being audited. Antle also points out that a repeated game setting may encourage collusion as it gives the manager and auditor a means of enforcing any private agreements.

3.3 Models of Major Directional Choice

Wilson [38] gives perhaps the first suggestion that the usefulness of game theoretic modelling extends beyond the principal agent model. In this paper, based on a presentation to the 1982 AAA annual meeting, Wilson speculates on directions for research in accounting theory. He begins by discussing some of the pros and cons of modelling auditing with game theory. The main drawback is the necessary assumption of super-rationality (that all players are perfectly rational and furthermore know that all the other players know that they are rational and so on .... and that no amount of evidence to the contrary during the game will change this). Also that the players are assumed to have almost unlimited powers of calculation. On the other hand, the players are considered to have limited foresight and it is recognised that some things may never be observable.

The principal-agent models of auditing have established a need for an auditor from purely informational requirements. Wilson suggests that game theory could also be used to investigate the influence of financial reporting requirements. For example, increasing the level of mandatory information disclosure may reduce the level of voluntary disclosure, with the result that the public becomes less informed under stricter reporting requirements. Such disclosure effects were later discussed in, for example, Dye [15]. Another suggested area of great potential interest is that of reputation, in which auditors may play sub-optimally in the short term to reap future benefits. Any model considering these reputation effects must be a multi-period model. Such models are either difficult to analyse or a trivial repetition of the single period game. To date no repeated game auditing model has been developed. Wilson also suggests that payoffs, rather than being set by the principal, could be jointly determined by the strategic interaction of the players. Such an approach lends itself to the analysis of strategic interaction between the auditor and manager.

Fellingham and Newman [16] develop the first model for analysing the strategic interaction between the manager (henceforth auditee) and the auditor during the audit. They mention the two existing methods of assessing audit risk. These are risk analysis, (such as the audit risk model) and modelling the audit using decision theory so that risk is only one element in the model. The former is often criticised for being incomplete and ad hoc whereas the latter suffers from implementation problems, although it is often considered better than risk analysis. One of the main flaws in decision theory is it's inability to take account of the auditee's behaviour, even though the behavioural influences of an audit (such as preventative sampling) have long been recognised. Because of this Fellingham and Newman suggest using game-theoretic models. They mention however that although conceptually more compelling, game theory is likely to prove just as intransigent in application as decision theory.

The model developed is a simple one, focusing on the auditor's ability to influence the auditee through the potential to observe his effort level before qualifying. The auditee first chooses an effort level to put into maintaining the internal controls. The auditor has a choice between observing the auditee's effort level (A_1) or not (A_2) before deciding whether to qualify. This gives six strategies for the auditor {A_2Q, A_2NQ, A_1Q/Q, A_1Q/NQ, A_1NQ/Q, A_1NQ/NQ} three of which are dominated (All of the A_1 test strategies apart from A_1NQ/Q). The auditee's effort level influences the chance of errors occurring and each player's costs are described as the expectation of the costs of 4 outcomes over the chance of errors occurring. They assume that for the auditor the cost of a correct audit opinion is zero. The auditee's cost is assumed to be zero if no error occurs and the auditor does not qualify (they point out that the estimation of these parameters is a major problem made worse by game theory since there are more costs to consider).

For some numerical examples the optimal solution involves mixed strategies whereas in decision theory a mixed strategy is never preferred. They mention that the only pure strategy that never occurs in equilibrium is A_1NQ/Q. In fact this can be proved if there is a positive cost associated with A_1. The next section of the paper investigates audit risk. It shows that the risk of type I / type II reporting errors depends critically upon both players' strategies. Decision theory is incapable of accommodating this observation. Fellingham and Newman point out that even this simple model captures the strategic interaction of an audit, is consistent with behavioural hypotheses regarding the influence of an audit and is consistent with observed audit phenomena. As areas for future work they suggest games of incomplete information and the development of optimal sampling strategies (although this is very difficult).

Nadeau [29] develops the model of Fellingham and Newman by considering a stage of substantive testing. The auditor therefore has three stages to his strategy. Firstly he chooses whether to observe the auditee's effort level (A_1) or not (A_2). Then, based on the results of the "A-test" he chooses a high (B_1) or low (B_2) level of substantive testing to actively search for errors. Finally he chooses whether or not to qualify his report based on the findings of his substantive testing.

The equilibrium behaviour of this model is used to consider various policy implications. By varying the cost of the outcome when an error goes undetected, Nadeau investigates the implications of raising (or lowering) each parties responsibility for limiting undetected errors. This can be used to consider the potential influence of a supervisory body setting these costs to encourage socially desirable behaviour. A second mechanism for encouraging desirable behaviour is a factor that can reduce the auditor's costs for failing to detect an error if he can show due diligence. This encourages the auditor to work hard to earn a discount factor in the event of subsequent litigation. In this model the discount is awarded if the auditor observes the auditee's effort level (the A-test).

Any regulatory influence to encourage certain behaviour will depend upon what is deemed "socially desirable". This in turn depends upon society's view of the auditing function. The most obvious desirable outcome is for both parties to work hard; the auditee reducing errors and the auditor detecting those that do occur. An interesting alternative is considered where the auditor does not have to work hard. He is considered to have fulfilled his obligation if his presence influences the auditee to work hard. The model is considered as both a cooperative and a non-cooperative game. It is shown that for some levels of costs the optimal behaviour is the same in both the cooperative and non-cooperative frameworks.

3.4 Auditing in a Cooperative Framework

Demski and Swieiringa [11] argue that the interaction between the auditee and auditor should be viewed in a cooperative setting. They put forward two observations to support this argument; Firstly the audit fee structure is cooperatively agreed upon (which effectively allows side payments between the players). In fact the negotiations over the fee structure can also be regarded as a mechanism for pre-play communication and commitment. Secondly, the legal responsibility for failing to detect errors will be shared between both parties. The player's motivations will therefore be similar since each wishes to limit the risk of errors going undetected.

The authors develop a model where some monetary outcome is the consequence of both the auditee's private action and some unobserved chance event. The auditee is required to issue a report of this monetary value. There are a choice of reporting methods, each of which introduce errors into the reported level. The auditee is obliged to use an acceptable reporting system and to provide a report with a tolerable level of errors. The auditor's responsibility is to provide independent certification that the auditee has fulfilled these two obligations. The auditor therefore chooses a final reporting scheme dependent upon the level of errors, the monetary value and the auditee's report and reporting method. It is then shown that if both players have similar utility functions and risk preferences and given a Pareto optimal fee structure, the players can be regarded as a single individual. This reduces the audit problem to a single-person Bayesian decision problem.

Hatherly, Nadeau and Thomas [24] consider a model of major directional choice in a cooperative setting. The model analysed is taken from Nadeau [29] where the auditee has a choice between high and low effort and the auditor can observe this effort, conduct substantive testing and subsequently qualify his report. The audit fee is the motivation for using a cooperative framework as it provides the necessary means for negotiation and pre-commitment. The model is used to consider the implications of setting costs to encourage certain behaviour. The penalties for failing to detect a material error are therefore not set with compensation in mind. The players' optimal behaviour as these penalties vary is then considered. The idea of using the fee schedule to enforce agreed behaviour has some interesting implications for the auditor/manager relationship. If, for example, the auditor can commit to a certain audit opinion before any testing is carried out his independence is called into question

In a cooperative setting the costly observation of the auditee's effort level is never optimal. If the auditee makes a binding commitment to put in high effort then any additional expenditure to "check up" on this action is wasted.. A second version of the model is analysed in which there is concern that the auditee might shirk after committing to a high effort level. It is therefore considered socially desirable for the auditor to observe the auditee's effort level. To encourage this Nadeau's [29] discount factor is introduced. This reduces the auditor's responsibility for undetected errors if he has shown due diligence, in this case the use of the A-test. This means that, in effect, the auditor can "buy" the discount by using the A-test which has no influence on the audit outcome in a cooperative setting.

3.5 Hypothesis Testing Models

Newman and Noel [30] develop a model in which the auditor's testing is considered to generate a noisy signal of the account balance. They consider the impact of changes in policy variables (cost of outcomes) on a range of results such as the probability of errors going undetected. This involves finding the equilibrium of the model and seeing how it changes as the payoff functions change. The auditee's strategy is to choose the level of material errors in a reported account $\{\theta_0 = NM , \theta_1 = M\}$ where $\theta_1 > \theta_0$. This choice reflects the situation where there is a material error in the account and the auditee must decide whether or not to go to the trouble of eliminating it. The auditor observes an imperfect signal from a normal distribution with mean θ_i. The auditor's choice is between accepting or rejecting this reported balance of the account which will be influenced by the auditee's action. It is assumed that the actual account balance is probabilistically revealed in a subsequent period (a similar assumption to the ex-post public information variable in Baiman, Evans and Noel). This gives 4 payoffs depending upon accept / reject and M / NM for each player.

A partial ordering is imposed on the player's payoffs to reflect their preferences. The auditor prefers to qualify if a material error is present whilst if there is not an error the auditor would prefer to not qualify. The auditee has the opposite preferences - so he benefits if an error goes unqualified and (unusually) if there is not an error he would prefer the report to be qualified. Furthermore it is assumed that if the audit report is to remain unqualified the auditee would prefer not to go to the trouble of removing the error, which means that the auditee doesn't have a dominant strategy. With these orderings the players' interests are clearly at odds. A critical value is developed for the reported balance - if this report is above a certain value the auditor rejects it. This value depends upon two factors. Firstly, the critical value depends on the auditor's prior beliefs about the auditee's strategy. Secondly the value depends on the ratio, L, of the benefits of not qualifying when there is no error over the benefits of qualifying when there is an error:

$$L = (NQ(NM) - Q(NM)) / (Q(M) - NQ(M))$$

The game is shown to have a mixed strategy equilibrium. The effects of changes in the payoffs on a range of results, such as the probability of a material error going unnoticed, are considered. These results are then compared with the predictions of a decision theoretic model. The two theories predict different reactions to an increase in L on the overall chance of rejection. Game theory says that there should be a positive relation, whereas decision theory predicts a negative effect. The authors suggest an empirical method could be used to determine which paradigm is more accurate. Newman and Noel mention several limitations of the model; The auditee is limited to two strategies, the auditor cannot issue a report without gathering evidence (although he is highly unlikely to do so in practice) and, more importantly, cannot vary the amount of testing done.

Fellingham, Newman and Patterson [17] overcome some of the limitations of the model developed by Newman and Noel by investigating sampling in a game theoretic setting. Classical Bayesian statistical techniques depend crucially on the idea that the environment is unconcerned with the outcome of the sampling. This is clearly not the case in auditing. The authors mention several ways in which the strategic effects of sampling have long been considered. For example, it was recognised as early as 1933 that sampling plans cannot be too rigidly systematic since a dishonest auditee could use this to his advantage. Also mentioned is the technique of preventative sampling, in which the impression is given that no area will be audit free (similar to a randomised sampling strategy).

A basic model is developed with similar assumptions to the previous paper about the relative sizes of the players' payoffs. The auditee chooses an error rate or "amount to divert" from a range of values (which improves on the situation where the auditee has only 2 strategies). The auditor then decides to accept or reject the accounting reports. Clearly since the auditee prefers a low error rate if rejected and a high error rate if accepted (whilst the auditor prefers the opposite) this game will have a mixed strategy equilibrium. This basic model is augmented by the inclusion of a signal observed by the auditor (as a result of substantive testing). The auditor's strategy in this model becomes a partition of the set of possible signals into "accept" and "reject" regions.

The authors consider four different signals covering perfect / imperfect and costly / free signals. They show that if the signal is costless and perfect then the auditor will always acquire information. If the signal is imperfect but costless then the auditee will never play a pure strategy in equilibrium because, as far as the auditee is concerned, the auditors response will randomise between accept and reject. If the signal is perfect and costly it will only be used if it is not too costly. Finally the case of costly imperfect information is considered. If the auditor decides to gather more information (through attribute sampling) he can choose the sample size n and a rejection strategy based upon the number of errors found. This leads to a huge number of potential strategies so attention is restricted to trigger strategies of the form "reject if the number of errors is greater than c* ". A numerical example is given where the auditor's optimal strategy involves randomising between two sample sizes. This suggests that such basic assumptions as the existence of an optimal sample size may not hold in a strategic environment. The authors suggest several avenues for future research including games of timing, of incomplete information and the division of the auditors tests into tests of controls and substantive tests.

Shibano [34] uses game theory to develop the theory of audit risk with imperfect audit technology. He identifies two types of risk; nonstrategic audit risk (NSAR) involving errors and strategic audit risk (SAR) involving fraud or irregularities. Recent statements on auditing standards from the American Institute of Certified Public Accountants increase the auditor's responsibility to include both error and fraud detection. SAS 53 recognises that audit procedures that are effective for detecting unintentional misstatements may be ineffective against those that are intentional. Shibano argues that a new testing theory is needed to account for these changes in the role of the auditor. He develops two models; one of testing for a hidden action and the other of testing of a report of hidden information.

In the first of these models the auditor is testing an exogenously determined null hypothesis. The auditee has an unobserved action, which is considered to be effort put into the internal control system in order to assess control risk. The auditor then observes a signal generated by his testing and decides whether or not to accept the null hypothesis. This evidence comes from a general distribution, rather than assuming that the evidence is, say, normally distributed. Thus the models are robust across a large range of populations. The solution concept used is Bayesian-Nash equilibrium. In the second setting the true account balance is determined probabilistically. The auditee observes this balance and chooses a balance to report to the auditor. Thus the auditee effectively chooses the auditor's null hypothesis. The auditor then observes a testing signal and decides whether or not to accept the null (i.e. the auditee's reported balance). This formulation is an advancement on previous models as it explicitly includes intentional misstatements. Since the actions occur in a strict sequence the solution concept used is sequential equilibrium.

Shibano then uses the existing audit risk formulation, AR = IR×CR×DR where the three terms are respectively inherent risk, control risk and detection risk. This risk assessment is enriched by distinguishing between the two situations given above. This gives expressions for both NSAR and SAR:

$$NSAR = IR^e \times CR^e \times DR^e$$
$$SAR = IR^i \times CR^i \times DR^i$$

There are therefore six risk components to consider, three for error audit risk and three for irregularity audit risk. Shibano assesses each of these components in the setting that he considers most appropriate. For example both CR^e and CR^i are assessed using the hidden action model. However, in order to make these assessments a number of simplifications are made. It is assumed that the internal control system operates after the auditee generates an irregularity (and that this system cannot be overridden by the auditee). In practice however any sufficiently large fraud must be able to do exactly this. To deal with the possibility that accounts may be affected by both errors and irregularities it is assumed that for irregularity-prone accounts the auditee knows the true account balance. It is also assumed that evidence gathered during tests of controls is uninformative during substantive testing.

As mentioned above, a particular interpretation of the hidden action model is used to derive control risk for both SAR and NSAR. Both inherent risk and detection risk for SAR are assessed using the hidden information setting. For NSAR inherent and detection risk are assessed using decision theory. This presents the auditor with two means of assessing audit risk. The choice of which to use will depend upon the auditor's judgement - if he believes that intentional misstatements are possible he should use the SAR model. However, this could lead to a further component of audit risk, namely the risk of incorrectly assessing the audit situation.

3.6 Models of Fraud Detection

Matsumara and Tucker [27] develop a model that is motivated by increases in the auditor's responsibility for detecting and reporting irregularities (including fraud) and illegal acts. Their approach is unique for two reasons. Firstly in this model the auditor designs an audit strategy to explicitly test for fraud. Secondly the analysis of the model uses both game theory and economic experimentation. The effects of 4 variables on the model are investigated - the auditors penalty for failing to qualify fraud, the requirements of auditing standards, the structure of quality control and the audit fee.

In this model unintentional errors are normally distributed with mean μ and variance σ^2, a distribution which is known to both players. The auditee's only choice is whether or not to commit fraud. If he chooses to do so this introduces more errors, $\mathbf{t}$, into the population. Thus after a fraud the errors are normally distributed with mean $\mu + \mathbf{t}$. The auditor has two tests. Test 1 is a test of transactions which can verify the percentage of errors in the population, after fraud has (or has not) occurred. Test 2, detailed tests of balances, can detect fraud. Two levels of this test, corresponding to large / small sample size, are available. Test 2 cannot give a false positive as it is assumed that the cost of avoiding such false accusations is included in the cost of the test. The auditor has three potentially optimal strategies {no test1/large test2, no test1/small test2, test1(if high % errors then large test2 / if low % errors then small test 2)}.

The auditee has a pure equilibrium strategy if the auditor doesn't use test 1 :- NF against a small test 2 sample size or F against a large test 2. If the auditor uses test 1 and a rationally conditional test 2 then the auditee is fraudulent with a probability that maximises his expected return. The solution concept used is Bayesian-Nash equilibrium. Where there are two equilibria the authors assume that the auditee plays to the equilibrium he prefers. From a numerical example the authors note that increasing the auditors penalty for failing to find fraud decreases the incidence of fraud. This is presumed to be because the auditee anticipates an increase in fraud detection on behalf of the auditor. An increase in the minimum testing requirements for test 2 effectively reduces the difference between large and small sampling sizes. This in turn means that test 1 is less important. If test 1 is used, the threshold value for large test 2 sample sizes increases and so the minimum level of testing occurs more often.

The model represents internal control as the percentage of clerical errors. The influence of the internal control system in the model is examined by considering 2 cases. For a high level of internal controls, test 1 is more effective as fraud is more likely to be noticed if there are few random errors. There are therefore more equilibria involving the use of test 1. Conversely with a poor level of internal controls test 1 becomes less effective and the auditor resorts to the more detailed (large sample) test 2. The effect of the size of the audit fee is also considered although this should make no difference to risk neutral players with complete information about utility functions in a game theoretic setting. However, it is considered because the model was also analysed through a multi-period experiment (and it is well known that the assumptions of rationality can lead to some unsatisfactory predictions in repeated games). In the experiment, an increase in the audit fee led to an increase in sampling and the percentage of fraud detected in the early games of a sequence, but towards the end of the repeated game the players did the opposite. This is perhaps due to end-game effects (a reputation for being "tough" is more valuable with more periods left to play) although the players were uncertain about exactly how many periods would be played. The differences between actual behaviour and that predicted by theory suggest that this could be an interesting area of future research.

In Patterson [33], Shibano's [34] "hidden-action" model is extended to include the auditor's sample size choice. For tractability it is assumed that the evidence is normally distributed. This assumption was also made in Newman and Noel [30], although Shibano mentions that this assumption may not be reasonable when taking a small sample from a skewed population. Also in contrast to Shibano, the control risk in this model is assessed to be 1, in other words the auditee can override the internal control system.

The hidden action in this model is assumed to be a choice of intentional material error arising from defalcation (an action that reduces asset value). The auditee chooses the mean of the sample evidence where {w_1=immaterial error, w_2=large error arising from defalcation}. A mixed strategy for the auditee is identified by a defalcation rate, the probability of choosing w_2. The auditor cannot distinguish between intentional and unintentional errors (an assumption that is supported by anecdotal evidence). He suspects fraud if the sampling error is large enough. Thus the auditor can be thought of as using variables sampling rather than discovery sampling. The auditor's strategy is a reporting decision rule similar to Newman and Noel [30] - reject if the sample error is greater than a critical value. However, in this model the auditor also chooses the sample size n. Having decided on the sample size n, the auditor is committed to performing all n samples, rather than using a more realistic but much less tractable Bayesian stopping rule. The game is equivalent to a simultaneous move game and thus the Nash equilibrium concept is used.

The players' payoffs are ordered in a similar way to previous models. In particular, the auditee prefers a material error if the auditor accepts and prefers no material error if the auditor rejects, whereas the auditor prefers to reject a material error and accept when there is no material error. These conditions are sufficient to ensure that neither "accept" nor "reject" can form a pure strategy equilibrium. Two conditions need to be satisfied for sampling to be worthwhile. Firstly, the cost of sampling must be low enough that the benefit from sampling outweighs the additional cost. Secondly the auditee's behaviour must be suitably unpredictable (a mixed strategy). Upper and lower bounds for the auditee's defalcation rate can be determined for sampling to be worthwhile. Assuming these conditions are met one of three equilibria will occur.

(1) The auditor randomises between sampling as in (2) and accepting without sampling. The auditee has a low defalcation rate.

(2) The auditor always samples and chooses his sample size and reporting rule to maximise his payoff. The auditee's defalcation rate lies between that in (1) and that in (3).

(3) The auditor randomises between sampling as in (2) and rejecting without sampling. The auditee has a high defalcation rate.

Patterson considers how changing the players' payoffs can affect the auditor's optimal sample plan, the auditee's defalcation rate and audit risk. As one would expect, increasing the auditee's payoffs for a material error increases the defalcation rate whereas increasing the payoffs for no material error decreases the defalcation rate. The sample size is effected by uncertainty - if a low defalcation rate gets lower the sample size decreases whereas if a high defalcation rate decreases the sample size increases. However since the defalcation rate lies strictly between 0 and 1 the audit risk cannot be eliminated. There is an increased interaction between the players in this game, it is pointed out, since a change in either players' payoffs results in a change in both equilibrium strategies.

Hansen [22] uses a different approach to motivate his model of the audit. He examines a more specific area (accounts receivable) so that he can analyse the "micro" characteristics of the items being tested, such as the size of the account or the error distribution. He assumes that the testing is being conducted by an internal auditor so that the incentives of the management and auditor are equivalent. He thus focuses attention on the strategic use of testing to deter employee fraud. He considers the auditor's optimal sampling strategy in three different settings.

Firstly, a decision theoretic model of non-strategic error detection is developed. The model considers a firm trying to prevent billing errors in N items (of differing value) of accounts receivable. There is a probability for each item that an error will occur in processing the bill - which can result in either over or under-billing the customer. It is shown that there may be no consistent link between the likelihood of testing and item value if the error is unrelated to the item value. If however there is such a relation (if for example the size of the error is a percentage of the value) then the firm will always test the higher value items.

The second setting includes the potential for employee fraud. Each item is processed by a different employee who has a non-zero chance of being dishonest. After any unobserved random billing error has occurred, a dishonest employee can introduce further reductions into the bill (for which he receives a percentage "kickback" from the customer). The auditor must therefore conduct testing to prevent both random and deliberate billing errors. If the auditor uses a mixed testing strategy this can look like Stratified Physical Units Attribute Sampling (in which the items are grouped according to value and each item in a group has the same chance of being tested).

The third setting considers a second mechanism for employee fraud. In this case the employee awards unearned discounts to his confederates (and subsequently gains a share of the profits). This differs from the previous setting as the amount stolen is a percentage of the bill's value. In this case the auditor's testing strategy resembles Dollar Unit Cell Width Sampling. The firm never tests small items, always tests large items and tests medium items with a probability proportionate to their size. The dishonest employee's strategy is similar - he always steals from the lowest value items because he will not get caught, he always steals from the highest value items because of the potential profit and he steals from intermediate value items with a probability that decreases in the value of the item.

Although the auditor's optimal strategy can resemble recognised sampling plans, the randomisation serves a different purpose in the model. In practice firms use statistical sampling in response to limited time and resources. In the model a mixed strategy is the least-cost way of limiting employee fraud. The auditor chooses a mixed strategy without any budget constraints. Furthermore, in practice statistical sampling is only used in large populations. In the model randomisation may take place even with a single item to be tested. This suggests that randomisation could be used in practice to deter theft.

3.7 Discussion

This work develops and analyses two models of the audit. The audit is considered to have a fundamental information asymmetry in so far as the auditor cannot know the auditee's motivations exactly. The audit risk model includes this in the inherent risk term which recognises that the basic chances of errors occurring (or the risk of fraudulent activity) differs between companies. This uncertainty can be modelled in a multi-person decision setting by regarding the audit as a game of incomplete information. In particular, since the auditor may infer the auditee's motives from his actions the audit will be modelled as a signalling game, described in section 2.5.

Chapter 4 develops a signalling game model of the audit as a means of error detection. The model is then used to consider the possible policy implications of varying the cost of some of the outcomes. Chapter 5 extends the analysis of the model to include mixed strategies and Pareto dominated equilibrium pairs. The model is also considered in a cooperative setting and a situation of proportionate liability. Chapter 6 develops a second, similar model that considers the occurrence of both unintentional and deliberate irregularities. The equilibrium set is characterised by considering the cost of the players' actions to be variables. This also has potential policy implications if the cost of actions are influenced by an external regulatory body.

The models to be developed below differ from the hypothesis testing models of auditing as they take a view of the audit, first suggested by Fellingham and Newman[16] , in which the players have major directional choices. One reason for this is that the strategic interaction between the players can be more clearly demonstrated with simple choices of actions. Returning to simple strategic choices may appear to be less realistic than regarding the auditor's actions as a hypothesis test. However, Duke et.al [12] argue that hypothesis testing may not be suitable for audit sampling because accounting populations tend to have low error rates and a highly skewed distribution. In a model of the audit where the auditor uses hypothesis testing, assumptions must be made either about the distribution of errors, or about the distribution of the testing report. By considering the audit in a more stylised setting we need only assume that the expected chance of a material error occurring (regardless of the distribution) is known.

The idea of inherent risk is included in the models through the auditor's uncertainty about the basic chance of errors occurring. This uncertainty is modelled by a probability distribution which determines, before play begins, the likelihood of errors (or irregularities) occurring. The error detection model developed in chapter 4 is the first of the simple directional choice models to have costly pure strategy information gathering strategies in equilibrium. It can be shown that in a game of perfect information, costly acquisition of information cannot form part of a pure strategy equilibrium. In a signalling game however, the auditor's beliefs about the other player's actions must be considered as part of the equilibrium and information acquiring strategies can form a pure strategy equilibrium.

A signalling game can be distinguished from a game with an observed signal such as the "hidden action" hypothesis testing models. Both models have imperfect information since an event occurs which is not observed by the auditor. In the hypothesis testing models however the auditor's testing generates a signal which is, in part, determined by the hidden chance event. In a signalling game the auditor has no direct means of assessing the result of this unobserved event. He must instead consider how the outcome of this unobserved event will influence the actions of the auditee and by observing these actions infer the outcome.

The model developed in chapter 6 considers the audit to be a means of detecting and preventing both intentional and unintentional errors. This provides an insight into the strategic interaction between a potentially dishonest auditee and the auditor against a background of random errors. This allows an increased interaction between the types of error and the auditor's opinion. For example, the auditor may incorrectly classify the results of fraud as merely a random error. The model also provides an alternative approach to the auditee's ability to override the internal control system. The auditee can do so (and must to conceal fraud) but there is a cost associated with such an action. This cost will depend on the level of internal controls in use. It is also assumed to differ between companies and the auditor will not know the level of this cost when planning the audit.

This model differs from previous work in the assumptions made about the players' motives. Patterson [33] , for example, assumes that the auditee successfully conceals the fact that an irregularity was intentional. In such a setting the only deterrent is that introducing further errors into the system increases the risk that the auditor will qualify his report. The model developed here gives the auditor a choice between qualifying for material error or qualifying with evidence of fraud. In such a setting the auditee can actually be "caught" having committed fraud. Matsumara and Tucker [27] consider a setting in which the auditee can be caught, although they assume that the auditor is rewarded for successfully detecting fraud (the auditor's "best" outcome is qualified fraud). In the model developed in chapter 6, successful fraud detection can be costly as it may lose the auditor future custom. The auditor's motivation for fraud detection is that, given fraud has occurred, non-detection may be even more costly.

Previous models in a fraud setting have been based on a "variables-sampling" approach in which the auditor's qualification decision is the result of hypothesis testing and his substantive testing is considered to generate a signal about the account balance. In such models the occurrence of random errors is modelled by considering a distribution (often normal) of errors which is known to both players. The occurrence of fraud creates additional errors which effectively shift the mean of this distribution. However, neither player's payoffs are affected by the occurrence of these errors. The model developed here is the first to recognise that the auditor has a responsibility to detect both kinds of error, and that these two requirements may sometimes be at odds. The auditor must therefore strike a compromise between fraud and error detection.

4 A MODEL OF ERROR DETECTION

4.1 Introduction

This chapter develops and analyses a signalling game model of the auditing process. The audit is considered here to be a report on whether or not the financial statements contain an unintentional material error. The issue of strategic interaction when the auditee has the ability to commit fraud is considered in chapter six. The sensitivity of the model's equilibrium pairs to changes in some of the payoffs is considered by regarding some of the costs to be variables. This permits a categorisation of the set of potential equilibrium pairs as these costs vary. In a similar approach to Nadeau [29] this can then be used to consider the policy implications of externally setting these costs to encourage certain behaviour.

The next section outlines a model of the auditing process. This describes the actions available to the participants and the sequence in which they occur. It also details how these actions interact to lead to one of four outcomes. The players' utilities from these outcomes are ordered (in terms of costs) to reflect the preferences of the auditor and manager. Section 4.3 takes the framework developed in 4.2 and expands it into a game theoretic model. This involves considering the information available to the players during the game and how they deal with any uncertainty. A number of the auditor's strategies are shown to be sub-optimal and can therefore be removed from any equilibrium considerations.

Section 4.4 analyses the auditor's cost structure in more detail. This leads to a classification of the auditor's optimal actions as his variable cost increases. Formulae are given that can generate all the necessary inequalities for categorising the equilibrium set. Section 4.5 then outlines a method for determining all the pure strategy equilibria as the two costs vary. A numerical example is considered to illustrate the characterisation of the equilibrium set. The example is used to consider how the changes in costs can influence the participants behaviour.

4.2 A Model of the Auditing Process

The "inherent risk" term of the audit risk model recognises that the chance of errors occurring varies from company to company. However, the assessment of this term can prove difficult. Dunn [13] for example states that: "The evaluation of inherent risk is the most demanding aspect of audit planning". Some of the factors that influence the inherent risk, such as the nature of the business and the levels of safeguards to prevent errors, can be observed by the auditor but others, such as the management's influence over the occurrence of errors, cannot. This is recognised in a research study into the extent of audit testing by the Canadian Institute of Chartered Accountants [10] which states: "The assessment of the system must take into consideration the degree of management influence,...... to the extent that this is determinable". The game theoretic papers to date do not consider the strategic effects of inherent risk in their models of the audit. This model incorporates inherent risk by considering the audit as a signalling game.

The two parties that have a primary interest in the outcome of the audit as well as the ability to directly influence it, are the auditors and the directors of the company. For this model each of these parties will be regarded as a single player since we cannot distinguish the motivations of individual members of each of these groups. For this analysis we will refer to the players as the auditor and the auditee.

In an attempt to reduce the frequency of errors in the financial statements measures are taken to prevent them as well as to catch them if they do occur. All such measures taken by a company are collectively referred to as internal controls. Clearly, the standards of internal controls in a company is of great interest to the auditor when planning an audit. The auditor's degree of certainty that he has not missed an error will depend on these internal controls and the amount of testing he carries out. If the internal controls are effective then the auditor needs to do less testing to achieve the same level of certainty. However, more effort is required by the auditee to instigate and maintain high levels of internal controls. This is the seed of the strategic interaction; Both players have incentives to detect or prevent any random errors but each player would prefer the other to do the necessary work.

The chance of errors occurring varies between companies. Some of the factors that influence the rate of errors occurring, such as management influence, cannot be seen by the auditor. The auditor will therefore have some uncertainty about the inherent chance of errors occurring. This uncertainty is modelled by a probability (known to both players) which determines the basic chance of errors occurring before play begins. In this model the auditee will be categorised into two types, where type 2 has a greater basic chance of errors occurring. The auditor's uncertainty about the risk of errors occurring is equivalent to an uncertainty about which type he is facing. The assumption that both parties know the probability of a type 2 auditee occurring means effectively that both parties know the percentage of businesses which are error-prone.

The auditee chooses how much effort to put into maintaining the internal control system. The auditor may regard this effort level as a signal and attempt to infer the auditee's type from it. The auditor's actions during the audit can be divided into three sections - observation, testing and qualification. Firstly, the auditor can choose to observe the standards of internal controls in the company, an action first considered in Fellingham and Newman [16]. To determine this will require some effort on the auditor's behalf. This decision is equivalent to a choice of tests (henceforth A-tests) which can reveal the auditee's effort level. Having considered the effectiveness of the internal controls the auditor can resort to substantive testing of accounts, that is to say searching for errors himself. The auditor's strategic decision here can be represented by a choice of tests (B-tests) of differing effectiveness.

Finally, the auditor will issue a report to the shareholders detailing his findings. The shareholders main concern here is whether the auditee's financial statements have been prepared in accordance with standard accounting procedures. If the auditor finds evidence of errors he may choose to qualify his report, a choice of accept/reject in Newman and Noel [30]. A qualified report is considered by the shareholders to be a refusal to issue an "all-clear" report. The sequence of events and actions can be summarised as:

1) Nature determines the auditee's type
2) The auditee chooses the effort to put in to maintaining the internal controls
3) The effort level influences whether an error occurs or not
4) The auditor can observe the effort level chosen in (2)
5) The auditor conducts substantive testing to detect errors
6) The auditor issues a report, which may contain a qualification, based on the results of his testing.

The strategic interaction can be seen clearly when each of the players has simple choices. The model developed here has the same strategy sets as Nadeau [29] although the informational structure is different. For the auditee this means a choice between high and low effort or $w \in \{H, L\}$. For the auditor there are choices between A_1 (observe effort level) or A_2 (don't observe) followed by a choice of tests B_1 (extensive substantive testing) or B_2 (limited testing). The auditor's decision to qualify can be based upon the results of the B-test in four ways:

Q - qualify all the time
R - reasonably qualify (qualify if the B-test finds an error, don't qualify if no errors are found)
NQ - never qualify
UR - unreasonably qualify (qualify if the B-test finds no error, don't qualify if errors are detected)

There are two chance events during the course of the game, the chance of an error occurring and the chance of it being detected by the auditor. These are modelled by the following probabilities:

p_{iw} = P(Error | Effort level w, auditee type i)
r_k = P(B-test reports an error | Error, test B_k)
t_k = P(B-test reports an error | No Error, test B_k)

Here t_k represents the risk of a type I sampling error (false positive) and $(1-r_k)$ represents the risk of a type II sampling error (false negative). As B_1 represents a more extensive test we will assume that $r_1 > r_2$ and $t_1 < t_2$. The probability p_{iw} represents a combination of both inherent risk and control risk, as it is the probability that an error will occur and go undetected by the internal control system. This approach is supported by Holstrum and Kirtland [25] when discussing potential interdependence in measuring components of audit risk:

> "[I]t does not seem feasible, in most practical audit situations, to assess inherent risk separately from control risk. Rather, if we desire to recognise the different levels of inherent risk in different audit situations, we likely would find it more practical to estimate the probability that material errors would exist after being processed through the internal accounting control system".

With two auditee types and two effort levels there are four values for $p_{iw.}$ For each type i, $p_{iH} < p_{iL}$ if there is to be any benefit in the auditee putting in high effort. If we assume that the auditee's effort has more effect on the error rate than his type we have the ordering $p_{1H} < p_{2H} < p_{1L} < p_{2L}$. We would expect that the effect of high effort is dependent upon the basic error rate. To include this consideration it will be assumed that $(p_{1L}-p_{1H}) > (p_{2L}-p_{2H})$ so that high effort makes more of a reduction in error rate in a low error environment.

There are four basic outcomes to this game {NQ(NE), Q(NE), NQ(E), Q(E)}. These outcomes depend on the existence of an error in the accounts and whether or not the auditor chooses to qualify his report. Both players will have a cost associated with each of these results denoted by C for the auditor and D for the auditee. These costs include the loss of reputation concerned with the outcome as well as any financial losses. As in Newman and Noel [30], it is assumed that the actual state of affairs is brought to light at some subsequent period. Thus NQ(E) is the expected future cost to each player of the error being brought to light. An ordering can be imposed on each player's costs to reflect the preferences of each of the players.

(1) Both players prefer an outcome where there are no errors and no qualification.

(2) The auditee's reputation is damaged by a wrong qualification; $D^{Q}(NE) \geq D^{NQ}(NE)$

(3) This reputation is further damaged by a true qualification; $D^{Q}(E) \geq D^{Q}(NE)$

(4) An incorrectly qualified report may result in litigation; $C^{Q}(NE) \geq C^{Q}(E)$

(5) The auditor's reputation is damaged if his opinion is found to be wrong;

$$C^{Q}(NE) \geq C^{NQ}(NE) \qquad C^{NQ}(E) \geq C^{Q}(E)$$

auditor:	$C^{Q}(NE) \geq C^{Q}(E) \geq C^{NQ}(NE)$	$C^{NQ}(E) \geq C^{Q}(E)$
auditee:	$D^{Q}(E) \geq D^{Q}(NE) \geq D^{NQ}(NE)$	$D^{NQ}(E) \geq D^{NQ}(NE)$

Relations **(1)**, **(2)** and **(5)** are equivalent to those used in Newman and Noel [30]. There are also costs associated with working hard for either player. Since each players' choices are between a high effort level and a low effort level we can without any loss of generality assume that the cost of low effort is zero. There are three actions that require effort:

D_H - Cost to the auditee of putting high effort into the internal controls

C_A - Cost to the auditor of test A_1 $\qquad$ C_B - Cost to the auditor of test B_1

The cost of a particular error level and substantive test for either player will be an expectation over the four basic outcomes listed above and the chance events that an error occurs and is detected. This structure is repeated for both auditee types i, auditee effort levels w and B-test levels k. For the auditor this gives:

$$C(i,w,B_kQ) = C_{Bk} + p_{iw}C^{Q}(E) + (1-p_{iw})C^{Q}(NE)$$

$$C(i,w,B_kUR) = C_{Bk} + p_{iw}(r_kC^{NQ}(E)+(1-r_k)C^{Q}(E))+(1-p_{iw})(t_kC^{NQ}(NE)+(1-t_k)C^{Q}(NE))$$

$$C(i,w,B_kR) = C_{Bk} + p_{iw}(r_kC^{Q}(E)+(1-r_k)C^{NQ}(E)) + (1-p_{iw})(t_kC^{Q}(NE)+(1-t_k)C^{NQ}(NE))$$

$$C(i,w,B_kNQ) = C_{Bk} + p_{iw}C^{NQ}(E) + (1-p_{iw})C^{NQ}(NE)$$

4.3 A Game Theoretic Analysis

The auditor is uncertain about the basic likelihood of errors occurring in the accounts. Thus the model is one of (asymmetric) incomplete information. The actions of the two parties occur sequentially, giving a dynamic game. The auditor can choose not to observe the effort level. In this case he must choose an action without knowing his opponents action. There are also "chance moves" such as whether or not the B-test finds an error which neither player can observe. The model is a dynamic game of one-sided incomplete information where the informed player moves first - a signalling game.

The solution concept for signalling games, discussed in section 2.5, is Perfect Bayesian equilibrium. At first attention in the analysis will be restricted to pure strategy equilibria. In equilibrium, both players strategies must be a best response to the other's strategy. To find such mutually stable pairs we can separately consider how each player can act optimally for each strategy which their opponent might use.

Firstly let us consider the auditees' strategies. Each auditee type has 2 strategies H or L. Equivalently we can consider the auditee strategies before classification into types. Clearly in this case the strategy may be contingent upon type. Using the notation type1 / type2 action this gives four strategies; {L / L, L / H, H / L, H / H }

If the auditor does not observe the auditee's action (A_2) he has 8 tests of the form{B_1 or B_2} followed by {NQ, UR, R, Q}. If the auditor does observe the effort level his subsequent actions can be contingent upon this observation giving 64 tests of the form A_1(X / Y) where X is one of the 8 tests above to be played after observing H and Y is similar after observing L. If the auditor uses A_2 and hence does not observe the auditee's action he can only be in a single information set. Alternatively, if the auditor does observe the auditee's action the auditor will be at one of two information sets. Thus the tests A_2X and A_1(X / Y) described above satisfy the criteria for a strategy outlined in chapter 2. Many of the auditor strategies are dominated.

Lemma 4.1 *If $r_k > 0.5 > t_k$ $\forall k$ then with the above ordering of costs the UR qualification strategy is dominated.*

Proof Assume that $r_k > 0.5 > t_k \; \forall k$ (1.1)

From above $C^Q(NE) \geq C^Q(E) \geq C^{NQ}(NE)$ and $C^{NQ}(E) \geq C^Q(E)$ (1.2)

Now consider $C(i,w,B_kUR) - C(i,w,B_kR)$ for some i,w, k

$= p_{iw}(2r_k-1)(C^{NQ}(E)-C^Q(E))+(1-p_{iw})(1-2t_k)(C^Q(NE)-C^{NQ}(NE))$

≥ 0 by (1.1) and (1.2)

$\Leftrightarrow C(i,w,B_kUR) \geq C(i,w,B_kR)$

Therefore qualification UR is dominated.

The condition $r_k > 0.5$ restricts our attention to B-test with at least a 50% chance of finding an error if one occurs. Similarly $t_k < 0.5$ gives tests with at least a 50% chance of reporting "No error" when none occurs. Clearly if these 2 conditions are not satisfied we can improve on the B-test by regarding each of its reports as the opposite (which is exactly what happens in the UR qualification strategy). We will therefore assume that for each test B_k $r_k > 0.5 > t_k$.

Lemma 4.2 *It is only worth using test B_1 with qualification strategy R.*

Proof Lemma 4.1 shows that the UR qualification strategy is never optimal. It remains to show that B_1Q and B_1NQ cannot be optimal actions. For any auditee type i and effort level w:

$C(i,w,B_1Q) - C(i,w,B_2Q) = C_B > 0$.

So B_1Q always costs the auditor more than B_2Q.

Similarly, $C(i,w,B_1NQ) - C(i,w,B_2NQ) = C_B > 0$.

So B_1NQ always costs the auditor more than B_2NQ.

If the auditor has decided to ignore the results of his testing (Q or NQ) there is no point in using the more expensive test. To find the equilibrium pairs of this game we will need to compare strategies to determine which are optimal. This task is simplified if the cost of each outcome is expressed in a standardised form. This can be done by considering the auditor's substantive testing and subsequent qualification as one test using the following observations:

(a) The Q strategy (always qualify) is equivalent to a B-test with

100% risk of a type I (false positive) error

0% risk of a type II (false negative) error

(b) The NQ strategy (never qualify) is equivalent to a B-test with

0% risk of a type I (false positive) error

100% risk of a type II (false negative) error

(c) The R qualification strategy does not change the risks of type I or type II error for either B-test

Thus the probabilities r and t can be thought of as functions of the auditor's substantive testing and qualification strategy. The definitions of section 4.2 can be extended to give:

$r_k^*(B_kQ_n)$ = P(Audit report qualified | Error, test B_k and qualification strategy Q_n)

$t_k^*(B_kQ_n)$ = P(Audit report qualified | No Error, test B_k and qualification strategy Q_n)

$r^*(B_2Q) = 1$	$r^*(B_2R) = r_2$	$r^*(B_1R) = r_1$	$r^*(B_2NQ) = 0$
$t^*(B_2Q) = 1$	$t^*(B_2R) = t_2$	$t^*(B_1R) = t_1$	$t^*(B_2NQ) = 0$

The auditor's cost function can now be expressed as:

$$C(i,w,K)=C_{Bk}+p_{iw}(r^*(K)C^Q(E)+(1-r^*(K))C^{NQ}(E)) \qquad \textbf{(4.3.1)}$$
$$+(1-p_{iw})(t^*(K)C^Q(NE)+(1-t^*(K))C^{NQ}(NE))$$
$$\text{where } K \in \{ B_2NQ, B_2R, B_1R, B_2Q \}$$

Lemma 4.3 *An auditor strategy $A_1(K / K)$ is dominated if $C_A > 0$.*

Proof Strategy $A_1(K / K)$ costs C_A + C(i,w, K) for each effort level w whereas strategy A_2K costs C(i,w, K). Clearly if $C_A > 0$ strategy $A_1(K / K)$ will cost more after either effort level. Therefore A_2K dominates $A_1(K / K)$.

In other words, there is no point in spending time and effort to learn the auditee's action if you are not going to use the information. The auditor therefore has 4 tests of the form A_2K and 12 tests of the form $A_1(K_H / K_L)$ where $K_H \neq K_L$ and K, K_H, $K_L \in$ { B_2NQ, B_2R, B_1R, B_2Q }.

The auditor's choice of optimal action will depend on his beliefs about the auditee's type and action since he does not know which type he is facing and he may choose not to observe the effort level. These beliefs are modelled as probability distributions over each of the auditor's information sets. The auditor's expected cost for a given strategy will depend on these beliefs. With 2 auditee types each with 2 identical actions the auditor's information set will have four nodes {1H, 1L, 2H, 2L} where 1H refers to a type 1 auditee playing H. If we draw the decision tree, the auditor is at one of four sub-trees, but he doesn't know which one. This information set will be referred to as the primary set since this is the state that the auditor finds himself in initially.

As the game progresses, the auditor may gather information (he may observe the auditee's action). Any subsequent information sets that are reached will be subsets of the primary information set. The beliefs for these secondary information sets can therefore be developed from the primary belief system. Whenever the auditor learns new information he finds himself, in extensive form terms, at a smaller information set. Initially the auditor does not know which of 4 possible states represents the actual state of play. If he observes the auditee's action he can rule out two of these states, that correspond to the auditee's other action. Thus the probability that he is at either of the remaining nodes will increase accordingly. This can be done using Bayes' rule :

$$P(A \mid B) = P(A \cap B) / P(B)$$

Bayes' rule satisfies the two basic criteria which we would expect of any method that we use to update beliefs. These are that the sum of the updated beliefs should be 1, and that the updated beliefs maintain the relative magnitudes which they had before the updating. So once the auditor has seen a particular action he allocates zero probability to the other action having been carried out instead. If the auditor feels that a type 1 auditee is twice as likely to put in high effort than type 2, then after observing high effort he feels it is twice as likely that the auditee will be of type 1. There are two methods of describing the primary belief system. Firstly we can simply assign a probability to each node in the information set. This would mean having 4 probabilities where for example:

$p_1 = P(\text{ type } t_1 \text{ H effort})$ $\qquad$ $p_2 = P(\text{ type } t_1 \text{ L effort})$

$p_3 = P(\text{ type } t_2 \text{ H effort})$ $\qquad$ $p_4 = P(\text{ type } t_2 \text{ L effort})$ $\qquad$ and $\sum_{i=1}^{4} p_i = 1$.

However, the primary information set is caused by uncertainty about two separate "events" that occur in sequence; the determination of the auditee's type followed by the auditee's action. Rearranging Bayes' rule gives:

$$P(\text{type and action}) = P(\text{action}|\text{type}) \times P(\text{type})$$

This suggests a second belief system concerned with each event separately. The advantage of this belief system is that it emphasises the difference between the auditor's belief about the auditee's type and his action. Define the following probabilities:

$P = P(\text{auditee is type 1})$ $\qquad$ $S_1 = P(H \mid \text{type 1})$ $\qquad$ $S_2 = P(H \mid \text{type 2})$

The following expressions will enable us to update P if the auditor chooses to observe the auditee's action:

$$P(\text{type } t_1 | H) = PS_1 / (PS_1 + (1-P)S_2)$$

$$P(\text{type } t_2 | H) = (1-P)S_2 / (PS_1 + (1-P)S_2)$$

$$P(\text{type } t_1 | L) = P(1-S_1) / (P(1-S_1) + (1-P)(1-S_2))$$

$$P(\text{type } t_2 | L) = (1-P)(1-S_2) / (P(1-S_1) + (1-P)(1-S_2))$$

There is a recursive element to any beliefs about the other player's actions. Let us suppose that the auditor can in fact predict precisely the auditee's strategy and weights his beliefs accordingly. If the auditee knows this then he may wish to change his strategy, in which case the auditor will change his beliefs and so on. Any equilibrium of this game needs to consider beliefs as well as strategies - in an equilibrium the auditor will know the auditee's strategy and thus will have "correct" beliefs but neither player will have an incentive to change.

Extending the equilibrium concept to beliefs offers a solution to the problems caused by imperfect information. In games of perfect information, we do not predict how the game will be played. Instead we look for mutually stable strategy pairs - where each player's strategy is a best response to his opponents. These stable pairs act as convenient focal points for rational players. Similarly, for signalling games we look for stable triplets of strategies and beliefs. We do not develop a method for choosing the subjective beliefs, rather we consider which beliefs would support a particular equilibrium. Thus we effectively create an "optimal strategy" function from belief systems to subjectively optimal strategies. This enables us to find equilibria for the game without specifying an optimal belief system. The auditor's expected cost for a given strategy can now be expressed as an expectation over these beliefs. Instead of comparing costs to find the optimal strategy, the auditor will compare these subjective expectations:

$$E(C(K_H / K_L)) = P\,(S_1C(1,H,K_H)+(1-S_1)C(1,L,K_L)) \\ + (1-P)(\,S_2C(2,H,K_H)+(1-S_2)C(2,L,K_L)) \qquad \textbf{(4.3.2)}$$

With the reduction of the number of strategies the extensive form is now more tractable. In figure 4.1 the sequence of play starts in the middle (the move by nature) and proceeds towards the edges. The auditor's beliefs cover those actions which take place earlier in the sequence of play. The information sets are shown by dotted lines and the grey highlighting.

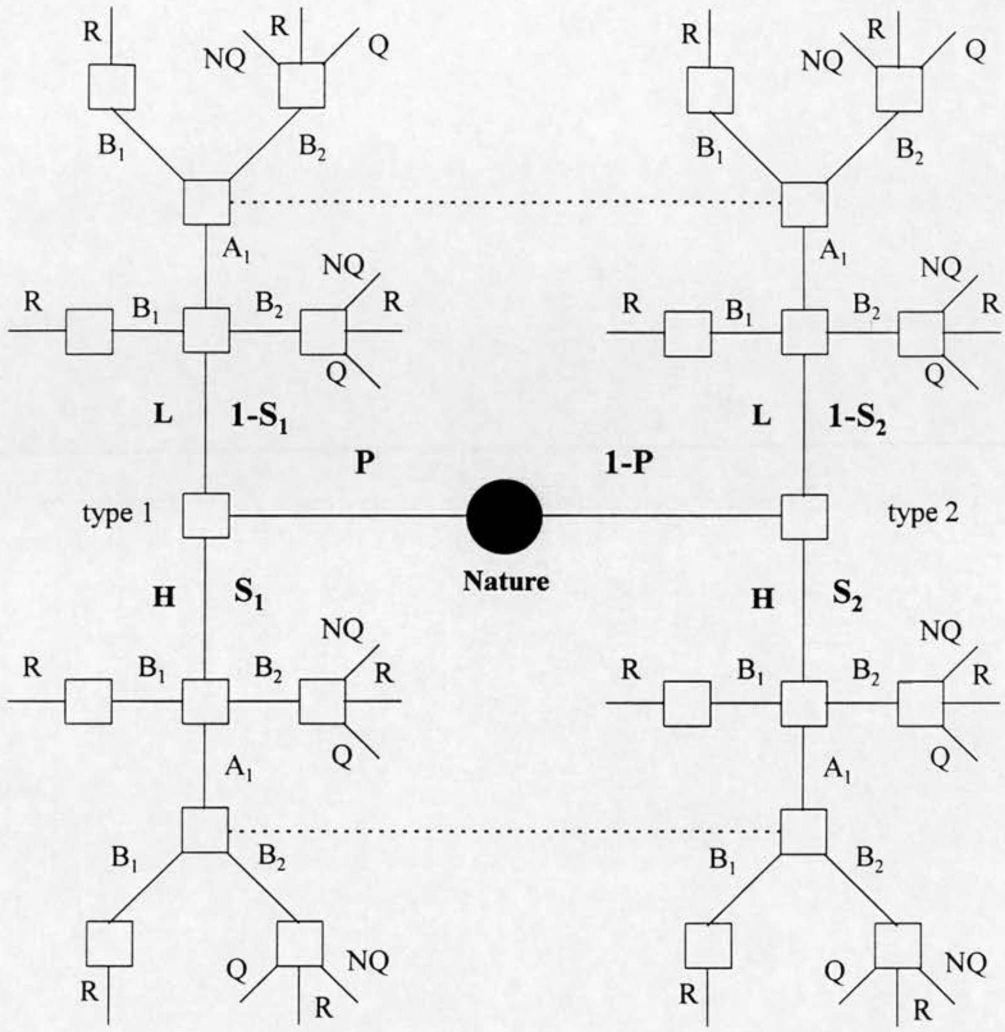

figure 4.1 - reduced extensive form

4.4 Equilibrium Pairs of the Signalling Game

Equilibria in signalling games are categorised by the sender's behaviour which can be considered by his opponent to be a signal of his type. An equilibrium in which both types send the same message is a pooling equilibrium. An equilibrium where each type sends a different message is a separating equilibrium. The auditor's initial belief about type, P = Prob(type 1), may be updated (to P' say) during the game through observation. However, if the auditor updates his belief using Bayes' rule, the values that P' can take are limited in the pure strategy case. Clearly, in a pooling equilibrium (where both types send the same signal) the auditor learns nothing from the signal and cannot therefore update his beliefs. Similarly, if the auditor chooses not to observe the signal he learns nothing and his belief remains unchanged.

Lemma 4.4 *In a separating equilibrium P' = 0, P or 1*

Proof If the auditor chooses A_2 he cannot update his belief and so P' = P. If the auditor uses A_1 there are 2 cases:

(1) **H / L** - t_1 plays H and t_2 plays L

If the auditor observes H then:

$P' = P(t_1 \mid H)$

$= P(t_1 \cap H) / P(H)$

$= P(H \mid t_1)*P(t_1) / P(H)$

$= 1 * P / P = 1$

If the auditor observes L then:

$P' = P(t_1 \mid L)$

$= P(t_1 \cap L) / P(L)$

$= P(L \mid t_1)*P(t1) / P(L)$

$= 0 * P / (1-P) = 0$

(2) **L / H** - t_1 plays L and t_2 plays H

As above after H $\quad P' = P(t_1 \mid H) = 0$

and after L $\quad P' = P(t_1 \mid L) = 1$

Both players' optimal behaviour will clearly depend upon the values of the costs associated with each of the four basic outcomes. There is the potential for some of the penalties to be set by an external body. If this were the case, the natural question to ask is "How should the penalties be arranged to encourage both parties to work hard?". We can consider one of the costs for each player as a variable when finding equilibria. We are then analysing a family of models, and the sensitivity of the equilibria to changes in this cost can be found. This will allow us to find penalty policies to encourage certain types of behaviour in the model as well as reducing the risk of equilibria being highly cost-specific.

Some of the outcomes correspond to one or both of the players doing their job correctly, for example NQ(NE) where neither player is in the wrong. The cost that varies in practice is associated with the result NQ(E). Here the auditee has failed to prevent an error and the auditor has failed to detect it. At present this penalty is determined through litigation or through out of court settlements as the shareholders' feel that both the auditor and auditee have failed in their duties. We will therefore consider the costs $C^{NQ}(E)$ and $D^{NQ}(E)$ to be variables in this model.

Each player's strategic choice can be summarised by one or more decision rules which describe how the players' actions change as $C^{NQ}(E)$ and $D^{NQ}(E)$ increase. The auditee's only decision is whether to play H or not. Against a general auditor strategy (K_H / K_L) this becomes: Play H if $D(i, H, K_H) < D(i, L, K_L) \Leftrightarrow$

$$D^{NQ}(E) > \big(D_H + (p_{iH}\, r^*(K_H) - p_{iL}\, r^*(K_L))D^Q(E) + ((1-p_{iH})t^*(K_H) - (1-p_{iL})t^*(K_L))D^Q(NE) + ((1-p_{iH})(1-t^*(K_H)) - (1-p_{iL})(1-t^*(K_L)))D^{NQ}(NE)\big) \,/\, (p_{iL}(1-r^*(K_L)) - p_{iH}(1-r^*(K_H))) \quad \textbf{(4.4.1)}$$

This will enable us to find a condition on $D^{NQ}(E)$ for an auditee type i to play H against any auditor strategy. However we need only consider those auditor strategies which can be optimal. Since the auditor has imperfect information his cost for a given strategy will be an expectation over his beliefs. For a general strategy $A_j(K_H / K_L)$ this expected cost is given by expression (4.3.2). These subjective expectations can be compared to determine which is optimal. This can be done in a similar way to the auditee's decision rule, by comparing actions pairwise. If the auditor has a set of n potentially optimal actions then for B_2NQ, say, to be optimal there are (n-1) inequalities that must be satisfied, $B_2NQ < K_i$ for each of the other actions. In general

$$E(A_j(W / X)) \le E(A_k(Y / Z)) \Leftrightarrow$$

$$C^{NQ}(E) \ge C^Q(E) + ((C^Q(NE) - C^{NQ}(NE))((t^*(W) - t^*(Y))(PS_1(1-P_{1H}) + (1-P)S_2(1-P_{2H})) + (t^*(X) - t^*(Z))(P(1-S_1)(1-P_{1L}) + (1-P)(1-S_2)(1-P_{2L}))) + C_{aj} - C_{Ak} + (PS_1 + (1-P)S_2)(C_{BW} - C_{BY}) + (P(1-S_1) + (1-P)(1-S_2))(C_{BX} - C_{BZ})) \,/\, (r^*(W) - r^*(Y))(PS_1P_{1H} + (1-P)S_2P_{2H}) + (r^*(X) - r^*(Z))(P(1-S_1)P_{1L} + (1-P)(1-S_2)P_{2L}) \quad \textbf{(4.4.2)}$$

where $W, X, Y, Z \in \{B_2NQ, B_2R, B_1R, B_2Q\}$

Looking at this decision rule shows that the absolute values of the four outcome costs does not influence the equilibria. What matters is the difference between qualifying and not qualifying, i.e. the expressions $(C^Q(NE) - C^{NQ}(NE))$ and $(C^{NQ}(E) - C^Q(E))$. Similarly, the auditee's action is influenced by the difference in the occurrence of errors between high and low effort rather than their absolute values.

These comparisons must take place before play starts since the auditor is deciding, amongst other things, whether to observe the auditee's action or not. Thus the beliefs P, S_1 and S_2 in the above expression are the auditor's initial beliefs. The auditors beliefs about the auditee's action will be either 0 or 1 in a pure strategy equilibrium. For each auditee strategy we need to find where each of the functions is a minimum in the $P \times C^{NQ}(E)$ plane, where P is limited to [0, 1].

The auditor's strategic decision can be divided into two distinct parts. Firstly, the auditor's belief P, and his beliefs S_i that a type i auditee will put in high effort, tell him what he expects the auditee's action (and type) to have been. If the auditor decides to observe the auditee's action he can adjust his beliefs S_i about the auditee's action to reflect the fact that he now knows what that action was. This observation may also alter his belief P about which type he is facing. If, for example, the auditor feels that only a type 1 auditee will put in Low effort then after observing Low effort the auditor is certain (P = 1) that he is facing a type 1 auditee. Secondly, given what he now believes about the auditee, the auditor must decide what actions he should take during substantive testing and qualification. This decision process will occur no matter what the auditor's beliefs are (the reduced extensive form has 4 identical sub-trees for the auditor's actions).

The sole influence that the auditee's type and action have over the outcome of the audit is through the basic chance of errors occurring p_{iw}. As the cost $C^{NQ}(E)$ increases the auditor's optimal behaviour will change. This can be represented by a sequence of actions for increasing $C^{NQ}(E)$. Each of the cost functions C(i,w,K) is linear and non-decreasing in $C^{NQ}(E)$. Therefore each cost function can be minimal (the action would be optimal) for at most one interval of $C^{NQ}(E)$. If we differentiate each of these cost functions with respect to $C^{NQ}(E)$ we get the following:

(i) $\dfrac{\partial C(i,w,B2NQ)}{\partial C^{NQ}(E)} = p_{iw}$ **(ii)** $\dfrac{\partial C(i,w,B2R)}{\partial C^{NQ}(E)} = p_{iw}(1-r_2)$

(iii) $\dfrac{\partial C(i,w,B1R)}{\partial C^{NQ}(E)} = p_{iw}(1-r_1)$ **(iv)** $\dfrac{\partial C(i,w,B2Q)}{\partial C^{NQ}(E)} = 0$

Lemma 4.5 *There are 4 action sequences as $C^{NQ}(E)$ increases:*

$B_2NQ \rightarrow B_2R \rightarrow B_1R \rightarrow B_2Q$ $\quad$ $B_2NQ \rightarrow B_2R \rightarrow B_2Q$

$B_2NQ \rightarrow B_1R \rightarrow B_2Q$ $\quad$ $B_2NQ \rightarrow B_2Q$

Proof We have $1 > r_1 > r_2 > 0$

$\Leftrightarrow 0 < 1\text{-}r_1 < 1\text{-} r_2 < 1 \Leftrightarrow$ **(i) > (ii) > (iii) > (iv)** = 0

So each of the cost functions is linear in $C^{NQ}(E)$ and increasing at a unique rate. This can be used to find intervals of $C^{NQ}(E)$ for which each action can be optimal. As $C^{NQ}(E)$ increases, the cost functions will be minimal in decreasing order of gradient. Of course one or more of these cost functions may never be minimal in which case they need not be considered. Since B_2Q is constant in $C^{NQ}(E)$, whereas the other costs are all increasing, it will always be optimal for sufficiently large $C^{NQ}(E)$. If we consider the case $C^{NQ}(E) = 0$ as a lower bound we find:

$C(i,E,B_2R) = p_{iw}\, r_2 C^Q(E)+(1\text{-}p_{iw})(t_2C^Q(NE)+(1\text{-}t_2)C^{NQ}(NE))$

$C(i,E,B_1R) = C_{Bk}+p_{iw}\, r_1 C^Q(E)+(1\text{-}p_{iw})(t_1C^Q(NE)+(1\text{-}t_1)C^{NQ}(NE))$

$C(i,E,B_2NQ) = (1\text{-}p_{iw})C^{NQ}(NE)$

$C^Q(NE) \geq C^{NQ}(NE)$ and $C^Q(E) > 0 \Rightarrow C(i,w,B_2NQ)$ is minimal for $C^{NQ}(E)=0$.

So B_2NQ is optimal for sufficiently small $C^{NQ}(E)$. This gives the first and last optimal actions as $C^{NQ}(E)$ increases. Also if both B_1R and B_2R occur then B_2R must occur before B_1R. This gives the following optimal strategy sequences:

$B_2NQ \rightarrow B_2R \rightarrow B_1R \rightarrow B_2Q$ $\quad$ $B_2NQ \rightarrow B_2R \rightarrow B_2Q$

$B_2NQ \rightarrow B_1R \rightarrow B_2Q$ $\quad$ $B_2NQ \rightarrow B_2Q$

We can also consider the values of $C^{NQ}(E)$ at which these changes of optimal actions occur. These conditions are found by comparing costs. For example, B_2R is a cheaper action than B_2NQ if $C(i,w,B_2NQ) > C(i,w,B_2R)$

$$\Leftrightarrow \quad C^{NQ}(E) > C^Q(E) + f(p_{iw})\times t_2 / r_2$$

Where $f(p_{iw}) = ((1\text{-}p_{iw})(C^Q(NE)\text{-}C^{NQ}(NE)) / p_{iw}$ $\quad$ $p_{iw} \in (0, 1)$

The function f is decreasing in p_{iw} since $C^Q(NE) > C^{NQ}(NE)$. We can develop similar expressions for the other potential changes in the optimal action sequences.

$\mathbf{B_2NQ \rightarrow B_2R}$	$\Leftrightarrow$	$C^{NQ}(E) > C^Q(E) + f(p_{iw})\times t_2 / r_2$	**(1)**
$\mathbf{B_2NQ \rightarrow B_1R}$	$\Leftrightarrow$	$C^{NQ}(E) > C^Q(E) + C_B / r_1 p_{iw} + f(p_{iw})\times t_1/r_1$	**(2)**
$\mathbf{B_2NQ \rightarrow B_2Q}$	$\Leftrightarrow$	$C^{NQ}(E) > C^Q(E) + f(p_{iw})$	**(3)**
$\mathbf{B_2R \rightarrow B_1R}$	$\Leftrightarrow$	$C^{NQ}(E) > C^Q(E) + C_B /(r_1\text{-}r_2)p_{iw} + f(p_{iw})\times(t_1\text{-}t_2)/(r_1\text{-}r_2)$	**(4)**
$\mathbf{B_2R \rightarrow B_2Q}$	$\Leftrightarrow$	$C^{NQ}(E) > C^Q(E) + f(p_{iw})\times(1\text{-}t_2)/(1\text{-}r_2)$	**(5)**
$\mathbf{B_1R \rightarrow B_2Q}$	$\Leftrightarrow$	$C^{NQ}(E) > C^Q(E) \text{ - } C_B /(1\text{-}r_1)p_{iw} + f(p_{iw})\times(1\text{-}t_1)/(1\text{-}r_1)$	**(6)**

These inequalities can be used to investigate which changeovers in optimal action sequences can occur. For example, for B_2NQ to be optimal none of the inequalities **(1)**, **(2)** and **(3)** must hold. The first of these inequalities to be satisfied as $C^{NQ}(E)$ increases tells us which action will replace B_2NQ as the optimal one.

Definition let A and B be inequalities involving the same variable y. There is an interval of y for which each of these inequalities holds; (A_L, A_H) and (B_L, B_H) say. Then A is *weaker* than B (B is stronger than A) if (B_L,B_H) is a subset of (A_L,A_H).

We do not need to consider **(6)** since if B_1R is optimal it can only be replaced by B_2Q. Since the function $f(p_{iw})$ is decreasing in p_{iw} the inequalities **(1)** to **(5)** become weaker as p_{iw} increases. These inequalities can be used to rule out one of the action sequences:

Lemma 4.6 *The optimal action sequence* $B_2NQ \rightarrow B_2Q$ *does not occur.*

Proof Consider inequalities (1) and (3). Now $r_k > 0.5 > t_k$
and $1 > r_1 > r_2 > t_2 > t_1 > 0$ since the B_1 test has a lower risk of both type I and type II errors than the less extensive alternative B_2.
$r_2 > t_2 \Leftrightarrow 1 > (t_2/r_2) \Leftrightarrow C^Q(E) + f(p_{iw})\times t_2 / r_2 < C^Q(E) + f(p_{iw})$
$\Leftrightarrow$ RHS of (1) < RHS of (3) $\Leftrightarrow$ (1) is weaker than (3)
So the conditions for B_2R to replace B_2NQ as the optimal action will always be satisfied for lower $C^{NQ}(E)$ than for B_2Q to replace B_2NQ. The optimal action sequence $B_2NQ \rightarrow B_2Q$ does not therefore occur.

We know that each sequence must begin with B_2NQ and end with B_2Q. We also know that there must be at least one other action in between these two. As $C^{NQ}(E)$ increases the auditor's choice of B-test will optimise the balance between the savings made by a more effective test against the additional cost of this test. There will come a point where the savings outweigh the cost C_B in which case the auditor will prefer B_1 to B_2. This gives the following:

Lemma 4.7 *One of three auditor action sequences occurs as $C^{NQ}(E)$ increases. Which occurs is determined by the cost C_B in the following way:*

$$B_2NQ \rightarrow B_1R \rightarrow B_2Q \quad \text{if} \quad C_B < L$$
$$B_2NQ \rightarrow B_2R \rightarrow B_1R \rightarrow B_2Q \quad \text{if} \quad U > C_B > L$$
$$B_2NQ \rightarrow B_2R \rightarrow B_2Q \quad \text{if} \quad C_B > U$$

where $L = (1-p_{iw})\times(C^Q(NE)-C^{NQ}(NE))\times(t_2r_1-t_1r_2)/ r_2$

and $U = (1-p_{iw})\times(C^Q(NE)-C^{NQ}(NE))\times((1-t_2)(r_1-1)+(1-t_1)(1-r_2))/ (1-r_2)$

Proof Consider the following 3 cases:

(i) C_B is very low. In this case the increased efficiency of the B_1-test will be worth using for even low $C^{NQ}(E)$. B_1R may become a cheaper option than B_2R whilst B_2NQ is the optimal action.

(ii) C_B is very high. In this case B_1R may not become a cheaper option than B_2R until after B_2Q is the optimal action (i.e. for high $C^{NQ}(E)$)

(iii) If C_B lies between these levels then B_2R will replace B_2NQ as the optimal action. For higher $C^{NQ}(E)$ B_1R will replace B_2R and for very high $C^{NQ}(E)$ B_2Q will replace B_1R as the optimal action.

Case (i) occurs when B_1R replaces B_2NQ as the optimal action - when **(1)** is stronger than **(2)**. Similarly case (ii) occurs when **(4)** is stronger than **(5)**.

Case (iii) occurs when **(2)** is stronger than **(1)** and **(5)** is stronger than **(4)**.

Now, **(1)** is stronger than **(2)** if RHS of **(1)** > RHS of **(2)**

$\Leftrightarrow C^Q(E) + f(p_{iw})\times t_2 / r_2 > C^Q(E) + C_B /r_1p_{iw} + f(p_{iw})\times t_1 /r_1$

$\Leftrightarrow C_B < p_{iw}\times f(p_{iw})\times (t_2\times r_1/r_2 - t_1)$

$\Leftrightarrow C_B < (1-p_{iw})\times(C^Q(NE)-C^{NQ}(NE))\times(t_2r_1- t_1r_2)/ r_2 = L$

(4) is stronger than **(5)** if RHS of **(4)** > RHS of **(5)**

$\Leftrightarrow C^Q(E) + C_B /(r_1-r_2)p_{iw} + f(p_{iw})\times(t_1-t_2)/(r_1-r_2) > C^Q(E) + f(p_{iw})\times(1-t_2)/(1-r_2)$

$\Leftrightarrow C_B > (1-p_{iw})\times(C^Q(NE)-C^{NQ}(NE))\times((1-t_2)(r_1-r_2)-(t_1-t_2)(1-r_2))/(1-r_2)$

$\Leftrightarrow C_B > (1-p_{iw})\times(C^Q(NE)-C^{NQ}(NE))\times((1-t_2)(r_1-1)+(1-t_1)(1-r_2))/(1-r_2) = U$

This gives conditions on C_B for cases (i) and (ii). For case (iii) consider U - L

$= (1-p_{iw})\times(C^Q(NE)-C^{NQ}(NE))\times\big(((1-t_2)(r_1-1)+(1-t_1)(1-r_2))/(1-r_2) -(t_2r_1- t_1r_2)/r_2 \big)$

Now $(1-p_{iw})\times(C^Q(NE)-C^{NQ}(NE)) > 0$. The remaining term can be simplified:

$= \big((1-t_2)(r_1-1)r_2+(1-t_1)(1-r_2)r_2 - t_2r_1(1-r_2) + t_1r_2(1-r_2)\big)/ r_2(1-r_2)$

$= (r_1r_2-r_2-r_1r_2t_2+t_2r_2+r_2-r_2r_2-r_2t_1+t_1r_2r_2-t_2r_1+t_2r_1r_2+t_1r_2-t_1r_2r_2)/r_2(1-r_2)$

$= (r_1r_2+t_2r_2-r_2r_2-t_2r_1)/r_2(1-r_2)$

$= (r_1-r_2)(r_2-t_2)/r_2(1-r_2) > 0 \quad$ since $1 > r_1 > r_2 > t_2 > 0$

Hence U > L and the interval for case (iii) is well defined.

We can categorise which of the three optimal action sequences occurs for a given auditee type and effort level in terms of the auditor's B-test cost C_B. Furthermore, we have shown that five of the inequalities that prompt a change in the auditor's optimal action are decreasing in p_{iw}. In other words, the change will occur for lower $C^{NQ}(E)$ if the auditee plays low effort (leading to a higher p_{iw}). Some of the twelve remaining observation (A_1-test) strategies can therefore be shown to be sub-optimal. Some of these strategies specify an action against H which is further along the optimal action sequence than for L, for example $A_1(B_2Q$ / $B_2NQ)$. These strategies cannot be optimal since if $C^{NQ}(E)$ satisfies the conditions for B_2Q to be optimal vs. H then clearly the weaker condition for B_2Q to be optimal against L will also be satisfied. This reduces the number of potential A_1-test strategies to six. The number of potentially optimal auditor strategies has been reduced from 72 to 10.

Lemma 4.7 classifies the auditor's optimal action for a given type and effort level. A strategy for the auditor must cover each eventuality - an optimal action after observing H and an optimal action after observing L. A strategy therefore can be determined by considering two optimal action sequences as $C^{NQ}(E)$ increases, one sequence for actions after observing H and another after observing L. The way in which these two sequences can change is limited by comparisons between U and L and the ordering of p_{iw} .

Lemma 4.8 U and L are decreasing functions of p_{iw}

Proof $L = (1-p_{iw})\times(C^Q(NE)-C^{NQ}(NE))\times(t_2r_1-t_1r_2)/ r_2$

We have $C^Q(NE) > C^{NQ}(NE)$ and $t_1, t_2, r_1, r_2 \in (0, 1)$. The increased efficiency of the B_1-test over B_2 gives $r_1 > r_2$ and $t_1 < t_2$

$\Rightarrow t_2r_1 > t_1r_2$

$\Rightarrow t_2r_1-t_1r_2 > 0$

$\Rightarrow$ L is a decreasing function of p_{iw}.

$U = (1-p_{iw})\times(C^Q(NE)-C^{NQ}(NE))\times((1-t_2)(r_1-1)+(1-t_1)(1-r_2))/ (1-r_2)$

$r_1 > r_2$ and $t_1 < t_2 \Leftrightarrow (1-t_1) > (1-t_2)$ and $(1-r_1) < (1-r_2)$

$\Rightarrow (1-t_1)(1-r_2) > (1-t_2)(r_1-1)$

$\Rightarrow (1-t_1)(1-r_2) - (1-t_2)(r_1-1) > 0$

$\Rightarrow$ U is a decreasing function of p_{iw}

Since the occurrence of each sequence is determined by the cost C_B the optimal action set can also be classified by C_B. For example, in table 4.1, optimal action set i occurs if $C_B < L(p_{2L})$. In this case B_2R is not optimal for either type or either effort level. So in category i we can rule out 4 of the 10 tests that are candidates for optimality as $C^{NQ}(E)$ increases, namely those involving B_2R. As C_B is increased the optimal action set changes from i to v.

Auditor's potentially optimal strategies for some $C^{NQ}(E)$											
C_B inc	Category	**A_2 test**						**A_1 test**			
		B_2NQ	B_2R	B_1R	B_2Q	NQ/2R	NQ/1R	NQ/Q	2R/1R	2R/Q	1R/Q
↓	**I**	√		√	√		√	√			√
	ii	√	√	√	√	√	√	√			√
	iiia or	√	√	√	√	√	√	√	√	√	√
	iiib	√	√	√	√	√		√			√
	iv	√	√	√	√	√		√		√	√
	v	√	√		√	√		√		√	

table 4.1 - categorisation of the potentially optimal action set

The only actions that can occur regardless of the cost C_B are B_2NQ, B_2Q and $A_1(B_2NQ\ /B_2Q)$ These "core" tests are the undominated strategies in the simpler model of Fellingham and Newman [16]. Clearly, for small enough $C^{NQ}(E)$ B_2NQ will always be the cheapest option, and similarly for large enough $C^{NQ}(E)$ B_2Q will become optimal. Each of these categories is quite complex. To illustrate how the optimal strategies are affected by varying $C^{NQ}(E)$ we will consider an example.

4.5 A Numerical Example

Any set of costs and probabilities that satisfies the payoff restrictions will reflect some of the motivations for the players. Many of these motivations will be in terms of comparisons between 2 costs or penalties. For example the benefit from using the B_1 test is the difference between r_1 and r_2 (and t_1 and t_2). If there is very little difference between the two tests then clearly the auditor will usually prefer the cheaper one.The auditor's optimal testing strategy depends fundamentally upon p_{iw}. The optimal action sequences have been analysed as functions of p_{iw} in the previous section. If the difference between p_{1H} and p_{1L} is negligible then as $C^{NQ}(E)$ increases the auditor's optimal actions will for the most part be the same against both H and L. The example chosen is from category iv. This gives a representative case, whilst at the same time avoiding any similarities between types, effort levels or B-tests:

$p_{1H} = 0.05$	$p_{1L} = 0.35$	$p_{2H} = 0.3$	$p_{2L} = 0.5$
$r_1 = 0.95$	$t_1 = 0.1$	$r_2 = 0.8$	$t_2 = 0.25$

$C^Q(NE) = 100$	$C^Q(E) = 60$	$C^{NQ}(NE) = 10$
$D^Q(NE) = 120$	$D^Q(E) = 220$	$D^{NQ}(NE) = 10$
$C_A = 5$	$C_B = 60$	$D_H = 55$

Auditee's optimal strategy for a given test

For this example the auditee's best response to each potentially optimal auditor test are described below. These inequalities were generated by condition (4.4.1) in section 4.4. For completeness, all ten of the auditor's tests have been considered even though two of the A_1-tests cannot be optimal in category **iv**

AUDITOR'S TEST	High Effort By type 1	High Effort By type 2
A_2B_2NQ	$D^{NQ}(E) > 193$	$D^{NQ}(E) > 285$
A_2B_2R	$D^{NQ}(E) > 224$	$D^{NQ}(E) > 683$
A_2B_1R	Always	$D^{NQ}(E) > 1740$
A_2B_2Q	Never	Never
A_1B_2NQ/B_2R	Always	$D^{NQ}(E) < 224$
A_1B_2NQ/B_1R	$D^{NQ}(E) < 686$	$D^{NQ}(E) < 193$
A_1B_2NQ/B_2Q	$D^{NQ}(E) < 1810$	$D^{NQ}(E) < 360$
A_1B_2R/B_1R	$D^{NQ}(E) > 1683$	Never
A_1B_2R/B_2Q	$D^{NQ}(E) < 5558$	$D^{NQ}(E) < 599$
A_1B_1R/B_2Q	$D^{NQ}(E) < 27840$	$D^{NQ}(E) < 2507$

table 4.2 - auditee's optimal response to each auditor test

The decision rule derived above can also be used to analyse the sensitivity of the equilibrium to the cost D_H. Since $D^{NQ}(E)$ is considered as a variable and all the other costs are fixed these conditions are of the form $D_H < f_i(D^{NQ}(E))$ for auditee type i. Furthermore each of these functions will be linear. The four types of pure strategy equilibrium are associated with these conditions as follows:

Pooling on H $D_H < \text{Min}\{f_1, f_2\}$ Signalling with t_1 playing H $f_1 > D_H > f_2$

Pooling on L $D_H > \text{Max}\{f_1, f_2\}$ Signalling with t_1 playing L $f_2 > D_H > f_1$

So for L / H we require $f_2 > D_H > f_1$. Condition 4.4.1 gives an inequality on D_H as a function of $D^{NQ}(E)$ for each optimal auditor strategy. For a particular example the inequalities for each auditee type can compared as $D^{NQ}(E)$ varies. The ten potentially optimal auditor strategies are considered in appendix A. Three of these can encourage a L / H separating equilibrium:

A type i auditee will put in high effort against A_1B_2NQ/B_2R

$$\Leftrightarrow \quad D_H < P_{iL}r_2D^Q(E) + (P_{iL}(1-r_2)-P_{iH})D^{NQ}(E) + (1-P_{iL})t_2D^Q(NE) + ((1-P_{iL})(1-t_2)-(1-P_{iH}))D^{NQ}(NE)$$

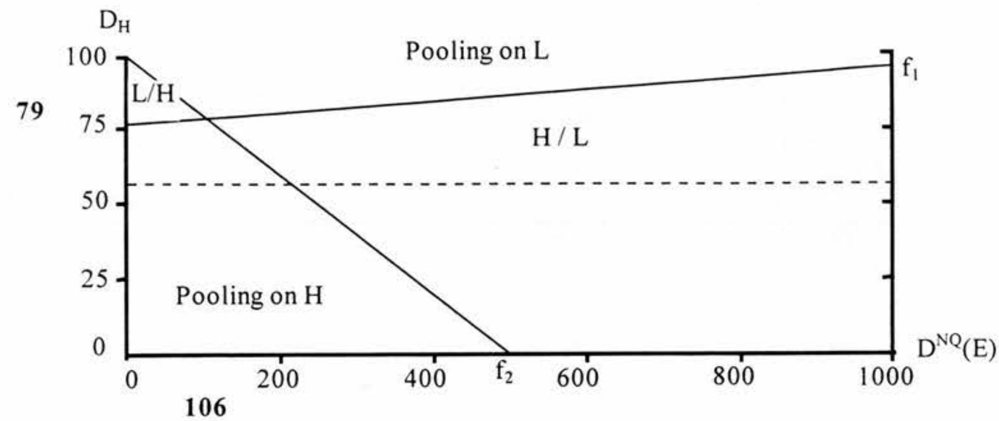

figure 4.2 :- auditee effort against A_1B_2NQ/B_2R

A type i auditee will put in high effort against A_1B_2NQ/B_1R

$$\Leftrightarrow \quad D_H < p_{iL}r_1D^Q(E) + (p_{iL}(1-r_1)-p_{iH})D^{NQ}(E) + (1-p_{iL})t_1D^Q(NE) + ((1-p_{iL})(1-t_1)-(1-p_{iH}))D^{NQ}(NE)$$

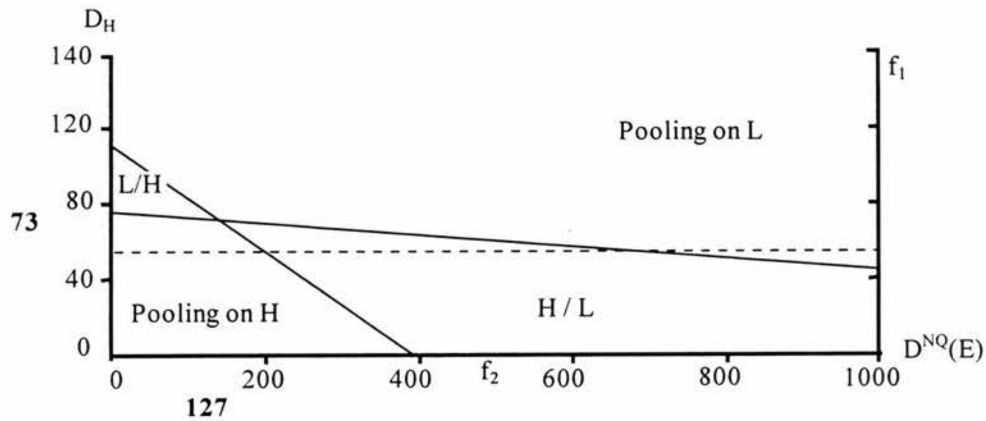

figure 4.3 :- auditee effort against A_1B_2NQ/B_1R

A type i auditee will put in high effort against A_1B_2NQ/B_2Q

$$\Leftrightarrow \quad D_H < p_{iL}D^Q(E) - p_{iH}D^{NQ}(E) + (1-p_{iL})D^Q(NE) -(1-p_{iH})D^{NQ}(NE)$$

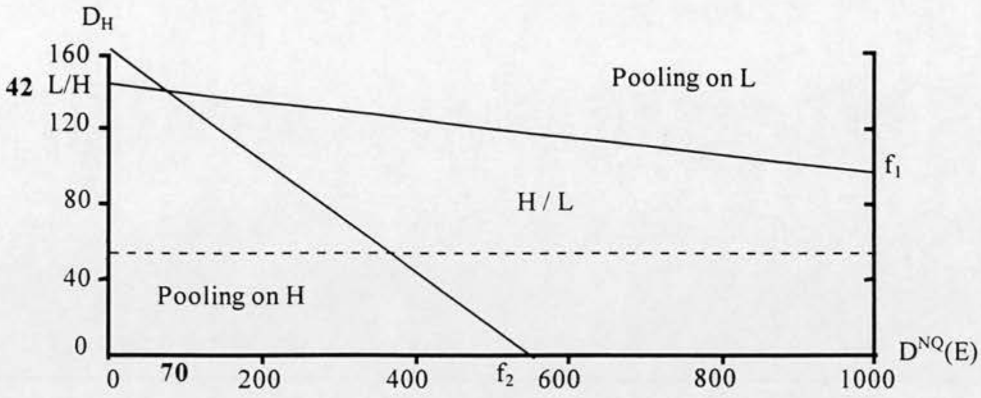

figure 4.4 :- auditee effort against A_1B_2NQ/B_1R

The three strategies that can encourage a L/H separating equilibrium are $A_1(B_2NQ/B_2R)$, $A_1(B_2NQ/B_1R)$ and $A_1(B_2NQ/B_2Q)$. This can give upper and lower bounds on D_H for the L/H separating equilibrium to occur. With auditor optimal strategy set (**iv**) this gives $D_H \in (76.675, 160)$. For $D_H = 55$ therefore, L / H does not occur.

Auditor's optimal strategy for a given auditee strategy

In category **(iv)** there are eight potentially optimal strategies $\{A_2B_2NQ,\ A_2B_2R,\ A_2B_1R,\ A_2B_2Q,\ A_1(B_2NQ/B_2R),\ A_1(B_2NQ/B_2Q),\ A_1(B_2R/B_2Q),\ A_1(B_1R/B_2Q)\}$. For the values chosen in this example the case where t_1 plays L and t_2 plays H does not occur. The inequalities for the three remaining cases are given in the following tables. If both auditee types put in the same effort (a pooling equilibrium) then lemma 4.4 tells us that the A_1 test cannot form part of an equilibrium pair. In these situations the auditor has only 4 potentially optimal tests. If both auditee types play H then (4.4.2) simplifies to $E(A_2W) \le E(A_2Y) \Leftrightarrow$

$$C^{NQ}(E) \ge C^Q(E) + \big((C^Q(NE) - C^{NQ}(NE))((t^*(W) - t^*(Y))(P(1-p_{1H}) + (1-P)(1-p_{2H}))) + (C_{BW} - C_{BY})\big) / (r^*(W) - r^*(Y))(Pp_{1H} + (1-P)p_{2H})$$

where $W, Y \in \{B_2NQ, B_2R, B_1R, B_2Q\}$

POOLING ON H $\Rightarrow S_1 = 1$ & $S_2 = 1$	
TEST COMPARISON	**INEQUALITY**
$E(A_2B_2NQ) \leq E(A_2B_2R)$	$C^{NQ}(E) \leq (30.15-6.375P) / (0.24-0.2P)$
$E(A_2B_2NQ) \leq E(A_2B_1R)$	$C^{NQ}(E) \leq (83.4-12P) / (0.285-0.2375P)$
$E(A_2B_2NQ) \leq E(A_2B_2NQ)$	$C^{NQ}(E) \leq (81+7.5P) / (0.3-0.25P)$
$E(A_2B_2R) \leq E(A_2B_1R)$	$C^{NQ}(E) \leq (53.25-5.625P) / (0.045-0.0375P)$
$E(A_2B_2R) \leq E(A_2B_2Q)$	$C^{NQ}(E) \leq (50.85+13.875P) / (0.06-0.05P)$
$E(A_2B_1R) \leq E(A_2B_2Q)$	$C^{NQ}(E) \leq (-2.4+19.5P) / (0.015-0.0125P)$

table 4.3 :- auditor's test comparisons if both auditee types play H

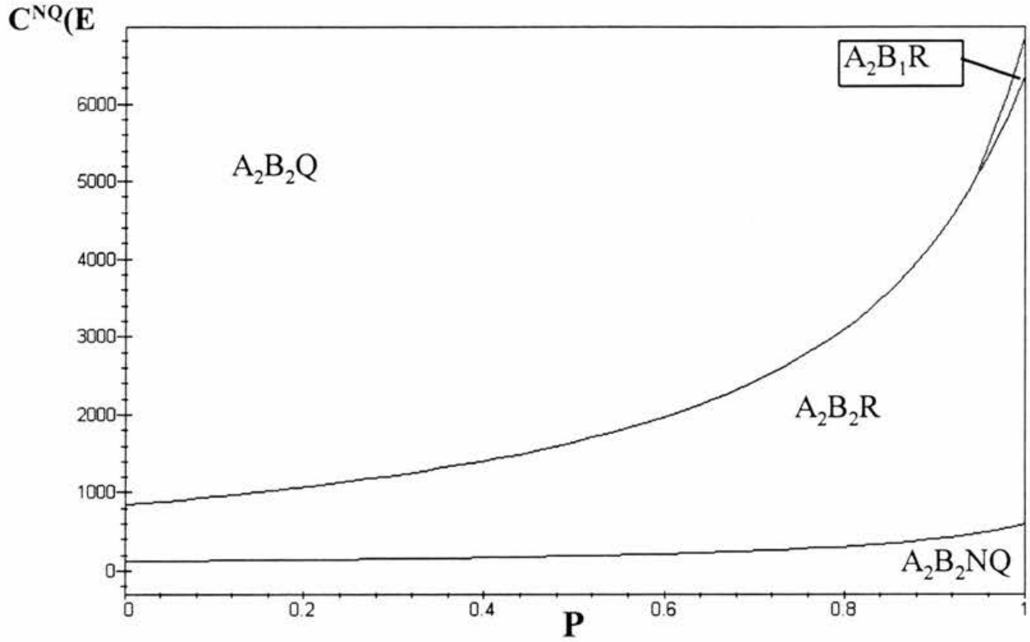

figure 4.5 :- optimal response to H/H

Figure 4.5 shows that not all of the inequalities given in table 4.3 are needed. In table 4.4 below, only the first and fifth comparisons are actually needed for this example. If both types play L then (4.4.2) gives $E(A_j(W / X)) \leq E(A_k(Y / Z)) \Leftrightarrow$

$$C^{NQ}(E) \geq C^{Q}(E)+((C^{Q}(NE)-C^{NQ}(NE))((t^*(X)-t^*(Z))((1-p_{2L})+P(p_{2L}-p_{1L})))$$
$$+(C_{BX}-C_{BZ})) / (r^*(X)-r^*(Z))(p_{2L}+P(p_{1L}-p_{2L}))$$

where $W, X, Y, Z \in \{B_2NQ, B_2R, B_1R, B_2Q\}$

POOLING ON L $\Rightarrow S_1 = 0$ & $S_2 = 0$	
TEST COMPARISON	**INEQUALITY**
$E(A_2B_2NQ) \leq E(A_2B_2R)$	$C^{NQ}(E) \leq (35.25-3.825P) / (0.4-0.12P)$
$E(A_2B_2NQ) \leq E(A_2B_1R)$	$C^{NQ}(E) \leq (93-7.2P) / (0.475-0.14252P)$
$E(A_2B_2NQ) \leq E(A_2B_2Q)$	$C^{NQ}(E) \leq (75+4.5P) / (0.5-0.15P)$
$E(A_2B_2R) \leq E(A_2B_1R)$	$C^{NQ}(E) \leq (770-45P) / (1-0.3P)$
$E(A_2B_2R) \leq E(A_2B_2Q)$	$C^{NQ}(E) \leq (39.75+8.325P) / (0.1-0.03P)$
$E(A_2B_1R) \leq E(A_2B_2Q)$	$C^{NQ}(E) \leq (-18+11.7P) / (0.025-0.0075P)$

table 4.4 :- auditor's test comparisons if both auditee types play L

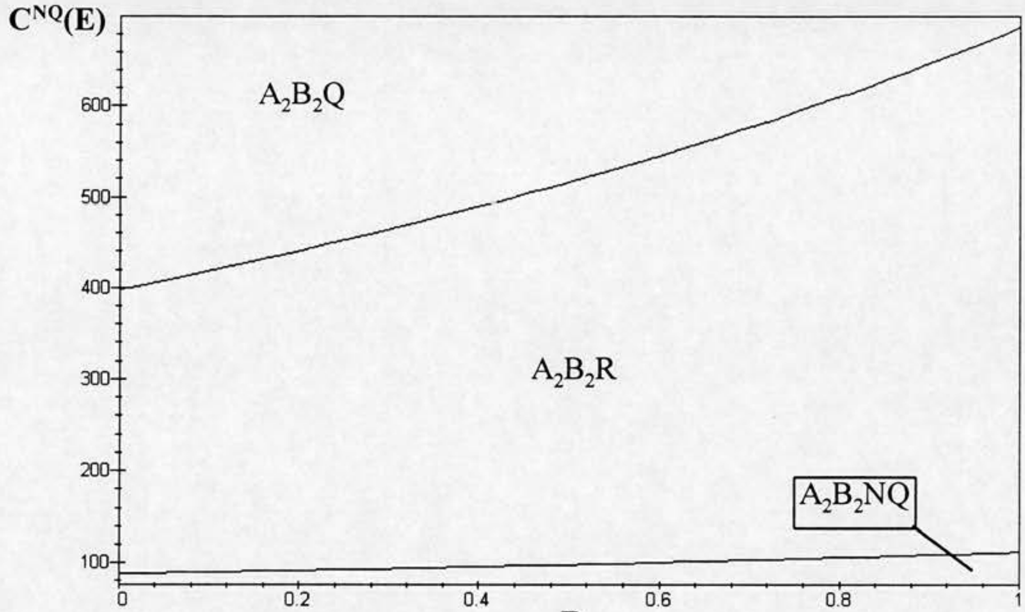

figure 4.6 :- optimal response to L/L

In a separating equilibrium the auditor must also consider A_1-test strategies. However, after observing the auditee's effort level the auditor will have an optimal action. The auditor therefore will never need to compare two A_1-test strategies. In the following table, the symbol $\geq\leq$ is used where the RHS of the inequality has an asymptote for $P \in (0, 1)$. However, we know that for sufficiently large $C^{NQ}(E)$ B_2Q will always be optimal (it is the end of every optimal action sequence). Thus whilst the boundaries may tend to the asymptote the discontinuity will never form part of a boundary. In this equilibrium the auditor's strategy comparison simplifies to the following:

$E(A_j(W / X)) \leq E(A_k(Y / Z)) \Leftrightarrow$

$$C^{NQ}(E) \geq C^{Q}(E)+((C^{Q}(NE)-C^{NQ}(NE))((t^*(W)-t^*(Y))P(1-p_{1H})+(t^*(X)-t^*(Z))(1-P)(1-p_{2H})) +C_{aj}-C_{Ak}+P(C_{BW}-C_{BY})+(1-P)(C_{BX}-C_{BZ})) / (r^*(W)-r^*(Y))(Pp_{1H})+(r^*(X)-r^*(Z))((1-P)p_{2L})$$

where $W, X, Y, Z \in \{B_2NQ, B_2R, B_1R, B_2Q\}$

SEPARATING WITH TYPE t_1 PLAYING H $\Rightarrow S_1 = 1$ & $S_2 = 0$	
TEST COMPARISON	**INEQUALITY**
$E(A_2B_2NQ) \leq E(A_2B_2R)$	$C^{NQ}(E) \leq (35.25-11.475P) / (0.4-0.36P)$
$E(A_2B_2NQ) \leq E(A_2B_1R)$	$C^{NQ}(E) \leq (93-26.1P) / (0.475-0.4275P)$
$E(A_2B_2NQ) \leq E(A_2B_2Q)$	$C^{NQ}(E) \leq (75+13.5P) / (0.5-0.45P)$
$E(A_2B_2R) \leq E(A_2B_1R)$	$C^{NQ}(E) \leq (770-130P) / (1-0.9P)$
$E(A_2B_2R) \leq E(A_2B_2Q)$	$C^{NQ}(E) \leq (39.75+24.975P) / (0.1-0.09P)$
$E(A_2B_1R) \leq E(A_2B_2Q)$	$C^{NQ}(E) \leq (-18+35.1P) / (0.025-0.0225P)$
$E(A_1B_2NQ/B_2R) \leq E(A_2B_2NQ)$	$C^{NQ}(E) \geq (40.25-35.25P) / 0.4(1-P)$
$E(A_1B_2NQ/B_2R) \leq E(A_2B_2R)$	$C^{NQ}(E) \leq (-5+23.775P) / 0.04P$
$E(A_1B_2NQ/B_2R) \leq E(A_2B_1R)$	$C^{NQ}(E) \leq (52.75+13.65P)/(0.075-0.0275P)$
$E(A_1B_2NQ/B_2R) \leq E(A_2B_2Q)$	$C^{NQ}(E) \leq (34.75+48.75P) / (0.1-0.05P)$
$E(A_1B_2NQ/B_2Q) \leq E(A_2B_2NQ)$	$C^{NQ}(E) \geq (80-75P) / 0.5(1-P)$
$E(A_1B_2NQ/B_2Q) \leq E(A_2B_2R)$	$C^{NQ}(E) \geq\leq (44.75-63.525P) / (0.1-0.14P)$
$E(A_1B_2NQ/B_2Q) \leq E(A_2B_1R)$	$C^{NQ}(E) \geq\leq (-13-53.4P) / (0.025-0.0725P)$
$E(A_1B_2NQ/B_2Q) \leq E(A_2B_2Q)$	$C^{NQ}(E) \leq (-5+88.5P) / 0.05P$
$E(A_1B_2R/B_2Q) \leq E(A_2B_2NQ)$	$C^{NQ}(E) \geq (80-51.225P) / (0.5-0.46P)$
$E(A_1B_2R/B_2Q) \leq E(A_2B_2R)$	$C^{NQ}(E) \geq (447.5-397.5P) / (1-P)$
$E(A_1B_2R/B_2Q) \leq E(A_2B_1R)$	$C^{NQ}(E) \geq\leq (-13-29.625P)/ (0.025-0.0325P)$
$E(A_1B_2R/B_2Q) \leq E(A_2B_2Q)$	$C^{NQ}(E) \leq (-5+64.725P) / 0.01P$
$E(A_1B_1R/B_2Q) \leq E(A_2B_2NQ)$	$C^{NQ}(E) \geq (80-3.6P) / (0.5-0.4525P)$
$E(A_1B_1R/B_2Q) \leq E(A_2B_2R)$	$C^{NQ}(E) \geq (44.75+7.875P) / (0.1-0.0925P)$
$E(A_1B_1R/B_2Q) \leq E(A_2B_1R)$	$C^{NQ}(E) \geq (-13+18P) / 0.025(1-P)$
$E(A_1B_1R/B_2Q) \leq E(A_2B_2Q)$	$C^{NQ}(E) \leq (-5+17.1P) / 0.0025P$

table 4.5 :- auditor's test comparisons if type 1 plays H and 2 plays L

For this case there are two areas of interest. Figure 4.7b shows the regions for low $C^{NQ}(E)$. The border between B_2Q and $A_1(B_2R/B_2Q)$ will curve across so that for all sufficiently high $C^{NQ}(E)$, B_2Q is always optimal. However there is a small region of interest for high costs given in figure 4.7a

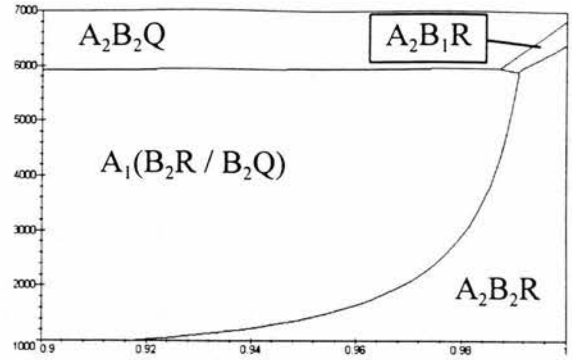

figure 4.7a :- large cost and high P

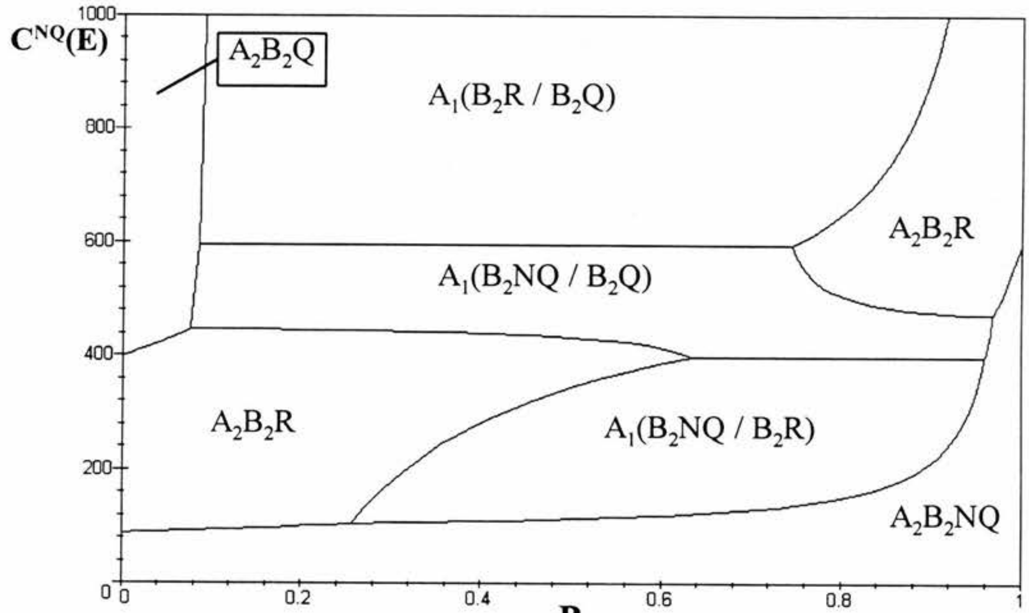

figure 4.7b :- optimal response to H / L

The optimal A_1 test strategy can be found by considering the two extremes P=0 or P=1 in figure 4.7b. This also shows clearly that observation is only optimal when the auditor is uncertain about which auditee type he is facing.

The equilibrium set

The equilibrium set can be seen more clearly for a particular starting probability P. Figures 4.5 to 4.7 then reduce to inequalities on $C^{NQ}(E)$. For each region of $C^{NQ}(E)$ this will give us the auditor's optimal response to each of the three cases of auditee behaviour. An equilibrium pair will occur if the the auditee's optimal behaviour to one of these strategies does not change the auditor's optimal strategy. For example, if P=0.9 then B_2NQ is an optimal response to H/H if $C^{NQ}(E) < 407$ and H/H is optimal against B_2NQ if $D^{NQ}(E) > 285$. Thus (B_2NQ, H/H) is an equilibrium pair if both $C^{NQ}(E) < 407$ and $D^{NQ}(E) > 285$. The equilibrium regions for each of the three cases are illustrated below for P = 0.9:

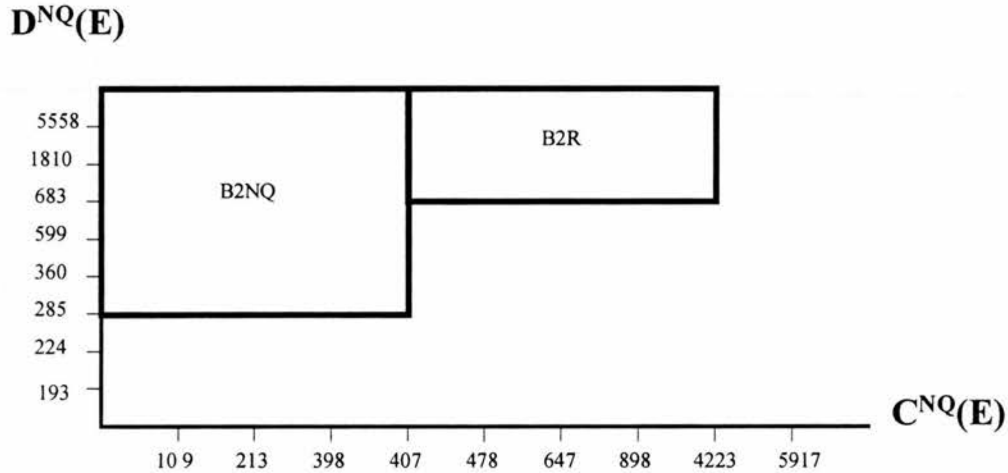

figure 4.8 :- pooling on H equilibrium

As we might expect, the auditee's best response to a testing strategy tends to be high effort for all sufficiently large penalties $D^{NQ}(E)$. The values of $D^{NQ}(E)$ for which high effort is optimal are given above in table 4.2. The levels of $C^{NQ}(E)$ which prompt a change in strategy depend on the auditor's belief P. These are illustrated above in figure 4.5. For $C^{NQ}(E) > 4223$ the auditors optimal response to H / H is B_2Q. However (B_2Q, H/H) is never in equilibrium since each auditee type prefers to put in low effort if the audit report is always qualified.

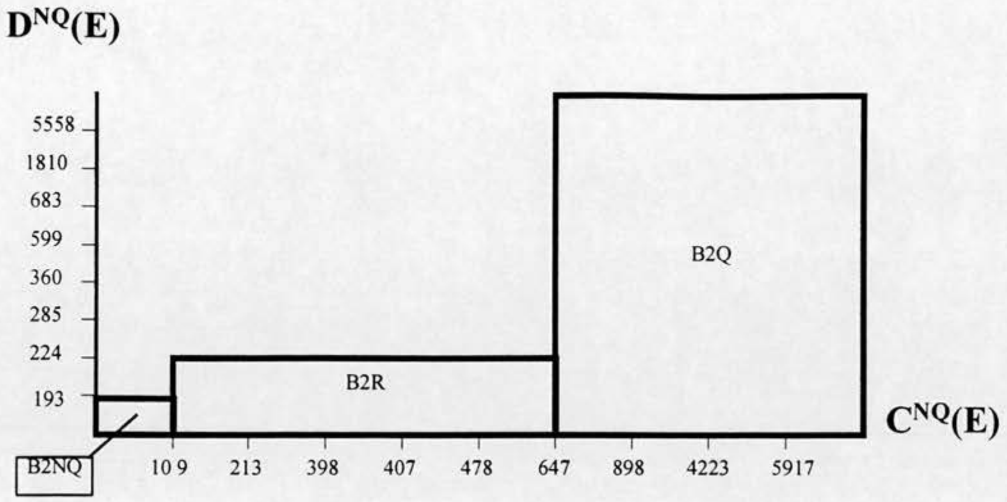

figure 4.9 :- pooling on L equilibrium

In contrast, both auditee types tend to play low effort if the cost $D^{NQ}(E)$ is sufficiently low. The boundaries of these equilibrium regions are therefore determined by two factors. As $C^{NQ}(E)$ increases the auditor's optimal response to L/L changes (shown in figure 4.6). These changes are one of the optimal action sequences described in section 4.4. There is also an upper limit on $D^{NQ}(E)$ for each auditor strategy. Above this level, the size of the penalty for an unqualified error makes it worthwhile for one auditee type to put in high effort. There is no upper limit for B_2Q since low effort is always the best response.

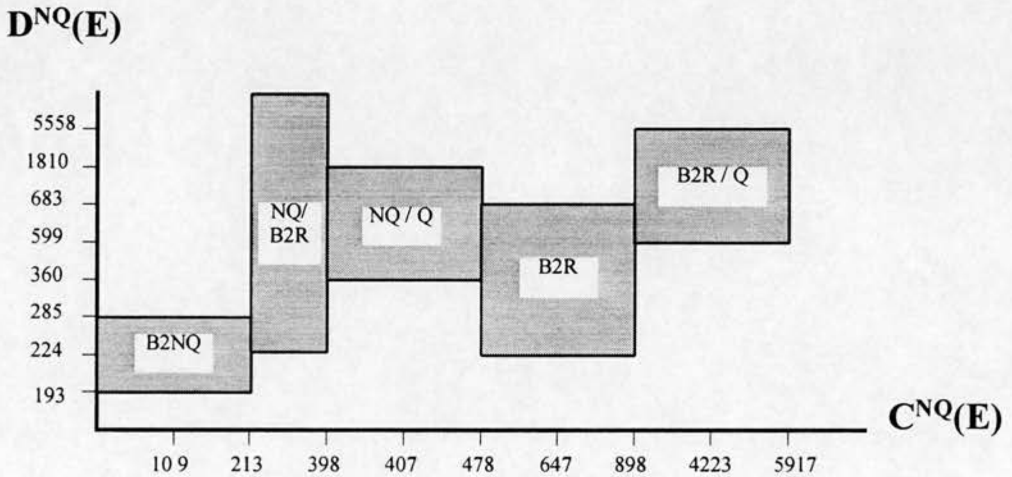

figure 4.10 :- separating H / L equilibrium

The regions of separating equilibria are a little more complex for two reasons. Firstly, the auditor's choice of optimal strategy must consider whether to observe the auditee's effort level or not. This depends critically on the value of P, as shown in figure 4.7. If the auditor is fairly sure which type of auditor he is facing (i.e. P close to 1 or 0) then there is less perceived benefit in becoming certain of the auditee's type by observing his effort level. The benefit from observation also depends on the conditional actions used after the effort level is observed. For $C^{NQ}(E) \in (478, 898)$ the auditor stops using the A-test. This is because B_2R is the best response to H and a "nearly best" response to L. Ideally, the auditor would prefer B_2Q after low effort. However the extra cost of using the A-test outweighs the savings made in the event (with a 10% chance) that the auditee is type 2. As $C^{NQ}(E)$ increases the difference between B_2R and B_2Q after L increases until it is worth using the A-test so that the subsequent tests are optimal after either effort level. Hence for $C^{NQ}(E) > 898$ the auditor uses $A_1(B_2R/B_2Q)$.

Secondly, there are two limits on the levels of $D^{NQ}(E)$ that support a separating equilibrium. A minimum level of $D^{NQ}(E)$ is required to encourage a type 1 auditee to put in high effort. At the same time if $D^{NQ}(E)$ is too high then a type 2 auditee will also put in high effort. These considerations give both a minimum and maximum level of $D^{NQ}(E)$ for each region. An exception to this is the region $(A_1(B_2NQ/B_2R), H/L)$. If a type 1 auditee puts in high effort he can reduce the chance of errors occurring to a low level (5%). In the worst case, where the auditor never qualifies, this leads to a 5% risk of an unqualified error (and correspondingly no risk of a qualified error or a false-positive). If the auditee puts in low effort then, even with a reasonable qualification strategy, the risk of an unqualified error is 7% (with a 28% risk of a qualified error and a 16% risk of a false positive). Therefore a type 1 auditee will play H against $A_1(B_2NQ/B_2R)$ for all $D^{NQ}(E)$.

Figures 4.8 to 4.10 describe the equilibrium regions for each of the three combinations of effort level that can occur in this example - L/L, H/L and H/H. To develop the full equilibrium set we must consider all three of these. The boundaries of the full equilibrium set for this example can be constructed by considering the boundaries of each of the three equilibrium regions simultaneously. We effectively overlay the three different effort level equilibrium regions. This idea of overlaying intuitively leads to two important questions. "is there an area outside all of the equilibrium sets?" and "what happens if two or more equilibrium regions overlap?".

Both these situations occur and lead to a consideration of mixed strategies and pareto dominance of equilibria in the next chapter. For P = 0.9 (so 10% of businesses are error prone) the full equilibrium set for the above example is given in figure 4.11

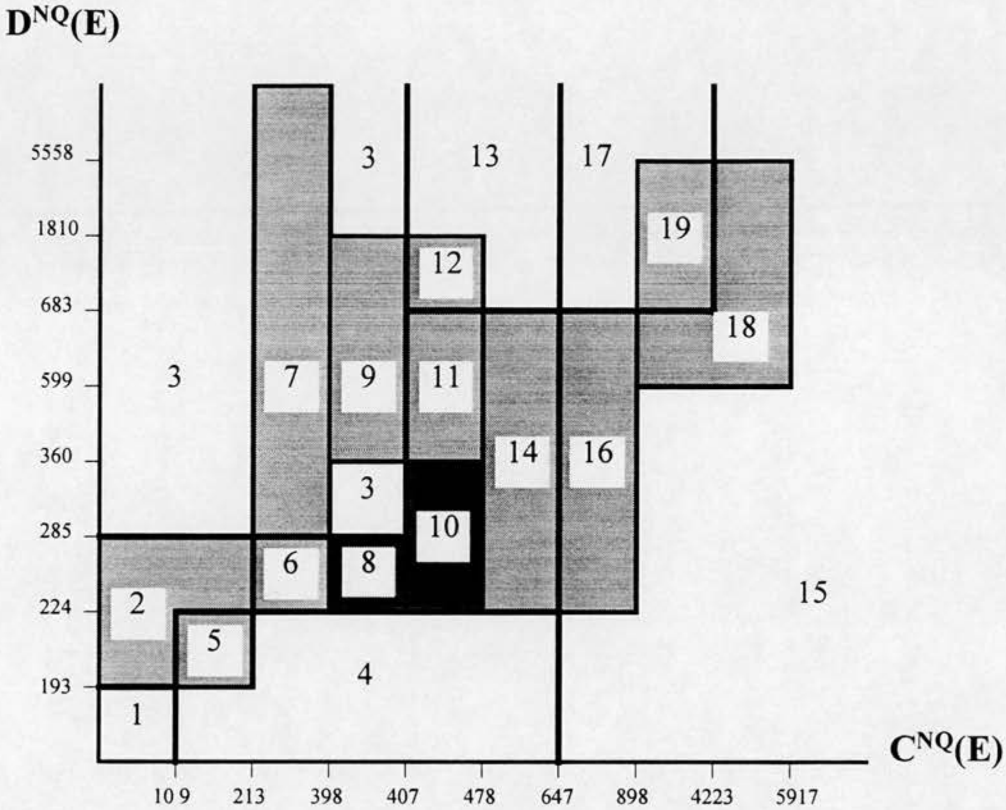

figure 4.11 :- equilibrium set for P = 0.9

1) (B_2NQ, L / L) **2)** (B_2NQ, H / L)
3) (B_2NQ, H / H) **4)** (B_2R, L / L)
5) (B_2R, L / L) or (B_2NQ, H / L) **6)** (A_1(B_2NQ / B_2R), H / L)
7) (A_1(B_2NQ/B_2R), H / L) or (B_2NQ, H / H) **8)** No pure strategy equilibrium
9) (A_1(B_2NQ/B_2Q), H / L) or (B_2NQ, H / H) **10)** No pure strategy equilibrium
11) (A_1(B_2NQ / B_2Q), H / L) **12)** (A_1(B_2NQ/B_2Q),H / L)or(B_2R,H / H)
13) (B_2R, H / H) **14)** (B_2R, H / L)
15) (B_2Q, L / L) **16)** (B_2Q, L / L) or (B_2R, H / L)
17) (B_2Q, L / L) or (B_2R, H / H) **18)** (A_1(B_2R/B_2Q), H / L) or (B_2Q, L / L)
19) (A_1(B_2R / B_2Q), H / L) or (B_2R, H / H) or (B_2Q, L / L)

It can be seen from figure 4.11 that if one of the players' costs is very low there tends to be a single equilibrium. If $C^{NQ}(E)$ is low (regions 1-3) the auditor has no incentive to work hard or qualify, regardless of the auditee's behaviour, and he therefore chooses A_2B_2NQ. If $D^{NQ}(E)$ is low (regions 1,4 and 15), the auditee has no incentive to reduce the occurrence of errors and will put a low level of effort into the internal control system. As $D^{NQ}(E)$ increases the auditee can reduce the risk of an error occurring (and the subsequent risk of NQ(E)) by putting high effort into the internal controls. The level of $D^{NQ}(E)$ when this option becomes cost effective will depend upon the auditor's testing policy. If $C^{NQ}(E) < 109$ both auditee types put in high effort if $D^{NQ}(E) > 285$. Since the auditor never qualifies in these regions any errors that the auditee fails to prevent will result in the NQ(E) outcome. In contrast if $C^{NQ}(E)$ is between about 400 and 800 the auditor uses the reasonable qualification strategy. In this case an unqualified error will only occur if both the auditee fails to prevent it and the auditor fails to detect it. With both parties limiting unqualified errors the penalties must be more extreme ($D^{NQ}(E) > 683$) before high effort becomes worthwhile.

If $C^{NQ}(E)$ is too high the auditor stops working hard and always qualifies. In this situation the threat of unqualified errors is large enough to overwhelm the other penalties. The auditor wishes to avoid this penalty at any cost - and this can only be done by always qualifying. Lemma 4.2 then tells us that there is no point in working hard with a pre-determined "always qualify" reporting strategy. The auditee is therefore facing a report which is qualified either correctly or incorrectly. By choosing high effort the auditee increases the likelihood of an incorrect qualification, which costs him less. However, for this example, this potential saving is smaller than the cost of putting in high effort. Thus once the auditor ceases to work hard the auditee ceases to work hard as well. If both players' costs are high the only equilibrium is (A_2B_2Q, L / L). With this equilibrium the audit can be regarded as a failiure - the auditor does as little work as possible, ignores the results of this work and always issues the same audit report. Furthermore, the auditors presence has no beneficial effect on the effort levels of either auditee type.

In many regions, it seems that hard work by one of the players sufficiently reduces the occurrence of unqualified errors. However each player would prefer the other to put in the necessary hard work. The hard worker will be the most concerned player (corresponding to a higher NQ(E) penalty). Thus in the top left of figure 4.11 the auditee puts in high effort whilst the auditor does nothing, and in the bottom right the auditee puts in low effort and the auditor does his best to limit the NQ(E) outcome. Regions of coordinated hard work, where both players contribute to the prevention of unqualified errors, arise when both players costs are fairly high (but not high enough to force the "no work / always qualify" equilibrium). This can be seen in regions 13,14 and 16-18.

Regions 8 and 10 in figure 4.11 have no pure strategy equilibrium. The following chapter extends the equilibrium analysis to mixed strategies to find the equilibria of these regions. Mixed equilibria tend to occur when the players interests are at odds. In these regions both players again have penalties of a similar size. However these regions fall into the area where hard work by one player suitably reduces the risk of an unqualified error. Since both player's costs are similar neither player will assume full responsibility for this level of work. The only compromise involves randomised strategies for both parties - so that each works hard with some probability.

There are two factors at work in determining the type of equilibrium that occurrs. If one player is considerably more concerned with the outcome NQ(E) he will work hard to reduce the chance of its occurrence whilst the other, relatively unconcerned player does not. To prevent this "shirking" both players must have similar penalties. In the case of very low penalties neither player will work hard, since neither party is particularly concerned by the outcome of the audit. With moderate penalties the players are unable to coordinate their hard work, since work by one of them is sufficient and each prefers the other to do the necessary work, so a mixed strategy "compromise" equilibrium ensues. For fairly high costs the players are driven to coordinate as both players work hard to prevent an unqualified error. For very high costs the players sole concern is preventing unqualified errors which leads to a "no work/always qualify" equilibrium.

5 EXTENSIONS TO THE MODEL

5.1 Introduction

This chapter considers in more detail the example developed in section 4.5. The equilibrium set of this example can be refined in two ways. Firstly, the analysis so far has been restricted to the pure strategy case. There are values of $C^{NQ}(E)$ and $D^{NQ}(E)$ for which there are no pure strategy equilibrium (regions 8 and 10 in figure 4.11). For these areas the equilibrium strategy must involve randomisation. There are also regions that have two or three pure strategy equilibria. In these cases further mixed strategies also exist. Ideally, we would like to be able to choose a single equilibrium for each value of $C^{NQ}(E)$ and $D^{NQ}(E)$ as the solution to the model. By considering the players' preferences over the potential equilibria the equilibrium set can be reduced. A similar approach can be used to determine when a mixed strategy equilibrium will be preferred to a pure strategy one.

It is shown that a consideration of the players' preferences will not always lead to a unique equilibrium. The nature of the game can range from non-cooperative (where the players prefer different equilibria) to implicitly cooperative (where they prefer the same outcome) as the outcome costs change. This motivates the analysis of section 5.4 where the model is considered as a cooperative game. In such a setting the player's incentives are identical and, if side payments and interpersonal comparisons of utility are permitted, the decision problem reduces to a single party utility maximisation. The cooperative game solution set can be compared to the non-cooperative set to consider when the players self interested behaviour resembles the group interest behaviour of the cooperative game.

Considering the players' outcome costs as independent variables reflects the current legal responsibility of the auditing profession. A change in policy in the US to proportionate liability suggests a move to more balanced penalties for the auditor and auditee. This would allocate a portion of the total penalty for failing to find an error to each party. The likely consequences of such a shift in policy are considered in section 5.5

5.2 Mixed Strategies in the Model

Parthasarathy and Raghavan [32] describe one of the algorithms which have been developed to find all the equilibrium pairs of a two person nonzero-sum non-cooperative game. However, the example of section 4.5 considered a family of games as two of the costs varied. The limited number of potential auditee strategies means that the assessment of mixed strategies can be carried out in a different fashion. We also wish to reduce the equilibrium set by pareto dominance. Both these aims can be achieved if we rank each potential outcome according to each player's preference (whilst taking into account how each player acts optimally) as the costs vary.

In a signalling game a mixed strategy equilibrium can be a less convincing equilibrium concept than a pure strategy one. Any mixed strategy equilibrium requires a certain amount of collaboration. Each player randomises to make the other players indifferent between some of their actions. Each player is willing to randomise in this way because he expects the others to randomise to make him indifferent. Thus the equilibrium depends upon the "willingness" of each player to randomise to obtain indifference. A player who is indifferent between some of his strategies has a continuum of mixed strategies - all of which, by definition, have the same expected cost. Since each player must actively collaborate in a mixed strategy equilibrium, we can consider the circumstances under which each player will choose the one mixed strategy that makes the other player indifferent and thus supports the mixed strategy equilibrium.

If there are no pure strategy equilibria then a mixed equilibrium is the only mutually stable prediction of how the game might be played. In these circumstances, the players might well be expected to randomise for the mixed strategy as the equilibrium becomes a focal point. If however, there are both pure and mixed strategies the players will be able to compare the benefits of each. If a mixed strategy costs a player more than any pure strategy equilibrium, he has no incentive to randomise. Even, if he knows that the other players are randomising, he loses nothing by choosing a different randomisation. Under these conditions it seems unreasonable that the other players should randomise in the hope that this player will also randomise. This motivates the following definition; An equilibrium is *motivationally* *unstable* if a player has no incentive to play the equilibrium strategy. A player will not participate in a mixed equilibrium if this represents his "worst" equilibrium outcome. A mixed strategy will therefore be motivationally unstable if it is pareto dominated by every pure equilibrium for some player i. For a signalling game this gives:

Definition Let G be a 2-player game with 2 equilibrium pairs (R_1, C_1), (R_2, C_2) and a mixed strategy equilibrium (R^*, C^*). Then the mixed strategy is motivationally unstable if (for utility in terms of cost):

$$U_i(R^*,C^*) > U_i(R_1, C_1)$$
$$U_i(R^*,C^*) > U_i(R_2, C_2) \qquad \text{for some player i}$$

The models developed in chapters four and six lend themselves to the analysis of mixed strategies since the auditor's costs explicitly include beliefs S_1 and S_2 about the chances of each auditee type playing H. In an equilibrium these beliefs represent the auditee's randomisation. The auditor's choice about whether to observe the auditee's action or not means that the model can exhibit both simultaneous move and sequential move behaviour. With the inclusion of mixed strategies the auditor can also choose to observe with a certain probability. The auditor's decision about whether to observe must be determined before play begins (using the auditor's ex-ante beliefs). If he chooses not to observe, then these ex-ante beliefs can be used to compare the expected cost of testing actions. However, if the auditor does observe the effort level this will change his belief about which type he is facing. These updated beliefs then need to be used to consider the expected cost of testing actions.

There are two kinds of mixed strategies in this model; These can be identified as mixed and observational strategies. In a mixed strategy the players randomise over strategies which would involve the auditor randomising between testing actions. Since the concept of a strategy effectively removes any player participation after the game begins, mixed strategies can be regarded as randomisation which takes place before play begins. In contrast, in an observational strategy the randomisation depends upon the information set reached. Thus the frequency with which a strategy occurs in equilibrium can depend on the outcome of previous actions. For example the auditor could decide to observe the effort level and then use a randomised testing strategy contingent upon the effort level observed.

In the error detection model there is the potential for the auditee to choose S_1 and S_2 so that the auditor will choose to observe and, after observing, be indifferent between testing actions for each effort level. After observing the effort level the only remaining uncertainty is about the auditee's type. Since this is represented by a single variable, P', the auditor can randomise over at most two best response actions. The auditor can then randomise between these two tests so that both auditee types are indifferent between high and low effort. Since the auditor is not indifferent until after observation he can have a different randomisation scheme for each effort level. However, we have the following result:

Lemma 5.1 *There are no motivationally stable observational strategy equilibria in the model of error detection.*

Proof For an observational strategy equilibrium to be motivationally stable the auditor must be indifferent between high and low effort. If for example the auditor's costs are greater in a low effort environment then the auditor could benefit if he can encourage either auditee type to put in high effort more frequently than in the observational equilibrium. This can be achieved by altering the randomisation after low effort to force a pooling on H equilibrium. In this case the observational strategy equilibrium is motivationally unstable.

There are therefore two requirements for an observational strategy equilibrium to exist and be motivationally stable. The auditor's expected costs must be the same for two actions after observing the effort level if he is to randomise between them, and before observation the expected cost from each randomisation must be the same if the auditor is to participate in the equilibrium. From above, after observing the effort level the auditor can be indifferent between at most two of his testing actions. The auditor can randomise after one effort level or both. If he randomises after both effort levels then either at least one action occurs in support of both mixed tests, or the auditor has 4 distinct actions. This gives three cases to consider:

(i) the auditor only randomises after one effort level.
Suppose the auditor randomises between actions W and X after observing high effort and uses Y after low effort. So the auditor is effectively randomising between the strategies $A_1(W/Y)$ and $A_1(X/Y)$. There are three cases to consider:

1) The auditor's costs for both $A_1(W/Y)$ and $A_1(X/Y)$ are increasing in S_i. Then the auditor prefers Y and low effort to either testing action after H. The mixed strategy is therefore motivationally unstable.

2) The auditor's costs for both $A_1(W/Y)$ and $A_1(X/Y)$ are decreasing in S_i. Then the auditor prefers either testing action after H to Y and low effort. The mixed strategy is therefore motivationally unstable.

3) If one strategy is increasing and the other decreasing, then the point at which the auditor's payoff functions intersect is at $S_i = 0$. Thus the auditee does not have a mixed strategy.

(ii) One action is in the support of both mixed testing strategies

The auditor's actions in this case are of the form (W,X) after H and (X,Y) after L. Now, the auditor's expected cost of an observational strategy equilibrium will be of the form:

$$C_A + P(S_1C(1,H,X)+(1-S_1)C(1,L,X)) + (1-P)(S_2C(2,H,X)+(1-S_2)C(2,L,X))$$

S_1 and S_2 are chosen so that

$PS_1C(1,H,X)+(1-P)S_2C(2,H,X) = PS_1C(1,H,W)+(1-P)S_2C(2,H,W)$ and

$P(1-S_1)C(1,L,X)+(1-P)(1-S_2)C(2,L,X)=P(1-S_1)C(1,L,Y)+(1-P)(1-S_2)C(2,L,Y)$

Thus the expected cost of the auditor's observational strategy is greater than the expected cost of A_2X. So the only possible observational strategy equilibrium will involve randomising between two actions (W,X) after observing H and two distinct actions (Y, Z) after observing low effort.

(iii) The auditor has 4 distinct actions

The auditor's expected cost (in terms of his updated belief P') can be expressed in terms of an expected error rate. Expected cost of test K after observing effort level w is (from 4.3.1) $P'C(1,w,K)+(1-P')C(2,w,K)$

$= C_{Bk} + (t^*(K)C^Q(NE)+(1-t^*(K))C^{NQ}(NE)) + (P'p_{1w}+(1-P')p_{2w})\times$
$(r^*(K)C^Q(E)+(1-r^*(K))C^{NQ}(E))-t^*(K)C^Q(NE)-(1-t^*(K))C^{NQ}(NE))$
$=C_{Bk}+p_w'(r^*(K)C^Q(E)+(1-r^*(K))C^{NQ}(E))$
$+(1-p_w')(t^*(K)C^Q(NE)+(1-t^*(K))C^{NQ}(NE))$ where $p_w' = P'p_{1w}+(1-P')\,p_{2w}$

Notice that $p_H' \in (p_{1H}, p_{2H})$ and $p_L' \in (p_{1L}, p_{2L})$ so $p_{2H} < p_{1L} \Rightarrow p_H' < p_L'$

Thus the analysis of section 4.3 can be used to consider an optimal action sequence after observing each effort level. Firstly, the proof of lemma 4.5 considers when changes in optimal action can occur. The first three conditions are repeated below:

$\mathbf{B_2NQ \rightarrow B_2R}$	$C^{NQ}(E) > C^Q(E) + f(p_{iw})\times t_2 / r_2$
$\mathbf{B_2NQ \rightarrow B_1R}$	$C^{NQ}(E) > C^Q(E) + C_B/r_1p_{iw} + f(p_{iw})\times t_1/r_1$
$\mathbf{B_2NQ \rightarrow B_2Q}$	$C^{NQ}(E) > C^Q(E) + f(p_{iw})$

Where $f(p_{iw}) = ((1-p_{iw})(C^Q(NE)-C^{NQ}(NE)) / p_{iw}$ $\qquad p_{iw} \in (0, 1)$

These show that B_2NQ is always optimal for $C^{NQ}(E) < C^{Q}(E)$. The ordering of the auditor's outcome costs in section 4.2 gives:

$$C^{Q}(NE) > C^{NQ}(NE) \qquad C^{NQ}(E) > C^{NQ}(NE) \qquad C^{Q}(NE) > C^{Q}(E) \qquad \textbf{(1)}$$

lemma 4.7 described the three potential action sequences:

$B_2NQ \rightarrow B_1R \rightarrow B_2Q$	if	$C_B < L$
$B_2NQ \rightarrow B_2R \rightarrow B_1R \rightarrow B_2Q$	if	$U > C_B > L$
$B_2NQ \rightarrow B_2R \rightarrow B_2Q$	if	$C_B > U$

where $L = (1-p_{iw})\times(C^{Q}(NE)-C^{NQ}(NE))\times(t_2r_1-t_1r_2)/\ r_2$
and $U = (1-p_{iw})\times(C^{Q}(NE)-C^{NQ}(NE))\times((1-t_2)(r_1-1)+(1-t_1)(1-r_2))/\ (1-r_2)$
Furthermore, lemma 4.8 showed that both U and L are decreasing functions of p_{iw} whilst lemma 4.5 showed that the value of $C^{NQ}(E)$ that prompts a change in action is decreasing in p_{iw} (except for $B_1R \rightarrow B_2Q$). Since $p_H' < p_L'$ we can consider the possible combinations of action sequences against H or L. Because the sequence $B_2NQ \rightarrow B_2Q$ does not occur the auditor cannot be indifferent between B_2Q and B_2NQ for any expected error rate. Thus (since all four testing actions are used) B_2NQ must be part of a mixed strategy after one effort level and B_2Q part of the other mixed strategy. This leaves four possible mixed strategies:

$(B_2R, B_2Q)\ /\ (B_2NQ, B_1R)$	$(B_1R, B_2Q)\ /\ (B_2NQ, B_2R)$
$(B_2NQ, B_1R)\ /\ (B_2R, B_2Q)$	$(B_2NQ, B_2R)\ /\ (B_1R, B_2Q)$

The auditor's cost function can be rearranged to find the values p_H^* and p_L^* that make the auditor indifferent after observing the effort level for (W, X) / (Y, Z):
$p_H^* = (C_{BX} - C_{BW} + (t(X)-t(W))(C^{Q}(NE)-C^{NQ}(NE)))\ /$
$((r(X)-r(W))(C^{NQ}(E)-C^{Q}(E))+(t(X)-t(W))(C^{Q}(NE)-C^{NQ}(NE)))$

where $C^{Q}(NE)-C^{NQ}(NE) > 0$ and $C^{NQ}(E)-C^{Q}(E) > 0$ by **(1)** above. A similar expression can be derived for p_L^* in terms of Y and Z. The terms (t(X)-t(W)) and (r(X)-r(W)) (or equivalently (t(Z)-t(Y)) etc.) will also be positive since either W is B_2NQ or X is B_2Q in each case and

$$r(B_2NQ) < r(B_2R) < r(B_1R) < r(B_2Q)$$
$$t(B_2NQ) < t(B_1R) < t(B_2R) < t(B_2Q)$$

Also, if $(C_{BX}-C_{BW})$ is negative and sufficiently large to make the entire numerator negative then $p_H^* < 0$ and the auditee cannot make the auditor indifferent between the two actions. If the numerator is positive however then clearly p_H^* decreases as $C^{NQ}(E)$ increases. The same argument shows that if the auditor can be made indifferent after observing low effort then p_L^* must be decreasing in $C^{NQ}(E)$. So for auditor indifference both p_H^* and p_L^* must be decreasing in $C^{NQ}(E)$

It can be shown that the auditee is unable to satisfy the requirements for the auditor to be indifferent after observing the effort level (for randomising) and make the auditor indifferent between his expected cost after high and low effort as $C^{NQ}(E)$ changes. Assume that the auditor is indifferent after high effort and low effort. Then both p_H* and p_L* must be decreasing in $C^{NQ}(E)$. Consider the actions B_2Q and B_2NQ; from the above one of these actions will be part of the mixed strategy after observing H and the other after L. For the auditor's expected cost to be the same after both effort levels we have 2 cases to consider:

(a) B_2Q occurs after observing H and B_2NQ after L
Then we require $C(p_H*,B_2Q) = C(p_L*,B_2NQ)$
$\Leftrightarrow C^Q(NE)-p_H*(C^Q(NE)-C^Q(E)) = C^{NQ}(NE)+p_L*(C^{NQ}(E)-C^{NQ}(NE))$
$p_H* = (C^Q(NE)-C^{NQ}(NE))-p_L*(C^{NQ}(E)-C^{NQ}(NE)) / (C^Q(NE)-C^Q(E))$
so (1) $\Rightarrow$ as p_L* increases p_H* decreases. Thus as $C^{NQ}(E)$ varies if an observational equilibrium exists it cannot be motivationally stable

(b) B_2Q occurs after observing L and B_2NQ after H
Then we require $C(p_L*,B_2Q) = C(p_H*,B_2NQ)$
$\Leftrightarrow C^Q(NE)-p_L*(C^Q(NE)-C^Q(E)) = C^{NQ}(NE)+p_H*(C^{NQ}(E)-C^{NQ}(NE))$
$p_L* = (C^Q(NE)-C^{NQ}(NE))-p_H*(C^{NQ}(E)-C^{NQ}(NE)) / (C^Q(NE)-C^Q(E))$
so (1) $\Rightarrow$ as p_H* increases p_L* decreases. Thus as $C^{NQ}(E)$ varies if an observational equilibrium exists it cannot be motivationally stable

Attention can therefore be focused upon mixed strategies, where the auditor can be considered to randomise before play begins. As a consequence of lemma 5.1 the auditor's mixed strategy must contain at least one A_2-test strategy as otherwise the auditor is effectively randomising after observing the effort level. Suppose the auditor randomises against a type i auditee using strategies S_1, S_2, ... , S_n with probabilities q_1^i, q_2^i, ..., q_n^i respectively. Since the sum of these probabilities must be 1 (the auditor always chooses some strategy) we can re-write $q_n^i = (1-q_1^i-q_2^i-..-q_{n-1}^i)$. To make a type i auditee indifferent these probabilities must satisfy:

$$q_1^iD_{iH}(S_1(H))+ q_2^iD_{iH}(S_2(H))+.....+ (1-q_1^i-q_2^i-...-q_{n-1}^i)D_{iH}(S_n(H)) = q_1^iD_{iL}(S_1(L))+ q_2^iD_{iL}(S_2(L))+.....+ (1-q_1^i-q_2^i-...-q_{n-1}^i)D_{iL}(S_n(L)) \quad \textbf{(5.2.1)}$$

Now, at least one of these strategies does not involve the A-test and hence uses the same test against both high and low effort. We can assume without loss of generality that this strategy is S_n (since the labelling of the strategies is unimportant). Expression 5.2.1 can be rearranged to find q_1^i.

$$q_1^i = (D_{iL}(S_n(L)) - D_{iH}(S_n(H)) \,/\, ((D_{iH}(S_1(H)) - D_{iH}(S_n(H)) - D_{iL}(S_1(L) + D_{iH}(S_n(L)) + \Sigma_{j=2..n-1}\, q_j^{\,i}\,(D_{iH}(S_j(H)) - D_{iH}(S_n(H)) - D_{iL}(S_j(L) + D_{iL}(S_n(L))) \quad \textbf{(5.2.2)}$$

Where each $D_{iw}(\,-\,)$ is a function of the form $x + yD^{NQ}(E)$ for some constants x and y. Since these are linear functions of $D^{NQ}(E)$, $q_1^{\,i}$ is of the form:

$$(a+bD^{NQ}(E)) \,/\, (c+dD^{NQ}(E))$$

We are interested in whether or not there can be an equilibrium in which both auditee types randomise. For this to occur, the auditor's mixed strategy must be identical for both types. In other words $q_1^{\,1} = q_1^{\,2}$, $q_2^{\,1} = q_2^{\,2}$ and so on. Since the equilibrium set is being considered as the cost of the outcome NQ(E) varies we require $q_j^{\,1} = q_j^{\,2}$ for each strategy Sj as $D^{NQ}(E)$ varies. Now,

$$(a+bD^{NQ}(E))/(c+dD^{NQ}(E)) = (e+fD^{NQ}(E))/(g+hD^{NQ}(E)) \text{ as } D^{NQ}(E) \text{ varies} \quad \textbf{(5.2.3)}$$

$\Leftrightarrow a/e = b/f = c/g = d/h = \alpha$ for some constant α.

A necessary condition for the auditor's randomising to be the same for both types is $q_1^{\,1} = q_1^{\,2}$ for a mixed strategy over any number of pure strategies.

Lemma 5.2 $q_1^{\,1} \neq q_1^{\,2}$ *as $D^{NQ}(E)$ varies.*

Proof $q_1^{\,1}$ and $q_1^{\,2}$ are of the form (a+bD)/(c+dD) and (e+fD)/(g+hD). From expression 5.2.2 for $q_1^{\,i}$ the numerator is $D_{iL}(S_n(L)) - D_{iH}(S_n(H))$ where strategy S_n does not involve the A-test. Thus S_n specifies the same test Z for both high and low effort levels where $Z \in \{B_2NQ, B_2R, B_1R, B_2Q\}$. Now, since there are only four testing actions, the denominator of the expression for $q_1^{\,i}$ will have a $D^{NQ}(E)$ term of the form:

$$(x_L(B_1R)p_{iL}+x_H(B_1R)p_{iH})(1-r_1) + (x_L(B_2R)p_{iL}+x_H(B_2R)p_{iH})(1-r_2) + (x_L(NQ)p_{iL}+x_H(NQ)p_{iH})$$

where $x_H(K)$ is the sum of those q_j (except q_1) where Sj(H) = strategy K.
Assume that the auditor's mixed strategy is the same for both types apart from for S_1. So $q_j^{\,1} = q_j^{\,2}$ for j=2,....,n. Then
$p_{1H} < p_{2H}$ and $p_{1L} < p_{2L} \Leftrightarrow$ the $D^{NQ}(E)$ term in the denominator of $q_1^{\,2}$ is greater than the same term in $q_1^{\,1}$. i.e. h > d.

if $Z \neq B_2Q$ then
$b = (p_{1L} - p_{1H}) \times (1 - r(Z))D^{NQ}(E)$
$f = (p_{2L} - p_{2H}) \times (1 - r(Z))D^{NQ}(E)$
$(p_{1L} - p_{1H}) > (p_{2L} - p_{2H}) \Leftrightarrow b > f$
Now $q_1^1 = q_1^2 \Rightarrow b/f = d/h$. But $d/h < 1 < b/f$
Therefore $q_1^1 \neq q_1^2$ and the auditor cannot make both auditee types indifferent.

if $Z = B_2Q$ then $b = f = 0$. $q_1^1 = q_1^2 \Rightarrow a/e = d/h$ where
$a = (p_{1L} - p_{1H})(C^Q(E) - C^Q(NE))$
$e = (p_{2L} - p_{2H})(C^Q(E) - C^Q(NE))$
$(p_{1L} - p_{1H}) > (p_{2L} - p_{2H}) \Leftrightarrow a/e > 1$
Now $q_1^1 = q_1^2 \Rightarrow a/e = d/h$. But $d/h < 1 < a/e$
So $q_1^1 \neq q_1^2$ and the auditor cannot make both auditee types indifferent.

Therefore for mixed strategies over n pure strategies if one auditee type is indifferent the other type will have a pure strategy best response. Geometrically this means that we can restrict our attention to the edges of the $S_1 \times S_2$ unit square. When randomising against one auditee type the auditor will never need to use more than two pure strategies - one that the auditee prefers to play H against and one that he prefers to play L against. We therefore need only consider mixed strategies with a support of two pure strategies. One of the following cases will occur:

- There are no pure strategy equilibria. In this case a mixed strategy can be regarded as the solution to the game.

- If there is one pure strategy equilibrium this will be considered the solution of the game.

- There are two or more pure strategy equilibria. Firstly, the player's preferences (which are needed to find mixed strategies) over the equilibria can be determined. If both players prefer the same equilibrium then this can be regarded as the solution to the game. Otherwise, the benefits of both pure and mixed equilibria need to be compared.

From lemma 5.2 we know that the only mixed strategies that need to be considered have a support of two pure strategies. The auditor can make one auditee type indifferent whilst the other type will have a pure strategy best response. The auditor will not randomise between observation strategies unless he is indifferent after observation of either effort level. Lemma 5.1 has shown that if S_1 and S_2 are chosen for this indifference then the auditor's expected cost will differ between effort levels and the resulting observational strategy is motivationally unstable. The only mixed strategies that can therefore occur will involve either two A_2 test strategies or one A_2 test and one A_1 test strategy. These mixed strategies will only be considered if there are two or more pareto optimal pure strategy equilibria.

A mixed strategy only needs to be considered if it is motivationally stable - the auditee will not choose a mixed signalling strategy if it costs him more than either pure strategy signal. There are two ways that two pure strategy equilibria can occur. Either one auditee puts in the same effort in both - for example (X, H/H) and (Y, H/L) or there are two pure strategy pooling equilibria; (X, H / H) and (Y, L / L). In a mixed strategy the auditee must consider the auditor's optimal responses as S_i varies. If the only optimal tests are part of the pure equilibria we have the following:

Lemma 5.3 *Any mixed strategy with a support of pure equilibrium strategies of the form* $A_1(X / Y)$ *and* $A_2(X)$ *is motivationally unstable*

Proof Suppose, without loss of generality, that the two pure strategy equilibria are $(A_1(X / Y), H/L)$ and $(A_2(X), H/H)$. Then since they are equilibria:
$D_2(A_1(X/Y))$ is increasing in $S_2 \Leftrightarrow D_{2L}(A_1(X/Y)) < D_{2H}(A_1(X/Y)) = D_{2H}(X)$
In a mixed strategy the auditee is indifferent between high and low effort
$\Leftrightarrow xD_{2L}(A_1(X/Y))+(1-x)D_{2L}(X) = xD_{2H}(X)+(1-x)D_{2H}(X)$
$\Leftrightarrow$ the mixed strategy costs the auditee $D_{2H}(X) > D_{2L}(A_1(X / Y))$. Thus the mixed strategy equilibrium costs the auditee the same as the pure strategy equilibrium which he least prefers.

The same argument shows that a mixed strategy involving $A_1(X / Y)$ and $A_2(Y)$ is motivationally unstable if both strategies a pure equilibrium strategies. Mixed strategy equilibria will only be compared to pure strategy equilibria if the pure strategy analysis does not lead to a unique pareto optimal equilibrium. The above lemmas show that there are only two cases where a mixed strategy could improve upon the pure strategy equilibrium payoffs; If the two equilibria both involve pooling then the auditee type that participates in a mixed strategy will depend on the auditor's optimal strategy set. If the two equilibria involve one pooling equilibrium and a separating equilibrium with a different testing strategy then the mixed strategy may be a good compromise. If the auditor uses an A_2 test strategy his expected cost can be expressed in terms of an expected error rate:

$$\text{Expected } C(A_2(K)) = C_{Bk}+p_w''(r^*(K)C^Q(E)+(1-r^*(K))C^{NQ}(E)) +(1-p_w'')(t^*(K)C^Q(NE)+(1-t^*(K))C^{NQ}(NE)) \quad \textbf{(5.2.4)}$$

$$\text{where } p_w'' = PS_1p_{1H}+(1-P)S_2p_{2H}+P(1-S_1)p_{1L}+(1-P)(1-S_2)p_{2L} \quad \textbf{(5.2.5)}$$

This can be used to classify the auditor's optimal strategy set as S_1 and S_2 vary. The auditor's A_2-test strategies will form one of three sequences for increasing $C^{NQ}(E)$ described in lemma 4.7. The points at which the optimal action changes are decreasing in p_w''. For auditor indifference the auditee can choose S_i to prompt such a change. This approach can also determine which A_1-test strategies can be optimal. If the auditor observes the effort level this will change his expected error rates. By Bayes' rule:

$$p_H' = (PS_1p_{1H}+ (1-P)S_2p_{2H}) / (PS_1 + (1-P)S_2) \quad \textbf{(5.2.6)}$$

$$p_L' = (P(1-S_1)p_{1H}+ (1-P)(1-S_2)p_{2H}) / (P(1-S_1) + (1-P)(1-S_2)) \quad \textbf{(5.2.7)}$$

Where $p_H' \in (p_{1H}, p_{2H})$ and $p_L' \in (p_{1L}, p_{2L})$. $p_{2H} < p_{1L} \Rightarrow p_H' < p_L'$

Equation 5.2.5 can be expressed in terms of p_H' and p_L':

$$p_w'' = ((PS_1+(1-P)S_2))p_H' +(P(1-S_1)+(1-P)(1-S_2))p_L' \quad \textbf{(5.2.8)}$$

so $p_H' < p_L' \Rightarrow p_H' < p_w'' < p_L'$. Thus the auditor's expected error rate is decreased after observing high effort and increased after observing low effort. Thus after observing H the auditor's optimal action will be earlier in the sequence, whilst after observing L it will be further on. The conditions U and L of lemma 4.7 can be used to determine all the auditor's optimal responses as S_1 and S_2 vary. The optimal responses for the example of section 4.5 are considered in the mixed strategy analysis of section 5.3

5.3 Refining the Equilibrium Set of the Example

Pareto dominance

The equilibrium set for the example of section 4.5 can be reduced by considering the pareto optimality of the equilibrium strategy pairs in each region. This can be achieved by considering the player's preferences. To compare the players' preferences the cost of each potential pure strategy outcome can be considered as a function of NQ(E). This can be done for a particular value of P. The discussion below will develop the equilibrium sets shown in figure 4.11, where there is a 90% chance that the auditee is type 1. Each outcome will be a linear function of the cost associated with NQ(E). As NQ(E) increases the minimum cost outcome can be found, which in the case of the auditor's outcome costs will be his optimal strategy. If all of the outcome costs are ordered a table can be generated to illustrate how the players' preferences change with the cost NQ(E). In the following tables 1 = best (the least cost)

L / L	109	126	217	230	250	264	579	647	716	908	999	1091	
A_2B_2NQ	1	2	3	4	7	8	10	10	10	10	10	10	10
A_2B_2R	2	1	1	1	1	1	1	1	2	5	5	6	8
A_2B_1R	8	8	8	8	8	7	7	7	7	7	6	5	5
A_2B_2Q	4	4	4	3	3	3	3	2	1	1	1	1	1
B_2NQ/ B_2R	3	3	2	2	2	2	2	3	6	6	7	9	9
B_2NQ/ B_1R	9	9	9	9	9	9	8	8	8	8	8	7	6
B_2NQ/ B_2Q	5	5	5	5	4	4	4	4	3	2	2	2	2
B_2R/ B_1R	9	9	9	9	9	9	8	8	8	8	8	7	6
B_2R/ B_2Q	5	5	5	5	4	4	4	4	3	2	2	2	2
B_1R/ B_2Q	5	5	5	5	4	4	4	4	3	2	2	2	2

table 5.1 :- auditor's action preference after L/L

Table 5.1 illustrates the auditor's optimal response to pooling on L. If $C^{NQ}(E) < 109$ his optimal response is B_2NQ, for $C^{NQ}(E) \in (109, 647)$ B_2R, and if $C^{NQ}(E) > 647$ B_2Q. The preferences for the remaining cases are given in appendix B. The auditor's best responses for each pure strategy effort level can also be compared to consider the auditor's preferences between pure strategy equilibria. This gives the following:

$C^{NQ}(E)$	< 1548	$\in(1548, 3449)$	$\in(3449, 4717)$	> 4717
H / H	1	2	3	3
H / L	2	1	1	2
L / L	3	3	2	1

table 5.2 :- auditor's equilibrium preference

For all sufficiently large $C^{NQ}(E)$ the auditor chooses B_2Q. In this case his preferred auditee strategy is L/L - giving an equilibrium of (B_2Q, L/L) since this reduces the risk that the auditors qualification will be incorrect. The auditee's preferences over auditor strategies can be considered in a similar way. However to consider the auditee's preferences between equilibria we also need to take into account the auditee's optimal action for each strategy. For this example this gives the following:

Type 1	193	224	440	873	1810	5558	27840	
A_2B_2NQ	L 1	H 1	H 1	H 3	H 5	H 7	H 7	H 7
A_2B_2R	L 6	L 6	H 6	H 6	H 3	H 3	H 6	H 6
A_2B_1R	H 4	H 4	H 4	H 1	H 1	H 1	H 1	H 5
A_2B_2Q	L 8	L 8	L 8	L 8	L 8	L 5	L 3	L 1
B_2NQ/ B_2R	H 2	H 1	H 1	H 3	H 5	H 7	H 7	H 7
B_2NQ/B_2Q	H 2	H 1	H 1	H 3	H 5	L 5	L 3	L 1
B_2R/ B_2Q	H 7	H 7	H 6	H 6	H 3	H 3	L 3	L 1
B_1R/ B_2Q	H 4	H 4	H 4	H 1	H 1	H 1	H 1	L 1

table 5.3:- type 1 auditee preferences and optimal responses to tests

Type 2	110	193	224	231	247	254	262	285	300	302	
A_2B_2NQ	L 1	L 1	L 1	L 1	L 2	L 2	L 4	L 5	H 5	H 6	H 6
A_2B_2R	L 4	L 5	L 5	L 3	L 3	L 3	L 2	L 2	L 2	L 2	L 3
A_2B_1R	L 5	L 4	L 2	L 2	L 1	L 1	L 1	L 1	L 1	L 1	L 1
A_2B_2Q	L 8	L 8	L 8	L 8	L 8	L 8	L 8	L 8	L 8	L 8	L 8
B_2NQ/ B_2R	H 2	H 2	H 3	L 3	L 3	L 3	L 2	L 2	L 2	L 2	L 3
B_2NQ/ B_2Q	H 2	H 2	H 3	H 5	H 5	H 6	H 6	H 6	H 5	H 6	H 6
B_2R/ B_2Q	H 7	H 7	H 7	H 7	H 7	H 7	H 7	H 7	H 7	H 5	H 5
B_1R/ B_2Q	H 6	H 6	H 6	H 6	H 6	H 5	H 5	H 4	H 4	H 4	H 2

table 5.4:- type 2 auditee preferences and optimal responses to tests

These tables can be used to reduce the equilibrium sets of figure 4.11 by pareto domination of equilibria. Consider for example region 5 of figure 4.11 where $C^{NQ}(E) \in (109, 213)$ and $D^{NQ}(E) \in (193, 224)$. There are two pure strategy equilibria in this region; (B_2NQ, H / L) and (B_2R, L/L). However table 5.3 shows that a type 1 auditee prefers (B_2NQ, H) to (B_2R, L) if $D^{NQ}(E) < 873$ whilst a type 2 auditee prefers (B_2NQ, L) to B_2R, L) if $D^{NQ}(E) < 285$. Table 5.2 shows that the auditor prefers a separating H/L equilibrium to pooling on L if $C^{NQ}(E) < 4717$. Since both auditee types and the auditor prefer (B_2NQ, H / L) the other equilibrium (B_2R L/L) is pareto dominated in this region. Similar comparisons can be made in other regions. The reduced equilibrium set for the example is given below:

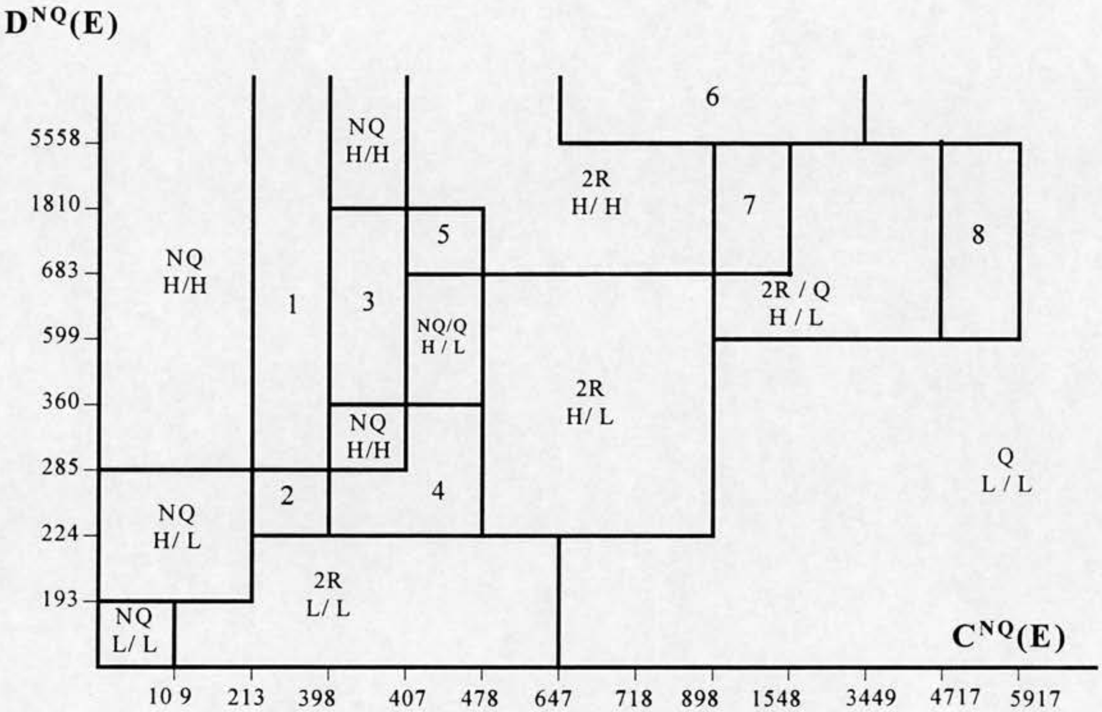

figure 5.1 - reduced equilibrium set for P = 0.9

1) (B_2NQ, H / H) or (A_1(B_2NQ/B_2R), H / L)
2) (A_1 (B_2NQ/B_2R), H / L)
3) (B_2NQ, H / H) or (A_1 (B_2NQ/B_2Q), H / L)
4) **RND**[B_2R / A_1(B_2NQ/B_2Q), H / H, H / L]
5) (B_2R, H / H) or (A_1(B_2NQ/B_2Q), H / L)
6) (B_2Q, L/L) or (B_2R, H/H)
7) (A_1 (B_2R/B_2Q), H / L) or (B_2R, H / H)
8) (A_1 (B_2R/B_2Q), H / L) or (B_2Q, L / L)

If one of the players' costs is very small there tends to be a single equilibrium - for example for low $C^{NQ}(E)$ the auditor will always play NQ regardless of the auditee's behaviour. If $C^{NQ}(E)$ is too high, the auditor stops working hard and always qualifies.

In this situation the auditee also stops working since the report will be qualified regardless of the effort put into the internal control system. In regions where there are two equilibria, one of which involves the A_1-test, one of the auditee types prefers to signal his type to the auditor. This need not be the "low error rate" type 1 auditee. In region 8 a type 1 auditee does prefer to show his type by putting high effort into the internal control system, in which case the auditor will qualify reasonably. However in region 1, where both players have low costs the auditor will prefer to never qualify since there is a chance that this is a false positive. When $C^{NQ}(E)$ is low the auditor is more concerned with avoiding incorrectly qualifying than failing to find errors. Because type 2 has a higher error rate he would prefer to play L (since high effort still leaves a fairly high chance of an error). This convinces the auditor that he is facing a type 2 auditee in which case he should qualify if his testing finds an error.

In figure 5.1 the regions with more than one equilibrium are those where the players prefer different outcomes. In this case, since the auditor can choose not to observe the auditee's action, we might expect the mixed strategy equilibrium to be a fair compromise.

Mixed strategies

Regions 1 and 3-8 in figure 5.1 have more than one pure strategy pareto optimal equilibrium. We therefore need to determine whether a mixed strategy can be a compromise equilibrium. With P = 0.9 the auditor's expected error rate pw" lies between 0.075 (both types play H) and 0.365 (both types play L). $C_B > U(0.075)$ so the optimal action sequence is $NQ \rightarrow B_2R \rightarrow Q$ and the optimal A_1 test strategies are $\{A_1(NQ/B_2R), A_1(NQ/Q)$ and $A_1(B_2R/Q)\}$. We can also determine when each auditor strategy has a decreasing cost as S_i increases. This occurs when $C(A_1(i, H, K_H/K_L) < C(A_1(i, L, K_H/K_L)$

$$\Leftrightarrow C^{NQ}(E) > \big(C_B(K_H)-C_B(K_L)+(p_{iH}\, r^*(K_H)-p_{iL}\, r^*(K_L))C^Q(E) + ((1-p_{iH})t^*(K_H)-(1-p_{iL})t^*(K_L))C^Q(NE) +((1-p_{iH})(1-t^*(K_H))-(1-p_{iL})(1-t^*(K_L)))C^{NQ}(NE)\big) / (p_{iL}(1-r^*(K_L))-p_{iH}(1-r^*(K_H))) \quad \textbf{(5.3.1)}$$

AUDITOR'S STRATEGY	Decreasing in S_1	Decreasing in S_2
A_2B_2NQ	$C^{NQ}(E) > 10$	$C^{NQ}(E) > 10$
A_2B_2R	Always	Always
A_2B_1R	Always	Always
A_2B_2Q	Never	Never
A_1B_2NQ/B_2R	Always	$C^{NQ}(E) < 166$
A_1B_2NQ/B_1R	$C^{NQ}(E) < 2548$	$C^{NQ}(E) < 331$
A_1B_2NQ/B_2Q	$C^{NQ}(E) < 1530$	$C^{NQ}(E) < 243$
A_1B_2R/B_1R	Always	$C^{NQ}(E) < 1739$
A_1B_2R/B_2Q	$C^{NQ}(E) < 5273$	$C^{NQ}(E) < 714$
A_1B_1R/B_2Q	$C^{NQ}(E) < 2040$	Always

table 5.5 - auditee's effort level influences auditor costs

The auditor's optimal strategies as S_1 and S_2 vary can be found from his preference tables given above and in appendix B. This can be used to find the points of intersection of the auditor's minimum cost hull. Each of these points will be a potential mixed strategy.

$C^{NQ}(E) \in$	**type 1 randomising** L / L ... H / L	**type 2 randomising** H / L ... H / H
< 109	NQ	NQ
(109, 213)	$B_2R \rightarrow NQ$	NQ
(213, 367)	$B_2R \rightarrow A_1(NQ/B_2R)$	$A_1(NQ/B_2R) \rightarrow NQ$
(367, 398)	$B_2R \rightarrow A_1(NQ/B_2R)$	$A_1(NQ/B_2R) \rightarrow B_2R \rightarrow NQ$
(398, 407)	$B_2R \rightarrow A_1(NQ/B_2R) \rightarrow A_1(NQ/Q)$	$A_1(NQ/Q) \rightarrow B_2R \rightarrow NQ$
(407, 478)	$B_2R \rightarrow A_1(NQ/B_2R) \rightarrow A_1(NQ/Q)$	$A_1(NQ/Q) \rightarrow B_2R$
(478, 647)	B_2R	B_2R
(647, 718)	$Q \rightarrow B_2R$	B_2R
(718, 898)	$Q \rightarrow A_1(B_2R/Q) \rightarrow B_2R$	B_2R
(898, 4065)	$Q \rightarrow A_1(B_2R/Q)$	$A_1(B_2R/Q) \rightarrow B_2R$
(4065, 4223)	$Q \rightarrow A_1(B_2R/Q)$	$A_1(B_2R/Q) \rightarrow Q \rightarrow B_2R$
(4223, 5917)	$Q \rightarrow A_1(B_2R/Q)$	$A_1(B_2R/Q) \rightarrow Q$
> 5917	Q	Q

table 5.6 :- auditor's optimal action set for mixed effort levels

In region 1 of figure 5.1 $C^{NQ}(E) \in (213, 398)$ and $D^{NQ}(E) > 285$. If $C^{NQ}(E) < 367$ the auditor's optimal responses as S_2 varies are $A_1(NQ/B_2R) \rightarrow NQ$. By lemma 5.4 this mixed strategy will be motivationally unstable as it will cost the auditee as much as his "worst" pure strategy equilibrium. If $C^{NQ}(E) > 367$ a third strategy, B_2R, is optimal for some S_2. However for $C^{NQ}(E)$ in this range $A_1(NQ/B_2R)$ is increasing in S_2 whilst B_2NQ is decreasing. Hence both the vertices $A_1(NQ/B_2R) \rightarrow B_2R$ and $B_2R \rightarrow NQ$ will cost the auditor more than either pure equilibrium. Any mixed strategy in this region is therefore motivationally unstable since either one or both players has no incentive to randomise.

In region 3 $C^{NQ}(E) \in (398,407)$ and $D^{NQ}(E) \in (360,1810)$. The auditor's optimal responses as S_2 varies are $A_1(NQ/Q) \rightarrow B_2R \rightarrow NQ$. Once again table 5.5 shows that for this interval of $C^{NQ}(E)$ $A_1(NQ/Q)$ is increasing and B_2NQ is decreasing. Any mixed strategy equilibrium will therefore cost the auditor more than either pure strategy equilibrium. Region 4 has no pure strategy equilibria and therefore a mixed strategy equilibrium is used. The auditor's responses to S_1 varying are such that the auditor prefers a pure strategy to any mixed strategy. The auditor will therefore randomise against type 2, where the mixed strategy minimises his costs. If the auditor plays B_2R with probability q_2 and $A_1(B_2NQ/B_2Q)$ with probability $(1-q_2)$ then:

$$q_2 = (-108 + 0.3D^{NQ}(E)) / (-135.3 + 0.34D^{NQ}(E))$$

$$S_2 = (C_A+39.75-0.1C^{NQ}(E)+\mathbf{P}(-63.525+0.14C^{NQ}(E))) / (1-\mathbf{P})(69.9 - 0.34C^{NQ}(E))$$

In region 5 $C^{NQ}(E) \in (407,478)$ and $D^{NQ}(E) \in (683, 1810)$. The auditor's optimal responses as S_2 varies are $A_1(NQ/Q) \rightarrow B_2R$, the same as in region 4. In region 4 the auditee's cost from $A_1(NQ/Q)$ was decreasing in S_2 and the cost from B_2R was increasing, so that there were no pure strategy equilibrium. In region 5 the opposite is true - $A_1(NQ/Q)$ has an increasing cost and B_2R decreasing, giving two pure strategy equilibria where type 2 prefers ($A_1(NQ/Q)$ H/L). However, $D_{2H}(A_1(NQ/Q)) > D_{2H}(B_2R)$ if $D^{NQ}(E) > 300$. This means that in region 5 the intersection of these two auditee cost functions (his expected return from the mixed strategy equilibrium) is greater than either pure strategy equilibrium. The auditee therefore has no incentive to randomise.

In region 6 $C^{NQ}(E) \in (647, 3449)$ and $D^{NQ}(E) > 5558$. Both auditee types prefer (B_2Q, L/L) whilst the auditor prefers the equilibrium (B_2R H/H). If $C^{NQ}(E) < 898$ the auditor has a single best response, B_2R, as S_2 varies so any mixed strategy must involve a type 1 auditee. However the auditor's optimal response as S_1 varies begins with B_2Q, which is increasing in S_1 and ends with B_2R which is decreasing in S_1. Thus any mixed strategy equilibrium will cost the auditor more than either pure equilibrium. If $C^{NQ}(E) > 898$ the auditor's optimal strategy as S_1 varies is $Q \rightarrow A_1(B_2R/Q)$ and as S_2 varies is $A_1(B_2R/Q) \rightarrow B_2R$. Any mixed strategy involving type 2 will cost the auditee the same as (B_2R H/H), the least preferred equilibrium. A type 2 auditee therefore has no incentive to randomise. Since B_2Q is increasing in S_1, any mixed strategy involving B_2Q and $A_1(B_2R/Q)$ must cost the auditor more than (B_2Q, L/L), the auditor's worst equilibrium and hence more than (B_2R, H/H). The auditor therefore has no incentive to participate in a mixed strategy equilibrium.

In region 7 $C^{NQ}(E) \in (898, 1548)$ and $D^{NQ}(E) \in (683, 5558)$. The pure equilibria are ($A_1(B_2R/Q)$, H/L) and (B_2R, H/H). The auditor's optimal responses as S_2 varies are the two pure equilibrium strategies and therefore by lemma 5.3 any mixed strategy equilibrium is motivationally unstable. Similarly, in region 8 the two pure strategy equilibria are ($A_1(B_2R/B_2Q)$, H/L) and (B_2Q, L/L) and the auditor's optimal responses as S_1 varies are the two equilibrium strategies. Lemma 5.3 therefore shows that this mixed strategy is unsatisfactory.

In this example the only mixed strategy equilibrium occurs when there are no pure strategy equilibria. There are regions of costs which have two pure strategy pareto optimal equilibrium pairs. In these regions mixed strategies are an unsatisfactory compromise as it costs one of the players more than either pure strategy equilibrium. In some regions therefore there is not a unique suggestion for how the game should be played. This highlights the strategic interaction in the model. In some cases it is impossible to determine the best action without knowing what the other party intends to do. Now that the equilibrium set has been categorised, this example can be used to consider how the assumptions of noncooperation and the allocation of liability influence the equilibrium set.

5.4 A Cooperative Version of the Game

The players' behaviour in a nonzero-sum game can vary between implicitly cooperative (where both players prefer the same outcome) to non-cooperative where the players' interests are at odds. It is interesting to see to what extent the players' collusion approaches the extreme case of cooperative games. In these the players make binding pre-play commitments and are able to compare utility and share out the payoffs. For the audit model detailed above, we need to consider the combined payoffs for both players and the actions that minimise this cost. This will enable us to determine strategy pairs that give points on the pareto efficient boundary of the cooperative payoff region. The Nash bargaining solution of the game is not considered here since we only wish to contrast the occurrence of strategy pairs in a cooperative and non-cooperative setting.

It will be assumed that once the game has started the players' actions are restricted to those available within the game. In other words the players are free to communicate before play begins but afterwards they can only communicate through their actions, in particular the auditee effort level. If this were not the case then the first step taken by the auditee would be to inform the auditor of his type. This would reduce the game to a game of cooperative costly perfect information which has been analysed in Hatherly, Nadeau and Thomas [24]

The costs and probabilities for the model will be the same as those considered in section 4.5. In a similar manner we can consider a family of games for varying $C^{NQ}(E)$ and $D^{NQ}(E)$. We can derive expressions for the expected costs of each of the potential outcomes. Some of the results for the auditor's strategies will still hold - for example it is in neither players interest for the auditor to use B_1 if he intends to qualify regardless (Q). If the joint costs of failing to find an error are $\Omega = C^{NQ}(E) + D^{NQ}(E)$ then:

$$C(1, H, B_2NQ) = 74 + 0.05\Omega \qquad C(1, L, B_2NQ) = 13 + 0.35\Omega$$
$$C(1, H, B_2R) = 132.7 + 0.01\Omega \qquad C(1, L, B_2R) = 123.9 + 0.07\Omega$$
$$C(1, H, B_1R) = 166.3 + 0.0025\Omega \qquad C(1, L, B_1R) = 179.1 + 0.0175\Omega$$
$$C(1, H, B_2Q) = 278 \qquad C(1, L, B_2Q) = 241$$

From these costs it can be seen that the action pairs (A_2B_2Q, H) and (A_2B_1R, L) are dominated. By comparing these costs the optimal actions are found to be the following:

B_2NQ, L	if $\Omega < 203$
B_2NQ, H	if $\Omega \in (203, 1468)$
B_2R, H	if $\Omega \in (1468, 4480)$
B_1R, H	if $\Omega \in (4480, 29880)$
B_2Q, L	if $\Omega > 29880$

Similar expressions can be derived for a type 2 auditee:

$C(2, H, B_2NQ) =$	$69 + 0.3\Omega$	$C(2, L, B_2NQ) =$	$11.5 + 0.5\Omega$
$C(2, H, B_2R) =$	$171.2 + 0.06\Omega$	$C(2, L, B_2R) =$	$147 + 0.1\Omega$
$C(2, H, B_1R) =$	$222.54 + 0.015\Omega$	$C(2, L, B_1R) =$	$213 + 0.025\Omega$
$C(2, H, B_2Q) =$	293	$C(2, L, B_2Q) =$	250

In this case only the action (B_2Q, H) is dominated. This leads to the following optimal strategies:

B_2NQ, L	if $\Omega < 288$
B_2NQ, H	if $\Omega \in (288, 390)$
B_2R, L	if $\Omega \in (390, 605)$
B_2R, H	if $\Omega \in (605, 1141)$
B_1R, H	if $\Omega \in (1141, 1831)$
B_2Q, L	if $\Omega > 1831$

This tells us the solution for each auditee type, we now need to consider the costs in terms of **P**, since the auditor will not know which type he is facing. In many cases the separate optimal strategies are sufficient - for example if $\Omega < 203$ then the auditor's best action is B_2NQ and both auditee types will play L. There is the possibility that for some P the optimal strategy will be neither of the separate optimal strategies. However, since the players' interests are no longer at odds there is no advantage to be gained by randomising. We can compare strategies pairwise to find regions of the plane $\mathbf{P}\times\Omega$ in the same way that was done for the auditor's non-cooperative strategies. The relevant strategy comparisons are given below:

COOPERATIVE VERSION OF THE GAME	
STRATEGY COMPARISON	**INEQUALITY ON $\Omega = C^{NQ}(E) + D^{NQ}(E)$**
$(B_2NQ, H/H) \leq (B_2R, H/L)$	$\Omega \leq (78-19.3P) / (0.2-0.16P)$
$(B_2NQ, H/H) \leq (B_2NQ/B_2R, H/L)$	$\Omega \leq (83-78P) / (0.2-0.2P)$
$(B_2R, H/L) \leq (B_2NQ/B_2R, H/L)$	$\Omega \geq (58.7P-5) / (0.04P)$
$(B_2NQ, H/H) \leq (B_2R, H/H)$	$\Omega \leq (102.2-43.5P) / (0.24-0.2P)$
$(B_2R, H/H) \leq (B_2NQ/B_2R, H/L)$	$\Omega \leq (19.2+34.5P) / (0.04)$
$(B_2R, H/H) \leq (B_1R, H/H)$	$\Omega \leq (51.34-17.74P) / (0.045-0.0375P)$
$(B_1R, H/H) \leq (B_2Q\ L/L)$	$\Omega \leq (27.46+47.24P) / (0.015-0.0125P)$
$(B_2R, H/H) \leq (B_2R/B_2Q, H/L)$	$\Omega \leq (83.8-78.8P) / (0.06(1-P))$
$(B_1R, H/H) \leq (B_2R/B_2Q, H/L)$	$\Omega \geq\leq (32.46-61.06P) / (0.015-0.0225P)$
$(B_2Q, L/L) \leq (B_2R/B_2Q, H/L)$	$\Omega \geq (-5+108.3P) / (0.01P)$
$(B_1R, H/H) \leq (B_1R/B_2Q, H/L)$	$\Omega \leq (32.46-27.46P) / (0.015(1-P))$
$(B_2Q, L/L) \leq (B_1R/B_2Q, H/L)$	$\Omega \geq (-5+74.7P) / (0.0025\ P)$

table 5.7 :- action comparisons for the cooperative game

These comparisons can be more clearly understood graphically

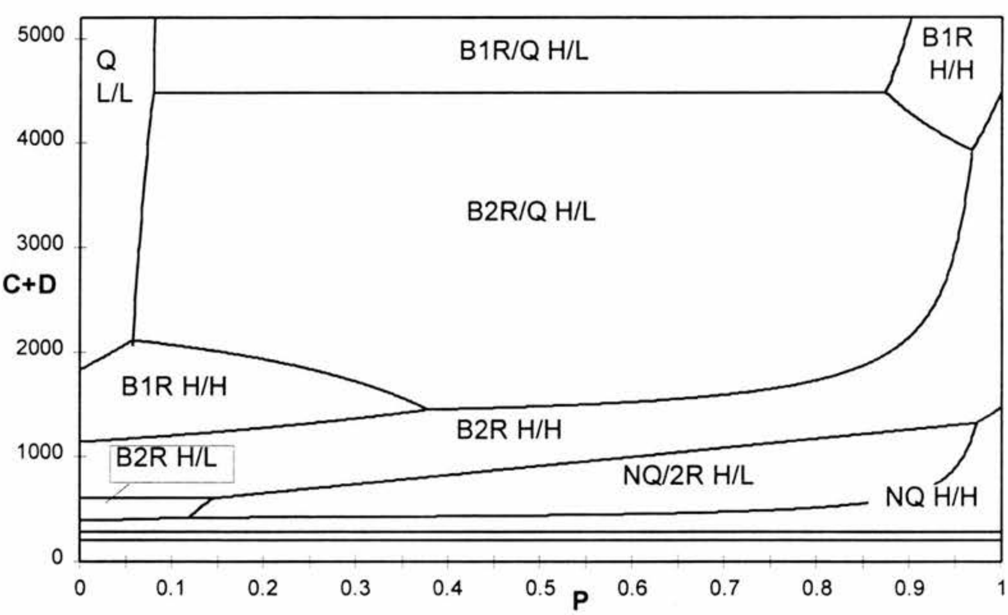

figure 5.2 :- optimal actions for the cooperative game

For very low Ω, B_2NQ is optimal. If $\Omega < 203$ then $(B_2NQ, L/L)$ is optimal, whilst if Ω is between 203 and 288 $(B_2NQ, H/L)$ is optimal. For very high values of Ω the boundary between $(B_2Q, L/L)$ and $(B_1R/Q\ H/L)$ curves across until for all sufficiently large Ω (around 28,000) B_2Q L/L is optimal for all P. For a specific starting probability P = 0.9 the optimal strategies are the following:

B_2NQ, L / L	if $\Omega < 203$
B_2NQ, H / L	if $\Omega \in (203, 288)$
B_2NQ, H / H	if $\Omega \in (288, 640)$
$A_1(B_2NQ/B_2R)$, H / L	if $\Omega \in (640, 1256)$
B_2R, H / H	if $\Omega \in (1256, 2147)$
$A_1(B_2R/B_2Q)$, H / L	if $\Omega \in (2147, 4285)$
B_1R, H / H	if $\Omega \in (4285, 5164)$
$A_1(B_1R/B_2Q)$, H / L	if $\Omega \in (5164, 27658)$
B_2Q, L / L	if $\Omega > 27658$

There are no optimal mixed strategies in the cooperative case. The costs must be extremely high before (B_2Q, L / L) becomes the solution ($\Omega > 27658$). In the non-cooperative case this equilibrium can occur if $C^{NQ}(E) > 647$. There are also a wide range of costs with the solution ($A_1(B_1R/B_2Q)$, H / L) in which the auditor (against type 1) and a type 1 auditee work hard and a type 2 auditee signals his type by playing Low. This can be regarded as the best outcome for the shareholders as "work hard" strategies will occur with probability P = 0.9. These payoff regions suggest that the policy of raising costs to encourage hard work is much more effective in a cooperative setting. There are also costs for which the equilibrium is the same in both the cooperative and non-cooperative cases. For example the solution ($A_1(B_2R/B_2Q)$, H/L) occurs in both cases if the following three conditions are met:

$2147 < C^{NQ}(E) + D^{NQ}(E) < 4285 \qquad C^{NQ}(E) > 898 \qquad D^{NQ}(E) > 599$

5.5 Limiting Auditor Liability

In the previous chapter, the impact of increasing the cost of not qualifying a material error on the equilibrium pairs of the model was considered. The level of this cost is driven by the courts; either directly as the result of legal action by the shareholders or indirectly through out of court settlements. A policy of joint and several liability has in many cases resulted in very high penalties for the auditor and relatively low penalties for the auditee. The example analysed suggests that setting these costs in this way may reduce the effectiveness of the audit since the auditor will be willing to qualify whenever possible. A change in policy in the US to proportionate liability, discussed in section 1.6, would suggest a move towards more balanced penalties for the auditor and auditee. In this section the likely consequences of such a shift in policy are explored.

To date the model of error detection has not considered as a constraint any interdependence between the penalties imposed upon the auditor and the auditee. A policy of proportionate liability will allocate responsibility for a proportion β of the losses to the auditor. If the total penalty is Ω this gives $C^{NQ}(E) = \beta\Omega$ and $D^{NQ}(E) = (1-\beta)\Omega$. The sets of equilibrium pairs for this game will be the same as in figure 5.1 above. However, the boundaries of the regions will now be determined by β and Ω. For example for $(B_2NQ, L / L)$ to be in equilibrium we had the following conditions:

$$C^{NQ}(E) < 109 \text{ and } D^{NQ}(E) < 193$$

These can be rearranged to give:

$\beta < 109/\Omega$ and $\beta > 1-193/\Omega$ these conditions intersect when $\Omega = 302$

Similar conditions can be derived for the other equilibrium regions of figure 5.1. Each boundary is a condition on either $C^{NQ}(E)$ or $D^{NQ}(E)$. Every boundary condition can therefore be re-expressed in terms of β. The regions of the equilibrium set are transformed as follows:

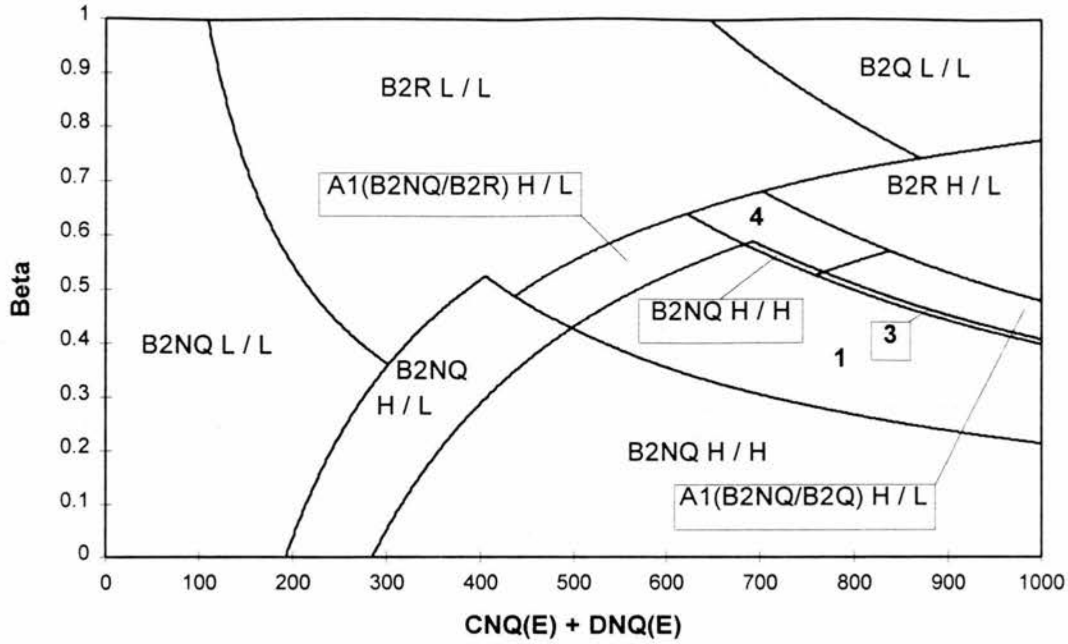

figure 5.3 :- equilibrium pairs for low Ω

In figure 5.4 the equilibrium pairs (B_2NQ H/H), **1**, (B_2NQ H/H), (B2R H/H) still occur for small β under region **6**. However the boundaries of these regions are all asymptotic to β=0.

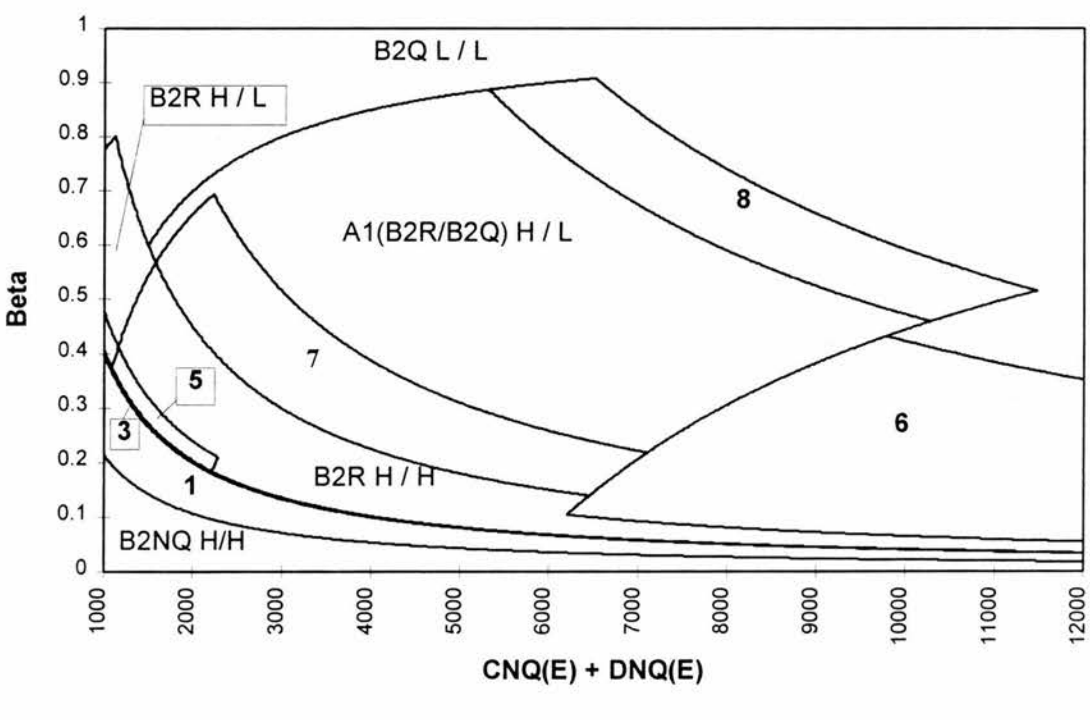

figure 5.4 :- equilibrium pairs for high Ω

The numbered regions in figures 5.3and 5.4 are regions where there is a choice of equilibrium.

1) (B_2NQ, H / H) or (A_1(B_2NQ/B_2R), H / L)
2) (A_1 (B_2NQ/B_2R), H / L)
3) (B_2NQ, H / H) or (A_1 (B_2NQ/B_2Q), H / L)
4) **RND**[B_2R / A_1(B_2NQ/B_2Q), H / H, H / L]
5) (B_2R, H / H) or (A_1(B_2NQ/B_2Q), H / L)
6) (B_2Q, L/L) or (B_2R, H/H)
7) (A_1 (B_2R/B_2Q), H / L) or (B_2R, H / H)
8) (A_1 (B_2R/B_2Q), H / L) or (B_2Q, L / L)

As we would expect from the analysis in the previous chapter, the two extremes do not motivate both players. For example if $\beta = 1$ (the auditor is solely responsible) then the auditee never uses high effort and the auditor acts to minimise his risk of incurring the NQ(E) penalty (this involves always qualifying for $\Omega > 647$).This reduces the usefulness of the audit. A qualified opinion from an auditor who will always qualify his opinion effectively tells the shareholders nothing about the state of the accounts. If $\beta = 0$ (the auditee is solely responsible) then the auditor never tests and never qualifies and the auditee always uses high effort to reduce the occurrence of errors. In this situation the auditor contributes nothing to the prevention and detection of errors. For $\beta > 0.91$ the auditor never uses the A_1 test. Here any considerations about auditee type are overwhelmed by the threat of NQ(E) and the auditor will concentrate on minimising this risk.

The only mixed strategy equilibrium (region 4) occurs for $\beta \in (0.53, 0.63)$ (see figure 5.3). This shows that even if the cost of not qualifying an error is shared equally amongst the two players then their interests can still be at odds. A region of particular interest is the "no work" equilibrium of (A_2B_2Q, L / L). This has been shown to occur for all sufficiently large $C^{NQ}(E)$ in the previous chapter. If liability is shared in this way the occurrence of the "no work" equilibrium depends upon both the size of the penalty Ω and the proportion β. It can be seen from figure 5.4 that if $\beta = 0.5$ (so both players share the penalty equally) this "no work" equilibrium occurs for very high penalties ($\Omega > 10\ 000$) whereas if the auditor is solely responsible this equilibrium occurs for low penalties ($\Omega > 647$). This suggests that both high penalties and a high β give the auditor the incentive to always qualify.

Even with an even division of the liability, $\beta = 0.5$, the equilibrium set in this setting differs from the solution set of the cooperative game. In particular, in the cooperative game the auditor resorts to the "no work" equilibrium when the combined costs Ω are above 28000. This illustrates that in the setting of proportionate liability the players are motivated by self interest, whereas in a cooperative setting they are concerned with the welfare of the group. As Ω increases, the auditor can limit his expected costs by choosing to always qualify, at the expense of the auditee. If both players are cooperating they are willing to work hard to reduce the occurrence of errors for a much wider range of penalties.

6 A SIGNALLING GAME COMBINING FRAUD AND ERROR DETECTION

6.1 Introduction

Some of the more recent models of the auditing process focus upon the strategic interaction between an auditor and a potentially fraudulent manager. There are two important factors that need to be included in this strategic interaction. Firstly, fraud and its detection do not occur in isolation. The auditor is also concerned with the detection of unintentional errors and both parties payoffs will be influenced by whether an error has occurred or not. Secondly, the auditee's willingness to commit fraud will differ between companies and will not be known to the auditor. A model of incomplete information is therefore developed in which the auditor must divide his resources between error detection and fraud detection.

This chapter develops a signalling game model of the audit where the auditee can be one of two "types" determined by his difficulty in committing a fraudulent act. The auditee then has a choice over the level of effort to put into maintaining the internal control system and whether or not to commit a fraudulent act. The auditor chooses the level of substantive testing and subsequent in depth testing to carry out before issuing an audit report. It is shown that no equilibrium exist where the manager always reveals his type to the auditor. The equilibrium set is shown to be dependent on the probability that the manager is the type who finds it easier to commit fraud.

The equilibrium behaviour of the model can be used to assess the components of the Audit risk model. This assessment of Audit Risk will depend upon the modelling assumptions. In particular the components of inherent risk and control risk will depend critically on the assumptions made about the nature of the fraudulent activity. Shibano [34] assumes that a dishonest employee cannot act outside the internal control system (as otherwise the control risk must be assessed as 1). Patterson [33] considers the alternative case where the auditee can override the controls. These two approaches can be considered to deal with two potential sources of fraud; management and employee. This ability to override the system permits serious fraud to go undetected and will therefore be of most concern to the shareholders. Attention here will be focused on the strategic interaction between an auditor and a manager with the potential to commit fraud that will go undetected by the internal controls.

This model considers the players' interaction in a setting where the occurrence of both random and intentional errors affect their outcomes. The term fraud is used in this model to describe the intentional introduction of an irregularity which will be of benefit to the auditee if undetected. The model developed requires only that this action introduces irregularities into the accounting system that can be detected by the auditor and that discovery of such an action will be to the detriment of the auditee. These requirements are sufficiently broad that the "fraud" action can be considered as either an intentional misstatement or a misappropriation of assets. The auditee must decide how much effort to put into the control system and whether to commit fraud. These two objectives can be at odds since a good internal control system will reduce the risk of errors going undetected but will make it more difficult to conceal irregularities. The auditor must divide his resources between substantive testing and in-depth investigation of suspect accounts.

The next section outlines the actions available to each player in more detail and considers the likelihood of each potential outcome. Section 3 then analyses this model using a game theory to find the equilibrium behaviour. Section 4 categorises the equilibrium set by considering the costs of the players' actions to be variables. Section 5 considers a numerical example to illustrate the results of the previous sections. The effects of varying the costs of actions on the equilibrium behaviour of the example are considered. Whilst lowering the cost of the in-depth testing will reduce the equilibrium fraud rate, a decrease in the cost of substantive testing may have the opposite effect. The components of audit risk are assessed and it is shown that measures to reduce the risk of errors going undetected may increase the risk of fraud going undetected.

6.2 A Model of the Audit with the Potential for Fraud

This model considers the interaction between two parties in an audit; the auditor and the manager of a company who has the potential to commit fraud. In a model of fraud detection the players' interests are clearly at odds. If fraud occurs then the auditor would prefer this to be found whereas the auditee would prefer it to go unnoticed. It is recognised that auditor tests that are designed to detect errors may be of little use in detecting fraud whereas in depth investigations of suspect accounts are a very costly means of searching for unintentional errors. The auditor therefore has to strike a balance between error detection and fraud detection. To incorporate this into a model we must consider both the occurrence of errors and the potential for fraud. There will be some effort involved on the behalf of the auditee in concealing fraudulent activity. The amount of effort required will depend on the difficulty in over-riding the internal control system. The auditee can be classified into two types by his difficulty in concealing this fraudulent activity. It is assumed that the two types have the same internal control system (and thus the same chance of errors occurring). Thus the extra difficulty experienced by, say, the type 1 auditee in concealing fraud will be undetectable to the auditor; it represents either a "conscience cost" or an "inexperience cost".

Inherent Risk in a fraudulent setting will be a function of the auditee's willingness to commit fraud. This term can be assessed by considering the probability that the auditee commits fraud in a mixed strategy equilibrium. The auditee's tendency to commit fraud will differ between companies and will be unobservable to the auditor. This uncertainty about the auditee's motivations will also influence the auditor's testing strategy and the assessment of inherent risk. This uncertainty is modelled here by considering the audit to be a game of incomplete information.

There are five actions during the game. The auditee has two action choices to consider. Firstly, a choice of effort level to put into maintaining the internal control system {H or L}. Secondly, he must decide whether or not to commit fraud {F or NF}. With two auditee types this gives 16 pure strategies. The effort level put in to maintaining the internal control system will affect the cost of concealing fraud. It will also be observed by the auditor and will be regarded as a potential signal of type. Thus it may be optimal to send a high effort signal when committing fraud, even though this will require more effort to conceal, as the effort level may convince the auditor of the auditee's honesty. In many cases fraud will only occur as part of a mixed strategy. The fraud decision can best be described by the associated probability of type i committing fraud f_{iH} after effort level H. The auditee strategies are contingent upon type and so will be expressed in the form (type 1 strategy / type 2 strategy). For example: H f_{1H} / H f_{2H}

The auditor has three action choices that occur in sequence. It is assumed that the auditor can costlessly observe the auditee's effort level and so these actions can be conditional on the observed effort level. Alternatively there may be a cost associated with this observation, but the observation forms a mandatory part of the audit. In which case, since the auditor will always incur this cost, it can be ignored. The auditor's first decision involves the amount of broad substantive testing to use. This can be represented by a choice of tests $\{B_1$ or $B_2\}$, where B_1 is more effective but also more time-consuming. These tests can detect irregularities (as a result of fraud) with a reduced effectiveness

Secondly the auditor must decide whether to carry out an in depth investigation for fraud $\{D_1\}$ or not $\{D_2\}$ based on the results of the B-test. We will assume that the B-test cannot distinguish between fraud and an error. The D-test on the other hand is only effective if it has some irregularity to focus upon so it is assumed that the auditor only has the D-test option if the B-test has reported an error. Thus the B-test can be regarded as "detection" whilst the D-test classifies into random errors, fraud and false-positives.

Finally the auditor issues an audit report. This report can be qualified in two ways. As in models of error detection such as Fellingham and Newman [16], the auditor may qualify his report due to the existence of large unintentional errors $\{Q_2\}$. In this setting the auditor may also make the more serious accusation that he has found evidence of fraudulent activity $\{Q_1\}$. It is assumed that the auditor will make this decision on the evidence available to him. In particular, the auditor is unable to issue qualification Q_1 without evidence to support this claim. The qualification decision is therefore entirely determined by the evidence found by the tests. If neither test finds any irregularity the audit report will not be qualified {NQ}.

With the qualification strategy determined from the results of the tests there are four auditor tests contingent upon the effort level observed - giving 16 strategies in all. The auditor's strategies can be expressed in the form (B_aD_b if H / B_cD_d if L). None of these strategies are dominated in the general case as each strategy can be a best response to some auditee fraud rate. There are also costs associated with effort for each player. As each choice is between two levels of effort we can assume that the lower level has a cost of zero. This gives the following:

D_H - the cost to the auditee of putting high effort into the internal control system

D_{iHF} - the cost to auditee type i of concealing fraud

C_B - the cost to the auditee of conducting test B_1

C_D - the cost to the auditor of conducting test D_1

An ordering can be imposed upon the auditee's cost of committing fraud. This cost will be higher in an environment with good internal controls so $D_{iHF} > D_{iLF}$. To emphasise the difference between the two auditee types we will assume that $D_{1LF} > D_{2HF}$. Finally, it seems reasonable to assume that the additional difficulty experienced by a type 1 auditee will be exascerbated in a high effort internal control environment. This can be modelled by assuming that $(D_{1HF}-D_{2HF}) > (D_{1LF}-D_{2LF})$. The other action costs can be considered as variables. A costly action will only be used if its cost can be offset against potential savings in the outcomes. For example, it costs the auditor time and effort to conduct a higher level of extensive tests B_1. However, this level of testing increases the chance of errors being found and thus reduces the chance of the costly outcome NQ(E) in favour of Q(E).

There is a clear link between the cost of productive actions and the differences between the costs of outcomes. Test B_1 can be made a more attractive option either by increasing the difference between the costs of outcomes or by decreasing the cost of the test. Treating these action costs as variables will allow us to consider how the optimal actions in the model are influenced by these costs.

With two stages of testing there will be an interplay between "detection" and "classification". For example the B-test may issue a false-positive report that subsequent in depth testing would reveal. Since the B-test is unable to distinguish between fraud and error occurrence it will be considered to report on the existence of some material irregularity "M". If this report is positive then the auditor has some area to focus his in-depth testing on. The D-test will then either confirm or disagree with the B-test report. Test D_2 (no in depth testing) will agree with the results of the B-test by default. It will be assumed that the D_1 test has no risk of a false negative since if the B-test has correctly found an error then subsequent investigation will not reverse this opinion. Furthermore, it will be assumed that the D_1 test will always detect a false positive since a spurious error will not stand up to detailed investigation. To model this interaction between the stages of testing define an "agreement" probability $\phi_j(A, B) \in [0, 1]$ where:

$\phi_j(A, B)$ = Prob (test D_j reports A given B-test report A and actual state B)

These probabilities only need to be considered if the B-test finds an irregularity. The above assumptions mean that:

$$\phi_2(M, NE) = 1 \qquad \phi_2(M, E) = 1$$
$$\phi_1(M, NE) = 0 \qquad \phi_1(M, E) = 1$$

There are a number of other chance events during the course of the game. The chance of unintentional errors being undetected by the internal control system will depend upon the level of internal controls. The B-test has a risk of both a false positive (t_a) and a false negative ($1-r_a$). The B-test can also collect evidence of fraudulent activity, although this cannot be identified as such without the D-test. However since some effort is taken to conceal this evidence the B-test will have a reduced chance (r_a') of such detection. It will be assumed that test B_2 is at least as affected by this as test B_1. Finally, even if the B-test finds evidence of fraud a subsequent more detailed investigation may fail to determine that the irregularity is intentional ($1-v_b$). Define the following probabilities for the auditor test B_aD_b after effort level w:

$p_w = P(\text{ Error} \mid \text{Effort level w})$

$t_a = P(M \mid NE, B_a)$

$r_a = P(M \mid E, B_a)$

$r_a' = P(M \mid F, B_a)$

$v_b = P(MF \mid F \text{ found by B-test}, D_b)$

Using this we can now develop expressions for the likelihood of each auditee qualification opinion for each of the four possible error-states. It will be assumed that the occurrence (and detection) of irregularities from fraud and errors are independent events. In other words the existence of an irregularity as a result of fraud will have no influence on existence of irregularities caused by errors. It is also assumed that, once a level of substantive testing is chosen, it is carried out in it's entirety. If this were not the case the auditor may stop after detecting an irregularity and incur only part of the cost of testing. Furthermore in situations where irregularities from both fraud and errors exist we would need to consider which irregularity is discovered first.

	Q_1	**Q_2**	**NQ**
NE	0	$t_a\phi_b(M, NE)$	$1-t_a\phi_b(M, NE))$
E	0	r_a	$(1-r_a)$
F	$r_a'v_b$	$r_a'(1-v_b)$	$(1-r_a')$
E&F	$r_a'v_b$	$r_a'(1-v_b) + (1-r_a')r_a$	$(1-r_a')(1-r_a)$

table 6.1 :- probability of each audit opinion for a given error state

For a given chance of error p and chance of fraud f_i the chances of each state occurring are:

$\text{Prob(NE)} = (1-p)(1-f_i)$ $\qquad$ $\text{Prob(E)} = p(1-f_i)$

$\text{Prob(F)} = (1-p)f_i$ $\qquad$ $\text{Prob(E\&F)} = pf_i$

Since Q_1 can only occur when evidence of fraud has been found there are 10 outcomes for each of the players. However, if both fraud and error occur then both players are more concerned with fraud. The auditee either gains more if the fraud is unnoticed or loses more if noticed. Each of the qualification opinions will cost the auditor more (or as much) if fraud occurs. The simplifying assumption will therefore be made that the outcomes for both fraud and error are the same as those for fraud. This leaves seven outcomes for each player:

Q2(NE), NQ(NE), Q2(E), NQ(E), Q1(F), Q2(F), NQ(F)

An ordering over these outcomes can be imposed to reflect the players' preferences.

- The auditee can benefit by successfully committing fraud; $D^{NQ}(NE) > D^{NQ}(F)$
- Fraud is partially successful if it is misclassified; $D^{NQ}(NE) > D^{Q2}(F) > D^{NQ}(F)$
- The auditee's reputation is damaged by a wrong qualification; $D^{Q2}(NE) > D^{NQ}(NE)$
- The damage to reputation of a correct qualification outweighs the potential damages of an error; $D^{Q2}(E) > D^{NQ}(E)$
- Damages are such that the auditee would prefer no errors; $D^{NQ}(E) > D^{Q2}(NE)$
- The auditee's costs are highest if fraud is detected; $D^{Q1}(F) > D^{Q2}(E)$

- The auditor's best outcome is no error and no qualification.
- Correctly qualifying may lose future custom from this client and this is more likely if fraud has been discovered; $C^{Q1}(F) > C^{Q2}(E) > C^{NQ}(NE)$
- The auditors reputation is damaged by incorrect qualification; $C^{Q2}(NE) > C^{Q1}(F)$
- Failing to find an error costs more than incorrectly qualifying; $C^{NQ}(E) > C^{Q2}(NE)$
- Incorrectly identifying fraud is worse than failing to find errors; $C^{Q2}(F) > C^{NQ}(E)$
- Detecting fraud but misclassifying it as an error is better than failing to notice anything; $C^{NQ}(F) > C^{Q2}(F)$

These imply the following orderings:

Auditor:

$C^{NQ}(F) > C^{Q2}(F) > C^{NQ}(E) > C^{Q2}(NE) > C^{Q1}(F) > C^{Q2}(E) > C^{NQ}(NE)$

Auditee:

$D^{Q1}(F) > D^{Q2}(E) > D^{NQ}(E) > D^{Q2}(NE) > D^{NQ}(NE) > D^{Q2}(F) > D^{NQ}(F)$

We can now express the type i auditee's expected cost of effort level H and fraud rate f_{1H} against a test B_aD_b

$$\begin{aligned} D_i\,(H\, f_{iH} \mid B_aD_b) = {} & D_H + f_{iH}D_{iFH} + p_H(1-f_{iH})(r_aD^{Q2}(E)+(1-r_a)D^{NQ}(E)) \\ & + (1-p_H)(1-f_{iH})(t_a\phi_b(M \mid M, NE)D^{Q2}(NE)+(1-t_a\phi_b(M \mid M, NE))D^{NQ}(NE)) \\ & + f_ir_a'v_bD^{Q1}(F) + f_i(r_a'(1-v_b)+r_a(1-r_a')p_H)D^{Q2}(F) + f_i(1-r_a')(1-p_Hr_a)D^{NQ}(F) \quad \textbf{(6.2.1)} \end{aligned}$$

We can impose some further restrictions on the costs to make the model more representative of the audit setting. Firstly, if the auditee's payoff against the D_1-test is decreasing in f_{iH} then the auditee never worries about getting caught. Either the penalty for getting caught is small or the chance of the auditor finding the fraud is negligible. In either case there is no strategic interaction between the players. Similarly, if the auditee's costs are increasing in f_{iH} even when the auditor doesn't use D_1 then fraud will never occur.

R1 $\quad \partial D_i(B_aD_1)/\, \partial f_{iH} > 0$ [Deterrent]

R2 $\quad \partial D_i(B_aD_2)/\, \partial f_{iH} < 0$ [Incentive]

A change in the level of substantive testing can effect a non fraudulent auditee's payoffs in two ways. B_1 has a higher chance of successfully finding error (which will increase the auditee's costs) and a lower chance of a false positive. Condition R3 ensures that the potential loss of reputation resulting from an incorrect qualification (and the chance of a false positive from the B-test) are such that the risk of a false positive is of the most concern to a non-fraudulent auditee. We would expect that there would be a substantial penalty to the auditee if fraud is noticed. Condition R4 ensures that there is a difference in the auditee's payoff between unsuccessful and partially successful fraud.

R3 $D^{Q}(NE) > D^{NQ}(NE) + p_w(r_1-r_2)(D^{Q}(E)-D^{NQ}(E))/(1-p_w)(t_2-t_1)$

R4 $D^{Q1}(F) > D^{Q2}(F) + ((1-r_2')(1-p_w r_2)-(1-r_1')(1-p_w r_1))(D^{Q2}(F)-D^{NQ}(F))/v_1 r_2'$

The auditor benefits from the D_1-test in two ways; through the ability to classify an irregularity as fraud and to re-classify false positive reports from the B-test. These two benefits are at odds; if the chance of fraud occurring increases then the chance of issuing a false positive decreases. If the ability to re-classify false positives is the more useful to the auditor we will have a model where test D_1 is used when there is no fraud. As the fraud rate increases the auditor will stop using the depth test. Condition R5 ensures that fraud detection is the primary motivation behind using the D-test.

R5 $$C^{Q2}(F) > C^{Q1}(F) + \frac{(1-p_w)t_a(r_a'+(1-r_a')p_w r_a)(C^{NQ}(E)-C^{NQ}(NE))}{v_1 r_a'(p_w r_a+(1-p_w)t_a)}$$

The timing of events in the model is illustrated by figure 6.1. The numbered nodes represent the following actions: (1) a casting move determines whether the auditee is type 1 (high conscience cost) or type 2. (2) the auditee (who knows his type) chooses an effort level $w \in \{H, L\}$ to put into maintaining the internal controls. (3) the auditee can act fraudulently {F} or honestly {NF}. (4) there is a chance that random errors go unnoticed by the internal controls. (5) the auditor observes the auditee's effort level w. He does not know the auditee's fraud rate or if there are random errors present (the auditor does not know at which of the four points marked "5" he is at). He chooses a level of substantive testing $\{B_1, B_2\}$which will report "M" if some irregularity is detected. (6) the auditor can investigate matters further $\{D_1\}$ or base his opinion on the results of the B-test. (7) if there are no intentional errors D_1 will confirm an accurate finding of the B-test (7a) or highlight a false positive (7b). If fraud has occurred (and is detected by the B-test), D_1 may gather enough evidence to warrant a more serious qualification Q_1.

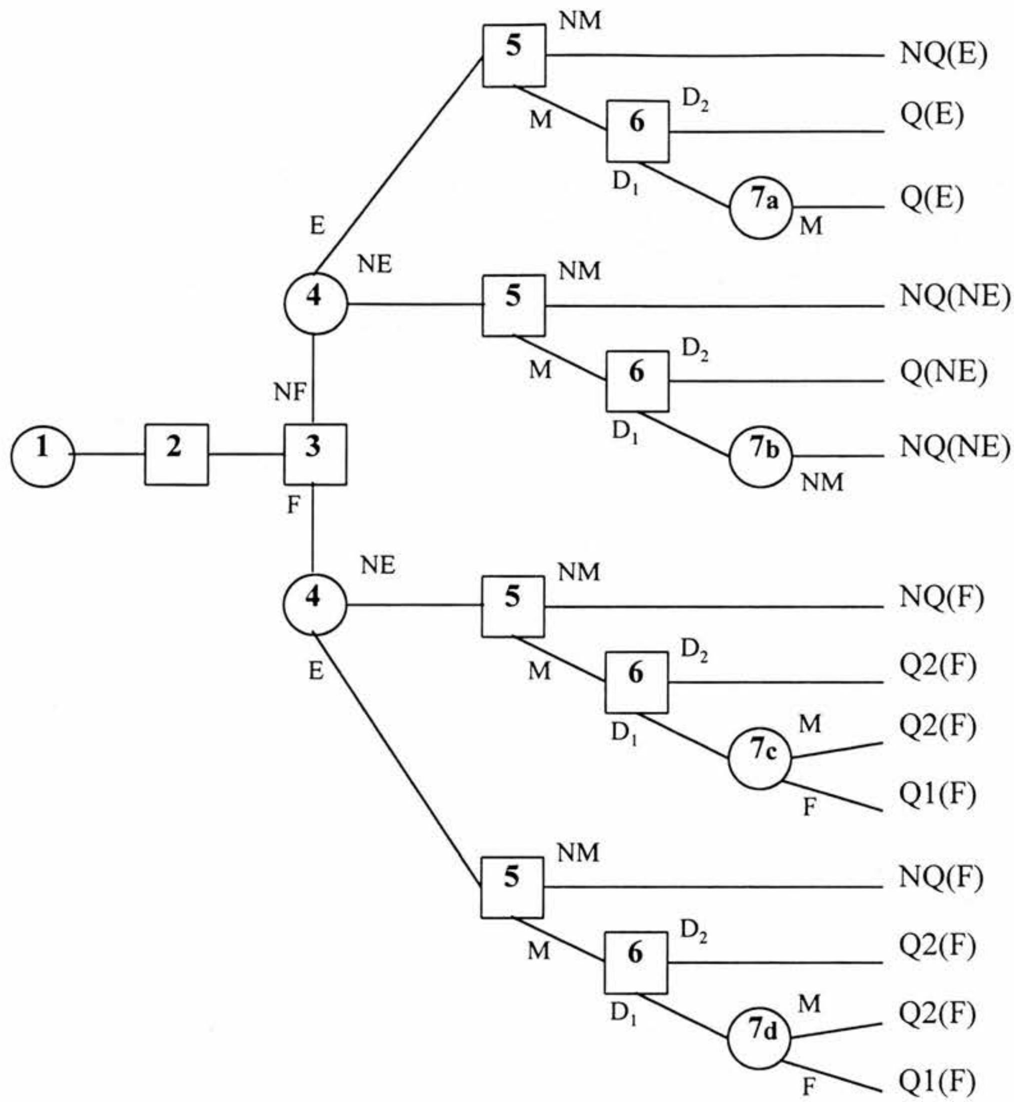

figure 6.1 :- sequence of actions / events

We can express the auditor's expected cost of test B_aD_b against auditee type i and effort level H. The expected D-test cost to the auditor will depend on the results of the B-test since the cost C_D will be incurred only if the B-test finds a material irregularity. The probability $\underline{Ma}$ that the B-test reports a material irregularity can be derived from table 6.1.

$$\underline{Ma} = p_w r_a(1-f_{iw} r_a') + f_{iw} r_a' + (1-p_w)(1-f_{iw})t_a \qquad \textbf{(6.2.2)}$$

The auditor's expected cost becomes:

$$C\,(B_aD_b,\, f_i \mid H\,) = C_{Ba} + \underline{Ma}C_D + p_H(1\text{-}f_i)(r_aC^{Q2}(E)+(1\text{-}r_a)C^{NQ}(E)) + (1\text{-}p_H)(1\text{-}f_i)(t_a\phi_b(M \mid M, NE)C^{Q2}(NE)+(1\text{-}t_a\phi_b(M \mid M, NE))C^{NQ}(NE)) + f_ir_a'v_bC^{Q1}(F) + f_i(r_a'(1\text{-}v_b)+r_a(1\text{-}r_a')p_H)C^{Q2}(F) + f_i(1\text{-}r_a')(1\text{-}p_Hr_a)C^{NQ}(F) \quad \mathbf{(6.2.3)}$$

The auditor can compare these expected costs to find the optimal test. The auditor will also need to consider what actions the auditee may have taken. Define the following probabilities to express the auditor's beliefs about the auditee's actions;

P = Prob(type 1) $\quad S_1$ = Prob(H | type 1) $\quad S_2$ = Prob(H | type 2)

F_{iw} = Prob(Fraud | effort level w, type i)

These are ex-ante beliefs that may change during the course of the game. In this model the auditor always observes the auditee's effort level. He will adjust his beliefs about the type of auditee he is facing after this observation. After observing the effort level, the auditor's concern will be his uncertainty about the fraud rate. His costs can therefore be described in terms of his expectation of the fraud rate $\underline{F}$. If the auditor updates his beliefs using Bayes' rule then this expected fraud rate is given by:

$$\underline{F}_H = f_{2H} + (f_{1H}\text{-}f_{2H})(PS_1 \,/\, (PS_1+(1\text{-}P)S_2)) \quad \mathbf{(6.2.4)}$$

$$\underline{F}_L = f_{2L} + (f_{1L}\text{-}f_{2L})(P(1\text{-}S_1) \,/\, (P(1\text{-}S_1)+(1\text{-}P)(1\text{-}S_2))) \quad \mathbf{(6.2.5)}$$

6.3 A Game Theoretic Analysis

The model described in the previous section is a sequential two-player non-zerosum non-cooperative game. The auditor's information is both incomplete (he does not know the auditee's motivations) and imperfect (events occur during the game that are unobserved). The auditee's information is complete but imperfect giving a signalling game. In this model the auditee is classified into two types, where type 1 finds it harder to conceal fraud. The auditor's uncertainty about the auditee's cost of concealing fraud is then equivalent to an uncertainty about the type of auditee he is facing. The auditor observes one of the auditee's actions and can try to infer the auditee's type from this.

Since the auditor always observes the auditee's action the mixed strategies in this model are Myersons [28] "behavioural strategies". The solution concept used is Perfect-Bayesian Nash equilibrium (PBE). A PBE of a signalling game is defined by Fudenberg and Tirole [19] as a strategy profile (s_1*, s_2*) and posterior beliefs P(• | a_1) such that:

$\forall\ t_i,\ s_1^*(\bullet \mid t_i) \in \text{ARGMAX}_{s1 \in S1}\ u_1(s_1, s_2^*, t_i)$

$\forall\ a_1,\ s_2^*(\bullet \mid a_1) \in \text{ARGMAX}_{s2 \in S2} \sum_{ti} P(t_i \mid a_1) u_2(a_1, s_2, t_i)$

$P(t_i \mid a_1) = P(t_i)s_1(a_1 \mid t_i) / \sum_{t' \in T} P(t')s_1^*(a_1 \mid t')$ if $\sum_{t' \in T} P(t')s_1^*(a_1 \mid t') > 0$

and $P(t_i \mid a_1)$ is any probability distribution if $\sum_{t' \in T} P(t')s_1^*(a_1 \mid t') = 0$

Consider first the auditee's optimal strategies. Each type has two decisions to make; whether or not to put a high level of effort into maintaining the internal controls and whether or not to commit fraud. As mentioned above, the conflicting interests of the players will often lead to a randomisation between fraud and no fraud. This fraud rate will depend upon the effort level chosen. The auditor may have different optimal responses if H is observed rather than L, with a correspondingly different optimal fraud rate for the auditee. For a given effort level the auditee can choose f_{iH} to minimise his costs (this will involve considering the auditor's optimal responses as f_{iH} varies). The auditee can then compare the costs (using the optimal fraud rates) of high or low effort.

The auditor's costs can be considered as functions of C_B , C_D and $\underline{F}$. For a particular $\underline{F}$ the $C_B \times C_D$ plane will be divided into four regions of optimal test; B_1D_1 if both C_B and C_D are small, B_1D_2 if C_B is small and C_D is large, B_2D_1 if C_B is large and C_D is small and B_2D_2 if both costs are large. The boundaries of these regions can be found by comparing testing strategies pairwise to derive inequalities on C_B and C_D . These inequalities will also depend on the value of $\underline{F}$. We can consider the behaviour of these inequalities (the boundaries) as $\underline{F}$ varies. This will enable us to partition the plane into regions of optimal test sets where each test that occurs in the set is an optimal action for some values of $\underline{F}$. These boundaries are determined by comparing the auditors costs for the two strategies. $C(B_aD_b)<C(B_cD_d)$ when

$$\begin{aligned} C_{Ba}-C_{Bd} &< (\underline{M}_b-\underline{M}_a)C_D \\ &+(1-\underline{F})\big(p_H(r_c-r_a)(C^Q(E)-C^{NQ}(E))+(1-p_H)(t_c\phi_d-t_a\phi_b)(C^Q(NE)-C^{NQ}(NE))\big) \\ &+\underline{F}\big((r_c'v_d-r_a'v_b)(C^{Q1}(F)-C^{Q2}(F))+((1-r_c')(1-p_Hr_c)-(1-r_a')(1-p_Hr_a))(C^{NQ}(F)-C^{Q2}(F))\big) \end{aligned} \qquad \textbf{(6.3.1)}$$

If $B_a=B_c$ this condition can be rearranged to give

$$C_D < \frac{\underline{F}(r_a'v_1)(C^{Q2}(F)-C^{Q1}(F)) + (1-p_H)(1-\underline{F})t_a(C^Q(NE)-C^{NQ}(NE))}{(\underline{F}(p_H(t_a-r_ar_a')+r_a'-t_a)+ P_e(r_a-t_a) + t_a)} \qquad \textbf{(6.3.2)}$$

Condition (6.3.2) can be considered in three parts; the expected reduction in costs of being able to correctly classify fraud with the D_1 test, the reduced risk of a false positive when there are no errors, and the expected cost of using test D_1. Condition (6.3.1) is more complex due to the interaction between the two stages of testing. For example changing from B_2 to B_1 will change the likelihood of the B-test reporting a material irregularity and thus the likelihood of using the D-test. In a similar vein changing from playing D_2 to D_1 after a B-test report "M" will reduce the risk of a false positive from the B-test whilst increasing the chance of detecting fraud. This improvement may mean that it is no longer worthwhile using test B_1 and changing from B_1 to B_2 will change the likelihood of "M" occurring.

These optimal test sets can then be used to determine the auditee's optimal fraud rate. For a fraud rate $\in (0,1)$ to be optimal it must make the auditor indifferent between two of his actions. With at most four testing actions this means that the auditee has at most five fraud rates to consider. With restriction R5 the auditor starts using the D_1 test if the occurrence of fraud is above a certain level. The auditee's optimal mixed strategy must occur when the auditor switches from a test which has a decreasing auditee cost to one that is increasing. R1 and R2 therefore restrict the auditee's potential fraud rates to 0,1 or the critical point at which the auditor starts using the D_1-test. Consider 6.2.1, the cost $D_i(.)$ to auditee type i of a test B_aD_b.

$$
\begin{aligned}
D_i\,(H\,f_{iH} \mid B_aD_b) = {} & D_H + f_{iH}D_{iFH} + p_H(1-f_{iH})(r_aD^{Q2}(E)+(1-r_a)D^{NQ}(E)) \\
& + (1-p_H)(1-f_{iH})(t_a\phi_b(M \mid M, NE)D^{Q2}(NE)+(1-t_a\phi_b(M \mid M, NE))D^{NQ}(NE)) \\
& + f_ir_a'v_bD^{Q1}(F) + f_i(r_a'(1-v_b)+r_a(1-r_a')p_H)D^{Q2}(F) + f_i(1-r_a')(1-p_Hr_a)D^{NQ}(F)
\end{aligned}
$$

It can be seen from this expression that the most obvious measure to deter fraud - namely increasing the penalty of getting caught, will not necessarily prevent fraud. If the auditor uses test D_2 in a pure strategy equilibrium then the "getting caught" outcome $Q_1(F)$ occurs with zero probability. Thus, if there is some incentive to commit fraud the auditee's best action if test D_2 is used is to commit fraud with certainty irrespective of the size of the penalty $D^{Q1}(F)$. If, on the other hand, there is a mixed strategy equilibrium then there must always be a positive chance of fraud occurring.

Lemma 6.1 $\partial/\partial f\,(xD_1(Hf \mid B_aD_b) + (1-x)D_1(Hf \mid B_cD_d)) >$
$\partial/\partial f\,(xD_2(Hf \mid B_aD_b) + (1-x)D_2(Hf \mid B_cD_d))$
for any auditor tests (B_aD_b, B_cD_d) and effort level H

Proof Consider $\partial/\partial f\,DAM_i(Hf \mid B_aD_b)$ for some effort level H.
$= D_{iHF} - p_H(r_aD^{Q2}(E)+(1-r_a)D^{NQ}(E)) - (1-p_H)(t_a\phi_bD^{Q2}(NE)+(1-t_a\phi_bD^{NQ}(NE)) +$
$r_a'v_bD^{Q1}(F) + (r_a'(1-v_b)+r_a(1-r_a')p_H)D^{Q2}(F)+ (1-r_a')(1-p_Hr_a)D^{NQ}(F)$
Where only the first term depends upon the auditee's type i. Since type 1 finds it more difficult to commit fraud we have $D_{1HF} > D_{2HF}$ and $D_{1LF} > D_{2LF}$.
Hence

$$\partial/\partial f\,DAM_1(Hf \mid B_aD_b) > \partial/\partial f\,DAM_2(Hf \mid B_aD_b) \tag{1.1}$$

for any test B_aD_b. Thus the same inequality will hold for B_cD_d giving

$$\partial/\partial f\,DAM_1(Hf \mid B_cD_d) > \partial/\partial f\,DAM_2(Hf \mid B_cD_d) \tag{1.2}$$

Both inequalities are unchanged by the multiplication of a positive constant so
x LHS(1.1) > x RHS (1.1) and (1-x)LHS(1.2) > (1-x)RHS(1.2) for $x \in [0,1]$
$\Rightarrow$ x LHS(1.1) + (1-x)LHS(1.2) > x RHS (1.1) + (1-x)RHS(1.2)
i.e. $(x\partial/\partial f DAM_1(Hf \mid B_aD_b) + (1-x)\,\partial/\partial f DAM_1(Hf \mid B_cD_d)) >$

$$(x\partial/\partial f\, DAM_2(Hf \mid B_aD_b) + (1\text{-}x)\partial/\partial f DAM_2(Hf \mid B_cD_d))$$
$$\Rightarrow \partial/\partial f\,(xDAM_1(Hf \mid B_aD_b) + (1\text{-}x)DAM_1(Hf \mid B_cD_d)) >$$
$$\partial/\partial f\,(xDAM_2(Hf \mid B_aD_b) + (1\text{-}x)DAM_2(Hf \mid B_cD_d)) \quad (1.3)$$

Lemma 6.1 considers how the auditee costs change as f_i changes. This can be used to consider mixed strategies. For an auditee mixed strategy to be optimal, the auditor must be using a randomisation that makes the auditee indifferent between F and NF. In other words the gradient of the auditee cost (with respect to f) must be zero. Since lemma 6.1 shows that the gradient of type 1's cost function is greater than the gradient of type 2's cost only one type can be indifferent to any mixed auditor strategy. This is used in section 6.4 to characterise the equilibrium set.

Consider the auditor's part of a mixed strategy. Lemma 6.1 shows that there will be two potentially optimal auditor mixed strategies for each effort level. The auditor will randomise between two strategies to make one auditee type indifferent. One of these strategies must have an auditee cost that is decreasing in f, whilst the other is increasing (if both are increasing, for example, then so is any positive linear combination of the two). We can therefore classify a mixed strategy for the auditor in terms of x, the probability that the auditor plays the test with a decreasing auditee cost. There will therefore be 4 values of x depending upon the effort level w and the auditee type i, x_{iw} say. Suppose the auditor randomises between B_aD_2 (which is decreasing in fi for the auditee) and B_bD_1 (which is increasing). Then the requirement of auditee indifference gives:

$$x_{iH} = \frac{(D_{iHF} + D_i(Hf=1 \mid B_bD_1) - D_i(Hf=0 \mid B_bD_1))}{(D_i(Hf=0 \mid B_aD_2)\text{-}\ D_i(Hf=0 \mid B_bD_1)\text{-}\ D_i(Hf=1 \mid B_aD_2)+ D_i(Hf=1 \mid B_bD_1))} \quad \textbf{(6.3.3)}$$

The denominator > 0 by R3 and R4. $D_{1HF} > D_{2HF} \Rightarrow x_{1H} > x_{2H}$ and $D_{1LF} > D_{2LF} \Rightarrow x_{1L} > x_{2L}$. Thus a lower occurrence of a deterrent (a test whose cost to the auditee is increasing in f_i) will limit type 1 auditee fraud. This expression can also be used to consider the impact of changing these costs D_{iHF} on the equilibrium fraud rates. Increasing D_{iHF} will reduce the occurrence of deterrent testing strategies in equilibrium. The equilibrium fraud rates are determined by both players costs. The set of potentially optimal fraud rates are determined solely by the auditor's costs. However, the costs D_{iHF} affect the gradient of the auditee cost functions. There will be critical points for each D_{iHF} where these cost gradients change from negative to positive. For example if the cost of committing fraud is sufficiently high all auditor strategies will have a cost to the auditee which is increasing in f_i. In this case the optimal fraud rate is f_i=0. So although a change in D_{iHF} can influence the choice of optimal fraud rate it does not directly influence the rate itself.

6.4 Categorisation of the Equilibrium Set

Equilibrium pairs in signalling games can be categorised by the senders behaviour. If different types send different signals (effort level) the auditor can infer which type he is facing. Such equilibria are therefore referred to as separating equilibria. If both types send the same signal, a pooling equilibrium, the auditor can infer nothing. There are also hybrid equilibria where the sender randomises between signals. If only one sender type randomises then the equilibrium will be referred to as partially hybrid. There can be several types of equilibrium as C_D increases. Firstly, for all sufficiently small (or large) C_D the auditor will have a single optimal test regardless of either the fraud rate or effort level. If this is the case after both high and low effort there will only be pooling equilibria. If the auditor has a mixed test after some effort level then partially hybrid equilibria can exist.

The auditor's part of a mixed strategy equilibrium must make the auditee indifferent between H and L effort whilst limiting the occurrence of fraud. This will involve a randomisation x_H* (or x_L*). If $x_H^* > x_{1H}$ then both types will play fraud with certainty after H (which is not in equilibrium). Similarly if $x_H^* < x_{2H}$ then neither type will play fraud after H, in which case a high level of depth testing is not optimal. These observations mean that

$$x_{1H} \geq x_H^* \geq x_{2H} \qquad \text{and } x_{1L} \geq x_L^* \geq x_{2L}$$

This in turn means that the auditees will tend to have pure fraud actions if they have mixed signalling actions, For example if S_1 and S_2 are such that the auditor plays x_H*

after observing H then the above inequalities and lemma 6.1 show that type 1's costs are increasing in f (an optimal rate of 0) whilst type 2's are decreasing (an optimal rate of 1). However, if the auditor has a mixed response after either effort level in a partially hybrid equilibrium then the randomising auditee must have a mixed fraud rate after one effort level.

Pooling equilibrium

If neither auditee type commits fraud then their costs are identical and thus only pooling equilibria can occur. There are two classes of pooling equilibria involving mixed fraud strategies. The auditor randomises to minimise either type 1's or type 2's fraud rate. Suppose the optimal fraud rate after some effort level H is F_H*. Then the auditees' optimal fraud rates can be found using equations 6.2.4 or 6.2.5. In a pooling equilibrium these expressions simplify to:

$$\underline{F}_H = f_{2H} + (f_{1H}-f_{2H})P \qquad \text{or} \qquad \underline{F}_L = f_{2L} + (f_{1L}-f_{2L})P$$

The auditor has two mixed strategy rates in a pooling equilibrium. x_{2H} will deter type 1 fraud and make type 2 indifferent. x_{1H} will limit type 1 fraud but encourage type 2. For a mixed strategy to be in equilibrium the auditor's expected fraud rate must be F_H*. There are two cases to consider:

If the auditor randomises to limit type 2 fraud after observing some effort level H then

$$f_{1H}=0 \quad \text{and} \quad f_{2H} = F_H^* / (1-P) \qquad \textbf{(6.4.1)}$$

If the auditor randomises to limit type 1 fraud after observing some effort level H then

$$f_{1H} = (F_H^*-(1-P))/P \quad \text{and} \quad f_{2H} = 1 \qquad \textbf{(6.4.2)}$$

We can also determine which class of mixed pooling equilibrium will occur.

Lemma 6. 2 *In a pooling equilibrium there are two types of mixed strategy;*

(a) if $F_H^* \leq (1\text{-}\boldsymbol{P})$ the auditor randomises to minimise type 2's fraud rate and type 1 never commits fraud.
(b) if $F_H^* > (1\text{-}\boldsymbol{P})$ the auditor randomises to minimise type 1's fraud rate and type 2 always commits fraud

Proof Lemma 6.1 shows that the gradient with respect to f of type 1's payoff for any auditor randomised test (B_aD_b, B_cD_d) and effort level H is greater than the gradient for type 2. Since auditee indifference to F requires a gradient of zero a particular randomisation x can only make one type indifferent. This gives two cases

(a) The auditor randomisation makes type 2 indifferent
$\Rightarrow$ RHS of (1.3) = 0 $\Rightarrow$LHS of (1.3) > 0. so type 1's cost function is increasing in f for this randomisation and thus type 1 can minimise his costs by choosing $f_{1H} = 0$
(b) The auditor randomisation makes type 1 indifferent
$\Rightarrow$ LHS of (1.3) = 0 $\Rightarrow$ RHS of (1.3) < 0. So type 2's cost function is decreasing in f for this randomisation and thus type 2 can minimise his costs by choosing $f_{2H} = 1$

If $F_H^* < (1\text{-}P)$
Auditor uses x_{1H} Then type 2 can minimise his costs with $f_{2H} = 1$
Thus $\underline{F} = f_{2H} + (f_{1H}-f_{2H})P \geq (1\text{-}P) > F_H^*$ This is not in equilibrium
Auditor uses x_{2H} Then type 1 can minimise his costs with $f_{1H} = 0$
This gives $\underline{F} = f_{2H}(1\text{-}P)$. Type 2 can minimise his costs by choosing f_{2H} so that $\underline{F} = F_H^*$. Hence $f_{2H} = F_H^*/(1\text{-}P) < 1$ since $F_H^* < (1\text{-}P)$. This is in equilibrium

If $F_H^* > (1\text{-}P)$
Auditor uses x_{1H} Then type 2 can minimise his costs with $f_{2H} = 1$
If type 1 plays NF this gives $\underline{F} = (1\text{-}P) < F_H^*$ so $f_{1H} > 0$ for equilibrium
Since type 1 is indifferent between F and NF he can choose $f_{1H} = (F_H^*\text{-}(1\text{-}P))/P$
giving $\underline{F} = F_H^*$. This is in equilibrium
Auditor uses x_{2H} Then type 1 can minimise his costs with $f_{1H} = 0$
This gives $\underline{F} = f_{2H}(1\text{-}P)$. Now type 2 can minimise his costs by choosing f_{2H} so that $\underline{F} = F_H^*$. However $f_{2H} \leq 1$ so $\underline{F} \leq (1\text{-}P) < F_H^*$. This is not in equilibrium

Therefore if $F_H^* <(1\text{-}P)$ the only equilibrium pooling strategy is for the auditor to use x_{2H}, in which case the auditee will behave as in (a). The auditor uses x_{1H} in equilibrium if $F_H^* > (1\text{-}P)$ and the auditee will behave as in (b).

The auditee's willingness to put in the pooling equilibrium effort level will depend on the auditor's testing action against the other effort level. In this model the auditor always observes the effort level before testing, and his strategy must therefore specify an action in response to both effort levels. Thus although low effort is a zero probability event in a pooling on H equilibrium (and vice-versa) the auditor's strategy must specify how he would respond to such an event, and this will influence the range of D_H for which the equilibrium holds. The equilibrium concept does not specify what the auditor may infer from observing a zero probability event - so the auditor will have a number of feasible responses. It will be assumed that the auditor's mixed strategy x in response to a zero probability event, by the argument above, will lie between x_1 and x_2. For each pooling equilibrium, a range of values for the zero probability response x that support the equilibrium will be given.

Separating equilibrium

The auditee's decision on whether or not to play H will depend upon the cost of doing so D_H. As D_H increases the auditee's optimal actions will change from H to L effort. However, the choice of effort level will also determine the level of testing that the auditor uses. If both types play H, for example, the auditor may test to deter fraud by a type 2 auditee. However, if a type 2 auditee prefers to play low effort the auditor will recognise H as a signal of type 1. In this case the auditor will randomise to deter fraud by type 1.

The auditee's cost function can be rearranged to give inequalities on D_H that prompt changes of effort level. To consider these inequalities in more detail it is convenient to refer to the auditee's cost against a randomised sampling scheme without the cost D_H. For example

$$\begin{aligned} D_i(H, f_i \mid B_aD_b) = D_H + F_iD_{iHF} + p_H(1-F_i)\big(r_aD^{Q2}(E)+(1-r_a)D^{NQ}(E)\big) \\ + (1-p_H)(1-F_i)\big(t_a\phi_bD^{Q2}(NE)+(1-t_a\phi_b)D^{NQ}(NE)\big) \\ + F_ir_a'v_bD^{Q1}(F) + F_i\big(r_a'(1-v_b)+r_a(1-r_a')p_H\big)D^{Q2}(F) + F_i(1-r_a')(1-p_Hr_a)D^{NQ}(F) \end{aligned}$$

Then $\rho_{iH}(x) = xD(H,F_i \mid B_aD_2) + (1-x)D(H,F_i \mid B_cD_1) - D_H$

These costs can then be ordered by comparing the two types' payoffs in a mixed strategy equilibrium. If the auditor uses x_{1L} then type 1 is indifferent whilst type 2 plays f=1. If the auditor uses x_{2L} then type 2 is indifferent and type 1 plays f=0 so in both cases we can consider $f_{1H} = 0$ and $f_{2H} = 1$. Consider two functions B_aD_2 and B_bD_1. Then by payoff restrictions R3 and R4 $D_i(H,0 \mid B_aD_2) > D_i(H,0 \mid B_bD_1)$ and $D_i(H,1 \mid B_aD_2) < D_i(H,1 \mid B_bD_1)$

$$\rho_{1H}(x) = D_1(H,0 \mid B_bD_1) + x\,(D_1(H,0 \mid B_aD_2) - D_1(H,0 \mid B_bD_1))$$
$$(D_1(H,0 \mid B_aD_2) - D_1(H,0 \mid B_bD_1)) > 0 \text{ so } \rho_{1H}(x) \text{ is increasing in x} \quad \textbf{(6.4.1)}$$

$$\rho_{2H}(x) = D_2(H,1 \mid B_bD_1) + x\,(D_2(H,1 \mid B_aD_2) - D_2(H,1 \mid B_bD_1))$$
$$(D_2(H,1 \mid B_aD_2) - D_2(H,1 \mid B_bD_1)) < 0 \text{ so } \rho_{2H}(x) \text{ is decreasing in x} \quad \textbf{(6.4.2)}$$

Now, both types payoffs are identical when the auditor randomises against type 2 since 2 is indifferent between F and NF while 1 plays NF

$$\rho_{1L}(x_{2L}) = \rho_{2L}(x_{2L}) \text{ and } \rho_{1H}(x_{2H}) = \rho_{2H}(x_{2H})$$

$x_{1H} > x_{2H}$ and (6.4.1) $\Rightarrow \rho_{1H}(x_{1H}) > \rho_{1H}(x_{2H})$
$x_{1H} > x_{2H}$ and (6.4.2) $\Rightarrow \rho_{2H}(x_{1H}) < \rho_{2H}(x_{2H})$

The same argument holds for low effort level giving

$$\rho_{1H}(x_{1H}) > \rho_{1H}(x_{2H}) = \rho_{2H}(x_{2H}) > \rho_{2H}(x_{1H}) \quad \textbf{(6.4.3)}$$

$$\rho_{1L}(x_{1L}) > \rho_{1L}(x_{2L}) = \rho_{2L}(x_{2L}) > \rho_{2L}(x_{1L}) \quad \textbf{(6.4.4)}$$

These can be used to consider the occurrence of separating and pooling equilibrium.

Lemma 6.3 *There are no pure strategy separating equilibrium*

Proof The occurrence of separating equilibria can be considered in terms of the auditee's cost of high effort D_H. For example suppose the auditor randomises against type 1 if both auditee's play H and against type 2 if both play L. For a separating equilibrium where type 1 plays H and type 2 plays L we require:

$$D_H < \rho_{1L}(x_{2L}) - \rho_{1H}(x_{1H})$$
$$D_H > \rho_{2L}(x_{2L}) - \rho_{2H}(x_{1H})$$

There will be four such inequalities for each player. Let $a_{ij} = \rho_{1L}(x_{iL}) - \rho_{1H}(x_{jH})$ with b_{ij} defined similarly for player 2. These eight conditions on D_H can then be referred to as the following:

$(\mathbf{a_{11}})\ \rho_{1L}(x_{1L}) - \rho_{1H}(x_{1H})$ $(\mathbf{b_{11}})\ \rho_{2L}(x_{1L}) - \rho_{2H}(x_{1H})$
$(\mathbf{a_{21}})\ \rho_{1L}(x_{2L}) - \rho_{1H}(x_{1H})$ $(\mathbf{b_{21}})\ \rho_{2L}(x_{2L}) - \rho_{2H}(x_{1H})$
$(\mathbf{a_{12}})\ \rho_{1L}(x_{1L}) - \rho_{1H}(x_{2H})$ $(\mathbf{b_{12}})\ \rho_{2L}(x_{1L}) - \rho_{2H}(x_{2H})$
$(\mathbf{a_{22}})\ \rho_{1L}(x_{2L}) - \rho_{1H}(x_{2H})$ $(\mathbf{b_{22}})\ \rho_{2L}(x_{2L}) - \rho_{2H}(x_{2H})$

These expressions can be partially ordered using (6.4.3) and (6.4.4) to give
$\mathbf{a_{12}} \geq \{\mathbf{a_{11}}, \mathbf{a_{22}}\} \geq \mathbf{a_{21}}$ and $\mathbf{b_{21}} \geq \{\mathbf{b_{11}}, \mathbf{b_{22}}\} \geq \mathbf{b_{12}}$ also $\mathbf{a_{22}} = \mathbf{b_{22}}$

For a separating equilibrium to occur one type must prefer to play H whilst the other plays L. This will occur for some range of D_H in between pooling on H and pooling on L. In a separating equilibrium the auditor will know the auditee's type after observing the effort level and will randomise accordingly. This will give the limits on the interval for which separating can occur. There are two cases to consider:

H/L in which case we have $(H / L\ x_{1H} / x_{2L})$. However for this to be a pure strategy equilibrium the following must also hold.
Type 1 must prefer $D_{1H}(x_{1H})$ to $D_{1L}(x_{2L}) \Leftrightarrow D_H < \mathbf{a_{21}}$
Type 2 must prefer $D_{2L}(x_{2L})$ to $D_{2H}(x_{1H}) \Leftrightarrow D_H > \mathbf{b_{21}}$
So $(H/L,\ x_{1H} / x_{2L})$ is a pure equilibrium for $\mathbf{b_{21}} < D_H < \mathbf{a_{21}}$.
However $\mathbf{b_{21}} \geq \mathbf{b_{22}} = \mathbf{a_{22}} \geq \mathbf{a_{21}}$ so there is no such interval.

L/H in which case we have $(L / H\ x_{2H} / x_{1L})$. However for this to be a pure strategy equilibrium the following must also hold.
Type 1 must prefer $D_{1L}(x_{1L})$ to $D_{1H}(x_{2H}) \Leftrightarrow D_H > \mathbf{a_{12}}$
Type 2 must prefer $D_{2H}(x_{2H})$ to $D_{2L}(x_{1L}) \Leftrightarrow D_H < \mathbf{b_{12}}$
So $(H/L,\ x_{1H} / x_{2L})$ is a pure equilibrium for $\mathbf{a_{12}} < D_H < \mathbf{b_{12}}$.
However $\mathbf{a_{12}} \geq \mathbf{a_{22}} = \mathbf{b_{22}} \geq \mathbf{b_{12}}$ so there is no such interval.

The non-occurrence of pure strategy separating equilibrium pairs is a result of the auditee's preferences. Each type prefers the auditor to test against the other type. The "honest" auditee benefits if the auditor conducts extensive in-depth testing as this reduces the risk of a false-positive result. On the other hand, the "fraudulent" auditee type clearly benefits if the auditor tests only to detect errors as any fraud is therefore unlikely to be found. These conflicting interests result in mimicking behaviour for both types. In some circumstances type 1 mimics type 2 while in other situations type 2 can benefit by mimicking type 1.

Partially hybrid equilibrium

These equilibria can occur if the auditor has a pure strategy response (and hence a pure fraud action) for one effort level. For a mixed fraud equilibrium the auditor's expected fraud rate $\underline{F}$ must make him indifferent between two of his strategies. Each vertex of the auditor's payoff hull will have a fraud rate associated with it. The auditee then chooses amongst the fraud rates to minimise his costs. Suppose the expected fraud rate that minimises the auditee costs after effort level H is F_H* and after L is F_L*. Then in a mixed strategy partially hybrid equilibrium the auditee types must choose fraud rates f_{1H} and f_{2H} signalling rates S_1 and S_2 so that the auditor's expected rate is F_H*.

Lemma 6. 4 *The only mixed strategy partially hybrid equilibrium is* $(S_1 f_{1H}=0, f_{1L}=F_H^* \,/\, Hf_{2H}=1, x_H^* \,/\, x_{1L})$ *and this occurs when* $F_H^* > (1-P)$

Proof We have $D_{1HF} - D_{2HF} \geq D_{1LF} - D_{2LF}$. This can be used to compare the a_{ij} and b_{ij} defined in the proof of the lemma 6.3. Consider $\mathbf{b_{11}} - \mathbf{a_{11}}$ for some auditor mixed strategies (W, X) / (Y, Z).

$\mathbf{b_{11}} - \mathbf{a_{11}} = \rho_{2L}(x_{1L}) - \rho_{2H}(x_{1H}) - \rho_{1L}(x_{1L}) + \rho_{1H}(x_{1H})$

Each of these costs can be considered at f=1 since type 1 is indifferent between f = 0 and f=1 and type 2 plays f=1. Therefore:

$\mathbf{b_{11}} - \mathbf{a_{11}} = D_{2LF} + x_{1L}\sigma(Y) + (1-x_{1L})\sigma(Z) - D_{2HF} - x_{1H}\sigma(W) - (1-x_{1H})\sigma(X)$
$- D_{1LF} - x_{1L}\sigma(Y) - (1-x_{1L})\sigma(Z) + D_{1HF} + x_{1H}\sigma(W) + (1-x_{1H})\sigma(X)$

where

$\sigma(W)= r_W v_W D^{Q1}(F)+(r_W'(1-v_W)+r_W(1-r_W')\mathbf{p_e})D^{Q2}(F)+(1-r_W')(1-\mathbf{p_e}r_W)D^{NQ}(F)$

$\Rightarrow \quad \mathbf{b_{11}} - \mathbf{a_{11}} = D_{2LF} - D_{2HF} - D_{1LF} + D_{1HF}$

$D_{1HF} - D_{2HF} - (D_{1LF} - D_{2LF}) \geq 0$ by assumption

$\Rightarrow \quad \mathbf{b_{11}} \geq \mathbf{a_{11}}$

In any hybrid equilibrium the auditor's randomisation x_H after some effort level H will lie between x_{1H} and x_{2H}. The auditee's decision of whether or not to play H will be an inequality on D_H similar to those developed above. These conditions were referred to in the proof of lemma 6.3 as $a_{ij} = \rho_{1L}(x_{iL}) - \rho_{1H}(x_{jH})$ for type 1 and b_{ij} for type 2. This can be extended to consider any x by defining $a_{i*} = \rho_{1L}(x_{iL}) - \rho_{1H}(x^*)$ for some x*. The inequalities (6.4.1) and (6.4.2) show that $\rho_{1H}(x^*)$ is increasing in x* and $\rho_{2H}(x^*)$ is decreasing. Now $x_{1H} > x^* > x_{2H}$ gives:

$a_{12} \geq a_{1*} \geq a_{11}$	$a_{22} \geq a_{2*} \geq a_{21}$	$a_{11} \geq a_{*1} \geq a_{21}$	$a_{12} \geq a_{*2} \geq a_{22}$
$b_{12} \leq b_{1*} \leq b_{11}$	$b_{22} \leq b_{2*} \leq b_{21}$	$b_{11} \leq b_{*1} \leq b_{21}$	$b_{12} \leq b_{*2} \leq b_{22}$

Since one type has a pure strategy effort level there are 4 potential partially hybrid equilibria. If type 1 has a randomised effort level then type 2 plays either H or L and vice versa.

1 Type 2 randomises and type 1 plays high :- auditor tests x^* / x_{2L}
type 2 indifferent if $D_H = b_{2*} \in (b_{22}, b_{21})$
type 1 plays high if $D_H < a_{2*} \in (a_{21}, a_{22})$
$b_{22} = a_{22} \Rightarrow$ there is no interval where both conditions hold.

2 Type 2 randomises and type 1 plays low :- auditor tests x_{2H} / x^*
type 2 indifferent if $D_H = b_{*2} \in (b_{12}, b_{22})$
type 1 plays low if $D_H > a_{*2} \in (a_{22}, a_{12})$
$b_{22} = a_{22} \Rightarrow$ there is no interval where both conditions hold.

3 Type 1 randomises and type 2 plays low :- auditor tests x_{1H} / x^*
type 2 plays low if $D_H > b_{*1} \in (b_{11}, b_{21})$
type 1 indifferent if $D_H = a_{*1} \in (a_{21}, a_{11})$
$b_{11} > a_{11} \Rightarrow$ there is no interval where both conditions hold.

4 Type 1 randomises and type 2 plays high :- auditor tests x^* / x_{1L}
type 2 plays high if $D_H < b_{1*} \in (b_{12}, b_{11})$
type 1 indifferent if $D_H = a_{1*} \in (a_{11}, a_{12})$
$b_{11} > a_{11} \Rightarrow$ both conditions hold for some interval of D_H
This corresponds to the equilibrium $(S_1 f_{1H}=0, f_{1L}=F_H^* / Hf_{2H}=1, x_H^* / x_{1L})$

In a partially hybrid equilibrium type 1 chooses S_1 so that type 2 commiting fraud with certainty leads to an expected fraud rate of F_H^*. If type 2 always plays H and commits fraud then type 1 must choose S_1 so that

$F_H^* = f_{2H} + (f_{1H}-f_{2H})(PS_1 / (PS_1+(1-P)S_2)$ (from 6.2.4)

where $f_{2H} = 1$, $f_{1H} = 0$ and $S_2 = 1$. This can be rearranged to give:

$$S_1 = (1-F_H^*)(1-P)/PF_H^* \qquad \textbf{(6.4.3)}$$

If type 2 always plays L and commits fraud then type 1 must choose S_1 so that

$F_L^* = f_{2L} + (f_{1L}-f_{2L})(P(1-S_1) / (P(1-S_1)+(1-P)(1-S_2))$ (from 6.2.5)

where $f_{2L} = 1$, $f_{1L} = 0$ and $S_2 = 0$. This can be rearranged to give:

$$S_1 = (F_L^*-(1-P)) / PF_L^* \qquad \textbf{(6.4.4)}$$

Hybrid equilibrium

In any hybrid equilibrium the optimal fraud rates will be $f_{1H} = f_{1L} = 0$ and $f_{2H} = f_{2L} = 1$. The conditions for auditor indifference involve choosing $S_1, S_2 \in (0,1)$ so that the auditor's expected fraud rate after observing H is F_H^* and after L is F_L^*. These can be used to derive expressions for S_1 and S_2:

$$S_1 = \frac{(1-F_H^*)(F_L^*-(1-P))}{P(F_L^*-F_H^*)} \qquad S_2 = \frac{F_H^*(F_L^*-(1-P))}{(1-P)(F_L^*-F_H^*)} \qquad \textbf{(6.4.5)}$$

Lemma 6.5 *A hybrid equilibrium can only exist if* $F_H < (1\text{-}P)$ *and* $F_L > (1\text{-}P)$

Proof $(1-P)S_2 / (PS_1+(1-P)S_2)= F_H^* \Leftrightarrow S_2 = (PF_H^*/(1-P)(1-F_H^*))S_1$ (5.1)
$(1-P)(1-S_2) / (P(1-S_1)+(1-P)(1-S_2))= F_L^*$
$\Leftrightarrow (1-S_2) = (PF_L^*/(1-P)(1-F_L^*))(1-S_1)$ (5.2)
These are of the form $S_2= g(F_H^*)S_1$ and $(1-S_2) = g(F_L^*)(1-S_1)$ where $g(x)=Px / (1-P)(1-x)$. So $g(x) > 1 \Leftrightarrow x > (1-P)$.
If $F_H^* > (1-P)$ and $F_L^* > (1-P)$ then $S_2 > S_1$ and $(1-S_2) > (1-S_1)$ which is a contradiction.
If $F_H^* < (1-P)$ and $F_L^* < (1-P)$ then $S_2 < S_1$ and $(1-S_2) < (1-S_1)$ which is a contradiction.

(5.1) and (5.2) can be solved to find S_1 and S_2:
$S_1 = (1-F_H^*)(F_L^*-(1-P))/P(F_L^*-F_H^*)$
$S_2 = F_H^*(F_L^*-(1-P))/(1-P)(F_L^*-F_H^*)$

If $F_H > (1-P)$ and $F_L < (1-P)$ then $(F_L^*-F_H^*) < (F_L^*-(1-P))$. S_2 is given by;
$S_2 = F_H^*(F_L^*-(1-P))/(1-P)(F_L^*-F_H^*) = (F_H^*/(1-P))\times((F_L^*-(1-P))/(F_L^*-F_H^*)) >1$
a contradiction

If $F_H \leq (1-P)$ and $F_L \geq (1-P)$ then $(F_L^*-F_H^*) \geq (F_L^*-(1-P))$. S_2 is given by;
$S_2 = (F_H^*/(1-P))\times((F_L^*-(1-P))/(F_L^*-F_H^*))$ which is ≥ 0 and ≤ 1
$F_H \leq (1-P) \Rightarrow (1-F_H) \geq P$
$S_1 = ((1-F_H^*)/P)\times((F_L^*-(1-P))/(F_L^*-F_H^*))$ which is ≥ 0 and ≤ 1(since $F_L^* \leq 1$)

Classification Three kinds of mixed fraud rate equilibrium can occur as D_H varies. For sufficiently low D_H both types will prefer to play H, giving a pooling equilibrium. Similarly for all high D_H both types will play L. For some intermediate values of D_H a mixed strategy separating equilibrium can also occur. In this case either one or both auditee types may have mixed effort levels. Which type of randomising occurs is then determined by F_H* and F_L* (which in turn are influenced by the costs C_B and C_D). This can be summarised as follows:

- The auditor has a single best response to both effort levels. In this case only pure fraud choice pooling equilibria will occur.

- The auditor has a single best response to one effort level. In this case partially hybrid equilibria can occur.

- The auditor's strategy involves randomisation for either effort level. The occurrence of hybrid equilibria now depends on F_H* and F_L*:

  if $F_H^* < (1\text{-}P)$ and $F_L^* < (1\text{-}P)$ then only pooling equilibria exist.

  if $F_H^* < (1\text{-}P)$ and $F_L^* > (1\text{-}P)$ then hybrid equilibria can exist.

  if $F_H^* > (1\text{-}P)$ then partially hybrid equilibria can occur in which type 1 never commits fraud and randomises between H and L effort and type 2 always commits fraud and always puts in effort level H.

6.5 A Numerical Example

The values below have been chosen to satisfy the payoff restrictions R1-R5 whilst satisfying the orderings on the outcome costs. The difference in the occurrence of error after high and low effort have been chosen so that there is a clear difference between effort levels. Although these values give what is felt to be a representative case, Fellingham and Newman [16] point out that an accurate estimation of these costs would prove very difficult. This problem is alleviated to some extent by considering the costs of actions to be variables - small changes in any of the outcome costs would change the points at which the optimal behaviour changes without altering the kinds of equilibrium behaviour that occur.

$P_H = 0.05$ $P_L = 0.35$ $C^{NQ}(F) = 600$ $D^{NQ}(F) = -500$
$D_{1FH}=300$ $D_{1FL}=150$ $C^{Q2}(F) = 400$ $D^{Q2}(F) = -300$
$D_{2FH}=100$ $D_{2FL}=25$ $C^{NQ}(E) = 300$ $D^{NQ}(E) = 150$
$C^{Q2}(NE) = 100$ $D^{Q2}(NE) = 120$
$r_1 = 0.95$ $r_1' = 0.65$ $t_1 = 0.1$ $C^{Q1}(F) = 80$ $D^{Q1}(F) = 700$
$r_2 = 0.8$ $r_2' = 0.5$ $t_2 = 0.2$ $C^{Q2}(E) = 60$ $D^{Q2}(E) = 220$
$v_1 = 0.9$ $C^{NQ}(NE) = 1\ 0$ $D^{NQ}(NE) = 10$

Auditee's optimal strategies

The auditee's strategic decision invloves comparing the cost of high effort (and optimal fraud rate F_H*) with the cost of low effort (and F_L*). Thus the first step is to determine the optimal fraud rates for high and low effort. This can be found by rearranging condition 6.3.1

$C\ (B_aD_b \mid H\ \underline{F}_H) = C\ (B_cD_d \mid H\ \underline{F}_H) \Leftrightarrow$

$$F = \big(t_c - t_a + p_w(r_c - t_c - r_a + t_a))C_D - C_{Ba} + C_{Bd} + p_H(r_c - r_a)(C^Q(E) - C^{NQ}(E)) + (1-p_H)(t_c\phi_d - t_a\phi_b)(C^Q(NE) - C^{NQ}(NE))\big) \,/\, \big(((1-p_H r_a)r_a' - (1-p_H)t_a - (1-p_H r_c)r_c' + (1-p_H)t_c)C_D + (r_c' v_d - r_a' v_b)(C^{Q1}(F) - C^{Q2}(F)) + ((1-r_c')(1-p_H r_c) - (1-r_a')(1-p_H r_a))(C^{NQ}(F) - C^{Q2}(F)) + p_H(r_c - r_a)(C^Q(E) - C^{NQ}(E)) + (1-p_H)(t_c\phi_d - t_a\phi_b)(C^Q(NE) - C^{NQ}(NE))\big) \quad \textbf{(6.5.1)}$$

This expression can be used to generate tables 6.2 and 6.3 below. Only comparisons between a D_1 test and a D_2 test are considered because the auditee wishes to minimise his costs. Under the payoff restrictions R1-R5, this minimising point will either be 0,1 or the critical point at which the auditor starts using the D_1-test.

Strategies	Critical fraud rate F_H*
B_2D_2 B_2D_1	$(-21.375 + 0.2775C_D) / (122.625 - 0.2425C_D)$
B_2D_2 B_1D_1	$(-23.175 + C_B + 0.1425C_D) / (193.35 - 0.524125C_D)$
B_2D_1 B_1D_2	$(-6.75 - C_B + 0.2775C_D) / (107.925 - 0.2425C_D)$
B_1D_2 B_1D_1	$(-8.55 + 0.1425C_D) / (178.65 - 0.524125C_D)$

table 6.2 :- auditor's critical fraud rate after observing H

Strategies	Critical fraud rate F_L*
B_2D_2 B_2D_1	$(-14.624 + 0.4425C_D) / (129.375 - 0.1975C_D)$
B_2D_2 B_1D_1	$(-27.224 + C_B + 0.3975C_D) / (185.25 - 0.368875C_D)$
B_2D_1 B_1D_2	$(6.75 - C_B + 0.4425C_D) / (125.475 - 0.19275C_D)$
B_1D_2 B_1D_1	$(-5.85 + 0.3975C_D) / (181.35 - 0.368875C_D)$

table 6.3 :- auditor's critical fraud rate after observing L

The auditee's strategic decision is simplified in two extreme cases where the auditee either always commits fraud or never commits fraud. For example if the auditee never commits fraud against an auditor strategy B_aD_b / B_cD_d then he will put in high effort if $D_H < D_i(Lf{=}0 \mid B_cD_d) - D_i(Hf{=}0 \mid B_aD_b)) = \Gamma(B_aD_b, B_cD_d)$ say. By rearranging expression 6.2.1 for the auditee's expected payoff. This gives:

$$\begin{aligned}\Gamma(B_aD_b/B_cD_d) = &(1-p_L)t_c\phi_d-(1-p_H)t_a\phi_b)D^Q(NE) \\ &+((1-p_L)(1-t_c\phi_d)-(1-p_H)(1-t_a\phi_b))D^{NQ}(NE)) \\ &+(p_Lr_c-p_Hr_a)D^Q(E)+(p_L(1-r_c)-p_H(1-r_a))D^{NQ}(E))\end{aligned} \quad \textbf{(6.5.2)}$$

And similarly, if the auditee always commits fraud against an auditor strategy B_aD_b / B_cD_d then he will put in high effort if $D_H < \Gamma_i(B_aD_b, B_cD_d)$ where

$$\begin{aligned}\Gamma_i^F(B_aD_b/B_cD_d) = &(D_{iFL}-D_{iFH})+(r_c'v_d-r_a'v_b)D^{Q1}(F) \\ &+((1-r_c')(1-p_Lr_c)-(1-r_a')(1-p_Hr_a))D^{NQ}(F) \\ &+(r_c'(1-v_d)+r_c(1-r_c')p_L-r_a'(1-v_b)-r_a(1-r_a')p_H)D^{Q2}(F)\end{aligned} \quad \textbf{(6.5.3)}$$

Clearly the only difference between the cost to each auditee type when committing fraud with certainty will be the first term of 6.5.3. For this example the conditions on D_H given in table 6.4 are generated by expressions 6.5.2 and 6.5.3. Γ_1^F can be found by subtracting 75 from Γ_2^F.

Play H if D_H < against	Γ^{NF}	Γ_2^F	**Play H if D_H < against**	Γ^{NF}	Γ_2^F
(B_2D_2 / B_2D_2)	50.55	-51	(B_1D_2 / B_2D_2)	65.7	- 80.325
(B_2D_2 / B_2D_1)	32.675	399	(B_1D_2 / B_2D_1)	47.285	369.675
(B_2D_2 / B_1D_2)	43.5	- 25.725	(B_1D_2 / B_1D_2)	58.65	- 55.05
(B_2D_2 / B_1D_1)	36.35	559.275	(B_1D_2 / B_1D_1)	51.5	529.95
(B_2D_1 / B_2D_2)	76.675	- 501	(B_1D_1 / B_2D_2)	76.15	- 665.325
(B_2D_1 / B_2D_1)	58.8	- 51	(B_1D_1 / B_2D_1)	58.275	- 215.325
(B_2D_1 / B_1D_2)	69.625	- 475.725	(B_1D_1 / B_1D_2)	69.1	- 640.05
(B_2D_1 / B_1D_1)	62.475	109.275	(B_1D_1 / B_1D_1)	61.95	- 55.05

table 6.4 :- critical values for D_H

In a mixed signalling equilibrium the auditee must be indifferent between H and L effort. The auditor chooses a mixed strategy to achieve this. The auditor mixed strategies therefore need to consider the difference in the auditee's costs between high and low effort for a fraud rate of 0 or 1. The auditor's randomisation can therefore best be described by considering the effort comparisons expressed in (6.5.2) and (6.5.3). However we also need to consider the cost to the auditee (for a fraud rate of 0 or 1) of different strategies after the same effort level. This can also been found from (6.5.2) and (6.5.3) by the introduction of a dummy variable. To compare the cost to the auditor of B_aD_b and B_cD_d after some effort level and a fraud rate of 0, for example, consider the following for any test "•"

$$\Gamma(B_aD_b / \bullet) - \Gamma(B_cD_d/\bullet) = D_L(\bullet) - D_H(B_aD_b) - (D_L(\bullet) - D_H(B_cD_d)) = D_H(B_cD_d) - D_H(B_aD_b)$$

$$\Gamma(\bullet / B_aD_b) - \Gamma(\bullet /B_cD_d) = D_L(B_aD_b) - D_H(\bullet) - (D_L(B_cD_d) - D_H(\bullet)) = D_L(B_aD_b) - D_L(B_cD_d)$$

Thus any comparisons between the auditee's payoffs can be re-expressed in terms of inequalities on D_H given in table 6.4. If the auditor has a single best response to both effort levels only pure fraud rate pooling equilibrium can occur. If there is no fraud then the auditees motivations are identical, and their optimal behaviour as D_H increases is determined by Γ^{NF}. If there is a pure fraud rate pooling equilibrium the optimal behaviour will be determined by either Γ_1^F or Γ_2^F. In a pooling equilibrium with a mixed strategy fraud rate by type 2 both types play H against (W,X) / (Y,Z) if

$$D_H + x_H D(Hf=0|W) + (1-x_H) D(Hf=0|X) < x_L D(Lf=0|Y) + (1-x_L) D(Lf=0|Z)$$

The costs can be considered at f = 0 since in equilibrium type 2 is indifferent between F and NF whilst type 1 prefers NF. Since f = 0 both types costs are identical. This condition can be rearranged to give $D_H <$
$D_L(Lf=0|Z) - D_H(Hf=0|X) + x_L(D_L(Lf=0|Y) - D_L(Lf=0|Z)) - x_H(D_H(Hf=0|W) D_H(Hf=0|X))$

$$D_H < \Gamma^{NF}(X / Z) + x_L(\Gamma^{NF}(\bullet / Y) - \Gamma^{NF}(\bullet /Z)) + x_H(\Gamma^{NF}(W/\bullet) - \Gamma^{NF}(X /\bullet)) \qquad \textbf{(6.5.4)}$$

Similar expressions could be found for comparing payoffs when fraud occurs with certainty after each effort level. However, for this example both types strictly prefer low effort if they can always commit fraud. In a partially hybrid equilibrium the bounds on D_H can also be found by using (6.5.4). However in a mixed fraud rate partially hybrid equilibrium (i.e. with a non zero fraud rate after both H and L) the point at which both types prefer L must consider type 2's costs for always committing fraud. The value of D_H which prompts a change to L/L satisfies the following two conditions:

$$D_H + (1-x_H^*)D_{1H}(Hf_{1H}=0|X) + x_H^*(D_{1H}(Hf_{1H}=0|W) = (1-x_{1L})D_{1L}(Lf_{1L}=0|Z)+x_{1L}(D_{1L}(Lf_{1L}=0|Y)$$

$$D_H + (1-x^*_H)D_{2H}(Hf_{2H}=1|X) + x^*_H(D_{2H}(Hf_{2H}=1|W) = (1-x_{1L})D_{2L}(Lf_{2L}=1|Z)+x_{2L}(D_{2L}(Lf_{2L}=1|Y)$$

These can be rearranged to give:

$$D_H = \frac{((\Gamma(X/\bullet)-\Gamma(W/\bullet))(\Gamma_2^F(X/Z)+x_{1L}(\Gamma_2^F(\bullet/Y)-\Gamma_2^F(\bullet/Z)))) - ((\Gamma_2^F(X/\bullet)\Gamma_2^F(W/\bullet))(\Gamma(X/Z)+x_{1L}(\Gamma(\bullet/Y)-\Gamma(\bullet/Z))))}{(\Gamma(X/\bullet)- \Gamma(W/\bullet) - \Gamma_2^F(X/\bullet)+\Gamma_2^F(W/\bullet))} \qquad \textbf{(6.5.5)}$$

In a full hybrid equilibrium the conditions on D_H can be found using (6.5.4) and considering the limits of the auditor's optimal test. As discussed in section 6.4 in a hybrid equilibrium the auditor's mixed strategy will be of the form x_H^* / x_L^* to make both auditee types indifferent between high and low effort. Furthermore $x_{1H} \geq x_H^* \geq x_{2H}$ for each effort level H. Now x_H^* and x_L^* will vary as D_H increases so that both auditee types remain indifferent. Thus limits on D_H can be found for which the associated x_H^*, say, falls between x_{1H} and x_{2H}. Formally, the requirements for a fully hybrid equilibrium become:

let $\mathbf{D} = \{d \in D_H : x_H^* \in [x_{1H}, x_{2H}] \cap x_L^* \in [x_{1L}, x_{2L}]\}$

then $\underline{d} \leq D_H \leq \bar{d}$ where $\underline{d} = \text{MIN}\,\mathbf{D}$ and $\bar{d} = \text{MAX}\,\mathbf{D}$

Auditor's optimal strategies

For a given effort level and rate of fraud occurrence the auditor will have 4 optimal strategies as C_B and C_D vary. In other words the $C_B \times C_D$ plane can be divided into 4 regions of optimal test. The boundaries of these regions, which will vary as the rate of fraud changes, can be generated from equations (6.3.1) and (6.3.2) above.

If $B_a \neq B_c$ then $C(B_aD_b) = C(B_cD_d) \Leftrightarrow$

$$\begin{aligned} C_{Ba}-C_{Bd} = {} & (\underline{M}_b-\underline{M}_a)C_D+(1-\underline{F}_H)\big(p_H(r_c-r_a)(C^Q(E)-C^{NQ}(E)) \\ & +(1-p_H)(t_c\phi_d-t_a\phi_b)(C^Q(NE)-C^{NQ}(NE))\big) \\ & +F_H\big((r_c'v_d-r_a'v_b)(C^{Q1}(F)-C^{Q2}(F))+((1-r_c')(1-p_Hr_c)-(1-r_a')(1-p_Hr_a))(C^{NQ}(F)-C^{Q2}(F))\big) \end{aligned}$$

if $B_a=B_c$

$$C_D = \frac{\underline{F}(r_a'v_1)(C^{Q2}(F)-C^{Q1}(F)) + (1-p_H)(1-\underline{F})t_a(C^Q(NE)-C^{NQ}(NE))}{(\underline{F}(p_H(t_a-r_ar_a')+r_a'-t_a)+ P_e(r_a-t_a) + t_a)}$$

Auditor observes high effort level		
$B_2D_2=B_2D_1$	$C_D = (122.625F+21.375) / (0.2775 + 0.2425F)$	**(1)**
$B_2D_2=B_1D_2$	$C_B = 14.7F + 14.625$	**(2)**
$B_2D_2=B_1D_1$	$C_B = 193.35F+23.175-(0.1425+0.524125F)C_D$	**(3)**
$B_2D_1=B_1D_2$	$C_B = -107.925F-6.75+(0.2775+0.2425F)C_D$	**(4)**
$B_2D_1=B_1D_1$	$C_B = 1.8+70.725F+(0.135-0.281625F)C_D$	**(5)**
$B_1D_2=B_1D_1$	$C_D = (178.65F+8.55)/(0.1425+0.524125F)$	**(6)**

table 6.5 :- boundaries of the optimal test regions after H

These conditions can be used to consider the auditor's set of optimal tests as $\underline{F}$ increases. We can determine the area of the $C_B \times C_D$ plane that is "swept" by each of these boundaries as $\underline{F}$ increases. For example the point ($C_B = 40$, $C_D = 100$) falls into the region where B_2D_2 is optimal if there is no fraud. As $\underline{F}$ increases condition (1) moves past so that the point now falls into the region where B_2D_1 is optimal. Finally, condition (5) moves past so the point ends up in the region where B_1D_1 is optimal. Thus the auditor's optimal test set is $\{B_2D_2, B_2D_1, B_1D_1\}$.

To find the optimal test set for any point we also need to consider the "corners" of the optimal test regions - where 2 of the boundaries intersect. The decision to stop using test D_1 (conditions (1) and (6)) is strongly influenced by $\underline{F}$. At $\underline{F} = 0$, the D-test ceases to be optimal at a lower level of C_D if test B_1 is being used since B_1 has less risk of a false positive. At $\underline{F} = 1$ the D-test ceases to be optimal at a higher level of C_D if B_1 is used since this test has a greater chance of picking up fraud for the D-test to focus on. The behaviour of the intersections as $\underline{F}$ increases can be found from conditions (1) to (6)

	If F < 0.03	
(2)=(4)=(1)	$C_B=14.625+14.7(-21.375+0.2775C_D)/(122.625-0.2425C_D)$	**(7)**
(5)=(6)=(4)	$C_B=1.8+0.135C_D+\frac{(70.725-0.281625C_D)(-8.55+0.1425C_D)}{(178.65-0.524125C_D)}$	**(8)**
	If F > 0.03	
(3)=(5)=(1)	$C_B=1.8+0.135CD+\frac{(70.725-0.281625C_D)(-21.375+0.2775C_D)}{(122.625-0.2425C_D)}$	**(9)**
(3)=(6)=(2)	$C_B=14.625+14.7(-8.55+0.1425C_D)/(178.65-0.524125C_D)$	**(10)**

table 6.6 :- points of boundary intersection after H

For this example the auditor's optimal test sets after observing H as F changes are shown in figure 6.2

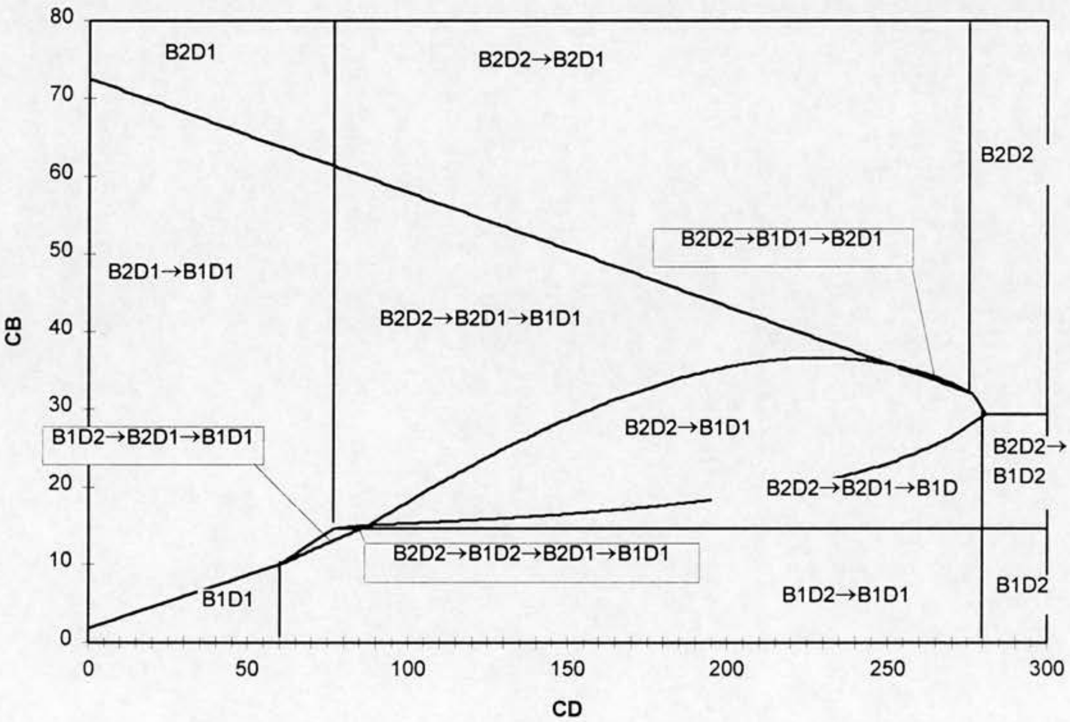

figure 6.2 :- auditor's optimal test after observing H as $\underline{F}$ increases

A similar set of conditions will apply after the auditor observes low effort.

Auditor observes low effort level		
$B_2D_2=B_2D_1$	$C_D = (129.375F+14.624) / (0.4425 + 0.1975F)$	**(11)**
$B_2D_2=B_1D_2$	$C_B = 3.9F + 21.374$	**(12)**
$B_2D_2=B_1D_1$	$C_B = 185.25F+27.224-(0.3975+0.368875F)C_D$	**(13)**
$B_2D_1=B_1D_2$	$C_B = -125.475F+6.75+(0.4425+0.1975F)C_D$	**(14)**
$B_2D_1=B_1D_1$	$C_B = 12.6+55.875F+(0.045-0.171375F)C_D$	**(15)**
$B_1D_2=B_1D_1$	$C_D = (181.35F+5.85)/(0.3975+0.368875F)$	**(16)**

table 6.7 :- boundaries of the optimal test regions after L

If F < 0.14		
(12)=(14)=(11)	$C_B=21.374+3.9(-14.624+0.4425C_D)/(129.375-0.1975C_D)$	**(17)**
(15)=(16)=(14)	$C_B=12.6+0.045C_D+\frac{(55.875-0.171375C_D)(-5.85+0.3975C_D)}{(181.35-0.368875C_D)}$	**(18)**
If F > 0.14		
(13)=(15)=(11)	$C_B=12.6+0.045C_D+\frac{(55.875-0.171375C_D)(14.624+0.4425C_D)}{(129.375-0.1975C_D)}$	**(19)**
(13)=(16)=(12)	$C_B=21.374+3.9(-5.85+0.3975C_D)/(181.35-0.368875C_D)$	**(20)**

table 6.8 :- points of boundary intersection after L

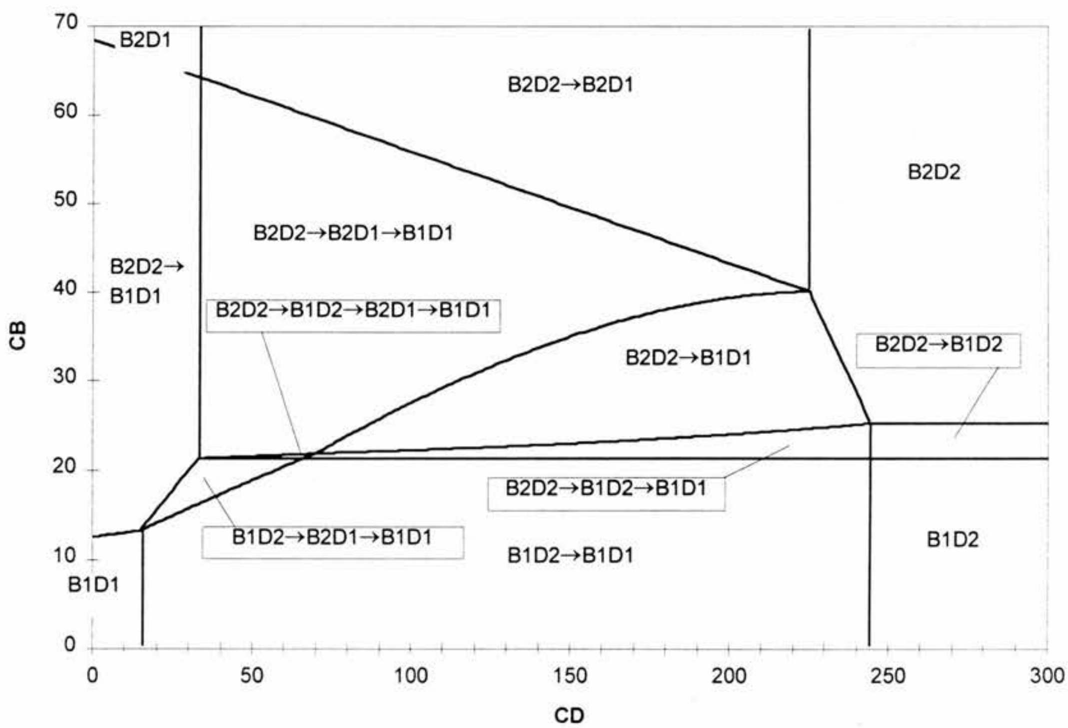

figure 6.3 :- auditor's optimal test after observing L as F increases

These two sets of conditions can be combined to give optimal test sets after either effort level. The auditee will choose the fraud rate that minimises his costs, which will be one of the points where the auditor's test changes.

If the auditor has a mixed strategy to limit fraud in a pooling equilibrium he must make one auditee type indifferent between F and NF. Suppose the auditor is using (B_aD_b, B_cD_d) after observing some effort level H. Then to limit the occurrence of fraud by auditee type i the should choose x_{iH} so that:

$$x_{iH} = D_{iHF} + \frac{\begin{array}{l}(-p_H(r_cD^{Q2}(E)+(1-r_c)D^{NQ}(E)) - (1-p_H)(t_c\phi_dD^{Q2}(NE)+(1-t_c\phi_dD^{NQ}(NE)) \\ + r_c'v_dD^{Q1}(F) + (r_c'(1-v_d)+r_c(1-r_c')p_H)D^{Q2}(F)+ (1-r_c')(1-p_Hr_c)D^{NQ}(F))\end{array}}{\begin{array}{l}(p_H(r_a-r_c)(D^{Q2}(E)-D^{NQ}(E))+(1-p_H)(t_a\phi_b-t_c\phi_d)(D^{Q2}(NE)-D^{NQ}(NE))+(r_c'v_d-r_a'v_b)D^{Q1}(F) \\ +(r_c'(1-v_d)+r_c(1-r_c')p_H-r_a'(1-v_b)-r_a(1-r_a')p_H)D^{Q2}(F) \\ +((1-r_c')(1-p_Hr_c)-(1-r_a')(1-p_Hr_a))D^{NQ}(F)\end{array}} \quad \textbf{(6.5.6)}$$

For this example this gives:

Strategies (x)	Strategies (1-x)	Pooling on H x_{1H} =	Pooling on H x_{2H} =	Pooling on L x_{1L} =	Pooling on L x_{2L} =
B_2D_2	**B_2D_1**	0.701917	0.281858	0.319316	0.052578
B_2D_2	**B_1D_1**	0.778216	0.465680	0.490012	0.289843
B_1D_2	**B_2D_1**	0.774238	0.310900	0.343015	0.056021
B_1D_2	**B_1D_1**	0.836342	0.500462	0.516761	0.305666

table 6.9 :- auditor fraud limitation randomisation for pooling equilibria

The conditions (6.5.2) and (6.5.4) above can be used to develop an expression for x_H* so that a type 1 auditee is indifferent between high and low effort against some auditor strategies (W,X) / (Y,Z) say. Similar expressions can be derived for type 2 indifference but the only mixed strategy hybrid equilibria involves type 1. If a type 2 auditee puts in high effort then:

$$x_H^* = (-D_H + x_{1L}(\Gamma(\bullet / Y)-\Gamma(\bullet / Z)) + \Gamma(X / Z)) / (\Gamma(X / \bullet)-\Gamma(W / \bullet)) \quad \textbf{(6.5.7)}$$

Or if a type 2 auditee puts in low effort then:

$$x_L^* = (D_H - \Gamma(X / Z)) + (\Gamma(X / \bullet) - \Gamma(W / \bullet))x_L) / (\Gamma(\bullet / Y) - \Gamma(\bullet / Z)) \qquad \textbf{(6.5.8)}$$

In a hybrid equilibrium the auditor's randomisation is given by:

$$x_H^* = \frac{(D_H - \Gamma_2^F(X/Z))(\Gamma(\bullet/Y) - \Gamma(\bullet/Z)) - (D_H - \Gamma(X/Z))(\Gamma_2^F(\bullet/Y) - \Gamma_2^F(\bullet/Z))}{((\Gamma(X/\bullet) - \Gamma(W/\bullet))(\Gamma_2^F(\bullet/Y) - \Gamma_2^F(\bullet/Z)) - (\Gamma_2^F(X/\bullet) - \Gamma_2^F(W/\bullet))(\Gamma(\bullet/Y) - \Gamma(\bullet/Z))} \qquad \textbf{(6.5.9)}$$

$$x_L^* = \frac{(\Gamma_2^F(X/Z) - D_H)(\Gamma(X/\bullet) - \Gamma(W/\bullet)) - (\Gamma(X/Z) - D_H)(\Gamma_2^F(X/\bullet) - \Gamma_2^F(W/\bullet))}{((\Gamma(\bullet/Y) - \Gamma(\bullet/Z))(\Gamma_2^F(X/\bullet) - \Gamma_2^F(W/\bullet)) - (\Gamma_2^F(\bullet/Y) - \Gamma_2^F(\bullet/Z))(\Gamma(X/\bullet) - \Gamma(W/\bullet))} \qquad \textbf{(6.5.10)}$$

The equilibrium set

The type of equilibrium that occurs has been classified in terms of the optimal fraud rates F_H^* and F_L^*. For a particular probability of type, $P = 0.9$, the equilibrium set can be seen more clearly:

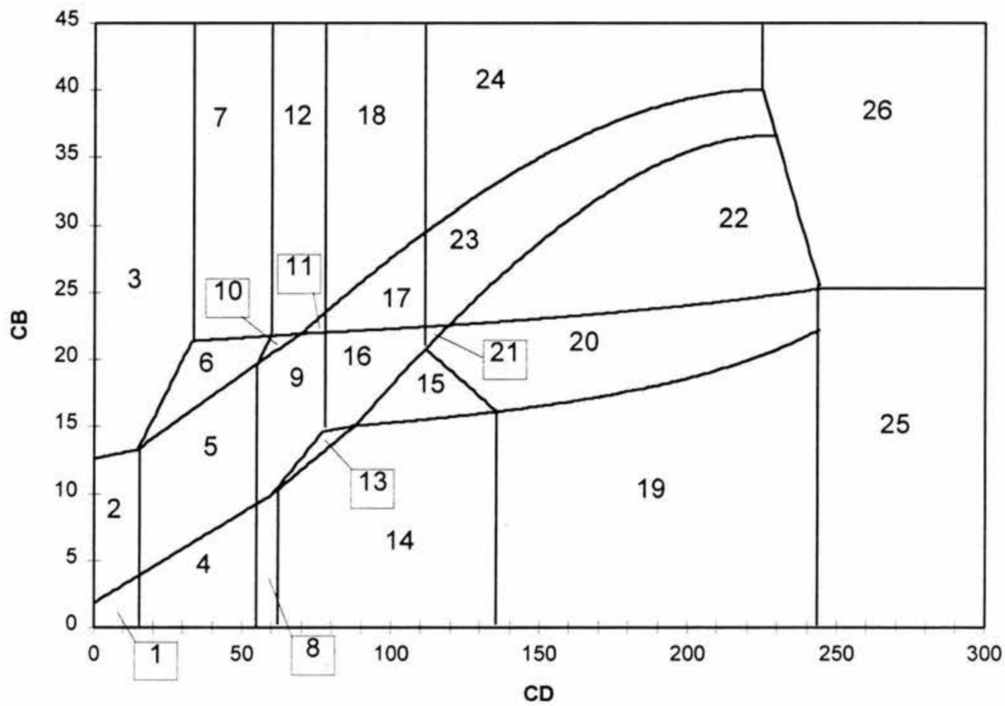

figure 6.4 :- auditor's equilibrium strategy set for P=0.9

Each of these regions will have different equilibria as D_H increases, changing the auditee's optimal behaviour. However the auditor's optimal testing strategies for each region are given below. These strategies are optimal responses to the auditee fraud rate. If the auditor has a sequence of strategies as F varies then, under payoff restrictions R1 to R5, the auditor need only consider the two "critical point" strategies where the optimal test changes from D_2 to D_1. The auditor's equilibrium strategies are given below:

1) B_1D_1 / B_1D_1	2) B_2D_1 / B_1D_1	3) B_2D_1 / B_2D_1
4) B_1D_1 / (B_1D_2, B_1D_1)	5) B_2D_1 / (B_1D_2, B_1D_1)	6) B_2D_1 / (B_1D_2, B_2D_1)
7) B_2D_1 / (B_2D_2, B_2D_1)	8) B_1D_1 / (B_1D_2, B_1D_1)	9) B_2D_1 / (B_1D_2, B_2D_1)
10) B_2D_1 / (B_1D_2, B_2D_1)	11) B_2D_1 / (B_2D_2, B_1D_1)	12) B_2D_1 / (B_2D_2, B_2D_1)

13) (B_1D_2, B_2D_1) / (B_1D_2, B_1D_1)	14) (B_1D_2, B_1D_1) / (B_1D_2, B_1D_1)
15) (B_2D_2, B_1D_1) / (B_1D_2, B_1D_1)	16) (B_2D_2, B_2D_1) / (B_1D_2, B_1D_1)
17) (B_2D_2, B_2D_1) / (B_2D_2, B_1D_1)	18) (B_2D_2, B_2D_1) / (B_2D_2, B_2D_1)
19) (B_1D_2, B_1D_1) / (B_1D_2, B_1D_1)	20) (B_2D_2, B_1D_1) / (B_1D_2, B_1D_1)
21) (B_2D_2, B_2D_1) / (B_1D_2, B_1D_1)	22) (B_2D_2, B_1D_1) / (B_2D_2, B_1D_1)
23) (B_2D_2, B_2D_1) / (B_2D_2, B_1D_1)	24) (B_2D_2, B_2D_1) / (B_2D_2, B_2D_1)
25) $\{(B_1D_2,B_1D_1)$or$(B_2D_2,B_1D_1)\}$ / B_1D_2	26) $\{(B_2D_2,B_1D_1)$or$(B_2D_2,B_2D_1)\}$ / B_2D_2

Regions 25 and 26 each contain more than region, as the optimal response to high effort varies. However, for this example, a strategy involving the D_2 test after low effort is always preferred to a randomised test after high effort. Thus, once D_2 becomes a best response to low effort the auditor's testing after high effort becomes irrelevant since both auditee types will always choose low effort. Figure 6.4 can be divided into six areas representing different kinds of equilibrium set. An example of each is given below:

Region 2 - Only pooling equilibria since the auditor always uses D_1

a) Pooling on H if $D_H < 62.48$	**b) Pooling on L if $D_H > 62.48$**
B_2D_1 / B_1D_1	B_2D_1 / B_1D_1
$S_1 = 1$ $f_{1H} = 0$	$S_1 = 0$ $f_{1L} = 0$
$S_2 = 1$ $f_{2H} = 0$	$S_2 = 0$ $f_{2L} = 0$

Region 6 - Only pooling equilibria as type 1 mimics type 2

$F_L^* = (6.75 - C_B + 0.4425C_D) / (125.475 - 0.1975C_D)$

a) Pooling on H if $D_H < 59.41$

$B_2D_1 / (B_1D_2, B_2D_1)$

$S_1 = 1 \quad f_{1H} = 0$

$S_2 = 1 \quad f_{2H} = 0$

b) Pooling on L if $D_H > 59.41$

$B_2D_1 / (B_1D_2, B_2D_1)$

$x_L = 0.05602$

$S_1 = 0 \quad f_{1L} = 0$

$S_2 = 0 \quad f_{2L} = 10F_L^*$

Region 10 - D_1 optimal vs. H and $F_L^* > (1\text{-}P)$. Partially Hybrid equilibria occur

$F_L^* = (6.75 - C_B + 0.4425C_D) / (125.475 - 0.1975C_D)$

Pooling on H if $D_H < 59.41$

Auditor	after H B_2D_1	after L $(B_1D_2\ B_2D_1)$
	$x_H = 0$	$x_L \in X$
Type 1	$S_1 = 1$	$f_{1H} = 0$
Type 2	$S_2 = 1$	$f_{2H} = 0$
		$X = (x_{2L}, 0.19589 - 0.00235D_H)$

Partially Hybrid equilibrium if $D_H \in (59.41, 62.51)$

Auditor	after H B_2D_1	after L $(B_1D_2\ B_2D_1)$	
		$x_L = (D_H - 58.8)/10.825$	
Type 1	$S_1 = (F_L^* - 0.1)/0.9F_L^*$	$f_{1H} = 0$	$f_{1L} = 0$
Type 2	$S_2 = 0$		$f_{2L} = 1$

Pooling on L if $D_H > 62.51$

Auditor		after L $(B_1D_2\ B_2D_1)$
		$x_L = 0.343015$
Type 1	$S_1 = 0$	$f_{1L} = (F_L^* - 0.1)/0.9$
Type 2	$S_2 = 0$	$f_{2L} = 1$

Region 16 - $F_H^* < (1\text{-}P)$ $F_L^* > (1\text{-}P)$ so hybrid equilibria occur

$F_H^* = (-21.375 + 0.2775C_D) / (122.625 - 0.2425C_D)$

$F_L^* = (-5.85 + 0.3975C_D) / (181.35 - 0.368875C_D)$

Pooling on H if $D_H < 57.3$

Auditor	after H $(B_2D_2\ B_2D_1)$	after L $(B_1D_2\ B_1D_1)$
	$x_H = 0.281859$	$x_L \in X$
Type 1	$S_1 = 1$	$f_{1H} = 0$
Type 2	$S_2 = 1$	$f_{2H} = 10F_H^*$
		$X = (x_{2L}, 0.40361 - 0.00171D_H)$

Partially hybrid equilibrium if $D_H \in (47.83, 51.95)$

Auditor	after H $(B_2D_2\ B_2D_1)$	after L $(B_1D_2\ B_1D_1)$	
	$x_H = (36.36984-D_H)/26.125$	$x_L = 0.516761$	
Type 1	$S_1 = (0.1(1-F_H*))/0.9F_H*$	$f_{1H} = 0$	$f_{1L} = F_L*$
Type 2	$S_2 = 1$	$f_{2H} = 1$	

Hybrid equilibrium if $D_H \in (51.95, 57.3)$

Auditor	after H $(B_2D_2\ B_2D_1)$	after L $(B_1D_2\ B_1D_1)$	
	$x_H = 3.09385-0.04908D_H$	$x_L = 2.56668-0.03946D_H$	
Type 1	$S_1 = (1-F_H*)(F_L*-0.1))/0.9(F_L*-F_H*)$	$f_{1H} = 0$	$f_{1L} = 0$
Type 2	$S_2 = F_H*(F_L*-(0.1))/0.1(F_L*-F_H*)$	$f_{2H} = 1$	$f_{2L} = 1$

Pooling on L if $D_H > 51.95$

Auditor	after H $(B_2D_2\ B_2D_1)$	after L $(B_1D_2\ B_1D_1)$	
	$x_H \in X$	$x_L = 0.516761$	
Type 1	$S_1 = 0$		$f_{1L}=(F_L*-0.1)/0.9$
Type 2	$S_2 = 0$		$f_{2L} = 1$

$X = (MAX(x_{2H}, 2.53282 - 0.03828D_H), MIN(x_{1H}, 0.42896 + 0.00222D_H))$

Region 19 - $F_H* > (1-P)\ F_L* > (1-P)$ so partially hybrid equilibria occur

$F_H* = (-8.55+0.1425C_D) / (178.65 - 0.524125C_D)$

$F_L* = (-5.85+0.3975C_D) / (181.35 - 0.368875C_D)$

Pooling on H if $D_H < 56.91$

Auditor	after H $(B_1D_2\ B_1D_1)$	after L $(B_1D_2\ B_1D_1)$	
	$x_H = 0.836342$	$x_L \in X$	
Type 1	$S_1 = 1$	$f_{1H} =(F_H*-0.1)/0.9$	
Type 2	$S_2 = 1$	$f_{2H} = 1$	

$X = (x_{2L}, 0.74224 - 0.00171D_H)$

Partially hybrid equilibrium if $D_H \in (56.91, 58.22)$

Auditor	after H $(B_1D_2\ B_1D_1)$	after L $(B_1D_2\ B_1D_1)$	
	$x_H = (65.64484-D_H)/10.45$	$x_L = 0.516761$	
Type 1	$S_1 = (0.1(1-F_H*))/0.9F_H*$	$f_{1H} = 0$	$f_{1L} = F_L*$
Type 2	$S_2 = 1$	$f_{2H} = 1$	

Pooling on L if $D_H > 58.22$

Auditor	after H $(B_1D_2\ B_1D_1)$	after L $(B_1D_2\ B_1D_1)$	
	$x_H \in X$	$x_L = 0.516761$	
Type 1	$S_1 = 0$		$f_{1L}=(F_L*-0.1)/0.9$
Type 2	$S_2 = 0$		$f_{2L} = 1$

$X = (MAX(x_{2H}, 6.28180 - 0.09569D_H), MIN(x_{1H}, 0.61086 + 0.00171D_H))$

Region 25 - Auditor never uses D_1 after observing Low effort - only pooling on L

Pooling on L $\forall$ D_H

B_1D_2

$S_1 = 0 \; f_{1L} = 1$

$S_2 = 0 \; f_{2L} = 1$

Figure 6.4 can be divided into six areas, shown in figure 6.5, representing the different kinds of equilibrium set. The full equilibrium set is detailed in appendix C. In area I (regions 1-3), the auditor always uses test D_1 and hence no fraud occurs. The equilibrium changes from pooling on L to pooling on H as D_H increases. In area II the auditor still uses D_1 after observing H, but a small rate of fraud does occur in a low effort internal control environment. The type 1 auditee does not commit fraud however, and thus benefits from a test involving D_1. This means that again the equilibrium changes from pooling on H to pooling on L as the type 1 auditee mimics the actions of type 2.

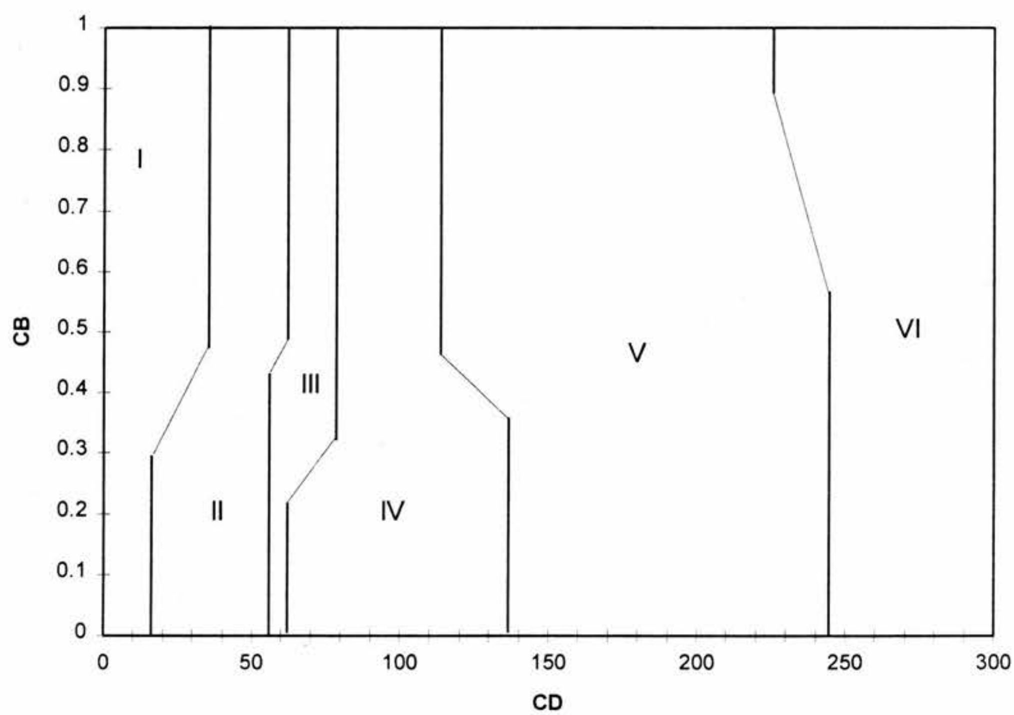

figure 6.5 :- equilibrium regions for P=0.9

In area III a higher level of fraud (involving both auditee types) occurs after low effort, although there is still no fraud after high effort. For low D_H, the advantages of reducing random errors outweigh the benefits of a limited occurrence of fraud and both types put in high effort (and never commit fraud). As D_H increases the cost of reducing errors increases to the point where both types abandon any attempts to prevent errors and resort to a mixed strategy fraud equilibrium. In these areas there is still no occurrence of fraud after high effort. The change to low effort and fraud becomes increasingly attractive as D_H increases. However, the type 2 auditee (who finds it easier to commit fraud) will stop putting in high effort for a lower level of D_H than type 1. This leads to a partially separating equilibrium (with a pure test after High effort) where type 2 commits fraud with certainty and puts in low effort. Type 1 never commits fraud and randomises between high effort (which identifies him as "honest") and low effort (where he benefits from the D_l test). The frequency with which the auditor uses test D_l after low effort decreases as D_H increases until for all sufficiently high D_H, both types choose low effort (and some positive fraud rate) and the auditor tests to limit fraud by type 1. In this situation the auditor reduces his use of the depth test to encourage the "honest" type 1 auditee to work harder (high effort) to prevent errors.

In area IV the auditor has a strategy that involves randomisation after observing either effort level. The auditee correspondingly has a non-zero fraud rate after either level, although the fraud rate is higher if low effort controls are in use. With high effort controls type 2 has a mixed fraud strategy whilst type 1 never commits fraud. If the auditee does not commit fraud then reducing the number of random errors will avoid potentially costly outcomes for the auditee. For small D_H therefore both types put in high effort. As D_H increases, the advantages of this error reduction are outweighed by the cost of the effort involved (and the associated lower fraud rates after high effort). Thus for all sufficiently high D_H both auditee types will put in low effort and the auditor will test to limit type 1 fraud whilst type 2 commits fraud with certainty. A second equilibrium can occur when the merits of high effort (low fraud rate) and low effort (high fraud rate) are comparable. In this case both auditee types randomise between high and low effort whilst type 1 chooses no fraud and type 2 always commits fraud. The auditor randomises to prevent type 1 also committing fraud and to limit the occurrence of low effort. The level of depth testing after observing either effort level increases as D_H increases until high effort is a more expensive option for either type. This hybrid equilibrium has a level of auditor uncertainty about auditee type that lies between pooling (where he learns nothing) and separating (where he always learns the auditee's type).

In area V there are high rates of fraud after both high and low effort. For sufficiently low D_H both types put in high effort to reduce the occurrence of errors whilst the auditor tests to limit type 1 fraud. As D_H increases, type 1 prefers to play L which in turn alters the auditor's testing after observing H. A partially separating equilibrium occurs in which type 2 puts in high effort and commits fraud with certainty whilst type 1 randomises between high effort (no fraud) and low effort (some fraud). The auditor tests to prevent type 1 fraud after H, to limit type 1 fraud after L, and to encourage type 1 to play low effort. In this situation the auditor is willing to put up with a higher occurrence of random errors in order to identify the auditee's type and limit fraud.

Finally, in area VI the cost of the D-test is quite high. After observing L there is a fair chance that any irregularity detected will be an error, in which case any further work (D_1) is of no benefit to the auditor. After observing H however there is a much better chance that a discovery by the B-test is a result of fraud, in which case the cost of the D_1-test is offset by the potential costs of failing to investigate fraud. The auditor therefore only uses test D_1 after observing high effort. In this case both auditee types will always put in low effort (and hence no D_1-test) irrespective of D_H and will commit fraud with certainty. For even larger C_D the auditor will not use test D_1 after either effort level. In this situation there is still some incentive for the auditee to use high effort and reduce errors as he would still prefer a completely unqualified audit report. However, reducing error occurrence involves effort D_H and also increases the difficulty of concealing fraud. In this example the increased difficulty in overriding a high effort internal control system outweighs the benefits of reduced error occurrence and both types will always choose L.

Assessing Audit Risk

The three main factors that determine the audit outcome are the expected fraud rate $\underline{F}$, the rate of occurrence of high effort S_i and the frequency with which the auditor uses the D-test (1-x). The high effort rate and fraud rate for certain kinds of equilibrium can be influenced by changes in C_B and C_D. On the other hand, if the auditee uses a mixed effort level strategy the auditor's use of the D-test can be influenced by either D_{iFH} (in a partially hybrid equilibrium) or D_H (in a hybrid equilibrium).

These three factors will in turn influence the components of the audit risk model. In an error detection setting inherent risk (IR_e) relates to the susceptibility of an account balance or class of transactions to material error. Control risk (CR_e) is the risk that material errors are not prevented or detected by the internal controls and detection risk (DR_e) is the risk that errors that are not prevented or detected by the internal controls are not detected by the auditor. Shibano [34] argues that this formulation can also be used to assess Audit Risk from irregularities. This requires a separate assessment of each component:

$$AR_e = IR_e \times CR_e \times DR_e \qquad AR_F = IR_F \times CR_F \times DR_F$$

This model considers inherent risk and control risk in both settings as one factor. In error detection we consider the probability p_w that an error occurs and goes undetected by the control system. For fraud detection we must assess the control risk to be 1, in which case the product of the first two terms will be IR, since it is assumed that the auditee can conceal fraudulent activity from the internal controls. The detection risk component is assessed after the auditor observes the effort level since the auditor's testing is contingent upon the standard of the internal controls. Suppose the auditor randomises between B_aD_2 and B_cD_1. Then from equation 6.2.3

$$IR_e \times CR_e = P(p_L - S_1(p_L - p_H)) + (1-P)(p_L - S_2(p_L - p_H))$$
$$DR_e = (1-\underline{F})((1-r_c) + x(r_c - r_a))$$
$$IR_F \times CR_F = PS_1 f_{1H} + (1-P)S_2 f_{2H} + P(1-S_1) f_{1L} + (1-P)(1-S_2) f_{2L}$$
$$DR_F = x + (1-x)(r_c'(1-v_1) + (1-r_c')r_a)$$

	F		**(1-x)**		**S_i**	
	$\frac{\partial}{\partial C_D}$	$\frac{\partial}{\partial C_B}$	$\frac{\partial}{\partial D_H}$	$\frac{\partial}{\partial D_{FH}}$	$\frac{\partial}{\partial C_D}$	$\frac{\partial}{\partial C_B}$
6a						
6b	+	-				
10a						
10b			-		+	-
10c	+	-		-		
16a	+			-		
16b	+		+		-	
16c	+		+	-/+[1]	+/-[2]	
16d	+			-		
19a	+			-		
19b	+		+		-	
19c	+			-		

table 6.10 :- the effects on strategies of varying the cost of actions

The auditor's mixed strategy in a hybrid equilibrium is influenced by D_{2FH}-D_{2FL} rather than the absolute values. Thus at the entry marked (1) the effect of changing one cost can have an ambiguous effect on the auditor's randomisation x. Also in a hybrid equilibrium the auditee's mixed effort level strategies are influenced by the critical fraud rate. At the entry marked (2) S_2 is increasing in F_H* whilst S_1 is decreasing in F_H*. The changes in mixed strategies as the action costs vary can be used to consider how the Audit risk terms described above are affected by changes in the action costs. This is shown in table 6.11 The two comments made above for region 16b will still hold. In addition, there are some regions where the decision to use D_1 does not affect the choice of B-test and hence the error-detection risk term is unchanged. These entries are marked (3) in the following table:

	$IR_F \times CR_F$		$IR_e \times CR_e$		DR_F		DR_e			
	$\frac{\partial}{\partial C_D}$	$\frac{\partial}{\partial C_B}$	$\frac{\partial}{\partial C_D}$	$\frac{\partial}{\partial C_B}$	$\frac{\partial}{\partial D_H}$	$\frac{\partial}{\partial D_{FH}}$	$\frac{\partial}{\partial D_H}$	$\frac{\partial}{\partial D_{FH}}$	$\frac{\partial}{\partial C_D}$	$\frac{\partial}{\partial C_B}$
6a										
6b	+	-							-	+
10a										
10b			-	+	+		-			
10c	+	-				+		-	-	+
16a	+					+		**0**[3]	-	
16b	+		+		-		**0**[3]		-	
16c			-/+[2]		-	+/-[1]	+		-	
16d	+					+		-	-	
19a	+					+		**0**[3]	-	
19b	+		+		-		**0**[3]		-	
19c	+					+		**0**[3]	-	

table 6.11 :- the effects on audit risk components of varying the cost of actions

These audit risk factors will be determined by each players' equilibrium strategy. Tables 6.10 and 6.11 consider a candidate equilibrium set from the areas of interest discussed above. The effects of varying the action costs on the three equilibrium determinants $\underline{F}$, (1-x) and S_i and the components of audit risk are considered.

These variations only occur in a mixed strategy and so candidates from regions 1-3 and 25-26 (which have pure effort levels and fraud rates) are not included. These variations in costs must be of limited size to ensure only localised changes. A large change in C_D, for example, will change the auditor's equilibrium strategies and thus the equilibrium set. The equilibrium within a region changes as D_H varies. Each region is considered as D_H increases so that, in 6 say, 6a occurs for low D_H whilst 6b occurs for higher D_H. Almost all the regions have a unique type of equilibrium for a given set of costs (although technically there are a continuum of pooling equilibria differing only in the action they prescribe after observing a zero probability event) with a unique optimal play prediction. However regions 16a, 16b, 16c and 16d overlap. In this case there is not a unique type of optimal strategy. This suggests that, as the assessment of audit risk depends upon the equilibrium, in some situations there may two or more equally valid assessment for the level of audit risk.

It can be seen from table 6.10 that a decrease in the cost of the D-test C_D will result in a decrease in the occurrence of fraud. Decreasing the cost C_B has an ambiguous effect on the occurrence of fraud. In some circumstances this also decreased the fraud rate as the more extensive B_1 test is more likely to find evidence of fraud which can subsequently be classified as fraud by the D-test. However, in other situations lowering C_B can raise the fraud rate. This will happen when the auditor is choosing between B_1 and D_1 . In this case an increase in C_B will result in D_1 being used for a lower fraud rate.

The frequency of D_1 in the auditors mixed strategy can be influenced either way by a change in D_H. In the partially hybrid equilibrium 10b the auditor's use of the D-test decreases as D_H increases. In this equilibrium the type 1 auditee does not commit fraud and is therefore only concerned with the prevention of unqualified errors. The auditor encourages the auditee to put in high effort by reducing his use of the D-test after observing low effort. As D_H increases high effort becomes a less attractive option to a type 1 auditee and the auditor must further reduce his use of the D-test after low effort to continue to encourage high effort. In region 16c, a hybrid equilibrium, the auditor adjusts his use of the D-test after high and low effort so that both auditee types are indifferent between high and low effort. As the cost of high effort increases the auditor increases his use of D_1 after both effort levels.

Table 6.11 shows that, as we might expect, a decrease in C_D will result in a decrease in the inherent risk of fraud occurring since this is determined by the equilibrium fraud rate. In region 10b however this will have the opposite effect on the inherent risk of error. In this case an "optimal" level for the cost C_D must strike a balance between the two kinds of inherent risk. Thus the need for the auditor to strike a compromise between error and fraud detection is reflected in the inherent risk terms.

This compromise can also be seen in the detection risk component. In region 10b an increase in the cost of high effort D_H increases the risk that fraud will go unnoticed. However the same increase will decrease the risk that an error goes undetected. In region 19b an increase in the cost of D_1 will lead to a correspondingly increased risk of errors going undetected by the internal control system. However an increase in C_D will also lead to a decrease in the error detection risk. In this region the auditor is once again trying to strike a compromise between using the B-test and the D-test. If C_D is increased the auditor will need to expect a higher fraud rate to start using test D_1. A higher chance of fraud occurring means a lower chance that the B-test will detect nothing. Thus the overall audit risk for errors may be reduced by increasing the D-test cost C_D.

7 CONCLUSIONS

7.1 Comparisons between the models

Both models considered the auditor to be uncertain about the auditee's motivations. In a setting of error detection this was modelled by uncertainty about the chance of errors occurring in the accounts. In a fraud prevention setting the auditor did not know how difficult the auditee would find it to commit fraud. In each case, to highlight the effects of the potential difference in auditee types, the auditee was considered to be either a "good" or a "bad" type by the auditor. In error detection a "good" type had a lower risk of errors occurring, whilst in a fraud prevention setting a "good" type found it more difficult to commit fraud. In both models the difference between the types of auditee was a question of degree - both types had some chance of errors occurring or, in the fraud prevention model both types could commit fraud.

There are four main differences between the models. In the fraud model the auditee has a second unobservable action choice (whether to commit fraud or not). This increased the number of potential outcomes, increased the auditor's uncertainty and led to an increase in the frequency of mixed strategies since even if the auditor knows the auditee's type and effort level there is still uncertainty about the fraud action. Secondly, in the fraud model the auditor had two areas of responsibility, to detect errors and prevent fraud. These two responsibilities can be in conflict, in which case the auditor must divide his resources between them. In a fraud setting the players preferences are more at odds. If the auditee commits fraud he would prefer this to go undetected whilst the auditor can limit his costs if he successfully detects fraud. One consequence of this difference is that mixed strategies are common in the second model, whilst they are quite rare in the first.

Thirdly, the auditor's testing actions are changed in the fraud detection model to include additional forms of qualification and a means of conducting an in-depth investigation to determine if an irregularity was deliberately introduced. The changes in the auditor's qualification, and the increased number of outcomes, meant that the auditor was restricted to qualifying on the evidence gathered. This effectively removes any strategic decision over the qualification. Finally, in the model of fraud detection the auditor is assumed to always observe the effort level since the auditor is uncertain about the auditee's type and fraud decision. This also makes the equilibrium analysis more tractable since, with observation, the model is a dynamic game.

In each model some of the costs were considered as variables to analyse the effects of changes in this cost on the optimal behaviour. In the first model one of the outcome costs NQ(E) was varied. This outcome occurs when an error exists and the auditor fails to appropriately qualify his audit report. The cost of this outcome is associated with the damages awarded to disgruntled shareholders. This permitted a consideration of how the damages awarded to shareholders can influence the outcome of the audit.

The cost of not qualifying an error is the expected discounted cost of the error being discovered at some future date. This is related to the cost of an error being discovered immediately Q(E). However the cost of an error subsequently being brought to light may be larger for two reasons; An unnoticed error may damage the business or generate further errors before it is found, and the cost may involve a punitive element since both parties are considered to have failed in their duty. The cost of the outcome NQ(E) can therefore be related to Q(E) by two factors; a punitive factor since the error went undetected for some time (and may have damaged shareholder's interests), and the likelihood that the error is subsequently brought to light. Thus when we consider increasing the penalty for NQ(E) in the first model, we are really considering increasing the punitive factor (so the shareholders are awarded greater damages if an error is subsequently found) which, in turn, increases the expected cost NQ(E).

In the second model there are more outcome costs as the model considers both error detection and fraud prevention. Thus the auditor has two conflicting areas of concern and he must divide his resources between them. Considering a single outcome cost as a variable is unsatisfactory in such a setting since a change in the cost will prompt either a change in the auditor's responsibility for fraud or his responsibility for error detection, whilst leaving the other unchanged. The relationships between the outcome costs also become more complex. The cost of failing to qualify fraud will be the expected discounted cost of the fraud being discovered at some future date. The likelihood of a fraud subsequently being discovered may be related to an error subsequently being discovered. Furthermore, an increase in the punitive factor for a fraud going undetected may also influence the factor for an error going undetected. Clearly, the relationship between these costs will have a strong influence on the behaviour in the model and thus the equilibrium behaviour will be sensitive to the particular relationship used. To avoid this problem it was decided that the costs of actions would be considered as variables. This looks at the motivation of the players from a different viewpoint. The auditor will use the B_1-test if doing so reduces his costs. This involves comparing the cost of the test with the change in expected outcome costs that the B-test will make. Since B_1 reduces the risk of an error going undetected, an increase in the cost $C^{NQ}(E)$ makes B_1 a more attractive option. Thus as $C^{NQ}(E)$ increases, a more expensive B-test can become cost effective. A consideration of how the cost of actions influences the players' actions can therefore be related to the cost of the players outcomes, without having to specify particular relationships between those outcome costs.

In both models, the inequalities derived for strategy comparisons show that the actual magnitudes of the outcome costs do not influence the behaviour. It is the differences between outcome costs (or the expected differences) that influence the players' actions. In the error detection model the cost of the auditor's strategy considers the difference between qualifying and not qualifying - the expressions $(C^{Q}(NE)-C^{NQ}(NE))$ and $(C^{NQ}(E)-C^{Q}(E))$. The auditee's decision to put in high effort is influenced by two considerations; Firstly, putting in high effort may lead to a different auditor testing strategy, with a different expected cost for the auditee. Secondly, the change in the chance of errors occurring that a better internal control system causes will also change the likelihood of the different outcomes being reached.

In the model of fraud detection, both players motivations are more complex. The auditee's decision to put in high effort can, as in the error detection model, change the auditor's strategy and reduce the chance of errors occurring. However in this model a high effort internal control system increases the difficulty of committing and successfully concealing fraud. The auditor's cost contains the two expressions of the error detection example, the difference between qualifying and not qualifying when there is (or is not) an error present. The possibility of fraud occurring gives two additional comparisons; The difference between correctly qualifying fraud and misclassifying it as an error and the difference in costs between misclassifying fraud and failing to detect it at all.

The model of error detection considered, in simplified form, the three main stages of an audit. The auditor could observe the internal control system, conduct substantive testing and choose to qualify based on the evidence gathered. The auditor's choice about observing the effort level leads to a game that is simultaneous if the auditor does not observe, or dynamic (sequential) if he does. This ambiguity lead to an interesting analysis of mixed strategies since the nature of a mixed strategy depends upon whether the game is simultaneous-move or dynamic. It was shown that if the auditor does observe in a mixed strategy equilibrium then he does so with a frequency strictly less than one. Thus if learning information is costly, the optimal use of this resource may involve probabilistic information gathering.

In the error detection model the auditor also had the choice of whether to qualify or not after considering the results of his testing. This permitted the strategies "always qualify" or "never qualify" that disregard the results of the testing. These strategies allowed the auditor to reduce the chance of one kind of sampling error to zero. If for example the auditor never qualifies then there is no chance of a false positive error occurring. Of course in practice an auditor will never qualify without any evidence. However, the amount of evidence required may vary - if failing to find an error will cost him dearly the auditor will be willing to qualify on the slimmest of evidence. The inclusion of qualification strategies gave a means of considering this variation in the amount of evidence collected without making the model over-complicated.

Qualification strategies for the auditor were not included in the model of fraud detection. In this model the auditor had two forms of qualification, Q_2 if an error had been detected and Q_1 if the error proved to be the result of fraud. Since Q_1 is a more serious accusation, it was assumed that the auditor could not issue Q_1 without sufficient evidence. If the auditor could qualify without evidence then a potential low-cost method of limiting fraud would be to conduct no testing and randomise over qualification opinions. Strategies such as "always Q_2" and "always NQ" are unlikely to be equilibrium strategies for the auditor if there is any chance of fraud occurring since they effectively guarantee that the auditee will never be caught committing fraud.

The auditing literature recognises that procedures for detecting errors may be of little use for detecting fraud. If the fraudulent activity has been concealed the auditor will need to conduct further investigations once an irregularity has been found to determine whether it occurred as the result of some fraudulent activity. This was modelled by the introduction of an in-depth test that can be used after broad substantive testing. Thus the two stages of testing can be regarded as detection and classification. It was assumed that the in-depth testing required some irregularity to investigate, so using the D-test was only an option if the B-test found something. This test was also of use in error detection since the B-test has a non-zero chance of making a false positive sampling error. A detailed investigation of a spurious error is likely to prevent the auditor issuing an incorrectly qualified report. In fact it was assumed that an in depth investigation would always reveal a false positive.

In both models the auditee could benefit, in some situations, from not revealing his type. In the error detection model, both players prefer errors not to occur but each would prefer the other to do the necessary work to achieve this. The auditor's expected chance of errors occurring will depend on his belief about the likelihood of each auditee type. If both types send the same signal his expected error rate will lie between the low error rate for a type 1 auditee and the high error rate for type 2. This can be of benefit to both auditee types since the auditor works harder as the error rate increases, until he resorts to "always qualify" to eliminate the risk of a false negative audit report. In a pooling equilibrium , type 1 can benefit as the auditor works harder than he would if he knew he was facing type 1. On the other hand since type 2 has a higher error rate, as $C^{NQ}(E)$ increases the auditor will resort to "always qualify", which leads to an expensive outcome for the auditee. If the auditor is uncertain about which type he is facing he may use a reasonable qualification strategy against type 2 for costs where he would prefer to qualify if he knew he was facing type 2.

In the model of fraud prevention it was shown that there are no pure strategy equilibria where the auditee reveals his type. Again, both types can benefit from the auditor's uncertainty. If the optimal fraud rate is low then a type 2 auditee will commit fraud whilst type 1 does not. In this case the non-fraudulent auditee can benefit from a mixed strategy involving the D_1-test by mimicking the signal of the fraudulent type. If the optimal fraud rate is high then type 2 commits fraud with certainty whilst type 1 has a non-zero fraud rate. In this case the auditor randomises to limit the occurrence of fraud by a type 1 auditee. This involves using the test D_1 with a lower probability than was needed to limit fraud by type 2. Thus a type 2 auditee mimics the signal of the less fraudulent type to benefit from a lower frequency of in depth testing.

7.2 Discussion of the Error Detection Model

Modelling the auditing process as a signalling game has two advantages. Firstly it includes the concept of "inherent risk" in a game theoretic setting. The equilibrium analysis shows how the auditors degree of uncertainty (P in this model) will influence his optimal strategy. Secondly, the greater uncertainty in an incomplete information setting increases the value of information acquisition. In particular pure strategies exist where costly information acquisition is optimal.

The model of error detection includes the auditior's uncertainty about the occurrence of errors. In a game of costly perfect information the auditor never uses his A-test (observation) strategy in a pure strategy equilibrium which would seem to be ignoring the value of the information. With the inclusion of uncertainty about the auditee's type the model implicitly recognises the value of learning the auditee's action. The observation can be used to infer the auditee's type. Even in the cooperative case, where the two players are no longer at odds but rather try to minimise their joint costs, it can be seen that observation plays an important part.

It was shown that observation can only occur if two conditions are met. Firstly, the auditee types must respond differently to the A-test strategy if observation is to be of use. If both types put in the same effort level then observing this tells the auditor nothing about which type he is facing. Secondly, the auditor must have different optimal testing strategies after learning the effort level. If this is not the case then the auditor will use the same test irrespective of his observation and there is no need to incur the cost of observation. Thus the A_1-test is only used in a separating equilibrium (where by observing the auditor learns which type of auditee he is facing and adjusts his testing accordingly) or as part of a mixed strategy. In a mixed strategy the auditor will not always learn the auditee's type through observation, but he is able to make a more accurate inference.

By considering the costs $C^{NQ}(E)$ and $D^{NQ}(E)$ to be variables the effects of these costs on the equilibria can been seen. This analysis has three advantages; Firstly, it reduces the risk that the equilibria of an example are the result of the particular values chosen. Secondly, these variable costs allow us to see if the model conforms to expectations. Thirdly, a consideration of the equilibrium set as these costs vary can suggest possible penalty levels to encourage certain behaviour. An analysis of the auditor's cost structure as $C^{NQ}(E)$ varies led to a categorisation of the auditor's set of potentially optimal strategies.

For low $C^{NQ}(E)$ and $D^{NQ}(E)$ neither player works hard, which is what we would expect to happen if in reality neither the auditor or the auditee were particularly concerned about the outcome of the audit. The high cost of the B_1 test and the degree of uncertainty P in the example considered meant that B_1 did not appear in any equilibrium, even though it was an optimal auditor strategy against a type 1 auditee playing H. In a pooling equilibrium, the auditor was sufficiently unsure about which type he was facing that a strategy such as B_2R or B_2Q had a lower expected cost than B_1R. With a separating equilibrium, the range of $C^{NQ}(E)$ for which B_1R was optimal against a type 1 auditee was sufficiently high that it was cheaper for the auditor to use B_2Q forcing the "no work" equilibrium (B_2Q, L / L).

Of more interest perhaps is the case of extremely high costs - in the model the auditor always qualifies, in which case the auditee always plays low effort. A related point is the area of joint and several liability. For high costs the no work equilibrium occurs. However, the level of costs that cause this equilibrium depend critically on how the costs are shared between the players. If the auditor is solely responsible the no work equilibrium occurs for low costs ($\Omega > 647$). If both players share the responsibility then this equilibrium will not occur until much higher costs ($\Omega > 10000$).

The inclusion of incomplete information complicates the equilibrium behaviour in two ways. The auditor can have compromise testing strategies as a result of uncertainty. For example the auditor may choose a strategy B_1R if both auditee types put in high effort even though he would prefer B_2R against type 1 or B_2Q against type 2. In this situation the auditor can only guarantee a second-best outcome. A second problem relates to encouraging hard work by the auditee. Because the two auditee types can respond differently, a penalty that encourages one to put in high effort may discourage the other. Another example of this motivation problem occurs with the use of the A-test. It might be expected that observation of the effort level would encourage the auditee to put in high effort so that the subsequent auditor strategy is reasonable. However in this model the A-test only occurs as part of a separating equilibrium in which one type puts in high effort and the other low. The observation positively influences the behaviour of one auditee type whilst encouraging the other not to work.

There are two influences on the nature of the equilibrium regions as $C^{NQ}(E)$ and $D^{NQ}(E)$ vary. If there is a large difference in the size of the penalties then, although both players prefer a low rate of errors, one player is much more concerned about this outcome. Thus the concerned player will do all he can to reduce the risk of errors going undetected whilst the unconcerned player will do no work. The unconcerned player can shirk since he can be sure that the other player cannot risk incurring the larger penalty. To give both parties an equal incentive to work hard, their costs for failing to qualify an error must be of a similar size. This point is further highlighted if the costs are apportioned according to a policy of proportionate liability. This setting clearly illustrates that both players will work hard only if they share the cost of failing to prevent errors.

If both players have a similar sized penalty for not qualifying errors then a second factor influences the equilibrium strategies. Both players have a similar incentive to work hard but in some cases hard work by one of them reduces the risk of unqualified errors to an acceptable level. The problem in this situation is one of coordination. For moderate penalty levels the players cannot coordinate their work and a mixed strategy compromise equilibrium occurs. For higher costs the players are driven to coordinate as both players must work to reduce the risk of errors going undetected. For even higher costs the auditor can eliminate the risk of incurring this penalty by always qualifying and the no work equilibrium occurs. For the example considered this equilibrium occurs, if both penalties are of a similar size, when the combined penalty is about 10,000. This can be compared with the cooperative game, where this equilibrium doesn't occur until the combined cost is greater than 28,000. If the players are concerned with minimising their combined cost then they can successfully work hard together for a large range of costs, whereas in the non-cooperative game the coordination between the players breaks down when the auditor can limit his own costs by always qualifying. This suggests that setting the costs to encourage hard work is a policy more suited to a cooperative setting.

There are two benefits to extending the equilibrium analysis to include mixed strategies. Firstly, the mixed strategy analysis involves considering the players' preferences over different outcomes. This can also be used to reduce the number of pure strategy equilibrium by pareto domination. If both players prefer the same equilibrium it is a potential focal point and, since the auditee moves first, this focal point is more compelling than in a simultaneous move game. When pareto domination of equilibria does occur the players motivations are similar as they agree on a best outcome. The extreme case where the player's motivations are identical is considered in the cooperative game. There are regions of the costs $C^{NQ}(E)$ and $D^{NQ}(E)$ where the equilibrium will be the same regardless of whether the audit is regarded as cooperative or non-cooperative. These areas occur when the penalties for both players are of a similar size and thus the players motivations are similar.

If the players prefer different equilibria then mixed strategies could provide a mechanism for compromise. It was shown that no behavioural strategies form a stable equilibrium and if a mixed strategy exists it will involve randomising by one auditee type. However if there are two pure strategy equilibria the players disagree about the best outcome. This means that mixed strategies will tend to be unstable. If the mixed strategy costs one of the players more than either pure strategy then he has no incentive to play his part in the mixed strategy and it fails to be in equilibrium. In the example considered in section 4.5 mixed strategies proved to be a poor compromise in every region with two pure strategies. This gives various regions where the game theoretic analysis does not give a unique solution for optimal play.

By considering pure strategy equilibria we are not greatly restricting the range of possible solutions - in this model randomising can be expensive. In those cases where no pure strategies exist the mixed strategy is the best suggestion as to how the game should be played. For the numerical example considered the auditor's stable mixed strategy involved randomising between an A_1-test strategy and an A_2-test. Thus the auditor has an optimal strategy that involves a random observation scheme and the frequency of observation increases as the cost $D^{NQ}(E)$ increases.

7.3 Discussion of the Fraud Prevention Model

This model captures the interaction between error detection and fraud prevention. Previous models of fraud detection have regarded random errors as noise which may obscure the results of the auditor's testing for fraud. However, neither players' payoffs are affected by the presence of random errors. It seems that one of the motivations behind committing fraud is that it is unlikely to be discovered. This is partly because the auditor may find it difficult to detect well concealed fraud and partly because the auditor also has a responsibility to detect errors. He cannot therefore devote all of his resources to the hunt for fraudulent activity.

The auditor's testing was divided into two stages; Broad substantive testing (B-test) that can bring irregularities to light and in-depth testing (D-test) that investigates the causes of the irregularity. Thus the B-test detects irregularities, and the D-test classifies them into errors, fraud or false positive sampling errors from the B-test. This concept of detailed investigation of the causes of an error can only be used strategically if random errors also occur. In a model that only considers fraud, the auditor will know the cause of any irregularity without any detailed investigation.

Even when the auditor's options are limited there can be some complex interactions between the two players. The players' interests are more at odds in this model, particularly over fraud prevention. For a wide range of costs the auditor will use D_1 if fraud is occurring and D_2 if not, whilst the auditee will commit fraud if test D_2 is used and will not if test D_1 is used. These preferences lead to a mixed strategy equilibrium. Because the auditee has two actions he could have a mixed strategy fraud rate and a mixed strategy effort level. However, it was shown that, in most cases, if he has a mixed fraud rate he has a pure effort level and with a mixed effort level he has a pure fraud rate.

It was assumed that both types had the ability to commit fraud. The "honest" type 1 auditee was simply more reluctant to commit fraud. This willingness to commit fraud is the sole difference between the auditee types. This was modelled by including a cost to the auditee of committing and concealing fraud. The auditee type who is reluctant to commit fraud will have a higher level of this cost. There are a number of factors that may increase the level of this cost in a way that is unobservable to the auditor. For example, a type 1 auditee may be more concerned with the prospect of fraud being detected, or he may find it more difficult to override the internal control system. For each type the cost is higher in a high effort internal control environment since a more effective internal control system is more difficult to override. The level of this cost determines which auditor strategies are worth committing fraud against, but does not affect the optimal fraud rates.

The auditee's optimal strategy is to choose a level of fraud occurrence that minimises his costs. To be in equilibrium this level of fraud must be part of a mutually stable pair - the auditee chooses a fraud rate so that the auditor is indifferent between two of his strategies and the auditor in turn randomises between these two strategies so that the auditee is indifferent between F and NF. Such a mixed strategy may at first appear to be collusion between the auditee and the auditor - after all in such an equilibrium the auditee commits fraud with positive probability whereas a sampling strategy with a higher occurrence of the D_1 test would deter all fraud. However such an equilibrium pair is the only efficient allocation of the auditor's resources. If the auditor decides to always use D_1 he will deter all fraud. However conducting in depth testing in a non-fraudulent environment is a waste of resources and such a strategy is not optimal. If the auditor never uses the D_1 test then this will encourage fraud - in which case the auditor is negligent if he never looks for fraud.

The auditee's optimal rate of fraud occurrence depends on the auditor's costs. In particular the occurrences will vary as C_B and C_D vary. We might expect that lowering these costs would encourage the auditor to use the associated test more frequently. This in turn should reduce the occurrence of fraud. In the example considered, lowering the cost C_D did decrease the fraud rate until for all sufficiently low C_D the auditor uses D_1 irrespective of the fraud rate. In these circumstances fraud will not occur. Lowering the cost C_B had a more ambiguous effect upon the optimal fraud rate. In some circumstances this also decreased the fraud rate as the more extensive test B_1 is more likely to find evidence of fraud (which can subsequently be classified as fraud by the D-test). However, in other situations lowering C_B can raise the fraud rate. In these situations the auditor is compromising by using either D_1 or B_1. As B_1 becomes a cheaper option, a higher expected fraud rate is needed for the auditor to consider changing to D_1. It was shown that measures to reduce the inherent risk of fraud (namely reducing C_D) may have the opposite effect upon the inherent risk of errors. This reinforces the idea that the auditor's testing must be a compromise between fraud and error detection.

Each auditee type had some incentives not to reveal their type to the auditor. Both types could benefit if the auditor focused his attentions on the other. This leads to behaviour where one type mimics the actions of the other. If the type 2 auditee commits fraud whilst the type 1 does not then it is in the interest of the "honest" type 1 to mimic the behaviour of type 2. By doing this he benefits from an auditor sampling strategy that contains the D_1 test with non-zero probability. This test reduces the risk of sampling errors (false positive) in a non-fraudulent environment. If type 2 plays fraud with certainty whilst type 1 plays fraud with positive probability then it is in the interests of 2 to mimic the less fraudulent type. If type 2 can be distinguished by his effort level the auditor will change his testing strategy to limit the occurrence of type 2 fraud by an increased use of the D_1 test.

It was shown that separating equilibria (in which the auditee effectively reveals his type) do not occur. However, since the auditee's payoffs are dependent upon type, situations arise which are partially separating (hybrid) in which the auditee's behaviour gives the auditor a better idea about which type he is facing. Two kinds of hybrid equilibrium were shown to occur; one where both types use each effort level with positive probability, and one in which type 1 has a mixed strategy effort level whilst type 2 always puts in high effort. In both situations the auditor cannot identify the fraudulent type 2 by his effort level.

In a hybrid equilibrium the auditor's mixed testing strategy limits both the occurrences of fraud and of mimicking behaviour. In these cases the auditor's optimal strategy uses the D_l test with a probability above that required to limit type 1 fraud but below that required to limit type 2 fraud. In some situations the auditor may reduce the frequency of the D_l-test as D_H increases to encourage a type 1 auditee to put high effort into the internal controls to reduce the occurrence of errors. In other situations the auditor is willing to put up with more random errors to identify the auditee's type and limit fraud.

It was also shown that the type of equilibrium that occurs will depend upon the auditor's belief about type **P**. This does not directly influence the overall rate of fraud which is determined in equilibrium. Both auditee types have incentives not to behave in a way that will reveal their private information (how difficult they find it to commit fraud) to the auditor. However, the auditee's optimal strategies involve a degree of "type discrimination" in a way that is unobservable to the auditor. If a low fraud rate F* is optimal then a type 2 auditee (who finds it easier to commit fraud) chooses a fraud rate so that the auditor's expected rate is F* whilst a type 1 auditee never commits fraud. When the optimal fraud rate is high (> (1-P)) this agreement is insufficient to reach the optimal fraud rate and type 1 also commits fraud. When both types are committing fraud with positive probability their preferences are most dissimilar. In these situations the auditor's optimal strategy can force the auditee to partially reveal his type.

For the example considered, as C_D increases the auditor stops using D_l after observing low effort. This result is somewhat counter-intuitive. Since it is assumed that high effort internal controls are more difficult to override we might expect the auditor to suspect fraud in a low effort environment where the controls can be more easily overridden. However, a poor system of controls gives a greater chance of errors occurring and hence a greater chance of incurring the D-test cost, whilst at the same time reducing the chance that a given error will actually be from fraud. The auditee has two potential signalling policies. It may be worth maintaining a high effort internal control system, even though this requires more effort to override, as the internal controls may convince the auditor that he does not need to search for fraud. On the other hand, maintaining a low effort internal control system makes it easier to commit fraud and any evidence may be lost amongst the random errors.

The assessment of the audit risk terms for the example highlights the conflict between fraud prevention and error detection. If the costs of the auditor's tests are changed to reduce the risk of fraud going undetected this can have the opposite effect on the risk of errors going undetected. This analysis followed Shibano [34] in assessing the audit risk from fraud and errors separately. Some mechanism is needed to connect these two assessments. In this model, for example, changing from B_2 to B_1 will reduce the chance of an error going undetected. Since the D-test requires an irregularity to focus upon this same change will increase the chance of fraud being detected as an irregularity and subsequently investigated in detail. Hence a change to B_1 may reduce the audit risk from both errors and fraud.

The number of equilibria raises an interesting question for the assessment of audit risk. In the example considered almost all of the payoff regions have a unique optimal mixed strategy, since the auditee has a unique optimal fraud rate. In one payoff region there are two equilibria, one in which the auditee has a pure effort level and mixed fraud rate and one in which he has a mixed effort level and pure fraud rate. This suggests that the idea of a unique assessment of audit risk might be inappropriate in a strategic setting.

7.4 Limitations of the Models and Areas for Future Work

In both models the number of auditee types, effort levels and auditor testing actions was limited to clearly illustrate the interaction between the auditor and auditee. This gave a stylised view of the audit that may overlook some interesting interaction in a more complex model. A more realistic approach to the auditor's uncertainty about the auditee's motivations might consider a continuum of types. In the error detection model for example this would involve considering an auditee type with every basic chance of error occurring within some interval. In the fraud prevention setting this could involve auditee types whose ability to commit fraud varies, so that some types find it impractical to be fraudulent. A signalling equilibrium would then involve some subset of the auditee types sending the same signal to the auditor. A further refinement could consider different preferences over outcomes for different auditees. In this case certain types may have incentives to signal their type.

A similar extension would give the auditee more effort levels to choose from. This could be modelled by a function determining the cost involved in reducing the error rate to a given level. The auditee's behaviour would depend critically upon the nature of this cost function. If it was linear for example then the auditee's strategies could resemble the high effort / low effort choice. If the outcome costs are low then the auditee puts as little effort into preventing errors as possible. Once costs increase so that it is worth reducing the error rate, the auditee puts as much effort into the internal controls as possible. The nature of this function would need to be considered in some detail to give the model a sense of realism.

Increasing the number of signals could also give rise to a situation where some signals are only available to one type. It would be interesting to investigate the circumstances under which an auditee would use a signal that uniquely described his type. If an increase in the number of signals was combined with an increase in the number of types then the auditor would be able to divide the set of auditees into as many subsets as there are feasible signals. In such a setting we might expect the use of costly observation of the signal to be more useful.

The auditor's B-test action could be extended to give a range of substantive tests. In the model of error detection the choice between B_2 and B_1 was often obscured by the choice of qualification strategy. The ability to choose to always qualify gave the auditor another means of reducing his risk of false positive / false negative sampling errors. Since the B_1 test reduces the risk of both sampling errors occurring, by using B_1 the auditor effectively buys a reduced sampling risk. A more complex approach could give the auditor a choice of a number B-tests of differing efficiency. These tests would have sampling risks of various sizes and a division of this risk between the two kinds of sampling error.

A consideration lacking from both models was the concept of materiality. Random errors in both cases were described by the probability that a material error occurred. To introduce the idea of materiality, the chance of a material error occurring would need to be re-expressed to consider the number of errors occurring. The auditor could then decide what level of error occurrence he deemed material. To analyse a model with this materiality choice it would be necessary to describe the probability of the B-test finding each error, and how the errors are distributed through the accounts. The chance of a sampling error from the B-test would depend upon both the extent of the test and the number of errors occurring. This would greatly complicate the assessment of the player's costs.

The setting of costs and probabilities is, to some extent, arbitrary. Costs are chosen to satisfy the outcome inequalities and to give a representative case. Since the equilibrium behaviour of the model depends on these outcome costs it would be interesting to try and estimate the values of these costs in practice. This problem is more pronounced in the model of fraud detection because there are more outcomes to consider.

A serious limitation in the fraud model is that the benefit from committing fraud is fixed. This benefit will clearly depend upon the size of the fraud, and this size will in turn affect the risk of the fraud being detected. The auditee would therefore need to strike a balance between the benefit of fraud and the risk of detection. This could be included as an auditee action before choosing the fraud rate. Again, this decision will be sensitive to the method of describing the relationship between fraud size and the probability of detection. Further investigation would need to be done to determine a fair estimate of how this relationship works in practice. It could well be the case that the auditee has a unique fraud size that optimises the trade-off between gain and risk. If this is the case the analysis would be equivalent to assuming that the size of fraud is fixed at this optimal size.

Another important consideration lacking from both models is the size of the audit fee. There are two ways that this could be included into the model. Firstly there could be a stage of pre-play audit fee negotiation. This would necessitate additional assumptions about the value which the auditee places on the audit and the availability and likely cost of other audit firms if agreement cannot be reached. This negotiation could be used to further refine the equilibrium set. In a payoff region with two equilibria, the audit fee could provide a mechanism for both players to coordinate on the same equilibrium. This could raise some interesting questions about auditor independence.

The model could be developed by considering a different action as a signal between the auditee and auditor. This is another way in which the audit fee could be included - the auditee could offer a certain fee and from this the auditor tries to infer the auditee's type. In a fraud detection setting a willingness to pay a large fee (which would allow the auditor to conduct detailed testing) could be regarded as a signal that the auditee has nothing to conceal. This could lead to an equilibrium in which the auditee conducts a large fraud whilst the auditor collects a large fee which convinces him not to test.

Another limitation of both models is that they regard the audit as a one-off event. A more realistic approach would be to regard the audit as a repeated game, although analysis of these can prove difficult. The models developed here could be used as the basic one period game to be repeated. Repeated games have the potential for a greater degree of cooperation between the players. In particular repetition could be used to achieve a compromise in settings where each player prefers a different outcome since the equilibrium play in the repeated game could involve alternating between the one-period equilibria. An important consideration in the area of repeated games would be how the duration of the auditor / auditee contract can affect the fraud and error rates in the model.

The setting of proportionate liability could be extended to consider two alternate allocations of liability; One, such as $\beta = 0.5$, where both parties have put in the same amount of effort and a second where one party has worked harder than the other and thus gets a much smaller share of any liability. This could be used to motivate hard work for relatively low penalty levels as working not only reduces the risk of an error going undetected but also shifts the burden for any liability to the other player. The equilibrium in such a setting would resemble the "Prisoner's Dilemma" in which the only Nash equilibrium is the least favourable outcome for both players. In an auditing context this would involve hard work by both players for low penalties since neither will wish to take the majority of the liability. Although such an outcome may not be favoured by the players it represents the most desirable outcome for the shareholders of the company. Thus a setting of proportionate liability could provide an alternative method of encouraging socially desirable outcomes to the audit.

In both models the equilibrium analysis did not necessarily lead to a unique prediction as to how the game should be played. Other equilibrium refinements could be considered to further reduce the equilibrium set. There are two approaches to this, considering either game theoretic refinements or considerations from auditing. One potential mechanism mentioned above would be to use the audit fee as a means of coordination. Alternatively, Antle's [2] concept of an independent auditor could be used or modified - for example the auditor could choose the equilibrium that minimises the risk to the shareholders.

On the other hand, a further game theoretic refinement could be used. Multiple equilibria occur when the player's prefer different outcomes and are unable to coordinate their equilibrium play. One method of reaching a compromise solution would be to use Aumann's correlated equilibria detailed in Fudenberg and Tirole [19]. This involves the players jointly randomising between equilibrium pairs by observing a publicly observable randomisation (so for example the players may agree to play one equilibrium if a tossed coin shows heads and the other equilibrium if it shows tails). Such an equilibrium will be a good compromise as each player's expected cost will lie between his costs for the two pure strategy equilibria. This approach raises interesting questions about what could be used as a public randomisation. Furthermore, an agreement to jointly randomise could be seen as weakening the auditor's independence.

7.5 Summary

This work has developed two models of the auditing process, considering situations of both fraud and error detection. The foundation for the formulation of both models is to view the audit as a process of incomplete information. In each setting, no matter how thorough the auditor's investigation, there will remain some residual uncertainty about the company being audited. Both these models showed that quite complex interactions between the two players can occur from fairly simple strategy sets. This suggests that these models could be used as the basis for further investigation into penalty regimes, or different ways of allocating liability, to encourage certain behaviour.

The model of error detection considered each of the three main stages of the audit; Investigation of the internal controls, substantive testing and qualification. Viewing the error detection process as a signalling game meant that costly information acquiring strategies could form part of a pure strategy equilibrium. This is an advancement over complete information models of auditing where observation can only form part of an equilibrium if it is costless. The equilibrium set for this model was developed as two of the outcome costs varied. This permitted the auditor's optimal strategy sets to be classified in terms of his B-test cost C_B. The variable costs were also used to consider the effects of increased litigation on the behaviour of the auditor and auditee.

A numerical example was considered in settings of cooperation, non-cooperation and proportionate liability. This example suggests that increasing the participants' penalties to encourage hard work may be counter-productive. A setting of proportionate liability shows that both players can most effectively be encouraged to work if they equally share the potential liability. The cooperative analysis suggests that high levels of litigation are more likely to encourage hard work in a cooperative setting. In the example considered there were regions of costs for which there exists an optimal solution irrespective of the assumptions made about cooperation

An analysis of mixed strategies showed that behavioural strategies, where the randomisation depends on the information set reached, are an unsatisfactory solution in this model since one player will have no incentive to participate in the randomisation. This means that in a mixed strategy, observation can only occur with a probability strictly less than one. It was also shown that only one auditee type could participate in a mixed strategy. This reduced the number of potential mixed strategy equilibria. For the example being considered it was shown that a mixed strategy was only a reasonable compromise if there were no pure strategy equilibria. If pure strategy equilibria exist then a mixed strategy equilibrium costs one of the players more than either pure equilibrium. Thus there can be costs for which this model does not have a unique solution.

The model of fraud prevention considered the occurrence of both fraud and unintentional errors. In this model the auditor had two stages of testing; Broad substantive testing that could detect irregularities and in-depth testing that can determine the cause of the irregularity. It was shown that no equilibrium exist in which the auditee reveals his private information to the auditor. The equilibrium set is classified in terms of the auditor's uncertainty about the auditee's type. The auditee's optimal fraud rate involved "type discrimination" where the two auditee types act as differently as possible without indicating their type to the auditor.

An example was considered to illustrate the equilibrium set. The effects of varying the costs of actions on the equilibrium behaviour were examined. Lowering the cost of in-depth testing was shown to reduce the equilibrium fraud rate, but a decrease in the cost of substantive testing could have the opposite effect. The components of audit risk were assessed for the example. It was shown that changes in the costs that reduce the risk of errors going undetected may increase the risk of fraud going undetected. This highlights the auditor's testing problem in which he must divide his resources between fraud and error prevention. For some costs there are two mixed strategy equilibria. This suggests that the assessment of audit risk may be non-unique in a strategic setting.

The analysis of the equilibrium behaviour of these two models shows that games of incomplete information can provide a rich setting in which to consider the interaction between the auditor and auditee. The auditor's uncertainty adds the consideration of inherent risk to his strategic planning. Although these models have quite stylised strategy sets the interaction between the players can be quite complex. The first model clearly demonstrated that information acquisition can play an important role in a setting of incomplete information.

By regarding one of each players' outcome costs as a variable a family of games was analysed. The effects of the variable costs on the equilibrium behaviour could then be seen. This could be used to consider the policy implications of externally influencing this cost to encourage certain behaviour. The error detection example illustrated that a change in one of the players costs will change his optimal strategy and thus the other player's optimal response. Hence a change in one player's penalty can influence the behaviour of both players. Such an interaction can only be accommodated by a strategic analysis, which suggests that game theory is a useful tool for considering regulatory issues.

The second model showed that fraud detection can be considered alongside error detection and that the players actions are influenced by their responsibilities for both. It was shown that no pure strategy separating equilibrium (in which the auditee reveals his type) can exist. The auditee's equilibrium behaviour is strongly influenced by his desire not to reveal his type. This led to equilibria in which the fraudulent auditee type mimicked the behaviour of the less fraudulent type to avoid a high level of testing designed to detect fraud. However, situations also arose in which a second, more surprising, form of mimicking occurred in which a non-fraudulent auditee mimics the behaviour of a fraudulent type to encourage the auditor to work harder. The example also showed that situations occur in which the auditor tests specifically to encourage the auditee to partially reveal his type. A model of complete information would be incapable of considering any of this interaction. This suggests that signalling games, and other games of incomplete information, can be used to consider a wider range of behaviour in an auditing setting. The discussion of the limitations of these models and the associated areas for expansion suggest that signalling games will be a fruitful area for the development of further models of the auditing process.

Game theory replaced decision theory as a modelling approach to auditing because it considered the strategic interaction between the manager and the auditor and led to a wider, more convincing, range of behaviour. This work suggests that considering only strategic interaction may also limit the range of behaviour in a model. Audit work is partly motivated by uncertainty about a company and it's managers and a convincing model of the auditing process must take account of this. In a strategic setting this leads to the description of the audit as a game of incomplete information. If attention is focused on the auditor's attempts to infer the auditee's motives then the result will be a signalling game.

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APPENDIX A
Sensitivity of Equilibria to Auditee's Effort Cost - D_H

A_2B_2NQ - H is optimal for t_i

$\Leftrightarrow \quad p_{iH}D^{NQ}(E)+(1-p_{iH})D^{NQ}(NE)+D_H < p_{iL}D^{NQ}(E)+(1-p_{iL})D^{NQ}(NE)$

$\Leftrightarrow \quad D_H < (p_{iL} - p_{iH})(D^{NQ}(E)-D^{NQ}(NE))$

For a given example this inequality will give us conditions on D_H for each type i. These conditions will be functions of $D^{NQ}(E)$, the cost that we consider varying. Let us denote these functions $f_i(D^{NQ}(E))$. For the example of section 4.5 we have:

$$f_1(D^{NQ}(E)) = -3 + 0.3\ D^{NQ}(E)$$
$$f_2(D^{NQ}(E)) = -2 + 0.2\ D^{NQ}(E)$$

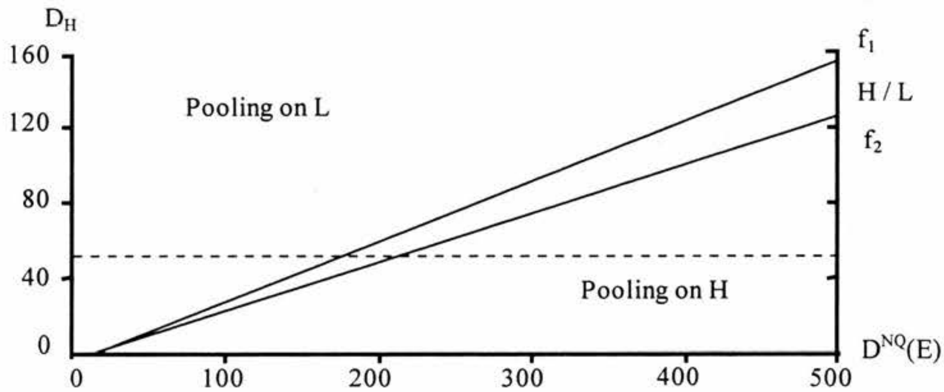

figure A1 :- auditee effort against A_2B_2NQ

A_2B_2R - H is optimal for t_i

$\Leftrightarrow \quad p_{iH}(r_2D^Q(E)+(1-r_2)D^{NQ}(E))+(1-p_{iH})(t_2D^Q(NE)+(1-t_2)D^{NQ}(NE))+D_H <$
$\qquad p_{iL}(r_2D^Q(E)+(1-r_2)D^{NQ}(E))+(1-p_{iL})(t_2D^Q(NE)+(1-t_2)D^{NQ}(NE))$

$\Leftrightarrow \quad D_H < (p_{iL}-p_{iH})(r_2D^Q(E)+(1-r_2)D^{NQ}(E)-t_2D^Q(NE)-(1-t_2)D^{NQ}(NE))$

For the example in 4.5 this gives us:

$$f_1(D^{NQ}(E)) = 41.55 + 0.06\ D^{NQ}(E)$$
$$f_2(D^{NQ}(E)) = 27.7 + 0.04\ D^{NQ}(E)$$

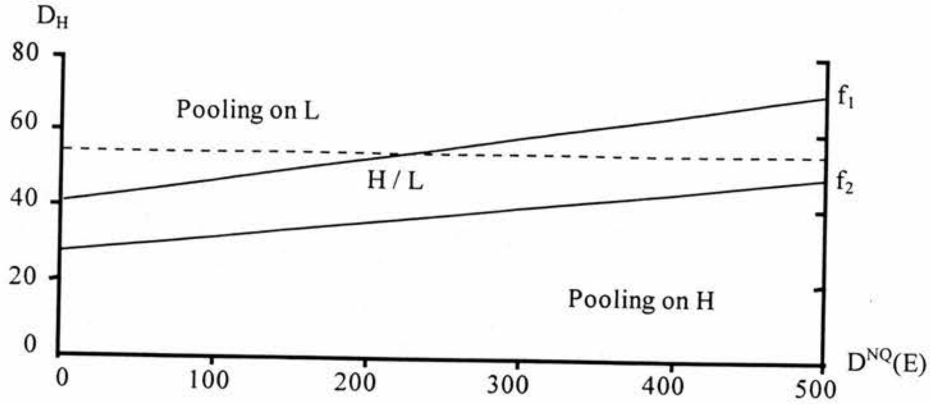

figure A2 :- auditee effort against A_2B_2R

A_2B_2R - H is optimal for t_i

$$\Leftrightarrow \quad p_{iH}(r_1D^Q(E)+(1-r_1)D^{NQ}(E))+(1-p_{iH})(t_1D^Q(NE)+(1-t_1)D^{NQ}(NE))+D_H < p_{iL}(r_1D^Q(E)+(1-r_1)D^{NQ}(E))+(1-p_{iL})(t_1D^Q(NE)+(1-t_1)D^{NQ}(NE))$$

$$\Leftrightarrow \quad D_H < (p_{iL}-p_{iH})(r_1D^Q(E)+(1-r_1)D^{NQ}(E)-t_1D^Q(NE)-(1-t_1)D^{NQ}(NE))$$

For the example in 4.5 this gives us:

$$f_1(D^{NQ}(E)) = 56.4 + 0.015\ D^{NQ}(E)$$

$$f_2(D^{NQ}(E)) = 37.6 + 0.01\ D^{NQ}(E)$$

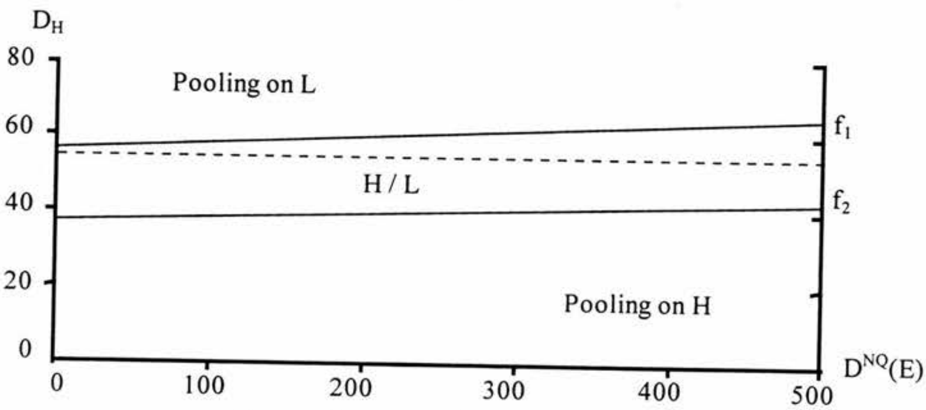

figure A3 :- auditee effort against A_2B_2R

APPENDIX A

A_2B_2Q - H is optimal for t_i

$\Leftrightarrow \quad p_{iH}D^Q(E)+(1-p_{iH})D^Q(NE)+D_H < p_{iL}D^Q(E)+(1-p_{iL})D^Q(NE)$

$\Leftrightarrow \quad D_H < (p_{iL} - p_{iH})(D^Q(E)-D^Q(NE))$

For the example in 4.5 we would have:

$$f_1(D^{NQ}(E)) = 30$$

$$f_2(D^{NQ}(E)) = 20$$

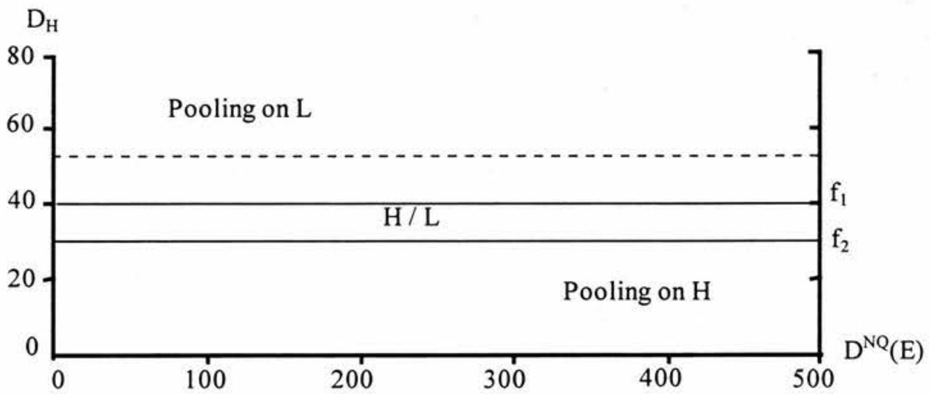

figure A4 :- auditee effort against A_2B_2Q

A_1B_2NQ/B_2R - H is optimal for t_i

$\Leftrightarrow \quad p_{iH} D^{NQ}(E)+(1-p_{iH})D^{NQ}(NE)+D_H <$

$$p_{iL}(r_2D^Q(E)+(1-r_2)D^{NQ}(E))+(1-p_{iL})(t_2D^Q(NE)+(1-t_2)D^{NQ}(NE))$$

$\Leftrightarrow \quad D_H < p_{iL}r_2D^Q(E) + (p_{iL}(1-r_2)-p_{iH})D^{NQ}(E) + (1-p_{iL})t_2D^Q(NE) +$

$$((1-p_{iL})(1-t_2)-(1-p_{iH}))D^{NQ}(NE)$$

For the example in 4.5 this gives us:

$$f_1(D^{NQ}(E)) = 76.475 + 0.02\ D^{NQ}(E)$$

$$f_2(D^{NQ}(E)) = 99.75 - 0.2\ D^{NQ}(E)$$

APPENDIX A

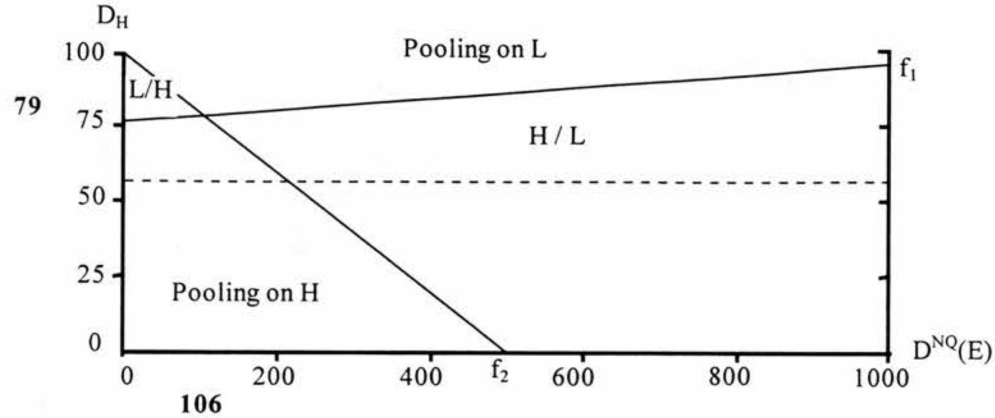

figure A5 :- auditee effort against A_1B_2NQ/B_2R

A_1B_2NQ/B_1R - H is optimal for t_i

$$\Leftrightarrow \quad p_{iH} D^{NQ}(E)+(1-p_{iH})D^{NQ}(NE)+D_H <$$
$$p_{iL}(r_1 D^Q(E)+(1-r_1)D^{NQ}(E))+(1-p_{iL})(t_1 D^Q(NE)+(1-t_1)D^{NQ}(NE))$$

$$\Leftrightarrow \quad D_H < p_{iL} r_1 D^Q(E) + (p_{iL}(1-r_1)-p_{iH})D^{NQ}(E) + (1-p_{iL})t_1 D^Q(NE) +$$
$$((1-p_{iL})(1-t_1)-(1-p_{iH}))D^{NQ}(NE)$$

For the example in 4.5 this gives us:

$$f_1(D^{NQ}(E)) = 77.3 - 0.0325\ D^{NQ}(E)$$
$$f_2(D^{NQ}(E)) = 108 - 0.275\ D^{NQ}(E)$$

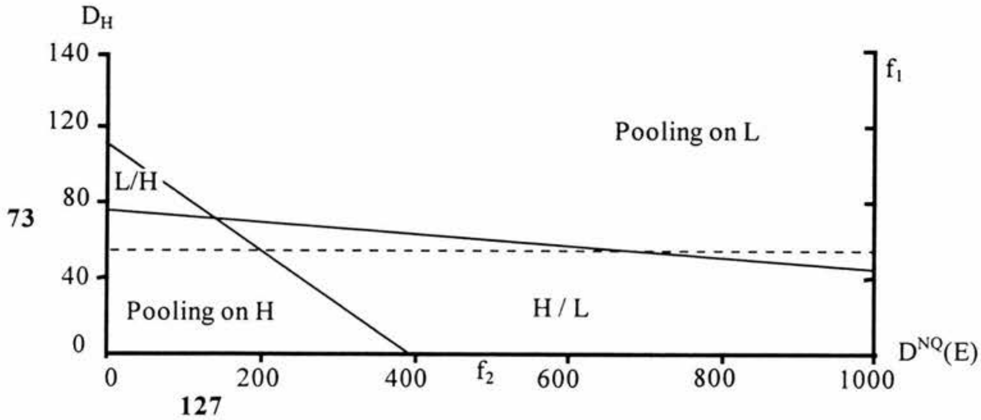

figure A6 :- auditee effort against A_1B_2NQ/B_1R

APPENDIX A

A_1B_2NQ/B_2Q - H is optimal for t_i

$$\Leftrightarrow \quad p_{iH} D^{NQ}(E)+(1-p_{iH})D^{NQ}(NE)+D_H < p_{iL} D^Q(E)+(1-p_{iL})D^Q(NE)$$

$$\Leftrightarrow \quad D_H < p_{iL} D^Q(E) - p_{iH}D^{NQ}(E) + (1-p_{iL})D^Q(NE) -(1-p_{iH})D^{NQ}(NE)$$

For the example in 4.5 this gives us:

$$f_1(D^{NQ}(E)) = 145.5 + 0.05\, D^{NQ}(E)$$

$$f_2(D^{NQ}(E)) = 163 - 0.3\, D^{NQ}(E)$$

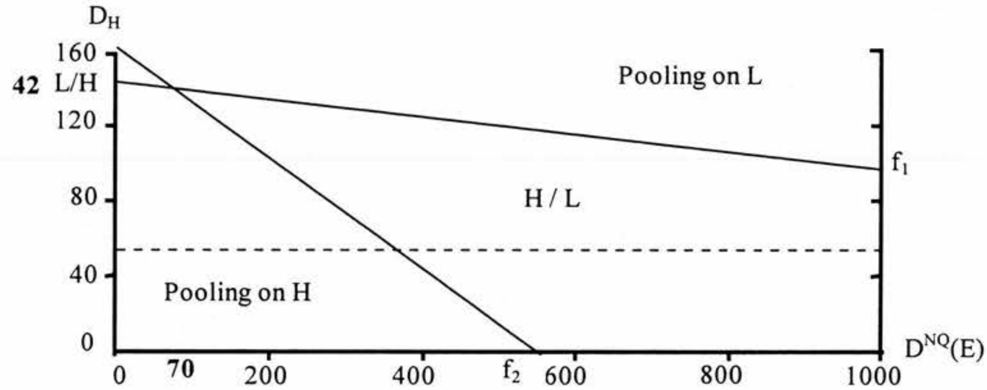

figure A7 :- auditee effort against A_1B_2NQ/B_2Q

A_1B_2R/B_1R - H is optimal for t_i

$$\Leftrightarrow \quad p_{iH}(r_2D^Q(E)+(1-r_2)D^{NQ}(E))+(1-p_{iH})(t_2D^Q(NE)+(1-t_2)D^{NQ}(NE))+D_H <$$
$$p_{iL}(r_1D^Q(E)+(1-r_1)D^{NQ}(E))+(1-p_{iL})(t_1D^Q(NE)+(1-t_1)D^{NQ}(NE))$$

$$\Leftrightarrow \quad D_H < (p_{iL}r_1-p_{iH}r_2)D^Q(E) + (p_{iL}(1-r_1)-p_{iH}(1-r_2))D^{NQ}(E)$$
$$+((1-p_{iL})t_1-(1-p_{iH})t_2)D^Q(NE) + ((1-p_{iL})(1-t_1)-(1-p_{iH})(1-t_2))D^{NQ}(NE)$$

For the example in 4.5 this gives us:

$$f_1(D^{NQ}(E)) = 42.375 + 0.0075\, D^{NQ}(E)$$

$$f_2(D^{NQ}(E)) = 35.95 - 0.035\, D^{NQ}(E)$$

APPENDIX A

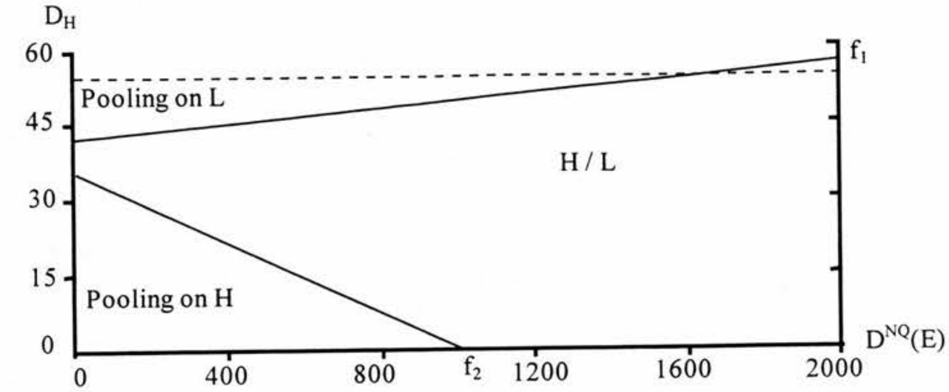

figure A8 :- auditee effort against A_1B_2R/B_1R

A_1B_2R/B_2Q - H is optimal for t_i

$$\Leftrightarrow \quad p_{iH}(r_2D^Q(E)+(1-r_2)D^{NQ}(E))+(1-p_{iH})(t_2D^Q(NE)+(1-t_2)D^{NQ}(NE))+D_H < p_{iL}D^Q(E)+(1-p_{iL})D^Q(NE)$$

$$\Leftrightarrow \quad D_H < (p_{iL}-p_{iH}r_2)D^Q(E) - p_{iH}(1-r_2)D^{NQ}(E) + ((1-p_{iL})-(1-p_{iH})t_2)D^Q(NE) - (1-p_{iH})(1-t_2)D^{NQ}(NE)$$

For the example in 4.5 this gives us:

$$f_1(D^{NQ}(E)) = 110.575 - 0.01\, D^{NQ}(E)$$

$$f_2(D^{NQ}(E)) = 90.95 - 0.06\, D^{NQ}(E)$$

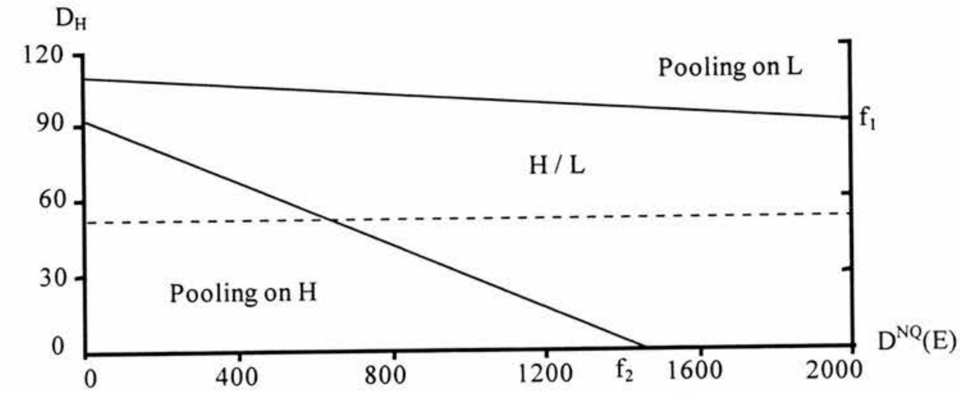

figure A9 :- auditee effort against A_1B_2R/B_2Q

APPENDIX A

A_1B_1R/B_2Q - H is optimal for t_i

$$\Leftrightarrow \quad p_{iH}(r_1D^Q(E)+(1-r_1)D^{NQ}(E))+(1-p_{iH})(t_1D^Q(NE)+(1-t_1)D^{NQ}(NE))+D_H < p_{iL}D^Q(E)+(1-p_{iL})D^Q(NE)$$

$$\Leftrightarrow \quad D_H < (p_{iL}-p_{iH}r_1)D^Q(E) - p_{iH}(1-r_1)D^{NQ}(E) +((1-p_{iL})-(1-p_{iH})t_1)D^Q(NE) - (1-p_{iH})(1-t_1)D^{NQ}(NE)$$

For the example in 4.5 this gives us:

$$f_1(D^{NQ}(E)) = 124.6 - 0.0025\ D^{NQ}(E)$$

$$f_2(D^{NQ}(E)) = 92.6 - 0.015\ D^{NQ}(E)$$

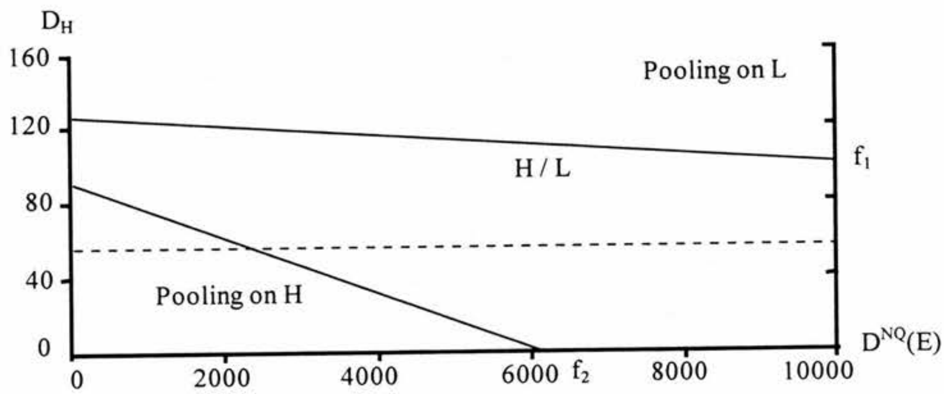

figure A10 :- auditee effort against A_1B_1R/B_2Q

From the above we can determine for which values of D_H each equilibrium can occur. In particular, we are interested in the occurrence of the second separating equilibrium, where type t_1 plays L. Only three of the auditor's strategies in this example allow the possibility of this equilibrium - namely A_1B_2NQ/B_2R, A_1B_2NQ/B_1R and A_1B_2NQ/B_2Q. From figures 5,6 and 7 we can see that this equilibrium can only occur if:

$$\text{Min }\{76.475, 77.3, 145.5\} < D_H < \text{Max }\{99.75, 108, 163\}$$

$$76.475 < D_H < 163$$

APPENDIX A

Since for this example D_H is set as 55, the L / H separating equilibrium will not occur. However, we this is only a local result that need not hold for other D_H or for other auditee costs.

APPENDIX B
Players' Preferences for the Numerical example

With a specific starting belief **P** = 0.9 Each of the potential outcomes can be regarded as a function of $C^{NQ}(E)$ or $D^{NQ}(E)$. For the example of section 4.5 this gives:

$C_B + C_{1H}(B_1R) = 80.9 + 0.0025C^{NQ}(E)$
$C_{1H}(B_2Q) = 98$
$C_{1H}(B_2R) = 33.275 + 0.01C^{NQ}(E)$
$C_{1H}(B_2NQ) = 9.5 + 0.05C^{NQ}(E)$

$C_B+C_{1L}(B_1R) = 92.3+0.0175C^{NQ}(E)$
$C_{1L}(B_2Q) = 86$
$C_{1L}(B_2R) = 37.925 + 0.07C^{NQ}(E)$
$C_{1L}(B_2NQ) = 6.5 + 0.35C^{NQ}(E)$

$C_B + C_{2H}(B_1R) = 90.14 + 0.015C^{NQ}(E)$
$C_{2H}(B_2Q) = 88$
$C_{2H}(B_2R) = 37.15 + 0.06C^{NQ}(E)$
$C_{2H}(B_2NQ) = 7 + 0.3C^{NQ}(E)$

$C_B + C_{2L}(B_1R) = 98 + 0.025C^{NQ}(E)$
$C_{2L}(B_2Q) = 80$
$C_{2L}(B_2R) = 40.25 + 0.1C^{NQ}(E)$
$C_{2L}(B_2NQ) = 5 + 0.5\,C^{NQ}(E)$

$D_H + D_{1H}(B_1R) = 85.4 + 0.0025D^{NQ}(E)$
$D_H + D_{1H}(B_2Q) = 180$
$D_H + D_{1H}(B_2R) = 99.425 + 0.01D^{NQ}(E)$
$D_H + D_{1H}(B_2NQ) = 64.5 + 0.05D^{NQ}(E)$

$D_{1L}(B_1R) = 86.8 + 0.0175D^{NQ}(E)$
$D_{1L}(B_2Q) = 155$
$D_{1L}(B_2R) = 85.975 + 0.07D^{NQ}(E)$
$D_{1L}(B_2NQ) = 6.5 + 0.35D^{NQ}(E)$

$D_H +D_{2H}(B_1R) = 132.4 + 0.015D^{NQ}(E)$
$D_H + D_{2H}(B_2Q) = 205$
$D_H + D_{2H}(B_2R) = 134.05 + 0.06D^{NQ}(E)$
$D_H + D_{2H}(B_2NQ) = 62 + 0.3D^{NQ}(E)$

$D_{2L}(B_1R) = 115 + 0.025D^{NQ}(E)$
$D_{2L}(B_2Q) = 170$
$D_{2L}(B_2R) = 106.75 + 0.1D^{NQ}(E)$
$D_{2L}(B_2NQ) = 5 + 0.5D^{NQ}(E)$

These functions generate the following preference tables (where 1 = best):

Type 1		193		224		440		873		1810		5558		27840		
A_2B_2NQ	L	1	H	1	H	1	H	3	H	5	H	7	H	7	H	7
A_2B_2R	L	6	L	6	H	6	H	6	H	3	H	3	H	6	H	6
A_2B_1R	H	4	H	4	H	4	H	1	H	1	H	1	H	1	H	5
A_2B_2Q	L	8	L	8	L	8	L	8	L	8	L	5	L	3	L	1
B_2NQ/ B_2R	H	2	H	1	H	1	H	3	H	5	H	7	H	7	H	7
B_2NQ/B_2Q	H	2	H	1	H	1	H	3	H	5	L	5	L	3	L	1
B_2R/ B_2Q	H	7	H	7	H	6	H	6	H	3	H	3	L	3	L	1
B_1R/ B_2Q	H	4	H	4	H	4	H	1	H	1	H	1	H	1	L	1

table B1 :- type 1auditee preferences and optimal responses to testing strategies

APPENDIX B

Type 2	110	193	224	231	247	254	262	285	300	302	
A_2B_2NQ	L 1	L 1	L 1	L 1	L 2	L 2	L 4	L 5	H 5	H 6	H 6
A_2B_2R	L 4	L 5	L 5	L 3	L 3	L 3	L 2	L 2	L 2	L 2	L 3
A_2B_1R	L 5	L 4	L 2	L 2	L 1	L 1	L 1	L 1	L 1	L 1	L 1
A_2B_2Q	L 8	L 8	L 8	L 8	L 8	L 8	L 8	L 8	L 8	L 8	L 8
B_2NQ/ B_2R	H 2	H 2	H 3	L 3	L 3	L 3	L 2	L 2	L 2	L 2	L 3
B_2NQ/ B_2Q	H 2	H 2	H 3	H 5	H 5	H 6	H 6	H 6	H 5	H 6	H 6
B_2R/ B_2Q	H 7	H 7	H 7	H 7	H 7	H 7	H 7	H 7	H 7	H 5	H 5
B_1R/ B_2Q	H 6	H 6	H 6	H 6	H 6	H 5	H 5	H 4	H 4	H 4	H 2

Type 2	360	599	633	683	1740	2507	
A_2B_2NQ	H 6	H 8	H 8	H 8	H 8	H 8	H 8
A_2B_2R	L 3	L 3	L 3	L 6	H 6	H 6	H 6
A_2B_1R	L 1	L 1	L 1	L 1	L 1	H 1	H 5
A_2B_2Q	L 8	L 6	L 5	L 3	L 3	L 3	L 1
B_2NQ/ B_2R	L 3	L 3	L 3	L 6	L 7	L 7	L 7
B_2NQ/ B_2Q	H 6	L 6	L 5	L 3	L 3	L 3	L 1
B_2R/ B_2Q	H 5	H 5	L 5	L 3	L 3	L 3	L 1
B_1R/ B_2Q	H 2	H 2	H 2	H 2	H 2	H 2	L 1

table B2 :- type 2auditee preferences and optimal responses to testing strategies

APPENDIX B

H / L	213	250	301	328	373	394	398	428	478	509	552	594	688	
A_2B_2NQ	1	2	3	4	5	5	6	6	7	7	7	7	7	7
A_2B_2R	5	5	5	5	4	3	3	3	3	1	1	1	1	1
A_2B_1R	8	8	8	8	8	8	8	8	8	8	8	8	8	8
A_2B_2Q	10	10	10	10	10	10	10	10	10	10	10	10	10	10
B_2NQ/ B_2R	2	1	1	1	1	1	1	2	2	3	3	4	4	5
B_2NQ/ B_1R	4	4	4	3	3	4	4	4	4	5	5	5	6	6
B_2NQ/ B_2Q	3	3	2	2	2	2	2	1	1	2	2	2	3	3
B_2R/ B_1R	7	7	7	7	7	7	7	7	6	6	6	6	5	4
B_2R/ B_2Q	6	6	6	6	6	6	5	5	5	5	4	3	2	2
B_1R/ B_2Q	9	9	9	9	9	9	9	9	9	9	9	9	9	9

H / L	692	770	815	827	898	917	1280	1294	1380	1386	1430	1436	1503	
A_2B_2NQ	7	7	7	8	9	9	10	10	10	10	10	10	10	10
A_2B_2R	1	1	1	1	1	2	2	2	2	2	2	2	3	3
A_2B_1R	8	8	8	7	7	1	7	8	7	7	6	6	6	6
A_2B_2Q	10	10	10	10	10	7	9	9	9	9	9	8	8	8
B_2NQ/ B_2R	5	5	6	6	6	10	6	6	8	8	8	9	9	9
B_2NQ/ B_1R	6	6	5	5	5	5	5	5	5	6	7	7	7	7
B_2NQ/ B_2Q	3	4	4	4	4	4	4	4	4	4	4	4	4	5
B_2R/ B_1R	4	3	3	3	3	3	3	3	3	3	3	3	2	2
B_2R/ B_2Q	2	2	2	2	2	1	1	1	1	1	1	1	1	1
B_1R/ B_2Q	9	9	9	9	8	8	8	7	6	5	5	4	5	4

H / L	1517	1534	1659	2861	3095	3275	3413	4439	4474	4618	5609	5917	6350	9332	
A_2B_2NQ	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10
A_2B_2R	3	3	3	3	3	4	5	6	6	6	6	6	6	6	6
A_2B_1R	6	5	5	5	6	6	6	5	5	5	5	4	4	4	3
A_2B_2Q	8	8	7	6	5	5	4	4	4	3	2	2	1	1	1
B_2NQ/ B_2R	9	9	9	9	9	9	9	9	9	9	9	9	9	9	9
B_2NQ/ B_1R	7	7	8	8	8	8	8	8	8	8	8	8	8	8	8
B_2NQ/ B_2Q	5	6	6	7	7	7	7	7	7	7	7	7	7	7	7
B_2R/ B_1R	2	2	2	2	2	2	2	2	3	4	4	5	5	5	5
B_2R/ B_2Q	1	1	1	1	1	1	1	1	1	1	1	1	2	3	4
B_1R/ B_2Q	4	4	4	4	4	3	3	3	2	2	3	3	3	2	2

table B3 :- auditor's strategy preference after H / L

APPENDIX B

L / H	60	66	103	113	116	126	132	231	235	243	247	261	264	
A_2B_2NQ	1	1	1	1	2	2	2	3	4	5	6	7	8	9
A_2B_2R	2	2	2	2	1	1	1	1	1	1	1	1	1	1
A_2B_1R	10	10	9	9	9	8	8	8	8	8	8	8	7	7
A_2B_2Q	5	5	5	4	4	4	4	4	3	3	3	3	3	3
B_2NQ/ B_2R	3	3	3	3	3	3	3	2	2	2	2	2	2	2
B_2NQ/ B_1R	7	8	8	8	8	9	10	10	10	10	10	10	10	10
B_2NQ/ B_2Q	4	4	4	5	5	5	6	6	6	6	5	5	5	5
B_2R/ B_1R	9	9	10	10	10	10	9	9	9	9	9	9	9	8
B_2R/ B_2Q	6	6	6	6	6	6	5	5	5	4	4	4	4	4
B_1R/ B_2Q	8	7	7	7	7	7	7	7	7	7	7	6	6	6

L / H	275	292	499	532	564	687	690	701	705	729	766	794	1036	
A_2B_2NQ	9	10	10	10	10	10	10	10	10	10	10	10	10	10
A_2B_2R	1	1	1	1	1	1	1	1	2	2	2	3	4	4
A_2B_1R	7	7	7	7	7	7	7	6	6	5	5	5	5	5
A_2B_2Q	3	3	3	2	2	2	2	2	1	1	1	1	1	1
B_2NQ/ B_2R	2	2	2	3	4	5	6	7	7	7	8	8	8	9
B_2NQ/ B_1R	10	9	9	9	9	9	9	9	9	9	9	9	9	8
B_2NQ/ B_2Q	5	5	6	6	6	6	5	5	5	6	6	6	6	6
B_2R/ B_1R	8	8	8	8	8	8	8	8	8	8	7	7	7	7
B_2R/ B_2Q	4	4	4	4	3	3	3	3	3	3	3	2	2	2
B_1R/ B_2Q	6	6	5	5	5	4	4	4	4	4	4	4	3	3

L / H	1048	1053	1142	1160	1178	2190	
A_2B_2NQ	10	10	10	10	10	10	10
A_2B_2R	4	5	5	6	7	7	8
A_2B_1R	5	4	4	4	4	4	4
A_2B_2Q	1	1	1	1	1	1	1
B_2NQ/ B_2R	9	9	9	9	9	9	9
B_2NQ/ B_1R	8	8	8	8	8	8	7
B_2NQ/ B_2Q	6	6	7	7	6	6	6
B_2R/ B_1R	7	7	6	5	5	5	5
B_2R/ B_2Q	2	2	2	2	2	3	3
B_1R/ B_2Q	3	3	3	3	3	2	2

table B4 :- auditor's strategy preference after L / H

APPENDIX B

L / L	109	126	217	230	250	264	579	647	716	908	999	1091	
A_2B_2NQ	1	2	3	4	7	8	10	10	10	10	10	10	10
A_2B_2R	2	1	1	1	1	1	1	1	2	5	5	6	8
A_2B_1R	8	8	8	8	8	7	7	7	7	7	6	5	5
A_2B_2Q	4	4	4	3	3	3	3	2	1	1	1	1	1
B_2NQ/ B_2R	3	3	2	2	2	2	2	3	6	6	7	9	9
B_2NQ/ B_1R	9	9	9	9	9	9	8	8	8	8	8	7	6
B_2NQ/ B_2Q	5	5	5	5	4	4	4	4	3	2	2	2	2
B_2R/ B_1R	9	9	9	9	9	9	8	8	8	8	8	7	6
B_2R/ B_2Q	5	5	5	5	4	4	4	4	3	2	2	2	2
B_1R/ B_2Q	5	5	5	5	4	4	4	4	3	2	2	2	2

table B5 :- auditor's strategy preference after L / L

H / H	324	407	490	948	1019	1089	1227	1293	2714	3837	3889	4047	4223	
A_2B_2NQ	1	1	2	4	4	5	6	6	7	7	7	7	7	7
A_2B_2R	5	2	1	1	1	1	1	1	1	1	1	1	1	2
A_2B_1R	8	8	8	8	5	4	4	4	4	4	2	2	3	3
A_2B_2Q	10	10	10	10	10	10	10	7	6	5	5	3	2	1
B_2NQ/ B_2R	2	3	5	5	6	7	7	8	8	8	8	8	8	8
B_2NQ/ B_1R	2	3	5	5	6	7	7	8	8	8	8	8	8	8
B_2NQ/ B_2Q	2	3	5	5	6	7	7	8	8	8	8	8	8	8
B_2R/ B_1R	6	6	3	2	2	2	2	2	2	2	3	4	4	4
B_2R/ B_2Q	6	6	3	2	2	2	2	2	2	2	3	4	4	4
B_1R/ B_2Q	9	9	9	9	9	6	5	5	5	6	6	6	6	6

H / H	4281	4725	
A_2B_2NQ	7	7	7
A_2B_2R	2	3	4
A_2B_1R	3	2	2
A_2B_2Q	1	1	1
B_2NQ/ B_2R	8	8	8
B_2NQ/ B_1R	8	8	8
B_2NQ/ B_2Q	8	8	8
B_2R/ B_1R	4	5	5
B_2R/ B_2Q	4	5	5
B_1R/ B_2Q	6	4	3

table B6 :- auditor's strategy preference after H / H

APPENDIX C
Equilibrium set for Fraud Model Example

1 - 3 Only pooling equilibria since the auditor always uses D_1

Region 1

Pooling on H if $D_H < 61.95$

Auditor	after H	B_1D_1			
	$x_H = 0$				
Type 1	$S_1 = 1$		$f_{1H} = 0$		
Type 2	$S_2 = 1$		$f_{2H} = 0$		

Pooling on L if $D_H > 61.95$

Auditor				after L	B_1D_1
			$x_L = 0$		
Type 1	$S_1 = 0$				$f_{1L} = 0$
Type 2	$S_2 = 0$				$f_{2L} = 0$

Region 2

Pooling on H if $D_H < 62.48$

Auditor	after H	B_2D_1			
	$x_H = 0$				
Type 1	$S_1 = 1$		$f_{1H} = 0$		
Type 2	$S_2 = 1$		$f_{2H} = 0$		

Pooling on L if $D_H > 62.48$

Auditor				after L	B_1D_1
			$x_L = 0$		
Type 1	$S_1 = 0$				$f_{1L} = 0$
Type 2	$S_2 = 0$				$f_{2L} = 0$

Region 3

Pooling on H if $D_H < 58.8$

Auditor	after H	B_2D_1			
	$x_H = 0$				
Type 1	$S_1 = 1$		$f_{1H} = 0$		
Type 2	$S_2 = 1$		$f_{2H} = 0$		

Pooling on L if $D_H > 58.8$

Auditor				after L	B_1D_1
			$x_L = 0$		
Type 1	$S_1 = 0$				$f_{1L} = 0$
Type 2	$S_2 = 0$				$f_{2L} = 0$

APPENDIX C

4 - 7 Only pooling equilibria as type 1 mimics type 2

Region 4 $F_L^* = (-5.85+0.3975C_D) / (181.35 - 0.368875C_D)$

Pooling on H if $D_H < 64.14$

Auditor	after H	B_1D_1	
	$x_H = 0$		
Type 1	$S_1 = 1$		$f_{1H} = 0$
Type 2	$S_2 = 1$		$f_{2H} = 0$

Pooling on L if $D_H > 64.14$

Auditor		after L $(B_1D_2\ B_1D_1)$	
		$x_L = 0.305666$	
Type 1	$S_1 = 0$		$f_{1L} = 0$
Type 2	$S_2 = 0$		$f_{2L} = 10F_L^*$

Region 5 $F_L^* = (-5.85+0.3975C_D) / (181.35 - 0.368875C_D)$

Pooling on H if $D_H < 64.66$

Auditor	after H	B_2D_1	
	$x_H = 0$		
Type 1	$S_1 = 1$		$f_{1H} = 0$
Type 2	$S_2 = 1$		$f_{2H} = 0$

Pooling on L if $D_H > 64.66$

Auditor		after L $(B_1D_2\ B_2D_1)$	
		$x_L = 0.305666$	
Type 1	$S_1 = 0$		$f_{1L} = 0$
Type 2	$S_2 = 0$		$f_{2L} = 10F_L^*$

Region 6 $F_L^* = (6.75-C_B+0.4425C_D) / (125.475 - 0.1975C_D)$

Pooling on H if $D_H < 59.41$

Auditor	after H	B_2D_1	
	$x_H = 0$		
Type 1	$S_1 = 1$		$f_{1H} = 0$
Type 2	$S_2 = 1$		$f_{2H} = 0$

Pooling on L if $D_H > 59.41$

Auditor		after L $(B_1D_2\ B_2D_1)$	
		$x_L = 0.05602$	
Type 1	$S_1 = 0$		$f_{1L} = 0$
Type 2	$S_2 = 0$		$f_{2L} = 10F_L^*$

APPENDIX C

Region 7 $F_L^* = (-14.624+0.4425C_D) / (129.375 - 0.1975C_D)$

Pooling on H if $D_H < 59.73$

Auditor	after H	B_2D_1	
	$x_H = 0$		
Type 1	$S_1 = 1$	$f_{1H} = 0$	
Type 2	$S_2 = 1$	$f_{2H} = 0$	

Pooling on L if $D_H > 59.73$

Auditor		after L (B_2D_2 B_2D_1)	
		$x_L = 0.052151$	
Type 1	$S_1 = 0$		$f_{1L} = 0$
Type 2	$S_2 = 0$		$f_{2L} = 10F_L^*$

8 - 12 D_1 optimal vs. H and $F_L^* > (1-P)$ so partially hybrid equilibria occur

Region 8 $F_L^* = (-5.85+0.3975C_D) / (181.35 - 0.368875C_D)$

Pooling on H if $D_H < 64.14$

Auditor	after H	B_1D_1	after L (B_1D_2 B_2D_1)
	$x_H = 0$	$x_L \in X$	
Type 1	$S_1 = 1$	$f_{1H} = 0$	
Type 2	$S_2 = 1$	$f_{2H} = 0$	
			$X = (x_{2L}, 0.41530 - 0.00171D_H)$

Partially Hybrid equilibrium if $D_H \in (64.14, 65.65)$

Auditor	after H	B_1D_1	after L (B_1D_2 B_2D_1)
		$x_L = (D_H-61.95)/ 7.15$	
Type 1	$S_1 = (F_L^*-0.1)/0.9F_L^*$	$f_{1H} = 0$	$f_{1L} = 0$
Type 2	$S_2 = 0$		$f_{2L} = 1$

Pooling on L if $D_H > 65.65$

Auditor		after L (B_1D_2 B_1D_1)	
		$x_L = 0.516761$	
Type 1	$S_1 = 0$		$f_{1L}=(F_L^*-0.1)/0.9$
Type 2	$S_2 = 0$		$f_{2L} = 1$

Region 9 $F_L^* = (-5.85+0.3975C_D) / (181.35 - 0.368875C_D)$

Pooling on H if $D_H < 64.66$

Auditor	after H	B_2D_1	after L (B_1D_2 B_1D_1)
	$x_H = 0$	$x_L \in X$	
Type 1	$S_1 = 1$	$f_{1H} = 0$	
Type 2	$S_2 = 1$	$f_{2H} = 0$	
			$X = (x_{2L}, 0.41520 - 0.00171D_H)$

APPENDIX C

Partially Hybrid equilibrium if $D_H \in (64.66, 66.17)$

Auditor	after H	B_2D_1	after L	$(B_1D_2\ B_1D_1)$
			$x_L = (D_H\text{-}62.475)/\ 7.15$	
Type 1	$S_1 = (F_L\text{*-}0.1)/0.9F_L\text{*}$		$f_{1H} = 0$	$f_{1L} = 0$
Type 2	$S_2 = 0$			$f_{2L} = 1$

Pooling on L if $D_H > 66.17$

Auditor		after L $(B_1D_2\ B_1D_1)$	
		$x_L = 0.516761$	
Type 1	$S_1 = 0$		$f_{1L}\text{=}(F_L\text{*-}0.1)/0.9$
Type 2	$S_2 = 0$		$f_{2L} = 1$

Region 10 $F_L\text{*} = (6.75\text{-}C_B\text{+}0.4425C_D)\ /\ (125.475 - 0.1975C_D)$

Pooling on H if $D_H < 59.41$

Auditor	after H	B_2D_1	after L	$(B_1D_2\ B_2D_1)$
	$x_H = 0$		$x_L \in X$	
Type 1	$S_1 = 1$		$f_{1H} = 0$	
Type 2	$S_2 = 1$		$f_{2H} = 0$	

$X = (x_{2L}, 0.19589 - 0.00235D_H)$

Partially Hybrid equilibrium if $D_H \in (59.41, 62.51)$

Auditor	after H	B_2D_1	after L	$(B_1D_2\ B_2D_1)$
			$x_L = (D_H\text{-}58.8)/10.825$	
Type 1	$S_1 = (F_L\text{*-}0.1)/0.9F_L\text{*}$		$f_{1H} = 0$	$f_{1L} = 0$
Type 2	$S_2 = 0$			$f_{2L} = 1$

Pooling on L if $D_H > 62.51$

Auditor		after L $(B_1D_2\ B_2D_1)$	
		$x_L = 0.343015$	
Type 1	$S_1 = 0$		$f_{1L}\text{=}(F_L\text{*-}0.1)/0.9$
Type 2	$S_2 = 0$		$f_{2L} = 1$

Region 11 $F_L\text{*} = (-27.224\text{+}C_B\text{+}0.3975C_D)\ /\ (185 - 0.368875C_D)$

Pooling on H if $D_H < 66.59$

Auditor	after H	B_2D_1	after L	$(B_2D_2\ B_1D_1)$
	$x_H = 0$		$x_L \in X$	
Type 1	$S_1 = 1$		$f_{1H} = 0$	
Type 2	$S_2 = 1$		$f_{2H} = 0$	

$X = (x_{2L}, 0.39896 - 0.00164D_H)$

APPENDIX C

Partially Hybrid equilibrium if $D_H \in (66.59, 69.4)$

Auditor	after H B_2D_1		after L $(B_2D_2\ B_1D_1)$	
			$x_L = (D_H-62.475)/14.2$	
Type 1	$S_1 = (F_L^*-0.1)/0.9F_L^*$	$f_{1H} = 0$		$f_{1L} = 0$
Type 2	$S_2 = 0$			$f_{2L} = 1$

Pooling on L if $D_H > 69.4$

Auditor		after L $(B_2D_2\ B_1D_1)$	
		$x_L = 0.490116$	
Type 1	$S_1 = 0$		$f_{1L}=(F_L^*-0.1)/0.9$
Type 2	$S_2 = 0$		$f_{2L} = 1$

Region 12 $F_L^* = (-14.624+0.4425C_D) / (129.375 - 0.1975C_D)$

Pooling on H if $D_H < 59.73$

Auditor	after H B_2D_1	after L $(B_2D_2\ B_2D_1)$
	$x_H = 0$	$x_L \in X$
Type 1	$S_1 = 1$	$f_{1H} = 0$
Type 2	$S_2 = 1$	$f_{2H} = 0$
		$X = (x_{2L}, 0.18489 - 0.00222D_H)$

Partially Hybrid equilibrium if $D_H \in (59.73, 64.51)$

Auditor	after H B_2D_1		after L $(B_2D_2\ B_2D_1)$	
			$x_L = (D_H-58.8)/17.875$	
Type 1	$S_1 = (F_L^*-0.1)/0.9F_L^*$	$f_{1H} = 0$		$f_{1L} = 0$
Type 2	$S_2 = 0$			$f_{2L} = 1$

Pooling on L if $D_H > 64.51$

Auditor		after L $(B_2D_2\ B_2D_1)$	
		$x_L = 0.319316$	
Type 1	$S_1 = 0$		$f_{1L}=(F_L^*-0.1)/0.9$
Type 2	$S_2 = 0$		$f_{2L} = 1$

13-18 $F_H^* < (1-P)$ $F_L^* > (1-P)$ so hybrid equilibria occur

Region 13 $F_H^* = (6.75+C_B-0.2775C_D) / (-107.925 + 0.2425C_D)$

$F_L^* = (-5.85+0.3975C_D) / (181.35 - 0.368875C_D)$

Pooling on H if $D_H < 61.25$

Auditor	after H $(B_1D_2\ B_2D_1)$	after L $(B_1D_2\ B_1D_1)$
	$x_H = 0.310900$	$x_L \in X$
Type 1	$S_1 = 1$	$f_{1H} = 0$
Type 2	$S_2 = 1$	$f_{2H} = 10F_H^*$
		$X = (x_{2L}, 0.41036 - 0.00171D_H)$

APPENDIX C

Partially hybrid equilibrium if $D_H \in (57.67, 59.58)$

Auditor	after H $(B_1D_2\ B_2D_1)$	after L $(B_1D_2\ B_1D_1)$	
	$x_H = 6.02914-0.09112D_H$	$x_L = 0.516761$	
Type 1	$S_1 = (0.1(1-F_H^*))/0.9F_H^*$	$f_{1H} = 0$	$f_{1L} = F_L^*$
Type 2	$S_2 = 1$	$f_{2H} = 1$	

Hybrid equilibrium if $D_H \in (59.58, 61.25)$

Auditor	after H $(B_1D_2\ B_2D_1)$	after L $(B_1D_2\ B_1D_1)$	
	$x_H = 10.93880-0.17352D_H$	$x_L = 8.05291-0.12649D_H$	
Type 1	$S_1 = (1-F_H^*)(F_L^*-0.1))/0.9(F_L^*-F_H^*)$	$f_{1H} = 0$	$f_{1L} = 0$
Type 2	$S_2 = F_H^*(F_L^*-(0.1))/0.1(F_L^*-F_H^*)$	$f_{2H} = 1$	$f_{2L} = 1$

Pooling on L if $D_H > 59.58$

Auditor	H $(B_1D_2\ B_2D_1)$	after L $(B_1D_2\ B_1D_1)$	
	$x_H \in X$	$x_L = 0.516761$	
Type 1	$S_1 = 0$		$f_{1L} = (F_L^*-0.1)/0.9$
Type 2	$S_2 = 0$		$f_{2L} = 1$

$X = (MAX(x_{2H}, 6.02914 - 0.09112D_H), MIN(x_{1H}, 0.45886 + 0.00238D_H))$

Region 14 $F_H^* = (-8.55+0.1425C_D) / (178.65 - 0.524125C_D)$
$F_L^* = (-5.85+0.3975C_D) / (181.35 - 0.368875C_D)$

Pooling on H if $D_H < 58.91$

Auditor	after H $(B_1D_2\ B_1D_1)$	after L $(B_1D_2\ B_1D_1)$	
	$x_H = 0.500462$	$x_L \in X$	
Type 1	$S_1 = 1$	$f_{1H} = 0$	
Type 2	$S_2 = 1$	$f_{2H} = 10F_H^*$	

$X = (x_{2L}, 0.40636 - 0.00171D_H)$

Partially hybrid equilibrium if $D_H \in (56.91, 58.22)$

Auditor	after H $(B_1D_2\ B_1D_1)$	after L $(B_1D_2\ B_1D_1)$	
	$x_H = (65.64484-D_H)/10.45$	$x_L = 0.516761$	
Type 1	$S_1 = (0.1(1-F_H^*))/0.9F_H^*$	$f_{1H} = 0$	$f_{1L} = F_L^*$
Type 2	$S_2 = 1$	$f_{2H} = 1$	

Hybrid equilibrium if $D_H \in (58.22, 58.91)$

Auditor	after H $(B_1D_2\ B_1D_1)$	after L $(B_1D_2\ B_1D_1)$	
	$x_H = 18.56884-0.30673D_H$	$x_L = 18.47474-0.30844D_H$	
Type 1	$S_1 = (1-F_H^*)(F_L^*-0.1))/0.9(F_L^*-F_H^*)$	$f_{1H} = 0$	$f_{1L} = 0$
Type 2	$S_2 = F_H^*(F_L^*-(0.1))/0.1(F_L^*-F_H^*)$	$f_{2H} = 1$	$f_{2L} = 1$

APPENDIX C

Pooling on L if $D_H > 58.22$

Auditor	$(B_1D_2\ B_1D_1)$	after L $(B_1D_2\ B_1D_1)$	
	$x_H \in X$	$x_L = 0.516761$	
Type 1	$S_1 = 0$		$f_{1L}=(F_L*-0.1)/0.9$
Type 2	$S_2 = 0$		$f_{2L} = 1$

$X = (MAX(x_{2H}, 6.28180 - 0.09569D_H), MIN(x_{1H}, 0.61086 + 0.00171D_H))$

Region 15 $F_H* = (-21.375+C_B+0.1425C_D) / (193.35 - 0.524125C_D)$
$F_L* = (-5.85+0.3975C_D) / (181.35 - 0.368875C_D)$

Pooling on H if $D_H < 52.21$

Auditor	after H $(B_2D_2\ B_1D_1)$	after L $(B_1D_2\ B_1D_1)$
	$x_H = 0.465680$	$x_L \in X$
Type 1	$S_1 = 1$	$f_{1H} = 0$
Type 2	$S_2 = 1$	$f_{2H} = 10F_H*$
		$X = (x_{2L}, 0.39492 - 0.00171D_H)$

Partially hybrid equilibrium if $D_H \in (45.72, 48.72)$

Auditor	after H $(B_2D_2\ B_1D_1)$	after L $(B_1D_2\ B_1D_1)$	
	$x_H = (65.64484-D_H)/25.6$	$x_L = 0.516761$	
Type 1	$S_1 = (0.1(1-F_H*))/0.9F_H*$	$f_{1H} = 0$	$f_{1L} = F_L*$
Type 2	$S_2 = 1$	$f_{2H} = 1$	

Hybrid equilibrium if $D_H \in (48.72, 52.21)$

Auditor	after H $(B_2D_2\ B_1D_1)$	after L $(B_1D_2\ B_1D_1)$	
	$x_H = 3.38705-0.05595D_H$	$x_L = 3.46274-0.06046D_H$	
Type 1	$S_1 =(1-F_H*)(F_L*-0.1))/0.9(F_L*-F_H*)$	$f_{1H} = 0$	$f_{1L} = 0$
Type 2	$S_2 = F_H*(F_L*-(0.1))/0.1(F_L*-F_H*)$	$f_{2H} = 1$	$f_{2L} = 1$

Pooling on L if $D_H > 48.72$

Auditor	$(B_2D_2\ B_1D_1)$	after L $(B_1D_2\ B_1D_1)$	
	$x_H \in X$	$x_L = 0.516761$	
Type 1	$S_1 = 0$		$f_{1L}=(F_L*-0.1)/0.9$
Type 2	$S_2 = 0$		$f_{2L} = 1$

$X = (MAX(x_{2H}, 2.56425 - 0.03906D_H), MIN(x_{1H}, 0.58170 + 0.00163D_H))$

Region 16 $F_H* = (-21.375+0.2775C_D) / (122.625 - 0.2425C_D)$
$F_L* = (-5.85+0.3975C_D) / (181.35 - 0.368875C_D)$

Pooling on H if $D_H < 57.3$

Auditor	after H $(B_2D_2\ B_2D_1)$	after L $(B_1D_2\ B_1D_1)$
	$x_H = 0.281859$	$x_L \in X$
Type 1	$S_1 = 1$	$f_{1H} = 0$
Type 2	$S_2 = 1$	$f_{2H} = 10F_H*$
		$X = (x_{2L}, 0.40361 - 0.00171D_H)$

APPENDIX C

Partially hybrid equilibrium if $D_H \in (47.83, 51.95)$

Auditor	after H (B_2D_2 B_2D_1)	after L (B_1D_2 B_1D_1)	
	$x_H = (36.36984-D_H)/26.125$	$x_L = 0.516761$	
Type 1	$S_1 = (0.1(1-F_H^*))/0.9F_H^*$	$f_{1H} = 0$	$f_{1L} = F_L^*$
Type 2	$S_2 = 1$	$f_{2H} = 1$	

Hybrid equilibrium if $D_H \in (51.95, 57.3)$

Auditor	after H (B_2D_2 B_2D_1)	after L (B_1D_2 B_1D_1)	
	$x_H = 3.09385-0.04908D_H$	$x_L = 2.56668-0.03946D_H$	
Type 1	$S_1 =(1-F_H^*)(F_L^*-0.1))/0.9(F_L^*-F_H^*)$	$f_{1H} = 0$	$f_{1L} = 0$
Type 2	$S_2 = F_H^*(F_L^*-(0.1))/0.1(F_L^*-F_H^*)$	$f_{2H} = 1$	$f_{2L} = 1$

Pooling on L if $D_H > 51.95$

Auditor	after H (B_2D_2 B_2D_1)	after L (B_1D_2 B_1D_1)	
	$x_H \in X$	$x_L = 0.516761$	
Type 1	$S_1 = 0$		$f_{1L}=(F_L^*-0.1)/0.9$
Type 2	$S_2 = 0$		$f_{2L} = 1$

$X = (MAX(x_{2H}, 2.53282 - 0.03828D_H), MIN(x_{1H}, 0.42896 + 0.00222D_H))$

Region 17 $F_H^* = (-21.375+0.2775C_D) / (122.625 - 0.2425C_D)$
$F_L^* = (-27.224 + C_B + 0.3975C_D) / (185.25 - 0.368875C_D)$

Pooling on H if $D_H < 59.23$

Auditor	after H (B_2D_2 B_2D_1)	after L (B_2D_2 B_1D_1)
	$x_H = 0.281859$	$x_L \in X$
Type 1	$S_1 = 1$	$f_{1H} = 0$
Type 2	$S_2 = 1$	$f_{2H} = 10F_H^*$
		$X = (x_{2L}, 0.38689 - 0.00164D_H)$

Partially hybrid equilibrium if $D_H \in (51.1, 55.21)$

Auditor	after H (B_2D_2 B_2D_1)	after L (B_2D_2 B_1D_1)	
	$x_H = (69.43317-D_H)/26.125$	$x_L = 0.490017$	
Type 1	$S_1 = (0.1(1-F_H^*))/0.9F_H^*$	$f_{1H} = 0$	$f_{1L} = F_L^*$
Type 2	$S_2 = 1$	$f_{2H} = 1$	

Hybrid equilibrium if $D_H \in (55.21, 59.23)$

Auditor	after H (B_2D_2 B_2D_1)	after L (B_2D_2 B_1D_1)	
	$x_H = 4.15334-0.06537D_H$	$x_L = 3.24162-0.03498D_H$	
Type 1	$S_1 =(1-F_H^*)(F_L^*-0.1))/0.9(F_L^*-F_H^*)$	$f_{1H} = 0$	$f_{1L} = 0$
Type 2	$S_2 = F_H^*(F_L^*-(0.1))/0.1(F_L^*-F_H^*)$	$f_{2H} = 1$	$f_{2L} = 1$

APPENDIX C

Pooling on L if $D_H > 55.21$

Auditor	after H $(B_2D_2\ B_2D_1)$	after L $(B_2D_2\ B_1D_1)$
	$x_H \in X$	$x_L = 0.490017$
Type 1	$S_1 = 0$	$f_{1L}=(F_L*-0.1)/0.9$
Type 2	$S_2 = 0$	$f_{2L} = 1$

$X = (MAX(x_{2H}, 2.65773 - 0.03828D_H), MIN(x_{1H}, 0.42170 + 0.00222D_H))$

Region 18 $F_H* = (-21.375+0.2775C_D) / (122.625 - 0.2425C_D)$

$F_L* = (-14.624+0.4425C_D) / (129.375 - 0.1975C_D)$

Pooling on H if $D_H < 52.37$

Auditor	after H $(B_2D_2\ B_2D_1)$	after L $(B_2D_2\ B_2D_1)$
	$x_H = 0.281859$	$x_L \in X$
Type 1	$S_1 = 1$	$f_{1H} = 0$
Type 2	$S_2 = 1$	$f_{2H} = 10F_H*$

$X = (x_{2L}, 0.16853 - 0.00222D_H)$

Partially hybrid equilibrium if $D_H \in (46.17, 50.29)$

Auditor	after H $(B_2D_2\ B_2D_1)$	after L $(B_2D_2\ B_2D_1)$	
	$x_H = (64.50777-D_H)/26.125$	$x_L = 0.319316$	
Type 1	$S_1 = (0.1(1-F_H*))/0.9F_H*$	$f_{1H} = 0$	$f_{1L} = F_L*$
Type 2	$S_2 = 1$	$f_{2H} = 1$	

Hybrid equilibrium if $D_H \in (50.29, 52.37)$

Auditor	after H $(B_2D_2\ B_2D_1)$	after L $(B_2D_2\ B_2D_1)$	
	$x_H = 6.88172-0.12603D_H$	$x_L = 6.76838-0.12825D_H$	
Type 1	$S_1 =(1-F_H*)(F_L*-0.1))/0.9(F_L*-F_H*)$	$f_{1H} = 0$	$f_{1L} = 0$
Type 2	$S_2 = F_H*(F_L*-(0.1))/0.1(F_L*-F_H*)$	$f_{2H} = 1$	$f_{2L} = 1$

Pooling on L if $D_H > 50.29$

Auditor	after H $(B_2D_2\ B_2D_1)$	after L $(B_2D_2\ B_2D_1)$
	$x_H \in X$	$x_L = 0.319316$
Type 1	$S_1 = 0$	$f_{1L}=(F_L*-0.1)/0.9$
Type 2	$S_2 = 0$	$f_{2L} = 1$

$X = (MAX(x_{2H}, 2.46920 - 0.03828D_H), MIN(x_{1H}, 0.43265 + 0.00222D_H))$

19-24 $F_H* > (1-P)\ F_L* > (1-P)$ so partially hybrid equilibria occur

Region 19 $F_H* = (-8.55+0.1425C_D) / (178.65 - 0.524125C_D)$

$F_L* = (-5.85+0.3975C_D) / (181.35 - 0.368875C_D)$

Pooling on H if $D_H < 56.91$

Auditor	after H $(B_1D_2\ B_1D_1)$	after L $(B_1D_2\ B_1D_1)$
	$x_H = 0.836342$	$x_L \in X$
Type 1	$S_1 = 1$	$f_{1H} =(F_H*-0.1)/0.9$
Type 2	$S_2 = 1$	$f_{2H} = 1$

$X = (x_{2L}, 0.74224 - 0.00171D_H)$

APPENDIX C

Partially hybrid equilibrium if $D_H \in (56.91, 58.22)$

Auditor	after H $(B_1D_2\ B_1D_1)$	after L $(B_1D_2\ B_1D_1)$	
	$x_H = (65.64484\text{-}D_H)/10.45$	$x_L = 0.516761$	
Type 1	$S_1 = (0.1(1\text{-}F_H^*))/0.9F_H^*$	$f_{1H} = 0$	$f_{1L} = F_L^*$
Type 2	$S_2 = 1$	$f_{2H} = 1$	

Pooling on L if $D_H > 58.22$

Auditor	after H $(B_1D_2\ B_1D_1)$	after L $(B_1D_2\ B_1D_1)$	
	$x_H \in X$	$x_L = 0.516761$	
Type 1	$S_1 = 0$		$f_{1L}=(F_L^*\text{-}0.1)/0.9$
Type 2	$S_2 = 0$		$f_{2L} = 1$

$X = (MAX(x_{2H}, 6.28180 - 0.09569D_H), MIN(x_{1H}, 0.61086 + 0.00171D_H))$

Region 20 $F_H^* = (-23.175{+}{+}C_B+0.1425C_D) / (178.65 - 0.524125C_D)$
$F_L^* = (-5.85+0.3975C_D) / (181.35 - 0.368875C_D)$

Pooling on H if $D_H < 45.72$

Auditor	after H $(B_2D_2\ B_1D_1)$	after L $(B_1D_2\ B_1D_1)$
	$x_H = 0.778216$	$x_L \in X$
Type 1	$S_1 = 1$	$f_{1H} =(F_H^*\text{-}0.1)/0.9$
Type 2	$S_2 = 1$	$f_{2H} = 1$
		$X = (x_{2L}, 0.72312 - 0.00171D_H)$

Partially hybrid equilibrium if $D_H \in (45.72, 48.72)$

Auditor	after H $(B_2D_2\ B_1D_1)$	after L $(B_1D_2\ B_1D_1)$	
	$x_H = (65.64484\text{-}D_H)/25.6$	$x_L = 0.516761$	
Type 1	$S_1 = (0.1(1\text{-}F_H^*))/0.9F_H^*$	$f_{1H} = 0$	$f_{1L} = F_L^*$
Type 2	$S_2 = 1$	$f_{2H} = 1$	

Pooling on L if $D_H > 48.72$

Auditor	after H $(B_2D_2\ B_1D_1)$	after L $(B_1D_2\ B_1D_1)$	
	$x_H \in X$	$x_L = 0.516761$	
Type 1	$S_1 = 0$		$f_{1L}=(F_L^*\text{-}0.1)/0.9$
Type 2	$S_2 = 0$		$f_{2L} = 1$

$X = (MAX(x_{2H}, 2.56425 - 0.03906D_H), MIN(x_{1H}, 0.58170 + 0.00163D_H))$

Region 21 $F_H^* = (-21.375+0.2775C_D) / (122.625 - 0.2425C_D)$
$F_L^* = (-5.85+0.3975C_D) / (181.35 - 0.368875C_D)$

Pooling on H if $D_H < 47.83$

Auditor	after H $(B_2D_2\ B_2D_1)$	after L $(B_1D_2\ B_1D_1)$
	$x_H = 0.701917$	$x_L \in X$
Type 1	$S_1 = 1$	$f_{1H} =(F_H^*\text{-}0.1)/0.9$
Type 2	$S_2 = 1$	$f_{2H} = 1$
		$X = (x_{2L}, 0.72673 - 0.00171D_H)$

APPENDIX C

Partially hybrid equilibrium if $D_H \in (47.83, 51.95)$

Auditor	after H $(B_2D_2\ B_2D_1)$	after L $(B_1D_2\ B_1D_1)$
	$x_H = (36.36984\text{-}D_H)/26.125$	$x_L = 0.516761$
Type 1	$S_1 = (0.1(1\text{-}F_H^*))/0.9F_H^*$	$f_{1H} = 0$ $f_{1L} = F_L^*$
Type 2	$S_2 = 1$	$f_{2H} = 1$

Pooling on L if $D_H > 51.95$

Auditor	after H $(B_2D_2\ B_2D_1)$	after L $(B_1D_2\ B_1D_1)$
	$x_H \in X$	$x_L = 0.516761$
Type 1	$S_1 = 0$	$f_{1L}=(F_L^*\text{-}0.1)/0.9$
Type 2	$S_2 = 0$	$f_{2L} = 1$

$X = (MAX(x_{2H}, 2.53282 - 0.03828D_H), MIN(x_{1H}, 0.42896 + 0.00222D_H))$

Region 22 $F_H^* = (-23.175+C_B+0.1425C_D) / (193.35 - 0.524125C_D)$

$F_L^* = (-27.224+C_B+0.3975C_D) / (181.35 - 0.368875C_D)$

Pooling on H if $D_H < 48.99$

Auditor	after H $(B_2D_2\ B_1D_1)$	after L $(B_2D_2\ B_1D_1)$
	$x_H = 0.778216$	$x_L \in X$
Type 1	$S_1 = 1$	$f_{1H} =(F_H^*\text{-}0.1)/0.9$
Type 2	$S_2 = 1$	$f_{2H} = 1$
		$X = (x_{2L}, 0.69318 - 0.00164D_H)$

Partially hybrid equilibrium if $D_H \in (48.99, 51.99)$

Auditor	after H $(B_2D_2\ B_1D_1)$	after L $(B_2D_2\ B_1D_1)$
	$x_H = (68.90817\text{-}D_H)/25.6$	$x_L = 0.490012$
Type 1	$S_1 = (0.1(1\text{-}F_H^*))/0.9F_H^*$	$f_{1H} = 0$ $f_{1L} = F_L^*$
Type 2	$S_2 = 1$	$f_{2H} = 1$

Pooling on L if $D_H > 51.99$

Auditor	after H $(B_2D_2\ B_1D_1)$	after L $(B_2D_2\ B_1D_1)$
	$x_H \in X$	$x_L = 0.490012$
Type 1	$S_1 = 0$	$f_{1L}=(F_L^*\text{-}0.1)/0.9$
Type 2	$S_2 = 0$	$f_{2L} = 1$

$X = (MAX(x_{2H}, 2.69173 - 0.03906D_H), MIN(x_{1H}, 0.57634 + 0.00163D_H))$

APPENDIX C

Region 23 $F_H^* = (-21.375+0.2275C_D) / (122.625 - 0.2425C_D)$
$F_L^* = (-27.224+C_B+0.3975C_D) / (185.25 - 0.368875C_D)$

Pooling on H if $D_H < 51.1$

Auditor	after H (B_2D_2 B_2D_1)	after L (B_2D_2 B_1D_1)
	$x_H = 0.701917$	$x_L \in X$
Type 1	$S_1 = 1$	$f_{1H} = (F_H^*-0.1)/0.9$
Type 2	$S_2 = 1$	$f_{2H} = 1$
		$X = (x_{2L}, 0.69663 - 0.00164D_H)$

Partially hybrid equilibrium if $D_H \in (51.1, 55.21)$

Auditor	after H (B_2D_2 B_2D_1)	after L (B_2D_2 B_1D_1)	
	$x_H = (69.43317-D_H)/26.125$	$x_L = 0.490017$	
Type 1	$S_1 = (0.1(1-F_H^*))/0.9F_H^*$	$f_{1H} = 0$	$f_{1L} = F_L^*$
Type 2	$S_2 = 1$	$f_{2H} = 1$	

Pooling on L if $D_H > 55.21$

Auditor	after H (B_2D_2 B_2D_1)	after L (B_1D_2 B_2D_1)
	$x_H \in X$	$x_L = 0.490017$
Type 1	$S_1 = 0$	$f_{1L} = (F_L^*-0.1)/0.9$
Type 2	$S_2 = 0$	$f_{2L} = 1$

$X = (MAX(x_{2H}, 2.65773 - 0.03828D_H), MIN(x_{1H}, 0.42170 + 0.00222D_H))$

Region 24 $F_H^* = (-21.375+0.2775C_D) / (122.625 - 0.2425C_D)$
$F_L^* = (-5.85+0.3975C_D) / (181.35 - 0.368875C_D)$

Pooling on H if $D_H < 46.17$

Auditor	after H (B_1D_2 B_1D_1)	after L (B_2D_2 B_2D_1)
	$x_H = 0.701907$	$x_L \in X$
Type 1	$S_1 = 1$	$f_{1H} = (F_H^*-0.1)/0.9$
Type 2	$S_2 = 1$	$f_{2H} = 1$
		$X = (x_{2L}, 0.58858 - 0.00222D_H)$

Partially hybrid equilibrium if $D_H \in (46.17, 50.29)$

Auditor	after H (B_2D_2 B_2D_1)	after L (B_2D_2 B_2D_1)	
	$x_H = (64.50777-D_H)/26.125$	$x_L = 0.319316$	
Type 1	$S_1 = (0.1(1-F_H^*))/0.9F_H^*$	$f_{1H} = 0$	$f_{1L} = F_L^*$
Type 2	$S_2 = 1$	$f_{2H} = 1$	

APPENDIX C

Pooling on L if $D_H > 50.29$

Auditor	after H	$(B_2D_2\ \ B_2D_1)$	after L	$(B_2D_2\ \ B_2D_1)$
	$x_H \in X$		$x_L = 0.319316$	
Type 1	$S_1 = 0$			$f_{1L}=(F_L*-0.1)/0.9$
Type 2	$S_2 = 0$			$f_{2L} = 1$

$X = (MAX(x_{2H}, 2.46920 - 0.03828D_H), MIN(x_{1H}, 0.43265 + 0.00222D_H))$

25-26 Auditor never uses D_1 after observing Low effort - only pooling on L
Region 25

Pooling on L $\forall$ D_H

Auditor		after L	B_1D_2
		$x_L = 1$	
Type 1	$S_1 = 0$		$f_{1L} = 1$
Type 2	$S_2 = 0$		$f_{2L} = 1$

Region 26

Pooling on L $\forall$ D_H

Auditor		after L	B_2D_2
		$x_L = 1$	
Type 1	$S_1 = 0$		$f_{1L} = 1$
Type 2	$S_2 = 0$		$f_{2L} = 1$