# THE OVERLOADING INTEGRATOR CORRELATOR

ΒY

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# DECLARATION

I hereby declare that all the work presented in this Thesis is my own, unless otherwise stated within the text, and that this Thesis has been composed by myself. To my Father and Mother with my gratitude.

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# LIST OF SYMBOLS USED IN THE TEXT

A	Function Area	
A <sub>+</sub>	Positive Area	
A_	Negative Area	
A <sub>n</sub>	Normalised Area	
A <sub>n+</sub>	Normalised Positive Area	
A <sub>n-</sub>	Normalised Negative Area	
A <sub>o</sub>	Sine Wave Amplitude	volt
a'	A Constant	
a <sub>o</sub> .	prbs Noise Amplitude	volt
a <sup>2</sup>	Signal-to-Noise Power Ratio	
В	Frequency Bandwidth	Hz
dB	Decibel	
CRO ·	Cathode-Ray-Tube	
E	Percentage Error (for load pulse generation)	
e	Percentage Error (for correlation functions co	omparison)
e <sup>2</sup>	Normalised Mean-Square Error	
f <sub>B</sub>	3 dB Frequency Bandwidth	Hz
f <sub>s</sub>	Sampling Frequency	Hz
G <sub>dc</sub>	dc Gain of a Function	
G(s)	Power Spectrum	
H(s)	Transfer Function	
HP	Hewlett Packard	
h(t)	Impulse Response Function	
h <sub>n</sub> (τ)	Normalised Impulse Response Function	
IP1	First Input Signal to the Correlator	
102	Second Input Signal to the Correlator	

•		(011)	
	LIST OF SYMBOL	S USED IN THE TEXT (continued)	
	К	A Constant	
	К <sub>а́</sub>	Area Scaling Factor	
•	k	Parameter Control	
	L	Spacing Between Two Noise Transducers	CM
	L <sub>opt</sub>	Optimal Spacing Between Two Transducers	
	LP(t)	Load Pulses of Linear Increment of Timing Counter Content	
	LP(t + t')	Pre-occurrence Load Pulses of Linear Incr of Timing Counter Content	rement
	LPr	Load Pulses Corresponding to Non-Linear D of Polarity Correlation Coefficient	ecrement
	LPp	Load Pulses Corresponding to Normalised C Correlation Function	ontinuous
	ms	Milisecond	10 <sup>-3</sup> sec
	n(t)	Noise Signal	volt
	OIP1	Output of First Input Signal	volt
	0IP2	Output of Second Input Signal	volt
•	OVL	Overload Pattern	
	Р	Joint Probability Density	
	P <sub>i</sub>	Performance Index	
	R	Relative Stability	
·	R <sub>o</sub>	Relative Stability Corresponding to the D Damping Ratio	esirged
	R(τ)	Continuous Correlation Function	(volt) <sup>2</sup>
	R <sub>xx</sub> (τ)	Auto-Correlation Function	(volt) <sup>2</sup>
	R <sub>xy</sub> (τ)	Cross-Correlation Function	(volt) <sup>2</sup>
	<b>r(</b> τ)	Polarity Correlation Coefficient	
	r <sub>xx</sub> (τ)	Polarity Auto-Correlation Coefficient	
	R <sub>xy</sub> (τ)	Polarity Cross-Correlation Coefficient	

LIST OF SYMBOLS USED IN THE TEXT (continued)

•	Fl., Noise Cirpal	•
5	Flow Noise Signal	-
SL	Saturation Level	
۶ <sub>۱</sub>	Up-Stream Signal	
s <sub>2</sub>	Down-Stream Signal	
S	Frequency Domain	Hz
т	Integration Period	sec
T <sub>c</sub>	Timing Counter Content	• •
Τ <sub>ω</sub>	Measure of the Width of the Function	sec
t	Time Domain	sec
ULA	Uncommitted Logic Array	
V	Voltage	volt
Vin	Input Voltage	volt
Vo	Output Voltage	volt
V	Velocity	metres/sec
x(t)	Input Random Variable Function	volt
y(t)	Output Random Variable Function	volt
y'(t)	Noise-Free Signal	volt
Z	Average Zero Crossing Rate	
Ŷ	Integration Period for Polarity Correlation	sec
μ <sub>f</sub>	Micro-farad	10 <sup>-6</sup> Farad
μ <sub>X</sub>	Mean Value of Input Signal	volt
μ <sub>y</sub>	Mean Value of Output Signal	volt
μs	Microsecond	10 <sup>-6</sup> sec
ρ	Damping Ratio	
ρ(τ)	Normalised Continuous Correlation Function	
ρ <sub>xx</sub> (τ)	Normalised Continuous Auto-Correlation Funct	ion
ρ <sub>χν</sub> (τ)	Normalised Continuous Cross-Correlation Funct	tion

LIST OF SYMBOLS US	ED IN THE TEXT (continued)				
ρ <sub>d</sub> (τ)	Desired Normalised Continuous Correlation Fun	ction			
ρ <sub>r</sub> (τ)	Normalised Continuous Correlation Function Corresponding to the Calculated Polarity Correlation Coefficient				
π	3.14159				
σ	Standard Deviation				
σ <sup>2</sup> n	Variance of Noise Signal				
τ΄	Time Delay Domain	sec			
τD	Peak Position Time Delay of the Correlation Function	sec			
vo	Mean Velocity	meter/sec			
<sup>¢</sup> 1,2	Data Clock Pulses				
<sup>ф</sup> 2А	Load Clock Pulses				
<sup>ф</sup> 2В	Shift Clock Pulses				
ζ	Rotation Axis				

 $(\mathcal{I}\mathcal{X})$ 

## SUMMARY

Advances in automatic system testing, adaptive and optimizing control systems and flow measurement transducers has led to a demand for a reliable, low cost, correlator. The cross-correlation technique is found to be one of the best methods in system identification to obtain a good knowledge about the plant process for improved control purposes. (Sriyananda, Towill, 1973). A system impulse response measured by means of the cross-correlation method can be used to evaluate system performance measures like the ratio of the areas of the positive and negative parts of the impulse response, the logarithmic decrement and the position of significant peaks. (Jones, 1967). A special purpose MOS integrated circuit implementing the Over-Loading Integrator correlator has been constructed (financed by the NRDC), for use in correlation flow meter systems. (Jordan and Kelly, 1975). By generating a frequency inversely proportional to the time delay peak position of the cross-correlation function, a signal directly proportional to flow velocity is obtained.

The possibility of using the LSI MOS integrated circuit polarity correlator in automatic measurement of the distinctive features (eg, area, single peak and multiple peak positions and magnitudes) of the normalised correlation function is the subject of this Thesis. The main objectives can be summarised as follows :

 To devise a control circuit that will enable the normalised continuous correlation function rather than the polarity correlation function to be obtained.

2. To modify the display mode of the original system by storing the

positive and negative correlation functions.

3. To devise a system automatically measuring the voltage proportional to the area, single peak and multiple peak positions and magnitudes of the normalised correlation function.

4. To present experimental evidence of the performance of the system.

Some aspects of the correlation function and its applications in flow measurements, automatic system testing and adaptive control systems are discussed in Chapter 1 with a brief review of polarity correlators and their applications.

The design of the LSI polarity correlator and the Overloading Integrator Correlator prototype are reviewed in Chapter 2.

The automatic measurement of the distinctive features of the normalised correlation function, using the LSI integrated circuit correlator, is discussed briefly with some circuit modifications and a clock generator design, a display mode system and a distinctive features detector is presented in Chapter 3. The experimental performance in dynamic system measurements, air flow measurements, and wave power application, is described and discussed in Chapter 4.

The conclusion and discussion of results are considered in Chapter 5 with some suggestions for further developments and applications of the LSI low cost polarity correlator.

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# CHAPTER 1 : CORRELATION BASED MEASUREMENT SYSTEM

### 1.1 INTRODUCTION

Correlation data analysis has become a powerful technique in many scientific and engineering fields. The theoretical background of correlation function analysis is applicable to a wide range of measurement applications. The proper choice of integration time and noise test signal is necessary in determining the correlation function of a system and, for example, evaluating the time delay position of the peak of the function. Application of correlation techniques in flow measurements, dynamic system testings, and control systems have been surveyed. The polarity coincidence correlation function has been investigated and its relation to continuous correlation function is studied.

## 1.2 COINCIDENCE POLARITY CORRELATION

The correlation function of noise signals can be derived from the polarity waveform using digital circuits. Two level quantisation is the simplest form and is achieved by passing the analogue noise signal through a clipper or zero crossing detector. The polarity waveform retains the same zero crossing as the original signals, but without information about the amplitude of the original signal. The polarity correlation function has a little distortion compared with the continuous correlation function and a serious disadvantage arises from the need to integrate for a longer time to achieve the same variance as the continuous correlation function. Figure 1.2.1 illustrates the amplitude distortion introduced by polarity correlation. It should be noted that there is no time axis distortion.

Van Vleck (1966) has derived a relation between the normalised continuous correlation function  $\rho(\tau)$  and the polarity correlation  $r(\tau)$ , which can be applied to Gaussian noise and sinusoidal waveforms. The equation is given by :

$$r(\tau) = \frac{2}{\pi} \arcsin(\tau)$$
 ... (1.2.1)

It is noticed that only normalised correlation functions can be obtained by this method. The mean square values must be calculated separately for use as scale factors.

Using equation (.1.2.1), a graph has been plotted of the polarity cross-correlation function,  $r_{\chi y}(\tau)$ , versus the continuous correlation function,  $\rho_{\chi y}(\tau)$ , as in Figure 1.2.2. It is clear from the graph that for  $\rho_{\chi y}(\tau)$  less than 0.7, that the shape distortion of  $r_{\chi y}(\tau)$ will be negligible compared with  $\rho_{\chi y}(\tau)$ . However, a considerable magnitude distortion is apparent.

#### 1.3 REVIEW OF CORRELATION TECHNIQUE MEASUREMENT

#### **1.3.1** Introduction

As the correlation technique is used in almost every field of science where measurement can be reduced to an electrical signal, it is important to review some of the applications which have been applied and the methods which have been used. The applications of correlation techniques in flow noise measurement, automatic system testings and control systems will be surveyed.



#### 1.3.2 Correlation Techniques Applied To Flow Measurement

Flow velocity can be derived from flow generated noise measured at two points in the flow stream by inserting suitable noise transducers depending on whether liquid - solid or air-solid mixtures are used. The delay time of the up-stream signal with respect to the down stream signal is the transport lag between the two measuring points. Transport velocity is given by

 $V_{\perp} = \frac{L}{\tau}$ 

where L is the spacing between two noise transducers, and

 $\tau$  is the time delay or transport lag.

Flow measurement of viscous liquids and turbulent flow of liquids under steady and pulsating flow conditions using the temperature crosscorrelation method has been found to be a reliable technique. Abesckera and Beck (1972) have applied this technique in steady and pulsating flow of liquids in the highly turbulent flow region. A controlled amount of heat was injected by a pseudo random noise type of heat modulator. The temperature variations were measured by thermocouples, amplified and applied to the inputs of the correlator. The time delay position of the most significant peak of the crosscorrelation function gives the transport lag and hence the transport velocity is determined. As the Reynold's number was increased, a good agreement was found with the true average velocity.

Beck et al (1969) have studied solids velocity and mass flow measurement in pneumatic conveyors using correlation techniques. Flow was conveyed in a pipe and the flow noise was detected by

capacitance transducers. Fluctuations in the number of particles passing through the sensing field of an electrode installed on the wall of the conveyor is converted into a voltage fluctuations and these are correlated by an on-line or off-line computer programme. The programme determines the time delay of the peak and also computes the mass flow simultaneously. The correlation technique gave an error of +2% full scale deflection over the mass flow range 45 lb/hr -An advantage of capacitance noise transducer is that its 800 lb/hr. output can be approximated to bandlimited white noise. Also, it can be either a part of the pipe wall without obstructing the flow of the material or according to the design arrangement of the pipe lines it can be made small to insert it in the flow system without disturbing the natural flow pattern.

The flow of slurries (liquid-solid mixtures) and suspensions (gas-solids) has been successfully monitored using the correlation technique with flow noise generated by capacitance and electrical conductivity transducers. Beck et al (1973) have applied this method to the evaluation of the particle size in a flowing turbulent fluid by measuring the area of the auto-correlation function; area is proportional to the scale of turbulence and hence the particle size. The solids try to follow the turbulent motion of the fluid, the large particles tend to follow only the large eddies, while the small particles can follow both the large and the small eddies.

Ong and Beck (1974) have applied correlation technique in flow velocity, total volume flow, concentration and particle size measurements of slurries. Ultrasonic and conductivity transducers were used to detect the flow turbulence. The ultrasonic beam was modulated in

phase and amplitude by properties of the turbulent flowing fluid and detected by an amplitude modulation or frequency modulation circuit. The two noise modulated waveforms were cross-correlated and the time delay was determined. The advantages of the ultrasonic transducers are that they are non-contacting and hence do not obstruct the flowing medium. Since the flow velocity profile is not uniform, a correlation factor must be taken into consideration when a cross-correlation is obtained from the ultrasonic transducers and it should be constant over a very wide range. Also the transducer gives a reading at zero delay time due to the turbulence of the conveying fluid. (Ong and Beck, 1975)

Mesch et al (1971) have used the cross-correlation technique in measuring the speeds of moving strips of sheet metal by non-contact method using optical scanning at two measurement points. This method also was applied to solids flow velocity measurement in pneumatic conveyors (Mesch and Kipphan, 1972). Optical and capacitive flow noise sensors were used and compared. The optimal spacing between the transducers has been predicted theoretically, assuming that the correlation function has Gaussian distribution and it was found that the spacing  $L_{opt}$  depends on the relative standard deviation of velocity ( $\sigma/\nu_0$ ) and the normalised cut-off frequency ( $\omega_0/\nu_0$ ). The equation is given by

$$L_{opt} = 0.88 \frac{v_0^2}{\sigma \omega_0}$$

where  $v_0 = \text{mean velocity}$ ,

 $\omega_0$  = cut-off frequency, and  $\sigma$  = standard deviation.

An automatic peak tracing correlator was used with an on-line read-

b

out. This method has been found satisfactory for small loading coefficient and vertical flow but some difficulties occurred for higher loading and horizontal flow.

#### 1.3.3 Correlation Techniques Applied To Automatic System Testing

The technique of measuring system dynamic responses by correlating the input and output signals is now a well-established method and can be briefly considered in three parts. First, an excitation signal must be fed to the system. Secondly, this signal and the response of the system under test must be measured and recorded and thirdly, the two signals must be correlated. From these correlation functions, information about the dynamic behaviour of the system can be obtained. For example, the system impulse response h(t), input auto-correlation function  $R_{xx}(\tau)$ , and the cross-correlation of the system output with the input  $R_{xy}(\tau)$ , are related by

$$R_{xy}(\tau) = \int_{-\infty}^{\infty} R(\tau) R_{xx}(t-\tau) dt$$
 ... (1.3.3.1)

For well-defined white noise (with constant power spectrum), the cross-correlation is approximated to the impulse response of the system as in the following equation

$$R_{xy}(\tau) = K h(\tau)$$
 ... (1.3.3.2)

where K is a constant.

Therefore, a knowledge of the impulse response can be used to

identify the system transfer function. The excitation noise signal is chosen so that  $R_{\chi\chi}(\tau)$  exists only for a delay time shorter than the significant time constant of the system under test.

Correlation analysis of dynamic systems using pseudo-random binary noise test signals has been studied by Godfrey (1969). Also, Briggs et al (1964-65) have applied the random binary signal to determine the dynamic characteristics in the form of impulse response. The pseudo random binary waveform has been found to be a very useful test signal in system identification since its spectral density has almost the same characteristics as white noise.

The Sperry Gyroscope Co Ltd has produced a go-no-go servomechanism tester based on an implementation of the cross-correlation analysis (Jones, 1967). The gain and the magnitude of a system impulse response were measured and automatically set by a programme unit. The signal representing go-no-go indicates whether the measured value of the impulse response is within or has exceeded the tolerance limits. A pseudorandom binary test signal was used. This tester has been applied to second and higher order active filters.

Sriyananda and Towill (1973) have reported a fault diagnosis and location technique utilising the deviation between features of 'good' and 'bad' correlation functions in conjunction with a voting technique giving maximum likelihood of fault location. Results obtained from a complex electro-hydraulic servo have shown that the fault detection method works successfully. Hence, automatic system testing can be achieved and components can be determined by using only the inputoutput cross-correlation function measured at suitable time delays. The excitation signal was a pseudo-random binary noise signal.

#### 1.3.4 Correlation Techniques Applied To Adaptive Control Systems

Most industrial processes are variable and unstable. This causes some problems and therefore control systems are required to adjust and give the optimum performance while the processes are changing.

An adaptive control system recognises the variation in process conditions, and corrects some parameters automatically to give the optimum operating condition for the process disturbances. Hence, a complete evaluation of the process is necessary so that the parameter adjustment can be designed to give the best possible system performance. An error signal is the basic signal which must be controlled and is found by comparing two signals. There are two major adaptive control systems (Beck, 1974). First, input adaptive control systems, in which the process response is measured and compared with the input signal to give an error signal which is used to compute the performance criteria used in the optimisation of the process. The second method, the model reference adaptive control requires direct measure of plant This response is compared with a model reference process response. (represents a desired process response) and then by computing the performance parameter the process can be optimised.

Aseltine et al (1958) have surveyed adaptive control systems of many classes, and pointed out a new self-optimising system by measuring the impulse response using the cross-correlation technique to get the figure of merit to control the damping ratio. The figure of merit is given by

 $F = A_{+} + R_{0} A_{-}$ 

У

where  $A_+$ ,  $A_-$  are the positive and negative areas respectively of the impulse response and  $R_0$  is the relative stability (area ratio) corresponding to the desired damping ratio.

Anderson et al (1959) used the cross-correlation technique in an adaptive control system using discrete-interval-binary noise to measure the impulse response of a dynamic system and thus evaluate the system performance. The dynamic system parameters were adjusted to restore the response of the system to an acceptable form. This technique was applied to a pitch damper flight control to adjust the relative stability of the system. The pitch damper is a predominantly secondorder function and therefore, the overshoot is related to the relative stability and defined as the ratio of the positive area to the negative area of the correlation function (Mishkin, 1961), ie,

Relative stability = R = 
$$\frac{A_+}{A_-}$$
.

By controlling an error signal based on the relative stability, the dominant damping factor is kept at a fixed value. The crosscorrelation method has been used in adaptive flight control and found to give satisfactory results.

A correlation technique for the adaptive control of solids flow transport measurement in pipelines has been reported by Beck (1974). Solids material have to be transported at a certain velocity over long distance so as to avoid as much as possible the risk of blockage of materials in the pipeline as well as to transport the materials with minimum power costs. This can be ensured by using an adaptive control system. A parameter defining the optimum performance of the transport process can be obtained by measuring the area of the normalised

correlation function (which gives the spread of velocities in the pipe and does not depend on the number of solids being conveyed in the pipe), and the peak position of the cross-correlation function. Beck has proposed the use of the parameter:

$$\beta_p = \frac{\tau}{A_n}$$

where  $\tau$  = the time delay of the peak of the function,

and A = the area of the cross-correlation function (normalised).

The process parameter  $\beta_p$  will be compared with the desired performance parameter  $\beta_d$  and the error signal is obtained and fed to the adaptive controllers to control the valve regulating the flow.

Recently, Henry and Beck (1976) published the results of experimental work on sand/water mixture which was recirculated continuously in a flow rig. An ARGUS 400 on-line control computer was tried first in conjunction with the LSI overloading polarity correlator. The system under closed loop adaptive control has been found fairly stable. But in general, the stability of the system is found likely to be very difficult to analyse because of the non-linearities of both controllers and the system. It has been suggested that LSI Overloading Integrator Correlator could be used in this control scheme by interfacing it to a microprocessor.

## 1.4 REVIEW OF COINCIDENCE POLARITY CORRELATORS

Special purpose digital correlators have become popular and effective in a wide range of measurement applications. Though the analogue correlators are in general likely to remain efficient in terms of operating speed and simplicity, they are designed for special applications. But it must be mentioned that their dynamic range and stability can not compete with digital correlation techniques. The polarity coincidence correlator can be realised by digitizing the two input analogue signals to binary (1,0), and then delaying one of the signals by shift register, multiplying by exclusive-OR gates and then integrating by means of counters. There are three types of polarity correlator designs (Hayes and Musgrave, 1973), serial (one-bit correlator), parallel (many-bits correlator) and serial/parallel correlator. It should be noticed that the polarity coincidence correlator normalises the correlation function and therefore it does not give information about the amplitude of the correlation function.

Veltman and Bos (1963) have used polarity coincidence correlator in dynamic system measurements. A Gaussian noise signal was used as a test signal as well as sine wave signal. The principle of adding auxiliary signal before clipping the analogue signals was discussed with a theoretical derivation given to show that the auxiliary signal does not affect the variance of the correlation function but in some cases will improve the function shape. A serial correlator is realised with shift registers, relays, inverters and clippers. The results indicated that the measurement method is satisfactory in system impulse response measurement.

Boonstoppel et al (1968) have reported on time delay flow measurement using polarity correlation techniques by the moment method. Time delay was defined by

$$\tau_{D} = \frac{\int_{-\infty}^{\infty} \tau r_{xy}(\tau) d\tau}{\int_{-\infty}^{\infty} r_{xy}(\tau) d\tau}$$

where  $\tau_D$  is the peak position time delay of the correlation function and  $r_{\chi\gamma}(\tau)$  is the polarity cross-correlation function.

This method has been applied to the down comer of a steam boiler to determine the natural circulation during start-up from several conditions. The temperature fluctuations were detected by chromelalumel thermocouples. Errors due to mismatched input channels were discussed and it was noted that a symmetrical correlation function is difficult to obtain and therefore great care must be taken in practical application of the moment method.

One-bit polarity coincidence correlation have been used to measure the impulse response of dynamic systems of first and secondorders using Gaussian noise and prbs noise test signals (McDonnel, Forrester, 1970). In one example, an auxiliary signal was added to the noise signal to be correlated to improve the measured impulse shape. The signal used was a triangular waveform. For long test signal sequences accurate results have been obtained without adding auxiliary signals.

Mesch et al (1971) have devised an automatic peak tracking correlator, to detect the peak time delay of the cross-correlation function based on the polarity coincidence technique. The principle of peak tracking correlation is to evaluate the difference between the delayed up-stream signal  $S_1$  and the down-stream signal  $S_2$  such that the mean square value of the difference  $\Delta S(t)$  is minimised, ie,

 $E[\Delta S^{2}(t)] = minimum$ 

or

 $\frac{dE\{[S_1(t - \tau) - S_2(t)]^2\}}{dt} = 0, \text{ and therefore,}$ 

2E { 
$$\Delta S : \frac{d\Delta S}{d\tau}$$
 } = -2E{[S<sub>1</sub>(t -  $\tau$ ) - S<sub>2</sub>(t)] . S<sub>1</sub>(t -  $\tau$ ) = 0

where  $\tau$  is a delay approximately equal to the width of the function. From the above equation, it follows that the peak tracking correlator differentiates one of the signals, multiplies it with  $\Delta S$  and forms the average values of the product signal. Analogue output of the peak position can be read out and is fed to a voltage-to-frequency convertor to control the time delay of the shift register. Hence the frequency output from the correlator is proportional to the inverse of the time delay and therefore to the flow velocity. The peak tracking system is less complex and less costly than the parallel correlator. The analogue frequency output allows a good resolution of the time delay measurement. But this method has the disadvantage of a slow response to flow rate changes. Also, it is necessary to have a broad smooth correlation function in order to provide adequate difference signals An assymmetric correlation function under changing flow condition. may also cause difficulties in finding the accurate peak position.

As long ago as 1963, Weinreb described the use of polarity coincidence correlator (one-bit) in the measurement of spectral lines of galactic deuterium. Many channels were used to get many points on the auto-correlation function. The correlator was very costly, but this parallel technique is useful for experimental work which demands accuracy. Weinreb evaluated the variance of one-bit correlation function and found that the variance of the autocorrelation function estimate is increased by a maximum factor of approximately  $\frac{\pi}{4}^2 \approx 2.5$ . Hence, in polarity correlation the integration time is required to be increased to get the same variance as the continuous correlation function.

#### 1.5 OVERLOADING INTEGRATOR POLARITY CORRELATOR

A new method of correlation function display and peak detection based on the principle of parallel polarity correlation method has The new system has been described and realised by Jordan (1973). been realised by the techniques of monolithic silicon integrated circuits using large scale arrays of digital circuits. This system was designed to be a low cost electronic system for use in correlation flow meters. A prototype correlation flow meter has been produced and successfully evaluated in a wide range of industrial situations. The object of the study reported in this Thesis is to investigate the more general purpose measurement applications of the system and to devise prototype circuits that will enable all the significant features of correlation functions to be automatically measured. In the next Chapter, the special purpose LSI correlator circuit will be described.

# CHAPTER 2 : OVERLOADING INTEGRATOR POLARITY CORRELATOR

#### 2.1 INTRODUCTION

The Overloading Integrator Correlator was originally designed to satisfy the requirements of correlation flow measurement. For this application a signal inversely proportional to the time delay position of the most significant peak of the correlation function is required. The Overloading Integrator Correlator automatically produces this output. The principle of this correlator will be reviewed and its implementation by LSI circuit techniques will be described. Subsequent Chapters will report circuit developments which will greatly extend the application range of this correlator.

### 2.2 THE PRINCIPLE OF THE OVERLOADING INTEGRATOR CORRELATOR

The Overloading Integrator Correlator was invented by Jordan (1973 Thesis) and realised by LSI MOS integrated circuit techniques by Jordan and Kelly (1975). The original idea arose from the observation that the magnitude of the correlation function increases with increments of integration time so that if a fixed level is chosen and correlation is permitted to grow through this level as shown in Figure 2.2.1, then the first part of the correlation function to reach this level is the peak position of the function. For multiple peak detection Figure 2.2.2 shows how lesser significant peaks can be monitored.

This method has been realised using a parallel array polarity correlator, as shown in Figure 2.2.3. The noise signals, [x(t), y(t)] are converted into binary signals (1,0) before the



FIGURE 2.2.2 MULTIPLE PEAK DETECTION .



Integration i 00011 Patterns I Patterns of One's Represents Overloaded Integrating Counters.

FIGURE 2.2.4

OVERLOAD PATTERN

correlation is performed. A time delay unit is realised by a shift register which shifts the binary signal [x(t)] when pulsed by the sample clock generator. The multiplication of two sampled signals  $[x(t-\tau), y(t)]$  can be performed by a coincidence gate which is described by the equation

 $C = A \cdot B + \overline{A} \cdot \overline{B}$ 

where C is the output of the gate. Logic 'l' indicates complete coincidence between the two binary signals, and logic 'O' means anti-coincidence. The output of the multiplier will be AND-ed with a pulse train and the output of the NAND gate will be integrated by a fixed capacity counter in each channel. Extra logic circuitry has been added so that the overload states can be stored. The first integrating counter that overloads gives the most significant peak position of the correlation functions. Figure 2.2.4 shows a typical overload pattern.

Figure 2.2.5 shows the block diagram of the Overloading Integrator Correlator. A timing counter controls the integration cycle by generating resetting control signals. By inspecting the content of the timing counter when an overload appears the correlation significance of the overload may be determined. The polarity correlation coefficient is given by (Jordan, 1973)

$$r(\tau) = \frac{2N}{T_c} - 1$$

where  $r(\tau)$  is the polarity correlation coefficient, N is the capacity of the integrating counter



FIGURE 2.2.5 OVERLOADING INTEGRATOR CORRELATOR BLOCK DIAGRAM

.

and  $T_c$  is the content of the timing counter when the overload appears.

The pattern of overloads will build up as shown in Figure 2.2.4 and the function is defined by the envelope of the pattern of ones.

Although this type of correlator has complex circuitry, its regularity makes it easy to realise by MOS integrated circuits. The realisation of an integrated circuit version of this correlator has been completed in the WOLFSON MICROELECTRONICS UNIT, UNIVERSITY OF EDINBURGH. The application of this integrated circuit correlator to the automatic measurement of the distinctive features of the correlation function is the principle concern of this study.

### 2.3 LSI INTEGRATED CIRCUIT CORRELATOR

This correlator is a digital large scale integrated circuit polarity correlator, designed specially for flow measurement applications. The integrated circuit chip can realise 12 points of correlation and can be connected in series with other chips to form complete polarity correlator systems. A description of the functional blocks used will be given to aid the description of the use of the special-purpose integrated circuit. Figure 2.3.1 shows the block diagram of the large scale integrated circuit chip with its 24 pin connections.

One of the binary signals is delayed by the input shift registers. The output of the 12th shift register can be connected to the shift register input of the next chip to realise any number of delay points by this series connections. It is noticed that two phase clock signals,  $\phi_1$  and  $\phi_2$  are required.  $\phi_1$  is the 'data lock out'pulse clock.  $\phi_2$ 



FIGURE 2.3.1 BLOCK DIAGRAM OF LSI CHIP
is the 'read in' clock signal. The complementary  $\phi_1$ ,  $\phi_2$  signals can be generated by simple circuitry shown in Figure 2.3.2, (Jordan and Kelly, 1975) which eliminates the possibility of the phases overlapping.

The multiplication of two digital signals can be performed by coincidence detector. The output of each delay shift register is compared with the second binary signal by means of exclusive-NOR logic gate. When there is coincidence between the two signals the output is logic '0' and for anti-coincidence, the output is logic '1'. The equation below represents the output

 $C = AB + \overline{AB}$ 

The output of the coincidence detector is AND-ed by the count clock. The output of the clocked coincidence detectors are counted by integrating counters of capacity  $(2^{15}-1)$  pulses, which is a satisfactory number to give a good approximation of the correlation function. The 'count clock' is the only external input required. Synchronisation between 'count clock' and  $\phi_2$  phase clock 'data lock out' must be ensured. To preset the integrating counters an external logic 'l' must be provided at the 'preset' input pin and it has to be synchronised related to the count clock.

Each counter will overload after (2<sup>15</sup> -1) clock pulses. The overloaded outputs are fed to 12 input-OR gate whose output is externally pulled up to +5V by a 10K resistor to give a wired-OR facility. An overload condition is indicated by logic 'O' and it is called the 'ICOL' output.









FIGURE 2.3.3 OUTPUT REGISTER CLOCK DRIVER

To form a correlation function from the evolving pattern of overloads during the integration cycle, the overloads of the integrating counters must be stored in R-S stores. External control signals are necessary to 'set enable' and 'reset' the store. They must not be removed simultaneously, otherwise hazard conditions will occur. The 'set' and 'reset' signals are provided by the timing counter.

It is convenient to shift the parallel overload pattern stored by the R-S stores serially out through the output shift registers which have parallel in/serial out facility. They have also serial input and output for external connection in series with other chips to form a complete correlator. There are three types of clock inputs to be provided externally to load ( $\phi_{2A}$ ) and shift ( $\phi_{2B}$ ) the output shift registers.  $\phi_{2A}$  pulses load the overload pattern from the store to the output shift register in parallel and  $\phi_{2B}$ pulses shift the overloads out serially and these two clock pulses must occur during  $\phi_2$  phase clock but not together. Figure 2.3.3 shows the timing diagram and the electronic circuit design (Jordan and Kelly, 1975) to generate  $\phi_1$ ,  $\phi_{2A}$ ,  $\phi_{2B}$  clocks.

The chip has been designed in accordance with the standard layout rules of the P-channel metal gate MTNS process of General Instrument Microelectronics Limited. For low operating speed requirement in some applications, a fully static circuit design was employed (Kelly and Jordan, 1974). The chip size (0.172 x 0.176 in) is likely to allow good yields in production. The LSI package has a total of 18 pin connections (inputs and outputs are TTL compatible) and power supplies +5V and -12V have to be provided.

## 2.4 OVERLOADING INTEGRATOR CORRELATOR PROTOTYPE SYSTEM

A prototype correlator was designed (Jordan and Kelly, 1975) using the integrated circuit correlator to show the usefulness of the system and to provide prototype systems for industrial evaluation. The prototype block diagram is shown in Figure 2.4.1.

The clock generator provides sample clock pulses (to generate  $\phi_1$  and  $\phi_2$  phase clock signals) for the input register and the zero crossing detectors, 'count clock' for the timing counter (to generate preset, set, reset signals) and the integrating counter, and 'output clock' to provide the output shift register with the clock pulses  $\phi_1$ ,  $\phi_{2B}$ ,  $\phi_{2B}$  to load the overload pattern into the stores and then shift the pattern out. Load/shift pulses (provided synchronously with the count clock) are used in conjunction with the output clock pulses to generate the signals needed for the output shift register.

The integration cycle is controlled by the timing counter which generates the necessary signals preset, set, reset respectively (the count clock is divided by two before it goes to the timing counter which has maximum capacity of 16-bits). The correlation coefficient significance of the reset can be chosen to prevent the system from detecting spurious noise by suitably setting the preset level of the timing counter. The overload pattern is shifted out during the integration cycle by regular intervals of the timing counter (load/shift pulses). The correlation function output defined by the envelope of the overload states is non-linear in this case (Jordan, 1973).

The input circuit channels (input buffer amplifiers and zero



FIGURE 2.4.I BLOCK DIAGRAM OF PROTOTYPE FLOWMETER SYSTEM crossing detectors) are identical to avoid the problem of mismatched channel dynamics which has been reported by Boonstoppel et al (1968).

The digital correlator contains the clock drivers circuitry to provide the chips with the clock pulses  $\phi_1$ ,  $\phi_2$  and  $\phi_1$ ,  $\phi_{2A}$ ,  $\phi_{2B}$ . In the prototype system ten chips have been used connected in series and two modes can be chosen manually by a switch, one for flow mode and the other for display mode.

### 2.5 SYSTEM TESTING APPLICATION OF THE IC CORRELATOR

It will be shown in the next Chapter that is is possible to generate overload patterns corresponding to equal decrements of the normalised continuous correlation function. Hence, in this case, the envelope of the pattern of overloads describes the normalised function. By generating a digital signal and a voltage corresponding to linear decrements of the correlation coefficient, a display system for a storage oscilloscope can be formed. It will be shown that the normalised correlation function overload pattern can be used to generate voltages proportional to function area and magnitudes and positions of peaks.

This modified system will find application in automatic system testing because use of appropriate test signals will result in overload patterns being obtained corresponding to equal decrement of the normalised impulse response of the system. Hence, automatic measurement of the distinctive features of the normalised impulse response will be possible.

Appendix (2) describes the effect of normalisation and dc decoupling on the measurements mode.

# CHAPTER 3 : NORMALISED CONTINUOUS CORRELATION FUNCTION FEATURE

DETECTOR

### 3.1 INTRODUCTION

Automatic measurement of the distinctive features (eg, area, single peak position, and multiple peak positions and magnitudes) of the normalised correlation function using the LSIC Overloading Integrator Correlator, is described. A TTL control circuit has been designed to obtain the normalised continuous correlation function making use of the Van Vleck relation. Some modifications are made to parts of the original system such as the clock generator and the display mode. The design of a system automatically producing a voltage proportional to the area, single peak position, and magnitudes and positions of the normalised correlation function is described.

#### 3.2 BLOCK DIAGRAM OF THE PROTOTYPE SYSTEM

The basic LSI polarity correlator is used in conjunction with a simple circuitry to measure the automatic distinctive features of the normalised correlation function. Figure 3.2.1 shows the block Digital correlator, timing counter diagram of the prototype system. and input circuit are the same circuitry designed for the Overloading Integrator Correlator flowmeter (Jordan and Kelly, 1975). A unity gain buffer amplifier has been used and modifications have been made to the clock generator. A circuit was designed generating 100, output register, load/shift pulses corresponding to the linear decrement of the normalised continuous correlation function. Circuits were built to measure the voltages proportional to area, single peak position, multiple peak position and peak magnitudes. Also, modification has been made to the display method so that positive and



FIGURE 3.2 IPROTOTYPE CORRELATOR SYSTEM BLOCK DIAGRAM.

negative values of the normalised correlation function can be simultaneously displayed.

It has been found that the prototype correlator and feature detector is working satisfactorily and the results obtained are acceptable in comparison with the Hewlett Packard correlator.

#### 3.2.1 Clock Generator

The clock generator provides sample clock pulse, count clock, and output clock to the system. Figure 3.2.1.1 represents the clock generation circuit diagram. A TTL 1 MHz crystal oscillator (ME106TC) is used to provide the basic clock signal of 100 kHz to the features detector prototype. The modification has been made (by constructing variable modulus counter, n-divider) in order to simplify the setting of the time delay range of the instrument.

Two units of the SN 74160 synchronous decade counter are used to divide by n=2 to n=99 using the synchronous parallel load facility. The thumbwheel switches preset the counter for a certain division ratio. It's circuit is arranged to that switch position '0' is connected to the decimal '9' of a 10-4 line decoder (SN 74147) and also '1' to '8', '2' to '7', etc. The basic principle of the divider is that, the counter counts up from the preset value 'X' to its maximum count (N=99) and recycles starting again from the preset number.

The operation of the two-stage divider can be described as follows. The unit counter,  $C_1$ , will start counting the units of the preset numbers, X. The carry will go high for one clock pulse enabling the tens counter,  $C_2$ , to count one clock pulse.  $C_1$  will



FIGURE 3.2.1.1 CLOCK GENERATOR CIRCUIT DIAGRAM



FIGURE 3.2.1.2 TIMING DIAGRAM FOR DIVIDE BY 25

count another ten clock pulses enabling  $C_2$  again by the carry signal. The counting will continue until the maximum count (99) is reached. At this point,  $Q_D$  of  $C_1$ , and  $Q_A$ ,  $Q_D$  of  $C_2$  stand at high level and the output from the 3-input NAND gate goes low. This logic change is used to load the preset number, X, and start the count sequence again. The output of the inverter, I, is AND-ed with the 100 kHz clock pulses to obtain a division of one and an output one clock pulse width. The timing diagram is shown in Figure 3.2.1.2 for division by 25. Note that the preset number will be 75 for this condition.

The output from the variable modulus counter drives a chain of decade counters. A wafer switch selects outputs from the counters to provide the count clock and sample clock waveforms. It should be noted that it is possible to set the count clock to faster than the sample clock. Hence, more than one pulse can be counted per coincidence and this results in a net reduction in the integration time.

### 3.2.2 Load Pulse Generator

To inspect the evolving overload patterns at times corresponding to equal decrement of the normalised continuous correlation function a special sequence of output register load/shift pulses must be generated. These load/shift pulses parallel load the stored overload pattern into the output shift register and, after the appropriate shift pulses are applied, the parallel pattern appears as a serial sequence at the output of the register.

The polarity correlation significance,  $r(\tau)$ , of an overload, is given by

$$r(\tau) = 2\left(\frac{N}{N+\alpha}\right) - 1$$

... (3.2.2.1)

where  $N = 2^{15} - 1 \approx 2^{15}$ ,  $0 \le \alpha \le N$ 

and  $N + \alpha$  is the timing counter contents when the overload occurs.

It should be noticed that the normalised polarity correlation coefficient,  $r(\tau)$ , is non-linear with respect to the timing counter contents,  $T_c = N + \alpha$ .

Two requirements are necessary for the generation of the correct sequence of the load/shift pulses :

 The normalised continuous correlation function instead of the polarity correlation function must be obtained. This can be achieved by using the Van Vleck relation (Appendix 3) when the signals have Gaussian statistics or they are sine waveforms. The relation is

$$\rho_{XY}(\tau) = \sin \frac{\pi}{2} r_{XY}(\tau)$$
 ...(3.2.2.2)

where  $\rho_{\chi y}(\tau)$  is the normalised continuous correlation function. Most practical signals can be described by Gaussian statistics and therefore the Van Vleck relation has a wide application range.

 Linear decrement of the normalised continuous correlation function is required.

The values of  $r_{\chi y}(\tau)$  for equal decrements of  $\rho_{\chi y}(\tau)$  were tabulated from the equation

$$r_{xy}(\tau) = \frac{2}{\pi} \sin^{-1} \rho_{xy}(\tau)$$
 ... (3.2.2.3)

Substituting the tabulated results of  $r_{xy}(\tau)$  into the equation (3.2.2.1), the equivalent contents of the timing counter, N +  $\alpha$ , were obtained. Table 3.2.2.1 shows the values of  $r_{xy}(\tau)$  and (N +  $\alpha$ ) [when N was set to  $(2^{12} - 1)$ ], resulting from equal decrements  $\rho_{xy}(\tau)$  of 0.01. By using N = 2047, it is possible to simplify the decoding requirements, ie, 12-bits of the timing counter must be used to generate the 100 load pulses corresponding to the equal decrement of  $\rho_{xy}(\tau)$ . For this purpose a programmable ROM could be used to decode the contents of the timing counter at the appropriate points during the integration cycle.

Finance was not available to construct a ROM decoder, hence, an alternative approach using TTL circuitry was investigated to generate the 100 load pulses.

From Table 3.2.2.1, it can be seen that the timing counter content ( $T_c$ ) changes incrementally. The increment remains constant over a certain range of  $T_c$  and then changes. For example, in the range 2144 to 2244, the increment is 40; in the range 2244 to 2349, the increment is 25. Hence, once the range for which the increment (n) remains constant has been defined, the counter number,  $T_c$ , can be decoded and used in conjunction with a divide by n counter. The load pulses generated by the counter correspond to equal decrement of the normalised continuous correlation function. The use of this arrangement leads to an error of  $\pm 1\%$  in the generated equal decrements of  $\rho_{xy}(\tau)$ . The error (E) is calculated by the following equation

$$E = \frac{\rho_d - \rho_r}{\rho_d} \times 100$$

	CORRI	ELATION COEFFIC	CIENT ρ(τ)	
ρ <sub>d</sub> (τ)	<b>r(</b> τ)	Т <sub>с</sub>	ρ <sub>r</sub> (τ)	ρ <sub>d</sub> (τ) - ρ <sub>r</sub> (τ)
		(approximated)		
*1.00	1.00000	2047	1.00000	0.00000
*0.99	0.90989	2144	0.98992	0.00008
0.98	0.87246	2184	0.98064	0.00064
*0.97	0.84367	2224	0.96890	0.00110
0.96	0.81933	2249	0.96045	0.00045
0.95	0.79783	2274	0.95122	0.00122
0.94	0.77835	2299	0.94129	0.00129
0.93	0.76038	2324	0.93070	0.00070
*0.92	0.74364	2349	0.91953	0.00047
0.91	0.72783	2369	0.91020	0.00020
0.90	0.71286	2389	0.90056	0.00056
<b>0.</b> 89	0.69859	2409	0.89062	<b>0.0</b> 0062
0.88	0.68491	2429	0.88041	0.00041
0.87	0.67176	2449	0.86996	0.00004
•0.86	0.65907	2469	0.85926	<b>0.</b> 00074
0.85	0.64679	2487	0.84947	0.00053
0.84	0.63489	2505	0.83952	0.00048
0.83	0.62332	2523	0.82943	0.00057
0.82	0.61205	2541	0.81921	0.00079
°0.81	0.60106	2559	0.80887	0.00113
<b>0.</b> 80	0.59033	2575	0.79959	0.00041
0.79	0.57984	2591	0.79023	0.00023
0.78	0.56956	2607	0.78081	0.00081
0.77	0.55949	2623	0.77132	0.00132
•0.76	0.54960	2639	0.76177	0.00177
0.75	0.53989	2656	0.75157	0.00157
0.74	0.53035	2673	0.74133	0.00133
0.73	0.52096	2690	0.73104	0.00104
0.72	0.51172	2707	0.72071	0.00071
0.71	0.50261	2724	0.71036	0.00064
0.70	0.49363	2741	0.69997	0.00003
<b>0.</b> 69	0.48477	2758	0.68975	0.00025

ρ <sub>d</sub> (τ)	r( <b>τ</b> )	Т <sub>с</sub>	ρ <sub>r</sub> (τ)	$ρ_d(\tau) - ρ_r(\tau)$	
		(approximated)			
*0.68	0.47604	2775	0.67916	0.00084	
0.67	0.46741	2791	0.66935	0.00065	
0.66	0.45888	2807	0.65953	0.00047	
0.65	0.45046	2823	0.64972	0.00028	
0.64	0.44213	2839	0.63991	0.00009	
0.63	0.43389	2855	0.63010	0.00010	
0.62	0.42573	2871	0.62030	0.00030	
0.61	0.41766	2887	0.61032	0.00032	
0.60	0.40966	2903	0.60075	0.00075	
*0.59	0.40174	2919	0.59100	0.00100	
0.58	0.39389	2936	0.58066	0.00066	
0.57	0.38611	2953	0.57035	0.00035	
0.56	0.37839	2970	0.56006	0.00006	
0.55	0.37074	2987	0.54981	0.00019	
0.54	0.36315	3004	<b>0.</b> 53960	0.00040	
0.53	0.34462	3021	0.52942	0.00058	
0.52	0.34814	3038	0.51927	0.00073	
0.51	0.34071	3055	0.50917	0.00083	
0.50	0.33333	3072	0.49911	0.00089	
0.49	0.32601	3089	0.48909	0.00091	
0.48	0.31873	3106	0.47912	0.00088	
0.47	0.31149	3123	0.46920	0.00080	
0.46	0.30430	3140	0.45933	0.00067	
0.45	0.29715	3157	0.44951	0.00049	
0.44	0.29004	3174	0.43973	0.00027	
0.43	0.28297	3191	0.43001	0.00001	
0.42	0.27594	3208	0.42034	0.00034	
*0.41	0.26894	3225	0.41073	0.00073	
0.40	0.26198	3243	0.40062	0.00062	
0.39	0.25505	3261	0.39056	0.00056	
0.38	0.24815	3279	0.38058	0.00058	
0.37	0.24128	3297	0.37065	0.00065	
0.36	0.23444	3315	0.36079	0.00079	

		50		
d(τ)	r(τ)	Tc	ρ <sub>r</sub> (τ)	$ρ_{d}(\tau) - ρ_{r}(\tau)$
-		(approximated)		u 1
1 34	0 22085	3352	0 34074	0 00074
0.34	0.21047	3371	0.33056	0.00056
0.32	0.20736	3300	0.33030	0.00045
0.32	0.20066	3409	0.32043	0.00045
0.30	0.10373	3429	0.31041	0.00041
n 29	0 18731	3420	0.20058	0.00040
0.29	0 18067	3467	0.28026	0.00036
0.20	0.17405	3487	0.23020	0.00020
0.27	0 16744	3507	0.25000	
0.25	0 16086	3527	0.23990	0.00016
0.23	0 15429	2547	0.23087	0.00013
0.24	0 14774	2547	0.23907	0.00013
0.23	0 14121	3587	0.22020	0.00001
0.21	0 13469	3607	0.20990	0.00010
0.21	0 12819	3627	0.20097	0.00010
0.20	0.12169	3649	0.19039	0.00039
0.15	0 11522	3671	0.19099	0.00003
0.17	0 10875	3693	0 16973	0.00027
0.16	0.10229	3715	0 15956	0.00044
0.15	0.09585	3737	0.14949	0.00051
0.14	0.08942	3759	0 13953	0 00047
0.13	0.08299	3781	0.12966	0.00034
0.12	0.07658	3803	0 11990	0.00010
0.11	0.07170	3825	0.11024	0 00024
0.10	0.06377	3849	0.09981	0.00019
0.09	0.05737	3873	0.08951	0.00049
0.08	0.05098	3897	0.07932	0.00068
0.07	0.04460	3921	0.06925	0.00075
0.06	0.03822	3945	0.05929	0.00071
0.05	0.03184	3969	0.04945	0.00055
0.04	0.02547	3993	0.03972	0.00028
0.03	0.01910	4017	0.03010	0.00010
0.02	0.01273	4041	0.02060	0.00060
0.01	0.00637	4065	0.01120	0.00120
0.00	0.00	4096	0.00000	0.0000
		· ·		

	TABLE	3.3.2.2		SELECTI	ON	OF	TI	MING	COUNT	ER	DE	CODE	NUMBERS			
	. T <sub>c</sub>			·			·		·							MSB
	2047		0	0	0		0	0	0		0	0	0	0	0	1
	2145		1	0	0		0	0	1		1	0	0	0	0	1
	2225		1	0	0		0	1	1		0	1	0	0	0	1
	2350		0	· 1	1		1	0	1		0	0	1	0	0	1
-	2470		0	1	٦		0	. 0	1		0	1	1	0	0	1
	2560		0	0	0		0	0	0		0	0	0	1	0	1
	2640		0	0	0		0	1	0		1	0	0	1	0	1
	2776		0	. 0	0		1	٦	0		1	1	0	1	0	۱
	2920		0	0	0		1	0	1		1	0	1	1	0	1
	3226		0	1	0		۱	1.	0		0	1	0	0	1	1
	3334		0	<b>1</b> .	1		0	0	0		0	.0	1	0	1	1
	3448		0	0	0		1	1	1		1	. 0	1	0	1	<b>]</b>
	<b>36</b> 28		0	0	1		1	0	1		0	0	0	1	1	1
	3826		0	1.	0		0	1	1		1	1	0	1	1	1

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Figure 3.2.2.1 LOAD PULSE GENERATOR BLOCK DIAGRAM



TRUTH TABLE					
R	S	Q	ā		
0	0	i	í		
0	1	1	0		
1	0	0	1		
1	1	NOC	ange		

FIGURE 3.2.2.2 R-S NAND LATCH AND THE TRUTH TABLE

where  $\rho_{d}$  is the desired correlation coefficient,

and  $\rho_r$  is the correlation coefficient obtained by the decoding method described above. Table 3.2.2.2 shows the numbers that have to be decoded with their corresponding binary conversion.

The block diagram for load pulse generator is shown in Figure 3.2.2.1. The last twelve bits of the timing counter are used for decoding purpose and 4-inputs AND gates are used in conjunction with 3-inputs NAND gates. The decoding pulses from the NAND gates are fed into 2-input dual NAND gate latches. The latches are of the R-S type in which the SET input sets the output Q to logic 'l' and the RESET input sets the output to logic '0'. Figure 3.2.2.2 represents the NAND R-S type latch and its truth table. The latch operation can be described as The first decoding point (eg, corresponding to  $\rho_{\chi\gamma}(\tau)$  = 1.0) follows. will set the Q output of the latch to high level (logic 'l') and the second decoding point (eg, corresponding to  $\rho_{\chi\chi}(\tau)$  = 0.99) will reset the output Q to logic 'O'. This operation provides a logic 'l' level for the interval between two decoded points of the timing counter The output of all the latches are fed to the logic gates sequence. shown in Figure 3.2.2.1 and eight bit parallel word is formed to set the count ratio of the frequency divider. A timing diagram for the circuit is shown in Figure 3.2.2.3.

The clock input to the divider is derived from the third output of the timing counter. In order to get a narrow load pulse this output was derived by decoding the three first outputs of the timing counter contents with the clock input of timing counter by a 4-input AND gate. The outputs of the divider are decoded by an 8-input NAND gate and the resulting signal is used to load the preset numbers into

Preset	Γ
	MSB
	2047
	A
<u>/</u>	<u>2145_</u>
J <u> </u>	D
<b></b>	2350
	<u>2330</u>
	2470
	E
Γ	2560
	<u>F</u>
	2640
	G
	2776
	<u> </u>
	2920
	3226
· · · · · · · · · · · · · · · · · · ·	П 3334
· · · · · · · · · · · · · · · · · · ·	К
	3448
	L
· · · · · · · · · · · · · · · · · · ·	
	M
•	
[]	
	Dha
$D = \frac{1}{12} \frac{1}{1$	$D_{\mathbf{Q}_{2}} = \mathbf{D} \cdot \mathbf{\Gamma} \cdot \mathbf{\Pi}$
$D_{c_1} = D_{a_1}A.M.N$	$U_{C1} = A$
$D_{d_1} = C \cdot F \cdot H \cdot A$	$D_{d_{2}} = Logic   (+5V)$
· · ·	

FIGURE 3.2.2.3 TIMING DIAGRAM FOR THE LOAD PULSE GENERATOR

the counters. The output from the inverter is AND-ed with the clock input to the divider to get the '100' load pulses with a width equal to the clock pulse width. The load pulse generator has been realised with TTL circuitry and found to operate satisfactorily.

#### 3.3 SINGLE PEAK DETECTION

Peak position (time delay) measurement is necessary to determine flow velocity by the correlation flow meter method (Beck, 1974) and the rise time of the dynamic system in control measurement (Truxal, 1953). Although the time to the peak is not the same as the rise time, it is still good measure of speed of response when the overshoot is not too small.

Figure 3.3.1 shows simple circuit for peak detection while the correlator is set up in the display mode. The output register shift pulses are counted by an 8-bit up-counter. A J-K flip-flop Q output is preseted to logic 'l' by the system preset control signal. When the peak appears, ie, the first overload occurs, the output of the AND gate will clock the flip-flop and Q goes to logic '0', thereby inhibiting the overload pattern. In the meantime, the number in the counter will be transferred to a buffer store and used to address the digital-to-analogue convertor to give a voltage proportional to (or inversely proportional to, depending on the convertion) time delay. The timing diagram for single peak detection is shown in Figure 3.3.2.

The DAC output voltage is given by the equation below

$$V_o = \frac{N_p}{N_{max}} V_{ref}$$

... (3.3.1)



FIGURE 3.3.1 VOLTAGE PROPORTIONAL TO THE TIME DELAY CIRCUIT DIAGRAM



# FIGURE 3.3.2 TIMING DIAGRAM FOR SINGLE PEAK DETECTION

where  $N_n =$  the peak position,

 $N_{max}$  = the capacity of the sotre,

and  $V_{ref}$  = the reference voltage to the DAC.

Equation (3.3.1) gives voltage directly proportional to the time delay. By connecting the DAC in the feedback path of an operation amplifier a voltage inversely proportional to peak position is obtained. This method has been found to operate reliably and has the advantage that the correlation function can be simultaneously displayed.

# 3.4 VOLTAGE PROPORTIONAL TO MULTIPLE PEAK MAGNITUDES AND POSITIONS

The cross-correlation technique has been applied in continuous gas chromatography to determine the peak positions and magnitudes of the response by using a pseudo random binary signal to modulate the input to the column (Godfrey and Devenish, 1969). The peak positions and magnitudes are a measure of the substances present and their relative proportions. Other multiple peak applications will probably be generated once a prototype system is available to assess the measurement possibilities. Figure 3.4.1 illustrates an evolving overload pattern for a multiple peaked function and shows that a peak position must be remembered to enable the next significant peak to be detected. The essential features of a system measuring multiple peak positions is shown in Figure 3.4.2. The overload pattern pulse sequence is counted by an up-counter  $(UC_1)$  and a store  $(S_1)$  holds the total number of peaks detected. A comparator (C) compares the contents of  $(S_1)$  and  $(UC_1)$  and







FIGURE 3.4.3 CONTROL SIGNALS FOR THE MULTIPLE PEAK DETECTOR .



FIGURE 3.4.2 MULTIPLE PEAK DETECTOR

generates a logic signal which when AND-ed with the overload pattern generates a pulse every time a new peak is detected. This pulse is used to transfer the contents of the UP counter  $(UC_2)$  (counting output register shift pulses) into the 3-stage store  $(S_2)$ . The digitalto-analogue convertor is addressed by the last stage of the store  $(S_2)$ and an output voltage proportional to the multiple peak position The same principle is applied to the magnitudes can be obtained. of the multiple peaked function by using the equal decrement load pulses to count down from a preset value of 100. An output voltage proportional to the magnitudes of the peaks of the function is obtainable from another DA convertor. Hence, at the end of the integration cycle the stores (S2,S3) will contain the positions and magnitudes of the multiple peaked function. By freezing the integration cycle, output voltages proportional to magnitudes and positions are obtainable from the DAC's by operating the manual clock to shift out from the stores  $(S_2, S_3)$  the number, related to the positions and magnitudes of the peaked function.

#### 3.5 CORRELATION FUNCTION AREA MEASUREMENT

The area measurement of the impulse response is relevant in control techniques. Van der Grinten (1963) shows that the slope of performance index  $P_i$  with respect to the parameter control k is given by

$$\frac{\partial P_{j}}{\partial k} = \frac{\int R_{xy}(\tau) d}{\int R_{xx}(\tau) d}$$

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... (3.5.1)

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where  $R_{\chi y}(\tau)$  is the cross-correlation between the random perturbation signal x(t) and the performance signal y(t). If the excitation signal is approximately white noise, then the denominator is a constant, unsubjected to the statistical variation. While the numerator is proportional to the area under the impulse response  $h(\tau)$  over the range where  $h(\tau)$  cannot be neglected. Hence, equation (3.5.1) can be written as follows :

$$\frac{\partial P_i}{\partial k} = \int_{-\infty}^{\infty} h(\tau) d\tau = \int_{0}^{\infty} h(\tau) d\tau \qquad \dots (3.5.2)$$

since  $h(-\tau) = 0$  for real system. This equation determines the dc gain between the parameter K and the performance index P at the set point. Also, the relative stability of a control system is related to the distribution of area in the impulse response of the system (Anderson, 1959).

The Overloading Integrator LSI polarity correlator can be used to determine the impulse response of a system under test. Because the measured function is automatically normalised, the measured impulse response will not be a function of the system dc gain. RMS values of the input and output signals are necessary to correct the normalised area of the impulse response. In Appendix (A2.2) it has been shown that the relative stability determined from the normalised impulse response is the same as for the area ratio of the continuous impulse response and is given by

$$R_0 = \frac{A_+}{A_-} = \frac{A_{n+}}{A_{n-}}$$
 ... (3.5.3)

where A+, A- are the areas of the continuous impulse response,



FIGURE 3.5.1 AREA MEASUREMENT CIRCUIT DIAGRAM

and  $A_{n+}$ ,  $A_{n-}$  are the areas of the normalised impulse response.

In the features detector prototype a circuit generating voltage proportional to the area of the normalised correlation function  $\rho_{XY}(\tau)$  has been constructed. The area is measured during the integration cycle of the correlator. Function area is measured by counting pulses generated by AND-ing output register shift pulses with the overload pattern. The total number stored in the counter at the end of the cycle is transferred by the preset control signal to a buffer store and the stored number is addressed to a digital-to-analogue convertor. The DAC output voltage is directly proportional to the area. A frequency divider was designed to give a scaling factor to ensure that the capacity of the up counter was not exceeded. Figure (3.5.1) shows the circuit diagram of the prototype circuit with its timing diagram.

Since the overload patterns can be obtained for both positive and negative correlation coefficients, the voltage proportional to positive or negative areas can be measured. The circuit has been found to operate satisfactorily.

### 3.6 MODIFIED DISPLAY OF THE CORRELATION FUNCTION

#### **3.6.1** Positive and Negative Correlation Function

It has been mentioned earlier that the correlation function has positive and negative values for  $\tau > 0$ . A complete knowledge about the function is required. Hence, the prototype correlation has been arranged such that the positive part of the normalised correlation function appears first followed by the negative part.



The positive part appears in the first integration cycle and then in the second integration cycle the negative part. This means that the digital input signals must be controlled by the integration cycle.

The timing diagram for the display of the positive and negative correlation function is shown in Figure (3.6.1.1). The preset signal is divided by two using a toggle flip-flop. The MSB of the timing counter goes high at a time corresponding to  $\rho_{XY}(\tau) = 1.0$  and it goes low at a time corresponding to  $\rho_{XY}(\tau) = 0$ . The correlation function of the positive part must be displayed when Q of the F-F goes high and the negative part when Q goes low.

A simple circuit for this purpose has been constructed (Figure (3.6.1.2)). The preset is fed to a toggle J-K flip-flop. The Q output is AND-ed with  $Q_1$  and  $Q_2$  outputs of the zero-crossing detectors.  $\overline{Q}$  is AND-ed with  $\overline{Q}_1$  and  $\overline{Q}_2$  outputs of the detectors. OR gates are used to perform the addition of  $(QQ_1, \overline{QQ}_1)$  and also  $(Q\overline{Q}_2, \overline{QQ}_2)$ .

The data inputs to the digital correlator are arranged such that the positive part will occur and then the negative correlation function. The following logic inputs must be provided to the digital polarity correlator.

Data 1 =  $QQ_1 + \overline{QQ}_1$ Data 2 =  $\overline{Q}_1$ 

Auto-Correlation mode

Data 1 =  $QQ_2 + \overline{QQ}_2$ Data 2 =  $\overline{Q}_1$ 

Cross-Correlation mode



FIGURE 3.6.1.2 DATA INVERTION CIRCUIT



# FIGURE 3.6.1.1 TIMING DIAGRAM

This method was found successful in displaying the positive and negative correlation function as well as choosing either autocorrelation or cross-correlation modes.

## 3.6.2 Correlation Function Display

The display of the developing overload pattern was originally used as a simple diagnostic aid for use when results obtained from the flow meter suggest a malfunction of the flow noise transducers. A twin beam storage scope CRO was used to store the horizontal lines when the overload pattern is logic '1'. Progressively smaller Y deflection were used as the integration time increased from the time corresponding to  $r_{XY}(\tau) = 1.0$ . This method has been discussed and illustrated by Jordan and Kelly, 1976. A modification has been made to this method to enable the normalised continuous correlation function to be displayed.

The circuit diagram of a circuit for generating a voltage proportional to the correlation coefficient is shown in Figure 3.6.2.1. The 100, equal decrement, load pulses are counted by a binary presetable synchronous down counter. The counter is preset to '100' when all the outputs reach zero and the borrow of the last counter goes low to load the number again while the preset signal (from  $T_c$ ) clears the counter. The digital-to-analogue convertor is addressed by the outputs of the down counter. The DAC output voltage is used as a vertical shift signal by the correlation function display system.

For the complete display positive and negative reference voltages are required. P-channel MOS FET electronic switches are used to







FIGURE 3-6-2-2 VOLTAGE DISPLAY TIMING DIAGRAM



gate positive and negative reference voltages to the digital-toanalogue convertor. Figure 3.6.2.2 shows the timing diagram to generate the voltage display. When the Q output of the toggling flip-flop goes high, it will inhibit the positive voltage at the same time  $\overline{Q}$  enables the switch selecting the negative voltage. When the Q output goes low the analogue switch will enable the positive reference voltage and inhibit the negative reference voltage.

A star resitance connection (Figure 3.6.2.3) is used to add the overload pattern output from the digital correlator with the voltage output of the DAC, which is proportional to the decreasing correlation significance. Hence, the overload pattern appears biased by a voltage decreasing from a positive to zero value for positive correlation and for negative correlation it appears biased by a voltage increasing from a negative to zero volts. By suitably adjusting the input amplifier of a storage CRO it is possible to write only bright lines corresponding to overloaded integrating counters and therefore the envelope of these bright lines will define The CRO must be triggered by the load pulses. the function. This method has been found to produce a satisfactory display as will be seen from the results presented in the next chapter.





FIGURE A 1 1 DROTOTYDE CODDELATOR SYSTEM
# CHAPTER 4 : PERFORMANCE OF EXPERIMENTAL WORK

## 4.1 EXPERIMENTAL SYSTEM

A prototype system (Figure 4.1.1) using the circuits described in Chapter 3 was demonstrated in various situations where the correlation technique can be usefully applied. The prototype correlator system has proved the versatility of the MOSIC Overloading Integrator polarity correlator. In the next sections, the experimental results of applying the prototype overloading correlator to dynamic systems testing, air flow velocity measurement, electric wave power generation will be presented.

#### 4.2 PERFORMANCE OF THE AUTO-CORRELATION FUNCTION DISPLAY

Experimental results have been obtained for a number of test signals and compared with the Hewlett Packard correlator (3721A model). The modified display of the normalised correlation function was stored on a storage CRO model (DM53A). Results for different conditions will be described.

#### 4.2.1 Auto-Correlation Function of a Sine Wave

The prototype was set to the auto-correlation mode. The normalised auto-correlation of a sine waveform signal was obtained by applying a sine wave signal (7.2 ms period) to the first input of the zero-crossing detector. The sample clock period is set to 0.06 ms and the range of the time delay was between ( $0 \rightarrow 120 \times$ 0.06 = 7.2 ms). Hence, one cycle of the auto-correlation function must be displayed. Figure 4.2.1.1 shows the sine wave signal and the expected result of the auto-correlation. It is noticeable that



the negative peak is not quite one. This is due to the slight difference between positive and negative reference voltage to the DAC of the voltage display circuitry (3.7.2).

The curved shape of the auto-correlation function is almost similar to the predicted theoretical normalised auto-correlation function. Another example of a sine wave is shown in Figure 4.2.1.2 with a period of 7.2 ms (display of  $1^{1}/_{3}$  period).

#### 4.2.2 Auto-Correlation of Gaussian Noise

A Gaussian noise signal was derived from the Hewlett Packard noise generator and connected to the input of the prototype system. The bandwidth of the noise signal was chosen to be 500 Hz with rms voltage of 1.5 V. The sample clock period was set to 0.07 ms. The displayed auto-correlation of the Gaussian noise is shown in Figure 4.2.2.1. The results are compared with the Hewlett Packard correlator and found almost similar.

As the bandwidth of the Gaussian noise was fixed and the sample clock period chosen to 0.15 ms, the peak of the auto-correlation function does not start at  $\rho_{XY}(\tau) = 1.0$ , Figure 4.2.2.2. This is due to the brightness of the storage scope which is not enough to store the first pulses and also the fly-back time of the storage scope has some effect on the start of the display. Hence, as the autocorrelation becomes narrower, the peak magnitude becomes less due to the above reasons.

#### 4.2.3 Auto-Correlation of Sine Wave Plus Gaussian Noise

The correlation of a signal plus noise has been studied and

O I





discussed by Bendat (1958). The normalised auto-correlation function of this mixture has been given by Lange (1967) as

$$\rho_{s+n}(\tau) = \frac{\rho_n(\tau) + a^2 \cos \omega_0 \tau}{1 + a^2} \qquad \dots (4.2.3.1)$$

where  $\rho_n(\tau)$  is the normalised auto-correlation of the Gaussian noise and is given by  $\rho_n(\tau) = \frac{\sin 2\omega_0 \tau}{2\omega_0 \tau}$ ;  $\omega_0$  is the angular frequency of the signal ;  $a^2 = \frac{A_0^2}{2\sigma_n^2}$ ; is signal-to-noise power ratio;  $A_0$  is the signal amplitude ;  $\sigma_n^2$  is the variance of the noise voltage ; and  $\tau$  is the time delay.

A mixture of a sine wave signal plus Gaussian noise signal was applied to the first input of the prototype correlator. Figure 4.2.2.1 shows the result when  $a^2 = 0$  (ie, without sine wave signal). A sine wave signal was added to the noise with power ratio  $a^2 = 0.072$ . The results for both Hewlett-Packard correlator and the prototype system are shown in Figure 4.2.3.1. As the signal-to-noise power ratio increased ( $a^2 = 0.5$ , 0.9) the amplitude of the periodic function was increased (Figure 4.2.3.2). These results are quite similar to the predicted theoretical results shown in Figure 4.2.3.3. It is also to be noticed that the variance of the display is satisfactory, though there are some problems with overwriting of successive correlation functions on the storage scope.

# 4.2.4 Auto-Correlation of PRB Noise

The normalised auto-correlation of a pseudo-random-binarysequence (prbs) was obtained by the prototype correlator (Figure 4.2.4.1).



The signal was derived from the Hewlett Packard noise generator model 3722A with long sequence (N = 524287) and clock period of 333  $\mu$ s. The sample clock for the prototype was set to be 0.05 msec. From the comparison between the Hewlett Packard correlator and the prototype correlator it will be seen that the results are almost similar apart from the amplitude for the reasons discussed previously.

Since the Van Vleck relation is applicable only for sine wave or Gaussian noise signals, it is noticed that the auto-correlation function of prb noise signal is approximately Gaussian shape for slow clock rate (1 ms) as shown in Figure 4.2.4.2. This means that the Van Vleck relation is applicable to the prbs when the clock rate is fast or the sample clock is chosen to give a narrow width autocorrelation (ie, impulse type function).

Hence, the pseudo random binary signal can be used as a test signal in system measurement in conjunction with the prototype correlator system.

#### 4.3 SYSTEM RESPONSE MEASUREMENT

Dynamic systems of first-order, second-order RC passive filters and second-order active RC filter have been tested and the normalised impulse responses have been obtained as cross-correlation function displays. The automatic measurement of the distinctive features of these responses was found to be satisfactory.

#### 4.3.1 First-Order RC Filter

The prototype correlator was used to determine the impulse response of the simple first-order RC filter shown in Figure 4.3.1.1.



Two test signals were applied to the filter.

1. Gaussian Noise Test Signal : A Gaussian noise signal was derived from the HP3722A noise generator to stimulate the simple RC filter. The filter time constant was chosen to be RC = 1.0 ms. The test signal has to be chosen so that the auto-correlation exists only for a time delay which is much shorter than the significant time constant of the filter under test. The bandwidth was chosen to be 1.5 kHz of a long sequence of length N = 524287. The normalised crosscorrelation function was displayed on the storage CRO, and also a comparison made with the Hewlett Packard correlator. As shown in Figure 4.3.1.2, the similarity between the two results is quite clear. The calculated peak of the normalised cross-correlation function was derived from the Hewlett Packard correlator result by applying the equation below

$$\rho_{XY}(\tau) = \frac{-R_{XY}(\tau)}{(R_{XX}(0)R_{YY}(0))^{\frac{1}{2}}}$$

This gives approximately  $\rho_{XY}(\tau) = 0.73$  which is the same as the value obtained by the prototype correlator system.

2. <u>Pseudo-Random-Binary-Noise Test Signal</u> : The low pass RC filter was also stimulated by prbs derived from the HP3722A noise generator. The impulse response of the system was measured with prbn test signals of length N = 524287 bits and clock period of 100 µsec. From Figure 4.3.1.3, the comparison with the Hewlett Packard correlator shows a good similarity between the two correlators. The calculated peak value of the normalised cross-correlation function from the Hewlett Packard correlator gives approximately  $\rho_{xy}(\tau) = 0.73$  which is

almost the same value obtained by the prototype system.

The digitally generated noise offers advantages in system response measurements (Korn, 1966). On the other hand, prb test signals requires judgement and caution in the design of the experiment (Godfrey, 1969).

It is noticeable from the photographs that the cross-correlation functions are similar for both Gaussian noise and prb test signals. This is to say that the well-defined auto-correlation function of the excitation signals (approximately impulse function) will give similar results. Hence, the Van Vleck relation can be applied to the prbs test signal as discussed in section 4.2.4.

Lamb (1970) discussed the filtering of system response modes of low damping using prbs and suggested that a careful choice of sequence length and clock period must be carried out to obtain an accurate estimate of a system impulse response. McDonnell and Forrester (1970) suggested that the long sequences of prbs and Gaussian noise test signals give better evaluation of impulse response using polarity coincidence correlator.

Short sequence length of both Gaussian noise and prb test signals were applied to the low pass filter. The results are shown in Figure 4.3.1.4 and 4.3.1.5 which are quite different from the Figures 4.3.1.2 and 4.3.1.3. The presence of high frequency components at the output of the RC filter is due to the use of short sequences. Hence, by choosing reasonable sequence length the highly oscillatory behaviour should be negligible as in Figure 4.3.1.2 and 4.3.1.3.



#### 4.3.2 Second-Order RC Filter

The dynamic impulse response of second-order RC simple filter (Figure 4.3.2.1) has been obtained by the prototype cross-correlation function using Gaussian and prb test signals. The two test signals were derived from the HP3722A noise generator. Long sequences have been used. The normalised cross-correlation obtained by the prototype system and the Hewlett-Packard correlator were obtained as shown in Figure 4.3.2.2 and 4.3.2.3.

The results from the Hewlett Packard correlator 3722A show that the normalised correlation function is similar to the cross-correlation function obtained by the LSI polarity correlator with peak value of  $\rho_{\chi y}(\tau) = 0.24$ . The Van Vleck relation is still applicable for both of the test signals used.

#### 4.3.3 Second-Order Active Filter

The prototype correlator was used to determine the impulse response of the active filter shown in Figure 4.3.3.1. Gaussian noise and prbs test signals were used. The normalised crosscorrelation functions were displayed on the screen of the storage CRO for different damping ratios positive and negative area, and peak positions were recorded. Results obtained by the prototype system were found to be similar to those obtained from Hewlett Packard correlator.

1. <u>Gaussian Noise Test Signal</u> : Gaussian noise was used to excite the dynamic system. The bandwidth was chosen to be 1.5 Hz with a long sequence N = 524287. Figure 4.3.3.2 shows the impulse response of the system under test for different damping ratio ( $\rho$  = 0.0, 0.3, 0.5,



FIGURE 4.3.1.1 FIRST\_ORDER RC FILTER ...



FIGURE 4.3.2.1 SECOND\_ORDER RC FILTER .



FIGURE 4.3.3.1 SECOND-ORDER RC ACTIVE FILTER WITH A TRANSFER FUNCTION  $H(s) = \frac{\omega_{s}^{2}}{S^{2} + 2\beta\omega_{s}S + \omega_{s}^{2}}$ 







FIGURE 4.3.3.2b CORRELATION FUNCTIONS OF A 2nd-ORDER ACTIVE FILTER

1.0). It is noticeable from the photograph at  $\rho = 0.5$  that although the sequence is long, the impulse response is not quite clear due to the presence of non-linearities. Moss and NG (1971) suggested that, to remove the effect of non-linearities, a proper choice of the amplitude of the test signal is to be considered. Hence, combining with the Lamb (1970) and Godfrey (1969) suggestions of proper choice of test signal, the system impulse response (obtained by crosscorrelation technique) can be improved. Figure 4.3.3.3 shows the impulse response for  $\rho = 0.5$  with N = 131071 and V<sub>rms</sub> = 1.2 V. It is clear that the sequence length is still long but not too long.

Pseudo-Random-Binary Test Signal : Prb test noise has been shown 2. to be the best perturbation signal (Briggs and Godfrey, 1966) in system It was used as an excitation signal to impulse response measurement. the input of the second-order RC active filter. The clock period was chosen to be 333  $\mu$ s and sequence length of N = 524287. The results of display were obtained quite similar to the impulse response obtained by the Gaussian noise. Figure 4.3.3.4 shows the crosscorrelation functions obtained by the prototype and the Hewlett-Packard correlator, for different damping ratios ( $\rho = 0.0, 0.2$ , Again, it is clear, that due to the presence of noise 0.5, 1.0). on the system, the impulse response of  $\rho = 0.5$  is not clear enough. As discussed earlier (Gaussian noise) the effect of non-linearities can be reduced by increasing the perturbation signal amplitude. The expected result is shown in Figure 4.3.3.5 which shows the improved impulse response.

From the results obtained by using the two test signals, it is clear that in general they have almost similar correlation shapes.





For both signals, the percentage error of the most significant peak magnitude (from the display) of the first overshoot obtained by the prototype and compared with the HP 3722A correlator is of the range 3-4%. The percentage error is given by

$$e = \frac{\rho_{xy}(\tau)[OVL] - \rho_{xy}(\tau)[HP] \times 100}{\rho_{xy}(\tau)[HP]}$$

where  $\rho_{\chi y}(\tau)[\text{OVL}]$  is the normalised cross-correlation function obtained by the prototype LSI correlator, and  $\rho_{\chi y}(\tau)$  [HP] is the calculated normalised cross-correlation obtained from the Hewlett Packard 3721A correlator.

This is not quite in agreement with the expected percentage error of equal decrement of  $\rho_{XY}(\tau)$  (using the L/S generator) which is an error of maximum 1%. The main reason is the error due to the brightness of the storage scope as well as instrumentation error of the HP 3721A correlator. Nevertheless, the results show that the LSI polarity correlator can be used as a general purpose correlator for impulse response measurement. Figure 4.3.3.6 shows the difference between the Hewlett Packard correlator and the prototype correlator for different values of damping ratio.

From the Appendix 2, equation (A2.1.14) shows that the normalised impulse response function is proportional to the damping ratio

$$ρ_{XY}(\tau) α \sqrt{\rho} h(\tau)$$
... (4.3.3.1)

The value of  $\sqrt{\rho}$  increases rapidly for  $\rho = 0.1$  to  $\rho = 0.5$  and the same time  $h(\tau)$  decreases. For  $\rho = 0.6$  to  $\rho = 0.9$ , increases



slightly with a decrease in  $h(\tau)$ . For  $\rho \ge 1$ , decreases with a decrease in  $h(\tau)$ . Hence, as  $\rho$  increases,  $\rho_{XY}(\tau)$  will increase and then show a decrease. This prediction is confirmed by Figure 4.3.3.6. The graph does not show zero value for zero  $\rho$ , indicating that the estimation of  $\rho$  is in error. An attempt was made to calculate the damping ratio by measuring the amplitudes of the first and second positive peaks of the impulse response (display mode). From Appendix A2 and equation (A2.3.4) we obtained the logarithmic decrement criteria using the normalised impulse response as shown in the following equation

$$\ln \frac{h_{n}(\tau_{1})}{h_{n}(\tau_{2})} = \frac{2\pi\rho}{(1-\rho^{2})^{\frac{1}{2}}} \qquad \dots (4.3.3.2)$$

Figure 4.3.3.7 shows the measured damping ratio and the experimental chosen values ( $\rho_{exp}$ ) (for a Gaussian noise test signal). The results are obtained for a suitably chosen test signal as discussed previously. The percentage error of the damping ratio is approximately 1-9% as can be seen from the graph. This is due to the problem of brightness of the storage scope, also for the higher damping ratios (ie, 0.7-0.9) the second peak is not clear enough. However, these results suggest that the LSI polarity correlator can be used in control systems to adjust the damping ratio and other performance features.

The impulse response area ratio (ie, relative stability) measurement has been obtained by measuring the positive and negative areas of the normalised cross-correlation function. From equation (A2.2.10), appendix (A2), it has been shown that the area ratios for the normalised impulse response is the same as for the continuous impulse response ratio, ie,



Area ratio = 
$$\frac{A_{n+}}{A_{n-}} = \frac{A_{+}}{A_{-}}$$

where  $A_{n+,-}$  is the normalised impulse response area, and  $A_{+,-}$  is the continuous impulse response area.

The area ratio determined by the prototype correlator is compared with theoretical values for different damping ratios (Figure 4.3.3.8). Percentage error in the range 2-5% is obtained. For higher damping ratios (0.8,0.9), the error is high due to the small, poorly defined, magnitude of the negative function.

These results show that the LSI polarity correlator can be used in automatic system testing and adaptive control systems. Better accuracy can be realised by generating load/shift pulses with less error as well as by taking care with the design, or selection of test signals.

### 4.4 AIR FLOW MEASUREMENT

Air flow velocity measurement by cross-correlation technique has been shown to give useful results (Annetts, 1976), Statutory requirements to provide adequate ventilation in coal working tunnels must be met by making regular air velocity measurement. The majority of velocity measurement in the underground tunnels are made using mechanical anemometers and stop watches. The application of correlation techniques is therefore worth consideration.

Air flow velocity measurement by cross-correlation is being investigated by the NCB Scottish Scientific Control Department using natural thermal fluctuations measured by two suitably spaced (10 cm)

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(4.3.3.3)







FIGURE 4.4.2 THERMISTOR TRANSDUCER CIRCUIT .

thermistors (Annetts, 1976). Figure 4.4.1 shows the small wind tunnel used with facilities for mounting the thermistors in the air stream along with an anemometer for calibration purposes. A variable speed fan provides a range of air velocities.

ITT P23 MP (matched pair) fast thermistors were used with nominal resistance at  $20^{\circ}C = 2 \ k\Omega$ . Figure 4.4.2 shows the thermistor transducer circuit for thermal fluctuation detection and amplification of the signal. The electrical noise signal from the transducer gives the thermal fluctuation pattern in wind tunnel (Figure 4.4.3). The prototype correlator system, Hewlett Packard correlator and the NRDC flow meter have been used to cross-correlate the outputs from the thermistor amplifiers.

The prototype was set in the auto-correlation mode. The bandwidth of the thermal noise is low, hence the input circuits (of the correlator) effectively differentiate the signals. Then the normalised auto-correlation function is therefore second order differentiated. Figure 4.4.4 shows the auto-correlation of the output from the upstream thermistor.

The output of the two thermistor amplifiers are fed to the inputs of the prototype correlator. The cross-correlation function is obtained and displayed on the storage CRO. Figure 4.4.5 shows the cross-correlation function generated by the air flow obtained by the Hewlett Packard correlator and the prototype correlator system. The comparison shows that the MOSIC Overloading Polarity Correlator operates satisfactorily and the results obtained are in good agreement with the Hewlett Packard correlator.



Vertical Scale = 2 V/cm Horizontal Scale = 0.5 s/cm Anemometer air velocity = 1.9 m/s





Prototype Correlator Anemometer air velocity = 0.8 m/s Sample clock period = 2 ms  $A_{n+} = 2 V$ ,  $A_{n-} = 1.1 V$ , K = 16

FIGURE 4.4.4 AUTO-CORRELATION FUNCTION OF UP-STREAM SIGNAL





The average magnitudes of the significant peaks of the normalised cross-correlation function were obtained from the screen of the storage CRO (average of ten readings for each fan speed) and compared with the average magnitudes of the significant peaks obtained by the Hewlett Packard correlator (ten readings for each auto-correlation, of up and down-stream transducers and the cross-correlation function, for each speed of air flow) for different velocities. A graph is plotted relating the Hewlett Packard results and the prototype correlator results as shown in Figure 4.4.6.

The percentage error of the normalised cross-correlation function obtained by the prototype system is a maximum of 7%. The error due to the brightness of the storage CRO has to be taken into consideration. The brightness has a great effect on the actual reading of the display cross-correlation function. Nevertheless, in comparison with the Hewlett Packard correlator (which has also some error) the results are satisfactory.

The voltage proportional to the time delay of the peak function was measured by the prototype system for different velocities of the air flow in the wind tunnel. Figure 4.4.7 shows the mean velocity measured by the anemometer and the measured velocity obtained by the Hewlett Packard correlator (from the screen) and the voltage proportional to the time delay of the peak function.

The low bandwidth of the thermal noise leads a broad correlation peak. This, when combined with the normal statistical errors, leads to the deviations indicated by Figure 4.4.7. However, results show (in general) that the prototype correlator is in good agreement with the Hewlett Packard correlator and that the circuit generating the





voltage proportional to the time delay operates satisfactorily.

The effect of the integration cycle on the normalised crosscorrelation function has been considered. It has been discussed in Appendix A1 that the proper choice of the integration cycle will give a good accuracy of the correlation measurement. This has been realised by the prototype correlator. Figure 4.4.8 shows the effect of short and long integration period in cross-correlation measurement. For short integration periods a multiple peaked function is obtained, but for a long integration cycle the normalised cross-correlation function has a smooth single peak. Since the cross-correlation measurement is to be used in Ventilation Control, it is necessary to consider the integration period in order to avoid the multiple peaked function.

In flow noise measurement the presence of spurious peaks will cause some problems. Overloads corresponding to small amplitude spurious peaks can be prevented from appearing by arranging for the correlator to reset before the  $r_{m}=0$  overload appears. In the prototype correlator system peaks with magnitude less than  $\rho_{m}=0.23$ ( $r_{m}=0.14$ ) can be eliminated. Figure 4.4.9 shows a function with spurious peaks eliminated.

## 4.5 ELECTRIC WAVE POWER GENERATION

In the University of Edinburgh, a new technique has been developed to generate electric power from the waves at sea. The amount of power in a wave train can be estimated by calculating the change of potential energy as the water in a wave above sea level falls into the trough in front of the wave, Figure 4.5.1, (Salter, 1974).



 $\lambda$  , Wavelength ; H\_{tc} , Trough to Crest Hight ; Cg  $\,$  , Centere of Gravity . PART OF PROGRASIVE TRAIN OF SINUSOIDAL FIGURE 4.5.1 GRAVITY WAVES IN DEEP WATER

-Cg

A Nodding Duck system has been designed for wave power generation. Its architectural model has been described in detail by Salter (1976). The duck string intercepts the sea wave front and motion of the duck is converted into an electrical power signal by generator mounted in the body of the duck. A tank has been established to produce very stable test conditions for scale model of the duck. Two wave level gauges are set in the tank to test the waveform of the generated waves. Figure 4.5.2 shows the output waveform detected by the gauge. The outputs of twenty oscillators (range 0.5-2.5 Hz) are mixed to generate the water waveform.

The outputs of the two gauges were fed to the inputs of the prototype correlator and the Hewlett Packard correlator. The normalised autocorrelation function was obtained by the prototype system and the continuous function obtained by the Hewlett Packard correlator (Figure 4.5.3). From this Figure, it will be seen that the autocorrelation indicates that the noise consists of Gaussian noise plus signal (may be sine wave) which is expected (Salter, private discussion). The prototype auto-correlation is acceptably similar to the HP 3721A correlator result. An attempt was also made to cross-correlate the outputs of the two gauges. The cross-correlation is shown in Figure 4.5.4. Again a good similarity has been obtained between the two correlators.

It is noticeable that the cross-correlation function does not show a smooth and clean single peak. From discussions with Salter and his team, it has been concluded that this is due to the dispersive nature of the water waveform in the tank. The higher frequencies will move slower than the low frequencies, hence, phase shift between the two frequencies will take place (see Figure 4.5.5). The cross-correlation function will


be obtained with different peaks either positive or negative magnitudes. This is the result obtained as can be seen from Figure 4.5.4. If the waveform is non-dispersive, then a clean single smooth peak would be expected.

# 4.6 MEASUREMENT OF DISTINCTIVE FEATURES

#### 4.6.1 Voltage Proportional to Time Delay

The circuit described in Section 3.4 for time delay measurement of the correlation function was checked by obtaining the auto-correlation of a sine wave signal. The auto-correlation function for a sine wave is a cosine waveform. The peak position of the negative display gives the half period of the auto-correlation function. The prototype correlator was set in the auto-correlation mode. The sample clock period was adjusted to 0.07 ms. The voltage proportional to the time delays were recorded and compared with the sine-wave periods (half period). Figure 4.6.1.1 shows the graph plotted for the time delay measurement. The results obtained for voltage proportional to the time delay shows the expected linear relation.

# 4.6.2 Voltage Proportional to Area

The circuitry described in Section 3.6 has been checked by monitoring sine waveform to obtain the area of the quarter period of the auto-correlation function. Simple analysis for the area measurement of the normalised auto-correlation is obtained. From the Figure 4.6.2.1, the polarity auto-correlation function is represented by the triangular shape with area  $(A_1)$ . The normalised auto-correlation function is represented by the cosine waveform with area  $(A_2)$ .





FIGURE 4.6.2.1 AUTO-CORRELATION FUNCTION OF SINE WAVE  $(\frac{1}{4} \text{ PERIOD})$ The area of the triangle is given by

$$A_1 = \frac{1}{2} \cdot \frac{1}{4f_0} \cdot 1 = \frac{1}{8f_0} \dots (4.6.2.1)$$

The area of the cosine wave is given by

$$A_{2} = \int_{0}^{1} \cos 2\pi f_{0} \tau d\tau = \frac{1}{2\pi f_{0}} \sin 2\pi f_{0} \tau \int_{\tau=0}^{\tau=\frac{1}{4f_{0}}} \sin 2\pi f_{0} \tau$$

$$A_2 = \frac{1}{2\pi f_0} \qquad \dots (4.6.2.2)$$

where  $\boldsymbol{f}_{0}$  is the signal (period) frequency.

The error correction  $\lambda$  between  $A_1$  and  $A_2$  is given by

$$\lambda_{\perp} = \frac{A_2 - A_1}{A_2}$$

Hence,  $A_2 = \frac{A_1}{1 - \lambda}$ 

$$\lambda = \frac{\frac{1}{2\pi f_0} - \frac{1}{8f_0}}{\frac{1}{2\pi f_0}} = \frac{4 - \pi}{4}$$

 $\lambda = 0.2146,$ 

since the auto-correlation function is built up, effectively, by 100 horizontal lines. Then the voltage proportional to the area of the triangle, when the base of the triangle is 2, is given by

$$A_{1} = \frac{100 \times \ell}{2 \times K_{a}} \cdot \frac{V_{ref}}{2^{n} - 1} \qquad \dots (4.6.2.3)$$

where  $K_a = A$  scaling factor,

l = maximum time delay number,

n = stages of the area store (ie, 8 bits),

and  $V_{ref}$  = reference voltage to the DA converter (5.1 V).

Hence the voltage proportional to the area of the normalised autocorrelation function (for a quarter period) is given by

$$A_2 = \frac{\ell}{0.7854 \text{ K}_a} \dots (4.6.2.4)$$

The equation (4.6.2.4) was checked by choosing a number of scaling factors,  $K_{z}$  Then by choosing a sine wave frequency giving a maximum time delay range display for a quarter period of the auto-correlation function a voltage proportional to the area was determined. The



frequency was changed to keep the quarter period display in a smaller time delay range. This was achieved by pulling out one package from the end of the series connection of digital correlator integrated circuits. A set of area measurements was obtained for & = 120, 108, 96 ... 12. The results obtained are plotted in Figure 4.6.2.1 for different scaling factors  $K_a$ . It shows that the best fit (between the calculated area  $A_2$  and the measured area) was obtained when  $K_a$  = 40. The circuit producing voltage proportional to the area of the normalised correlation function has been found to operate successfully.

#### 4.6.3 Multiple Peak Measurement

Although the prototype circuit appeared to function correctly no time was available to carry a detailed assessment on its performance.

# CHAPTER 5 : CONCLUSION AND FURTHER WORK

# 5.1 CONCLUSION

The preceding chapters have summarised the results of investigations into the feasibility of applying the LSI Overloading Integrator Correlator to the automatic measurement of the distinctive features of the normalised correlation function. Some aspects of the correlation technique and its applications have been considered in Chapter 1. The principle of the Overloading Integrator Correlator and the description of the LSI circuit has been described in Chapter 2. The design of a system capable of automatically measuring the distinctive features of the normalised correlation function has been discussed in Chapter 3. The performance of a prototype system has been discussed and described in Chapter 4.

The results obtained by the prototype correlator system show that the LSI polarity correlator is a versatile device in correlation analysis. Reliable measurement of system impulse response, the time delay of the peak position, the area and the multiple magnitudes and positions of the correlation function has been demonstrated. All these measurements can be achieved with simultaneous display of the function.

Field trials of the prototype correlator have been successfully completed at the University of Bradford, Mechanical Engineering Department (University of Edinburgh) and in the Laboratories of the National Coal Board, Edinburgh. At the University of Bradford open channel water flow was measured by the correlation method using optical flow noise transducers. Air flow by the correlation method was

demonstrated in a NCB wind tunnel. The prototype correlator was also applied to signals derived from Salter's wave power generation system. Function display and feature detection was successfully demonstrated.

The application of the prototype correlator to dynamic system testing was successfully demonstrated. The amplitude of the normalised cross-correlation function was found to be very similar to the Hewlett Packard correlator (3721A) results. The area ratio, of the second order active filter, has been found to be very similar to theoretical predictions. The calculated damping ratio from the displayed correlation function has been shown to be similar to the Gaussian noise and Pseudo Random Binary experimental values. test signals were used for impulse response determination where it was noted that a proper choice of signal amplitude, bandwidth (clock -rate) sequence length and sampling periods have to be made for. accurate measurements of the impulse response. It is concluded that the Van Vleck relation can be applied to the prb noise test signal when the auto-correlation is approximated to delta-function.

Hence, it can be concluded that the LSIC Overloading Integrator Polarity Correlator can be used as a general purpose correlator since it has been applied to a variety of applications without the need to calibrate. The results show that the Integrated Circuit Correlator is easy to use as well as reliable and it can compete with other correlators (eg, the Hewlett Packard correlator) by its low cost. Though the general purpose prototype correlator was constructed with additional complex electronic TTL circuitry this requirement can be eliminated by using the logic array technique which will be discussed in the next Section.

#### 5.2 FURTHER WORK

The prototype correlator was realised by the LSIC polarity correlator in conjunction with extra TTL circuitry. These TTL circuits were designed to perform the control functions of the LSIC Timing counter, clock drivers to the LSI correlator and the chip. load pulse generator have been put on several printed circuit boards. One way to minimise the use of additional TTL circuits will be to fabricate a special purpose integrated circuit to provide all the necessary control signals required by the LSIC polarity correlator. In the Department of Electrical Engineering, University of Edinburgh, the design of the Uncommitted Logic Array (ULA) (Figure 5.2.1) chip has recently been completed (Thomson 1976). The ULA is a large scale integrated circuit making the use of P-channel MOS technology. It contains NAND gates , NOR gates and shift registers. It is -estimated that two-ULA's will be required to meet all the requirements of the prototype correlator system. The use of the ULA in conjunction with the LSIC Overloading Integrator Correlator will allow instrument manufacturers to obtain a simple, reliable and low cost general purpose correlator.

#### 5.3 APPLICATIONS

The work described in this Thesis has shown that the LSIC correlator can be used to construct a general purpose digital correlator. By using ULA techniques a low cost system can be manufactured. Two more applications of correlation analysis will now be described which will benefit from the availability of a low cost correlator and feature detector (Jordan and Manook, 1976).



FIGURE 5.1.1 LAYOUT OF THE UNCOMMITTED LOGIC ARRAY

# 5.3.1 Fire Detection

Noltingk and Robinson (1970) have shown that the normalised cross-correlation function analysis (using optical signals) can give a clear indication of whether or not a flame is alight in large boilers. If the normalised cross-correlation coefficient is below a certain threshold, the flame can be taken as absent, while higher correlation coefficients indicate that the flame is present. They have concluded that a three-level output corresponding to normal flame (strong correlation), abnormal flame (weak correlation) and no flame (no correlation) can be obtained by this method.

Mr M Annetts (private communication) has shown that the area of a cross-correlation function obtained from ventilation shaft flows changes due to changes in the nature of the thermal fluctuations of the air flow. Hence if the area is set up to a certain threshold then a fire can be detected before it spreads by monitoring changes of the area signal. It has been shown in this study that the LSIC polarity correlator can be easily arranged to measure the area of the normalised cross-correlation function. Since a large number of detection points will be required in coal mining it is clear that a low cost correlator will be required.

## 5.3.2 Blood-Velocity Measurement

Blood-velocity measurements have been Ochieved by placing two ultrasonic probes on the femoral and popliteal sites (Morris et al, 1975). The impulse response for a segment of diseased superficial femoral artery was obtained. By calculating the transfer function

relating the two signals a model can be produced of the diseased arterial segment which can be used to interpret changes produced in the circulation by conservative therapy, and to establish the early Woodcock et al (1975) have shown that on-set of arterial disease. the impulse response (calculated from blood-velocity/time waveforms recorded transcultaneously using ultrasonic Doppler-shift flowvelocity meters) and the cross-correlation function of the input and output waveforms (of a segment of diseased superficial femoral artery) provide information about the state of the collateral circulation which may be clinically useful. The significance of the absolute values, initial peaks and the shape of the impulse response and the cross-correlation function are important in clinical analysis. The LSIC correlator has been shown to determine the normalised impulse response accurately as well as the peak position and also the multiple peak positions and magnitudes of the cross-correlation function. Hence, the low cost integrated circuit correlator is particularly suitable for this application.

# APPENDIX 1 : CORRELATION THEORY

# A1.1 AUTO-CORRELATION FUNCTION

For a stationary random process x(t) the auto-correlation function is obtained by taking the product of two values of the signal x(t) separated by an interval  $\tau$  and averaging the product over the integration period T. Thus,  $R_{\chi\chi}(\tau)$  is a function of  $\tau$  only and can be described as follows :

$$R_{XX}(\tau) = \lim_{T \to \infty} \frac{1}{T} \int_{0}^{T} x(t)x(t+\tau)dt \qquad \dots \text{ Al.l.1}$$

where  $\tau$  is the time delay between two values of the signal x(t).

The auto-correlation function  $R_{\chi\chi}^{}(\tau)$  has a number of interesting properties :

1. The mean-squared value  $(\sigma_{X}^{2})$  of the random process is given by  $R_{_{XX}}(0) \ :$ 

$$R_{XX}(0) = \lim_{T \to \infty} \int_{0}^{T} [x(t)]^{2} dt = \sigma_{X}^{2} \qquad \dots \text{ A1.1.2}$$

2. The auto-correlation function is even

$$R_{xx}(\tau) = R_{xx}(-\tau)$$
 ... A1.1.3

3. The auto-correlation function is a maximum at the origin :

$$R_{XX}(0) \ge |R_{XX}(\tau)|$$
 ... A1.1.4

# A1.2 CROSS-CORRELATION FUNCTION

For two sets of random stationary processes [x(t) and y(t)] the cross-correlation function  $R_{\chi\gamma}(\tau)$  is obtained by averaging the product of the two random signals over the integration period T. Thus,  $R_{\chi\gamma}(\tau)$  is a function of  $\tau$  only and can be described as follows :

$$R_{XY}(\tau) = \lim_{T \to \infty} \frac{1}{T} \int_{0}^{T} x(t) y(t+\tau) dt \qquad \dots \text{ Al.2.1}$$

where  $\tau$  is the time delay between the two signals.

 $R_{\chi y}(\tau)$  is a real-valued function which may be either positive or negative and does not necessarily have a maximum at  $\tau = 0$  (except when x(t) = y(t)). However, it does obey a certain symmetry relationship as follows (when x and y are interchanged) :

1. 
$$R_{xy}(\tau) = R_{yx}(-\tau)$$
 ... A1.2.2

2. 
$$|R_{\chi\gamma}(\tau)| \leq \frac{1}{2}[R_{\chi\chi}(0) + R_{\gamma\gamma}(0)]$$
 ... A1.2.3

3. 
$$[R_{xy}(\tau)]^2 \le R_{xx}(0) R_{yy}(0)$$
 ... Al.2.4

Cross-correlation function analysis can be applied in time delay measurements, detection and recovery of signals and the determination of impulse response functions.

#### A1.3 NORMALISED CORRELATION FUNCTION

In the classical statistics, the normalised correlation function is given by :

$$\rho_{XY}(\tau) = \frac{C_{XY}(\tau)}{\left[C_{XX}(0) \ C_{YY}(0)\right]^{\frac{1}{2}}}$$

where  $\boldsymbol{C}_{\boldsymbol{\chi}\boldsymbol{y}}(\tau)$  is the cross-covariance function and given by

$$C_{xy}(\tau) = R_{xy}(\tau) + \mu_x \mu_y$$
 ... A1.3.2

... A1.3.1

and the auto-covariance functions are given by :

$$C_{xx}(\tau) = R_{xx}(\tau) + \mu_x$$
  
... A1.3.3  
 $C_{yy}(\tau) = R_{yy}(\tau) + \mu_y$ 

where  $\mu_{\boldsymbol{X}}$  and  $\mu_{\boldsymbol{Y}}$  are mean values.

If  $\mu_x = \mu_y = 0$ , then equation (Al.3.1) becomes

$$\rho_{xy}(\tau) = \frac{R_{xy}(\tau)}{\left[R_{xx}(0) R_{yy}(0)\right]^{\frac{1}{2}}} \dots A1.3.4$$

and satisfies for all  $\tau - 1 \leq \rho_{\chi \gamma}(\tau) \leq 1$ .

The normalised correlation function is a measure of the coherence between the two stationary random signals. For example in flow measurement, when the distance (L) between the transducers (which are the source of the noise signals) is zero, then  $\rho = \pm 1$ , and when  $L = \infty$ ,  $\rho = 0$ . Hence, a proper choice of the spacing between the transducers must be achieved, Well-defined and large correlation peak function with small (L) enables an accurate measurement of the

peak position, but for small time delays the percentage error may be larger than it would be if a larger (L) were used. The peak function becomes broad and less distinct if L is very large and hence considerable errors in estimating the peak position will occur. (Mesch, Fritsche and Kipphan, 1974)

#### A1.4 VARIANCE OF THE CORRELATION FUNCTION

Due to the choice of the integration time T some error will be introduced into the estimates of correlation function. Also errors will be introduced if the functions of time have non-stationary statistical characteristics. Bendat and Piersol (1971) show that for two stationary random processes, the variance of the estimate is given by :

$$Var[R_{xy}(\tau)] = \frac{1}{T^2} \int_{0}^{T} \int_{0}^{T} [E(x(u)y(u+\tau)x(v)y(v+\tau)) - R_{xy}^2(\tau)] dv du$$
Al. 4.1

where  $R_{\chi y}(\tau)$  is the cross-correlation obtained by a finite choice of T.

For fourth order Gaussian stationary random processes with zero mean values, the equation (Al.4.1) becomes :

$$Var[R_{xy}(\tau)] = \frac{1}{T^2} \int_{0}^{T} \int_{0}^{T} [R_{xx}(v-u)R_{yy}(v-u) + R_{xy}(v-u+\tau)R_{xy}(v-u-\tau)] dv du$$
... Al.4.2

By rotating the axis of the above integration between  $\zeta$  and  $\nu$  the equation (A1.4.2) becomes :

$$\operatorname{Var}[R_{Xy}(\tau)] = \frac{1}{T} \int_{-T}^{T} [1 - \frac{|\zeta|}{T}] R_{XX}(\zeta) R_{yy}(\zeta) + R_{Xy}(\zeta + \tau) R_{yX}(\zeta - \tau)] d\zeta$$
... A1.4.3

where  $\zeta = v - u$ .

For large values of T (ie,  $T \ge 10\tau),$  equation (Al.4.3) becomes :

$$\operatorname{Var}[R_{Xy}(\tau)] = \frac{1}{T} \int_{-\infty}^{\infty} [R_{Xx}(\zeta)R_{yy}(\zeta) + R_{Xy}(\zeta-\tau)R_{Xy}(\zeta-\tau)]d\zeta \dots A1.4.4$$

Equation (A1.3.4) gives the uncertainty in  $R_{\chi y}(\tau)$ , when the integration time is finite.

For bandlimited white noise with zero mean and bandwidth B, the variance is given by :

$$Var[R_{xy}(\tau)] \approx \frac{1}{2BT} [R_{xx}(0)R_{yy}(0) + R_{xy}^{2}(\tau)] \dots A1.4.5$$

It is noticed that the variance can be reduced by increasing BT.

# A1.5 NORMALISED MEAN-SQUARE ERROR

The normalised mean-square error is given (Bendat and Piersol, 1971) by :

$$e^2 = \frac{Var[R_{xy}(\tau)]}{R_{xy}^2(\tau)}$$
 ... A1.5.1

Using equation (A1.4.5) of Appendix (A1.4) into equation (A1.5.1) we find that :

$$e^2 \approx \frac{1}{2BT} \left[1 + \frac{R_{xx}(0)R_{yy}(0)}{R_{xy}^2(\tau)}\right] \dots A1.5.2$$

Using equation (A1.3.4) into (A1.5.2) then :

$$e^2 \approx \frac{1}{2BT} \left[1 + \frac{1}{\rho_{xy}^2(\tau)}\right]$$
 ... A1.5.3

From the above equation it is noticed that the bandwidth of the noise signal and the integration period must be kept reasonably large to reduce the variance and hence the normalised mean-square error.

Bendat and Piersol (1968) suggest that for practical applications,  $T \ge 10\tau$  and  $BT \ge 5$  are reasonable.

Since the output of the system under test consists of a noisefree output plus spurious noise as shown in Figure Al.5.1, then (assuming that the spurious noise n(t) is not correlated with the test signal) the output auto-correlation function is given by :

$$R_{y}(0) = R_{y}'(0) + R_{n}(0)$$
 ... A1.5.4

where  $R_y'(0)$  is the auto-correlation function of the noise-free output at zero time delay,

and  $R_n(0)$  is the spurious noise auto-correlation at  $\tau = 0$ .

Substituting equation (A1.5.4) into equation (A1.5.2) gives :

$$e^{2} \approx \frac{1}{2BT} \left[1 + \frac{R_{xx}(0)R_{yy}'(0)}{R_{xy}^{2}'(\tau)} + \frac{R_{xx}(0)R_{nn}(0)}{R_{xy}^{2}'(\tau)}\right] \dots A1.5.5$$

Considering the perturbation signal to be a prbs noise signal with two level amplitude  $(\pm a)$ , then

$$R_{XY}(\tau) = a_0^2 \overline{R}_{XY}(\tau)$$

$$R_{XX}(0) = a_0^2 \overline{R}_{XX}(0)$$

$$R_{YY}(0) = a_0^2 \overline{R}_{YY}(0)$$

where the barred terms refer to the correlations for a = unity, and equation (A1.5.5) becomes :

$$e^{2} \approx \frac{1}{2BT} \left[1 + \frac{\overline{R}_{xx}\overline{R}_{yy}^{(1)}(0)}{\overline{R}_{xy}^{2}(\tau)} + \frac{\overline{R}_{xx}(0)R_{nn}(0)}{a_{0}^{2} \overline{R}_{xy}^{2}(\tau)}\right] \qquad \dots A1.5.6$$
  
Ist term 2nd term

From equation (A1.5.6), the first term represents the error due to the statistical inaccuracy and the second term is the error due to the presence of the spurious noise. Hence, beside the bandwidth of the test signal and the integration period, the amplitude of the \_prbs\_will\_reduce\_the\_effect of the\_spurious\_noise\_(Beck, 1974)

# A1.6 CORRELATION FUNCTION OF SIGNAL PLUS NOISE

The detection of a periodic signal in noise can be obtained from the normalised auto-correlation function of the signal plus noise (s + n) which is given by :

$$\rho_{s+n}(\tau) = \frac{R_{s+n}(\tau)}{R_{s+n}(0)} \qquad \dots \text{ A1.6.1}$$

Consider two signals  $x_1(t)$  and  $x_2(t)$  at the respective times t and t +  $\tau$ , where

$$x_1(t) = y_1(t) + A_s \sin \omega_0 t$$

$$x_2 = y_2(t + \tau) + A_0 \sin \omega_0(t + \tau)$$

where  $A_h$  is a constant,

and  $\omega_0$  is the angular frequency of the periodic signal.

 $y_1(t)$  and  $y_2(t + \tau)$  are noise fluctuations obeying the bivariate normal distribution with zero mean, equal variance  $\sigma_n^2$ , and with a normalised correlation  $\rho_n(\tau)$ .

For the auto-correlation function of sine wave signal, we have :

$$R_{s}(\tau) = \frac{A_{0}^{2}}{2} \cos \omega_{0} \tau$$
 ... A1.6.3

For the noise signal, the auto-correlation function is given by :

$$R_n(\tau) = \sigma_n^2 \rho_n(\tau)$$
 ...A1.6.4

where  $\rho_n(\tau)$  is the normalised auto-correlation function of the noise signal and is given by :

$$\rho_n(\tau) = \frac{\sin 2\omega_0 \tau}{2\omega_0 \tau}$$
 for Gaussian noise.

The auto-correlation function of the signal plus noise is given by Goldman (1953):

$$R_{s+n}(\tau) = R_{s}(\tau) + R_{n}(\tau)$$
 ...A1.6.5

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... A1.6.2

Using equations (A1.6.3), and (A1.6.4) into equation (A1.6.5), then

$$R_{s+n}(\tau) = \frac{A_0^2}{2} \cos \omega_0 \tau + \sigma_n^2 \rho_n(\tau)$$
 ... A1.6.6

for  $\tau = 0$ , equation (A1.5.6) becomes

$$R_{s+n}(0) = \frac{A_0^2}{2} + \sigma_n^2$$
 ... A1.6.7

Using equations (A1.6.6) and (A1.6.7) into equation (A1.6.1), then

$$\rho_{s+n}(\tau) = \frac{\frac{A_0^2}{2} \cos \omega_0 \tau + \sigma_n^2 \rho_n(\tau)}{\frac{A_0^2}{2} + \sigma_n^2}$$

The normalised auto-correlation function of the sine wave signal plus noise is :---

$$\rho_{s+n}(\tau) = \frac{a^2 \cos \omega_0 \tau + \rho_n(\tau)}{1 + a^2} \qquad \dots \ A1.6.8$$

where  $a^2 = \frac{A_0^2}{2\sigma_n^2}$  is the signal-to-noise power ratio.

McFadden (1956) has shown that the auto-correlation of a sine wave signal plus noise changes little when the mixture is extremely clipped.



where y(t) = y'(t) + n(t)





FIGURE A.2.1.1 IMPULSE RESPONSE FOR  $H(s) = \frac{\omega_{h}^{2}}{s^{2} + 2\beta \omega_{h} s + \omega_{h}^{2}}$ 

....

# APPENDIX 2 : SYSTEM FUNCTIONS MEASURED BY CORRELATION ANALYSIS

# A2.1 SYSTEM IMPULSE RESPONSE

Consider the linear dynamic system with an impulse response h(t) and an input test signal x(t) shown in Figure A2.1.1. The response of the system to x(t) is given by the convolution integral :

$$y(t) = \int_{0}^{\infty} h(t') x(t - t') dt'$$
 ... A2.1.1

The weighting function h(t') is the response to a unit impulse applied t' time units earlier.

Substituting equation (A2.1.1) into equation (A1.1.1) of Appendix A1.1, the following equation is obtained :

$$R_{xy}(\tau) = \lim_{T \to \infty} \frac{1}{T} \int_{0}^{T} \{x(t - \tau) \int_{0}^{\infty} h(t) x(t - t')dt'\} dt$$

Interchanging the integrations

$$R_{xy}(\tau) = \int_{0}^{\infty} h(t) \{ \lim_{T \to \infty} \frac{1}{T} \int_{0}^{T} x(t - \tau) x(t - t') dt \} dt' \dots A2.1.2$$

The auto-correlation function is defined by :

$$R_{xx}(t' - \tau) = \lim_{T \to \infty} \frac{1}{T} \int_{0}^{T} x(t - \tau) x(t - t') dt \qquad \dots \quad A2.1.3$$

Hence, equation A2.1.2 becomes :

$$R_{xy}(\tau) = \int_{0}^{\infty} h(t) R_{xx}(\tau - t) dt$$
 ... A2.1.4

The input noise is chosen so that the input auto-correlation function exists only for a delay time that is much shorter than the significant time constants of the linear system under test. The prbs noise signal is approximate band limited white noise and its auto-correlation function is triangular shape and can be viewed as an approximation to the required impulse function. Since the clock rate (and hence bandwidth) of the noise is much greater than the system pass band, then h(t) is almost constant over a small range of values around t =  $\tau$  and equation (A2.1.4) becomes :

The specific relations between the power density spectrum  $G_{XX}(\omega)$ and the auto-correlation function (Weiner-Khinchin), (Korn, 1966),

$$G_{XX}(\omega) = 2 \int_{0}^{\infty} R_{XX}(\tau) \cos \omega \tau d\tau \qquad \dots \quad A2.1.6$$

for  $\omega = 0$ 

$$G_{XX}(0) = 2 \int_{0}^{\infty} R_{XX}(\tau - t) dt$$

then becomes

$$R_{xy}(\tau) = \frac{1}{2}h(\tau) G_{xx}(0) = \text{constant x impulse response}$$
$$R_{xy}(\tau) = Kh(\tau) \qquad \dots \text{ A2.1.7}$$

An example of determining the impulse response of second-order active filters is given. Since the normalised correlation function is of interest in this work, it is necessary to determine the normalised impulse response.

The second-order active filter which has a basic differential equation of motion in which the highest derivative is the second. The roots will determine the dynamic characteristics of the system. Many problems in system analysis can be studied using this system and the solutions can be applied to higher order systems. The equation of second-order is given by :

$$\frac{d^2y}{dt^2} + 2\rho\omega_n \frac{dy}{dt} + \omega_n^2 y = \omega_n^2 x(t)$$

The Transfer function is given by :

$$H(s) = \frac{-\frac{2}{\omega_n}}{s^2 + 2\rho\omega_n s + \omega_n^2} \dots A2.1.7$$

where  $\rho$  is the damping ratio, and  $\omega_n$  is the natural undamped pulsatance.

The characteristic equation for equation (A2.1.7) is (when  $0 \le \rho < 1$ ) :

$$s^2 + 2\rho\omega_n s + \omega_n^2 = 0$$

$$[s_{1} + \rho\omega_{n} - j\omega_{n}(1-\rho^{2})^{2}][(s_{2} + \rho\omega_{n} + j\omega_{n}(1-\rho^{2})^{2}] = 0$$

Hence, the roots are :

$$s_{1} = -\rho\omega_{n} + j\omega_{n} (1-\rho^{2})^{\frac{1}{2}}$$
$$s_{2} = -\rho\omega_{n} - j\omega_{n} (1-\rho^{2})^{\frac{1}{2}}$$

Then the impulse response is  $(0 \le \rho < 1)$  underdamped :

$$h(t) = \frac{\omega_n}{(1 - \rho^2)^{\frac{1}{2}}} e^{-\rho\omega_n t} \sin \omega_n (1 - \rho^2)^{\frac{1}{2}} \dots A2.1.8$$

For overdamped ( $\rho$  > 1) the impulse response is given by :

h(t) = 
$$\frac{\omega_n}{(\rho^2 - 1)^2} e^{-\rho\omega_n t} \sinh \omega_n (\rho^2 - 1)^{\frac{1}{2}t}$$
 ... A2.1.9

For the condition when  $\rho = 1$  (critically damped) :

$$h(t) = \omega_n^2 t e^{-\omega_n t}$$
 ... A2.1.10

From the root locus (Figure A2.12) the path of the roots as the damping ratio is varied indicates the change in relative stability of the system and hence the character of the impulse response.

If we follow the form of roots for a variation of  $\rho$  from zero to infinity, we will recognise a locus of roots in the complex plane for  $\rho = 0$ ,

$$s_1, s_2 = \pm j\omega_n,$$

that is, the roots are purely imaginary, the impulse response is one of constant amplitude oscillations at a natural pulsatance  $\omega_n$  rad/sec. For  $\rho < 1$ , the roots are complex conjugates as :



FIGURE A.2.1.2 ROOT LOCUS OF 2nd ORDER SYSTEM AS  $\nabla$  VARIES FROM O TO  $\infty$  .



FIGURE A.2.3.1 IMPULSE RESPONSE OF 2nd - ORDER SYSTEM.

$$s_1, s_2 = -\rho\omega_n \pm j\omega_n (1 - \rho^2)^{\frac{1}{2}} = \sigma + j\omega$$

 $\sigma$  is the real axis of the s-plane, and j $\omega$  is the imaginary axis.

The results become oscillatory at a damped natural pulsatance

$$\omega_{n}(1 - \rho^{2})^{\frac{1}{2}}$$

For  $\rho = 1$ , the roots come together (repeated) and have real value

$$s_{1}, s_{2} = -\omega_{n}$$

For  $1 < \rho < \infty$ , there are two negative real roots, one close to the origin and the other is well displaced from the origin. The closer to the imaginary axis, the more it will effect the impulse response, but the other root is an exponential and dies out relatively quickly.

For the case when  $0 \le \rho < 1$ , the cross-correlation function is given by (when the input is white noise) :

$$R_{xy}(\tau) = \frac{a'\omega_n}{(1 - \rho^2)^{\frac{1}{2}}} e^{-\rho\omega_n t} \sin \omega_n (1 - \rho^2)^{\frac{1}{2}} t \dots A2.1.11$$

The mean square value R(O) of the output signal of the system under test is given by Bendat and Piersol (1971) :

$$R_{yy}(0) = \frac{a'\omega_n}{8\rho}$$
 ... A2.1.12

Using equations (A2.1.11), (A2.1.12) and (A1.3.4), the normalised cross-correlation function is given by :

$$\rho_{xy}(\tau) = 2\sqrt{2\rho\omega_n} e^{-\rho\omega_n \tau} \sin \omega_n (1 - \rho^2)^{\frac{1}{2}\tau} \dots A2.1.13$$

Assuming  $\boldsymbol{\omega}_n$  is constant, then

$$ρ_{xy}(\tau) α \sqrt{\rho} h(\tau)$$
 ... A2.1.14

# A2.2 AREA OF THE IMPULSE RESPONSE

It is known in control theory that the area (A) of the impulse response determines the system dc gain and is given by :

$$A = G_{dc} = \int_{0}^{\infty} h(\tau) d\tau \qquad \dots A2.2.1$$

where  $G_{dc}$  = system dc gain,

and  $h(\tau)$  is the system impulse response obtained by the crosscorrelation analysis. Assuming that  $R_{\chi\gamma}(\tau) = Kh(\tau)$ , and using

$$R_{XY}(\tau) = (R_{XX}(0)R_{YY}(0))^{\frac{1}{2}}_{XY}(\tau)$$

Equation (A2.2.1) becomes :

$$G_{dc} = \frac{1}{K} (R_{xx}(0)R_{yy}(0))^{\frac{1}{2}} \int_{0}^{\infty} \rho_{xy}(\tau) d\tau \qquad \dots A2.2.2$$

$$= \frac{1}{K} (R_{xx}(0)R_{yy}(0))^{\frac{1}{2}}A_n \qquad \dots \quad A2.2.3$$

where  ${\rm A}_{\rm n}$  is the area of the normalised cross-correlation function of

the system under test. It is noticeable that due to the normalisation, the gain,  $G_{dc}$ , is no longer dependant of the input and output signal levels. Hence, it is necessary to measure the RMS values of the input and output noise signals and multiply them with  $A_n$  to get the dc gain dependance.

Some cases, the measurement of  $A_n$  is wanted as in the adaptive control scheme described by Beck (1974). Also for relative stability measurement it can be shown that the area ratio of the continuous impulse response is equal to the area ratio of the normalised impulse response.

The area ratio is defined as (Mishkin, 1961) :

$$R = \frac{A_{+}}{A_{-}}$$

where K is the relative stability,

 $A_+$  is the positive area of the impulse response, and  $A_-$  is the negative area of the impulse response.

From equation (A2.2.3)

$$A = \frac{1}{K} (R_{xx}(0) R_{yy}(0))^{\frac{1}{2}} A_{n}$$

Hence,

$$A_{+} = \frac{1}{K} (R_{xx}(0) R_{yy}(0))^{\frac{1}{2}} A_{n+}$$
 ... A2.2.5

... A2.2.4

where  ${\rm A}_{\rm n+}$  is the positive area of the normalised function and

$$A_{n-1} = \frac{1}{K} (R_{xx}(0) R_{yy}(0))^{\frac{1}{2}} A_{n-1}$$
 ... A2.2.6

where  $\boldsymbol{A}_{n-}$  is the negative area of the normalised impulse response function.

Substituting equations (A2.2.5), (A2.2.6) into equation (A2.2.4) the following equation is obtained :

$$R = \frac{A_{+}}{A_{-}} = \frac{A_{n+}}{A_{n+}}$$
 ... A2.2.7

Hence, for the second-order active filter, relative stability obtained from the normalised correlation function is the same as the value obtained from the continuous impulse response and is given by :

$$R = \frac{A_{+}}{A_{-}} = \frac{A_{n+}}{A_{n-}} = e^{\pi \rho / (1 - \rho^{2})^{\frac{1}{2}}} \dots A2.2.8$$

# A2.3 THE AMPLITUDE OF THE IMPULSE RESPONSE

Using analogue correlators, the amplitude of the impulse response,  $h(\tau)$ , does give information about the noise signals amplitudes. But the amplitude information will be lost if the correlator normalises the correlation function. This can be understood using equation (Al.3.4), ie,

$$h(\tau) = \frac{1}{K} (R_{xx}(0) R_{yy}(0))^{\frac{1}{2}} \rho_{xy}(\tau) \qquad \dots A2.3.1$$

Hence, correlation must be made to the measured function by measuring the RMS values of the input and output noise signals. In some applications, the normalised amplitude can be used. For example, the logarithmic decrement criteria uses the natural logarithm of the ratio of amplitudes of the impulse response measured one period apart, gives information about the stability of the system and is given by :

$$\ln \frac{h(\tau_1)}{h(\tau_2)} = \text{Log Decrement} \qquad \dots \text{ A2.3.2}$$

1

Referring to Figure A2.3.1, this can be stated for second-order active filters as :

$$\ln \frac{h(\tau_1)}{h(\tau_2)} = \frac{2\pi\rho}{(1-\rho^2)^{\frac{1}{2}}} \qquad \dots \quad A2.3.3$$

Substituting equation (A2.3.1) into equation (A2.3.3), the following equation is obtained :

Hence, Log Decrement of  $h(\tau) = \text{Log Decrement of } h_n(\tau)$ .

# APPENDIX 3 : POLARITY CORRELATION

# A3.1 THE VAN VLECK THEOREM

This theorem states that if x(t) is a sample function of a Gaussian random noise with zero mean, and the function  $x_c(t)$  is the function formed by infinite clipping of x(t), then the normalised auto-correlation function of x(t) and  $x_c(t)$  are related by :

$$\rho_{XX}(\tau) = \sin \frac{\pi}{2} r_{X_{c}X_{c}}(\tau)$$
 ... A3.1.1

where  $\rho_{XX}(\tau)$  is the normalised continuous auto-correlation function, and  $r_{XX}(\tau)$  is the normalised clipped (polarity) auto-correlation cc function.

The derivation of (A3.1.1) based on the use of the joint probability density function which is a bivariate Gaussian distribution. It is noticeable that the joint probability density function is even and symmetrical. This means that :

P<sub>++</sub> = P<sub>--</sub>

 $P_{+-} = P_{-+}$ 

and

where  $P_{++}$  is the joint probability that  $x_c(t) = +1$  and  $x_c(t + \tau) = +1$ , and  $P_{--}$  is the joint probability that  $x_c(t) = -1$  (or zero in binary case) and  $x_c(t + \tau) = -1$ .

 $P_{+-}$  is the joint probability that  $\chi(t) = +1$  and  $\chi(t + \tau) = -1$ , and  $P_{-+}$  is the joint probability that  $\chi(t) = -1$  and  $\chi(t + \tau) = +1$ . It should be remembered that equation (A3.1.1) is true only for Gaussian random noise and sine waveform. McFadden (1953) has shown that it is approximately true for a weak sine wave signal in Gaussian noise.

# A3.2 VARIANCE AND INTEGRATION TIME

In Appendix Al.4, it is shown that the variance of estimated correlation function of a Gaussian signal with zero mean is proportional to  $\frac{1}{BT}$ , where B is the bandwidth of the power spectra of the signal and T is the integration period.

In order to sample the data variables, the sampling frequencies are chosen to be greater than the Nyquist rate which is  $2f_B$ . In practice, real systems do not have band limited spectra, hence, a sampling rate in the range  $2f_B < f_S < 10f_B$  is often chosen, where  $f_B$ is the 3 dB bandwidth and  $f_S$  is the sampling frequency. In practice,  $5f_R$  is often used.

In a polarity correlation, the zero crossings are the only distinct events and these are determined by the average zero crossing rate, Z, hence, the expected number of correlatable events in a time  $\gamma$  is given by  $\gamma$ Z.

Hence, comparing the correlatable events of continuous correlation and polarity correlation, the following relation is obtained :

$$5f_B T = \gamma Z$$

Hence, the integration time  $\gamma$  required for polarity correlation is given by :

$$\gamma_{\perp} = 5 f_{B} T/Z$$

Jordan(1973) has shown that the average zero crossing rate for non-exponential auto-correlation cases is given by :

$$Z = \frac{1}{\pi} \left( - \frac{\dot{R}(0)}{R(0)} \right)^{\frac{1}{2}}$$

where R(O) is the mean value of the output signal.

Hence, the integration time is given by :

$$\gamma = 5\pi f_{B}T \left(-\frac{R(0)}{R(0)}\right)^{\frac{1}{2}} \dots A3.2.2$$

It is clear that longer integration period is required by the polarity correlation method, since the zero crossing operation does not give information about the high frequency. Jordan (1973) has summarised the integration time requirements for polarity correlator as shown in Table A3.2.1. He has shown that the integration time can be reduced by differentiating the input noise signals before entering the zero crossing detector. Then, applying equation (A3.2.2) the following result is obtained :

# $\gamma = 1.7 T$

The time constant, RC, of the differentiator network must be sufficient to ensure differentiation over the effective bandwidth of the noise signal.

TABLE A.3.2.1

The Integration Time requirements of Polarity Correlators .

Discription of Noise	$\frac{\aleph}{T} = \frac{\text{Polarity Correlation Integration Time}}{\text{Continuous Correlation Integration Time}}$
Bandlimited Noise.	2.5
2nd-Order Filtered White Noise	2.5
Gaussian Noise.	2.94
Differentiated Gaussian Noise .	1.7
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