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RISK-BASED DESIGN OF STRUCTURES FOR FIRE

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In the memory of my parents Mejbas and Halleh, my grandmother Erbeideh,
my beloved Ayesha and Aunt Noufah
and to
Seham, Amani, Mohammad, Tala & Abd Al-Rahman
and
Mahmoud, Sarah, Ahmad, Abdullah & Raiyah

Declaration

This thesis and the research described and reported within have been completed solely by Ahmad Mejbas Al-Remal under the supervision of Professor Asif Usmani and Professor José Torero. Where other sources are quoted, full references are given.

Ahmad Mejbas Al-Remal

November, 2012

Abstract

Techniques of performance-based design in fire safety have developed notably in the past two decades. One of the reasons for departing from the prescriptive methods is the ability of performance-based methods to form a scientific basis for the cost-risk-benefit analysis of different fire safety alternatives. Apart from few exceptions, observation of past fires has shown that the structure's contribution to the overall fire resistance was considerably underestimated.

The purpose of this research is to outline a risk-based design approach for structures in fire. Probabilistic methods are employed to ascertain uniform reliability indices in line with the classical trend in code development.

Modern design codes for complex phenomena such as fire have been structured to facilitate design computations. Prescriptive design methods specify fire protection methods for structural systems based on laboratory controlled and highly restrictive testing regimes. Those methods inherently assume that the tested elements behave similarly in real structures irrespective of their loading, location or boundary conditions. This approach is contested by many researchers, and analyses following fire incidents indicated alarming discrepancy between anticipated and actual structural behaviour during real fires.

In formulating design and construction codes, code writers deal with the inherent uncertainties by setting a ceiling to the potential risk of failure. The latter process is implemented by specifying safety parameters, that are derived via probabilistic techniques aimed at harmonising the risks ensuing different load scenarios. The code

structure addresses the probability of failure with adequate detail and accuracy. The other component of the risk metric, namely the consequence of failure, is a subjective field that assumes a multitude of variables depending on the context of the problem. In codified structural design, the severity of failure is implicitly embodied in the different magnitudes of safety indices applied to different modes of structural response.

This project introduces a risk-based method for the design of structures in fire. It provides a coherent approach to a quantified treatment of risk elements that meets the demands of performance-based fire safety methods.

A number of proposals are made for rational acceptable risk and reliability parameters in addition to a damage index with applications in structural fire safety design. Although the example application of the proposed damage index is a structure subjected to fire effects, the same rationale can be easily applied to the assessment of structural damage due to other effects.

Publications

Usmani, A., Röben, C., Al-Remal, A., *A Very Simple Method for Assessing Tall Building Safety in Major Fires*, International Journal of Steel Structures, 2009, Volume 9, Issue 1, Pages 17-28.

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"وما بكم من نعمة فمن الله"

"*And whatever of blessing is upon you, it is from Allah*" (The Holy Kor'aan)

Indeed I claim no capacity for any achievement without the grace of Allah. Here, I thank Him for the people I have been privileged with in the course of my studies.

I have had the privilege of working under two eminent scientists. My principal supervisor, Professor Asif Usmani, has been more than a mentor. He went the extra mile to provide unwavering support and encouragement, and kept me in check whenever I seemed to divert off course. Not only have I learnt a lot from his outstanding work, but he also taught me how to learn and how to write. I enjoyed the thorough scrutiny and the challenges he made to my every claim. I am also grateful to Professor José Torero for the constructive advice and support, and for the enjoyable courses I was fortunate to attend. Both professors set an example for me, in terms of knowledge and in character.

I am deeply indebted to my brother Mahmoud, whose faith in me never diminished, for the words of encouragement and support. Mahmoud has never been far despite the distance.

The pursuit of achievement takes a great deal of selfishness. Often is the case that those closest to us choose to share the burden, but none of the credit. My children, Amani, Mohammad, Tala and Abd Al-Rahman endured the brunt of the many times I took for myself. I am humbled by their capacity to return my shortcomings with yet more smiles and encouragement.

I cannot find the words to thank my wife and my best friend, Seham. She taught me a different purpose to life and the true meaning of giving. No achievement compares to having her by my side.

Ahmad Mejbas Al-Remal
November 2012

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Chapter 1

Introduction

This project introduces a risk-based method for the design of structures in fire. It provides a coherent approach to a quantified treatment of risk elements that meets the demands of performance-based fire safety methods.

During the past two decades, the subject of structural response to fire gained unprecedented momentum. Interest among the research community in the UK surged markedly following Broadgate fire in 1990 [10], an incident that came to expose the lagging status of fire research in structural engineering. The extensive investigation that followed Broadgate suggested that the structural behaviour in fire was not adequately understood. Together with other building fires as well as large full-scale fire tests in Australia and Germany, Broadgate provided motivation for the landmark Cardington test at the Building Research Establishment (BRE) Laboratory in Cardington. Evidence from the Cardington test provided a valuable database for increased research activity in structural fire engineering.

The time-honoured prescriptive methods for fire protection showed alarming inconsistency with regards to the performance of structures in real fires. While the structure performed beyond expectation in the case of Broadgate, it failed tragically in the World Trade Center on 11 September 2001. All WTC buildings as well as Broadgate were designed using prescriptive methods that deemed to satisfy the relevant building regulations. With the advent of revolutionary forms in architecture, design for fire is becoming increasingly challenging.

Additionally, the considerable volume of research work in structural response to fire requires an organisational platform to extract practical design methods. The presence of a directive environment should help channel the efforts of researchers and regulators towards meaningful fire safety solutions. As part of this project, a quantitative parameter for assessing disproportionate collapse associated with the localised nature of fire is proposed. The *damage index* can be incorporated into the risk metric to provide a quantified risk estimate. Although the general treatment focuses on the design of structures for fire conditions, it retains a broad spectrum and can be adopted for other loading types.

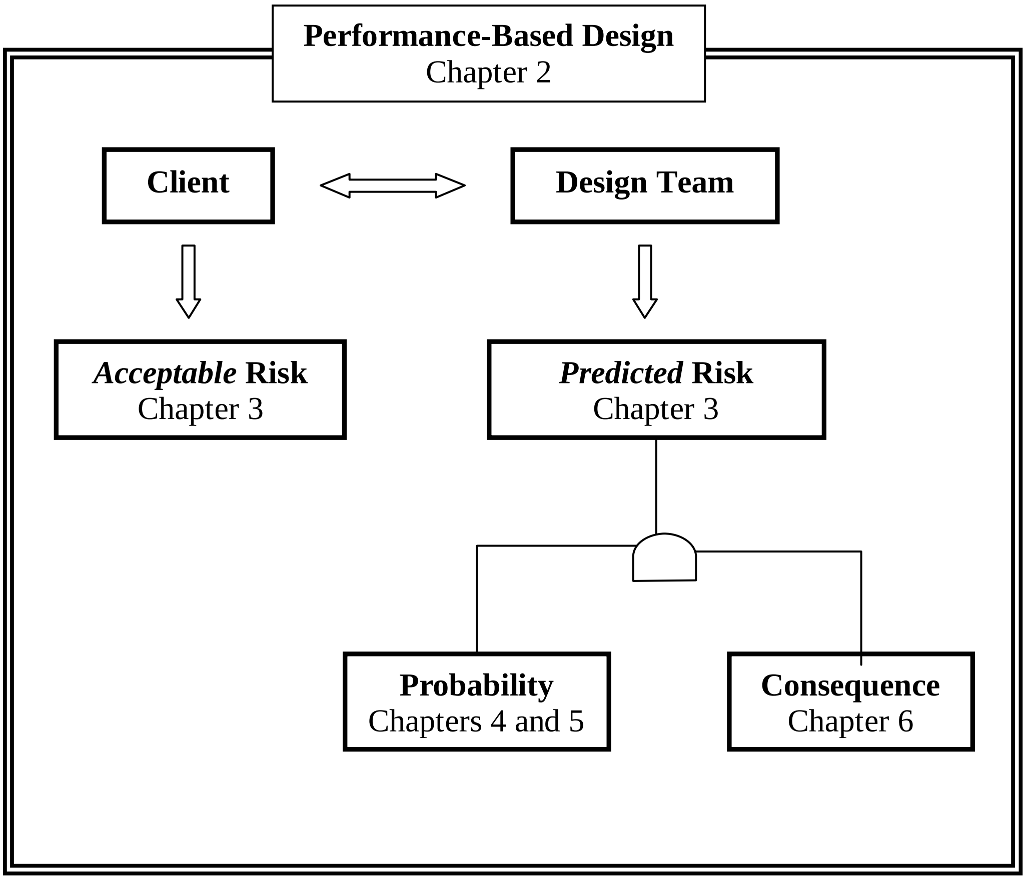
This thesis builds on the recent advancement in fire safety engineering applications, revisiting the architecture of approach and introducing a rational treatment based on performance-based design. The relationship between fire, structural engineers as well as clients takes a far more interactive shape than ever; an interaction that is more of need than of novelty.

The methodologies presented herein provide the engineering community with a design tool that enables extended freedom in selecting or developing fire-resisting systems whilst maintaining uniform safety levels. The design of structures can be calibrated to meet a performance criterion by a trade-off between “systems” and “systems-reliability”. The acceptance criterion is straightforward: the *risk* associated with failure is equal for all conceivable failures.

In current codes, certain modes of failure, such as in yielding, are preferred to other modes, owing to the development of such failures over longer periods of time. This is the reason for specifying higher target safety indices to unfavourable failure modes in an attempt to instigate potential failure in more favourable modes. The selection of the different safety indices, however, is subjective and based on the behaviour of individual members rather than that of the whole structure. Despite the success of the codes, examples can be drawn from actual practice where the code intent is easily contravened, as is shown in chapter 5.

The implementation of quantitative risk assessment in design codes requires a holistic treatment of structures wherein different members assume different damage indices related to how their failure affects the global structure. A proposal for such index is presented in chapter 6.

The organisation of the thesis is shown in the following diagram.



Chapter 2: Performance-Based Design

The evolution of industrial economies necessitated the development of standard practices. Standardisation provided the industry with uniform performance standards, but created considerable bureaucratic barriers to innovation. Fire engineering is one example where the industry is governed by standard tests and specifications that in many cases do not relate to the problems they are intended to solve.

Designers as well economists have realised the need to depart from rigid prescriptive requirements to more rational solutions that are tailored to the objectives of the design. It is also essential to develop quantitative models of the performance of fire safety systems that may be incorporated in decision formulae.

Performance-based is objective-based design. A discussion of the common fire safety objectives along with analysis and verification tools is presented. The rationale of performance-based design is open to deterministic and probabilistic models. Probability-based models however offer a basic component the quantitative risk assessment which is the logical approach for decision making in performance-based design. A number of decision methodologies are duly described.

The achievement of safety during fire is accomplished by integrated systems of fire safety products and installations, in addition to the naturally-existing capacity of the structure. The common objective of safety systems is providing sufficient time for evacuation and for retention of structural capacity until fire extinction. A three-tier reliability index system can meet the different functional requirements of the structure during the different stages of fire development.

Chapter 3: Risk Analysis

Design is essentially a plan for the future. The uncertainties surrounding future demands and capacities create an environment of risk.

The focus of the design is to minimise potential losses that may arise from projected influences. The process is a balance between the two main components of the risk: probability of an event and its consequence.

The analysis of risk involves a significant number of social parameters that cannot be described in a crisp manner. Engineering projects on the other hand, require quantitative analysis that enables comparison and ranking of design options. The arrival at fit-for-purpose engineering solution must account for societal expectations, but should be carried out within the correct context. A number of risk assessment criteria are described.

Consistency in design can be achieved by unifying risk of events influencing the structure. The chapter present a simple formula for unified acceptable risk.

Chapter 4: Safety and Reliability Engineering

Gauss showed that "*the mean of reported locations may be thought of as the true position, because an unbiased measurement taken by an unbiased observer is just as likely to be slightly above as slightly below the true value*." [11]

Design is based on predictions for both demand and capacity. Both are projected in the future surrounded by an array of blurring uncertainties.

Uncertainty is encountered by extra allowances and margins that are intended to offset its effect. Risk is a function of the uncertainty and consequence of an event.

For unfavourable events, safety is the opposite of risk; hence safety factors that do not account for uncertainties are meaningless.

Uncertainty modelling and reliability techniques are introduced. The main focus of the chapter is level II reliability methods due to its wide application and reasonable computational demand.

Structural behaviour in fire is inherently nonlinear and potentially dynamic if local or global collapse ensues. The implications of applying level I reliability methods to structural fire design are assessed, especially with regards to converting fire and the corresponding structural response to time-invariant variables.

Two proposals are made at the end of the chapter. One is for a risk-based target reliability index, which fits well within the framework of performance-based design. The second is for an optimisation method to calculate the reliability index that is capable of detecting multiple design points.

Chapter 5: Limit State Design in Practical Situations: Is your structure safe enough?

The chapter provides a brief introduction to system reliability and analyses the relationship between the failure of members and the global structures for different system types. The reliability of series, parallel, redundant and damaged systems is introduced.

The formulation for level I reliability method is member-based. Examples of standard design practice are give to illustrate the impact of ignoring system reliability. Recommendations are made in the aim of improving reliability in practical design and construction.

Chapter 6: The Strength Loss Method

Current design codes are based on reliability. The reliability or safety indices which form the basis of design formulas are derived such that target probabilities of failure are not exceeded.

The risk associated with different modes of failure (buckling or yielding for example) is treated subjectively by specifying different reliability indices to respective modes. This approach holds for the design of isolated members and if member sizes are optimised. In actual practice, the failure of members of the same failure modes, such as columns, may have significantly different impact on the whole structure. A simple example could be the failure of a column in the ground or tenth storey of a 15 storey building.

In the most advanced codes, the treatment of failure is confined to predominantly prescriptive measures to prevent disproportionate collapse. These provisions do not require consideration of the characteristics of the specific structure and their adequacy cannot be verified by calculation. More importantly, there exists no quantitative damage parameters that permit the computation of the risk associated with failure.

Chapter 6 presents a formulation for a global damage index that can be used to quantitatively assess the effect of damage on a structure; hence it is suitable for application in performance-based designs. An example is given for a multi-storey structure under fire.

Chapter 7: Conclusions and Recommendations for Further Research

In addition to emphasising the key messages of the thesis, the final chapter points out in the direction of promising developments related to this research. It includes a proposal for the development of a level I structural fire code, FiRel (Fire Reliability Calculation); a programme for reliability calculation for structural performance in fire using response surface modelling and the use of the strength loss method in the design of structures for fire following earthquakes.

Like any other work, this thesis has built on a vast amount of previous high quality research. It is hoped that it would complement existing knowledge and serve as reference for related work in the future.

Chapter 2

Performance-Based Design

Performance-based design (PBD) is not new to civilisation [12]. Two examples of PBD follow.

Case 1: When a tailor designs a shirt for a customer, he starts by taking the customer's measurements. Next he displays different textile materials, buttons, threads and designs. Both the tailor and the customer discuss the cost of making the shirt. Occasionally, the customer argues that a certain textile is expensive, but the tailor explains that the shirt would not need ironing and should last longer if that material is selected. The deal succeeds if they both agree on design and price.

Case 2: Mass-producing workshops have a different view. For cheaper and faster production, *generic* sizes are taken. The available sizes would depend on the *frequency distribution* of relevant sizes in the target market. *Deemed to satisfy* patterns are used, and all what the worker in the workshop has to do is use the right pattern to cut the shape, use a specified textile and buttons. Everything should work, without having to think about it.

Case 1 is a true example of performance-based design. The *objective* is set around specific requirements (taking the actual customer's measurements reduces the *uncertainty* in the shirt size). The *reliability* of various *design alternatives* are assessed in view of benefit and cost (different materials are available, however, the tailor feels that the running cost of maintenance (ironing) and the *risk* of faster depreciation of the investment (cheap textiles fade quicker) outweigh the saving in the initial cost). Reliability is sustained; the probability that the tailor makes a mistake in designing two different shirts for two different customers is small and generally decreases with time and experience. Moreover, the continued interaction during the design and execution process provides a channel for modification and refinement.

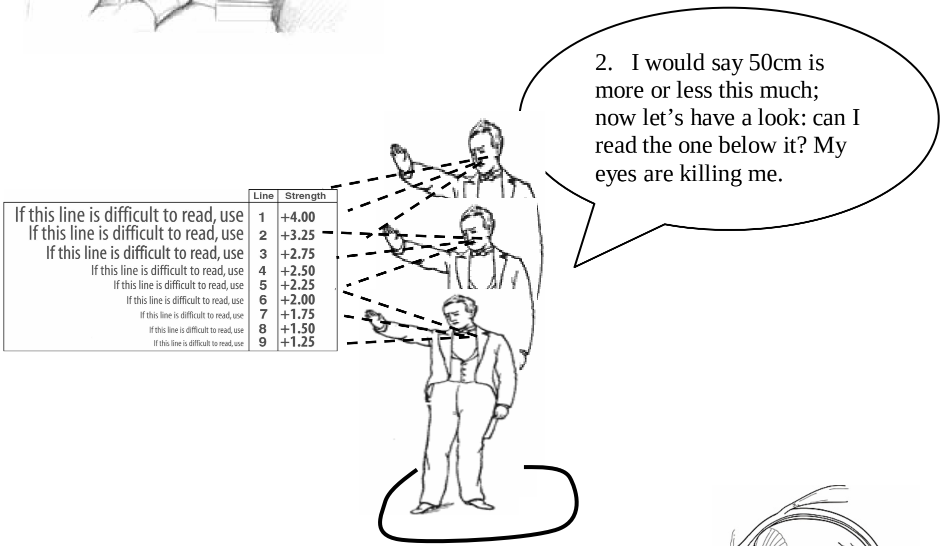
Case 2 is a compromise of case 1. The objective is compromised; the specific sizes are unknown but the produced sizes are thought to fit most people. The alternatives are limited and fixed, *x* number of models with *y* number of textiles. *The reliability is unknown!* People may like certain designs or materials and not others, and the manufacturer does not speak to the user. This case is a typical example of prescriptive design.

It is worth noting, however, that prescriptive designs provide cheaper and time-saving alternatives with little need for specialist expertise.

Figure 2.1 analyses two alternatives in terms of rationale and possible consequences. It contrasts informed risk assessment based on calculated prognosis to uninformed decision making.

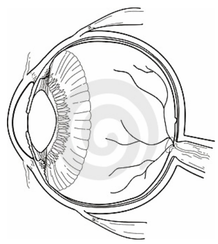
Q.1: Would the person's *height or position* affect the test? What if the examinee is *long-sighted*?

1. Examinee is always seated with head positioned at a measured distance from the reference device point. The examination is by a qualified person.



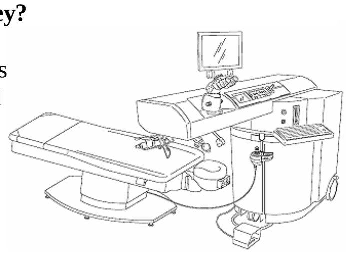
Q.2: Can non-vision related medical problems be detected and would they affect the test result?

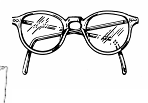
1. Yes
2. No



Q.3: What are my options and how reliable are they?

1. Laser surgery or custom- made spectacles – Life-cycle-cost vs. benefit well-defined
2. Generic off-the-shelf spectacles – might or might not work!





Q.4: Success rate?

1. Calculated from sound statistical data
2. Unknown

Figure 2.1 Shopping for Spectacles

2.1 Elements of Performance-Based Design

The three main components of Performance-Based design (PBD) are:

- Definition of the design objectives
- Investigation of the alternative designs available to meet the objectives
- An informed decision making process utilising *reliability* and *risk assessment* of alternatives as tools to select the most *efficient* solution.

Prescriptive design could be considered as *one form of PBD*, with some concessions in the above three criteria. The objectives are mostly generic and not job-specific, the alternatives are pre-set and limited, and the reliability analysis is almost completely missing. A generic procedure for performance-based design is outlined in figure 2.2.

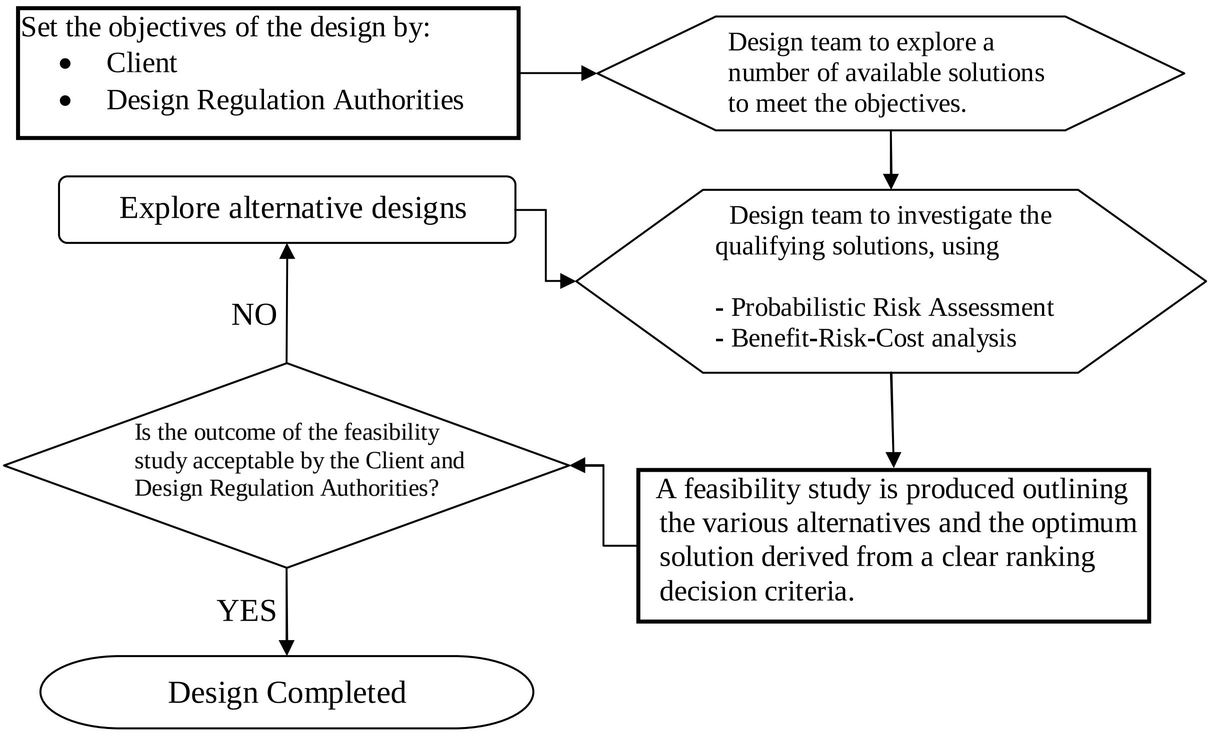


Figure-2.2 Generic Flowchart of Performance-Based Design Process

2.2 Why Performance-Based Design?

The motivation behind performance-based design for structural fire safety engineering is twofold. The engineering community has realised the inadequacy of prescriptive techniques by evidence of recent failures, including the collapse of the twin towers in New York in September 2001. The problem is compounded by the inability to relate performance of structural elements in real fires to the code-approved ratings and fire protection measures.

The other side of the story relates to the restrictions placed on industry, especially with regards to innovation. New technologies have to undergo series of standardised tests, which are neither cheap nor provide any insight on how systems may be improved. This realisation was reflected in the World Trade Organisation Agreement on Technical Barriers on Trade (clause 2.8) which encourages members to “specify technical regulations based on product requirements in terms of performance rather than design or prescriptive characteristics [13].”

Performance-based framework encourages a wider range of alternatives that support optimisation of project cost and resources. More importantly, it facilitates better understanding of the impact of fire protection solutions on the actual fire safety objectives.

2.3 Performance-based design in Fire

Focus on performance-based design (PBD) for fire safety has grown rapidly over the past two decades. The adoption of the approach, whether called performance-based or objective-oriented, has evolved in marked changes in regulations in many countries. Starting in the UK and Japan in mid 1980's and Australia in the late 1980's, the approach is gaining greater acceptance and its application is growing on an international scale.

Whether applied to fire safety engineering or in any other contexts, the principles of PBD do not change: quantitative assessment of design alternatives that meet the job-specific objectives is the main approach. In fire safety engineering, the procedure of design is as follows:

1. *Required performance of output variable*: Identification of the project specific needs in terms of fire safety and definition of the design objectives.
2. *Input variables*: Selection of the most realistic fire scenarios
3. *Prospective output variables*: Determination of the various design alternatives achieving the objectives for every fire scenario.
4. *Balance formula*: Quantitative assessment of the design alternatives on the basis of *Benefit-Risk-Cost* comparison. This step has so far been performed by deterministic approaches. Extension to a probabilistic formulation is computationally cumbersome but is feasible.

2.3.1 Objectives of Fire Safety Engineering

There most certainly exists a consensus on the key objectives of fire safety design. These has been summarised in CIB Report: "Rational Fire Safety Engineering Approach to Fire Resistance of Buildings" as follows [14]:

1. Protection of health and safety, that of the building occupants and of the fire-fighter's in addition to other people in the vicinity who might be affected by the spread of fire or smoke.
2. Protection of property, by minimising damage to structure and fabric, safeguarding property and preserving public image.

3. Protection of the environment, by mitigating the impact of gaseous, liquid and solid waste.
4. Protection of architectural, historic and cultural value
5. Protection of infrastructure. This is particular to buildings with broad functional activities, such as telecommunication towers or public internet servers, where the cost of interruption exceeds the direct fire damage by many orders of magnitude.

2.3.2 Fire Safety Strategy

The main phenomena that pose risk to safety, and hence compromise the above objectives are [14]: *propagation of smoke and gases* within the building or to adjacent spaces, *fire spread* within the building and *structural failure* particularly if it initiates *progressive collapse*.

The above conditions set the *demand* on design alternatives and thereby stipulate respective failure modes or limit states. The aim of fire safety design is to engineer a satisfactory performance for the building whereby capacity exceeds demand by an acceptable margin.

The principal limit states applicable to building fire safety engineering are:

- Smoke Leakage
- Thermal Insulation
- Integrity
- Load bearing capacity

Smoke is the main killer in most building fires. In addition to toxicity and visibility impairment, smoke propagating at high temperature can initiate fire beyond the compartment of origin. Smoke damage assumes the larger proportion of repair cost as compared to heat-related damage [15]. To mitigate the risk of developing untenable conditions or ignition in adjacent spaces, walls and floors of the fire compartment must possess adequate thermal insulation. Most standard fire tests define an upper limit of the temperature rise on the cold side; for example: in the standard BS 476 fire test, 140 ^{o}C average and 180 ^{o}C maximum at any one point [15]. Integrity of the fire enclosure can be compromised by cracking failure of special fire sealants which allows heat and smoke to infiltrate to adjacent areas. Load-bearing capacity addresses the two main criteria of ultimate limit state and serviceability as appropriate. Excessive deflection hampers rescue operations and can result in serious damage to compartment walls or floors. Design must ensure that structural elements continue to carry applied loads in the variety of mechanisms developing throughout fire. In particular, the probability of structural failure initiating beyond the origin of fire or progressive collapse must be kept low.

One distinction between the above limit states is that their relevance to design follows the temporal evolution of fire. The most obvious example is that the effect on the structure becomes significant only in the post-flashover stage. This property is a powerful tool in the development of design alternatives. Using a quantified risk parameter, design optimisation can be performed through a trade-off that can easily be established between active and passive fire protection systems.

Limit states are merely surrogate formulations that serve to formalise design procedures. This important fact must always be remembered, especially when special projects are at hand. In some cases, conventional limit states may fall short of capturing all potential critical conditions in which case designers need to revert to ad-hoc design methods. Nuclear facilities, historic buildings and communication centres are a few examples. Table 2.1 contains a brief summary of risk control criteria in building fire safety design [16].

Table 2.1: The Fire Safety System – Subsystems & Possible Measures

(Reproduced from [16])

Subsystem *(Objectives)*	Possible Measures *(Design Criteria)* Hardware	 Software
Control of fire initiation and development in early stages	• Earth leakage devices • Surveillance systems • Materials of construction • Alarm and detection systems plus hose reels and extinguishers • Sprinklers • Other automatic fire suppression hardware	• Regular maintenance of electrical and mechanical systems • Human monitoring of surveillance systems • Presence of occupants within the building • Presence of occupants trained in early fire fighting in building • Management and maintenance of alarm and detection systems • Maintenance of hose reels and extinguishers • Management and maintenance of sprinkler systems
Control of flame spread	• Physical barriers • Materials of construction including linings • Alarm and detection systems plus fire brigade	• Maintenance of barriers • Management and maintenance of alarm and detection systems
Control of spread of smoke and toxic products	• Physical barriers • Smoke exhaust systems (purging) • Pressurization systems (e.g., stairs or zones)	• Maintenance of barriers • Management and maintenance of Smoke exhaust and pressurization systems
Provision of means to allow occupant avoidance	• Signage • Exits	• Presence of trained wardens • Evacuation drills
Provision of structural adequacy	• Size of structural members • Overall structural behaviour • Fire protective coatings, concrete cover	• Maintenance of coatings

2.4 Quantitative Assessment of Fire Safety Design

2.4.1 The Risk Triangle

Risk has three main aspects: *event, consequences and context* [17]. Failure of a building under the event of an earthquake, wind, fire or any load is a *risk*. The *consequences* of failure play a major part in risk assessment. Although the probability of an earthquake with an intensity of 8.0 degrees on the Richter scale is quite low, the consequences can be catastrophic. Codes have sometimes assigned, though implicitly, risk-related factors to some loads, especially where buildings housed a large number of people or for hospital and emergency buildings. An example of those is the importance factor for wind loads in the United States. Response of people to the consequences of an event is paramount to design. Although death is an indisputable certainty, perception for death in a fire is far different from that in a car accident. The impact of a large toll of fatalities in a single incident is far greater than to the same number over a number of accidents. Finally, by *context*, we mean who is preparing the assessment, for whom and for what purpose. Compromising the environment is a typical example of most development schemes. The argument suggests that the benefit of creating jobs outweighs the loss due to health problems, while obviously, assigning a monetary value for health.

Structures can fail in more than one way. Failure is the state where the structural resistance falls below the load effects. Different failure modes have different probabilities and consequences depending on factors such as those listed below.

1. Probability of load occurrence (dead, live, wind, fire, etc.)
2. Type of load (static, dynamic, cyclic, time-dependent like creep)
3. Relative magnitude of the load
4. Response of the structure to the load (sway, cracking, vibration, falling glass, collapse)

5. Mode of structural response (sudden like brittle fracture or buckling, or prolonged like yielding, , excessive deflection)
6. Human response to the load and consequences (panic in fires and nuclear attacks, discomfort to floor vibration)
7. Cost versus benefit in reducing the probability of event occurrence.
8. Cost of remedial action
9. Acceptable failure rate.

The first three factors represent the event, the next three the consequences and the last three the context.

To increase the chances of obtaining an accurate analysis, consideration should be given to the following points.

- Understanding the System: Components of the system are the constituents of the demand-capacity formula (loads, material behaviour, structural integrity) and attributes affected by its performance (human lives, environment, economy).

- Establishing a representative model of the system for *the risk* study: This includes the identification of *basic variables and their uncertainties.* The basic variables in structural engineering could be the structure and loads, human factors and a careful examination of previous failures and successes. Attention should be given to uncertainties in material properties, section dimensions, connections relative stiffness, the computational model for the structure and loads, uncertainty in statistical modelling, fabrication tolerances, various construction techniques, and *quality control* policies.

- Identification of relationship between design variables, such as the interaction of failure modes. How would certain failures affect the whole structure, or adjacent structures? *Event Trees* are valuable instruments in this respect.

2.5 Statistical Decision Theory

Decision makers are often faced with the prospect of choosing the design approach to the project at hand. Inherent in the decision process is the *uncertainty* surrounding the success or failure of the design attributes within the economic context. The technicalities of design are handled by the design team and are of no material interest to stakeholders.

Statistical decision theory provides a viable ranking tool to extract *gain* or *loss indices* from design options [18]. Subsets of the latter are *payoff analysis*, *Hurwicz* and *Bayes'* criteria and *utility theory*.

By constructing a *payoff table*, decision makers are presented with quantified *expected values* for profit or loss that incorporate the effects of uncertainty.

Table 2.2: Typical Payoff Table

Decisions, D_j	Events, E_i				
	E_1	E_2	E_3	…………	E_n
D_1	C_{11}	C_{12}	C_{13}	………….	C_{1n}
D_2	C_{21}	C_{22}	C_{23}	………….	C_{2n}
D_3	C_{31}	C_{32}	C_{33}	…………..	C_{3n}
…..	……….	…………	…………	………….	……..
D_n	C_{n1}	C_{n2}	C_{n3}	…………	C_{nn}

In the above table, C_{ij} represents the *payoff* or *consequence* of decision D_j should event E_i take place. The matrix form of the above table and a graphical representation by a tree diagram are depicted in figure 2.3.

$$\begin{bmatrix} D_1 \\ D_2 \\ \vdots \\ \\ D_n \end{bmatrix} \begin{bmatrix} E_1 & E_2 & \cdots & E_n \end{bmatrix} = \begin{bmatrix} C_{11} & C_{12} & \cdots & C_{1n} \\ C_{21} & C_{22} & \cdots & C_{2n} \\ \vdots & & \ddots & \\ C_{n1} & C_{n2} & \cdots & C_{nn} \end{bmatrix}$$

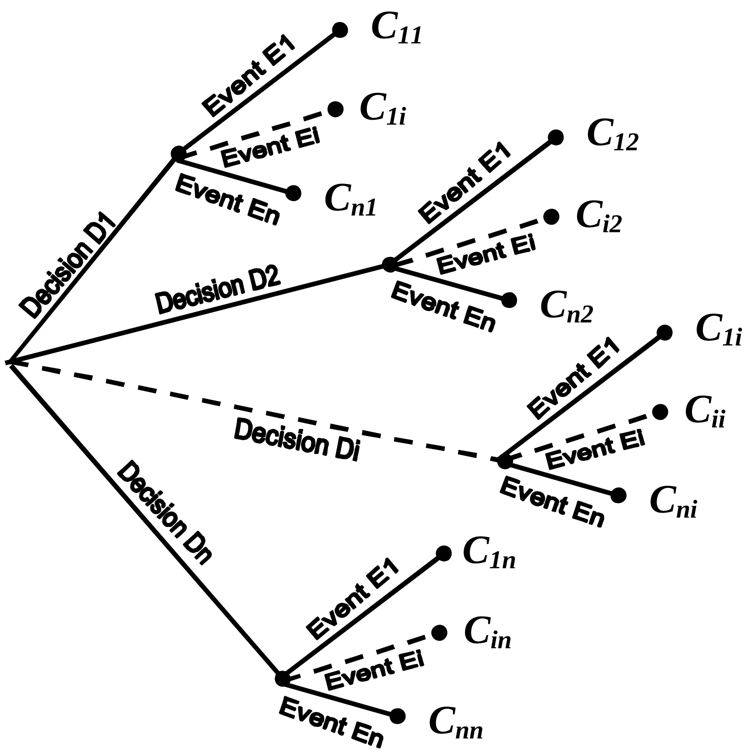


Figure 2.3: Payoff Matrix and Event-Decision Tree Diagram

A number of different criteria can be applied to arrive at a decision based on payoff tables. These include the *maximax*, *maximin*, *minimax*, *Hurwicz* and *Bayes'* criteria [18, 19]. The *maximax* (maximum of maxima) criterion represents the largest, or most "optimistic", value of payoff of any one of all rows, namely max.[*Cij*].

Maximin (maximum of minima) selects the maximum value of the row containing minimum payoff corresponding to any one event (or row), E_i, i.e.; max [min [C_{ij}, E_i]], i = *1,2,…,n*. To apply the *minimax* criterion, a *regret matrix* (also referred to as *opportunity loss*) is first constructed from the difference between the maximum payoff, C_{i_max}, and other payoffs corresponding to an event, E_i. For each row corresponding to event E_i, this difference stands for the loss associated with decisions D_i to D_n. A column of the maximum regret of each row is constructed and the optimum decision opts for the minimum value thereof (minimum of regret maxima), that is min [max [(C_{i_max} - C_{i_max}), E_i].

Table 2.3a: Payoff Table based on *Maximax* (*most optimistic*) criteria

Decisions, D_j	Events, E_i					*Maximum* Payoff	**Optimum Decision**
	E_1	E_2	E_3	……	E_n		
D_1	C_{11}	C_{12}	C_{13}	……	C_{1n}	Max [C_{1j}]	**D_i with Maximum value**
D_2	C_{21}	C_{22}	C_{23}	……	C_{2n}	Max [C_{2j}]	
D_3	C_{31}	C_{32}	C_{33}	……	C_{3n}	Max [C_{3j}]	
…..	……	……	……	……	…	……	
D_n	C_{n1}	C_{n2}	C_{n3}	……	C_{nn}	Max [C_{nj}]	

Table 2.3b: Payoff Table based on *Maximin* (*most pessimistic*) criteria

Decisions, D_j	Events, E_i					*Minimum* Payoff	**Optimum Decision**
	E_1	E_2	E_3	……	E_n		
D_1	C_{11}	C_{12}	C_{13}	……	C_{1n}	Min [C_{1j}]	**D_i with Maximum Value**
D_2	C_{21}	C_{22}	C_{23}	……	C_{2n}	Min [C_{2j}]	
D_3	C_{31}	C_{32}	C_{33}	……	C_{3n}	Min [C_{3j}]	
…..	……	……	……	……	…	……	
D_n	C_{n1}	C_{n2}	C_{n3}	……	C_{nn}	Min [C_{nj}]	

Table 2.3c: Payoff Table based on *Minimax* (*least regret*) criteria

Decisions, D_j	Events, E_i					*Regret*	**Optimum**
	E_1	E_2	E_3	……	E_n	$(C_{ij\_max} - C_{ij})$	**Decision**
D_1	C_{11}	C_{12}	C_{13}	……	C_{1n}	$C_{1j\_max} - C_{1j}$	
D_2	C_{21}	C_{22}	C_{23}	……	C_{2n}	$C_{2j\_max} - C_{2j}$	
D_3	C_{31}	C_{32}	C_{33}	……	C_{3n}	$C_{3j\_max} - C_{2j}$	$\boldsymbol{D_i}$ **with Minimum Value**
…..	……	……	……	……	…	……	
D_n	C_{n1}	C_{n2}	C_{n3}	……	C_{nn}	$C_{nj\_max} - C_{nj}$	

A forward development of the above criteria enables the design alternatives to be analysed in terms of payoff and the associated reliability.

Risk is a function of probability of an event and its consequence. Assuming that both the probability and the consequence are of the same order, it can be liberally defined as the product of probability and consequence of an event.

$$\begin{aligned} Risk &= f\left(\mathrm{Pr}\,obability\,, Consequence\right) \\ &= f\left(p\,, C\right) \end{aligned} \tag{2.1}$$

By applying Hurwicz or Bayes' criterion [18], *prior* (or subjective) probabilities, $p_{r\text{-}i}$, are superimposed on the payoff table to deduce *weighted averages* or *expected values* for the risk, *R*, associated with decision, *D*. The probability, $p_{r\text{-}i}$, is the probability of occurrence of event E_i, i.e; $p_{r\text{-}i} = p(E_i)$.

$$\begin{bmatrix} D_1 \\ D_2 \\ \vdots \\ D_n \end{bmatrix} \begin{bmatrix} E_1 & E_2 & \cdots & E_n \end{bmatrix} \begin{bmatrix} p_{r-1} & 0 & \cdots & 0 \\ 0 & p_{r-2} & \cdots & 0 \\ \vdots & 0 & p_{r-i} & \vdots \\ 0 & 0 & \cdots & p_{r-n} \end{bmatrix} = \begin{bmatrix} R_{11} & R_{12} & \cdots & R_{1n} \\ R_{21} & R_{22} & \cdots & R_{2n} \\ \vdots & & \ddots & \\ R_{n1} & R_{n2} & \cdots & R_{nn} \end{bmatrix} \quad (2.2)$$

Hurwicz's method employs weighted averages according to the following formula:

$$R_{ij-prior} = \sum_{i=1}^{n} p_{r-i}\, C_{i\,j} \Big| D_j \,, \quad \text{for } \forall\, \mathrm{D_j} \qquad (2.3)$$

The optimum decision (also called the *optimal act*), $D_{j\text{-}optimum}$, is the one with the most desirable (minimum or maximum) consequence or utility; that is:

$$D_{j-optimum} = D_j : R_{ij-prior} = \min \textit{ or } \max \left[R_{ij-prior}\right] \qquad (2.3\text{-a})$$

Minimum or maximum values indicate a *negative* or *positive* risk representing an opportunity loss or payoff respectively. Since the probabilities attached to the events are subjective, the expected value of risk, $R_{ij-prior}$, in the above formula is *the expected value of risk under uncertainty*.

2.6 Performance-Based or Prescriptive design?

Is it worth spending time and money to carry out a higher order analysis? This question is often posed to the design team.

Economists frequently use the *expected value of perfect information* (EVPI) as a parameter to estimate the benefit gained from further investigation [18, 19]. Given perfect information, the client always opts for optimal acts, and the prior probabilities, p_{r-i}, are interpreted as *relative frequencies* or *weight-values*, w_i,.

Therefore, the risk with perfect information is calculated as the union of probabilities:

$$R_{ij-perfect\ information} = \sum w_i \ C_{ij-optimum} \qquad (2.4)$$

where,

$w_i = p_{r-i}$ (values of prior probabilities are unchanged)

The above equation resembles the hypothetical case where the client makes the same decision when faced by the same problem an *infinite* number of times.

The EVPI can then be calculated as the difference between the risk (gain or loss) with perfect information and the expected value of risk under uncertainty.

$$EVPI = R_{ij-perfect\ information} - R_{ij-prior} \qquad (2.4\text{-a})$$

The EVPI sets the upper bound on expenditure related to gaining further knowledge through *sampling* information, whether experimental or via simulation.

In fire safety engineering design, EVPI can be used to aid the decision whether to adopt prescriptive or performance-based design. Prescriptive solutions intrinsically yield designs under uncertainty. The life-cycle-*cost* of each prescriptive solution is evaluated under a number of fire scenarios (single-floor, severe multi-storey fire, etc.). Prior probabilities are assigned to each scenario and the risk is calculated under uncertainty and with perfect information. The EVPI (or EVSI discussed in the next section) is the maximum cost including design fee that the client should pay for performance-based design. It is imperative to remember that the *cost* must include provision for the value of human life so as to account for fatalities or injuries. Values and models for the frequencies of occurrence in addition to reliability of fire-protection systems can be found in references [16, 20].

2.7 Bayes' Criterion

Bayes' theory provides a tool for analysing prospective decisions in a multistage manner. The initial stage utilises *prior* analysis and mimics Hurwicz criterion. The second stage, the *pre-posterior* analysis, is an upward refinement of prior probabilities. Yet again it assigns subjective probabilities to events but these are based on past information about similar problems.

It is especially beneficial when statistical information is limited and needs to be supplemented by value judgement and intuition in order to infer the probability of an event from sample observations.

As discussed in the above section, the value of EVPI is calculated on the presumption that an infinite number of samples have been assessed. In other words, the uncertainty inherent into inferring from a sample to the target population is nullified. EVPI is true if, and only if, the full range of events has been examined. This is almost impossible in practical terms.

Pre-posterior analysis enables the calculation of a *point estimate* of EVPI, namely the *value of sample information*, EVSI. As it might have already been concluded, EVSI embodies estimates of events derived from existing previous sample surveys. In the payoff calculation, prior probabilities are replaced by the conditional probability of the estimates, X_j, given that the event, E_i, is the true state of affairs, $p\left(X_j \middle| E_i\right)$. To illustrate, a consultant may give the following advice to the client regarding fire load in an office:

Based on the surveys we carried out over a number of years, we estimate the following probabilities of a high fire load:

- A 30% probability a high fire load, and this is 80% likely to be the case, or
- A 45% probability of a high fire load but this is 15% likely, or
- A 65% probability a high fire load that is only 5% likely.

These are all prior probabilities all from past experience. They can be viewed as *relative weights* of the individual probabilities, i.e., P(30%) = 80% for example. They are probabilities of "guessed or estimated" probabilities

The above inference suggests the probability that the *true* office fire is high, is:

$P\text{-}_{prior\_high}$ = (0.3)(0.80) + (0.45)(0.15) + (0.65)(0.05) = 0.34, or 34%

The probability that the event is true and that the sample estimates it as true is calculated by the joint probability of the event and the sample evidence [18].

$$\begin{aligned} p\left(E_i \cap X_j\right) &= p\left(E_i\right) p\left(X_j \middle| E_i\right) \\ &= p\left(X_j\right) p\left(E_i \middle| X_j\right) \end{aligned} \qquad (2.5)$$

The above gives:

$$p\left(E_i \middle| X_j\right) = \frac{P\left(E_i\right) p\left(X_j \middle| E_i\right)}{P\left(X_j\right)}$$

Which, by using the total probability theorem, becomes what is known as Bayes' theorem: [21]

$$p\left(E_i \mid X_j\right)=\frac{P(E_i)\,p\left(X_j \mid E_i\right)}{\sum_{j=1}^{n} P(E_i)\,p\left(X_j \mid E_i\right)} \qquad \text{(2.5-a)}$$

In the above equation:

$P(E_i| X_j)$: *Posterior probability* of event E_i, that is the probability that the outcome is E_i if the observation is X_j

$P(E_i)$: *Prior probability* of event E_i, which could be based on value judgement, past experience or intuition

$P(X_j| E_j)$: The *probability* that the observation is X_j if the event is E_i

Now, suppose that *one* sample office was analysed and it was found that it had high fire loads; what is the probability of that the *true* fire load is high?

The posterior probabilities are first calculated as follows:

$P\text{-}_{posterior\_30\%}$ = (0.3)(0.80) / [(0.3)(0.80) + (0.45)(0.15) + (0.65)(0.05)] = 0.70

$P\text{-}_{posterior\_45\%}$ = (0.45)(0.15) / [(0.3)(0.80) + (0.45)(0.15) + (0.65)(0.05)] = 0.20

$P\text{-}_{posterior\_65\%}$ = (0.65)(0.05) / [(0.3)(0.80) + (0.45)(0.15) + (0.65)(0.05)] = 0.10

Then the posterior probability that the true fire load is high is:

$P\text{-}_{posterior\_high}$ = (0.70)(0.30) + (0.20)(0.45) + (0.10)(0.65) = 0.37, or 37%

The above procedure can be repeated as many times as required to incorporate new sample information.

The *expected* value of payoff *if* the survey is carried out and the *optimal act* (or *decision*) is made accordingly is:

$$R_{ij-sample\ \inf ormation} = p\left(\bigcup_{i=1}^{n_i}\left(E_i \bigcap_{j=1}^{n_j} X_j\right)\right) C_{ij-optimum}$$

$$R_{ij-sample\ \inf ormation} = \sum_{i=1}^{n_i}\prod_{j=1}^{n_j} p(E_i)\ p\left(X_j \middle| E_i\right) C_{ij-optimum} \qquad (2.6)$$

The *expected value of sample information*, EVSI, is the difference between the *expected* value of payoff *with* sample information and the expected payoff *without*.

$$EVSI = R_{ij-sample\ \inf ormation} - R_{ij-prior} \qquad (2.6\text{-a})$$

For the last example, the value of sample information is (0.37-0.34) times the act under consideration (to spend £10000 on fire alarms, say).

The EVSI presents a more accurate measure to aid decision makers which design methodology to adopt.

If the study or survey is commissioned and once it is completed, the sample information become available and *posterior analysis* can be performed. Posterior analysis is a subset of pre-posterior analysis since a particular sample evidence, $X_{j\text{-}pos}$, is determined hence only $p\ (X_{j\text{-}post} \mid E_i)$ need to be considered.

Posterior probabilities may be used as prior probabilities in potential further analyses, as part of the process of sequential decision making.

2.8 Role of Society in Engineering Design: Input from Psychology, Law and Insurance Providers

Although unanimously conceded, acceptable failure rate is an issue of long-standing controversy. Man-made designs cannot be perfect. Some might fail and cause loss of life and property. The difficult question is: how much are we willing to lose?

The compromise to safety is not of choice; it is essential to achieve practical design. In modern structural codes, safety indices are calculated through a trade-off between safety and economy. And here comes a more difficult question: how do we put a value to human life in the economic formula?

The answer to the first question will involve psychologists in the process. Human perception of death, especially in fire, needs to be examined. Though death is universally accepted as inevitable, individual social characteristics, like religion, standard of living or life expectancy, contribute to the acceptance criteria. Public surveys, designed by both engineers and psychologists, are needed for this purpose.

To the second, the answer potentially comes from insurance providers. Insurance companies implement rigorous risk assessment techniques in calculating premiums [22]. As providers of *life* insurance policies, they are best placed to resolve issues like the *theoretical* monetary value of life. Other sources could be implicit values from consumer expenditure or court rulings on compensation [4, 23].

The legislative environment in different countries differs by approach only. Various examples can be cited for legal instruments aimed at reducing risk to health and safety via imposition of control on certain aspects of public behaviour. In the USA, the Cigarette Fire Safety Act was introduced to reduce fire incidents due to smoking [24]. The verdict on any engineering scheme is made by society and societal input is essential to promoting engineering standards.

2.9 Value of Human Life

Consensus on a parameter or model to estimate human life does not exist even among experts in law and sociology.

The underlying principle in placing a value on life (so called *value of statistical life*, VSL) is to develop a situation where an individual *accepts risk* at a *certain price*. In the most simplistic form, economists compute the difference in wage between two jobs versus the difference in risk to life. If a job that involves an additional 2% risk of death for an extra £1000, he/she is implicitly placing a value of (1000/0.02 = £50000) on their life. This method is called the *revealed preferences method*. The *contingent valuation method* is another where a sample population are *asked* a series of questions about the amount they will accept to assume a higher risk. The point where the subject refuses the more money defines the highest risk. The calculation of VSL is similar to that of the former method. Both methods are criticised on grounds of subjectivity.

Other methods exist, such as the *consumer market behaviour method* and the *meta analysis method*. A brief yet informative discussion of the various approaches can be found in Brannon [25] and a comprehensive critical review for VSL evaluation worldwide was published by Viscusi and Aldy [26].

Other formulations relate the value of life to the cost of risk reduction. One such formula is [23],

$$V = \frac{L}{P} = \frac{E}{P} \qquad (2.7)$$

In the above, L is the expected loss due the risk of death, P is the probability of death and E is the expected loss due to acceptable protection expenditure. An example can be given for the risk reduced by using a pedestrian subway [23]. The probability of being killed while crossing the road was 1.225×10^{-8} (UK figures in 1971), and it was estimated that using the subway would have been considered if the additional time was less than 16 seconds. People put a value of their time at £0.24/hour according to U.K. transport studies in 1971. The above results in a value of life of:

$$\begin{aligned} V &= 0.24 \times \left({}^{16}\!/_{3600} \right) \div (1.225 \times 10^{-8}) \\ &= £87{,}000 \ (1971 \ \textit{figures}) \end{aligned}$$

The above equation suggests that the estimated value of life increases with the decrease of the probability of death, hence suggests a variation in the estimate of value depending on risk level.

Other figures were also derived from the *implicit value of consumption activity*. A figure of $351,000 for the estimated value of life (US 1980) was based on the purchase price of smoke detectors, running cost of batteries and the changes of the probability of fire-related death or injury[23]. More details are available in references [23, 26].

2.10 Acceptable Risk - Utility theory

The preceding sections perpetuate the notion that *descriptive* (the-how) decisions and *normative* (the-how should) decisions are the same. It assumes a *uniform* attitude towards risk perception across the population. This forms the backbone of the risk definition that amalgamates the probability and consequence of an event in a single risk factor.

Quite naturally, different individuals place different values to money. The extent to which a loss or gain makes on assets, individual psychology and experience play their role in decision making. Monetary value alone is not necessarily an adequate decision parameter.

Conceived by Daniel Bernoulli in 1738, the *utility theory* (sometimes called the *preference theory*) postulates that a value of an item is determined by its utility rather than its price. The utility an item yields is subject to the choice of the person taking the decision under conditions involving risk, and is a measure of the pleasure they derive from the item. It provides people a formula combining asset value and probabilistic risk indicators.

The primary use of the utility theory is to derive utility functions as indicators for *acceptable risk*.

To ignore the variability of attitude towards risk would be erroneous; to impose the intensive technical content of fire safety design on clients renders the design process circular. In almost all cases in fire safety design, the *utility index*, that apportions values reflecting preference, is implicitly assigned by the design team.

Normalising the design alternatives to an audience of stakeholders requires the elimination of the attitude towards risk element from the decision formula. The simplest method to achieve that is *insurance*. Most stakeholders generally require insurance of buildings and contents in addition to public liability insurance as prerequisite to committing any investment.

2.10.1 Utility Functions

Utility functions are common scales to which design variables are mapped so as to facilitate comparison of alternatives. Common utility functions are:

- Monetary value, including value of statistical life (VSL)
- Time
- Life Quality Index (LQI)
- Custom functions (structural damage in building design for instance)

Once the values of design alternatives are expressed in terms of a utility, the design process becomes that of *optimisation* of the utility function under constraints of *choice variables*. For example, a design may require the cost of a fire protection system to be minimised subject to providing minimum evacuation time, limiting maximum structural damage, etc. The cost is in this case the *objective function* that is to be minimised and the *constraint* functions are those enforcing conditions of minimum evacuation time or maximum structural damage. A typical optimisation problem is shown in figure 2.4.

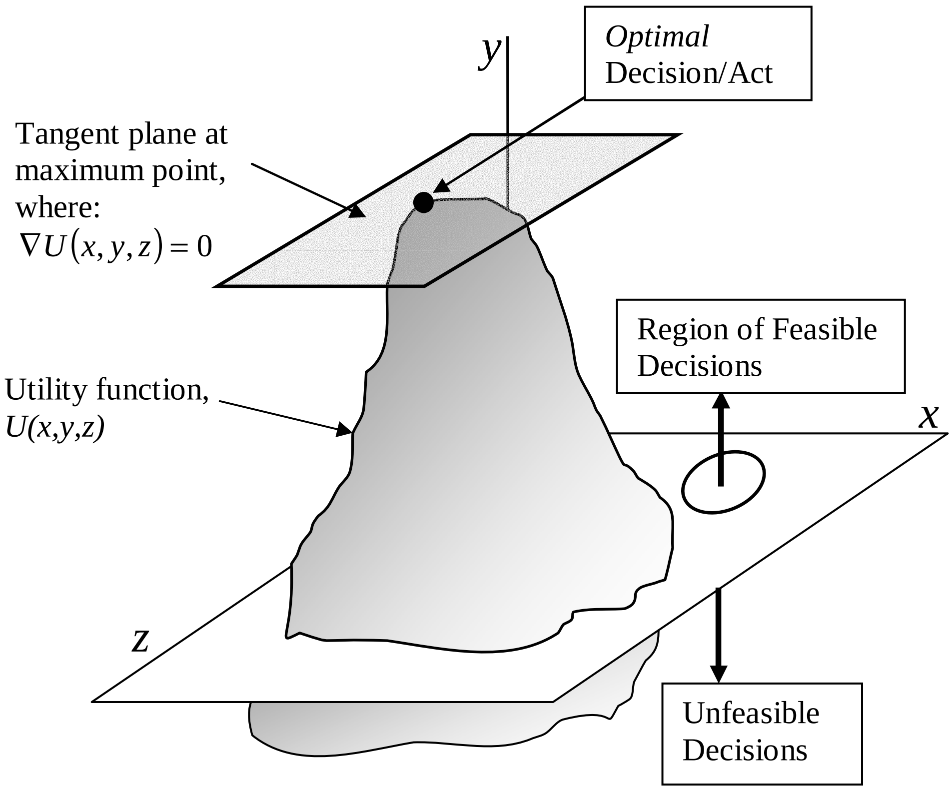


Figure 2.4: Example of a Utility Function

2.10.1.1 Time

How much will you pay to go back a few years in time? What do you need a holiday for? Why do we hate waiting? Why do they want a big garden?

Time is a utility like no other. It is a measure of the dynamics of life as we know it. On the sociological level, vast resources are expended to increase the amount of time apportioned to pleasure. Cutting down journey times, less working hours, minimum hourly wage and increasing number of recreational facilities are evidence to the value we place on time. An interesting discussion of time as a source of utility was made by Zeckhauser [27].

Within the economic context, it is not difficult to extract design life or investment life and apply it as an objective function. It facilitates integrating design with average age of population since constraint function would be the estimated age of two (or any chosen number of) generations. Engineering design practices contain examples of subjective provisions to that effect. Residential buildings are traditionally designed for 50 years whereas bridges for 100 years. Both types are static structures in the physical sense, but the impact of bridges on the dynamics of the economy surpasses that of buildings by orders of magnitude.

A clear distinction should be made between the use of time as a utility function in this section and in section 2.14 where time is a choice variable within the context of the fire safety formula.

2.10.1.2 Life Quality Index, LQI

The life quality index, LQI, is a measure of societal welfare incorporating three factors: Gross Domestic Product (*G*), Life Expectancy (*E*) and the proportion of time spent for pleasure (1-c). The mathematical formulation was developed by Pandey et al [28] as follows:

$$L_Q = G^{c/(1-c)} E \qquad (2.8)$$

In the above formula, *c* is the proportion of life spent on generating *G*, hence *(1-c)* is the time spent on non-productive activities, including pleasure. It follows that the power $c/(1-c)$ is the proportion of time spent on generating wealth.

The LQI is commonly used to determine the maximum expenditure a society is willing to make to reduce risk to life, commonly known as the *Societal Willingness to Pay* (SWTP). The SWTP is capped by the constraint that life quality must not be compromised; that is, LQI must increase or be kept constant. The first condition implies:

$$d\,L_Q \geq 0 \qquad (2.8\text{-a})$$

The second condition requires that any increment of G, *dG*, required to increase E by *dE*, must correspond to an increase in L_Q:

$$\frac{dL_Q}{L_Q} = \left(\frac{c}{1-c}\right)\frac{dG}{G} + \frac{dE}{E} \geq 0 \qquad (2.8\text{-b})$$

Taking the critical point where $dL_Q = 0$, yields:

$$SWTP = dG = -G\left(\frac{1-c}{c}\right)\frac{dE}{E} \qquad (2.9)$$

It follows directly that an estimate of what is known as a *societal value of statistical life*, SVSL, can be arrived at as [29]:

$$SVSL = \int_E dG = G\left(\frac{1-c}{c}\right)\overline{E} \qquad (2.10)$$

where $\overline{E}$ is the average life expectancy.

The above concept can be easily extended to arrive at *WTP* amounts for utility functions other than human life, for example, cost or design life. It has been used to determine safety levels in civil engineering facilities and life-cycle-cost of structures [28].

2.10.1.3 Custom Utility Functions

Of particular interest to structural and fire engineers is the expected resulting from a possible hazard. Several models have been developed to represent damage [30-34]. Discussion of the latter and other models is left to Chapter 6, The Strength Loss Method.

2.11 Design Optimisation

Mathematical optimisation techniques are widely used in economics, engineering and operational research. Depending on the type of the problem, *linear or nonlinear programming* is used to arrive at *stationary points* of an *objective function* under *equality* or *inequality constraints* [35, 36].

The subject matter is the *objective function* for which extreme points are desired. The constraint functions represent the boundaries of *choice variables*.

A function, f(**x**), is said to have a *local extreme value* (*minimum, maximum or saddle point*) at x_0, if either [37]:

$$\nabla f(\mathbf{x_0}) = 0 \qquad \text{or} \quad \nabla f(\mathbf{x_0}) \ \text{ does not exist}$$

For a *n* number of choice variables, x_i , the objective function, U(**x**) [36]:

$$U = f\ (x_1, x_2, \ldots\ldots, x_n)$$

with first partial derivatives,

$$f_1\ ,\ f_2\ , \ldots\ldots\ldots,\ f_n\ (\ f_1 \text{ denoting } \partial U/\partial x_1\ ,\ f_n \text{ denoting } \partial U/\partial x_n\ )$$

Then, to have an extremum (maximum or minimum), the *first-order necessary condition* is:

$$f_1 = f_2 = \ldots\ldots\ldots = f_n = 0$$

The second-order partial derivatives can be expressed as:

$$d^2U = |H|\ \mathbf{x}$$

where, $|H|$ is the *Hessian* determinant:

$$|H| = \begin{vmatrix} f_{11} & f_{12} & f_{13} & \cdots & f_{1n} \\ f_{21} & f_{22} & f_{23} & \cdots & f_{2n} \\ f_{31} & f_{32} & f_{33} & \cdots & f_{3n} \\ \vdots & & & \ddots & \vdots \\ f_{n1} & f_{n2} & f_{n3} & & f_{nn} \end{vmatrix}$$

and, **x** is the vector:

$$\mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ \vdots \\ x_n \end{bmatrix}$$

The *second order sufficient condition* for a *maximum* is:

$$|H_1| > 0\,;\, |H_2| < 0\,;\, |H_3| > 0\,;\, \ldots\ldots\ldots;\, (-1)^n |H_n| > 0$$

For a *minimum*, it is:

$$|H_1|\,,\, |H_2|\,,\, |H_3|\ ,\ldots\ldots\ldots,\ |H_n| > 0$$

$|H_n|$ is the n^{th} *principal minor* of $|H|$

The *constraint function* enforces mutual dependence between variables, x_i, and has the effect of narrowing the domain of the objective function, as shown in figure 2.5. It typifies the requirement that the range of utilities obey available limited resources, and takes the form of *equality* or *inequality* constraint. Typical constraint functions are total working hours, total budget, available floor area, evacuation time greater than 10 minutes, etc.

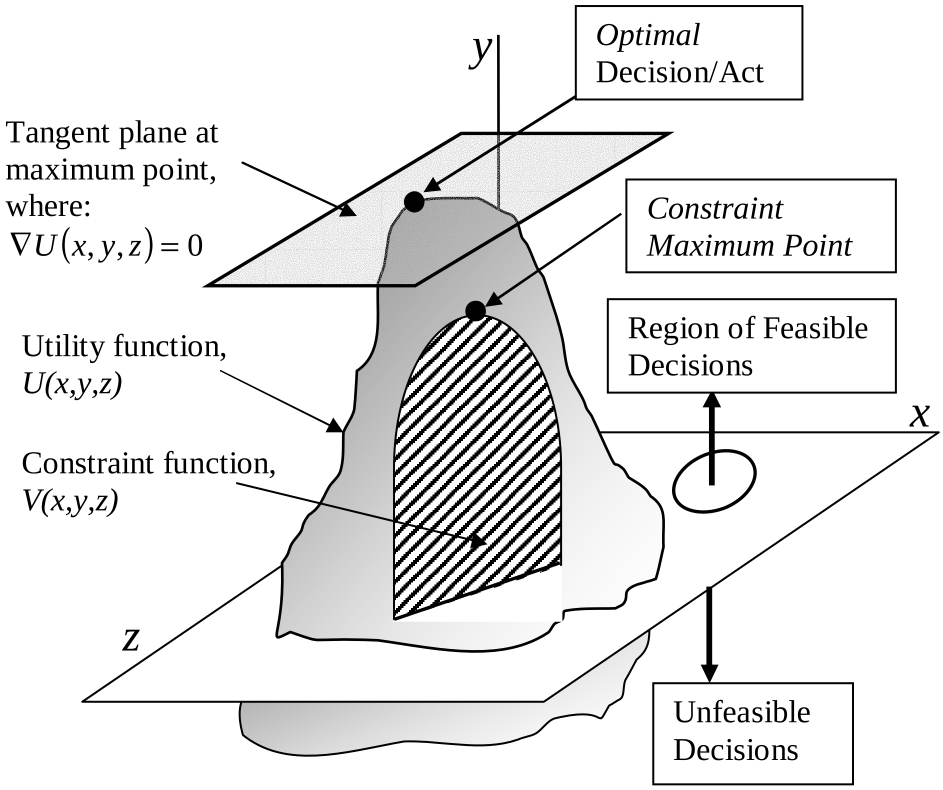


Figure 2.5: Utility Function with Constraint

2.11.1 Solution of Optimisation Problems

A number of methods exist to solve the above problems, including those with the most general form of multiple objective functions under multiple constraints. These include linear programming techniques such as the *Simplex* and *Gradient methods*. *Integer programming* enables the solution of discrete linear programming problems, where the extremum must take an integer value. For *non-linear* problems, *dynamic programming* offers an efficient tool for solving discrete and continuous value problems.

Solution of *constrained* optimisation problems is most commonly performed by the *Lagrange-Multiplier method*.

Details of applications of the above methods to economic problems can be found in most mathematical economics textbooks, such as Chiang [36]. Majid's textbook, [35], presents clear and comprehensive treatment of applications in structural engineering.

2.12 Deterministic or Probabilistic Models?

Both deterministic and probabilistic models can be used to evaluate candidate designs. The fundamental difference between the two approaches is analogous to prescriptive versus performance-based design.

Deterministic models assign fixed nominal values for design parameters based on past observations. The input is processed by a range of tools; from simple models suitable for hand calculation to finite element analyses. The acceptance criteria are guaranteed by amplifying the demand through a safety factor. Selection of safety factors is *subjective* and usually relates to previous experience.

Probabilistic models on the other hand address the uncertainty of design parameters by treating them as random variables or processes to derive safety factors or indices. Control is imposed by setting target reliability or safety indices that are directly related to the probability of failure. In addition, sensitivity analysis identifies those parameters whose variation produces little effect on the failure probability, and thus can be objectively classed as deterministic.

It can be seen that deterministic methods are compromised versions of probabilistic techniques. They are, however, computationally affordable and less intractable to practicing engineers. Practical design can be arrived at by a combination of both approaches with a satisfactory level of accuracy. Early probabilistic simulation can produce sensitivity information to truncate the number of random variables and reduce computational expense.

Figures 2.6 [8] and 2.7 illustrate the general process of modelling in fire safety engineering It is confined to *non-structural* aspects of fire safety design.

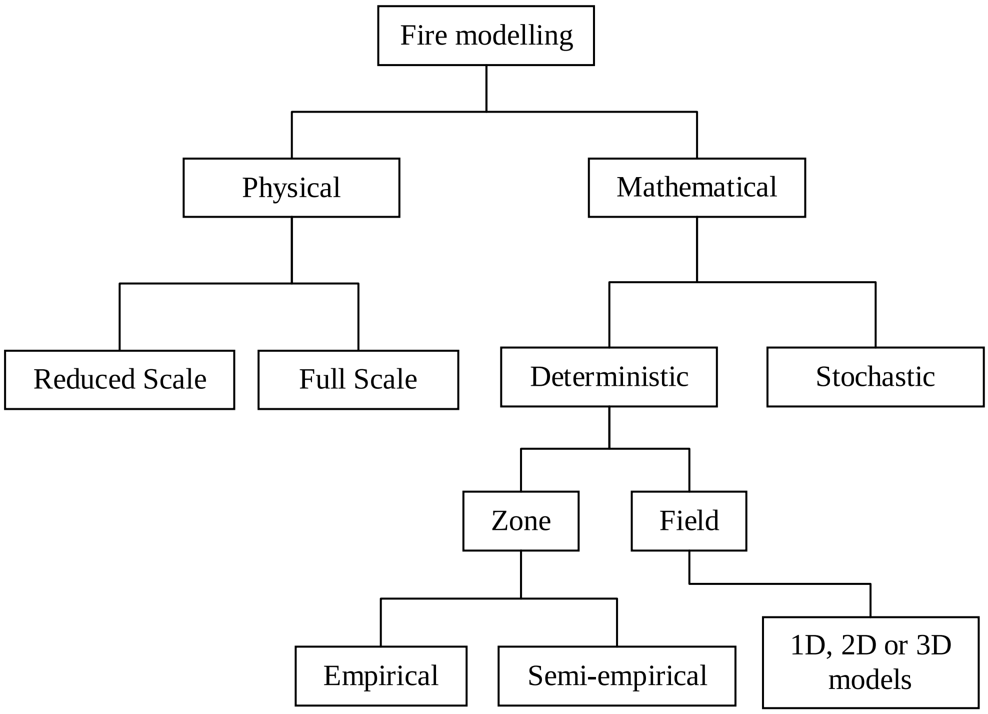


Figure 2.6: Modelling in Fire Safety Engineering (excludes *structural* fire safety design) – (Reproduced from [8])

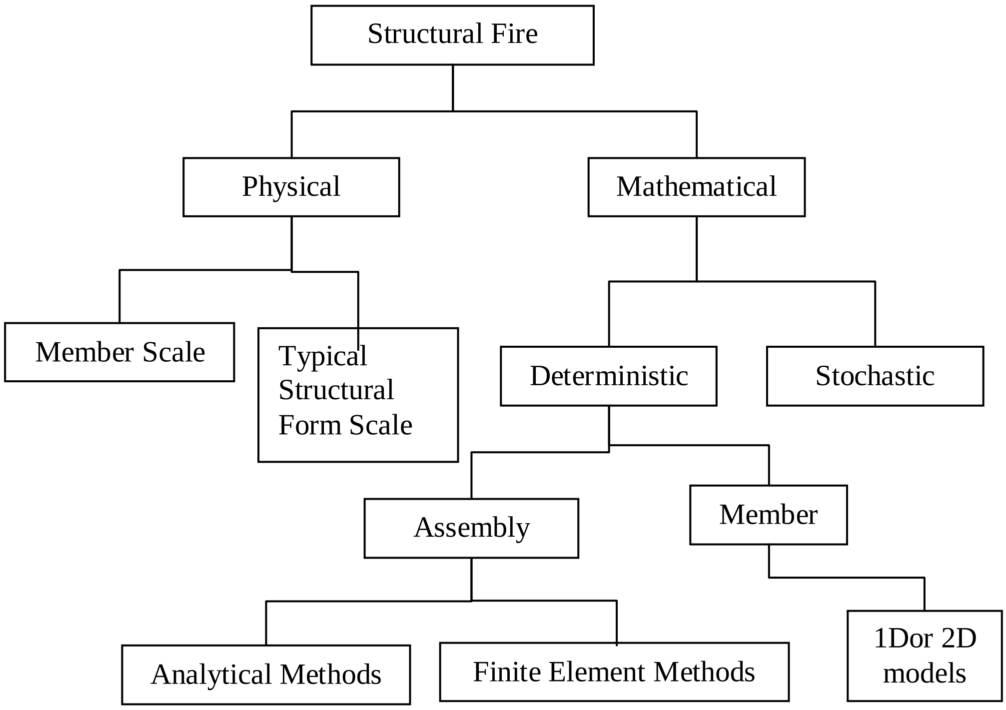


Figure 2.7: Structural Fire Modelling in Fire Safety Engineering

It is worth noting that a recent *round-robin* study of fire modelling indicated considerable discrepancy between modelling and experimental results [38]. The study employed deterministic models with well-defined fire compartment characteristics. The latter would have greatly reduced epistemic uncertainties, yet significant variation between predicted modelling results and actual fire behaviour was reported. The random uncertainty in fire behaviour is the likely cause of such anomaly. Robust models that account for aleatory and epistemic uncertainties need to be explored to arrive at values of safety indices with the ability to capture most probable conditions for design.

2.12.1 Advantages of Performance based design

- Performance-based design reduces the uncertainty in one important design variable: *client's needs*. The objectives address the specific project aspects and the client's desired performance. Since code objectives set the lower bound of acceptance criteria, they sometimes fall short of meeting specific requirements.
- It provides great flexibility in design. This opens the door to various options, where the client and the designer have the opportunity to evaluate different schemes.
- It provides a valuable tool for sound scientific assessment of design. Accurate and logical probabilistic-based comparisons between alternatives can be made. The *decision criteria* are aided by quantitative feasibility studies of the different alternatives.
- Since the decision criteria are based on a study of objectives and qualifying solutions, the impact of changing the objectives (for example, the change of building occupancy) on the benefit-risk-cost formula can be evaluated precisely. Changing a paper warehouse to a theatre reduces the fuel, for instance, but the risk of casualties due to fire increases. The balance is always retained since the design is tuned to a minimum target safety index.
- The reliability of the design option can be kept constant.

2.12.2 Disadvantages of Performance-based design

- Designers must be highly qualified as they carry the burden of providing design alternatives. In contrast, prescriptive approaches are pre-qualified and may be used by designers with modest experience.
- Design time and cost are higher. This may be balanced by potential savings in fire-protection materials and systems or reduced insurance premiums.
- Higher quality control is required on design. As opposed to checking that a designer selected the correct detail of a wall-floor junction, a thorough review of

the fire development and control model may be required to verify the safety of the same junction.

2.13 Case Studies

2.13.1 Design of a new building in Sweden

The following case study is by [39]. A 20-storey office building with each storey having a 2000 m^2 net floor area is being designed. The specific features of the building are omitted here since they are not relevant to this discussion.

Four strategies to fulfil the Swedish building code were proposed. Based on risk assessment of the building, *life-cycle-cost* (LCC) analysis was prepared for the four strategies. The following design criteria were used:

- Context: LCC calculation for Building Owner and Contractor
- Cost of Fire damage: Material Damage, interruption to business and hidden cost of fire
- Cost of human life was *not* considered

The calculations included allowance for otherwise investing the building cost in other ventures. To account for the net present value, a real interest rate that accounts for inflation was implemented as follows:

$$r = \left(\frac{1 + r_c}{1 + I}\right) - 1$$

where:

r: real interest rate calculated for costing purposes (accounts for inflation)

r_c: interest rate calculated for costing purposes

I: inflation

Both the building owner and contractor would be insured against fire, and are therefore *risk-neutral* since the uncertainty in fire damage is covered by insurance. This precludes consideration of uncertainty in the calculated cost of fire damage, and fire damage can be evaluated deterministically. Thus, the *Life-Cycle-Cost (LCC)* was calculated using the following equation:

$$LCC = A_{inv} + \sum_{i=0}^{n} \frac{RM_i}{(1+r)^i} + \frac{A_r}{(1+r)^n} + \sum_{i=0}^{n} \frac{S_{Fi}}{(1+r)^i}$$

where:

A_{inv}: Initial investment (building cost)

RM_i: Running and maintenance cost in year *i*

r: Real interest rate

A_r: Reinvestment costs

S_{Fi}: Average fire damage cost per year with insurance

The average cost of fire damage with insurance, S_{Fi} was calculated as:

$$S_{Fi} = IP + A \cdot p_{fire} + \left(\overline{S}_h + E\right) + (1 - A) \cdot p_{fire} \cdot \left(\overline{S}_p + \overline{S}_i + \overline{S}_h\right)$$

where:

IP: Annual insurance premium

E: Excess

p_{fire}: Average number of fires per year

A: Proportion of fires where damage is less than the excess

$\overline{S}_p$: Mean value of property damage per fire which is less than the excess

$\overline{S}_i$: Mean cost due to interruption of operations (downtime) per fire which is less than the excess

$\overline{S}_h$: Mean value of hidden cost per fire

For this project, the real interest rate, r, was taken as 5% in accordance with analyses carried out in Sweden in 1990s, and the mean value of hidden cost per fire, $\overline{S}_h$, was considered small and therefore ignored. The running and maintenance cost was assumed to remain constant. Moreover, the insurance was found to cover the damage cost, hence the annual cost of fire damage, $\sum_{i=0}^{n} \frac{S_{F\,i}}{(1+r)^i}$, was replaced by a fixed fee.

One LCC formula was developed for the building owner and another for the contractor, as given in the following equations.

$$f(x)_{owner} = LCC = A_{inv} + \sum_{i=0}^{n} \frac{RM_i}{(1+r)^i} + \frac{A_r}{(1+r)^n}$$

$$f(x)_{contractor} = LCC = A_{inv}$$

The results of the LCC analyses using *relative* costs are presented in Table 2.4.

Table-2.4 *Relative* Life-Cycle Cost (LCC) and Construction Cost for fire-protection strategies (Reproduced from [39]

Office Building	Building Owner		Contractor	
	LCC	Ran k	Construction Cost	Rank
Strategy 1 *Prescriptive* design with evacuation alarm	US$ 169,200	4	US$ 110,500	2
Strategy 2 *Prescriptive* design with sprinkler system	US$ 140,900	3	US$ 170,500	4
Strategy 3 *Performance-Based* Design Active fire protection with automatic fire alarm	US$ 75,700	2	US$ 103,00	1
Strategy 4 *Performance-Based* Design Active fire protection with sprinkler system	US$ **(-)** 1,900	1	US$ 134,500	3

The negative LCC of strategy 4 indicates an increase in revenue to the client that surpasses the building cost. In this particular strategy, the number of escape stairways was reduced thus increasing the total rentable office area.

2.13.2 Appraisal of an existing building in the USA

American fire safety codes contain *equivalency clauses* that permit the use of alternative methods if their equivalency to prescriptive designs can be proven to the authority having jurisdiction [40]. The US Congress included an equivalency clause in the Federal Fire Safety Act 1992. Subjectivity in the decision process is precluded by implementing fire safety calculations to produce a set of acceptable solutions which are prioritised on the basis of their predicted impact on the risk associated with fire.

The General Services Administration (GSA) is the business agent for the US government that operates the federal government real property, and is the body responsible for the fire and life safety of employees and visitors occupying space under its control. Since federal government buildings are *not* covered by insurance, the GSA must implement risk management regimes that optimise fire safety design to ensure *life safety*, *property protection* and *mission continuity*. The following case study is of an existing federal government building that underwent a fire safety evaluation by the GSA as commissioned by the building owner.

- John W. Peck Federal Building - Cincinnati, Ohio
- Constructed in 1963
- 10 Floors above grade, basement and sub-basement
- 6173 m2 per floor
- Limestone masonry external walls, reinforced concrete floors and roof.
- Fire safety provisions:
    - Egress: 6 stairwells constructed of masonry walls with 90 minutes fire rating
    - Discharge: Through structurally-unprotected corridors and lobbies on the first floor and a sprinkler-protected skywalk on the second leading to an adjacent building.
    - Sprinklers: "Standard-type" sprinklers in sub-basement, most of basement, skywalk, second floor south wing, computer rooms, 6th and 7th floors.
    - Fire Alarm: Selective evacuation type where instructions are delivered via recorded tapes or live communication. Occupants of the fire floor are instructed to evacuate to the floor below, those one floor above the fire floor two floors down and occupants of the floor below the fire floor are notified to expect evacuees.
    - Elevators are automatically recalled to the second floor upon activation of first floor devices.

Tools for Verification:

1. <u>Prescriptive:</u> Fire Safety Evaluation System (FSES) – Chapter 7 of NFPA 101A: "Guide on alternative approaches to life safety". FSES required sprinklers throughout, so the building failed.

2. <u>Performance-Based Design</u>: This comprised *fire modelling* and *occupant evacuation modelling.*

Fire modelling was performed using NIST-BFRL FASTlite to estimate the rate of development of hazardous conditions. Research by NIST and GSA indicated that, for typical office building fuel packages, the time histories of the *heat release rate*, $\dot{q}$, grew at a medium rate proportional with time-squared, t^2:

$$\dot{q} = \alpha t^2$$

For 3-sided office workstations, α was found to be 0.117 kJ/s^3.

Design assumptions were further corroborated by site visits and examination of previous fire reports. Calculations were performed for typical self-contained offices or open spaces with doors open or closed. Fire was modelled to grow till flashover or oxygen starvation, through ventilation restriction or depletion.

Elements of Equivalency Analysis:

- Variables of Interest: Fire growth, Occupant awareness & response as well as time to reach safety.

- Verification: This is mainly:
  - Capability (capacity per system)
  - Adequacy (sufficient number of systems)
  - Reliability of systems (not covered in reference). In particular, the impact of sprinklers on developing hazardous conditions had to be assessed.

- Design Options:
  - Calculation of a margin of safety for a number of alternatives and compare it to that of a code-complying building
  - Complete sprinkler protection: Prevents flashover at fire origin, limits fire size to less than 1 MegaWatt and prevents flame spread beyond room of origin.
  - Any other technical analysis procedure subject to approvals

Margin of Safety Approach:

Design was constrained by either of the below two conditions:

$$Margin\ of\ Safety = ASET - RSET$$

$$\Rightarrow$$

$$ASET \geq Safety\ Factor \times RSET$$

$$or$$

$$ASET_{alternative} \geq ASET_{sprinkler}$$

where:

$ASET$:	Available Safe Egress Time
$RSET$:	Required safe Egress Time

Acceptance Criterion:

According to the regulations, an analysis must indicate that the existing or proposed safety systems provide a period of time equal or greater than the amount of time available for escape from a similar building compliant with a prescriptive solution. The following survival conditions for a typical sprinklered building (since the comparison was with a sprinklered building) were taken from the NFPA Fire Protection Handbook, 17th edition (applicable at the time):

- Gas Temperature at Eye Level ≤ 93 °C
- Maximum Ceiling Temperature 260 °C
- CO Concentration ≤ 0.15% by Volume

Interaction of multiple effects was not considered. The results of the analysis are shown in Table 2.5.

Table 2.5: Time in seconds for Sprinkler Activation and Untenability *(ASET)*

<table>
<tr><td colspan="3">Sprinkler</td><td colspan="4">Time to Untenability (seconds)</td></tr>
<tr><td rowspan="2">Type</td><td colspan="2">Activation Time (sec.)</td><td rowspan="2">Office</td><td rowspan="2">Open Plan</td><td colspan="2">Corridor</td></tr>
<tr><td>Office</td><td>Open Plan</td><td>Office</td><td>Open Plan</td></tr>
<tr><td>None (Un-sprinklered)</td><td>X</td><td>X</td><td>180</td><td>300</td><td>360</td><td>400</td></tr>
<tr><td>Standard</td><td>260</td><td>425</td><td>180</td><td>300</td><td>∞</td><td>400</td></tr>
<tr><td>Quick Response</td><td>171</td><td>299</td><td>∞</td><td>∞</td><td>∞</td><td>∞</td></tr>
</table>

Several models use hydraulic flow approximation to model egress time. The FASTlite suite contains such model which was used to calculate RSET. The occupant load used was 650 people per floor as per the Life Safety Code, and the possibility of two out of the six exits being blocked was considered. Results are shown in Table 2.6.

Table 2.6: Calculated Egress Time in seconds *(RSET)*

Number of Exits Available	Number of Occupants	Time to Clear Floor (seconds)	Time to move One Floor (seconds)
6	300	70	90
	650	150	200
4	300	100	140
	650	220	290

Conclusions:

The available and required times were shown to be quite close (ASET around 30 seconds higher). Standard sprinklers had no benefit in terms of increasing the available time, ASET. Their only benefit would be to protect people on other floors, hence the use of selective evacuation since people on other floors may not need to evacuate. The use of Quick Response sprinklers was recommended.

A number of factors such as pre-movement time and limited mobility were not considered. Consequently, GSA recommended a safety factor of 2,

$$ASET \geq 2 \times RSET$$

The value of the safety factor was based on judgement and past experience.

2.14 Utility Analysis of Structures in Fire – *Time* Concept

To compute the probability of an event resulting from the interaction of different variables, all the variables should be dimensionally consistent.

What do we use to measure failure or success of a member in fire? What kind of damage is caused to the structure by fire? What measure is likely to be most important to people? Fire accidents have been rare which may have contributed to inadequacies in fire research and limitations on its funding. Despite the scarcity of statistical data, structures sustained their integrity over a long period of time in most major fires. The rise in steel temperature alone cannot be correlated to the limit state of performance. Structural failure could vary according to different restraint conditions or the actual load on the member during fire, which involves other parameters in addition to temperature.

The proposed approach comes from the answer to the question: What is most important to people? In a fire, people need sufficient time to leave the building safely. Time is crucial.

2.14.1 Failure modes

Almost all failures fall in one of the two following categories:

- Ductile failures: Failure is progressive at macro-scale, and is a function of *observable history* of deformation or degradation parameters. A typical example is yielding of a tension member.

- Brittle failures: Failure here is a function of *non-observable history* of deformation or degradation parameters. These are mainly two types: brittle failures as in fracture and stability failures like buckling.

The core acceptance criterion for failure is *time scale*. The significance of time in the failure equation stems from the ability to impose sufficient control on failure progression. With almost all today's mechanics derived in a macroscopic or observable scale, engineers so understandably aim to guide structural failure to a path that can be monitored and assessed within available resources. In practical terms, analysing collected data from a structural health monitoring system, for example, requires a minimum time space. If a particular failure spans a time interval smaller than this time space, it occurs without sufficient pre-warning.

Time is proposed as a measure of safety. The conditions where the structure is required to develop a certain capacity represent the *limit states*. We then need to ensure that these limits are not exceeded.

The choice of time as a measuring unit, or safety thermometer, is made for various reasons:

- The behaviour of fire is dynamic, and thus time-dependent
- Time carries the *highest utility* for people in the event of fire
- A vast amount of statistical data is available about the behaviour of fire and people affected by fire, and it is all linked to time.

Temperature is an important factor in fire. However, the response of structure to temperature effects varies drastically according the composition of the structural system, boundary conditions and restraint conditions to the members. One fire with a relatively low temperature could cause more damage to the structure than another more severe fire if it affects a more important part of the structure. Therefore, a single value of temperature cannot define the limit state. Structural deformations or over-stress are excluded for one important reason: the structure's behaviour cannot be judged without reference to its temporal evolution. One will *not* accept excessive deflection during the evacuation time for instance, but is likely to accept it after the people and firemen evacuate.

2.14.2 Safety Thresholds

In Japan, building design for earthquakes follows two criteria. In a frequent earthquake, the structure should be designed to satisfy strength and serviceability limit states. In a severe (less frequent) earthquake, the design must satisfy strength limit state requirement but not necessarily the serviceability requirements. The approach is obviously probability-based since higher risk of loss of serviceability (second threshold) is accepted with the lower probability of occurrence. The safety is not compromised at either threshold, which is the least the society can expect.

In the UK and USA, allowable stress increase factors with wind or earthquake in the allowable stress methods are another example of probability-based safety concepts. They were specified to account for the fact that those loads are of a transient nature, and therefore a relaxation to safety measures is warranted.

For structures in fire, three safety thresholds are proposed:

- Time to evacuation
- Time for firemen to save any remaining occupants and save property
- Time to fire extinction

First Threshold: Time to evacuation

The *demand* is established through fire modelling. This time is normally the time required for *awareness, pre-movement and movement* plus a safety margin. This time should consider the proximity of the building to fire brigade, hospitals and emergency buildings and special cases where elderly people or children are involved,.

The *capacity* combines *safety* and *serviceability* for this case. The structure shall remain safe for evacuation throughout this time, the deflections and floor and ambient temperature limited to allow people movement.

Second Threshold: Time for Firemen to evacuate remaining occupants and save property

Extra time justified for firemen for the following reasons:

- Their risk of being injured is lower than ordinary people (no panic, protective gear, experience in dealing with fire)
- They could prevent progress of fire and thus reduce further damage, or overall collapse.
- They could save buildings of high cultural or strategic value.

The structure should remain safe for evacuation throughout this time. Deflections and floor and ambient temperature may exceed those in the first threshold since firemen are trained and equipped with protective gear.

Third Threshold: Time to Fire Extinction

This is normally the time estimated for full fuel consumption, oxygen starvation or the successful suppression of fire. Again the safety may not be compromised, but the deflections and temperature may reach any level as long as they do not initiate collapse. Key elements (*tree trunks*) should be designed to a smaller probability of failure, while other members may have lower reliability indices, in line with the practice in earthquake engineering.

Proposed Design Procedure:

1. Identification of the *structural*-fire scenarios: These are the fire scenarios most onerous to the structure, and do not necessarily include all the possible realistic fire scenarios.

2. Establishment of the computational model that incorporates the structure and the structural-fire scenarios.

3. Reliability analysis of the limit states, and establishment of *objective (performance) functions constrained* by safety requirements. *Safety constraints* are traditionally delivered by *target reliability indices.*

The design converges to a constrained optimisation problem. For example, if we use life-cycle-cost, LCC, as an objective utility and the probability of failure as constraints, we may define:

$$g_1 = t_{1-available} - t_{1-required} \qquad \text{Threshold-1}$$

$$g_2 = t_{2-available} - t_{2-required} \qquad \text{Threshold-2}$$

$$g_3 = t_{3-available} - t_{3-required} \qquad \text{Threshold-3}$$

The design is then optimised as:

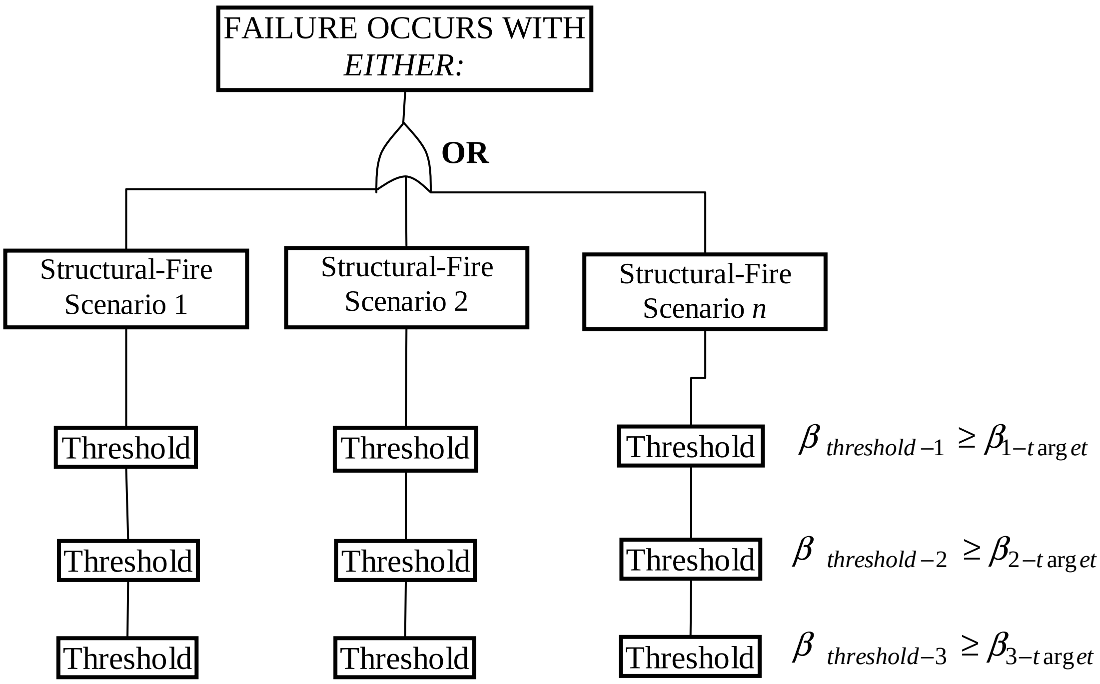
Minimise: LCC,

<u>subject to</u>:

$$p(g_1 < 0) \leq p_{1-target}$$
$$p(g_2 < 0) \leq p_{2-target}$$
$$p(g_3 < 0) \leq p_{3-target}$$

In the above inequalities, $p_{1\text{-}target}$ is the target probability of failure for threshold-1.

For all load scenarios, the reliability index must be kept constant for the relevant limit state (or threshold). This is better explained in the diagram of the Failure Event Tree in figure 2.8.



<u>**Figure-2.8** Failure Tree of Structure under Fire depicting Target Reliability Indices</u>

2.15 Concluding Remarks

A coherent approach to performance based design for structures in fire was introduced in this chapter. Statistical decision theory manifests itself as the core engine for calculated decision making. Following the final iteration of the design process, optimisation techniques are implemented to achieve an extremum of utility.

The case studies strongly support the implementation of performance based techniques; albeit neither study utilised probabilistic approaches in the analysis. Fire behaviour imposes significant epistemic and random uncertainties making irrational to rely solely on deterministic methods. Figure 2.9 is a generic outline of the performance-based process.

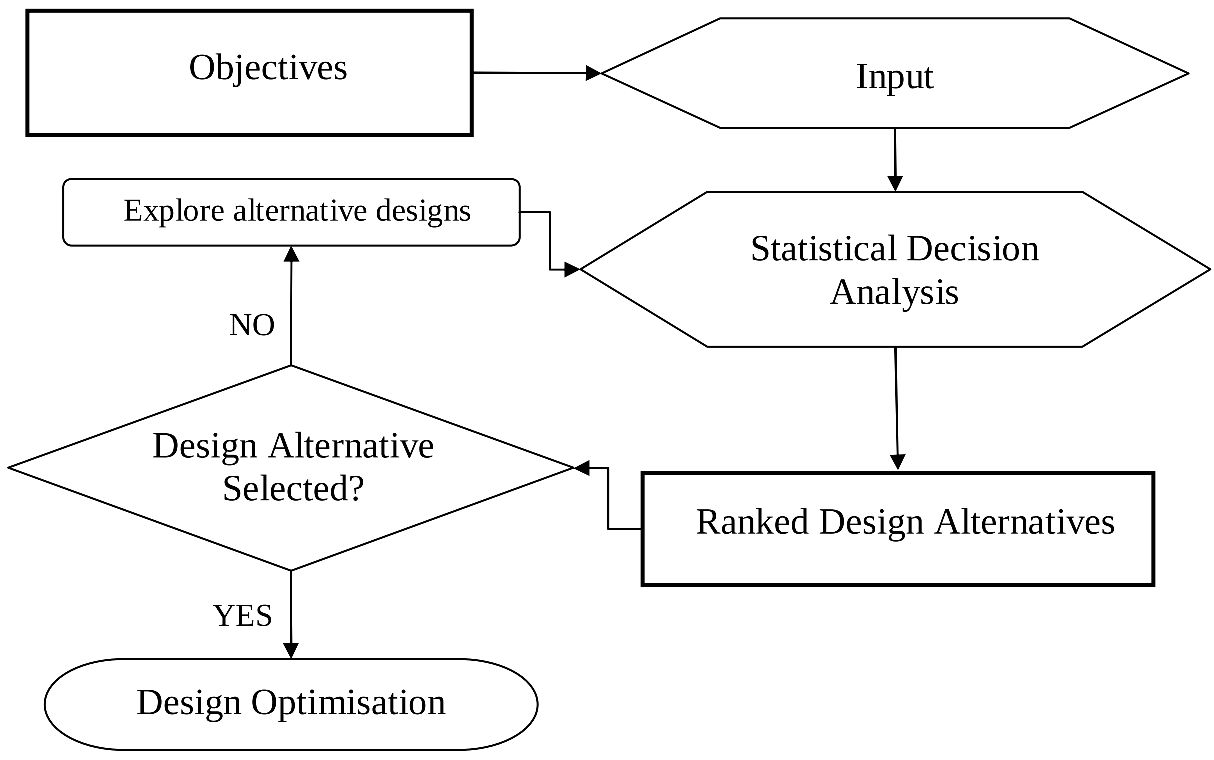


Figure-2.9 Core Engine of Performance-Based Design Process

Chapter 3

Risk Analysis

The construction of two 30m diameter by 10m high steel tanks in Angern, Germany, was described by Terzaghi [5, 41] and Selvadurai [42].

In 1930, two molasses storage tanks were to be relocated. They were previously supported on heavily reinforced concrete slabs resting on conical piles. The substrate for the new foundation was a stratum of soft clay containing small quantities of fine sand and silt 2m below ground, underlain by very stiff clay at 4m depth. Molasses load could be classed as fluid pressure, which resulted in a calculated uniform base load of 162 kN/m^2. Settlement calculations revealed an estimated maximum settlement of 10cm at the centre of a concave deformed base. The tank base was therefore designed for the curvature of 10cm over a 30m circular slab. A 10cm thick concrete slab was constructed as proposed by Terzaghi. Later observations of soil settlement were consistent with the initial expectation of a dish-shaped deformation albeit with few local irregularities.

In a neighbouring site with similar soil conditions, a similar tank was constructed, but on a 0.5m thick reinforced concrete slab supported by 1.1x0.48m ribs. Due to the rigidity of the base, the contact stresses were minimum in the centre and maximum at the edges. This induced large moment and shear forces in the tank's base slab which were overlooked in the initial design, leading to raft failure and the base rivets shearing. The tank contents flowed out and were lost.

Terzaghi commented that: "the heavy expenditure in constructing the rigid bottom merely lead to failure"[41].

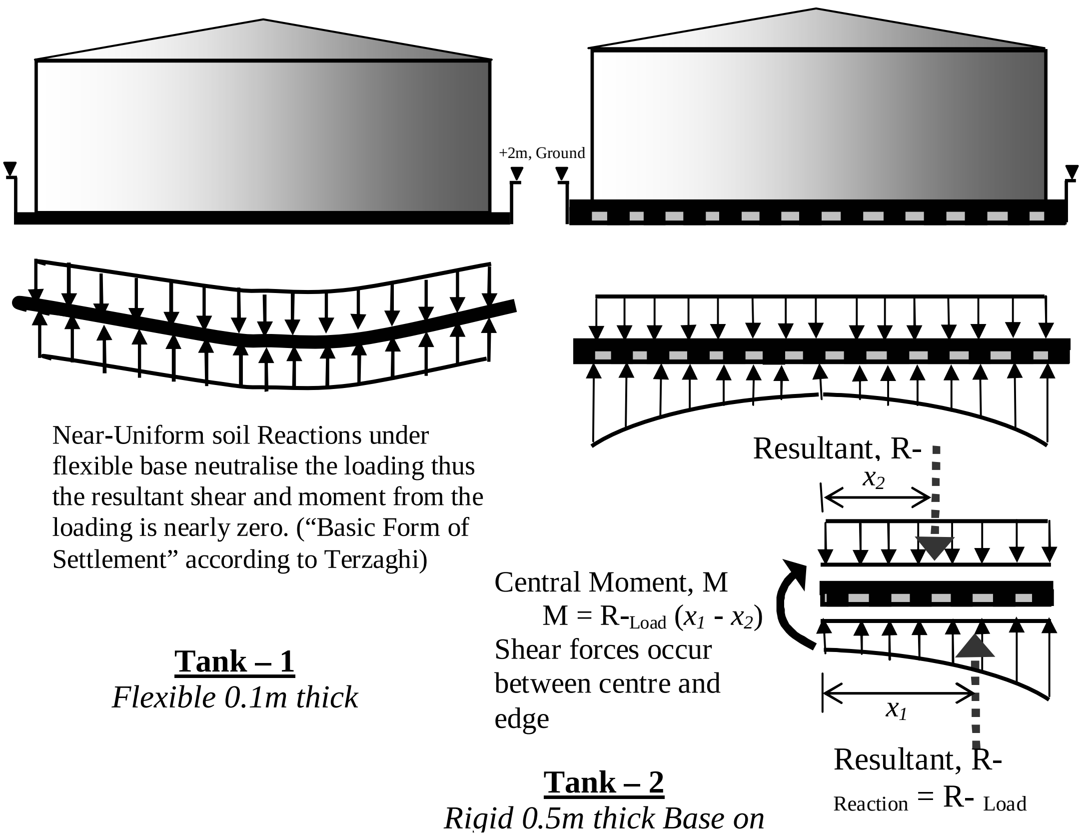


Figure 3.1: Generic Illustration of Tank and Soil Forces
(This illustrative figure is by this author)

The above example highlights the influence of uncertainty in design and construction. Two sources of uncertainty can be identified: one that relates to the uncertainty in soil parameters and site conditions (*aleatory* uncertainty), and the second to the methods of analysis that were used to carry out the design (epistemic uncertainty).

The work for the first tank followed an *a priori*, *posteriori* probabilistic logic. A priori soil investigative samples were used to infer general soil conditions, and rigorous analytical methods for the interaction between the tank base and supporting ground were used to calculate the base thickness. Post construction monitoring and observation of actual settlement were used to verify initial expectations.

The designers of the second tank presumed that extra expenditure could offset the uncertainty in soil properties. The problem was further compounded by their lack of understanding of the behaviour of slabs on elastic foundations (illustrated in figure 3.1). Such human error, or cognitive uncertainty, contributes to the majority of failures in structural engineering [2].

3.1 Risk Analysis

Risk arises from uncertainty. If one can be 100% certain of an outcome, decisions can be made entirely on merits without fear of whether or not or how a hazard may materialise. It would be possible to build facilities to exact specifications and without recourse to any safety factor.

The notion of risk expresses fear of the unknown. The two questions of essence are: is it, or is it not, going to happen, and how might it impact the venture? Thus that the classical formulation of risk discussed in the preceding chapter is:

$$Risk = f\,(Probability\,, Consequence) \qquad (3.1)$$

Risk is traditionally represented by the product of probability and consequence, or as sets of ordered pairs of probability and consequence on a graph [43], as in figure 3.2.

$$Risk = Probability \times Consequence \tag{3.2}$$

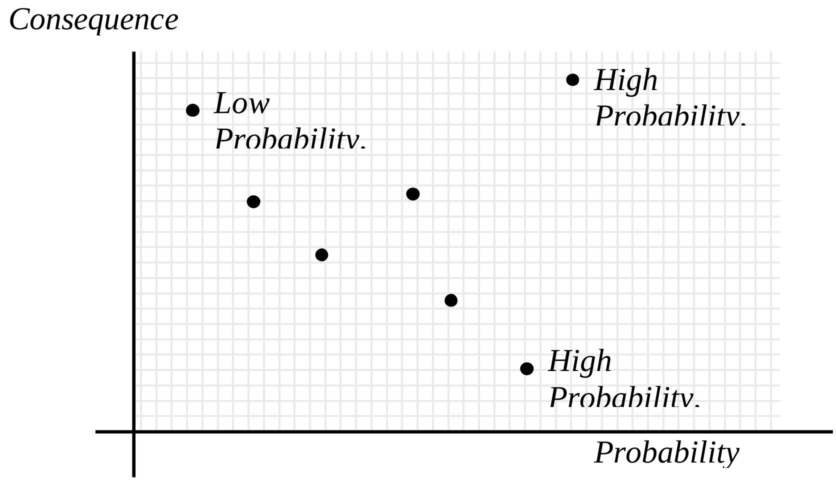


Figure 3.2: Risk as a Function of Probability and Consequence

Table representation is also used, commonly with a column ranking risk from low to high. Any of the above formulations falls within what is called *probabilistic risk assessment*, as described in figure 3.3.

If the consequence is undesirable, the risk is defined as a *negative* risk and *safety* is represented by its reciprocal or complement [44]:

$$Safety = \frac{1}{Risk} = \frac{1}{Probability \times Consequence} \tag{3.3-a}$$

or

$$Safety_N = 1 - Risk = 1 - \left(\Pr obability \times Consequence_N\right) \tag{3.3-b}$$

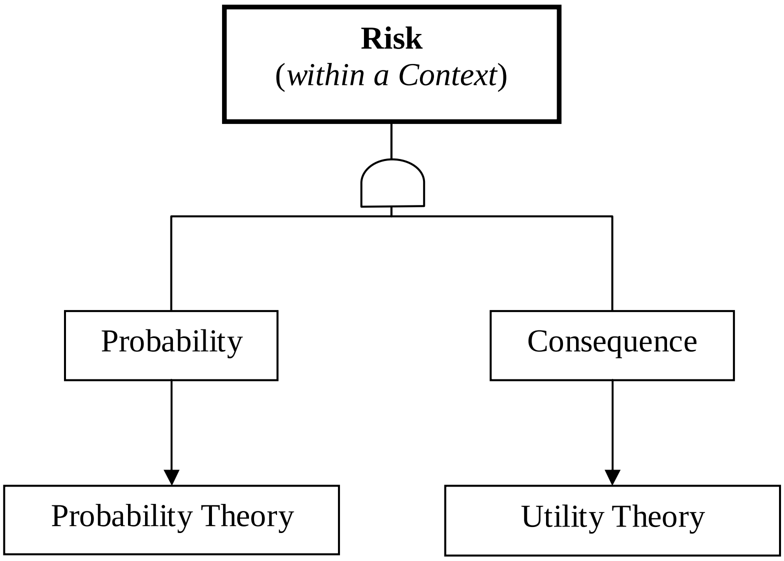


Figure 3.3: Risk Components and Mathematical Models

The subscript, N, in equation (3.3-b) indicates normalised values; i.e., full safety, S_N, or full consequence is equal to 1.Whichever format is used, safety increases if either or both of the probability and the consequence are reduced. In the hypothetical case of perfect safety, the argument holds that either the event is improbable or its consequence is not undesirable. Throughout this manuscript, the term “risk” refers to *negative* risk that is associated with undesirable consequences.

The former two components of risk can be defined within the realm of objectivity. *Context* is a third aspect of risk. It is easier to decide on a £100 purchase than £1 million project. This aspect is highly subjective and intertwines personal psychology of the decision maker with value assessment of the venture. It is also specific to the project under consideration. It is essential that the context of risk (people, utilities, everything affected by it) is established for the risk assessment to be meaningful. Defining *acceptable risk* would be irrelevant without a clear understanding of the context. A concise yet informative treatment can be found in references [4, 17].

The main purpose of this chapter is to introduce probabilistic methods, and more specifically reliability theory. It deals with the first element of the risk formula. The question of the second element of consequence can be generally addressed using utility theory which was briefly described in the last chapter. A more specific treatment of the consequence in terms of structural damage is presented in chapter 6.

3.2 Types of Risk

Risk can be categorised into four types: individually-perceived risk, collectively-perceived risk, calculated risk and real risk [17], as shown in figure 3.4.

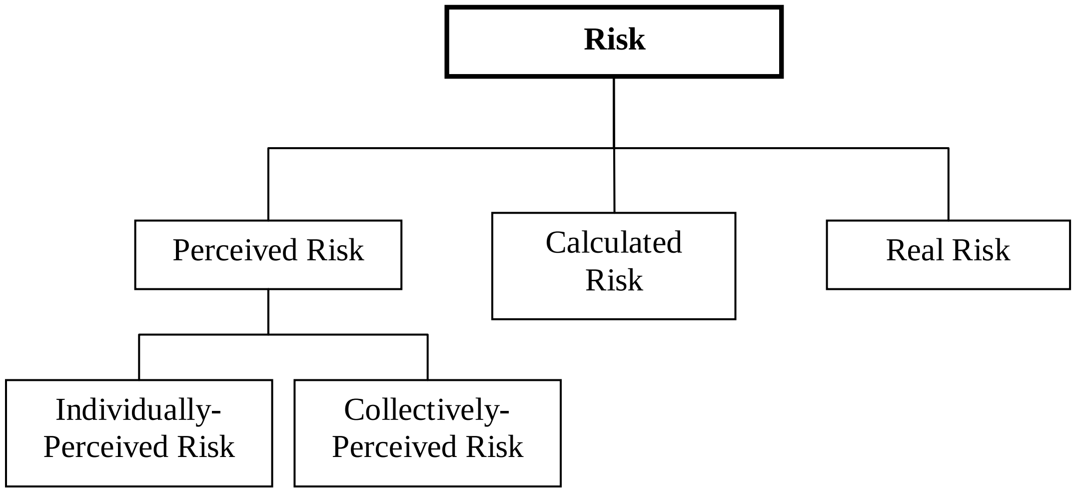


Figure 3.4: Risk Types

Individually-perceived risk describes one's personal understanding and feeling about risk. It is highly subjective and depends on the magnitude of risk, personal attitudes and individual attributes. For identical ventures, decisions can differ between risk-averse and risk-inclined individuals, and for persons with different levels of assets or experience.

Collectively-perceived risk represents a consensual evaluation of a group (usually a social group) towards a certain risk. It is a highly contentious matter as it depends on the political situation within the group and how information about risk is intercommunicated.

Calculated risk is the mathematical modelling of the impact of a future event. Structural design codes are one premise for calculated risk which is the main subject of this chapter.

The fourth type is real risk, which can be defined as the hypothetical calculated risk in the absence of epistemic uncertainty. The underlying assumption is that the measurements and cognitive methods used are sufficient and correct. It may be used where a significant amount of data can be gathered, such as traffic information for a road design, but is of little benefit for highly random and widely spaced occurrences such as earthquakes and tsunamis.

Making a decision is fundamentally a cognitive process whose input is the data from risk evaluation. It would be rather simplistic to assume that psychological factors can be neutralised. Equally, it is unrealistic to expect that decisions can be rationally made in the presence of the fuzziness associated with heuristics. The de-convolution of the two risk types is not natural, but is necessary to allow clear boundaries between design options.

3.3 Acceptable Risk

Evaluating risk depends on the perspective through which risk elements are viewed. The manner is which risk is represented and communicated has a direct impact on risk acceptance. Evaluating is *not* calculating risk. It is rather using the quantitative results to extract qualitative information that make decisions easier. It is about how the information is represented, akin to viewing it from different angles. People find ranking tables, for example, far easier to understand than abstract reliability indices.

3.3.1 Representation of Risk Data and Risk Evaluation

Three methods exist for risk evaluation: risk comparison, cost effectiveness and risk-cost-benefit analysis.

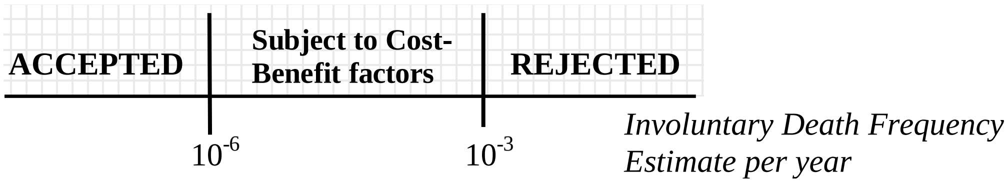
3.3.1.1 Risk Comparison

Risk comparison is based on comparing the risk of a project to recorded statistical risks of common hazards, such as wind, earthquakes or road traffic accidents. The idea is to argue that risk can be acceptable since a precedent of acceptance was established for a comparable event.

The above criterion has been applied with a clear distinction between voluntary and involuntary risks. People have different reactions to the nature of the hazard in question. Starr [4] postulated three hypotheses to describe public attitude towards death. The first concluded that the public are 1000 times less willing to accept death due to involuntary risk than voluntary risk. Secondly, the psychological upper bound for level of acceptability of risk can be taken as the statistical death rate due to disease. Thirdly, the acceptability of risk can be “crudely” [4] proportionate to the third power of associated benefits.

The first hypothesis underlines the mitigating effect of prior knowledge and understanding on the general perception of death. Death on the battlefield or that of stuntmen has less impact than that due to sudden strong earthquake. It is evident from the second hypothesis that the public have two views about death from disease, albeit subconsciously. One is that disease is not fully understood, no more than humans themselves, and the second is that the public have no option but to accept the existing disease fighting measures. Not unexpectedly, the third hypothesis highlights the different weights assigned to risk and benefit in the risk-benefit formula.

There is a general consensus on limits of risk acceptance for risk comparison purposes. These limits are from reference [4], and are summarised in Figure 3.5. The risk under consideration is the risk of death.



- The 10^{-6} compares to death from natural hazards
- The 10^{-3} similar to that from diseases for a 30 year old individual.

Figure 3.5: Acceptability Limits of Involuntary Death

The psychological impact of multiple-fatality events is significantly different from that of single or small number of fatalities. An account can be made to the public attitude to multiple deaths by using frequency-consequence curves, such as that in Figure 3.6. A suggested acceptance zone is shown below the dotted line for meteorite impacts, which is the shaded area in the figure. Statistics are for US population.

It must be noted however, that the frequency-consequence curves are based on statistics of natural hazards which contains no societal input. As a result, they ignore the general tendency of the public to avoid high-fatality single accidents. Its use without modification may therefore be un-conservative. A number of researchers suggested a modification by reading N^m from the abscissa instead of N for the number of fatalities, where $m > 1$. For example, for a frequency of 10^{-5} on the meteorite line, the maximum number of fatalities is 100. Road accidents are more frequent than air travel accident, but they result in small individual numbers of fatalities. Hence, for the yearly fatalities in road accidents with a 10^{-5} frequency, acceptable fatalities would also be 100. For an aircraft accident with the same 10^{-5}

frequency however, the acceptable number of fatalities would be $(100)^{1/m}$. If m is chosen as 1.1, then the acceptable risk for the aircraft accident would be 66 deaths.

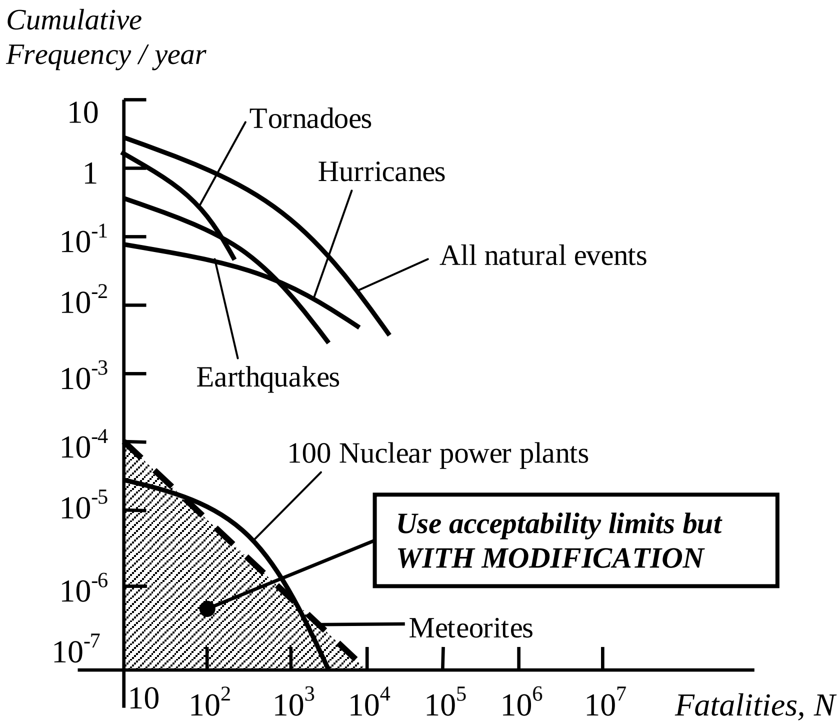


Figure 3.6: Frequency-Consequence Curve for Death Risk in the USA
(Adapted from ref. [4] and modified)

More rational methods based on empirical studies were also used. A study by Allen [2] on Canadian buildings concluded that an estimated 100 failures per year of the total 5 million structures in 1981, in Canada. Human error attributed to 90% of the failures, leaving only about 10% of the failures due to other causes. This yields an annual failure rate of 2 x 10^{-6}, or 10^{-4} in a 50-year design life.

In an interesting proposal to involve the number of persons at risk in a variable failure rate, Allen [2] suggested the following formula for the *acceptable* failure probability:

$$P_f = \frac{T\,A}{W\sqrt{n}} P_0 \tag{3.4}$$

where:

- P_f : Target failure Probability
- P_0 : *Basic* annual failure probability, 10^{-5}
- T : Lifetime of Structure in Years
- A : Activity factor (1.0 for buildings, 0.3 post-disaster, etc)
- W : Warning Factor
- n : Number of Persons at Risk

The above formula was simple and sufficiently accurate as it was derived solely from empirical data.

3.3.1.2 Cost Effectiveness

The acceptance criterion in this method is based on the cost of reducing risk. In PRA, the marginal costs of reducing risk are compared for number of potential alternatives. Cost effectiveness is achieved when the funds are allocated to the option with the lowest marginal cost.

The method obviously suffers from inconsistency in providing the cost of risk reduction for similar objectives. The expenditure on raising awareness of preventive measures of household fires is far less than that allocated to fire and rescue personnel for woodland fires. Yet, raising awareness targets a much greater number of people than would potentially be affected by wildfires.

3.3.1.3 Risk-Cost-Benefit Analysis

Combining risk, cost and benefit in a single model requires that all three elements are dimensionally consistent. Monetary value is the traditional unit used to represent all three.

Calculating the monetary value for property or service is simple. Assigning a monetary equivalent to life is a different matter. Several explicit approaches to establishing a statistical value of life are available and are covered in more detail in section 2.9 of chapter 2. Other methods refer to court rulings, especially regarding the level of compensation offered, as an implicit way of estimating the monetary equivalent of life.

3.3.2 Acceptable Risk Criteria

Perceiving risk is quite complex and can differ significantly for the same risk depending on factors that include qualitative and quantitative elements. Risk-based decisions are influenced by the following risk characteristics: nature of the hazard, exposure, and the consequences and benefits of accepting the risk [4].

The analysis of personal and societal attitudes towards risk is the domain of sociologists and psychologists [4, 45, 46]. The main scope is the study of the non-quantifiable heuristic factors that reflect social preferences.

Acceptance of risk is based on an amalgamation of qualitative and quantitative factors. From the societal or heuristic point, it depends on three factors: need, control and fairness. Does the risk need to be taken? Are there sufficient controls on the process that produces risk? And, are the risk, costs and benefits equally distributed among the public?

We need firemen. Hence they must be properly trained and equipped. And the whole public benefits from their service and are thus happy to share the cost.

In contrast, the primary function or PRA is the evaluation of costs and benefits associated with a probable event. Calculated risk can unavoidably be deficient as it does not account for intuitive risk, or when it does, it is usually implicitly embedded within the quantitative models.

The above disparity in approach led to a number of conflicted opinions. In quantitative risk assessment for example, a risk is acceptable if it is small. In intuitive risk assessment (IRA), a small risk may not necessarily be acceptable. Why should there be more deaths in road accidents than in rail accidents? Why can't the same control be exercised on both?

The most contentious issue is the treatment of death. While death is universally accepted in PRA, with equal statistical values of life, intuitive methods place different values to death depending on context. The public would look less favourably at the death of a victim in fire than that of a fireman who was attempting to save his life. The fireman was aware of and accepted the risk, and was simply doing his job. His death was tragic but most likely unavoidable, and could be classed *involuntary* considering his training and equipment. The victim's death may have been avoided had fire brigade arrived earlier or in larger numbers.

Quantitative risk analysis can demonstrate the efficiency of the fire service by simply comparing the cost of running the service plus statistical probability of firemen death to the statistical fatality rate of the public times the statistical value of life. Presenting such arguments, however, can be viewed as insensitive and sometimes politically incorrect.

Table 3.1 shows a comparison of intuitive and calculated risk in terms of approach, influential factors, purpose and analysis of risk evaluation data.

Table 3.1: Risk Assessment Methods		
Criterion	**Intuitive/Heuristic Risk Assessment, *IRA***	**Probabilistic Risk Assessment, *PRA***
Approach	▪ *Active* approach that averts risk *unless* it is needed and applied under satisfactory constraints. *Don't drive in snow. If you have to, use winter tyres. If nobody is going, there is no point in you going!* ▪ Based on *risk perception*	▪ *Passive* approach that accepts *all* risk equally, and ranks them in terms of associated economic cost and benefit. ▪ Based on *quantitative analysis*
Factors	*IRA = f (need, control, fairness)*	*PRA = f (probability, consequence)*
Purpose	To represent a societal perspective on risk in terms of value and impact	To ensure consistency of risk evaluation
Acceptance Criteria for: Risk Comparison	Risk is acceptable if it unavoidable (like involuntary), controls are dependable and costs and benefits are equally distributed among the public.	Risk is acceptable if it is small
Acceptance Criteria for: Cost-effectiveness & Risk-Cost-Benefit	▪ Risk compares well to risk characteristics (hazard, exposure, consequences, benefits), societal values and different mitigation options can be practically applied. ▪ Not all deaths are equal; it depends on circumstances. ▪ Statistical value of life can only be consistent for comparable risks	▪ The best option draws the least funds. ▪ All deaths are equal ▪ Statistical value is used to assign a monetary value to life

3.3.2.1 Acceptable Risk Criteria for Structural Design

Risks with a probability of occurrence between 10^{-7} and 10^{-6} are insignificant for legislative purposes [13]. (The source appears to relate to US legislation). This is consistent with figure 3.6.

The above rules may be used to examine which effects or loads need to combined for structural design. The theoretical background for the development in load combinations for structural design is briefly described in chapter 4. The following calculations are mainly excerpted from reference [13] to illustrate an example case.

Assume for example that fire, F, and wind, W, are considered for a building. Fire and wind are intermittent and can be modelled using Poisson pulse processes. The probability of coincidence for two intermittent Poisson processes can be inferred as [13]:

$$P(F \cap W) = R_F \times R_W \left(\tau_F + \tau_W\right) \tag{3.5}$$

, where τ_F and τ_W are pulse periods in years, for fire and wind; R_F and R_W are the mean annual rate of occurrence of fire and wind respectively.

Take a typical fire duration for 4 hours/year [13], or (4/(24x12x30) = $4.6\text{x}10^{-4}$ years. The rate of ignition can be taken as 10^{-6} /m^2/year (based on number of fires per the total number of buildings), and assuming that sprinklers are present, the probability of development to flashover of 10^{-2}. Therefore, for a typical 100m^2 apartment, the mean rate of flashover-fire is 10^{-6} /year. Statistics for non-tropical windstorms indicate a duration of 4 hours ($4.6\text{x}10^{-4}$ years) occurring 4 times a year.[13]

The above gives a probability of coincidence of fire and wind of $3.68\text{x}10^{-8}$ from equation (3.5). This is significantly less than the 10^{-7} threshold; hence no need to combine the two actions for structural design.

3.3.3 A Proposed Simple Formula for Acceptable Risk

The above criteria for risk acceptance are strongly related to the general attitude of the public to involuntary risks. Events such as earthquakes are beyond human control, and the public would submit to their consequences as unavoidable.

A reference accepted probability of an undesired event can be taken from statistics of an involuntary (e.g., natural) event. Let that be $P_{ref.}$. The risk associated with the reference event is $R_{ref.}$.

The difference in public attitude towards involuntary and voluntary risks can be established by statistical studies. Starr's hypotheses are an example. However, it is probably more realistic to establish criteria based on *comparative preference*. It is easier for a person to state how much more they're willing to accept one risk than another than to give abstract figures. The ratio of *public acceptance* of a risk under investigation, R_i, to the reference risk is v_i. The consequence of the reference risk and the investigated risk are C_{ref} and C_i respectively.

It is reasonable to design any utility to a *uniform acceptable risk*. The acceptable risk reflects public preference, and therefore includes the preference factor c_i. Hence,

$$R_{ref} = R_i \text{ , these are "Acceptable" Risks} \qquad (3.6)$$

This can be expanded into:

$$P_{ref}\ C_{ref} = \frac{P_i\ C_i}{v_i} \qquad (3.7)$$

If $C_{ref.}$ And C_i can be established from past statistics, this equation can be used in two different ways:

- To derive a maximum acceptable probability of occurrence for event *i*, $P_{i\text{-}acceptable}$, (say of fire), which can be implemented in the design of the process or building (say fire detection and active protection measures), or
- If the probability of occurrence, P_i, is accepted as fact from past statistics, to derive a relative value of acceptable cost as a consequence of event *i*, which may be suitable for use in overall cost and in compensation (except for acts of deliberate malice).

The above formula is used as basis for a propose target reliability index for use in performance-based design for structures in fire. This is described in more detail in chapter 4.

Example

Take earthquakes as the reference event, and use estimated annual figures 10^4 fatalities with 10^{-2} frequency. For a population of 300 million, the fatality rate is 3.3×10^{-5} per person/year. The consequence in this example is limited to the number of fatalities only. Assume that a survey indicated that the public are 1000 times less willing to accept casualties due to fire compared to earthquakes; i.e. v = 1/1000. The probability of occurrence of a developed fire for a $100 m^2$ apartment can be taken as 10^{-6}/year. [13]. Assuming an average of 4 persons per apartment, the probability of a developed fire is 2.5×10^{-6} per person/year. This yields an estimated maximum number of fire fatalities, C_i, that the public would accept as:

$$P_{ref}\ C_{ref} = \frac{P_i\ C_i}{v_i}$$

$$\Rightarrow\ C_i = \frac{v_i\ P_{ref}\ C_{ref}}{P_i} \quad = \quad \frac{(1/1000) \times 10^{-2} \times 3.3 \times 10^{-5}}{2.5 \times 10^{-6}} = 1.32 \times 10^{-4}\ \text{/person/year}$$

For the whole population, the above gives an acceptable number of fatalities due to developed fire as 4×10^4/year. It is larger than the fatalities due to earthquakes, mainly because fire is significantly less frequent, and its damage is limited to smaller areas confined to the vicinity of the initial fire. The figure must not be interpreted as the number of fatalities the public would accept every year, nor should the same logic be applied to the annual fatalities due to earthquake. The expected annual number of fatalities is the risk of death, which is the product of the annual probability and consequence. In the case of fire, that is (2.5×10^{-6} x 4×10^4 = 0.1 fatality per year), compared to (10^{-2} x 10^4 = 100 fatality per year) for earthquakes. The ratio of 0.1 to 100 is the acceptance ratio, v.

It would be interesting to compare the above figure with the statistics of the annual number of deaths due to fire. This can give an indication of the performance of the national fire safety strategy.

The public acceptance is largely influenced by past statistics. It is possible that fatality rates higher that the above are accepted if they are reasonable lower than the corresponding rates of previous years.

3.4 Safety Legislation

Legislating for safety is limited to defining the minimum functional requirements of elements or processes. The function of law is not to impose the specific aspects of acceptable solutions. It is more to define the forms of acceptable processes that may be applied to arrive at acceptable solutions. Thus conformity is achieved if it is established that an acceptable process had been used in carrying out the work, even if failure had ensued.

In most cases, legal instruments such as the building regulations contain sets of pre-qualified options that achieve conformity. Such means are deliberately left generic so as to permit a variety of solutions, and to prevent legal instruments from becoming a barrier to innovation. The detailed qualitative and quantitative procedures reside appropriately in technical documents, such as design codes and standards. In a way, the building law defines the manner of an acceptable design, and the technical standards facilitate its realisation, as illustrated in figure 3.7.

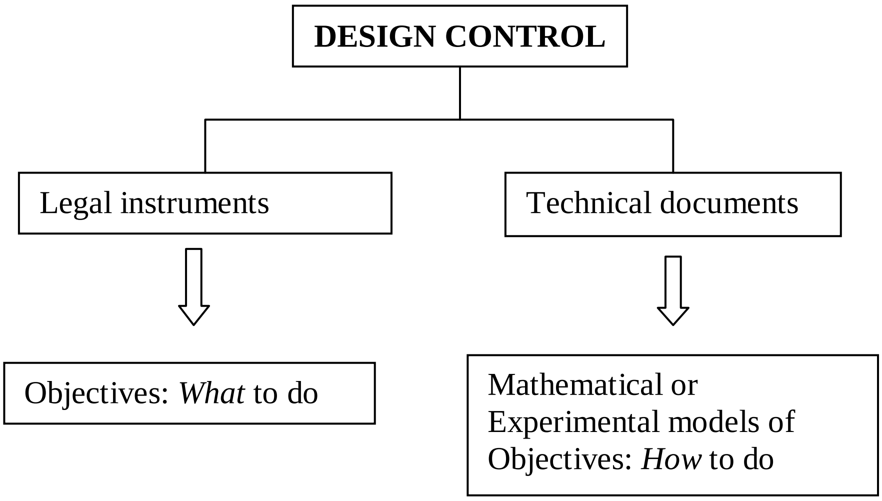


Figure 3.7: A Schematic Representation of the Structure of *Design Control*

The legal framework governing engineering works sets two acceptance criteria: prescriptive and performance-based. Both approaches serve the fundamental purpose of facilitating decision making.

The specific background objectives for prescriptive designs are obscured. Despite their technical context, the use of prescriptive solutions as deemed-to-satisfy solutions presents itself as a means of compliance with the law. It provides a simple decision-making formula: accept or reject. Owing to this binary property, all acceptable alternatives have exactly the same value in terms of functional performance. The factor that influences decisions simply becomes cost. The application of the risk formula is not possible, due to the absence of the probability element.

The primary distinction of performance-based design (PBD) lies in the *explicit* formulation of the design objectives. When evaluating a design alternative, a limit state function separating the demand objectives from solutions is established. In practical terms, these are the sets of functional requirements that must be met under predefined actions, such as deflection under load or available safe egress time during fire. The two element of the risk formula are handled separately: the uncertainty content in design parameters is managed uncertainty modelling techniques, whereas utility theory provides a suitable paradigm for the consequence.

The accountability process can be affected depending on the route taken. Ruling on whether or not the set of prescriptive rules are met is straightforward and relatively inexpensive. Arbitration is PBD, on the other hand, involves expert opinion, or opinions, can be lengthy and significantly more expensive.

Decisions made on grounds of coherent probabilistic risk assessment can be defended with ease. It is also vital that risk information is communicated clearly to individuals at risk and those with the responsibility of mitigating its effect. A common acceptable rule in arbitrations is: "*If the decision is the result of an acceptable decision process, then it is not necessary to agree with the decision to find it acceptable* [4]."

3.5 Conclusion:

It is unlikely that the complexity of risk can be captured in its entirety in an informative framework. For most economically-driven projects, even at the corporate level, probabilistic risk assessments remains the method of choice for decision making. Intuitive risk reflects the societal aspects of risk and is unfeasible to integrate in quantitative models.

Codes are not intended to manufacture optimum designs. Their main function is to ensure *uniformity* and *efficiency* in safety-driven engineering practice. IThey unify the language of design which simplifies communication and accelerates production in addition to providing measures for quality control. The task of producing cost-effective buildings or products is the responsibility of designers.

Where performance-based design is opted for, probabilistic analysis becomes the logical choice. It is the natural paradigm for a structure that is to be constructed in the future, for loads and actions that are forecast in the future.

Probabilistic risk assessment is a rational tool that *aids* (just *aids*) informed decision making. It should *not* be construed as a method that can completely evaluate risk, but rather a means of facilitating comparison and examining the effect of increasing risk or cost. It is neither entirely objective, nor absolutely accurate. A number of omissions, approximations and value judgements would be exercised in its preparation. It would remain valid as long as the assessment process is acceptable and consistent.

The use of probabilistic assessment without intuitive risk components puts the assessment out of context and of little, if any, value. Both approaches can be applied simultaneously by enforcing compromises on each. The extent of the realism of risk and the process of cost benefit analysis are better handled by probabilistic risk calculation, while the upper limit of acceptable risk should be set via a comprehensive heuristic study. The application is mostly at the national level involving code writers, the law establishment, psychologists, sociologist and economists.

Chapter 4

Structural Safety and Reliability

Structures and engineering systems are designed to respond to imposed actions in a manner that does not diminish their functional characteristics. The mode of response may be sustained, as in plastic deformation, or transient, such as vibration under wind or earthquake, or a mixture of both, as in the temporal evolution of deflection during fire. Satisfactory performance is achieved whenever the response does not diminish the fitness of the structure to perform the function for which it was built, for the length of its design life.

The above concept has long created controversy, a very productive one indeed. How many actions does one really *expect*? What data should be collected? How long should one observe to establish periodicity or aperiodicity? Can actions and effects be quantified? How *certain* are the calculated or tested design resistances? How accurate are the deterministic methods? *Who* sets the limits for *acceptance* of designs and *how*? Simple as they may seem, attempts to answer these questions have resulted in the birth of a vast array of interconnected disciplines of science and engineering.

Reliability engineering evolved from an ancillary technique embedded in safety evaluation to a highly sophisticated independent discipline.

The academic and public interest in safety was largely stimulated by unexpected failures. The word unexpected is underlined as it directs this discussion to its core. Had failures been expected, perhaps some mitigation would have been possible. When failures are expected, they are easier to understand, easier to avoid in the future, and more importantly, easier to accept.

The notion of expectation expresses an understanding of the uncertainty surrounding future ventures. Such understanding makes calculated values interpret as expected values, and leads directly to the methods of calculated predictions, more commonly known as probability theory.

4.1 Uncertainty Modelling

Uncertainty is a natural property of our knowledge sphere. It is why we accept tolerances and make extra allowances. The models we use in different branches of science are based on our knowledge of past observations. Within the relevant errors, they can be true representatives only of the measured samples of past phenomena. As we choose to extend their applicability to future phenomena, we assume the latter inherit the same characteristics. In addition, our knowledge is an assembly of tests and analyses, which contain a vast array of assumptions, approximations and errors.

The two contributors to uncertainty are the phenomenon itself, and the methods of measurement and analysis. Aleatory, non-cognitive, and random uncertainty are common term for the former, and epistemic, subjective, or cognitive uncertainty relates to prediction errors, and describe the latter [3, 47].

4.1.1 Sources of Uncertainty

Aleatory uncertainty refers to the non-cognitive elements of the phenomenon which characterise the inherent *randomness* that results in a range of observed values.[48] It is intrinsic to the physical phenomenon or parameter, and as such is naturally objective. The objectivity makes it easier to reduce by improving sampling and observation and increasing measurement accuracy.

The main sources of aleatory uncertainty are [3, 47]:

- The random characteristics of the physical process
- The use of sample statistics to infer information about the physical parameters
- Lack of knowledge

Epistemic uncertainty relates to the cognitive part of studying a process. For example, how certain can one be that all the parameters of interest have been included in a performance function? Evidently, cognition is a personal attribute, which leads to one important characteristic of epistemic uncertainty: subjectivity. The associated vagueness results in fuzzy sets which are beyond the practical capabilities of probability analysis, and better treated in the premise of fuzzy logic. [48, 49]

Epistemic uncertainty is associated with the following sources [3, 47]:

- The definition of parameters, such as objective or performance functions, indicators of quality such as skill of workers.
- Human factors, such as errors in measurement and mishandling of data
- Modelling errors that result from approximations and assumptions
- Definition of interrelations between parameters.

The mathematical models used to evaluate uncertainty depend on which of the two above types is considered, as is shown in figure 4.1. It must be noted the two types are heavily interconnected, and the fragmentation is for the purpose of convenience in analysis.

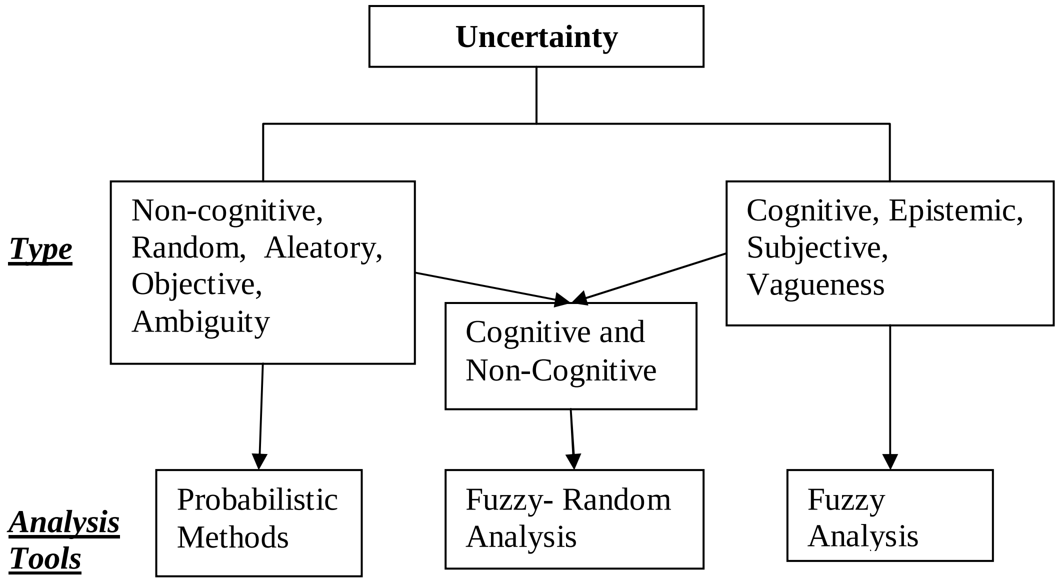


Figure 4.1: Uncertainty Type and Analysis Tools

Probability theory forms the basis of reliability techniques and is applied to assess non-cognitive or random uncertainty. It is based on a binary logic with elements either belonging or not belonging to a set. For element Z with a state variable x of 0 or 1, set S can be defined as:

$$S = \{Z : x = 1\} \qquad (4.1)$$

Fuzzy logic is used for cognitive uncertainties. Elements in fuzzy logic belong to a set with varying degrees of belief, or grade membership values ranging from 0 to 1 [48]. For a fuzzy element Y with membership function α_y, in set A:

$$A = \{Y \in A : \alpha_y \in [0,1]\} \qquad (4.2)$$

Any analysis involving fuzzy variables can be performed by using permutations of variables modified by their respective membership values, most commonly maxima and minima [47, 48].

4.1.2 Modelling Errors

Prediction errors, or modelling errors, can be incorporated within the random variable under consideration. The true "state of nature" [3] value of a single random variable X, can be represented by the product of the prediction error, ε, and the model value X_m.

$$X = \varepsilon X_m \qquad (4.3)$$

The first two moments of X_m can be obtained from the mean and variance of samples. Both the model values, X_m and ε, are random and can be assumed statistically independent, with means μ_{Xm} and μ_ε and coefficients of variation, δ_{Xm} and δ_ε. Hence, the mean value of X can be calculated as:

$$\mu_X = \mu_\varepsilon \mu_{X_m} \qquad (4.4)$$

A measure of its uncertainty of X is its coefficient of variation, δ_X, which is:

$$\delta_X = \sqrt{\delta_{X_m}^2 + \delta_\varepsilon^2} \qquad (4.5)$$

The above results may be generalised for a function, $Z = \varepsilon_f f(X_i)$, of multiple variables, X_i, yielding the following results [3]:

$$\mu_Z = \varepsilon_f \ f\left(\mu_{X1}, \mu_{X_2}, \ldots, \mu_{Xn}\right) \tag{4.6}$$

and

$$\delta_Z = \delta_f^2 + \frac{1}{\mu_f^2}\sum_i \sum_j \rho_{ij} c_i c_j \sigma_{X_i} \sigma_{X_j} \tag{4.7}$$

where,

$$\mu_{X_i} = \varepsilon_i \ X_{m_i} \ ,$$

ε_f is the mean prediction error, which is the bias of f

$$c_i = \frac{\partial f}{\partial X_i} \text{, at } \left(\mu_{X1}, \mu_{X_2}, \ldots, \mu_{Xn}\right) \text{, and}$$

$$\rho_{ij} = Correlation\ Coefficient\ between\ X_i\ and\ X_j$$

It is also possible to combine non-cognitive and cognitive uncertainties by incorporating membership functions of fuzzy variables. The resulting *fuzzy-random probability density function* contains the random distribution function, PDF, of random variables and normalised weight or membership functions, PMF, of fuzzy variables. More detailed description can be found in references [47, 49].

Decision is inherently a binary operation. It therefore requires a boundary surface separating acceptable from non-acceptable solutions, which must belong to crisp sets. By its mere position, a boundary surface sets the lower bound of acceptable solutions. A *limit state* in structural engineering is one such boundary surface.

Integrating fuzzy logic in design codes poses a difficult question to lawmakers and design professionals: how can design liability be identified in the absence of a clear cut-off between accepted and rejected solutions? The fundamental barrier to its application is the *subjective* nature of fuzzy elements. The verdict in any dispute is binary and requires rigid rules to enable its application. On the other hand, truth values, or membership grades, are subjective: what is good for one person may not be for another. If a decision based on a truth value results in failure, how can blame be apportioned if no clear lines were crossed? It appears that this is one reason why modern design codes are based on probabilistic methods, which exclusively deal with random uncertainty. Cognitive uncertainty due to lack of knowledge or skill, etc., is adequately catered for by insurance, such as professional indemnity insurance. Law enforcement becomes considerably easier when the safe and unsafe regions are clear.

The following treatment relates to structural design codes and therefore is restricted to reliability analysis.

4.2 The Framework of Probabilistic Design

Design is a rational assessment of an entity that is forecast in the future. It is a *process of prediction*. What we know about design is no different from what we know about the future: it is merely *expected.*

Probabilistic design extrapolates past observations a few steps in time so as to project the future condition of a utility. It thus presumes that the forecast utilities inherit similar demands and capacities.

The greatest benefit of probabilistic analysis is in how it effects realism in design. Studying the stochastic history of loads, for instance, reveals which loads need to be considered, which types should be combined, and how. It also directs research to areas most influential on the process at hand, as is evident in medical research. One may look at probabilistic analysis as a means of optimising the use of knowledge and our limited resources. A recent paper [13] analysed the coincidence of fire with other loads, and concluded that fire need not be combined with wind, non-sustained live, roof or earthquake loads, apart from when the fire is caused by the earthquake.

The overall process of probabilistic calculations is simple. It begins with selecting the parameters of interest and collecting data using any of the widely available measurement techniques. The data is analysed and subsequently refined into statistical models. The statistical models are extended to probabilistic models which are used to assess design, as illustrated in figure 4.2.

Figure 4.2: Probabilistic Design
(Reproduced from reference [1])

Random data collection and analysis is quite an involved subject and specialised sources should be consulted for further details [21, 50-52]. It is also of no relevance to this work. Statistical analysis is also beyond the scope of this chapter.

The above framework is strictly limited in scope. No means are available to establish that data is *sufficient*, since sufficient can only mean sufficient if *all* data is known or can be estimated. Records of fifty years of wind data cannot ensure that the next fifty years will witness the same weather conditions. The real scope is to have an environment where decisions about the future can be made by judging the past. Events that display higher uncertainty may then receive more effort to reduce the uncertainty, higher allowance to deal with variability or better social tolerance.

Uncertainty is intrinsic to all branches of human knowledge, not only engineering. This chapter however, concentrates on the uncertainty areas of relevance to structural engineering. It introduces reliability theory, which deals with the first element of the risk formula. The question of the second element of consequence can be generally addressed using utility theory which was briefly described in chapter 2. A specific proposal for treatment of the consequence in terms of structural damage is presented in chapter 6.

It must be emphasised that reliability is the subject of *neither* this chapter nor indeed this thesis. Its importance stems from the fact that it provides the mathematical foundation which describes the uncertainty in whether or not the design objectives are met, hence it fits appropriately in the framework of PBD. This chapter is at an introductory level, and readers interested in the subject have a wealth of excellent references at their disposal [2, 3, 43, 48, 49, 53-57]. A number of authors have kindly provided materials free of charge on the worldwide web. The second edition of Ditlevsen and Madsen's book "Structural reliability Methods", the reliability course notes at the Indian Institute of Science, publications of the Joint Committee on Structural safety (JCSS), NASA's publications, two of which are in references [58, 59], are some examples. "(http://www.web.mek.dtu.dk/staff/od/books/OD-HOM-StrucRelMeth-Ed2.3.7-June-September.pdf), (http://nptel.iitm.ac.in/courses/Webcourse-contents/IISc-BANG/Reliability%20Engg/New_index1.html), (http://www.jcss.byg.dtu.dk/).

4.3 Brief Review of the History of Structural Reliability Engineering

The oldest record of building codes dates back to Hammurabi (1750 BC) [60] in ancient Babylon in Iraq. Having confined the scope to punitive measures for unsatisfactory performance, they were more of a legal instrument than a technical design code. Hammurabi's code could be classed as a single-objective *performance-based design* code specifying the "what" but not the "how": the building must not collapse, nor cause harm to users. With that in mind, all options were open to designers to achieve the design intent. The magnificent structures they built are but a testimony to what profuse inspiration was afforded by that culture of freedom.

Unfortunately, we have little or no records of any design codes that may have been used by other civilisations.

The evolution of building code as an independent entity started in 1189 with the party wall specification in London's building regulations. The code specified 3 ft thick by 16 ft high party walls aimed at controlling the spread of fire. The set of regulations grew to include prohibiting combustible roof covering and a 9 ft limit on the minimum height of building projections over streets so as to allow the movement of people on horses [60].

Codes continued to grow during the sixteenth and seventeenth centuries in the UK. The desire to control the spread of disease in cities resulted in regulating the density of people by banning infill and subletting.

Failure is an established motivator for development. This has typically been the case in structural design codes. The 1666 great fire of London stimulated the development of a comprehensive suite of codes, focusing on the control of fire spread. Structural design requirements were stipulated for masonry walls and timber in the first appearance of distinct structural codes. Health and safety rules emerged following death and injuries due to building collapse. The technical requirements for quality control of brick and mortar were introduced during the seventeenth to the nineteenth centuries.

The introduction of reinforced concrete and steel in the nineteenth century paved the way to emergence of the Working Stress Design (WSD) code in the early twentieth century.

The earliest building codes in the United States date back to the New Amsterdam 1625 regulations on roof types [61]. Restrictions on wooden chimneys and thatched roofs were introduced in Boston in 1630, and New York adopted the first code in the US in 1850. The proliferation of codes in the Us began in the early parts of the twentieth century by a number of trade organisations, like the Building Officials Conference of America (BOCA), who published its first code in 1950. BOCA's code was used in the Northeast and Midwest of the US but large cities opted for their own codes owing to their unique building systems and political influence.[61]

Codes have always been written for the purpose of reducing risk. The development of mathematical models in probability theory resulted in a natural progression of codes from a product of experience and judgement to reliability-based technical standards. Most reliability engineering research and development occurred in the twentieth century. The following is an abbreviated version of some noteworthy work, and more comprehensive reviews can be found in references [2, 60, 62]. Dates in brackets for the following two paragraphs are from reference [2].

In the twentieth century, the implications of uncertainty on structural safety were realised as early as the 1920s. Forssell (1924) proposed the principle of optimality, stating that the purpose of design is to minimise the total cost which comprises the initial cost plus the cost of failure. Probabilistic-based design framework appeared as early as 1926, when Mayer suggested design based on *Mean* & *Variance* as safety measures which is similar to later proposals by Basler (1960) and Cornell (1974). Mayer's work, however, found no application in design offices. Pierce (1926), Tucker (1927) and later Weibull (1939) introduced the *weakest link theory* that stated that a chain is as strong as its weakest link. Weibull (1939) presented a comprehensive treatment of statistical methods in strength of materials. The work of Freudenthal (1947) on the fundamental problem of safety under random loads was the first to initiate acceptance within the design community. Plum (1950) noted the discrepancy between observed low failure rate of reinforced concrete slabs and the economically-optimum safety levels in design. Johnson (1953) introduced the theory of *structural reliability* and *economical design*, including statistical methods developed by Weibull (1939). Baker (1956) derived the weighted safety factors shown in the following section. In 1960, Basler proposed safety measures based on mean and variance akin to Mayer's (1926) work. Lind *et al* (1964) characterised rational design in a code as a process of selecting a set of *best* values of loads and resistance. They suggested an iterative procedure for the load and resistance values, with the code being a control "black-box" to optimise safety and cost.

The period from 1967 to 1974 witnessed a surge in the academic interest in structural reliability theory. Design professionals on the other hand, were still reluctant to use what seemed to be too radical at the time. Their argument rested on a number of issues: imperfect mathematical models were used to arrive at near-perfect rational design paradigm, the computational onus was cumbersome and therefore prohibitive, and insufficient data was available to perform statistical analysis with reasonable accuracy. Very few felt the need for change in the first place; when deterministic methods performed so well with very few failures, usually attributed to human error or "Acts of God".

Cornell proposed the second-moment *reliability index* [2]. His index was based on a linear approximation at the mean values of load and resistance; hence it suffered from inconstancy to equivalent mechanical formulations of safety. Further refinements by Hasofer and Lind resulted in the development of Hasofer-Lind reliability index, which the most widely used safety index today. The relation between the reliability index method and practical design was confirmed by Ravindra *et al* [2]. Further modification followed which lead to the evolution of a number of codes, such as the CSA (Canada, 1974), NKB (Norway, 1977) and OHBDC (Canada, 1983).

Some researchers questioned the viability of reliability-based methods in describing real life processes. Human error, the main suspect in every case for failure, is not accounted for in the formulation of those methods. Understandably, human behaviour is far too complex to model, and any such attempt would have to involve psychologists and sociologists: an option that is not only intractable to the concerned parties but hardly feasible in practical terms. A survey by Matousek [2] on 800 failures concluded that structural failures were almost always due to gross human error. As a result, such failures do not necessarily occur at strength levels close to the mean strength. In other cases, failure was caused by exceptionally high loads, again far from the mean design loads. Brown [2] showed that the failure rates predicted by the theory were too small. As an example, failures of suspension bridges were at a 1:40 rate in the twentieth century.

The former arguments no doubt devalue reliability methods as tools for modelling “real structural behaviour”. This is very true. The main purpose of a code is to draw policies that “control” the design process by implementing a trade-off between safety and economy. It can be argued that reliability methods provide a tool for such “control”. More understanding of real performance is vital for code development; but how can the “more understanding” be input to the code without a rational framework?

Reliability methods present a balance formula between load and strength. Each side is carefully segregated from the other, so that additional knowledge of either side can be input in the corresponding probabilistic model. The balance mechanism does the rest: more knowledge about loads leads to smaller *load factors* and thus greater loads are allowed, and the same logic applies to the strength side.

Silo structures present an excellent example of the importance of rationally-based design standards. According to Rotter [63], defective standards for loading and design contributed to the high rate of silo failures (over 1000 failures in the 1980s in North America). Oversimplified loading regimes coupled with simplistic behaviour models were the main causes of the exceptionally high failure rate.

4.3.1 Working (Allowable) Stress Design (WSD/ASD)

This method has been by far the most popular in the engineering design offices. The basis of design is quite simple; estimated actual values of actions are selected, then the structure is designed so that the effect of the actions is within the *assumed* linear response, with the capacity in excess of the failure effect by a pre-determined safety factor. Baker, in 1956, proposed a simplified method for calculating the safety factor based on probabilistic evaluation [64]. The method accounts for the different weights of factors affecting the capacity, as shown in Table 4.1 below. The onus of selecting the appropriate weight was left on the design engineer.

Table 4.1 - Baker's Weighted Safety Factor [**Safety Factor = 1.0 + Σ Weight / 10**]

Weighted Failure Effect	**Minimum Weight**	**Maximum Weight**
1. Results of Failure:	1.0 (less serious)	4.0 (serious)
2. Workmanship:	0.5 (cast in place)	2.0 (factory-manufactured)
3. Load Conditions:	1.0 (e.g. load cases including wind)	2.0 (simple spans or sustained loads)
4. Importance of member in structure		0.5
5. Warning of Failure		1.0
6. Depreciation of Strength		0.5

WSD specifications incorporated parameters to cater for the frequency of occurrence for loadings or undesirable events. Stress increase factors were prescribed to amplify the allowable stresses for transient loads such as wind or earthquake loads. Evidently, whether explicitly employed or not, a probabilistic dimension mobilised the thinking process behind the categorisation.

One of the fallacies that designers had to contend with was that the true factor of safety remained unknown. Design was acceptable as long that the structure *appeared intact*, even though it may be on the verge of failure [41]. The link between the predicted value of actual factor of safety and the target value could only be established by probabilistic techniques. An appreciation of this concept, further motivated by accidents such as the NASA challenger accident in 1986 [59], resulted in elevating safety treatment to Limit State Design.

4.3.2 Limit State Design (LSD)

The shortcomings of WSD insofar a lack of clear connection to the failure state, paved the way to Limit State Design (LSD), also known as load and resistance factor Design (LRFD) in the United States. Satisfactory design must attain a *capacity* that equals the demand as minimum, and is conventionally amplified by a safety factor. The critical condition at which capacity equals demand is called the *limit state*.

This method was first used in the former Soviet Union and other East European countries in the 1940's [2], and formed the format of the American Concrete Institute code in the early 1960's. Research work based on probabilistic techniques and statistics on behavioural models for the limit states followed rapidly, and limit state methods are now the norm in most countries. Reliability engineering evolved from a method embedded in safety analysis to an advanced engineering discipline.

The format for limit state design varies between codes but the underlying methodology is the same. The *capacity* of the structure, R, is calculated, and then reduced using a *factor*, φ, to account for probabilities of under-strength due to the use of nominal section properties, approximations in computational models, generic material properties and defects in construction. This factored capacity is then compared to the load effects, Q, that are amplified by the respective load factors, γ, that account for uncertainties of loads. Thus, for the structure to be safe and serviceable, it should satisfy the following condition

$$\varphi R \geq \gamma Q \Rightarrow Structure\ is\ Safe \tag{4.8}$$

The values of the capacity reduction factor, φ, and the load factor, γ, depend on the uncertainty in the structural response to certain actions (tension, flexure, shear, torsion) and the load effect (dead, live, snow, wind). The objective of modern LSD codes is to ensure that the probability of failure is *acceptably* small, and a trade-off between safety, economy and practicality is employed to justify acceptable failure probabilities. This concept forms the backbone of the determination of the above capacity and load factors.

4.4 Reliability Analysis

Reliability is defined as the ability of a structure or an engineering system to sustain successful performance over a certain period of time, e.g.; design life.

In terms of design approach, reliability is no different a process from any other. Both *prescriptive* and *performance-based designs* can be implemented to perform reliability calculations.

Failure can occur whenever the demand, Q, exceeds the capacity, R, any point of time during the design life. The probability of failure, P_f, can then be defined as:

$$P_f = P(Q > x) \big| (R \leq x) \qquad (4.9)$$

, where x is a dimensionally-consistent random variable common to R and Q.

The reliability or safety set is the complement of the failure set; hence the probability of safety, or reliability, P_s, is:

$$P_s = 1 - P_f \qquad (4.10)$$

The principal focus of reliability analysis is the determination of the probability of failure, P_f. If $f(\mathbf{x})$ is defined as a joint probability density function (pdf) of a vector, $\mathbf{x}$, of random variables representing demand (Q) and capacity (R), the probability of failure can be expressed as: [3, 43, 62, 65]

$$P_f = \int_{R<Q} f(\mathbf{x})d\mathbf{x} \qquad (4.11)$$

The target of all reliability engineering methods is the computation of the above integral. Apart from very few practical problems, neither analytical nor computational solutions are conceivable for the above integral in its basic form. It is therefore customary to de-convolute the above integral by applying proper transformations to normalise and de-correlate the random variables thereof. The result is a finite number of independent or multi-normal integrals representing the marginal probability density functions. [66]

$$P_f = \int_{R<Q} f(\mathbf{x})dx = \int_{\Omega} \prod_{i=1}^{n} f(x_i)dx_i \qquad (4.11\text{-a})$$

In the last expression, Ω is the set of the intersection of domains of functions, $f(x_i)$. In performance-based design, the solution is carried out explicitly via direct computation of the probability of failure, P_f, using simulation-based methods. Prescriptive methods, on the other hand, adopt an implicit approach by using approximate methods to compute surrogate parameters such as the reliability index, β.

Table-4.2: Reliability-Oriented Design Methods

Design approach	Order of Accuracy		Qualify as:	Remarks	
Optimisation of Utility	Level IV		Performance-Based	Client specifies acceptable risk	Most Accurate
Simulation-Based Methods	Level III		Performance-Based (with prescribed risk) – Full (or partial) reliability spaces used	Code prescribes acceptable risk via target reliability indices	Compromise of Level IV
First-Order Reliability Method (FORM or FOSM) & Advance Second Moment (ASM)	Level II	Decreased Accuracy	Prescriptive – *representative moments* of random variables (e.g.; mean and variance) used	Code prescribes acceptable risk, but full probability distribution of variables is not considered	Compromise of Level III
LSD, LRFD, ASD or WSD * (Note: Old WSD employed a *non-probabilistic* safety factor)	Level I		Prescriptive – Partial Load & Resistance factors imposed on *nominal* values to achieve target indices	Target indices specified by codes – No explicit probabilistic design – FORM or SORM applied on *generic* probability distributions	Compromise of Level II

* LSD: Limit State Design, LRFD: Load & Resistance Factor Design, ASD: Allowable Stress Design, WSD: Working Stress Design

In simulation-based methods, computation of the full (or partial) probability space is carried out using the classical definition of probability [58, 67]. The integral can be evaluated directly by numerical integration or asymptotic expansion [62]or indirectly using probabilistic techniques such Monte Carlo simulation [2, 3, 43, 53]. In a Monte Carlo analysis, if R is the subset of random variables representing the resistance, and Q is that of load, and a pseudo- random set of N analyses of a performance function $g(Q,R)$ yield a failure subset of $n_{R<Q}$, then:

$$P_f = \lim_{N \to \infty} \frac{n_{R<Q}}{N} \qquad (4.12)$$

The limit state surface lies at the set where $R=Q$. The technique is conceptually simple but computationally burdensome since a large N (usually one order of magnitude higher than $1/P_f$) is needed for an accurate estimate of small values of P_f [53, 58]. However, since most failures are observed in the tail region away from the mean, variance reduction techniques, such as importance sampling, have been developed to confine sampling to the areas of highest likelihood of initiating failure, which reduces the number of samples required in the simulation.

Approximate methods target optimum extreme probabilities, so the use of full probability functions is not necessary [57]. They are computationally affordable as they only use representative moments of probability distributions. Examples are the First Order Reliability Method (FORM/FOSM or the Advanced Second Moment, ASM) and the Second Moment Reliability Method (SORM) [2, 3, 43, 53].

4.4.1 The Reliability Index, β

For a vector, **x**, of basic design variables, the following is arbitrarily defined: $g(\mathbf{x})$ is the performance function where $g(\mathbf{x}) > 0$ represents the safe set, $g(\mathbf{x}) < 0$ is the failure set and limit state is at $g(\mathbf{x})=0$.

Formulating a solution for a joint probability distribution for real-life engineering problems is practically unfeasible. The alternative approach has therefore been to rewrite the basic performance problem in a format where the above formulation applies. In essence, **x** is mapped to **u**, and as corollary, $g(x)$ is mapped to a mirror thereof, say $h(u)$.

If we had a standard normal joint pdf of design variables **u**, then the probability of safety, P_s, can be calculated as $\Phi\ (u^*)$, where Φ is the joint cumulative probability density function, CDF of **u**. By definition, u^* is the number of standard deviations from the expected value, $E(\mathbf{u})$, to some point, u_i.. The condition for safety is that the coordinates of u_i are such that the performance function, $h(u_i)$ > zero. *Let* u^* be β.

Since β is directly proportional to safety, it is called the *reliability* or *safety index*. Furthermore, as the purpose of safety analysis is to determine the minimum level of safety of a system, the target of reliability calculation is usually β_{min}. Logically, β_{min} occurs at the boundaries of the safe set; i.e, at $h(u) = 0$, or the *limit state* surface.

Appropriate transformations are used to transform the component random values in **x** into a vector of uncorrelated standard normalised variables, **u**, such that $E(\mathbf{u})=0$ and $Cov(\mathbf{u},\mathbf{u}^T)=\mathbf{I}$, with $E(\mathbf{u})$, $Cov(\mathbf{u},\mathbf{u}^T)$ being the expected value and the covariance matrix of **u**, and **I** the identity matrix [2, 3, 16, 43, 68]. Since the mean-value point of **x** is mapped into the origin of the transformed variables **u**, the reliability index is the minimum distance from the origin of **u** to the mirror of the limit state surface in **u**, $h(\mathbf{u})$, as shown in figure 4.3.

$$\beta_{\min} = \beta = \min \sqrt{\mathbf{u}^T\mathbf{u}}, \text{ subject to } h(\mathbf{u}) = 0 \tag{4.13}$$

The method of Lagrange's multipliers can be used to convert the above constrained to an unconstrained optimisation problem, and gradient-based optimisation techniques usually form the basis of the solution algorithm described in the subsequent section. The limit state surface is iteratively approximated around design points using Taylor expansion and either the first or the second term of the expansion is used depending on whether FORM/FOSM or SORM are implemented. [43, 69] Several algorithms exist for solving the above problem, such as Hohenbichler and Rackwicz algorithm discussed in the following section [3].

The above reliability index is known as Hasofer-Lind reliability index, β_{HL}, and is reasonably accurate as long as the radius of curvature of the limit state surface is sufficiently large compared to β_{HL}.

Hasofer-Lind's index supersedes earlier formulation by Cornell that produced inconsistent β values under equivalent mechanical formulations. Because the approximation was made at the mean values in Cornell's method, changes in the formulation of the *same* limit state surface (for example: from Margin = R - Q to Margin = R/Q - 1) resulted in inconsistent values for the reliability index , since the *same* mean values of R and Q are used to calculate the first and second moments for the Margin in *different* formulations. In Hasofer-Lind's case, the calculation of the reliability index is that of a *geometric* distance, which remains consistent since mapping variables into another coordinate system does not affect the minimum distance from the origin to the limit surface.

Other formulations for *a generalised reliability index* exist in literature [2, 54] where the case is such that the radius of curvature is not large relative to β_{HL}. These methods are however computationally intensive and may not be justified since simulation techniques have verified the accuracy of β_{HL} for most practical applications [2].

The probability of failure, P_f is the complement of the probability of safety, P_s. Hence:

$$\begin{aligned} P_f &= 1 - P_s \\ &= 1 - \Phi(\beta) \\ &= \Phi(-\beta), \quad \text{since } 1 - \Phi(x) = \Phi(-x) \end{aligned} \tag{4.14}$$

Due to the approximations in linearization and the use of moments instead of the full probability distribution functions, it is more appropriately expressed as:

$$P_f \cong \Phi(-\beta) \tag{4.15}$$

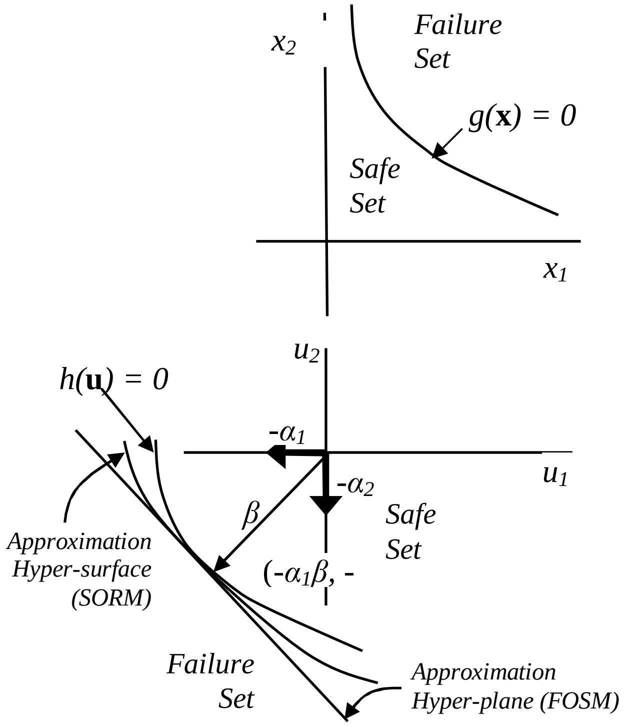


Figure-4.3: Geometric Representation of Reliability Index, β
(Reproduced and modified from reference [6])

A direct conclusion of the last formulation is that β is less sensitive than P_f to changes of the basic random variables, by virtue of the properties of Φ. It is therefore suitable for comparing designs (as in figure 4.4), which has been invaluable to the development of limit state or LRFD codes based on value judgement of existing designs.

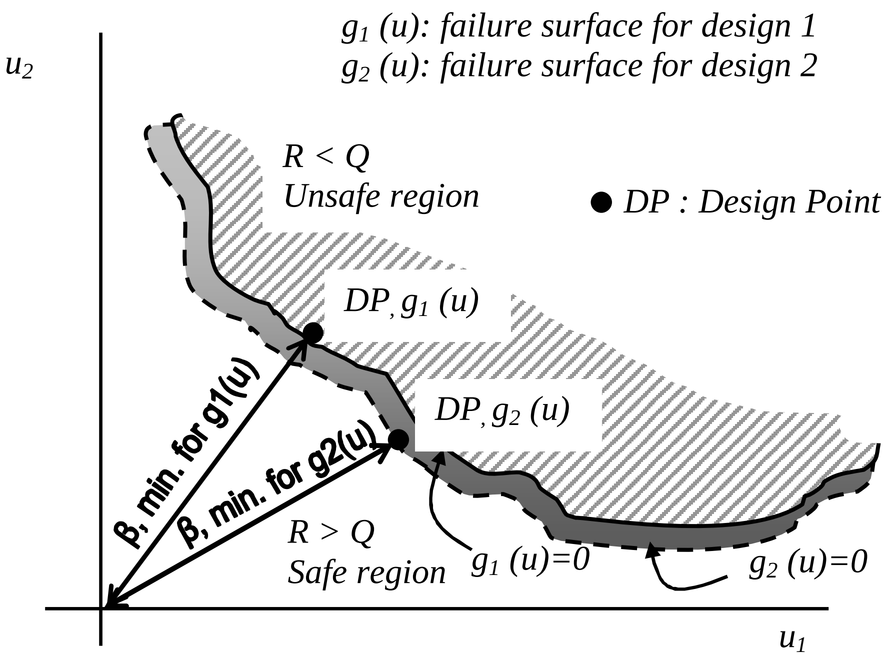


Figure 4.4 Comparing Alternative Designs

4.4.1.1 Algorithm for Calculating the Reliability Index, β

The following details of the Hohenbichler-Rackwicz algorithm are taken from Appendix B.2 of reference [3], apart from the note on the mean values in step 1). The Rosenblatt transformation is used to transform non-normal correlated variables to uncorrelated standard normal variables.

1- Assume a failure point, $\mathbf{x}_\circ^* = \mathbf{x}_\circ$ [usually the mean values are used [43]]

2- Using Rosenblatt transformation, transform the *x* variables in the original space to *u* variables and obtain the corresponding point in the *u*-space (Appendix B.1 of the same reference).

3- Calculate the Jacobian matrix, **J**:

$$\mathbf{J} = \frac{\partial\left(u_1, u_2, \ldots\ldots, u_n\right)}{\partial\left(x_1, x_2, \ldots\ldots, x_n\right)}$$

$$= \begin{bmatrix} \frac{\partial u_1}{\partial x_1} & \frac{\partial u_1}{\partial x_2} & \cdots & \frac{\partial u_1}{\partial x_n} \\ \frac{\partial u_2}{\partial x_1} & \frac{\partial u_2}{\partial x_2} & \cdots & \frac{\partial u_2}{\partial x_n} \\ \vdots & \vdots & \vdots & \vdots \\ \frac{\partial u_n}{\partial x_1} & \frac{\partial u_n}{\partial x_2} & \cdots & \frac{\partial u_n}{\partial x_n} \end{bmatrix}$$

, evaluated at $\mathbf{x}_\circ$

The partial derivatives can be calculated using implicit differentiation as:

$$\frac{\partial u_i}{\partial x_j} = \frac{\partial \Phi^{-1}\left[F\left(x_i \middle| \cdots\right)\right]}{\partial x_j} = \frac{1}{\phi(u_i)} \frac{\partial\left[F\left(xi \middle| \cdots\right)\right]}{\partial x_j}$$

F is the joint cumulative distribution function of the original variables, $\mathbf{x}$

Knowing that for $i < j$, $\frac{\partial u_i}{\partial x_j} = 0$, the Jacobian would be a lower triangular matrix whose inverse can be easily obtained using back substitution.

4- Evaluate the performance function and gradient vector at $\mathbf{u}_\circ$:

$$g\left(\mathbf{u}_\circ\right) = g\left(\mathbf{x}_\circ\right)$$

$$\mathbf{G}_{\mathbf{u}\circ} = \left(\mathbf{J}^{-1}\right)^t \mathbf{G}_{\mathbf{x}\circ}$$

5- Obtain a new failure point:

$$\mathbf{u}^* = \frac{1}{\mathbf{G}_{\mathbf{u}\circ}^t \mathbf{G}_{\mathbf{u}\circ}} \left[\mathbf{G}_{\mathbf{u}\circ}^t \mathbf{u}_\circ - g(\mathbf{u}_\circ) \right] \mathbf{G}_{\mathbf{u}\circ}$$

The corresponding failure point, $\mathbf{x}^*$ in the *x*-space (of the original variables), for the above point $\mathbf{u}^*$ can be calculated using first order approximation as:

$$\mathbf{x}^* \cong \mathbf{x}_\circ + \mathbf{J}^{-1}\left(\mathbf{u}^* - \mathbf{u}_\circ\right)$$

6- Calculate $\beta = \left(\mathbf{u}^{*t}\mathbf{u}_\circ\right)^{1/2}$

7- Using the new failure point, $\mathbf{x}^*$, from step 5, repeat steps 2 through 6 until convergence of β ($\beta_{i-1} \approx \beta_i$, *i* is the number of iteration).

4.4.1.2 Sensitivity of the Reliability Index, *β*

The problem of calculating the reliability index is an optimisation problem as detailed in the above two sections, aimed at the minimum value of the reliability index since it corresponds the estimate of the maximum probability of failure. The failure point in the *u*-space, $\mathbf{u}^*$, corresponds to the minimum value of the reliability index, and hence is the *most probable failure point.*

The coordinates of $\mathbf{u}^*$, $\left(u_1^*, u_2^*, \ldots, u_n^*\right)$ can be related to the reliability index, β, by using scalar notation as [3, 6, 43]:

$$\mathbf{u}_i^* = -\boldsymbol{\alpha}_i^* \beta \qquad (4.16)$$

where $\boldsymbol{\alpha}_i^*$ is the vector of direction cosines, $\alpha_{u_i}^*$, of the scalar distance β along the axes u_i, and the negative sign appears since $\boldsymbol{\alpha}_i^*$ is in the direction of decreasing $h(\mathbf{u})$ [6].

The individual values, $\alpha_{u_i}^*$, are “direction cosines”; that is:

$$\alpha_{u_i}^* = \frac{u_i^*}{\beta} \qquad (4.17)$$

Therefore, an $\alpha_{u_i}^*$ value can be viewed as *a proportion of the length of β*. The larger an $\alpha_{u_i}^*$, the larger its contribution to β, hence they are commonly used as *sensitivity factors.*

The basic design variables, x_i, that correspond to small values of $\alpha_{u_i}^*$ can be used as deterministic variables (the mean value may be used) without significant compromise on the accuracy of β [2, 68].

4.4.1.3 Modelling or Prediction Errors in the Reliability Index, *β*

The calculation of the reliability index amalgamates a range of approximations and assumptions. Hence, it itself is a random variable, and it would be unrealistic to report a *single* value from a probabilistic calculation.

A reliability index should be reported with an estimate of the expected error therein. The modelling error can be calculated using techniques such that explained in the Modelling Error section above.

4.4.2 Interpretation of the Reliability Index, β

Engineer A designs a column with a capacity of 1.2 times the load, but is only 75% certain about it. Engineer B designs the same column to a capacity of 1.1 times the load and is 93% certain. Whose design is better?

The expected capacity design A is 0.75x1.2 = 0.9 of the load; that of design B is 0.95x1.1 = 1.02 of the load. Design A fails, while design B is adequate and more economical (assuming the knowledge cost is the same).

The central safety factor does not relate to the uncertainty in the design parameters. Hence, its use would have made design A (the failure) safer than design B.

Proper assessment requires the examination of the safety margin coupled with uncertainty, as shown in the above simple example. The logical candidate that describes the relative uncertainty in a random variable is the coefficient of variation, COV. The reciprocal of COV is β. Moving from β = 2 to 3 means that we are 1.5 times *more certain* of safety; we either increased the capacity, or decreased the uncertainty 1.5 times.

For design A, the central factor of safety is 1.2; for B it is 1.1, indicating that A is even safer than B. The COV for A is however (uncertainty/(capacity-demand) 0.25/0.2 = 1.25); for B, it is (0.07/0.1 = 0.7). The reliability indices for A and B are 0.8 and 1.4 respectively, which reflect that correct ranking.

One of the drawbacks of representing designs in terms of P_f is that changes of the basic random variables produce disproportionate changes in the probability of failure. On a normal distribution, for example, a shift from 3.0 to 2.0 standard deviations from the mean corresponds to a change of probability from 0.0014 to 0.0228; an increase of almost1500% in $P_{f.}$

If the central safety factor were 2.0 at 2 standard deviations, it would be difficult to relay to a practicing engineer that his designs would have a safety factor of 30 if a section with a mean strength 1 standard deviation further is used.

The cause of the problem in the above representation is that calculating P_f from $P_f = \Phi(-\beta)$ is practically performed for β_{min}. Hence the corresponding P_f is the maximum P_f. In actual fact, P_f and β are mere predictions, simply as they are the product of probabilistic calculations. Real failures have in most cases occurred due to human error or unusual extreme conditions. The “real” P_f is variable; the single value we calculate is an estimate of its maximum predicted value.

The correct purpose of representing reliability data is the “expression of certainty” about design. One cannot state that a structure is 99% safe. What can be said is that one is 99% sure that it would survive the set of rationally predicted loads.

4.4.3 Weaknesses of the Reliability index, *β*

The strength of the reliability index concept lies in that the failure probability can be estimated without knowledge of the full probability distributions. The full shape of the limit sate function is not needed since linearization is performed only at the design points. Depending on the degree of nonlinearity, linearization may yield conflicting estimates of the actual probability of failure. Figure 4.5 shows two different performance functions with an equal reliability index.

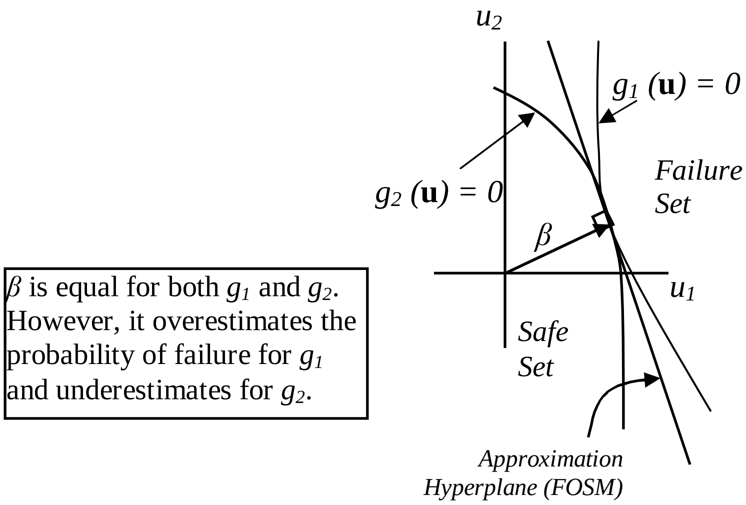


Figure 4.5: Discrepancy in Estimating P_f from Reliability Index, β
(Reproduced and modified from reference [3])

4.5 Load Combination

The stochastic models loads are established using statistics of extremes. Maximum load and minimum strength values are modelled using extreme value distributions such as Gumbel and Weibull probability distributions. The detailed treatment of this class of probability distributions is available in references [2, 3, 5, 70].

For an N number of loads over a design period, T, the maximum load effect at any point of time t, $U_{max}(Q_{(t)})$ of loads Q_i, can be represented as [71]:

$$U_{\max}\left(Q(t)\right)=\max\left(\sum_{i=1}^{N}U\left(Q(t)\right)\right),\quad t\in\left(0,T\right) \tag{4.18}$$

Static loads can be easily described by random variables since they are almost time independent. Transient loads on the other hand are modelled as stochastic or random processes. Considerable effort went into the combination of quasi-static loads and loads with a transient nature, by converting time-variant stochastic processes to time-invariant processes using their extreme value distributions [62]. Outcrossing techniques are typically used to extract the extreme value distribution for the probability that a stochastic process crosses a level in the time interval (0,*T*) [54, 62]. This is followed by an optimisation process for the maximum effect of point-in-time combined values [2, 3, 53, 62, 68, 71]. Figure 4.6 illustrates.

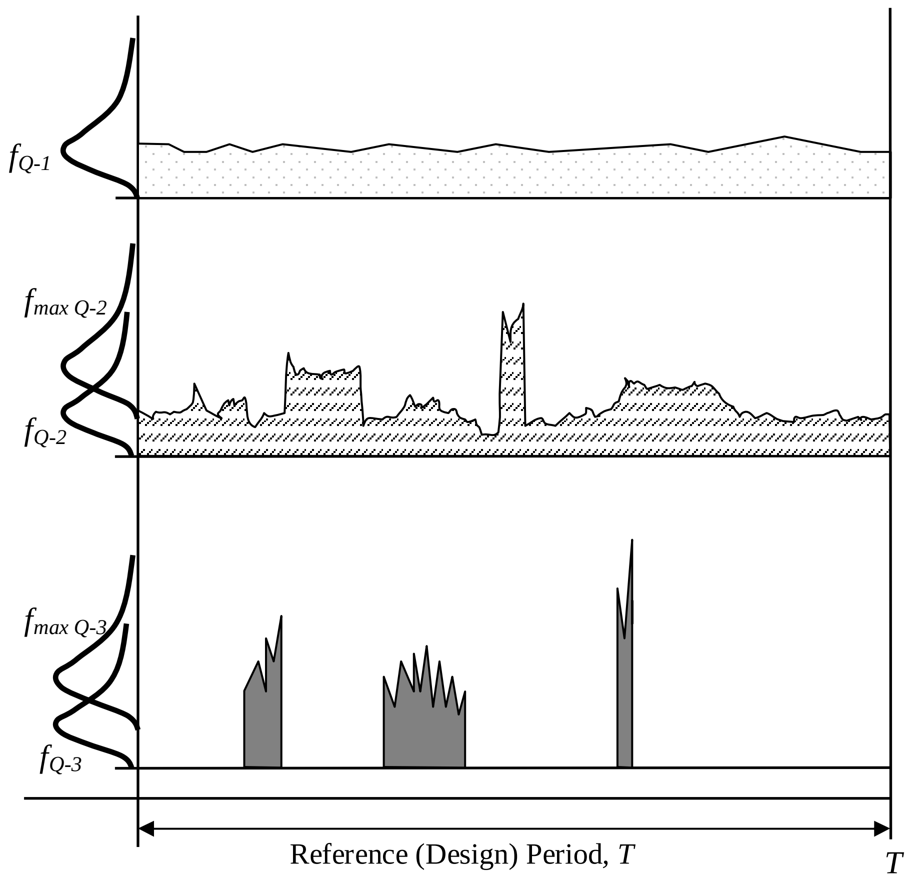


Figure 4.6: Combination of Loads
(Reproduced and modified from reference [5])

The design period *T*, is typically 50 years for strength limit states and 10 years for serviceability design for building structures [70].

Several statistical studies indicated that the maximum load effect does not occur at the coincidence of the peak values of all loads in a combination [5, 13, 71-73]. Combining extreme values of loads leads to uneconomical design and may not be necessary. A commonly used rule for the linear combination of loads is the maximum load-companion load rule, which is based on studies of coincidence of loads. The maximum load is assumed to occur at a peak value of one load, Q_i, combined with arbitrary values of other loads, $\sum_{j \neq i}^{n} Q_j$:

$$Q_{\max} \approx \underset{i}{\overset{N}{Max}} \left[\max_{T} Q_i(t) + \sum_{j \neq i}^{n} Q_j(t) \right] \tag{4.19}$$

Loads, obviously, interchange roles in peak value, and the combination producing the maximum load effect usually governs.

Critics of the above method describe it as inadequate as it ignores cases where the maximum combined effect results from a number of loads at values near the peak, especially when loads are correlated [5], as in fire following earthquake. However, using advanced simulation techniques, the method has been proven to be sufficiently accurate for most design situations [5].

4.6 Code Calibration

The main attraction of LSD/LRFD format is its simplicity and appeal to practising engineers [3]. It was realised in the early 1990s that the use of level II reliability methods in the design office was not practical.[74] LSD and LRFD codes are calibrated by enforcing level II reliability methods (FORM & SORM) in level I format (Limit State, LRFD, ASD).

In practical terms, if the reliability index has been calculated, the most probable failure point, $\mathbf{x}^*$, would be known. The coordinates of $\mathbf{x}^*$ contain the basic design variables, such as geometry, loads, material properties. The specific load and strength variables are also included in $\mathbf{x}^*$. The problem of find the load factor, γ, reduces to evaluating the ratio of load coordinate of $\mathbf{x}^*$ to the nominal value of the load, Q_n. The same process is used to calculate the strength reduction factors, φ. The nominal values of the load and strength, called the characteristic values, are certain quantiles of their respective probability distributions, usually the mean values. Hence [3],

$$\gamma_i = \frac{x^*_{Q_i}}{Q_{n_i}} \text{ and } \varphi_j = \frac{x^*_{R_j}}{R_{n_j}} \tag{4.20}$$

In the above equation, γ_i and Q_{ni} are the partial load factor and nominal value for load i, and φ_j and R_{nj} are the strength reduction factor and nominal strength value for strength j.

4.6.1 The Process of Load Calibration

The methods of code calibration fall into three categories: value judgement, fitting and code optimisation [54, 74].

The improvement of quality control procedures resulted in consistent measurements of characteristic values for material and dimensional parameters. This consistency was taken as an indicator of success of the code in force. For example, values for the factor of safety in ASD steel design in the US converged to 5/3 since 1936 [74]. Designs using subsequent codes, therefore, were calibrated to achieve the same level of safety. This was the standard practice till the 1960s [54].

Fitting takes a reverse approach to that described in the above section. Load and resistance factors are pre-selected so as to give the member sizes given by the previous code. This methodology was applied in the development of load factors in the 1989 AASHTO LRFD specifications for highway bridges [74].

Load and resistance factors can be used as objective functions under the constraint of a reliability index in an optimisation process. This method is known as code optimisation. Depending on the format, objective of the code and the choice of target reliability indices, a range of load and strength factors are obtained.

A degree of arbitrariness and subjectivity exist, particularly in the selection of code formats and reliability indices, hence the variation of different national and regional codes.

The above approach was used in the development of the 1977 Canadian code for buildings [54] and the 1982 ANSI A58.1 Load Code [74]. More details are available in references [2, 53, 54, 74, 75].

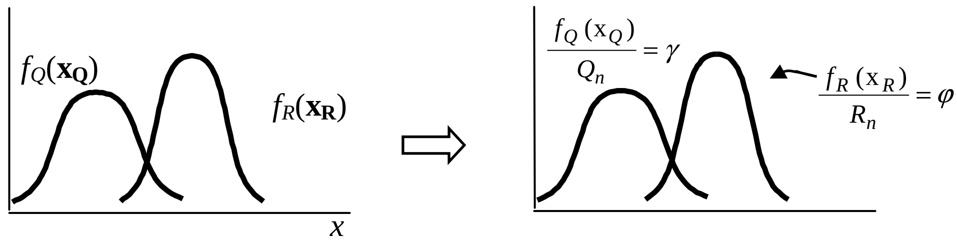
Load and resistance factors, γ and φ, may be visualised as probability density functions, *pdf*, of the quotient of the load and resistance by their respective nominal values. The aim of the optimisation process becomes the specific values of γ and φ that correspond to the target reliability index, β_t. The basic variables in performance function, $g(x)$ are segregated into a product of the nominal load and resistance values by partial load and resistance factors.

$$g(\mathbf{x}) = g(\mathbf{R}, \mathbf{Q}) \qquad (4.21)$$

$$g(\mathbf{R},\mathbf{Q}) = g(\varphi R_n, \lambda Q_n) \qquad (4.21\text{-a})$$

The optimisation process becomes straightforward:

- $g(\mathbf{x}) = g(\mathbf{R},\mathbf{Q}) = g(\varphi R_n, \lambda Q_n)$
- Minimise $g(\varphi R_n, \lambda Q_n)$ subject to a target reliability index β_t



$\gamma \equiv$ *pdf of* $\frac{f_Q(x_Q)}{Q_n}$, $\varphi \equiv$ *pdf of* $\frac{f_R(x_R)}{R_n}$

Figure 4.7: Partial Factors, γ and φ, in Code Calibration

The same logic can be followed in the formulation of a probability-based *single factor of safety* that achieves similar reliability indices [3].

4.7 Maintenance of Code through Quality Control

Successful engineering practice necessitates the application of a strict regime of quality control procedures. If sample describing the characteristic strength of materials deviate appreciably from the domain used in code development, the validity of the code is compromised.

A number of techniques employing statistical and probabilistic concepts are used in acceptance of samples. These are collectively called *acceptance sampling techniques* [21].

Acceptance sampling by attributes is one technique where a sample of the total is selected and tested. The test results in a *good* or *bad* classification, and the total is rejected if a specified proportion of the sample is bad. The number of rejected items in the sample can be used to infer the probability of accepting the total.

The above sampling sets a threshold for acceptance which disregards the actual distance from the acceptance limit. Yield strength measurements of 238, 239 and 240 MPa would be good while 233 and 234 MPa would be bad if the acceptance limit is 235 MPa. An alternative method that accounts for the actual weights of the measurements is the *acceptance sampling by variables*. The actual measurements are analysed and their statistical data (such as the mean and variance) are compared to acceptable standard data. The total is accepted if the sample data compares well with the standard data. This method requires a smaller number of samples than sampling by attributes, since the whole data is analysed as opposed to counting good or bad items. More details and examples can be found in references [2, 21].

Sampling procedures are the scope of construction and quality control standards and form part of quality certification requirements.

4.8 Load Combination for Structural Fire Design

The appeal of LSD and LRFD format to practicing engineers has been a main driver for shaping the current design codes. If codes are not practical, they’re not usable. It is therefore desirable to aim to include the fire effect on structures as another load type within the existing LSD/LRFD format.

Realistic combination of different loads depends on the history of their co-occurrence. Methods such as the load coincidence (LC) [72] method are derivatives of the latter concept.

A study of the coincidence of various loads, such as gravity and wind loads, concluded that fire need not to be combined with most loads due to the low probability of their coincidence [13]. More details are in section 3.3.2.1 of chapter 3.

The scope of the above study is limited to deciding whether or not it is reasonable to combine fire with another load. It acknowledges that more extensive analysis is required in the assessment of the structural behaviour under fire, but stops short of providing details on the actual analysis under combined loading.

4.8.1 The Fire Problem

In traditional load combination, the ability to lump loads in linear combinations is attributed to the *linear elastic static* response of the structure. Controls are placed so that the maximum expected stress levels do not result in significant deformations beyond which linear analysis becomes invalid. By evading nonlinearity and precluding time from the explicit formulation, the influence of the whole structure on the behaviour of single members is reduced to the level where linear analysis methods can be applied with sufficient accuracy. As a result, *single members can be designed individually to linear combinations of static loads*. Ingenious and convenient; this type of analysis applies to the majority of structure classes encountered in the design office. It is also commensurate with the level of education offered at first degree courses in most universities. The success of LSD/LRFD in practice is behind the desire to extend the same treatment to fire action.

A beautiful term describes the ranking of elements in galvanic series. Elements that assume the anode role sacrifice themselves to protect cathodes and are therefore called “noble”. Those becoming cathodes within the reactive environment are “less noble”.

Codes specify a hierarchy of target reliability indices depending on the member’s mode of response. Tension members, for example, have lower reliability indices than compression members since they usually fail in yielding. Yielding takes place over a reasonable period of time which gives sufficient warning and allows rescue and mitigation. By specifying different target reliability indices, codes aim at initiating the failures in tension members (the *noble*) so as to allow time to protect other parts of the structure (the *less noble*).

Fire presents a unique counterintuitive problem in structural design. Code-favourable elements can become less noble in the response hierarchy. Members tend to interchange roles in the resistance chain due to the effect of nonlinearity. Tension members can be the last to lose any resistance as heat amplifies elongation thereby relieving applied forces or causing them to buckle prematurely due to their large slenderness. Buckling at early heating stages reduces the chances of developing significant plastic strains, so they are likely to regain capacity as they return to shape in the cooling stage.

Compression members suffer as the heat-induced elongation induces pre-stress that increases the compression stiffness, thereby attracting extra loading. Slender compression members exhibit good performance during cooling as the heat loss imposes a straightening effect, while short compression members can undergo irrecoverable plastic strains that degrade their capacity [76].

Tests have also shown the significance of restraint effect on the behaviour of structural elements [7, 77-80]. Figure 4.8 shows the variation of the behaviour of a composite beam with and without axial restraint. Any treatment of uncertainty in a probabilistic design framework must allow for the bifurcation exhibited after certain temperatures, which suggests a range of load factors rather than a single value. It also highlights the necessity for a holistic analysis of the structure in assessing the behaviour of individual elements, since the transformation in the beam behaviour is influenced by the stiffness of the parent structure.

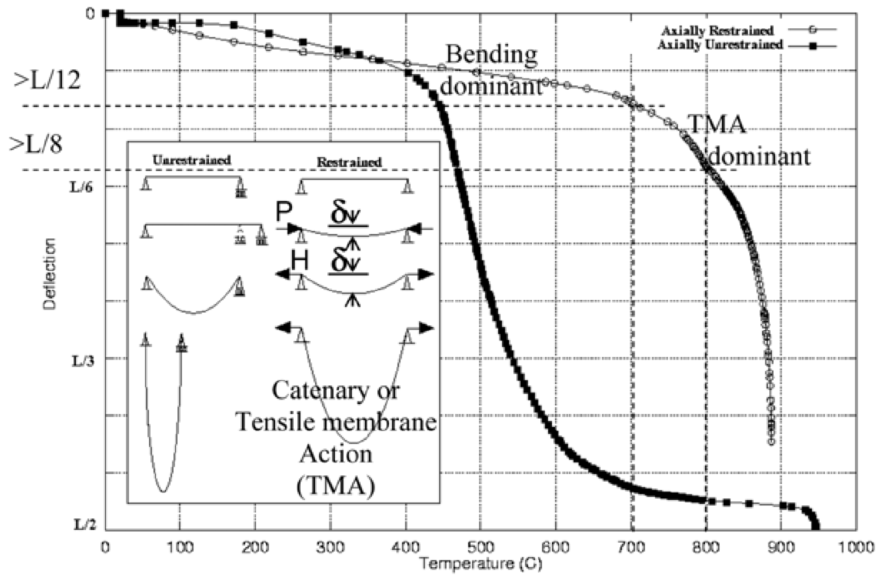


Figure 4.8: Runaway behaviour of a composite floor beam
(Reproduced from reference [9]*)*

Moreover, the heating regime has a significant impact on the stresses induced in the structure. It is difficult to construct rules for the use of short-hot or long cool fires in structural design, since it is not possible to assess the impact of the specific fire without including the restraint effects of the particular structure.

The framework of LSD/LRFD codes is centred on combining a multitude of stochastic load processes, using a procedure that eliminates time from the design equation. There are two problems in applying this approach to fire. One is that the constraints (such as maximum deflection or stress) are dependent on time in fire since they influence evacuation, and the second is that the performance of other active and passive fire engineering systems is measured in time.

In fire, permanent damage to the structure is most likely inevitable. The functional requirement of the structure is the control of the *temporal rate of damage*. Since repair is expected after fire, the extent to which the structure may deform is relaxed. The only constraint on deformations is that they evolve at a rate that does not impede evacuation or progress to collapse. The acceptance criterion for structural performance is directly linked to the functional requirements for evacuation and is a function of the fire evolution.

Another fundamental difference between fire and other loads is caused by nonlinearity. Spatial properties, connection details, relative stiffness and heating regime, all play a major role in the behaviour of structures in fire. Whereas a beam remains a beam with the variation of service loading, it might transform into a catenary under fire.

4.8.2 Anatomy of Fire Protection Systems

The interrelationship between the components of fire protection systems, active and passive, can be used to build an event tree diagram for potential fire scenarios. Statistics exists for the performance of the various fire protection components. The consequence of the events can be integrated into the failure tree to extract the risk associated with each scenario.

Good engineering design utilises the event tree to attain equal or near-equal risks for all possible scenarios. The design as such would be directed into optimising the use of different systems in the most economical manner. It is essentially a simple optimisation process that has a uniform target reliability index for all design alternatives.

As an example, the structural capacity during fire can be modelled as a standby component when using passive structural fire protection. If the passive fire protection is eliminated, the structural capacity becomes part of the overall fire protection system. By examining the risk with and without passive protection, the actual value of having passive fire protection can be estimated.

If the maximum risk for the whole fire safety system (on any branch of the failure tree) is greater than that without passive protection, the latter may be, and in fact should be eliminated. An example event tree diagram for the progression of fire beyond origin is shown in figure 4.9.

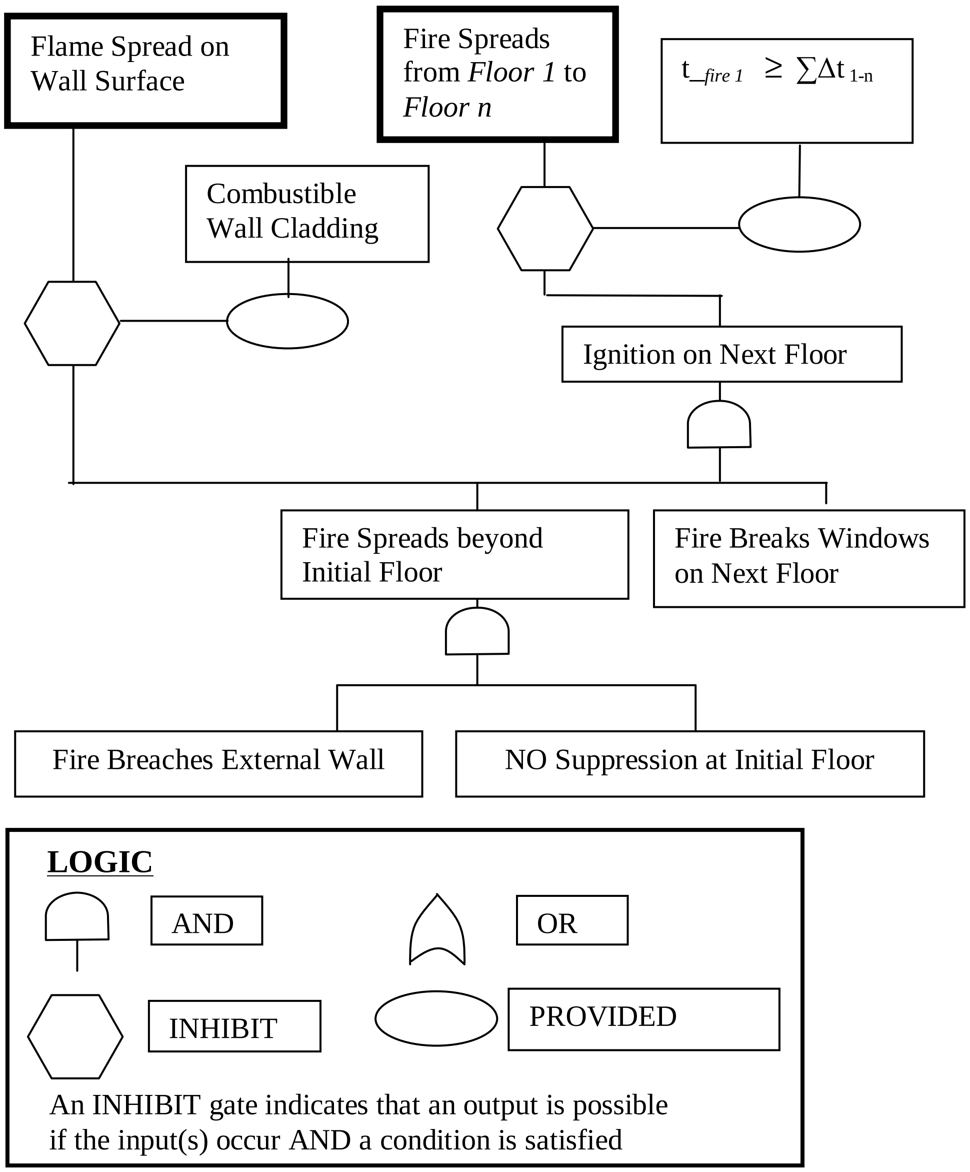


Figure 4.9: Fault Tree Example for Fire Progression to Next Floors

The resultant probabilities of the outcomes shown on the fault tree can be easily calculated using probability relations for AND, OR, or Conditional events.

4.8.3 A Proposal for an LSD/LRFD Fire Effect Combination

The recipe for LSD/LRFD includes the coincidence of loading described above [13] with time-independent loads and response models. This may be achieved by defining bounds of member behaviour during fire, thereby subdividing the response of members according to the prevalent mode. For example, in Figure 4.8 above, beams can be designed for two stages:

1. During egress and evacuation: Maximum deflection is limited to L/30. Hence, for the above specific beam, the maximum steel temperature within egress time can be checked, and if need be, either beam redesigned or passive protection applied.
2. After egress and evacuation: No limit on maximum deflection or stress levels as long as structural response does not induce total collapse.

The resistance is a function of the section and material properties, geometry and boundary conditions. The demand comes from the magnitude and duration of the heat flux imposed on structural members. This depends on the compartment geometry, fuel and ventilation conditions.

For stage 1, a performance function can be constructed as the difference between limiting temperature and actual steel temperature.

$$g(\mathbf{x}) = T_{limiting} - T_{steel-actual} \qquad (4.22)$$

Extreme probability distributions of the maximum limiting temperature *before runaway* ($T_{limiting}$) and the maximum actual steel temperature can be derived. Temperatures can then be statistically correlated to the stress and strain levels of different steel sections. The idea is to use maximum stress as *surrogate* for temperature. By using its maximum value distribution, time becomes implicit and LSD/LRFD formats may be used to combine the fire with other loads. The behaviour during the above stage is nearly linear, so fits well within the framework of LSD/LRFD.

The stress and strain levels corresponding to the actual steel temperatures are then used to back-calculate an equivalent static load with a suitable load factor. Those corresponding to the limiting temperatures can be used to compute the resistance reduction factor. Again this needs be performed within a strict framework of building types, geometry and boundary conditions.

Stage 2 can be implemented as an independent check for structural stability. Usmani *et al* [9] proposed a simple method for calculated pull-in forces on columns during fire. The main strength of method is derived from the fact that time is precluded from its formulation. The additional checks suggested in [9] can be made implicit by using a system of *building classes* and defining characteristics that ensure the validity of its application. The characteristics most influential to the behaviour are by-products of the sensitivity analysis associated with reliability computations.

4.9 Performance Functions for Structural Fire Design & Response Surface Modelling

Often is the case that most performance functions in structural fire modelling are nonlinear dynamic finite element models. Simplified calculation models exist, but they are not suited for complex structures where performance-based design has been opted for.

Running a reliability analysis using direct simulation techniques or FOSM/SORM involves calculating the performance function at every design point in the iteration. Certainly in the case of analysing structures for fire effects, the computational effort involved in a single analysis of a moderate size structure can be in the order of days on an ordinary personal computer. To provide a meaningful reliability estimate, a large number of simulations is needed. It may therefore be impractical to link a reliability model to a finite element model, especially that most computers used in design offices are typical PC's.

Response surface modelling is a viable technique that can be used to circumvent the repetitive calculation of $g(x)$ at the design points [53, 81]. It could therefore be a more reasonable approach to reliability calculations for structures in fire.

Since the value of $g(x)$ is only relevant at the design point in FOSM/SORM calculations, the idea is to substitute the original $g(x)$ with a function that is simpler and accurate enough at the design point. The principle is to evaluate $g(x)$ at arbitrarily selected points, and to use these values to fit a function, say $g_R(x)$, thereto. The substitute function, $g_R(x)$, is an n^{th} order polynomial, most commonly of the second order [53]. The reliability calculations proceed by evaluation of the design point and reliability index using $g_R(x)$. An improved response surface may be obtained by sampling points close to the design point just obtained, and new reliability index and design point can be calculated. The procedure can be repeated until convergence to a design point and reliability is achieved [81].

4.10 A Proposed Risk-based Target Reliability Index for Performance-Based Design

Several models have been proposed for including the cost of failure in the overall project cost.

One such proposal is: [82]

$$L_i = I_i + (1 - p_i) C_i \, E[S_i] + p_i F_i \tag{4.23}$$

In the above:

L_i: The expected total cost

I_i: Initial cost

p_i: Probability of occurrence of total failure

F_i: The estimated cost of total failure, including structural failure and any associated losses

C_i: The estimated cost of partial failure

$E[S_i]$: The probability of occurrence of partial failure

The probability of total failure is statistically very small and hence it can be set to zero [82]. Moreover, past total building failures were accepted as due to exceptional events that could not be avoided by design. The above equation becomes:

$$L_i = I_i + E[S_i] C_i \tag{4.23-a}$$

Equation 4.23-a can be generalised to include the running cost, R_i, as:

$$L_i = I_i + C_i \, E[S_i] + R_i \tag{4.23-b}$$

The use of performance-based design requires a parallel risk framework that has a commensurate flexibility. It should be possible to vary the acceptable probability of failure depending on the actual risk level. Current probability-based codes deal with the probability part of the risk and set target reliability indices accordingly. A more complete treatment should include the consequence of events. This has been used successfully in earthquake engineering in the US, and is behind the methodologies developed by the Pacific Earthquake Engineering Research Center (PEER).

Good design should aim at a *uniform risk* due to all predicted events. This is the underlying principle in the *risk comparison method* detailed in chapter 3.

From chapter 3, (equations 3.6 and 3.7)

$$R_{ref} = R_i \tag{4.24}$$

giving:

$$P_{ref}\ C_{ref} = \frac{P_i\ C_i}{v_i} \tag{4.24-a}$$

In the above, R_{ref} and R_i are the reference risk and risk under consideration, P and C are the respective probability and consequence of events. The ratio of *public acceptance* of a risk under investigation, R_i, to the reference risk, R_{ref} is v.

The reference risk should be taken as an *involuntary* risk, whose probability of occurrence is beyond human control.

For structural damage assessment, the consequences C_{ref} and C_i can be replaced by target damage indices, D_{Tref} and D_i.

The maximum acceptable values of the damage indices, D_{ref} and D_i are set by the functional requirements of the building during the event. These obviously vary from one event to another. The maximum deformation during fire is different from that during an earthquake. Additionally, the principal function of the building depends on the event. During an earthquake, the building is the “safe refuge” and people have no time to escape. In fire, on the other hand, the building becomes a “safe egress route” that must allow people to leave safely.

The most important conclusion from the last paragraph is that D_{ref} and D_i are *specific to each building*. They would vary to ensure that the building performs its intended function during the event.

A *risk-based target reliability index* for event *i*, β_{Ti}, can be obtained by rewriting equation 4.24-a as:

$$\beta_{T_i} = \Phi^{-1}\left(1 - \frac{v_i\ P_{ref}\ D_{T\,ref}}{D_i}\right) \tag{4.25}$$

The above index is specific to the building and ensures consistent risk levels. It is therefore suitable for performance-based design.

The projected total cost can be calculated using equation 4.23-b as follows:

$$L_i = I_i + \Phi\left(-\beta_{T_i}\right)D_i + R_i \tag{4.25-a}$$

Chapter 6 presents a method for quantifying structural damage.

4.11 A Proposed Optimisation Technique for Calculating the Reliability Index

The calculation described in section 4.4.1 for the reliability index described is a *gradient-based* technique. The algorithm tracks the most probable point, u, by minimising the angle between the vector, $\mathbf{u}$, and the unit vector normal to the performance function, $g(\mathbf{u}) = 0$ [2], as depicted in figure 4.10.

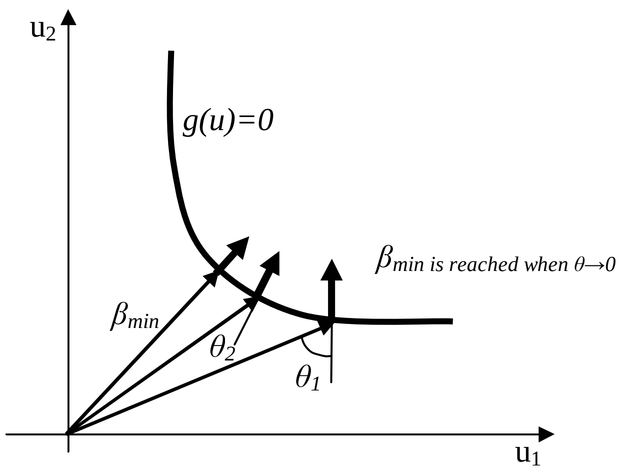


Figure 4.10: Iterations to Calculate β_{min}
(Reproduced & modified from reference [2]*)*

The termination criterion for the algorithm is the convergence of β which is marked by angle θ approaching zero.

The main problem in the above approach is that the algorithm stops at a minimum, without knowing whether it is a *local* or *global* minimum. If *g(u)* has multiple minima, they are unlikely to be detected. Moreover, the detected minimum is the *first* minimum encountered along the trajectory, so it depends of the choice of the *starting point*, $P_\circ$, in the simulation, as shown in figure 4.11.

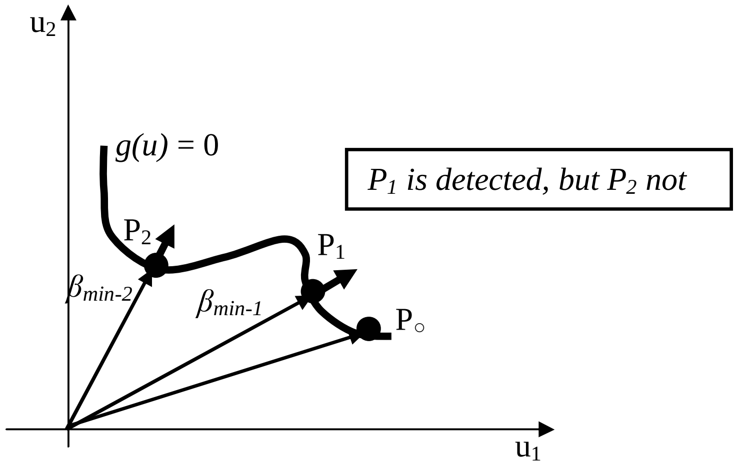


Figure 4.11: Performance Function with Multiple Minima

The following is a proposed method for capturing extrema, minima and maxima. The concept is explained with the aid of the following figures, using a two-variable performance function.

Two circles with centre at the origin of u_1u_2 are drawn, as in figure 4.12. The radii r_1 and r_2 are arbitrary.

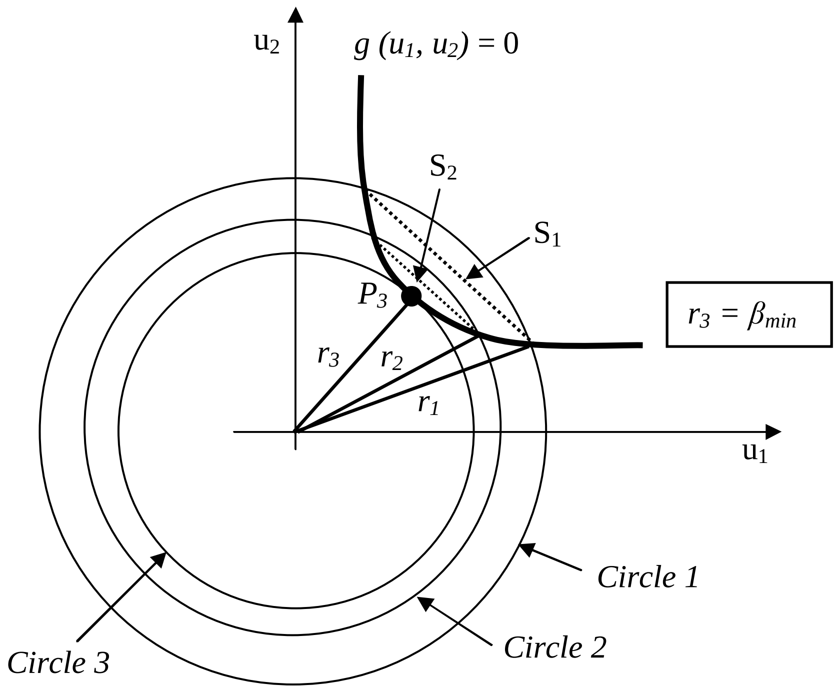


Figure 4.12: Capturing β_{min} using Circles

The distances S_1 and S_2 determine the direction of the minimum distance from $g(u_1,u_2)$ to the origin. The point where the distance S is nearly zero occurs at the minimum radius, r_3, and is the minimum reliability index, β_{min}.

The same procedure applies for multidimensional surfaces. The calculation starts at an arbitrary plane, i_1i_2 say, and the minimum distance is calculated as above. The closest point to the origin of i_1i_2 (P_3 in the above figure) becomes the starting point for the calculation in the i_2i_3 plane; i.e., the centre of *circle*-$_1$-i_2i_3. The same procedure is continued through the $i_{n-1}i_n$ plane.

The method has the following advantages:

- Simple algorithm – Start at one point on $g(u) = 0$ and search for the points on the circle where $g(u)=0$, stop search when you return to the starting point. Repeat for second circle. If $S_2 < S_1$, then $r_3 < r_2$. Start circle 3 and continue drawing circles until $S \rightarrow$ zero.
- The choice of starting point is not important since the algorithm follows a circle.
- Because it follows the *circle,* it is guaranteed to converge since it must return to starting point.
- Easy to capture any number of design points. Multiple points are detected by the number of intersection points. (Points P_3 and P_4 in figure 4.12-a). Using regression analysis, these points can be used to construct a response surface which enables the calculation of P_f.

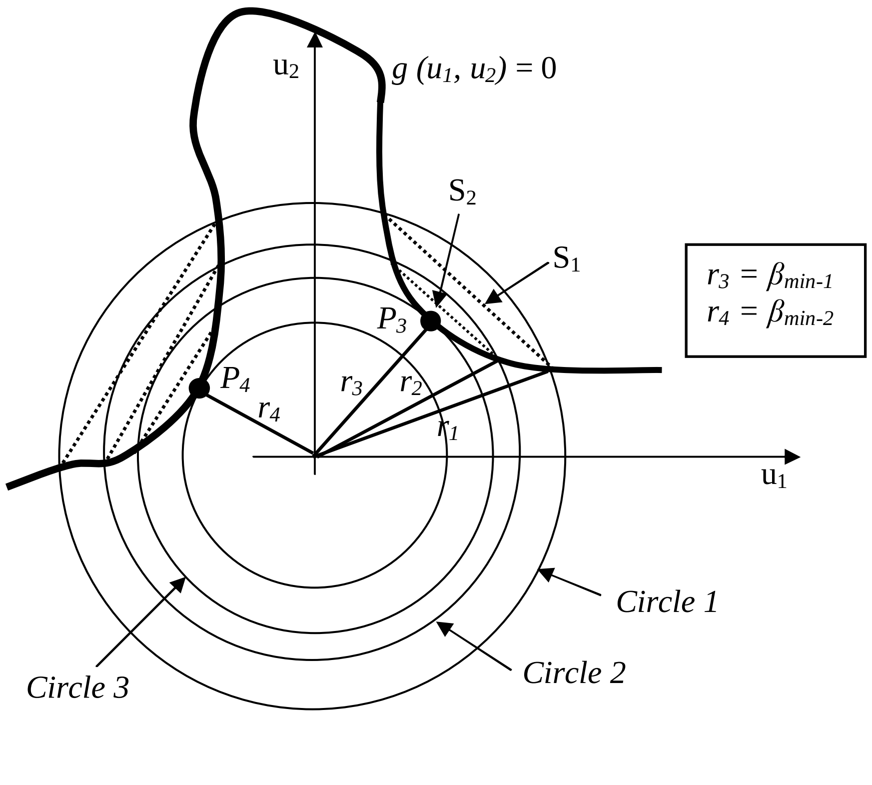


Figure 4.12-a: Detecting Multiple Design Points

- The relative difference between S_1 and S_2 with respect to r_1 and r_2 gives an indication of the degree of curvature or nonlinearity of $g(u)$.

$$\frac{\Delta S}{\Delta r} \propto \text{radius of curvature of } g(u)$$

The above can be utilised in estimating the difference in radii, Δr, to accelerate convergence. It can also be useful in estimating the error in calculating the probability of failure, P_f, from the reliability index, β. Figure 4.12-b illustrates an estimate for the correction to P_f. The hatched area is a function of S and r.

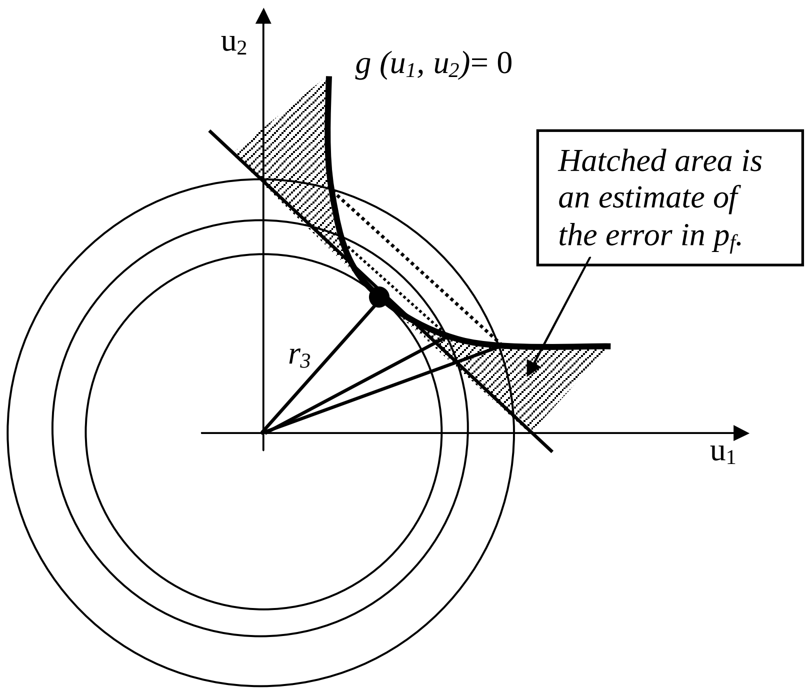


Figure 4.12-b: Estimating the Error in P_f

The method can be used with circular or near-circular performance functions. A check can be made by using circles with different origins at the beginning of the calculation.

4.12 Conclusion

The assessment of risk is as complex a process as risk itself. It is unfeasible to carry it out without making significant compromise to risk parameters.

The primary objective of risk analysis for engineering projects is the calculation of the probability and consequence of events, including failures. Other aspects of risk such as social impact are taken into account qualitatively, primarily due to their subjective nature.

Probabilistic techniques in engineering were introduced with a main focus on reliability. The structural response to elevated temperatures exhibits strong nonlinearity and the acceptability limits for deformations are dependent on time. The two features place restriction on the applicability of limit state or LRFD format to structural design for fire effects. A proposal for the design for fire effects using a two-stage approach was presented.

Reliability calculations are essentially optimisation processes of performance functions under the constraint of target reliability indices. Current level I codes (LSD/LRFD, ASD) are calibrated using subjectively- selected reliability indices. The flexibility inherent in performance-based design is its main distinguishing characteristic. The choice of target reliability indices can be improved by incorporating the risk involved in the specific project. A simple method for a risk-based target reliability index was also introduced.

Finite element method (FEM) is the preferred tool for analysis of structures for fire conditions which is characterised by a nonlinear behaviour. Reliability computations can be almost unfeasible if commonly used algorithms are linked to FEM simulations. Response surface models are a reasonable alternative that can be used for reliability calculation.

The use of gradient optimisation techniques in level II reliability algorithms poses limitations on the ability to capture multiple failure points. An optimisation method with the ability to detect multiple design points is introduced in the final part of the chapter.

Chapter 5

Limit State Design in Practical Situations: Is your Structure Safe Enough?

Engineering design is carried out mostly by commercially driven organisations. The design office is managed through a process of optimisation that allocates resources in proportion to the forecast benefit. Like any practice, engineering design is regulated by law. Legal requirements, nonetheless, set minimum performance standards for a wide range of applications, but do not necessarily capture every foreseeable situation.

Structural engineers often find themselves part of a design delivery process which is limited in time and budget. The main aim of conventional practice is to meet the client's brief while satisfying the building regulations. In general, it is deemed sufficient to ensure that design complies with the applicable codes and standards. As a matter of tradition, expert code writers carry the onus of mapping elaborate load-structure behaviour to straightforward design formulation. Therefore, the task of the engineer is truncated to collating actions in code-specific formats and ensuring that resistances exceed their respective effects.

This chapter examines some common design situations where reliability differs for members of the same type in the same structure.

5.1 Acceptance Criteria - Time Scale and Failure Modes

Natural systems pursue minimum energy content. When a system is perturbed by an applied action or change of conditions, elements deform in the mode dissipating the maximum energy. If a number of deformation modes are physically possible, the system would pursue the mode with the highest capacity for energy disposal.

Depending on the geometric characteristics and boundary conditions, elements dissipate internal energy through either ductile *modes* such as elongation or deflection, or *brittle* modes as in fracture or buckling.

In a similar manner to natural systems, structures favour the brittle option having the higher rate of energy disposal. On the hand, design codes and designers favour ductile failure modes. Failures of the latter type progress over an *observable* period of time, thereby providing warning of imminent risk and allowing emergency operations and remedial works. This is the reason codes assign lower safety indices to members exhibiting ductile failures in an attempt to initiate failure therein.

A necessary condition for the validity of the code approach is *optimality*. The fact that actions on structures are mostly natural and modelled by random processes, optimum design can only be achieved from a continuous space of solutions. In engineering design, however, it is impractical to use different sections for similar members with relatively small difference in forces. In fact, the feasibility of optimisation is largely hampered by industry-standard discrete sets of available materials, as in standard steel and timber sections.

As discussed in chapter 2, time is the main element in failure acceptance criteria. The ability to impose sufficient control on failure development can only be assessed using time scale. Structural codes are written with the aim to guide structural failure to a path that can be monitored and assessed within available resources. Failures that cannot be observed occur without warning and mitigation efforts can be seriously hindered. The below generic formulation illustrates the concept.

$$\left.\begin{array}{l} t_{-action} + t_{-observation} \leq \quad t_{-failure} \\ \Rightarrow \\ \qquad\qquad t_{-action} \leq \quad t_{-failure} - t_{-observation} \end{array}\right\} \Rightarrow \text{System is Safe}$$

5.2 Structural Failure Modes

As mentioned in chapter 2, failures generally fall in the two categories:

- Ductile failures: Failures that are progressive at macro-scale, exhibiting an *observable history* of deformation or degradation parameters. A typical example is yielding of a tension member.

- Brittle failures: Failure is characterised by *non-observable history* of deformation or degradation parameters. These are mainly two types: microscopic failures as in fracture and stability failures like buckling.

Despite the rigour in reliability methods, account for the consequence of failure is still subjective [53]. Table-5.1 shows typical values of the reliability index used in the AISC-LRFD specifications.[64] It is clear that target reliability indices differentiate between ductile and brittle failures. The intention of differentiation is to impose a degree of control on structures in an attempt to instigate failure in ductile modes.

Table 5.1 A Sample of Target Reliability indices, β_t, used in AISC LRFD Code for L_n / D_n = 1.0 (L_n & D_n are Nominal Live & Dead Loads) [64]

Type of Element	**Target Reliability Index, β_t**
Tension Member, yield limit state	3.0
Tension Member, Fracture limit state	4.1
Rolled Beam, flexure limit state	2.5 - 2.8
Rolled Beam, shear limit state	3.4
Columns	2.7 – 3.6
Fillet Welds	4.4

5.3 What is wrong with LSD/LRFD Codes?

Nothing is wrong with LSD/LRFD! The format of codes was designed with the engineering office practice in mind. The extent of rigour required in code documents was influenced by many factors, such as the general level of education of designers, the characteristics of common building types and the record of past design specifications. Earlier codes such as the working stress design (WSD) had a simpler format backed by an impressive success record. Code committees came to an agreement in the 1990s [74] that level II reliability methods were to be implicitly applied in LSD/LRFD format.

The foundation of LSD/LRFD is based on the ability to superimpose different actions in linear static combinations. Such approach enabled the continued application of *member-based* design and was welcomed by practising engineers. Allowance for the consequence of failure is treated by separate prescriptive provisions such as recommendations for sufficient redundancy and alternative load paths. The provision for a minimum tensile capacity for connections is steel buildings in BS 5950-1, 2000 is an example of the latter. It is intended to ensure adequate connection resistance if the member becomes subject to membrane-like forces during collapse. This type of structural behaviour is typical in fire conditions.

Many practicing engineers realised the importance of global structural treatment, but were forced to use judgement and intuition in the absence of rational code guidance. Fazlur-Rahman Khan and El Nemieri [83] highlighted the need to advance structural codes to a stage where the relative significance of members to the global structural integrity is incorporated in the design formula. Using the same rationale behind reducing column loads in multi-storey buildings, Khan and El Nemieri suggested that designers modify the strength reduction factor, φ, to account for redundancy or uniqueness of structural elements. Though not explicitly proclaimed, the authors' suggestion could have well marked an early adoption of performance-based design codes. Moses [84] investigated the failure mechanisms of parallel and series structural systems verifying the variability of the partial safety factor with merely the number of elements in a structural system.

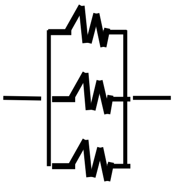
5.4 System Reliability

Failure of a member within a structure does not necessarily result in collapse of the whole structure. Structural systems are usually designed to account for the eventuality of partial failures by redistributing loads to other members.

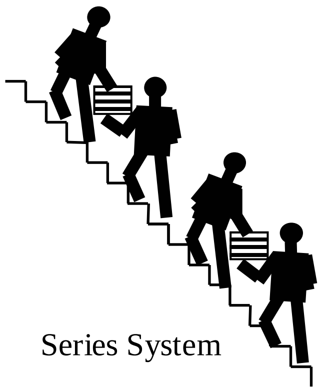
Structural systems are not unlike other engineering systems. They can be idealised to identify the impact of failure of individual members on the global structure.

5.4.1 Reliability of Series & Parallel Structural Assemblies

In simple terms, parallel systems can be described as those that share the demand, as in rowers in a regatta. Series systems on the other hand relay the *full* demand from one to another; as in relay races, transport systems generally or chains, as shown in figure 5.1. It logically follows that the capacity of parallel systems is the collective capacity of all members, whereas that of series systems limited by the smallest capacity of any individual member, or the weakest link.



Parallel Electric Circuits



Series System

Figure 5.1: Examples of Parallel and Series Systems

It is unlikely that any engineering system is designed without allowance for the failure of one or more components. For the purpose of this discussion, we shall call the system of components that represent the minimum requirements for operation without allowance for partial failure as the *base* system.

If a system of components is an *optimally* designed base system, then whether the components are in parallel or series, failure would occur upon the loss of any individual member. The main difference between the two is the mode of failure. Series systems lose total capacity *at the instant of failure*. Parallel systems on the other hand undergo a *transition* state where the additional demand propagates through the remaining components. Should the demand diminish or the capacity be enhanced, progressive failure can be halted.

Owing to its collective property, parallel systems provide designers with the ability to utilise *parallel redundancy*. By either enhancing the individual capacities of some (not necessarily all) components, or inserting additional ones, the system can be designed to operate following the failure of any number of components. The design of aircraft engines involves contingency for possible failure of one or more engines. Engines are designed to operate at full capacity only if some engines are damaged, since the anticipated time of such conditions is limited. During normal service, engines operate at a proportion of the capacity to maintain acceptable performance for the length of the design life [56]. Parallel systems can also encompass *standby* systems where the added components are idle during normal service.

The minimum probability of failure of a parallel system, $P_{f\_parallel}$, is:

$$P_{f\_parallel} = \bigcap_{i=1}^{n} P_{f-i} \tag{5.1}$$

, or

$$P_{f-parallel} = \int_{\bigcap g_i<0} f(\mathbf{x})\, d\mathbf{x} \tag{5.1-a}$$

When the failure conditions of members of the parallel system are mutually exclusive, $P_{f\_parallel}$ can be written as:

$$P_{f\_parallel} = \prod_{i=1}^{n} P_{f\,i} \tag{5.1-b}$$

The upper bound of the probability of failure for a parallel system is the failure probability of any single component.

Failures of all members in a series system are fully positively correlated since the failure of any member induces the failure of the remaining members. The survival of members on the other hand, is independent from other members. The lower bound for the probability of failure is therefore the largest failure probability of any single member. The upper bound is the complement of the survival set of all members, i.e.; the systems fails *if not all* members survive. The survival necessitates the survival of all members of the series; hence the survival or safe set, $L_{\text{-}safe\text{-}series}$, is the intersection of safe sets of all members.

$$L_{-safe-series} = \bigcap_i g_i > 0 \qquad (5.2)$$

The failure set, $L_{\text{-}failure\text{-}series}$, is the complement of the safe set:

$$L_{-safe-series} = 1 - \bigcap_i g_i > 0 \qquad (5.2\text{-a})$$

The upper bound for the probability of failure of a series system is [3, 43, 56, 68, 85]:

$$P_{f\_series} = 1 - P_{survival\_series} \qquad (5.3)$$

The above result can be generalised by straightforward deduction from of Figure 5.2 to:

$$P_{f-series} = \int_{\bigcup g_i < 0} f(\mathbf{x})\, d\mathbf{x} \qquad (5.3\text{-a})$$

For a series of discrete members, the probability of survival and failure, $P_{survival}$ and P_f, can be simplified to:

$$P_{survival\_series} = \prod_{i=1}^{n} P_{survival-i} \qquad (5.3\text{-b})$$

Thus the probability of failure is:

$$P_{f\_series} = 1 - \prod_{i=1}^{n} P_{survival\_i} \tag{5.4}$$

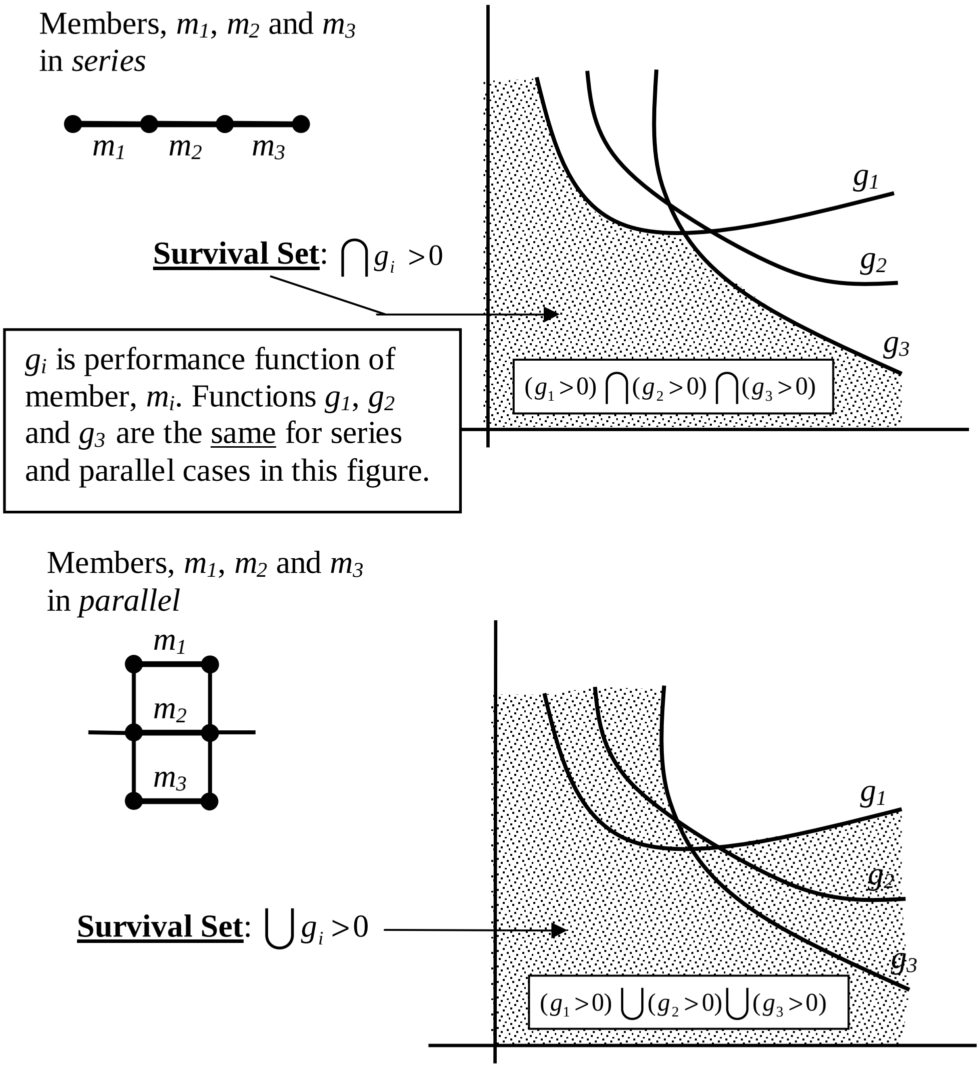


Figure 5.2 Series and parallel System Reliability

It is easily seen from Figures 5.2 that parallel systems offer greater safety than series systems under similar performance conditions. The extra safety is the additional area offered by any one member; i.e. area below any of the performance functions g_1, g_2 or g_3. This is further illustrated in Figure 5.3.

Safety Difference between Parallel and Series Systems

= $\left(\bigcup g_i > 0\right) - \left(\bigcap g_i > 0\right)$, denoted by

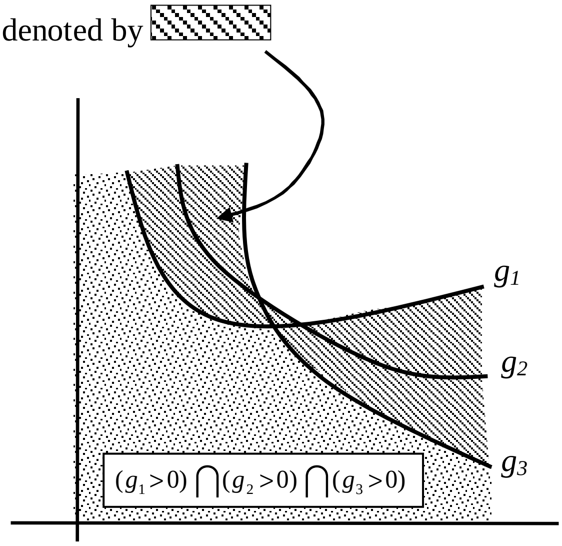


Figure 5.3 Difference of Safety between Series and Parallel Systems

Purely series systems cannot be theoretically designed with any binary redundancy; i.e.; if members are either perfectly operational or failed, then adding members to the series to make it longer does not enhance its overall reliability. The capacity of *the weakest* member or members must be amplified to allow for any margin of tolerance.

5.4.2 Redundant Systems

When a system contains a component that is idle during normal operation conditions, but becomes active on the incidence of failure, it is called a redundant system. The definition is quite oversimplified since the components many engineering systems are quite interrelated and contain some reserve capacity.

Regardless of the particular mechanism of load or demand redistribution, redundancy exists if the demand can be sustained after partial failure for a specified period of time.

In probabilistic terms, providing alternative routes translates into displacing the failure bounds in the negative direction; i.e., the lower and upper bounds of the failure probability are decreased.

The series systems of figure 5.4 are taken as examples. Assume that the failures of all members are statistically independent. Then, the lower bound of the probability of failure for the system, m_1-m_3, is:

$$P_{f\,\min-(m_1-m_3)} = \max\left(p_{f-m_1}, p_{f-m_2}, p_{f-m_3}\right) \qquad (5.5)$$

Assume that $P_{f\text{-}min(m1\text{-}m3)}$ is $P_{f\text{-}m2}$; that is m_2 is the weakest link. It is decided to supplement m_2 with a standby member, m_4 that is activated if m_2 fails. Therefore, the system would fail in the region of m_2 and m_4 only if both fail. The lower bound of the probability of failure for the system becomes:

$$P_{f\,\min-(m_1-m_3\,with\,m_4)} = \max\left(p_{f-m_1}, p_{f-(m_2\ AND\ m_4)}, p_{f-m_3}\right) \qquad (5.5\text{-a})$$

And $P_{f\min}$ of the series is decreased since,

$$P_f\left[m_2\ AND\ m_4\right] = P_f\left[m_2 \cap m_4\right] = P_f\left[m_2\right] \times P_f\left[m_4\right] \qquad (5.5\text{-b})$$

A calculation for the upper bound should yield a similar trend since the survival of the system follows either route m_1-m_2-m_3 OR m_1-m_4-m_3, as shown in figure 5.4.

Members, m_1, m_2 and m_3 in *series*

Members, m_1, m_2 and m_3 in *series* with a *standby* member, m_4, in case m_2 fails

Figure 5.4 Series and Redundant Systems

It is evident from the above analyses that series systems are more sensitive to failure events than parallel systems, and are likely to fail with little if any warning. This knowledge should be utilised while designing series-type structural systems such vertical wall bracing, or columns in multi-storey buildings.

Enhancing the capacity of some members in a series has been successfully used to hinder failure propagation. In the United States, power transmission towers are designed for “security loads” [86, 87]. One in every 10 to 20 transmission towers in a series is designed to withstand the effect of failure of intermediate towers, thereby preventing domino-like (cascading) progressive collapse, albeit that the principal function of power transmission is disrupted. Figure-5.5 illustrates the concept.

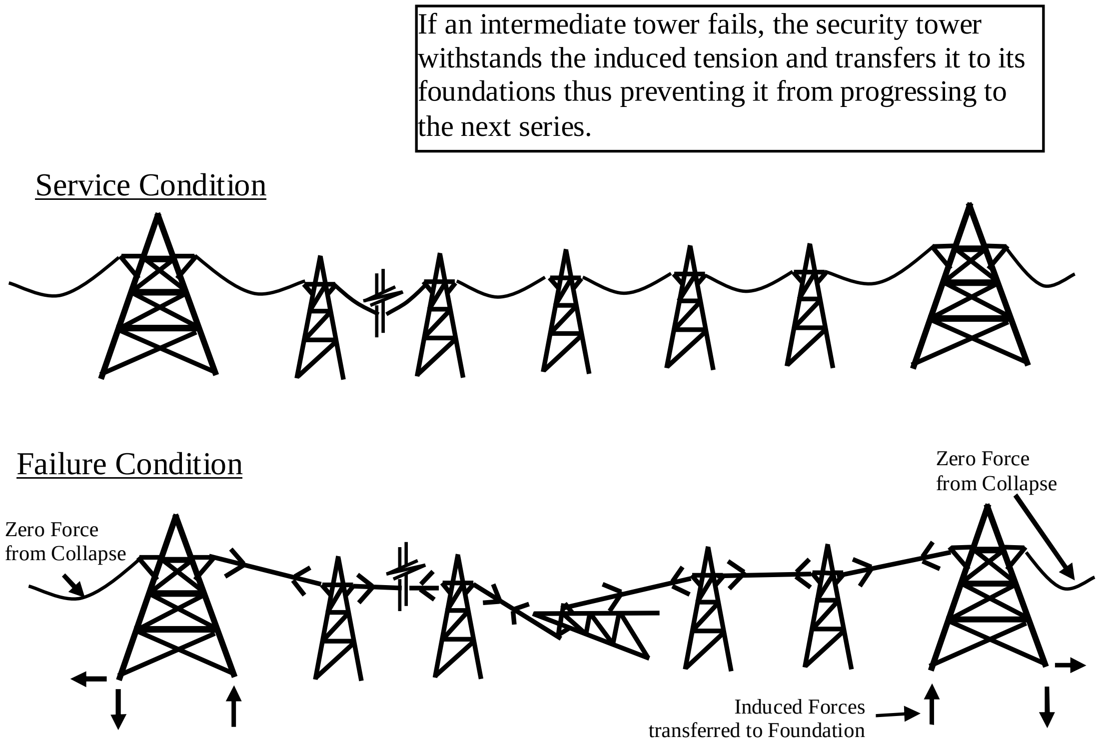


Figure 5.5: Security Design to prevent Cascading of Power-Transmission Towers

5.4.3 Reliability of Damaged Structures

The probability that a structure fails after damage can be easily modelled using conditional probability rules. If an event *A*, results in damage, D_A, the probability of the development of damage, D_B, due to a subsequent or correlated event, *B*, can be expressed as:

$$P[D_B|B] = P[A]P[D_A|A]\,P[B|A] \qquad (5.6)$$

5.4.4 System Reliability Index

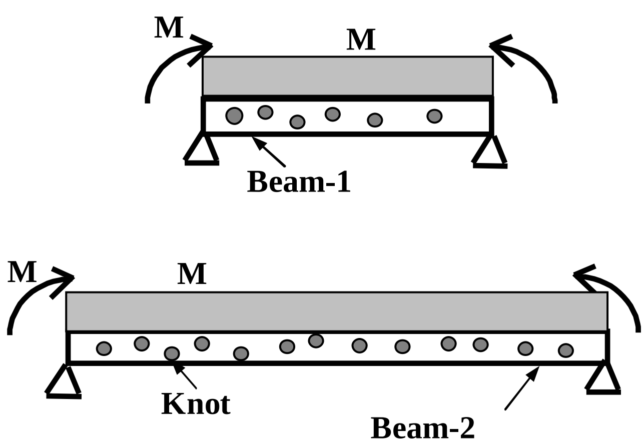
The reliability index of any system type can be calculated using the basic relationship:

$$\beta_{system} = \Phi^{-1}(1 - P_{f-system}) \tag{5.7}$$

The minimum $P_{f\text{-}system}$ is obviously the value of most interest.

Example – 1

The failure of a chain is governed by its weakest link. The longer the chain is, the higher its probability of failure. Timber members are a traditional example where chain-like failure mode applies [88]. The reduction in their strength is a function of timber defects (knots, etc) which generally increase as the length of the member increases. Two beams of different length will therefore have different probabilities of failure even when subjected to the same bending moment as shown in Figure-5.6.



The probability of failure of Beam-2 (the longer beam) is greater than Beam-1 since the number of knots is greater, although they have the *same* section, and are subject to the same forces.

Figure 5.6 Probability of Failure versus Beam Length

Example - 2

A bracing system incorporating tension & compression members is shown in Figure-5.7. The elements in the individual bays are assumed of equal stiffness and therefore share the load equally. The shown structure represents a parallel system. The compression member was optimised. Since wind can act in any two opposite directions of any arbitrary plane, the designer has had to upgrade the tension member to the same section of the compression member. With properly-designed connections, the tension member is assumed to fail in yielding (β = 3.0). The compression member is assumed to fail in buckling (β = 3.6).

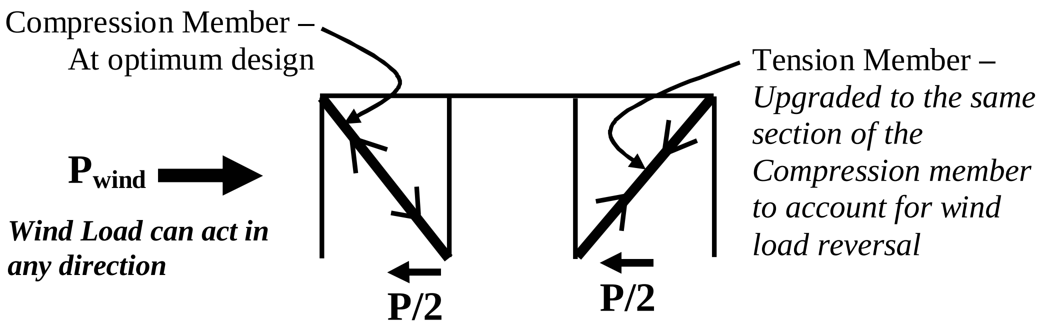


Figure 5.7: Example of a Bracing System

The above inevitable upgrade yields a *realised* reliability index for the tension member of a minimum of 3.6.

$$\beta_{tension} \geq 3.6$$

The critical compressive stress can reach the yield stress in a purely hypothetical condition when the slenderness ratio, kl/r, of the member is zero.

$$\lim_{kl/r \to 0} f_{cr} = f_y$$

If the realised reliability index, $\beta\_{compression}$, is equal to 3.6, $\beta\_{tension}$ would be greater than 3.6, since the tension member does not buckle and therefore has a larger capacity.

$$\text{If } \beta_{compression} = 3.6 , \Rightarrow \beta_{tension} > 3.6$$

The last equation indicates that the compression member is the more likely to fail before the tension member.

Example – 3

A structural steel multi-storey frame for an office building is designed for a governing load combination of Dead + Imposed load (DL + LL). Since the overall height of the internal column is just under12m, the designers opted for a uniform section for the entire height. The design has thus simplified the connection details and insured the same beam length and details throughout all three floors. The saving in detailing, fabrication and erection outweighs the slight increase in column weights. Should the client accept the upgrade?

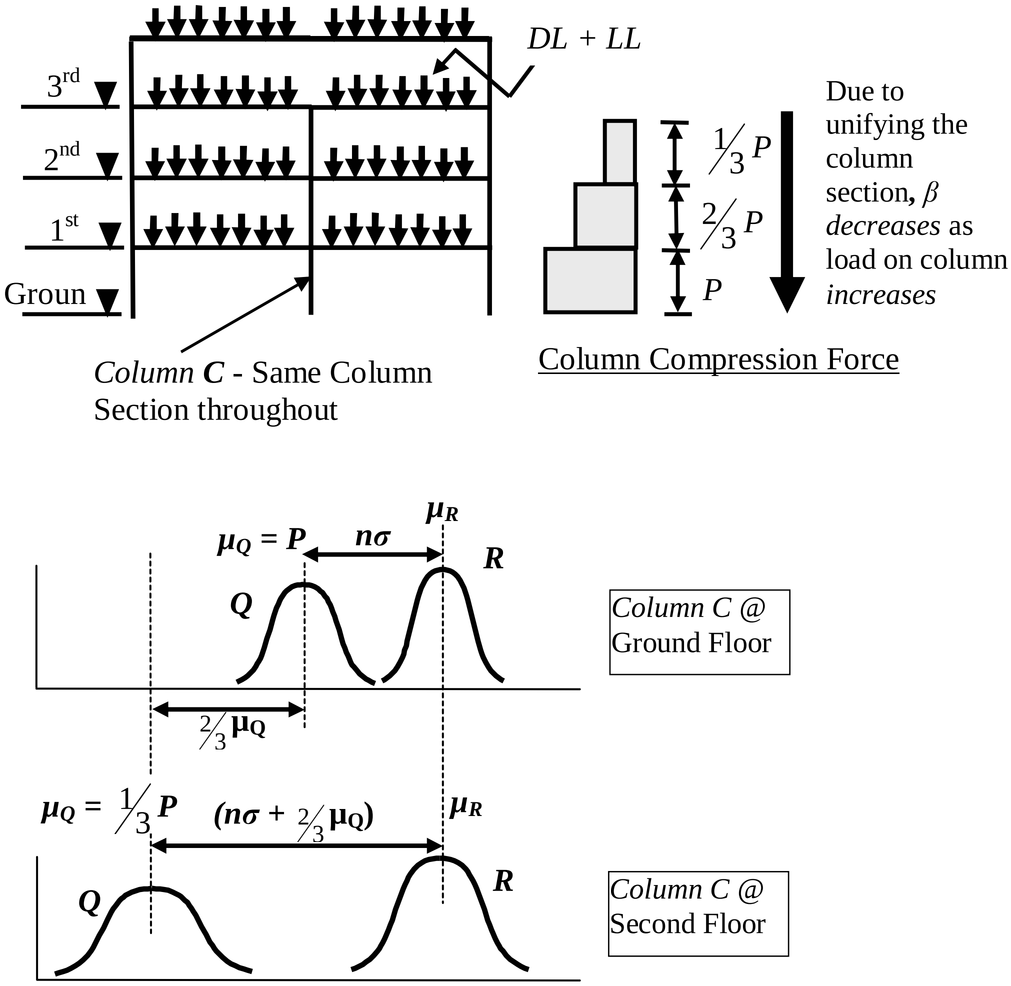


Figure 5.8: Code Objective contradicted by Unifying Column Section

Assuming that the column is a short column, β can be taken as 2.7, and applies to the ground floor section that is presumably optimised for the maximum load.

The column section at the second floor carries 1/3 of the ground floor column load. For the purposes of this example, typical values of the mean and variance for dead and live loadings are taken from reference [89].

Load	Bias factor ,λ (λ = Mean, μ_Q / *Nominal*)	Coefficient of Variation, *COV*
Dead	1.05	0.10
Live Load	1.0	0.25

Assume the ratio of Live/Dead load (*LL/DL*) of 2.0. Then,

$$\mu_Q = \mu_{QDL} + \mu_{QLL}$$
$$= 1.5\mu_{QLL}$$

COV_{DL} = 0.1, and COV_{LL} = 0.25

$$\Rightarrow \quad \sigma_{DL} = 0.1\mu_{QDL} = 0.1\lambda_{DL}\,DL$$

For DL/LL= 2.0,

$$\sigma_{DL} \quad = 0.1\,\lambda_{DL}\,\frac{LL}{2}$$
$$= 0.1\,\frac{\lambda_{DL}}{\lambda_{LL}}\,\frac{\mu_{QLL}}{2}$$
$$= 0.053\,\mu_{QLL}$$

We also have,

$$\sigma_{LL} = 0.25\,\mu_{QLL}$$

Hence,

$$\sigma_{DL+LL} = \sqrt{\sigma_{DL}^2 + \sigma_{LL}^2}$$
$$= \sqrt{(0.053\,\mu_{QLL})^2 + (0.25\,\mu_{QLL}})^2$$
$$= 0.256\,\mu_{QLL}$$

The offset between μ_Q and $\mu_Q/3$ is $2/3\mu_Q$, which corresponds to a number of standard deviations, *n*: (*n* as shown in the above figure)

$$n = \frac{2/3\,\mu_Q}{\sigma_{DL+LL}} = \frac{2/3 \times 1.5\,\mu_{QLL}}{0.256\,\mu_{QLL}}$$
$$= 3.91$$

The reliability index for the second floor column is 2.7 + 3.91 = 6.61, compared to 2.7 of the ground floor column.

5.5 What can Engineers do to Complement LSD/LRFD Designs?

The design situations in the above examples are frequently encountered in practice. An understanding of the basic principles of reliability design can help engineers improve safety of structures.

Safety can be increased by reducing uncertainty. A simple application of this principle can be used to deal with the situations described in the above examples. Uncertainty can be reduced by imposing more stringent quality control procedures on the more important elements.

Some simple qualitative procedures are cheap and very efficient. In the above cases, an engineer would request more thorough check for wood defects on longer beams, or specify smaller tolerances on the ground floor column. In reinforced concrete in particular, more test cubes should be taken for columns or shear walls (especially at lower floors) than for beams or slabs. Moreover, the uncertainty arising from human error can be decreased by assigning the more critical members to better qualified workers with more frequent direct supervision.

5.6 Conclusion

This chapter describes the current treatment of structural design by limit state codes. It highlights the need for more in-depth thinking of whole structure safety.

LRFD should not be perceived as a tool of to optimise design. It is an organisational platform that serves three purposes: to unify design practice, to provide measures for quality control and to provide a space that allows systematic improvement to the code with the arrival of new knowledge.

Until formal risk-based guidelines evolve, designers should take note of the following recommendations:

- It must be remembered that Limit State or LRFD codes are based on reliability which relates to probability of failure but not its consequence. The mode and result of failure are implicitly taken into account but only at the member level. Designers should give thought to the risk associated with failure and put in place ad-hoc mitigation measures.

- Thought must be given to the overall structural system. Well-designed structures are resilient. They have sufficient redundancy to confine the effects of localised failures and adapt to damage.

- Series systems such as columns in multi-storey buildings or bracing are more susceptible to disproportionate collapse than parallel systems. They should be optimised as much as possible, and redundancy should be provided by parallel components. Connections should be designed to withstand localised damage and redistribute the load to undamaged members.

- Designs should be optimised as much as practically possible. Increasing member sizes might have the opposite effect on whole structure safety. If upgrade is necessary, it should be done consistently and proportionately throughout: connections as well as beams, foundations with columns, etc. Conditions where member capacity is higher than its connections can lead to catastrophic failures. Therefore, a minimum capacity for connections equal to the maximum capacity of the upgraded member should be maintained. Case 3 in reference [90] is quite insightful. An upgrade can be as much a malignant *overdose* as a benign act of generosity.

- Reducing uncertainty is key to increasing safety. Quality control procedures can be utilised to enhance the safety of more critical members.

Chapter 6

Strength Loss Method

"Have you heard of the wonderful one-hoss shay,
That was built in such a logical *way,*
It ran a hundred years to a day,
And then, of a sudden, it-ah, but stay,
……………………………
Now in building of chaises, I tell you what,
There is always somewhere a weaker spot,
………………………
And that's the reason beyond a doubt,
A chaise breaks down, but doesn't wear out,
…………………………..
………………….
But the Deacon swore (as Deacons do),
………………………
It should be so built that it couldn't break daown,
"Fur," said the Deacon, '"t's mighty plain
Thut the weakes' place mus' stan' the strain;
'N' the way t' fix it, uz I maintain,
Is only jest
T' make that place uz strong uz the rest.'"

……………………..

………….

First of November, the Earthquake day,

There are traces of age in the one-hoss shay,

A general flavor of mild decay,

But nothing local, *as one may say,*

There couldn't be, - for the Deacon's art

Had made it so like *in every part*

……………………….

First of November, 'Fifty-five'!

This morning the parson takes a drive.

…………………………..

………………………

The parson was working his Sunday text,

Had got to fifthly and stopped perplexed

…………………….

- First a shiver, and then a thrill,

Then something decidedly like a spill, -

…………………

What do you think the parson found,

When he got up and stared around?

The poor old chaise in a heap of mound,

As if it had been to the mill and ground!

You see, of course, if you're not a dunce,

How it went to pieces all at once, -

All at once, and nothing first, -

Just as bubbles do when they burst.

End of the wonderful one-hoss shay,

Logic is logic. *That's all I say."*

In 1858, Oliver Wendel Holmes, Professor of Anatomy at Harvard Medical School, wrote the "one-hoss shay", of which the above excerpts are taken [91]. Holmes describes an imaginary piece of manufacturing genius that is designed to last one hundred years, exactly; then all parts fail, all at once.

The one-hoss shay was a vehicle in its time. Crushable parts in today's cars and trains and reduced steel beam section in seismic design can be viewed as realisations of Holmes' "weak spots". Another important feature of the one-hoss shay was the elimination of maintenance cost due to all the components failing simultaneously. The reduction of maintenance cost is a fundamental aim of reliability design in many modern engineering disciplines.

The major advances in the automobile industry with regards to reliability and damage control are not paralleled in structural engineering. Unlike structures, vehicles are built in batches that follow rigorous refinements to prototypes [49]. A series of tests to extreme conditions is applied before a model is released for production.

Structural engineers do not enjoy the same luxury. Car design and manufacturing are strictly controlled by manufacturers. In contrast, building design is the result of an architectural embodiment of a personal vision, and the relationship between people and buildings is far more intimate. Full scale tests are neither inexpensive, nor conclusive due to the variety of architectural designs.

The global departure to PBD codes necessitates the use of compatible quantitative risk assessment. The failure of individual members should be mapped to the global structure, so that the relative importance of various members to the overall structural stability is properly represented. Consequence indices that can be integrated with probabilities to produce the risk metric are fundamental ingredients in performance-based design.

The consequence of failure in present codes is not treated as an integral part of risk assessment, but implicitly in the varying reliability indices for different failure modes. This is partly due to member-based design approach adopted by the codes which intrinsically lacks the ability to assess the relative importance of members in the parent structure. The code treatment for failure is confined to the almost generally prescriptive rules for the prevention of disproportionate collapse. This is described in more detail in the following section.

The implementation of performance-based design requires the elevation of today's reliability-based codes to risk-based codes. Risk evaluation forms a basic part of decision making in a performance-based project. The missing ingredient is a *quantitative damage or loss parameter* that can be integrated with the probability of failure to produce risk.

This chapter is devoted to the treatment of structural damage. The Strength Loss method presents a simple quantitative index to evaluate damage within a risk-based metric.

6.1 Structural Engineering Approaches to Prevention of Disproportionate Collapse

The design to mitigate disproportionate collapse is a basic requirement of building codes and regulations. Many national codes stipulate generic analyses to ensure that potential collapse remains at a comparable scale to the cause of damage. Great emphasis is placed on ensuring redundancy in addition to recommendations for connection design to resist the load redistribution that follows damage [92].

In 2011, a comprehensive research of existing methods for structural robustness and prevention of disproportionate collapse was commissioned by the UK Department of Communities and Local Government – Centre for the protection of National Infrastructure [93]. The report provides an extensive review of building code requirements for robustness in the UK, Europe, the USA and Canada.

Following is a brief description of some popular approaches to structural robustness. More details are available in references [92, 93].

Prescriptive rules for structural robustness are the most popular methods in practice. They are simple, quick and as a result quite inexpensive. Two common approaches are *tie-force design* and *key element design* methods [92, 93]. Tie-force design is based on specifying a minimum tensile capacity for connections to ensure that catenary action can take place on the onset of a column removal. As a result of specifying the connection capacity, this method is restricted to buildings with a maximum of five storeys in the UK [92]. Structural members whose risk of removal exceeds certain limits are designed as key elements. In the UK, such members are required to resist a 34 kN/m^2 pressure applied in any direction.

More advanced techniques, such as *alternative load-path* methods, focus on the dynamic response of the structure upon the sudden loss of a member or sudden application of load. These employ dynamic analysis following a scenario-dependent and scenario-independent approach. When the hazard that initiates damage (the scenario) is not modelled, the analysis is called scenario-independent. This is the more popular option, where the effect of removing a member, for example, is modelled by applying its loads suddenly to the structure. Scenario-dependent methods on the other hand, use the specific hazard to evaluate the damage caused to the structure.

Performance-based rules concern certain classes of buildings and involve the use of *risk-based* methods. The building classes relate to the number of people at risk in the event of collapse, and include buildings over 15 storeys high or grandstands accommodating more that 5000 spectators [92]. These methods require a probability-based analysis of the hazard and its consequence, but are not currently used in the main body of codes and standards [93].

The application of risk-based methods requires the use of damage indices that can be integrated with the probability of occurrence in the risk metric. Some damage assessment models are reviewed in the following sections.

6.2 Existing Damage Assessment Models

Damage models fall in two main categories: empirical and analytical. The common purpose of the two approaches is the development of a “value” parameter that can be applied within a life-cost model.

6.2.1 Empirical damage Models

The focus of experimental investigation has been existing structures. Some examples are given below and more details can be found in reference [94].

A point system was used to grade buildings in Long Beach, California in 1971. Points are assigned to different features of each building. These include: Framing system, Bracing system, Partition, Special hazards such as un-reinforced masonry, and the Physical condition (signs of deterioration such as bowing or cracking). The total points are algebraically summed to produce a damage index.

Other methodologies were adopted to evaluate the relative damage resulting from inter-storey drift. In one model, the damage to the i^{th} storey, D_i, is calculated as:

$$D_i = F\left(\frac{\Delta_i}{\left(\Delta_y\right)_i}\right) \tag{6.1}$$

in which, Δ_i is the calculated inter-storey drift of the i^{th} storey, $(\Delta_y)_i$ is the inter-storey drift of the i^{th} storey at yielding and F is a distribution function.

The cumulative damage in structures was described by a number of researchers [94]. In seismic structures, the following expression was proposed:

$$D = \sum_{i=1}^{n} a_i \left\langle \frac{Z}{y} - 1 \right\rangle^{b_i} \qquad (6.2)$$

where Z is the maximum displacement response, y is the yield displacement, and a_i and b_i are empirical constants. The $\left\langle \frac{Z}{y} - 1 \right\rangle^{b_i}$ is a singularity function defined as follows:

$$\left\langle \frac{Z}{y} - 1 \right\rangle^{b_i} = \begin{cases} b_i \geq 0, \left\langle \frac{Z}{y} - 1 \right\rangle^{b_i} = 0 \\ b_i = 0, \left\langle \frac{Z}{y} - 1 \right\rangle^{b_i} = \left(\frac{Z}{y} - 1 \right)^{b_i} \end{cases}$$

Another model was developed for the damage resulting from a predefined series of events k, as:

$$D_k = \frac{\sum_{i=1}^{m} w_{ik}\, d_{ik}}{\sum_{i=1}^{m} w_{ik}} \qquad (6.3)$$

where,

$$d_{ik} = d_{ik} + \left(\frac{Z_{ik} - y_{ik}}{c_{ik} - y_{ik}} \right) \left(1 - d_{ij} \right) \qquad (6.3\text{-a})$$

In the above expressions,

w_{ik}: cumulative importance factor for the i^{th} member and event k

d_{ik}: local damage index, element i and event k

d_{ij}: local damage index, element i and events $j < k$

Z: demand in terms of displacement or other parameters

c: capacity in units consistent with demand

y: threshold f or limit state, such as yielding.

In another research, laboratory test data was analysed for arbitrarily defined damage states [94]. The damage states were for example: yielded, cracked or failed. The variations of the threshold values of damage states were described using normal or lognormal distributions. They defined a "central damage factor", γ as the ratio of the estimated repair cost to the replacement cost of an element. The expected damage was thus calculated as:

$$E(D) = \sum \gamma P(D_i|v) \qquad (6.4)$$

where $P(D_i|v)$ is the conditional probability of damage state D_i given demand v.

6.2.2 Analytical damage Models

The adoption of performance-based design in earthquake engineering resulted in an increase of research activity towards the definition of damage indices. The Pacific Earthquake Engineering Research Center (PEER) methodology was a precursor to the development of some damage indices [30, 95].

PEER's formula is a description of the probable damage associated with one event "*or*" another. The "*or*" operator becomes a summation of probabilities which is generalised into the multidimensional integral in the formula, shown below [30]:

$$v(DV) = \iiint G\langle DV \mid DM \rangle \left| dG\langle DM \mid EDP \rangle \right| dG\langle EDP \mid IM \rangle \left| d\lambda(IM) \right. \qquad (6.5)$$

In the above, *DV* is a Decision Variable (failure, loss), *DM* is a Damage Measure (e.g.; repair), *IM* is an Intensity Measure (spectral acceleration) and *EDP* is an Engineering Demand Parameter (such as inter-storey drift, buckling load or stiffness). The left side of the equation, $v(DV)$ is the probability of *DV*, which could be the probability of failure, P_f, or that of loss, P_{-loss}. Equation 6.5 is a representation of an event tree of all probable events (hazards). It simply reads: the probability of loss (*risk*) is the probability of *all* possible hazards with intensity *IM*, causing an *EDP* (buckling load), that results in a *DM* (repair).

Damage indices are the different parameters suitable for representing *EDP*. Some examples are presented below.

One proposal for the damage index, *EDP*, was the eigenvalue buckling load factor, λ_{cr} [30]. The corresponding probability of failure is calculated as:

$$P_f = P(\lambda_{cr} < 1) \qquad (6.6)$$

The above method is computationally efficient and the choice of the buckling load is appropriate when stability is the main consideration. The authors [30] however acknowledge the limitation of linear buckling analysis used in the method.

Disproportionate collapse becomes “disproportionate” when indirect risks contribute significantly to the system risk. For example, failure of five floors (indirect risk) that follows the failure of a column on another floor (direct risk) is disproportionate. An index of robustness, I_{Rob}, was defined as the ratio of the system direct risk to the total direct and indirect risk [95].

$$I_{Rob} = \frac{R_{Dir}}{R_{Dir1} + R_{Indir1}} \qquad (6.7)$$

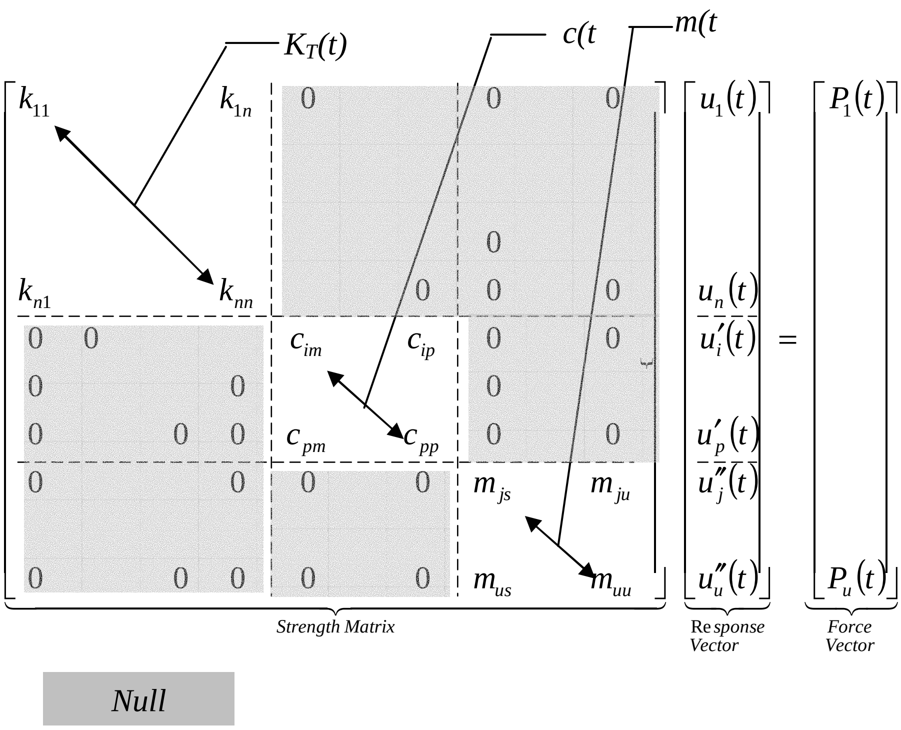
The index takes a value of 1 for a robust system with no indirect risk, and zero when all risks are indirect [95].

6.3 The Strength Loss Method

Under arbitrary external effects, **P(**t**)**, at any point in time, t, **P(**t**)** is constant and equal to **S(**t**)R(**t**)**, where **S(**t**)** is the strength matrix assembled from the structure tangent stiffness matrix (Thermo-elastic plastic matrix [96]), $\mathbf{K_T}(t)$, the damping matrix, **c(**t**)** and the mass matrix, **m(**t**)**,

$$\mathbf{P}(t) = \mathbf{S}(t)\,\mathbf{R}(t) \qquad (6.8)$$

R(t) is a response vector formed by grouping the displacement vector, *u(t) that includes thermally-induced displacements,* the velocity vector, *u'(t),* and the acceleration vector, *u''(t)* .



At any point in time, *t*, under a specific design condition, P(t) is constant. Hence,

$$\mathbf{S_1}\,\mathbf{R_1} = \mathbf{S_2}\,\mathbf{R_2}\,,$$

Then,

$$\mathbf{I}\,\mathbf{R_1} = \mathbf{S_1}^{-1}\,\mathbf{S_2}\,\mathbf{R_2}$$

Define $\mathbf{R_k} = \mathbf{S_1}^{-1}\mathbf{S_2}$. Since $\mathbf{I}\,\mathbf{R_1} = \mathbf{R_1}$, we have:

$$\mathbf{R_1} = \mathbf{R_k}\,\mathbf{R_2}$$

Inversing both side yields:

$$\mathbf{R_1}^{-1} = (\mathbf{R_k}\,\mathbf{R_2})^{-1}$$

or,

$$\mathbf{R_1}^{-1} = \mathbf{R_2}^{-1}\mathbf{R_k}^{-1}$$

Multiplying both sides by $\mathbf{R_2}$:

$$\mathbf{R_2}\mathbf{R_1}^{-1} = \mathbf{I}\,\mathbf{R_k}^{-1}$$

Inverse both sides,

$$\mathbf{R_k} = \mathbf{R_1}\mathbf{R_2}^{-1} \qquad (6.9)$$

The dot product of $\mathbf{R_1}$ and $\mathbf{R_2}^{-1}$ is achieved by transposing $\mathbf{R_2}^{-1}$ and pre-multiplying by $\mathbf{R_1}$ and taking the trace of $\mathbf{R_1}$ and $\mathbf{R_2}^{-1}$, $tr(\mathbf{R_1}\,\mathbf{R_2}^{-1})$, which is equal to $tr(\mathbf{R_2}^{-1}\,\mathbf{R_1})$.

Therefore, the product, $\mathbf{R_1}\,\mathbf{R_2}^{-1}$, is a 1x1 matrix, the trace of which is a single value, R_k.

$$R_k = tr\,(\mathbf{R_1}\mathbf{R_2}^{-1}) \qquad (6.9\text{-a})$$

If $\mathbf{R_1}$ is associated with an initial state of the system, and $\mathbf{R_2}$ with a subsequent state as affected by an additional load or a decrease of strength, then R_k would range from zero to unity, where unity signifies no loss of strength and zero total collapse.

The damage can be defined as the ratio of the remaining (post-event) strength to the original (pre-event) strength is the strength loss ratio, *DSL*.

$$DSL = \mathbf{S}^{-1}(\mathbf{S}_1 - \mathbf{S}_2) \qquad (6.10)$$

Using equation (6.9), equation (6.10) becomes:

$$DSL = 1 - R_k \qquad (6.11)$$

The calculation of the strength loss ratio, *DSL*, revolves around calculating an inverse for vector $\mathbf{R_2}$. The concept of inverse for a scalar quantity can be illustrated in Figure 6.1.

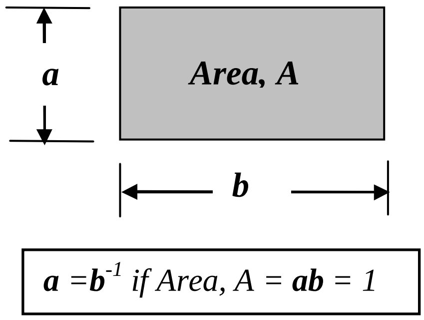


Figure 6.1: Inverse of scalar *b*

It is possible to obtain the same area, *A*, using *b* with more than one value of *a*, if *b* and *a* were associated with direction cosines.

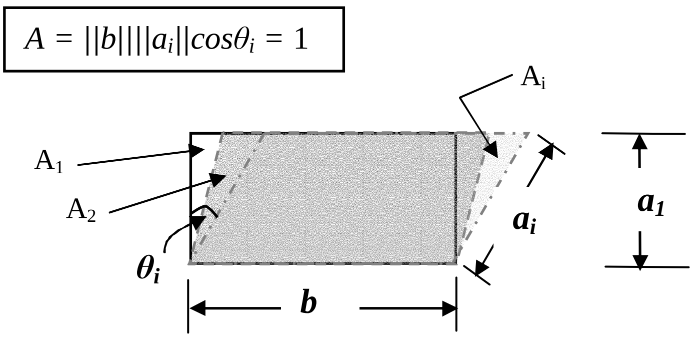


Figure 6.2: (Inverse) vectors for vector **b**

One can conclude that there exist an infinite number of (*inverse*) vectors, $\mathbf{a}_i$, whose dot product of **b** yields an area of 1, as shown in Figure 6.3.

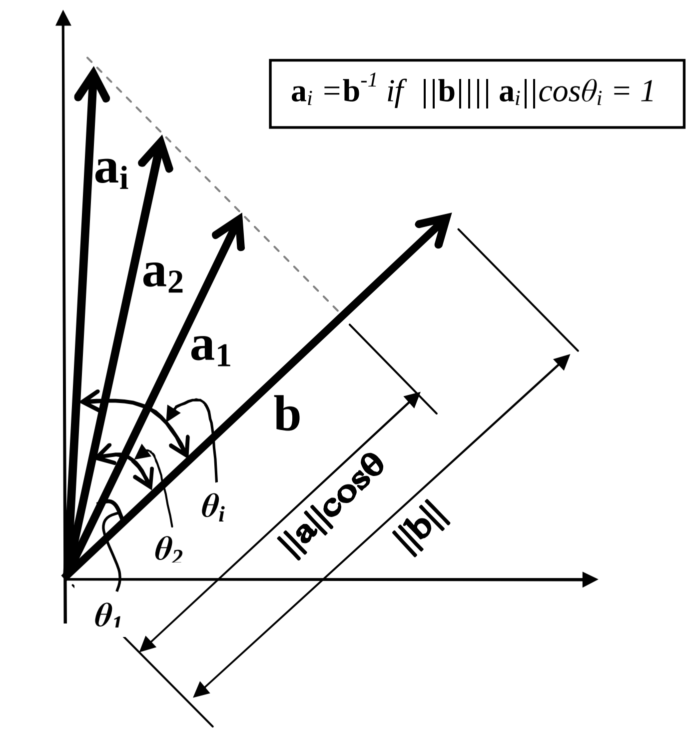


Figure 6.3: (Inverse) vectors for vector **b**

If **a** is *co-linear* with **b**, then the magnitude of **a** would be:

$$\|\mathbf{a}\| = \frac{1}{\|\mathbf{b}\|} \tag{6.12}$$

Defining **η** as the vector of direction cosines for **a** and **b** (being co-linear), vector **a** may be written as:

$$\mathbf{a} = \boldsymbol{\eta}\frac{1}{\|\mathbf{b}\|} \tag{6.13}$$

But **η** is the unit vector of direction cosines for **b**, which can be written as:

$$\boldsymbol{\eta} = \frac{\mathbf{b}}{\|\mathbf{b}\|} \qquad (6.13\text{-a})$$

Hence, vector **a** becomes:

$$\mathbf{a} = \frac{\mathbf{b}}{\|\mathbf{b}\|^2} \qquad (6.13\text{-b})$$

Since **a** and **b** are co-linear, their dot product is determined as:

$$\mathbf{a} \bullet \mathbf{b} = tr\,(\mathbf{a}^T\,\mathbf{b}) = 1 \qquad (6.14)$$

Substituting (6.13-b) in (6.14), the co-linear inverse vector, $\mathbf{b}^{-1}$, is:

$$\mathbf{b}^{-1} = \frac{\mathbf{b}^T}{\|\mathbf{b}\|^2} \qquad (6.15)$$

It is important to reiterate that $\mathbf{b}^{-1}$ in *not a unique inverse* to **b**. There is an infinite number of inverses for any vector, hence the common convention that vectors cannot be inversed. Vector **a** is however the only inverse that coincides with **b**. The reason for selecting this specific vector in calculating R_k is as follows. The calculation of R_k reflects the relative change in the “magnitude” of response parameters at their respective degrees of freedom. To enable a valid comparison of the individual pre-event and post-event response parameters, both response vectors must have the same direction cosines; hence the choice of a co-linear inverse vector.

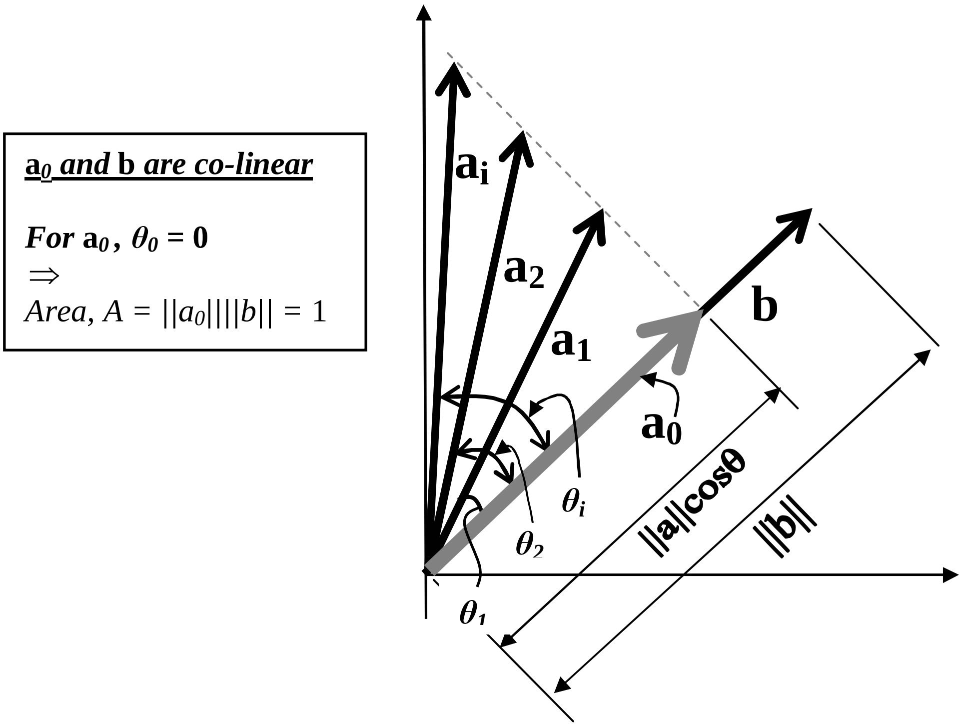


Figure 6.4: Co-linear (Inverse) vector for vector **b**

The inverse vector in the above is similar in concept to reciprocal vectors used in crystallography [66]. The use of the term vector "inverse" is probably better substituted by "vector reciprocal".

6.3.1 Application of the Strength Loss Method in Structural Design for Fire

Understanding the way fire affects buildings is important for proper fire protection design. Unlike many design loads, such as wind or earthquake, fire has a local rather than global effect on the structure which must be considered when extrapolating wind or earthquake solutions to fire safety design. Moreover, the response of structure to fire is highly nonlinear, but the dynamic influence on mechanical stresses is generally negligible. Heating causes relatively slow degradation of material properties coupled with large deformations. As a result, equation (6.8) can be truncated to:

$$\mathbf{P}(t) = \mathbf{K_T}(t)\,\mathbf{u}(t) \qquad (6.16)$$

The Strength Loss ratio, *DSL*, can then be expressed as:

$$\begin{aligned} DSL &= 1 - R_k \\ &= 1 - \mathbf{u}_2^{-1}\mathbf{u}_1 \end{aligned} \qquad (6.17)$$

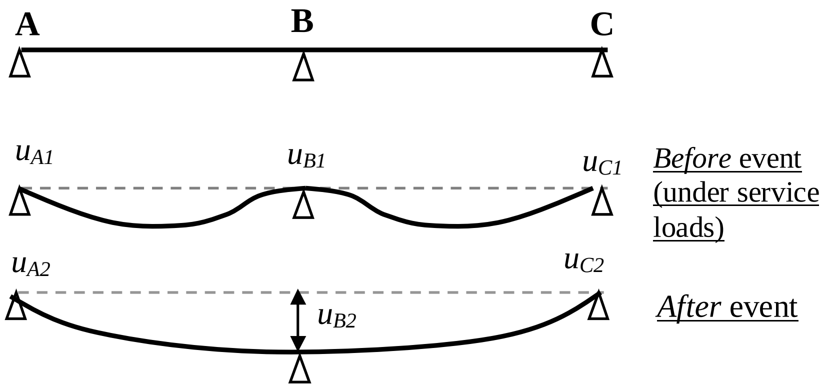
where $\mathbf{u}_1$ and $\mathbf{u}_2$ are the deformation vectors before and after the event (fire) causing damage.

6.3.2 Tips for Calculating the Strength Loss ratio, *DSL*

6.3.2.1 Choice of Degrees of Freedom

The choice of degrees of freedom for structural analysis is arbitrary. The structure can be solved as long as the number of independent DOF's is at least equal to the degrees of kinematic indeterminancy. For convenience, the degrees of freedom, DOF's, at the junctions of members are typically used. Coordinates of the junctions have to be input to define the geometry, and as a result, it is easier to define the degrees of freedom and boundary conditions at these coordinates. It also simplifies the definition of constitutive relations and computer programming.

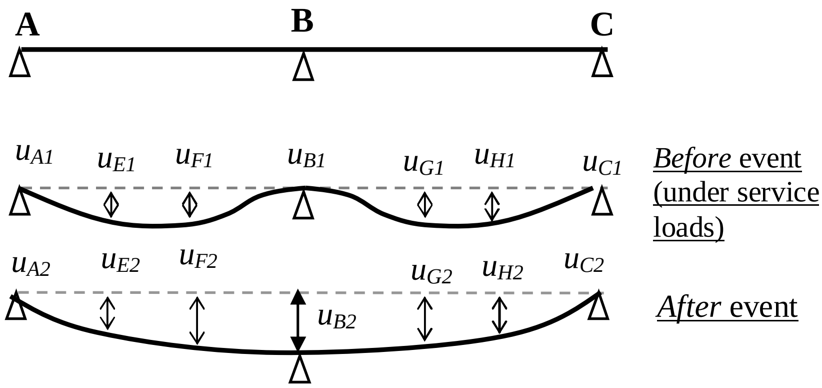
In calculating the strength loss ratio, *DSL*, one is essentially dealing with the output of the structural analysis. It is therefore not necessary to restrict the number of deformations in the response vectors, u_1 and u_2, to those defining the degrees of freedom. Such procedure is not necessary and can sometimes lead to misleading *DSL* values. The beam in Figure 6.5 can be defined using the DOF's at points A, B and C. When the deflections at A, B and C are used to calculate *DSL*, the resulting *DSL* is 1 (total collapse), since the relative change in deflection at B is equal to infinity ($(u_{B2} - u_{B1})/u_{B1}$, $u_{B1} = 0$). This is incorrect as the support displacement does not necessarily indicate collapse.



$$\left.\begin{aligned}\mathbf{u}_1^T &= [u_{A1} \quad u_{B1} \quad u_{C1}]\\ \mathbf{u}_2^T &= [u_{A2} \quad u_{B2} \quad u_{C2}]\end{aligned}\right\} \Rightarrow DSL = 1 - {u_{B1}}/{u_{B2}} = 1, \text{ since } u_{B1} \text{ is zero.}$$

Figure 6.5: Strength Loss for a two-span Beam (Set 1 of $\mathbf{u}_1$ and $\mathbf{u}_2$)

In Figure 6.5-a, *DSL* for the same above beam is calculated with different components of response vectors. The calculated *DSL* in this case is more accurate as it captures more points on the profile of the beam.



$$\left.\begin{aligned}\mathbf{u}_1^T &= [u_{A1} \quad u_{E1} \quad u_{F1} \quad u_{B1} \quad u_{G1} \quad u_{H1} \quad u_{C1}]\\ \mathbf{u}_2^T &= [u_{A2} \quad u_{E2} \quad u_{F2} \quad u_{B2} \quad u_{G2} \quad u_{H2} \quad u_{C2}]\end{aligned}\right\} \Rightarrow DSL < 1$$

Figure 6.5-a: Strength Loss for a two-span Beam (Set 2 of $\mathbf{u}_1$ and $\mathbf{u}_2$)

The above calculations are estimates of *DSL*. Strictly speaking, the precise value of DSL can only be calculated if points at infinitesimally small increments along the entire profile of the structure are used.

6.3.2.2 Calculation of the Strength Loss ratio, *DSL*

In some cases, the structure can deform in a way that the deformations cancel each other in their algebraic sum. One case is shown in Figure 6.6.

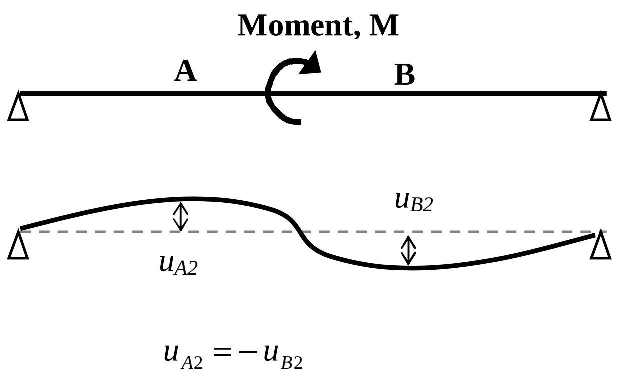


Figure 6.6: Strength Loss for a Beam with an Applied Central Moment

The direct calculation of R_k from equation (6.17) would indicate zero damage. This situation is circumvented by taking the "square root of the square of $\mathbf{u}_1$ and $\mathbf{u}_2$ components" in calculating R_k. For example,

$$\left.\begin{aligned}\mathbf{u}_1^T &= \begin{bmatrix} u_{A1} & u_{B1} & u_{C1} \end{bmatrix} \\ \mathbf{u}_2^T &= \begin{bmatrix} u_{A2} & u_{B2} & u_{C2} \end{bmatrix}\end{aligned}\right\}$$

$$\Rightarrow R_k = \frac{1}{\left\|\mathbf{u}_2\right\|^2}\left(\sqrt{\left(u_{A1}u_{A2}\right)^2} + \sqrt{\left(u_{B1}u_{B2}\right)^2} + \sqrt{\left(u_{C1}u_{C2}\right)^2}\right)$$

This procedure was used in calculating R_k for the example in the next section.

Example

A section in simple structure of a multi-storey building is depicted in Figure 6.7 [7]. It consists of a line of columns connected to a stiff inner core by 12m composite beams. The structure was subjected to a standard time-dependent temperature curve, representing a severe fire scenario at the fifth, sixth and seventh floor. The material properties for steel and concrete are nonlinear and are functions of the temperature evolution. The beams are analysed without passive fire protection and are subjected to a maximum temperature of 800 °C. The columns are assumed to have passive fire protection whose influenced is modelled by limiting the maximum temperature to 400 °C. The analysis was performed using the general-purpose finite element programme, Abaqus. The model data was kindly provided by Dr. Charlotte Röben [7].

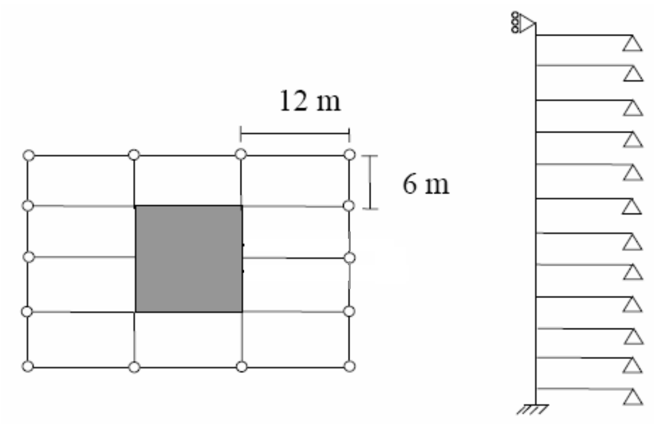


Figure 6.7: Plan & Section of the Structural Model
(Reproduced from reference [7])

The displacements resulting from temperature rise are used to calculate the strength loss ratio at different time increments. The evolution of strength loss with time is shown in Figure-6.8, and Figure-6.9 shows the relative change of the strength loss ratio with respect to time.

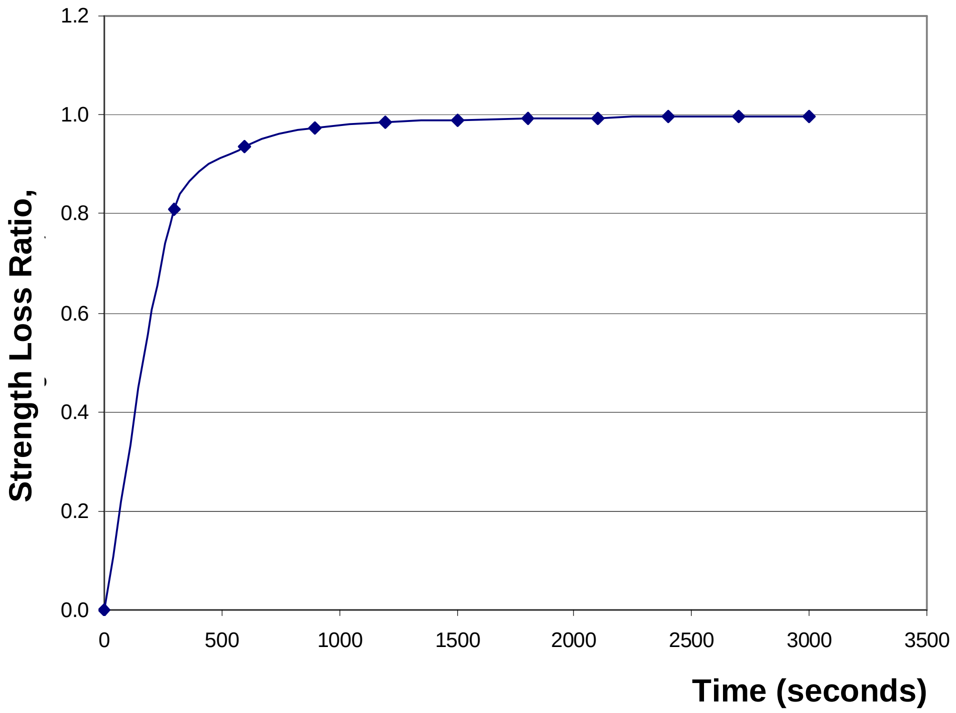


Figure 6.8: Strength Loss Ratio, *DSL*, vs. Time

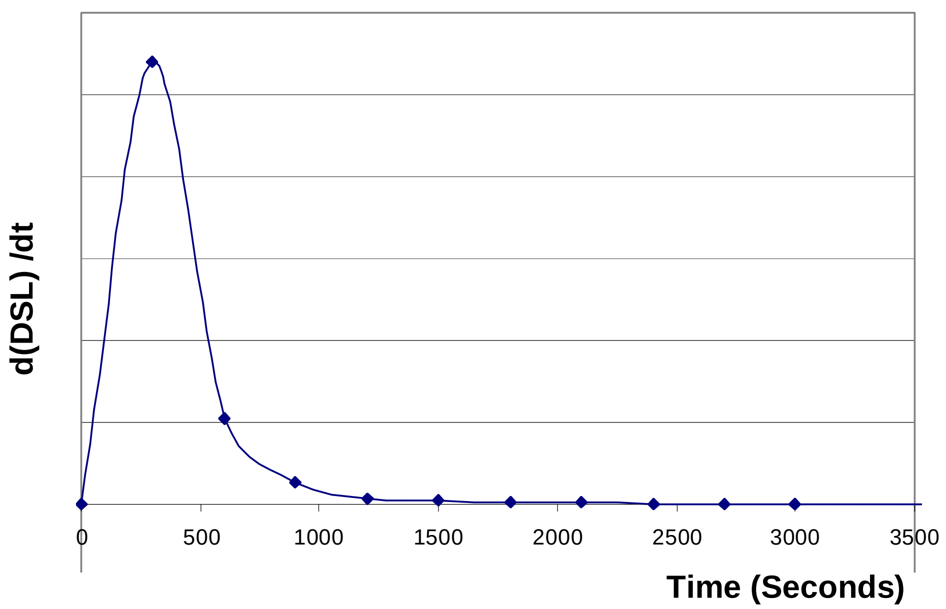


Figure 6.9: Change in Strength Loss with respect to Time

The zone in the region of 400 seconds signifies the maximum rate of strength loss, which can be a sign of runaway behaviour. After the 1000th second, the structure undergoes a transition into a new load-carrying mechanism, as indicated by a near constant rate of strength loss.

6.4 Application of Strength Loss Method in Performance-Based Design

Logical design apportions risk uniformly to the elements of a structure, and structural members are proportioned to ensure a consistent consequence of failure.

A proposal for the expected total cost of the structure was given in equation (4.25-a) in chapter 4 as:

$$L_i = I_i + \Phi\left(-\beta_{T_i}\right)D_i + R_i$$

The above can be rewritten as:

$$L_i = I_i + P_{fT}\, D_i + R_i \qquad (6.18)$$

where P_{fT} is the target (acceptable) probability of failure.

The strength loss ratio, *DSL*, can be used as the damage index, D_i, in the above formula.

For a multi-storey building, the probability of fire occurrence is equal for all compartments of similar fire-pertinent conditions. That is P(Fire 1) = P(Fire 2) = … = P(Fire i), where i is the index describing the location of the compartment.

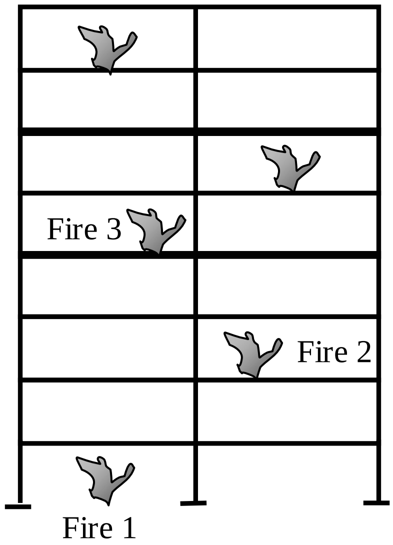


Figure 6.10: Fire Scenarios for a Multi-Storey Building

Under any number of credible fire scenarios, the total cost, L_i, should be kept constant. Suppose that the strength loss ratio, DSL, was calculated for two example scenarios, Fire 1 and Fire 2 say, as $DSL_{\text{-Fire 1}}$ and $DSL_{\text{-Fire 2}}$. The total cost corresponding to Fire 1 is calculated as:

$$L_1 = I_1 + P_{fT}\, DSL_{-Fire1} + R_2$$

For Fire 2, L_2 is:

$$L_2 = I_2 + P_{fT}\, DSL_{-Fire2} + R_2$$

For a constant cost (or risk),

$$L_1 = L_2$$

giving,

$$I_1 + P_{fT}\, DSL_{-Fire1} + R_1 = I_2 + P_{fT}\, DSL_{-Fire2} + R_2$$

The above equation can be written as:

$$(I_1 + R_1) - (I_2 + R_2) = P_{fT}\,(DSL_{-Fire1} - DSL_{-Fire2})$$

Defining the initial and running cost, C_{IR}, as $C_{IR} = I + R$, the above equation can be generalised as:

$$\Delta C_{IR} = P_{fT}\,\Delta DSL \qquad (6.19)$$

Understanding the way fire affects buildings is important for proper fire protection design. Fire action is fundamentally different from other design loads, such as wind or earthquake, since its effect can be alleviated by active and passive fire protection systems. Different fire safety options have different initial and running cost. Statistics for the probability of failure of fire safety systems (alarms, sprinklers, passive fire protection) is also available. Finally, the calculated damage (DSL) can be reduced by enhancing the structural strength. It is therefore possible to vary the parameters in equation (6.19) to compare different fire safety options while keeping the risk constant.

Suppose that sprinklers with passive fire protection were used for Fire 1 compartment, with a combined probability of failure of P_{F1}. Sprinklers may then be excluded from Fire 2 compartment if the following condition is met:

$$C_{IR1} - C_{IR2} = P_{\mathrm{F1}}\, DSL_{-Fire1} - P_{\mathrm{F2}}\, DSL_{-Fire2}$$

where P_{F2} is the probability of failure of fire protection systems without sprinklers, and $DSL_{\text{-Fire 1}}$ and $DSL_{\text{-Fire 2}}$ are the strength loss rations for compartment 1 (with sprinklers), and compartment 2 (without sprinklers).

The risk associated with multiple floor fires can be assessed in a similar manner by taking the probability of fire progressing to other floors. Details of calculating the probability of multiple floor fires can be found in reference [9].

6.5 Conclusion

The application of risk in performance-based design requires the use of a damage (or loss) index coupled with the probability of hazard under consideration. A number of existing empirical and analytical damage models were reviewed in this chapter.

PEER's formula yields the expected risk to a facility under a multitude of probable actions. It takes as such a *passive* approach to risk assessment, providing the client with the expected risk to any candidate design. Different designs may have *different risks*.

The proposed PBD design procedure in section 2.14.2 (chapter 2) for optimising the life-cycle-cost (*LCC*) on the other hand, is an *active* method of calculating the optimum LCC for different designs *under the constraint of equal risk*. The client thus gets a number of design options, all carrying the *same risk*.

Whether PEER's criterion or LCC optimised criterion is used, the strength loss ratio, DSL, provides a viable consequence parameter that aids the computation of expected risk.

Chapter 7

Conclusions and Suggestions for Further Research

The development of performance-based techniques promises revolutionary changes to the fire safety profession. The pursuit of rational fire safety solutions provided great momentum of fire engineering research in the past two decades.

The main purpose of engineering is to provide service to society. The link between engineering codes and societal preferences are often buried under an array of complex legal and scientific regulations.

This thesis emphasises the importance of implementing societal considerations in design using rational objective-based engineering methodologies.

As a plan for a predicted process, design is surrounded by an amalgam of uncertainties that invariably result in risk. A review of the impact of uncertainty on risk and the decision process has been presented.

Modern structural design codes are based on reliability. The codes are designed using probabilistic techniques and are aimed at ensuring minimum safety indices. The consequence of events is not considered in current limit state codes; hence the associated risk is not evaluated.

Risk-based methods are becoming increasingly popular to the ability of providing risk indices that facilitate informed decision making. The risk metric is a function of the probability and consequence of a hazard. Hence it is necessary to integrate a damage or loss parameter with probability to arrive a risk index. A review of some existing empirical and analytical damage indices was made, and a new damage index was proposed.

There exist many opportunities for future research activities that might benefit from the large volume of existing fire safety knowledge. Some ideas are proposed in the following sections.

7.1 Level I Limit State/LRFD Structural Design Code

It is important to acknowledge the need of design methods in LSD/LRFD formats. A wide range of design codes contain recommendations for structural design in fire, but are limited to individual member design.

The aim of this proposal is to enable fire effects to be included in the linear LSD/LRFD load combinations.

As detailed in section 4.7.3 of chapter 4, a safety function can be defined as the difference between limit state and actual steel temperature. (Equation 4.22)

$$g(\mathbf{x}) = T_{\lim iting} - T_{steel-actual}$$

The limit state conditions define the maximum acceptable limits of deformations, which are the values that permit egress and evacuation. These limits cannot be generalised to all structures. However, it might be possible to develop general rules by defining a table of building classes and taking into account the factors that influence fire development.

The load and resistance factors may be calculated by identifying the stress level that corresponds to corresponding to the limit state and actual steel temperatures.

The LSD/LRFD codes can be extended to permit the inclusion of pertinent design factors, such as member dimensions and boundary conditions, heating regime and fire-pertinent factors such as the ventilation ratio. A *nomogram* similar to that used by AISC [97] in column design seems to be a good candidate. The influence of the constituent factors may then be developed deterministically and probabilistically on an individual basis, and the code can be refined with the build-up of new information.

In figure 7.1, the terms *demand* and *capacity* (rather than load and resistance) are intentionally used to highlight the possibility of applying the concept to traditional load-strength check or for more comprehensive total fire protection system design.

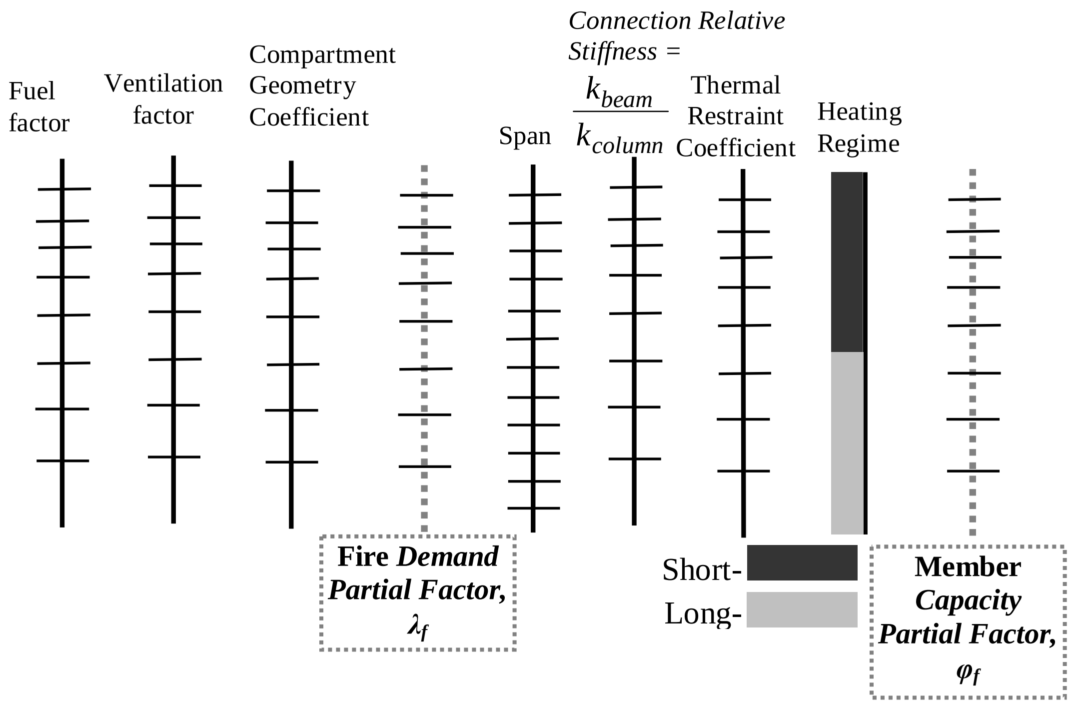


Figure- 7.1: Envisaged (*illustrative-only*) Nomogram for Beam Design in Fire

7.2 Structural Fire Reliability Calculation Programme, FiRel

FiRel is a proposed computer programme for reliability calculation for structural performance in fire using response surface modelling.

The programme is designed to analyse specific building classes by including factors that describe spatial properties, connection rigidity, floor restraint, ventilation factors, etc.

The programme is linked to a finite element code where a small number of analyses are made for selected fire scenarios. The strength loss ratio can then be calculated for the different scenarios, and a suitable (building-specific) probability distribution thereof is derived. Response surface modelling is then used to calculate a reliability index based on the difference between the expected and acceptable values of the strength loss ratio.

$$g(\cdot) = DSL_{acceptable} - E(DSL)$$

The acceptable strength loss ratio is specific to the building or building class and is defined as the value corresponding to the deformation limit that allows emergency operations or does not cause progression to total collapse, as detailed in section 2.15.2.

7.3 A Simple Method for Assessing Structural Safety from Fire following Earthquake

Fire following earthquake can cause large fatalities and severe losses to properties and infrastructure. It is the subject of a current collaborative research between the University of Edinburgh and the Indian Institute of Science.

Statistics exist for the probability of coincidence of fire and earthquake, such as those cited in reference [13]. The strength loss ratio, *DSL*, proposed in chapter 6 is calculated from the remaining strength ratio, R_k. The probability of failure may be defined as the probability of the remaining strength after earthquake being less than what is required for performance in fire. This is represented by:

$$P_f\,(F|E) = P\,(R_k|E < R_{k_T-F})$$

The target remaining strength for fire, $R_{kT\text{-}F}$, is “pre-calculated” as the value corresponding to the maximum deformations that do not inhibit emergency operations nor lead to a total structural collapse.

7.4 Development of the Proposed Optimisation Technique for Calculating the Reliability Index

A technique for calculating the reliability index was presented in section 4.10 of chapter 4. The method is conceptually simple but no application was made since it falls beyond the scope of this thesis.

Validation is planned using example application with verification by established methods and Monte Carlo simulation.

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