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FORECASTING STOCK MARKET VOLATILITY: EVIDENCE FROM FOURTEEN COUNTRIES

Abstract

This paper evaluates the out-of-sample forecasting accuracy of eleven models for weekly and monthly volatility in fourteen stock markets. Volatility is defined as within-week (within-month) standard deviation of continuously compounded daily returns on the stock market index of each country for the ten-year period 1988 to 1997. The first half of the sample is retained for the estimation of parameters while the second half is for the forecast period. The following models are employed: a random walk model, a historical mean model, moving average models, weighted moving average models, exponentially weighted moving average models, an exponential smoothing model, a regression model, an ARCH model, a GARCH model, a GJR-GARCH model, and an EGARCH model. We first use the standard (symmetric) loss functions to evaluate the performance of the competing models: the mean error, the mean absolute error, the root mean squared error, and the mean absolute percentage error. According to all of these standard loss functions, the exponential smoothing model provides superior forecasts of volatility. On the other hand, ARCH-based models generally prove to be the worst forecasting models. We also employ the asymmetric loss functions to penalize under/over-prediction. When under-predictions are penalized more heavily ARCH-type models provide the best forecasts while the random walk is worst. However, when over-predictions of volatility are penalized more heavily the exponential smoothing model performs best while the ARCH-type models are now universally found to be inferior forecasters.

Key words: Stock market volatility, forecasting, forecast evaluation
JEL Classification: C22, C53, G12, G15

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1. INTRODUCTION

Forecasting return volatility is of great importance to many financial decisions including portfolio selection and option pricing. Various methods by which such forecasts can be achieved have been developed in the literature and applied in practice. Such techniques range from the extremely simplistic models that use naïve (random walk) assumptions through to the relatively complex conditional heteroskedastic models of the GARCH family. Without question GARCH models have secured a vast following in the academic literature – indeed, their general use has become so widespread that there now exists several survey papers which document the properties and empirical applications of the ARCH class of models (see for example, Bollerslev, Chou and Kroner (1992), Bera and Higgins (1993), and Bollerslev, Engle and Nelson (1994)).[1]

However, despite the appeal of complexity and despite their popularity, it is by no means agreed that complex models such as GARCH provide superior forecasts of return volatility. Dimson and Marsh (1990) is a notable example in which simple models have prevailed – although it should be pointed out that ARCH models were not included in their analysis. Specifically, Dimson and Marsh apply five different types of forecasting model to a set of UK equity data, namely, (a) a random walk model; (b) a long-term mean model; (c) a moving average model; (d) an exponential smoothing model; and (e) regression models. They recommend the final two of these models and, in so doing, sound an early warning in this literature that the best forecasting models may well be the simple ones.

[1] As these survey articles show, the ARCH family of models has been extended well beyond the simple specification of the initial ARCH model of Engle (1982) and GARCH model of Bollerslev (1986). These additions to the family have attempted to refine both the mean and variance equations to better capture the stylized features of high frequency data.

Other papers in this literature however spell out a mixed set of findings on this issue. For example, Akgiray (1989) found in favour of a GARCH (1,1) model (over more traditional counterparts) when applied to monthly US data. Brailsford and Faff (1996) investigate the out-of-sample predictive ability of several models of monthly stock market volatility in Australia. In the measurement of the performance of the models, in addition to symmetric loss functions, they use asymmetric loss functions to penalize under/over-prediction. They conclude that the ARCH class of models and a simple regression model provide superior forecast of the volatility. However, the various model rankings are shown to be sensitive to the error statistics used to assess the accuracy of the forecasts.

In contrast, Tse (1991) and Tse and Tung (1992) investigated Japanese and Singaporean data and found that an exponentially weighted moving average (EWMA) model produced better volatility forecasts than ARCH models. Evidence with respect to foreign exchange markets includes West and Cho (1995), Andersen and Bollerslev (1998), Brooks and Burke (1998), Andersen, Bollerslev and Lange (1999) and Balaban (1999) – for example, West and Cho (1995) can not show superiority of any forecasting models.

In the finance literature, generally the existing evidence concerning the relative quality of volatility forecasts is related to an individual country's stock market: the USA (Akgiray, 1989), the UK (Dimson and Marsh, 1990 and McMillan, Speight and Gwilym, 2000), Japan (Tse, 1991), Singapore (Tse and Tung, 1992), Australia (Brailsford and Faff, 1996), Switzerland (Adjaoute, Bruand and Gibson-Asner, 1998), the Netherlands, Germany, Spain and Italy (Franses and Ghijsels, 1999), Turkey (Balaban, 1998). Furthermore, the range of forecasting models is often restricted to a narrow set of the most popular models that have been explored in the literature.[2] Moreover, most of the previous researches focus on the forecasting over a single horizon – commonly monthly stock market volatility.

[2] For example, Franses and Ghijsels (1999) limit their focus to a narrow set of GARCH models.

The current paper seeks to extend and supplement this existing evidence by, in a single unifying framework, analyzing a wide range of volatility forecasting approaches across fourteen countries. Specifically, in the context of volatility forecasting we consider more countries than ever before evaluated in a single paper – namely, fourteen countries comprising Belgium; Canada; Denmark; Finland; Germany; Hong Kong; Italy; Japan; Netherlands; Philippines; Singapore; Thailand; the UK and the US. Moreover, a considerable range of forecasting models are used – a random walk model, a historical mean model, moving average models, weighted moving average models, exponentially weighted moving average models, an exponential smoothing model, a regression model, an ARCH model, a GARCH model, a GJR-GARCH model, and an EGARCH model. Furthermore, we provide analysis that involves both weekly and monthly volatility forecasts, thus allowing a comparison of the forecasting interval to be made. Also, following Brailsford and Faff (1996), we compare the forecasting techniques based on both symmetric (mean error, the mean absolute error, the root mean squared error and the mean absolute percentage error) and asymmetric error statistics.

The main results of our study can be summarized as follows. First, based on the conventional symmetric loss functions, we find that the exponential smoothing model provides superior forecasts of volatility. Second, the ARCH-based models generally prove to be the worst forecasting models in the context of these symmetric measures. Third, when under-predictions are penalized more heavily ARCH-type models provide the best forecasts while the random walk is worst. Finally, when over-predictions of volatility are penalized more heavily the exponential smoothing model performs best while the ARCH-type models are now universally found to be inferior forecasters.

The remainder of this paper is organized as follows: in the second section, the data and methodology are described, in the third section the empirical results are presented, and finally in the fourth section the paper is concluded.

2. EMPIRICAL METHODOLOGY

2.1 Data and Sample Description

We employ daily observations of stock market indices of fourteen countries covering the period December 1987 to December 1997. The data are sourced from *Datastream*. The investigated countries (indices) are Belgium (Brussels All Shares Price Index); Canada (Toronto SE 300 Composite Price Index); Denmark (Copenhagen SE General Price Index); Finland (Hex General Price Index); Germany (Faz General Price Index); Hong Kong (Hang Seng Price Index); Italy (Milan Comit General Price Index); Japan (Nikkei 500 Price Index); the Netherlands (CBS All Share General Price Index); the Philippines (Philippines SE Composite Price Index); Singapore (Singapore All Share Price Index); Thailand (Bangkok S.E.T. Price Index); the UK (FTSE All Share Index) and the US (NYSE Composite Index).

Our analysis involves both weekly and monthly volatility forecasts.[3] Continuously compounded weekly returns are calculated as follows:

$$R_{w,t} = \ln(I_{w,t} / I_{w,t-1}) \qquad (1)$$

where $I_{w,t}$ and $R_{w,t}$ denote the value of stock market index and continuously compounded return on trading day t in week w, respectively. We define weekly realised volatility as the within-week standard deviation of continuously compounded weekly returns as follows:

$$s_{a,w} = [\frac{1}{(n-1)} \sum_{t=1}^{n} (R_{w,t} - \mu_w)^2]^{0.5} \qquad (2)$$

$$\mu_w = (1/n) \sum_{t=1}^{n} R_{w,t} \qquad (3)$$

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[3] The discussion in the text outlines the case for weekly forecasts only. The monthly forecasting follows a similar process, but details are suppressed to conserve space.

Mean daily index return and within-week standard deviation of daily returns in week w are respectively shown by μ_w and $\sigma_{a,w}$. The number of trading days in a week is given by n. In the data set for each country, there are 522 weekly volatility observations. Of these, the first 261 of the observations (from December 1987 to November 1992) are used for estimation, while the second 261 observations (from December 1992 to December 1997) are used for forecasting purposes.[4]

In the Table 1 summary statistics for within-week standard deviations of returns in the full sample period, the estimation period and in the forecast period are presented. The table shows that in only four countries – namely, Canada, Finland, Hong Kong and Italy, standard deviations in the forecast period are higher than in the estimation period. Thus in the majority of our sample countries, standard deviations decline from the first to the second subperiod.[5]

2.2 Forecasting Techniques

The following models are employed as forecast competitors.

a) Random walk model

This model says that the best forecast of this week's volatility is the last week's realised volatility viz.:

$$\sigma_{f,w}(RW) = \sigma_{a,w-1} \qquad (4)$$

where w = 262, ..., 522.

[4] In the case of the monthly analysis, there are 120 monthly volatility observations which are split evenly between estimation and forecasting.

b) Historical mean model

According to this model, the best forecast for this week's volatility is an average of all available past observations of weekly volatility.

$$\sigma_{f,w}(HM) = \frac{1}{(w-1)} \sum_{j=1}^{w-1} \sigma_{a,j} \qquad (5)$$

where w = 262, ..., 522.

c) Moving average (MA-α) model

This model says that the best forecast of this week's volatility is an equally weighted average of realized volatilities in the last α weeks.

$$\sigma_{f,w}(MA(\alpha)) = \frac{1}{\alpha} \sum_{j=w-\alpha}^{w-1} \sigma_{a,j} \qquad (6)$$

where w = 262, ..., 522, and α = 4, 6, 12, 24, 36, 52. The (arbitrarily) chosen values of α represent different horizons from the very short, (α = 4), to the long term, (α = 52).

d) Weighted moving average (WMA-α) model

In the WMA-α model, the weight of each observation is not equal in contrast to the MA-α model (Liljeblom and Stenius (1997)). Specifically, in our analysis the weight of each observation, λ_i, is chosen to decline by 10%, giving the highest (lowest) weight to the newest (oldest) information.

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[5] This contrasts other studies such as Brailsford and Faff (1996) which considered a forecasting period that encompassed the stock market crash of October 1987 and, hence, the forecasting models were asked to predict

$$\sigma_{f,w}(WMA(\alpha)) = \sum_{j=w-\alpha}^{w-1} ?_i \sigma_{a,j} \quad (7)$$

where w = 262, ..., 522, and α = 4, 6, 12, 24, 36, 52.

e) Exponential smoothing (ES) model

In the ES model, the forecast of volatility is a function of the immediate past forecast and the immediate past observed volatility (Dimson and Marsh (1990); Brailsford and Faff (1996)).

$$\sigma_{f,w}(ES) = \theta\sigma_{f,w-1}(ES) + (1-\theta)\sigma_{a,w-1} \quad (8)$$

where w = 262, ..., 522.

The smoothing parameter (θ) is restricted to lie between zero and one. Following the previous researchers, we determine the optimal value of θ empirically using mean absolute error, root mean squared error, and mean absolute percentage error statistics separately. To this end, we start with an initial value of θ, zero in our case, and increment by 0.01 each time until we obtain unity. We select the optimal value of θ that produces the lowest error according to each error statistic (Brailsford and Faff (1996)).[6]

volatility over a period in which actual volatility was relatively high.

[6] Since the choice of the optimal smoothing parameter is quite consistent across the different error statistics, all the reported results are based on the Root Mean Squared Error (RMSE) criteria. Using RMSE criteria we find the following optimal values of θ; Belgium=0.89, Canada=0.87, Denmark=0.88, Finland=0.88, Germany=0.87, Hong Kong=0.66, Italy=0.91, Japan=0.74, the Netherlands=0.82, Pilippines=0.80, Singapore=0.85, Thailand=0.84, the UK=0.85, and the US=0.85.

f) Exponentially weighted moving average (EWMA-α) model:

In this model, the past observed volatility is replaced by the α-week moving average forecast; ie., the forecast of the MA-α model (Tse, 1991; Tse and Tung, 1992; and Brailsford and Faff, 1996).

$$\sigma_{f,w}(EWMA-\alpha) = \lambda\sigma_{f,w-1}(EWMA-\alpha) + (1-\lambda)\sigma_{a,w}(MA-\alpha) \qquad (9)$$

where w = 262, ..., 522, and α = 4, 6, 12, 24, 36, 52. Similar to the MA-α models, the (arbitrarily) chosen values represent different horizons from the very short to the long term.[7] For the calculation of optimal values of λ, the same process as used for θ described above, is employed.[8]

g) Regression (REG) model

First we run the simple autoregression of the observed weekly volatility on its own lagged value (over the sample w = 1 to 261) viz.:

$$\sigma_{a,w} = c + \beta\sigma_{a,w-1} + u_{w-1} \qquad (10)$$

Then we construct the forecast for the first week of the forecast period (w = 262) using the estimated regression parameters:

$$\sigma_{f,w}(REG) = c + \beta\sigma_{a,w-1} \qquad (11)$$

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[7] In the moving average, weighted moving average, and exponentially weighted moving average models, we find the best forecasts are obtained when α is equal to 12, so in all tables only the results for α =12 are presented.

[8] When α=12, according to the RMSE criteria, we find the following optimal values for λ; Belgium=0.80, Canada=0.92, Denmark=0.60, Finland=0.20, Germany=0.81, Hong Kong=0.41, Italy=0.81, Japan=0.00, the Netherlands=0.80, Philippines=0.01, Singapore=0.27, Thailand=0.00, the UK=0.75, and the US=0.09.

We update the regression equation weekly, using a rolling sample of 261 observations – ie., each week we drop the oldest observation and add the last or newest observation. Hence, for each country the total estimation procedure requires estimation of 261 regressions to obtain out-of-sample forecasts of weekly volatility. Note that this procedure effectively lets us utilize time-varying parameters for each forecast.

h) ARCH(1) model

Following the basic ARCH model of Engle (1982) we estimate an ARCH (1) model, in which the conditional mean function is modeled as a first order autoregression:

$$R_t = c + ?\, R_{t-1} + e_t \qquad (12)$$

and the conditional variance equation is modeled as:

$$h_t = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 \qquad (13)$$

The daily forecast errors (ε_t) are assumed to be conditionally normally distributed with a zero mean and variance h_t based on the information set Ψ available at time t-1.

$$\varepsilon_t|\Psi_{t-1} \sim N(0, h_t^2)$$

Similar to the case of the regression analysis, in all of the ARCH-type models (ARCH (1), GARCH (1,1), GJR-GARCH (1,1), and EGARCH (1,1) models) we update the model weekly. At each run, we drop the last five observations, and add the new five observations.

i) GARCH (1,1) model

In a daily GARCH (1,1) model (Bollerslev (1986)), the conditional volatility today depends on yesterday's conditional volatility and yesterday's squared forecast error.

$$h_t = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \beta_1 h_{t-1} \quad (14)$$

j) GJR-GARCH(1,1) model:

Following Glosten, Jagannathan and Runkle (1993) this model allows asymmetry in the conditional volatility equation.

$$h_t = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \alpha_2 \varepsilon_{t-1}^2 D_{t-1}^- + \beta_1 h_{t-1} \quad (15)$$

where D^-_{t-1} is a dummy variable taking the value of 1 if $\varepsilon_{t-1}<0$, and 0 otherwise.

k) EGARCH (1,1) model

Finally, we use Nelson's (1991) EGARCH (1,1) model as follows.

$$\ln(h_t) = \alpha_0 + ?(\frac{\varepsilon_{t-1}}{h_{t-1}}) + ?[(\frac{|\varepsilon_{t-1}|}{h_{t-1}}) - (\frac{2}{\pi})^{0.5}] + \beta \ln(h_{t-1}) \quad (16)$$

3. FORECAST EVALUATION AND EMPIRICAL RESULTS

Following Brailsford and Faff (1996), we compare the forecast performance of each model through both symmetric and asymmetric error statistics.

3.1 Symmetric Error Statistics

Four commonly used loss functions or error statistics: the mean error (ME), the mean absolute error (MAE), the root mean squared error (RMSE), and the mean absolute percentage error (MAPE) are employed to measure the performance of the forecasting models.

$$ME = \frac{1}{261}\sum_{m=262}^{522}(\sigma_{f,w} - \sigma_{a,w}) \quad (17)$$

$$\text{MAE} = \frac{1}{261}\sum_{m=262}^{522}|\sigma_{f,w} - \sigma_{a,w}| \qquad (18)$$

$$\text{RMSE} = [\frac{1}{261}\sum_{m=262}^{522}(\sigma_{f,w} - \sigma_{a,w})^2]^{0.5} \qquad (19)$$

$$\text{MAPE} = \frac{1}{261}\sum_{m=262}^{522}|\frac{(\sigma_{f,w} - \sigma_{a,w})}{\sigma_{a,w}}| \qquad (20)$$

In the above equations, $\sigma_{f,W}$ and $\sigma_{a,W}$ denote the volatility forecast and the realised volatility in week w , respectively.

First, it should be recognised that the mean error (ME) metric suffers from the fact that large errors of positive and negative sign may offset each other and, hence, may lead to an unreliable ranking device across the various forecasting models. Accordingly, we provide only a brief discussion of the mean error measures across the various volatility forecasting models and markets.[9] Perhaps, the most useful guide that the ME provides is the degree of average under- or over-prediction of volatility. On this score we generally found that across all countries, the ARCH-type models tend to over-predict volatility, while the non-ARCH-type models under-predict volatility. Moreover, the degree of over-prediction of the former is considerably more pronounced than the under-prediction of the latter.

Tables 2, Table 3, and Table 4 provide results of actual and relative forecast error statistics for each model according to the remaining symmetric error measures, (MAE, RMSE, and MAPE, respectively). The relative forecast error is obtained by taking the ratio of the actual error statistic of a given model divided by the actual error statistic of the worst-performing model for that country. The tables also show the ranking of forecasting models, for each country, from 1 (best forecast) to 11 (worst forecast).

In the case of the MAE we can identify a number of key features from Table 2. First (and most notably), the Exponential Smoothing method clearly produces the most accurate volatility forecasts – for 12 out of the 14 countries, ES is ranked number one. Second, notwithstanding the general dominance of ES, it is found that the ranking of the non-ARCH based methods is very compact. For example according to MAE, we find that the relative difference in forecasting performance across the non ARCH-based models range between 4.2 % (Japan – ES versus RW and UK – ES versus HM) to 8.7 % (Finland – ES versus RW). Third, more generally we see that the non-ARCH based models (led by ES) consistently outperform their ARCH based rivals. Indeed, the four ARCH-based models systematically rank as the four worst forecasting approaches according to the MAE. Moreover, the best ARCH-based model (ranked eighth overall) tends to be considerably worse than the most inferior of the non ARCH-based models (ranked seventh). For example, in the case of Belgium the relative difference amounts to 64.2 % (random walk versus GJR-GARCH), while in the case of the Netherlands the relative difference is 60.6 % (historical mean versus ARCH).

Fourth, of the ARCH-based models, EGARCH tends to be preferred (highest ranking amongst ARCH-based models for eight out of the fourteen countries), whereas the standard ARCH model is most often ranked bottom (for six countries). Interestingly, the relative difference in forecasting performance (according to MAE) across the ARCH-based models ranges considerably – from a minimum of 1.8 % (Canada – EGARCH versus GARCH) to a maximum of 31.7 % (Japan – EGARCH versus ARCH). Fifth, a comment regarding the performance of the moving average group of models is also worthwhile. Generally, we

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[9] In order to conserve space, we only report the details of the weekly analysis in tables. However, we offer comments on the unreported monthly analysis at each stage as we progress. Full details are available from the

observe that, in terms of MAE, these models provide the second best volatility forecast behind ES. Moreover, the weighted moving average model (WMA-12) performs best achieving the top overall rank on one occasion (Belgium) and the second overall ranking in eight instances.

Sixth, a final comment on the unreported monthly results compared to the weekly counterparts (as discussed above) is warranted. Generally, all of the basic features identified for the weekly forecasts are evident in the monthly case. In one sense the dominance of the ES model is even more decisive for the monthly forecasts as the method only fails to be ranked number 1, on one occasion (Finland – where the MA model wins). However, in another sense the preference for monthly ES forecasts is weaker due to a generally smaller percentage gap between ES and other methods (compared to the weekly forecast analysis). Often the relative MAE measure for the monthly (weekly) ES forecast is above 0.5 (below 0.25). For example, in the case of Italy its monthly relative MAE is 0.752 compared to a value of 0.233 in the counterpart weekly case. Also it worthy to note that generally while the ARCH-based models still perform worse than their non-ARCH based rivals, the degree of difference is much less pronounced in the monthly analysis. However, it is found that standard ARCH model forecasts of monthly volatility are often ranked last of the eleven models (this occurs for eight of the countries).

Turning our attention to the results based on the RMSE metric displayed in Table 3, several features are evident – mostly reinforcing the results gained from the MAE analysis. First, as was the case above with MAE the Exponential Smoothing approach dominates – gaining a number one ranking for 11 out of the 14 countries. Second, again notwithstanding the general dominance of ES, based on RMSE we observe a relatively small difference in accuracy between the non-ARCH based methods. Third, the non-ARCH based models again are consistently superior to the ARCH based models.

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authors upon request.

Fourth as was the case using MAE, based on RMSE, EGARCH tends to be preferred from the ARCH-based models as it ranks highest amongst ARCH-based models for eight out of the fourteen countries. Fifth as was the case in MAE, we generally observe that, in terms of RMSE, the MA models (particularly WMA-12) provide the second best volatility forecast behind ES. Indeed, the weighted moving average model (WMA-12) performs best achieving the top overall rank three times and the second overall ranking on eight occasions. Sixth, with regard to the RMSE measures for unreported monthly forecasts, ES still dominates and again the non ARCH-based models are generally superior.

Table 4 reports the outcome of the MAPE metric across the eleven weekly forecasting models and fourteen countries. An analysis of the table reveals that the major patterns identified for MAE and RMSE are predominantly intact and so only brief further comment will be made. While it is found that the MAPE still favours the ES model, now five out of fourteen of the country index return volatilities are better forecast by other models. For example, based on the MAPE the historic mean model is superior in forecasting volatility for Finland and Italy. Furthermore, MA models continue to perform quite well while ARCH-based models remain as the poorest volatility forecasters. Finally, with regard to the MAPE applied to monthly forecasts similar comments as stated above are appropriate here as well.

3.2 Asymmetric Error Statistics

The conventional error statistics used in the previous subsection, ME, MAE, RMSE, and MAPE, are symmetric; ie., they give an equal weight to under-and-over-predictions of volatility of similar magnitude. However, many investors do not give equal importance to under and over prediction of volatility, especially, in the pricing of options, while under-prediction of volatility is undesirable for a seller, over-prediction of it is undesirable for a buyer. Following Pagan and Schwert (1990) and Brailsford and Faff (1996), to penalize under/over-predictions more heavily, the following mean mixed error statistics, MME, are constructed: [10]

$$MME(U) = \frac{1}{261}\left[\sum_{t=1}^{O}\left|s_{f,w} - s_{a,w}\right| + \sum_{t=1}^{U}\sqrt{\left|s_{f,w} - s_{a,w}\right|}\right] \qquad (21)$$

$$\mathrm{MME}(O) = \frac{1}{261}\left[\sum_{t=1}^{O}\sqrt{\left|s_{f,w} - s_{a,w}\right|} + \sum_{t=1}^{U}\left|s_{f,w} - s_{a,w}\right|\right] \qquad (22)$$

where O is the number of over-predictions, and U is the number of under-predictions. MME(U) and MME(O) penalize the under-predictions and over-predictions more heavily, respectively.

In Table 5 we report the results of the eleven volatility forecasting methods across the fourteen countries when assessed by the MME(U) and the MME(O) error metrics. The key features of this analysis in the context of MME(U) which penalises under-prediction more heavily can be summarised as follows. First, given that we have already established that the ARCH-based models are more heavily prone to (average) over-prediction, it is no surprise that these models do particularly well according to MME(U). Indeed, in stark contrast to the previous analysis (based on the symmetric measures) ARCH-based models are ranked number 1 in all fourteen countries. Second, it is interesting to note that of these models the EGARCH variation fairs best by providing the supreme volatility forecast for eight of the fourteen countries. Next best of the ARCH-based models is GJR-GARCH which achieves a number 1 ranking in the case of three countries. Third, we find that the difference between the forecasting ability of the ARCH-based models is typically quite small. For example, in the

[10] Since the absolute values of all forecast errors are less than one, taking their square root increases the penalty for under/over-prediction.

case of Denmark there is just 1 % difference between the relative MME(U) measures of all four ARCH-based models.

Fourth, it is found that the best non ARCH-based model (ranked fifth overall) tends to be considerably worse than the most inferior of the ARCH-based models (ranked fourth). For example, Canada produces an MME(U) for the 4th ranked standard ARCH model that is 44.7 % better than its closest rival (WMA-12). Fifth, we observe that the previously preferred ES method has a highest ranking of fifth (Finland) according to MME(U) and that mostly it is ranked 8 or 9 across the different countries. Clearly, the inferior ability of ES (relative to the ARCH-based models) to over-predict volatility drives this result.

Sixth, similar to the now poor showing of the ES, the MA models also rate quite badly – often ranked at 7 or worse according to MME(U). Seventh, according to the MME(U) metric, the worst performing model at forecasting weekly volatility is the random walk model – it achieves the worst ranking in twelve of the fourteen countries. This is not surprising when it is recognised that RW typically produces the highest incidence of under-estimation of weekly volatility. Eighth, a very similar pattern of results to those just listed are also applicable to the unreported monthly analysis based on the MME(U) metric.

Turning now to the MME(O) results reported in Table 5, we see a return to the same sort of pattern discussed earlier with regard to the symmetric error measures. Specifically, the key features of this analysis (which penalises over-prediction more heavily) can be summarised as follows. First, similar to all the symmetric measures, ES dominates – although, perhaps to a lesser extent here as it achieved supreme ranking for 7 out of the 14 countries. Second, again (this time based on MME(O)) we see a relatively small gap between the accuracy of the non-ARCH based methods. Third, the non-ARCH based models again are consistently superior to the ARCH based models.

Fourth, somewhat in contrast to the earlier symmetric analysis, EGARCH is not so clearly preferred amongst the ARCH-based models in the context of MME(O) as it ranks highest amongst ARCH-based models six out of the fourteen countries. Fifth as was the case in the earlier symmetric analysis, we generally observe that, in terms of MME(O), the MA models provide the second best volatility forecast behind ES. Sixth, with regard to the MME(O) measures for unreported monthly forecasts, ES still dominates and again the non ARCH-based models are generally superior.

4. CONCLUSION

Volatility forecasting is a widely researched area in the finance literature. The performance of forecasting models of varying complexity has been investigated according to a range of measures and generally mixed results have been recorded. On the one hand some argue that relatively simple forecasting techniques are superior, while others suggest that the relative complexity of ARCH-type models is worthwhile. In this paper we seek to extend and supplement this existing evidence by, in a single unifying framework, analyzing a wide range of volatility forecasting approaches across fourteen countries. Specifically, our analysis encompasses the ten-year period 1988 to 1997 for the market returns of Belgium; Canada; Denmark; Finland; Germany; Hong Kong; Italy; Japan; Netherlands; Philippines; Singapore; Thailand; the UK and the US.

In our analysis, a considerable range of forecasting models are used – a random walk model, a historical mean model, moving average models, weighted moving average models, exponentially weighted moving average models, an exponential smoothing model, a regression model, an ARCH model, a GARCH model, a GJR-GARCH model, and an EGARCH model. Furthermore, we provide analysis that involves both weekly and monthly volatility forecasts, thus allowing a comparison of the forecasting interval to be made. Also, we compare the forecasting techniques based on both symmetric (mean error, the mean absolute error, the root mean squared error and the mean absolute percentage error) and asymmetric error statistics.

The key thrust of our results can be summarised into two parts as follows. The first set of conclusions relates to the outcome of our analysis when employing the standard symmetric error metrics to assess volatility forecasting performance. First, we consistently found that the Exponential Smoothing approach dominates in providing superior forecasts of weekly volatility. Second, notwithstanding the general dominance of ES, we observe a relatively small difference in accuracy between the non-ARCH based methods. Third, the non-ARCH based models are consistently found to be superior to the ARCH based models. Fourth, EGARCH tends to be preferred from the ARCH-based models. Fifth, we generally observe that the MA models provide the second best weekly volatility forecast behind ES. Sixth, with regard to monthly volatility forecasts, the features just noted still generally apply – in particular, ES still dominates and again the non ARCH-based models are generally superior.

The second set of conclusions relates to the outcome of our analysis when employing the non-standard asymmetric error metrics to assess volatility forecasting performance. Interestingly, our results change when the asymmetric loss functions, that penalize under/over-prediction, are employed. Specifically, when under-predictions are penalized more heavily ARCH-type models provide the best forecasts while the random walk is worst. However, when over-predictions of volatility are penalized more heavily the exponential smoothing model performs best while the ARCH-type models are now universally found to be inferior forecasters.

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Table 1: Summary Statistics: Within-Week Standard Deviations

	Belgium	Canada	Denmark	Finland	Germany
Full Period					
Mean	0.0051	0.0048	0.0050	0.0084	0.0086
Standard Dev	0.0039	0.0030	0.0035	0.0059	0.0062
Skewness	2.6991	3.2658	3.8511	2.6860	4.9616
Kurtosis	13.4987	25.1900	30.5222	18.9146	45.4618
Estimation Period					
Mean	0.0054	0.0048	0.0051	0.0065	0.0095
Standard Dev	0.0047	0.0026	0.0037	0.0052	0.0073
Skewness	2.3840	2.2490	2.9370	2.3570	5.0010
Kurtosis	10.2340	12.4650	17.2990	10.9550	41.0710
Forecast Period					
Mean	0.0048	0.0049	0.0048	0.0103	0.0078
Standard Dev	0.0029	0.0033	0.0034	0.0060	0.0047
Skewness	2.6700	3.6600	5.0000	3.4800	3.1600
Kurtosis	15.9900	28.3600	49.0000	27.0300	22.0200

	Hong Kong	Italy	Japan	Netherlands	Philippines
Full Period					
Mean	0.0118	0.0097	0.0089	0.0071	0.0126
Standard Dev	0.0107	0.0057	0.0067	0.0043	0.0076
Skewness	5.2704	2.3184	2.4076	2.4407	1.3473
Kurtosis	45.1534	14.1141	11.0324	12.3680	5.3678
Estimation Period					
Mean	0.0105	0.0087	0.0094	0.0071	0.0142
Standard Dev	0.0104	0.0059	0.0080	0.0044	0.0081
Skewness	6.1310	2.3500	2.1890	2.4080	1.1380
Kurtosis	55.3760	11.2360	9.0240	11.5750	4.6120
Forecast Period					
Mean	0.0132	0.0107	0.0083	0.0070	0.0110
Standard Dev	0.0109	0.0053	0.0052	0.0043	0.0067
Skewness	4.7000	2.6400	2.2100	2.4700	1.5900
Kurtosis	38.9700	20.2300	9.9600	13.1400	6.6500

	Singapore	Thailand	UK	US
Full Period				
Mean	0.0074	0.0128	0.0063	0.0064
Standard Dev	0.0056	0.0091	0.0032	0.0039
Skewness	3.1548	2.0638	2.2687	2.9443
Kurtosis	17.6716	8.5456	11.9167	20.3649
Estimation Period				
Mean	0.0074	0.0129	0.0071	0.0073
Standard Dev	0.0062	0.0103	0.0036	0.0041
Skewness	3.2390	2.1110	2.2840	2.4200
Kurtosis	17.8920	8.2760	10.5610	13.8160
Forecast Period				
Mean	0.0073	0.0127	0.0055	0.0056
Standard Dev	0.0051	0.0078	0.0024	0.0036
Skewness	2.8500	1.7400	1.6300	4.0100
Kurtosis	14.9000	6.8700	9.7100	35.5700

The full period includes the whole sample (522 weeks from 1988 to 1997); the estimation period covers the first 261 observations, and the forecast period covers the second 261 weeks.

Table 2: Mean Absolute Error (MAE) of Forecasting Weekly Volatility

	Belgium			Canada			Denmark			Finland			Germany		
	Actual	Relative	Rank	Actual	Relative	Rank	Actual	Relative	Rank	Actual	Relative	Rank	Actual	Relative	Rank
RW	0.236	0.259	7	0.248	0.348	7	0.248	0.262	7	0.523	0.321	7	0.346	0.232	6
HM	0.208	0.229	6	0.207	0.290	5	0.213	0.225	6	0.415	0.255	6	0.351	0.236	7
MA-12	0.185	0.203	3	0.207	0.290	5	0.202	0.213	3	0.387	0.237	2	0.277	0.186	4
WMA-12	0.184	0.202	2	0.202	0.283	3	0.199	0.210	2	0.389	0.239	4	0.271	0.182	2
EWMA-12	0.186	0.204	4	0.204	0.286	4	0.203	0.214	4	0.388	0.238	3	0.274	0.184	3
ES	0.181	0.199	1	0.195	0.273	2	0.195	0.206	1	0.381	0.234	1	0.262	0.176	1
REG	0.195	0.214	5	0.194	0.272	1	0.203	0.214	4	0.389	0.239	4	0.297	0.199	5
ARCH(1)	0.888	0.976	10	0.703	0.986	9	0.905	0.956	8	1.472	0.903	8	1.416	0.950	9
GARCH(1,1)	0.854	0.938	9	0.713	1.000	11	0.947	1.000	11	1.589	0.975	9	1.490	1.000	11
GJR-GARCH(1,1)	0.820	0.901	8	0.708	0.993	10	0.943	0.996	9	1.616	0.991	10	1.443	0.968	10
EGARCH	0.910	1.000	11	0.700	0.982	8	0.945	0.998	10	1.630	1.000	11	1.354	0.909	8

	Hong Kong			Italy			Japan			Netherlands			Philippines		
	Actual	Relative	Rank	Actual	Relative	Rank	Actual	Relative	Rank	Actual	Relative	Rank	Actual	Relative	Rank
RW	0.641	0.249	7	0.482	0.313	7	0.406	0.212	7	0.269	0.282	6	0.529	0.229	6
HM	0.612	0.238	6	0.374	0.243	4	0.381	0.199	6	0.290	0.304	7	0.578	0.250	7
MA-12	0.529	0.205	2	0.369	0.239	3	0.342	0.179	3	0.244	0.255	5	0.443	0.192	3
WMA-12	0.523	0.203	1	0.375	0.243	5	0.334	0.174	2	0.238	0.249	3	0.434	0.188	2
EWMA-12	0.535	0.208	3	0.362	0.235	2	0.342	0.179	3	0.237	0.248	2	0.445	0.193	4
ES	0.561	0.218	4	0.360	0.233	1	0.326	0.170	1	0.233	0.244	1	0.430	0.186	1
REG	0.563	0.219	5	0.380	0.246	6	0.352	0.184	5	0.240	0.251	4	0.484	0.210	5
ARCH	2.575	1.000	11	1.526	0.990	9	1.915	1.000	11	0.869	0.910	8	2.310	1.000	11
GARCH	2.093	0.813	9	1.528	0.991	10	1.417	0.740	10	0.955	1.000	11	1.898	0.822	9
GJR-GARCH(1,1)	2.100	0.816	10	1.485	0.963	8	1.321	0.690	9	0.928	0.972	9	1.929	0.835	10
EGARCH	1.977	0.768	8	1.542	1.000	11	1.307	0.683	8	0.932	0.976	10	1.887	0.817	8

	Singapore			Thailand			UK			US		
	Actual	Relative	Rank	Actual	Relative	Rank	Actual	Relative	Rank	Actual	Relative	Rank
RW	0.384	0.287	7	0.612	0.264	7	0.211	0.199	6	0.260	0.264	6
HM	0.331	0.247	6	0.574	0.247	6	0.221	0.209	7	0.278	0.283	7
MA-12	0.316	0.236	4	0.525	0.226	3	0.173	0.164	3	0.203	0.207	4
WMA-12	0.314	0.234	3	0.518	0.223	2	0.169	0.160	2	0.199	0.202	2
EWMA-12	0.319	0.238	5	0.525	0.226	3	0.174	0.164	4	0.202	0.205	3
ES	0.312	0.233	1	0.505	0.217	1	0.166	0.157	1	0.194	0.197	1
REG	0.313	0.234	2	0.542	0.233	5	0.187	0.177	5	0.232	0.236	5
ARCH	1.340	1.000	11	2.322	1.000	11	1.058	1.000	11	0.916	0.932	10
GARCH	1.237	0.923	9	2.090	0.900	8	0.918	0.868	10	0.872	0.887	9
GJR-GARCH(1,1)	1.243	0.928	10	2.104	0.906	9	0.910	0.860	9	0.983	1.000	11
EGARCH	1.194	0.891	8	2.115	0.911	10	0.879	0.831	8	0.844	0.859	8

The mean absolute error, (MAE), actual figures must be multiplied by 10^{-2}. *Actual* is the calculated error statistic. *Relative* is the ratio between the actual error statistic of a model and that of the worst-performing model for that country. The best performing model has a rank of 1.

Table 3: Root Mean Squared Error (RMSE) of Forecasting Weekly Volatility

	Belgium			Canada			Denmark			Finland			Germany		
	Actual	Relative	Rank	Actual	Relative	Rank	Actual	Relative	Rank	Actual	Relative	Rank	Actual	Relative	Rank
RW	0.333	0.348	7	0.389	0.516	7	0.390	0.369	7	0.743	0.429	7	0.510	0.325	7
HM	0.296	0.310	6	0.327	0.434	6	0.338	0.319	6	0.656	0.379	6	0.484	0.308	6
MA-12	0.266	0.278	3	0.324	0.430	5	0.326	0.308	4	0.589	0.340	3	0.424	0.270	3
WMA-12	0.265	0.277	1	0.319	0.423	3	0.324	0.306	3	0.588	0.340	2	0.420	0.268	2
EWMA-12	0.269	0.281	4	0.322	0.427	4	0.328	0.310	5	0.590	0.341	4	0.427	0.272	4
ES	0.265	0.277	1	0.313	0.415	1	0.321	0.303	1	0.584	0.337	1	0.415	0.264	1
REG	0.282	0.295	5	0.315	0.418	2	0.323	0.305	2	0.600	0.347	5	0.443	0.282	5
ARCH	0.937	0.980	10	0.741	0.983	9	0.945	0.893	8	1.553	0.897	8	1.490	0.950	9
GARCH	0.909	0.951	9	0.754	1.000	11	1.058	1.000	11	1.707	0.986	9	1.569	1.000	11
GJR-GARCH(1,1)	0.876	0.916	8	0.750	0.995	10	1.056	0.998	10	1.731	1.000	11	1.532	0.976	10
EGARCH	0.956	1.000	11	0.729	0.967	8	1.008	0.953	9	1.714	0.990	10	1.431	0.912	8

	Hong Kong			Italy			Japan			Netherlands			Philippines		
	Actual	Relative	Rank	Actual	Relative	Rank	Actual	Relative	Rank	Actual	Relative	Rank	Actual	Relative	Rank
RW	1.080	0.363	6	0.675	0.410	7	0.563	0.282	7	0.378	0.356	6	0.720	0.298	7
HM	1.108	0.372	7	0.548	0.333	6	0.522	0.262	6	0.432	0.407	7	0.708	0.293	6
MA-12	0.970	0.326	3	0.515	0.313	2	0.487	0.244	3	0.345	0.325	3	0.608	0.252	3
WMA-12	0.959	0.322	1	0.517	0.314	3	0.476	0.238	2	0.339	0.319	2	0.600	0.249	1
EWMA-12	0.972	0.327	5	0.518	0.314	4	0.487	0.244	3	0.347	0.327	4	0.613	0.254	4
ES	0.966	0.325	2	0.511	0.310	1	0.466	0.233	1	0.333	0.314	1	0.601	0.249	2
REG	0.971	0.326	4	0.536	0.325	5	0.489	0.245	5	0.352	0.331	5	0.628	0.260	5
ARCH	2.975	1.000	11	1.603	0.973	8	1.996	1.000	11	0.929	0.875	8	2.413	1.000	11
GARCH	2.522	0.848	10	1.648	1.000	11	1.547	0.775	10	1.062	1.000	11	2.079	0.862	9
GJR-GARCH(1,1)	2.443	0.821	9	1.614	0.979	9	1.451	0.727	9	1.044	0.983	10	2.116	0.877	10
EGARCH	2.243	0.754	8	1.643	0.997	10	1.413	0.708	8	1.035	0.975	9	2.043	0.847	8

	Singapore			Thailand			UK			US		
	Actual	Relative	Rank	Actual	Relative	Rank	Actual	Relative	Rank	Actual	Relative	Rank
RW	0.561	0.399	7	0.845	0.345	7	0.275	0.252	7	0.394	0.398	7
HM	0.507	0.361	6	0.784	0.320	6	0.272	0.250	6	0.377	0.381	6
MA-12	0.473	0.337	2	0.725	0.296	4	0.232	0.213	3	0.321	0.324	4
WMA-12	0.473	0.337	2	0.713	0.291	2	0.228	0.209	2	0.319	0.322	2
EWMA-12	0.482	0.343	5	0.725	0.296	4	0.233	0.214	4	0.320	0.323	3
ES	0.476	0.339	4	0.707	0.288	1	0.225	0.206	1	0.317	0.320	1
REG	0.471	0.335	1	0.724	0.295	3	0.242	0.222	5	0.346	0.349	5
ARCH	1.405	1.000	11	2.451	1.000	11	1.090	1.000	11	0.990	1.000	11
GARCH	1.316	0.937	9	2.341	0.955	9	0.957	0.878	10	0.959	0.969	9
GJR-GARCH(1,1)	1.317	0.937	10	2.344	0.956	10	0.951	0.872	9	0.983	0.993	10
EGARCH	1.259	0.896	8	2.320	0.947	8	0.922	0.846	8	0.915	0.924	8

The root mean squared error, (RMSE), actual figures must be multiplied by 10^{-2}. *Actual* is the calculated error statistic. *Relative* is the ratio between the actual error statistic of a model and that of the worst-performing model for that country. The best performing model has a rank of 1.

Table 4: Mean Absolute Percentage Error (MAPE) of Forecasting Weekly Volatility

	Belgium			**Canada**			**Denmark**			**Finland**			**Germany**		
	Actual	Relative	Rank	Actual	Relative	Rank	Actual	Relative	Rank	Actual	Relative	Rank	Actual	Relative	Rank
RW	0.561	0.203	6	0.577	0.263	7	0.625	0.208	6	0.691	0.290	7	0.569	0.208	6
HM	0.593	0.214	7	0.528	0.241	6	0.651	0.217	7	0.422	0.177	1	0.667	0.244	7
MA-12	0.476	0.172	4	0.522	0.238	5	0.543	0.181	3	0.553	0.232	4	0.471	0.172	4
WMA-12	0.474	0.171	3	0.515	0.235	4	0.535	0.178	2	0.553	0.232	4	0.465	0.170	3
EWMA-12	0.470	0.170	2	0.499	0.228	3	0.547	0.182	4	0.553	0.232	4	0.455	0.166	2
ES	0.459	0.166	1	0.489	0.223	2	0.530	0.177	1	0.535	0.225	3	0.443	0.162	1
REG	0.522	0.189	5	0.479	0.219	1	0.609	0.203	5	0.499	0.209	2	0.536	0.196	5
ARCH	2.765	1.000	11	2.190	1.000	11	2.947	0.982	8	2.163	0.908	8	2.674	0.978	10
GARCH	2.573	0.931	9	2.138	0.976	9	2.979	0.992	10	2.336	0.981	9	2.735	1.000	11
GJR-GARCH(1,1)	2.488	0.900	8	2.112	0.964	8	2.959	0.986	9	2.382	1.000	11	2.625	0.960	9
EGARCH	2.763	0.999	10	2.142	0.978	10	3.002	1.000	11	2.358	0.990	10	2.480	0.907	8

	Hong Kong			**Italy**			**Japan**			**Netherlands**			**Philippines**		
	Actual	Relative	Rank	Actual	Relative	Rank	Actual	Relative	Rank	Actual	Relative	Rank	Actual	Relative	Rank
RW	0.549	0.182	7	0.557	0.268	7	0.609	0.174	6	0.460	0.239	6	0.573	0.174	5
HM	0.524	0.174	6	0.427	0.206	1	0.662	0.189	7	0.519	0.270	7	0.839	0.255	7
MA-12	0.474	0.157	2	0.442	0.213	4	0.507	0.145	3	0.406	0.211	4	0.510	0.155	4
WMA-12	0.468	0.155	1	0.447	0.215	5	0.501	0.143	2	0.400	0.208	3	0.503	0.153	2
EWMA-12	0.482	0.160	3	0.438	0.211	3	0.507	0.145	3	0.385	0.200	1	0.504	0.153	3
ES	0.491	0.163	4	0.431	0.208	2	0.491	0.141	1	0.387	0.201	2	0.490	0.149	1
REG	0.494	0.164	5	0.463	0.223	6	0.599	0.171	5	0.418	0.217	5	0.643	0.196	6
ARCH	3.020	1.000	11	2.077	1.000	11	3.494	1.000	11	1.922	1.000	11	3.286	1.000	11
GARCH	2.284	0.756	10	2.010	0.968	9	2.489	0.712	10	1.899	0.988	10	2.497	0.760	8
GJR-GARCH(1,1)	2.211	0.732	9	1.957	0.942	8	2.321	0.664	9	1.861	0.968	8	2.526	0.769	10
EGARCH	2.203	0.729	8	2.027	0.976	10	2.319	0.664	8	1.865	0.970	9	2.508	0.763	9

	Singapore			**Thailand**			**UK**			**US**		
	Actual	Relative	Rank	Actual	Relative	Rank	Actual	Relative	Rank	Actual	Relative	Rank
RW	0.608	0.216	7	0.596	0.208	7	0.462	0.180	5	0.548	0.216	6
HM	0.586	0.208	6	0.589	0.206	6	0.576	0.225	7	0.726	0.287	7
MA-12	0.533	0.189	4	0.521	0.182	3	0.383	0.149	4	0.448	0.177	4
WMA-12	0.529	0.188	2	0.516	0.180	2	0.377	0.147	2	0.437	0.173	3
EWMA-12	0.532	0.189	3	0.521	0.182	3	0.381	0.149	3	0.432	0.171	2
ES	0.522	0.185	1	0.498	0.174	1	0.368	0.143	1	0.415	0.164	1
REG	0.538	0.191	5	0.556	0.194	5	0.471	0.184	6	0.543	0.214	5
ARCH	2.819	1.000	11	2.863	1.000	11	2.565	1.000	11	2.533	1.000	11
GARCH	2.566	0.910	9	2.398	0.838	9	2.222	0.866	10	2.246	0.887	9
GJR-GARCH(1,1)	2.576	0.914	10	2.390	0.835	8	2.179	0.850	9	2.305	0.910	10
EGARCH	2.509	0.890	8	2.419	0.845	10	2.097	0.818	8	2.214	0.874	8

The mean absolute percentage error, (MAPE), actual figures. *Actual* is the calculated error statistic. *Relative* is the ratio between the actual error statistic of a model and that of the worst-performing model for that country. The best performing model has a rank of 1.

Table 5: Mean Mixed Error (MME) Statistics from Forecasting Weekly Volatility

	Belgium								**Canada**							
	MME(U)			MME(O)			%	%	MME(U)			MME(O)			%	%
	Actual	Relative	Rank	Actual	Relative	Rank	Underestimaton	Overestimation	Actual	Relative	Rank	Actual	Relative	Rank	Underestimaton	Overestimation
RW	2.269	1.000	11	2.347	0.251	6	48.3	51.7	2.308	1.000	11	2.308	0.280	4	50.6	49.4
HM	1.682	0.741	5	2.674	0.287	7	36.8	63.2	1.958	0.848	8	2.318	0.281	5	40.2	59.8
MA-12	1.949	0.859	8	2.104	0.225	4	45.6	54.4	1.953	0.846	7	2.357	0.286	7	40.2	59.8
WMA-12	1.922	0.847	7	2.099	0.225	3	44.4	55.6	1.915	0.830	5	2.321	0.282	6	40.6	59.4
EWMA-12	2.024	0.892	10	2.064	0.221	2	45.6	54.4	2.029	0.879	10	2.251	0.273	3	42.1	57.9
ES	2.003	0.883	9	1.983	0.212	1	47.9	52.1	1.935	0.838	6	2.209	0.268	2	41.8	58.2
REG	1.876	0.827	6	2.328	0.249	5	41.4	58.6	2.026	0.878	9	2.082	0.253	1	45.6	54.4
ARCH	1.034	0.456	4	9.091	0.974	10	3.1	96.9	0.885	0.383	4	8.059	0.978	8	3.1	96.9
GARCH	0.913	0.402	2	9.020	0.966	9	1.1	98.9	0.782	0.339	2	8.242	1.000	11	1.5	98.5
GJR-GARCH(1,1)	0.878	0.387	1	8.725	0.935	8	1.1	98.9	0.791	0.343	3	8.209	0.996	10	1.5	98.5
EGARCH	0.968	0.427	3	9.333	1.000	11	1.1	98.9	0.780	0.338	1	8.173	0.992	9	1.5	98.5

	Denmark								**Finland**							
	MME(U)			MME(O)			%	%	MME(U)			MME(O)			%	%
	Actual	Relative	Rank	Actual	Relative	Rank	Underestimaton	Overestimation	Actual	Relative	Rank	Actual	Relative	Rank	Underestimaton	Overestimation
RW	2.280	1.000	11	2.374	0.250	5	48.3	51.7	3.453	0.805	10	3.639	0.293	7	48.7	51.3
HM	1.632	0.716	6	2.740	0.288	7	33.3	66.7	4.291	1.000	11	1.817	0.146	1	65.1	34.9
MA-12	1.893	0.830	9	2.320	0.244	4	41.8	58.2	2.724	0.635	7	3.272	0.264	5	42.9	57.1
WMA-12	1.874	0.822	8	2.286	0.241	2	42.9	57.1	2.693	0.628	6	3.323	0.268	6	41.0	59.0
EWMA-12	1.920	0.842	10	2.302	0.242	3	40.6	59.4	2.783	0.649	8	3.214	0.259	3	43.7	56.3
ES	1.869	0.820	7	2.222	0.234	1	42.1	57.9	2.676	0.624	5	3.229	0.260	4	41.0	59.0
REG	1.608	0.705	5	2.629	0.277	6	34.9	65.1	3.232	0.753	9	2.776	0.224	2	49.0	51.0
ARCH	0.986	0.432	1	9.299	0.979	8	1.1	98.9	1.645	0.383	1	11.748	0.947	8	1.9	98.1
GARCH	1.007	0.442	2	9.469	0.997	10	0.8	99.2	1.745	0.407	2	12.209	0.984	9	1.9	98.1
GJR-GARCH(1,1)	1.008	0.442	3	9.437	0.993	9	0.8	99.2	1.769	0.412	3	12.313	0.992	10	1.9	98.1
EGARCH	1.009	0.443	4	9.499	1.000	11	0.8	99.2	1.775	0.414	4	12.407	1.000	11	1.5	98.5

MME(U) and MME(O) are the mean mixed error statistics that penalise the underpredictions and overpredictions more heavily, respectively. *Actual* is the calculated error statistic. MME(U) and MME(O) actual figures must be multiplied by 10^{-2}. *Relative* is the ratio between the actual error statistic of a model and that of the worst-performing model. The best performing model has a rank of 1.

Table 5 (cont.): Mean Mixed Error (MME) Statistics from Forecasting Weekly Volatility

	Germany								Hong Kong							
	MME(U)			MME(O)			%	%	MME(U)			MME(O)			%	%
	Actual	Relative	Rank	Actual	Relative	Rank	Underestimaton	Overestimation	Actual	Relative	Rank	Actual	Relative	Rank	Underestimaton	Overestimation
RW	2.776	1.000	11	2.806	0.235	5	49	51	3.792	1.000	11	3.735	0.245	6	48.7	51.3
HM	1.845	0.665	5	4.004	0.336	7	27.2	72.8	3.769	0.994	10	3.701	0.243	4	43.7	56.3
MA-12	2.349	0.846	9	2.585	0.217	4	45.2	54.8	3.192	0.842	6	3.643	0.239	2	43.3	56.7
WMA-12	2.313	0.833	7	2.560	0.215	3	45.2	54.8	3.132	0.826	5	3.637	0.238	1	42.9	57.1
EWMA-12	2.403	0.866	10	2.482	0.208	2	44.8	55.2	3.241	0.855	7	3.676	0.241	3	43.7	56.3
ES	2.345	0.845	8	2.406	0.202	1	46.4	53.6	3.303	0.871	8	3.784	0.248	7	44.4	55.6
REG	1.960	0.706	6	3.241	0.272	6	32.6	67.4	3.405	0.898	9	3.733	0.245	5	41.8	58.2
ARCH	1.587	0.572	3	11.510	0.965	9	2.3	97.7	2.878	0.759	4	15.250	1.000	11	2.7	97.3
GARCH	1.592	0.573	4	11.923	1.000	11	1.5	98.5	2.363	0.623	3	13.678	0.897	10	3.1	96.9
GJR-GARCH(1,1)	1.571	0.566	2	11.676	0.979	10	1.9	98.1	2.212	0.583	2	13.444	0.882	9	3.1	96.9
EGARCH	1.499	0.540	1	11.293	0.947	8	2.2	97.7	2.198	0.580	1	13.397	0.878	8	2.7	97.3

	Italy								Japan							
	MME(U)			MME(O)			%	%	MME(U)			MME(O)			%	%
	Actual	Relative	Rank	Actual	Relative	Rank	Underestimaton	Overestimation	Actual	Relative	Rank	Actual	Relative	Rank	Underestimaton	Overestimation
RW	3.443	0.993	10	3.358	0.277	7	51.3	48.7	3.151	1.000	11	2.982	0.220	3	51.7	48.3
HM	3.467	1.000	11	2.419	0.199	1	56.3	43.7	2.139	0.679	5	3.890	0.287	7	30.7	69.3
MA-12	2.746	0.792	5	3.154	0.260	5	44.8	55.2	2.588	0.821	8	3.024	0.223	4	41.4	58.6
WMA-12	2.809	0.810	7	3.190	0.263	6	45.6	54.4	2.554	0.811	7	2.960	0.219	2	42.1	57.9
EWMA-12	2.746	0.792	5	3.082	0.254	4	44.8	55.2	2.588	0.821	8	3.024	0.223	4	41.4	58.6
ES	2.813	0.811	8	3.020	0.249	3	45.2	54.8	2.618	0.831	10	2.808	0.207	1	45.6	54.4
REG	3.073	0.886	9	2.908	0.240	2	50.2	49.8	2.175	0.690	6	3.532	0.261	6	34.9	65.1
ARCH	1.615	0.466	4	12.050	0.994	9	1.5	98.5	2.025	0.643	4	13.535	1.000	11	1.5	98.5
GARCH	1.575	0.454	2	12.054	0.994	10	0.4	99.6	1.495	0.474	3	11.517	0.851	10	1.9	98.1
GJR-GARCH(1,1)	1.541	0.444	1	11.849	0.977	8	0.8	99.2	1.385	0.440	2	11.121	0.822	9	1.1	98.9
EGARCH	1.591	0.459	3	12.126	1.000	11	0.4	99.6	1.378	0.437	1	11.102	0.820	8	1.1	98.8

MME(U) and MME(O) are the mean mixed error statistics that penalise the underpredictions and overpredictions more heavily, respectively. *Actual* is the calculated error statistic. MME(U) and MME(O) actual figures must be multiplied by 10^{-2}. *Relative* is the ratio between the actual error statistic of a model and that of the worst-performing model. The best performing model has a rank of 1.

Table 5 (cont.): Mean Mixed Error (MME) Statistics from Forecasting Weekly Volatility

	Netherlands MME(U)			MME(O)			%	%	Philippines MME(U)			MME(O)			%	%
	Actual	Relative	Rank	Actual	Relative	Rank	Underestimaton	Overestimation	Actual	Relative	Rank	Actual	Relative	Rank	Underestimaton	Overestimation
RW	2.446	1.000	11	2.458	0.260	6	48.7	51.3	3.560	1.000	11	3.525	0.261	6	50.6	49.4
HM	2.383	0.974	9	2.745	0.290	7	41.8	58.2	2.412	0.678	4	5.280	0.391	8	28.7	71.3
MA-12	2.357	0.964	7	2.364	0.250	4	48.3	51.7	3.059	0.859	9	3.399	0.252	5	43.3	56.7
WMA-12	2.288	0.935	5	2.364	0.250	4	46.7	53.3	2.999	0.842	7	3.375	0.250	4	42.9	57.1
EWMA-12	2.431	0.994	10	2.179	0.230	1	47.9	52.1	3.093	0.869	10	3.372	0.250	3	43.7	56.3
ES	2.342	0.957	6	2.254	0.238	2	48.7	51.3	3.030	0.851	8	3.285	0.243	2	44.1	55.9
REG	2.368	0.968	8	2.260	0.239	3	45.6	54.4	2.468	0.693	6	4.433	0.328	7	32.2	67.8
ARCH	1.105	0.452	4	8.850	0.934	8	4.2	95.8	2.453	0.689	5	1.481	0.110	1	2.3	97.7
GARCH	0.990	0.405	3	9.472	1.000	11	1.1	98.9	1.943	0.546	2	13.377	0.990	10	0.4	99.6
GJR-GARCH(1,1)	0.965	0.395	1	9.295	0.981	9	1.1	98.9	1.974	0.554	3	13.507	1.000	11	0.4	99.6
EGARCH	0.978	0.400	2	9.344	0.986	10	0.8	99.2	1.932	0.543	1	13.374	0.990	9	0.4	99.6

	Singapore MME(U)			MME(O)			%	%	Thailand MME(U)			MME(O)			%	%
	Actual	Relative	Rank	Actual	Relative	Rank	Underestimaton	Overestimation	Actual	Relative	Rank	Actual	Relative	Rank	Underestimaton	Overestimation
RW	2.977	1.000	11	2.921	0.261	5	51.3	48.7	3.862	1.000	11	3.741	0.254	2	51	49
HM	2.298	0.772	5	3.203	0.286	7	36.4	63.6	3.222	0.834	7	4.268	0.290	7	37.2	62.8
MA-12	2.488	0.836	9	2.847	0.255	3	42.9	57.1	3.231	0.837	9	3.824	0.260	3	43.3	56.7
WMA-12	2.426	0.815	7	2.874	0.257	4	41.8	58.2	3.187	0.825	5	3.828	0.260	5	41.0	59.0
EWMA-12	2.527	0.849	10	2.814	0.252	1	44.4	55.6	3.231	0.837	9	3.824	0.260	3	43.3	56.7
ES	2.477	0.832	8	2.821	0.252	2	42.5	57.5	3.226	0.835	8	3.675	0.249	1	42.5	57.5
REG	2.393	0.804	6	2.935	0.262	6	42.5	57.5	3.214	0.832	6	4.080	0.277	6	39.5	60.5
ARCH	1.553	0.522	4	11.181	1.000	11	3.1	96.9	2.476	0.641	4	14.736	1.000	11	2.7	97.3
GARCH	1.376	0.462	3	10.763	0.963	9	1.9	98.1	2.164	0.560	1	13.939	0.946	8	1.5	98.5
GJR-GARCH(1,1)	1.363	0.458	2	10.825	0.968	10	1.5	98.5	2.211	0.573	3	13.981	0.949	9	1.9	98.1
EGARCH	1.361	0.457	1	10.547	0.943	8	1.5	98.5	2.200	0.570	2	14.070	0.955	10	1.9	98.1

MME(U) and MME(O) are the mean mixed error statistics that penalise the underpredictions and overpredictions more heavily, respectively. *Actual* is the calculated error statistic. MME(U) and MME(O) actual figures must be multiplied by 10^{-2}. *Relative* is the ratio between the actual error statistic of a model and that of the worst-performing model. The best performing model has a rank of 1.

Table 5 (cont.): Mean Mixed Error (MME) Statistics from Forecasting Weekly Volatility

	UK								US							
	MME(U)			MME(O)			%	%	MME(U)			MME(O)			%	%
	Actual	Relative	Rank	Actual	Relative	Rank	Underestimaton	Overestimation	Actual	Relative	Rank	Actual	Relative	Rank	Underestimaton	Overestimation
RW	2.260	1.000	11	2.177	0.215	5	52.9	47.1	2.407	1.000	11	2.406	0.262	5	50.2	49.8
HM	1.210	0.535	5	3.387	0.334	7	26.1	73.9	1.601	0.665	5	3.585	0.391	7	28.7	71.3
MA-12	1.868	0.827	8	2.088	0.206	4	44.8	55.2	2.048	0.851	8	2.163	0.236	3	46.4	53.6
WMA-12	1.840	0.814	7	2.040	0.201	2	44.8	55.2	1.981	0.823	7	2.170	0.236	4	44.1	55.9
EWMA-12	1.912	0.846	10	2.083	0.205	3	44.4	55.6	2.086	0.867	10	2.119	0.231	2	46.4	53.6
ES	1.904	0.842	9	1.956	0.193	1	47.1	52.9	2.069	0.860	9	2.015	0.220	1	46.7	53.3
REG	1.410	0.624	6	2.785	0.275	6	33.7	66.3	1.952	0.811	6	2.631	0.287	6	40.2	59.8
ARCH	1.090	0.482	4	10.14	1.000	11	0.4	99.6	1.034	0.430	4	9.179	1.000	11	2.3	97.7
GARCH	0.930	0.412	3	9.440	0.931	10	0.4	99.6	0.948	0.394	2	9.032	0.984	9	1.1	98.9
GJR-GARCH(1,1)	0.917	0.406	2	9.395	0.927	9	0.4	99.6	0.972	0.404	3	9.097	0.991	10	1.1	98.9
EGARCH	0.897	0.397	1	9.220	0.909	8	0.4	99.6	0.927	0.385	1	8.870	0.966	8	1.5	98.5

MME(U) and MME(O) are the mean mixed error statistics that penalise the underpredictions and overpredictions more heavily, respectively. *Actual* is the calculated error statistic. MME(U) and MME(O) actual figures must be multiplied by 10^{-2}. *Relative* is the ratio between the actual error statistic of a model and that of the worst-performing model. The best performing model has a rank of 1.