

A MATHEMATICAL
and
EXPERIMENTAL INVESTIGATION
on
MINE VENTILATION AND FANS.

by

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D. Sc. 1925.



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Mining is being carried on at steadily increasing depths, and as mines go deeper, ventilation presents problems of increasing complexity and importance. The working of mineral deposits at moderate depths has led to the sinking of numerous shafts, each serving for the winning of the minerals from a small area. But with greater depths, the heavy cost of sinking and fitting shafts has made larger mineral concessions a necessity; and with modern progress in the construction of roadways supported by circular steel girders and brick or cement linings, it is becoming evident that in the future still more extensive concessions will be worked from a single pair of shafts. Money which in the past has been spent in sinking new shafts and erecting extensive surface works will be found to be more economically expended in making larger roadways with permanent linings, which will not only serve for the winning of the minerals from an area much more extensive, but will also lead to increased safety and economy, especially by reducing the accident rate and the cost of haulage below their present day values.

The Science of Mine Ventilation is faced with the problem of ventilating these extended areas in an effective manner, and of securing, if possible, that there shall not be set an effective limit to the size of the area of economical exploitation by a failure to supply an adequate /

adequate volume of air. This problem divides itself into a mining problem which has to do with the provision and maintenance of suitable shafts and airways, and an engineering problem which has to do with the provision of suitable ventilating appliances for the production of the necessary currents of air.

The solution of the mining problem is found in the provision of large airways in parallel arrangement with each other, well secured by strong linings, as smooth as possible to prevent excessive friction and as tight as possible to prevent serious leakage of air from the intake to the return. The power required to circulate the necessary volume of air will depend upon how well this mining problem has been solved. Large roadways not only increase the facilities for safe and efficient transport, but also reduce very considerably the bill for power required to produce adequate ventilation.

The following table shows how rapidly the power required to circulate a given volume of air decreases with increase of dimensions of the airway.

Table Showing the Effect of Increase of Size of Airway in Reducing the Power Required for Ventilation.

Diameter of Airway	Percentage Power Consumption.
D x 1	100
D x 1.1	62.1
D x 1.2	40.2
D x 1.3	26.9
D x 1.4	18.6
D x 1.5	13.2
D x 1.6	9.5
D x 1.7	7.1
D x 1.8	5.3
D x 1.9	4.0
D x 2.	3.1

For the solution of the engineering problem, the volume of air required and the pressure necessary to circulate that volume must be known. As the efficiency obtained from the ventilator will depend largely on the accuracy of this estimate, the determination of the P-Q relation for the mine becomes a matter of import. The task, however, is one which presents considerable difficulty.

The Resistance of the Mine.

The pressure-Volume relation, of profound importance when selecting a suitable fan, depends upon the value of the mine resistance, and this is not only difficult to predetermine, but there is no general agreement upon the best methods of its estimation, or even upon the units which should be employed. To indicate suitable methods and units will be an objective of the present investigation.

The motion of a current of air in a mine airway bears some analogy to that of the complex motion of water in a pipe. The form, size, and degree of roughness of the boundaries of an airway are not characterised by that degree of uniformity which is met with in a pipe, and, unlike water, air is a highly compressible fluid, which adds considerably to the difficulty of the problem.

The roughness of the boundary impedes the flow of air and causes the velocity at any given point to vary in magnitude and direction. Air has

no /

no transverse elasticity and therefore no tendency to recover any definite shape after having been distorted; but when a layer of air moving with a greater velocity streams past a layer of retarded air near the boundary, there is a rapid diffusion of the molecules across the surface separating the two layers considered, and this continually tends to equalise the motion. The laws of this lateral communication of motion are not exactly known, but it causes the boundary friction to take effect, not only in retarding the layers of air more immediately in contact with the boundary, but also in retarding the motion of the whole stream of air.

The rate of this lateral diffusion of momentum is, however, greatest near the boundary, because after collision with the rubbing surface, air particles are deflected with a lateral component of velocity which increases the rate of mixing of layers of air near the boundary. Further inwards where the action does not reach, there will be a further lateral diffusion of momentum and equalisation of velocity between stream lines possessing a relative velocity. For very small velocities, the motion of the air is approximately regular, and at any moment from any point in the fluid to a neighbouring point the change of velocity will be gradual, so that the relative velocity of successive layers will be nearly constant and could be represented by a shear; that is, from near the boundary to the centre the distribution of velocity could be taken as linear, the velocity at the centre being greatest and the velocity falling at any point in proportion to the distance of the /

the point from the centre.

Increase of velocity soon produces eddies which traverse the air in all directions, giving rise to what is known as turbulent flow. This turbulence leads to a rapid interchange of momentum over the cross-section. The movements which give rise to the friction of the air in an airway, must thus be regarded as rapidly and even suddenly varying from one point to another, and the effect of the viscosity of the air on the internal resistance is quite negligible in comparison with that due to those movements.

Velocity at a Point.

While the actual motion at any point in a current of air flowing in an airway is very complex, it is found that over a sensible period, the velocity at the point as measured by an anemometer or other suitable instrument tends towards a definite or average value which varies very little in magnitude or direction. The variations of direction and magnitude of the velocity at any point may thus be regarded as being periodic, and if for each point the mean value of the velocity as determined by measurement were substituted for the actual complex motion at the point, the motion of the air current could be treated as steady stream line or steady laminar motion.

Volume of Flow.

The volume q which passes through an area over which the velocity v is normal to the area is given by $q = v \cdot da$. If the velocity makes an angle A with the normal to the area, $q = v \cdot \cos A \cdot da$.

The total volume Q flowing through the whole cross-

section /

cross-section of the airway is $Q = \int v \cdot \cos \theta \, da.$ (1)

Mean Velocity of Flow.

The mean velocity of flow over the airway is equal to the volume divided by the area, or

$$V = Q/a = (1/a) \int v \cos \theta \, da. \dots\dots\dots(2)$$

The mean velocity is spoken of simply as the velocity of the air current.

The Frictional Resistance of the Boundary.

The results of experiments made by many workers show that except for very small velocities the frictional resistance of an airway is:-

1. Proportional to the extent of the rubbing surface
2. Very nearly proportional to the square of the velocity.

Let k = the frictional resistance estimated in pounds per square foot at a velocity of 1 foot per second; s = the extent of the rubbing surface in square feet;

V = the velocity of flow in feet per second;

r = the resistance of the airway.

$$\text{Then } r = ksv^2 \dots\dots\dots(3)$$

The value of k is found to be approximately constant for any given surface. In hydraulics it is customary to take as the coefficient of friction a quantity f such that $f = k \cdot (1g/G)$ where G is the weight of 1 cubic foot of water.

In mine ventilation k is taken as the coefficient of friction, and not f and values of k have been calculated for a standard velocity of 1,000 cubic feet per minute. For a standard velocity of 1 foot per second, those values would require to be multiplied by .0036.

The Effect of the Diameter of the Airway.

It has been observed that the diffusion of
momentum /

momentum goes on most rapidly close to the boundary, and since in a small airway the region of this increased activity is relatively greater than in a large airway, it is to be expected that the value of the coefficient of friction would be greater for smaller airways.

Values of the coefficient of friction for airways of small and of normal size have been given by Raux and Murgue, but they do not appear to have attempted to derive a law which would enable values to be calculated for intermediate sizes.

For the coefficient of friction f in hydraulics, Darcy has proposed the expression

$$f = \left\{ a + (a/d) \right\} + \left\{ b + (b_1/d^2) \right\} / v$$

where

a	=	.004346
a ₁	=	.0003992
b	=	.0010182
b ₁	=	.000005205

Although this expression involves four constants, it cannot be regarded as entirely adequate, for the conditions of flow are really more complicated than can be expressed by any rational formula.

It is necessary for practical purposes to be able to discriminate the probable value of the coefficient of friction, and Darcy's method which clearly indicates how the value of the coefficient decreases for a larger airway, and increases with the roughness of the surface is very helpful. Later workers have shown that a fair approximation may be obtained by omitting a part of Darcy's formula and writing $f = a + (a_1/d)$

Thus in a paper on "The Coefficient of Friction of Air Flowing in Long Pipes," Prof. Unwin

has /

has reviewed the observations made by Stockalper on the flow of air in pipes in the St. Gothard's tunnel, and has shown that a coefficient of friction of the form

$$k = k^1 \left\{ 1 + (3/10d) \right\} \dots\dots\dots(4)$$

where d = the diameter of the pipe in feet, gives results which agree very closely with experimental data.

If this form were adopted for mine airways, the resistance could be expressed by

$$r = k^1 \left\{ 1 + (3/10d) \right\} sV^2 \dots\dots\dots(5)$$

For an airway of uniform cross-section and neglecting the effect of expansion of the air, the total force acting along the axis of the airway may be equated to the resistance, giving the relation

$$Pa = k^1 \left\{ 1 + (3/10d) \right\} sV^2$$

where P = the difference of pressure in pounds per square foot on the two ends of the airway,

a = the cross-section of the airway in feet.

Writing K for $k^1 \left\{ 1 + (3/10d) \right\}$ and multiplying by a^2 gives

$$Pa^3 = KsV^2 \cdot a^2 = Ksq^2$$

or

$$P = (Ks/a^3) \cdot q^2$$

The quantities K , s , and a , are all peculiar to the given airway, and writing R for (Ks/a^3) in the above relation gives

$$P = Rq^2 \dots\dots\dots(6)$$

where R is a measure of the resistance of the airway.

In the formula (6) all units are in pounds, feet and seconds. Thus P is in pounds per square foot; q is in cubic feet per second or in cusecs.

It has been found that it would be useful to fix a unit value of resistance, and a Committee of the Institute of Mining Engineers have recommended that the unit value of R should be such that in an airway having unit resistance, a volume of one thousand cubic feet per second or one Kilo-cusec, would flow when a ventilating pressure of one pound per square foot is maintained. The unit volume is thus taken as the Kilo-cusec, and is represented by Q.

The unit resistance has been called the Atkinson. Transposing (6) gives

$$R = (P/Q^2) \text{ Atkinsons} \dots\dots\dots (7)$$

and for unit values of P and Q

$$R = (1/1^2) = 1 \text{ Atkinson.}$$

In an airway in which there is no leakage the relation expressed by (7) is found to be satisfied very nearly.

The Effect of Leakage.

In every mine, leakage of air occurs at many points of the ventilating circuit. It is convenient to divide this leakage into two parts; one which occurs at or near the fan, and the other which occurs between the intake and return airways of the mine. These may be called surface and underground leakage respectively.

The legal enactment that provision shall be made for the reversal of the air-current, has necessitated the introduction of a system of doors at which leakage invariably occurs. The amount of this leakage and the difficulty of preventing it have not been fully realised. Tests made on two modern

forcing fans, showed 10 per cent leakage at these doors. The use of iron doors is not sufficient. Doors should close on a soft packing of rubber or felt and upon a strongly constructed frame and be held tight by a sufficient number of thumb screws. Leakage at this point could thus be eliminated entirely.

When a separate ventilating shaft is not provided, and the shaft must be used for winding purposes, there will always be some surface leakage. The full pressure difference created by the fan acts here and it is very difficult to keep the leakage so low as 10 per cent.; values between 30 and 40 per cent were found in the tests made in the present investigation.

In its action, surface leakage is equivalent to the addition of an airway in parallel with the mine. The value of the resistance upon which the fan operates is thus lowered below that of the mine resistance. The effect of underground leakage is variable and depends upon where it occurs as well as upon the resistance of the leakage path.

A Particular Case Considered.

Consider the case of a ventilating circuit in which a volume of 1,000 cubic feet per second is passing, with a ventilating pressure of 6 pounds per square foot. If a volume of 100 cubic feet per second leaks from the intake to the return airway at a point where the pressure difference is 3 pounds per square foot, and there are no other leakage paths so that the remaining 900 cubic feet of air traverse the full circuit, then, estimating resistances in Atkinsons and assuming the square law, the resistance

of the part of the air circuit in-by the leak
 $= 3/(0.9)^2 = 3.7$ Atkinsons. The resistance of the
 part outside the leak $= 3/1^2 = 3$ Atkinsons, and
 the total resistance of the air circuit $= 6/1^2$ or
 6 Atkinsons.

The leakage path is in parallel with that
 part of the circuit beyond the leak, and these
 parallel circuits have a joint resistance of 3
 Atkinsons, since the total resistance is only 6
 Atkinsons. If there had been no leakage, the
 resistance of the circuit would have been $3+3.7$ or
 6.7 Atkinsons. The effect of leakage is thus to
 lower the resistance of the air circuit in which it
 occurs. For an airway in which there is no leakage
 the volume of air passing may be taken as proportional
 to the square root of the ventilating pressure. But
 the volume which passes through a leakage path has
 been found to be approximately proportional to the
 pressure difference.

Again, in the case considered above, if the
 ventilating pressure were increased until there was a
 pressure difference of 6 pounds per square foot where
 the leak occurs, the leakage volume would become
 200 cubic feet per second, the volume passing beyond
 the leak $900 \cdot \sqrt{2}$, or 1,272 cubic feet per second,
 and the total volume would be 1,472 cubic feet per
 second. The pressure required to overcome the
 frictional resistance of the airway on the shaft
 side of the leak would be 3×1.472^2 or 6.5 pounds
 per square foot and the resistance /

resistance of the airway expressed in Atkinsons, assuming the square law would be $12.5/1.472^2$ or 5.769 Atkinsons.

Thus the airway considered would have a resistance of 6.7 Atkinsons if there were no leakage; with leakages of 100 and 200 cubic feet per second occurring as indicated, the apparent resistance would have become 6 and 5.796 Atkinsons respectively. If the leakage current had followed the square law, then the increase of leakage due to the increase of pressure would have left the value of the resistance unchanged and equal to 6 Atkinsons. But since the leakage current follows a different law, error would result if the square law were assumed when, for a new ventilating pressure, an estimate is made of the resistance of an airway in which leakage occurred.

These considerations of the effect of leakage, indicate that the resistance of an airway in which leakage occurs is a definite quantity for a definite pressure only. The leakage air does not flow against the resistance of the full length of the airway, and as the pressure increases, a disproportionate volume of leakage air passes without overcoming the resistance of the full length.

If, therefore, the convenience of the use of a value R as a measure of the resistance of the mine, as in formula (6), be retained, it becomes necessary to define exactly what is meant by R , and also to obtain an expression involving P , Q and R which can be relied upon to give the pressure-volume relations for the airway with a sufficient degree of accuracy for practical purposes.

It will be shown that a formula $P = RQ^n$ may be used, in which R is the resistance of the airway in Atkinsons when a volume of 1,000 cubic feet per second is flowing.

The effect of leakage has been shown to be such as to lower the apparent resistance of the airway, and therefore the pressure which overcomes that resistance. The pressure-volume relation for the airway would thus be more correctly expressed in ascending powers of the volume, and experience shows that there is no need to include terms of higher degree than the third. The relation thus becomes:-

$$P = AQ + BQ^2 + CQ^3 \dots\dots (8)$$

where A = a constant depending upon the resistance of the leakage paths, and correcting also for any departure of the frictional resistance from the "square law"; C = a constant, generally negative in value. B a constant depending upon the resistance of the airway.

A less close degree of approximation may be obtained by including no terms of degree higher than the second; or

$$P = AQ + BQ^2 \dots\dots\dots (9)$$

in which form the constants B and A are both positive.

In any real airway the leakage will occur at numerous points along the airway, and whatever the values of the leakage paths may be, those which are nearest to the fan will be subject to the greatest pressure difference, and the leakage across them will have the greatest influence on the result.

If the ventilating pressure on the airway were increased in order to secure an increase of volume, the distribution of pressure along the airway would be greatly changed, the leakages on the outbye portion of the airway would be much increased in proportion to those inbye, and a disproportionate part of the total pressure would be spent in overcoming the friction of the outbye part of the airway in which the stronger current would flow. This effect would be allowed for by the term of the third degree.

Expressions in the form proposed for hydraulics by Hagen and Reynolds and suitable for logarithmic computation, give an approximation nearly as good, and are more easily applied than formula (8). Hagen proposed the form $h/l = mv^n/d^x$ involving three constants m , n and x . He found that when the values $n = 1.75$ and $x = 1.25$ were substituted, the value of m was then nearly independent of variations in v and in d .

Reynolds proposed the form $h/l = c(v^n/d^{3-n}) P^{2-n}$ where $n = 1$ for low velocities and 1.7 to 2 for ordinary velocities. P is a function of the temperature and corrects for changes in the viscosity of the fluid due to changes of the temperature. When variations in the temperature are neglected, Reynolds' formula is identical with Hagen's if x is put equal to $3 - n$, and for practical purposes Hagen's form is the more convenient. Adapted to the purposes of mine ventilation Hagen's formula would become

$$P = (m/d^x) \cdot v^n \dots\dots (10)$$

Formula (8) and (9) involving as they do three

constants /

constants, while very useful for special purposes, are too complex for general use.

St. Venant was the first to propose a formula involving two constants, viz., $d/h = mV^n$ where m and n are experimental constants. For use in Mine Ventilation this would take the form

$$P = mV^n \dots\dots\dots (11)$$

or expressed in terms of the volume flowing, formula (11) becomes

$$P = RQ^n \dots\dots\dots (12)$$

which form has been adopted by the Ventilation Committee of the Institute of Mining Engineers.

The values of the two constants, R and n involved in formula (12) will depend upon the frictional resistance of the airway, and also upon the amount and distribution of the leakage. The values of R and n for an airway can be easily determined by subjecting the airway to a series of ventilating pressure and determining the volume corresponding to each pressure.

Then taking logs. in formula (12) gives $\log. P = \log. R + n \log. Q$. The graph of this expression is a straight line, and $\log R$ is the intercept at the origin, and n is the slope of the line. But the value $\log. P = \log. R$, or $P = R$ at the origin is the value when $Q = 1$, or $\log. Q = 0$. Hence:-

The value of R in formula (12) is that of the apparent resistance of the circuit when it is conducting one unit of volume or 1 Kilocusec, and this value is thus a perfectly definite one.

Application of Formulae to Case already Considered.

In the case considered, where leakage occurred, with a ventilating pressure of 6 pounds per square foot the volume flowing was 1,000 cubic feet per second, and with a pressure of 12.5 pounds per square foot the volume was 1,472 cubic feet per second. To apply formula No. (8), another set of values is required. If the pressure at the leakage point were reduced to $1\frac{1}{2}$ pounds per square foot, the leakage volume would fall to 50 cubic feet per second, and the total volume flowing would be 686 cubic feet per second, with a ventilating pressure of 2.91 pounds per square foot.

In $P = AQ + BQ^2 + CQ^3$, the three sets of relations are,

$$6 = A \times 1 + B \times 1^2 + C \times 1^3.$$

$$12.5 = A \times 1.472 + B \times 1.472^2 + C \times 1.472^3.$$

$$2.91 = A \times .686 + B \times .686^2 + C \times .686^3.$$

These equations are satisfied by the values

$$A = 0.013, \quad B = 6.483, \quad C = -0.496.$$

The general relation is thus

$$P = 0.013Q + 6.483Q^2 - 0.496Q^3.$$

To apply formula No. (9), two sets of values are required, and the first two sets above may be used.

$$P = AQ + BQ^2$$

$$6 = A \times 1 + B \times 1^2$$

$$12.5 = A \times 1.472 + B \times 1.472^2.$$

$$\text{Solving gives } A = 0.723, \quad B = 5.177.$$

If these values are substituted for the third relation, the pressure obtained is found to be 3.08 pounds per square foot, which differs from the

true value 2.91 by 0.17 pounds per square foot, an error of 5.5 per cent.

To apply formula No. (12) two sets of relations are required and three sets are known. In one set of relations $Q = 1$ and the value of R can therefore be immediately written down since in

$$P = R \cdot 1^n$$

$$R = P = 6$$

Using the other two sets to determine R and n the relations are:-

$$\log 2.91 + n \log 0.686 = \log R.$$

$$\log 12.5 + n \log 1.472 = \log R.$$

Solving gives $n = 1.815$ and $R = 5.76$.

Thus, in estimating the pressure required to pass the intermediate volume of 1,000 cubic feet per second by means of a curve:-

$$P = 5.76 \times Q^{1.815}$$

which passes through the other two known positions, the error involved is $6 - 5.76 = 0.24$, or 4.2 per cent. It is thus clear that of the formulae which involve only two constants the formula

$$P = RQ^n$$

is at least as accurate as

$$P = AQ + BQ^2, \text{ and as the}$$

logarithmic form is the more convenient, in general its use is preferable.

Pressure Volume Graph of the Mine Resistance.

It is convenient to plot the pressure-volume relations for a mine of given resistance, adopting P as ordinates and Q as abscissae. When 2 is the value of the index n , in $P = RQ^n$, the curve is a parabola.

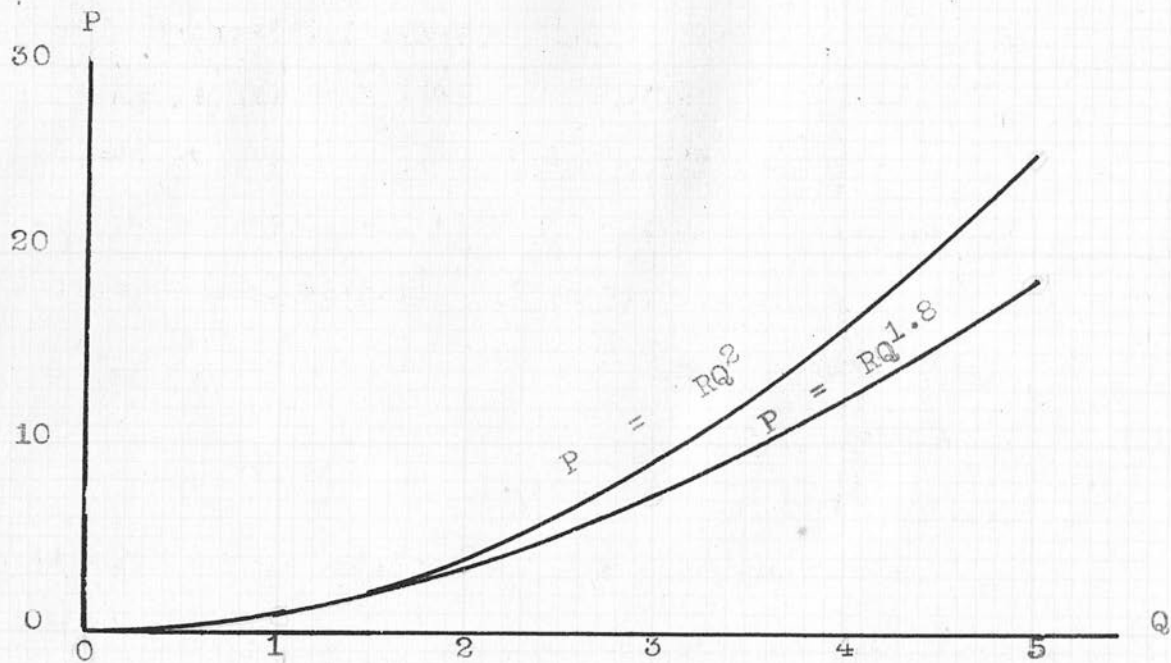


Figure 1. Graph of a Resistance of 1 Atkinson with index values of 2 and 1.8 respectively.

Figure 1 shows the graph for a mine having a resistance of one Atkinson; the upper curve being drawn for $P = RQ^2$ and the lower curve for $P = RQ^{1.8}$. It is seen that a reduction in the value of n has the effect of flattening the curve. The value of P for any given value of Q will be directly proportional to the value of R , and hence the curves of figure 1 may be called "Resistance" curves.

Fan Laws.

Experiments with fans have shown that for all practical purposes the following laws are applicable to fans, viz:-

(1) The depression produced by a fan which operates on a constant resistance is proportional to the square of the speed of the fan; or if P = the depression in pounds per square foot, and X = the number of revolutions of the fan per minute, then

$$P \propto X^2 \quad \dots\dots\dots (13)$$

(2) The volume of air delivered by a fan when working on a mine of constant resistance is proportional to the speed of the fan; or, if Q = the volume of air in cubic feet per minute, then

$$Q \propto X \quad \dots\dots\dots (14)$$

(3) The power required to drive a fan which is working on a constant external resistance varies as the cube of the volume of air passed per minute, and therefore as the cube of the speed of the fan; or if I = the power input to the fan, then

$$I \propto X^3 \quad \dots\dots\dots (15)$$

(4) When a fan is driven at a constant speed on a variable resistance, the depression acting on the

external /

external resistance is a fraction of the total or hypothetical depression produced by the fan. The internal resistance of the fan is a function of the square of the volume of air which is passing, and the depression available for overcoming the external resistance, is the total depression diminished by the depression required to overcome the internal resistance of the fan. If R = the external resistance, and r = the internal resistance arising from friction, eddy currents, etc.; p = the total depression produced by the fan, and P = the depression which acts upon the external resistance, then

$$P = p \cdot R/(R + r) \dots\dots\dots (16)$$

(5) The ratio $P : p$ is the ratio of the pressure usefully applied to the total pressure developed by the fan, and is thus a measure of the efficiency of the fan. Hence,

$$\eta = P/p = R/(R + r) \dots (17)$$

If the external resistance R is constant the ratio $R/(R + r)$ will be constant, and this gives an additional law of great importance, viz.:-

The efficiency of a fan which works on a constant external resistance is given by

$$\eta = R/(R + r), \text{ and is independent of the speed of the fan.}$$

If a fan is run at a constant speed on a series of resistances of descending value, and the volumes passed plotted as abscissae against the corresponding depressions as ordinates, a smooth curve drawn through the points so obtained is found to be a parabola. When the experiments are repeated for a higher constant speed of the fan

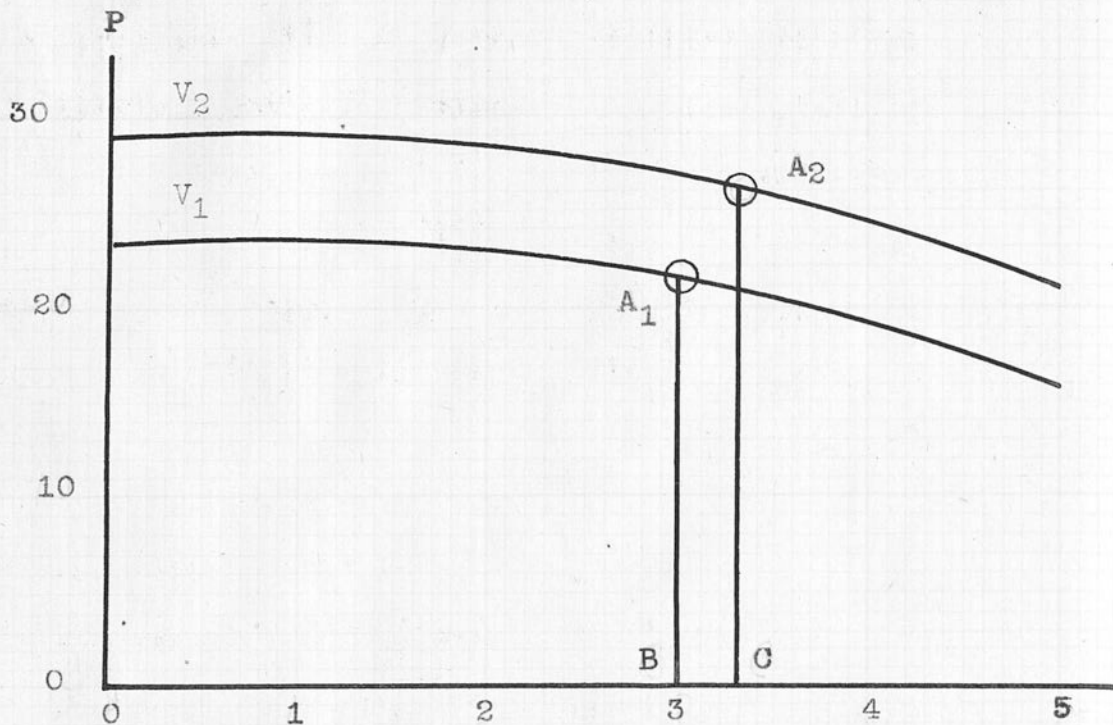


Figure 2. Graph of the pressure-volume relations of a fan for speeds V_1 and V_2 .

A_1 and A_2 are corresponding points.

the curve obtained is a similar parabola. Since for a constant resistance, the volume passed is proportional to the speed of the fan, and the depression to the square of the speed, for every point A_1 (figure 2) on the curve for a speed V_1 , there must exist a point A_2 on the curve for a speed V_2 , such that $OC : OB = V_2 : V_1$ and $A_2C : A_1B = V_2^2 : V_1^2$

These points A_1, A_2 are conveniently defined as "Corresponding Points." Corresponding points occur at the points of intersection of any parabola $P = RQ^2$ drawn through the origin, and the fifth law of a fan is most generally expressed thus:-
 "At all corresponding points on any series of pressure-volume curves of a fan, the fan will operate with the same efficiency."

It is therefore sufficient to plot one set of curves giving the pressure-volume-efficiency relation for a definite speed V_1 of the fan. The efficiency of any point A_2 not lying on the pressure-volume curve, can be at once determined by finding on the curve the point A_1 which corresponds to A_2 .

The Characteristic Curves of a Fan.

If the input, I , to the fan be determined for each set of values of pressure and volume during the tests on a fan, a curve of efficiencies can be plotted from the relation, Efficiency $= \eta = \text{useful work} / \text{total work} = PQ/I \dots\dots\dots (18)$

Such a curve has been drawn in figure 3, for

the /

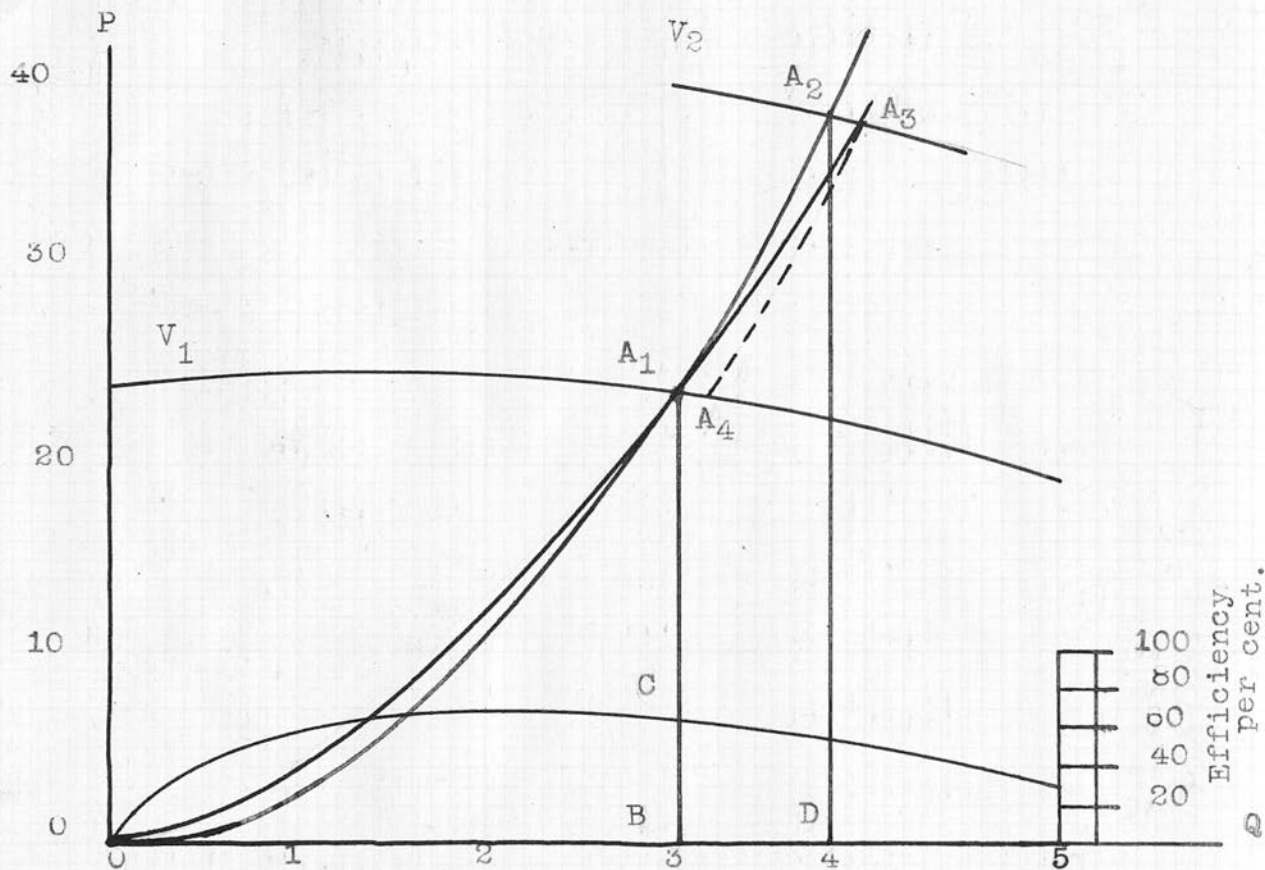


Figure 3. P - Q and Efficiency Graphs showing how efficiency for higher speeds is obtained.

Resistance Curve OA_1A_2 is $P = 2.6 Q^2$

Resistance Curve OA_1A_3 is $P = 4.139 Q^{1.6}$

the pressure-volume curve V_1A_1 obtained for a speed V_1 of the fan. The efficiency for the point A_1 , i.e. when the fan is passing a volume OB at a depression A_1B , is given by CB , the intercept between the volume axis and the efficiency curve on the vertical drawn through A_1 .

If the fan were run at a higher speed V_2 on a constant resistance, and passed a volume OD at a depression A_2D , the efficiency for the point A_2 would be the same as that for the point A_1 and would therefore be given by CB .

If the mine resistance were represented by $P = RQ^n$ where $n = 1.6$, the pressure-volume and resistance curves would intersect in a point A_3 and not in A_2 and the efficiency for the point A_3 would be that of the corresponding point A_4 . It will be seen that, except for great variations in the speed of the fan, or for steep parts of the pressure-volume curve, the error would be comparatively small if the efficiency of the point A_3 were taken as being the same as that for the point A_1 .

Application to the Choice of a Fan.

By means of the characteristic curves of a fan, the limits of the resistance on which the fan would operate with an efficiency of not less than a definite minimum value can be readily determined. Suppose it is desired to choose a fan which will operate with an efficiency of not less than 60 per cent. on a mine having a certain normal resistance, and that this resistance is liable to increase or diminish throughout a certain range of variation; and suppose that a fan having the characteristics

shown /

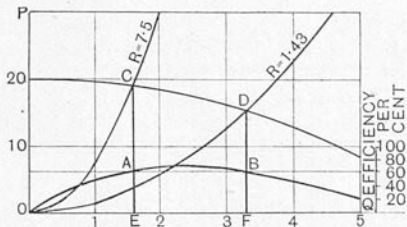


FIG. 4.— SHOWING THE CHARACTERISTIC CURVES OF A FAN, AND HOW TO DETERMINE THE LIMITING VALUES OF THE RESISTANCES WITHIN WHICH THE FAN WILL GIVE AN EFFICIENCY OF NOT LESS THAN 60 PER CENT. THE VALUES FOR THIS CASE SHOW THAT R MUST NOT BE GREATER THAN 7.5 *Atkinsons* NOR LESS THAN 1.43 *Atkinsons*.

shown in figure 4 has been offered. The horizontal line which represents an efficiency of 60 per cent. cuts the efficiency graph of the fan in the points A and B, so that these points mark out the range within which the fan would operate with the desired efficiency. Vertical lines EAC and FBD are drawn through A and B respectively and these cut the pressure-volume curve in the points C and D. If now resistance curves are drawn through the points C and D, these curves mark out the limits of the mine resistance within which the fan will give an efficiency of 60 per cent; and, further, the desired efficiency will be maintained within this range for speeds of the fan, higher or lower than the speed for which the characteristic curves have been drawn.

These resistances are numerically equal to CE/OE^2 and DF/OF^2 units respectively, and if the value of the mine resistance and the variations of that resistance arising from any cause whatever fall within these limiting values, the fan will be suitable for the work to be done and will work with the required efficiency. If not, a fan having the characteristics which satisfy the conditions of the case must be sought.

Variations of the Resistance of the Mine.

Variations of the mine resistance arise from changes (a) in the length and cross-section of the airways traversed by the air currents; (b) in the number of airways in parallel with each other; (c) in the number and distribution of tubs and other obstructions in the airways; (d) in the leakage paths

between intake and return airways, and (e) in the natural ventilating pressure caused by variations of temperature, etc.

The resistance increases with increase in the length of the air passages, when there is no increase in the number of splits in parallel. Increase in the number of parallel splits may balance any increase of resistance due to increase of length, or may even lower the resistance. In cases where new seams are opened out, and where for purposes of ventilation they are directly connected to the shafts, an increase in the number of passages in parallel leading from the shafts will lower the resistance of these passages, and as all the air will not be required to traverse all the length of the shafts the effective resistance of the latter will also be reduced. A careful analysis of the values of the component parts of the resistance and of their probable changes should enable the variation in the resistance of the mine to be determined approximately. In most cases where this analysis has been carefully made and where the general lay-out of the ventilation system has been carefully planned, the variation should not be more than about 30 per cent. above or below the computed value of the normal resistance.

Referring to figure 4 it is seen that a fan having the given characteristics would give a satisfactory efficiency over a range of resistances varying from 7.5 to 1.43 Atkinsons. If such a fan were chosen to work on a mine having a normal resistance of about 4 Atkinsons it would probably give satisfaction for any new conditions that would

be likely to arise.

Effect of Natural Causes.

In deep mines seasonal changes of temperature are likely to produce greater changes in the effective resistance against which the fan operates, than all other causes together. The natural depression may, and generally does, assist the fan to maintain the ventilating currents. It may in some cases oppose the fan. But whether it opposes or assists the fan it always exerts a very important influence upon the conditions of operation of the fan.

If a series of tests are made with a fan in a laboratory, the pressure-volume and efficiency curves secured for the fan are obtained under conditions in which the fan alone operates on a series of diminishing resistances. When a fan operates on a mine it rarely works alone on the mine resistance. The presence of a natural depression or other ventilating agency may either make it more difficult or more easy for the fan to discharge a given volume of air. But, while these agencies alter the effective resistance against which the fan operates, the characteristic curves of the fan can still be applied to determine the output and the efficiency of the fan, for if the actual resistance against the fan can be determined it is immaterial how that resistance has been produced.

The Equivalent Resistance.

It is only in the rare case when a single ventilating appliance operates alone on a mine, that the resistance against which the appliance works is identical with the resistance of the mine. When a

second or third appliance operates the conditions are radically changed, and the resistance, now called the equivalent resistance, must be calculated before any clear view of the situation is possible. The equivalent resistance is not an artificial or subjective quantity, it is the actual or objective resistance against which the appliance operates and is thus of more direct importance than the mine resistance in the study of the fan. The resistance of the mine is indirectly important because it is one of the factors which determine the value of the equivalent resistance.

The Equivalent Resistance for a Fan in Series with a Natural Depression.

In summer, the temperature of the air in the downcast shaft of a mine may be as high as that in the upcast shaft in which case there is no natural depression. The resistance against the fan is then identical with the mine resistance, R , and is such that $R = P/Q^n$, where P is the ventilating pressure produced by the fan, and Q is the volume of air circulated in Kilo-cusecs.

The temperature in the downcast in colder weather may fall considerably below that of the upcast, and give rise to a natural ventilating pressure P_0 . This natural pressure adds itself to the pressure produced by the fan, and to cause the same volume of air to circulate it would be sufficient if the fan created a ventilating pressure $P - P_0$. The natural depression would thus reduce the effective resistance against the fan to the value

$$(P - P_0) / Q^n \dots\dots\dots (19)$$

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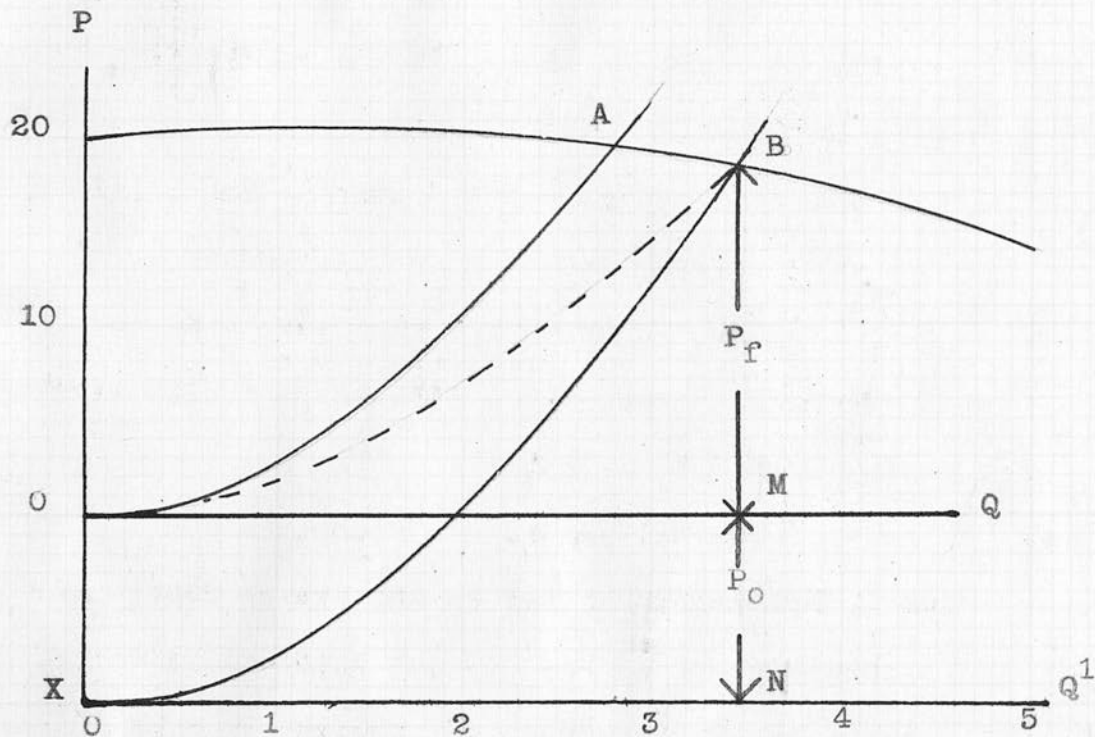


Figure 5. Fan and Natural Pressure in Series.

This may be illustrated graphically as in figure 5, in which the natural depression P_0 is shown plotted below the volume axis OQ of the fan characteristic while the pressure P_f due to the fan extends from the volume axis upwards to the characteristic curve. The mine resistance curve is drawn from the axis XQ^1 , so that it lies along XB and cuts the pressure-volume characteristic of the fan in the point B . If there had been no natural pressure, the resistance curve would have been referred to the axis OQ and would have cut the characteristic curve in the point A .

The effect of the action of the natural pressure so far as the fan is concerned, is the same as if the fan had been put to work on the smaller resistance represented by the dotted curve OB in figure 5, instead of on the mine resistance represented by the curve OA .

The value of the "Equivalent Resistance" for the fan, R_{ef} , is given by

$$R_{ef} = BM/OM^n \dots\dots\dots (20)$$

Similarly the equivalent resistance for the natural agency is given by

$$R_{en} = MN/OM^n \dots\dots\dots (21)$$

Obviously the sum of these two equivalent resistances is equal to the mine resistance or

$$R_{ef} + R_{en} = BN/OM^n \dots (22)$$

When there is a natural ventilating pressure it acts in series with the fan, either assisting or opposing. It is clear that when ventilating appliances are in series each passes the whole volume of air, but shares the ventilating pressure/

pressure in direct proportion to the useful work it performs. The total work done is

$$PQ = Q(P_1 + P_2 + P_3 + \dots) \dots\dots (23)$$

The relations expressed in 20, 21, and 22 thus apply generally to ventilating appliances in series with each other.

Equivalent Resistance for Appliances in Series.

The equivalent resistance R_{e1} for a fan F_1 which operates in series on a mine having a resistance of R Atkinsons, and which develops $1/N$ th of the total ventilating pressure P , and passes a total volume of air of 1 Kilo-cusec, is given by

$$R_{e1} = (P/N)/Q^n = (1/N) \times (P/Q^n) = R/N \dots (24)$$

Equivalent Resistances for Appliances in Parallel.

Fans in parallel must each give the full ventilating pressure but divide the volume in proportion to the useful work done by each. The total work done is

$$PQ = P(Q_1 + Q_2 + Q_3 + \dots) \dots\dots\dots (25)$$

The equivalent resistance R_{e1} for the fan F_1 which operates in parallel with other ventilating appliances on a mine having a resistance of R Atkinsons, and which passes $1/N$ th of the total volume of air Q , at a ventilating pressure P , is

$$R_e = P/(Q/N)^n \quad \text{or} \quad \frac{P}{N^n R} \dots\dots (26)$$

Equivalent Resistance for a Series Parallel Arrangement.

In the case of a series-parallel arrangement as when two fans F_1 and F_2 operate on a mine having a resistance of R Atkinsons, and in which there is a natural ventilating pressure P_0 , the equivalent resistance for the combination of fans, by (19), is

$$R_e = (P - P_0) / Q^n.$$

If /

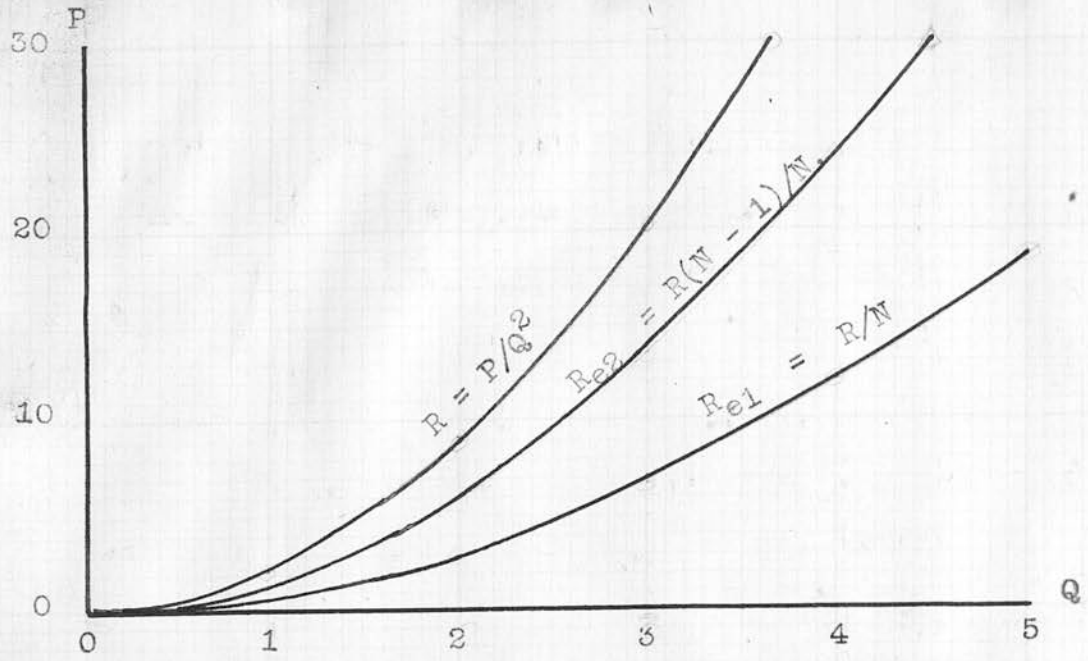


Figure 6. Graphs of the mine resistance, $R = P/Q^2$, and of the equivalent resistances for two fans, F_1 and F_2 in series. F_1 develops 1/3rd and F_2 develops 2/3rds of the total pressure, P .

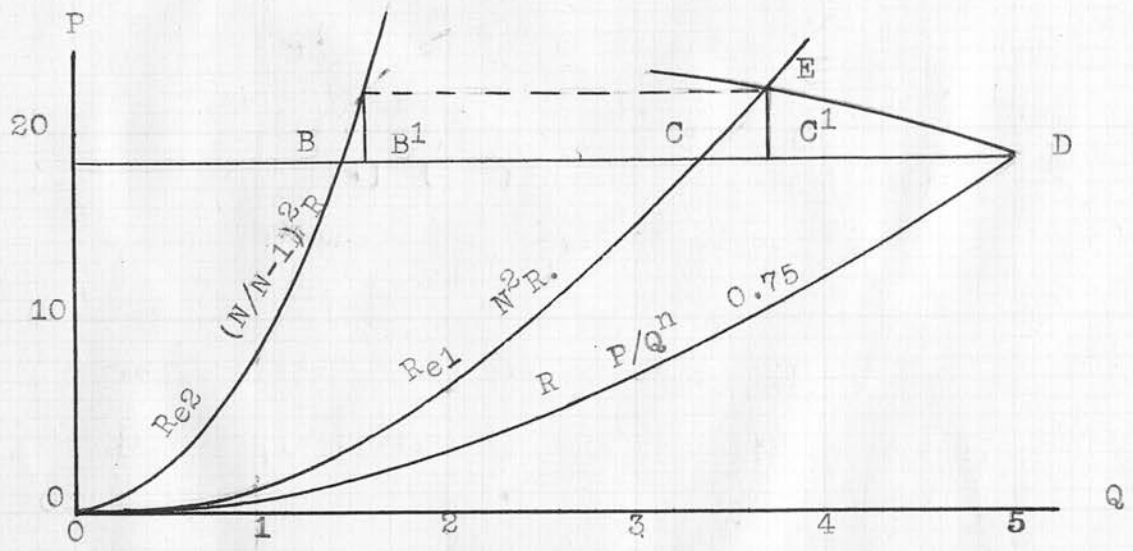


Figure 7. Showing the equivalent resistance and the mine resistance for a parallel arrangement.

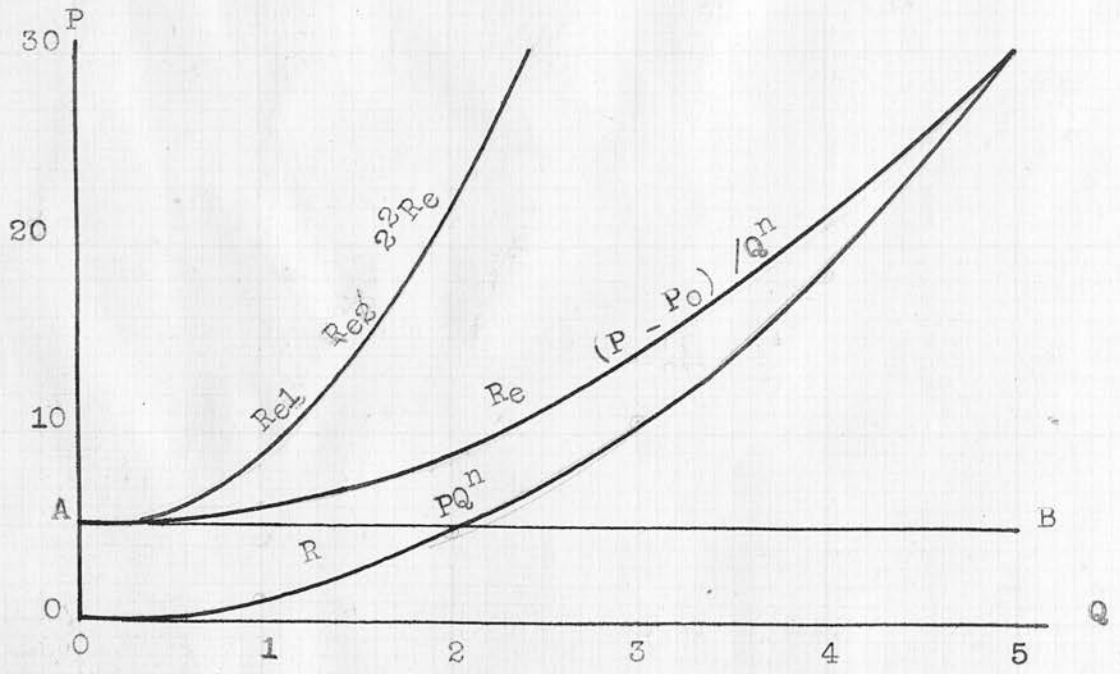


Figure 8. Graph showing the equivalent resistance against two parallel fans which share the volume equally and are in series with a natural pressure of 5 pounds per square ins.

If the fan F_1 passes $1/N$ th of the total volume of air Q , and thus does $1/N$ th of the total useful work, the equivalent resistance R_{e1} for the fan F_1 is

$$R_{e1} = N^n R_e \quad \text{or} \quad (N/Q)^n (P - P_0) \quad \dots (27)$$

The fan F_2 would pass a volume $Q - Q/N$, or $Q(N - 1) / N$, and the equivalent resistance R_{e2} for F_2 will be

$$R_{e2} = \left[\frac{N}{(N - 1)Q} \right]^n (P - P_0) \quad \dots (28)$$

These relations are shown graphically in figures 6, 7, and 8. Figure 6 shows the equivalent resistance against two fans in series, where F_1 develops $1/3$ and F_2 develops $2/3$ of the total ventilating pressure. The sum of the two series resistances is equal to the mine resistance, or $R_{e1} + R_{e2} = R$.

Figure 7 shows the effect of the addition of a fan F_2 in parallel with another fan F_1 . When F_1 acts alone on the mine resistance its operating point for a volume of 5 Kilo-cusecs is the point D on the resistance curve. When F_2 also operates and passes a volume AB, equal to 1.465 Kilo-cusecs the operating point of F_1 which passes a volume of AC equal to 3.535 Kilo-cusecs, goes along to C. The relation between the equivalent resistances and the mine resistance is expressed by the rule for the addition of parallel resistances, viz.,

$$\sqrt{1/R_{e1}} + \sqrt{1/R_{e2}} = \sqrt{1/R} \quad \dots (29)$$

Figure 8 shows the graph of the equivalent resistance against two parallel operating fans when sharing a volume of 5 Kilo-cusecs equally and working in series with a natural ventilating pressure, P_0 .

Experimental Determination of the Characteristic Curves of a Fan.

The characteristic curves are obtained from tests made on the fan. In these tests the resistance against which the fan works, is varied, and for each value of the resistance the volume of air passed, the ventilating pressure, and the input to the fan, must be carefully determined.

The difficulty of determining the true value of the ventilating pressure arises immediately a serious attempt is made to measure it. The velocity of the air varies considerably over the cross-section of the fan drift, and the depression varies with the velocity. To obtain the dynamic depression the "velocity" head must be subtracted and this is done automatically by pointing the tube leading to the manometer against the direction of the air current. The result obtained, however, depends upon the position in the drift where the tube is placed, and to secure reliable results a position must be selected which will give the mean value of the depression.

If the relative velocity of the air at different planes be considered as equivalent to the production of a shear the velocity would increase from the periphery toward the centre in accordance with a straight line law. Would such a variation in velocity be in accordance with the results obtained from most carefully conducted experiments, and if so what are the positions of the mean velocity and of the mean dynamic water-gauge ?

Very elaborate experiments were made in 1884 at the Breslau Gas Works by the Prussian Mining Commission /

Commission.* In these tests a spare gas holder 85.3 feet diameter and containing 70,634 cubic feet was used and the rate of fall of the gas holder was determined electrically and served as a check on the various methods of measurement of the air velocity in a pipe 14.3 inches diameter. It was found that there was a considerable difference of speed at different parts in the same vertical plane. The centre gave the maximum speed and pressure and the circumference the minimum; the mean speed was found to occur at $2/3$ of the radius from the centre of the pipe. The Casella anemometer was found to give variable errors, the excess ranging between 7 per cent. and 13 per cent.

Messrs. Stanton and Pannell have shown that the mean velocity divided by the velocity at the centre varies but little and is approximately equal to 0.82.

Consider the case of a linear distribution of velocity over a circular airway.

Let R = the radius of the duct

r = the distances measured from the centre of the duct

v = the velocity of the air at points distant r from the centre of the duct

KR = the velocity of the air at the centre of the duct

n = a constant to be determined from Messrs. Stanton and Pannell's results.

\bar{v} = the mean velocity of the air

Q = the volume of air discharged from the duct.

Assume /

*

Assume $v = K(R - r/n) = C - kr$

where $C = KR$ and $k = K/n$

Then

$$Q = K \int_0^R 2\pi r(R - r/n) dr$$

$$Q = K\pi \left[R^3 - (2R^3/3n) \right]$$

$$Q = \pi R^2 \times K \left\{ R - (2R/3n) \right\}$$

or $Q = \text{area of duct} \times \text{velocity at the point where the mean velocity occurs.}$

The assumption that $v = K(R - r/n)$ or $C - kr$, is thus found to be consistent with the results obtained by the Prussian Mining Commission, viz., that the mean velocity occurs at $2/3$ of the radius from the centre of the duct.

The velocity at the periphery is

$K(R - R/n)$ or $KR(n - 1)/n$, i.e., it is equal to the velocity at the centre multiplied by $(n - 1)/n$.

Messrs Stanton and Pannell's results give mean velocity / velocity at centre = $0.82/1$ or $(1 - 0.18) / 1$. But since the drop in the velocity is taken as proportional to the distance from the centre, the drop at the periphery will be one-half more than that at the point where the mean velocity occurs, i.e., the drop will be 0.27 . Thus the velocity at the periphery / the velocity at the centre is $0.73/1 = (n - 1)/n$, whence $n = 3.7$. The distribution of velocity across the vertical is therefore conical and the distribution of volume is a paraboloid.

To determine the position where the root mean square value occurs.

Let /

Let \bar{r} be the position where the root mean square value, \bar{v} , occurs, so that $\bar{v} = C - k\bar{r}$

$$\text{Then } \bar{v}^2 = \frac{\int_0^R 2\pi r v^2 dr}{\pi R^2} = \frac{2}{R^2} \int_0^R v^2 r dr$$

But $v = C - kr$, so that $v^2 = C^2 + k^2 r^2 - 2kCr$

Thus

$$\bar{v}^2 = (2/R^2) \int_0^R (C^2 r + k^2 r^3 - 2kCr^2) dr$$

$$\text{i.e. } C^2 + k^2 \bar{r}^2 - 2kC\bar{r} = C^2 + (k^2 R^2/2) - (4kCR/3)$$

or

$$k^2 \bar{r}^2 - 2kC\bar{r} + (4kCR/3) - (k^2 R^2/2) = 0.$$

Therefore

$$\bar{r} = (C/k) - \sqrt{(C^2/k^2) + (R^2/2) - (4CR/3k)}$$

$$\bar{r} = (KR/K/n) - \sqrt{(K^2 R^2 / K^2 / n^2) + (R^2/2) - (4KR^2/K/n)}$$

$$\bar{r} = Rn - R \sqrt{n^2 + \frac{1}{2} - (4n/3)}$$

Substituting the value for n found above gives

$$\bar{r} = 0.66 R.$$

The position where the mean square value of the velocity occurs is thus the same as that where the mean velocity occurs. This is obviously the case where a linear distribution of velocity from the centre outwards, for while the velocity falls off in proportion to the distance from the centre, the area of the elementary ring over which any definite velocity acts increases in proportion to the radius.

The mean velocity and the mean dynamic water-gauge can therefore be read off at points two-thirds along the radius from the centre or at points distant $D/6$ from the sides of the airway.

Tests to Obtain the Characteristic Curves of a Sirocco Fan.

The results of a series of tests made on

TABLE II.—TESTS MADE WITH AN 18-INCH SIROCCO FAN AT THE FIFE MINING SCHOOL, COWDENBEATH, ON AUGUST 16TH, 1922.
 Note.—The tests shown bracketed together were made with the same resistance in the airway. The area where the velocity of the air was measured was 20 × 21 inches, or 2·92 square feet.

Number of test.	Motor input, in watts.	Depression of the water-gauge, in inches.		Speed of the fan, in revolutions per minute.	Velocity of the air, in feet per minute.	Volume of air passed, in cubic feet per minute.	Average value of the depressions obtained by Pitot and muffled tubes, in inches of water-gauge.	Average depression corrected for a speed of fan of 1,000 revolutions per minute.		Average value of each set of depression, in square foot.	Volume corrected for a speed of 1,000 revolutions in cubic feet per minute.	Average of corrected volumes for each set of tests, in cubic feet per minute.	Useful work in the air, in foot-pounds per second.	Watts input corrected for a speed of 1,000 revolutions per minute, in watts.	Average input for each set of tests for a speed of 1,000 revolutions per minute, in watts.	Combined efficiency of the motor fan and motor.	Efficiency of fan alone, assuming the efficiency of the motor to be 80 per cent.
		Muffled tube.	Pitot tube.					Water-gauge, in inches.	Pounds per square foot.								
1	615	2·715	—	1,070	0	0	2·7150	2·37	12·32	12·32	0	0	0	502	0	0	
2	726	2·775	2·770	1,092	284	830	2·7725	2·32	12·08	12·08	760	760	153	556	37·2	46·5	
{ 3 4 5 }	794	2·440	2·485	1,045	470	1,360	2·4625	2·26	11·72	11·72	1,310	1,310	242	696	689	47·7	59·6
	642	2·075	2·013	965	405	1,180	2·0440	2·16	11·42	11·42	1,222	1,222	242	712			
	590	2·025	2·080	963	417	1,215	2·0525	2·18	11·32	11·32	1,262	1,262	242	660			
{ 6 7 8 }	1,015	2·305	2·255	1,035	655	1,900	2·2800	2·11	10·95	10·95	1,835	1,835	349	916	939	50·4	63·0
	920	2·255	2·255	990	578	1,690	2·2375	2·29	11·90	11·90	1,708	1,708	349	948			
	910	2·195	2·235	985	578	1,690	2·2150	2·29	11·90	11·90	1,716	1,716	349	954			
{ 9 10 11 }	1,150	1·855	1·890	903	1,140	3,300	1·8725	2·31	12·01	12·01	3,680	3,680	667	1,560	1,568	57·8	72·2
	1,278	1·960	1·900	933	1,041	3,035	1·9300	2·22	11·53	11·50	3,290	3,290	667	1,570			
	1,275	1·955	2·005	932	1,013	2,962	1·9800	2·28	11·85	11·80	3,180	3,180	667	1,575			
{ 12 13 14 }	1,404	1·525	1·515	888	1,333	3,890	1·5200	1·93	10·03	10·03	4,380	4,380	743	2,028	2,020	49·7	62·1
	1,485	1·585	1·570	904	1,407	4,108	1·5775	1·93	10·03	10·04	4,550	4,550	743	2,015			
	1,632	1·675	1·690	933	1,408	4,110	1·6825	1·935	10·06	10·04	4,400	4,400	743	2,018			
{ 15 16 17 18 }	1,647	0·240	0·160	740	2,004	5,840	0·2000	0·366	1·91	1·91	7,900	7,900	218	4,065	3,992	7·42	0·9
	1,750	0·140	0·145	755	2,076	6,050	0·1425	0·250	1·30	1·65	8,010	8,010	218	4,065			
	1,720	0·230	0·140	760	1,933	5,650	0·1850	0·322	1·67	1·65	7,430	7,430	218	3,920			
1,720	0·235	0·145	760	2,205	6,440	0·1900	0·329	1·71	1·71	8,450	8,450	218	3,920				

an 18-inch diameter Sirocco fan are given in table II. In making these tests the mean dynamic water-gauge was read off from a U tube inclined at an angle $\tan^{-1} \theta = 0.1$, and a Pitot tube facing the air current and placed at a point $D/6$ from the side of the airway was connected to one limb. A tube filled with cotton wool and having a piece of flannel wrapped over the end was also placed at $D/6$ from the side and this tube could be put into communication with the U tube by turning a cock. In the table this tube is referred to as a muffled tube and the readings obtained from it agreed very closely with those obtained from the Pitot tube.

The resistance against the fan was varied from a large value to a very small value and for each resistance a series of three or four values of all the quantities to be determined was taken, and the mean values of each series were used in calculating the results. In the case of the depression the average values of the readings of the Pitot and muffled tubes were taken. In calculating the final results the average values of each series were corrected for a standard fan speed of 1,000 R.P.M.

The fan was driven by ^a/direct current electric motor coupled directly and the input to the motor was carefully determined. The total pressure, p , multiplied by the volume, Q , gives the total work done by the fan and this must be equal to the actual input to the fan. Thus $pQ = I$, or $p = I/Q$.

The results have been plotted in figure 9, in which test points on the curve of useful pressures P_u are marked by small circles. Points on the

input /

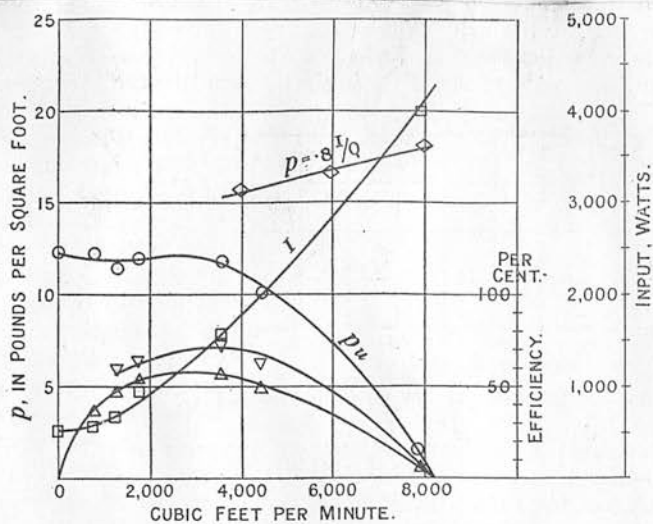


FIG. 9.—RESULTS OF TESTS ON AN 18-INCH SIROCCO FAN AT THE FIFE MINING SCHOOL, COWDENBEATH, FIFE.

TABLE III.—RESULTS OF MEASUREMENTS OF AIR AT THE MOUTH OF THE ÉVASÉE OF AN 18-INCH SIROCCO FAN.

No. of square.	Velocity of air, in feet per minute.	No. of square.	Velocity of air, in feet per minute.
1	+515	7	+2,582
2	-477	8	+2,791
3	-581	9	+2,453
4	+1,153		<hr/>
5	+1,068		10,938
6	+1,434		<hr/>
		Average velocity ...	1,215

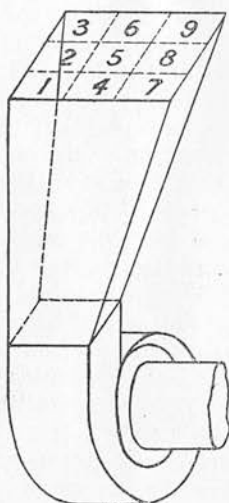


FIG. 10.—ARRANGEMENT OF THE ÉVASÉE OF THE FAN IN THE TESTS.

input graph I, are marked by squares. Test points on the line of total pressure, p , have been calculated on the assumption that the efficiency of the motor was 80 per cent. The lower curve marked with Δ points gives the efficiency of the fan and motor; and the curve marked by ∇ points gives the efficiency of the fan alone.

The curve of input is concave upwards and this form is characteristic of fans which have vanes with forward curvature. Where the curvature of the vanes is backwards the input curve is concave downwards, and for radial vanes the input is shown by a straight line.

Experiments with the Fan Évasée.

Where the évasée of a fan diverges too quickly serious eddying of the air may be produced. The Sirocco fan at the Fife Mining School has been found to throw its air sideways against the direction of entry of the air to the fan. An évasée was constructed for this fan as sketched in figure 10. One side was vertical and the other side was inclined at an angle of 14 degrees in the direction in which the fan discharged most air. The results obtained when an anemometer was held successively in each of the squares marked are shown in Table III. A negative velocity indicates that the air was passing inwards at the corresponding square. The eddy currents were more pronounced when an évasée was used having a symmetrical taper about the fan outlet, and also when the angle of divergence was greater than 14 degrees. The experiments indicated that even 14 degrees was too great an angle of divergence.

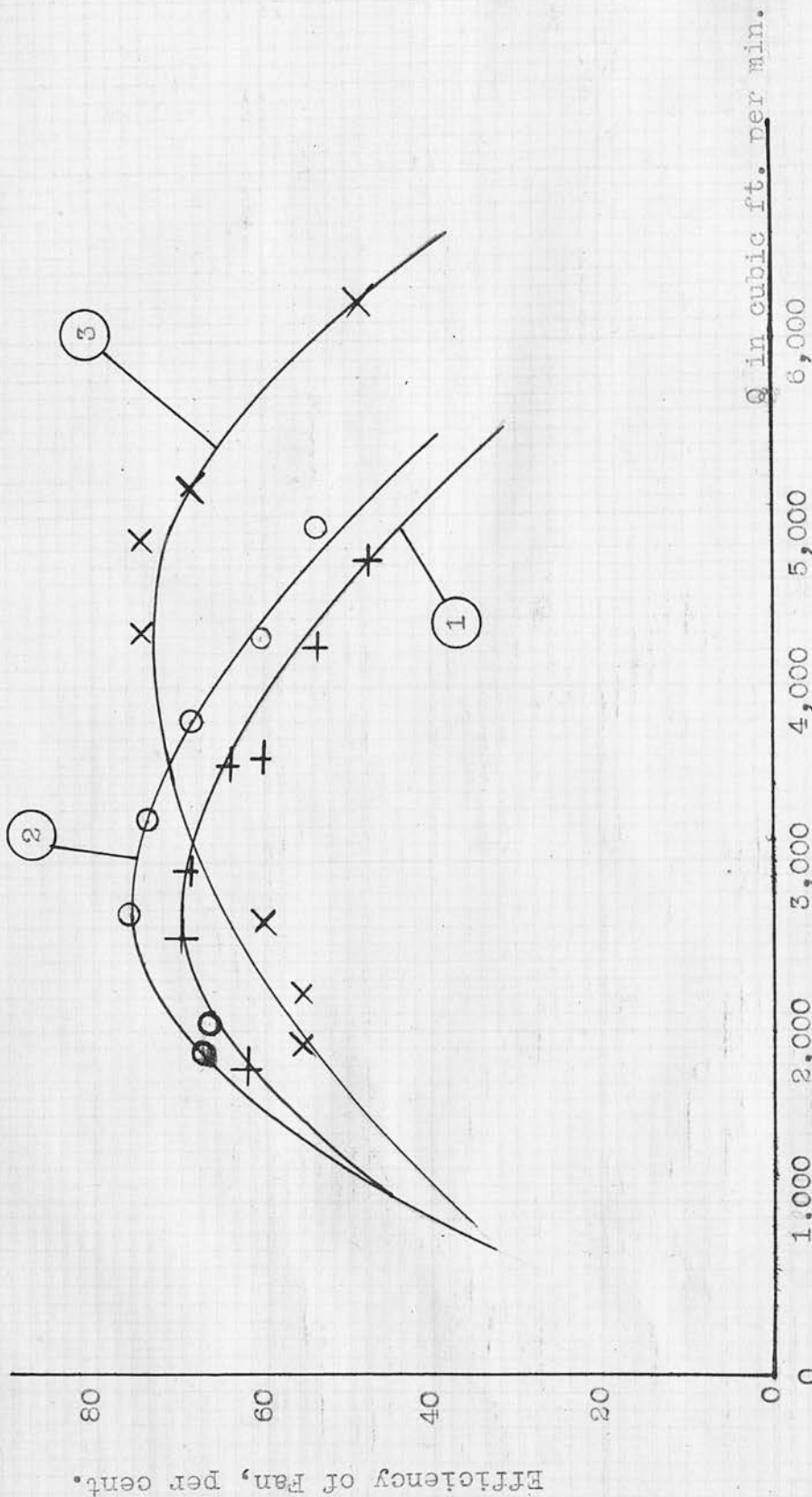


Fig. 11. Efficiency Curves of Ordnance Fan.

- (1) Without induction tube or well-defined start of evolute.
- (2) With induction tube but no well-defined start of evolute.
- (3) With induction tube and well-defined start of evolute.

TABLE IV.

Tests with Ordnance Fan Runner in a Sirocco Fan Casing.

No. of Test	Motor Input. Wats.	Inches Pitot Tube	of W.G. Muffled Tube.	Speed of Fan R.P.M.	Volume of air in cub. ft./min.	Average W.G.	Average W.G. corrected to 1,000 R.P.M.	Press: ure in pds. per sq. inch.	Volume correc: ted for 1,000 R.P.M.	Useful work in the air in ft. lbs. per sec.	Watts Input correc: ted to speed of 1,000 R.P.M.	Combined Efficiency	Efficiency of Fan.
1.	998	2.420	2.457	1018	1770	2.428	2.340	12.18	1739	353	945	50.7	63.4
2.	1174	2.240	2.257	990	2510	2.246	2.292	11.92	2535	504	1210	56.5	70.6
3.	1326	2.130	2.170	978	2903	2.150	2.248	11.68	2970	578	1418	55.3	69.1
4.	1445	1.870	1.923	952	3380	1.896	2.090	10.88	3550	644	1676	52.1	65.1
5.	1622	1.513	1.573	917	3867	1.543	1.835	9.55	4220	671	2105	43.3	54.1
6.	1758	1.340	1.417	912	4318	1.378	1.657	8.62	4730	680	2318	39.8	49.8

TABLE V.

Tests with Ordnance Fan Runner in Sirocco Casing with Induction Tube Fitted.

No. of Test	Motor Input Watts.	W.G. Inches		Speed of Fan R.P.M.	Volume of air in c.ft. per min.	Average of W.G. readings Ins.	Average W.G. correct: :ed to 1,000 R.P.M.	Pressure in pounds per square inch.	Volume correct: :ed for 1,000 R.P.M. c.ft./min.	Use: :ful work in the air in ft. lbs per sec.	Input corr: :ected to 1,000 R.P.M	Comb. Effic: :ency	Effic. of Fan.
		Pitot Tube	Muffled Tube										
1.	938	2.358	2.360	994	1857	2.359	2.388	12.42	1868	387	955	54.9	68.6
2.	1020	2.350	2.363	990	1995	2.356	2.402	12.50	2016	420	1052	54.2	67.8
3.	1147	2.297	2.330	980	2555	2.313	2.410	12.53	2610	546	1218	60.9	76.1
4.	1320	2.137	2.220	957	3064	2.178	2.378	12.36	3204	661	1505	59.6	74.5
5.	1454	1.913	1.967	935	3525	1.940	2.320	11.54	3770	726	1780	55.3	69.1
6.	1598	1.673	1.747	915	3920	1.710	2.043	10.64	4280	758	2087	49.3	61.6
7.	1736	1.427	1.513	889	4370	1.470	1.860	9.67	4920	793	2470	43.5	54.5

TABLE VI.

Tests with Ordnance Fan in Sirocco Casing with Induction Tube Fitted and Evolute Separated
From the Casing Outlet.

No. of Test.	Motor Input Watts.	W. G. Inches Pitot Tube	Speed of Fan.	Volume of air in cub. ft. per min.	W. G. Corr: :ected for 1,000 R.P.M.	Pressure in pds. per square foot.	Volume :ted for 1,000 R.P.M.	Useful work in the air ft. lbs. per sec.	Watts Input :ted for 1,000 R.P.M.	Combined Efficiency: :cy.	Fan Efficiency.
1.	1030	2.137	972	1840	2.260	11.75	1893	371	1122	44.8	56.0
2.	1160	2.077	966	2139	2.223	11.57	2210	426	1286	45.0	56.3
3.	1238	2.010	965	2542	2.160	11.22	2635	493	1378	48.5	60.6
4.	1362	1.870	952	3402	2.065	10.74	3575	640	1580	55.0	61.3
5.	1495	1.823	946	4175	2.040	10.60	4410	779	1765	59.8	74.8
6.	1562	1.740	937	4580	1.980	10.30	4880	838	1900	59.8	74.8
7.	1788	1.697	954	4920	1.860	9.70	5160	835	2060	55.0	68.8
8.	2010	1.167	926	5620	1.366	7.07	6076	716	2533	38.4	48

Tests with an Ordnance Fan 18-Inches Diameter.

In these tests the runner of the Sirocco fan was replaced by an Ordnance fan runner, which differs from the Sirocco fan mainly in having corrugated vanes. The tests were made in a manner similar to those carried out with the Sirocco fan. Three series of tests were made. In the first series the casing was exactly as in the tests with the Sirocco fan. The second series was made to ascertain what effects would follow when a parallel sided induction pipe 19-inches long was inserted between the casing outlet and the evasee. The third series was made to determine the effect of separating the start of the casing evolute from the outlet by the insertion of a V-shaped piece of wood in the space above the runner where the evolute freely communicated with the outlet. In these tests the induction pipe was retained.

The results of the three series are given in Tables IV., V., and VI. respectively, and they are shown plotted in figure 11. It will be seen that the induction tube improved the efficiency, but that when the evolute was cut off from the outlet the efficiency was reduced unless when the fan was working on a very low resistance and passing a large volume of air. The points obtained in the third series of tests were also found to be very erratic.

Mathematical Expression for the Efficiency of a Fan.

Practical purposes would be materially helped if a method could be evolved which would enable the characteristics of a fan to be determined with a high degree of accuracy and with the minimum

of trouble and expense. The following considerations indicate a method of doing this.

Murgue has shown that the theoretical motive column H , in feet, for a fan running with a peripheral velocity of V feet per second is given by the formula:-

$$H = (V^2/g) - (VV_2 \cos \theta) / g - W^2 / 2g; \quad \text{where}$$

θ = the angle between the vanes and the backward extension of the tangent where the vanes meet the periphery.

W = the velocity in feet per second of the air leaving the *évasée*.

V_2 = the velocity of the air passing through the fan relative to the vanes of the fan.

The second term of this expression supplied the correction for variation in the angle between the vanes and the periphery in different fans. When $\theta = 90^\circ$, this term vanishes; and when, as in the case of the Sirocco fan, $\theta > 90^\circ$, this term changes its sign and becomes positive. The third term makes the correction for the kinetic energy carried away into the atmosphere by the air escaping from the *évasée*.

It is more convenient if the expression be made in terms of V_r , the radial velocity of the air, when the formula becomes:-

$$H = (V^2/g) - (VV_r \cot \theta) / g - W^2 / 2g \quad \dots (30)$$

Beside losses arising from kinetic energy carried into the atmosphere there are frictional and eddy current losses. These losses may be assumed to be proportional to the square of the volume of air passed by the fan, and if p_i is the loss of pressure caused ^{by}, and r is the internal resistance of the fan, then $p_i = rQ^2$.

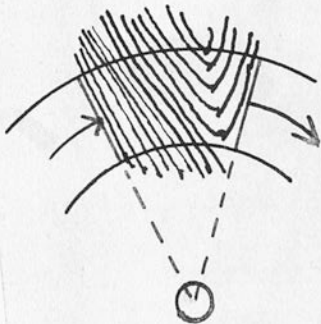


Fig. 12.

Murgue's formula for the motive column of a fan assumes that all the air particles expelled from the fan attain the same velocity as the particles in immediate contact with the vane tips. This obviously is not the case. Immediately in front of a moving vane the air streams moving radially relative to the vanes are more highly compressed and attain a higher tangential velocity than streams farther away. Indeed, behind the vanes there may even be a re-entry of air with a reversal of radial and tangential velocities. (Figure 12)

The total actual motive column H_t produced by the fan is therefore equal to KH , where K is a fraction called by Murgue the "manometrical yield."

The total ventilating pressure p , in pounds per square foot equals wH_t where w is the weight of a cubic foot of the air. The useful pressure which overcomes the resistance of the mine is the total pressure diminished by the pressure p_1 spent in overcoming the internal resistance of the fan.

Hence, adopting P for the useful pressure in pounds per square foot, Murgue's formula may be written:-

$$P = (KwV/g) \times (V - V_r \cot \theta) - rQ^2 \dots \dots \dots (31)$$

Important Expression giving the Efficiency of a Fan.

The efficiency of a fan is the ratio of the useful to the total pressure of the fan, or $\eta = P/p$.

In the above expression for P the quantity V_r , the radial velocity of the air in the fan is equal to the volume in cubic feet discharged per second divided by the surface perimeter, a , of the fan, or $V_r = Q/a$.

Substituting this value for V_r in (31) gives

$$P = (KwV/g) \times (V - Q \cot \theta / a) - rQ^2 \dots \dots \dots (32)$$

To apply this formula to a fan, K and r must be determined by experiment. Since there are two unknowns two sets of relations between P and Q are necessary for this purpose. The values of θ and a are obtained by measurement; V is determined from the diameter of the fan and its rotational speed.

Rapid Method of Determining the Characteristics of a Fan

These considerations show that it is possible to determine the characteristic curves of a fan by running it at a constant speed on two different external resistances, and making careful measurements of the volume of air passed and of the ventilating pressure for each of the resistances. In order to smooth out small errors in the readings taken, it is desirable to make at least three determinations of the pressure and volume for each case. It is not necessary to hold the speed exactly constant for each of these three determinations, but the speed must be measured as well as the pressure and volume, and the volume and pressure corrected to the selected speed. The mean of the corrected values thus obtained should be taken; in this way accurate results will be obtained. In carrying out the tests the fan would first be run on the mine resistance and afterwards on a smaller resistance obtained by short-circuiting some air between the two shafts. An alternative would be to increase the resistance by erecting an obstruction such as a regulator.

The following example explains the procedure.

Example:- A fan having a peripheral outlet of 70 square feet and fitted with vanes having a backward curvature of 45 degrees, when run at a peripheral speed of 120 feet per second on a mine resistance was found to pass 140,000 cubic feet of air per minute with a water-gauge of 2.98 inches. The resistance was then increased by inserting /

inserting a regulator in the main airway, and when running at the same speed the fan passed 120,000 cubic feet of air per minute with a water-gauge of 3.383 inches.

Determine the values of K and r for the fan, and plot the characteristic curves.

The known data is:-

$$P_1 = 2.98 \text{ inches W.G., or } 15.50 \text{ pds. per sq. ft.}$$

$$P_2 = 3.383 \text{ inches W.G., or } 17.59 \text{ pds. per sq. ft.}$$

$$Q_1 = 2,333 \text{ cubic feet per second.}$$

$$Q_2 = 2,000 \text{ cubic feet per second.}$$

w = weight of 1 cubic foot of air, or 0.075 lb. (say)

$$P = KwV/g(V - Q \cot \theta/a) - rQ^2$$

Substituting in this equation the two sets of values obtained from the experiments gives:-

$$15.50 = (K \times .075 \times 120/32.2) (120 - 2,333 \times 70) - r \times (2,333)^2.$$

$$15.50 = K \times 24.223 - r (49/9) 10^6 \dots\dots\dots (a)$$

$$17.59 = K \times .075 \times 120/32.2 (120 - 2,000 \times 70) - r \times (2,000)^2$$

$$17.59 = K \times 25.554 - r \times 4 \times 10^6 \dots\dots\dots (b)$$

Multiplying (a) by 36 and (b) by 49 and solving gives

$$K = 0.8 \quad \text{and} \quad r = 0.713 \times 10^{-6}$$

But by definition, resistance $R = P/Q^2$ and the resistance is 1 Atkinson when a volume of 1,000 cubic feet of air per second passes with a pressure difference of 1 pound per square foot. Thus 1 Atkinson = $1/(1,000)^2 = 10^{-6}$ foot-pound-second unit.

The value of r as found is thus equal to 0.713 Atkinson. Referring to equations (a) and (b) it is clear that the first term on the right hand of

each /

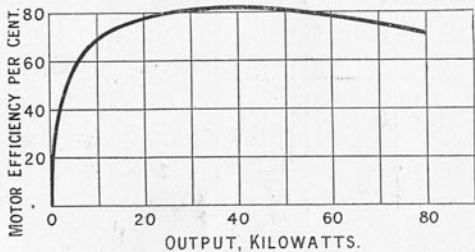


FIG. 13. - MOTOR-EFFICIENCY GRAPH.

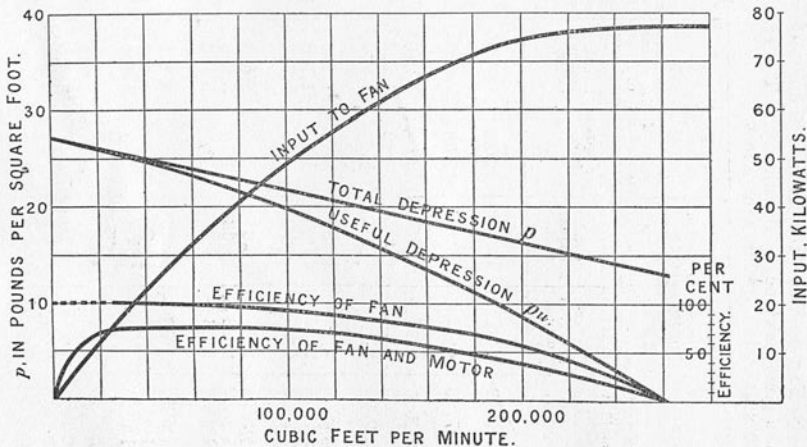


FIG. 14. - GRAPHS OF THE RESULTS OBTAINED BY THE RAPID METHOD OF TESTING A FAN.

each equation, viz., 24.223K and 25.554K represents the theoretical or total depressions for the respective conditions. Substituting the value of K found, these quantities are found to be 19.38 and 20.44 pounds per square foot respectively.

The expression giving the total depression of a fan is $p = KwV/g(V - Q \cot \theta /a)$ and it is clear that when a fan is run at a constant speed on a variable resistance, the only quantity on the right hand of this equation that can vary is Q, and since Q occurs only in the first degree, the graph of the total depression is a straight line. If θ is 90° the graph is horizontal; it dips as the volume is increased when θ is less than 90° and rises for values of θ greater than 90° .

But since the graph of the total depression is a straight line, and two points on this line are known, the graph can be immediately drawn.

To complete the work it is necessary to calculate the value of the term pQ^2 for several values of Q and to subtract these from the corresponding values of the total depression. The results are given in Table VII. The last row of figures gives the efficiency of the fan alone. In practice the very large efficiencies indicated for small volumes delivered by the fan would not be secured since under these conditions considerable losses would arise from eddy currents and re-entry of air to the fan. But over the range of practical interest, the results would be very servicable. If the efficiency curve of the motor which drives the fan is as shown in Figure 13, the characteristic curves of the fan will be as plotted in Figure 14.

TABLE VII.

Values of Q , rq^2 , P , P and n .

Q	$.6 \times 10^3$	$2 \times .6 \times 10^3$	$3 \times .6 \times 10^3$	$4 \times .6 \times 10^3$	$5 \times .6 \times 10^3$	$6 \times .6 \times 10^3$
rq^2	0.3166	1.267	2.851	5.069	7.92	11.405
P	24.8	22.7	20.5	18.4	16.3	14.0
P	24.48	21.43	17.65	13.33	8.38	2.595
per cent.	98	94.4	86	72.4	51.4	18.5

The importance of the method described lies in the fact that it would give a clear view of the performance of a fan in cases where it is inconvenient to measure the input to the fan motor and to carry out a full series of observations of the values of the volumes and depressions given by the fan on a fairly long series of resistances.

Should Regulators be put in Fan Drifts ?

Attempts have sometimes been made to improve the efficiency of a fan by putting a regulator in the fan drift. Is this practice advisable ? In the case just considered the insertion of a regulator increased the resistance so that instead of 140,000 cubic feet of air passing per minute with a pressure of 15.5 pounds per square foot, only 120,000 cubic feet passed with a pressure of 17.6 pounds per square foot. The power inputs to the fan are thus found to be 61.5 and 55.4 Kilowatts respectively, so that the volume of air has been reduced to 85.5 per cent and the power input to 90 per cent of their former values. If this reduced volume of air is sufficient for the requirements of the mine, the insertion of a regulator would thus effect a saving of 10 per cent in the power bill.

The same results, however, could have been obtained by reducing the speed of the fan from 120 to 103 feet per second, so as to give a depression of 11.4 pounds per square foot, when the power input would have been approximately $61.5 \times (103/120)^3$ or 39 Kilowatts. Thus with a speed regulator the expenditure of power is only 63.5 per cent of its former value, whereas, with the resistance regulator method, it is 90 per cent.

The initial expense of installing a
regulator /

regulator would be small compared with the cost of a variable speed motor, but if the regulation is likely to obtain for a few years a reduction of 16 Kilowatts on the load, which represents a saving of £300 per annum, would justify the extra cost of such a motor.

Saving Effectuated by Reducing the Volume of Air in Circulation during Idle Hours.

It is worth while enquiring whether the volume of air in circulation could be reduced during idle hours. During the hours of working, the movements of tubs adversely affects the ventilation and there are times when the airways are obstructed by races of full tubs, by empty races conveying timber, etc., and by the presence of men, horses, and, in some cases, locomotives. The movement of the cages in the shafts during winding hours increases the resistance of the shafts, especially when the cages pass each other, and so retards the air current. The frequent opening of doors increases the leakage of air and reduces the volume which passes into the working places. All these causes tend to reduce the volume of air where it is required and at a time when it is most needed, the greater quantity of noxious and inflammable gases being liberated during the hours of coal getting.

If then, conditions are to be as good during working hours as idle hours, it will be necessary to run the fan at a greater speed and thus increase the volume of air in circulation. If the fan is run fast enough to maintain adequate ventilation during working hours there will be an unnecessary waste of power if its speed be maintained during /

during idle hours. Since the volume circulated varies as the cube root of the power, if the power is reduced to one-half its value, the volume will fall to 79.37 per cent of its full value, neglecting the effect of natural ventilation. If this assist the fan^{it}/would raise the volume to well over 80 per cent of the full value. If, therefore, it is found that during idle hours 80 per cent of the current for working hours is sufficient for practical purposes, a saving of one-half of the power bill for ventilation can be effected during those periods. In nearly all well ventilated mines a drop of 20 per cent in the working-hours current would leave the workings sufficiently well ventilated for idle hours. If, however, the mine is but poorly ventilated during working hours no reduction of the air current can be allowed, since the case may be one which calls for a greater, and not a lesser, ventilating current at all times.

Variable-Speed-Alternating-Current Motors for Driving Fans.

Speed variation of induction motors can be obtained by the cascade arrangement; by varying the number of poles of the stator; by rheostatic control of the rotor currents; and by the application of secondary motors or motor-generator sets, driven by the rotor currents, which either assist the main motor or return energy to the mains. Speed regulation may also be secured by the use of three-phase commutator motors having either series or shunt characteristics.

With regard to the saving effected by the use of variable-speed motors, the results obtained

by the Pittsburgh Coal Company, Pa., U.S.A., are of especial interest. The figures were given in the discussion of a paper read before the Association of Iron and Steel Electrical Engineers on March 18th., 1922.

At the Harrison Mine a 15 by 5 foot fan was driven by a 250 horse-power slip-ring induction-motor with a Sherbius speed-regulator. Under ordinary working conditions the speed ~~is~~ varies about 26 per cent., or from 128 to 95 revolutions per minute. In order to ^{have /}run the fan under existing conditions at full speed, the energy required would have been 1,037,184 Kilowatt-hours. The power actually used was 706,056 Kilowatt-hours, and the saving effected by lowering the speed during idle hours was 331,128 Kilowatt-hours, which represents a saving of about £690 for power supplied, assuming a rate of 0.5^d per Kilowatt-hour. The power factor was 70 per cent. at 95 revolutions per minute, and 94 per cent. at 128 revolutions.

The results for 1921 at Montour No. 8 mine, where a 150-Horsepower brush-shifting motor was used for driving the fan were:- total energy used, 262,855 Kilowatt-hours; energy that would have been required if the fan had been kept running at full speed all the time, 569,400 Kilowatt-hours, representing a saving of 306,545 Kilowatt-hours, or about £640 per annum.

The Use of Fans in Combination.

The use of booster fans in combination with a main fan is a very common occurrence in coal mines. Where the resistance of one or more of the

airways /

airways in parallel with each other is much greater than the average value, it is necessary to choose between a booster fan for the high resistance splits or regulators for those of lower resistance.

The booster fan raises the ventilating pressure of the section to which it is applied by an amount sufficient to ensure the circulation of the required volume in the section. Thus extra pressure is applied only where it is wanted and there is no waste of pressure.

Where regulators are used, the main ventilator must impart to the whole volume of air circulated a pressure equal to the greatest pressure required by any of the ventilating districts in parallel with each other. There is thus waste of energy, and in order to prevent this waste becoming still greater from the circulation of more air than is necessary in some of the districts, regulators are applied to these districts. The regulator is a dissipator of energy, the energy lost in foot-pounds per minute being equal to the product of the drop of pressure in pounds per square foot at the regulator and the volume of air in cubic feet per minute which passes through the regulator.

From these considerations it might at first sight be concluded that the booster fan would in general be preferable to the regulator. The problem, however, cannot be disposed of so easily, and each case must be considered upon its merits. Where only a small part of the total volume of air in circulation must pass through a
regulator /

regulator, or where the necessary drop in pressure at the ventilator is small, it will generally be found that the regulator is preferable to the booster fan. On the other hand, where a large volume of air would be required to undergo a pressure-drop at the regulator, or where a large pressure-drop would be sustained by a moderately large volume of air, the booster fan could be economically applied. Between these limiting conditions careful discrimination is necessary.

Position of a Regulator or of a Booster Fan.

The best position of a regulator is near where the split to be regulated joins the main return airway. The pressure-difference between the intake and return roadways of the split is then a minimum, and the leakage of air will also be a minimum.

If the split should require an increased instead of a reduced volume, a booster fan would be applied, and for the same reason the best position for the regulator is found to be the best for the booster fan. This fan, however, may be equally well placed either where the split joins the main intake or where it joins the main return airway. If the booster fan is put to work near the face, transmission losses will be greater owing to the greater distance between the generator and the fan motor, but a greater evil will be the excess of air leakage between the intake and return airways, since then the greatest pressure-difference between the intake and return will be near the workings, where the stoppings are usually most defective, and the ground is least consolidated. Further, there is the

serious objection that the greater pressure would be on the return side so that the leakage would be from the return to the intake. It would not be difficult to visualise a case where, the booster fan placed near the working face and with rather broken ground or defective stoppings, there might be a good circulation of air near the face, but very little fresh air entering the section, the circulation for the most part consisting of air which had leaked from the return into the intake airway. Such an arrangement would not only be inefficient, but it might also give rise to grave danger and be accompanied by a false sense of security. If the airways serving a section of the workings were leaking badly, and a booster fan were put to work near the junction with a main airway, the increased pressure given by the fan would fail to produce the desired volume of air at the working faces, and attention would thus be directed to the need for improvement of the stoppings.

The selection of a booster fan suitable for any given work is a simple matter. The volume of air required in the section and the increase of pressure necessary would be estimated. A fan having suitable characteristics to pass the volume at the necessary pressure could then be chosen quite independently of the action of the main ventilating pressure.

Utilisation of Excess Energy of a Regulated Split.

The volume of air passing in any split in which there is an excess of ventilating pressure could be effectively regulated by /

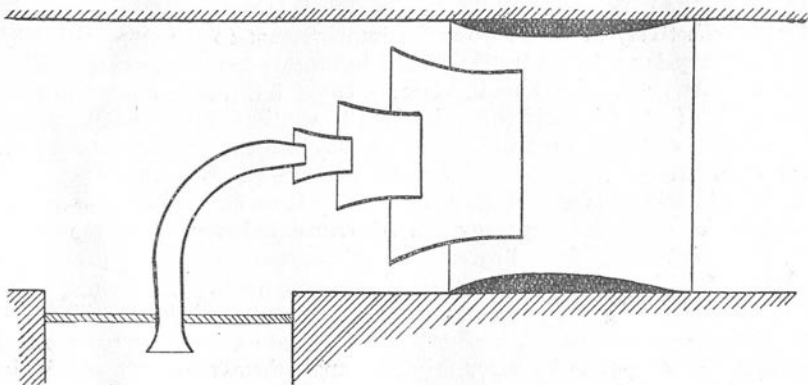


Fig. 15.—NOZZLE ARRANGEMENT FOR UTILIZING EXCESS ENERGY OF AIR.

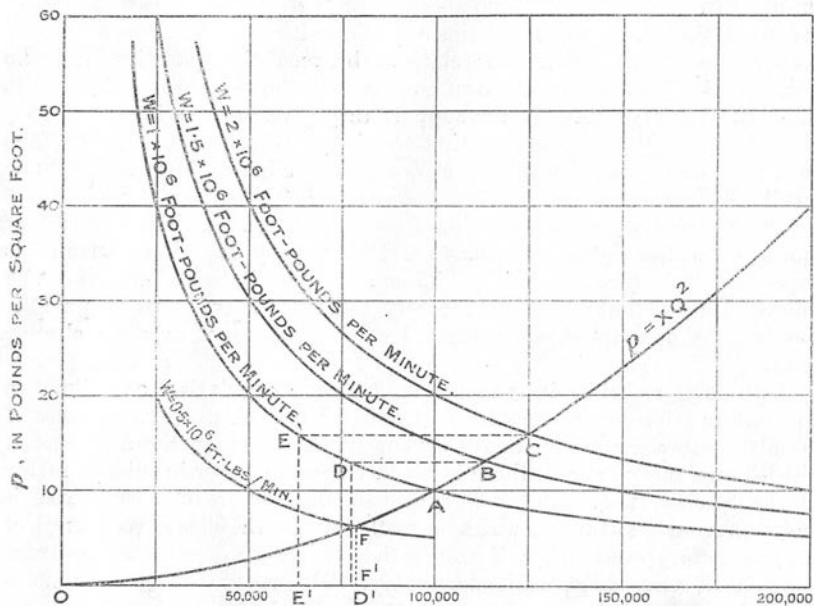


Fig. 16. Q , IN CUBIC FEET PER MINUTE.

by causing the air to pass through a suitable nozzle which would waste very little pressure. A part of the kinetic energy in the air escaping from such a nozzle could be usefully applied if it were arranged for the air to escape into a non-regulated split through a device somewhat as sketched in Figure 15.

The Operation of Fans in Parallel.

When two fans operate together on the same mine in parallel or in series with each other, it is clear that the total useful work done by the combination is equal to the sum of the useful work done by each fan. If a fan F_1 operates on a mine alone and does 10^6 foot-pounds of useful work per minute then since the useful work is pQ , where p is the pressure in pounds per square foot, and Q is the volume of air in cubic feet per minute, the work done can be represented by the hyperbola EDA, Figure 16. Similarly useful works of 0.5×10^6 and 2×10^6 foot-pounds per minute can be represented by hyperbolae passing through the points F, B, and C respectively.

If the mine resistance is represented by the curve $p = XQ^2$, the points F, A, B, C, where this curve intersects the hyperbolae would determine the pressure-volume relation for the respective useful outputs. A single fan developing 2×10^6 foot-pounds of useful work per minute would operate at the point C. But two fans operating together on the same mine and each developing 10^6 foot-pounds of useful work per minute, would necessarily operate at the same point C on the mine resistance graph. If the fans worked in series they would divide the ventilating pressure equally, and if they worked in

parallel /

parallel they would divide the volume of air equally.

Hence, for two fans in parallel, each developing 10^6 foot-pounds of useful work per minute, the point of operation for each fan will be the point E obtained by drawing a horizontal line through C to cut the hyperbola EDA. The pressure given by each fan will be equal to EE^1 and the volume passed will be given by OE^1 .

Similarly a single fan developing 1.5×10^6 foot-pounds of useful work per minute, would operate at the point B, while if one fan developing 10^6 foot-pounds per minute works in parallel with another which develops 0.5×10^6 foot-pounds per minute, the point of operation for the fan which develops the larger useful work would be D, the intersection of the horizontal line drawn through B, with the hyperbola EDA. The point of operation for the other fan would be obtained by continuing the horizontal BD until it intersected the hyperbola drawn through F.

It is thus clear that a fan developing useful work equal to 10^6 foot-pounds per minute would operate at the point A on the mine resistance curve, provided that no other ventilating appliance worked on the mine at the same time. The starting of another fan which developed 0.5×10^6 foot-pounds per minute, would make it necessary for the first fan to operate at the point D if it continues to develop the same useful power. Similarly if the second fan develops 10^6 foot-pounds per minute, the point of operation for the first fan moves to E.

In general, unless the speed of the first fan is increased when the second fan is brought into
action /

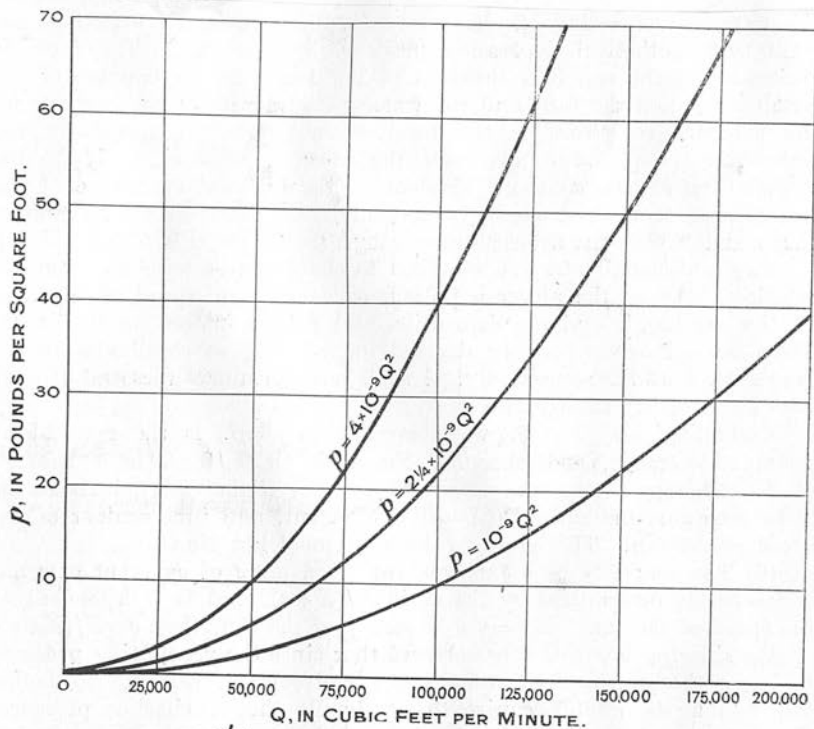


FIG 17.—PRESSURE-VOLUME-RESISTANCE GRAPHS.

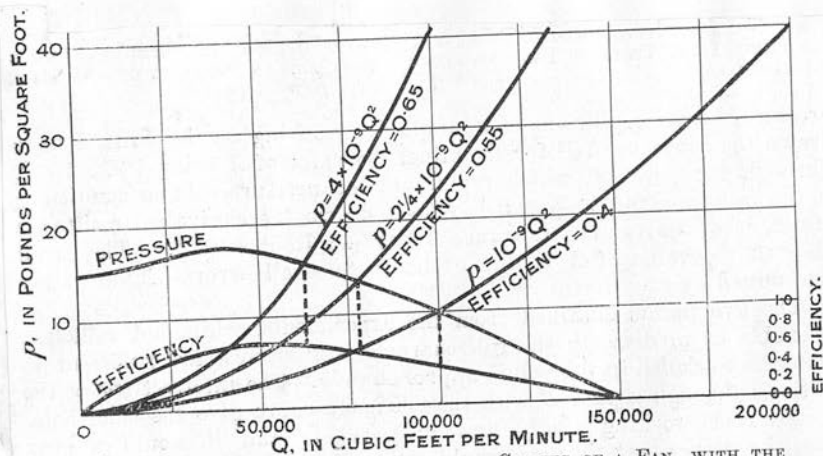


FIG 18.—PRESSURE-VOLUME-AND-EFFICIENCY CURVES OF A FAN, WITH THE RESISTANCE GRAPHS OF FIG 17 SUPERPOSED.

action, these points of operation are not reached, and the load on the first fan decreases. If, however, the first fan happens to be operating on a steep portion of its pressure-volume curve, it may maintain the load fairly constant.

The points D and E do not lie on the graph of the mine resistance, but they determine the effective or equivalent resistance upon which the fan must operate under the conditions indicated. Graphs of the resistances corresponding to the points A, D, and E have been drawn in figure 17. The values of these resistances expressed in foot-pound-minute units are 4×10^{-9} , 2.25×10^{-9} and 10^{-9} respectively, or expressed in Atkinsons 4×3.6 , 2.25×3.6 and 3.6 respectively.

Superposition of Resistance and Equivalent Resistance
Graphs of the Characteristic Curves of a Fan.

In figure 18, upon the pressure-volume and efficiency curves of a fan, the graphs of figure 17 have been superposed. If this fan works alone on a mine having a resistance of 3.6 Atkinsons, represented by the graph $p = 10^{-9} Q^2$, the figure shows that the fan would pass a volume of 100,000 cubic feet per minute, at a ventilating pressure of 10 pounds per square foot, with an efficiency of 0.4.

The application of a fan in parallel which would pass one-third of the total volume, and leave the existing fan to deal with two-thirds of the volume, would cause the equivalent resistance for this fan to rise to $R_{e1} = R \times (3/2)^2$ or to 8.1 Atkinsons, and the efficiency would rise to 0.55. The addition of a second fan which would deal with one-half of the total volume of air, would raise the

equivalent /

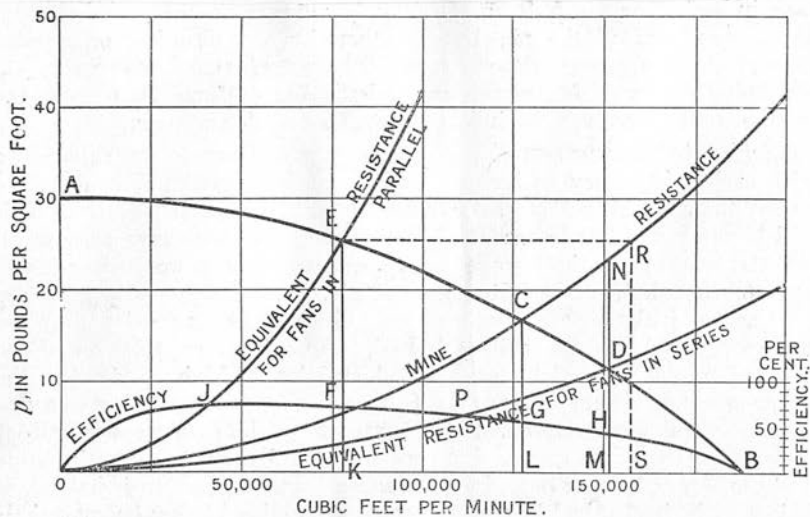


FIG. 19—GRAPHS TO SHOW THE RELATION OF THE MINE RESISTANCE AND THE EQUIVALENT RESISTANCES FOR FANS IN SERIES AND IN PARALLEL, AND TO OBTAIN THE EFFICIENCY AND LOAD AT WHICH A FAN WOULD OPERATE AT VARIOUS CONDITIONS.

equivalent resistance from 3.6 to $2^2 \times 3.6$ or 14 Atkinsons, and the efficiency of the fan would rise to 0.65.

In the case of the addition of a fan to pass one-third of the volume of air, the equivalent resistance for this smaller fan would be $3^2 \times 3.6$, or 32.4 Atkinsons; whereas a second fan which should divide the volume equally would work on an equivalent resistance of 14 Atkinsons.

The laws of the equivalent resistance are

- (1) For fans in parallel $R_e = RN^2$
 (2) For fans in series $R_e = R/N$

In figure 19 the graph of a mine resistance R , equal to 3.6 Atkinsons has been superposed on the characteristic curves of a fan. The fan working alone on the mine would operate at the point C and would thus pass a volume OL at a pressure CL, and would work with an efficiency GL, equal to 50 per cent.

If the speed of the fan were maintained constant, and another fan were run in parallel, its speed being regulated until the fans divided the load equally, the point of operation would move up to the point E, where the equivalent resistance graph OJE equal to $4R$ cuts the pressure-volume characteristic of the fan. The first fan would operate with an efficiency FK, equal to 65 per cent, and the two fans would pass a volume equal to 2OK at a pressure equal to EK. If a second fan were run in series and the speed of the fans adjusted until they divided the ventilating pressure equally, the fans would operate on an equivalent resistance represented by the curve OPD, and the existing fan would then work with an efficiency HM, equal to 40 per cent.



In the case considered the addition of a fan in parallel would improve the efficiency, but a fan in series would lower the efficiency. It is, however, important to observe that by a suitable parallel or series arrangement, the position of the operating point on the fan characteristic curves can be moved until it reaches the point at which the fan would operate with ~~the~~ maximum efficiency.

Possible Overload on Constant-Speed Motors which drive Series or Parallel Arrangements of Fans.

If a fan is driven by a motor which is developing its full load when the fan is acting alone on the mine, there will be a heavy overload on the motor when a second fan is run up in series, unless the speed of the motor can be reduced when both fans are running. The actual load can be read off direct if the input curve for the fan has been drawn. In other cases it may be estimated.

Referring to figure 19, which has been drawn for a fan having radial vanes, when the fan is operating alone at the point C the motor is carrying a full load. If the speed be ^{now/} maintained constant while a fan in series is started up so as to divide the pressure equally, the operating point on the mine resistance will pass to N, and that on the pressure-volume curve of the fan to D. The load on the fan would thus be in direct proportion to the volume passed and equal to the full-load multiplied by OM/OL , which gives an over-load of 19 per cent. Similarly, if two fans are running in parallel, and one is then isolated from the mine while the speed of the other is maintained constant, there would be a large increase of the load on the motor.

Referring /

Referring again to figure 19, if two fans in parallel are sharing the load equally and working on the equivalent resistance OJE, the stopping of one of the parallel fans will cause the point of operation to move from E to C, and the volume passed by the fan to increase from OK to OL, i.e. from 79,000 to 128,000 cubic feet per minute. But, since the characteristic curve is that of a fan having radial vanes the total pressure developed by the fan will remain constant for a constant speed and the load on the motor will increase by 62 per cent.

Rectification of the Equivalent Resistance for Improved Efficiency.

It has been observed that by a suitable parallel or series arrangement, the point of operation of a fan can be brought to coincide with the point at which the fan operates with maximum efficiency. The method of calculating the equivalent resistance is as follows:-

1. For a Series Arrangement:- $R_e = R/N.$

Let two fans F_1 and F_2 operate in series on a mine, and each develop a pressure of 12.5 pounds per square foot and pass 5,000 cubic feet of air per second. The total pressure acting on the mine will be 25 pounds per square foot, and the resistance of the mine will be $25/5^2$ or 1 Atkinson. The equivalent resistance for each fan will be $R_e = R/(25/12.5)$ or 0.5 Atkinson; alternatively it is P/Q^2 or $12.5/5^2$ or 0.5 Atkinsons, i.e. one-half of the resistance of the mine. If, again, while the same volume of air circulates, the fan F_1 give a pressure of 15 and the fan F_2 a pressure of 10 pounds per square foot, while the mine resistance remains unchanged and equal to

1 Atkinson /

1 Atkinson, the equivalent resistance R_{e1} for the fan F_1 will be $R/(25/15)$, or 0.6 Atkinsons. Alternatively it is P_1/Q^2 or $15/5^2$ or 0.6 Atkinson, while the equivalent resistance for the fan F_2 will be R_{e2} or $R/(25/10)$ or 0.4 Atkinson.

2. For a Parallel Arrangement:- $R_e = RN^2$

If a fan F_1 act alone on a mine resistance and pass 5,000 cubic feet of air per second at a ventilating pressure of 25 pounds per square foot, the resistance against the fan is the resistance of the mine, and is equal to $25/5^2$ or 1 Atkinson. If a second fan F_2 be run in parallel with F_1 and the speeds are regulated until each fan passes 2,500 cubic feet of air per second, the volume of air circulating in the mine remaining unchanged, the ventilating pressure will remain constant at 25 pounds per square foot, and the resistance of the mine will be the same. The equivalent resistance against which each fan would operate, however, will be $R_{e1} = R_{e2} = RN^2 = 1 \times 2^2$, or 4 Atkinsons; alternatively it is $25/2.5^2$ or 4 Atkinsons, i.e. four times as large as the resistance of the mine.

Should the speeds be regulated until F_1 pass 3,000 and F_2 2,000 cubic feet per second, and the conditions of the mine remain unchanged, the equivalent resistance for the fan F_1 will be $R_{e1} = RN_1^2$, or $R(5/3)^2$ or 2.7 Atkinsons; alternatively it is $R_{e1} = P/Q_1^2$ or $25/3^2$ or 2.7 Atkinsons; for the fan F_2 it will be $R_{e2} = RN_2^2$ or $R(5/2)^2$ or 6.25 Atkinsons; alternatively it is $R_{e2} = P/Q_2^2$ or $25/2^2$ or 6.25 Atkinsons.

The following examples indicate how the
method /

method of rectification of the equivalent resistance is applied to particular cases which require a parallel and a series arrangement respectively.

Example. A fan is working on a mine in which there is no natural water-gauge, and passes 3,000 cubic feet of air per second at a ventilating pressure of 12 pounds per square foot, with an efficiency of 30 per cent. The characteristic curves of the fan show that it would pass 2,000 cubic feet per second at a pressure of 16 pounds per square foot with an efficiency of 60 per cent. What is the nature of the parallel arrangement required to make the fan work with an efficiency of 60 per cent, and what changes would this involve in the volume of air circulated and in the power input required to ventilate the mine ?

The mine resistance $R = P/Q^2$ or $12/3^2$ or 1.3 Atkinson. The equivalent resistance R_e for an efficiency of 60 per cent is $R_{e1} = RN_1^2$. Therefore, $N_1 = \sqrt{R_{e1}/R}$ or $\sqrt{4/1.3}$ or 1.732. The fan F_1 would deal with $1/1.732$, i.e. with $100/173$ of the total volume of air passed. Hence the fan F_2 would pass 73.2 per cent of the volume passed by the fan F_1 .

The volume that would pass against the resistance of the mine with a ventilating pressure of 16 pounds per square foot is $\sqrt{P/R}$ or $\sqrt{16/1.3}$ or 3.464 Kilo-cusecs. The fan F_1 would thus pass 2,000 and the fan F_2 would pass 1,464 cubic feet of air per second.

The useful work developed before paralleling is $P \times Q$ or $12 \times 3,000$ or 36,000 foot-pounds per

second /

second. The power input to the fan is the output divided by the efficiency or 36,000 divided by 0.3 or 120,000 foot pounds per second. The useful work after paralleling is $3,464 \times 16$ or 55,424 foot-pounds per second. The equivalent resistance against the fan F_2 is $16/1.464^2$ or 7.46 Atkinsons, and if a fan has been chosen to work with an efficiency of 60 per cent on this resistance, the power-input to the two fans after paralleling will be $55,424/0.6$ or 92,373 foot-pounds per second.

There would thus be a saving of 27,627 foot-pounds per second in the fans and an addition of about 15 per cent to the volume of air circulated due to the new arrangement. If an extra volume of air is not required, the speed of both fans could be lowered by about 15 per cent, when the original volume would be circulated and the power input to the fans would fall to about one-half its first value, as would be expected since the fans would be working at double the former efficiency.

Example:- A fan is passing 3,000 cubic feet of air per second at a ventilating pressure of 18 pounds per square foot, with an efficiency of 40 per cent, and the characteristic curves show that it could pass 4,000 cubic feet per second at a pressure of 16 pounds per square foot with an efficiency of 60 per cent. What is the nature of the series arrangement that would cause the fan to operate with an efficiency of 60 per cent, and what changes would this involve in the power input required to ventilate the mine ?

The mine resistance R is P/Q^2 or $18/3^2$

or /

or 2 Atkinsons. The equivalent resistance for an efficiency of 60 per cent is $R_e = P/Q^2$ or $16/4^2$ or 1 Atkinson; and since $R_e = R/N$, therefore $N = R/R_e$ or $2/1$ or 2.

The additional fan must therefore develop the same useful output as the existing one. A fan would therefore be sought to pass a volume of 3,000 cubic feet per second against a resistance of 1 Atkinson, with an efficiency of 70 per cent. In order to pass 3,000 cubic feet per second against a resistance of 1 Atkinson, a ventilating pressure $P = RQ^2$ or 1×3^2 or 9 pounds per square foot will be required.

The existing fan can pass a volume of 4 Kilo-cusecs when working on a resistance of $16/4^2$ or 1 Atkinson; it will therefore pass 3 Kilo-cusecs at a pressure of 9 pounds per square foot when running at a lowered speed, ~~and~~, and the efficiency being independent of the speed, will still be 60 per cent. The speed of each fan would therefore be regulated to give a pressure of 9 pounds per square foot.

The average efficiency of the two fans will be 65 per cent, and the input will be $18 \times 3,000/0.65$ or 8,3000 foot pounds per second; whereas the input to the single fan was $18 \times 3,000/0.4$ or 135,000 foot-pounds per second.

In the case of rectification of the equivalent resistance by means of a parallel arrangement the additional fan deals with only a fraction of the volume of air and is relatively small; for a series arrangement a fan to deal with the whole volume would

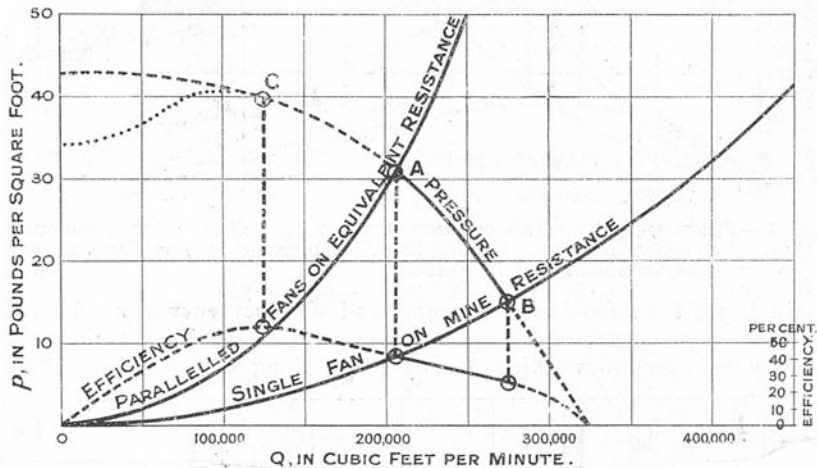


Figure 20, in which the results of Seymour-Wood's tests, Table VIII have been plotted.

TABLE VIII.

The First Fan Trial Run on the Pit. Resistance 0.71 Atkinsons.

Description	Volumes of air in cub. ft. per min.	Water-gauge. Inches.	Speed of Fan R.P.M.	Electrical Horse-power input to Motor	Horse-power in the air.	Overall Efficiency	Increase of volume of air with fans in parallel.
No. 1 fan	276,687	2.90	226	462.4	126.4	27.3	--
No. 2 fan	291,937	3.25	226	562.8	149.4	26.5	--
Both fans in parallel	410,877	6.00	226	945	388.4	41.0	40 per cent.

TABLE IX.

Fans running at 200 R.P.M. on the Pits. Resistance 1.34 Atkinsons.

Fan	Volume of air in c. ft. / min.	Water-gauge Inches	Elect. Horse-Power to Motor.	Horse-power in the air.	Fan Efficiency Per Cent.	Increase of volume of air with fans in parallel.
No. 2 fan	235,850	4	369	149	46.7	--
do.	235,200	4	369	148	46.5	--
Both fans in parallel	338,000	6	619	320	59	43.8 per cent.

TABLE X.

Fan.	Volume of air in cub. ft./min.	Water-gauge Inches.	Fan Speed R.P.M.	Electrical Horse-power to Motor.	Horse-power in the air.	Fan Efficiency	Increase of volume of air with fans in parallel.
No. 2	279,000	3.7	200	389	162	48	--
do.	280,188	4.0	200	356	177	53	--
Both Fans running in parallel.	326,716	6.10	203	289 289 <u>578</u>	313	62	16.6 per cent

TABLE XI.

Test Results from Double-Inlet Fan, 14½ feet in Diameter, and 5 feet, 10 inches Wide, Running on the Pit.

Test.	Volume of air in cub. ft. per min.	Water-Gauge. Inches.	Fan Speed R.P.M.	Electrical Horse-Power to Motor	Horse-power in the air.	Overall Efficiency.
No. 1	328,848	6.3	202	545	326	60
No. 2.	339,527	6.3	202	546	337	61.7

be necessary. For this reason it is only in cases where the horse-power of ventilation is large that rectification by a series arrangement would be considered. Where parallel rectification is required the saving in capital expenditure is decidedly more promising.

Results obtained from the Parallel Operation of Fans.

Experiments with Capell fans operating singly and in parallel have been made at the Murton Colliery by Seymour-Wood.* Several experiments were first made with the fans working in artificial resistances. The first trial made with the fans working on the pits, after the furnace and boiler fires had been allowed to go out, gave the results shown in Table VIII.

These results have been plotted in figure 20. Through the point B, which represents the pressure-volume condition for No. 1 fan working alone on the mine, the graph of the mine resistance $P = RQ^2$, has been drawn. The value of R is 0.71 Atkinson. The point A represents the experimental conditions when both fans were working in parallel on the mine. Through this point an equivalent resistance graph, $P = R_e Q^2$ has been drawn. The value of R_e is found to be $2^2 \times 0.67$ Atkinsons. The formula for the equivalent resistance $R_e = 2^2 R$, gives $R_e = 2^2 \times 0.71$ Atkinsons which differs from the experimental value by only 5.5 per cent, and this figure is within the experimental error.

The two points obtained from these experiments enable the characteristic curves of the fan to be sketched approximately. The intersections of the verticals through A and B with the efficiency graph

* Trans. I.M.E., LXIII., p. 92.

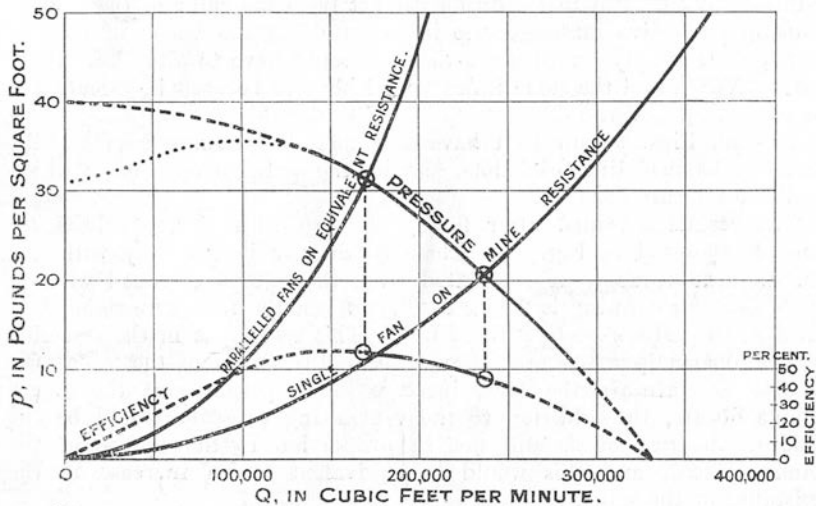


FIG. 21.—PRESSURE-VOLUME-AND-EFFICIENCY CURVES OF MR. E. SEYMOUR WOOD'S TESTS AFTER THE INTRODUCTION OF REGULATORS INTO THE MINE.

give the efficiency for the respective conditions of operation. Since these values are known the graph can be plotted. The full-line curves join the points obtained from the experiments, and on the assumption that the complete curves are as shown by the hatched lines, a combined efficiency of 60 per cent could be obtained, if the point of operation moved along to C. The fact that the fans tested would work with a higher efficiency was shown by further tests in which the resistance of the mine was increased by the addition of regulators.

Seymour-Wood repeated the fan tests on the mine, with the resistance increased by regulators from 0.71 to 1.34 Atkinsons. The results which are given in Table IX. have been plotted in figure 21.

There is some difficulty about the results obtained in this test with the fans running in parallel. If the resistance had followed the square law, then since a water-gauge of 4 inches was required to pass a volume of 236,000 cubic feet per minute, a pressure of 8.2 inches would have been necessary to pass 338,000 cubic feet of air per minute. The difficulty may in part be explained by greater leakage due to the increase of pressure inside the regulator, or a door may have been ajar. The fans were afterwards fitted with sixteen additional vanes and the tests repeated, with the results given in Table X.

The average value of the volume for the two tests with No. 2 fan running alone was 4,659 cubic feet per second, and the average water-gauge was 3.85 inches, or 20.02 pounds per square inch. The resistance is $R = P/Q^2$ or $20.02/4.659^2$ or 0.92 Atkinson. For the fans running in parallel the volume was

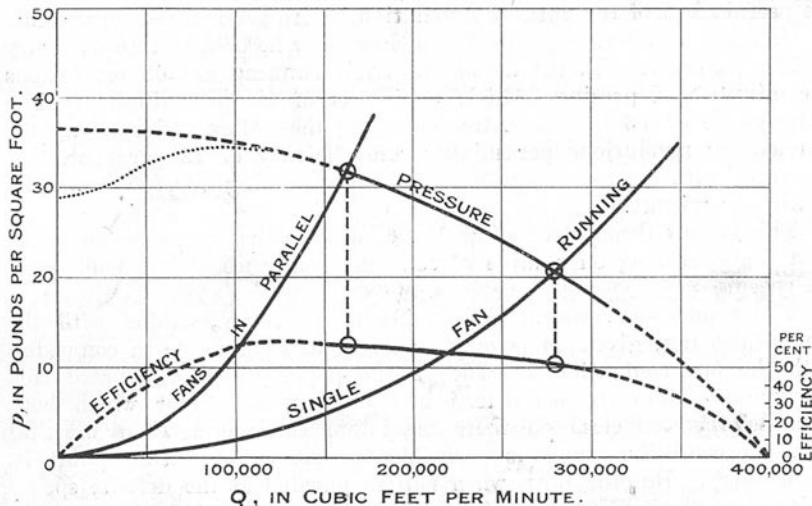


FIG. 22—PRESSURE-VOLUME-AND-EFFICIENCY GRAPHS OF MR. E. SEYMOUR WOOD'S TESTS AFTER FITTING THE FAN WITH SIXTEEN ADDITIONAL VANES.

5,445 cubic feet per second, the pressure 6.1 inches of water-gauge or 31.72 pounds per square inch, and the resistance $R = P/Q^2$ or $31.72/5.445^2$ or 1.09 Atkinsons. The equivalent resistance for the individual fans would therefore be $2^2 \times 1.09$ Atkinsons.

The anomaly of the previous tests has not recurred here, as the increase in the resistance could be anticipated from the effect of a natural water-gauge. These results have been plotted in figure 22, and the complete form of the characteristic curves shown by hatched lines.

From a knowledge gained by these tests a new double-inlet Capell fan was designed and put to work on the mine resistance without the addition of artificial resistances, and the test results obtained from this fan are given in Table XI. and these show an overall average efficiency of 60.85 per cent. A test was made with this fan running at 180 revolutions per minute, when 309,705 cubic feet per minute passed at a water-gauge of 5.15 inches, with an overall efficiency of 61 per cent.

These tests demonstrate clearly the validity of the law which states that the efficiency of a fan is independent of the speed of the fan.

A further experiment of running the single inlet fan and the double inlet fan in parallel was tried, and gave the following results, viz.:-

Single-Inlet and Double-Inlet Fans Working Together.

Double-Inlet Fan, speed 180 R.P.M.

Single-Inlet Fan, speed 200 R.P.M.

Double-Inlet Fan, water-gauge 5.9 inches.

Single-Inlet Fan, water-gauge 5.7 inches.

Mean water-gauge 5.8 inches.

Volume of air, 342,953 cubic feet per minute.

Electrical Horse-Power to Motor:- Double-Inlet Fan, 257, Single-Inlet Fan, 295. Total Electrical Horse-Power = 552,

Horse-Power in the air, 312.

Overall Efficiency, 56 per cent.

In these tests, the conditions had been found under which the fans attained to near the maximum efficiency when working alone. But the fans, when run in parallel at the definite speeds which they attained when working singly, passed together only 342,953 cubic feet of air per minute - an increase of about 10 per cent with a water-gauge of 5.8 inches, and a combined overall efficiency of about 56 per cent. The total horse-power rose from 407 to 502. If it had been possible to allow the speed of the fans to rise freely, the motors could have developed their combined horse-power of 800 and would have added still further to the volume of air circulated without lowering the overall efficiency below 56 per cent.

Fan Tests at the Wellesley Colliery, Denbeath.

At the Wellesley Colliery two fans have been installed to ventilate the workings. One is a Waddle Fan 21 feet diameter, driven by compound tandem steam engines coupled directly. The other is a double inlet Sirocco fan 119 inches diameter and 7 feet wide, driven by Belliss and Morcom compound vertical steam engines coupled directly.

The fans are connected by drifts, fitted with isolation doors, to a 14 feet diameter vertical shaft, and from the foot of this shaft there is a drift which enters the upcast shaft at a point 102

feet /

feet below the surface. Either fan can be placed in connexion with the mine, or be shut off by means of the isolation doors operated from the top of the fan drift from the outside.

A series of tests was made with these fans to elucidate the laws which govern the action of fans when working in parallel. The results of these tests are shown in Table XII.

In the first test the Waddle fan was run at a speed of 70 R.P.M. with the isolation doors closed, and the indicator cards of the steam engine were taken. The engines indicated 10.5 horse-power, and it was found that ~~the~~ strong eddy currents of air circulated through and near the fan.

The fan was then run up to its normal speed of 125 R.P.M., and the isolation doors were opened, while the Sirocco fan was isolated from the mine and stopped running. The engines indicated 190.8 horse-power, the volume of air circulated was 131,000 cubic feet per minute, and the water-gauge determined by means of a tube placed at D/6 from the side of the fan drift and facing against the air current was 4.584 Atkinsons.

The Sirocco fan was then run up to its normal speed of 172 R.P.M. and put to work on the mine while the Waddle fan was isolated and stopped running. A volume of 120,000 cubic feet of air circulated with a water-gauge of 4.12 inches, and the calculated R_s was 5.105 Atkinsons. It was not found practicable to indicate the Belliss and Morcon Engines.

The Waddle fan was then started and the isolation doors opened, and the fans were thus placed in parallel on the mine resistance. The results

are /

TABLE XII.

Fan Tests at Wellesley Colliery, Denbeath, by Joseph Parker, February 4th., 1922.

No. of Test	Waddle Fan.						Sirocco Fan.						
	R.P.M.	Volume of air passed by fan in cubic feet per min.	Water-gauge Inches.	Horse-Power in the air passed by fan	I.H.P. of Engines driving fan	Overall Efficiency of fan & engines	Effy. of fan allowing 7 per cent for frictional losses.	Resist: ance against the fan $R = P/Q^2$ R_w	Remarks.	R.P.M.	Volume of air passed by fan in cub. ft./min.	Water gauge Inches.	Restnctance against fan $R P/Q^2$ R_s
1.	70	0	1.6	0	10.5	0	0	∞	Doors in fan drift closed	-	-	-	-
2.	125	131,000	4.2	85.2	190.8	44.57	47.92	4.584	Waddle fan working alone on mine.	-	-	-	-
3.	-	-	-	-	-	-	-	-	Sirocco fan do.	172	120,000	4.12	5.105
4.	133	136,000	4.3	92.15	223.8	40.25	43.28	5.026	Fans in parallel on the mine	172	0.	4.25	∞
5.	115	0	4.2	0	21.8	0	0	∞	do. do.	172	126,000	4.25	5.011
6.	88	92,575	2.2	35	84.95	38.84	41.74	4.910	Waddle fan working alone. Steam cut off at $3/8$ the stroke	-	-	-	-
7	115	25,700	3.55	14.4	79.45	17.9	18.	100.6	Fans in parallel on the mine.	160	97,000	3.65	7.3

are shown in test No. 4, Table XII. The speed of the Waddle fan rose to 133 R.P.M.; it gave a water-gauge of 4.3 inches and drew from the mine a volume of 136,000 cubic feet of air per minute. The engines indicated 228.8 horse-power.

The Sirocco fan gave a water-gauge of 4.25 inches but drew no air from the mine. At both ears of the fan there were strong eddy currents, but farther back in the fan drift there was a region of perfectly still air in which a match burned with a flame erect. The calculated resistance, R_w , against the Waddle fan was 5.026 Atkinsons, while the resistance, R_s , against the Sirocco fan was infinite.

In test No. 5, the admission of steam to the engines driving the Waddle fan was checked by wire-drawing at the throttle valve until the speed was 115 R.P.M., when the conditions were found to be reversed. The Sirocco fan drew 126,000 cubic feet of air per minute from the mine, while the Waddle fan drew none. In the drift leading to the Waddle fan there was a repetition of the quiescent region and of eddy currents near the fan ear. The power indicated by the Waddle fan engine dropped to 21.8 horse-power. The calculated resistance, R_s , against the Sirocco fan was 5.011 Atkinsons, and R_w , the resistance against the Waddle fan was infinite. By varying the admission of steam to the engines driving the Waddle fan it was found that the fan could be made to carry any portion of the load from zero to full load. During the tests the speed of the Sirocco fan was maintained constant by the action of a crank shaft governor.

The total volume of air passed by both

fans /

fans when working in parallel was little more than the volume passed by either fan when working alone on the mine. When a fan is operating on a flat portion of its pressure-volume curve, a reduction in the volume passed by the fan would not lead to much increase in the pressure while the fan was driven at constant speed. But without an increase in the pressure an increased volume cannot be circulated in a mine having a definite resistance.

The engines were not adapted to be run at higher speeds, and to test how fans in parallel would behave when the speed could rise freely, the eccentric sheave of the engine driving the Waddle fan was set to cut off the steam at $\frac{3}{8}$ of the stroke, and the Waddle fan was put to work alone on the mine resistance, with the throttle valve full open. The results are given in test No. 6. The fan attained ~~the~~ a speed of 88 R.P.M. and passed 92,575 cubic feet of air per minute with a water-gauge of 2.2 inches, while the engines indicated 84.95 horse-power. The calculated resistance against the fan was 4.91 Atkinsons.

The shaft governor of the Belliss and Morcom engines was then disconnected and the Sirocco fan was run up to a speed of 160 R.P.M. and put in parallel with the Waddle fan, with the results shown in test No. 7. The Waddle fan speeded up to 115 R.P.M. and drew 25,700 cubic feet of air per minute from the mine, while the Sirocco fan drew 97,000 cubic feet of air per minute. The volume of air circulated had thus increased by one third when the speed was free to rise within the limits indicated.

The efficiency of the Waddle fan, however, fell away very much, because this fan is not adapted to work on a resistance of the order of 100.6 Atkinsons.

These tests clearly indicate that when fans operate in parallel on a mine, the resistance against the fans is something quite distinct from the resistance of the mine. Tests Nos. 4, 5 and 7 show, that the resistance against any one of two fans which work in parallel, may have any value between that of the mine resistance and infinity. The results conform to the relation between the equivalent resistances and the mine resistance expressed in formula 29.

The value of the effective resistance of the mine, calculated from the results given in test No. 2 when the Waddle fan worked alone on the mine is 4.584 Atkinsons. The value obtained similarly from test No. 3 with the Sirocco fan is 5.105. In test No. 6 the value for the Waddle fan working alone is 4.910. The true value would lie between 4.584 and 5.105.

In each of tests Nos. 4 and 5, one of the values of the equivalent resistance is infinity, and the remaining equivalent resistance is then identical with the effective resistance of the mine, the values being 5.026 and 5.011 respectively. In test No. 7 the equivalent resistance R_w for the Waddle fan is 100.6 Atkinsons and R_s , that for the Sirocco fan is 7.3 Atkinsons.

Substituting these values in equation 29 the value obtained for the effective resistance of

the /

the mine is 4.585 Atkinsons, a value which lies within the limits of the resistance as determined from experiments with a single fan --

$$\begin{aligned} 1/\sqrt{100.6} + 1/\sqrt{7.3} &= 1/\sqrt{R_e} \\ \text{or } 0.097 + 0.037 &= 1/\sqrt{R_e} \\ \text{therefore, } R_e &= 4.585 \text{ Atkinsons,} \end{aligned}$$

where R_e is the effective resistance of the mine.

Each of the fans at the Wellesley Colliery has a rated capacity of 200,000 cubic feet per minute with a water-gauge of 4 inches. The fans have thus been designed to work on a resistance of $20.8/(200/60)^2$ or 1.872 Atkinsons. Since the mine resistance is about $2\frac{1}{2}$ times this value, this Colliery furnishes a good example of a case where the fans would work in series with an increased efficiency.

Natural Ventilation at Wellesley Colliery.

When the fans were stopped running, an air current of 12,260 cubic feet per minute was measured in the fan drift. The barometer read 30.12 inches at the surface and 32.10 inches at the bottom of the downcast shaft. The thermometer read 35° Fah. at the surface, 55° at the bottom of the downcast shaft, and 68° at the bottom of the upcast shaft. The hygrometer readings were:- at the downcast shaft bottom, dry bulb 55° , wet bulb, 53° ; and at the upcast shaft bottom, dry bulb 68° , wet bulb 68° .

The natural ventilating pressure, P_0 , and the mine resistance R may be determined thus:-
The natural volume of air, Q_0 is 0.2043 cusecs. In test No. 2 the pressure P_1 is 4.2 inches of water-gauge or 21.84 pounds per square foot, and the volume Q_1 is 2.185 cusecs. In this test the natural
pressure /

pressure P_0 would be added to the pressure P_1 observed on the water-gauge. Hence, if R is the resistance of the mine, R_w and R_n the resistances overcome by the fan and by the natural causes respectively:-

$$R = R_w + R_n \quad \dots \quad \dots \quad \dots \quad \text{I.}$$

$$P_0 = RQ_0^2 \quad \dots \quad \dots \quad \dots \quad \dots \quad \text{II.}$$

$$P_0 + P_1 = RQ_1^2 \quad \dots \quad \dots \quad \dots \quad \dots \quad \text{III.}$$

$$\text{therefore, } 2P_0 + P_1 = R(Q_0^2 + Q_1^2)$$

$$2P_0 + 21.84 = R(0.2043^2 + 2.183^2)$$

$$P_0 = 2.4042 R - 10.92.$$

Substitute this value for P_0 in (II.),

$$2.4042R - 10.92 = 0.4175 R$$

$$\text{Therefore, } R = 4.622 \text{ Atkinsons}$$

$$\text{and } P_0 = 0.19 \text{ pounds per square foot.}$$

If in a similar manner the values given in tests Nos. 3, 4, 5, and 6 are used in the determination of the R and P_0 the values obtained are as shown in Table XIII.

Table XIII.

No. of Test	Value of R	Value of P_0
II.	4.622	0.190
III.	5.159	0.215
IV.	4.386	0.183
V.	5.011	0.209
VI.	4.875	0.203
Average values $R = 4.81$		$P_0 = 0.2$

In each of the tests the natural pressure was in series with the gauge pressure, and the equivalent resistance R_n overcome by the natural pressure was in series with the effective resistance of the mine, R_e , i.e. with the equivalent /

equivalent resistance overcome by the fan. Hence

$$R = R_e + R_n.$$

R_e and R_n are proportional to the gauge pressure and the natural pressure respectively. The estimated value of R_n for each test is shown in Table XIV.

Table XIV.

No. of Test.	P	P_0	R_e	R_n	$R = R_e + R_n$
II.	21.84	0.195	4.584	0.043	4.627
III.	21.424	0.215	5.105	0.045	5.150
IV.	22.36	0.183	5.026	0.046	5.072
V.	21.84	0.209	5.011	0.045	5.056
VI.	11.44	0.203	4.910	0.088	4.998

Average values $R_e = 4.929$

$R = 4.98.$

The value of R_e as calculated from the results of test No. VII. is 4.585. The value of R_n in this test would be about 0.045 and the value for R as found from test No. VII. would thus be $4.585 + 0.045$ or 4.63.

The value thus obtained is almost identical with the value obtained in test No. II. when the Waddle fan was working alone on the mine. It is difficult to know whether the resistance of the mine may have been temporarily reduced by the opening of a door underground when tests Nos. II and VII. were being made.

Recapitulation of Results of Tests.

1. The value of the mine resistance has been estimated from the results obtained from fans working singly and in parallel on a mine and the

value /

value obtained from the parallel test has not diverged more from the average value than has some of the values obtained directly from the test when a single fan worked on the mine.

The validity of the method of equivalent resistances is thus clearly established.

2. In test No. II. the Waddle fan passed 131,000 cubic feet per minute when running at a speed of 125 R.P.M. against a resistance estimated at 4.584 Atkinsons; in test No. VII the Waddle fan was running at 115 R.P.M. and the two fans in parallel passed 122,700 cubic feet per minute against a resistance estimated at 4.585 Atkinsons. Taking the volume of air passed by a fan when working on a constant resistance as being proportional to the speed, the Waddle fan which passed 131,000 cubic feet per minute at 125 R.P.M. would have passed $131,000 \times 115/125$ or 120,500 cubic feet per minute at a speed of 115 R.P.M. Hence, the volume passed by the two fans when working in parallel was not quite 2 per cent more than the volume which the Waddle fan alone would have passed when running at the same speed.

Referring to Tables VIII., IX. and X which give the results of Seymour-Wood's experiments, fans working in parallel gave percentage increases in the volume of air circulated of 40, 43.8 and 16.6 respectively.

The principle of equivalent resistances explains these apparently perplexing results.

Thus, referring to figure 7, if for a definite speed of a fan F_1 the pressure-volume curve were a straight line throughout the range of volume represented by the points B, C, and D, then when a fan F_2 is run in parallel the operating point would move to C, and the total volume of air circulated by the two fans would be exactly the same as that circulated by F_1 alone. In general, however, the pressure-volume curve of a fan F_1 would be a curve such as DE, which intersects the equivalent resistance curve in E and not in C. In such a case the volume circulated by the two fans when working in parallel would be greater than that circulated by F_1 alone by the volume represented by $BB^1 + CC^1$.

When F_1 is working on a steep part of the curve DE, this increase of volume would be substantial.

3.

The principle of equivalent resistances can clearly be applied in cases where the rectification of the point of operation of a fan is necessary to obtain maximum efficiency.
