UNIVERSITY OF EDINBURGH

"ALLOCATION METHODS FOR STUDENT MIDWIVES"

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THESIS PRESENTED FOR Ph.D. IN BUSINESS STUDIES, 1978

DECLARATION

This thesis has been composed by the undersigned, and represents his own research.

ACKNOWLEDGEMENTS

I wish to thank the following people for their help:

Dr. R.C. Daniel - for his help and guidance throughout the project.

Miss K. Rowe
Professor H.P. Williams
Dr. J.O. Evans
Mr. R.M. McKenzie
Mr. R. Black
Mr. J.J. Glen

- for their advice relating to different aspects of the research.

Miss Grant (Education Officer, C.M.B. for Scotland)
Miss Jamieson (Principal Nursing Office, S.M.M.P.)
Miss Tomlin (Senior Nursing Officer, S.M.M.P.)
Miss Barr (Senior Midwife Tutor, S.M.M.P.)

- for their patience in explaining the allocation system at the Simpson Memorial Maternity Pavilion, and for permitting access to their files.

Mrs. S. Niven - for typing the thesis.

ABSTRACT

The thesis makes an analysis of methods available for scheduling nursing staff on a week by week basis. As illustration it considers the situation pertaining at the Simpson Memorial Maternity Pavilion between 1973 and 1978, where student midwives have to be allocated in order to satisfy staffing requirements on each ward, while simultaneously ensuring that each nurse receives the necessary experience on different wards in the course of her year's training.

Section I analyses the constraints governing nurse scheduling at the S.M.M.P. under two separate systems used between 1973 and 1978, and provides an exhaustive survey of alternative course structures and solution formulation methods.

Section 2 details the existing solution in 1973 and describes two models of that situation which were formulated in order to permit computer simulation of the problem.

In Section 3 the scheduling problem at the S.M.M.P. is put into the context of generalised allocation methods. The suitedness of existing mathematical techniques to this problem is considered, and that of sub-gradient optimisation is tested extensively, with modifications to published techniques being detailed where an improvement has been made in the applicability to the present problem. The method is found to be weak when applied to problems of this scale, so a new method is developed which uses a heuristic algorithm to allocate nurses to a set of acceptable schedules. This approach

is more powerful and may have applications in other fields.

Section 4 describes changes in the training constraints which make it possible to adopt a cyclically repetitive standard schedule at the S.M.M.P. Drawbacks in the present allocation pattern are pointed out, and a new scheduling system developed which eliminates these.

In the Conclusions comparison is made between the above methods of allocation with regard to their suitability to the real situation as typified by that at the S.M.M.P.

GLOSSARY

It has been necessary to use some terms and abbreviations, for various concepts, which have a meaning specific to this thesis. While deprecating the use of jargon, it is felt that the use of these terms helps to simplify the stating of the problem and to prevent ambiguity, and is thus justified.

WARD:	This is used to refer to the group or place in which a girl's training takes place each week. In most cases it will be an actual hospital ward, but in some cases it may be a unit or a department. For instance out-patients is a department rather than a ward, but still comes under that general classification. The following are all classified as wards:
51, 52, 53:	Post-natal wards. These are sometimes referred to as GENERAL wards, since the staffing levels required are often less exacting than those for the special wards. They are referred to individually by their numbers, or collectively by the abbreviation GEN.
54:	This ward is used for segregation, patients with temperatures, and terminations (abortions). It is referred to by its number.
49:	This is the pre-natal ward, where expectant mothers stay prior to the delivery.
LW:	Abbreviation for labour ward.
SC, SCU:	Abbreviations for special care unit.

OP:	Abbreviation for out-patients' ward.
SPECIAL WARDS:	These are the five wards above, as opposed to the GENERAL wards.
SET:	A monthly intake of around 12 girls. Referred to by its month of arrival e.g. The June Set. Divided into half-sets. Also any other group arriving together.
ROSTER:	The chart or computer matrix which shows when each half-set will be on night duty, day duty, district work, and leave.
PLAN:	The chart or computer matrix which shows which ward each girl is on or has been on for each week. It is made up of individual schedules for each girl.
EXPERIENCE:	(EXP) The list of wards which a girl has already visited in the relevant half of her course.
CHOICE:	The list of wards which a nurse is free to visit next.
SLACK:	The number of weeks a girl has to do on special wards deducted from the number of weeks she has available to do them in. Slack weeks are done on general wards.
URGENCY:	A rating based on the number of special wards a girl still has to visit and the amount of slack she has.
ALLOCATED:	Placed on a specific ward for a number of weeks.

PRE-ALLOCATED: Used to describe the situation when during the week being considered, a girl is not free to move, but is already performing a spell of duty on a ward.

SCHEDULE: A one-year plan for one girl, showing which ward the girl will be on for each week of her course. More than one girl may be put on the same schedule.

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INTRODUCTION

0.1 The setting of the problem - The Simpson Memorial Maternity Pavilion

The practical work which forms a basis for this thesis was carried out using the course structure, staffing levels and nursing requirements which existed at the Simpson Memorial Maternity Pavilion in 1973. Most real examples will refer to this period and to the constraints which were relevant at this time. Subsequent reorganisation in December 1976, which included the amalgamation of this hospital with the Elsie Inglis Memorial Maternity Hospital, has altered the framework of the problem, and permitted a considerable degree of simplification, the most striking example of which is that it is now possible to plan schedules using blocks of six girls rather than having to allocate them individually. Some flexibility is lost, but implementation is greatly simplified. The hospital's existing solution to the new form of the problem is discussed in Chapter 3, along with the recommendations of this thesis for its improvement. The Nurses Information Booklet[1] contains the following:

> "The Simpson Memorial Maternity Pavilion is named after Sir James Young Simpson who died in May, 1870 after making outstanding achievments in the field of medicine. He was Professor of Midwifery, House Surgeon, Manager, Gynaecologist and Obstetrician. It was Sir James' studies of the problems of administering other anaesthesia in Obstetrical practice that led to his use of chloroform. As a man he had an overwhelming ambition to relieve pain, and it is true to say that every pregnant woman owes this great genius some respect.
>
> After his death, a memorial fund was set up and it was agreed that the monies so raised should be used to establish a specially

planned Maternity Hospital to be known as the Edinburgh Royal Maternity and Simpson Memorial Hospital. This was founded at 79, Lauriston Place in 1879 and functioned successfully for over 40 years. By 1926, however, the old hospital was proving to be quite inadequate in space and inconvenient for the increasing demands put on it. It was also undesirable to extend on the same site. Accordingly, the directors accepted an offer from the Managers of the Royal Infirmary to build a new Maternity Hospital in conjunction with that institution.

The new hospital and present Pavilion opened in 1939. It has today a bed complement of 208 with 28 Special Care Baby Cots. As a unit, it has proved to be very functional and because of its flexibility for modernisation the plans are to develop the existing building rather than to build afresh. The first stage of a £1m development has already commenced in our Labour Suite and it is hoped that once complete, the facilities so provided will maintain the Hospital's position in the forefront of modern maternity units."

This hospital performs a general function for the Edinburgh region by taking care of a proportion of normal straightforward deliveries, but in addition to this it has facilities for special care. Cases may be referred to the Simpson from other hospitals, particularly those in the Border region, if they promise to have any complications which would be better dealt with there, for instance in the case of diabetics. Up to 4000 deliveries are performed at the Simpson each year, of which about 25% are instrumental deliveries.

As well as providing this service to the community, the Simpson is a training hospital, teaching girls from all over the world. Its good reputation extends to most countries, with a particularly large proportion of its intake coming from countries which are, or used to be, a part of the Commonwealth. The girls who come to be trained are already qualified nurses, and as a result the course can afford to be quite intensive. As well as attending a course of lectures and theoretical teaching the girls were expected, under the previous system, to start nursing in the hospital from the first week they arrived. The wards which they started on were ones where their general nursing training was sufficient for them to cope, while they were acquiring additional skills in midwifery.

The qualifications required on entry to the programme of training are:

Registered General Nurse/State Registered Nurse or
Registered Sick Children's Nurse

The aim of the course[2] is

"To prepare Registered Nurses to become proficient in the practice of midwifery and to function as a member of a team caring for the mother and child during pregnancy, labour, postnatal and neonatal periods."

The qualification obtained is:

State Registered Midwife.

Recruitment and application are the responsibility of the Senior Midwife Tutor, South Lothian College of Nursing and Midwifery, and she shares the responsibility for selection with the Divisional Nursing Officer (Midwifery), South Lothian District.

0.2 The nature of the training course

The course which the girls take lasts for a year. During this time they will work on a variety of different tasks, gaining practical experience of different aspects of the work as they go along. It can be seen that the Simpson is particularly well suited to this function, dealing as it does with a large variety of unusual cases which a nurse in an ordinary hospital might encounter only infrequently.

The sort of work which each girl does takes several different forms. For part of her course she will work on pre- and post-natal wards where the need for general nursing care predominates. She will work on the Labour Ward where as well as performing nursing duties she will also have to perform deliveries herself. Some of the departments which she works with are not actually wards, such as the out-patients department and the special care unit. For a few weeks she will work as a district nurse outside the hospital. The aims and objectives of the course are stated in appendix 1.

0.3 The trainee's role

While the girl is being trained she is expected to work in a number of different wards which are mentioned above. While she is doing this work she will be contributing to the level of service provided by that ward. Since the number of trainees approaches the number of trained staff at the hospital, each ward depends on having a certain minimum number of trainees working there every week.

During the period with which this study concerns itself, the number of girls being trained at any one time was around 140, while ~~trained~~ nursing staff* made up around 180. The proportions are shown in detail in tables 0-1 to 0-4. The required staffing levels have been determined by past experience, and for a given rate of deliveries per year they should remain within known limits - an assessment of the validity of the accepted required staffing levels is contained in Chapter 2. Figure 0-1 shows the staffing levels graphically.

0.4 The nature of the problem

The basic problem is to plan the training courses for a number of trainee midwives in such a way that

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* All trained nurses plus auxilliaries, but excluding student midwives.

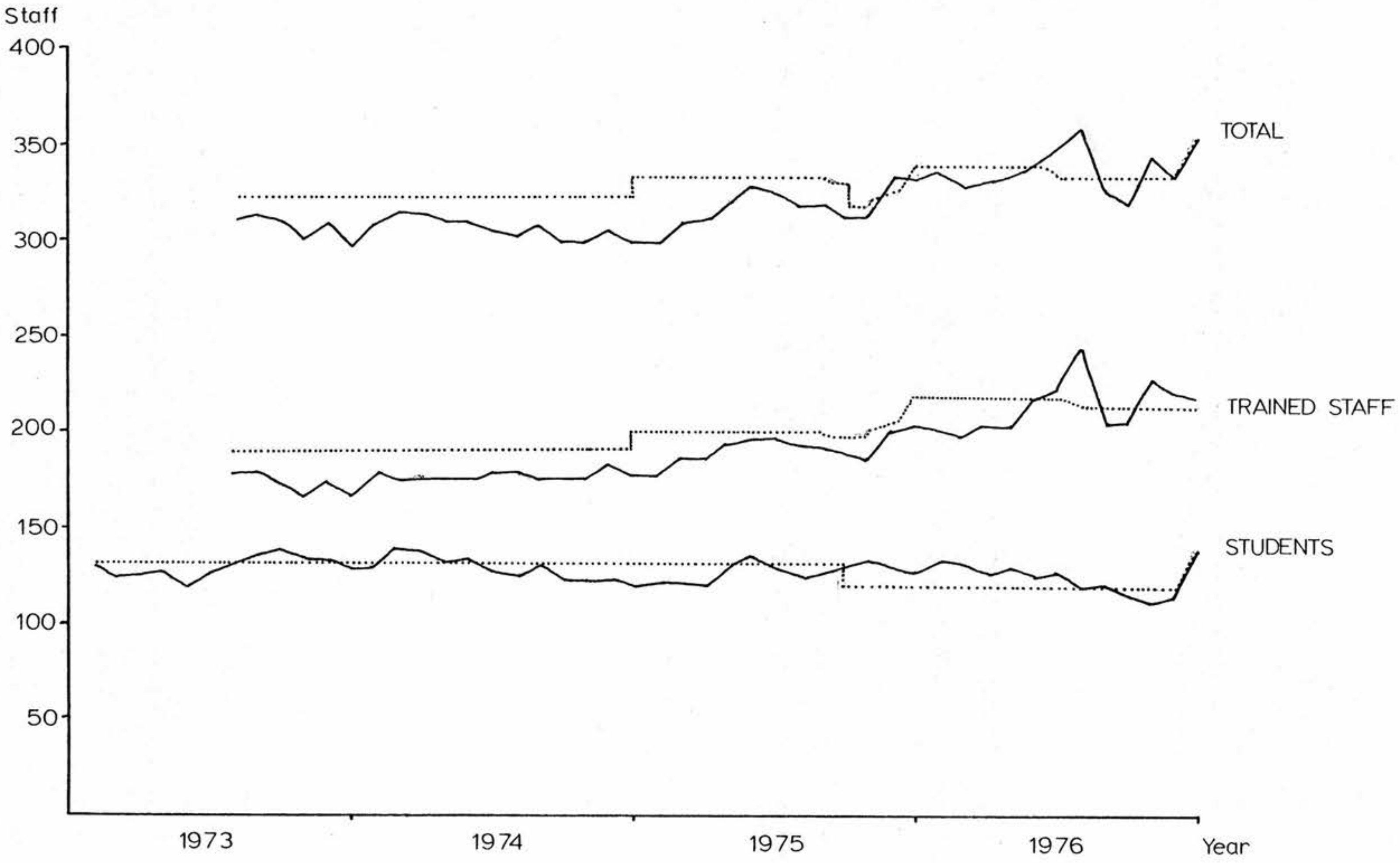

FIGURE O-1

Staffing levels, 1973 - 1976   Dotted lines show desired levels

TABLE O-1

S.M.M.P. ESTABLISHMENT COMPOSITION, 1973

1973	JAN	FEB	MAR	APR	MAY	JUN	JUL	AUG	SEP	OCT	NOV	DEC
Senior Nursing Officers	4	4	5	4	5	4	4	4	4	5	5	5
Nursing Officers (Teaching)	4	4	4	4	4	4	4	4	4	4	4	4
Clinical Tutors				1	1	1	1	2	2	2	2	2
Night Superintendant	1	1	1	1	1	1						
Nursing Officers (Service)	5	5	5	6	8	8	8	8	8	7	7	7
District Nursing Sisters												
Ward Sisters	29½	29	31	31	30	31	30	31½	31	31	33	34
Staff Midwives	39	40	42	41	34	32	33	32	32	33	29½	30½
Staff Nurses	5	6	13½	6	5	2½	6	6	3½	5½	10½	3½
Enrolled Nurses	12	12	12	12	11	10	11	10½	10½	10	9½	9½
Nursery Nurses	13	12½	12½	14	15½	16	19	17½	19½	16	16	14
Nursing Auxilliaries	57	59	58	57	61	63	63	64	61½	54½	60	59½
Student Midwives	130	125	126	130	120	126	132	136	138	134	134	130
Extra Student Nurses							8	21	14	11	17	10
Research	1	1	2	3	3	3	0	0	2	2	2	2
O.T. Attendants							1	1	1	1	1	1
Dietitian	1	1	1	1	1	1						

Fractions represent part-time staff

TABLE O-2

S.M.M.P. ESTABLISHMENT COMPOSITION, 1974

1974	JAN	FEB	MAR	APR	MAY	JUN	JUL	AUG	SEP	OCT	NOV	DEC
Senior Nursing Officers	5	5	5	4	5	5	5	5	5	5	5	5
Nursing Officers (Teaching)	3	3	3	4	4	4	4	4	4	3	4	4
Clinical Tutors	2	2	2	3	3	3	3	3	3	3	3	3
Nursing Officers (Service)	7	7	7	8	8	8	8	8	8	8	8	8
District Nursing Sisters									30	29	30	33
Ward Sisters	33½	33½	33½	33½	33½	34	33	33				
Staff Midwives	36	33½	38	37	37½	38	38	36	38	43	46	38
Staff Nurses	11½	11½	7½	8½	10½	4	7	7				
Enrolled Nurses	9	9	9	9	9	9	9	9	9	9	9	10
Nursery Nurses	14½	14	15	15	15	15	15	15	15	16	15	15
Nursing Auxilliaries	59	60	59½	59	59	62	61	60	61	59	60	60
Student Midwives	132	140	138	133	135	127	126	132	123	124	123	120
Extra Student Nurses	9	13	14	15	28	29	18	6	0	24	14	16
Research	2	2	2	2	2	2	3	3	3	2	2	2
O.T. Attendants	1	1	1	1	1	1	1	1	1	1	1	1

TABLE O-3

S.M.M.P. ESTABLISHMENT COMPOSITION, 1975

1975	JAN	FEB	MAR	APR	MAY	JUN	JUL	AUG	SEP	OCT	NOV	DEC
Senior Nursing Officers	5	5	5	5	5	4	4	4	4	5	5	5
Nursing Officers (Teaching)	4	3	3	3	3	3	3	3	3	3	3	3
Clinical Tutors	3	3	3	3	3	3	3	3	2	2	3	2
Nursing Officers (Service)	8	8	8	8	8	8	8	8	8		7	7
District Nursing Sisters	33	33	32	35	35	36	37	39½	34½	34½	41	8
Ward Sisters												32½
Staff Midwives	40	45	45	43	44	49	46	43	43	46	49½	38
Staff Nurses												5
Enrolled Nurses	8	8	10	10	11	11	10	10	9	10½	9	9
Nursery Nurses	17	17	17	17	17	15½	15½	18	18½	19	19	19
Nursing Auxilliaries	56	61	61	65	66	64	63	62	60	60½	62	59½
Student Midwives	122	123	123	128	133	129	125	127	129	132	130	127
Extra Student Nurses	21	25	12	24	26	36	17	19	14	12	17	16
Research	3	3	3	3	3	3	2	2	2	3	3	0
O.T. Attendants	1	1	1	1	1	1	1	1	1	1	1	1

TABLE 0-4

S.M.M.P. ESTABLISHMENT COMPOSITION, 1976

1976	JAN	FEB	MAR	APR	MAY	JUN	JUL	AUG	SEP	OCT	NOV	DEC
Senior Nursing Officers	5	5	2	2	5	5	5	5	5	5	5	4
Nursing Officers (Teaching)	3	3	3	3	3	3	3	3	3	3	3	4
Clinical Tutors	2½	2½	2½	2½	2½	2½	2½	2½	2½	2½	2½	2½
Nursing Officers (Service)	7	7	7	7	7	7	8	8	8	8	8	8
District Nursing Sisters	8	8	8	7	7	7	7	7	6	6½	5½	5½
Ward Sisters	33½	31½	32	35	35	34	32½	32½	33½	34½	39	38
Staff Midwives	43	44	49	49	49	52½	51½	47½	47	45	40½	53½
Staff Nurses	11	10	11	12	10½	6	12	11	3	13	20	2
Enrolled Nurses	10	9	9	9	9	10	10	10	10	10	10	10
Nursery Nurses	19	21	20	19	18	17½	18	18½	19	19½	19½	20½
Nursing Auxilliaries	60	59½	60	59	57	60	61	59	58	58	56	56
Student Midwives	133	130	128	129	124	127	121	122	116	115	115	139
Extra Student Nurses	28	26	22	16	13	19	30	14	11	13	13	14
Research												
O.T. Attendants	1	1	1	1	1	1	3	4	4	4	4	4

(1) Each girl has completed certain minimum training requirements by the end of her year's course.

(2) Each ward is fully staffed every week.

These two requirements should be numerically compatible.

Certain structural parallels allow one to express this problem in a similar fashion to the formulation of job-shop scheduling problems. If we assume that each nurse is an item which needs to be processed by different machines (wards) in order to emerge after a certain time as a finished article, then we can state that each machine has a minimum as well as a maximum capacity.

The constraints in the problem now divide themselves into two categories; process constraints and loading constraints. If the problem is re-stated in these terms, then the following formulation is arrived at:

Schedule the processing of a number of items in such a way that

Process constraints:

(A) Each item will spend a certain time duration being processed by each machine.

(B) The order in which each item is processed by different machines is one of a set of pre-determined acceptable sequences.

Loading constraints:

(C) No machine will be over-loaded at any time.

(D) No machine will be under-utilised at any time.

Since there is no standard solution method for job-shop scheduling problems with more than a very few variables this approach is of limited usefulness as far as applying a known technique is concerned.[3] However, it is necessary to explore all similarities with standard methods even if only for the sake of completeness, permitting the elimination of certain approaches. In many cases, as in the formulation above, a greater insight into the structure of the problem can be attained by such comparisons.

The two aspects, patient care and midwife training, which affect the problem may be broken down into other component aspects:

Patient care

(1) The number of trainee midwives available at a given time on a given ward must not fall below a stipulated minimum or rise above a stipulated maximum, since understaffing will cause a drop in the standard of patient care provided, while overstaffing in one area indicates inefficient distribution of staff overall.

(2) The girls present must be sufficiently trained and experienced to perform the job - certain jobs require skills and knowledge which the girl only acquires while on the course.

Midwife training

(1) A stipulated number of weeks must be done on each of a number of different duties in the course of a year.

(2) In order to achieve the best balance between theoretical teaching and practical experience it is advisable for each ward to be visited more than once in the course of the year.

(3) To avoid unnecessary disruption each ward must not be visited too often; the number of switches from one ward to another must be kept within limits. In practice rules (2) and (3) give rise to the situation where ideally each ward is visited once in the first half of a girl's course and once in the second.

(4) A set number of weeks must be worked on night duty.

These constraints combine to produce a problem of considerable complexity. In Chapter 6 it will be shown that a mathematical programming approach can be useful, but that the size of the problem presents problems in some cases. First, however, some manual approaches to the problem will be studied. In order to place this work in its appropriate setting the next section will give a brief resume of other work in the field of nurse scheduling.

0.5 Operations Research in hospital planning and scheduling

O.R. has been used in some studies in order to construct an economic model of the hospital. Attempts have been made to quantify factors such as the general need for care which has to be met[4] and the costs of supplying nursing staff with the correct levels of training and experience[5].

It is difficult to locate any work which is relevant to this problem among the large volume of medical journals. As yet computer allocation of nursing staff has not been particularly successful, nor is it widely used. Perhaps typical of a section of the published work is "Allocating Student Nurses by Computer" by Davis and Saunders[6]. In this example, the Oxford Regional Hospital board is allocating student nurses in a teaching hospital. Between 8 and 12 weeks can be spent on each duty, relatively flexible requirements, and the 500 student nurses form a smaller percentage of the total staff than do those at the Simpson. Also the different duties can be done in any order. The result of these factors is that both the staffing and the training are much less critical than at the Simpson. The computer programme has been designed merely to keep a record of each nurse's training, and to warn the allocator each time a nurse is due to move to a new ward. The allocation is performed manually - the computer is just used as an extensive filing system.

In some cases, more use is made of the computational power of existing O.R. techniques. Gupta et al[7] have used queueing theory to determine the optimal size for the messenger staff in a hospital; Williams et al[8] and Haussmann[9] have applied the same methods to out-patient service and to service requirements in a burn unit respectively.

Some work has been done on the problem of designing cyclical repeating schedules to be allocated manually. Maier-Rother and Wolfe[10] have looked at this subject, considering what number of cycles is needed relative to the numbers of girls employed on one ward at a time in order that staffing be even enough. However this only deals with twelve nurses, and only two states are considered, on or off duty, making it trivial by comparison with the problem at the Simpson. Francis[11] and Howell[12] have considered a similar problem, but in this case they deal only with up to seven personnel over a seven week period. Although day, night, evening and split shifts are considered, the alternatives are still only on or off duty. Banks[13] has published his work on manual allocation, and this is used extensively by British hospital staff concerned with allocation.

More relevant to the present problem are those papers which relate mathematical programming to hospital work. Rothstein[14] tries to minimise the personnel required for day to day staffing while maximising the number of weekly schedules with the two days off paired rather than separate. This is a fairly trivial problem, dealing with a seven day period and a small number of people to be allocated. Again only two states (on or off duty) are considered. A system is used by Newcastle Regional Hospital Board which has been described by Barlow[15]. Up to 50 nurses are considered for periods of up to 208 weeks. Acceptable arrangements of holiday, study blocks and fixed periods are fed in for each nurse. The computer then tries out "every possible arrangement of experience within the restraints set by the information given by the user" to "give the most even staffing possible for the whole three year period of training". It is stated that an average training scheme usually involves trying hundreds of possible arrangements which takes the computer only a few minutes. It is hard to see how an explicit enumeration scheme could actually consider all of the combinations of schedule provided by a 50 girl, 208 week scheme anything like as fast as this - it can only be assumed that a very large number of options are ruled out in advance for reasons of personal preference or otherwise. Certainly the staffing must not be critical in this hospital, for the results quoted are very uneven. For instance the staffing on Specialty B changes from 16 to 5 over the period which they cover, that of Specialty C from 11 to 1, and that of Specialty O from 14 to zero. Thus, although the problem is large, a very wide range of results seems to be acceptable. This would not be acceptable in the case of the Simpson.

Warner and Prawda[16] develop an algorithm which attemps to minimize "shortage costs" of nursing care over the scheduling period, while assigning nurses of the correct skill class among the wards and shifts of a scheduling period. This uses a powerful computational approach to a problem which concerns the same order of magnitude of nurses as that at the Simpson, but while considering skill classes, it has no sequencing constraints of any kind to satisfy. Finally Warner[17] has considered a heuristic approach to scheduling nursing staff which takes account of individual preferences. While the problem is not of immediate relevance, the method bears some similarities to that developed in Chapter 7 of this thesis, so a resume is given in Appendix D.

To sum up, it is evident that very little work has been done on the particular kind of problem discussed in this thesis. As a result tools which have been employed to tackle this problem have been taken from the more general field of mathematical programming - there seems to be no precedent within the nursing field.

CHAPTER 1

1.1 Introduction

In this Chapter, some different approaches to the problem are described, dealing in general terms with the nature of the questions which must be asked in order to arrive at a meaningful formulation of the problem. In the long term there are no uncontrollable variables in this problem. However at any given time the Central Midwives Board for Scotland, or through them the E.E.C., impose a series of rules external to the decision process at the Simpson[1]. These rules define values which certain variables have to assume, making them uncontrollable variables in the context of the present problem.

Most of these uncontrollable variables relate to training requirements. However when staffing levels are considered the situation is not so easily defined, since many short-term factors affect staffing levels from day to day. In many cases, the desired minimum number of student nurses required on a given ward may be open to dispute, particularly when the number is relatively large (- more than two on duty simultaneously). Even when a minimum requirement is determined, though, the scheduled number of nurses is not automatic. The question needs to be asked: in order to ensure that there is always one girl on duty on a given ward, how many nurses have to be scheduled nominally to that duty? Because of the fact that service has to be provided for twenty four hours a day, the shift work structure has to be taken into account.

Unlike that fixed factor, short-term absenteeism can not be calculated precisely, and instead sufficient extra staffing has to be provided so that, given the prevalent absence levels, it will always be possible to provide one nurse of sufficient experience at all hours of the day, or that failure to do so will occur sufficiently infrequently that the level of disruption caused by short-notice re-arrangements of the schedule is kept within acceptable limits. Thus discretion and judgement are needed to establish a realistic range of desired staffing levels.

For a given status quo a realistic way of approaching the problem is to start by fixing the values of certain variables and to see if a solution can be achieved. If no acceptable answer is found, the variables which have been taken as uncontrollable can be re-assessed and altered, and the procedure repeated. So, with regard to staffing levels, the first approach to the problem is to try to design a model which provides solutions which do not violate the minima currently used in the manual method. Only if this fails is it necessary to re-examine the minima in order to see if they can be altered. This is not to say that minimum staffing levels quoted by the Hospital are accepted without question, rather it makes a distinction between long and short-term planning. This is clearer when an example is considered.

If the staff of the Simpson say that twelve girls are needed on a given ward for any given week, then it is reasonable to enquire whether ten or eleven would suffice. Because of the shift system a scheduling of twelve will provide two or three girls on duty at any given time[2]. (See Chapter 2.11 : Determination of staffing requirements.) However if the Simpson says that the minimum number of girls actually on duty on a given ward is one, then this figure brooks no argument - it is fixed in the short-term, and has to be accepted for the purposes of constructing any model.

In the long-term, however, it is reasonable and necessary to ask the question: If ward A was moved geographically to be adjacent to ward B (whose minimum on duty was also one) could a single nurse keep an eye on both wards simultaneously without letting the level of patient care suffer? If the answer is yes, then a change could be made to the physical location of resources and facilities which would alter the constraints affecting any model. Thus a short-term uncontrollable variable can become a decision variable in the long-term.

1.2 General solution structures

Let the following factors be taken as fixed:

(1) Each girl is present for one year.[3]

(2) During that time she is sometimes available to work on the wards, but not always.

The factors which could cause her absence are:

Holidays
Lectures
Community work
Illness or indisposition

The first three are predictable and can be pre-planned; their structure will be built into any model. The last factor, absenteeism, is a variable which for most purposes can be treated as random. Its affect on planning is allowed for in two ways

(A) A safety margin must be left when overall minima are determined, so that there are always enough girls who can be re-allocated at short notice.

(B) Provision must be made for girls who have been ill to catch up on the experience which they have missed. Commonly this is allowed for by planning for the course to end with a holiday period. Any girl who has missed out on some clinical experience can work during this final holiday, without having to extend the period during which she is nominally at the Simpson. This prevents disruption to any plans she has made for the time after she is due to leave the Simpson, possibly to take on a new job.

Holidays and community work have remained significantly unchanged over the period concerned in this study, but lectures have been altered (see Table 8-3). Under the old system a girl would spend one day a week studying even when nominally on a given ward. She would also miss the occasional hour or two's work by attending lectures. These factors were taken into account when establishing the old minimum staffing levels (i.e. pre - 1977). This course structure had to be accepted as an internal uncontrollable variable; the decision to alter it could only come from authorities external to the Simpson. In October 1975 the Central Midwives Board for Scotland issued a recommendation[4] that the study time should be re-arranged to fit into specific weeks during which no ward work would be done. These would be arranged into blocks of one or two weeks at intervals throughout the year's course. It was this structural change which permitted the approach described in Chapter 8.

(3) The number of girls being trained at any one time is fixed (within defined limits).

(4) The total number of all staff, trained and student, is fixed. Once again this total is within defined limits, since the actual fixed figure is an overall budget for salaries for the establishment, set externally. Thus it would be possible to employ more nurses of lower grades and wage scales, or fewer nurses of higher grades. However, as stated earlier, for the sake of constructing a model it is realistic to start by taking the size and quality of the present establishment as being fixed. Thus we can say that the number of staff is fixed.

(5) Minimum staffing requirements are fixed. To repeat, these levels are to some extent discretionary, but at this stage, as with establishment size, we can say let the levels be fixed, without specifying at what level. In other words, in order to investigate the structure of the problem, we are really labelling these variables as short-term uncontrollable variables; we will investigate and define their specific levels and values later.

Now let us consider the main decision variables. These are as follows:

(A) How many intakes do we have each year ?

(Defining: What is the size of each intake?)

(B) How many different schedules do we use?

1.3 How many intakes per year?

To understand the conflicting factors involved in this problem, some extreme solutions will be considered:

(1) Have one intake per year of N girls, where N is the total complement of staff at any given time. There are two main reasons why this is not practicable. Firstly in order for staffing to be even on all wards at least some of the girls would have to start on the Labour Ward in their first week, without any training, which would obviously put the patients at risk. Secondly, the level of experience would fluctuate wildly - at the end of each year the Hospital would be staffed by practically fully trained midwives; the next week they would be replaced by complete beginners. Again this would produce a less satisfactory service for the patients.

(2) Have N intakes every 365/N days (N still defined as above). With a training complement of over 100 this is impracticable, since with girls arriving individually every two or three days throughout each week it would produce a situation of unneccessary complexity - it would be impossible to plan a coherent pattern of study for them or even to keep track of who was doing what.

Obviously both of these extremes are ludicrous, but they enable us to establish between what limits the best solutions will lie. Table 1-1 lists the multiples of days which are most likely to be useful.

TABLE 1-1

USEFUL MULTIPLES OF INTAKE AND COURSE LENGTHS

INTAKES × TIME PERIOD = TOTAL DAYS	BEST FIT	SIMP-LICITY	EVEN-NESS	ADMINISTRATIVE SIMPLICITY	NOTES
1 × 365 = 365	***	***	***	↑	Annual
2 × 180 = 360		***			
3 × 120 = 360		***			
4 × 90 = 360		***	***		
4 × 91:(13 × 7) = 364	**	***			Quarterly
6 × 60 = 360		***			
7 × 52 = 364	**	***		EITHER EXTREME	
10 × 36 = 360				IS UNACCEPT-	
12 × 30 = 360		*		ABLE: CENTRAL	
12 × (4 or 5) × 7 = 364	**	***	***	REGION IS BEST	Monthly
13 × 28:(4 × 7) = 364	**	**	***	COMPROMISE	4-Weekly
18 × 20 = 360					
20 × 18 = 360					
28 × 13 = 364	**				
26 × 14 = 364	**	***			
30 × 12 = 360					
36 × 10 = 360					
52 × 7 = 364	**	***	***		Weekly
365 × 1 = 365	***	***		↓	

CONSTANT
EXPERIENCE

A blank in any column indicates that the intake category concerned possesses no advantage with respect to that characteristic. A rating of one to three stars indicates a particular advantage with respect to that characteristic, with more stars showing a greater advantage.

In order to make a decision between these possibilities there are several factors which have to be considered. The decision will be based mainly on practical considerations, but if implementation of the scheme is being considered it is as important to design a scheme which is easy to employ as one which is mathematically elegant. The following criteria were considered:

(A) Simplicity of pattern

Ideally the system should fit neatly into the calendar year, so that its logic is easily understood and remembered. Given this requirement certain decisions still have to be made; for instance is it neater to have girls arriving on the same date each month, or on the same day of the week each month?

(B) Evenness

There is a relationship between the number of weeks spent away from ward duty and the number of weeks between each intake. If the former are almost multiples of the latter, then an even overall staffing will be possible. For instance, to take arbitrary figures, if there was an intake every ten weeks, giving just over five intakes per year, and if each girl spent ten weeks away from the wards, then, as long as these weeks were distributed in some even and interlocking fashion, at any given time there would be exactly four intakes on ward duty.

In the example above, if twenty weeks were spend off the wards, the staffing could still be arranged evenly, but any figure between multiples, or less than the number of weeks between intakes, would cause large variations in the staff available for ward duty.

(C) Experience levelling versus administrative complexity

As mentioned above, if the number of annual intakes is small, the drop in overall average experience level will be great at each intake, whereas if the number of intakes is large, there will be a large number of small groups of girls to plan for and keep track of, leading to frequent personnel changes on each ward. (The senior staff of the Simpson are of the opinion that increasing turnover of staff on the ward tends to lead to lack of stability and continuity.)[5] The best solutions will occur between these two extremes.

Let us examine these factors one by one to see which solutions they tend to suggest in conjunction.

(a) Simplicity

The most elegant solutions would be to have intakes either weekly, monthly, or quarterly. The arrival of each new set would tie in neatly with the calendar, and be easily remembered. Weekly arrivals have the advantage that each intake would always start on the same day of each week, a bonus not shared by the other two systems. A quasi-monthly system could be used which, by ignoring a discontinuity at the end of the year, would also ensure arrival on the same day of the week each time;

the thirteen intake system at four-week intervals. This system however loses one benefit of the monthly and quarterly systems - they can be referred to by the name of the month or season of arrival, whereas it would have to be labelled in some arbitrary fashion, thereby losing its quality of simplicity in application; not a critical point, but worth noting.

The next point is that for the purpose of organising the movement from one duty to another it is essential that any given period can be sub-divided into an exact number of weeks, since a week is the smallest unit which this plan is designed to cope with. No girl is ever placed on a ward for less than a week, and the training requirements are all in multiples of weeks. Any period, defined by an overall scheme, which involved numbers of days rather than even numbers of weeks would be unsuitable for the purpose for which it was intended. This eliminates most of the alternatives listed, but highlights two in particular as being suitable:

(i) The quarterly intake divides the year into four thirteen week periods.

(ii) The four-weekly intake divides the year into thirteen four-week periods.

By assigning an arbitrary length of four weeks to each month other than March, June, September and December which are five weeks long, it is possible to use a monthly intake. This keeps each quarter a constant length of thirteen weeks, but has several disadvantages, not least of which are the problems caused by the irregularity of the course structure.

(b) Evenness/

(b) Evenness

Under the two systems, the number of weeks when a girl was not available for nursing duties were as follows :

	Year	Study	Holiday	Total
Monthly :	1973		5	5
Quarterly :	1977	9	5	14

Table 1-2

Under the 1973 system, if the schedule was the same for each girl in an intake, at least ten intakes would be needed before the five weeks of holiday were spread out evenly throughout the year. Under the 1977 system, with fourteen weeks out of 52 spent on activities other than ward duties, it is possible to spread out the weeks of ward attendance by using only four intakes.

With quarterly intakes, and one overall repeating schedule, once every thirteen weeks there would be two different intakes on holiday simultaneously, with only one intake being on holiday for the other twelve weeks. By employing more than one schedule, it will of course be possible to spread out this peak of absence, but not to achieve complete evenness of availability until the number of schedules increased to equal the number of weeks between intakes. Still considering a quarterly intake, it can be seen that, although with thirteen different schedules being used it would be possible to spread the periods of absence through study and holiday out evenly, there is still no guarantee that a system which causes one girl to take a certain holiday thirteen weeks later in the course than her fellow student will be acceptable.

Although the pattern can be adjusted till it fits mathematically, there is no guarantee that the training schedules which result will be good ones. The question of the number of schedules will be discussed later in this Chapter. Thus on the criterion of evenness the simplest regular staffing levels will be achieved when the number of intakes is equal to, or a multiple of, the number of weeks in a year divided by the number of weeks when a girl is unavailable for nursing under any given system:

$$\text{minimum number of intakes} = \frac{52}{\text{unavailable weeks}}$$

So, under the 1973 system, there must be over 10.4 intakes per year, and under the new system there must be over 3.7 intakes per year in order for it to be possible to achieve an even staffing level with the minimum number of schedules.

(c) Experience levelling versus administrative complexity

As mentioned above, in the extreme case of one intake per year, the average level of experience drops from 51½ weeks just before the end of the year to half a week one week later, an unacceptable drop. Let us consider the variation which occurs with three other systems, the weekly intake, the monthly and the quarterly.

Weekly intake: From the middle of one week to the middle of the next there is no alteration in average experience levels, since both points in time are half way through a cycle. The maximum fluctuation will always be less than a week, so this can be taken as one extreme case with the of annual intakes as the other.

Monthly intake: In order to compare regular systems we will consider here the four-weekly intake, values for which will be very similar to the monthly intake. Half a week before the end of a cycle (i.e. half a week before the next intake), and half a week after a new intake are the two points in time compared. The average number of weeks completed by the students drops from 27½ to 24½, a fall in average experience level of 10.9%.

Quarterly intake: If the same two points, half a week before and half a week after an intake, are compared the drop is from an average experience of 32 weeks to 20 weeks, or 37.5%.

So, to tabulate the drops in experience level as measured by average number of weeks per girl already completed, the results are as follows:

Intakes	Drops in experience level
Annual	100. %
Quarterly	37.5 %
Four-weekly	10.9 %
Weekly	0.

Table 1-3

This factor is thus quantifiable in its simplest form, but it must be borne in mind that the quality of the overall schedule affects the acceptability of any given solution. For example, if a schedule spreads the experienced girls in the latter parts of their course evenly throughout the wards, then more supervision is available for the less experienced girls, thereby mitigating to some extent overall variations in experience level. However if the experienced girls tend to move from ward to ward together in a group, then the average experience level of the girls who are on other wards will become critical.

More difficult to quantify is the degree of administrative complexity. There are several factors involved here.

(i) Selection. This is simpler and fairer if intakes are large, since all of the candidates' qualifications can be compared side by side. This would tend to avoid the situation where a less well qualified girl might be accepted because she was able to start at a time of the year which was unpopular and poorly subscribed.

(ii) Variations in numbers of actual arrivals. Not all of the girls who are awarded a place at the Simpson actually arrive, and allowance has to be made to permit some degree of oversubscription to counteract this. There are points to be made for and against frequent and infrequent intakes on this account. With infrequent large intakes, the Law of Large Numbers will ensure a smaller proportional variation.

To illustrate this point consider the actual intake sizes for 1973. The set sizes were as follows:

Jan.	11	July	14
Feb.	6	Aug.	12
Mar.	12	Sep.	17
Apr.	13	Oct.	12
May	10	Nov.	12
June	12	Dec.	9

Table 1-4

The preferred size of set at this time was 12, so the average absolute deviation from the correct number for both halves of the year is 13.8% (both are the same only by co-incidence).

If the first six months are grouped together and taken as a single intake, and similarly with the second six months, then the desired number is 72 in each case. The actual numbers are 64 and 76, giving deviations respectively of -11.2% and + 5.5%, an absolute average of 8.4%.

The second point in favour of infrequent intakes is that, once the girls have arrived, the size of the student body is then known precisely for a reasonable period ahead* and plans can be made accordingly. On the other hand, if for some reason there is a critical drop from the anticipated number of arrivals, then it is impossible to make adjustments till the arrival or at least recruitment of the next intake. With frequent intakes a low figure one period can be rectified sooner, but a time lag still

* barring drop-outs and long term absences

exists.

(iii) Theoretical teaching. Where large groups with the same level of clinical experience are a disadvantage from the nursing point of view, because of the irregularity in average experience levels discussed above, they are an advantage from the teaching point of view. With relatively few intakes it is possible to have larger classes all with the same level of practical experience and all at the same stage of their course. On the other hand with frequent intakes it is either necessary to teach girls of different experience simultaneously or to deliver the same lectures again and again to new groups as they reach the required level.

A summary of how these criteria affect different intake patterns is given in Table 1-1.

It can be seen from the above assessment that there is no one system which is entirely satisfactory on all criteria. The insistance that the periods between intakes should be an exact number of weeks limits the choice to annual, quarterly, four-weekly and weekly intakes. The monthly intake can also be adjusted to work out in exact numbers of weeks by assigning the appropriate number to each nominal month. Of these five alternatives, all are neat and the sets easily labelled except for the four-weekly intake.

The concept of evenness which dictates that the interval between intakes should ideally not be greater than the total number of weeks that each girl is unavailable for nursing rules out annual intakes under both the 1973 and 1977 requirements. The annual intake system is also ruled out by the average experience level criterion. Under the 1973 system, a quarterly intake is too infrequent for the number of unavailable weeks to be spread out evenly, so we are left with the weekly, four-weekly and monthly system for the 1973 requirements.

The weekly intake system is then elimated because of the administrative and teaching problems it would introduce. This leaves a choice of monthly or four-weekly for the 1973 system and quarterly, monthly or four-weekly for the 1977 system. At this stage it is a question of assessing priorities, since each system has its drawbacks. The choices that were made by the Simpson were as follows:

<u>1973-1976</u>

During this period the monthly system was used. The actual system is described in detail in Chapter 3, but suffice it to say here that this system was probably adopted for reasons which were not specifically examined by the Simpson. Many of the scheduling problems, caused by the complexity of the irregular system, were no doubt not anticipated when the system evolved. Its advantages were neatness, in that each set was labelled with the name of the month of its arrival, and a capability of providing very even staffing on most wards with a basic plan (referred to as the roster in this thesis) which was very similar for every girl, and was highly suitable in terms of the training pattern which was provided.

Its disadvantages were the administrative complexity of coping with twelve intakes each year from both the planning and teaching points of view, and the fact that, since the lengths of service on each ward did not fit neatly into the number of weeks available for them, it was impossible to use a standard repeating schedule which would ensure even staffing. As a result it was necessary to invent a unique schedule for each girl to spread out the staff availability, a task of great complexity whose time requirements were excessive. More will be said on this subject later.

Post 1977

From the beginning of 1977 a change to a series of quarterly intakes was commenced. The advantages of this are great from the administrative point of view. It is now possible to design standard schedules which can be repeated to give the required staffing, obviating the need for individual scheduling. Administratively it is simpler, and above all, the teaching system is now much more coherent and easily planned. However, there are disadvantages. The flexibility of scheduling only a few weeks ahead is lost, since it is harder to make short-term allowances for unpredictable alterations in staffing levels caused by absences or nurses leaving and the evenness of staffing will rarely match that under the old system. The training programmes are also less satisfactory, since the schedules have had to be changed from a training-oriented point of view to one relating more to staffing requirements. For instance, under the new system it is essential for some girls to visit the Labour Ward first, after only two weeks of theory, whereas in the past they always visited either the Ante- or Post-natal Wards first. Not only is this less satisfactory from the girl's point of view, giving her less chance to settle in and learn her way around before being given such responsible work, but also it is inevitable that the patients will be attended by some girls less experienced than the girls were previously.

There is another point which should be cleared up here: it will have been noticed that the convenient multiples discussed earlier in this Chapter never added up to exactly one year - in fact with leap years occurring it is impossible for them to do so. The quarterly system does have an advantage here. Arrivals occur on the 1st of March, June, September and December, giving quarterly lengths in 1977 of 12 weeks 6 days to 1st March, 13 weeks 1 day to 1st June, 13 weeks 1 day to 1st September and 13 weeks exactly to 1st December. Thus although the quarters are almost exactly divisible into weeks, any odd days can be allowed for by altering the exact time spent on study each time at the start of the course. Although problems may still arise if the adjustments are not made judiciously, the discrepancy will probably be maintained at a lower level than under the old system, where the intake for one month arrived earlier and earlier in the previous month as the years passed. It still may be necessary at some time in the future to have a long week of perhaps ten days in order to bring the plans back into line with the calendar.

1.4 How many schedules are needed?

Let us again start by considering the extreme cases. The first is to have a unique schedule invented for each girl. This method obviously involves a lot of work, but will give in the end, if successful, the most even staffing possible by any method, possessing the flexibility to take care of even short-notice discrepancies. It can be used regardless of how many intakes there are each year.

The other extreme is to have one standard schedule which is applied to all arrivals. Unlike the other extreme, this one is dependent on there being sufficient intakes to even out the staffing. An example will best illustrate this point. Suppose that the standard schedule placed a girl on community work for four weeks starting in week 26. In order for there to be continuity of staffing, one group would have to start its twenty-sixth week at the same time as the previous one started its thirtieth. Thus with a unique schedule the number of intakes must be such that the time between each is no greater than the time of the shortest duty required for the training of each nurse. If we now consider a ward which has to be visited for a period of five weeks it can be seen that, with one schedule and the required four-weekly arrivals, the number of girls on this ward will double for one week in every four as two groups arrive there simultaneously.

This leads to the conclusion that if differing lengths of time have to be spent on different wards it will be impossible to achieve even staffing with only one schedule. Indeed it will still be extremely difficult even with more than one schedule, perhaps impossible, but at least the variation in staffing levels will have the potential to be progressively reduced as the number of schedules increases.

The relationship between length of duty on a given ward and intake frequency can be taken further. Assume again that community work is done for four weeks, but that there is a twelve week period between intakes. It is easily seen that in order to spread the work out to provide even staffing for the community section it will be necessary to have three different schedules. This introduces the drawback mentioned in Section 1.3 while factor (b) was being discussed. If it was already decided that the best schedule which could be devised placed a girl on community work for four weeks from week 26, then we are now faced with the problem of inventing two more schedules which are less satisfactory in order to maintain even staffing. These two new schedules might put girls on community work for four weeks starting on weeks 22 and 30 respectively. Thus, although the three schedules put girls on this duty in such a way that they are consecutive to each other, there is still an eight week difference between the commencement week for community work between two of the schedules, over 15% of the total length of the course.

The situation is complicated still further when the thirteen week gap between quarterly intakes is considered, instead of the twelve week period used above. The number of schedules required to give completely even staffing now jumps to thirteen, providing four out of the thirteen groups on duty at any one time. Compare this fraction of four-thirteenths with the fraction of one third occurring in the previous example. The difference in commencement time between the extreme groups will now have to rise from eight weeks to twelve weeks, almost one quarter of the total course length.

Once again we can see that compromise will be necessary. The more schedules that are employed, the more even is the staffing provided. Unfortunately here we have two factors in opposition to this - as the number of schedules increases, not only does the complexity of administration increase, but also the quality of the training supplied to each girl falls further and further on average below that provided by the one or the few ideal schedules, since girls may be forced to go on to certain wards at inappropriate times during their courses, where there has been insufficient chance to cover the necessary ground in their theory course. The decision as to how many schedules to use is affected by these factors and also by the compatibility of the relationships between the length of visits to each ward and the length of time between intakes as described above. Thus two approaches are possible. Firstly, it will be evident that some duties will be more difficult to deal with than others; for instance a ward or duty which is visited only once by each girl and for a short number of weeks will be more difficult to schedule in order to provide even staffing than one which is visited more than once, for a total number of weeks which is greater. A problem ward like this might dictate a certain minimum number of schedules, as in the case of community work described above. This minimum number may still have to be increased in the light of the evenness of staffing provided on the wards.

The second approach is to work out the best achievable staffing for any given structure and number of schedules. Let us take as an example the quarterly intake system. The mechanics of this calculation will be explained in more detail in Chapter 8 , but suffice it to say at this stage that, if each girl goes on a certain ward for four weeks once during the year, and if there is an intake every thirteen weeks, then using the most even spread possible the number of girls on that ward at any time will be four thirteenths of the size of each intake. This can be achieved when the number of schedules is unlimited, and if fractions of a nurse are permitted. However if nurses and sets of nurses are taken as being indivisible, then completely even solutions will no longer be possible in most cases. Moreover, if the number of schedules is restricted the distribution of staffing will become more uneven still, since a spread of staffing can only be achieved by sufficient variety of schedules. Let the example be taken further to illustrate:

During any given quarter of thirteen weeks, one intake will do its duty on the ward under consideration for four weeks. We can say that the total staffing available for the quarter is 4 intake weeks.

If we use six schedules, we divide the intake up into six separate sets. Total staffing available is now 24 set weeks.

We have thirteen weeks in the quarter, so given a completely even spread we would have 1 11/13ths sets on duty each week. However, since by definition each set cannot be broken up into subsets, thereby increasing the number of schedules, we must reject this fractional answer. What it tells us is that for eleven weeks out of the thirteen there will be two sets on duty, while for two weeks each quarter there will only be one. This is the best possible staffing achievable in this situation with six schedules. There is no guarantee that a plan can be devised which will produce this in practice, since the staffing for other wards may conflict, but at least we know what the target is that the scheduler is aiming for.

Thus, without having to produce and design a plan, we still know what its best possible results will be, and we can press on or reject it on this criterion. Let us say that the prospect of halving the staffing on this ward eight weeks in the year was not acceptable. We could then repeat the calculation for another number of schedules, say twelve:

Staffing = 4 intake weeks
= 48 set weeks if there are 12 schedules
Staffing/wk = 48/13 throughout each quarter
= 3 9/13

It is possible to demonstrate this algebraically:

Let N = the number of intakes per year

tn = the time spent on ward n

S = the number of sets in each intake

The fraction of an intake on ward n each week = $tn/\frac{52}{N}$

$\therefore$ The number of sets on ward n each week $= (tn/\frac{52}{N}) \times S$

$$= \frac{tn \times N \times S}{52}$$

In the case quoted above the number of sets on ward n each week will be $\frac{4 \times 4 \times 12}{52} = 3\frac{9}{13}$

The implication of this vulgar fraction is that the minimun number of sets for any week will be 3 (the answer ignoring the remainder), while the remainder indicates that there will be an extra set available for 9 weeks out of every 13. So there will be 4 sets on duty for nine weeks and 3 sets on duty for four weeks in each quarter. The drop now is only one quarter, but it happens for four weeks each quarter instead of two. This will no doubt be more acceptable on balance.

Each different ward can be examined in this way, and the number of different schedules eventually decided upon would be the minimum one which still gave an acceptably small variation in staffing levels. The next step would be to attempt to design a set of schedules which attained these optimum levels, bearing in mind that they might not be simultaneously possible for all wards. In the example above, for instance, it might be that the pattern needed for the other wards forced a solution where for one week there were only two sets on the ward considered above, and for another week five.

There is yet another compromise available when the number of schedules needed is being assessed. It is possible to standardise some elements of the schedules and to leave others variable. For instance, in the case where each girl had an individual schedule which was only worked out for a few weeks in advance it would still be advisable to pre-plan which weeks each girl would go on holiday. Here we differentiate between what we have called "roster" and "plan" when discussing the type of formulation which schedules individually. We have used "roster" to label the pre-decided elements of each schedule and "plan" to describe the week by week allocations. This is described in more detail in Chapter 3, but the system which was in use up to 1977, which was incorporated into our model, used a roster which pre-determined holiday, community work and night duty times. There were six different types of roster, and between them they achieved even staffing on community work and night duty, but rather variable numbers of sets on holiday at any given time, from one to four.

1.5 Continuous versus discrete allocation

In this context these terms are to be defined as follows. It is possible to allocate for a short period ahead, continually updating the scheduling in real time as the weeks pass. A decision made now will affect planning in weeks to come, but its implications are not necessarily determined explicity. This is referred to as continuous allocation. An alternative method is to take a period of time in the future and make allocations within that period which are self-contained and do not affect other time periods. This is referred to here as discrete allocation.

Let us assume that a decision has been made as to how many intakes and schedules there should be in a given context. These are now taken as uncontrollable variables, and the problem they define has to be formulated in such a way as to permit values to be assigned to the controllable variables. A different approach has to be made in the case of two different types of formulation. In one, where a relatively small series of schedules can be devised so that cyclical repetition will provide even staffing, the allocation will be done once and for all by manual methods involving pattern manipulation with some degree of trial and error. In the other, where individual schedules are invented from week to week a decision has to be made as to the approach to take.

1.5.1 Continuous allocation

It is possible, both manually and by computer, to design a continuous system which allocates one week ahead at a time. It can be seen that a heuristic system has to be used here, since without the facility for looking ahead to assess the after-affects of each decision it is impossible to minimise overall staff shortages. In one approach the primary objective is to provide even staffing for the week being considered (one week ahead) by using the resources currently available, i.e. the nurses who are so far not allocated for that week. As a direct result of this, it becomes a secondary objective to allocate these nurses in such a way that they will complete all of the duties laid down in their training requirements by the time they leave the course.

A different approach is to allocate for the week ahead in such a way that each girl is likely to be able to complete her training programme, relegating the staffing requirements to the status of a secondary objective. By this method neither aim will actually be guaranteed, although usually schedules will be arrived at which are satisfactory. This method can only be used with the present allocation problem at the expense of some degree of inefficiency through overstaffing.

1.5.2 Discrete allocation

In order to introduce optimisation techniques it is necessary to isolate discrete portions of the problem. To take a very simple example, if there was a yearly intake there would be no overlap between groups, so it would be possible to solve the scheduling problem for one year ahead at a time. An iteration scheme could be designed which improved the regularity of staffing each time, and testing for optimality would be easy, using two main criteria.

(1) By one method the optimum levels for staffing would be known in advance by a series of simple calculations as described in the section of this Chapter relating to the choice of numbers of schedules. All that would be required would be a check at each iteration to see if these levels had been achieved.

(2) The other most useful criterion would be to assess the variations between the required and actual staffing levels achieved at each iteration. If the objective was to elimate all under-staffing, with over-staffing being ignored, then the optimum point would be recognised when the sum of all shortfalls equalled zero.

Similarly the absolute deviation from the required level could be summed and progressively reduced towards zero. These objectives would be pursued on the assumption that satisfying the training programmes of each girl was guaranteed by the structure of the formulation. This could be achieved for instance by supplying a list of feasible schedules and devising an algorithm which would choose a suitable combination of them.

This method of isolating a discrete portion of the problem, for a specific time ahead, will also work even when there are overlapping intakes. It is possible to start at a certain point where, for example, one intake is starting and others are partially completed, and to work ahead for a quarter or a year. As long as the schedules and portions of schedules used are feasible then the problem is still to minimise some measure of deviation from the stipulated staffing levels. Thus it is a property of the series of methods of solving scheduling problems for a fixed period that they may also be applied to sections of a continuing problem, either consecutively or with an overlap between time periods which ensures some form of updating in order to take account of changing circumstances such as girls leaving half way through their course.

If the discrete unit being considered represents only part of the total staffing for a period, then overall optimisation will not be achieved, but in some circumstances it would be an advantage to ascertain the optimal way of distributing the nurses whose schedules to date were known, then to add to that the schedules for more recent arrivals.

This property has proved useful when investigating the applicability of different solution methods as in Section Three, since one which is inefficient can be ruled out after a trial with a reduced and simplified problem, saving the waste of time which would be associated with formulating each type of approach to the problem in a way which would cope with the complexity and scale of of the real problem.

This does not imply that a full scale continuous problem can be solved by decomposing it into smaller parts; rather it means that any method which fails to work at the simple level will certainly not work at the more complex.

CHAPTER 2

2.1 Introduction

In this Chapter the constraints affecting the aspects of the problem discussed in Chapter 1 are discussed in greater detail, with differentiation being made between the situation as it existed in 1973 and as it has evolved since 1977.

At this stage it is useful to list the different types of ward or department which a girl is likely to visit during her course, since their individual characteristics determine many of the scheduling rules.

WARD 49 :	This is the Pre-natal Ward where patients stay before the birth. The reason that some mothers come in for a while before the birth has already been referred to: this hospital takes a number of cases where complications are expected. The seriousness of the complications varies, but many (high blood-pressure in the mother for instance) might make it advisable to monitor the progress of mother and child before the birth. About a quarter of the trainee nurses start their training on this ward.
WARDS 51,52,53 :	These are the Post-natal Wards where the patients receive after-care. Girls who do not start their course on Ward 49 will start on one of these wards.
WARD 54 :	This ward is for patients who need to be kept in isolation, for patients with temperatures and for

terminations of pregnancy.

LABOUR WARD: While each girl is here she is expected actually to perform a certain number of deliveries as well as assisting at others.

SPECIAL CARE UNIT: This is not actually a ward, but is the department where all critical cases are handled - babies who are premature or are not well for some reason or another.

All of these wards require a certain minimum staffing level for night duty as well as for day duty. Their requirements will be discussed in more detail in Section 2.6.

OUT-PATIENTS: This department needs a certain number of trainees during the day but is not open at night.

DISTRICT WORK also known as COMMUNITY WORK: For a spell of four or five weeks each girl will do some district nursing outside the hospital. Despite the fact that it is outside it is still a necessary part of the course.

2.2 Generation of a simple course structure

Now that we know more about the types of work which a girl will do on the course we can start to define some simple criteria governing the order in which she visits each ward. Before we start to generate a structure, however, two points should be made clear:

(1) The criteria which will be discussed and the descriptions of the training which will be given, in the rest of this Chapter relate to the external constraints which obtained in January 1973. An attempt will be made to generalise and to explain the rationale behind the 1973 course

structure as implemented by the staff of the Simpson in order to facilitate understanding of the various influences which governed the form of the training programme. Later, when dealing with the computer model, many of the rules of thumb discussed here will be adopted as fixed constraints.

(2) This Chapter is not intended to be a complete description of the problem but instead should act as a general introduction to a series of concepts which will be discussed in more detail, and from a more quantitative viewpoint, in the next Chapter. A deeper assessment of the validity of these concepts will be made in Chapter 3.

Let us now set out a few simple rules which will affect the order in which a girl visits the various wards. These are not necessarily binding, but are factors which helped to shape the course in 1973.

2.2.1 Some simple rules governing course structure

(1) A trainee should only visit a certain ward when she has learnt enough from her lectures and her experience on other wards to perform her job competently.

(2) A trainee should only do night duty once she has achieved a sufficient general understanding of the work, since at night there are fewer other people about to help her.

(3) If a trainee visits a certain ward early on in her course she is likely to learn less from the experience than she would do later, with the aid of more theoretical knowledge. On the other hand, the theoretical training will be more easily absorbed if she knows what conditions are like in

practice.

(4) A trainee is likely to learn less if she is moved frequently from ward to ward - she will have less opportunity to settle in and learn the routine in any one place. The standard of service which she provides is also likely to suffer. Let this stand, therefore as a proviso to rule three - that a trainee should be moved as few times as possible.

(5) A trainee should not start a spell of night duty at the same time as starting a spell of work on a new ward - the move to the new ward should occur at least one week before the commencement of night duty in order to let the girl find her way about. This is for the same reason as (4) but is more specific in its application. The rule is broken sometimes in practice, but this will be dealt with later.

These rules can be divided into two categories:

NEVER BROKEN - (1) Practice should be preceded by sufficient theory. (See 1 above.)
(2) Night duty should be undertaken only when the nurse is sufficiently experienced to work alone.

DESIRABLE - (3) Each ward should be visited more than once.
(4) As few changes as possible should be made.
(5) When night duty is to be done on a certain ward, the move to that ward should be at least one week prior to the change to night duty.

2.2.2 The implications of these rules

Let us take these rules one by one and see what concrete ideas are suggested by each.

Rule 1: The first type of ward to be visited is critical, since it represents the first practical training of the course. At this stage the student nurse's practical experience will be least. An inspection of the list of wards indicates that Ward 54, the Labour Ward and the Special Care Unit require specialised skills. Ward 49 is slightly more complex than the three Post-natal Wards, but is still within the capabilities of a trained nurse. The course should therefore start with a session on any one of Wards 49, 51, 52 or 53.

District work should be undertaken only once a girl has visited every type of ward at least once, so it will need to be done about half way through the course, if not later.

Rule 2: This rule would prevent us from putting a girl onto night duty as soon as she arrives at the hospital, but would let her do such work after she had been there for a few weeks.

Rule 3: The problem can be alleviated by arranging for each girl to visit each type of ward more than once.

Rule 4: The effect of this rule is to limit the number of visits to each ward to two if possible.

Rule 5: This prevents us from starting a block of night duty immediately after a block of district work or holiday, since it would then be impossible to do one week of day duty on the same ward prior to the change.

It has now been established that each ward should be visited twice if possible, with the two visits as far apart as possible to allow for the absorption of theoretical knowledge between the two periods. Each duty should therefore be performed once early, and once late, in the course. The course requirements[1] provide for a three week holiday during the course. The logical conclusion to be drawn from the above rules is that each ward should be visited once before the holiday and once after it.

The criterion in Rule 1 relating to district work places that duty sometime after the end of the first half. Since there is a natural break in the course there anyway it would seem reasonable to do the district work either immediately prior to or after the holiday.

2.3 A suggested general course structure

Using these rules we can now draw up a schedule which looks like Table 2-1.

Several points arise at this stage which need further explanation. Extra information needs to be supplied to explain a differentiation between Wards 51,52 and 53, and the others. This is as follows:

(1) Wards 51, 52 and 53 are all Post-natal Wards and are thus similar in most respects. For this reason only one needs to be visited although all three could be. We can therefore classify the three together as being one type of job, a visit to a Post-natal Ward being viewed in the same light in terms of training as a visit to one of the other single wards.

(2) Although special midwifery skills are needed in these wards, a fair proportion of the work could be described

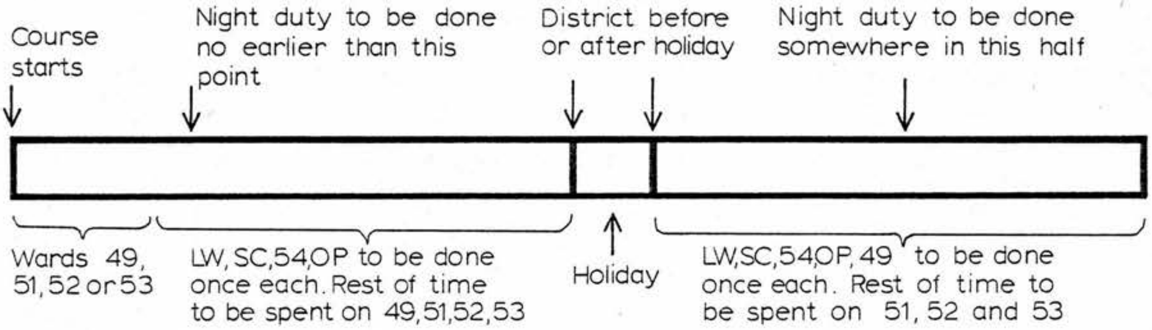

TABLE 2-1

Simple block schedule

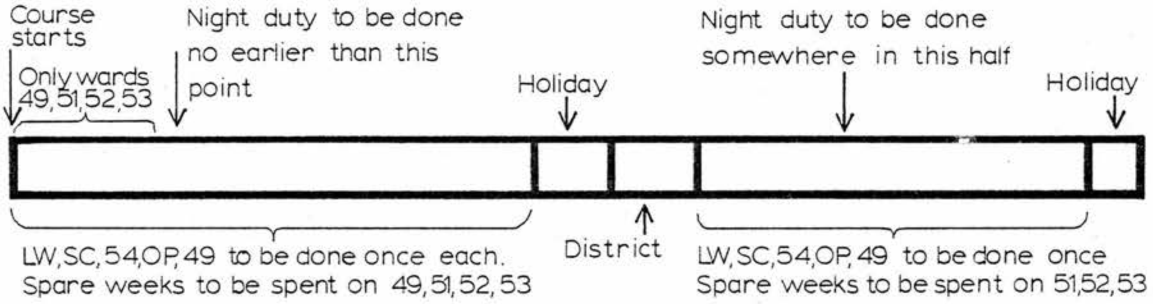

TABLE 2-2

More specific block schedule

All unmarked sections are day duty

TABLE 2-3

Actual block schedule, 1973

as general nursing, and therefore these wards can be treated less stringently than the others. Although it would be extremely inadvisable from the point of view of continuity to do a single week on say Labour Ward and then move on again, it is more feasible for this to be done with a Post-natal Ward. This puts Wards 51, 52 and 53 into a category of greater flexibility than the other wards. In practice the spells of duty on the other wards (referred to here as the Special Wards) are shuffled about within the available time, with the spare weeks remaining being spent on the Post-natal Wards. This explains the legend on Table 2-1. In the first half, all the Special Wards need to be visited once - Labour Ward, Special Care, Out-patients, 54 and 49. Ward 49 is unique among the Special Wards in that a girl can start her course on it, but if she starts instead on a Post-natal Ward she will still need to visit Ward 49 at some time in the first half.

2.4 General course structure used by the Simpson in 1973

This Section will describe the general course structure which was in use in 1973. Although several variations could be devised which would be consistent with the rules stated above, the system actually used was at least as simple and logical as any other. The rationale behind each non-arbitrary decision will be explained in order to assist with the description of the system rather than to attempt to justify it.

The Central Midwives Board required there to be five weeks of holiday on the course[2]. It was decided by the Simpson that two of these weeks should be at the end of the course. The reasoning behind this is that any girl who has missed some of the course through illness or for any other reason can work on through her final holiday to cover the training which she has missed. This leaves three weeks holiday for the midle of the course.

The C.M.B. (Central Midwives Board) asked for four or five weeks of district work[3]. It was decided arbitrarily by the Simpson that these weeks should follow the holiday.

On the basis of the above extra information it is now possible to draw up a more specific general structure for the course - see Table 2-2.

The training regulations[4] stipulate that each girl should do thirteen weeks altogether on night duty, and the organisers of this course have decided that the most suitable method of breaking this up is to do it in three sessions, two of four weeks and one of five. Since the period of district work is included in the second half it might seem reasonable to do only one spell of night duty in the second half, with two in the first. The way that this has been done is shown in Table 2-3.

At this stage one might ask why it is necessary to determine in advance the parts of the course where night duty should be done. Why not leave the system in a more flexible state? The answer is that this structure of the course has to dovetail with the programme of lectures which the girls attend. If a girl is on night duty she is unable to attend theoretical study so it is better if the whole class goes on night duty together and takes a break from lectures.

2.5 Fixed versus flexible course structure

Using the definitions as defined in the previous Chapter, the type of allocation practised by the Simpson was continuous allocation; decisions would be made and updated from week to week, only a short time, typically one to ten weeks, before their implementation. As a result it would not be possible to tell a new trainee in advance what ward she would be on three months hence. This system was very flexible with regard to the way in which it could cope with short-term fluctuations in staffing. However, in order to plan ahead for such factors as holidays and district work, which affect the overall numbers available for ward duty at any given time, and night duty and lectures, which have to inter-relate, it is necessary to have an underlying course structure which is pre-determined.

Henceforth we shall refer to the fixed course structure as "roster". This term has been adopted because the hospital staff did not have a specific name for it. Table 2-3 shows the general form of the roster for one girl. Later we will be combining these for a number of girls, and will be more specific as to the durations of the different sections of the course.

The more specific type of schedule, showing which actual ward a girl is on for a given week will be called the "plan".

2.6 Summary of general staffing requirements

We have now covered the general structure which a girl's course will follow. We have looked at some of the basic pressures which help to shape the course. In the next Chapter a specific description will be given of the way in which the hospital actually runs the course, including details of the staffing levels achieved in practice.

So far we have looked mainly at process constraints - those which affect the training which each girl receives. Now let us look at the loading constraints - the factors limiting the staffing levels within the hospital.

Although this is a description of the general situation and problem of the hospital's solution, it is apparent that it is worth introducing specific figures at this stage, since they relate to staffing levels which have been discovered from experience to be needed (See Section 2.11 of this Chapter). Thus they are constraints belonging to the situation rather than to the hospital's method of scheduling. In all cases but out-patients (and district work, which is a special category, being external) there are two sets of figures, one for day duty and one for night duty. These sets are subdivided to give a minimum and an optimum figure for each ward:

	WARD	49	51,52,53	54	SC	LW	OP	DISTRICT
DAY	Optimum	12	8	4	10	15	8	12
	Minimum	11	7	4	9	14	7	6
NIGHT	Optimum	5	5	2	6	6	-	-
	Minimum	4	4	2	5	5	-	-

Table 2-4

Required staffing levels for student nurses as determined by the staff of the Simpson as at 1973. (For a discussion of the process by which these staffing levels are determined, see Section 2.11).

Several points are raised by this Table.

(1) No maximum figures are given. There are limits to the number of girls who can work on a ward, set by a number

of factors. The first is the fact that wards could be overcrowded with staff. This is not going to be critical unless the overstaffing was very great, causing a shortage in staff facilities for that ward or creating a situation where nurses get in each other's way. The second is that, since each girl is meant to be learning from her experience she will need a realistic share of the workload. This will be most critical in the Labour Ward where each girl actually has to perform a number of deliveries.* Overstaffing there will mean that there are fewer deliveries for each girl, with the possible consequence of some failing to complete the minimum training requirements.

However, it can be seen that in general the maximum staffing loads will be less critical than the minimum. In the case of an understaffed ward there is going to be a reduction in service to the patients, with a possible risk of danger to life, either of the mother or the child. In practice, therefore, no maxima are quoted. However this does not cause the system to go out of control, for two reasons:

(i) The allocators attempt to keep the figures as near to the optimum as possible.

(ii) Because of the general balance of the system a case of overstaffing at one point would be matched by understaffing elsewhere. Since the latter case is critical it would be rectified or prevented, thereby correcting the original overstaffing.

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* and is also in competition with medical students

(2) The minimum is usually only one below the optimum. This suggests that the task of obtaining the exact number of girls wanted in each ward each week is quite critical. In practice the actual number placed on a ward will exceed the optimum by one or two girls in quite a high percentage of cases. This will be discussed in Chapter 3 as part of the assessment of the hospital's solution.

(3) In the case of Ward 54 the minimum and optimum figures are the same, both by day and by night. In fact these figures also count as maxima in practice, so for the purposes of the formulation of the problem they can be taken as fixed staffing requirements.
The reasons for this are mainly ethical. Since most of the work on this ward concerns abortions, the staff in charge of the Simpson feel that as few student nurses should work there for as little time as possible. This kind of work is unpopular with nursing staff as it goes against not only the raison d'être of the hospital but also the sense of vocation of the nurses. As a result, the staffing level of four is the minimum number which will permit this ward to be staffed on a continuous basis, and is not exceeded in practice.

Since the service to the patients at the Simpson needs to be provided on a continuous basis it is necessary to spread out the weeks of holidays, district work and other ward duties throughout each year. The merits and demerits of using greater or small numbers of intakes each year have been discussed in Chapter 1, where it was explained that, although the use of twelve intakes a year possessed some qualities of neatness, the irregularity of the schedule structure precluded the use of standard repeating solutions to the problem. In that Chapter the calculations governing the number of intakes and schedules required to give even staffing were explained, and it was mentioned that, in the case of a system where twelve intakes a year were used, it would be a good idea to fix some aspects of the course in advance and leave others flexible.

It can be seen that, although even staffing can be achieved either by increasing the number of intakes or by subdividing each into a great enough number of schedules, a satisfactory solution can really only be achieved by a blend of the two aspects.

To illustrate this point, it would be possible to obtain fairly even staffing with regard to holidays with only one intake each year, but in order to do so, some girls would have to go on holiday for the first few weeks of their course! It was also demonstrated in Chapter 1 how the experience levels of the staff would fluctuate wildly from one period to the next.

With a greater number of intakes each year it is possible to make the structures of each schedule more similar to each other while still giving an even spread of staffing. This was one of the priorities of the 1973 system - the training patterns followed fairly strict rules, but the use of twelve intakes allowed even staffing to be provided with only six basic patterns of roster. These are given in Table 3-2 in the next Chapter.

The fact that many of the blocks of duty contained within the Roster have durations of four or five weeks would tend to suggest that even staffing might be achieved with one type of roster for each intake, so that twelve different groups were on duty at a time. In fact at that time it was considered desirable to have some overlap each time that a new group arrived on a given ward and another left. Generally it was wished that the old girls, or some of them, stayed on the ward concerned for a week after the new girls had arrived in order to smooth the transition and help the new girls. To this end it was necessary to subdivide the twelve intakes each into two half sets. The mechanism of this arrangement is explained in the next Chapter where the hospital's 1973 solution is explained.

2.7 Controllable and uncontrollable variables

In Chapter 1 it was explained how some aspects of the problem could be treated either as controllable or uncontrollable variables depending on the priorities and external influences on the problem at any given time. The factors affecting the problem in 1973 have now been assessed, permitting the following distinctions to be made:

2.7.1 Uncontrollable variables

The number of intakes per year.

The number of groups per intake.

The total number of girls.

The number of girls per intake.

The length in weeks of the course.

The number of weeks to be done on each ward in each half of the course.

The number of weeks of night duty to be done.

The number of weeks of district work to be done.

The number of weeks of holiday to be done.

As well as these variables there are a series of precedence rules governing the sequences of training permitted, affecting both the structural aspects such as roster and the decision aspects such as plan. Henceforth these words ("roster" and "plan") must be distinguished from two programs "ROSTER" and "PLAN". These programs are part of the large-scale simulations of the problem, defining the range of options available in each category. In the case of ROSTER, the precedence rules are specified in a fixed form by defining the six patterns of roster which are permitted but with PLAN the rules serve merely to modify the vast range of possible course patterns which are possible. The rules will be summarised below.

2.7.2 Controllable variables

(1) The ward which each girl will be placed on each week. This deceptively simple statement disguises the actual approach which is usually adopted:

Is this girl already allocated this week?

If not, which ward should she go to?

How long should she stay there?

On the other hand it is also possible to approach the problem by defining a set of feasible schedules, in the same way as the rosters were defined. The disadvantage of this is that whereas there are six rosters, each of which is used in rotation with the others, in the case of PLAN there will be an extremely large number of possible schedules. There are roughly forty thousand sequences in which the wards can be visited, and each sequence can give rise to more than one schedule since the length of each visit to a ward can vary. This figure is based on explicit listings of schedules which were devised for the two halves of the course during the design of an early computer model of the scheduling system. These are discussed in Section 4.5.1.

Thus two girls can perform their visits in an identical order, yet one may spend three weeks on a certain Post-natal Ward at the start of her course and the other four, doing one less week of ante-natal during the second half of her course. The schedules for the two girls will be radically different in that they will only be on the same ward together for about thirty five weeks in the year.

Despite this problem it is possible to define a set of feasible schedules which are selected to be compatible with one another, or to use some sort of system for generating feasible schedules which suit the problem at a given stage in its solution. The approach will be described in Chapter 7. Using this sort of method the other decision variable is:

(2) The schedule which each girl will be placed on.

2.8 Summary of data for the 1973 system

It is now possible to summarise the rules, constraints and data for the 1973 problem, although specific values for some of the constraints will not be stated here but will be explained in more detail in the next Chapter.

Summary of the constraints for 1973 system

Number of trainee midwives present at any time - 144 approximately
Number of intakes per year - 12
Number of rosters per intake - 2
Number of girls per half set - 6 approximately
Length of course - 52 weeks

Training requirements (- weeks per nurse):

Ward	1st half of course	2nd half of course	Total
49	⩾ 4	⩾ 3	⩾ 7
Post-natal			⩾ 13
54	⩽ 2*	⩽ 2*	⩽ 4
OP	⩾ 2	⩾ 1	⩾ 3
LW	⩾ 4	⩾ 4	⩾ 8
SC	⩾ 4	⩾ 2	⩾ 6
Holidays	3	2	5
District	-	4 or 5	4 or 5
Night Duty	⩾ 8	⩾ 4	13

Staffing levels:

	DAY		NIGHT	
Ward	OPT	MIN	OPT	MIN
49	12	11	5	4
51, 52, 53 (each)	8	7	5	4
54	4	4	2	2
OP	8	7	-	-
LW	15	14	6	5
SC	10	9	6	5
District	12	6	-	-

ROSTER

Roster pattern as described in Section 3.3. and in Table 3-3.

Sequencing rules

These are investigated in Chapter 3 and summarised in Section 3.7.

* The requirement here is that no nurse should stay on Ward 54 for more than two weeks in either half. Work on this ward is not compulsory and may be refused on ethical or religious grounds. See Staffing Requirements, point (3), in Section 2.6.

Table 2-5

2.9 A comparison of 1973, 1977 and proposed future staffing levels

Appendix 2 gives in full the text of the October 1975 recommendations for the revision of training for student midwives. This revision was made necessary by the policy of the E.E.C. of unifying standards throughout the member countries. It is this requirement which has heralded the change of emphasis towards a training - rather than service-based system. The old system struck a balance between the two which permitted both aims to be met simultaneously. The E.E.C. directives are solely aimed at providing the required amount of clinical experience for each girl, and take no account of the contribution which she normally makes to staffing levels. As a result, the changes recommended have had a tendency to place girls for longer periods on the Special Wards, where they may not be needed, and take them away from Ante- and Post-natal Wards, where their general nursing training allows them to contribute soonest and most safely to the staffing of the hospital.

Because of this incompatibility, the senior staff of the Simpson have so far resisted pressures to comply in total with the E.E.C. recommendations, but have instead moved only as far towards them as they feel they can without jeopardizing the levels of care provided to patients, although the staffing levels in some wards have in fact been forced down.

The main change is that the theory teaching is now done in blocks of study weeks rather than interspersed with the practical training work. Nine weeks of the course are now devoted entirely to theory. That the number of weeks each girl is available to work on the wards appears to have been reduced by nine is slightly mitigated by the fact that the girls do not have to spend a day per week while on ward duty in going to lectures and studying. However the actual number of days spent on study has been increased from thirty five to forty five, so in effect the hospital has two ward weeks per girl less to staff the hospital with. Under the new system the Out-patients department is considered together with the Ante-natal Ward, so in the following tables this adjustment has been made throughout for consistency.

The E.E.C. recommendations[5] as to the length of time each trainee midwife should spend on each ward compared to that under the old system is as follows:

	Theory (days)	LW	OP+ 49	SC	Post-natal	Nights	District
1973 System	35	8	10	6-7	13	13	4-5
E.E.C. recommendations	45	10	10	6-7	8	8	4
					weeks		

Table 2-6

From these figures, knowing the number of girls who will be present in total, it is possible, using the methods outlined in Chapter 1, to calculate how many girls will theoretically be available each week for each ward. Before doing so, we shall introduce the set of figures which the hospital wants to aim for during a transition period:

	LW	OP+ 49	SC	POST-NATAL	NIGHTS	DISTRICT
WEEKS:	9	8	5	12	8-9	4

Table 2-7

These figures will be used to produce the "theoretical best" figures for the 1977 system, but another set labelled the "real figures" for 1977 are those achieved by a scheme proposed by Black[6]. The 1973 "real average" figures are compiled from the Simpson's training records for that year. The following tables show by comparison the implications of the training scheme proposed by the E.E.C.[7]

STAFFING LEVELS

LABOUR WARD:	DAY	NIGHT	TOTAL
1973 system optimum	15.	6	21.
1973 system real average	15.3	6	21.3
E.E.C. recommendations	21.7	6	27.7
1977 system real figures	(11) 18-24 (2)*	6	(11) 24-30 (2)
1977 system theoretical best	18.9	6	24.9

Table 2-8

SPECIAL CARE:	DAY	NIGHT	TOTAL
1973 system optimum	10.	6	16.
1973 system real average	10.5	6	16.5
E.E.C. recommendations	12.	6	18.
1977 system real figures	(9) 6-12 (4)	6	(9) 12-18 (4)
1977 system theoretical best	7.8	6	13.8

Table 2-9

ANTE-NATAL (+OP):	DAY	NIGHT	TOTAL
1973 system optimum	20.	5	25.
1973 system real average	24.1	5	29.1
E.E.C. recommendations	22.7	5	27.7
1977 system real figures	(4) 12-18 (9)	6	(4) 18-24 (9)
1977 system theoretical best	16.2	6	22.2

Table 2-10

POST-NATAL:	DAY	NIGHT	TOTAL
1973 system optimum	28.	17	45.
1973 system real average	29.5	17	46.5
E.E.C. recommendations	17.2	5	22.2
1977 system real figures	(6) 24-30 (7)	6	(6) 30-36 (7)
1977 system theoretical best	27.2	6	33.2

Table 2-11

*Where two figures are shown for the real figures under the 1977 system these refer to the staffing levels achieved by the scheme devised by Black[8]. This scheme is described in full in Chapter 8. Under this system the staffing varies, and is either one or the other of the figures given, but never in between. The figures in brackets on either side of the two figures linked by the dash indicate the number of weeks out of thirteen that each applies.

In the cases of Labour Ward and Special Care the E.E.C. recommenations produce a rise in the staffing supplied by student midwives. These figures have all been arrived at by calculating what change in staffing would occur as a direct result of putting each nurse onto each ward for the number of weeks they recommend. The Black system falls on either side of the level inferred from E.E.C. proposals in the case of Labour Ward, but below the inferred level in the case of Special Care. The drastic fluctuation between six and twelve girls will be discussed further in Chapter 8. In the case of Ante-natal, the reduction is even more than the E.E.C. would require, but the reason why these reductions have had to be made become clear when the figures for Post-natal are examined. If the E.E.C. recommendations were carried out, with the inherent shift from post-natal training to the more specialised duties the result would be that staffing supplied by student midwives would drop from an average of 29.5 during the day to 17.2, and from 17 at night to 5. The hospital has chosen only to go part way with this, mainly by reducing the staff on night duty, but in order to keep enough girls on post-natal duties it has had to make some reductions elsewhere; in the case of Special Care, where they can ill be afforded. The reason why shortages are so critical will be explained in the next section, but basically a certain minimum number of girls have to be on a given duty for a given week before round-the-clock staffing can be provided - any shortfall below the critical number will not just cause fewer girls to be available, it will cause there to be none on duty for a shift instead of one.

Although actual figures have been quoted from the Black system, the derivation of these does not concern us at this stage; the point being made is that the E.E.C. rules have not yet been accepted in full. While certain concessions have been made to the E.E.C. norm as part of a gradual transition the hospital is hoping that it will not be forced to change all the way. This means that staffing levels are not fixed and uncontrollable. Instead, discretion has to be used when choosing suitable values for staffing levels and then trying to design a set of schedules which achieves this target. Since the hospital has defined a new set of required lengths of training as an interim proposal, and these are the parameters used by the Black scheme, we too shall adopt them for our proposals.

2.10 Summary of constraints for the 1977 system

Thus the proposed alterations to the training scheme have a direct repercussion on the theoretically best achieveable staffing levels. The other main effect of the changes has already been assessed in Chapter 1 - the separation of the theory work into exclusive weeks of study has so altered the balance of the problem that it now becomes possible to achieve even staffing with a scheme using quarterly intakes. Despite having some drawbacks this system would seem to be an improvement in many ways over the monthly intake scheme. As a result of this several uncontrollable variables under the 1973 system become controllable variables under the new scheme, and the problem becomes less closely defined in many ways. The following summary indicates what rules have been established so far.

Summary of constraints for the 1977 system

Number of trainee midwives present at any time - 144 approximately

Number of intakes per year - 4

Length of course - 52 weeks

The number of sub-groups per intake and the size of each is now a decision variable.

Training requirements/

Training requirements:

Ward	Weeks
49 + OP	8
Post-natal + 54	12
LW	9
SC	5
Holidays	5
District	4
Night duty	8-9

Staffing levels:

Ward	Day	Night
49 + OP	16	6
Post-natal + 54	27	6
LW	19	6
SC	8	6
District	10	-

ROSTER As defined in the Revision of Training recommendations, October 1975 (Appendix 2).

Sequencing rules Will be discussed and defined in Chapter 8.

Table 2-12

2.11 Determination of staffing requirements

The determination of staffing levels is an extremely difficult subject at the Simpson. Attempts have been made to establish a set of rules which govern the levels of staffing required on different wards, notably by McFarlane[9]. One system commonly applied is the Aberdeen formula[10] named after the region where it was developed. The rationale behind these systems is that the level of nursing care which is needed is closely linked to a series of identifiable factors. For instance patients are classified with regard to the amount of care they need and the seriousness of their case - are they bed-ridden, in wheelchairs, or able to walk? Are they incontinent? Are they able to turn over in bed without assistance? Factors such as these can be assessed, then the formula determines an overall likely work load for the entire ward, enabling a decision to be made as to the number of staff needed, and their required skills.

Unfortunately the Simpson has not been able to obtain or develop a model which gives meaningful results for a maternity hospital, and prefers therefore to use knowledge gained from past experience.[11] If adequate numbers of staff are available this method works very efficiently, but there is still the underlying problem that the work load is unalterable and the budget for the overall staff wage bill is fixed. In fact pressures are applied from time to time by the government for the Simpson and other hospitals to cut back on the size of the establishment. As a result, if extra nursing care is required in one department it can only be at the expense of a reduction elsewhere, or by reducing the average level of experience and qualification of the establishment as a whole, to obtain more staff for the same money. This tends to eliminate the usefulness of any system which produces a series of ideal staffing levels, since they will often be impractical.

As a rule of thumb there should be five staff to every child in an intensive care unit and three staff to every child otherwise[11].

As mentioned in the previous Section, it is easier to identify a critical number for staffing if, for instance, at least one nurse has to be on duty round the clock. When the lengths of shifts are taken into account plus the effects of working only five days out of seven it becomes necessary to have five girls allocated to a ward to ensure that at least one is constantly on duty. Thus five is the absolute minimum for any given staffing unit. However, as mentioned above, in many cases staffing levels are dictated by necessity rather than by choice, so in this thesis the staffing levels which have been aimed for are those quoted as being satisfactory by the senior staff of the Simpson. Obviously extra staff cannot be produced if they are not employed by the Simpson, and if the money is not available to increase the size of the establishment. A much more realistic aim is to plan the scheduling so that the best use is made of the staff available. The best way to do this is to produce a system which provides the most even staffing possible rather than alternating between over- and under-staffing. Since the theoretical best levels of staffing can be calculated from the number of staff present and the number of weeks worked on each ward according to the training schedules, as described in Chapter 1, the target is already known, and the objective of any scheme must be to approach this theoretical target as closely as the integer restraints of dealing with nurses permit.

CHAPTER 3

THE EXISTING SOLUTION AT JANUARY 1973

3.1 Introduction

In the previous Chapter the general situation has been described, and the framework of the problem, partly determined by external factors, partly by the conscious decisions of the hospital, has been established. In this Chapter the 1973 solution will be dealt with in a more specific fashion.

(A) This Chapter will describe the rules used by the staff of the Simpson to solve the problem, and the methods by which they apply those rules.

(B) Actual numbers will be introduced.

3.2 Practical use of Roster

In the last Chapter the need for an overlap between different groups on a ward was explained. The choice of twelve intakes in a year, and the use of blocks of work of four and five week durations tends to lead to a situation where the staffing pattern has no gaps. There is no overlap since an entire group leaves a certain duty at once and is replaced for the next week by a totally new group.

It was also pointed out that the fewer the intakes in a year, the greater the number of different rosters needed for each intake would be, in order to obtain even staffing levels.

We have now demonstrated that twelve subdivisions in a year gives even staffing, but the hospital found that to get an overlap at each changeover a further subdivision was needed. Each group was therefore divided into two half sets. The following table demonstrates this point:

	23	24	25	26	27	28	29	WEEK
NOVEMBER INTAKE	N	N	N					
	N	N	N	N				
DECEMBER INTAKE				N	N	N	N	
					N	N	N	N = NIGHT DUTY

Table 3-1

This diagram shows only the changeover point and not the whole spell of night duty for each group. It shows how half the November set leaves after week 25, leaving the other half set to help the newcomers from the December set along. These people in turn can help the second half of their own set. The above example also serves to illustrate the terminology used by the hospital. Each intake group is called a SET, and is referred to by the month in which it arrived.

The intervals between the arrival of successive groups are not even since twelve does not divide evenly into fifty-two weeks. Every third interval will be for five weeks instead of four. This suggests that the minimum number of sets which can be represented in an overall roster is three. To put this another way, there will need to be three different roster patterns, but this block of three can then be repeated for successive intake groups. Since the sets are divided into half sets to provide continuity at the changeovers, it is apparent that in fact we will have six basic roster patters, combining to form a unit which is repeated every thirteen weeks.

3.3 The actual roster

The roster in use over the period being considered is

1972						1973																																																				1974				
DEC						JAN				FEB				MAR				APR					MAY				JUN				JUL					AUG				SEP					OCT				NOV				DEC					JAN				
26	3	10	17	24	31	7	14	21	28	4	11	18	25	4	11	18	25	1	8	15	22	29	6	13	20	27	3	10	17	24	1	8	15	22	29	5	12	19	26	2	9	16	23	30	7	14	21	28	4	11	18	25	2	9	16	23	30	6	13	20	27	3
			L	L	▨					N	N	N	N														N	N	N	N	L	L	L	D	D	D	D	D	N	N	N	N	N												L	L	▨					N
			L	L	▨						N	N	N	N														N	N	N	N	L	L	L	D	D	D	D	D	N	N	N	N	N											L	L	▨					
								L	L					N	N	N	N														N	N	N	N	N	L	L	L	D	D	D	D		N	N	N	N													L	L	
								L	L						N	N	N	N														N	N	N	N	N	L	L	L	D	D	D	D		N	N	N	N												L	L	
N												L	L					N	N	N	N	N														N	N	N	N	L	L	L	D	D	D	D		N	N	N	N														
N	N											L	L						N	N	N	N	N														N	N	N	N	L	L	L	D	D	D	D		N	N	N	N													
	N	N	N	N												L	L						N	N	N	N																	N	N	N	N	L	L	L	D	D	D	D	D	N	N	N	N	N							
D		N	N	N	N											L	L							N	N	N	N																N	N	N	N	L	L	L	D	D	D	D	D	N	N	N	N	N					
D	D	D	D		N	N	N	N													L	L					N	N	N	N																	N	N	N	N	N	L	L	L	D	D	D	D		N	N	N	N		
L	D	D	D	D		N	N	N	N												L	L						N	N	N	N																	N	N	N	N	N	L	L	L	D	D	D	D		N	N	N	N	
N	L	L	L	D	D	D	D		N	N	N	N													L	L					N	N	N	N	N																		N	N	N	N	L	L	L	D	D	D	D		N	N
N	N	L	L	L	D	D	D	D		N	N	N	N												L	L						N	N	N	N	N																		N	N	N	N	L	L	L	D	D	D	D		N
	N	N	N	N	L	L	L	D	D	D	D	D	N	N	N	N	N												L	L						N	N	N	N														N	N	N	N	L	L	L	D	D	D
		N	N	N	N	L	L	L	D	D	D	D	D	N	N	N	N	N											L	L							N	N	N	N														N	N	N	N	L	L	L	D	D
					N	N	N	N	N	L	L	L	D	D	D	D		N	N	N	N													L	L					N	N	N	N														N	N	N	N	N	L
						N	N	N	N	N	L	L	L	D	D	D	D		N	N	N	N												L	L						N	N	N	N														N	N	N	N	N
										N	N	N	N	L	L	L	D	D	D	D		N	N	N	N													L	L					N	N	N	N	N														N
											N	N	N	N	L	L	L	D	D	D	D		N	N	N	N													L	L						N	N	N	N	N													
N														N	N	N	N	L	L	L	D	D	D	D	D	N	N	N	N	N												L	L	▨					N	N	N	N										
N	N														N	N	N	N	L	L	L	D	D	D	D	D	N	N	N	N	N											L	L	▨						N	N	N	N									
	N	N	N	N														N	N	N	N	N	L	L	L	D	D	D	D		N	N	N	N															L	L						N	N	N	N						
		N	N	N	N														N	N	N	N	N	L	L	L	D	D	D	D		N	N	N	N														L	L							N	N	N	N					
L					N	N	N	N	N														N	N	N	N	L	L	L	D	D	D	D		N	N	N	N															L	L						N	N	N	N	N	
L						N	N	N	N	N														N	N	N	N	L	L	L	D	D	D	D		N	N	N	N														L	L							N	N	N	N	N

TABLE 3-2

Actual 1973 roster

N = Night duty

D = District

L = Leave

Note irregularities caused by uneven lengths of years reproduced in Table 3-2. The number of weeks spent on each duty for each of the six basic roster types is shown in the following table:

		DAY	NIGHT	DAY	NIGHT	LEAVE	DISTRICT	DAY	NIGHT	DAY	LEAVE	: TYPE
Sets 1,4,7 etc.	1st half set	4	4	13	4	3	5	0	5	12	2	1
	2nd half set	5	4	13	4	3	5	0	5	11	2	2
Sets 2,5,8 etc.	1st half set	4	4	13	5	3	4	1	4	12	2	3
	2nd half set	5	4	13	5	3	4	1	4	11	2	4
Sets 3,6,9 etc.	1st half set	4	5	13	4	3	4	1	4	12	2	5
	2nd half set	5	5	13	4	3	4	1	4	11	2	6

Table 3-3

The differing numbers of weeks done on the first batch of day duty establish a staggered effect between the two halves of each set which is eventually compensated for in the last session of day duty. All girls do thirty weeks on day duty except those in the first set (types 1 and 2) who replace one week with an extra week of district work. This longer session on district for one set in three is needed to provide the overlap and continuity of staffing throughout the year. The following table illustrates this point:

SCHEDULE TYPE :	EACH COLUMN REPRESENTS 1 WEEK																	TOTAL WEEKS DISTRICT DONE BY EACH SCHEDULE TYPE
5	D																	4
6	D	D																4
1		D	D	D	D	D												5
2			D	D	D	D	D											5
3							D	D	D	D								4
4								D	D	D	D							4
5											D	D	D	D				4
6												D	D	D	D			4
1															D	D	D	5
2																D	D	5
	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	TOTAL NUMBER OF SETS ON DUTY

Table 3-4

Not only do we have a constant overlap, but we also have a constant level of staffing with two half sets on district work at any time. The actual variation in the lengths of time spent on any duty from one type to another is never more than one weeks duration at any stage of the course, and in fact the totals are identical for all categories, regardless of the type of roster, with the exception of the one week district/day swop mentioned above.

It can be seen from Table 3-2 that there are always six half sets on night duty. To be more specific there are always two half sets on their first spell of night duty, two on their second and two on their third. The reason why this has worked out so evenly is explained by Table 3-3. Each set does two spells of four weeks and one of five, and the five week blocks have been enclosed in rectangles to make the pattern clearer.

3.4 The scale of the problem

At any one time during the period 3rd June, 1973 to 28th October, 1973 there were between 126 and 142 girls receiving training. From this it can be seen that the average size of an intake is somewhere around eleven girls. This is a very small number, since each month a certain number of girls fail to arrive. This occurs more frequently with girls from distant countries. If two or three fail to turn up there could be a critical staff shortage. There are two aspects of the course which held to combat this:

(1) At most times there will be more than one set available to perform a certain duty. When sets are added together they will tend to balance out any variation in numbers. An exception to this is district work, where for most of the time all of the girls will come from the same set (apart from overlap weeks). As a result this duty shows a higher variation in numbers than any other, with a minimum of 6 and a maximum of 14 for the period quoted above.

(2) Since the drop-out rate prior to arrival is relatively constant the hospital tends to overbook for the places available. This tends to prevent acute shortages, but it can also lead to the situation where all of the girls turn up one month, causing a problem of overstaffing.

From the figures in Table 3-17, the smallest average set size for any week was 10.5 and the largest 11.8. In the same period, the largest set contained seventeen girls and the smallest six. These gave rise to the largest and smallest half-set sizes of nine and three, (Table 3-5).

This demonstrates that the subdivision of the total number of girls into a great many smaller units is likely to cause a large variation in the sizes of the units.

TABLE 3-5

SET SIZES IN 1972/3

	SET	START OF COURSE	END OF COURSE	(If different)
1972	July	8		
	August	9		
	September	15		
	October	13		
	November	14	13	
	December	7		
1973	January	11		
	February	6		
	March	12		
	April	11	9	
	May	10		
	June	12	11	
	July	12	13	
	August	14	12	
	September	17	16	
	October	12		
	November	12		
	December	9		

The following Table shows the actual staffing situation for those aspects of the course controlled by the basic plan

			ACTUAL FIGURES (From training records, JAN - DEC 1973)		
	OPTI-MUM	MINI-MUM	MIN	AV	MAX
DAY DUTY	73	65	67	79	100
NIGHT DUTY	32	28	28	32	36
DISTRICT	12	6	6	10	14
HOLIDAY	-	-	2	10	21

Table 3-6

These figures show a successful overall picture, with the only evident problem being overstaffing on day duty. As well as creating some problems from the service point of view, the main disadvantage of this is that girls may not be getting sufficient experience in certain aspects of the work. To quote a specific example, each girl has to perform a certain number of deliveries to receive her qualification, and if there are too many girls being trained, then there may not be sufficient deliveries to go round. If the girl does not get through her quota while she is working in the Labour Ward then she might have to be called at short notice from another ward at a later date to perform a delivery.

On the other hand, it may be this very overstaffing which permits the present allocation system to work without violating the minimum restraints too often, a situation which is obviously more critical from the point of view of service to the patient. This implies that a more efficient system of allocation could manage with fewer girls.

3.5 Training requirements

In this section we will look at the specific training requirements and training pattern of the nurses. Some of the rules are stipulated by the Central Midwives Board[1], others are dictated by the specific situation at the Simpson. However it should be emphasised that the series of rules described in this section represents the overall problem as tackled by the Simpson Maternity Hospital, not as we have formulated it.

The requirements of the course are described chronologically, starting from week one, and the description is split into two parts, corresponding to the two halves of the course.

The training requirements stipulated by the **Central Midwives** Board[2], are reiterated below:

		1st half of course	2nd half of course	Total	
Pre-natal Ward	49:	4	3	7	(minimum)
Terminations etc.	54:	2	2	4	(maximum)
Out-patients	OP:	2	1	3	(minimum)
Labour Ward	LW:	4	4	8	(minimum)
Special Care Unit	SC:	4	2	6	(minimum)

Table 3-7: Weeks of different duties to be performed during the year's course.

District training is taken care of by the roster, so any weeks not covered by the above are spent in Post-natal Wards (51,52 and 53). The minimum number of weeks to be spent on these wards is theoretically 13, but in effect they are used as slack variables to be used when the girl is not needed on any of the other wards.

3.5.1 First half of the course/

3.5.1 First half of the course

When a new girl arrives we have a series of requirements to be met in the first half of her training. The following rules of thumb are a summary of the rules and reasons which determine the structure of the course as implemented by the Simpson.

(1) At this stage the girl will have a general nursing training, but no experience of midwifery. The first ward she is put on is one where she can cope and be useful, while at the same time learning more specific skills in practice and from the lectures she is receiving. What is done is to put roughly 3 girls out of 4 on the Post-natal Wards (51, 52 and 53) and one in four on the Pre-natal Ward (49). The exact proportions are of course subject to overall demand for staff on each ward. She is put on this ward for a number of weeks one less than the number of weeks she is doing on her first session of day duty. This is because the next ward she will work in will involve night duty, so the change is made one week before her session of night duty starts to give her a chance to familiarise herself with the set up in the new ward during the day when there are more people around to help her.

(2) If the girl has started on a Post-natal Ward, then when she changes ward one week before her session of night duty, she has a choice of three wards to work on. (By this it is meant that there are three wards which she is permitted to work on. The choice is made by the person working out the allocation, not by the girl. We shall

continue to use this meaning of the word choice for the sake of convenience, since no ambiguity should now arise). The Out-patient Department is closed at night, and the student midwife will not yet be experienced enough to work on Ward 54, so she is left with the possibility of working on Ward 49 (Pre-natal), Labour Ward or the Special Care Unit.

If, however, the girl has started on Ward 49 then she must spend at least three weeks on one of the Post-natal Wards before going on to one of the more specialised wards. Thus the first few weeks of the course for a half-set of four might look like this:

		DAY DUTY				NIGHT DUTY				
GIRL	WEEK:	1	2	3	4	5	6	7	8	9
1		51	51	51	LW	LW	LW	LW		
2		52	52	52	SC	SC	SC	SC		
3		53	53	53	49	49	49	49		
4		49	49	49	51	51	51	51		

Table 3-8

From this point onwards, until one week before the next spell of night duty, she has a free choice of wards, with one exception; Out-patients can only be done on day duty. A spell on Ward 49 from this point onwards in the first half will be for four weeks. The lengths of duty on the other special wards will be as shown in the Table on page 62. The minimum session for a Post-natal Ward is one week - the maximum is determined by the amount of slack for that ward which can be defined as the number of weeks before the next spell of night duty minus the number of weeks needed to complete the training on all the wards other than 51, 52 and 53. To determine this one needs to look ahead to the night duty to be performed at the end of the first half of the course.

This night duty is done exclusively on Post-natal Wards and on Ward 54. Preferably, but not always, the change to a ward for night duty is again made one week before the commencement of night duty. For brevity, and to distinguish them from the special wards such as Labour Ward, Out-patients etc., I have called the Post-natal Wards "GENERAL" Wards. This is contrary to the terminology used in the hospital, where the word implies something different, but for the purpose of this study it is useful as it suggests the concept of a non-specialist ward where the girl is placed to fill up the slack in her training program. In print-out from the simulation programs I have used the letter G to represent the General or Post-natal Wards.

Ward 54 has a maximum constraint with regard to number of weeks training, but no individual minimum. That is to say that a girl can decline to work on it at all if she has religious or moral objections to dealing with abortions, but from the overall planning situation there is a minimum number of girls who must work on it to maintain the staffing at an acceptable level. Thus as far as an individual girl's plan is concerned Ward 54 tends to behave in some ways as a secondary slack variable, since it is not complusory but can be used to fill in time if needed.

To exemplify the previous few paragraphs let us take a hypothetical case of a girl who has no objection to working on Ward 54. Let us assume (in advance) that she will work there for two weeks immediately prior to her first set of holidays. Her roster is type one,(see Table 3-3) and her training has started like that of girl 1 in the previous example. Her incomplete plan for part 1 will look like this :

DAY NIGHT DAY NIGHT LEAVE

.G.G.G.LW.LW.LW.LW. . . . . .G.G.G.54.54.L.L.L.

Table 3-9

There are thirteen weeks to be filled in, and in this period she has to do two weeks of OP, four weeks of 49 and four weeks of SC. That adds up to ten weeks, leaving three slack weeks to be filled with G.

Since OP is for days only, the next week to be filled in must be 49, SC or G. The session of G can be from one week to the maximum slack of three. There is some advantage in putting the girl on for all three weeks together, but the pressures from other constraints within the problem are much more critical, and this cannot always be achieved. In the examples below A is preferable to B :

DAY NIGHT DAY NIGHT LEAVE

A: G.G.G.LW.LW.LW.LW.G. G. G.SC.SC.SC.SC.49.49.49.49.OP.OP.G.G.G.54.54.L.L.L.

B: G.G.G.LW.LW.LW.LW.G.SC.SC.SC.SC. G.49.49.49.49. G.OP.OP.G.G.G.54.54.L.L.L.

Table 3-10

Although the second spell of night duty has to be done on G or 54, 54 does not necessarily have to be done at the second spell of night duty. Thus the plan could equally well look like either of these two examples :

DAY NIGHT DAY NIGHT LEAVE

.G.G.G.LW.LW.LW.LW.SC.SC.SC.SC.54.54. G.49.49.49.49.OP.OP.G.G.G.G.G.L.L.L.

.G.G.G.LW.LW.LW.LW.54.54.SC.SC.SC.SC.49.49.49.49. G.OP.OP.G.G.G.G.G.L.L.L.

Table 3-11

These, then, are the rules for the first half of the course. Some are dictated by the need to make sure that a girl has the necessary experience to tackle certain wards at certain stages in her course, some (like the session of G on night duty at the end of the first half) because they help to provide an even availability of staff throughout the year, and some because of the nature of the work (no OP duty at nights).

3.5.2 Second half of the course

Similar rules exist for the second half of a girl's training, but in this case they are simpler than for the first half. This is to be expected, as by now a girl will have six months experience behind her, so she can really go to any ward at any time.

She will start the second half with a spell of four or five weeks night duty, possibly preceded by one week's day duty. Ideally she will start that week of days on the same ward which she will be on for nights, but since she has probably been to each ward this constraint is not so critical as earlier in the course. For instance she could do one week on a General Ward on day duty and then start straight away on night duty in the Labour Ward, since she will already be familiar with it. Again the constraint applies that she cannot do OP work on night duty, and theoretically she should do her night duty on a special ward, but in practice the requirements of the different wards are more important week by week and she might end up doing her night duty in a Post-natal Ward if it is short-staffed.

In some cases the rules may have had to be violated in the first half. For instance a girl may have done five instead of four weeks on Labour Ward because of a temporary shortage of staff. In cases like this the requirement for the second half would be altered to compensate. In fact in the second half each girl does two two-week sessions on Labour Ward. This gives greater flexibility to the overall staffing plan. (Bearing this in mind there is no reason why the two sessions should not be done consecutively.)

If a girl has been ill and has missed some training during the year then she can do up to two extra weeks' work during her final spell of leave to complete her minimum training requirements.

The allocation rules used by the Simpson in 1973 have now been defined in sufficient detail for us, using the same system, to invent a training schedule for one nurse which will fulfil all requirements as far as her experience is concerned.

(1) The girl might decide to leave before her course is completed. Since some advance planning is essential it can be seen that the disruption caused will vary as a function of how much notice the girl gives.

(2) If a girl is ill or falls behind for some reason by more than the three weeks allowed for in her final leave then it is sometimes necessary to have her drop to a later set, i.e. leave her intake group and join the one which started a month after her. This will make the pattern which her training follows look odd, but causes very little disruption to the overall set-up, unless she was originally a member of an understaffed set.

(3) Some girls return to the Simpson to do a refresher course. They are nominally attached to the monthly intake which they coincide with, but their training is simplified and more flexible. Typically they might do a week or two on a Post-natal Ward, two weeks on LW, a week on OP, SC and 49, three or four weeks on district and one week's holiday.

NOVEMBER 1973 SET	NOVEMBER				DECEMBER					JANUARY				FEBRUARY				MARCH					APRIL				MAY	
	4	11	18	25	2	9	16	23	30	6	13	20	27	3	10	17	24	3	10	17	24	31	7	14	21	28	5	12
J. BARKER	51	51	51	49	49	49	49	SC	SC	SC	SC																	
R. DUNSMUIR	52	52	52	49	49	49	49	52	52																			
J. HEAD	51	51	51	LW	LW	LW	51	51	LW	OP	OP																	
J. HAYCOCK	53	53	53	LW	LW	LW	LW	53	53																			
S. SIMMONS	52	52	52	SC	SC	SC	SC	49	49	49	49																	
F. SMITH	53	53	53	SC	SC	SC	SC	49	49	49	49																	
A. McLEOD	49	49	49	51	51	51	LW	LW	LW	LW																		
A. MACPHERSON	52	52	52	52	SC	SC	SC	SC	52																			
S. NAUGHTON	53	53	53	53	LW	LW	LW	LW	49	49	49	49																
R. URCH	51	51	51	51	49	49	49	49	SC	SC	SC	SC																
R. MOGRIDGE	49	49	49	52	52	52	52	LW	LW	LW	LW																	
A. McNIGHT	49	49	49	49	53	53	53	SC	SC	SC	SC																	

TABLE 3-12
Actual planning table, November 1973 set

OCTOBER	OCTOBER				NOVEMBER				DECEMBER					JANUARY				FEBRUARY				MARCH					APRIL	
1973 SET	7	14	21	28	4	11	18	25	2	9	16	23	30	6	13	20	27	3	10	17	24	3	10	17	24	31	7	
					NIGHTS																	NIGHTS						
A.E.M.BROWN	52	52	52	49	49	49	49	52	52	LW	LW	LW	LW	SC	SC	SC	SC	OP	OP							L	L	
M.DELAHUNTY	49	49	49	52	52	52	52	LW	LW	LW	LW	54	54	OP	OP	SC	SC	SC	SC							L	L	
L.HINNELL	53	53	53	LW	LW	LW	LW	49	49	49	49	OP	OP	54	54	SC	SC	SC	SC							L	L	
A. HOGAN	51	51	51	49	49	49	49	51	SC	SC	SC	SC	51	51	OP	OP	LW	LW	LW	LW		54	54			L	L	
J. WADDELL	~~49~~	~~49~~	~~49~~	~~51~~	~~51~~	TERMINATED ~~51~~	~~SC~~	~~SC~~	~~SC~~	~~SC~~																L	L	
E. WESTE	51	51	51	SC	SC	SC	SC	51	51	51	LW	LW	LW	LW	49	49	49	49	OP	OP	5	51	51	54	54	L	L	
G.BENNETT	52	52	52	52	49	49	49	49	49	52	52	SC	SC	SC	SC	LW	LW	LW	LW	OP	OP						L	
C. DUNCAN	49	49	49	49	52	52	52	52	SC	SC	SC	SC	LW	LW	LW	LW											L	
J. JOHN	51	51	51	51	LW	LW	LW	LW	51	54	54	SC	SC	SC	SC	49	49	49	49	OP	OP						L	
A. LEWIS	52	52	52	52	SC	SC	SC	SC	52	49	49	49	49	54	54	OP	OP	LW	LW	LW	LW						L	
S. QUAYLE	53	53	53	53	LW	LW	LW	LW	51	51	49	49	49	49	5	5	OP	OP	SC	SC	SC	SC	54	54			L	
M. SPRIGGS	53	53	53	53	SC	SC	SC	SC	LW	LW	LW	LW	OP	OP	49	49	49	49	54	54					54	54	L	

TABLE 3-13
Actual planning table, October 1973 set

3.6 Allocation in practice

Given the rules as defined above, it is possible to consider the status quo at any point in 1973, and to take each girl in turn and say which options were open to her for the next week of her course. Some would be on a Pre or Post-natal Ward on the first month of their course, some would be nearing the end of their first or second halves and would have a very limited choice of wards to squeeze in by a deadline, and others would be about a third of the way through either half with up to six possible wards to choose from. Some girls will already be placed for the week we are considering. For instance if a girl was put on a session of four weeks LW last week then she will be spoken for for the next three - her case is not therefore under consideration this week. Table 3-12 is a copy of the actual planning book entry for the November set as at the week commencing 6th January, 1974, and should make this point clear. This group is at the most flexible point in the first half of their course, so no forward planning is required at this point. For instance, on the week of the 6th, R. Dunsmuir could go onto LW, SC, OP, 54 or G (51, 52 and 53), a very wide choice. Similary Haycock has a choice of SC, OP, 49, 54 or G.

However, if we look at a group which is nearer to a deadline, it becomes evident that more precise forward planning is required. Table 3-13 is a copy of the page for the previous set, OCTOBER, as it looked at 6th January, 1974. E. Weste, for instance, has been tentatively scheduled for three months ahead, up till the end of the first part of her course at the week starting March 24th.

There are several points of interest which this diagram illustrates.

First, consider the case of Ward 54. This ward is critical since the daytime staffing requirement of four is a maximum as well as a minimum - there is no slack, and the same applies to the night-time requirement of two. Clearly the night duty must be planned in advance to give the exact numbers and correct overlap. This has been done on the chart where the relevant entries have been underlined.

Consider also the block for the first two girls (outlined). Both girls have the same two special wards to visit before their next spell of night duty, so these have been pre-planned to interlock in a balanced fashion.

Another ward which illustrates careful pre-planning is Outpatients. At any week in the year there will be between 5 and 6 sets available to do OP duty, i.e. on the thirteen week spell of day duty in the first half or any time after the night duty in the second. In the case of Jan 6 for instance there are five sets available. Over a thirteen week period the weekly sequence of number of sets available for OP looks like this:

5½, 6, 6, 5½, 5½, 5½, 6, 5, 5, 5½, 6, 6, 5

OP requirements are: Minimum - 7
Optimum - 8
Actual average - 8.8 } For period June 3 to October 28th 1973
Observed maximum - 11 }

It is not usual for girls to go onto OP immediately after their first spell of night duty for two reasons:

(1) They are often in the middle of a three or four week session of LW, SC or 49, started a week before the end of night duty.

OCTOBER	OCTOBER				NOVEMBER				DECEMBER					JANUARY				FEBRUARY				MARCH					APRIL	
1973 SET	7	14	21	28	4	11	18	25	2	9	16	23	30	6	13	20	27	3	10	17	24	3	10	17	24	31	7	
A.E.M. BROWN																		OP	OP									
M. DELAHUNTY														OP	OP													
L. HINNELL												OP	OP															
A. HOGAN															OP	OP												
J. WADDELL																												
E. WESTE																			OP	OP								
G. BENNETT																				OP	OP							
C. DUNCAN																												
J. JOHN																				OP	OP							
A. LEWIS																OP	OP											
S. QUAYLE																	OP	OP										
M. SPRIGGS													OP	OP														

TABLE 3-14

Actual planning table, October 1973 set, OP only

(2) At this stage they are still not very experienced.

From the point of view of getting accumtomed to a ward for a week before going onto night duty there, it is also unusual for a girl's one week on OP in the second half to be fitted into the one week of day duty which sometimes occurs between her finishing her district work and her starting her third spell of night duty.

So making allowance for these and similar idiosyncracies of the system the average number of sets available in any week to do OP drops from around 5½ to around 4½. One way to obtain an even overall staffing is to obtain local solutions for each set. If you can organise a constant supply of two girls from each set in advance, overlapping at the changeovers, then when each week's final allocation has to be made a few adjustments with one or two girls who have a relatively free choice that week should solve the problem.

Diagram 3-14 is a copy of diagram 3-13 omitting all but the OP allocations.

It can be seen that a good sub-optimal solution for this set has been achieved. It satisfies both local criteria:

(A) The number of girls from this set doing OP in the relevant period is relatively constant, and

(B) there is a week's overlap at each changeover.

Duncan's spell of OP has not been scheduled yet, but as the self-imposed minimum limits from this set have already been met, her contribution is no longer vital, and can be left till the actual week involved. In this way she might be able to compensate at shorter notice for a shortfall in the other groups. In the same way the girls who are on the second half of their course are liable to be allocated only one week in advance, as they only do a one week visit to OP in the second half, and overlap then is impossible (and also less necessary since by then they are more experienced).

So this process of local optimisation in advance serves two purposes:

(a) It helps to ensure a regular staffing level over an indefinite period.

(b) It helps to keep the blend of girls of different levels of experience relatively constant on that ward. There are likely to be, at any one time, two girls in the fourth month of their course, two in the fifth, two in the tenth and two in the eleventh.

It should perhaps be pointed out at this stage that the pre-planning is only a guide, and can be changed when any individual week's requirements become critical. For instance if the week starting Jan 27th had a shortage of girls for SC it might be decided to put Quayle on SC instead of OP that week. Duncan could then do her fortnight's OP. (She could not go onto SC as she had already done that ward.)

OP is a relatively simple example, being a days-only ward, with planning durations of only two and one weeks in each half, but the principles illustrated by the analysis of the pre-planning of this ward should also apply to the other special wards. Whether or not these principles work effectively is discussed in Section 3.8.

So far we have looked at service-based allocation. That is to say we have pre-planned to ensure that each ward will be correctly staffed (within the framework of each girl's possible training programme). However, problems can arise this way. For instance one might pre-plan to satisfy the minimum requirements for OP for say three months ahead. It could then occur that six extra girls reach the last week of their course not having done their second half's one week there. One would have no option but to put them on that ward along with the other eight who had been pre-planned, causing gross overcrowding. Obviously it would have been better to have one too many on the ward for the past six weeks. This contingency is difficult to check for. It might be possible to run a computer simulation forward for six months or so at a time, but this would have to work on assumed future numbers of monthly arrivals which might be inaccurate. If the excess at any one week was caused by more girls arriving than the average then the model would not be able to predict it.

It is also possible that there could be long-term imbalances in the system. Girls might, on the average, be doing their first session on OP a fraction of a week later as the months passed, and this might not cause a critical build-up of girls needing trained on that ward for several years.

A possible way of avoiding this sort of build-up is to consider every girl each week, looking at how much slack there is in her schedule and how many special wards she still has to visit in that part of her course. This way it is possible to avoid the situation where some girls have their options restricted near the end of their course; the position will be recognised in advance.

This point of view could be called training-based allocation. Both approaches however rely on there being a balance in the system between staffing requirements, and availability of nurses. If the system is in equilibrium then either will work. If it changes then problems will occur, in the first case with girls receiving insufficient training, and in the second with wards being under- or over-staffed. The second problem can be prevented in the long run by maintaining a relatively steady number of girls going through the system. Although the number in a set has reached 50% above or below the ideal, the number present in total at any week between June 1973 and Oct 1973 never varied by more than ± 7½%.

The first problem is more difficult to control. The presence of too many trainees is not the only factor which might lead to each getting insufficient experience. A drop in the number of deliveries at the hospital also causes difficulties, and is more difficult to control, especially if it is part of an overall drop in birth rate. At the moment some girls are having to spend a week out in another hospital to perform the necessary number of deliveries. It would seem then that the policy at the Simpson is to allocate from the training point of view, as the general pattern is of over-staffing, and to rely on this to ensure that there will always be enough girls available each week for each ward.

In this Chapter the allocating procedure used by the staff of the Simpson has been described. When considered along with the fixed rules affecting staffing levels and training requirements we have a complete definition of the 1973 problem. In the next Section these different aspects are combined to give a complete list of rules and preferences affecting the scheduling at that time.

3.7 The 1973 system - a summary of rules and objectives

The rules governing the 1973 system were summarised in Section 2.8 with regard to the training requirements and staffing levels needed. In this Chapter the procedures used by the staff of the Simpson when allocating have been described. Some are hard and fast rules; others are merely preferences, but when summarised they define the influences which affect the sequencing of each girl's training course. In the next Chapter, when dealing with computer models it is necessary to decide which of the Simpson's rules will be adopted as constraints on the models, but here they can be quoted in their entirety.

The following list summarises the points raised in this Chapter:

(1) First ward of training is 49, 51, 52 or 53.

(2) Length of visit is one week less than length of initial spell of day duty.

(3) If first ward is 51, 52 or 53, second ward is 49, LW or SC.

(4) If first ward is 49, second ward is 51, 52 or 53.

(5) If second ward is 49, LW or SC, length of visit is 4 weeks.

(6) If second ward is 51, 52 or 53, length of visit is at least 3 weeks.

(7) If second ward is 51, 52 or 53, length of visit is preferably 4 weeks.

(8) OP can only be done on day duty.

(9) The second spell of night duty is done on 51, 52, 53 and/or 54.

(10) The ward chosen will be started one week ahead of second spell of night duty.

(11) By that week the girl must have visited every special ward once.

(12) Every spare week up till then will be done on 51, 52 or 53.

(13) The girl can start her second half on any ward except if it starts on night duty in which case OP is impossible.

(14) The first spell of night duty in the second half will preferably

be done on 49, LW or SC.

(15) In the second half she will visit LW twice and every other special ward once.

(16) She will spend any spare weeks on 51, 52 or 53.

(17) These spare weeks will preferably be consecutive.

(18) They will preferably all be done on the same ward.

These rules cover the normal sequence of training for a student midwife as applied until 1977. The following list should be added for completeness:

(19) If a girl is ill or for any reason misses more than three weeks' work she may be moved to the set which started one month later than she.

(20) If a girl does a refresher course it will comprise:

Post-natal: 2 weeks
Ante-natal: 1 or 2 weeks
SC: 1 week
LW: 2 weeks
OP: 1 week
District: 1 week optional

(21) These can be done in any order as long as District is not the first duty.

The problem as approached by the staff of the Simpson in 1973 has now been completely defined. The model described in the next Chapter uses these rules as a basis for its structure, while the simulation described in Chapter 5 drops many of the arbitrary rules in an attempt to simplify the detail of the problem while preserving its essential structure and nature.

TABLE 3-15

1974 system - a comparison of training and staffing requirements

	STAFFING*			STAFFING - NURSE/WKS/YR (x52)			TRAINING		TRAINING - NURSE/WKS/YR (x130)	
WARD	MIN	OPT	MAX	MIN	OPT	MAX	MIN	MAX	MIN	MAX
51	7	8	13	364	416	676				
52	7	8	13	364	416	676				
53	7	8	12	364	416	624				
Post-natal	21	24	38	1092	1248	1976	15		1950	
54	4	4	4	208	208	208		4		520
49	11	12	18	572	624	936	7		910	
SC	9	10	14	468	520	$\boxed{728}$	6		$\boxed{780}$	
OP	7	8	11	364	416	572	3		390	
LW	14	15	18	728	780	$\boxed{936}$	8		$\boxed{1040}$	
DISTRICT	6	12	14	312	624	728	4	5	520	650

The figures in boxes show cases where there is a conflict between the two requirements, making overstaffing inevitable. In practice, since 2 girls do 4 weeks of District for every 1 who does 5 weeks, the training needs will supply 563 nurse weeks per year, less than the optimum staffing.

*Figures quoted by Senior Nursing Officer in 1974

3.8 Assessment of the Simpson's solution

The system described was used for some years, and worked successfully enough. Rules were broken from time to time, but this was not really crucial. The main problem was the amount of time which was taken each week to work out the allocation manually. However for the sake of obtaining a precise picture of the status quo for comparison with proposed alternative solutions let us look at the different aspects of the hospital's solution one by one.

(1) The roster is of necessity clumsy, since it is meant to be based on a monthly intake. In order to have each set arriving on the same day of the week the months have had to be assigned arbitrary four and five week lengths. This is satisfactory for the first few years, but after six years the set for one month will be arriving three quarters of the way through the previous month. What has happened in the past is that someone has had to make out a new roster every few year, incorporating a transition period which eliminated the fact that the old roster was out of step with the months. One solution to this would seem to be to abandon the idea of labelling each set with the name of a month. If they were referred to by colours, or letters of the alphabet then the problem wouldn't arise, and the roster could be allowed to slip behind by a day or so each year. At the moment it seems that the effort to keep each intake in line with a month is causing problems to arise and possibly errors to be made. For instance on the hospital's roster the type 1 and type 2 half sets (see Table 3-3) appear to be starting with five and six weeks of day duty respectively instead of four or five. At the end of the course they are doing eleven and ten

weeks respectively instead of twelve and eleven. This system would still work, although it would distort the time spent on different wards. For instance if a type two nurse started on Ward 49 she would do five weeks on it, instead of the normal three weeks half way through the first half. However when the plan sheets are consulted showing the actual weeks worked for each set one discovers that the Jan 73 group started normally but ended with the shorter second half session on day duty, giving them only 51 weeks training instead of 52. The April 73 group started with the longer start and finished with the shorter, giving them the correct length of course of 52 weeks. The July group had the long start, but the October group used the normal roster again.

To sum up, this uncertainty could be removed by retaining roughly twelve intakes every year, but by disassociating them from the months of the year.

(2) In the vast majority of cases the training schedule for each girl is adhered to, but in some cases the rules are broken. Take for instance the first half's training for the second half set of the January 73 intake. This is reproduced in Table 3-16. Non-standard allocations have been marked. In some ways, this deviation may be an advantage. Why did Young stay on LW when Wilson was due to start there, and could equally well have done so? In this case it seems to have been done to avoid putting Wilson on night duty without prior experience on LW. In that case the flexibility of the plan is an advantage. (She did only three weeks of LW instead of four in her second half to compensate). On the other hand the case of Hammer doing five

JANUARY	JANUARY				FEBRUARY				MARCH				APRIL					MAY				JUNE				JULY	
1973 SET	7	14	21	28	4	11	18	25	4	11	18	25	1	8	15	22	29	6	13	20	27	3	10	17	24	1	8
J. JAMES																											
E. REES																											
L. MAYES																											
A. MULHOLLAND																											
J. BRAND																											
E. WYLIE																											
G. WILSON	49	49	49	49	52	52	52	52	52	LW	LW	LW	SC	SC	SC	SC	54	54	LW	OP	OP	52	52	52	52	52	
S. YOUNG	51	51	51	51	LW	LW	LW	LW	LW	54	54	49	49	49	49	OP	OP	SC	SC	SC	SC	51	51	51	51	51	
L. WHITE	52	52	52	52	52	LW	LW	LW	LW	52	52	OP	OP	SC	SC	SC	SC	52	49	49	49	49	54	54	53	53	
E. LINTON	53	53	53	53	SC	SC	SC	SC	49	49	49	49	LW	LW	LW	LW	53	53	OP	OP	52	52	53	53	54	54	
N. HAMMER	52	52	52	52	49	49	49	49	49	SC	SC	SC	SC	OP	OP	LW	LW	LW	LW	54	54	53	53	53	53	EX	

TABLE 3-16

Actual planning table, January 1973 set, second half-set only weeks of 49 seems to indicate a breakdown of the system. She stayed on, not because she needed the training there, but because the ward was short-staffed. Once again this was allowed for in the second half of her course.

Overall it would seem that the training schedule for one nurse in six or seven breaks the rules normally adhered to, usually by keeping a girl on a certain ward longer than she should be. In the week before the second spell of night duty however, about two girls out of five fail to start on the ward with which they will continue on night duty. So usually it seems that deviations from the standard patterns of training schedule happen because the staffing situation has become critical on a certain ward for that week. Obviously the adequate staffing of wards week by week should take priority over the training schedules, but it would be better if both were satisfactory.

(3) How well are the wards staffed from week to week? Table 3-17 shows the actual numbers on each ward in the period June to October 1973. Weeks where the staffing has fallen below the minimum requirement are shown in red. Weeks where the staffing is at the optimum level are underlined. Most of the other weeks are over-staffed. It can be seen from this that under-staffing of the ward is a very rare occurence, and to this extent the system seems to be working well. However there are certain costs associated with this success. One of them, the disruption to girls' training schedules, has already been discussed and been found relatively unimportant. Another, though, is this:

With a relatively inefficient allocation system, satisfying

TABLE 3-17

ACTUAL STAFF PRESENT JUNE TO OCTOBER 1973

	1	2	W E E K S																				3	4	5	6
OP	8	7	9	9	9	9	9	10	8	8	8	11	11	11	10	8	8	8	8	6	7	9	176	8.8	11	55
49	12	11	13	12	17	16	16	17	16	12	13	16	18	16	17	16	17	16	13	14	16	14	305	15.3	18	90
LW	15	14	16	16	14	15	15	16	18	16	15	17	16	17	14	14	16	15	13	13	15	15	306	15.3	18	45
51	8	7	9	8	8	8	8	8	10	8	7	9	12	13	10	8	8	10	8	7	8	9	176	8.8	13	40
52	8	7	8	7	8	8	8	8	10	8	7	8	13	12	9	9	8	10	8	7	7	9	172	8.6	13	35
53	8	7	8	7	7	7	7	7	9	7	7	9	11	12	9	8	8	10	7	7	7	8	162	8.1	12	30
54	4	4	4	5	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	81	4.1	5	5
SC	10	9	9	9	10	9	9	11	12	10	9	11	14	13	11	11	13	11	10	9	9	10	210	10.5	14	45
NIGHT	32	28	35	33	33	32	32	32	32	36	33	33	28	28	31	31	31	32	33	33	34	33	645	32.3	36	50
DAY	73	65	76	73	86	76	76	81	87	73	70	85	100	89	83	79	82	86	71	67	73	78	1591		100	70
HOLIDAY			8	13	4	19	21	12	6	18	21	6	3	2	7	14	11	6	11	13	7	3	206	10.3	21	
DISTRICT	12	6	9	12	14	9	6	6	6	8	11	11	11	11	10	7	7	7	10	13	14	14	196	9.8	14	20
TOTAL			132	135	137	136	135	131	131	135	135	135	142	142	131	131	131	131	126	127	128	128				
AVERAGE SET SIZE			11	11.3	11.4	11.3	11.3	10.9	10.9	11.3	11.3	11.3	11.8	11.8	10.9	10.9	10.9	10.9	10.5	10.6	10.7	10.7				

Columns :
1 Optimum
2 Minimum
3 Total
4 Average
5 Maximum
6 % of cases overstaffed

minimum criteria can only come at the expense of general overstaffing. Column 6 of Table 3-17 shows the percentage of cases in which a ward is overstaffed in this period. It can be seen that the figures are very high. As mentioned earlier, this overstaffing can be critical on the Labour Ward where girls have to get a certain amount of experience on actual deliveries, as well as causing general overcrowding. The costs of overstaffing may not be very high, but the real disadvantage is the inefficient use of resources.

CHAPTER 4

4.1 Introduction

The previous Chapter describes the drawbacks associated with the manual allocation system in use in 1973. Of these the most serious was the time which the scheduling took and the mental strain caused by the task of having to settle the allocations only a week prior to the time when they came into operation. The member of staff who performed this task used to spend up to four hours outside her normal working period on a Friday evening in order to establish the next week's schedules, and confessed to living in permanent fear of discovering one week that an allocation was just not possible within the rules[1].

It appeared that to improve on that solution method a more efficient allocation system was required. It also seemed advantageous to set up a system which could be run for several weeks into the future, to identify possible future bottlenecks.

For simplicity a constant average number of arrivals for each new set was assumed, but this number could be altered to see what variations were caused.

To this end a computer model was designed which would mimic the real-life situation as closely as possible, the idea being that different systems could be tried out on the model to see which gave the best long-term results. The resulting program was, of necessity, rather long and consisted almost entirely of logical rather than arithmetic operations.

This Chapter sets out to describe the model and to indicate how it was developed. It can be divided into sections whose specific tasks can be carried out independantly of the other sections. This simplified the design task and also provides a scheme by which the operation of the model can be described in a step-by-step fashion.

The four main divisions in this Chapter are:

4.5: What choices are open to each girl? This describes the development of various methods, and explains the adoption of one of them.

4.6: What criteria should be used to decide between these choices? This introduces the concept of "urgency", which is used to decide which nurse to allocate first.

4.7: Allocation. In this section some allocation heuristics are evaluated.

4.8: Data handling. This describes the scheme by which data are stored, and indicates how they are modified by the program.

Section 4.9 outlines the basic differences between the model and the manual approach.

4.2 General aims of the model

An attempt was made to design a computer model which duplicated the characteristics of the real situation in as many ways as possible. The main objective was to draw general conclusions from the solution of the problem which might lend themselves to formulation as heuristics to be used with similar problems. For the purposes of generating the model, the rules of allocation and optimal staffing levels were taken directly from those used by the hospital, with no attempt being made to adapt, improve or criticize at this stage. The results produced by the model should then cast some light on the structure of the real-life problem.

Ackoff and Sasieni describe this approach to problem solving in "Fundamentals of Operations Research"[2]. Scientific method usually entails experimentation, but in operations research it may not be practical to manipulate variables in an organisation; for instance a company cannot risk failure in order to carry out a successful experiment. The answer to this is to construct representations of the system (models) on which the research is carried out. In most cases it is possible to find the optimal values of the controllable variables - those values that produce the best performance of the system for specified values of the uncontrolled variables. The solution may be derived by conducting experiments on the model (i.e. simulation) or by mathematical analysis. In some cases the values of the variables must be known (concrete or numerical analysis) while in others it may not be necessary (abstract or symbolical analysis). For certain types of function classical mathematics provides powerful tools for finding the best values of the controlled variables, and new techniques have been developed in recent years for those cases where the constraints are too numerous for the traditional methods.

On the other hand the function may consist of a set of computational rules (an algorithm) which permits us to compute the utility of performance for any specified set of values for the controlled and uncontrolled variables. This does not provide the optimum values directly, but usually it is possible to devise a system whereby successive values of the controlled variables are chosen which converge on an optimal solution. With some systems a good solution can be found relatively quickly and easily, while the cost of continuing till an optimal solution has been found would outweigh the benefits which this improvement would bring.

4.3 Nature of the model

The model which was designed to represent the scheduling problem at the Simpson was a concrete or numerical model. A procedure was incorporated which tended to produce a good initial set of values for the controlled variables. The utility of the model was then evaluated, and, if this was acceptable, the process was repeated for the variables corresponding to the next time period. Several models were designed, each of which performed a part of the total task, with varying degrees of simplicity or sophistication. These models were then combined. The usual relationship was that the output produced by some parts was used as input by others. The largest part of the combined model at this stage was concerned with data handling rather than allocation. For instance, one part of the model would determine what the roster was for all girls, while another would discover which girls were free to be moved in the week under consideration. Another would find out which wards each girl could move to, while yet another would decide which girl's situation was the most urgent. All of this processing would have to occur before an attempt was made actually to allocate any of the girls for that week.

When the model had been developed to this stage, a very simple heuristic was used to allow it to make allocations, and its performance was assessed. The model could not mimic faithfully every aspect of the real life situation in every respect. For instance the case of girls arriving at random intervals to do refresher courses was not included. Neither was the situation where a girl would miss some weeks because of illness and then drop back to another set. The reason for this is that the information to be gleaned from the program by making it slightly more realistic was minimal in relation to the amount of extra programming which would be required. There are other examples than those mentioned, and they will be dealt with under the relevant section in the description of the development of the model.

4.4 Description of its development

It is felt that the best way to describe the form and functions of the model would be to describe its component parts in the order in which they were developed. If the aim of each part of the model is understood, along with the method by which it produces its results, then the structure of the full scale model will be much more readily apparent. In the full model, some of the earlier parts are present as untouched portions of text, others as subroutines.

For the purposes of this model the rules and values quoted in Section 2.8 were adopted, with the roster as shown in Table 3-3. The sequencing rules used were those described in Section 3.7 with the following modifications:

> Rule (6) is dropped and rule (7) is upgraded from a preference to a fixed course of action.
>
> Rules (14), (17) and (18) are dropped since they tend to complicate the programming without making any appreciable difference to the capability of the program. If the model worked well they could be introduced subsequently as extra constraints, and if it didn't work well they would be irrelevant. Similarly the rules (19) to (21) describe situations which do

not affect the majority of trainees, so these events, together with that of girls leaving during the year, were not built into the structure of the model. As in the cases above they could be added subsequently if the model proved to have a useful practical application.

In "Fundamentals of Operations Research"[3], Ackoff and Sasieni point out that a solution to a model can only be of use if the model copies faithfully enough the characteristics of the real-life situation. The procedure described in the previous paragraph is an attempt to copy all of the salient aspects of the real problem as approached by the staff of the Simpson while leaving out any irrelevant detail.

The model's function can be divided into four main parts; the first determines what wards a girl can move to at each juncture, while the second compares the choices which are available according to various criteria. In the third part the individual allocation is made according to the results of part two. The fourth part is the data manipulation and storage. This division is not sequential - the four aspects of the model overlap and inter-relate, but a division of this nature makes analysis and description of the model more coherent.

4.5 What choices are open to each nurse?

This was the first section to be tackled when the model was being designed. The problem can be stated:

(1) The number of weeks of her course which have been done already.

(2) The nature of the roster she is on.

(3) Her past experience i.e. which wards she has already

visisted, in what order, and for how long.

The first move was to attempt to draw up a decision tree which would show all possible combinations of training schedules.

4.5.1 Decision trees

If a girl's experience is traced along successive branches from the start, each node representing duty on a new ward, then a node will be arrived at which represents the ward she is presently on. Branches will lead from it to a number of other nodes, each of which represents a ward to which she is permitted to move to at her next change. A section of this type of system representing the first half of the course is shown in Figure 4-1. This section represents all possible schedules for a student nurse who has started with Ward 49 as her first position. There are two main drawbacks with the method of following a decision tree to determine the choices open to a girl at a given stage. The first is that, in itself, the system is not completely authoritative. The list of choices which it gives at each node is still subject to further conditions. For instance the occurrence of G as a possibility at a node is still subject to the amount of float remaining in the schedule. That is to say that the choice of G might still be open at this juncture, so long as too many weeks have not been spent on G earlier in the course. As another example, take the case of the end of the first half where a girl has the choice of OP —— G or G —— OP —— G. If there are only two weeks on day duty left before the final first half spell of night duty then the first case is the only possible one, since the second would involve doing OP on night duty, which is impossible. However, if there were three or four weeks left before the weeks of night duty then it would be possible for a girl to do

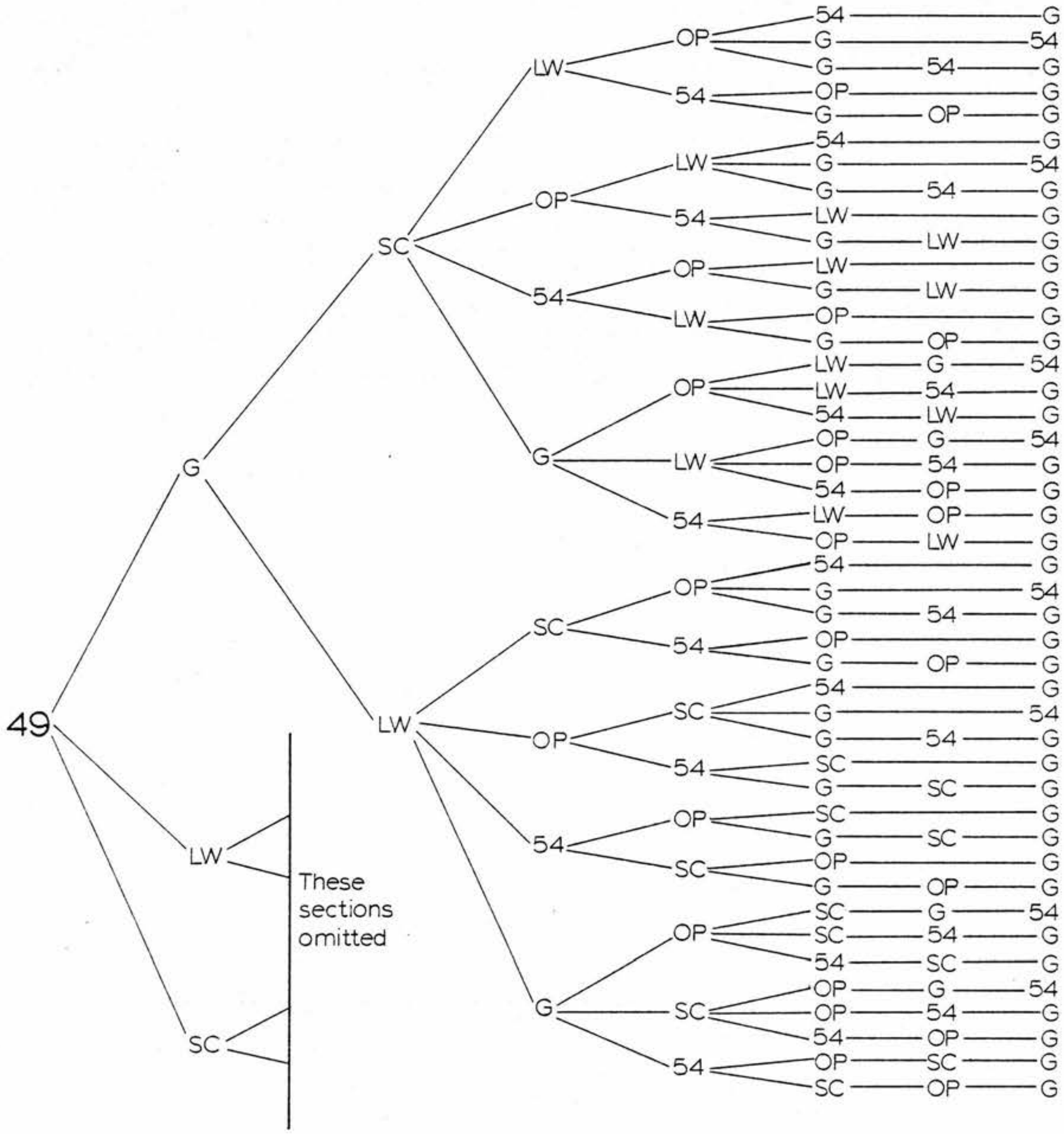


FIGURE 4 - 1

Branch diagram showing possible course sequences starting on Ward 49 one week of G before her spell of OP.

This then is the first objection, that the choices defined at each stage are all possible under some conditions, but are not definitive in all cases. The second objection is that of size. To list the complete set of all possible schedules involves the use of an unnecessary amount of computer storage space. When the branch system is drawn out in full the number of possible routes is approximately nine hundred for the first half and seven hundred for the second. If it is permitted for a schedule to split the work on Postnatal Wards in the second half to more than one visit then the potential number of schedules rises to 5,000 with two visits to antenatal, and over 40,000 with three. It would seem an improvement if an algorithm was designed which would generate parts of the decision tree, rather than storing all the information contained in the whole tree. The advantage of this approach is that in many cases the system can be entered somewhere in the middle, without having to trace the relevant girl's course right from the beginning.

4.5.2 The advantages of an algorithmic approach to choice determination

What is it about this method which gives it greater flexibility than the tree network? The answer is this: The tree network is totally divergent. At each node, the number of choices open to a girl is represented by the number of branches shown. As a result the total number of ways of completing the course can be found merely by counting the number of branches which exist as a representation of the last week of the course. It can be seen that in order to discover the choices open to a girl in any one week it is necessary to identify precisely which node she is at. The only way to do this is to trace her course right through from the start.

However a tree is only a specialised form of net work in which no loops occur. It is possible to modify the existing divergent network so that it converges in places, by identifying nodes which are common to more than one branch, as demonstrated by the following example:

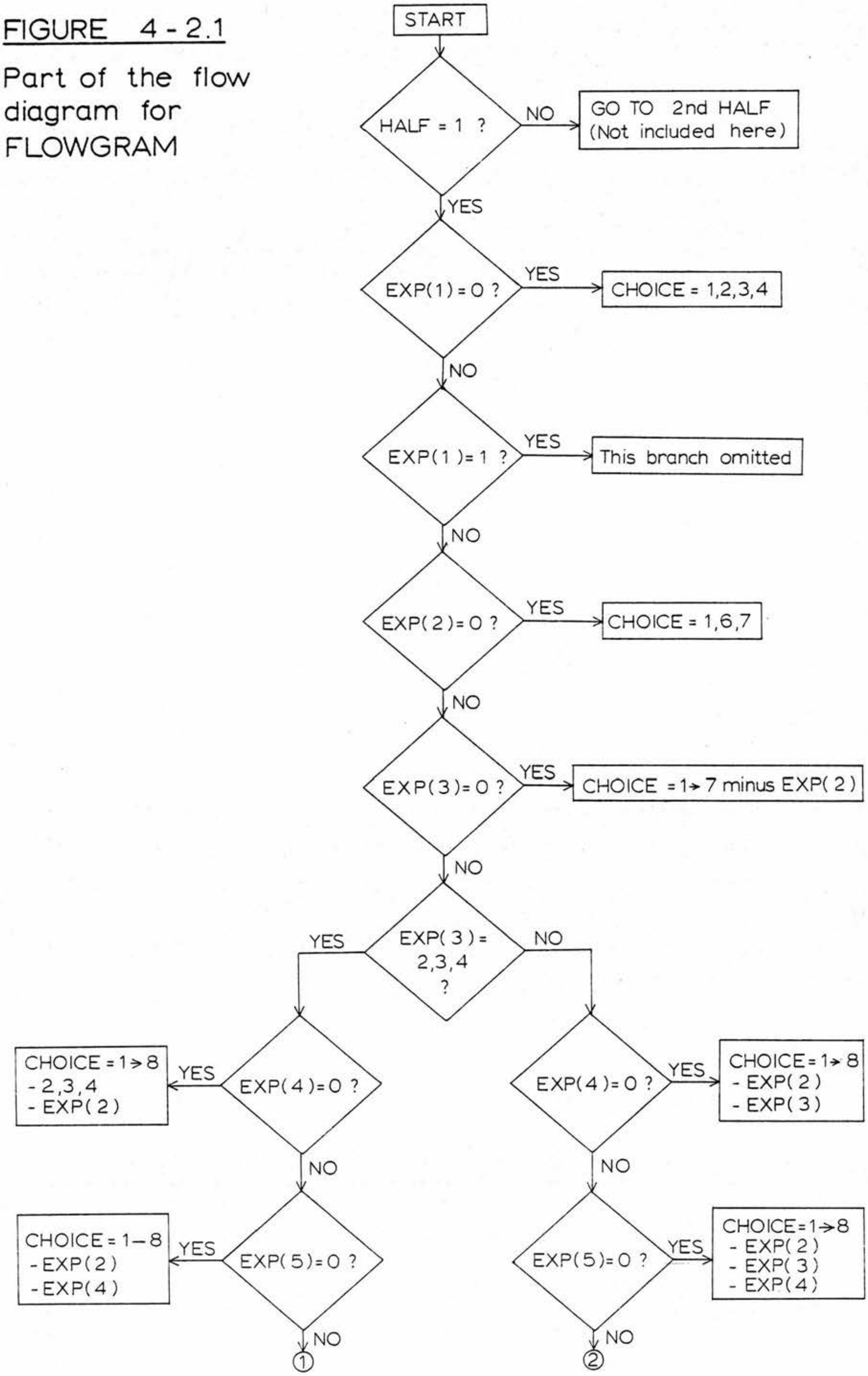
If a girl starts her course with Ward 49 instead of a general ward then the next two or three choices she has will be affected. At this point therefore the system is diverging. However, by the time she gets to the end of the first half of her course, with only a couple of wards left to visit, the identity of the first ward she visited is no longer relevant. It can be seen that somewhere during her first half the system has become convergent again, giving a final part to her course which is to all intents and purposes linear.

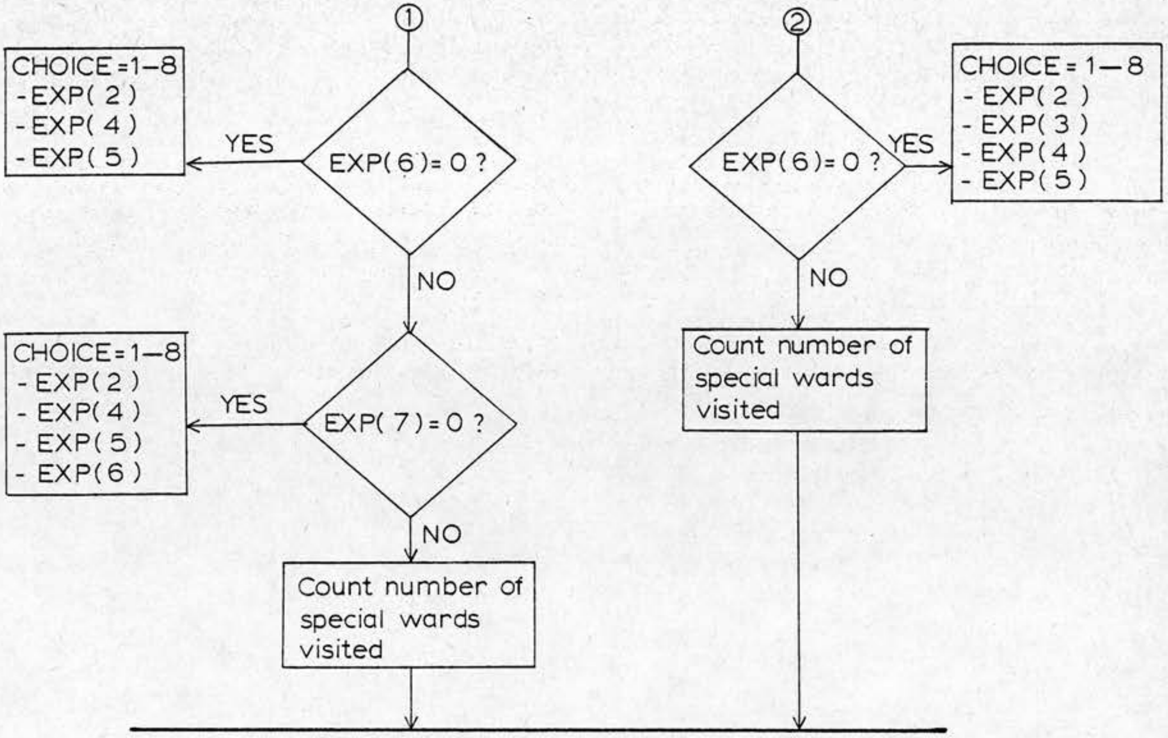
It may be necessary to design an algorithm which analyses the options at relevant nodes in order to achieve this convergence.

The first algorithm to be constructed was still largely divergent except in two places (Figure 4-2). At each node a number of choices may be offered. These depend on the part of the algorithm's flow diagram which the girl has reached, and also on the wards which she has visited on the way. A choice between wards at any stage does not necessarily imply divergence. In fact the only time that this is the case is right back at the first week where the choice between Ward 49 and the general wards determines which half of the flow diagram will be followed. All other divergences in the system are a result of simple tests such as "Are there more than two weeks left?" and "How many special wards still have to be done?" (After choices 9 and 13). This algorithm was not entirely satisfactory and was improved on later, but it is worth describing its operation

FIGURE 4 - 2.1

Part of the flow diagram for FLOWGRAM





Rest of flow diagram omitted

FIGURE 4 - 2.2

Continuation

EXP(n) indicates the nth ward to be visited.
The numerical values assigned to the right hand sides of each expression are as follows:

CODE	WARD
1	49
2	51
3	52
4	53
5	54
6	SC
7	LW
8	OP

here for comparison with later alternative methods.

4.5.3 Flowgram

Flowgram is the name given to the computer program based on the flow diagram part of which is shown in Figure 4-2. If the whole flow diagram had been included it would have extended to several pages - the section shown is sufficient to show the method. A glance is sufficient to indicate that it is still inordinately long for the job which it does. It takes as input a roster for one girl, and a list of the wards which she has worked on at each week of her course so far (Plan). It processes "plan" to give a shorter list (EXP) which stores the identity of each ward she has visited, regardless of the duration of that visit.

If Plan so far looked like this:

G.G.G.G.LW.LW.LW.LW.G.OP.OP.SC.SC.SC.SC

then EXP would look like this:

G.LW.G.OP.SC.

It then processes this information and prints out what choices are open to the girl for her next week's work. Although this program was based on the flow diagram shown in Figure 4-2, various modifications were made in order to make the programming easier. In effect the flow diagram traces the possible paths through the decision tree, part of which is shown in Figure 4-1. In most cases it identifies specific nodes of the tree, but where there are sections which are shared by more than one route, it uses a subroutine to attain some convergence.

For instance there is one in Flowgram called "allocate night duty" which deals with the last five or six weeks of the course. This can be called from three different points in the program, in the first and the second half, so in effect it provides a common ending for more than one branch of the flow diagram.

4.5.4 Flowgram's faults

To re-iterate, flowgram is unnecessarily long, and is not definitive enough. That is to say that its final selection of choices might still have to be modified with reference to the point on the roster occupied by the girl in question. Luckily both of these problems can be solved by the same methods. These are:

(1) Pay attention to the week of the course which the girl in question is on, with reference to her roster type.

(2) Use the concept of float more extensively to determine whether a girl still has the Post-natal Wards as a choice.

The effect of these two measures can be simply described. Originally we started with a branch diagram, which was totally sequential i.e. to identify a node one had to trace a girl's course right through from her first week. Flowgram continued to be sequential, but started to achieve some convergence towards the end of the first half which at least allowed some sub-routines to be used to reduce the size of the program. Now, by introducing the two points above, we have a situation where choice can be determined half-way through the first half of the course without having to trace through every ward which the trainee being considered has visited.

4.5.5 Choice

A new program was written embodying principles (1) and (2) above. In fact a certain degree of sequentiality was preserved in parts where it seemed simpler to tackle the question in that fashion. For instance a different route will be followed at first depending on whether the girl started on a pre- or post-natal ward. However, since the routes converge again so soon, the main part of the program is independant of the early sequence. A flow diagram for the first half of this program is shown in Figure 4-3.

Choice takes as input the following information:

(1) the roster type for the girl in question.

(2) The week of the course which she is on.

(3) Her plan to date, showing which wards she has visited and for how long.

It processes "plan" to give "exp" (experience to date i.e. wards visited regardless of duration).

As output, it produces the following information:

(A) The list of choices open to her for her next week's work.

(B) The half of the course which she is on.

(C) Her float (the number of weeks of post-natal still to be done).

This program is much shorter than flowgram, and the results which it gives are totally definitive, since reference to roster and float has been made from the earliest stages. The second half of this program is shown in Figure 4-4 and it can be seen that, thanks to the greater simplicity of the second half of the course, total linearity has been achieved i.e. there is no divergence.

In Section 3.7 a list of rules was given covering the required training pattern as adopted by the hospital. Choice satisfies all of the rules one to eighteen, with the exception of two, five, six and seven which refer to duration of stay rather than the order in which wards should be visited. This aspect of the

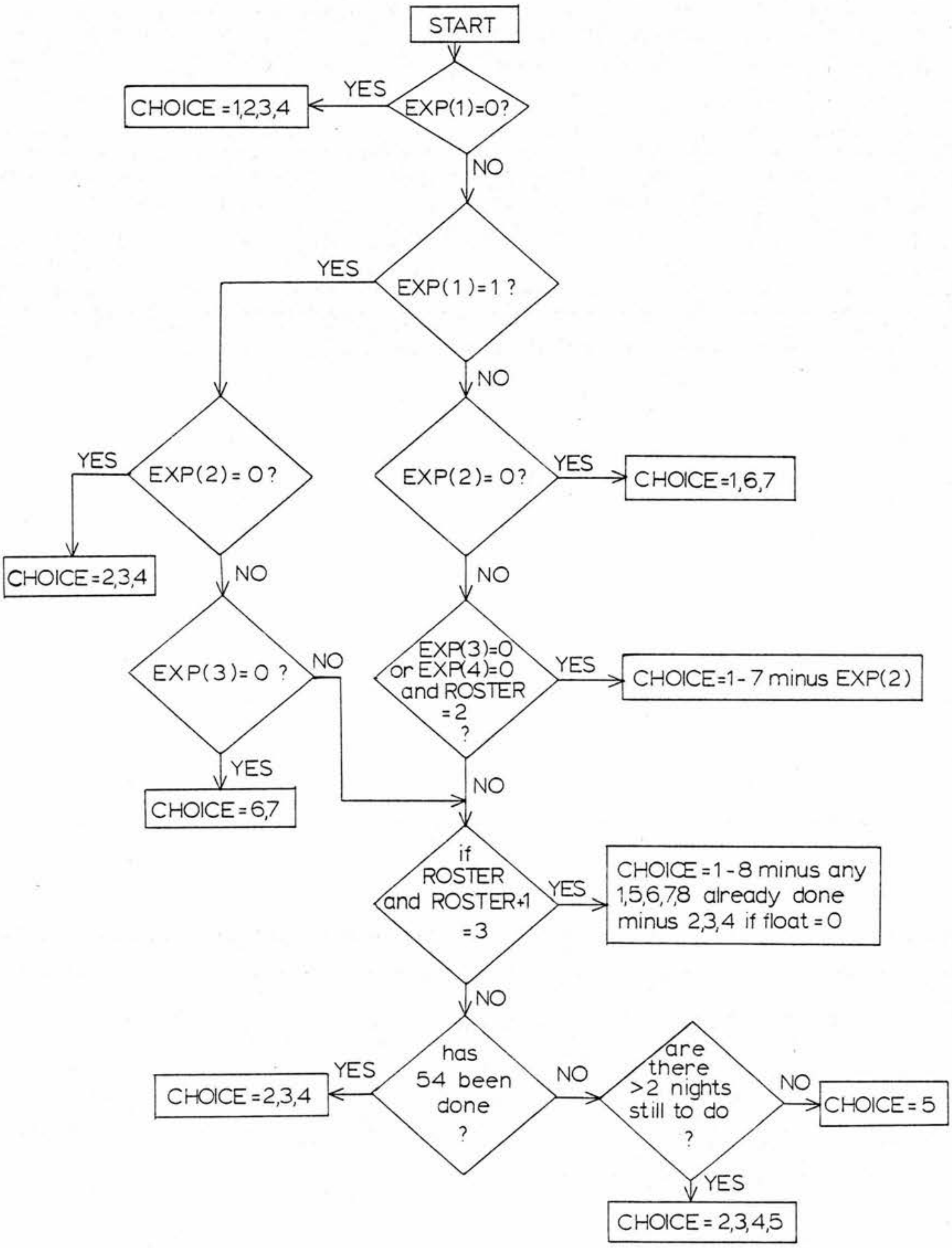


FIGURE 4-3

Flow diagram of CHOICE (First half)

See Figure 4-2 for key

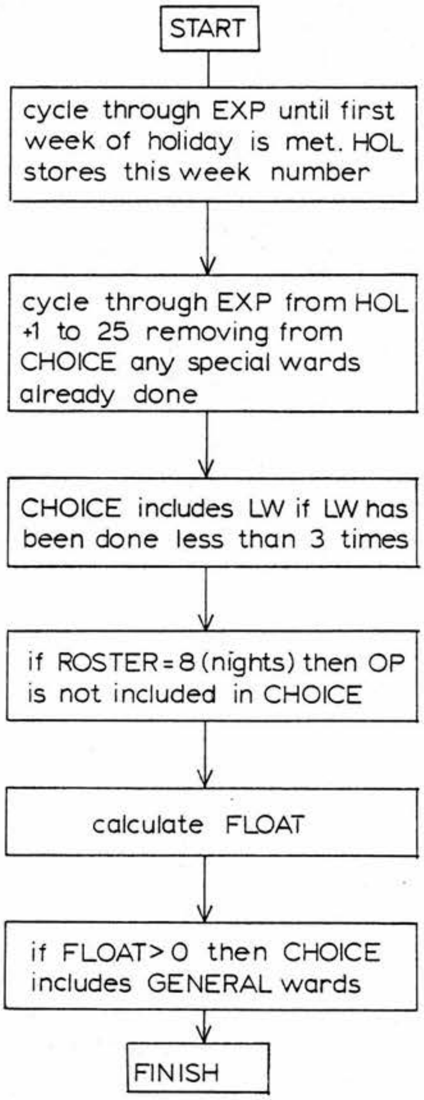


FIGURE 4-4

Flow diagram of CHOICE (Second half)

problem will be dealt with in a different part of the program, to be discussed later.

A copy of the output for Choice is included in Appendix E. It is designed to be run as an interactive program. That is to say that the program asks for certain data as it is running, producing a form of conversation with the operator through the medium of a teletype machine.

4.6 What criteria should be used to decide between these choices?

4.6.1 Urgency

Urgency is a model which is designed to mimic the human decision - making process involved in determining which girls should go on to which wards. Because of this derivation it is a heuristic rather than an optimisation technique. There are several reasons why this problem as it stands does not lend itself to optimisation, other than locally for specific totally defined sections, the most important of which is the fact that it is too cumbersome in terms of store requirements.

As stated in General Aims at the start of this Chapter, it is possible to design an algorithm for many problems whose initial or early solutions are sufficiently good that the additional benefits which would be gained by optimising would be outweighed by the extra computational time involved. Also, in order to optimise using the technique of re-solving the problem with slightly different values of the controllable variables each time, it is usually necessary to devise a system whereby an improvement in overall utility is achieved at each iteration. If this is not guaranteed then it is not always possible to ensure that the solutions will eventually converge on the optimum. In Section 3, iterative techniques are discussed, but the intention here is to assess what degree of success can be achieved using a non-optimising simulation of the human decision - making progress.

In general a satisfactory solution is being sought to a set of constraints which, when found, determines the allocations for successive weeks, i.e. we are merely looking for a feasible solution. Having found one there is no need to search for alternatives, for, although they exist, they will be no better than the one we already have, according to the set of rules which shape the problem. For instance if we have put girl X on Ward 51 and girl Y on Ward 52 we may have a feasible solution. Changing it so that X is on 52 and Y is on 51 is not going to alter the situation for better or worse as long as both girls are free to be put on these wards.

The observant reader will have noted that it was stated earlier that a set of solutions was being sought which would serve to minimise the absolute deviations from the staffing levels named by the hospital as being optimal. This is quite true, and the problem will be formulated in this way mathematically, but when the problem is being tackled manually, or using a set or heuristics built into a computer model, a different approach suggests itself.

Let us assume that the total number of staff available is exactly sufficient to provide the optimum staffing level for each ward, no more, no less. It was seen in Table 1-4 that the minimum staffing level for each ward is always one less than the optimum level, with the exception of Ward 54, where the required level is fixed, with no deviation permitted, and district work, which depends on the size of successive sets. If we have exactly the right number of girls then a deficiency on one ward will always be accompanied by an overstaffing on another.

Because of this relationship, not only is understaffing constrained against, but so, indirectly, is overstaffing. As a result our solution will be feasible only if it falls within narrowly defined limits. In practice there are more girls being trained than would be needed if the allocation was 100% efficient, so hopefully any deviations from the optimum will be in a positive direction, i.e. causing over- rather than under-staffing, a situation which is obviously not so critical from the point of view of patient care.

These three points would seem to suggest that a heuristic approach is justified. It is now necessary to decide which criteria will determine the allocations of different girls. Let us assume that the problem is in the process of being solved - last week and its predecessors had satisfactory solutions, and the immediate task is to allocate girls for the coming week. The program CHOICE has provided the information as to which girls can go to which wards, so how should the decisions be made between the alternatives provided?

The following is a list of situations chosen to represent the whole range of urgency from critical to non-critical allocation decisions. They are presented in random order so that each can be assessed on its own merits:

A girl may be:

(a) Starting the course this week.

(b) Somewhere in the middle of the second half - choice of two wards - float zero.

(c) On second spell of night duty.

(d) In middle of first half - choice of four wards.

(e) Two weeks of course left - two weeks of LW still to do.

(f) Already on a ward which was chosen last week.

(g) In second half - choice of two wards - float three.

It is obvious that some are at more critical parts of their courses than others, but which would we decide to look at first? Which cases are the most urgent?

No mechanism exists at the present state of the model for removing a girl from a ward once she has been placed on it, so it is fairly evident that the highest priority situation for this week is the one where a girl has already been allocated in a block starting on a previous week, case (f). Henceforth such a girl will be described as being "pre-allocated" for the relevant week.

Setting this case aside, which remaining situation presents the smallest range of choice? A glance at the list will show that case (e) must be most urgent. Here there is only one choice - the girl must go onto LW - so her schedule is as fixed and as high on the list still to be allocated as it could be. The remaining cases are not as simple as these two, however. From among a series of apparently disimilar cases it is necessary to extract a number of common characteristics which can be used as criteria when the decision is made as to which case to consider for scheduling first.

Obvious points are these:

(1) How many wards does the girl have left to do in this half of her course?

(2) How long will it take to do them?

(3) How many weeks does she have left to do them in?

(4) Are all her choices special wards?

(5) If not, how many weeks of general wards does she still

have to do?

However, the girl might be at a point in her course where these questions are not applicable; for instance she may just have finished the first four weeks of her course, performed on Ward 49. She has a large portion of the first half of her course to complete, has many wards to visit in that time, and has the maximum amount of float (weeks of general wards still to do), yet she is constrained by the stipulations of her course to visit a general ward for at least four weeks of her fifth week onwards. Her urgency will be greater than the general criteria above would suggest, so we must classify her as a special case.

Let us then take another look at the questions which must be asked in order to determine the urgency for any particular girl. It is possible to simplify the five above by combining them.

(A) Is the girl at a special point in her course?
    YES: Give her a pre-determined urgency appropriate to that case.
    NO: Continue.

(B) How many special wards does she still have to do?

(C) What is her float? (How many weeks of General does she have left?)

Her float is determined by a subroutine which finds out the number and identity of the special wards she still has to visit, counts ahead to the next natural break in the routine of the course (the second spell of night duty in the first half of the course: the holidays at the end of the second half of the course), and subtracts from the number of weeks left till that point the number of weeks needed for special wards. The questions (1) to (5) are answered as follows:

(i) Answer is the number of special wards left (counted by subroutine) plus one if float is greater than zero. (Remember that she does not have to visit all three general wards)

(ii) The minimum time is now known for the special wards still to be done. (The subroutine has access to the information relating to lengths of stay on wards done at different parts of the course)

(iii) The subroutine has found this out by a straightforward count. To be strictly accurate the subroutine counts ahead in the first half until it reaches one week prior to the second spell of night duty, since the change over to G and 54 work starts from then.

(iv) Yes, if float equals zero, no otherwise.

(v) Answer equals float.

It can be seen that our original five questions have been reduced to a check for generality followed by a two-criterion assessment.

Common sense, supported by experimentation, tells us that the first criterion - "How many special wards still to do?" - is a stronger one than the second - "Size of float?" - since the nature of the problem dictates that we are trying to allocate first those girls with the most limited choice. The second criterion can still serve a function, as a tie-breaker in the case of two girls having the same number of special wards each still to visit. Let us examine the implications of this decision by considering two examples:

(1) Girl A has four special wards to visit and her float is

one week. Girl B has one special ward to visit and her float is four weeks. Girl A can go onto any one of four special wards next week plus three general wards, giving her a total number of choices of seven wards. Girl B can go onto one special or three general wards giving a total of four choices.

In the case of Girl A, all four special wards have to be visited at some time or another, but only one general need be. Since the girl can be put onto any of the general wards (the one with the greatest shortage) they can be classed together, for the purposes of comparison in this example, as one category. Using this basis for comparison, A has a genuine choice of five wards, and B only two. B should therefore be more urgent.

(2) Girl A has one special ward to visit and her float is one week. Girl B has one special ward to visit and her float is five weeks. Both have a choice of two categories (four if the individual general wards are considered). They are both therefore equally easily allocated this week, so we must look to the future. If we put Girl B on the special ward this week, then by next week Girl A (who has been doing one week of G - her only week of float) will be reduced to one choice with zero float - the highest urgency other than pre-allocated. However if A were to be put on the special ward and B on General then it would take B five weeks to reach that state or urgency, during which time another place on the special ward will quite possibly have become vacant.

Looked at another way the number of schedules available to A is two, while the number available to B is six. Thus the minimum number of weeks before starting the special ward will be zero, if special is next week, while the maximum number will be equal to the amount of float. If each different schedule is considered independently, then the number of possible schedules in the one-special-ward case will be Float + 1.

Therefore, the lower the float, the higher the urgency.

Let us now define RESIDUE as being the number of special wards still needing to be visited at any point in a girl's course.

Using our first and second level criteria we can construct a hierarchical order covering most general cases, giving each one a number to represent its urgency. This is shown in Table 4-1. Omitted from this first table are two important cases, those for the row RESIDUE = 0 and the column FLOAT = 0. Values for these, and for the special cases, can only be arrived at by trial and error. The rationale behind the assignment of the first set of values was as follows:

(a) Treat the zero float situation as a special case, assuming that the absence of G as an alternative makes a lot of difference. The reason for this is that the optimum staffing level asked for by the hospital for the three general wards combined is twenty four, nine greater than the optimum for LW, the next largest. The minimum satisfactory staffing level for the three general wards, once again as stipulated by the hospital, is twenty one, compared to fourteen for LW. It is obviously easier to find a place for a girl in a category where the requirement is 21 - 24 rather than one where it is 14 - 15, since a greater week by week variance is acceptable.
For these reasons the zero float situation was given a higher set of urgencies.

TABLE 4-1

Simple Urgency Table

		FLOAT							
		1	2	3	4	5	6	7	8
	1	31	30	29	28	27	26	25	24
RESIDUE	2	23	22	21	20	19	18	17	16
	3	15	14	13	12	11	10	9	8
	4	7	6	5	4	3	2	1	0

TABLE 4-2

Modified Urgency Table, 54 done

		FLOAT								
		0	1	2	3	4	5	6	7	8
	0		37	37	37	37	37	37	37	37
	1	37	31	30	29	28	27	26	25	24
RESIDUE	2	36	23	22	21	20	19	18	17	16
(54 DONE)	3	35	15	14	13	12	11	10	9	8
	4	34	7	6	5	4	3	2	1	0

TABLE 4-3

Modified Urgency Table, 54 not done

		FLOAT								
		0	1	2	3	4	5	6	7	8
	0		36	36	36	36	36	36	36	36
	1	37	33	32	31	30	29	28	27	26
RESIDUE	2	36	25	24	23	22	21	20	19	18
(54 NOT DONE)	3	35	17	16	15	14	13	12	11	10
	4	34	9	8	7	6	5	4	3	2

(b) In the residue = 0 situation the only choices are from among the general wards. This limits the number of choices considerably, and although it should be relatively easy to find a place on a general ward, the limitation would seem to merit a high urgency rating. This is further justified by the fact that the needs of a girl who can only go onto a general ward tend not to clash with those of girls who have special wards still to do. In this case it will not matter how many weeks of G the girl still has to do, since the ranges of her choices for subsequent weeks will remain unchanged, suggesting that all girls with residue = 0 should have the same urgency regardless of float.

The modified table is represented in Table 4-2.

4.6.2 Special cases - Ward 54

It can be seen that only four special wards are being considered in the section above. This is because Ward 54 is a special case. It is not compulsory for a girl to visit Ward 54, but out of each batch of girls, a certain proportion will have to do it to maintain the staffing level there. The model simulates this situation by making Ward 54 non-compulsory, but by increasing the urgency of a girl who has still not visited that Ward, increasing the likelihood of her being picked for it. For all of the values in Table 4-2 other than the row and column for residue = 0 and float = 0 a figure of 2 is added to the urgency if the girl has not visited Ward 54.

On the other hand, 54 behaves in some ways as an alternative to the general wards, in that it can be done during the second spell of night duty. Also its non-compulsory nature makes it in effect increase the value of the float, especially for the row residue = 0. Since it is increasing the choice for that row, without introducing any elementof compulsion, the row residue = 0 has all of its values reduced by 1 if 54 still has not been done.

The column float = 0 has its values unchanged, since the performance of 54 affects float, a float of zero implying that 54 has already been done.

Table 4-3 shows the figures for a case where 54 has not yet been done, as opposed to 4-2 where 54 is assumed to have been done.

4.6.3 Further special cases

(1) On the first week of a girl's course, 54 has not been done, and there is a choice of one special ward (49) or G (so float must be greater than zero). Looking at Table 4-3 , the most appropriate urgency is 33. The tables cannot be used directly for this case, as the first spell of night duty is still to be done, and the concept of float here must be meaningless.

(2) If a girl has started on G, and now awaits allocation to her next ward, she has a choice of three special wards, with no G. This gives an urgency of 35.

(3) A girl waiting to start her second ward who has just finished on Ward 49 will have a choice of G only. Here the urgency is somewhat arbitrary. We know that it should be high, to be compatible with other cases where the choice is G only, but in this case the girl is being placed on the ward for at least four weeks, as opposed to the one week minimum in other cases. For the sake of simplicity the figure of 35 was chosen, to be the same as case (2) above.

(4) If the week being considered is the week before the second spell of night duty starts, then the girl will be about to start on

a general ward. Her situation will be the same as that in case (3) above, so the same urgency of 35 is awarded.

At this stage it was decided to distinguish between a girl who had a choice of only G and who was already on a general ward, and a girl with a choice of only G who was doing something different up till this point. The reason for this is that it was intended to incorporate a preference later in the program for a girl to stay on one general ward rather than moving from one to another. In order to facilitate this it is advisable to make the case of a girl who is already on G more urgent than that of a girl just starting G. The difference chosen was 2, taking priority over the difference of 1 produced by consideration of whether or not Ward 54 had been done. Consideration of this point gives us a value for the next special case.

(5) If the week being considered is during the second spell of night duty then the girl is in the same situation as in case (4) above, except that she is already on a general ward. The urgency is thus 37. Similarly, in the cases of residue = 0, where the girl is not already on a general ward, the urgency can be reduced by two.

In case (4) and (5) above, if Ward 54 has already been done, then the urgency can be reduced, since, as described in the previous Section, the urgency rating should make it likely that 54 will be visited, without making it compulsory. The equivalent cases to (4) and (5), in the cases where 54 has already been done, are as follows.

(6) One week before the second spell of night duty, 54 already done : urgency = 34.

(7) During second spell of night duty, 54 already done : urgency = 36.

TABLE 4-4

Complete Urgency Table

1st WEEK: 33 PRE-ALLOCATED: 38

2nd WEEK: 35

54 DONE -

		FLOAT								
		0	1	2	3	4	5	6	7	8
ON G -	0		37	37	37	37	37	37	37	37
NOT ON G -	0		35	35	35	35	35	35	35	35
	1	37	31	30	29	28	27	26	25	24
RESIDUE	2	36	23	22	21	20	19	18	17	16
	3	35	15	14	13	12	11	10	9	8
	4	34	7	6	5	4	3	2	1	0

54 NOT DONE -

		FLOAT								
		0	1	2	3	4	5	6	7	8
ON G -	0		36	36	36	36	36	36	36	36
NOT ON G -	0		34	34	34	34	34	34	34	34
	1	37	33	32	31	30	29	28	27	26
RESIDUE	2	36	25	24	23	22	21	20	19	18
	3	35	17	16	15	14	13	12	11	10
	4	34	9	8	7	6	5	4	3	2

ONE WEEK BEFORE 2ND NIGHTDUTY TILL END OF 1ST HALF -

ON G	NOT ON G	
36	34	54 DONE
37	35	54 NOT DONE

4.6.4 Justification of urgency values

The full set of urgency ratings, including special cases, is laid out in Table 4-4. These urgencies are relative, being based on an assessment of the human scheduler's decisions and on the analysis of the problem detailed above. If the model produces good results then the choice is vindicated; otherwise it will be seen to be a poor simulation, in which case some relative values may be altered. The hypothesis is that the scheduling is performed according to the general principles outlined above, but not necessarily as represented by the relative values given in the urgency tables (4-4).

4.7 Allocation in the model

We are now in the situation of knowing for each girl the wards which she is free to visit and the urgency which is associated with her present situation. It is also assumed at this stage that we know how many girls are on each ward at the present and how many are needed for each in the next week. How then do we allocate the nurses to the wards?

The first method to be adopted was a simple heuristic procedure. The hospital has stipulated the staffing levels which they regard as being most suitable for each ward. These figures are adopted as target values. First the girls who are already allocated (urgency = 38) are considered. When they are allowed for, a "shortage" figure is arrived at for each ward.

The girls with urgencies of 37 are now considered. These are girls who can only go onto a general ward, or who have a choice of only one special ward with no float left. If a girl has a choice between more than one ward then she is allocated to the ward with the greatest shortage. Once she is put on that ward, the shortage is reduced by one. This process continues until we reach the girls with the lowest urgencies. By now the choice of available wards will be more restricted than it was originally, but the girls we are looking at are the ones with the widest choice, so an appropriate matching should still be possible.

These procedures determine which girls should be considered first, but they do not decide which wards they should be put on when there is a choice. A number of criteria are possible, the simplest relating to the shortages on the possible wards. It would seem reasonable that, if the girl under consideration is able to move on to either of two similar wards, she should be put on the one which is furthest below its optimum staffing level. However this is a crude way to decide the allocation, and many improvements will spring to mind. For instance, if there are two choices, Ward X whose optimum is 10 and whose present staffing 7, or Ward Y whose optimum is 5 and whose present staffing is 3, which of the two should be selected? Ward X has the greatest absolute shortage, but Ward Y has the greater percentage shortage. It can be seen that the lower the optimum number, the greater will be the effect of any understaffing. Similarly, because of the varying nature of the duties on different wards, a shortage of one girl may be more critical on some wards than on others. Different criteria were experimented with in the small scale simulation described in the next Chapter, but for the purposes of this full-scale model these different aspects of the critical nature of shortages from ward to ward were reflected by introducing a weighting system. The choice would now be made as follows :

Of the choices available select the one whose present shortage multiplied by its weighting factor is greatest. In the case of a tie, the first ward encountered with the tied value was selected. The program accomplishes this by adopting the first possible ward as its choice, but then replacing it by any subsequent ward whose shortage × weight is higher.

Different values were then tried for the weights, the most efficient being those based on the optimum number for each ward, with the smallest wards having the greatest weighting. The figures giving the best results were as follows:

	DAY DUTY		NIGHT DUTY	
WARD	OPTIMUM	WEIGHTING	OPTIMUM	WEIGHTING
49	12	5	5	12
P	8	7	5	12
54	4	15	2	30
LW	15	4	6	10
SC	10	6	6	10
OP	8	8	0	0

Table 4-5

The product of the optimum staffing value and the weighting is nearly constant i.e. weighting $\propto \frac{1}{\text{optimum}}$, with the exception of Post-natal and Out-patients, where Out-patients is given a slight advantage over Post-natal, since the importance of full staffing on the former duty is likely to be greater.

The use of these weights provides a neat way of applying the rule that the optima of 4 for Ward 54 on day duty, and 2 for Ward 54 on night duty, are maxima as well as minima. As soon as the optimum figure is achieved, the appropriate weighting is altered to zero, thereby ensuring that no girl is put on that ward henceforth.

4.8 Data manipulation and storage

The status quo may be stored in three arrays, roster, plan and exp, which will be described below. These three arrays are the only data external to the program which have to be read in at the start of each run. As a result of the updating which occurs within the run to each it is possible to produce the three arrays as output, and then read them in their revised form at the beginning of the next run. For simplicity the same format will be used to describe these arrays here as is used in the program. The general case for a two-dimensional array can be represented as follows:

ARRAYNAME $(m_1 : m_2 , n_1 : n_2)$

This describes an array whose first dimension has values from m_1 to m_2 inclusive, by increments of one, and whose second dimension is from n_1 to n_2 inclusive by increments of one. The lower and upper values can be either positive or negative integers, so long as the latter is greater than the former. The arrays used here are as follows:

Roster (1 : 280 , 1 : 110) - roster has been described earlier; it contains the basic schedule for each girl relating to holidays, community work and night duty. In this program it will accomodate up to 280 girls and will continue into the future for up to 110 weeks. Different spells of night duty, holiday, etc. are represented by single digit numbers, which are contained in a location in the array identified by the two parameters. For instance the term Roster (20, 5) might have the value 2, which represents the first spell of night duty. This then stores the information that nurse number 20 is on her first spell of night duty at week 5.

Plan (1 : 280 , -1 : 110) - plan has the same dimensions as roster plus an additional two colums corresponding to week labels of -1 and 0. The zero column was used as a spare column in the event of some labelling being required to differentiate between different categories of plan in the future; the -1 column contains a code number indicating each girl's working status. It will be 112 if she has not started or has finished the course at the week being considered, 111 if the file is a blank, and zero if the girl is on the course at the relevant week. This requires further explanation. When the data is read in the first few files are occupied by girls whose entire course has been planned, and whose courses end on week 48. Week 49 is the first week for which planning is required. The next set's plan starts on week 1 of our plan, and is four weeks from completion at week 48. The third set's plan does not start till week 4, and is eight weeks from completion at week 48. If the array were printed two-dimensionally it would have the structure shown in Table 4-6.

The vertical line between weeks 48 and 49 divides the period already planned (on the left of it) from the period to be planned by the program. The diagram represents the nature of the data as read in at the start of a computing run. As the run progressed, the hypothetical vertical line would move to the right as more of the plan was filled in. At any given week the program works with the relevant data which it selects from the preceding part of the plan - in other words the allocations which it makes for one week become data for subsequent weeks. The code numbers contained in column -1 refer to the status of each girl at the week being considered, so they are updated as the line moves to the right.

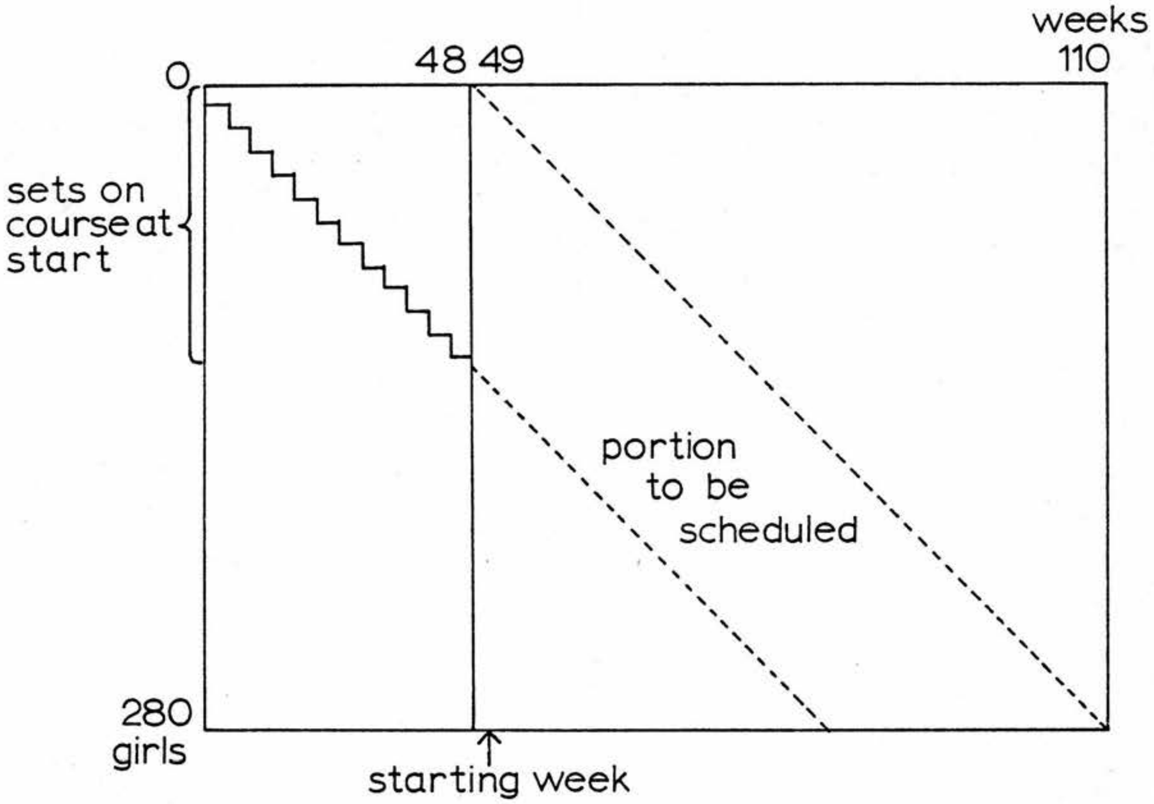


TABLE 4-6

Structure of plan array

Some of the rows will remain blank as the sets are of different sizes in the real situation. Enough spare capacity is included to cope with anything up to the largest set size occuring in the period since 1973.

Plan is completed by filling in single digit numbers corresponding to the different wards which each girl is to visit.

EXP (1 : 280 , 1 : 25) - Exp is an abbreviation of experience, and is explained in detail in Section 4.5.3 of this Chapter. To reiterate, it stores the identity of each ward visited in turn, regardless of the duration of each visit. Each time that an allocation is made, the exp array for that girl is altered by scanning the row relating to that girl's code number (between 1 and 280), and replacing the first zero encountered with the code number relating to the ward she is being placed on. The code numbers used in the model (whose final form was labelled X280 in order to refer to the number of girls it could cope with) were as follows:

Code:	1	2	3	4	5	6	7	8
Ward:	49	51	52	53	54	LW	SC	OP

Thus if a girl were put on Ward 49 as her first duty, her experience file would be a number one followed by twenty-five zeros.

Although this is a programming consideration it is mentioned here in order to clarify the meaning of some of the sample printout in Appendix E.

Thus, using these three arrays it is possible to store the status quo at any given stage of running the model. If it is wished to carry on allocating when the storage of the program is full it is merely necessary to read the status quo into a file, and re-run the program, instructing it to read in its data from the new file rather than the original file. In practice it is necessary to terminate the first run at a point where the roster structure matches that of the original data, so that the same procedure may be used for reading it in again.

These arrays are fairly large, so to reduce the amount of time required for processing the data it was stored in direct access form.

There are many other ways in which this program could have been modified to make it more efficient to run and more elegant in structure, but the construction of the original crude model gave enough information to indicate that it would be more fruitful to progress to a smaller scale model, which would be cheaper and faster to run which behaving in the same way as the full scale model, rather than attempt to perfect the original scheme.

Appendix E contains the complete text of the program X280. To aid understanding of its method of operation a resume is also given. This is done in preference to the common method of including a flow diagram, since the sequence of operations mainly follows one single route, with only a few loops or alternative courses of action. A definition of the meaning of certain variables is included before the resume; most have already been discussed in earlier sections of this Chapter.

4.9 Full-scale model versus manual allocation - a comparison

At this stage it is intended merely to point out some of the basic differences between the two approaches - a full appraisal will be found in the conclusions in Section 5.

The intention behind designing a model such as this was to find out whether the human decision - making process in this context could be simulated effectively by a computer model. The rules of the problem were adopted largely intact, with the exception of some minor simplifications, and the method of approaching the week by week solution of the problem was designed to copy as far as possible the solution techniques adopted by the existing allocators. There were however some shifts of emphasis which were almost unavoidable by the nature of the process being used.

The human allocator starts with a set of rules governing the structure of the training course, some explicitly stated and others adopted by common sense or by virtue of their past suitability in similar situations. Within the framework of these rules she attempts to arrange a set of schedules so that the required staffing levels are attained. If an impasse is reached and it seems to be impossible to provide enough staff for a certain ward then the allocator breaks or modifies the training rules in order to obtain the requisite number of staff for the week under consideration. In the real-life situation the week being considered is likely to be the week immediately after that in which the allocation is being done, so there is a greater feeling of urgency than if the week being planned were some time in the more distant future, with the likelihood of there being future alterations to be made prior to the implementation of the plan. It is conceivable that from time to time the rules might be broken unneccesarily if a solution existed which the allocator had failed to find. This situation is more likely to occur under the conditions of urgency mentioned above.

It would be exceedingly complicated to program a model so that it too could break certain rules. The nature of this problem dicates that the normal approach would be to simulate the staffing situation and adopt the real training rules, then to minimise the deviations from the correct staffing. It would be extremely difficult to design a model which guaranteed the achievement of even staffing and attempted to minimise the deviation from the normal rules. This may sound ludicrous, but is not in fact so, on closer examination. The above approach would be possible if all the constraints were of the type such as $t_{LW} = 8$ where t_{LW} represents the numberof weeks spent on the Labour Ward. With constraints of this nature it would be possible to relax them and to permit t_{LW} to equal seven or nine weeks. However it is these constraints which we least wish to violate. The Rules which could conceivably be broken are the sequencing rules, since their effect on the quality of service or of training is not absolute but is one of degree. Thus, if a girl only did seven weeks on LW she would fail to achieve the required training which would allow her to be awarded her qualification. If on the other hand she had to do only two weeks of LW in the first half and six in the second instead of four and four, it might be inconvenient as far as her theory training was concerned but it certainly would not be critical.

Unfortunately since these sequencing rules are not concerned with mere numerical values it would be very difficult to design ways in which they could be broken in a controlled fashion. It is not possible to relax them to a degree - they would have to be dropped entirely and replaced by other rules which, though less desirable, produced an acceptable different result. A series of penalties would have to be assigned to violations of the preferred rules.

Thus there is a different emphasis demonstrated by the model.

The next problem is that although the model copies many of the human thought processes it does not have the ability to produce new techniques in answer to new problems. The human allocator, when faced with a situation which has not occurred previously will try to apply known techniques to its solution. If these fail then he or she will examine the new situation for characteristics which might suggest a new technique or approach. There may, for instance, be a type of pattern or symmetry about the situation which indicates a new method of resolving it. It is this flexibility which gives the human an advantage over the computer when unfamiliar problems occur, since the human will search out new patterns and relationships while the computer will only be able to recognise those which have been anticipated and which it has been programmed to identify.

It was hoped that this inability to grapple with some of the more testing aspects of the scheduling would be compensated for by its ability to apply the simple rules very quickly and totally methodically, and thereby to work further into the future than the manual method. It was hoped also that by working for several months ahead rather than just one week it would be able to predict future bottlenecks in training, allowing manual intervention, and so removing one aspect of uncertainty which worried the human allocators who did not have the time available to work more than one week ahead at a time. It was possible for them to do so with regard to perhaps one ward, as described in Section 3.6, but not to work out the entire schedule in advance.

However it was this type of insight, allowing the human brain to recognise patterns in the scheduling of future weeks, which the model could not imitate. In effect, although the human allocator was normally only planning the week ahead, she would always do so with an implicit understanding of likely future effects whereas the model allocates one week at a time with only the underlying structure of the training course to ensure that feasible solutions will exist for future weeks. As a result it is possible for an underlying imbalance to develop, and for the solutions to deteriorate week by week. This is what occurred with the full scale program - its initial solutions were good, but as the weeks were tackled one by one it deteriorated progressively. To take an example of the kind of situation which the model was unable to handle at its existing stage of development, consider the case of an allocation which has to be made immediately prior to a session of night duty. Since the model has no facility for scanning ahead it is capable of making decisions whose future implications can lead to infeasible situations occurring. Let us assume that the week being considered was for day duty in the case of a number of the nurses, with night duty starting the following week. If there was a large number of places still to be filled on the Labour Ward, it is conceivable that eight of these girls be placed on that ward for four weeks each. This will solve the present week's staffing problems, but consider what will happen in the next week. Each of these girls will go onto night duty, giving a total of at least eight on that ward, whose optimum number is six. Regardless of what is done for the next weeks allocations there are bound to be at least two girls too many on that ward, and it is possible that this will also create a shortage on other wards.

The prospect of eight girls from the same set all starting on the same ward simultaneously is extreme, but the principle applies, and situations like this caused the solutions to deteriorate. It was difficult to experiment with the large model, since its size dictated that it be run as a batch job, which entailed a delay between each run and any subsequent modifications to the program. Because of this restriction it was decided that it would be more realistic to construct a small scale model, possessing all the main characteristics of the full scale one, which could then be used for experiment on the more rapid interactive basis. If this could be made to work, then the findings which resulted could be applied to the large scale model.

The next Chapter describes this model and the results which it produced.

CHAPTER 5

5.1 Introduction

It was explained in the previous Chapter that the small model was developed in order to ease the computational burden. This made it possible to run the program interactively, thus eliminating the time delay between making a modification to the program and seeing its results.

In this Chapter the small scale simulation will be described, with particular attention being paid to the characteristics which it shares with the full-scale simulation. Where there are differences between the nature of the two models these will be explained. Flow diagrams are provided to illustrate the decision and operating sequence of the small-scale model, and it will readily be seen that they are very similar in structure to those reproduced in Chapter 4 which describe the program X280, the large scale simulation. The program of the small-scale simulation is labelled X108, since it cycles through 108 nurses when allocating. That is to say that the label is a measure of the maximum capacity of the program in terms of the number of nurses it can allocate.

Some sample printout is included to show what information the program provides; the results of several runs are then illustrated graphically.

5.2 Description of the small-scale simulation

It was essential that the smaller scale model retained the main characteristics of the full-scale model, so the new program

Weeks:	4	8	12	16	20	24	28	32	36	40	44
	Day	Night	Day			Hol					
		Day	Night	Day			Hol				
			Day	Night	Day			Hol			
				Day	Night	Day			Hol		
					Day	Night	Day			Hol	
						Day	Night	Day			Hol

TABLE 5-1

Simplified model - roster suggestion

was developed merely by editing the larger program. Alterations would be made to specific values (for instance the figure 280 relating to the total number of nurses would be deleted each time it appeared and be replaced by the figure 108) but no change would be made to the sequence of operations. Thus the logical structure of the smaller scale model would still be identical to that of the larger. The roster was greatly simplified and the length of the course shortened. This was achieved not by reducing the number of weeks spent on each ward, but by removing a portion equivalent in length to the second half of the course. For neatness, the course was divided into six portions of four weeks each. Originally the plan was to have been structured as in Table 5-1, but it was discovered that the use of only one four-week spell of night duty gave an unrepresentative allocation problem, so the roster was simplified yet again. By dropping the spell of night duly the roster became merely a twenty week spell of duty followed by a four week holiday. The lengths of each visit are almost identical to those required for the first half of the course under the 1973 system:

CODE	1	2	3	4	5	6
WARD	49	G	54	LW	SC	OP
WEEKS	3	5	2	4	4	2

Table 5-2

Once again the abbreviation of G for general wards (51, 52 and 53) has been used to distinguish their characteristics within the model as being different from the other special wards. All wards other than G are visited once only; G is visited for a three-week spell if it is the first ward of the course, with either one or two visits thereafter, or else it may be visited for as many as five one-week spells. The first duty is always 49 or G, thereafter the wards may be visited in any order. The required staffing for each ward was derived from the availabilities of nurses produced by the above training scheme i.e. the figures were devised so that it would be possible, given the number of nurses available, always to provide the correct number for each ward without having to deal with fractions of nurses. Similarly it is possible to allocate sufficient to each ward without needing extra girls to cover fluctuations, with the consequent overstaffing which this entails. The staffing requirements are as follows:

CODE	1	2	3	4	5	6
WARD	49	G	54	LW	SC	OP
OPT	5	8	3	6	6	3
MIN	4	7	3	6	6	3

Table 5-3

With the exception of wards 49 and the general wards, the minimum and optimum figures are the same, so the problem has very little slack. An overstaffing on one ward will most usually be accompanied by an understaffing on another. In order to ensure that feasible solutions would be possible an initial set of data was constructed which would produce even staffing henceforth merely by repetition. This is shown in Table 5-4.

All figures on the left of the line are data supplied to the model. The tens in the top right hand corner are the code for the holiday, and are determined by the roster. The line is not straight since the wards are allocated for up to four weeks ahead. At the bottom of the table are vertical totals for each of the six wards. Note that the plan for each intake is identical as far as

```
**UPDATED PLAN**

2 2 2 5 5 5 5 4 4 4 4 1 1 1 6 6 3 3 2 2 10 10 10 10
2 2 2 4 4 4 4 3 3 5 5 5 5 1 1 1 2 2 6 6 10 10 10 10
2 2 2 1 1 1 2 2 6 6 3 3 4 4 4 4 5 5 5 5 10 10 10 10
2 2 2 3 3 2 1 1 1 6 6 2 5 5 5 5 4 4 4 4 10 10 10 10
1 1 1 2 2 2 2 5 5 5 5 4 4 4 4 6 6 3 3 2 10 10 10 10
1 1 1 2 2 3 3 2 6 6 2 5 5 5 5 4 4 4 4 2 10 10 10 10
        2 2 2 5 5 5 5 4 4 4 4 1 1 1 6 6 2
        2 2 2 4 4 4 4 3 3 5 5 5 5 1 1 1 2
        2 2 2 1 1 1 2 2 6 6 3 3 4 4 4 4 5 5 5 5
        2 2 2 3 3 2 1 1 1 6 6 2 5 5 5 5 4 4 4 4
        1 1 1 2 2 2 2 5 5 5 5 4 4 4 4 6 6
        1 1 1 2 2 3 3 2 6 6 2 5 5 5 5 4 4 4 4
                2 2 2 5 5 5 5 4 4 4 4 1 1 1
                2 2 2 4 4 4 4 3 3 5 5 5 5
                2 2 2 1 1 1 2 2 6 6 3 3 4 4 4 4
                2 2 2 3 3 2 1 1 1 6 6 2 5 5 5 5
                1 1 1 2 2 2 2 5 5 5 5 4 4 4 4
                1 1 1 2 2 3 3 2 6 6 2 5 5 5 5
                        2 2 2 5 5 5 5 4 4 4 4
                        2 2 2 4 4 4 4 3 3
                        2 2 2 1 1 1 2 2 6 6
                        2 2 2 3 3 2 1 1 1
                        1 1 1 2 2 2 2 5 5 5 5
                        1 1 1 2 2 3 3 2 2
                                2 2 2 5 5 5 5
                                2 2 2 4 4 4 4
                                2 2 2 1 1 1
                                2 2 2 3 3
                                1 1 1 2 2
                                1 1 1 2 2
                                        2 2 2
                                        1 1 1
                                        2 2 2
                                        2 2 2
                                        1 1 1
                                        1 1 1
```

	Ward	Code	1	2	3	4	:Week
	49	1	5	5	4	4	
Staffing	G	2	7	7	8	8	
levels	54	3	3	3	3	3	
provided	LW	4	6	6	6	6	
	SC	5	6	6	6	6	
	OP	6	3	3	3	3	

TABLE 5-4

Simplified model - starting plan

the starting data are concerned, so mere repetition of each would cause the staffing levels indicated to repeat every four weeks. However the model cannot recognise such a pattern, and is obliged to allocate each week in isolation as in the case of the full-scale model. There is no point in programming the model to recognise this sort of pattern since it is an artificial device created in order to guarantee the existence of an ideal solution, and would not occur in real life. This table also shows an unsuccessful first week's run by an early version of the model. It should be noted that OP is not done during the second four weeks of the course. This means that the data could also have been used if a trial had been wished with the night duty session included. The plan shows that each intake comprises six girls. It might be assumed that this simplifies the problem since there are half as many girls to allocate in each intake as in the large scale simulation, but in fact the opposite is the case. With six girls available there is much less flexibility, with fewer girls ready to change wards at any given week of the course. Also, since there are now only six sets present at any time rather than twelve, flexibility will be further reduced, and the solution of the problem will become harder still.

5.3 Small-scale simulation - a summary of constraints

The following summary is in the same form as for the large-scale problem:

No. of trainees present at any time	-	36
No. of intakes per course length	-	6
No. of rosters per intake	-	1
Length in weeks of course	-	24

WARD	TRAINING REQUIREMENTS	STAFFING LEVELS OPTIMUM	MINIMUM
49	3	5	4
Post-natal	5	8	7
54	2	3	3
LW	4	6	6
SC	4	6	6
OP	2	3	3

ROSTER: 20 weeks of duty + 4 weeks of holiday

SEQUENCING RULES: 49 or Post-natal first, then other wards in any order. All wards but Post-natal to be visited once only; Post-natal minimum spell of duty is one week.

Since this model is a modified form of the full-scale model, the flow-diagrams describing it are almost identical to those for the larger model. A simple one and one slightly more detailed are shown in Figures 5-1 and 5-2.

Roster and Plan have the same format as those in the large-scale model, X280, but Exp is different. In X108 (again named with reference to its maximum capacity for girls), Exp is no longer an array which stores the identities of wards visited in the appropriate sequence. Because of the simpler nature of the training sequence requirements it is now only necessary to record whether each ward had been visited yet or not. The array is EXP (1:108, 1:6). Its initial value is all zeroes, but as each girl visits a new ward the number corresponding to that ward is changed to a one. Thus if girl No. 1 visits Ward 5, then exp (1, 5) is altered from zero to one. The Exp column corresponding to ward code 2 is redundant but is retained to make the labelling simpler. As a result if all of the Exp values for one girl are added it will give the total number of special wards which she has visited so far.

GEN (1:108) is used to accumulate the number of weeks of G which each girl has done.

5.4 Small-scale model - urgency values

The urgencies are calculated on the same basis as with the full-scale model:

Pre-allocated: 99

First week: 28

		5	4	3	2	1	0	- No. of weeks of G already done
Total Number of Special Wards Visited	5		30	29	28	27	26	
	4	30	29	28	27	26	25	
	3	24	23	22	21	20	19	
	2	18	17	16	15	14	13	
	1	12	11	10	9	8	7	
	0	6	5	4	3	2	1	

Table 5-5

5.5 Small-scale model - allocation system

All of the aspects of the model discussed so far have been handled in the same way as in the full-scale model. The advantage is that because of the reduced scale it is possible to experiment with the allocation rules and get immediate feed back of results. Under the time restrictions imposed by the size of the full-scale model it was only possible to run for thirteen weeks ahead at a time, but with the smaller scale model a run of forty-eight weeks is possible. By using the same technique as with X280 of reading the status quo out into a data file, then using it as input for the next run, it is possible to examine interesting algorithms far into the future. Again the method of Direct Access input and output is used.

The original versions worked on the same basis as X280 when allocating. To recap, the rule was:

Of the choices available for a girl, select the one whose present shortage multiplied by its weighting factor is greatest. In the case of a tie, the first ward encountered with the tied value is selected. The program accomplishes this by adopting the first possible ward as its choice, but then replacing it with any subsequent ward whose shortage × weight is higher.

A series of runs was then made as before trying out different values for the weights for each ward. The results of these runs, and subsequent ones described below, are given in the next section.

The next factor to be taken into consideration was the number of alternative ways that a ward could be staffed. To illustrate, let us consider an extreme example. Let us assume that the first girl to be considered has a choice of two wards, 49 and G. Let us further assume that there are four other girls who could do G but none who could do 49. If the shortage on G was 4 and on 49 was 1, then the previously described algorithm would tend to put the girl on G. Thus it reacts to the shortage there without taking into account the fact that it has left itself with no possible girl for Ward 49. To attempt to rectify this situation the algorithm was modified so that the weights each time were multiplied by the inverse of the number of girls currently available to go onto the relevant ward. Thus the fewer girls available as an alternative to the one being considered, the greater the weighting tending to place her on the ward in question.

It was noticed that this new algorithm caused an imbalance in the rare circumstance of two wards becoming overstaffed during allocation. The results were good, but the logic of the algorithm was improved by causing the alternative staffing factor to be multiplied with the weighting in its non-inverted form if the shortage on a given ward became negative, i.e. if it became over-staffed.

For each of the above runs the weight attached to Ward 54 was reduced to zero as soon as that ward was correctly staffed, in order to ensure that it was never overstaffed. This was to mirror the situation in the real-life problem whereby the optimum for Ward 54 was also a maximum. In subsequent runs the maximum staffing was maintained by an arithmetic manipulation which caused Ward 54 to exhibit a weight of -50 as soon as it became correctly staffed.

A modification was made to the urgency ratings to take into account how many weeks of the course had been completed. This involved either adding this number to the urgency, adding one half of the number, or adding one tenth of the number. All of these measures detracted from the results.

In some cases, the first few weeks were predetermined so that the model did not have to work them out. It was felt that, by automatically starting four girls on G each time and two on Ward 49, the model would be assisted towards producing a balanced set of solutions with better overall staffing. In fact the reduction of the models choice seemed to be the critical aspect, and the solutions worsened.

Finally there were a number of trials carried out where minor adjustments were made between different wards with regard to the models behaviour when they were almost correctly staffed, but perhaps one under or over. The effects of this fine tuning did not

FIGURE 5 - 1
Simple flow diagram of X108

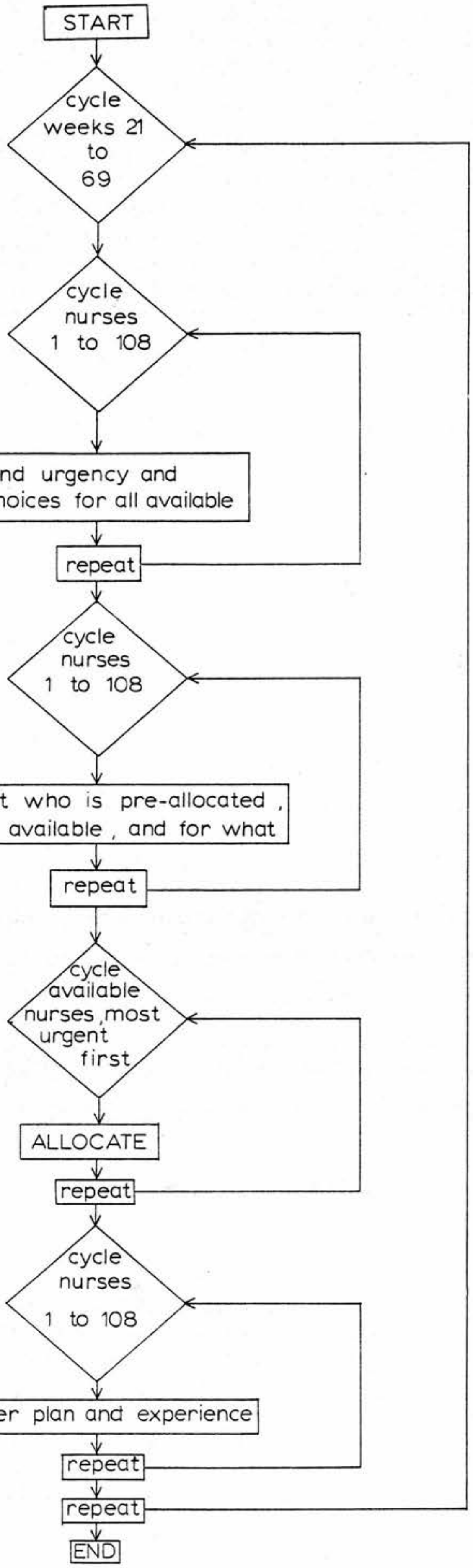
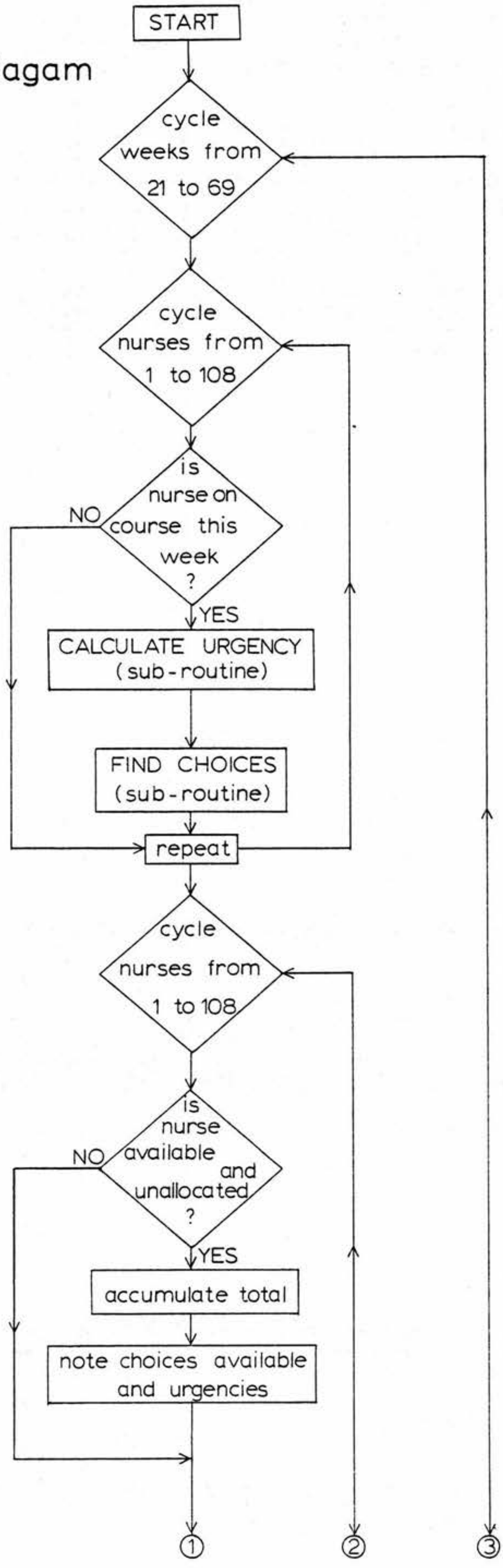


FIGURE 5 - 2.1

Detailed flow diagam of X108



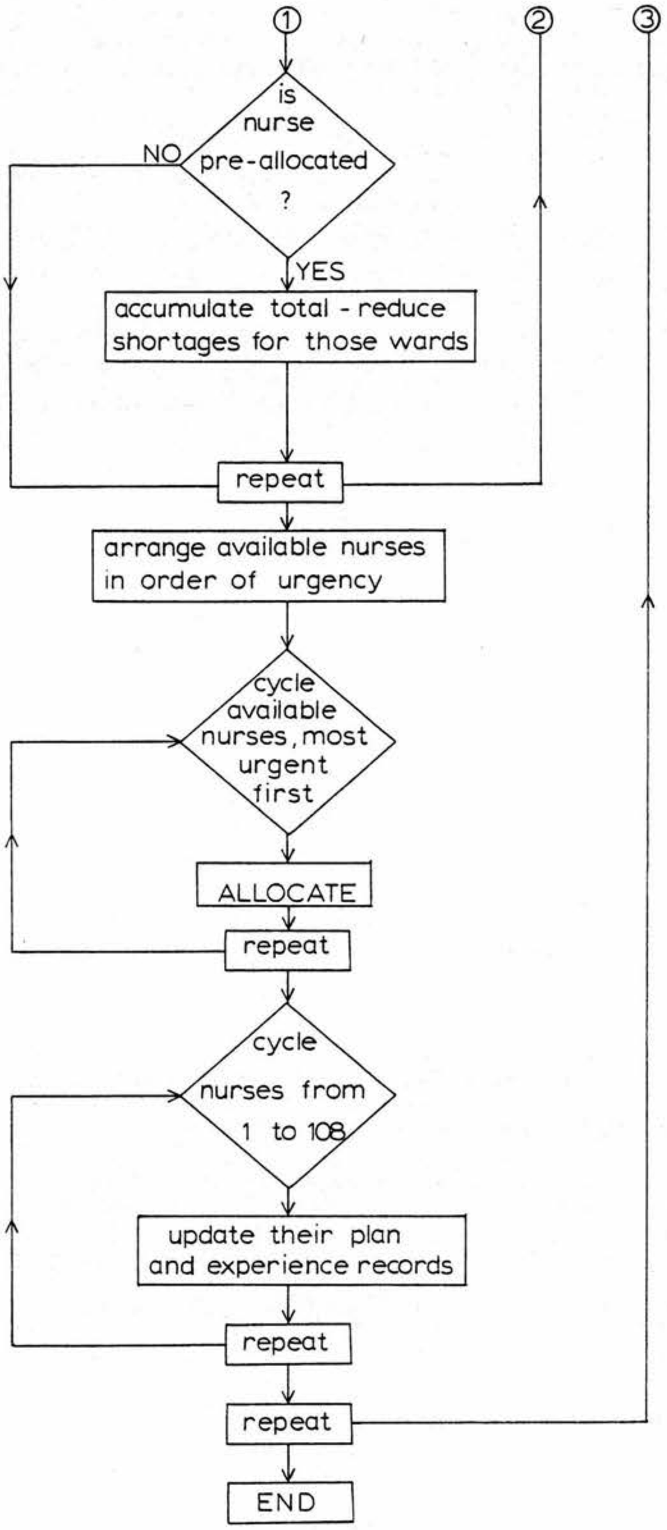


FIGURE 5 - 2.2

Continuation

produce any noticeable improvement.

Detailed results of these experiments are given in Section 5.7.

5.6 Output and assessment of results

The format of the output was at first fairly similar to that of X280, with the same optional output of different information. Sample output for X108 is shown in Appendix E. At the end of each week's printout the computer analyses its own results and prints them out. Once the system had been properly tested a further reduction in computational burden could be achieved by instructing the program to print an analysis of the results, but to omit the results themselves.

Two cumulative measures of assessment are used:

(1) Mean weekly deviation. This is the average deviation, both positive and negative, from the optimum range. For the purposes of this measure both the minimum and optimum figures for 49 and G are taken as being a deviation of zero. The absolute value of all other deviations is summed and an average determined. Mean weekly deviation can therefore be represented by

$$\frac{\sum_{t=1}^{T} \sum_{j=1}^{J} \left| d_j \right|_t}{T}$$

where T is the number of weeks being considered and $\left| d_j \right|_t$ is the absolute value for deviation from the optimum in week t on ward j.

(2) Root mean squared deviations. A measure of the variance

could have been obtained by calculating

$$\sqrt{\frac{\sum_{t=1}^{T} \sum_{j=1}^{J} (\bar{x}_j - x_{jt})^2}{T}}$$

where $\bar{x}_j$ was the average number of staff on ward j; x_{jt} was the actual number on ward j at week t.

However this is not ideal for two reasons: Firstly, $\bar{x}_j$ will always tend to be the optimum figure, or near enough to make its inclusion unnecessary. Secondly the above expression is looking at the variance ward by ward as well as week by week, whereas what is more relevant here is some measure of how good or bad the average week's total allocation is. This means that it is no longer possible to use the device of squaring to eliminate neatly the positive and negative signs of the deviations.

However, the method of squaring can still be of use in order to emphasise any individual bad weeks. The expression which was used was as follows

$$\sqrt{\frac{\sum_{t=1}^{T} \left(\sum_{j=1}^{J} \left| d_j \right|_t \right)^2}{T}}$$

where $\left| d_j \right|$ is the absolute value of d_j

It was considered better to have a series which consistantly erred from the optimum by a small amount than one which allocated correctly for most of the time but then produced the occasional unacceptably bad figure. Consider the two series below, representing deviation figures for ten weeks each:

		MEAN	SUM OF SQUARES	MEAN OF SUM OF SQUARES
A	: 0 0 0 0 1 0 0 4 1 0	0.6	18	1.8
B	: 1 0 1 1 1 0 1 0 1 0	0.6	6	0.6

Table 5-6

It can be seen that series A and B have the same arithmetic mean, although series A would seem to be less acceptable because of the one bad week which it contains. The mean of the sums of the squares are 1.8 and 0.6 respectively, while the 'root mean squared deviations' are 1.34 and 0.77 for A and B. Thus by this method it is possible to distinquish between two such series, and to select one which spreads any deviations it contains as evenly as possible between the weeks.

For the abbreviated runs a different format was used, printing out only the staffing levels achieved each week for the six wards, then after sixty-eight weeks printing out the relevant diagnostics. If the program was run for successive batches of forty-eight weeks each then the printout for subsequent ones could be made to include all of the deviation figures from week twenty-one, so providing a complete record for the equivalent of several years forward run. Samples of this printout are given after the normal printout.

5.7 Results of experiments with the small-scale model

It is usual practice in problems of this nature to try a large number of different data sets, in order to ensure that the results obtained are representative, and not determined atypically by some chance configuration in the first data set used. In order to guard further against the possibility of there existing some favourable idiosyncracy in the data sets used, these are often published along with the results to permit future researchers to test their own algorithms on the same sets[1]. In this way it is possible to achieve a direct comparison with previous work, for instance in terms of the average number of iterations needed for an integer program to reach an optimal point, or the machine time needed to solve a given problem. Commonly these are averaged for a number of data sets which is sufficiently large for the findings to be statistically valid.

However the model under consideration creates its own data; only the starting point is common to all runs, and if run for enough weeks ahead (in terms of the model) the algorithm being tested will tend to stabilise at a particular average deviation level. This quality of solution will by then be independant of the original data pattern, but as long as each algorithm is given the same starting point the results will be comparible at an even earlier stage.

Unless otherwise stated, the cumulative figures quoted will be those achieved by week sixty-eight, i.e. forty-eight weeks after the starting point. By this point they all tend to settle down to a steady level of deviation, since there will have been two complete changes of personnel by then.

The first experiments were made using differing weights, in order to explore the emphasis which should be given to the different wards in order to achieve the best overall staffing balance. It will be noticed that the Post-natal Wards (labelled G) are often singled out for a different weight from the others - this is because the optimum and minimum figures for that category differ by a figure of one. The same applies to Ward 49, but the aim was merely to give different treatment to one of the wards with a degree of slack, so G was chosen since its overall staff requirements are the greater. The weights tested, with results were as shown in Table 5-7.

	WEIGHTS							
VERSION	49	G	54	LW	SC	OP	MEAN DEVIATION	ROOT MEAN SQUARED DEVIATIONS
1	2	2	2	2	2	2	2.0	2.69
2	2	1	2	2	2	2	2.5	3.32
3	4	1	4	4	4	4	3.7	4.14
4	10	1	10	10	10	10	4.3	5.13
5	5	3	8	4	4	8	3.2	3.75
6	10	3	16	8	8	16	3.2	3.68

Table 5-7

When the shortage for 49 or G reaches 1 the weight for that ward is reduced to 1 if it is not already at that value, since the minimum figure has been achieved. The criterion used when selecting which of the possible wards a given girl should move to was to select the one whose product of weight and present shortage was the greatest.

The figures for version 5 were chosen because they give very nearly a constant when multiplied by the optimum staffing levels:

49	G	54	LW	SC	OP	:	WARD
5	8	3	6	6	3	:	OPTIMUM

If a given ward requires a large number of girls, as in the case of G, then by the nature of the model there will be more girls available to perform that duty, so it will be easier to achieve the correct level of staffing, or one very near it. Conversely if the required number is small it may be more difficult to provide the correct staffing, so the weights in version 5 were intended to

VERSION	WEIGHTS 49	 G	 54	 LW	 SC	 OP	$\times \frac{1}{\text{AVAILABILITY}}$	MEAN DEVIATION	ROOT MEAN SQUARED DEVIATIONS
7	1	1	1	1	1	1	√	1.6	2.06
8	2	1	2	2	2	2	√	0.9	1.65
9	4	1	4	4	4	4	√	0.7	1.50
10	10	1	10	10	10	10	√	1.4	2.31
11a	5	3	8	4	4	8	√	1.2	2.00
12a	15	13	18	14	14	18	√	1.2	2.27

Table 5-8

VERSION	WEIGHTS	$\times \frac{1}{\text{AVAILABILITY}}$	× AVAILABILITY	MEAN DEVIATION	ROOT MEAN SQUARED DEVIATIONS
11	As Version 9	+ve shortage	-ve shortage	0.9	1.78

Table 5-9

counteract this tendency.

The next hypothesis to be tested was that a given girl should be encouraged to go to a certain ward if the number of alternative girls who could perform the same duty was small, and discouraged from being placed on a ward if there were many other nurses who could do the same. This was achieved by using the inverse of the number of nurses available for each ward as the weighting factor. This brought a marked improvement over the previous weightings. The inverse availability weighting was then combined with the other weightings previously tried, and a further improvement was achieved - see Table 5-8.

An improvement was made to the logical consistency in cases of overstaffing by causing the availability figure to be used as it stood rather than inverted in cases where the staffing figure for a given ward had a negative shortage. Version 11 is the same as Version 9, but with this modification made. The result was marginally worse, but only by an amount which could be attributed to a random variation caused by this alteration. This is shown in Table 5-9.

The next question to be considered was whether the allocation could be improved by altering the urgency figures to take more account of the number of weeks left in any girl's course. In order to increase the urgency of any girl nearing the end of her course, some proportion of the number of weeks already done was added to the existing urgency figure. For brevity the figure representing the number of weeks done will be referred to in these tables by the label used in the program - SLACKINV. If a fraction of SLACKINV was required, any remainder was ignored. This operation, known as integer division, was represented in the program by two oblique lines, and this convention will also be used in these tables.

Another option to be considered was whether the allocation would be improved by dictating how each girl's course should start. It was felt that by insisting that four girls out of six should start on G and two on 49, a balance might be established which would then be maintained by the program. The results of these modifications are given in Table 5-10. The changes are all made to Version 11, since that is the best version so far with the exception of Version 9 which contained the logical inconsistency discussed above.

Version 11, without starting constraint -

VERSION	URGENCY MODIFICATION	MEAN DEVIATION	ROOT MEAN SQUARED DEVIATIONS
12	+ Slackinv	1.3	2.16
13	+ Slackinv //2	1.4	2.41
14	+ Slackinv //10	1.4	2.21

Version 11, with starting constraint -

VERSION	URGENCY MODIFICATION	MEAN DEVIATION	ROOT MEAN SQUARED DEVIATIONS
15	—	2.2	2.89
16	+ Slackinv	1.7	2.22
17	+ Slackinv //2	1.8	2.45
18	+ Slackinv //10	2.0	2.71

Table 5-10

The conclusion seems to be that an attempt to introduce an extra constraint, even if it should theoretically be a move which helps the algorithm towards a good solution, will in fact limit the range of choice of that algorithm and thus cause the solutions to

	WEIGHTS								MEAN	ROOT MEAN SQUARED
VERSION	49	G	54	LW	SC	OP	+ve shortage	-ve shortage	DEVIATION	DEVIATIONS
11	4	1	4	4	4	4	$\times \frac{1}{\text{Availability}}$	× Availability	0.9	1.78
22	8	1	4	4	4	4	$\times \frac{1}{\text{Availability}}$	× Availability	1.1	2.22
23	4	1	4	4	4	4	$\times \frac{1}{\text{Availability} \times 10}$	× Availability × 10	1.1	2.11

Table 5-12

deteriorate in quality overall, as measured by deviations from ideal staffing levels.

Further experiment was made with fine adjustment of the weights. Version 9 was taken as a basis, and the weight for Ward 49 was altered to varying values. In Table 5-11, Version 9 is repeated for comparison.

	WEIGHTS								ROOT MEAN
VERSION	49	G	54	LW	SC	OP	x $\frac{1}{\text{AVAILABILITY}}$	MEAN DEVIATION	SQUARED DEVIATIONS
9	4	1	4	4	4	4	✓	0.7	1.50
19	6	1	4	4	4	4	✓	0.7	1.52
20	8	1	4	4	4	4	✓	0.7	1.51
21	10	1	4	4	4	4	✓	0.7	1.52

Table 5-11

More of a difference was observed when Version 11 was taken as the basic version - see Table 5-12.

So far Wards 49 and G have had optima of 5 and 8 respectively, and minima of 4 and 7. As an experiment the requirements were tightened up so that the minimum concept was dropped and the optimum for 49 was 4 for two weeks out of four and 5 for the other two. Similarly G had optima of 7 and 8. The resulting increase in deviation was not as great as might be expected from the effects of the modification made in Version 15. This is shown in Table 5-13.

VERSION	DESCRIPTION	MEAN DEVIATION	ROOT MEAN SQUARED DEVIATIONS
24	As 9 but with optimum requirements altered	1.1	1.74

Table 5-13

The next factor to be altered was the reduction of the weights to 1 when Ward 49 or G reached the minimum value. The results of this are contained in Table 5-14

VERSION	DESCRIPTION	MEAN DEVIATION	ROOT MEAN SQUARED DEVIATIONS
25	Same as 20 but Ward 49 stays at 8 when on minimum	0.8	1.72
26	Same as 24 but Wards 49 and G stay at 4 when on minimum	1.2	1.93
27	Same as 11 but Ward 49 stays at 4 when on minimum	0.6	1.39
28	Same as 11 but Wards 49 and G stay at 4 when on minimum	1.8	2.28

Table 5-14

Finally two more experiments were carried out on the weighting values. These are described in Table 5-15.

VERSION	DESCRIPTION	MEAN DEVIATION	R.M.S. DEVIATIONS
29	Same as 9 but weight for G becomes 0 when G has negative shortage	2.3	3.16
30	Same as 9 but weights for 49 and G become 0 at shortage of 1, then 1 again at shortage of 0	2.4	3.19

Table 5-15

To recap; each week 30 nurses are being allocated to 6 wards. In the case of Wards 49 and G there is a small tolerance in that staffing at the optimum and the minimum levels will count for zero deviation, but the other four wards have to be on the optimum level to avoid registering any deviation. This is a fairly exacting requirement, yet the simple heuristics outlined in this Chapter have been able to achieve patterns of staffing whose average weekly

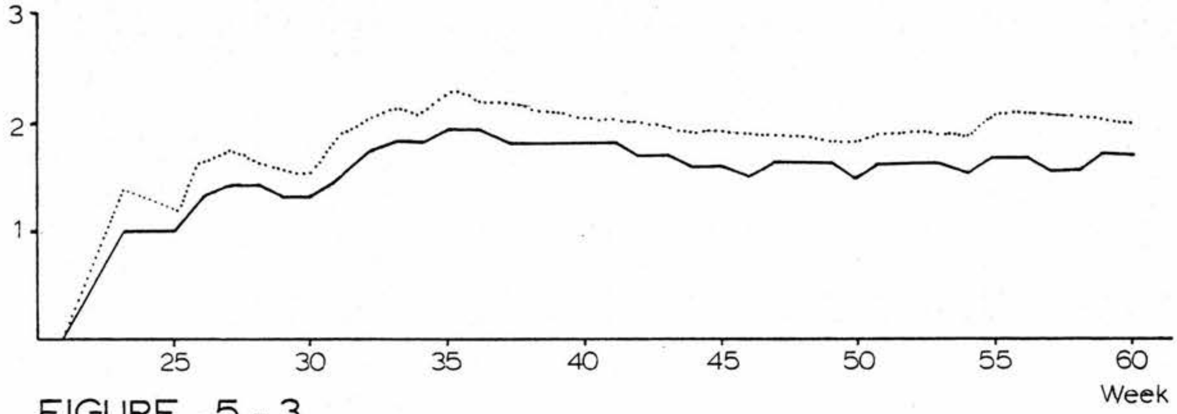


FIGURE 5 - 3

Version 7

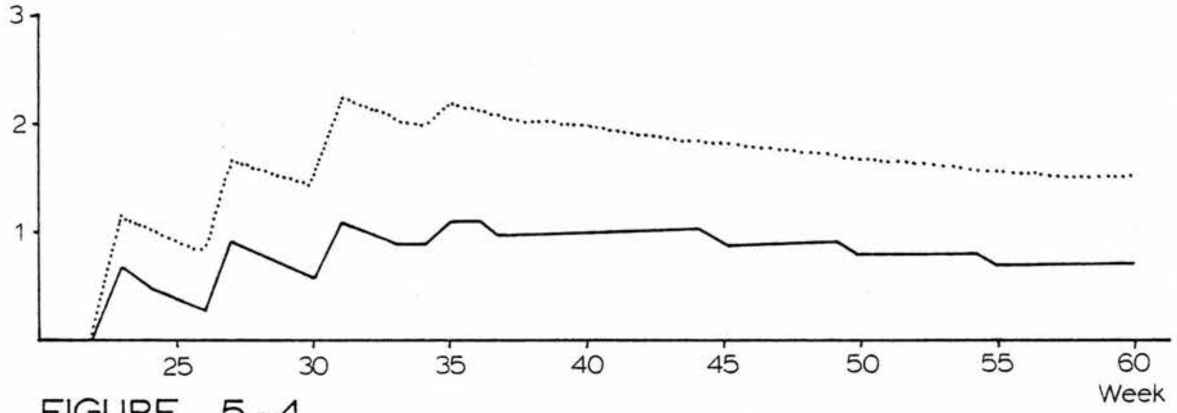


FIGURE 5 - 4

Version 9

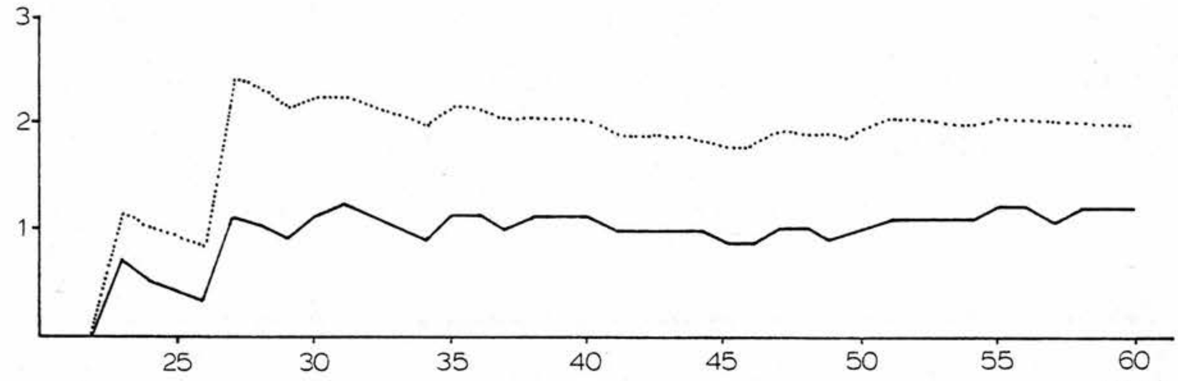


FIGURE 5 - 5

Version 11a

Graphical representation of results

———— : Mean deviation

………… : Root mean squared deviations

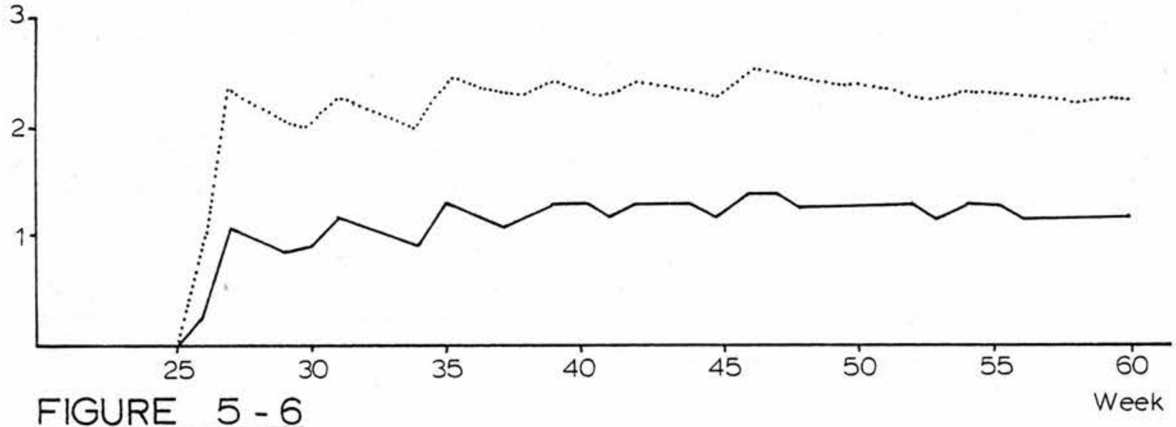


FIGURE 5 - 6

Version 12a

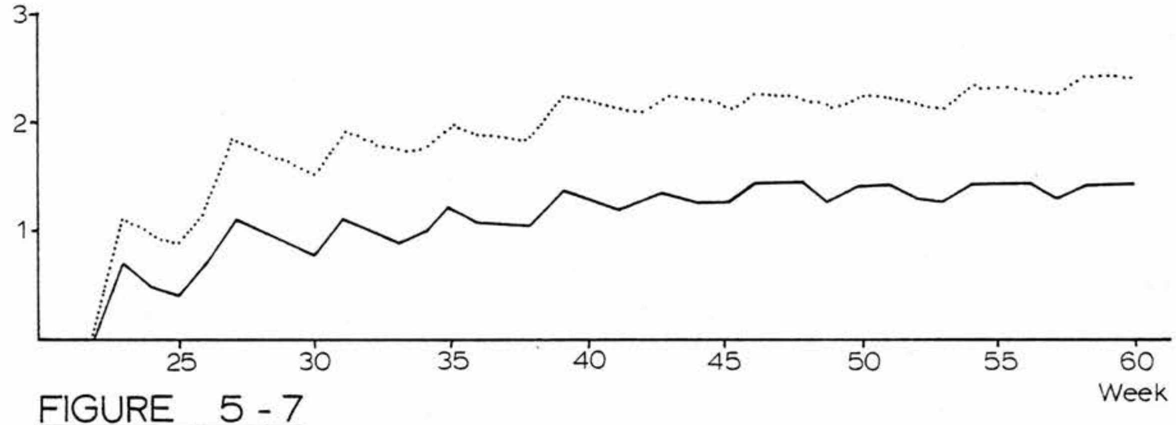


FIGURE 5 - 7

Version 13

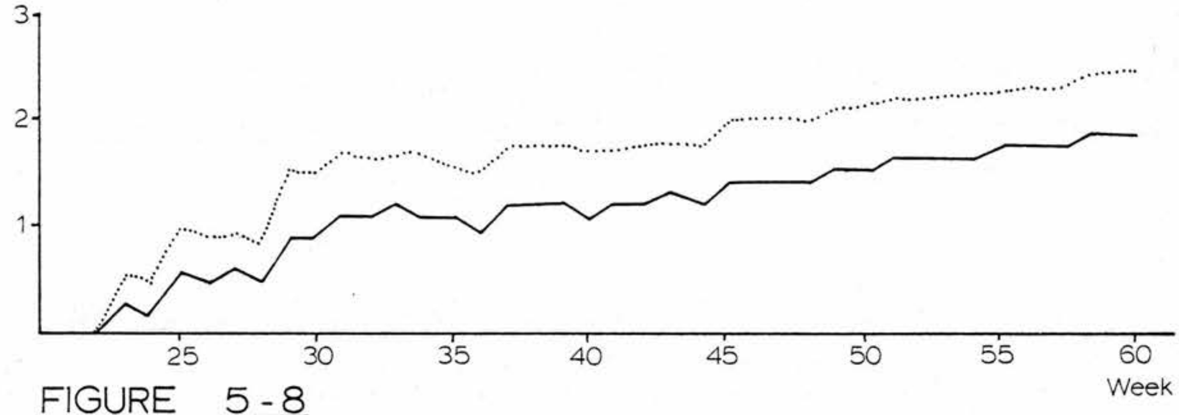


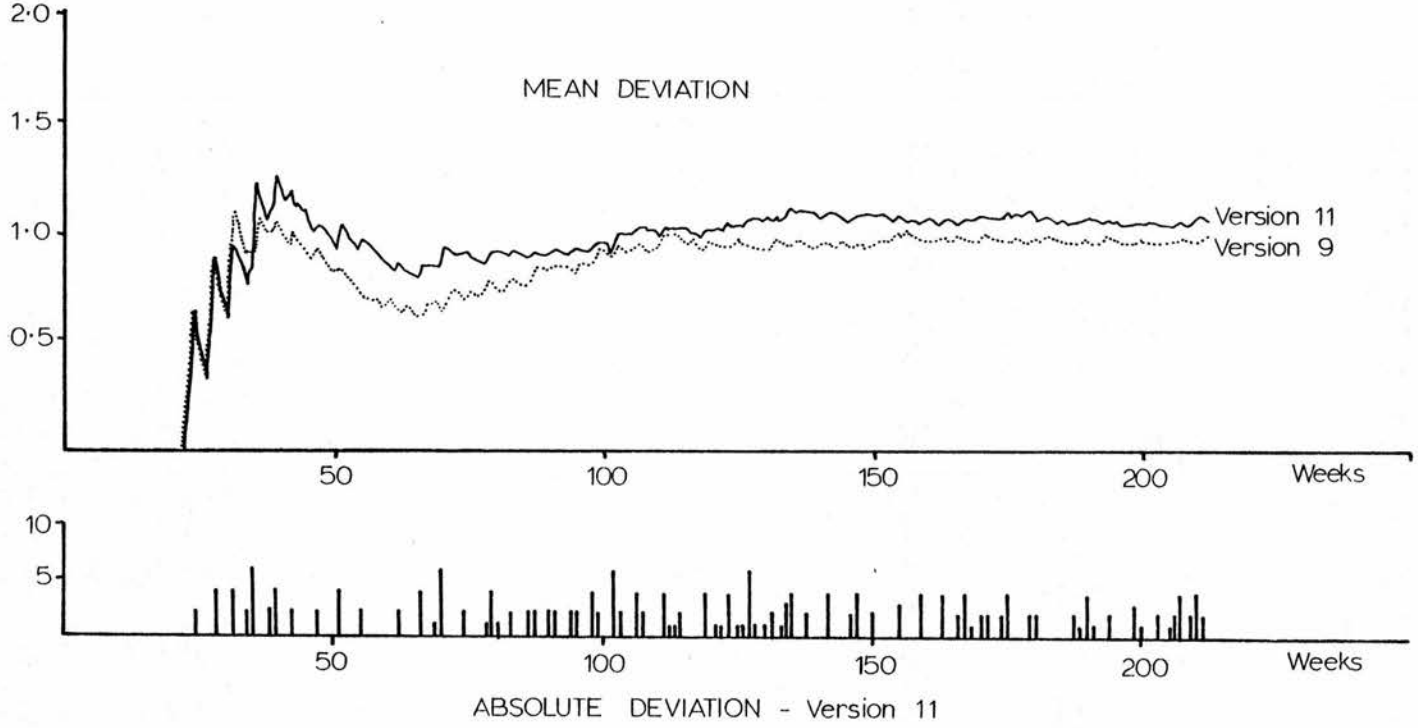
FIGURE 5 - 8

Version 17

Graphical representation of results

———— : Mean deviation

·········· : Root mean squared deviations



MEAN DEVIATION

ABSOLUTE DEVIATION - Version 11

FIGURE 5-9

Graphical representation of results - long run

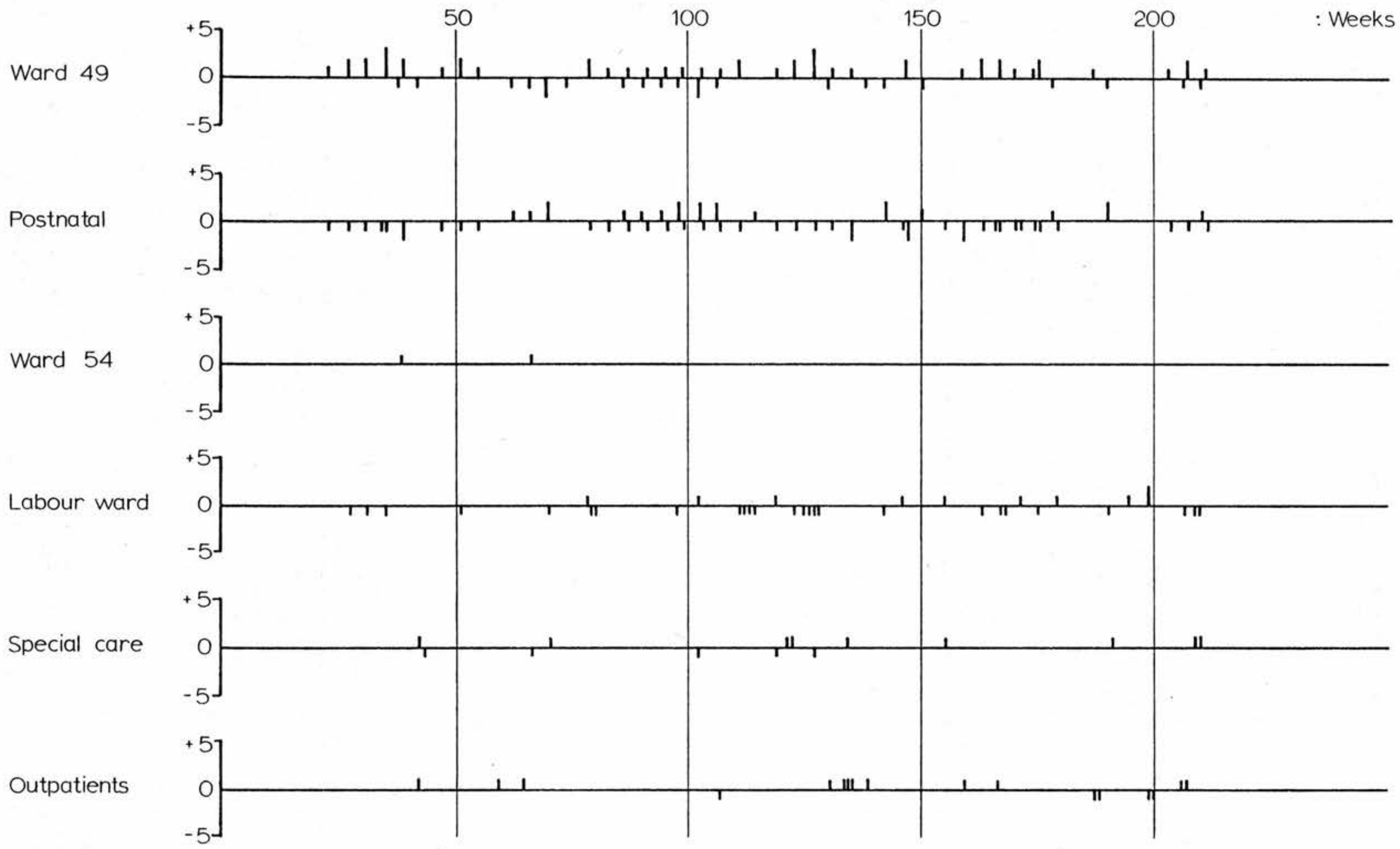


FIGURE 5 - 10

Long run results ; deviations for individual wards - Version 11 deviation is around 0.7 girl wards. That is to say that the total of over- and under-staffing averages out at less than one girl out of thirty per week.

Figures 5-3 to 5-11 illustrate some of the sets of results which provided the tables in this section. Whereas in the tables, the deviation figures quoted are for Week 68 each time, Figures 5-3 to 5-8 plot the cumulative values of the deviations week by week up till Week 60. Figures 5-9 and 5-10 consider a longer run forward to Week 212, and examine patterns of absolute as well as cumulative deviation values.

CHAPTER 6

6.1 Introduction

In this Chapter a survey will be made of existing general solution methods applicable to the nurse scheduling problem. Several of the more commonly used linear and integer programming techniques prove to be unsuitable, and this is explained in Sections 6.3 and 6.4. In Section 6.5 the concept of Lagrangian adaptation is introduced, and its relevance explained. It is shown how this technique allows some of the constraints to be temporarily relaxed and the advantages of this approach are outlined.

Sections 6.6 to 6.8 describe Fisher's[1] method of using Lagrangian relaxation, while Sections 6.9 to 6.12 give an account of a relation of this method proposed by Held and Karp[2]. The original papers restrict themselves to fairly formal mathematical presentations of the techniques so in each case we attempt to provide an intuitive analysis of the algorithms being considered rather than merely recounting extracts from published sources. The insight gained from this is valuable in its own right, and has aided considerably in the application of these powerful techniques to the problem studied here.

Sections 6.13 and 6.14 describe the application of the sub-gradient algorithm explained in the previous Sections, while Section 6.15 details some alternative procedures which were tested. Sections 6.16 to 6.21 evaluate the performances of each of the methods used.

6.2 The approach to computational methods/

6.2 The approach to computational methods

It was felt that a problem of this complexity might be amenable for a more powerful computational approach. In order to facilitate the testing of a number of methods, a series of approximations to the real-life problem were established, in some cases equal in scale to the real problem, and of the same degree of structural complexity, but with some of the incidental idiosyncracies removed to simplify the initial formulation.

The general problem was taken to be:

$$(1.1) \quad \min Z = \sum_{iq} c_{iq} x_{iq}$$

$$(1.2) \quad \text{s.t.} \sum_{q} x_{iq} = 1 \qquad , \forall i$$

$$(1.3) \quad \sum_{iq} a_{ijqt} x_{iq} \geqslant d_{jt}, \ \forall j, t$$

$$(1.4) \quad x_{iq} = 1 \text{ or } 0 \qquad , \forall i, q$$

Where x_{iq} = 1 if girl i is on schedule q (the decision variables),

c_{iq} is the cost of putting girl i on schedule q,

a_{ijqt} = 1 if schedule q puts girl i on ward j at time t.

d_{jt} is the demand for girls on ward j at time t.

Constraint (1.2) permits each girl to be placed on only one schedule.

Constraint (1.3) ensures that demand will be satisfied for each ward at each unit of time.

One of the problems encountered in the real problem was that of generating permissible schedules. Here we will treat schedule-generation as a separate sub-problem. As a result, we eliminate from this formulation the set of precedence constraints, since permissible schedules will be stated explicitly. This highlights the difference in approach from the model-building or simulation method discussed earlier. In that case, each week was dealt with as it was reached, and limits as to how far ahead the program could be run were only dictated by external factors, such as the amount of machine time available, or the amount of storage needed.

In the case of the formulation above, however, it is necessary to consider a discrete unit of time. The model will produce a solution for that period, but could not consider a subsequent period without further data. This differs from the simulation's method, which creates its own data as it runs. Against the drawback of being forced to consider a discrete time period can be set the advantage of obtaining a better overall view within that period. Whereas the simulation could make a decision at one point which would present bottlenecks later, the general mathematical model treats the time units within its prescribed period simultaneously, the equivalent of looking ahead within that limited period.

The criterion used in the objective function is cost. This may be approached in two ways. In some cases a realistic set of costs would be established, relating to the suitability of different schedules to each girl. However, in most cases there would be no difference between the needs of different girls - a schedule which was a good one for one girl would also be good for another.

The cost are still useful, however, in permitting the allocation mechanism to distinguish between two girls or more.

In the case of a stalemate between two equally suitable choices, the differing artificial costs will determine in a random fashion which of the two should be selected, thus permitting a valid selection to be made. By using this method, one is assured that any solution arrived at will automatically satisfy both the precedence constraints which are implicit in the permitted schedules and the demand constraints which are stated explicitly in the problem.

In order to avoid a situation where the model continues to work at obtaining the cheapest solution unneccesarily, it is possible to stop calculation as soon as a feasible solution is arrived at.

6.3 Linear programming

Constraint (1.4) was relaxed and some sets of data were run as linear programs, using a standard package[3] in order to see if integer solutions were produced. Small sets of data, up to the order of 10 girls × 10 weeks × 5 schedules, generally produced integer solutions, but exceptions were found to this. Problems of a greater magnitude would seem to produce non-integer solutions even more frequently, but this was not investigated more closely since larger problems caused the package being used to exhaust its available store. This drawback is not an insoluble one, but it does indicate that a more efficient tool than the linear program is needed. Clearly integer programming methods which incorporate linear programming stages, such as the Lang-Doig branch and bound algorithm[4], or R.J. Dakin's subsequent modification[5], will also be unwieldy in this case.

6.4 Integer solution methods[30]

The best method would seem to be one which leads rapidly to the establishment of an initial feasible solution, since the computation can be stopped at that point if the costs being used are artificial. Because this stage of the problem is difficult in this type of allocation problem, it seemed logical to choose a method which would allow the problem to commence with a less binding set of constraints.

6.5 Lagrangian relaxation[6]

Lagrangian relaxation is a method of relaxing some of the constraints temporarily and it seemed particularly suitable in this case, for reasons which will be explained after a description of the method.

According to this idea, some of the constraints governing the problem are removed, and incorporated in a modified form in the objective function. In effect, any violation of the original constraint is penalised by having the deviation from the permitted equality multiplied by a factor and added to the cost of the objective function. The factor is called the Lagrangian multiplier, and there is one for each constraint which has been relaxed. At each iteration, an alteration is made to the values of the Lagrangian multipliers in order to increase the penalty attached to constraints whose desired limits are being exceeded most consistently. Minimisation of the Lagrangian objective function with fixed multiplier values yields a lower bound on the cost of an optimal solution to the scheduling problem, and these bounds become stronger as the multipliers are adjusted iteratively[7]. This method is generally used when one set of constraints is "harder" to satisfy than another, in the sense that they complicate an otherwise simple problem. The "easier" constraints are left intact, and therefore have to be satisfied at each iteration, whereas the tighter constraints contained in modified form in the objective function are accounted for more gradually, iteration by iteration. In this case it is the set of demand constraints which is "harder" to satisfy.

Define π_{jt} to be the cost of a violation of demand constraint d_{jt}, $\pi_{jt} \geqslant 0$.

If $\sum_{iq} a_{ijqt} x_{iq} \geqslant d_{jt}, \ \forall j, t$ (1.3)

are the constraints being considered then any violation of a constraint will be of amount $\left(\sum_{iq} a_{ijqt} x_{iq} - d_{jt}\right)$ only if +ve.

The cost of each violation will hence be

$$\pi_{jt} \left(\sum_{iq} a_{ijqt} x_{iq} - d_{jt}\right) \quad , \forall j, t.$$

If we incorporate this into our objective function we arrive at the Lagrangian formulation of the problem:

$$(2.1) \quad \min L (\underline{x}, \underline{\pi}) = \left[\sum_{iq} c_{iq} x_{iq} - \sum_{jt} (\pi_{jt} (\sum_{iq} a_{ijqt} x_{iq} - d_{jt}))\right]$$

$$(2.2) \qquad \text{s.t.} \sum_{q} x_{iq} = 1, \forall_{i}$$

$$(2.3) \qquad x_{iq} = 1 \text{ or } 0, \ \forall i, q$$

If this were solved with the π values set at 0, then the solution arrived at would be the solution to the original problem (1) with the constraints (1.3) omitted. In effect each girl would be put on the schedule which was cheapest for her, regardless of the effect on staffing. If the π values are increased selectively and the process repeated, then a new solution will be arrived at. This minimisation of the Lagrangian provides a new lower bound on the cost of an optimal solution to the original scheduling problem.[31]

At this stage a system has to be developed by which the π values are altered in such a way that the primal problem tends towards its optimum objective function value. First an existing method developed by Fisher[8] will be described, then an adaptation of an alternative method which the author believes is more relevant to this specific problem.

6.6 An algorithm for solving resource-constrained network scheduling problems

In this Section a brief resume will be given of Fisher's paper, "Optimal Solution of Scheduling Problems Using Lagrange Multipliers: Part I"[9] which bears many similarities to the present problem. Fisher also uses Lagrange multipliers to dualise the resource constraints, forming a Lagrangian problem is which the network constraints appear explicitly, while the resource constraints appear only in the Lagrangian function. Fisher's network constraints are more complex than those in the nurse scheduling problem since he is considering individual jobs week by week, whereas here the permitted schedules are stated explicitly, but the general method is of relevance.

6.6.1 A resume of Fisher's method

Fisher considers a scheduling problem that meets these six conditions:

1. A set of I jobs must be performed, where the i th job consists of n_i tasks numbered from 1 to n_i and the time taken to perform each task is a known integer represented by p_{ij} for the j th task of the i th job.
2. A set of K resources is available, where R_{kt} is the amount of the k_{th} resource available in time period t, and r_{ijk} is the amount of the k th resource required by task ij during its processing. In the case of the job-shop scheduling problem, the resources would correspond to the available machines, and each task would require only a single machine while it was being processed.

3. Once a task ij is started, at start time t_{ij}, it must continue, uninterrupted, for p_{ij} time units, to its finish time f_{ij}. In other words, $f_{ij} = t_{ij} + p_{ij}$ $(j = 1, \ldots, n_i \; ; i = 1, \ldots, I)$.
4. The start times of the tasks on a given job are constrained by a cycle-free precedence network which is determined by a set of constraints for each job of the form

   $t_{ij} \geqslant f_{il} = t_{il} + p_{il}$, $L \in P(ij)$, where $P(ij)$ is the predecessor set for task kj and contains all tasks of job i that must be completed before ij is started.
5. Define $f_i = \max_{j = 1, \ldots, n_i} (f_{ij})$ to be the completion time of job i.

   $f_i \leqslant T(i = 1, \ldots, I)$ where T is an integer upper bound on the number of time periods until all the jobs are completed.
6. We are required to minimise $Z = \sum_i g_i (f_i)$, where g_i is some specified non-decreasing function of f_i, by finding values of t_{ij} $(j = 1, \ldots, n_i \; ; i = 1, \ldots, I)$ that satisfy conditions 1 - 5.

   Depending on the function chosen for g_i it might be decided to minimise total job tardiness, or to minimise a more complicated function such as the square of job tardiness.

   Fisher's formulation[32] in its most general form is almost identical to (1):

$$(3.1) \quad \min \sum_{iq} c_{iq} x_{iq}$$

$$(3.2) \quad \text{s.t.} \sum_{q} x_{iq} = 1 , \quad (i = 1, 2, \ldots, I)$$

$$(3.3) \quad \sum_{iq} a_{iqkt} x_{iq} \leqslant R_{kt}, \quad (t = 1, 2, \ldots, T; k = 1, 2, \ldots, K)$$

(3.4) $x_{iq} = 0 \text{ or } 1. \quad (q = 1, 2, \ldots, Q_i \; ; i = 1, 2, \ldots, I)$

where c_{iq} is a function of job tardiness,

$x_{iq} = 1$ implies that the q th job schedule is being used for job i

a_{iqkt} is the amount of resource k used during time period t by job $_i$ under schedule q

R_{kt} is the amount of the k_{th} resource available in time period t.

From this formulation, Fisher develops the Lagrangian problem: Let π_{kt} (t = 1, ..., T ; k = 1, ..., K) denote a non-negative multiplier associated with the Kt th constraint in (3.3). We then have

(4.1) $$w(\pi) = \min_{x_{iq}} \left[\sum_{iq} c_{iq} x_{iq} + \sum_{kt} \pi_{kt} \left(\sum_{iq} a_{iqkt} x_{iq} - R_{kt} \right) \right],$$

(4.2) $$\text{s.t.} \sum_{q} x_{iq} = 1, \quad (i = 1, \ldots, I)$$

(4.3) $$x_{iq} = 0 \text{ or } 1 \quad (q = 1, \ldots, Q_i \; ; i = 1, \ldots, I)$$

Positive values of π_{kt} will tend to impose constraints (3.3) by penalising solutions which violate these constraints. Fisher uses $w(\pi)$ as a lower bound on the optimal objective value of (3). Proof of this relationship, the weak duality theorem, may be found in sub-section 6.6.2.

In order to apply this relationship two questions must be answered:

(1) How to solve (4) given π

(2) How to set π to obtain stong bounds

6.6.2 The weak duality theorem

Let Z* denote the optimal objective value of (3). Then

$w\ (\Pi) \leqslant Z^*$ for any $\Pi \geqslant 0$

Proof. Let x^*_{iq} be optimal in (3). Then we have

$$Z^* = \sum_{iq} c_{iq}\ x^*_{iq} \geqslant \sum_{iq} c_{iq}\ x^*_{iq} + \sum_{kt} \Pi_{kt}\ (\sum_{iq} a_{iqkt}\ x^*_{iq} - R_{kt}) \geqslant w(\Pi)$$

6.7 Solution of Fisher's Lagrangian problem

Let Π be specified. Now let us reformulate (4.1)

$$(4.1)\ w\ (\Pi) = \min_{x_{iq}} \left[\sum_{iq} c_{iq}\ x_{iq} + \sum_{kt} \Pi_{kt}\ (\sum_{iq} a_{iqkt}\ x_{iq} - R_{kt}) \right]$$

$$= \min_{x_{iq}} \left[\sum_{iq} c_{iq}\ x_{iq} + \sum_{kt} \Pi_{kt} \sum_{iq} a_{iqkt}\ x_{iq} \right.$$

$$\left. - \sum_{kt} \Pi_{kt}\ R_{kt} \right]$$

$$= \min_{x_{iq}} \left[\sum_{iq} (c_{iq} + \sum_{kt} \Pi_{kt}\ a_{iqkt})\ x_{iq} - \sum_{kt} \Pi_{kt}\ R_{kt} \right]$$

Let $b_{iq} = c_{iq} + \sum_{kt} \Pi_{kt}\ a_{iqkt}$

Then $w\ (\Pi) = \min_{x_{iq}} \left[\sum_{iq} b_{iq}\ x_{iq} - \sum_{kt} \Pi_{kt}\ R_{kt} \right]$

Let $b_i = \min_q b_{iq}$. It is now evident from the above reformulation of (4.1) that an optimal solution of (4) may be obtained by setting an x_{iq} for which $b_{iq} = b_i$ to 1 for each i and all other x_{iq} to zero. The value of the objective function will then be

$$w\ (\Pi) = \sum_{i} b_i - \sum_{kt} R_{kt}\ \Pi_{kt}$$

Fisher then presents two algorithms for finding a job schedule with minimum value of b_{iq}, one for precedence networks of general form, and one for a special network. Explanations of these algorithms are omitted from this thesis as being irrelevant to the development of the approach in question.

6.8 Determining values for Fisher's Π

Fisher's method of adjusting Π values uses a linear programming problem, equivalent to the general formulation of this stage in the process of solving the present problem which may be stated as

(5.1) $\max w(\pi)$

(5.2) s.t. $\pi \geqslant 0$

The new formulation is given by

(6.1) $\max w = \sum_{i} b_i - \sum_{kt} R_{kt} \pi_{kt}$

(6.2) s.t. $b_i \leqslant c_{iq} + \sum_{kt} a_{iqkt} \pi_{kt}$, $(q = 1, \ldots, Q_i \; ; \; i = 1, \ldots, I)$

(6.3) $\pi_{kt} \geqslant 0$ $(k = 1, \ldots, K \; ; \; t = 1, \ldots, T)$

(6.4) w and b_i $(i = 1, \ldots, I)$ unrestricted in sign.

Problem (6) is exactly the LP dual of (3) with integrality relaxed. The simplex method is applied to (6), dealing explicitly with only a subset of the constraints in (6.2), since these constraints are too numerous to be considered in their entirety. This selection of critical constraints is valid since only a few are binding at any given extreme point solution. The non-explicit constraints are checked for violation at each iteration by solving a sequence of job subproblems, the most violated constraint being added to the explicit set of constraints. In other words, Fisher examines some of the job sequences which the present solution provides, and has discovered that it is usually only necessary to enumerate two or three in order to determine which of the implicit constraints is being most violated and should therefore enter the basis. Fisher uses this method as a bounding procedure to be used with a branch and bound algorithm.

6.9 An alternative method of determining values for π

Held & Karp[10] (1970) devised a function w which acted as a "pseudo-dual" for the problem from which it was derived, its maximum value providing excellent bounds for their branch-and-bound algorithm. The general form of this type of problem can be stated as:

(7.1) $\max w(\pi),\ \pi \in S$

(7.2) where $w(\pi) = \min \left\{ c_m + \pi . v_m : m = 1, \ldots, M \right\}$

where c_m is a scalar, $v_m = (v_{m1}, \ldots, v_{mn})$ is a real n-vector for all m, S is a closed convex subset of E^n, ($\pi . v_m$ denotes the inner product), and M is very large.

They tried a steepest-ascent procedure and a simplex method using column generation in order to maximise w, but found these prohibitively slow. To get round this problem they devised a new "sub-gradient" method, which turned out to be highly effective. Subsequently it was pointed out to Held & Karp that their method bore similarities to one discussed by Agmon[11] and Motzkin & Schoenberg[12] for the solution of linear inequality systems.

6.10 Held and Karp's[13] sub-gradient method

In (7) they assume that w has an upper bound. Since it is piecewise linear there exists at least one point π^* such that $w(\pi^*) = w^* = \max w$. They denote by $V(\pi)$ the set of all n-vectors v_m such that the minimum is assumed for the index m;

(7.3) $V(\pi) = \left\{ v_m : c_m + \pi . v_m = w(\pi),\ m = 1, \ldots, M \right\}$

Figures 6-1 and 6-2 present this graphically.

At π_1, the minimum of $w(\pi)$ is assumed for m = 2, i.e. $V(\pi) = \left\{ v_1 \right\}$

At π_2, however, the values of two vectors coincide, both sharing the value of $w(\pi_2)$

At this point $V(\pi) = \left\{ v_1, v_2 \right\}$

When, as is usually the case, $V(\pi)$ is a singleton, the function

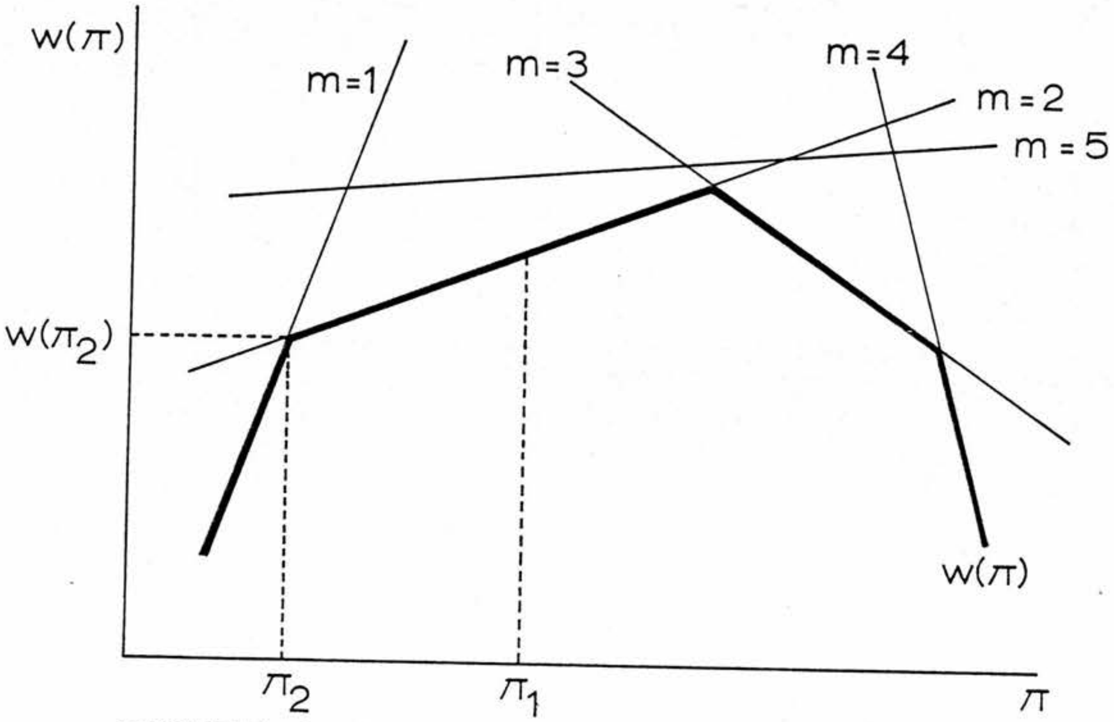


FIGURE 6-1

Graphical representation of w(π)

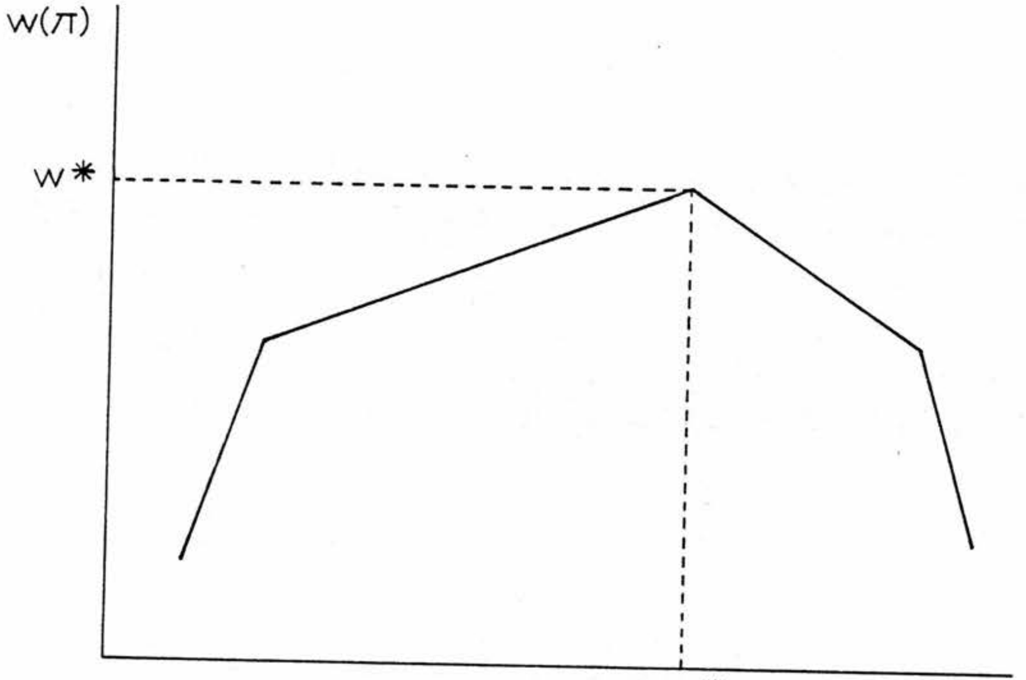


FIGURE 6-2

Graphical representation of w*

w is differentiable at π, and the single member of V (π) is the gradient ∇ w (π). To cope with the situation at π_2 (and indeed cases where more than two vectors have equal values at a point) a new concept has to be introduced. At a point like w (π_2) we consider the sub-gradient rather than the gradient.

At a point like w (π_2) where two vectors coincide any gradient whose value lies between the gradients of the two intersecting functions of π is called a sub-gradient. More precisely, the set of all sub-gradients at π is the compact, convex set ∂ w (π) called the subdifferential[14].

In graphical form this may be represented as in Figure 6-3. At π_1 in Figure 6-3, all sub-gradients are less steep than v_1, steeper than v_2, or they equal v_1 or v_2 in gradient i.e. they lie in the cone indicated.

Put more precisely:

The n-vector u is a sub-gradient at $\pi_o \in E^n$ of the concave function w on E^n if

$$w\ (y) - w\ (\pi_o) \leqslant u.\ (y - \pi_o) \quad \text{for all } y \in E^n, \quad (7.4)$$

where E^n is the set of all real numbers. Figure 6-4 shows this graphically.

For the present application, only linear cases will occur, since w is piecewise linear.

6.11 Resume

Let us consider what we have so far established:

We have a function w (π) which is equal to the minimum values of the expression $c_m + \pi.\ v_m$ over all values of m. In the cases we shall consider it will be possible to determine a maximum value for w (π). This value can be attained for more than one value of π, if

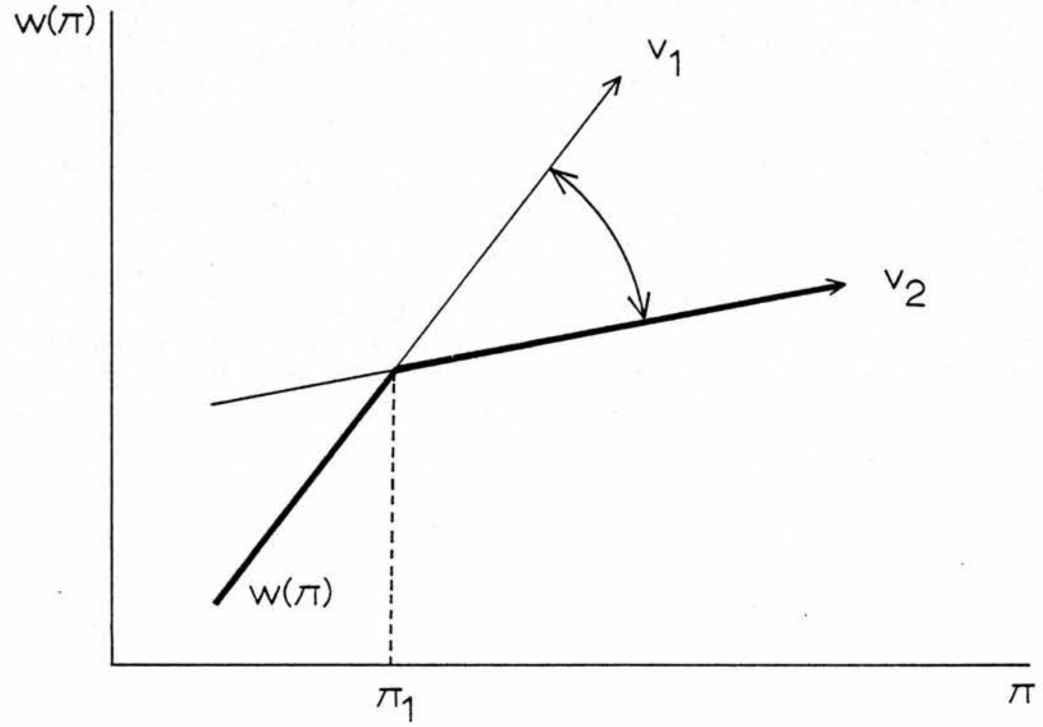


FIGURE 6-3

Graphical representation of subgradients

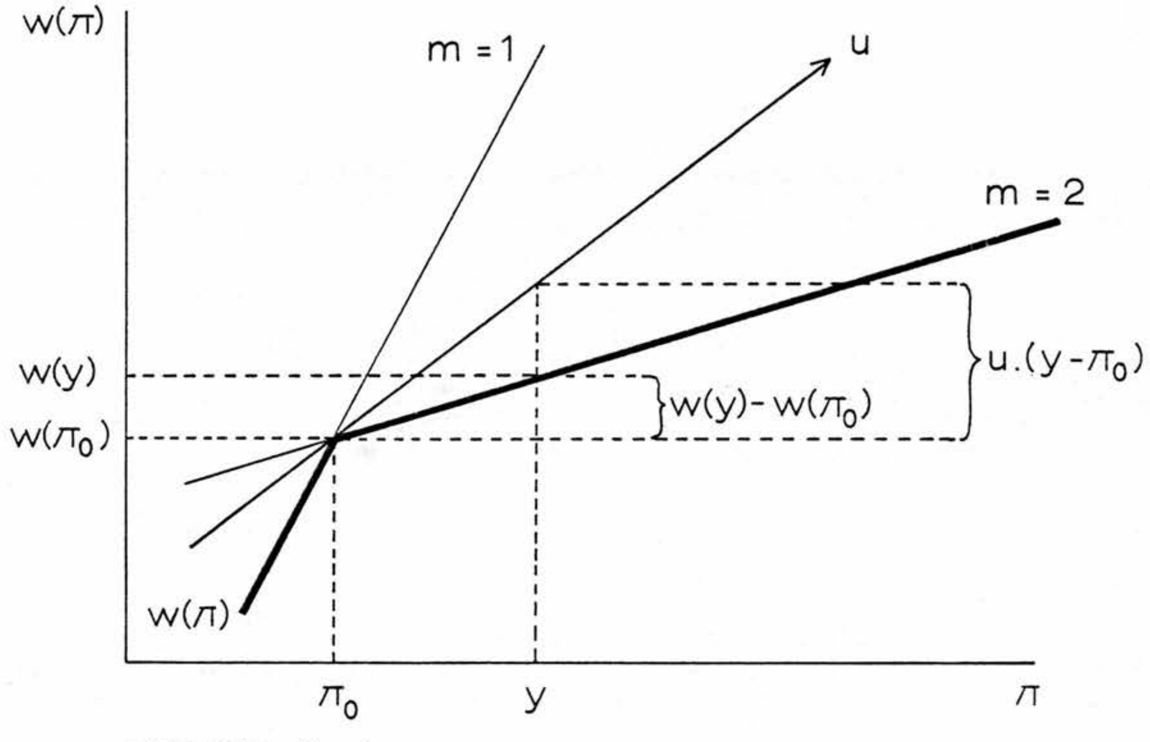


FIGURE 6-4

Graphical representation of a subgradient we have a situation like the one depicted in Figure 6-5, but generally it will reach its maximum value for one value π^* of π. We wish to identify the value of π for which this maximum value w* is attained. We can determine the gradient of w (π) for each value of π where V (π) is a singleton. In the next Section it will be explained that more than one approach may be taken in the event of V (π) having more than one member.

6.12 Method

Given a value of π, we know two things by calculation:

(1) The value of w (π)

(2) The gradient ∇ w (π) or subdifferential ∂ w (π) depending on the cardinality of the set V (π).

We need to know two more things:

(1) Is w (π) already at its maximum?

(2) If it is not, how should we alter the value of π to increase the value of w (π)?

(1) is easy to answer: If the gradient ∇ w (π) or one of the sub-gradients comprising ∂ w (π) is equal to zero, then we are at the maximum value of w (π). If not, a movement along the function w (π) in the correct direction will produce an increase in value.

In answer to (2), the positive or negative value of the present gradient tells us which way to alter π, and the steepness or otherwise gives an indication of how close we are to the max w (π).

Since the specific algorithms to be described work in slightly different ways, the general solution method will first be explained. Once again an intuitive analysis will be given using a graphical approach.

It is reasonable to start with the value of π set to zero.

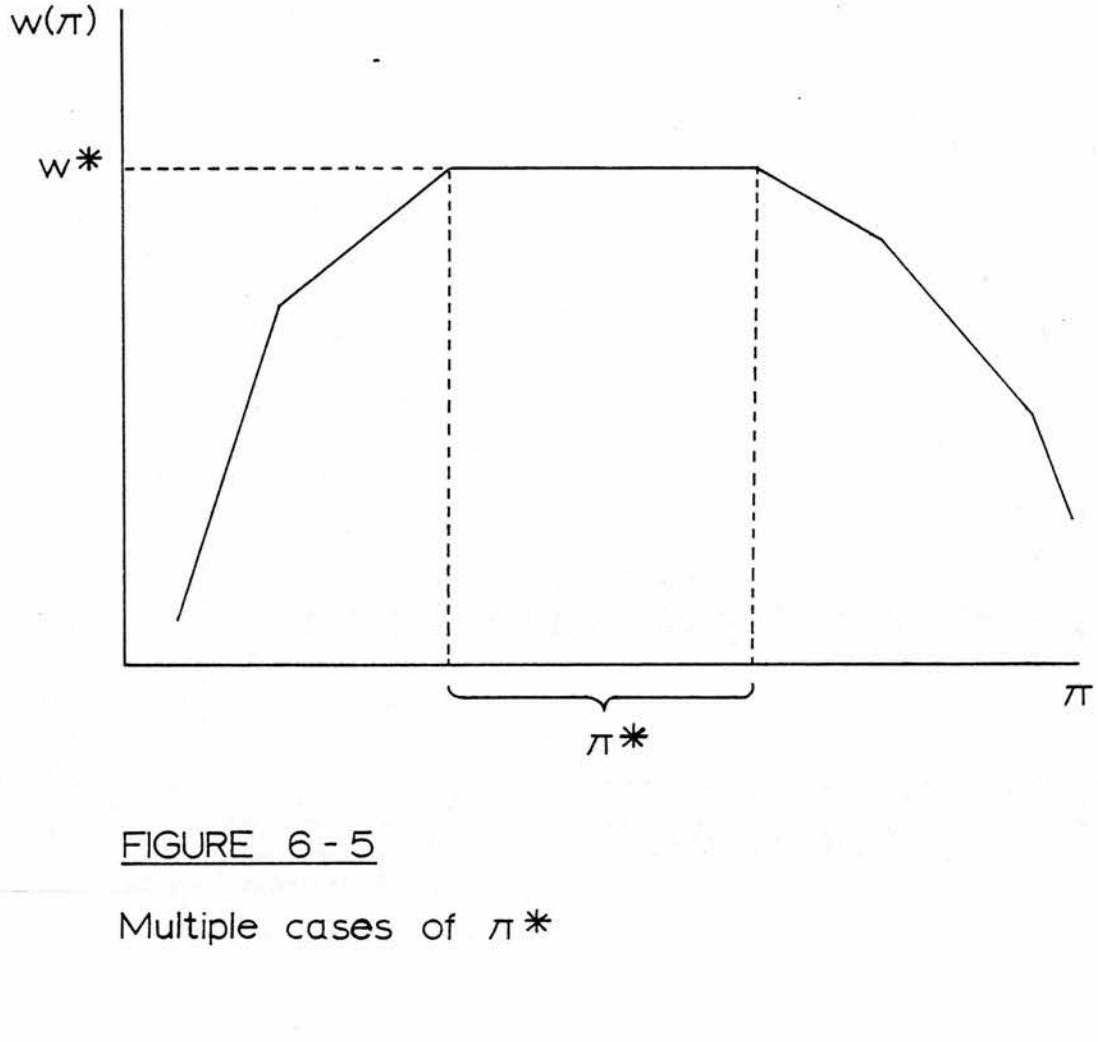


FIGURE 6 - 5

Multiple cases of π^*

Figure 6-6 shows our present state of knowledge. We know the value of w (π_0) and we know that we have at this point a positive gradient. This means that an increase in π will lead to an increase in w (π) within certain limits. If we go too far, the value of w (π) will decrease again. However, rather than play safe we want to make quite large changes in π at first with smaller ones later in order to home in fastest on the maximum value of w (π). The move from π_0 to π_1 is called a step, and the problems connected with determining the best step sizes will be discussed at length later on. For the time being, let us assume that we have decided what the step size multiplier (a positive scalar, S) shall be. We re-evaluate w (π) and ∇ w (π) for the new value of π, giving the situation shown in Figure 6-7.

The gradient ∇ w (π_1) is negative, so we have come too far, past the peak of the function w (π). By luck, however, w (π_1) is greater than w (π_0). We know now to make a smaller step in the direction of the new gradient. Since the step size, S, is always positive, and the gradient w (π_1) is negative, we will end up reducing the value of π. Figure 6-8 shows the situation at the next iteration.

∇w (π_2) is still slightly +ve, but less so than before. (It must be $\leqslant \nabla$ w (π_0)). π is therefore increased again, but by a smaller still amount. It would appear that, given the correct step size sequence, each iteration would provide a value for π nearer to π^* (the value for which the maximum value of w (π), w* is attained) even though the value of w (π) would not necessarily increase at each iteration. In Section 6.18 it will be demonstrated why this property is not sufficient to ensure convergence on π^* in a finite

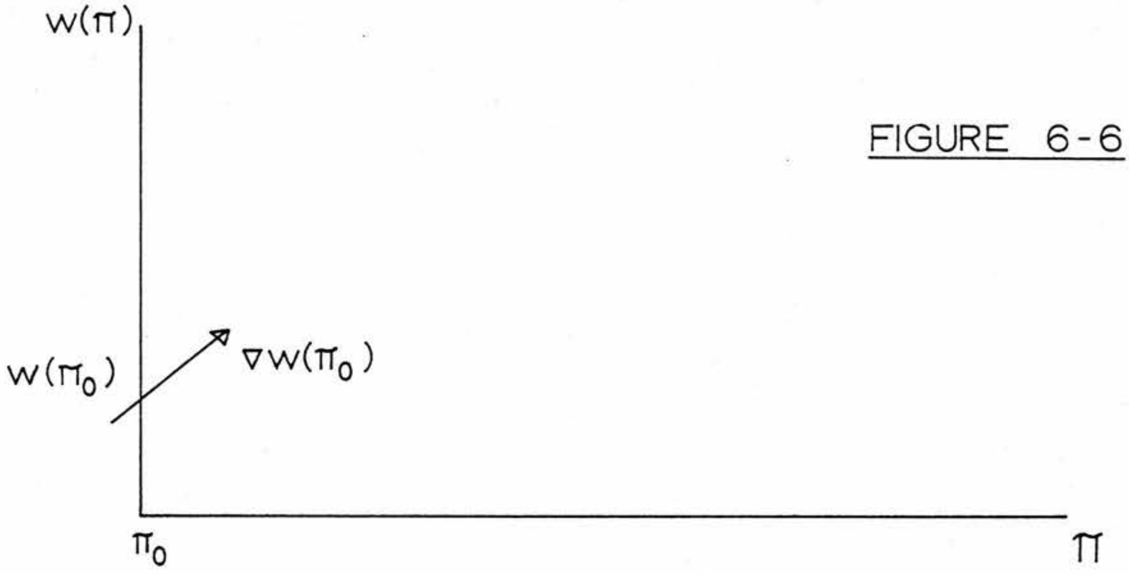


FIGURE 6-6

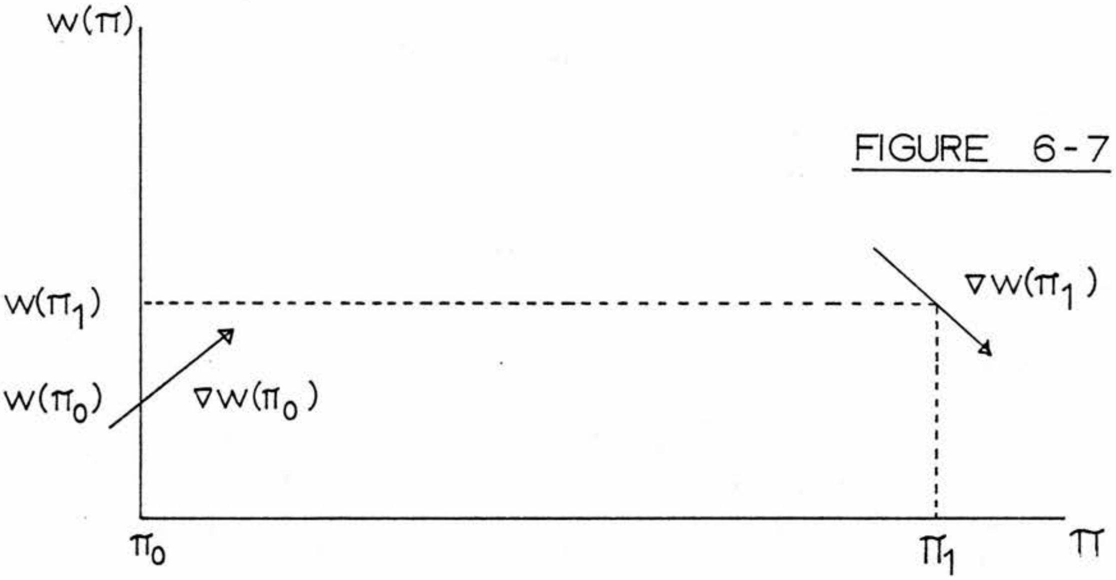


FIGURE 6-7

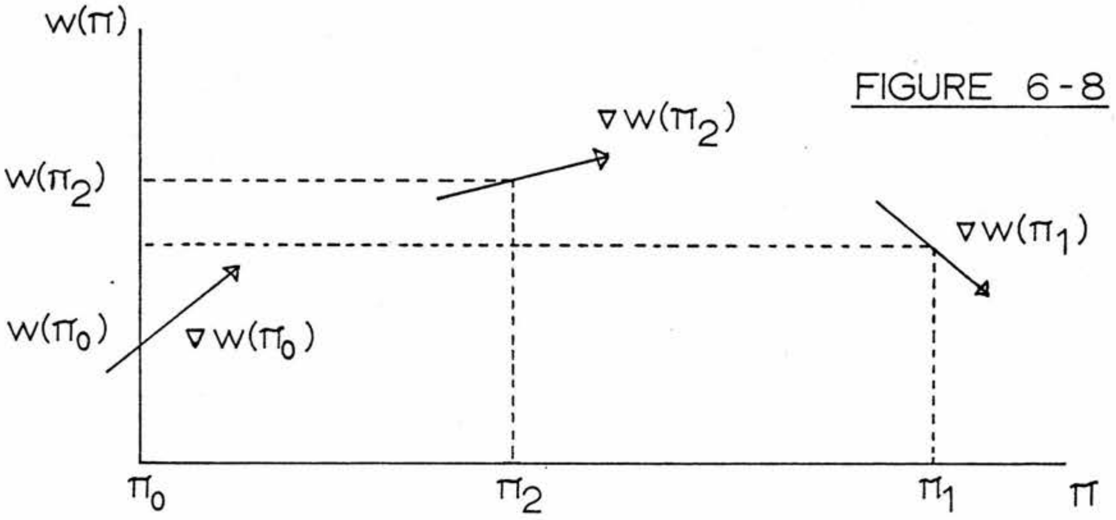


FIGURE 6-8

Graphical representation of sub-gradient method number of steps.

In the general example above, each iteration has produced a singleton V (π), with w differentiable at π. What happens when the set V (π) has more than one member? It is possible to use ∂ w (π) to define a direction of steepest ascent for a non-optimal point, in order to make use of existing steepest ascent procedures, as suggested by Bertsekas and Mitter[15], Dem'janov[16], Geoffrion[17] and Grinold[18], but the work done by Held[19] suggests that this is unneccesary. He takes the view that the identification and evaluation of the entire set V (π) in order to get a direction which is only locally preferable requires too much computation to justify its usefulness. Instead he elects to obtain only a unique

$v\ (\pi) = v_m \in V\ (\pi)$ (7.5)

as a result of the evaluation (7.2) of w (π).

To illustrate the effects of this let us consider the function

$w\ (\pi) = \min\ (-\pi_1,\ 2\pi_1 - 3\pi_2,\ \pi_1 + 2\pi_2)$

Figure 6-9 indicates the contours of this function.

Let us consider first a simple situation, that which pertains at the point (3,0). From this point, the function slopes "uphill" to the origin, the gradient being $-\pi_1$. Movement towards this intersection will eventually lead one to the point (0,0) where w (π) is maximised.

Now consider the situation at the point (3,2). The set V (π) is still the singleton $\{(-1,0)\}$. However if π_1 is reduced as before, then after a reduction of one we arrive at the point (2,2). Any further reduction of π_1, according to the gradient $\{(-1,0)\}$ will lead to a decrease in w (π). Since we are on one of the three rays shown in Figure 6-9 we are at a point where V (π) has two members.

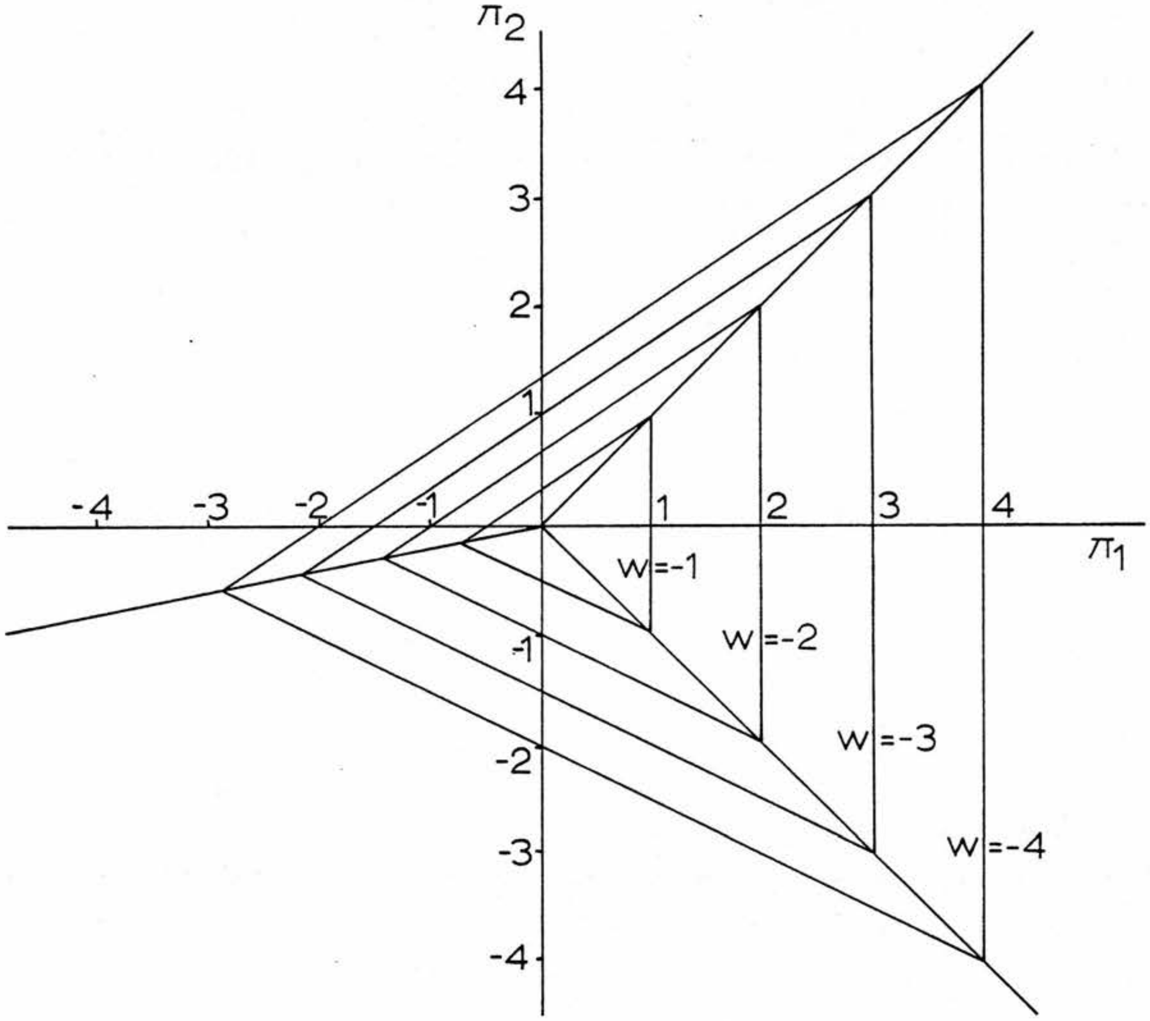


FIGURE 6-9

Contours of the function $w(\pi)=(-\pi_1, 2\pi_1-3\pi_2, \pi_1+2\pi_2)\min$

In this case $V(\pi) = \left\{(-1,0), (2,-3)\right\}$. Unfortunately any movement in the direction (2,-3) from point (2,2) will also lead to a decrease in $w(\pi)$. In fact at any point on any of the three rays w will be nondifferentiable, and $v_m < 0$ for all $v_m \in V(\pi)$. As stated above, however, one $v(\pi)$ will be evaluated and used despite this drawback, rather than spend the computational effort required to find a locally best direction:

$v(\pi) = v_m \in V(\pi)$ (7.5).

This decision leads to a further simplification. Since the choice of direction is not always locally optimal, there is no point in trying to maximise $w(\pi)$ in that direction. The computational requirement is again reduced by this decision, although more iterations may be required in total.

6.13 Held's sub-gradient algorithm[20]

Given $\pi^0 \in E^n$ and the sequence $\left\{S_n\right\}$ of positive scalars as step sizes, define the sequence $\left\{\pi^n\right\}$ by

$\pi^{n+1} = \pi^n + S_n \, v(\pi^n)$ for n = 0, 1, ... (7.6)

Now we have to establish a set of conditions under which $w(\pi^n)$ will converge to its maximum w*. The most general result, according to Poljak[21] is that this will occur under the conditions

$$S_n \longrightarrow 0, \sum_{n=0}^{\infty} S_n = \infty \qquad (7.7)$$

A number of more specific conditions was used by Held, each of which was a variation on the Agmon-Motzkin - Schoenberg procedure[22] for the determination of a point π such that $w(\pi) \geqslant \hat{w}$, (where $\hat{w}$ is an underestimate of the maximum value $w(\pi^*)$) i.e.

$\pi \cdot v_m \geqslant \hat{w} - c_m$, m = 1, ..., M (7.8)

In general terms, convergence is assured thus:

Let π^* be any point in the optimal set. Since $v(\pi^n) \in \partial w(\pi^n)$, relation (7.4) says

$$w^* - w(\pi^n) \leqslant v(\pi^n) \cdot (\pi^* - \pi^n) \qquad (7.9)$$

Assuming π^n to be non-optimal, then the direction $v(\pi^n)$ makes an acute angle with the ray from π^n through π^*. As long as S_n is sufficiently small, the point π^{n+1} will be closer to π^* than was π^n. The sequence will thus approach the optimal set, though the objective values $w(\pi^n)$ are not monotonic.

As illustration, consider the function

$$w(\pi) = \min(-2\pi_1 + \pi_2, 2\pi_1 - 4\pi_2, \pi_1 + 2\pi_2)$$

Let us assume that we are at the point (2,1), giving $w(\pi) = -3$ (Figure 6-10). In this case $\pi^* = (0,0)$ for simplicity. A step size of $S_n = \frac{1}{2}$ produces the new $\pi^{n+1} = (1,1\frac{1}{2})$. π^{n+1} is nearer to π^* than was π^n as predicted, even though $w(\pi^{n+1})$ is now equal to -4, where $w(\pi^n)$ equalled -3. As the step size is increased from zero, π^{n+1} becomes closer to π^*, until the point where its direction of change is at right angles to a line between its present position and π^*. From there onwards it recedes from π^* until, beyond a certain point, it is further from π^* than was π^n.

It can be seen from this that the choice of step size is absolutely critical to the algorithm. Convergence is not guaranteed overall by the relationship expressed in (7.9); it only occurs within limits.

6.14 Choice of step size

To complicate the matter further, although we know that there is an upper limit on the step size which will permit convergence to occur, we do not know what this upper limit is. (7.7) gives us a

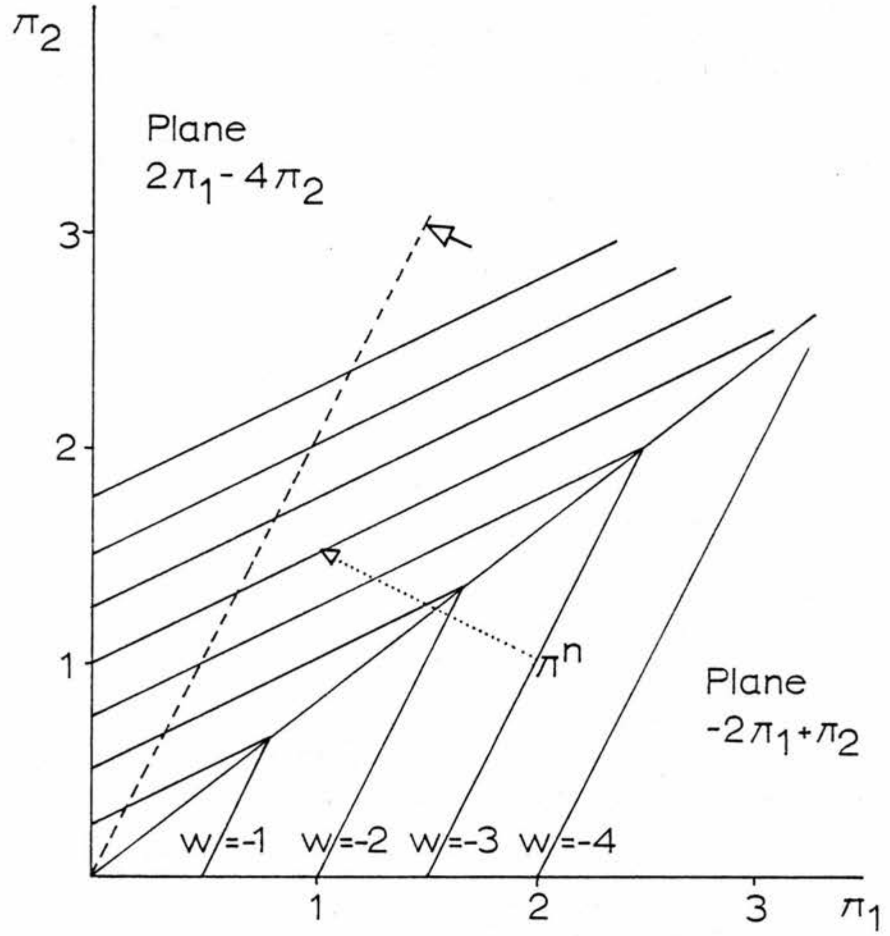


FIGURE 6 - 10

Effects of step size; maximisation problem.
-------- Indicates the locus of all π^{n+1} where π^n lies on the plane $-2\pi_1+\pi_2$, and $\lambda=1$. In this case it passes through the optimum point since the estimate is equal to w*. In future this line will be referred to as the $\lambda=1$ step limit.

general set of conditions under which convergence will occur, but it is fairly obvious that a more precise but flexible set of conditions is required in order for a solution to be arrived at within a sufficiently small number of iterations.

(7.8) employed an underestimate, $\hat{w}$, of the maximum value of w^* of $w(\pi^n)$. The choice of step size adopted by Held[23] is as follows:

$$S_n = \lambda_n \frac{\hat{w} - w(\pi^n)}{|v(\pi^n)|} 2 \qquad (7.10)$$

where $|\cdot|$ denotes the Euclidean norm. For each n, $\epsilon < \lambda_n < 2$ for some fixed $\epsilon > 0$. The sequence $w(\pi^n)$ either converges to $\hat{w}$, or a point π^n is obtained such that $w(\pi^n) \geqslant \hat{w}$. This procedure has obvious drawbacks, not the least of which are (1) the difficulty of obtaining a good underestimate, and (2) the fact that the procedure stops when the underestimate is exceeded.

Attempts were made to devise a method less sensitive to the accuracy of the estimate, and these will be described later. First, however, let us take a brief look at the derivation of (7.10).

The numerator of the expression, $\hat{w} - w(\pi^n)$, is self evident. If $w(\pi^n)$ is homing in on $\hat{w}$ then one wants to take smaller steps to avoid overshooting by too much. On the other hand if the difference between $\hat{w}$ and the current value of $w(\pi^n)$ is great then relatively larger steps can be risked with less danger of going beyond the critical point where π^{n+1} is further from π^* than was π^n.

Next let us consider the denominator, $|v(\pi^n)|^2$. This relates to the fact that the gradient of a plane on which π^n sits not only determines the direction that a step will take us but also how far it will move π. As illustration, consider Figure 6-11. This diagram represents the contours of the function

$$w(\pi) = \min(-\pi_1, 2\pi_1 - 3\pi_2, \pi_1 + 2\pi_2)$$

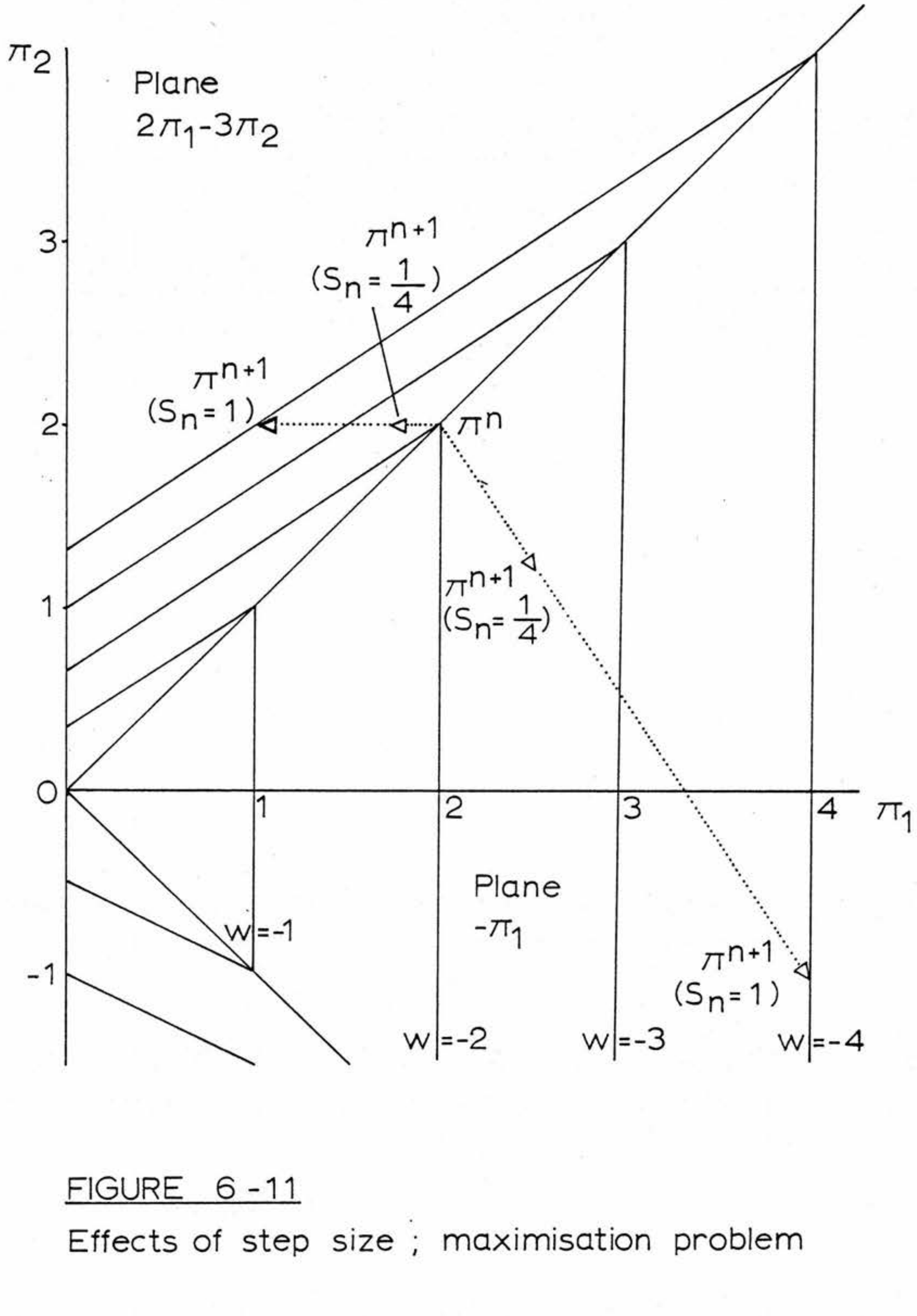


FIGURE 6 -11

Effects of step size ; maximisation problem with only positive values of π_1 being considered.

π^n is the point (2,2). Let us take a step size of $\frac{1}{4}$ in the direction suggested by each plane in turn.

A step of $S_n = \frac{1}{4}$ in the direction $(-\pi_1)$ gives us $\pi^{n+1} = (1\frac{3}{4},2)$.

A step of $S_n = \frac{1}{4}$ in the direction $(2\pi_1 - 3\pi_2)$ gives us $\pi^{n+1} = (2\frac{1}{2},1\frac{1}{4})$.

w (π^{n+1}) is the same in both cases, at $2\frac{1}{2}$, but it can be seen that a step in the direction of the steeper gradient, $(2\pi_1 - 3\pi_2)$ causes a greater movement across the plane of the axes.

It must be remembered that the critical property for convergence to occur is that π^{n+1} must be nearer to π^* than was π^n, the value of w (π^{n+1}) being of limited relevance. To illustrate the effect let us now consider a step size of size $S_n = 1$.

From $\pi^n = (2,2)$, in the direction $(-\pi_1)$ we find $\pi^{n+1} = (1,2)$

From $\pi^n = (2,2)$, in the direction $(2\pi_1 - 3\pi_2)$ we find $\pi^{n+1} = (4,-1)$.

In both cases, w $(\pi^{n+1}) = -4$, but it is fairly evident that the point (1,2) is nearer to π^* at the origin than is (4,-1).

In general terms, then, it can be seen that movement in the direction of a small gradient will result in a decrease in the distance between π^n and π^* over a much greater range of step sizes, whereas movement in the direction of a steep gradient causes π^n to recede from π^* as soon as S_n exceeds a relatively low value.

Held, then, chooses to divide the step size by the square of the Euclidean norm of the direction v (π^n), so that, the larger the gradient, the smaller the step size S_n.

Finally we have the factor λ_n. The determination of values for λ is somewhat arbitrary, but Held uses a positive value less than two. The upper limit is set by investigation into a suitable order of magnitude for S_n, while the lower limit must have a fixed value greater than zero, since if it ever reached zero the whole solution process would stop. Between these limits, the best sequence seemed to Held to be a decreasing series, for the following reasons:

Held adopted a modification of the Agmon - Motzkin - Schoenberg procedure, in that he replaced the underestimate $\hat{w}$ with an overestimate $\overline{w}$. This avoids the problem that the solution procedure stops as soon as $\hat{w}$ is exceeded, but it introduces a new problem i.e. the sequence S_n will no longer diminish to zero. To overcome this problem it is necessary to adopt a sequence λ_n which tends towards zero. Held chose a number of series with this characteristic, typical of which was the following:

Set $\lambda = 2$ for 2p iterations, where p is a measure of the problem size.

Halve the value of λ and the number of iterations until the number of iterations reaches some threshold value Z.

Halve λ every Z iterations until the resulting S_n is sufficiently small.

Held notes that this procedure violates the condition $\sum S_n = \infty$, causing the possibility to occur of convergence to a non-optimal print, but claims that this almost never happened.

The various series chosen for this research will be explained and evaluated in Section 6.18.

6.15 Alternative ascent procedures

6.15.1 Steepest ascent procedure

Grinold[24] outlines a steepest ascent procedure which is equivalent in this case to exploring the directional derivatives of the function w (π). The directional derivative of w (π) in the direction v is defined as :

$$\nabla w\ (\pi : v) = \lim_{\alpha \to 0^+} \frac{w\ (\pi + \alpha v) - w\ (\pi)}{\alpha} \qquad (8.1)$$

Suppose $\pi \in \Pi$ and w (π) is the dual of a bounded primal of the form :

min u x (8.2)

subject to $Ax = b,\ x \geqslant 0$ (8.3)

Grinold defines the dual problem as

max w (u) (8.4)

s.t. $u\ D \leqslant g$ (8.5)

Let D^1 represent the tight constraints : $d^k \in D^1$ if and only if $ud^k = g_k$.

A direction v is feasible if $u + \alpha\ v \in V$ for some $\alpha > 0$. Thus v is feasible if and only if $v\ D^1 \leqslant 0$.

Grinold defines the steepest ascent problem as :

max ∇ w (u : v) (8.6)

s.t. $v\ D^1 \leqslant 0,$ (8.7)

$-v\ R \leqslant 0,$ (8.8)

$(-1, -1, \ldots, -1) \leqslant v \leqslant (1, 1, \ldots, 1)$ (8.9)

where R is the matrix of extreme tight rays : ur = 0 if and only if $r \in R$. Grinold demonstrates how the inclusion of the contraint (8.8) eliminates the consideration of directions of infinite decrease.

Grinold's procedure was not tried for the present problem, since it was considered by Held[25] to be unnecessarily costly in terms of computation time for the limited benefits which it brought, i.e. determining a locally optimal direction.

6.15.2 Modified gradient direction

The method considered here was that proposed by Camerini, Fratta and Maffioli[26]. Their problem is defined in the same form as 7.1-2),

$$\max_{\pi} w\ (\pi) = \min_{m} \left\{ c_m + \pi . v_m \right\} \qquad (9.1)$$

$$= c_{m(\pi)} + \pi . v_{m(\pi)} \qquad (9.2)$$

and they adopt the same iterative scheme,

$$\begin{cases} \pi^0 = 0 \\ \pi^{n+1} = \pi^n + S_n\ g^n, \qquad (9.3) \end{cases}$$

$\left\{ S_n \right\}$ being a sequence of scalars, the step sizes in (7), and g^n the gradient of w (π^n) such that

$$g^n = \nabla w\ (\pi^n) = v_{m(\pi^n)} \qquad (9.4)$$

They make the point that g^n can be a unique gradient, or one of a set of sub-gradients if the gradient of $m(\pi)$ is undefined at the point under consideration, in which case any of the sub-gradients can be used. They justify this by pointing out that the purpose of the iteration scheme is to come closer and closer to the optimum region, rather than to improve the value of the objective function at each step.

They then claim that the efficiency of the procedure can be improved by selecting the modified gradient direction

$$g^n = v^n + \beta_n\ g^{n-1} \qquad (9.5)$$

where $v^n = v_{m(\pi^n)}$ and β_n is a suitable scalar ($S^{n-1} = 0$ for $n = 0$).

Several policies were tested by Camerini et.al. for choosing S_n and g^n, and the one which they decided was most successful was the following:

$g^n = v^n + \beta_n\ g^{n-1}$ where

$$\beta_n = \begin{cases} -\gamma \dfrac{g^{n-1}\ .\ v^n}{\left\| g^{n-1} \right\|^2} & \text{if } g^{n-1}\ .\ v^n < 0, \\ 0 & \text{otherwise} \end{cases} \qquad (9.6)$$

and/

and $S_n = \frac{w^* - w_n}{\|g^n\|^2}$ (9.7)

where w* is a good estimate of $\max_{\pi} w(\pi)$.

In their experience, a value of 1.5 was found to be best for γ. The modified gradient direction method was easily incorporated into the program used to assess the sub-gradient algorithm, since the basic procedure is identical. In fact if γ is given a value of zero the program works with unmodified gradient direction, since (9.5) is in fact a weighted sum of all previous gradient directions. It is claimed that Crowder[27] has successfully used the method to avoid troublesome effects due to the "sub-gradient's alternating components". By this they mean that in cases where there is a zig-zag effect from one plane to another and back again, the modified gradient will tend to lead each successive value of $w(\pi)$ in the direction of the line of intersection of the two planes until another plane is encountered.

6.16 Implementation and results

Held's sub-gradient method and choice of step size were used as a basis for experimentation with the scheduling problem. At each iteration values were printed out for the following variables:

(1) Dual

(2) Step size

(3) Deficit

(4) Primal

Let us see how these variables relate in practice to their theoretical values as discussed earlier in this Chapter.

6.16.1 Dual

The value of the dual problem, $w(\pi)$, was defined as

$$\min \left\{ c_m + \pi . v_m \ : m = 1, \ \ldots, \ M \right\} \qquad (7.2)$$

Successive values of π were selected to maximise the minimum value of w (π), by decreasing the distance between π^n and π^* (the maximum value of π). Although the optimal set will be approached, the increase in value of w (π) is not monotonic.

The definition of w (π) which was used was obtained by transforming the Lagrangian formulation (2).

$$(2.1) \quad L\ (\underline{x}, \underline{\pi}) = \left[\sum_{iq} c_{iq}\, x_{iq} - \sum_{jt} (\pi_{jt}\ (\sum_{iq} a_{ijqt}\, x_{iq} - d_{jt})) \right]$$

$$(10.1) \quad = \left[\sum_{iq} c_{iq}\, x_{iq} - \sum_{jt} (\pi_{jt} \sum_{iq} a_{ijqt}\, x_{iq}) + \sum_{jt} \pi_{jt}\, d_{jt} \right]$$

$$(10.2) \quad w\ (\pi) = \min_{\sum_q x_{iq} = 1, \forall i} \ L\ (\underline{x}, \underline{\pi})$$

$$(10.3) \quad = \min_{\sum_q x_{iq} = 1} \left[\sum_{jt} \pi_{jt}\, d_{jt} + \sum_{iq} x_{iq}\ (c_{iq} - \sum_{jt} \pi_{jt}\, a_{jqt}) \right]$$

$$(10.4) \quad = \sum_{jt} \pi_{jt}\, d_{jt} + \min_{\sum_q x_{iq} = 1} \left[\sum_{iq} x_{iq}\ (c_{iq} - \sum_{jt} \pi_{jt}\, a_{jqt}) \right]$$

$$(10.5) \quad = \sum_{jt} d_{jt}\, \pi_{jt} + \sum_{i} \min_{q}\ (c_{iq} - \sum_{jt} \pi_{jt}\, a_{jqt})$$

Thus (10.5) was the expression which was evaluated to give the current value of the dual at each iteration.

6.16.2 Step size

Let us consider the gradient $\nabla w\ (\pi)$. This can be defined as

$$(10.6) \quad \frac{\partial w\ (\pi)}{\partial \pi_{jt}} = d_{jt} - \sum_{i} a_{jq*t}$$

if girl i is allocated to schedule q*, i.e. $c_{iq*} - \sum_{jt} \pi_{jt}\ a_{jq*t}$

gave the minimum value in (10.5). It is a point worthy of note that the gradient is equivalent to a meaningful value when applied to the real problem, since (10.6) defines the shortage or deficit on a given ward during a given week. Now consider how this affects the step size in practice. Held's choice of step sizes was

$$(7.10) \quad S_n = \lambda_n \frac{\hat{w} - w\ (\pi^n)}{\left| v\ (\pi^n) \right|^2}$$

where $\left| \,.\, \right|$ denoted the Euclidean norm, and for each n, $\epsilon < \lambda_n < 2$ for some fixed $\epsilon > 0$. $\hat{w}$ was the underestimate, S_n the step size for iteration n.

In practice this becomes:

$$(10.7) \quad S_n = \lambda_n \frac{\hat{w} - w\ (\pi^n)}{\sum_{jt} (d_{jt} - \sum_{i} a_{jq*t})^2}$$

d, i, j, q, t are used as defined in (1.1-4)

If we let s_{jt} represent the shortage $\left| d_{jt} - \sum_{i} a_{jq*t} \right|^+$ on ward j in week t then the expression becomes

$$(10.8) \quad S_n = \lambda_n \frac{\hat{w} - w\ (\pi^n)}{\sum_{jt} s_{jt}^2}$$

Negative shortages, i.e. overstaffing, are not included

The sequence $\{\pi^n\}$ was defined by

$$(7.6) \quad \pi^{n+1} = \pi^n + t_n\ v\ (\pi^n) \text{ for } n = 0, 1, \ldots .$$

By substituting the expression in (7.10) we arrive at

$$\pi^{n+1} = \pi^n + \lambda_n \frac{\hat{w} - w\ (\pi^n)}{\left|v\ (\pi^n)\right|^2} \times \frac{v\ (\pi^n)}{1}$$

$$(7.11) \qquad = \pi^n + \lambda_n \frac{\hat{w} - w\ (\pi^n)}{v\ (\pi^n)}$$

For the present let us assign a value of one to λ_n, so that it may be dropped from the expression. It can now be seen that $\frac{\hat{w} - w\ (\pi^n)}{v\ (\pi^n)}$ is the increase required in π^n, in order that $w\ (\pi^{n+1})$ should equal $\hat{w}$, if and only if $w\ (\pi^n)$ lies on the same plane as $\hat{w}$.

This may be illustrated graphically as in Figure 6-12.

This situation will only occur on the last iteration, and even then only if $\hat{w} = w*$. More commonly the plane that $w\ (\pi^n)$ lies on will not pass through the maximum point on $w\ (\pi)$. Keeping $\hat{w} = w*$ the situation shown in Figure 6-13 might occur.

Here the same increase in π has been made, but a new plane has been met, and the maximum value of w* has not been reached. It can be seen from the convex nature of $w\ (\pi)$ that any move which fails to reach w* will fall short rather than overshoot, and that the precise move illustrated in Figure 6-12 will only be needed on the last iteration. This is the reason for Held's use of the variable λ. By assigning to it any value greater than one the step made will be proportionately greater along the π axis. He used a starting value of two and decreased it during the solution of the problem. More will be said later on this subject in Section 6.18.

6.16.3 Deficit

The value printed out was the sum of the shortages on all

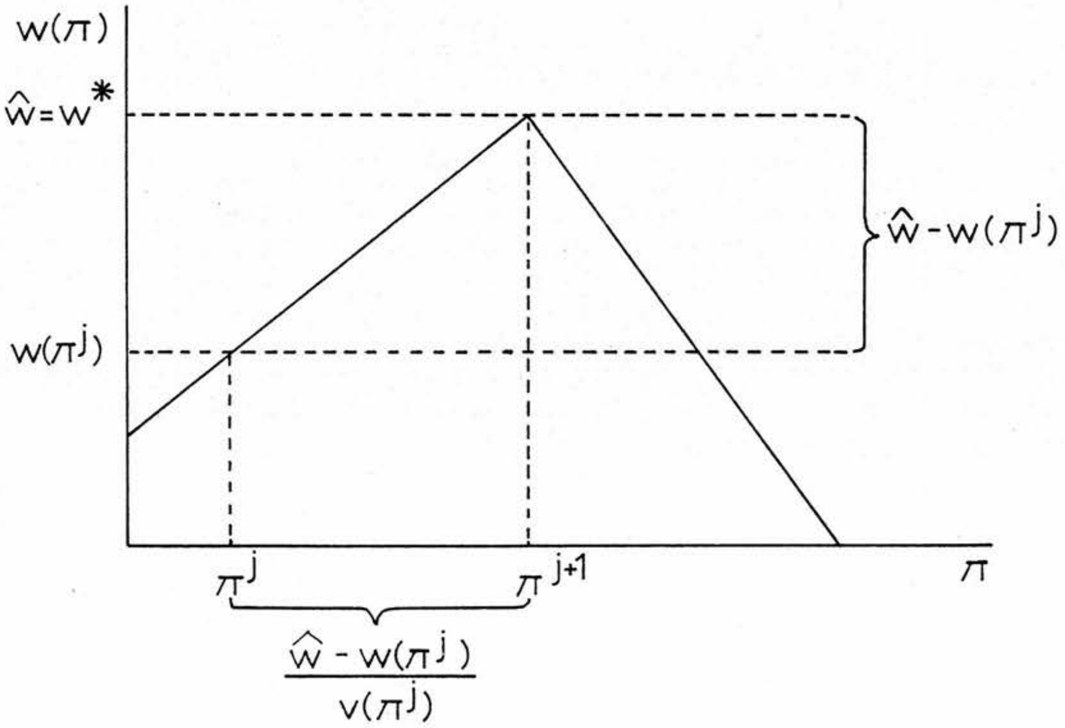


FIGURE 6-12

Determination of step size

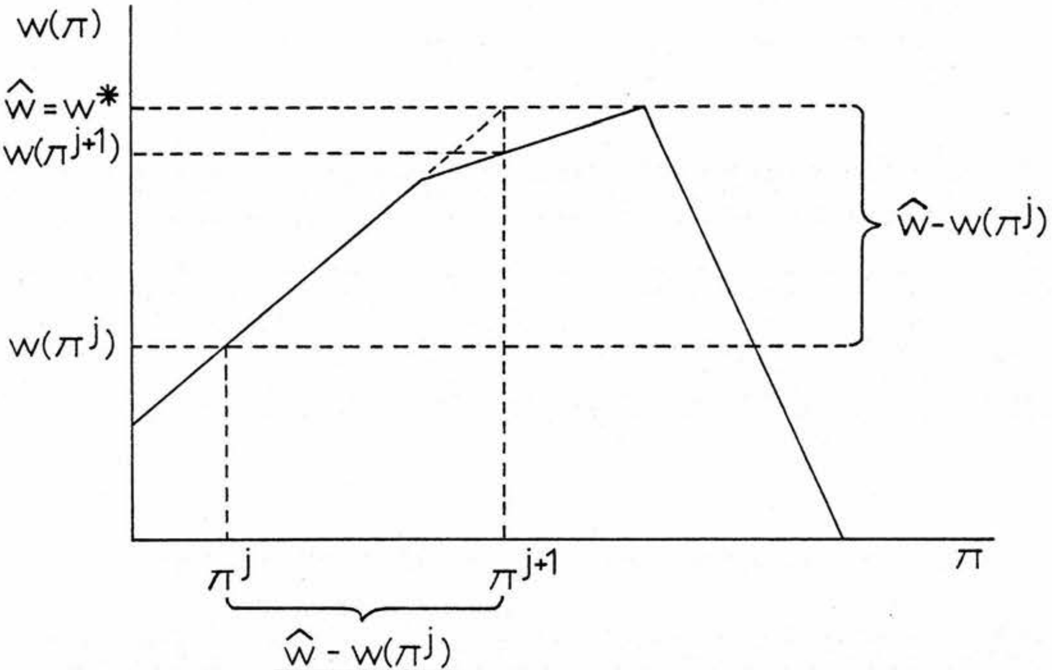


FIGURE 6-13

Effects of step size

weeks and wards : $\sum_{jt} s_{jt}$. As defined at (10.8) s_{jt} only includes positive shortages.

6.16.4 Primal

The value of the primal, as given in (1.1) is $\sum_{iq} c_{iq} x_{iq}$. On the final iteration, when w* is attained, Z (the value of the primal) will equal w (π). Any change in either the deficit or the primal value from one iteration to the next indicates that the value of w (π) is now defined by a new plane.

6.17 Choice of estimate

Mention was made in Section 6.14 of the problems associated with the use of an underestimate. First, it is difficult to obtain a good underestimate, and second, the procedure stops when the underestimate is reached or exceeded. For some types of problem this may not be critical. In most types of calculation it is the value of the objective function in the primal which is of interest. If the procedure was being used to set up a series of schedules where the cost being considered was a monetary one, it might in some cases be possible to state in advance that any solution which had an objective function value of less than a pre-determined level was acceptable, even if some of the Lagrangianised constraints were being violated. Here, however, the Lagrangian constraints are critical and the value of the objective function, and hence the stated underestimate $\hat{w}$, are irrelevant since they deal with artificial costs. It is not even necessary to minimise these costs, since the objective of the procedure is to obtain a feasible solution which does not violate the constraints which have been Lagrangianised. As a result the solution procedure is terminated as soon as the value of $\sum_{jt} s_{jt}$ is reduced to zero, regardless of whether the value of the dual, $w(\pi)$ could be further increased, or the value of the primal decreased.

Experiments were made using over- as well as underestimates, and these proved to be fairly successful. In order to reduce $\sum_{jt} s_{jt}$ to zero it is only necessary to arrive on the correct plane of $w(\pi)$, not to find the maximum value of $w(\pi)$ on that plane, so even a poor overestimate ($\overline{w}$) could achieve reasonable results in some cases. It was felt that the overestimate should be generated automatically by the data, since the algorithm could then be assessed on its own performance, rather than being subject to the initial value selected by the programmer.

The first method used was to cycle through each girl, i, in turn, selecting the schedule, q, whose cost was greatest for her:

$$\sum_{i} (\max c_{iq} \ \forall q)$$

This gives the maximum possible value for the primal, given the present set of costs. Although this sets an upper limit it may still be an order of magnitude too great, and the over-large series of step sizes which result are likely to cause the algorithm to fail.

Experimentation with different estimates revealed the relationship between the estimate and the step sizes selected. If the estimate was much too high, then the numerator of the step size expression, $\overline{w} - w(\pi^n)$, would be correspondingly great, and too large a step would be taken. In many cases the value of $w(\pi^{n+1})$ would be negative and would become progressively more negative in subsequent iterations as the algorithm took $w(\pi)$ further away from $\overline{w}$, owing to increasing step sizes. With a good estimate, however, the step size would be within the correct range and the value of

$w(\pi^1)$ would be higher than that of $w(\pi^0)$. This property suggested a heuristic which could be used to arrive at a reasonable overestimate. If the initial value of $w(\pi^1)$ was lower than $w(\pi^0)$ then the overestimate could be reduced and a new value for $w(\pi^1)$ calculated using a new step size. As the estimate was successively reduced, and the step size got smaller, eventually one would be arrived at which produced a $w(\pi^1)$ near to w^* than was $w(\pi^0)$. At this point the current estimate would be retained, and a new step could be taken from $w(\pi^1)$ to $w(\pi^2)$. Although it was necessary to require that $w(\pi^1)$ was greater in value than $w(\pi^0)$, this condition could now be dropped, and the normal iterative scheme used from $w(\pi^2)$ onwards. It was found by experiment that a reduction of 10% in the estimate at each iteration which failed to improve $w(\pi^0)$ gave the best results. Sometimes this produced a slight underestimate, and this could have caused the algorithm to converge below the required value of w^*. Rather than wait until this convergence had occurred, the estimate was increased each time that the current value of $w(\pi)$ came within a certain percentage of the value of $\hat{w}$. If $w(\pi^n)$ became equal to or greater than 0.95% of $\hat{w}$ then $\hat{w}$ would be increased again by 10%. The primary reason for changing the estimate is to cause an associated change in the step size. However, the step size can be changed independently without recourse to an alteration in the estimate. In the next section such changes in step size will be discussed.

6.18 Choice of step size - implications

As described in 6.16.2 the main determinants of step size are (1) the distance between the current value of the dual and the estimate and (2) the gradient of the current solution plane. However it was mentioned that the factor λ was used by Held[28] to vary the step size at different stages of the problem's solution. The scheme used by Held to vary λ was as follows:

$\lambda = 2$ for 2p iterations
$\lambda = 1$ " p "
$\lambda = \frac{1}{2}$ " p/2 "
$\lambda = \frac{1}{4}$ " p/4 "
.
.
.
until Z iterations, then halved every Z iterations.

p is a measure of the problem size.

A decreasing series of this nature is needed in order to cause convergence to occur. Perusal of Figures 6-12 and 6-13 would tend to suggest that a value of $\lambda = 1$ would suffice, since the optimal point would be arrived at accurately as soon as the correct plane is reached. However this only applies in the one-dimensional cases illustrated, where w (π) only varies with one π. Figure 6-14 shows the moves made at each iteration in a two-dimensional case. The dotted lines show the positions which should be moved to from each plane in an attempt to attain the estimate, which in this case is equal to the optimum point of value zero. The dotted lines are plotted to show the step size in the case of $\lambda = 1$. It can be seen from this that an infinite number of iterations will be required to reach w*, even when $\hat{w}$ = w*. This seems to be a fundamental weakness in the system. However it can be seen from Figure 6-14 that at least the search will tend to head in the right direction and can be terminated at any point considered to be acceptably near the optimum. Figure 6-15 illustrates another two-dimensional case. This

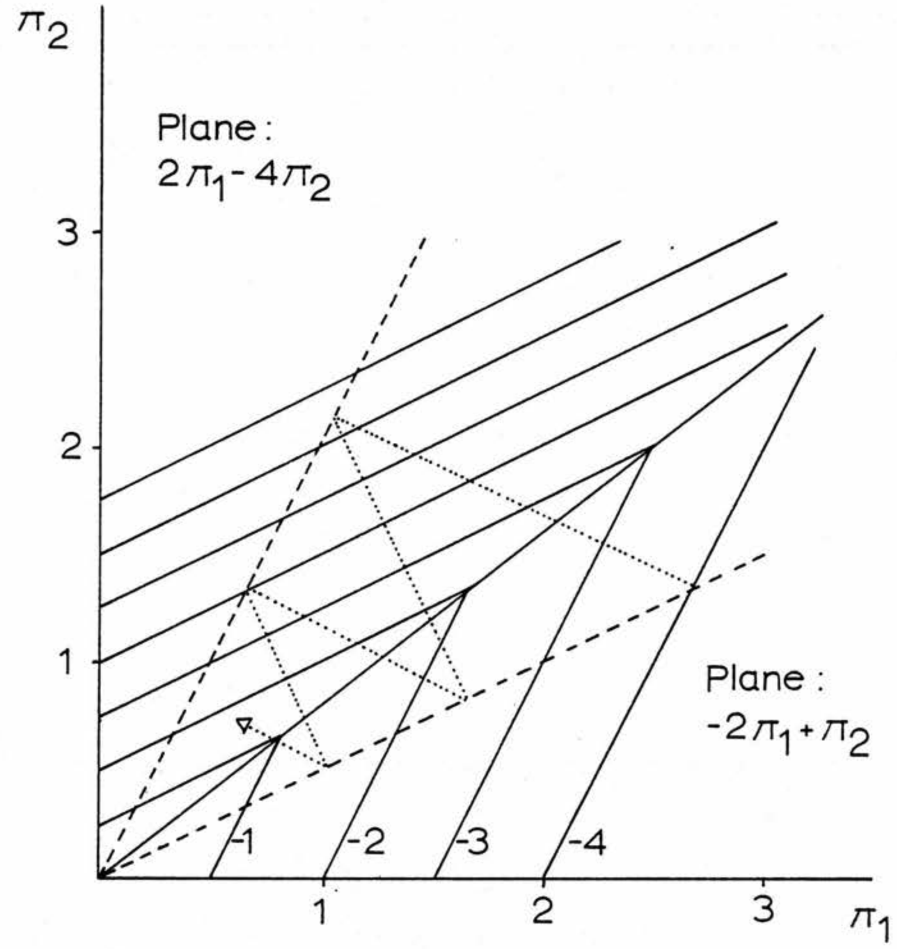


FIGURE 6-14

Route taken to reach the optimum value, given correct estimate; maximisation problem.
------: $\lambda = 1$ step limits

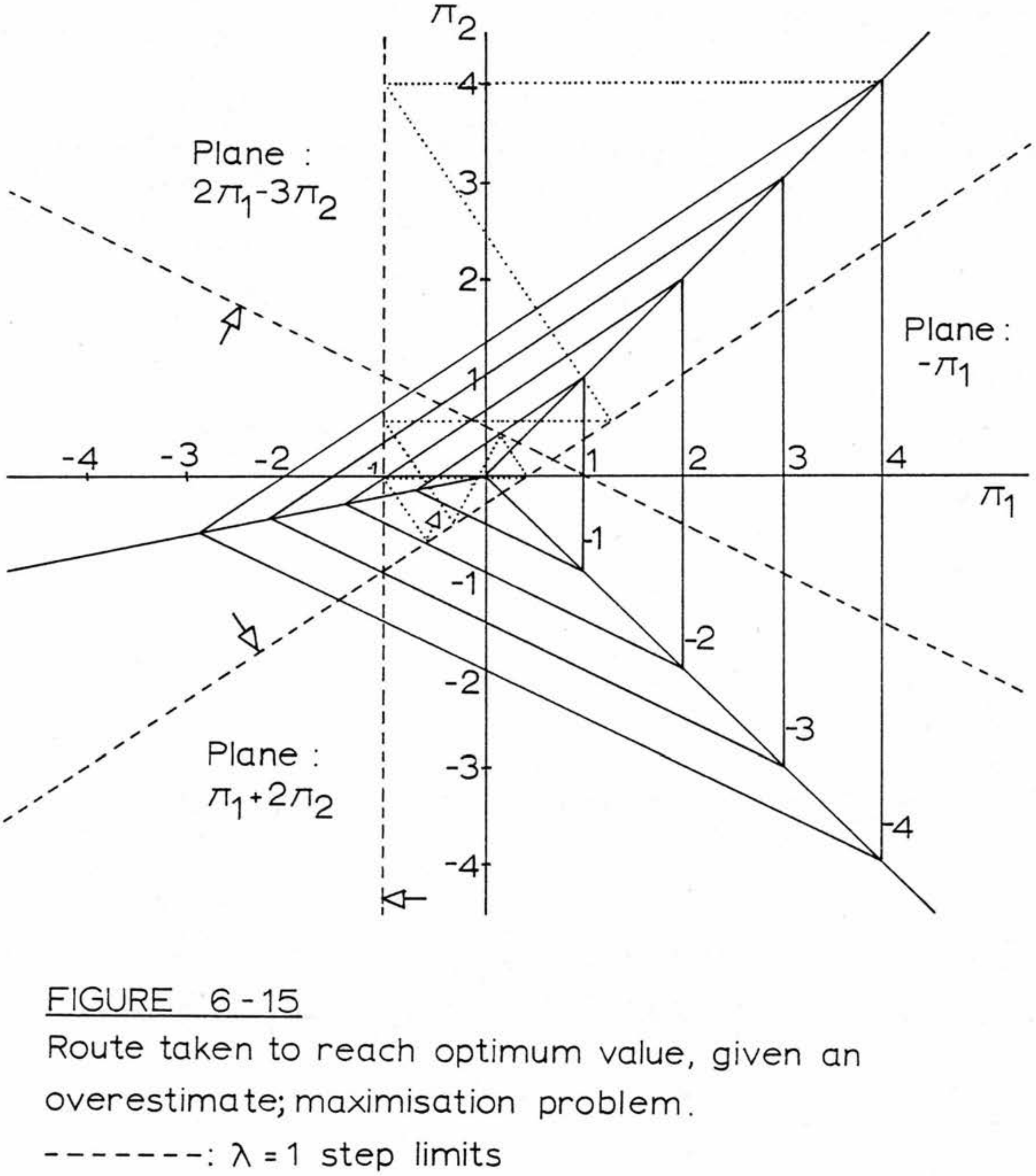


FIGURE 6-15

Route taken to reach optimum value, given an overestimate; maximisation problem.

-------: $\lambda = 1$ step limits

time the optimal point is still zero, but the estimate is now $\hat{w} = 1$. Each time that a step is taken, from any point on the three planes indicated, the new value of w (π) will lie on one of the three dotted lines. They represent the respective contour lines, for each plane, whose value is equal to one, and thus will be arrived at after any step where $\lambda = 1$. Since none of the lines pass through the optimal point, no convergence will occur. As the series of values allocated to λ decreases below one, a new mechanism will come into play. This is illustrated in Figure 6-16. The locus of w (π) is shown when λ is very small. It may take several iterations to reach a new plane, but, when one is met, a zig-zag procedure starts along the line of intersection of the two planes. Eventually the third plane will be reached, and, if the values of λ are reduced, then w (π) will converge on the optimum point, but once again only after an infinite number of iterations. It can be seen that the value of λ is critical. If it is reduced too far too soon it will cause the step size to decrease to a value at which no further useful progress will be made. This was found to occur fairly often, and manifested itself by an alternation between two pairs of values for deficit and primal value indicating a direction of progress along the edge between two planes.

Various methods of altering the choice of step size were tried, with the aim of (1) avoiding unneccesarily large steps and (2) avoiding premature reduction of the step size. The next two subsections describe these attempts.

6.18.1 Improving the dual value

Held's algorithm caused w (π) to approach $\hat{w}$ without necessarily requiring its value to increase on each iteration. As an experiment

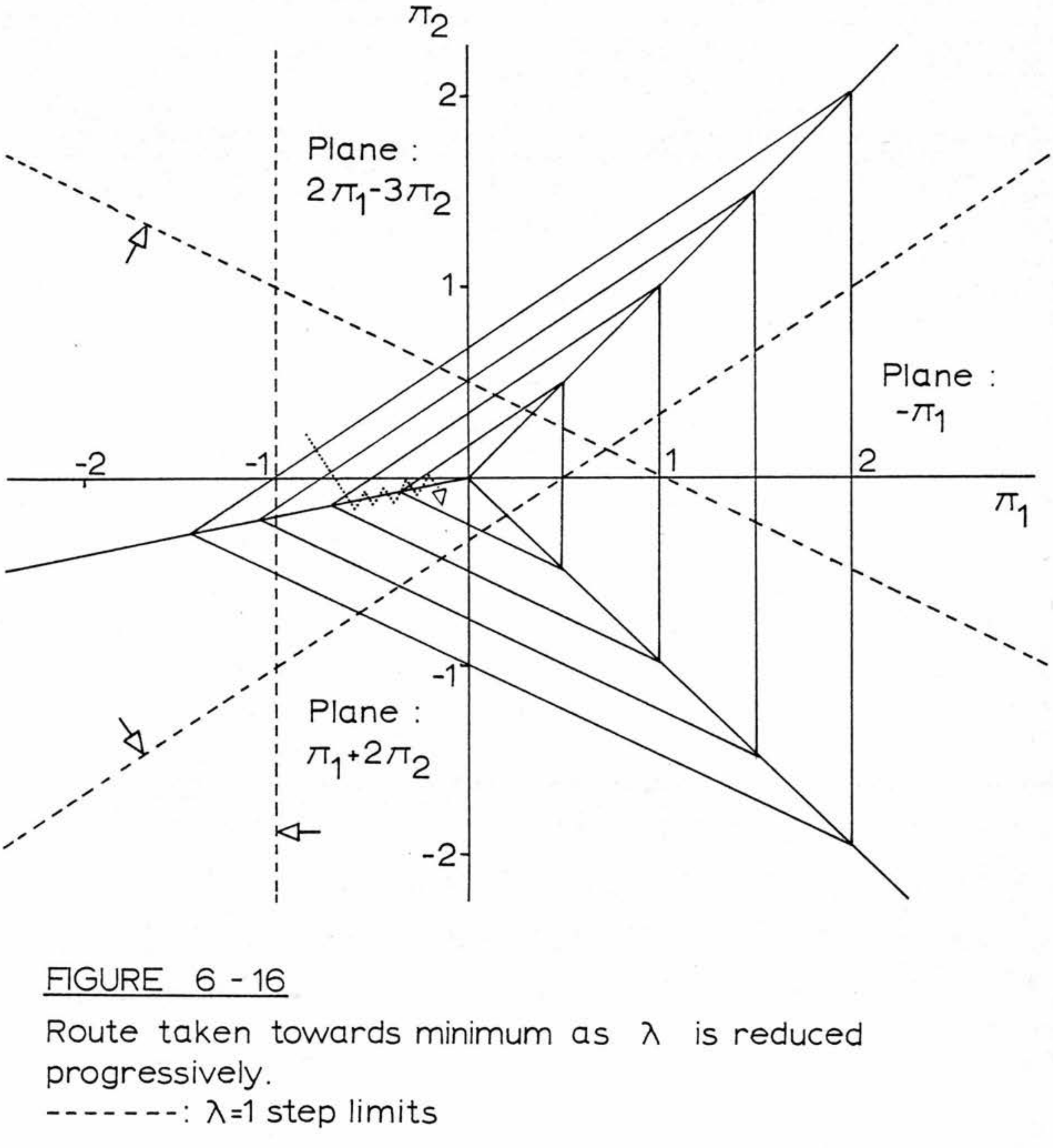


FIGURE 6 - 16

Route taken towards minimum as λ is reduced progressively.

-------: λ=1 step limits

a condition was inserted into the algorithm causing it to repeat an iteration if the value of w (π) had decreased. The next iteration was performed with the size of the step halved. Unless w (π) is currently on one of the rays of intersection of two planes it will always be possible to increase its value by taking a sufficiently small step up the plane it currently lies on, or on to an adjacent plane. The step size was successively halved each time a move failed to increase w (π), and each step was taken from the same point until an increase was achieved. This iteration was then adopted, and the next series commenced. As a result of this procedure a distinction was made between iterations (each improvement in w (π)) and evaluations (each attempt to improve w (π)). In effect this procedure causes the small step size strategy illustrated in Figure 6-16 to take place without the same risk of premature reduction in step size below a useful value. In general, movement will be made along successive rays towards the optimal point, making the accuracy of the overestimate almost totally irrelevant. The system does have one drawback, however; progress will become very slow if at any iteration a ray is approached too closely, since the insistence on an improvement in the dual at each iteration will set an upper limit on the possible step sizes.

Despite this drawback the system did prove to be fairly successful at getting close to the optimum early on, and in later iterations most often an improvement in the dual was achieved every time with only one evaluation.

6.18.2 Increasing the step size

As mentioned in Section 6.18 it could happen that the step size decreased prematurely, causing a repetitive alternation between

Table 6-1

Sub-gradient method - experimental results for one data set

Program: FAST 6 Data set: REAL 3

Estimate: 200 reduced to 45.75 after 16 evaluations

ITERATION	0	1	2	3	4	5	6	7
EVALUATION	0	16	17	18	19	20	21	22
DUAL	25.71	26.15	27.03	27.49	27.72	28.00	28.15	28.28
PRIMAL	25.7	29.9	25.9	27.7	26.0	27.7	27.5	27.3
DEVIATION	47	3	44	5	27	5	14	14

ITERATION	8	9	10	11	12	13	14	15
EVALUATION	23	25	26	28	29	32	33	34
DUAL	28.31	28.42	28.71	28.75	29.90	28.94	29.01	29.14
PRIMAL	27.7	28.2	26.6	27.7	27.5	28.2	27.4	28.4
DEVIATION	21	3	26	5	14	3	10	3

ITERATION	16	17	18	19	20	21	22	23
EVALUATION	35	36	37	38	39	30	41	42
DUAL	29.17	29.29	29.31	29.35	29.37	29.39	29.40	29.43
PRIMAL	28.2	28.4	28.2	28.4	28.2	29.1	28.2	28.9
DEVIATION	3	3	3	3	3	3	3	1

ITERATION	24	25	26	27	28	29	30	31
EVALUATION	43	43	45	40	47	48	49	50
DUAL	29.44	29.46	29.47	29.48	29.49	29.51	29.54	29.60
PRIMAL	28.9	28.9	28.9	28.9	28.9	28.9	28.9	28.9
DEVIATION	1	1	1	1	1	1	1	1

ITERATION	32	33	34	35	36	37	38
EVALUATION	51	52	53	57	60	63	65
DUAL	29.73	29.98	30.03	30.03	30.11	30.18	30.24
PRIMAL	28.9	28.9	30.9	31.3	30.9	28.9	30.2
DEVIATION	1	1	2	4	2	1	0

Table 6-2

Sub-gradient method - experimental results for one data set

Program: FAST 6 Data set: REAL 3

Estimate: Self-generated 279.20 reduced to 46.56 after 18 evaluations

ITERATION	0	1	2	3	4	5	6	7	8
EVALUATION	0	18	19	20	21	22	23	24	25
DUAL	25.71	26.11	27.04	27.49	27.74	28.00	28.16	28.28	28.53
PRIMAL	25.7	29.9	25.9	26.0	27.7	27.5	27.7	27.1	27.7
DEVIATION	47	3	44	5	27	5	14	5	20

ITERATION	9	10	11	12	13	14	15	16	17
EVALUATION	27	28	30	31	34	35	36	37	38
DUAL	28.59	28.75	28.76	28.92	28.96	29.06	29.12	29.20	29.30
PRIMAL	27.7	26.6	28.2	26.6	27.9	27.7	28.4	28.2	28.4
DEVIATION	5	26	3	26	5	8	3	3	3

ITERATION	18	19	20	21	22	23	24	25	26
EVALUATION	39	40	41	42	43	44	45	46	47
DUAL	29.32	29.36	29.39	29.40	29.42	29.44	29.45	29.47	29.48
PRIMAL	28.2	28.4	28.2	29.1	28.2	28.9	28.9	28.9	28.9
DEVIATION	3	3	3	3	3	1	1	1	1

ITERATION	27	28	29	30	31	32	33	34	35
EVALUATION	48	49	50	51	52	53	54	55	60
DUAL	29.48	29.49	29.51	29.54	29.61	29.74	30.00	30.02	30.02
PRIMAL	28.9	28.9	28.9	28.9	28.9	28.9	28.9	30.9	31.3
DEVIATION	1	1	1	1	1	1	1	2	4

ITERATION	36	37	38	39	40
EVALUATION	64	67	71	75	78
DUAL	30.06	30.13	30.21	30.23	30.24
PRIMAL	30.9	30.6	30.9	28.9	30.2
DEVIATION	2	4	2	1	0

two planes with only slow progress being made along the ray of intersection towards the optimum. The system outlined in 6.18.1 is also susceptible to this fault. To counter against this, the program being used for the sub-gradient algorithm was modified to keep a memory of the past few values of the deficit and primal. If these were found to be repeating in cycles then the step size was increased by a factor of at least two with no improvement in the dual being required. This would usually serve to remove the iteration sequence from the rut which it had got into. In practice this was not often needed. Experience seemed to suggest that if a given problem was going to be capable of solution then the optimum would be found after less than 100 and most often less than 50 iterations. If it continued beyond that region it most often converged to a point short of the optimum. As a result it was usually evident after a few iterations whether the program was going to work with a given set of data.

6.19 Practical comparisons between two modified solution methods

Tables 6-1 and 6-2 show the values of dual, primal and deficit at each iteration for two runs of the modified sub-gradient program described above. Note that not every evaluation is listed, only those which led to an improvement in the dual and hence a new iteration number. In both cases the same set of data has been used. The problem has been to allocate 30 girls to 5 wards over 20 weeks with 10 schedules to choose from. The values of Z and the interval at which λ halved are not relevant since the step size alteration procedure outlined in 16.18.2 was used. If the solution lay on the same plane then the step size was doubled at each iteration, as long as the dual was being increased each time. It was then halved

TABLE 6-3

Effects of varying the estimate

Programs: LAG 5 or FAST 5 (algorithms are identical, only printout format differs)

Data set: REAL 3

	REDUCTION	ESTIMATE	ITERATIONS	EVALUATIONS	STANDARD SUB-GRADIENT
1.	0.95	60	58 fail	140 fail	127 fail
2.	0.95	50	19	29	34
3.	0.9	272	20	68	109 fail
4.	0.9	60	29	88	78
5.	0.9	50	17 fail	71 fail	62 fail
6.	0.9	40	16	31	89 fail
7.	0.9	35	35	56	91
8.	0.85	272	22	64	67 fail

at each evaluation which failed to bring an improvement. In the problem shown in Table 6-1 a bad estimate of 200 was inserted for a set of data whose optimum was 30.24. After 16 evaluations of the first iteration the estimate had been reduced to 45.75, and improvement in the dual was obtained, and the algorithm went on to improve the dual in the next seven succesive evaluations, reaching the optimum eventually after 38 iterations and 65 evaluations. In Table 6-2 the program has been instructed to create its own estimate. It has done so by starting with the greatest primal cost possible with the given data set. When this has been reduced after 18 evaluations the final estimate has come down from 279.20 to 46.56. By comparing the deficits and primal values in the two tables it can be seen that many iterations have arrived at the same planes in succession, demonstrating that the solution process has become relatively independant of the estimate size. In each case the reduction in the estimate size has been 10% i.e. the new estimate has been 0.9 of the previous one. Table 6-3 gives the number of iterations needed to solve the same problem with different estimates and reduction factors. It can be seen that the procedure is highly variable in terms of its chances of success and solution time. By comparison the figures for the number of iterations taken by the standard sub-gradient method as described by Held reveal it to be even more unreliable.

6.20 Evaluation of modified gradient technique

Section 6.15.2 described a technique for modifying the gradient direction at each iteration. Table 6-4 shows a series of runs performed with a set of data of the same dimensions as the previous test. With gamma equal to zero, the gradient is unmodified,

TABLE 6-4

Modified gradient direction results

Program: LAG 4OPT Data sets: REALCOST, REAL 3 to REAL 10

GAMMA	AVERAGE ITERATIONS
0	111
-1	106
0.1	109
0.5	106
1.5	110
2.0	115

so this provides a basis for comparison. It would seem that the improvement gained is not great in this case, probably because the unmodified method was already fairly efficient at approaching the optimum. The problem of step size and the accuracy with which the algorithm homes in on the optimum is the critical part of the operation, and this has not been improved by the modification of the gradient.

In order to assess the effectiveness of Grinold's[29] ascent procedure described in 16.5.1 a modification was made to several runs of the program. The direction of steepest ascent is obvious if w (π) lies on one plane; it is only when an edge is encountered that a sub-gradient needs to be considered and a direction of steepest ascent could be of benefit. The program being used was altered so that it printed out a message whenever a tie occurred between two planes. Apart from deliberate trials that message was never printed out on any subsequent run, demonstrating what was intuitively obvious, that the probability of landing on the line of intersection of two planes after a step whose length was based on other factors was extremely low. It would seem in fact that the commonly used term "sub-gradient optimisation" is a misnomer, since the sub-gradients never need to be calculated. If it were possible to move to an intersection point by calculation, as in the Simplex method, then it might be possible to move along the line of intersection, but so far this has not been achieved.

6.21 The concepts of size and looseness - results

As improvements were made to the algorithm, attempts were made to solve larger problems. Experimentation revealed that the tightness or looseness of the data was as important as the size of the problem. If one set of data required every girl for each week of the time under consideration in order to achieve full staffing then the problem was said to be tight. Under these circumstances a solution was rarely found by the sub-gradient method, and took a great number of iterations when it was found. However much larger problems were solved easily when the requirements were loose. The percentage tightness was defined as

$$\frac{\sum_{jt} d_{jt}}{IT} \times 100$$

This definition was the most representative, although others would have been possible. For instance the requirements of one ward might dictate a unique solution in terms of the total number of girls on each schedule even if there were surplus girls each week as far as the requirements of other wards were concerned. Thus it might have been useful to assess how many alternative solutions there were in the case of each set of data and use this as a measure of looseness, but since in many cases it was very hard to determine this, this approach was found to be unsuitable. In each case the data was constructed so that at least one solution was possible. This was done by designing the schedules, allocating an arbitrary number to each so that the total number of girls was correct, and then basing the ward demands on the number supplied by that blend of schedules. Table 6-5 gives an analysis of the results obtained from sets of data with varying numbers of girls, wards, weeks and schedules and within different ranges of tightness. It can be seen that these are very unimpressive. The looser sets of data could be solved trivially by many methods, even manually. The tighter ones are the critical sets, where an algorithm could be of most use if efficient. However this one seems to be of extremely limited usefulness for any

SIZE							TIGHTNESS				
No.	I	J	T	Q	IQ	IJTQ	50%+	60%+	70%+	80%+	90%+
1	30	5	20	20	600	60,000	2/3 = 66·7 23·5 76	0/2	0/1	0/2	0/2
2	30	5	20	10	300	30,000	7/8 = 87·5 13·3 44·1	3/6 = 50 27 81·3	1/7 = 14·3 72 150	0/6	0/6
3	20	5	10	6	120	6,000	1/1 = 100 11 18	2/2 = 100 20·5 34	0/1	0/1	0/1
4	10	5	10	6	60	3,000	4/4 = 100 9·8 16·5	2/3 = 66·7 21 37	1/6 = 16·7 39 93	0/2	0/3
5	8	5	10	5	40	2000	3/3 = 100 6·3 13·7	1/1 = 100 16 33	0/1	0/1	0/1
6	8	5	5	4	32	800	2/2 = 100 11 15·5	2/3 = 66·7 13 16·5	0/2	0/3	1/3 = 33·3 37 96
7	8	3	5	4	32	480	3/3 = 100 5 7·3	3/3 = 100 9·7 18·3	2/4 = 50 14·5 22	1/3 = 33·3 23 57	0/2
8	6	3	4	3	18	216	2/2 = 100 3 7	2/2 = 100 3 9 5	1/1 = 100 9 20	1/2 = 50 11 40	0/2
9	4	2	3	4	16	96	2/2 = 100 2·5 4	3/3 = 100 1·3 7·7	2/2 = 100 4 17·5	2/2 = 100 15·5 31	0/2
10	4	2	4	2	8	64	3/3 = 100 1 4	3/3 = 100 1·3 3·3	3/3 = 100 2·3 8·7	2/3 = 66·7 5·5 12	1/3 = 33·3 9 17

KEY

A/B =	C
D	E

A : Successes

B : Trials

C : % Success rate

D : Average number of iterations

E : Average number of evaluations

Both averages are for successful runs only

TABLE 6 - 5

Overall table of results for subgradient method sets above 60% tightness.

Linear programming might work better on tight programs (perhaps over 80 - 90%) but the drawbacks mentioned in Section 6.3 still apply, and the area within which most sets of data might be expected to fall (60 - 90%) would remain uncovered.

CHAPTER 7

7.1 Introduction

In view of the unsatisfactory results obtained from the use of standard approaches to this type of problem, in particular from subgradient optimisation, it was felt that it might be possible to design an algorithm which could deal with tighter problems more satisfactorily. A heuristic approach was adopted which was tailored to suit the special characteristics of this problem. The task was simplified by omitting the artificial costs and an approach was developed which ensured far better initial solutions. This Chapter describes the algorithm and its implementation.

Its structure in similar to one described by Warner[1] which is a cyclic co-ordinate descent algorithm (explained in Section 7.6 and in Appendix D). In this case, as in his, a set of acceptable schedules exists, and an initial solution is obtained by allocating each nurse to one of the set of schedules according to a system which tends to produce a fairly good staffing arrangement at the start. At each iteration one or more nurses is moved from her present schedule to one which, from explicit calculation, is known to improve the overall solution, or if that is not possible, to improve the staffing on the ward and week with the worst deficit. An iterative system of this sort does not guarantee convergence on the optimal region, but is found in most practical cases to work well. A method of dealing with local optima is described, and it is also demonstrated that this drawback only occurs infrequently. A table of results is given which compare favourably with those obtained for the sub-gradient methods.

7.2 A heuristic approach to the simplified scheduling problem

In practice, as can be seen from the tables of results 6-3 to 6-5, the modified and simple sub-gradient methods took an excessively large number of iterations, most fairly lengthy, to arrive at the optimum point or even one close to it. The results also seemed to be too dependent on an accurate estimate of the value of w*, the maximum value of w. If an underestimate, $\hat{w}$, of w* is used, then one of two events will occur:

(1) The sequence $w(\pi^n)$ will converge to $\hat{w}$. Using Held's choice of step size (7.10), S_n will equal zero when $\hat{w} = w(\pi^n)$, so the iteration will cease.

(2) A point π^n is obtained such that $w(\pi^n) > \hat{w}$. Again the procedure will terminate.

If an overestimate is used then it is very difficult to ensure convergence within the correct range of values. A point $w(\pi^n)$ may be reached which is very close to w*, but the step size is based on the difference between $w(\pi^n)$ and the overestimate, which may still be large. As a result a series of large steps will be made which will cause a hunting or divergent sequence to commence. Thus, with an overestimate, the correct range of values for $w(\pi^n)$ will be found relatively quickly, but convergence, even with reductions in λ, may fail to occur.

It was felt that a heuristic method would be at least as efficient at arriving at a near-optimal point, and might be an improvement. The reasons for this are as follows:

(a) It is possible to devise a method whereby the artificial costs, c_{iq}, inserted merely to perturb the problem and prevent a continuous series of ties between numerous alternatives, could be dropped, so simplifying the problem.

(b) The best starting solution for the sub-gradient method involved putting each girl on to the schedule which incurred the least cost for her; standard procedure for this type of problem, but one which, considering the arbitrary nature of the costs, was unlikely to achieve even an approximation to the best solution in this particular case.

If the artificial costs, c_{iq}, are dropped, the following formulation is arrived at:

$$\min \sum_{jt} \left[D_{jt} - \sum_{q} x_q \, a_{jqt} \right]_+ \qquad (11.1)$$

$$\text{s.t.} \sum_{q} x_q = I \qquad (11.2)$$

$$x_q \geqslant 0 \ \forall \ q \qquad (11.3)$$

$$\text{where} \quad [\alpha]_+ = \alpha \text{ if } \alpha > 0$$
$$= 0 \text{ if } \alpha \leqslant 0$$

x_q = the number of girls on schedule q (Integer $\forall$ q)

I = the number of girls available

D_{jt} = the demand for girls on ward j in week t

a_{jqt} = 1 if schedule q puts a girl on ward j in week t; 0 otherwise.

It can be seen that in this formulation the simplification arises because no distinction is made between different girls. This distrinction was only imposed partly arbitrarily in (1); x_q in (9) is equal to $\sum_i x_{iq}$ in (1) for all q.

In this case, the objective function seeks to eliminate all shortages, paying no regard to overstaffing. Overstaffing was considered during the development of the algorithm; although it would be possible to try to minimise overstaffing as well as understaffing, perhaps with a lesser weighting attached, such an endeavour would be unproductive, since the overall balance of the staffing availabilities and requirements within the problem ensures that any algorithm which minimises one will automatically minimise the other.

With regard to (b), the starting solution in this case is just as obvious, but is much more likely to be close to the eventual optimum i.e. start with a 'roughly' even number of girls on each schedule. If the schedules are complementary to each other then in many cases the starting solution will be an optimum solution.

7.3 The heuristic algorithm

The simplest method of trying to improve on the starting solution is to move one girl off her present schedule and put her on another which improves the solution. This can be done according to a number of different criteria.

Let q^- be the schedule from which the girl is going to be removed and q^+ be the one on which she will be placed. Let $\partial q^- q^+_{jt} = 1$ if a move from q^- to q^+ increases the number of girls on ward j in week t, -1 if it reduces it, and 0 if it leaves it the same.

$\partial q^- q^+_{jt}$ will be 1 or -1 if one or other schedule has an $a_{jqt} = 1$ for that j and t, or 0 if neither or both has.

The following criteria were considered:

Select the change which makes the greatest reduction in overall costs, the costs being defined as:

(1) The sum of the +ve shortages over all D_{jt}'s.

(2) The number of D_{jt}'s with a +ve shortage.

(3) The sum of the +ve shortages plus half the sum of the -ve shortages (overstaffings).

(4) The sum of all +ve and -ve shortages i.e. all deviations from the D_{jt}'s.

(5) The sum of the squares of the +ve shortages for all D_{jt}'s.

All of the above criteria were found to be very demanding in terms of required computational time, for the following reason: $\partial q^- q^+_{jt}$ has to be worked out for every permitted q^- and q^+, e.g. for every q^- where $x_q > 0$, related to every other q. In the usual case where all $x_q > 0$ this involves considering $Q \times (Q - 1)$ changes, where Q is the number of permitted schedules. Since this is of order Q^2 it can be seen that the computation required increases as the square of the number of available schedules. This imposes a drastic computational penalty on the type of problem often encountered in a real-life situation.

Despite this drawback, all the above methods were considered, and the problem resolved itself as will be explained below. The other main shortcoming was that even the better of the above approaches tended to arrive very rapidly at local optima, this being an accepted pitfall of this sort of combinatorial approach.

In an attempt to resolve these local optima a system of progressive weightings was introduced which related to evaluation of individual wards and weeks. When assessing potential changes according to the criteria above the amount of over- or under-staffing on each ward and week would be taken into account. It was felt that those D_{jt}'s which were consistently not met could be singled out by comparing a weighted average of staffing figures rather than the figures which applied only to the iteration in question. It was hoped that the weightings could be adjusted so that a persistent offender with a small current deviation from the ideal staffing level would take precedence over a case which was only temporarily badly staffed.

It was intended that temporary stalemates between two schedules would be resolved by this procedure, but in practice the method proved to be unsatisfactory. Local minima still occured which involved two or more alternative schedules, and the algorithm caused the solution to cycle between them. Rather than gradually giving more and more weight to the wards which were being neglected by the cycling schedules, the weights merely accentuated the shortages which were already determining the cycling behaviour.

In order to resolve this problem, and simultaneously to reduce the computational task, an attempt was made to analyse the sort of heuristic approach which one would adopt if one was to try to solve a problem manually.

7.4 Comparison with manual methods

A common procedure can be stated as follows:

(1) Scan through schedules to see if any allocates uniquely to a given $_j$ and $_t$. This is to say, if D_{jt} is > 0 for any $_j$ and $_t$, and only one schedule q allocates to that $_j$ for that week $_t$, then the minimum number of girls on that schedule to permit a "feasible" solution (where min $\sum_{jt} \left[D_{jt} - \sum_q x_q . a_{jqt} \right]_+ = 0$) must be D_{jt}. That is to say,

$$x_{q*} \geqslant D_{jt} \text{ if } \sum_{q} a_{jqt} = 1 \text{ and } a_{jq*t} = 1, \ \forall\, j, t. \qquad (11.4)$$

$$\text{Alternatively, } \min x_{q*} = D_{jt} \text{ if } \sum_{q} a_{jqt} = 1 \text{ and } a_{jq*t} = 1$$

$$= 0 \text{ otherwise} \qquad , \forall\, j, t.$$

From this, if $\sum_{q} \min x_{q} > I$, then the problem is infeasible. More obviously, if, for any j and t, $D_{jt} > 0$ and $\sum_{q} a_{jqt} = 0$ then the problem is also infeasible. These operations are simple to perform, and as well as checking for two conditions for infeasibility, they will also, in any cases where there are unique a_{jqt}'s, reduce the size of the problem. The minimum number required is pre-allocated to each appropriate schedule, and the D_{jt}'s and total girls available, I, are reduced accordingly. In practice this occurs quite frequently, and the resulting simplification of the problem is considerable.

(2) Adopt a starting solution, let us say by placing as near an equal number as possible of girls on each schedule.

(3) Total up the shortages relating to each D_{jt}. If total = 0 then stop.

(4) Look to see where the greatest shortage occurs.

(5) Select a schedule that will alleviate this shortage, and put an extra girl on it, taking her from one which seems to have the greatest number of allocations to weeks and wards which do not have shortages.

(6) Return to (3).

Although this set of heuristics is crude, it has an approach which characterises human solution methods; that is to say that it relies more on locally obvious improvements than on an exhaustive analysis of all possible solutions. In this case, some computation is needed to define the new shortages, but having got these, the normal approach would seem to be to eliminate the greatest shortage rather than to make a move which causes the best overall improvement. The advantage of this strategy is that the number of changes which has to be assessed is drastically reduced. There may be only two schedules which have an $a_{jqt} = 1$ for the j and t with the greatest shortage - there cannot be only one or the D_{jt} would already have been satisfied in step (1), but also there cannot be many or that D_{jt} would not be associated with the largest deficit.

Thus the potential schedules for q^+ will be small in number, and commonly one would be chosen at random.

To find q^-, one would tend to scan the other schedules where $x_q > 0$ to find the one whose staffing pattern had the least correspondence with the wards and weeks with shortages. In other words, computation would be minimised, and selection would be done instead on the basis of pattern matching.

Obviously if some of these heuristics are being adopted as part of an algorithm for computer use, one can afford to be slightly more exhaustive in terms of the arithmetic required to determine which q^- could best be used, with minimum disruption of the existing solution, but the same basic approach will greatly reduce the amount of computation required.

7.5 An algorithm based on the manual method

With this in mind, a program was designed to adopt the following rules for swopping:

(1) Find the maximum shortage on any j and t. Label its location j*, t*. If maximum shortage is 0 then stop.

(2) Identify the schedules where $a_{j^*qt^*} = 1$. Try each in turn as

permissible q^+.

(3) Consider all permissible $\partial q^- q^+_{jt}$, totalling in each case: The sum of all +ve and -ve changes made to existing deficits where

(a) the deficit before the change was +ve

or (b) the deficit after the change was +ve. See below for explanation.

(4) Select the $\partial q^- q^+_{jt}$ which causes the greatest overall reduction in deficit i.e. where the sum of the changes totalled according to the conditions in (3) was the most negative or least in value.

(5) Reduce x_{q^-} by one, and increase x_{q^+} by one. Return to (1).

The following examples should clarify rule (3):

If a given deficit before a potential change is 5, and after it is 4, then the value of the change is -1.

If it is changed from 1 to 0 then the change of deficit = -1
" " " " " 0 to -1 " " " " " = 0
" " " " " -1 to 0 " " " " " = 0
" " " " " 0 to 1 " " " " " = +1
" " " " " 2 to 3 " " " " " = +1
" " " " " -6 to -5 " " " " " = 0

In other words any change which increases or decreases the total of +ve deficits is counted; a change which only affects the amount of overstaffing on a given ward and week is ignored.

In brief the strategy can be described thus:

Find the change which will cause an improvement in the maximum deficit. If there is more than one candidate, choose the one which causes the greatest overall improvement in staffing deficits, or the least deterioration.

7.6 Initial assessment

Three methods were used, within this overall strategy, with regard to the number of changes made at each iteration. In order to give the algorithm its most testing conditions for operation, sets of data were constructed with known feasible solutions, usually requiring the placing of an equal number of girls on each schedule, and the problem was commenced with all the girls on one schedule, one of the worst possible solutions. Three rules tried were as follows:-

(1) Move one girl from q^- to q^+ at each iteration.

(2) Move from q^- to q^+ half the number of girls presently on q^- (rounded down in the case of odd numbers).

(3) Move from q^- to q^+ a number of girls equal to the identified maximum shortage on any j and t.

Method of (1) is going to be capable of homing in on a specific solution more accurately than (2) and (3), since in a near-optimal situation, (2) and (3) are likely to move too many girls at a time, possibly swapping back and forwards past the optimal solution. (2) will be much worse than (3) in this respect, but there are circumstances in which (3) will move too many as well. On the other hand, method (1) is a priori going to require many more iterations to reach the optimum point. With I girls and Q schedules, starting with all of the girls on the first schedule, and with the solution requiring an equal number of girls on all schedules, the minimum number of iterations will be equal to $I - I/Q$.

With both methods (2) and (3) the minimum will be $Q - 1$, but this minimum is far more likely to be achieved with (3) than with (2), since it is sensitive to specific maximum shortages.

With these points in mind it seemed advisable to employ a mixed strategy, using method (3) until the maximum shortage dropped below a set level, then, having got near to the optimum, changing to method (1). The level which turned out to be most suitable was a maximum shortage of three; on arriving at this the change was made to method (1). (This level would change for different problem sizes). Using method (1) gives an algorithm which bears similarities to a method proposed by Warner[2], described in Appendix D. The problem which he considers only bears superficial similarities to this one, since they are not having to meet requirements for a number of different wards; they are only concerned with whether a girl is on or off duty during a specific time period, in their case a day rather than a week. This makes their problem considerably simpler, and allows them to consider individual nurse preferences.

Initial results were extremely promising - the algorithm was able to solve much larger problems than the sub-gradient method, and in the case of data which both could cope with, it solved the problem much faster. For an analysis of results see Table 7-9.

7.7 An analysis of the problem structure

7.7.1 Geometric analysis - introduction

Although it is possible to analyse the structure of this type of problem in different ways there are various characteristics of the type which lend themselves to geometrical analysis. It was seen in Section 6.12 that both the one- and two-dimensional cases of the sub-gradient method could be represented on paper. However they failed in most cases to illustrate the drawbacks which occur in multi-dimensional cases. It will be explained below that a constraint on the value of the sum of all co-ordinates permits us to map the feasible region of a three-dimensional problem in two dimensions. Similarly a three dimensional model may be built to represent the feasible region of a four-dimensional case. The values of the objective function may be plotted on the diagram or model, along with figures for the maximum shortage on any given ward and week, for each feasible point. When this is done, relationships appear which were not otherwise apparent, permitting an extra insight to be gained, not just into the iteration procedures which would produce the fastest convergence to the optimal region, but also the nature of cases which might be awkward to solve. The geometrical approach promoted new ideas for algorithms which could then be tested on multi-dimensional cases. If a new procedure had an unexpected effect it was possible to return to the three or four-dimensional cases to determine the reasons.

7.7.2 Geometric analysis - explanation

In the case of a problem with two schedules, the set of possible solutions can be shown as in Figure 7-1.

They will range from I girls on schedule 1 and none on schedule 2 to the opposite extreme. It must be noted that only integer points on the line indicated are feasible. If a third schedule is introduced, a three dimensional representation of the problem can be drawn. See Figure 7-2.

At all integer points on the plane indicated (that which passes through the points I, 0, 0 ; 0, I, 0 ; 0, 0, I) the sum of the values of q_1,q_2 and q_3 will be equal to I, therefore each such point will be a feasible solution. The algorithm swops from one point to an adjacent one at each iteration. Since the sum of the three co-ordinates is constrained to a constant for any point on the plane,

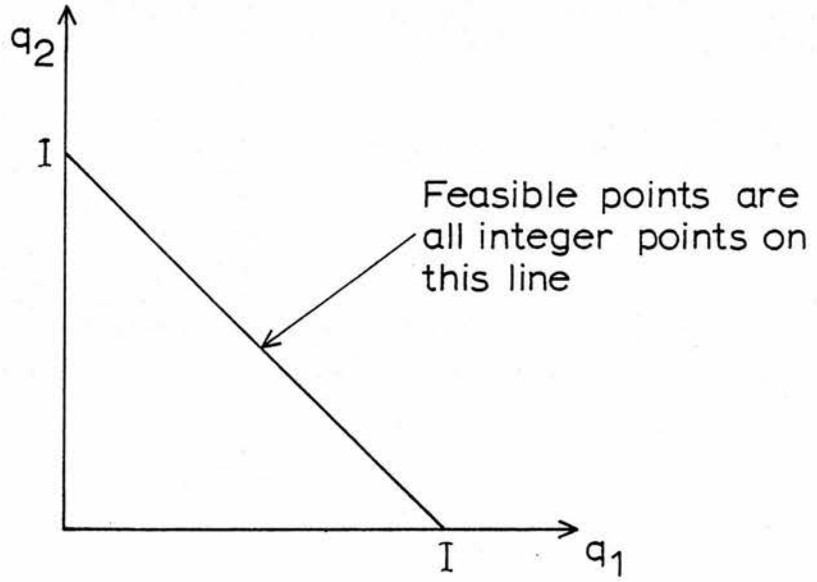


FIGURE 7-1

Set of feasible solutions in two-dimensional scheduling problem

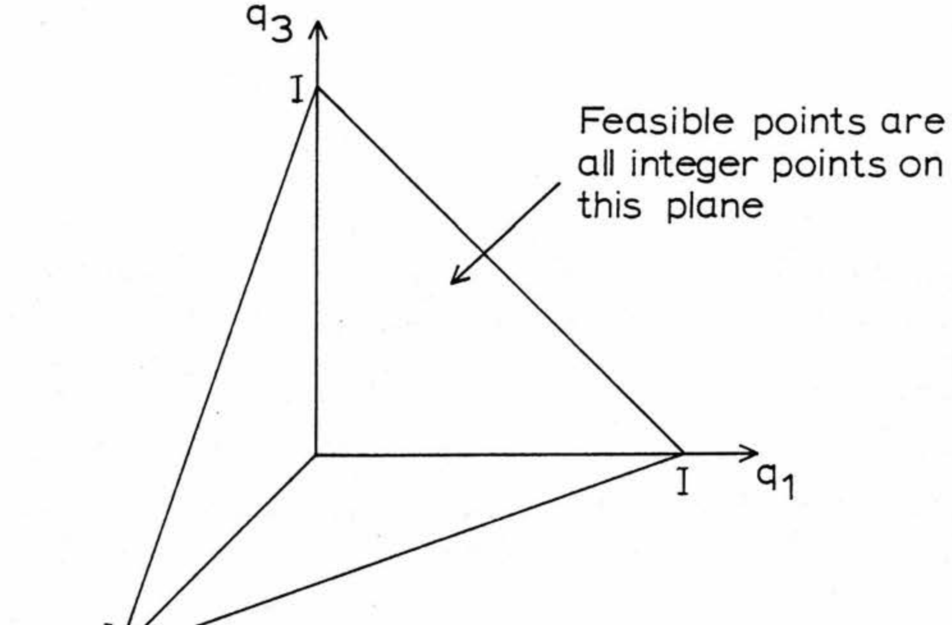


FIGURE 7-2

Set of feasible solutions in three-dimensional scheduling problem

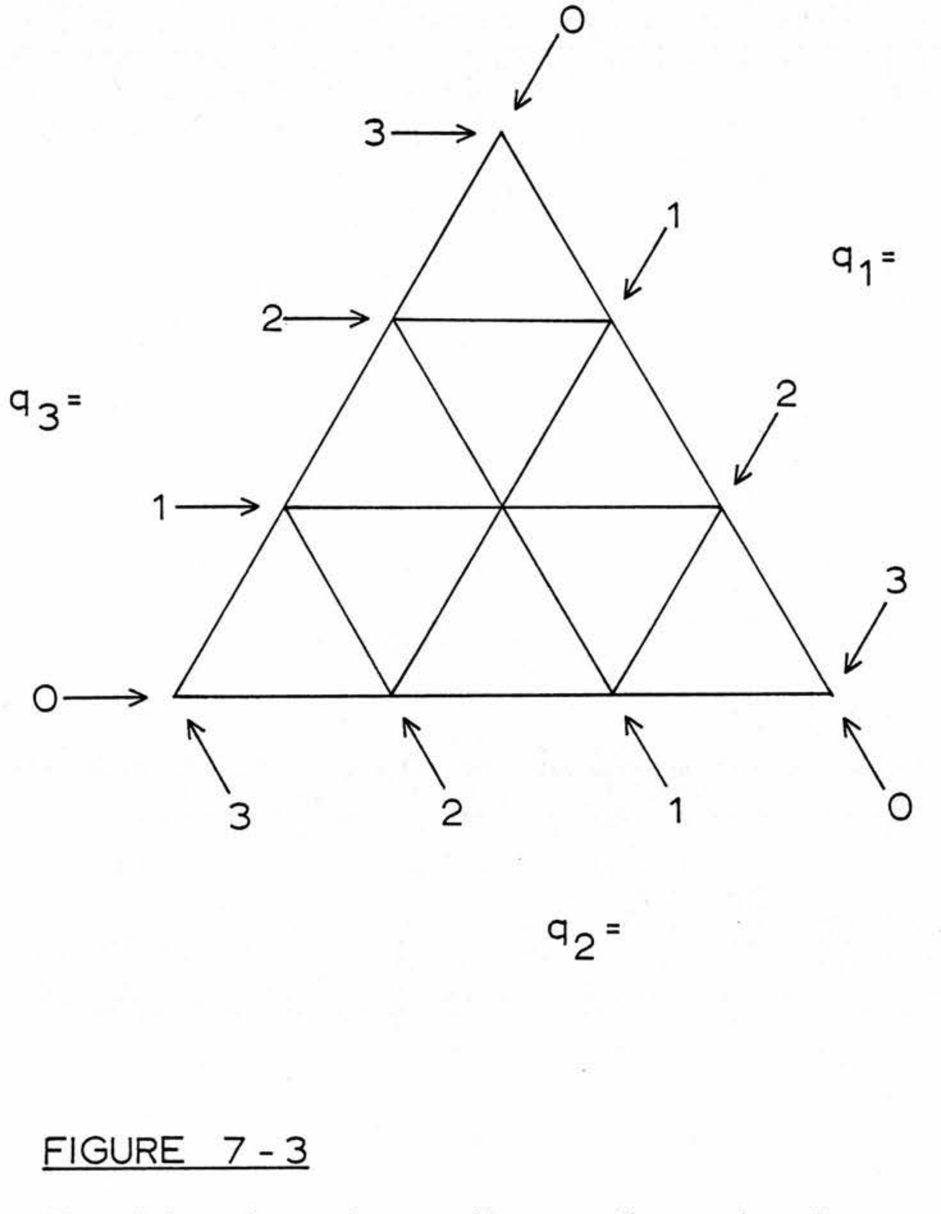


FIGURE 7-3

Feasible plane for a three-dimensional scheduling problem it is possible to represent the three-dimensional case in two dimensions, by drawing the feasible plane and mapping on it the positions of the integer points. For the problem size Q = 3 and I = 3 the plane would look like Figure 7-3.

It can be seen that there are ten feasible points. The lines represent moves which can be made by taking a girl off one schedule and putting her on another. Thus it is possible to move from 1, 0, 2 to 1, 1, 1 in one move, but not from 1, 0, 2 to 1, 2, 0 or to 2, 1, 0.

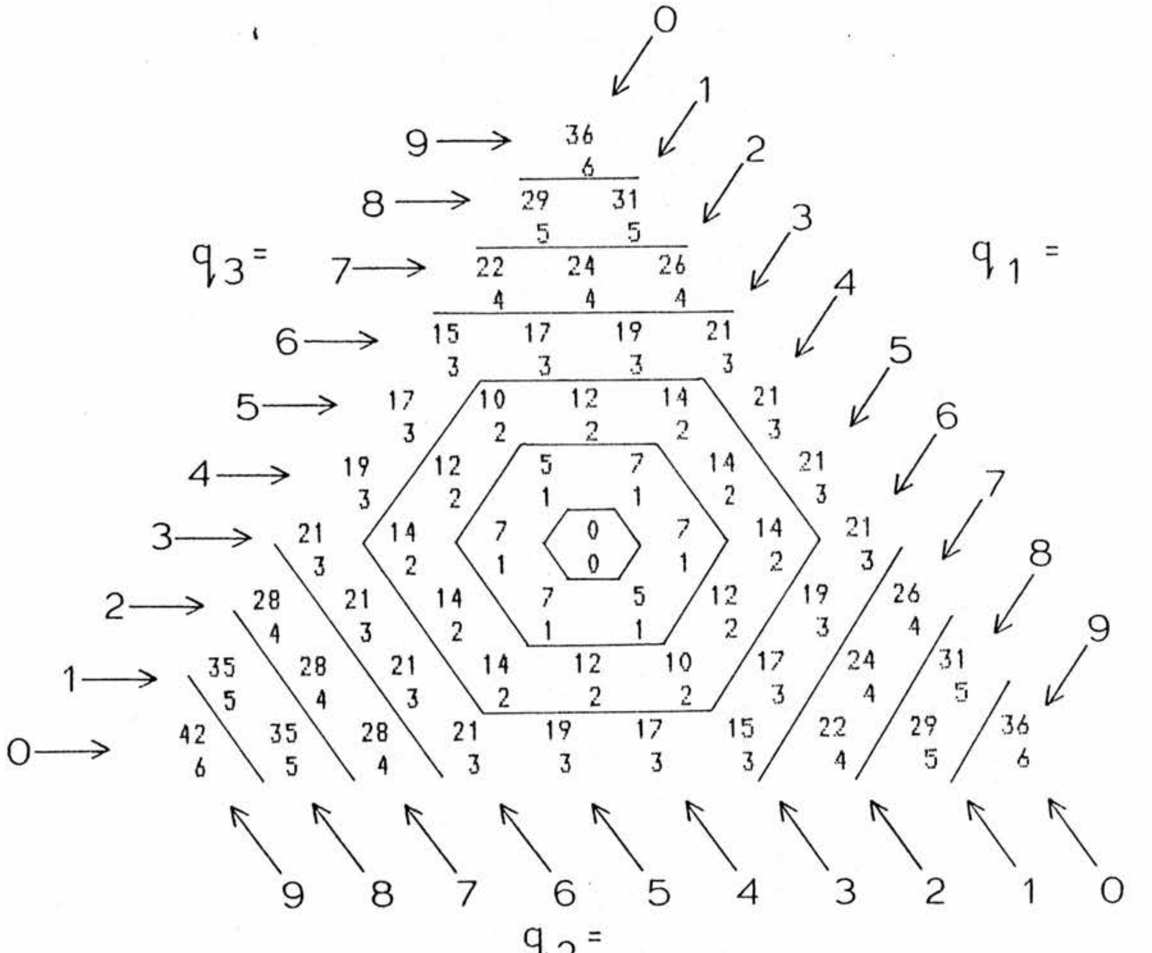
7.7.3 Analysis of some typical cases

Figures 7-4 to 7-11 give examples of a number of sets of data with the problem analysis which results from them. At each point, two figures are given: the upper is the total shortage at that point, the lower is the maximum shortage. The contour lines indicated separate groups of points with the same maximum deficit, and it can be seen that, in the three-dimensional case at least, changes from point to point according to a reduction in the maximum gradient will cause a very efficient arrival at the optimum point or points. In the cases shown here, a reduction in the total sum of shortages is also effected at each iteration, but as will be seen later, this is not always the case with problems of more than three dimensions. Note also that a series of iterations according to greatest reduction in overall shortage will often take a different route from a series based on reduction of the maximum shortage, with overall deficits being the secondary criterion. In Figure 7-6 for instance, starting at the point 0, 0, 9 (the upper point of the triangular plane in the diagram) the latter rule will move through the solution points shown below:

```
GIRLS:      9
WARDS:      4
WEEKS:      8
SCHEDULES: 3


DEMAND:
          1 2 3 4 5 6 7 8 : WEEKS

WARD 1 - 3 3 0 3 6 0 0 3
WARD 2 - 0 0 6 6 0 0 3 3
WARD 3 - 3 3 3 0 3 6 0 0
WARD 4 - 3 3 0 0 0 3 6 3

                 WARDS
                 -----
SCHEDULE 1 - 1 1 2 2 3 3 4 4
SCHEDULE 2 - 4 4 3 1 1 3 2 2
SCHEDULE 3 - 3 3 2 2 1 4 4 1
```

FIGURE 7-4

Data set and analysis of its feasible plane



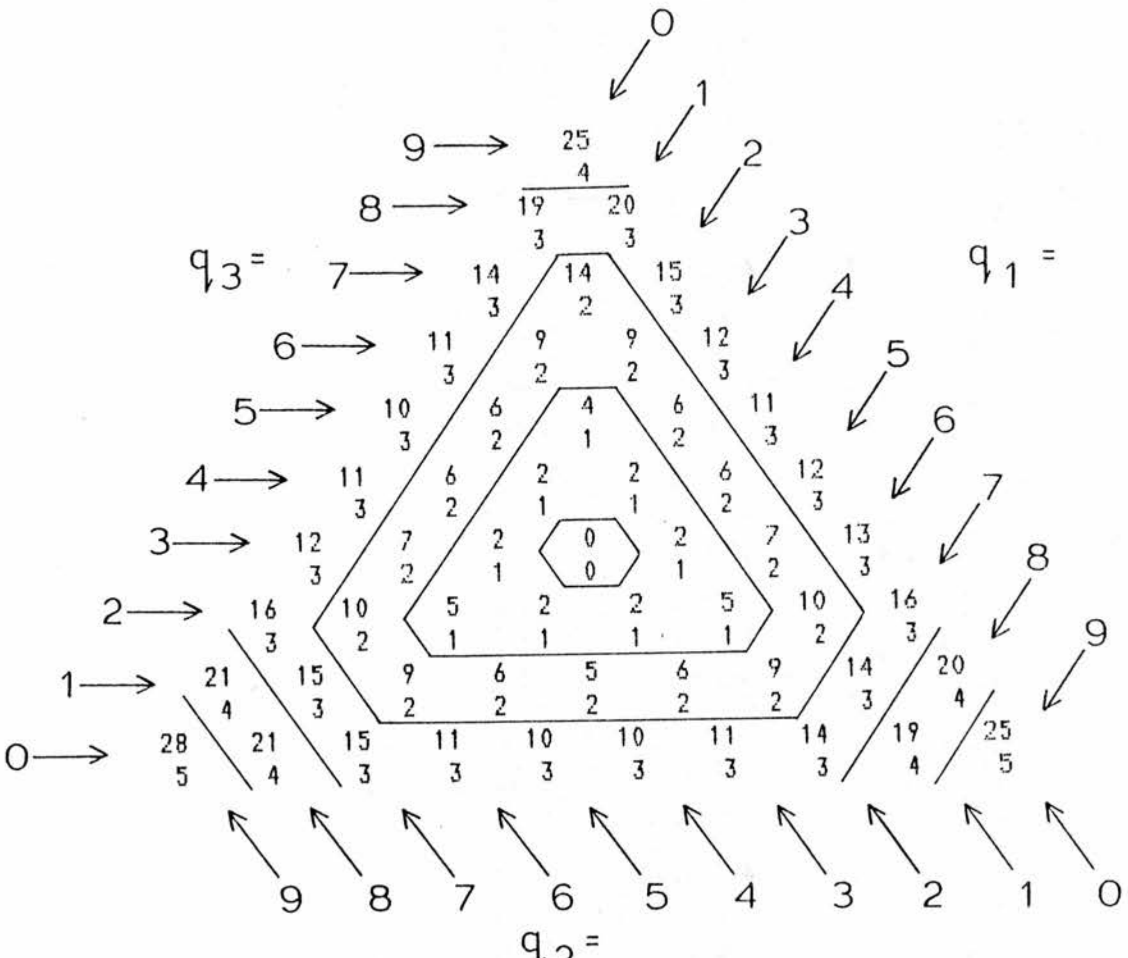
The upper figure of each pair is the total deficit at that solution point - the lower is the maximum for any single ward and week

```
GIRLS:      9
WARDS:      4
WEEKS:      8
SCHEDULES:  3


DEMAND:
           1 2 3 4 5 6 7 8 : WEEKS

WARD 1 - 2 2 0 0 5 0 0 1
WARD 2 - 0 0 1 3 0 0 2 3
WARD 3 - 3 3 1 0 3 4 0 0
WARD 4 - 2 3 0 0 0 2 5 3

                 WARDS
                 -----
SCHEDULE 1 - 1 1 2 2 3 3 4 4
SCHEDULE 2 - 4 4 3 1 1 3 2 2
SCHEDULE 3 - 3 3 2 2 1 4 4 1
```

The upper figure of each pair is the total deficit at that solution point - the lower is the maximum for any single ward and week

FIGURE 7-5

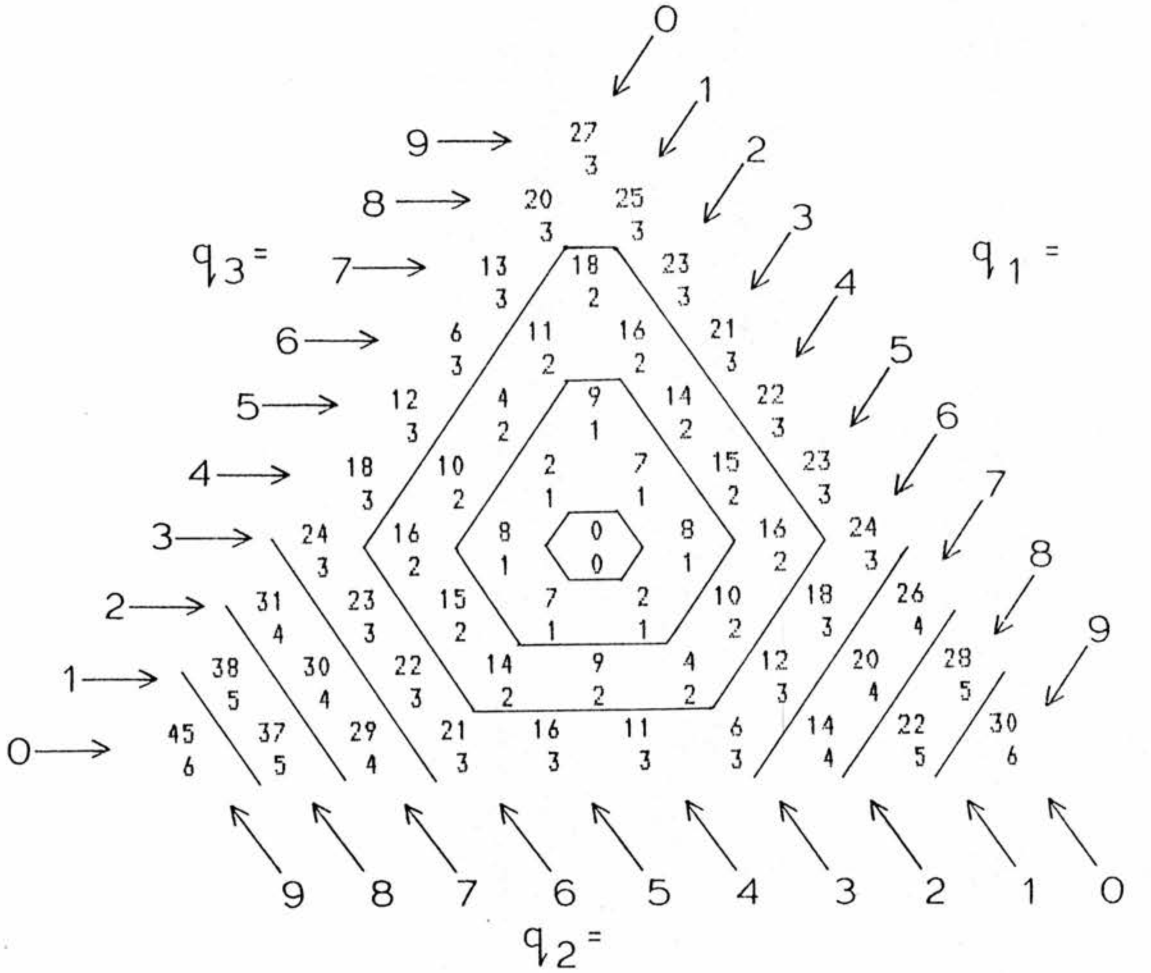
Data set and analysis of its feasible plane



```
GIRLS:      9
WARDS:      4
WEEKS:      8
SCHEDULES:  3


DEMAND:
           1 2 3 4 5 6 7 8 : WEEKS

WARD 1 - 0 0 0 3 6 0 3 6
WARD 2 - 0 0 6 6 0 0 3 3
WARD 3 - 6 6 3 0 0 3 0 0
WARD 4 - 3 3 0 0 3 6 3 0

                 WARDS
                 -----
SCHEDULE 1 - 3 3 2 2 4 4 1 1
SCHEDULE 2 - 4 4 3 1 1 3 2 2
SCHEDULE 3 - 3 3 2 2 1 4 4 1
```

FIGURE 7-6

Data set and analysis of its feasible plane



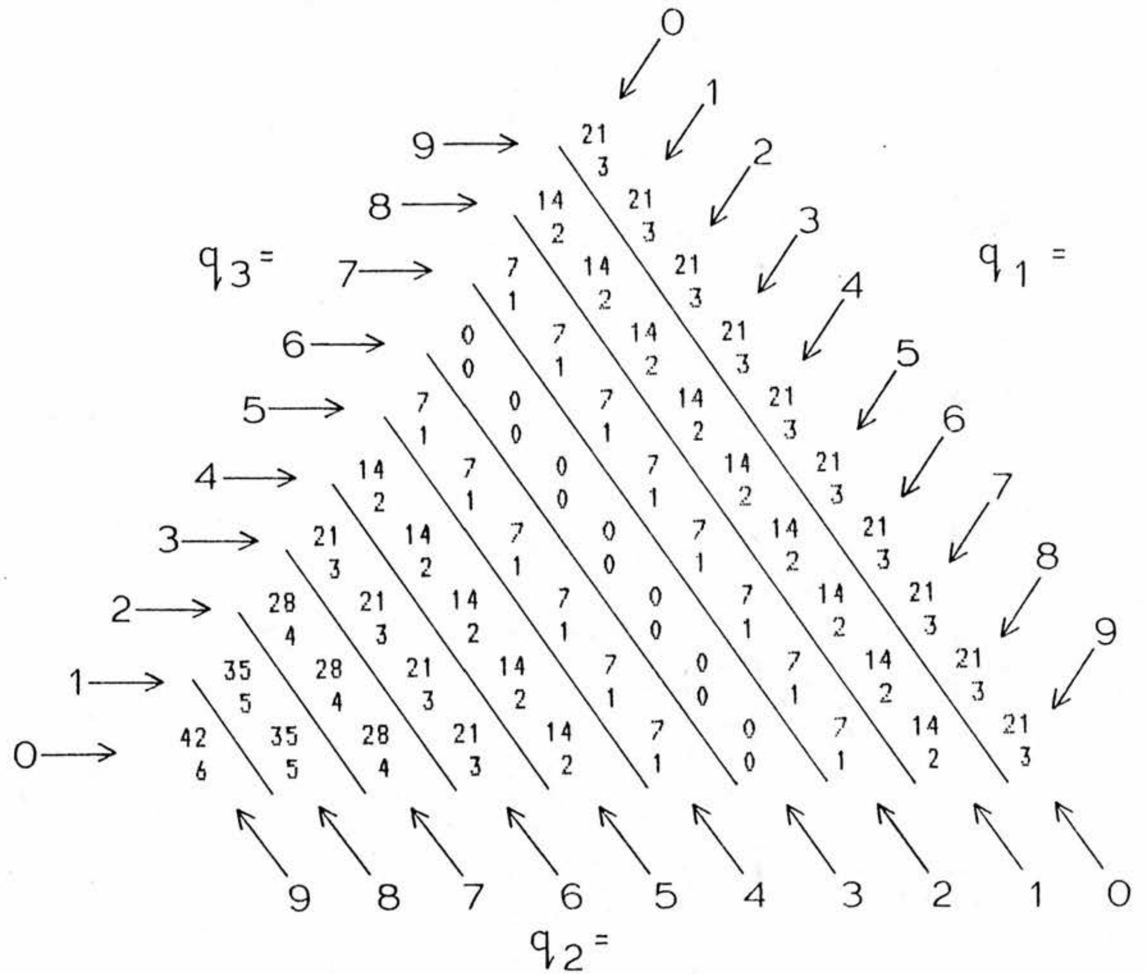
The upper figure of each pair is the total deficit at that solution point - the lower is the maximum for any single ward and week

```
GIRLS:     9
WARDS:     4
WEEKS:     8
SCHEDULES: 3


DEMAND:
         1 2 3 4 5 6 7 8 : WEEKS

WARD 1 - 0 0 0 3 9 0 0 6
WARD 2 - 0 0 6 6 0 0 3 3
WARD 3 - 6 6 3 0 0 3 0 0
WARD 4 - 3 3 0 0 0 6 6 0

                  WARDS
                  -----
SCHEDULE 1 - 3 3 2 2 1 4 4 1
SCHEDULE 2 - 4 4 3 1 1 3 2 2
SCHEDULE 3 - 3 3 2 2 1 4 4 1
```

The upper figure of each pair is the total deficit at that solution point - the lower is the maximum for any single ward and week

FIGURE 7-7

Data set and analysis of its feasible plane

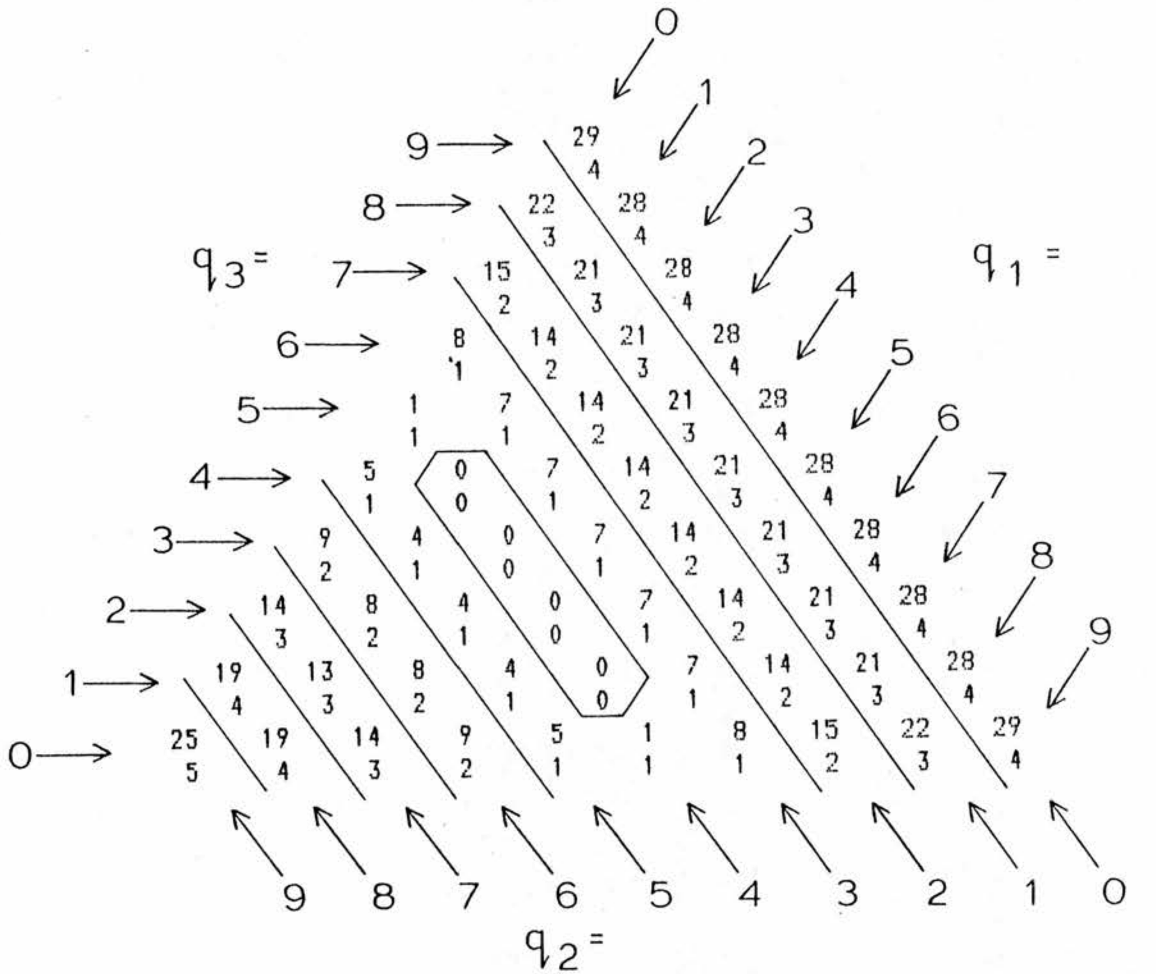


```
GIRLS:      9
WARDS:      4
WEEKS:      8
SCHEDULES: 3


DEMAND:
         1 2 3 4 5 6 7 8 : WEEKS

WARD 1 - 0 0 0 4 9 0 0 0
WARD 2 - 0 0 5 5 0 0 4 4
WARD 3 - 5 5 4 0 0 4 0 0
WARD 4 - 4 4 0 0 0 1 3 1

                 WARDS
                 -----
SCHEDULE 1 - 3 3 2 2 1 4 4 1
SCHEDULE 2 - 4 4 3 1 1 3 2 2
SCHEDULE 3 - 3 3 2 2 1 1 4 4
```

The upper figure of each pair is the total deficit at that solution point - the lower is the maximum for any single ward and week

FIGURE 7-8

Data set and analysis of its feasible plane

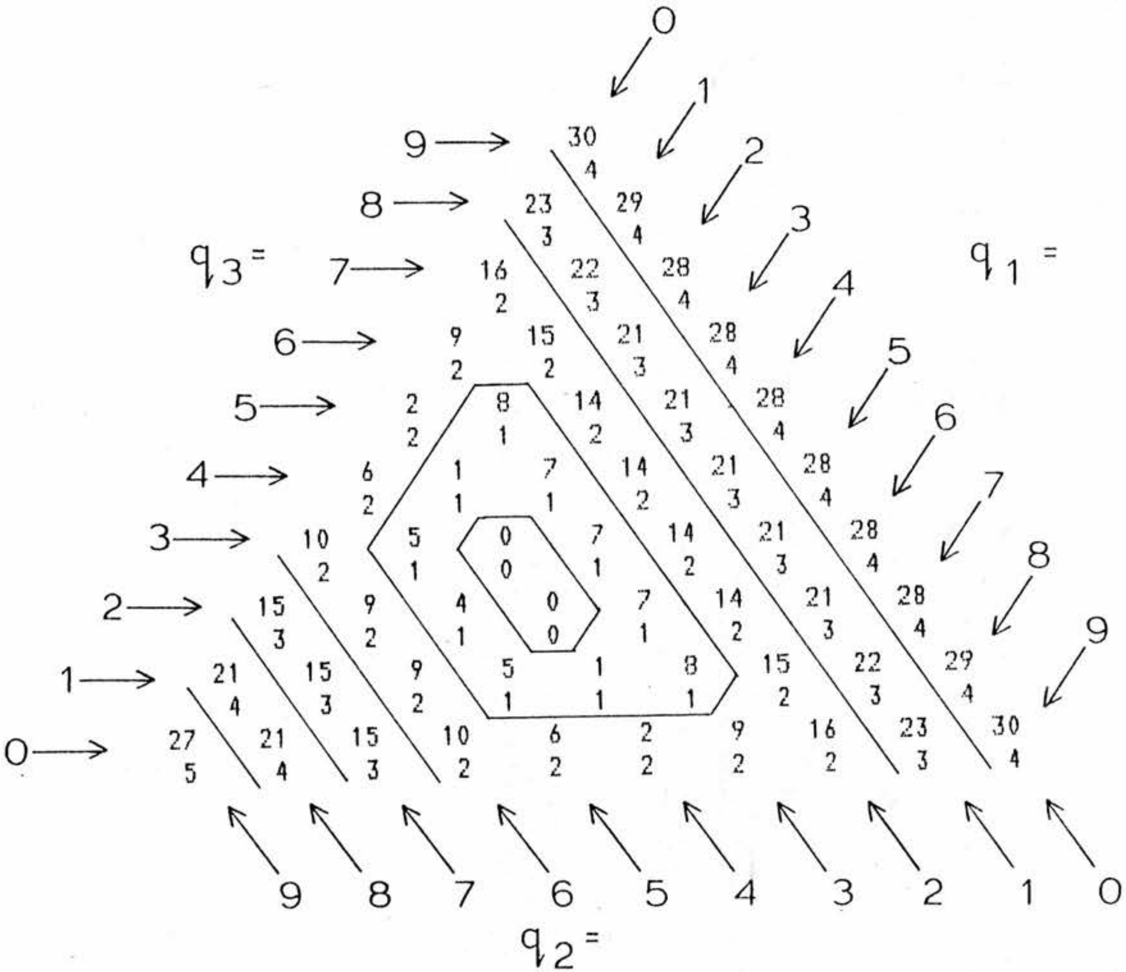


```
GIRLS:     9
WARDS:     4
WEEKS:     8
SCHEDULES: 3


DEMAND:
         1 2 3 4 5 6 7 8 : WEEKS

WARD 1 - 0 0 0 4 9 0 0 0
WARD 2 - 0 0 5 5 0 0 4 4
WARD 3 - 5 5 4 0 0 4 0 0
WARD 4 - 4 4 0 0 0 2 3 2

                  WARDS
                  -----
SCHEDULE 1 - 3 3 2 2 1 4 4 1
SCHEDULE 2 - 4 4 3 1 1 3 2 2
SCHEDULE 3 - 3 3 2 2 1 1 4 4
```

The upper figure of each pair is the total deficit at that solution point - the lower is the maximum for any single ward and week

FIGURE 7 - 9

Data set and analysis of its feasible plane

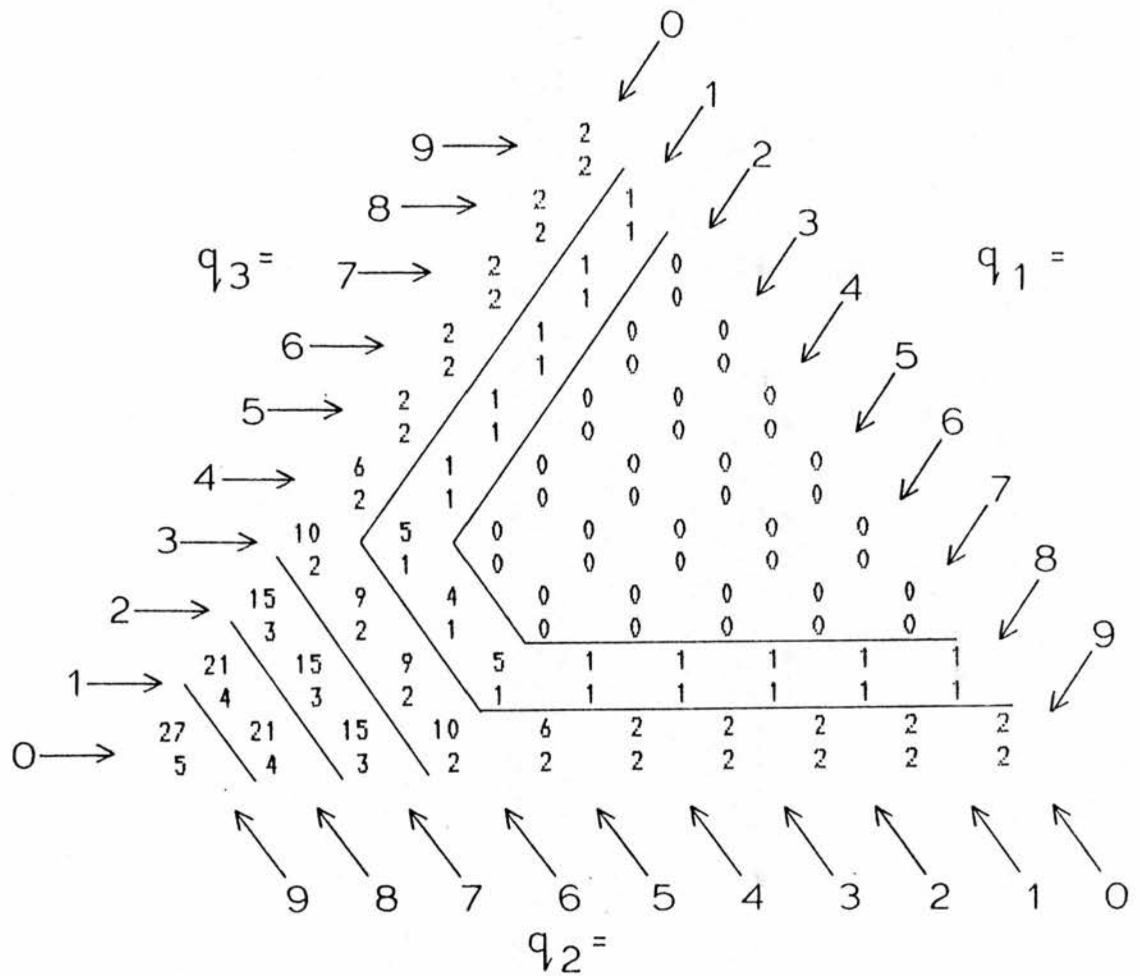


```
GIRLS:     9
WARDS:     4
WEEKS:     8
SCHEDULES: 3


DEMAND:
           1 2 3 4 5 6 7 8 : WEEKS

WARD 1 - 0 0 0 0 9 0 0 0
WARD 2 - 0 0 5 5 0 0 0 0
WARD 3 - 5 5 0 0 0 0 0 0
WARD 4 - 0 0 0 0 0 2 3 2

                   WARDS
                   -----
SCHEDULE 1 - 3 3 2 2 1 4 4 1
SCHEDULE 2 - 4 4 3 1 1 3 2 2
SCHEDULE 3 - 3 3 2 2 1 1 4 4
```

The upper figure of each pair is the total deficit at that solution point - the lower is the maximum for any single ward and week

FIGURE 7-10

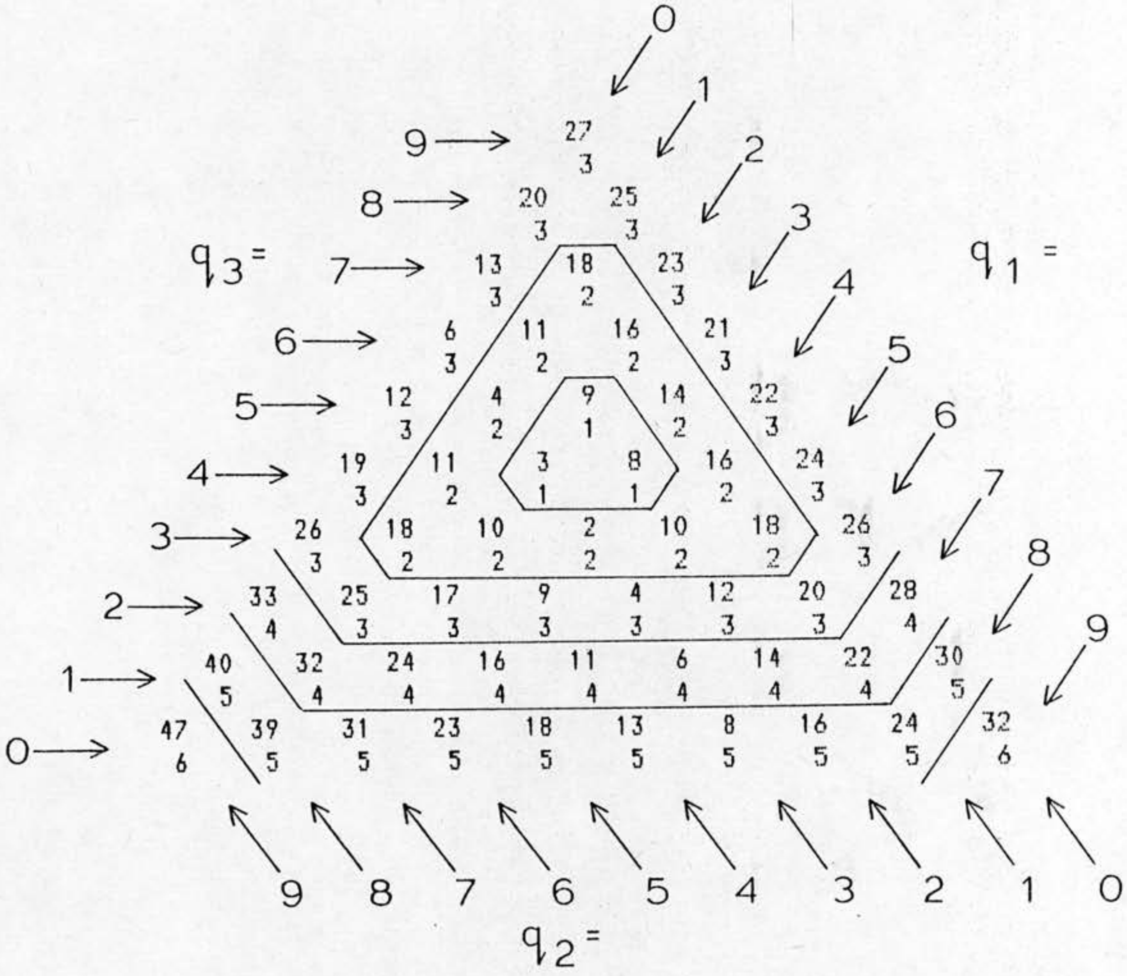
Data set and analysis of its feasible plane



```
GIRLS:      9
WARDS:      4
WEEKS:      8
SCHEDULES: 3


DEMAND:
        1 2 3 4 5 6 7 8 : WEEKS

WARD 1 - 0 0 0 3 6 0 3 6
WARD 2 - 0 0 6 6 0 0 3 3
WARD 3 - 6 6 3 0 0 3 0 0
WARD 4 - 3 3 0 0 3 6 5 0

                 WARDS
                 -----
SCHEDULE 1 - 3 3 2 2 4 4 1 1
SCHEDULE 2 - 4 4 3 1 1 3 2 2
SCHEDULE 3 - 3 3 2 2 1 4 4 1
```

FIGURE 7-11

Data set and analysis of its feasible plane



The upper figure of
each pair is the total
deficit at that solution
point - the lower is
the maximum for any
single ward and week

Co-ordinates :	0,0,9	0,1,8	1,1,7	1,2,6	2,2,5	2,3,4	3,3,3
Total deficit :	27	20	18	11	9	2	0
Max deficit :	3	3	2	2	1	1	0

Table 7-1

whereas the former will take this route:

Co-ordinates :	0,0,9	0,1,8	0,2,7	0,3,6	1,3,5	2,3,4	3,3,3
Total deficit :	27	20	13	6	4	2	0
Max deficit :	3	3	3	3	2	1	0

Table 7-2

The number of iterations is the same in both cases.

Analysis of this kind also reveals the reasons for the relative efficiencies of the three rules concerning the number of girls to be moved at each iteration. Consider Figure 7-12 starting at the point 9,0,0 (the bottom right hand corner). Taking first the least efficient rule, where half the number of girls on q^- (rounded down in the case of an odd number) are moved to q^+. The sequence of iterations is this:

Co-ordinates :	9,0,0	5,4,0	5,2,2	3,4,2	3,2,4	
Total deficit :	36	17	12	7	7	. . .
Max deficit :	6	3	2	1	1	

Table 7-3

These moves are shown on Figure 7-13 which represents the same problem. The solution then swops indefinitely from point 3,2,4 to 3,4,2 and back, failing to arrive at the optimum point.

The one swop at a time rule gives us this:

Co-ordinates :	9,0,0	8,1,0	7,2,0	6,3,0	5,3,1	4,3,2	3,3,3
Total deficit :	36	29	22	15	10	5	0
Max deficit :	6	5	4	3	2	1	0

Table 7-4

```
GIRLS:      9
WARDS:      4
WEEKS:      8
SCHEDULES: 3


DEMAND:
         1 2 3 4 5 6 7 8 : WEEKS

WARD 1 - 3 3 0 3 6 0 0 3
WARD 2 - 0 0 6 6 0 0 3 3
WARD 3 - 3 3 3 0 3 6 0 0
WARD 4 - 3 3 0 0 0 3 6 3

                  WARDS
                  -----
SCHEDULE 1 - 1 1 2 2 3 3 4 4
SCHEDULE 2 - 4 4 3 1 1 3 2 2
SCHEDULE 3 - 3 3 2 2 1 4 4 1
```

The upper figure of each pair is the total deficit at that solution point - the lower is the maximum for any single ward and week

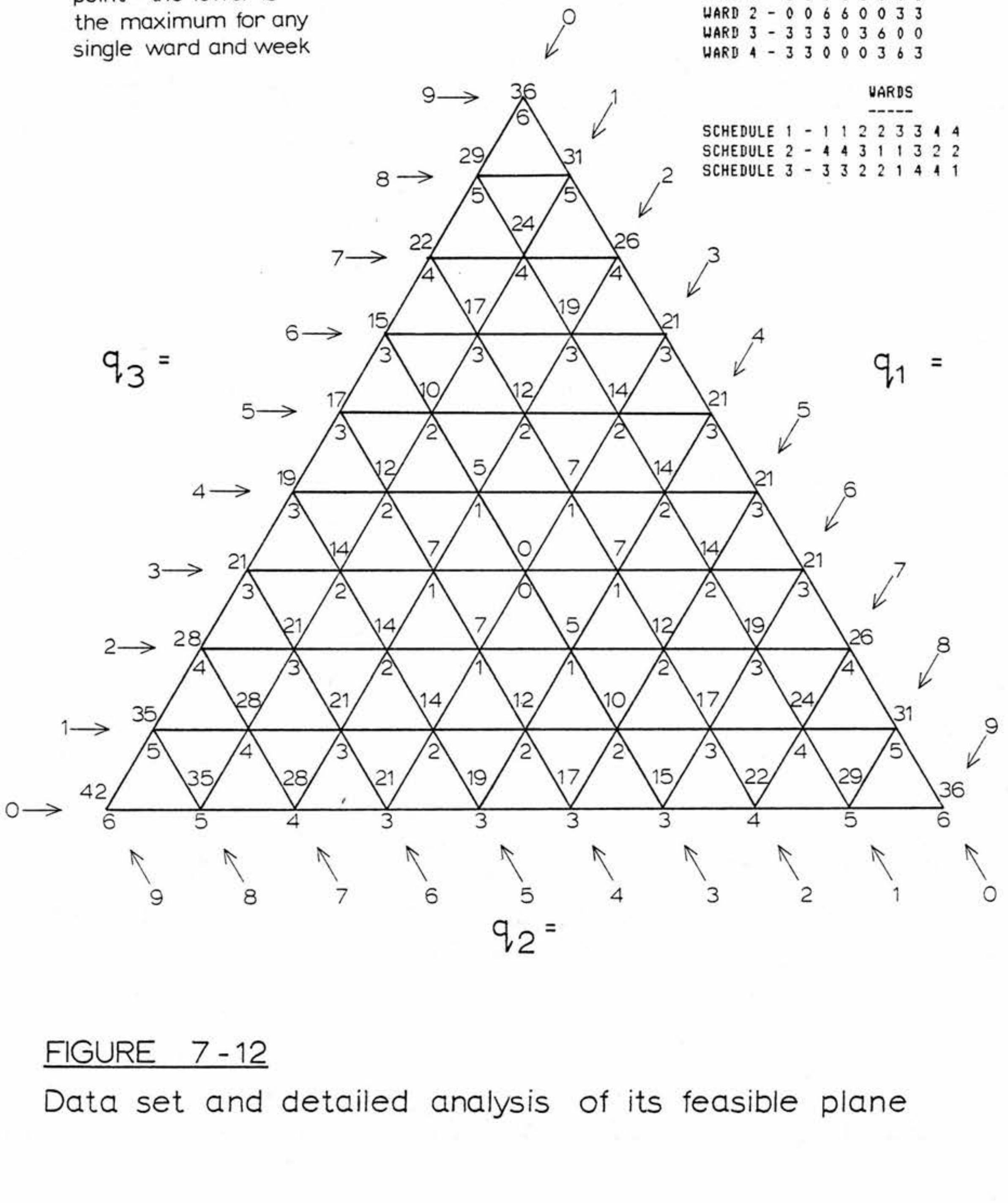


FIGURE 7-12

Data set and detailed analysis of its feasible plane

```
GIRLS:     9
WARDS:     4
WEEKS:     8
SCHEDULES: 3


DEMAND:
         1 2 3 4 5 6 7 8 : WEEKS

WARD 1 - 3 3 0 3 6 0 0 3
WARD 2 - 0 0 6 6 0 0 3 3
WARD 3 - 3 3 3 0 3 6 0 0
WARD 4 - 3 3 0 0 0 3 6 3

                    WARDS
                    -----
SCHEDULE 1 - 1 1 2 2 3 3 4 4
SCHEDULE 2 - 4 4 3 1 1 3 2 2
SCHEDULE 3 - 3 3 2 2 1 4 4 1
```

The upper figure of each pair is the total deficit at that solution point - the lower is the maximum for any single ward and week

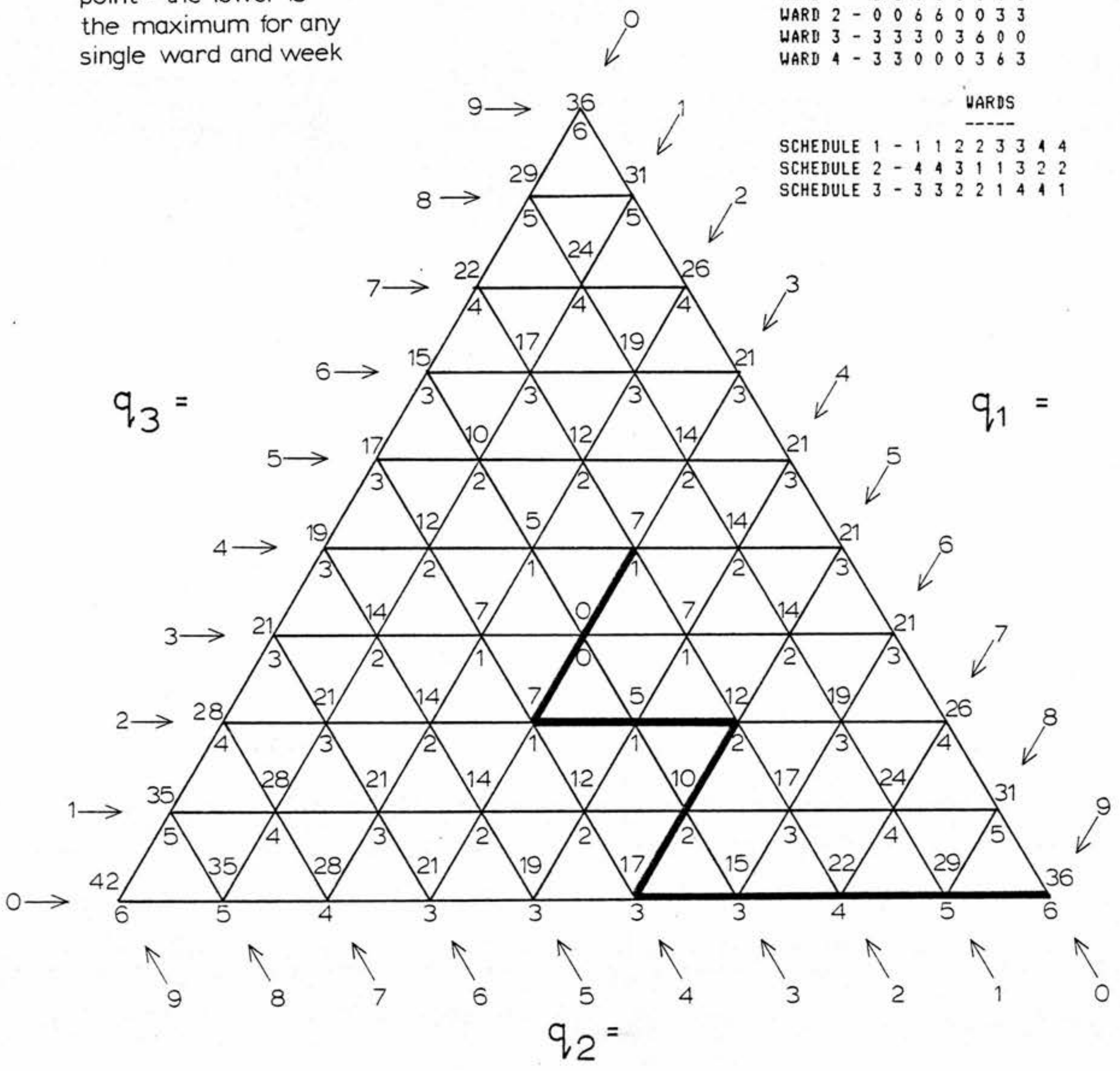


FIGURE 7-13

Data set and detailed analysis of its feasible plane

Six moves is the shortest possible route which can be taken one step at a time from an extreme point to the centre. This can easily be seen from inspection of the table. Note that this agrees with the formula for this type of data set quoted above: I - I/Q.

Finally, consider the route taken by the third method, which states that the number of girls moved should be equal to the value of the greatest deficit. This method guarantees that the greatest deficit on a given ward and week, or at least one of the greatest deficits if there is a tie, will be reduced to zero. Although this method is extremely efficient given the appropriate conditions, it is not always possible to apply it. For instance, the number of girls at present on q^- may be less than the maximum deficit, and it will be impossible to apply this rule. This is why a mixed strategy must be employed. However the efficiency of this rule in the right circumstances can be seen by applying it in the problem represented in Figures 7-12 and 7-13.

Iteration	:	0	1	2
Co-ordinates	:	9,0,0	3,6,0	3,3,3
Total deficit	:	36	21	0
Max deficit	:	6	3	0

Table 7-5

So far the maximum deficit algorithm has not done anything which the total deficit algorithm could not do. To consider the matter further we must examine some four-dimensional problems. Since the total values of each of the four schedules are still constrained to a fixed sum, the four-dimensional case can be built as a three-dimensional model whose shapewill be pyramidal. Some interesting relationships come to light when viewed in this sort of structure. Figure 7-14 represents such a structure, with adjacent

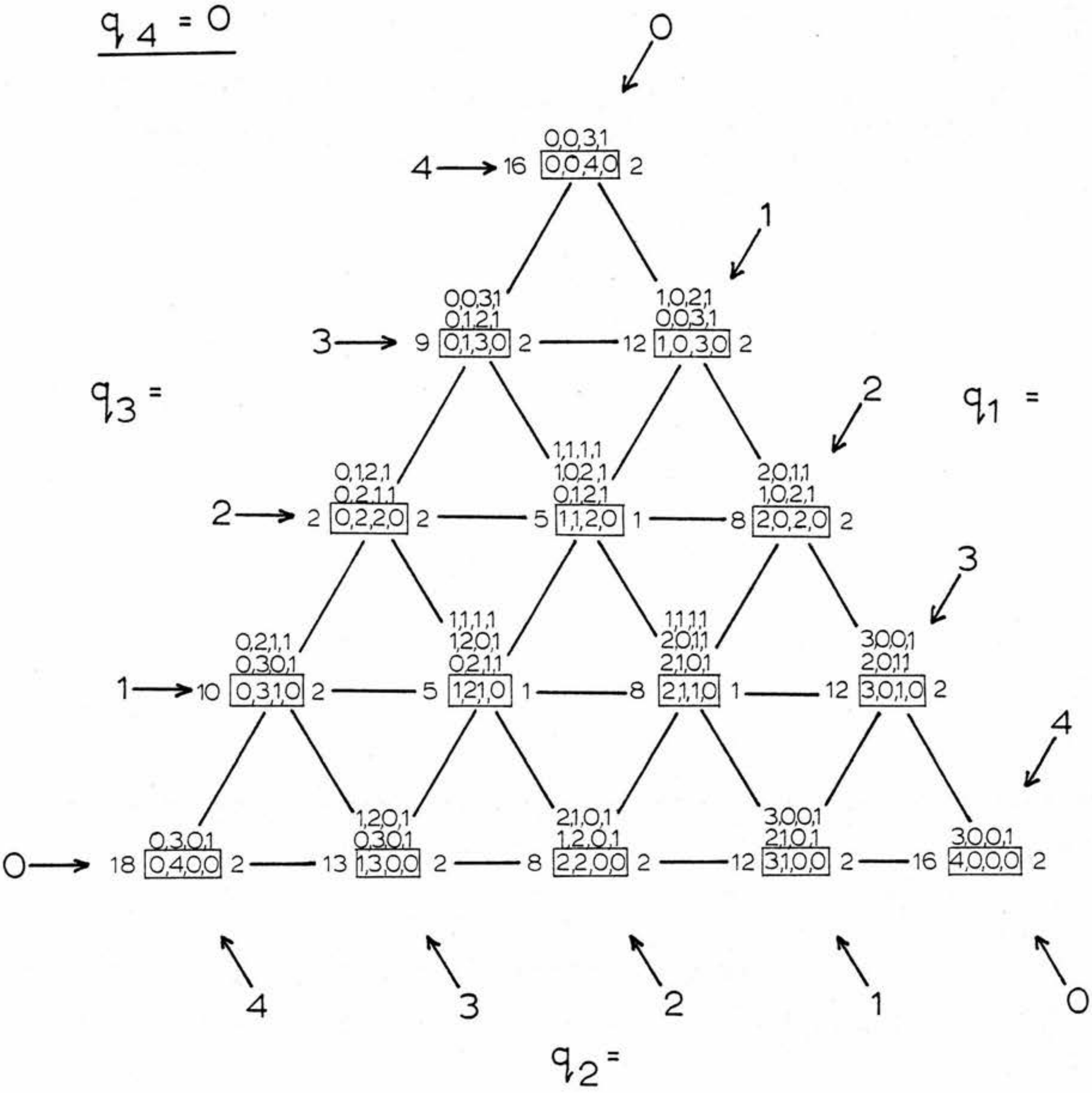


FIGURE 7-14.1

Feasible planes of four-dimensional case

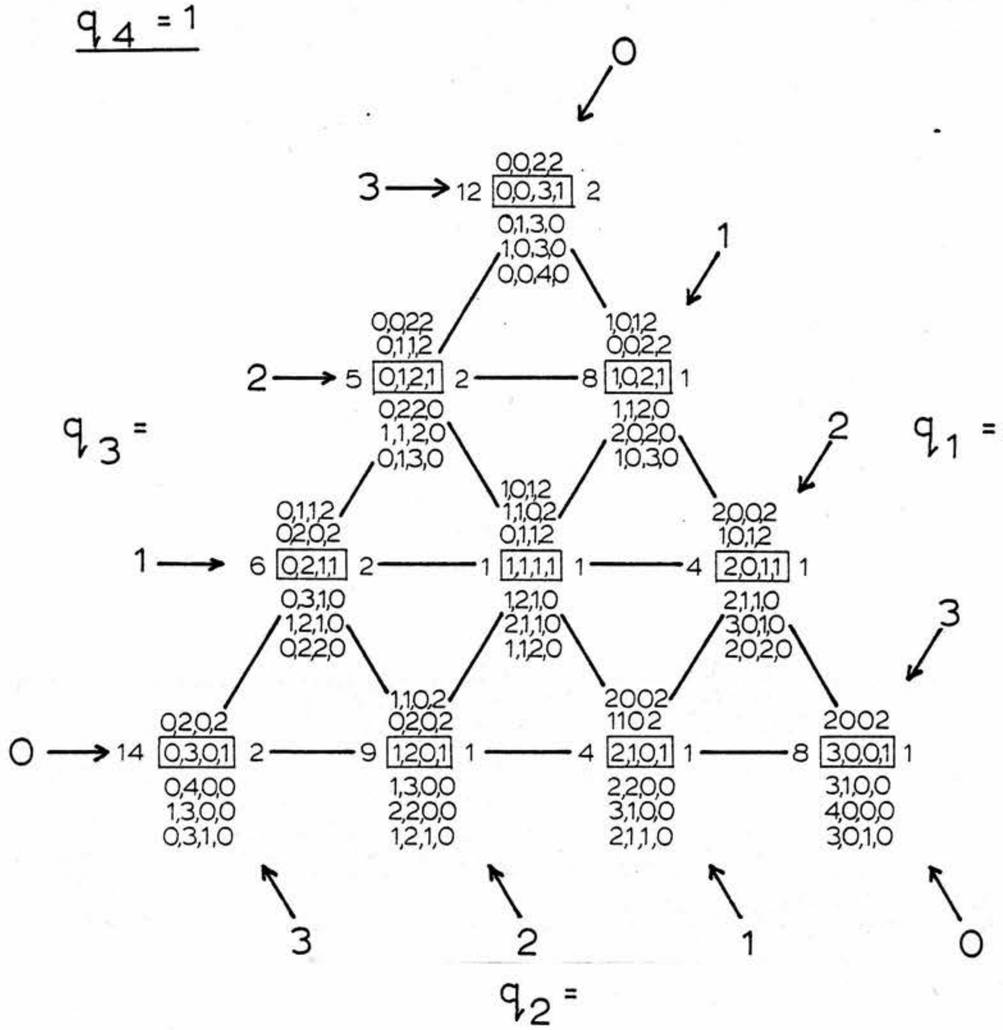
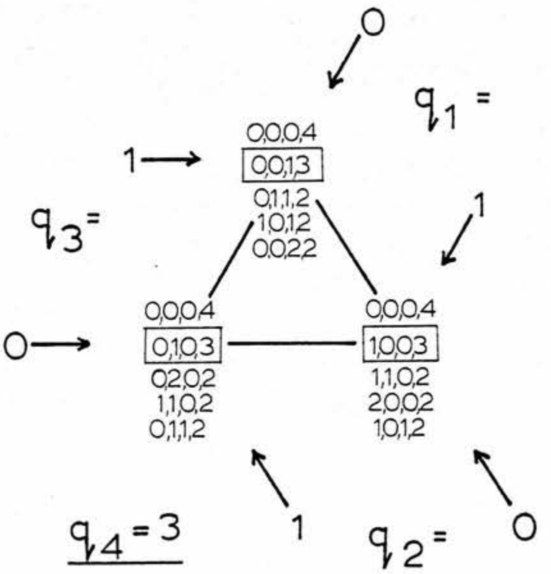
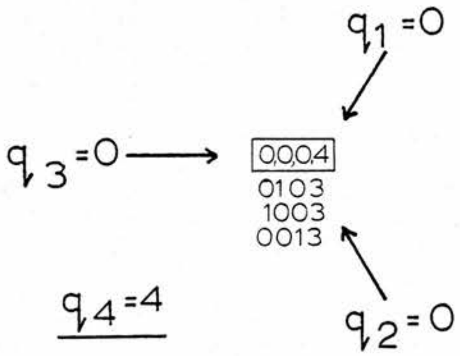


FIGURE 7-14.2

Continuation





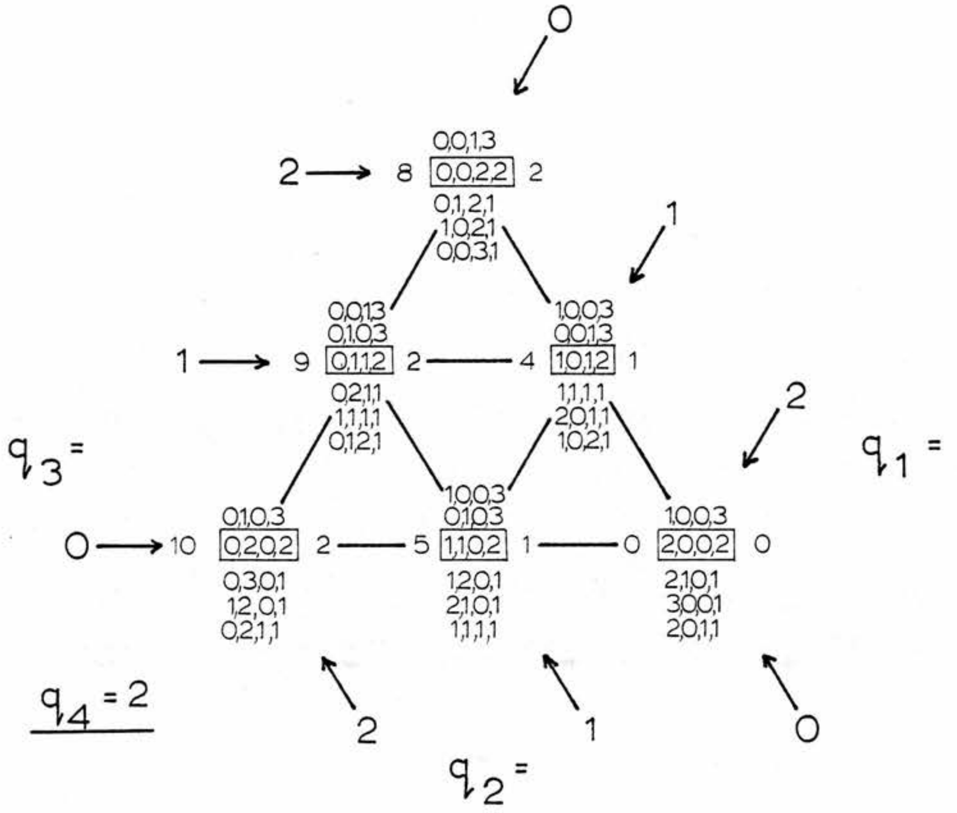


FIGURE 7-14.3

Continuation

layers in the vertical sense depicted side by side. Q = 4 and I = 4 in this case. At each node is given the following information:

adjacent
points in level
above

Total Deficits | CO-ORDINATES | Maximum Deficit

adjacent
points in level
below

Permissible steps can be ascertained either by following a line on the same horizontal plane, or by referring to the co-ordinates given at the present point for adjacent levels.

This problem differs from many of those previously considered in that the optimal solution point no longer occurs at the point representing equal numbers on all schedules (1,1,1,1) but at the point 2,0,0,2, half-way up one edge of the pyramid. Let us consider what happens when we start at the point 0,4,0,0 (the bottom left hand corner of the base of the pyramid) and employ the maximum shortage algorithm, moving one step at a time. The sequence is this:

Iteration :	0	1	2	3	4	5
Co-ordinates :	0,4,0,0	0,3,1,0	1,2,1,0	1,1,1,1	2,0,1,1	2,0,0,2
Total deficit :	18	10	5	1	4	0
Max deficit :	2	2	1	1	1	0

Table 7-6

There are many points of interest here; first it is not possible to obtain a reduction in the value of the maximum deficit at each iteration. This also was true in the three dimensional case. This time, however, it is not possible to secure a reduction in the total deficit at each iteration either. Taken together, though, the combined algorithm manages to reach the optimum point. At iteration 4 there are actually three candidate points, each with identical values for the total and maximum deficits, but the choice is irrelevant, since all are adjacent to the optimal point. One has been chosen at random. The route taken requires one more iteration than is theoretically necessary for this particular problem; the shortest route would have been used if the first move had been to 1,3,0,0 instead of 0,3,1,0.

Consider what takes place at iteration 4. No improvement is made to the maximum deficit, and the total deficit actually deteriorates. The move is made in this case only because a move has to be made unless the optimal point has been found. Thus a local minimum would be characterised by a series of iterations cycling away from the local minimum and back to it, the first move constrained and the second according to the criteria of the algorithm.

In this example there is one noticeable local minimum in terms of total deficits; that with a total of two at 0,2,2,0. No adjacent point has a value less than five. However a drop in the value of the maximum deficit is obtained by moving to either of two adjacent points, 1,2,1,0 and 1,1,2,0. The solution route from 0,2,2,0 would look like this:

Iteration	0	1	2	3	4
Co-ordinates	0,2,2,0	1,2,1,0	1,1,1,1	2,0,1,1	2,0,0,2
Total deficit	2	5	1	4	0
Maximum deficit	2	1	1	1	0

Table 7-7

Once again there are alternative moves, this time at iterations 1 and 3. This route possesses an interesting characteristic when examined on a three dimensional model; 0,2,2,0, 1,1,1,1, and 2,0,0,2 are in a straight line. The movements away from this straight line which result in an increase in the total deficit are caused by the prohibition of non-integer solutions.

7.8 Implementation and results

The algorithm used was that described in 7.4. Two aspects of the method were amenable to different strategies:

7.8.1 The starting solution

Many different criteria could be used to provide a starting solution.

(1) The simplest one was merely to place all available girls on the first schedule. This would normally provide a very bad starting solution, but subsequent improvement would be rapid.

(2) The nurses could be distributed as evenly as their integer nature would allow between all the schedules. This usually gave a fairly good initial solution, but sometimes this led to a local minimum which caused the algorithm to fail. For reasons which will be explained in 7.8.2, progress from an even starting solution is often slow.

(3) Another heuristic tried was:

Find which D_{jt} is greatest.

Allocate half the girls to a schedule which alleviates this demand.

Alter demands accordingly, and locate new maximum deficit.

Allocate half remaining girls to a schedule which helps this.

Repeat previous two steps till all girls have been placed on a starting schedule.

This provides a better starting solution than (1) but avoids the problems caused by the even spread given by method (2).

(4) A similar method to (3) is:

Find which D_{jt} is the greatest.

Allocate a number of girls corresponding to half of that demand to a schedule which alleviates it.

Alter demands accordingly, and locate new maximum deficit.

Allocate a number of girls corresponding to half that demand, or the remaining girls, whichever is least, to a schedule which helps this deficit.

Repeat previous two steps till all girls have been placed on a starting schedule.

This provides a starting solution similar to that given by (3).

7.8.2 The step size

Section 7.5 describes three different rules which could be used to determine the step size, and outlines the relative advantages and drawbacks of each. A mixed strategy is described, as follows:

(1) Move from q^- to q^+ a number of girls equal to the identified maximum shortage on any j and t.

If this exceeds the number of girls currently on q^-, then only move that number of girls.

If the number arrived at by either of the above criteria is less than or equal to three then only move one girl.

Another mixed strategy which was used was this:

(2) Move from q^- to q^+ a number of girls equal to the identified maximum shortage on any j and t.

If this exceeds half the number of girls currently on q^-, then only move half that number of girls.

If the number arrived at by either of the above criteria is less than or equal to three then only move one girl.

Both of the above strategies have the effect of moving relatively large numbers of girls at each iteration until the maximum shortage drops to three. From that point on they home in one move at a time, a procedure which is likely to be more accurate, if more time-consuming. The decision to move a number equal to the current greatest shortage whenever reasonable is self-explanatory. It was originally introduced when it was noticed that, using the single step method, the same step was often repeated until one bad deficit was reduced sufficiently.

It can now be seen why an even spread will not necessarily be the best choice for the initial solution. In many cases an even distribution will cause there to be sufficiently few nurses on each schedule that a single step only will be possible at each iteration. If the solution demands a larger number on one or two schedules than on the others, then it may take a large number of iterations to achieve this, moving girls one at a time from various other schedules. Conversely, if the initial solution is an uneven one, it is much simpler for the algorithm to even them out if that is what is required, since, several may be moved at a time. The following example illustrates this point; the numbers represent the number of girls on each of the 3 schedules. In the first case an even solution is desired, but the initial solution is to put all the girls on the first schedule. In the second case the required solution is for all the girls to be on the first schedule, and the starting solution is an even one:

CASE 1:

Iteration	0	1	2
Schedule 1	9	6	3
" 2	0	3	3
" 3	0	0	3

CASE 2:

Table 7-8

Iteration	0	1	2	3	4	5	6
Schedule 1	3	4	5	6	7	8	9
" 2	3	3	2	2	1	1	0
" 3	3	2	2	1	1	0	0

It takes two iterations in case 1, and six iterations in case 2. Balanced against the fact that it is easier to smooth out the distribution between schedules, than to make it uneven from an even start, is the observation that the even starting solution is often a better one. However it may be a good solution but near to a local minimum, and in some cases the local minimum would cause the solution procedure to get stuck in a problem which could successfully have been solved given an uneven initial solution with a bad initial value.

When analysing the performance of this algorithm different starting solutions were tried with the same sets of data in some cases, and sets of data were chosen which exhibited widely varying solutions, some with an even spread, some very uneven. The number of iterations taken, together with the number of trials where a solution was obtained is shown in Table 7-9.

It is seen that the size of problem tackled by this algorithm is far greater than that coped with by the sub-gradient algorithm as shown in Table 6-5. Trials for the heuristic

SIZE							TIGHTNESS				
No.	I	J	T	Q	IQ	IJTQ	70%+	80%+	90%+	95%+	100%
1	200	5	20	60	12,000	1,200,000	4/4 = 100 22·3	4/4 = 100 29	4/4 = 100 37·3	1/4 = 25 46	0/4
2	190	5	20	60	11,400	1,140,000	4/4 = 100 20	4/4 = 100 27·5	4/4 = 100 39	2/4 = 50 54	0/4
3	180	5	20	60	10,800	1,080,000	4/4 = 100 19·5	4/4 = 100 23	4/4 = 100 31·8	2/4 = 50 44·5	0/4
4	120	5	20	30	3,600	360,000	4/4 = 100 15·8	4/4 = 100 18	4/4 = 100 26·3	3/4 = 75 30·3	0/4
5	60	5	20	30	1,800	180,000	4/4 = 100 12	4/4 = 100 12·8	4/4 = 100 14	4/4 = 100 14	2/4 = 50 15

KEY

A/B = C
D

A : Successes
B : Trials
C : % Success rate
D : Average number of iterations - successful cases only

TABLE 7-9

Overall table of results for heuristic method algorithm were performed with sets of data up to the size which might be encountered in a real problem, the upper limit being set by practical considerations relating to the time taken to run such large problems. The size of I only affects the number of iterations required indirectly, whereas J, T and Q, because of the cycle structure, tend to affect the time taken for each iteration directly. This is an improvement on the sub-gradient algorithm, where the deepest-nested cycle involved I, J, Q and T, causing the value of I to be far more critical. It is interesting to note that each iteration of the heuristic algorithm is relatively fast (about 3 seconds CPU time) when the number of schedules with girls on them is small, since fewer comparisons have to be made between swops from potential donor schedules. The time taken for an iteration rises to over 15 seconds when there are girls on every schedule. (These figures are for a 190 girls, 5 ward, 20 week, 60 schedule problem). With more efficient coding these timings would be expected to speed up considerably.

7.9 Schedule generation

It is advantageous to use as few schedules as possible defined explicitly in the data, in order to keep the problem size to a minimum. However it will sometimes happen that no feasible solution is found with the given range of schedules. At this point it would be reasonable to define a further schedule or group of schedules which could be added to the list of feasible schedules in order to facilitate solution of the problem.

Two method of doing this suggested themselves.

(1) When looping occurs, indicating a local minimum, or non-optimal overall minimum, record which wards and weeks are causing the

greatest problem, and which are non-critical. Feed these desired characteristics into an algorithm which generates permissible schedules, and add the newly generated schedule or schedules to the explicitly defined set of schedules. For instance, if there had been consistent difficulty in finding a set of schedules which provided girls for ward 1 in week 1 at the same time as providing girls for ward 5 in week 20, then a schedule would be generated which did these two things simultaneously. Sometimes it might be necessary to produce two or three schedules in order to achieve a desired blend.

(2) A simpler method, the one which was tried in practice, was to store a much larger list of permissible schedules which would not be considered until the original set caused the solution procedure to stick. At this point the larger set would be referred to. It was only necessary to scan the large set for certain wards and weeks where conflicts had occurred. One or more extra schedules could then be added to the set considered explicitly at each iteration. A typical size of problem to be tried had a set of sixty starting schedules in its explicit set, and a set of two hundred potential extra schedules.

It was found in practice that it was best policy to adopt several new schedules if looping started. Although this would increase the problem size by a certain extent, this was found to be preferable to adding new schedules one at a time. The reason was that several iterations were needed to detect looping - sometimes the same five solutions would occur in a repetitive cycle, so the identity of at least six previous solutions had to be kept and updated at each iteration. If a new schedule were introduced, unsuccessfully, then it might take another six iterations before the looping was detected again, or more if it did not start at once. For this reason it was found to be worth the slight extra increase in problem size to achieve the best modification each time the external schedule list had to be consulted.

In practice the method of keeping a store of extra schedules proved to work, allowing solutions to be found for problems which were previously insoluble, but it was a fairly time-consuming process to introduce and test new schedules at a near-optimal point. Whether or not the system would be of practical use would depend on how critical it was that a zero-deficit solution was found, rather than one which was near-optimal.

CHAPTER 8

8.1 Introduction

It would be appropriate to pause at this point to re-iterate the meanings of some previously encountered terms and to define any new ones necessary for the understanding of this Chapter. The Chapter's contents will then be outlined.

So far the systems which have been considered have shared many features. They have had a roster which was designed in advance of individual schedules and was repeated unchanged time after time. The roster contained information relating to the fixed aspects of a nurse's schedule, such as which weeks would be spent on leave, on night duty or on district work. Within the framework of the roster nurses would have their plan designed - this would be an individually allocated schedule dictating the ward to be visited on any given week of the year's course. Although many of the plans may have been similar from one nurse to the next it would only be by coincidence if the plan followed by one nurse was identical in structure and content to one which had previously been used for another girl, or which was being used concurrently by another. The plan would not usually be determined in full at the start of a girl's course; it would be scheduled continuously as the course progressed.

In Chapter 8 the concept of a cyclical standard schedule will be discussed. This consists of plans which are used repeatedly by different nurses who have consecutive courses. These plans are fixed in advance like the roster in the previous systems and may be used simultaneously by groups of girls. Together with other such plans, the individual components are part of an overall schedule. This standard schedule may be so designed that the staffing is even at all points, and the individual plans are all satisfactory. The standard schedule is then repeated at the appropriate intervals and the staffing is thus taken care of indefinitely.

Such an attractive system must have drawbacks, or else it would have been adopted earlier by the staff of the Simpson. Firstly it is not always possible to design one which achieves the ideals of staffing and training mentioned above. Secondly there may not be an appropriate time interval which repeats evenly. Thirdly the problem of fluctuating arrivals has to be overcome.

One attempt to design such a system for the Simpson is described in this Chapter. It has many advantages and is cleverly designed, but in our view it fails to satisfy some of the crucial criteria relating to staffing levels which such a plan must take as a sine qua non. We then detail our own proposals for an amended version which in our view makes use of the advantages of the first described system, but eliminates its worst faults. An analysis of a proposed transitional period is included.

8.2 Cyclical standard schedules for use with the real problem

A number of changes introduced at the start of 1977 made it feasible to consider the design of a series of standard schedules, i.e. ones which could be applied repeatedly to provide even staffing levels. Previous to these changes there had been several obstacles to the introduction of such a system.

The manual system in use in 1973 had caused a great deal of hard work and strain for those people involved in its running[1], but the results which they produced by that method were very good - staffing levels were kept very even, and a great degree of flexibility was maintained.

One of the basic obstacles to using a repeating set of schedules is the uneven length of the year - it can not be divided exactly into a number of weeks or even days, so some sort of compensation factor has to be introduced every once in a while like the system of leap years. In designing a standard schedule which will fit each year, allowance has to be made for such variation. If the system is tied rigidly to repeating sets of thirteen weeks then monthly arrivals will occur earlier each year, and at some point it will become necessary to make some major re-adjustment. Obviously a better system is to have some section of the course which can be stretched if necessary so that the schedules fit the year's length exactly; for instance a week could be deemed to last for eight days at a suitable point. Under the old system this would have been very difficult to achieve since girls started work on the wards right from the first week, but since the new system commences each time by putting an intake on lectures for two weeks there is now a greater degree of flexibility as to the actual day that the girls start on the wards; in other words alterations of one day to the length of the initial study period will be sufficient to keep each intake's work pattern in alignment with those of its predecessors, a necessary condition for the standard schedules to produce even staffing. Because of the extra latitude which this gives it is possible with the new system to fix arrival dates for each intake which are unchanged from one year to the next; the first days of March, June, September and December.

The second obstacle to the use of standard schedules under the old system was that of unevenness in arrivals. With standard schedules one loses the flexibility which individual scheduling provides, and as a result it becomes more important to have a regular number of girls available month by month in order to be able to maintain the staffing levels which a pre-planned schedule can achieve given a constant supply of nurses. Under the old system the variation was too great to allow this system to be used. For instance in 1973 the minimum size of an intake was six and the maximum seventeen. Obviously if these girls had been slotted into pre-planned schedules then there would have been gross over- and under-staffing which would have accompanied each set from ward to ward. The old individual allocation system was able to cope with this however, and could spread any shortfall or over-supply between different wards, perhaps balancing one against the other.

So it can be seen that steady numbers of available nurses are a pre-condition for using standard schedules. Variability is too great when there are twelve or thirteen intakes per year, but becomes acceptable with a quarterly intake system owing to the smoothing effect of grouping larger numbers of girls together.

These then are two basic reasons why a standard repeating system should be feasible under the new system but not under the old. There is one more reason which, although less tangible, still has a powerful effect on the decision to opt for a set of standard schedules. One of the drawbacks of such a system, as discussed in Chapter 1, is that staffing levels will be less even on the whole than those achieved with individual scheduling. On the other hand, good training plans will be provided, and in fact guaranteed, for each girl. It is notable, then, that the people involved with planning the schedules are also most concerned with the teaching side of the hospital. Their priorities, understandably enough, are to schedule the girls with the minimum of effort compatible with giving each her required training. Problems which the resulting schedules may cause for the other people whose job it is to provide round-the-clock service with the fluctuating numbers of nurses provided by these schedules do not affect the scheduler directly.

In fact in one case one of the people connected with scheduling and training denied that a staffing problem existed. Until recently the Assistant Matron worked out the schedules, and her priorities lay more in favour of providing even staffing even if it meant deviating from each nurse's ideal training course. In other words there has been a shift in emphasis which permits the prospect of standard schedules to be considered, notwithstanding the drawbacks which this entails.

8.3 The 1977 schedule - Black system

In October 1975 a set of recommendations relating to student midwife training were put forward by a committee set up to consider the subject following the issue of a new set of E.E.C. directives. (See Appendix B). Although the recommendations were approved in principle by the Central Midwives Board for Scotland, the proposed programme was not one which would have given even staff availability from week to week, since the blocks of study would have overlapped with those from other intakes so that in some weeks all girls would have been on the wards whereas in others there would have been two intakes (i.e. half the total complement of student midwives) away doing their theoretical studies. At this stage Roy Black of

1	2	3	4	5	6	7	8	9	10	11	12	13
▨	▨	A				P			▨	▨	L	
▨	▨	P				L			▨	▨	A	
▨	▨	A				L			▨	▨	P	
▨	▨	L			P				▨	▨	A	
▨	▨	L				A			▨	▨	P	
▨	▨	P				A			▨	▨	L	

14	15	16	17	18	19	20	21	22	23	24	25	26
L		H		▨	▨	P			H	N/A		
A		H		▨	▨	A			H	N/L		
P		H		▨	▨	N/A				P		H
A		H		▨	▨	N/L				S		
P		H		▨	▨	P		S			H	P
L		H		▨	▨	S			P			

27	28	29	30	31	32	33	34	35	36	37	38	39
N/A	P		S			▨	▨	N/P		N/S		C
N/L	S			P		▨	▨	N/S		N/P		A
S		P		L		▨	▨	P		L		N/S
H	P			A		▨	▨	L		P		N/P
P	N/A			N/L		▨	▨	C				P
H	N/L			N/A		▨	▨	C				L

40	41	42	43	44	45	46	47	48	49	50	51	52
C			L					▨	P		H	
C				P				▨	L		H	
N/S		N/P		C				▨	L		H	
N/P		N/S		C				▨	A		H	
P	L			N/P		N/S		▨	A		H	
L	A			N/S		N/P		▨	P		H	

2	3	3	3	3	3	3	3	3	2	2	2	3	A)	
4	4	5	4	4	5	5	5	4	4	5	5	5	P)	
3	3	3	4	4	3	3	3	3	3	3	3	3	L)	DAY
1	2	1	2	1	1	1	1	2	1	2	1	1	S)	
2	2	2	1	2	2	2	2	2	2	2	2	1	C)	
1	1	1	1	1	1	1	1	1	1	1	1	1	N/A)	
1	1	1	1	1	1	1	1	1	1	1	1	1	N/P)	
1	1	1	1	1	1	1	1	1	1	1	1	1	N/L)	NIGHT
1	1	1	1	1	1	1	1	1	1	1	1	1	N/S)	
16	18	18	18	18	18	18	18	18	16	18	17	17		

A = Antenatal
P = Postnatal
L = Labour Ward
H = Holiday
C = Community
S = Special Care Unit
N/ = Night Duty on Relevant Unit

▨ = Study Block

TABLE 8-1

The Black scheme

Strathclyde University[2] was consulted and he devised four schemes, each of which was a repeating set of standard schedules within the framework of the new E.E.C. directive. One of these was accepted and approved by the Central Midwives Board for Scotland for implementation as from the 1st December 1976. The scheme which was accepted is reproduced in Table 8-1. The presentation of the scheme is reproduced in Appendix C.

The first point to be made is that, under the new requirements, there are five weeks of holiday per year, and nine weeks of study, giving a total of fourteen weeks during which girls will not be available for ward work. Since there are thirteen weeks between intakes it is possible to stagger the weeks-off in such a way that, apart from one week, there will always be a constant number of girls available for practical nursing duties. Under the Black plan the odd fourteenth week of absence from ward duty is not spread out on to a different week for each of the six sets, but instead two of the sets have a week's holiday on week twenty-three, and another two on week twenty-seven. This indicates a conflict between two aims. On one hand there are administrative advantages to having all the girls taking this one week's holiday on the same week, but on the other hand this obviously will not provide even staffing. Having accepted that the sets must take their holidays on different weeks it would seem better to give the six sets six different weeks rather than just four. The week by week staffing position in terms of number of sets on duty for each week of a quarter is shown below:

WEEK:	1	2	3	4	5	6	7	8	9	10	11	12	13
NO. OF SETS:	16	18	18	18	18	18	18	18	18	16	18	17	17

Table 8-2

Under this system it has become necessary to put some sets on to the Labour Ward for their first spell of clinical duty. This is a result of the decision to put every girl on the Labour Ward at some time before her first holiday. Having decided that, it becomes necessary to start some girls on that ward in order to provide even staffing. Under the 1973 system every girl had a spell on post- or ante-natal first, duties which she should be capable of performing with the standard of training which she has already reached. The hope is that enough of the ground will be covered in the initial two weeks of study under the new system to prepare the girls for such a duty early on.

The other problem which this plan might introduce is also inherent in its structure - will it be satisfactory to have gaps of up to thirteen weeks between the blocks of theoretical study? Under the previous system the teaching was continuous throughout the year, and some doubts have been raised as to the advisability of having these large gaps, despite the fact that the actual number of study hours is similar to that under the previous system. Apart from the fact that the clinical and the theoretical experience will tend to inter-relate less, there is also the consideration that there will be less feedback on the student midwife's performance during the periods between study blocks. Only time will show whether these fears have any foundation, but on balance the present indications are that the advantages of a stable coherent system of theoretical teaching made possible by the simplification of the teaching time-table will outweigh these disadvantages. Apart from these two points the schedules provided for each set are very suitable.

8.4 Fluctuations in staffing levels/

8.4 Fluctuations in staffing levels

The main drawback to this system is the fluctuation in staffing levels which it causes. In the interests of simplification of the scheduling problem several sources of variation in staffing levels have arisen which could be considered to violate some of the basic requirements dictated by the service needs. The following two tables serve to illustrate some of these points. Two columns are given for the new system, the first on the basis of there being 160 girls present, the second referring to a trainee establishment of 140 nurses.

	OLD SYSTEM	NEW SYSTEM	
TOTAL TRAINEES	140	160	140
HOLIDAY WEEKS/GIRL	5	5	5
DISTRICT WEEKS/GIRL	4-5	4	4
STUDY WEEKS/GIRL	-	9	9
	9-10	18	18
WARD DUTY WEEKS/GIRL	42-43	34	34
WARD DUTY GIRL WEEKS/YEAR	5973	5440	4760

Table 8-3

The drop in apparent staffing levels will not be quite as great as this suggests, since when each girl is on a ward she will now be working there for more complete days each week. However there is still an overall drop in the level of staffing which will be provided by students. Under the old system the average number of hours worked on the wards each week that a girl was allocated to ward duty was thirty four. Under the new system the figure is forty hours. Multiplying these figures by the total number of girl-weeks available per year gives us:

Nurse-hours on ward duty per year (140 girls present)

1973 system - 203,082 hours
1977 system - 190,400 hours
Reduction - 12,682 hours

This is equivalent to having eight or nine girls fewer than under the old system for the same size of establishment. Put another way, that is approximately £22,000 worth of nursing care which is no longer available for staffing, that being the cost of hiring eight more nurses to make up the number of hours of staffing needed. Since however the budget for the hospital is fixed, any shortfall of this kind must necessarily result in reduced standards of care for the patients. A comparison of optimum and actual staffing levels under the old system and as provided by the new, is shown in Table 8-4. It is compiled under the assumption that there are 6 girls in each section, a total of 144 being trained. The old figures are compiled from a 20-week period when the average total number of nurses was just under 134; maximum = 142, minimum = 126.

The figure for girls working on Out-patients is now included with the figure for Ward 49. Apart from this there are some noticeable differences in the staffing levels provided. The greatest concerns the Post-natal Wards, 51 to 54, where the day-time staffing level has dropped from an actual average of 29.5 to a proposed level of 24 to 30 and the night-time level has dropped from 17 to 6. This difference will need to be made up by employing more trained staff on these duties. Since the total number of staff in the establishment is more or less fixed by the amount of money available in the overall budget, this means that there will be fewer trained staff

	OLD SYSTEM 160 GIRLS			NEW SYSTEM 144 GIRLS	
WARD	OPTIMUM		ACTUAL	ACTUAL	
49	12		15.3	12 (4 wks), 18 (9 wks)	DAY DUTY
51.3	24	28	29.5	24 (6 wks), 30 (7 wks)	
54	4				
SC	10		10.5	6 (9 wks), 12 (4 wks)	
LW	15		15.3	18 (11 wks), 24 (2 wks)	
OP	8		8.8	-	
DIST	12		9.8	6 (2 wks), 12 (11 wks)	
49	5		5	6	NIGHT DUTY
51.3	15	17	17	6	
54	2				
SC	6		6	6	
LW	6		6	6	

(The figure in brackets shows the number of weeks in each quarter that the given staffing level will obtain)

Table 8-4

available elsewhere, which may reduce levels of patient care. Where are the girls who would previously have been used to a greater extent on the post-natal wards now deployed? Apart from general reductions in numbers of girls, the other answer lies in the number of girls on Labour Ward. Here the average complement has risen from 15.3 girls on day-duty to 18.5. Thus part of the problem is that in order to satisfy training requirements the Labour Ward is staffed by a greater number of student midwives, while staffing levels in the less demanding Post-natal Wards must either be made up using fully trained staff, or must be allowed to drop.

The use of more student midwives on the Labour Ward could produce problems when trying to ensure that each girl was able to perform the required number of deliveries, were it not for the fact that the requirement has dropped from 20 to 15.

So the first problem associated with the new schedule is the necessary redeployment of trained staff. The second problem, which exacerbates the first, is the amount of fluctuation which is caused in staffing levels from week to week.

The problem is greatest on the Special Care Unit, where for four weeks out of thirteen the staffing level doubles from six to twelve. Since the previous staffing level averaged just over ten it seems likely that this unit will be understaffed for over two thirds of the year. Figure 8-1 shows variation in staffing levels in graphical form; it can be seen that the Special Care Unit is not the only duty with a very large fluctuation. For instance, for nine weeks out of thirteen, Ward 49 increases its complement by 50%, and for two weeks out of thirteen, the number of nurses on community work halves.

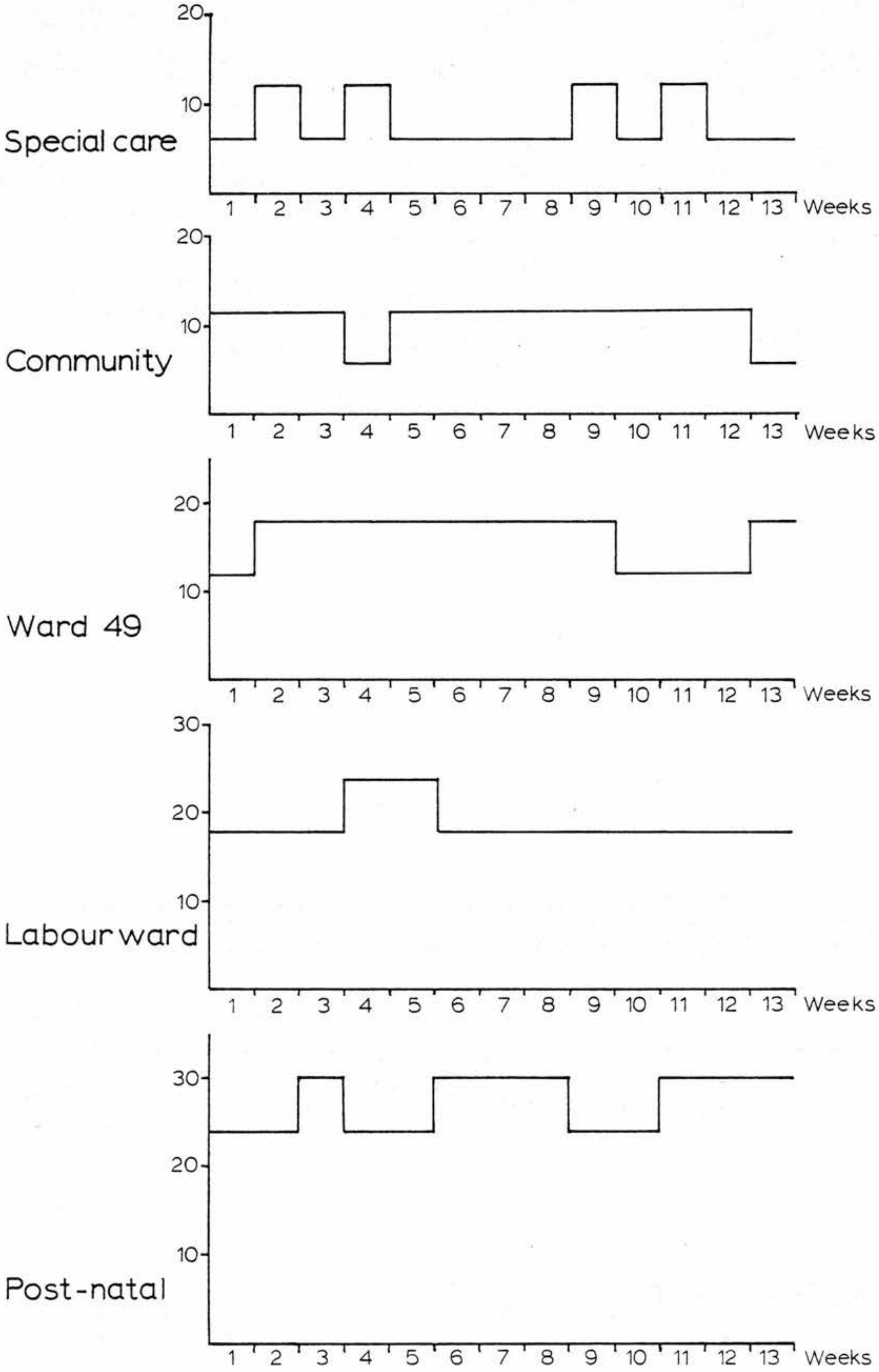


FIGURE 8-1

Staffing levels under Black system
Pattern is repeated each quarter - vertical axes show number of student nurses allocated to each ward

It is pertinent at this point to assess whether these large fluctuations are a result of poor design. In fact it is the choice of system which is at fault, not the design within the system. Black has chosen to work with each intake divided into six sets, making the number of girls in each set about six (exactly six if the complement of trainee midwives is 144). Rapid inspection, using the methods outlined in Section 1-4 shows that it is impossible to achieve completely even staffing with this number of sets - the availabilities are not even multiples of the requirements. Thus, for each week when the staffing provided by the system deviates from the ideal, the difference will be at least six girls too few or too many. Similarly, the number provided must always be a multiple of six except in the case of the Post-natal Wards where the allocation is split between four wards. Because of this, some of the old optima can no longer be achieved.

8.5 Justification of the Black system

We must consider here the possible justifications for developing this system. In order to assess these, one has to consider whether the payoff of increased evenness of staffing balances the problem of increased complexity. The critical ward here seems to be the Special Care Unit. This is the one with the greatest fluctuation, and also the one where for over two-thirds of the year the staffing drops to a level which is causing concern among the top staff at the Simpson[3]. If the situation here could be improved then it definitely would merit the complexity of having more than six intakes. It is evident from the symmetry of the arrangement of SC on the Black plan that this ward was filled in first, the others being fitted in around it. Ignoring the other wards for the time

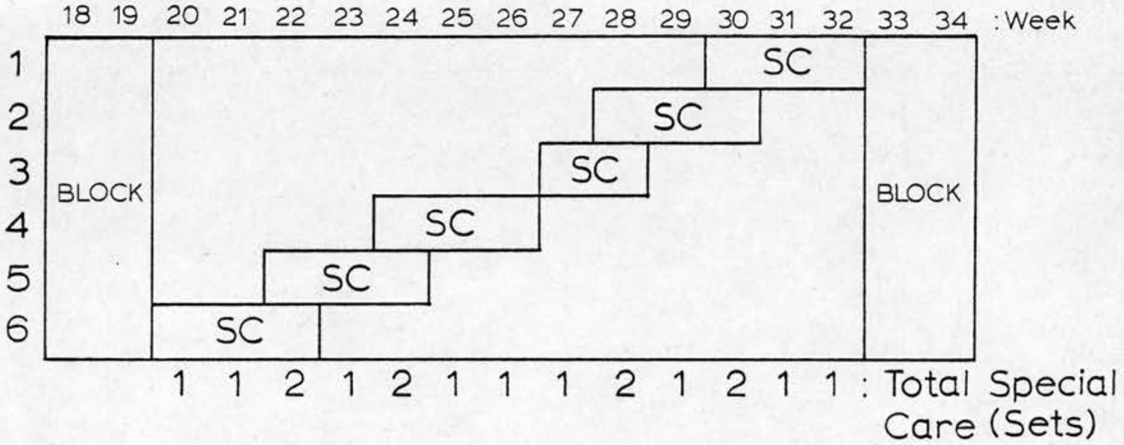


FIGURE 8-2

Pattern of SC allocation under the Black system

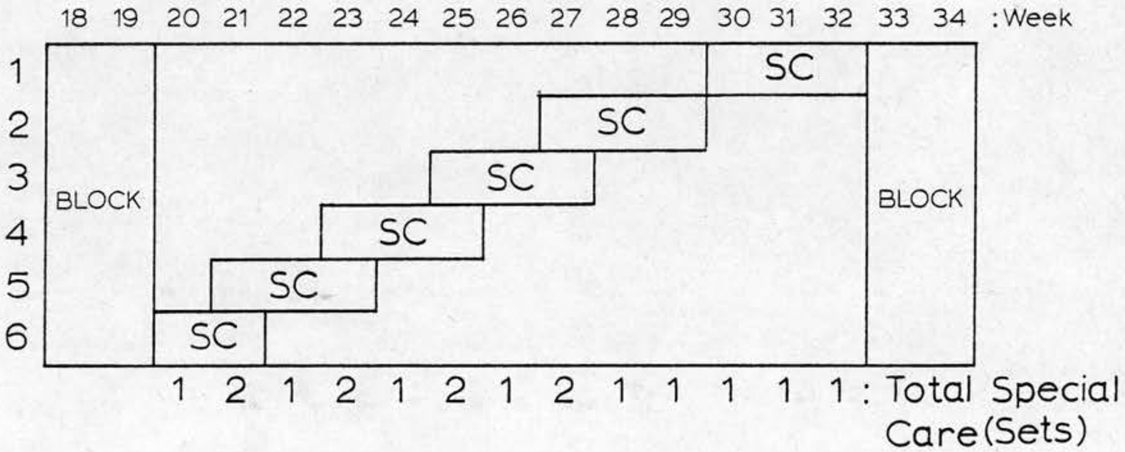


FIGURE 8-3

Pattern of SC allocation under a new system being, the pattern is as shown in Figure 8-2.

All of the Special Care duty is done between weeks twenty and thirty-two of a girl's course, apart from night duty which is done later. Apart from the beginning and the end and the point in the middle between weeks 26 and 27 Black has managed to achieve an overlap between successive groups. The problem is that although this overlap is desirable from a continuity point of view, it causes the number of staff to double four times in the thirteen week cycle. It will be noticed that set 3 only does two weeks of duty at this point. This is so that they have a week of SC in hand which can be done later as an extra week of night duty, a necessary condition in order to attain evenness of staffing then. Since overall numbers are smaller at night and there are fewer trained staff about this requirement is of the utmost priority and is achieved admirably by the Black system.

Still considering only the staffing on SC we must now ask: If the number of sets is increased would it be possible to make the staffing more even? We can shuffle the weeks of duty around, retaining one two-week block out of six, in order to try to make the overlaps occur at different weeks. It is not feasible to move a girl from SC to another ward and then back again since this would disrupt her training too much.

First consider the possible improvement which could be achieved if the number of sets was doubled. Let us keep the first six sets as they are above and then add a further six whose overlaps occur at different points, such as in Figure 8-3.

Comparison of the six-set system (six girls per set) with the twelve-set system (three girls per set) produces the figures in

Table 8-5.

REPEATING 13 WEEK CYCLE:	1	2	3	4	5	6	7	8	9	10	11	12	13
NUMBER OF GIRLS 6 SETS:	6	6	12	6	12	6	6	6	12	6	12	6	6
12 SETS:	6	9	9	9	9	9	6	9	9	6	9	6	6

Table 8-5

The peaks of twelve girls are spread out more evenly under the second system, but there are still five weeks during which the minimum number of six girls applies. This is obviously still not satisfactory. It is possible to assess what the best spread would be with eighteen sets - at the most even it would provide eight girls for twelve weeks and six girls for only one; a much better solution. However, it does not seem to be possible to design an eighteen set pattern so that this is achieved. Figure 8-4 is the best spread that has been achieved in practice, and it is possible to demonstrate that this is in fact the best that can be done, since inevitably two groups of six sets out of the eighteen must share an overlap.

Thus it would be possible at this stage to discard the idea of using a greater number of sets than six, since the extra complexity does not seem to be balanced by the solution of the staffing problem. This is the point at which the rules defining the problem were reconsidered. As stated earlier, the staff held it to be unacceptable for a girl to do two weeks of special care, move to another ward, and then return for a single week, since the disruption would be too great. However in answer to an inquiry it was stated[4] that it would be reasonable for a girl to take a week's holiday during a period of special care, since this would not involve the disruption of moving from one ward to another. This altered the problem radically, and

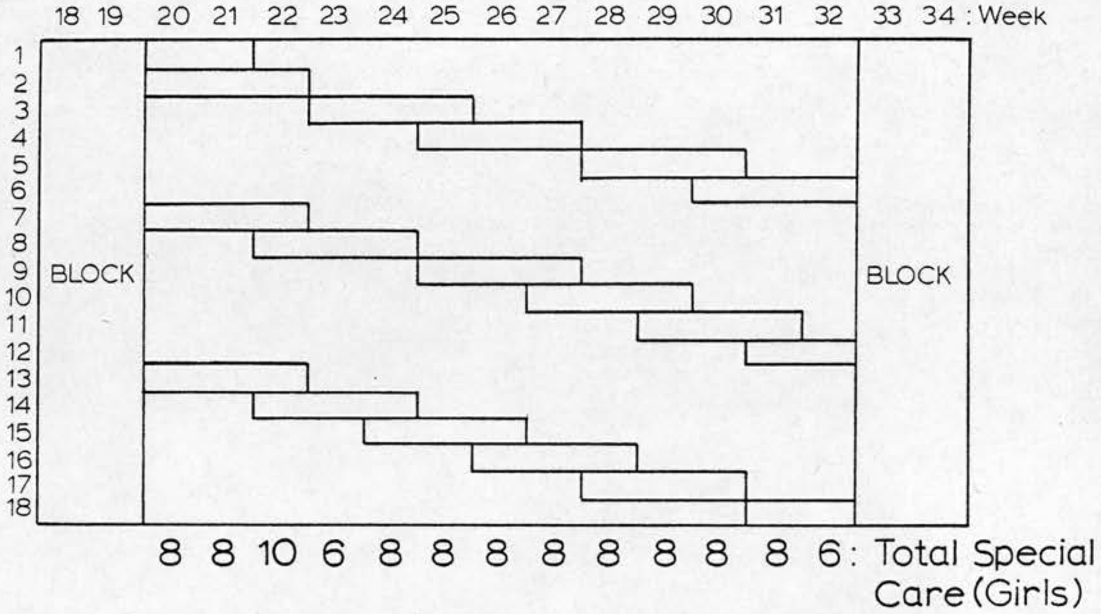


FIGURE 8 - 4

Best spread of SC staffing using an eighteen set system

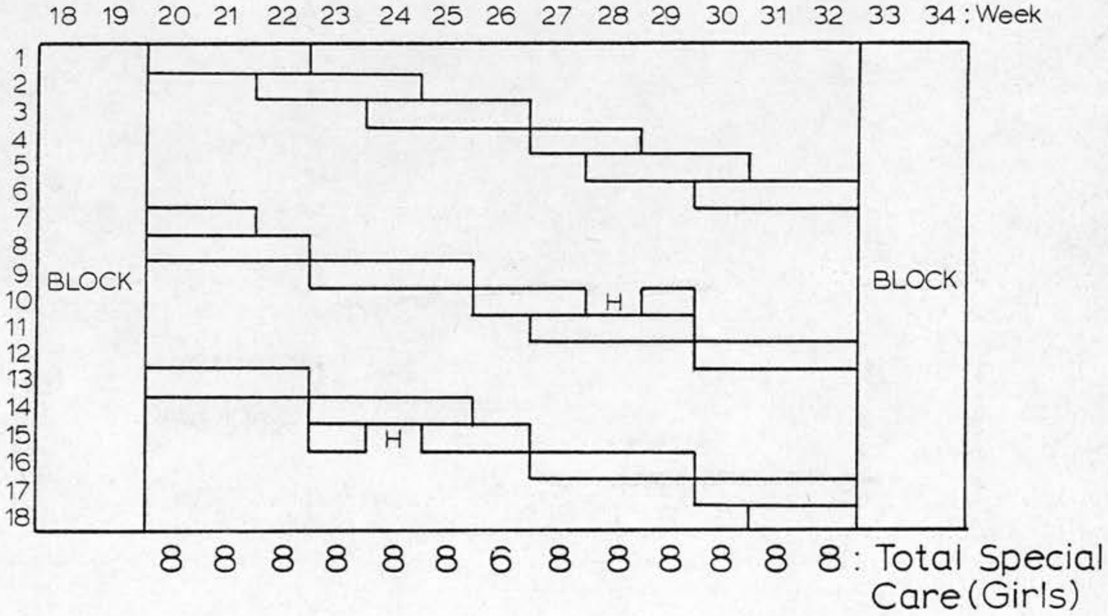


FIGURE 8 - 5

Best spread of SC staffing using a modified eighteen set system. H = Holiday made it possible to design an eighteen set system which achieved the theoretical best staffing, with eight girls for twelve weeks and six for only one. This is shown in Figure 8-5.

It would seem then that the systems involving larger numbers of sets were not considered because no way of making the best use of their potential had been found. Unfortunately the problem seems to have been oversimplified when the proposed scheme was presented to the staff of the Simpson. The solution with only six sets has the advantage of looking simple and easy to use, and at first glance appears to be a surprisingly simple solution to a complex problem. This neatness and simplicity will obviously appeal to the person who has to implement the procedure. However this still leaves the fact that the staffing levels provided, especially for the Special Care Unit, should have been inadequate enough to merit the rejection of the scheme. Why were they accepted? The answer would seem to lie in the way that the scheme was presented. Nowhere on the duplicated plan of the scheme are actual numbers of girls mentioned; instead the plan refers to numbers of sets. With an establishment of 160 student midwives being reduced to 140, it would seem reasonable to take a figure of 144 as being a suitable one to work with in order to compare different schemes, since all of the numbers of sets being considered will divide into it. This figure even flatters a bad solution, since a plan which allows one set on SC at a time will let the minimum drop from six girls to five from time to time if the establishment falls to 140. (Remember that the previously accepted optimum number was ten.)

Using the figure of 144, then, each set will comprise six trainee midwives. By presenting the staffing levels in terms of numbers of sets on duty (See Table 8-1) the variation is never more than plus or minus one set from the norm, and on superficial examination this gives the impression of being fairly even. It is only when each figure is multiplied by six that the position becomes clear, with both Special Care and Community section varying between twelve and six girls, and Ante-natal being eighteen and twelve, to name the wards which have the greatest proportional variation.

The staff at the Simpson were happy to adopt any system which gave some relief from the complexity of individual week by week planning, so a plan of this form which gave the impression of extreme simplicity combined with albeit illusory evenness of staffing would appear most attractive, even if the transition period from one system to another promised to be chaotic.

8.6 Proposal for an improved standard schedule

The Black system is admirable in its simplicity, and provides good training schedules, but falls down badly in the daytime provision of staff. The analysis detailed above with regard to the Special Care Unit suggested that it would be worth trying to construct a system with a greater number of sets, each of a smaller size. SC was considered since it had tighter constraints than any other type of ward within the framework of the type of system used by Black, and had the worst staffing pattern as a result. Having shown that the problem could be resolved with regard to this ward by increasing the number of sets it seemed likely that the other wards would benefit as well.

The task as it now appeared was to design a system that, while smoothing out the irregularities of the Black system, still preserved its advantages. It would be an advantage to keep the structure as similar to the Black system as possible to facilitate the transition from one system to the other, causing as few variations from the optimum as possible during the interim.

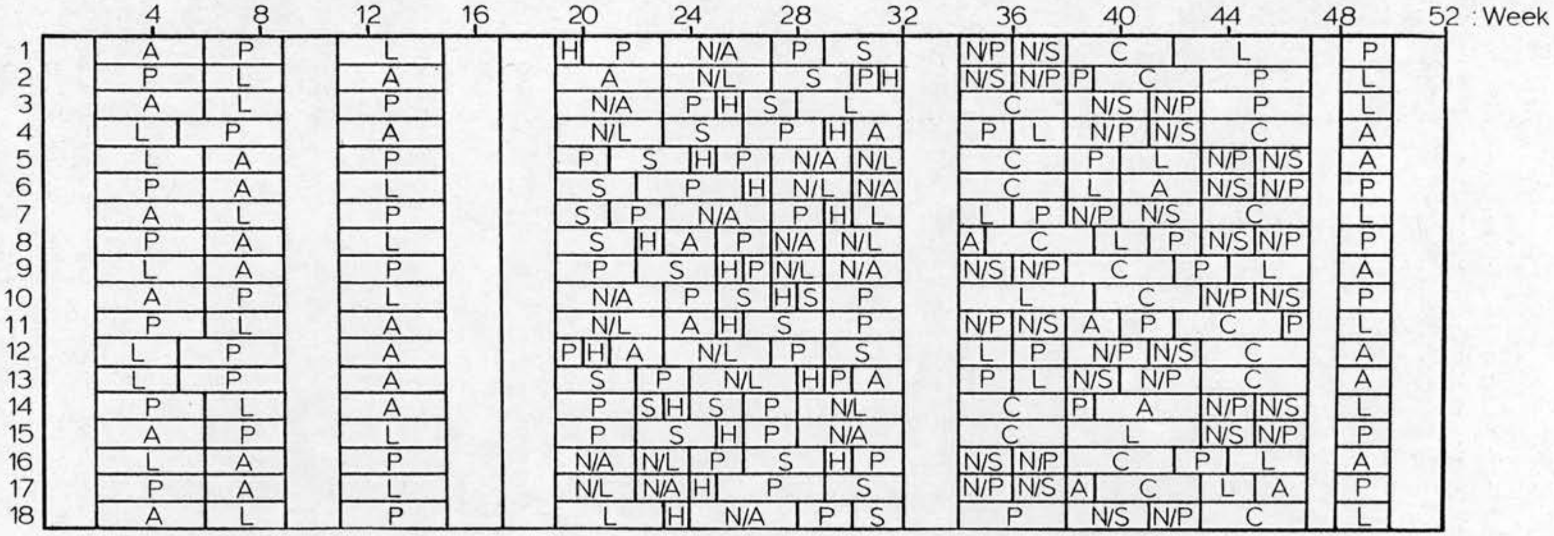
In the light of what had been discovered while investigating the SC problem it was decided to adopt an eighteen set instead of a six set system. The blocks of study have been retained unchanged, as have the first and third periods of leave. The patterns of night duty are directly related to those in the Black system, as these are successful features of that method. The balance of the parts of the course when each girl visits each ward are kept very similar, to aid a possible transition from one method to the other. In other words, if under the Black system community work was always done between weeks 35 and 47, then this rule would be retained.

At first, Black's six sets were adopted as the first six of the eighteen, and even after subsequent modification to relate to the other twelve they have remained very similar.

In designing the system the SC weeks and night duty were blocked into the framework first, along with community schedules. Weeks 3 to 9, 12 to 15, and 49 and 50, are used exclusively for Ante-natal, Post-natal, and Labour Ward, and moreover they are thirteen in number. It is possible to plan them so that they are self contained, since between them they cover each week of a quarter exactly once and once only. Having designed a system for them which is internally balanced and provides completely even staffing the scheduler is then left with the gaps in the two thirteen week periods from weeks 20 to 32 and weeks 35 to 47 inclusive. If these proved to be impossible to fill evenly then it was necessary to return to the thirteen week group of Ante-natal, Post-natal and Labour Ward and make changes, and once or twice it was even necessary to change some of the arrangements for night duty and community work. Each stage was carried out in close consultation with Miss Jamieson, the Divisional Nursing Officer of the Simpson (formerly called the Matron). Finally a system was devised which reached the optimum staffing levels theoretically possible with an eighteen set plan. The training schedules provided are similar to and at least as good as those under the Black system. Table 8-6 shows the plan which was eventually arrived at.

8.7 Comparison of the Black system and the proposed system

The comparitive figures for staffing under the Black system and the proposed new one are given in Table 8-7 , and are represented graphically in Figure 8-6. The improvement in the regularity of the staffing does not have to be discussed point by point - the figures speak for themselves. In response to an enquiry from Miss Jamieson as to whether the plan caused each girl to change more frequently from one ward to another, the average number of times each girl started a new ward in the course of the year was calculated. For the Black system the figure is 13.0; for the proposed system 13.3. However, this figure does not take into account one or two subtleties of the proposed system. To quote an example; In the Black system, set 3, the girl is on LW for weeks 31 and 32, block for two weeks, Post-natal for two weeks, then LW for two more weeks. A similar situation obtains for the same weeks for set 7 under the proposed system. Here the pattern is LW for weeks 31 and 32, block for two weeks, then straight back to LW, without an intervening visit to another ward. This is followed by Post-natal for two weeks, but instead of yet another change of ward, the next change is to night duty on the same ward. This benefit applies to eight of the eighteen



1	2	3	4	5	6	7	8	9	10	11	12	13	: Week
16	16	16	16	16	16	16	16	18	16	16	16	16	: A = Antenatal
28	28	28	26	28	28	26	28	28	26	26	26	28	: P = Postnatal
18	18	18	20	20	18	20	20	20	20	18	18	18	: L = Labour Ward
8	8	8	8	8	8	8	8	8	8	8	8	6	: S = Special Care
12	12	12	8	12	12	12	10	10	12	12	12	8	: C = Community
6	6	6	6	6	6	6	6	6	6	6	6	6	: N/A (Nights)
6	6	6	6	6	6	6	6	6	6	6	6	6	: N/P (Nights)
6	6	6	6	6	6	6	6	6	6	6	6	6	: N/L (Nights)
6	6	6	6	6	6	6	6	6	6	6	6	6	: N/S (Nights)

TABLE 8-6

Proposed new system

Table shows pattern for each quarterly intake, divided into eighteen sets. Figures on the left show staffing levels with two nurses per set.

Antenatal

B	12	18	18	18	18	18	18	18	18	12	12	12	18
P	16	16	16	16	16	16	16	16	18	16	16	16	16

Postnatal

B	24	24	30	24	24	30	30	30	24	24	30	30	30
P	28	28	28	26	28	28	26	28	28	26	26	26	28

Labour Ward

B	18	18	18	24	24	18	18	18	18	18	18	18	18
P	18	18	18	20	20	18	20	20	20	20	18	18	18

Special Care

B	6	12	6	12	6	6	6	6	12	6	12	6	6
P	8	8	8	8	8	8	8	8	8	8	8	8	6

Community

B	12	12	12	6	12	12	12	12	12	12	12	12	6
P	12	12	12	8	12	12	12	10	10	12	12	12	8

TABLE 8-7

Comparison of staffing between Black system (B) and proposed system (P) -weekly levels for one quarter

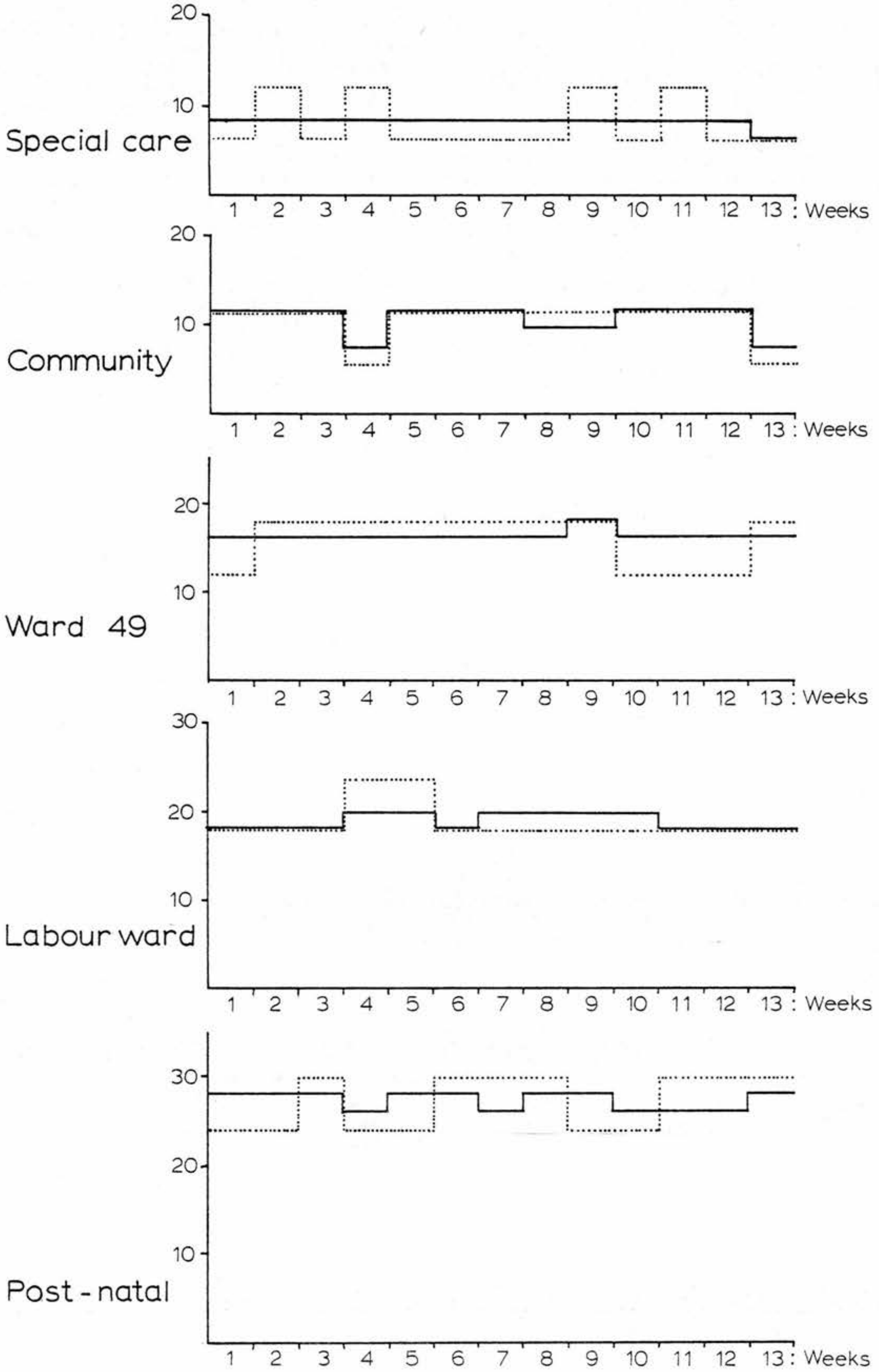


FIGURE 8-6

Comparison of staffing levels between Black system (··········) and proposed system (———)

Vertical axis = Number of student nurses on duty each week sets, and twice in two cases. When this is taken into account the revised figures are:

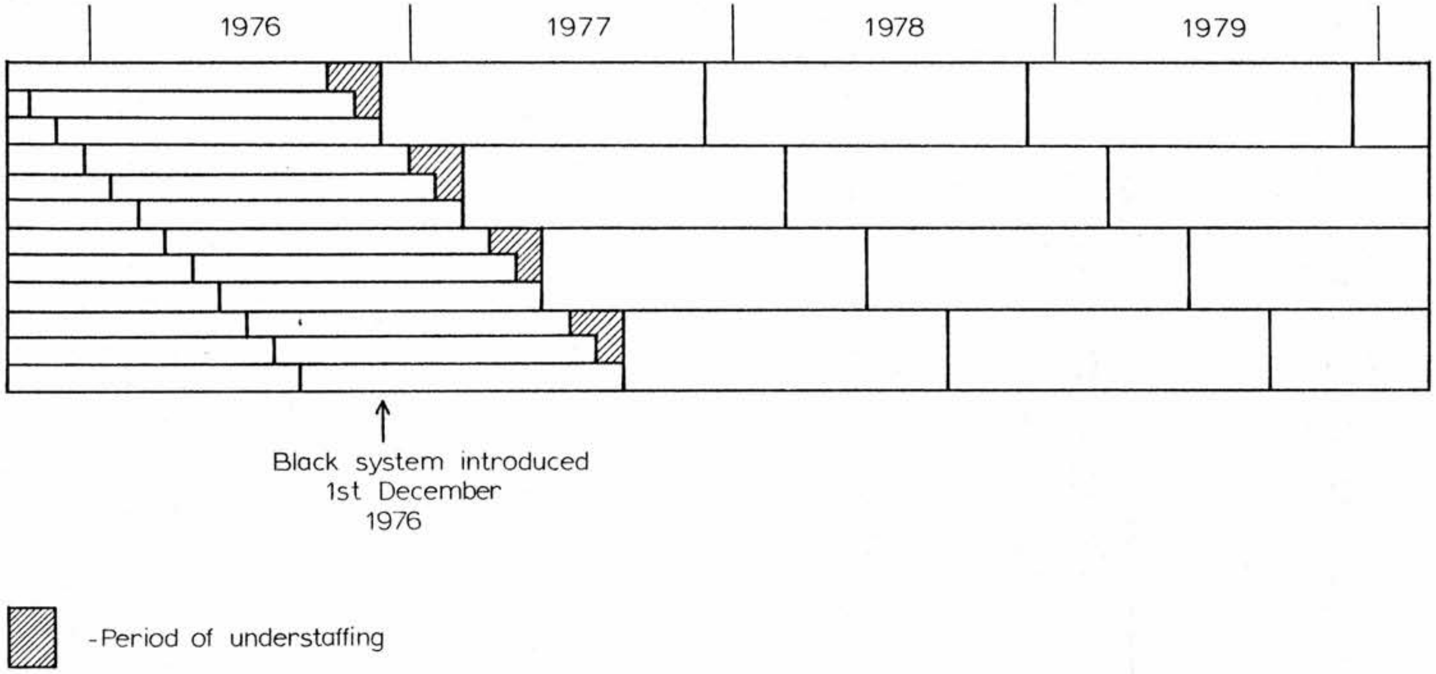
Average number of changes under the Black system - 13.0

Average number of changes under the new system - 12.7

Thus it can be seen that the new system has a slight advantage over the Black system in terms of quality of schedules as well as having a great advantage in terms of evenness of staffing. The only issue remaining to be considered is whether the disruption caused in the changeover period would be great enough to outweigh the advantages of the proposed system.

8.8 Changeover to the new system

In fact the two systems are very similar. It was stated in Section 8.6 that the Black system was adopted with minor modifications as the schedule for the first six sets of the eighteen. As a result the two systems are highly complementary and a change from one to the other would provide the minimum of disruption. Figure 8-7 shows the reason for the great amount of disruption caused when the Black system was adopted. Instead of replacing each group as it left it was necessary to wait for the start of each new quarter, when a larger group was admitted. This caused great under-staffing prior to each quarterly intake as can be seen from the diagram. It can also be seen that a change from the Black system to the proposed system would provide no problem of this nature since both systems involve quarterly intakes. The next point to be considered is the week by week staffing provided for each ward during the transition period. It is proposed that the change would occur as each new intake arrived - they would start on the new system, but girls presently on the Black system would complete their courses according to it. The change would



Small blocks represent intakes under the 1973 system ; the large blocks represent quarterly intakes under the revised system. The proposed alternative system would fit in without a break from any of the quarterly intake starts. See Table 8-8 for staffing levels during the transitional period.

FIGURE 8 - 7

Changeover from 1973 to Black system

Antenatal

B	12	18	18	18	18	18	18	18	18	12	12	12	18
3/4 B+1/4 P	12	18	18	18	18	18	18	18	18	12	12	12	18
1/2 B+1/2 P	12	18	18	18	18	18	14	14	16	16	16	16	18
1/4 B+3/4 P	12	18	18	18	16	16	14	14	18	18	18	16	16
P	16	16	16	16	16	16	16	16	18	16	16	16	16

Postnatal

B	24	24	30	24	24	30	30	30	24	24	30	30	30
3/4 B + 1/4 P	24	24	30	24	24	30	30	30	24	24	30	30	30
1/2 B + 1/2 P	24	24	30	24	24	30	28	28	28	26	26	26	28
1/4 B + 3/4 P	32	26	24	20	26	30	28	28	28	26	26	26	28
P	28	28	28	26	28	28	26	28	28	26	26	26	28

Labour Ward

B	18	18	18	24	24	18	18	18	18	18	18	18	18
3/4 B+ 1/4 P	18	18	18	24	24	18	18	18	18	18	18	18	18
1/2 B+1/2 P	18	18	18	24	24	18	20	20	20	20	18	18	18
1/4 B+ 3/4 P	18	18	20	26	22	16	20	20	20	20	18	18	18
P	18	18	18	20	20	18	20	20	20	20	18	18	18

Special Care

B	6	12	6	12	6	6	6	6	12	6	12	6	6
3/4 B +1/4 P	6	12	6	12	6	6	6	6	12	6	12	6	6
1/2 B +1/2 P	6	12	6	12	6	6	8	8	8	8	8	8	6
1/4 B +3/4 P	8	8	8	8	8	8	8	8	8	8	8	8	6
P	8	8	8	8	8	8	8	8	8	8	8	8	6

Community

B	12	12	12	6	12	12	12	12	12	12	12	12	6
3/4 B +1/4 P	12	12	12	6	12	12	12	12	12	12	12	12	6
1/2 B + 1/2 P	12	12	12	6	12	12	12	12	12	12	12	12	6
1/4 B+ 3/4 P	12	12	12	6	12	12	12	12	10	12	12	12	8
P	12	12	12	8	12	12	12	10	10	12	12	12	8

TABLE 8-8

Staffing levels during transition period

P = Proposed system, B = Black system. Each row shows weekly staffing for one quarter, changeover lasting for one year. The figures marked are the only cases where staffing is higher or lower than the extremes of the Black system.

therefore take place over three quarters. Table 8-8 shows the staffing levels for all wards over the transitional year. Of 154 ward/weeks there are only four where the staffing level is higher or lower than that under the Black system. In every other case the staffing stays the same or improves at each new transitional quarter. All of these figures are impressive, but once again the critical case of the Special Care Ward stands out.

This would seem to negate the only criticism which could be levelled at the scheme - that it would be a nuisance to change to it after the disruption caused during the previous change. These figures amply demonstrate that, far from being a nuisance, it would improve the staffing situation progressively from one quarter after its implementation.

CHAPTER 9

CONCLUSIONS

The conclusions presented here will offer comparisons between the different solution methods available according to the same scheme as used in the body of the thesis. The main classification to be made will be between continuous and discrete methods of solution, and each of these will be assessed under a number of categories. In the case of continuous solution methods, manual and heuristic computer model approaches will be compared. The categories of discrete problem methods are manual, computer heuristic and computer optimising.

9.1 Continuous solution methods

A short comparison of manual and computer methods has already been made at the end of Chapter 4, and the main points it makes will be reiterated here. In certain circumstances the overall pattern of scheduling is too varied from one year to the next to permit the use of a repeating fixed schedule. If this is the case then it is necessary to deal with the problem on a continuous basis. It was hoped that a model could be constructed which would take over the manual task of scheduling, and by virtue of its speed and efficiency could be made to run for a duration representing a timescale of many weeks ahead, thereby identifying future bottlenecks in training in sufficient time for action to be taken to prevent them. A model was constructed which tackled the full-size scheduling problem, and although it started by producing tolerable solutions for a few weeks it gradually deteriorated, designing progressively less satisfactory solutions. This indicated that there was some sort of imbalance in the scheduling rules, but one whose subtle nature had escaped definition or quantification. In other words if a human allocator and the model were given the task of allocating for one week ahead, the model's solution would appear to be at least as good as the human one. However if both had to allocate for up to ten weeks for instance, the model would tend to be trapped by bottlenecks which resulted from its own past allocations, while the human would tend to be aware of an overall pattern even while doing the week by week allocations, and would so avoid this problem. To quote an example, it could be that staffing figures for a group of wards were coming predominantly from one set, but that none of that set had been on OP to date. A human allocator would tend to notice this coincidence, but the model might continue to use girls from the same set for duties on a selection of other wards. Staffing figures might be ideal, and all would be well until the set in question came to the end of whatever half of the course they were on. At that point, the model would be obliged to put all of them on OP at once, regardless of the overstaffing which that might cause.

It is interesting to note that the very complexity of the problem, with regard to variable set size and unexpected drop-outs, which makes the problem such a chore for the human allocator, is also the characteristic which makes the poor allocator's manual methods of solution the most efficient and successful.

Experiment with the small scale model led to an improvement in the week by week allocation methods, and the algorithms used managed to achieve a state of steady staffing whose deviation from the ideal was not excessive. However these did not have to deal with the conflict caused by a girl changing from day to night duty while on the same ward, and so their improved week-by-week methods would still have failed to combat the overall imbalance found when using the full-scale data.

The ability to work ahead for some time was one of the aspects of the model which was anticipated to be an advantage. However this proved not to be the case in practice. If the program did identify a bottleneck in the future it was not capable of identifying the cause or of re-allocating in such a way as to prevent it from happening. At this stage a human allocator would have to step in and start altering the program's solutions. In other words, at the last resort, it would be necessary to fall back on manual methods. Although the model could perform a run forward for several weeks it would have to be re-run each week to take account of variations in the expected data such as

(1) Absenteeism

(2) Drop-outs

(3) Variations in the expected number of arrivals for new sets.

(4) Applications for short refresher courses.

As a result most of its ability to run ahead on projected data would be lost.

Consider now its ability on a week-to-week basis. As far as scheduling for one week ahead is concerned it could on occasion be more efficient than a human allocator. It would be faster, and would be incapable of overlooking any aspects of any trainee's course. For instance it would not be able to put a trainee into a situation where insufficient weeks were left for that girl to visit all of the required wards. However any short term advantages of this nature that it might possess are overridden by the problems of the long term imbalance.

The second adverse aspect of its short term operation is that the data manipulation involved with running the program is complex. This is no great disadvantage if the program only had to be run once a year, but since a weekly run would be needed to take account of unexpected fluctuations in nurse supply as listed above, the time taken to set the program up does assume some importance. Even in normal circumstances it would almost be as quick to allocate manually as it would be to feed into the computer all of the information required to update the attendance status. Any on-the-spot alterations made to the schedule to take account of last minute crises would also have to be amended on the computer's files, and a listing would have to be given of the staffing at the time to check that all alterations had been made correctly. If the computer did make allocations which were unsatisfactory, then it could be a very lengthy business to replace them with a better manual allocation.

On balance, it would seem that a computer model which attempts to solve a problem of this nature by emulating the proccesses carried out by its human allocator can only improve on the previous allocator in terms of short-term efficiency and speed. If these advantages are negated, as described above, then its lack of flexibility is a real drawback, and for this reason it was decided not to pursue this line of enquiry any more but to consider what could be achieved by a different approach.

9.2 Discrete solution methods

By selecting a discrete period of time reaching for a certain number of weeks into the future it is possible to define the problem in such a way as to give the computer a greater advantage over a human allocator. Here the problem is considered in its entirety instead of one week at a time, and this increased combinatorial complexity favours the speed and thoroughness of the computer. It is now possible to introduce the possibility of optimising the solution according to certain criteria for this fixed time period. Heuristic methods can still be used, and were tried with some success, but they can be of a much more elegant nature when the problem is being considered as a whole than they were when only portions of it were being considered at a time. What of the previously stated arguments against the use of the computer? Firstly, in this case time is not an object, since the program would only need to be run infrequently - for instance it could be designed to deal with thirteen weeks at a time. How then would the objection be countered that variation in staffing levels caused unexpected fluctuations? With the previous type of model, of a continuous nature, every girl had to be included, and a variation one week would cause repercussions in future weeks. Any manual intervention would then have to be fed into the computer to allow it to compensate. With the discrete system it would be possible to produce a schedule for the future few weeks which would be a starting point for manual alterations. If a girl did leave during her course, manual changes could be made to the overall schedule to compensate, but it would not be necessary to feed this information back into the computer since the scheduling would not be interactive. It would merely mean that a slightly different set of data would be fed into the computer for its next run. One policy might be to run the program as soon as the number of arrivals was known for each new intake, or alternatively several runs could be made in advance for each of a possible range of arrival numbers.

The question at this stage was whether techniques existed which were powerful enough to deal with a problem of this nature and size. The problem as defined by (1.1-4) has $(I \times J \times Q \times T)$ "a" variables, describing the schedules, and $(J \times T) + (I \times Q) + I$ constraints. If we consider the case of a problem where there are 140 girls, 8 jobs, 100 schedules and 20 weeks being considered we find that the dimensions of the problem are as follows:

Number of variables: $x = 14,000$

Number of constraints: One schedule per girl constraints = 140

Demand constraints = 160

300

Before comparing results obtained by different methods let us consider some of their relative merits and demerits. With reference to his research into the Travelling Salesman problem, Olav Solem[1] quotes the following as characteristic properties for the different solution methods considered here:

<u>Combinatorial programming</u>

Good: Great flexibility in constraint and object function formulation.
Computation can be stopped (resulting in a feasible solution).
Initiation solution can be used.

Bad: Data dependent with large scattering in computer times from one case to another.
Difficult to estimate computer times.
High computer times.

Heuristic programming

Good: Low computer times.

Easy to program.

Time independent.

Bad: No guarantee of discovering a specified solution.

Most of these characteristics apply here as well, with the notable exception of the second point relating to combinatorial programming. Because of the artificial nature of the costs a feasible solution is all that is required for this problem, but even the sub-gradient optimisation method had great difficulty in finding any. Also with this problem the advantage of being able to supply a starting solution applies equally to the heuristic method.

Let us consider first the results which were achieved using combinatorial programming.

9.3 Computer optimisation methods

Several methods were considered, and the one chosen as being most suitable for the present nurse scheduling problem was that of sub-gradient optimisation. The methods used are described in detail in Chapter 6. Held, Wolfe and Crowder's algorithm[2] was used with a variety of sets of data then modifications were made to it and their usefulness assessed. It was found that Held's use of an underestimate was inappropriate to the present problem, since it caused premature termination of the solution procedure. Further investigation revealed that Held's method would only work in cases where feasible solutions were easily attainable, and where it was possible to state an objective function value which, though non-optimal, was satisfactory as a solution. These conditions impose grave limitations on the usefulness of Held's approach. In an attempt to surmount these difficulties various procedures were adopted. The use of an overestimate proved to be effective in causing rapid convergence to the general vicinity of the optimal region, but further convergence was not assured. Several systems were then assessed which made use of a variable estimate. A method was devised whereby an overestimate was calculated automatically from the given data, eliminating the need for an informed guess. This procedure was able to work because the first crude overestimate was subsequently reduced to a fairly accurate figure in the light of the initial trial iterations. This procedure is described in Section 6.17, and is likely to be of use in the case of any problem where an underestimate is unsatisfactory.

Attempts to base the choice of λ (the step size multiplier) on various factors which related to the current position of π^n proved ineffectual and investigation of the reasons for this led to the demonstration of some fundamental weaknesses in the entire subgradient method. It was shown that only one gradient ever needed to be calculated, and that, in the extremely rare event of an iteration arriving at a value of π which lay on the line of intersection of two planes, a random decision as to which gradient to use would suffice. Thus the method is far less subtle than would appear from many published sources. Furthermore, examination of the properties required for convergence on the optimum to occur revealed the following facts:

(1) The method in practice relies critically on the accuracy of the estimate. In many kinds of problem this is impossible to determine precisely enough.

(2) It is extensively quoted that use of the correct sequence of λ will guarantee convergence despite the use of an

estimate which differs from the optimum. In fact it can be proved that convergence will occur after an infinite number of iterations if not before, a property which, though mathematically sound, is of extremely limited practical usefulness.

Some variations on Held's method were assessed, notably Grinold's steepest ascent procedure[3] and Camerini, Fratta and Maffioli's modified gradient direction[4]. These were all limited by the basic flaws in the system. In order to make an evaluation of the basic system, ignoring the accuracy of the estimate, a large number of trials were carried out where the estimate was exactly equal to the optimum value for each set of data. This information makes the problem considerably simpler to solve, since the step size should always be of exactly the right magnitude. However practical trials supported the view described above, that any number of iterations up to infinity might still be needed. For the optimum value to be reached any sooner than that depended to a large degree on chance, and the probability of its being found exactly after any given step decreases in inverse proportion to the number of solution planes associated with any given set of data.

Gue et al[5] stated in 1968 that computational results up to then offered little hope of solving zero-one problems with more than 100 variables. Although progress has been made since then, that limit seems to apply to this particular method, as seen by Table 6-5. The number of x variables is given by the product IQ and it can be seen that in cases where this figure exceeds 100 a solution is rarely found. Where a solution is achieved this is because the looseness of the problem causes the feasible region to the sufficiently large that the iterative procedure can arrive at it without any necessity for precise convergence.

9.4 Suggestions for further study

It has been demonstrated in this thesis that the sub-gradient method has two fundamental weaknesses:

(1) It is over-dependent on the accuracy of the estimate.

(2) It cannot guarantee convergence on the optimum in a finite number of iterations.

However the method does possess properties which it might be possible to exploit in a more efficient fashion than has been achieved to date. So far, sub-gradient methods have relied upon the gradient of the plane on which π^n lies in order to determine the direction and extent of the next move. The modified gradient direction method will tend to lessen the zig-zag effect in the case of a series of iterations alernating between two adjacent planes, and the net effect of this is that a direction is chosen which tends to be closer to that of the line of intersection of the planes. This is a slight improvement, but does not help the problem of convergence occurring only in an infinite number of iterations. The only way that this can be surmounted is by identifying the actual gradient and direction of a line of intersection of two planes and travelling along it until a new plane is encountered. At this point two or more new rays will be met, and the steepest of these should be selected. In simple terms this procedure could be described as moving along the edges of the multi-dimensional function rather than from point to point on its surface planes. Since any given ray is joined, by other rays, in a finite distance, to the optimum point or plane, it would seem that this procedure would be more specific in terms of the iterative path which it prescribed. Computationally it would be more demanding, but there must be many types of problem where the guarantee of achieving the optimum point in a finite number of iterations would justify the extra burden of calculation at each iteration. It should also be noted that this procedure would be able to work with a precise determination of the step size at each iteration, and would therefore require no estimate of the optimum value to be provided in advance.

9.5 Heuristic programming

Attempts were made to devise a heuristic system which could cope more satisfactorily with tight problems, and which was less dependent on the number of variables. The method which was adopted was similar in some ways to the cyclic co-ordinate descent algorithm described in Appendix D. To re-iterate the procedure involves the choice of an initial solution according to various criteria which will tend to make it a relatively good one. At all stages the current solution is feasible with regard to training patterns for each nurse and is meaningful in practical terms (i.e. it does not involve fractions of nurses or other impossible concepts). The factor which changes at each iteration is the staffing level on each ward for each week. Iterative schemes were tested which, in general, tended to ensure that the overall staffing pattern at each iteration was better than that at the previous iteration. Many different criteria were tested, but the basic scheme remained unchanged - that at each iteration one or more girls were moved from a schedule which was making an adverse contribution to the staffing pattern to one which benefitted the staffing levels. It was not possible to ensure an improvement in total deficits at each iteration, but gradually a system was evolved which tended towards the optimum region. The system is described in detail in Chapter 7, but to put it briefly it can be stated thus:

Find the change which will cause an improvement in the maximum deficit on any ward and week. If there is more than one candidate, choose the one which causes the greatest overall improvement in staffing deficits, or the least deterioration. If there is no candidate which improves the maximum deficit then choose that one which keeps it the same, while causing the greatest improvement in staffing deficits, or the least deterioration.

At first sight this seemed slightly arbitrary, in its insistence on a move being made, even to the detriment of overall shortage with no associated improvement in maximum shortage. It was only when detailed analysis was made, using graphs and three-dimensional models as well as experiment with varied data sets, that it became apparent that it possessed qualities not exhibited by other superficially similar iterative procedures.

As is the case with most heuristic methods, it cannot guarantee that a solution will be found - it is possible to construct sets of data which can trap the iterative procedure at a local minimum of overall and maximum staffing deficits. Although such sets of data can be contrived, it is surprising how infrequently the heuristic fails when using data sets more typical of real-life scheduling problems. Procedures are described which will permit the solution procedure to be continued. It is interesting to note that a very simple one, that of giving the problem a new starting solution after a local minimum has been encountered, will still guarantee that the optimal solution is eventually found in a finite number of trials, which is better than can be claimed for the sub-gradient method!

Finally, there is one parameter which can still be varied, and that is the number of girls who should be moved at each iteration - loosely equivalent to the concept of step size. Trials with different values are compared in Section 7.8.2.

9.6 Comparison of results obtained by combinatorial and heuristic methods

In theoretical terms the sub-gradient method will reach the optimal point after an infinite number of iterations if not before. The abandonment of the solution procedure and adoption of a new starting point will, at worst, lengthen the solution procedure.

The heuristic procedure will almost always take no more iterations than would be required by an explicit enumeration process, since it is possible to choose a new starting point in such a way as to benefit from the information gained in the previous attempt. However, the important comparison is not between the theoretical worst performances of each method, but between the general quality of solutions which they achieve in practice. Here the difference between the two systems is most telling: compare Tables 6-5 and 7-9.

The sub-gradient method was unable to solve any problem of over 80% tightness which had more than 32 "x" variables.

The heuristic method solved all problems which it was presented with of up to 90% tightness, the largest of which had 12,000 "x" variables. All of the problems which had been solved by the sub-gradient method were solved trivially by the heuristic method; typically in roughly one tenth of the number of iterations.

Within the field of scheduling there are many variations - problems which have superficial similarities may have to be formulated differently from each other, some being easily solved and others being effectively impossible within our present framework of knowledge. However the heuristic which has been described within this paper deals with a problem which is fairly general in its nature. In wide terms the problems which it could solve possess the following characteristics:

(1) Certain processes, j, have to be performed on each item, i.

(2) Certain sequences, q, determining order and duration of processes exist.

(3) There is one processing unit for each process. Each unit has a processing level d_{jt} which determines a preferred limit to the number of items it may process simultaneously in time t. This may be a minimum or a maximum level.

(4) It is desired to allocate a sequence, q, to each item, i, so that the overall shortage below the minima d_{jt} (or the overall excess above the maxima d_{jt}) may be minimised over all j and t.

It can be seen that this formulation describes many recognised problems, and it is felt that useful progress might be made in studying the applications to which it is suited.

9.7 Repetitive standard schedules

While this research was under way, a change in the external constraints upon the Simpson was made, as described in Chapter 2. A new system was adopted which uses a standard set of six schedules. There are now four intakes a year, and each girl on arrival is placed on one of the standard schedules, which is designed to overlap at quarterly intervals with the same schedules which are already being used by the girls who are part way through their courses. Administratively this is much simpler than the previous system, but the staffing levels which are provided by this method are inferior in many ways to those achieved by the more laborious system which used to be used of planning an individual schedule for each girl.

In Chapter 8 the drawbacks of the present system are detailed. A proposed replacement system is described which was designed to provide better staffing while maintaining the simplicity of the current system and providing a trouble-free transitional period. Its advantages are manifest, and it was greeted with enthusiasm by Miss Jamieson, the Divisional Nursing Officer. If sanctions are brought to bear to force the hospital to adopt the exact E.E.C. regulations then it will be necessary in the near future for the system to be altered again to satisfy the revised constraints. If this proves to be necessary then it is hoped that the proposed system will form a basis for the new solution.

9.8 Summary

As far as the specific problem at the Simpson is concerned, a repeating standard schedule is by far the easiest to operate, and in most ways the most satisfactory solution, but in designing one great care must be taken to ensure that the endeavour to simplify the problem does not cause previously accepted staffing levels to suffer. Simplicity can only be bought at a price, and unfortunately the people who benefit from the ease of using such a programme are not usually the ones who suffer from the staffing drawbacks. With these reservations, such a system must be regarded as the best available.

If the structure of the problem is such that, because of irregularity and variations in staffing supply, it is not possible to use such a system, then the manual method of allocating week by week, although time-consuming and difficult, still seems to produce the best results.

The heuristic scheduling method described in this thesis could be used to design a set of repetitive standard schedules as required by the Simpson, where the problem is well defined. However it is too difficult, in the light of unexpected variations in available staffing levels, and because of the uneven nature of our calendar, to use a computerised system like this as a continuing process. It would be necessary to employ a permanent member of staff to update the data in order to make best use of the method.

The implications and potential uses of the heuristic method extend to a much wider and more general field than is represented by this specific problem. The sub-gradient method has been extensively studied and widely used, but has been found wanting for certain classes of problem because of their size or complexity, or because of the difficulty of providing an accurate estimate as to the optimal value. It has been shown that the heuristic method will prove to be of particular use in these cases.

APPENDIX A

AIMS AND OBJECTIVES OF MIDWIFERY TRAINING

C.M.B. FOR SCOTLAND

AIM:

To prepare registered nurses to become proficient in the practice of midwifery and to function as a member of a team caring for the mother and child during pregnancy, labour, postnatal and neonatal periods.

OBJECTIVES:

1. To promote an understanding of the physiological, psychological, pathological and social events relating to pregnancy, labour, puerperium, and the newborn child.

   Course content to include -

   (a) reproductive and fetal anatomy.
   (b) physiology of reproduction - genetics.
   (c) physiological changes in pregnancy, labour, puerperium.
   (d) neonatal adaptation.
   (e) disorders of pregnancy.
   (f) complications of labour.
   (g) complications of puerperium.
   (h) neonatal disorders.
   (i) family planning.
   (j) teaching of parentcraft.
   (k) anaesthesia - general and regional.

2. To prepare the student to develop knowledge and skills in all aspects of care during pregnancy, labour, postnatal and neonatal periods.

   Skills include -

   (a) practical skills.

(b) observational skills.
(c) recording skills.
(d) skills to deal with emergency situations.
(e) communication skills.
(f) teaching skills.

The student should be able to -

IN PREGNANCY:

(a) give an acceptable standard of care to patients in pregnancy.
(b) take a complete history from the patient.
(c) perform a selective physical examination of the patient.
(d) receive and admit a patient to the antenatal ward.
(e) understand and assist in investigative procedures.
(f) understand and assist in therapeutic procedures.
(g) be aware of needs of pregnant woman and be able to meet these needs.
(h) be able to teach parents.

IN LABOUR:

(a) give an acceptable standard of care to patients in labour.
(b) receive and admit a patient in labour.
(c) evaluate the condition of the mother and fetus and take appropriate action.
(d) understand the use of specialised techniques and equipment.
(e) take necessary action in emergency situations.
(f) list the measures to relieve pain in labour.
(g) be proficient in the use of inhalational analgesia.
(h) perform an episiotomy under local analgesia.
(i) deliver a patient safely.
(j) conduct, under supervision, a spontaneous delivery.
(k) prepare the patient for operative procedures and assist in such procedures when required.
(l) document all necessary records and be aware of recognised lines of communication.

IN POSTNATAL WARD:

(a) give an acceptable standard of care specific to postnatal patients.
(b) receive and admit a mother and baby to postnatal ward.
(c) perform daily examinations of the mother and baby and take appropriate action.
(d) understand and assist in investigative and therapeutic procedures.
(e) take necessary action in emergency situations.
(f) provide an optimum environment for the mother and baby.
(g) understand the importance of infant nutrition.
(h) be aware of needs of postnatal women and be able to meet these needs.
(i) be able to teach postnatal mothers.
(j) recognise the need for ensuring continuity of care of mother and baby after discharge.

(k) list methods of family planning.
(l) initiate discussion on family planning.

IN NEONATAL UNIT:

(a) give an acceptable standard of care.
(b) preparation for reception of infants to the unit.
(c) evaluate the condition of the infant and take appropriate action.
(d) assess behaviour of infant and detect deviations from normal.
(e) understand infant nutrition and hydration.
(f) know alternative methods of feeding babies.
(g) understand the use of specialised equipment in the unit.
(h) understand and assist in investigative procedures.
(i) understand and assist in therapeutic procedures.
(j) know long term prognosis and recognise the need for ensuring continuing care following discharge.
(k) be aware of the importance of communication with parents, and exchange of information.

3. To prepare the student to acquire a knowledge of legislation applicable to midwifery practice and an understanding of social and environmental factors affecting the childbearing woman and her infant.

Course content to include -

Statutory and professional organisations.
Acts and regulations relevant to maternity services.
Notification and registration of births.
The unsupported mother.
Financial and social benefits; voluntary services.
Adoption; foster parents; child minders.
Genetic counselling.
Non-accidental injury.
Handicapped Children.
Vital statistics.

4. To assist the student to develop skills in teaching.

Course content to include -

Instruction in use, value and limitations of teaching methods and teaching aids.
Parentcraft teaching.
Participation in parentcraft teaching.

5. To stimulate interest in research techniques -

(a) initiate midwifery research.
(b) participate in research projects.

LECTURES

OBSTETRICS:	
Reproductive physiology; genetics; maternal adaptation to pregnancy	3
Antenatal pathology including sexually transmitted diseases	7
Labour - physiology and pathology	6
Puerperal complications	1
Family planning	1
Infertility	1
PAEDIATRICS:	10
1. Neonatal adaptation	
2. Resuscitation of newborn	
3. Congenital abnormalities	
4. Metabolic disorders	
5. Respiratory disorders	
6. Neonatal infections	
7. Neonatal jaundice	
8. Low birth weight	
9. Normal and abnormal patterns of development	
10. Non-accidental injury	
ANAESTHESIA:	
General and regional; Resuscitation of patients	3
COMMUNITY HEALTH and relevant LEGISLATION	4
	36

APPENDIX B

CENTRAL MIDWIVES BOARD FOR SCOTLAND

REVISION OF TRAINING

These recommendations of the ad hoc Committee set up to consider student midwife training have been approved in principle by the Central Midwives Board for Scotland.

LENGTH OF TRAINING

Following registration as a nurse the length of midwifery training should extend to one year. If the EEC directives require two years for acceptance throughout the community, the second year should consist of a planned in-service programme of consolidation, but midwifery training should not be extended to two years.

REVISION OF CURRICULUM

1. All programmes should follow a similar plan with conformity of timing and content to a suggested programme (Attached).
2. There should be an introductory block of at least one week.
3. Subsequently the choice of "blocks" or study days should be left to the individual training school. If the block system is adopted nine weeks should be devoted to theoretical instruction. The two final blocks should be used for revision. 45 study days would equate to 9 weeks of block system.

   Total Hours of Instruction - 300

   The instruction day should be from 0900 - 1630 hrs. but this could be varied slightly so long as the number of hours remained the same.

4. Consultants' Participation

   Consultants' periods of instruction may be in lectures or clinical teaching sessions, a total of 36 hours.

5. Community Experience

   Four weeks community experience should be retained. Programmes must be well organised and submitted to the Board for approval. The experience must be related to obstetrics and paediatrics and use made of suitable General Practitioner Units and Health Centres.

6. Night Duty

   The time spent on night duty should be reduced to eight weeks. Preferably the first experience of night duty should not be given before the 12th week. The longer period should be in the second half of training when greater responsibility may be given.

7. Holidays

   Students cannot be given a choice of holidays. They should, however, be informed of the dates of holidays at the beginning of the course.

8. Starting Dates

   Courses should commence three monthly - on 1st March, 1st June, 1st September and 1st December each year.

9. Clinical Experience

   Antenatal - 12 weeks

   4 weeks to be spent in antenatal outpatient departments to cover all aspects of outpatient care.

   Labour Ward - 10 weeks

Neonatal Intensive Care Unit - 6 weeks

Attendance at Clinics

Attendance at a clinic should be for
20 hours at antenatal clinics
6 hours at parentcraft classes
4 hours at child health clinics
2 hours at family planning clinics

All clinics must be approved by the Board. They may be in hospital or at a Health Centre.

Labour Ward Experience

1. Students should attend a patient in labour on at least 20 occasions, if possible being present at the actual delivery.
2. The number of patients personally delivered must be not less than 15 and preferably exceed this requirement.
3. In the last twelve weeks of training each student must conduct the delivery by another student of five patients. Supervision of this procedure must be carried out by the midwife in charge of the Labour Ward or by sister midwives approved by her.

10. EXAMINATIONS

Continuous Assessment

Assessment by the teaching and clinical staff of the training school should take place throughout the training period and should include -

(a) assessment of clinical performance
(b) performance in theoretical work including test marks
(c) clinical/oral examination in last twelve weeks

If in the opinion of the teaching and clinical staff the student is not making progress she must be counselled about the advisability of repeating certain experience or of discontinuing training. Only those who reach a satisfactory standard as a result of this continuous assessment can be presented for the State Examination, which will consist of -

Written Examination

No change from the present pattern.

Oral Examination

This should be of 30 minutes duration and be in part clinically orientated.

Dates of Examinations

These will be held as at present, namely commencing on the third Tuesday of February, May, August and November.

SUGGESTED PROGRAMME - MIDWIFERY TRAINING

(47 weeks exclusive of holidays)

1st		6th		11th		18th		23rd	
Orientation - 1 week	Antenatal and Postnatal Wards - 4 weeks	Block 1 week	Labour Ward and Special Care Unit - 4 weeks	Block 1 week	Clinical Experience - All departments - 6 weeks to include night duty - 3 weeks	Block 1 week	Clinical Experience - 4 weeks	Block 1 week	Clinical Experience - 3 weeks Holidays - 3 weeks

30th		39th		45th		49th		50th
Block 1 week	Community - 4 weeks Night Duty - 4 weeks	Block 1 week	Clinical Experience - 5 weeks	Block 1 week	Clinical Experience - 3 weeks	Block 1 week	Clinical Experience - 2 weeks	Holidays - 2 weeks + 5 days

CLINICAL EXPERIENCE - 40 weeks - 1600 hours

1.	Antenatal including antenatal outpatient department -	12 weeks -	480 hours
2.	Labour Ward	10 weeks -	400 hours
3.	Neonatal Intensive Care Unit	6 weeks -	240 hours
4.	Postnatal	8 weeks -	320 hours
5.	Community	4 weeks -	160 hours
			1600 hours

STUDY BLOCKS - 9 weeks - 300 hours
(including 36 hours instruction from consultants)

(Instruction Day - 9.00 - 16.30).

APPENDIX C

LOTHIAN HEALTH BOARD

SOUTH LOTHIAN COLLEGE OF NURSING AND MIDWIFERY

MIDWIFERY TRAINING PROGRAMME

Agreement in principle has already been reached with the Central Midwives Board for Scotland for the grouping of the Midwifery Schools at the Simpson Memorial Maternity Pavilion and the Elsie Inglis Memorial Maternity Hospital. The date of implementation is to be 1st December, 1976. The enclosed paper outlines the proposed new programme.

SUBMISSION

1. AIM

"To prepare Registered Nurses to become proficient in the practice of midwifery and to function as a member of a team caring for the mother and child during pregnancy, labour, postnatal and neonatal periods".

2. PROFESSIONAL QUALIFICATION

Registered Central Nurse or State Registered Nurse

Registered Sick Children's Nurse

3. RESPONSIBILITY FOR RECRUITMENT

Senior Midwife Tutor

South Lothian College of Nursing and Midwifery

4. APPLICATION AND SELECTION

Application - Senior Midwife Tutor

South Lothian College of Nursing and Midwifery

Selection - Joint responsibility between the Senior Midwife Tutor

South Lothian College of Nursing and Midwifery

Divisional Nursing Officer (Midwifery),

South Lothian District

5. NUMBER OF INTAKES AND STUDENT MIDWIVES

Intakes 4 - 1st March, June, September and December

6. ALLOCATION

This will be the responsibility of the Senior Midwife Tutor in consultation with the Divisional Nursing Officer (Midwifery).

Experience will be divided between the two hospitals, each student having experience in both. Approximately 25% of the Student Midwives available for clinical experience will be placed at the Elsie Inglis Memorial Maternity Hospital.

7. THEORETICAL TEACHING

This will be carried out at the Simpson Memorial Maternity Pavilion.

8. EDUCATIONAL PROGRAMME

This is based on the requirements as laid down by the Central Midwives Board for Scotland, outlined below:

Course content -

(a) reproductive and fetal anatomy
(b) physiology of reproduction - genetics
(c) physiological changes in pregnancy, labour, puerperium
(d) neonatal adaptation
(e) disorders of pregnancy
(f) complications of labour
(g) complications of puerperium
(h) neonatal disorders
(i) family planning
(j) teaching of parentcraft
(k) anaesthesia - general and regional

Lecture time - 36 hours:

OBSTETRICS

Reproductive physiology; genetics; maternal adaptation to pregnancy	3 hours
Antenatal pathology including sexually transmitted diseases	7 hours
Labour - physiology and pathology	6 hours
Puerperal complications	1 hour
Family planning	1 hour
Infertility	1 hour

PAEDIATRICS/

PAEDIATRICS

Neonatal adaptation	1 hour
Resuscitation of newborn	1 hour
Congenital abnormalities	1 hour
Metabolic disorders	1 hour
Respiratory disorders	1 hour
Neonatal infections	1 hour
Neonatal jaundice	1 hour
Low birth weight	1 hour
Normal and abnormal patterns of development	1 hour
Non-accidental injury	1 hour

ANAESTHESIA

General and regional; resuscitation of patients	3 hours
COMMUNITY HEALTH and relevant LEGISLATION	4 hours
	36 hours

Tutorial time - 264 hours

Biological sciences related to Obstetrics including applied Anatomy and Physiology	25 hours
Obstetrics	125 hours
Paediatrics	30 hours
Community and Social Services	10 hours
Parentcraft	6 hours
Analgesia and Anaesthesia	3 hours
	199 hours

Remaining time - 65 hours

Seminars

Symposia

Discussion

Projects and Research

Clinical demonstrations - e.g. Ultrasound

Case presentation

Revision

Assessments and examinations

STUDY BLOCKS

These will consist of 9 weeks or 300 hours of instruction including consultants' lectures. The Study Blocks will occur at weeks 1, 2, 10, 11, 18, 19, 33, 34 and 48. Each instruction day will run from 0830 hours until 1600 hours.

9. CLINICAL EXPERIENCE

Diagram 1

Demonstrates the sequence of theoretical instruction and clinical experience. Each class is divided into six sections or streams which progress through the clinical areas. The diagram shows one complete student midwife programme as well as the overall distribution of the students throughout the clinical areas in each week. As each class is divided into six sections the totals indicated at the foot of the diagram represent 'sixths' of classes, e.g.

Week 1:

Day Duty = 12/6ths or two complete classes including -

2/6ths in antenatal
4/6ths in postnatal
3/6ths in labour ward
1/6th in special care unit
2/6ths in community

Night Duty = 4/6ths

Diagram 1/

DIAGRAM 1

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<tr><td>N/A</td><td colspan="2">P</td><td colspan="3">S</td><td colspan="2" rowspan="6">▨</td><td colspan="2">N/P</td><td colspan="2">N/S</td><td>C</td></tr>
<tr><td>N/L</td><td colspan="3">S</td><td colspan="2">P</td><td colspan="2">N/S</td><td colspan="2">N/P</td><td>A</td></tr>
<tr><td colspan="2">S</td><td colspan="2">P</td><td colspan="2">L</td><td colspan="2">P</td><td colspan="2">L</td><td>N/S</td></tr>
<tr><td>H</td><td colspan="3">P</td><td colspan="2">A</td><td colspan="2">L</td><td colspan="2">P</td><td>N/P</td></tr>
<tr><td>P</td><td colspan="3">N/A</td><td colspan="2">N/L</td><td colspan="4">C</td><td>P</td></tr>
<tr><td>H</td><td colspan="3">N/L</td><td colspan="2">N/A</td><td colspan="4">C</td><td>L</td></tr>
</table>

<table>
<tr><td>40</td><td>41</td><td>42</td><td>43</td><td>44</td><td>45</td><td>46</td><td>47</td><td>48</td><td>49</td><td>50</td><td>51</td><td>52</td></tr>
<tr><td colspan="3">C</td><td colspan="5">L</td><td rowspan="6">▨</td><td colspan="2">P</td><td colspan="2">H</td></tr>
<tr><td colspan="4">C</td><td colspan="4">P</td><td colspan="2">L</td><td colspan="2">H</td></tr>
<tr><td colspan="2">N/S</td><td colspan="2">N/P</td><td colspan="4">C</td><td colspan="2">L</td><td colspan="2">H</td></tr>
<tr><td colspan="2">N/P</td><td colspan="2">N/S</td><td colspan="4">C</td><td colspan="2">A</td><td colspan="2">H</td></tr>
<tr><td>P</td><td colspan="3">L</td><td colspan="2">N/P</td><td colspan="2">N/S</td><td colspan="2">A</td><td colspan="2">H</td></tr>
<tr><td>L</td><td colspan="3">A</td><td colspan="2">N/S</td><td colspan="2">N/P</td><td colspan="2">P</td><td colspan="2">H</td></tr>
</table>

2	3	3	3	3	3	3	3	3	2	2	2	3	A)	
4	4	5	4	4	5	5	5	4	4	5	5	5	P)	
3	3	3	4	4	3	3	3	3	3	3	3	3	L)	DAY
1	2	1	2	1	1	1	1	2	1	2	1	1	S)	
2	2	2	1	2	2	2	2	2	2	2	2	1	C)	
1	1	1	1	1	1	1	1	1	1	1	1	1	N/A)	
1	1	1	1	1	1	1	1	1	1	1	1	1	N/P)	
1	1	1	1	1	1	1	1	1	1	1	1	1	N/L)	NIGHT
1	1	1	1	1	1	1	1	1	1	1	1	1	N/S)	
16	18	18	18	18	18	18	18	18	16	18	17	17		

A = Antenatal
P = Postnatal
L = Labour Ward
H = Holiday
C = Community
S = Special Care Unit
N/ = Night Duty on Relevant Unit

▨ = Study Block

Diagram II

Indicates the Total Clinical Experience obtained in each area, including Night Duty.

DIAGRAM II TOTAL CLINICAL EXPERIENCE

Section	Antenatal	Postnatal	Labour Ward	Special Care Unit	Community	Total Weeks
1	8 weeks (includes 4 weeks night duty)	12 weeks (includes 2 weeks night duty)	9 weeks	5 weeks (includes 2 weeks night duty)	4	38
2	8 weeks	12 weeks (includes 2 weeks night duty)	9 weeks (includes 4 weeks night duty)	5 weeks (includes 2 weeks night duty)	4	38
3	8 weeks (includes 4 weeks night duty)	12 weeks (includes 2 weeks night duty)	9 weeks	5 weeks (includes 3 weeks night duty)	4	38
4	8 weeks	12 weeks (includes 3 weeks night duty)	9 weeks (includes 4 weeks night duty)	5 weeks (includes 2 weeks night duty)	4	38
5	8 weeks (includes 3 weeks night duty)	12 weeks (includes 2 weeks night duty)	9 weeks (includes 2 weeks night duty)	5 weeks (includes 2 weeks night duty)	4	38
6	8 weeks (includes 2 weeks night duty)	12 weeks (includes 2 weeks night duty)	9 weeks (includes 3 weeks night duty)	5 weeks (includes 2 weeks night duty)	4	38

APPENDIX D

A brief resume of D. Michael Warner's paper "Scheduling Nursing Personnel According to Nursing Preference: A Mathematical Programming Approach", Operations Research 24, 1976 pp 842-856.

Warner's preface reads:

> "This paper formulates the nurse scheduling problem as one of selecting a configuration of nurse schedules that minimise an objective function that balances the trade-off between staffing coverage and schedule preferences of individual nurses, subject to certain feasibility constraints on the nurse schedules. The problem is solved by a cyclic co-ordinate descent algorithm. We present results pertaining to a 6 month application and compare hospital with computer results."

The algorithm finds near-optimal solutions. It starts with an initial configuration of nurse schedules, one each. It fixes all but one, i, and then searches Π_i (the set of feasible solutions for nurse i). The lowest present cost and best schedule configuration are updated if one of Π_i results in a lower schedule configuration cost than to date. When all Π_i have been tested, the cost has either dropped or stayed constant. This process cycles through all I nurses, and stops when I iterations are carried out without an improvement. Each set Π_i will always contain at least one feasible schedule, because of the way in which the feasibility sets are constructed.

At first those schedules are used which have the lowest dissatisfaction cost. If the feasibility region is viewed as the cartesian product of the feasibility regions $\Pi_1, \Pi_2, \Pi_3, \ldots, \Pi_I$, the algorithm is simply a cyclic co-ordinate descent algorithm along the co-ordinate directions Π_i. When 4 days are given off every

14 day pay period, π_i contains at most $\binom{14}{4} = 1001$ schedules. The number is reduced considerably when previous schedules, special requests and other feasibility set constraints are considered. Convergence is assured since the cartesian product contains a finite number of points, namely

$\prod_{i=1\to I} \|\pi_i\|$, where $\|\pi_i\|$ is the number of schedules in the set π_i

The procedure is as follows:

1. Determine π_i for each i
2. Calculate schedule pattern costs for each schedule $x^i \in \pi_i$ for $i = 1, \ldots, I$
3. Choose initial mix, let BEST be its cost. (Lowest cost schedule for each π_i)
4. Let $i = 1$, $K = \|\pi_i\|$, $k = 1$ and CYCLE = 0
5. Try k^{th} candidate schedule, x^{ik} in mix by removing present schedule for i. Let cost = TEST
6. If TEST < BEST go to 8
7. Let $k = k + 1$. If $k = K + 1$ go to 9 else go to 5
8. Let CYCLE = 0 and BEST = TEST. Insert x^{ik} in place of i's current schedule. Schedule mix now has best found so far. Go to 7
9. If CYCLE = I, stop. Otherwise $i = i + 1$ (if $i > I$, let $i = 1$) and let $K = \|\pi_i\|$, $k = 1$, and CYCLE = CYCLE + 1. Go to 5.

The problem is defined as:

$$\text{find } x^1, x^2, \ldots, x^I \text{ to min } \lambda \left[\sum_k f_k \left(\sum_i x_k^i \right) + \sum_k \sum_j h_{jk} \left(\sum_{i \in B_j} x_k^i \right) \right]$$

$$+ (1 - \lambda) \sum_i A_i \sum_n \alpha_{in} g_{in} (x^i)$$

$$\text{s.t. } x^i \in \pi_i \text{ , } i = 1, \ldots, I$$

where costs are: g_{in} (x^i) = cost of violating nonbinding constraint $n \in N_i$ of schedule x^i

α_{in} = weight given by nurse to violation of nonbinding constraint $n \in N_i$ (aversion co-efficient)

A_i = Aversion index (historical comparison of past schedules with nurse i's preferences)

k = 1 to 14 days

Group staffing level costs are f_k $(\sum_i x_k{}^i)$, where $x^i = (x_1{}^i, \ldots, x_{14}{}^i)$

B_j = index set of nursing subgroups j; J is index set of all subgroups

$m_k{}^j$ = min. no. of nurses on day k for subgroup j

$d_k{}^j$ = desired no. of nurses on day k for subgroup j

Staffing cost for violating these constraints is h_{jk} $(\sum_{i \in B_j} x_k{}^i)$

With a 4 nurse, 20 schedule problem the figures were as follows

Initial cost = 239.45

cost = 12.3 achieved in a CPU time of 0.367 on a CDC 6400

Optimum cost = 7.55 " " " " " " 10.509

With a 5-7 nurse problem with 200 schedules on average, CPU times for an acceptable reduction in cost were around 5 seconds.

It can be seen that this problem is of a much smaller scale than the scheduling problem considered in the body of the thesis. It is also much simpler in that the only states that each nurse can be in on a given day are on duty/off duty (vacation, request, Birthday, weekend, etc)/meeting/class. As far as costs are concerned the only states considered are on or off duty. This contrasts with the eight or more wards which are being simultaneously considered for 140 girls in the present problem.

APPENDIX E

Contents :

CHOICE - Program
CHOICE - Printout
X280 - Variables
X280 - Résumé
X280 - Program
X280 - Output
X108 - Program
X108 - Output
X108 - Abbreviated output
FASTNEW7(Heuristic method) - Program
FASTNEW8(" ") - Output

FILE IDENTIFIER: CHOICE

CHOICE - Program

```
%BEGIN
%INTEGER %ARRAY MATRIX(1:13,0:110), CHOICE(0:8), EXP(1:13,1:25)
%INTEGER I,J,COUNT,Y,Z,NO,COMP,A,HALF,CODE,LW,SC,OP,FORTYNINE
%INTEGER FLOAT,X,HOL,WEEK,K,K1,NIGHT,FIFTYFOUR,FN0,FN1,FN2,FN3,FN4
%INTEGER NEWVALUE
A=20


%INTEGERFN ROSTER (%INTEGER CODE,WEEK)
   %RESULT=MATRIX(CODE,WEEK)-(MATRIX(CODE,WEEK)//100)*100
%END

%INTEGERFN PLAN (%INTEGER CODE,WEEK)
   %RESULT=MATRIX(CODE,WEEK)//100-(MATRIX(CODE,WEEK)//10000)*100
%END

%INTEGERFN URGENCY (%INTEGER CODE,WEEK)
   %RESULT=MATRIX(CODE,WEEK)//10000
%END

%ROUTINE CHANGE ROSTER (%INTEGER CODE,WEEK,NEWVALUE)
   MATRIX(CODE,WEEK)=(MATRIX(CODE,WEEK)//100)*100+NEWVALUE
%END

%ROUTINE CHANGE PLAN (%INTEGER CODE,WEEK,NEWVALUE)
   MATRIX(CODE,WEEK)=(MATRIX(CODE,WEEK)//10000)*10000%C
   +(MATRIX(CODE,WEEK)-(MATRIX(CODE,WEEK)//100)*100)%C
   +NEWVALUE*100
%END

%ROUTINE CHANGE URGENCY (%INTEGER CODE,WEEK,NEWVALUE)
   MATRIX(CODE,WEEK)=(MATRIX(CODE,WEEK)-(MATRIX(CODE,WEEK)%C
   //10000)*10000)+NEWVALUE*10000
%END

%ROUTINE ROSTERS(%INTEGER L)
%CYCLE I=K,1,K1
   CHANGE ROSTER(J,I,L)
%REPEAT
%END



%ROUTINE ALLOCATE ROSTER(%INTEGER A,B,C,D,E,F,G,H)
K=A+1;K1=K+B+3
   ROSTERS(1)
K=K1+1;K1=K+C+3
   ROSTERS(2)
K=K1+1;K1=K+12
   ROSTERS(3)
K=K1+1;K1=K+D+3
   ROSTERS(4)
K=K1+1;K1=K+2
   ROSTERS(5)
K=K1+1;K1=K+E+3
   ROSTERS(6)
%IF F=1 %THEN CHANGE ROSTER(J,K1+1,7)
K=K1+F+1;K1=K+G+3
   ROSTERS(8)
K=K1+1;K1=K+H+9
   ROSTERS(9)
K=K1+1;K1=K+1
   ROSTERS(10)
%END


%ROUTINE FIND HALF(%INTEGER X)
COUNT=0;HALF=0
%CYCLE I=X,1,X+26
FN0=ROSTER(CODE,I)
   %IF FN0=5 %THEN ->35
```

```
   %IF FNO=10 %THEN ->40
   COUNT=COUNT+1
%REPEAT
35: HALF=1
40: HALF=2 %UNLESS HALF=1
%END


%ROUTINE CALCULATE FLOAT(%INTEGER X)
FIND HALF(X)
%IF HALF=1 %THEN%C
      %START
         NIGHT=0
         %CYCLF I=X,1,X+COUNT
            %IF ROSTER(CODE,I)=4 %THEN NIGHT=NIGHT+1
         %REPEAT
         LW=4;SC=4;OP=2;FORTYNINE=4
         %CYCLE I=1,1,25
            %IF EXP(CODE,I)=1`%THEN FORTYNINE=0
            %IF EXP(CODE,I)=6 %THEN LW=0
            %IF EXP(CODE,I)=7 %THEN SC=0
            %IF EXP(CODE,I)=8 %THEN OP=0
         %REPEAT
         FLOAT=COUNT-NIGHT-1-FORTYNINE-LW-SC-OP
      %FINISH
%IF HALF=2 %THEN%C
      %START
         LW=8;SC=6;OP=3;FORTYNINE=7
         %CYCLE I=1,1,25
            %IF EXP(CODE,I)=1 %AND FORTYNINE=7 %THEN FORTYNINE=3 %AND ->50
            %IF EXP(CODE,I)=1 %AND FORTYNINE=3 %THEN FORTYNINE=0 %AND ->50
            %IF EXP(CODE,I)=6 %AND LW=8 %THEN LW=4 %AND ->50
            %IF EXP(CODE,I)=6 %AND LW=4 %THEN LW=2 %AND ->50
            %IF EXP(CODE,I)=6 %AND LW=2 %THEN LW=0 %AND ->50
            %IF EXP(CODE,I)=7 %AND SC=6 %THEN SC=2 %AND ->50
            %IF EXP(CODE,I)=7 %AND SC=2 %THEN SC=0 %AND ->50
            %IF EXP(CODE,I)=8 %AND OP=3 %THEN OP=1 %AND ->50
            %IF EXP(CODE,I)=8 %AND OP=1 %THEN OP=0 %AND ->50
            50:
         %REPEAT
         FLOAT=COUNT-FORTYNINE-LW-SC-OP
      %FINISH
%END




COUNT=0
READ(CODE)
READ(NO)
WEEK=A+NO+1
X=WEEK
%CYCLE I=1,1,13
   %CYCLE J=0,1,110
      MATRIX(I,J)=0
   %REPEAT
%REPEAT
%CYCLE I=1,1,NO
      READ(NEWVALUE):CHANGE PLAN (CODE,I,NEWVALUE)
%REPEAT
%CYCLE I=0,1,8
      CHOICE(I)=0
%REPEAT
%CYCLF I=1,1,25
      EXP(CODE,I)=0
%REPEAT
Z=1;COMP=0;I=0
%CYCLE Y=1,1,110
      FN1=PLAN(CODE,Y);FN2=PLAN(CODE,Y-1)
      EXP(CODE,Z)=FN1 %UNLESS FN1=0 %OR FN1=FN2
   %IF EXP(CODE,Z)=6 %THEN COMP=COMP+1
   %IF COMP=2 %THEN %START
                       I=I+1
                       %IF'PLAN(CODE,Y+2)=6 %AND I=1 %THEN%C
                       EXP(CODE,Z+1)=6 %AND%C
                       Z=Z+1
                     %FINISH
      Z=Z+1 %UNLESS EXP(CODE,Z)=0
      %IF FN1=0 %AND FN2#0 %THEN->10
%REPEAT
```

```
10:

J=CODE
ALLOCATE ROSTER(A,0,1,0,0,1,0,2)
CALCULATE FLOAT(X)
FN3=ROSTER(CODE,WEEK)
%IF FN3=5 %THEN CHOICE(1)=10 %AND->15
%IF FN3=6 %THEN CHOICE(1)=9 %AND->15
PRINT STRING('HALF IS:')
WRITE(HALF,5)
NEWLINE

%IF HALF=1 %THEN %START


%IF EXP(CODE,1)=0 %THEN %START
                     %CYCLE I=1,1,4
                       CHOICE(I)=I
                     %REPEAT
                      ->15
                   %FINISH
%IF EXP(CODE,1)=1 %THEN%C
%START
%IF EXP(CODE,2)=0 %THEN CHOICE(1)=2 %AND%C
                   CHOICE(2)=3 %AND%C
                   CHOICE(3)=4 %AND->15
%IF EXP(CODE,3)=0 %THEN CHOICE(1)=6 %AND%C
                   CHOICE(2)=7 %AND->15
%FINISH %ELSE %START
               %IF EXP(CODE,2)=0 %THEN CHOICE(1)=1 %AND%C
                                        CHOICE(2)=6 %AND%C
                                        CHOICE(3)=7 %AND->15
               %IF EXP(CODE,3)=0 %OR(EXP(CODE,4)=0 %AND%C
                                      FN3=2)%C
               %THEN %START
                        %CYCLE I=1,1,7
                          CHOICE(I)=I
                        %REPEAT
                        CHOICE(EXP(CODE,2))=0
                        ->15
                     %FINISH
            %FINISH
%IF FN3=ROSTER(CODE,WEEK+1)=3 %THEN %C
     %START
        %CYCLE I=1,1,8
          CHOICE(I)=I
        %REPEAT
        %CYCLE I=1,1,15
          %IF EXP(CODE,I)=1 %THEN CHOICE(1)=0
          %CYCLE J=5,1,8
            %IF EXP(CODE,I)=J %THEN CHOICE(J)=0
          %REPEAT
        %REPEAT
        %UNLESS FLOAT>0 %THEN %START
                                %CYCLE I=2,1,4
                                  CHOICE(I)=0
                                %REPEAT
                              %FINISH
                                 ->15
    %FINISH
FIFTYFOUR=0
%CYCLE I=1,1,15
   %IF EXP(CODE,I)=5 %THEN FIFTYFOUR=1
%REPEAT
%IF FIFTYFOUR=1 %THEN %START
                        %CYCLE I=2,1,4
                          CHOICE(I)=I
                        %REPEAT
                      %FINISH %ELSE%C
                      %START
                        %IF NIGHT>2 %THEN %START
                                         %CYCLE I=2,1,5
                                           CHOICE(I)=I
                                         %REPEAT
                                       %FINISH %ELSE%C
                                       CHOICE(5)=5
                      %FINISH
%FINISH %ELSE %START
%CYCLE I=1,1,15
   HCL=I
```

```
   %IF EXP(CODE,I)=9 %THEN->70
%REPEAT
70:
%CYCLE I=1,1,8
   CHOICE(I)=I
%REPEAT
%CYCLE I=HOL+1,1,25
   CHOICE(EXP(CODE,I))=0 %UNLESS 1<EXP(CODE,I)<5
%REPEAT

LW=0
%CYCLE I=1,1,25
   %IF EXP(CODE,I)=6 %THEN LW=LW+1
%REPEAT
%IF LW<3 %THEN CHOICE(6)=6
%IF FN3=8 %THEN CHOICE(8)=0
CALCULATE FLOAT(WEEK)
%UNLESS FLOAT>0 %THEN %START
   %CYCLE I=2,1,4
      CHOICE(I)=0
   %REPEAT
%FINISH
%FINISH


15:PRINT STRING('ROSTER IS:')
%CYCLE I=1,1,110
   FN4=ROSTER(CODE,I)
   %UNLESS FN4=0 %THEN WRITE(FN4,2)
%REPEAT
NEWLINE

PRINT STRING('CHOICES AT THIS POINT ARE:')
%CYCLE I=1,1,8
   %UNLESS CHOICE(I)=0 %THEN WRITE(CHOICE(I),5)
%REPEAT
NEWLINE
PRINT STRING('FLOAT IS:');WRITE (FLOAT,5);NEWLINE
COUNT=25
%IF HALF=2 %THEN COUNT=0
20:%UNLESS COUNT=25 %THEN PRINT STRING('THE FIRST HALF IS COMPLETE')
NEWLINE
PRINT STRING('EXPERIENCE SO FAR IS:')
%CYCLF I=1,1,25
   %UNLESS EXP(CODE,I)=0 %THEN WRITE(EXP(CODE,I),5)
%REPEAT
%ENDOFPROGRAM
```

Sample run of CHOICE :

```
COMMAND:RUN(OB2)
DATA:1 46 2 2 2 6 6 6 6 1 1 1 1 8 8 5 5 7 7 7 7 2 2 2 2 2 2 2
DATA:10 10 10 9 9 9 9 2 6 6 6 6 2 2 2 2 5 5 7 7
HALF IS:     2
CHOICES AT THIS POINT ARE:     1     8
THE FIRST HALF IS COMPLETE
EXPERIENCE SO FAR IS:     2     6     1     8     5     7     2    10
    9     2     6     6     2     5     7
```

ORDER (comp, 1): Nurse's code number if on days.

ORDER (comp, 2): Nurse's urgency if on days.

COMP: Total of nurses on days with free choice.

COMP 2: Total of nurses on days with one choice.

TOTAL (1-8): Number of COMP nurses available to each ward.

TOTAL 2 (1-8): Number of COMP 2 nurses available to each ward.

ROSTER:

PART OF COURSE IN SEQUENCE	ROSTER CODE
DAY	1
NIGHT	2
DAY	3
NIGHT	4
HOLIDAY	5
DISTRICT	6
DAY	7
NIGHT	8
DAY	9
HOLIDAY	10
END	11

PLAN (X, -1):

CODES: 0: Girl on course

112: Not yet on course or finished

111: Blank nurse file

PLAN (X, 1-110):

WARD	PLAN CODE
49	1
51	2
52	3
53	4
54	5
LW	6
SC	7
OP	8

Variables used in X280

X280 - 7.10.74 VERSION

RÉSUMÉ

The lines marked with an asterisk perform calculations or alter data in some way. The others are concerned with data input and output.

```
START

 Declarations
 Request for style or printout
 Request for mode of output

 Routines:
   *Quicksort
   *Find half
   *Calculate float
   *Find urgency (Subroutine of C.U. below)
   *Calculate urgency


 Read quarter
*Initialise ALLOCATION, HALF, AUG, EXP, PLAN, URGENCY
 If quarter = 1, then read (direct access) initial data for ROSTER,
   PLAN and EXP
*Otherwise read them in from output of previous run and
 Put codes for PLAN (221-280, -1) to 112 for six and 111 for 4 in each 10.
 Copy holidays & district work from ROSTER to PLAN
*Establish weights (8+8: wards for day & night)


 Cycle for 13 weeks (49-61 inclusive)
 Print headline: Week X
 Put ALLOCATION, HALF and CHOICE to zero
*Put SHORTAGE 1 & 2, WARD 1 & 2, TOTAL 1-4, ORDER 1 & 2 to zero
*Re-establish weights for ward 5 at original values


*Cycle code = 1→280 ←
*If any nurse is on her first week then alter code to include her
*If girl is not on course or is a blank then return to cycle →
*If girl is just finished, alter code and go on to next nurse →
*We are now considering one girl on her course:
*  CALCULATE URGENCY
*The next section calculates what choices of ward are available to this
*  nurse for the next week (the one under consideration)


 On request prints out urgency and choices for each girl
```

```
*TOTAL 1 & 2 and COMP 1 & 2 put to ZERO
 Print headline: DAY DUTY
 On request give number of each available girl, her urgency, and put
   an asterisk under the column for each ward she can go on, unless
   she has already been allocated in a previous week, or if she is on
   night duty (if ROSTER is an even no.)
*Increase COMP by 1
*Store day nurses code no. & urgency in ORDER
*Put choices available to that nurse into comulative TOTAL (1-8)
*If girl is pre-determined (+day duty, still on course) then put her
*  choice into TOTAL 2 (1-8)
*COMP 2 increases by 1
 OUTPUT: Print COMP (total available days) then total available for
   each ward
 Print pre-allocated totals - overall then ward by ward
 Print optimum               -             "
 Print minimum               -             "
 Print shortage (opt - pre-allocated)      "
*QUICKSORT - sorts available nurses (not pre-allocated) into order of
*  urgency
 List printed on request
*Ward (1-8) put to zero
*Cycles through each available girl, from most urgent first <--------
*Takes ward which she can go on to whose combined weight & shortage  |
*  are ≥ any other ward she can go to - puts her on it (ALLOCATION   |
*  (girl) = WARD (1-8)) and that reduces that shortage by one -------
 Print day allocations on request
*Cycles through all girls; if allocation has been made, calculates
*  how long the girl will be on her new ward and changes PLAN & EXP
*  accordingly
 LIST new shortage


*Repeat section between brackets for night duty


 On request prints updated PLAN


 On completion of one quarter (13 cycles) reads out status quo  into
   DA file


END
```

FILE IDENTIFIER: X280

X280

```
%BEGIN
PRINT STRING('OFF WE GO!');NEWLINE
%STRING(10) REPLY1,REPLY2,REPLY3,REPLY4,REPLY5,REPLY6,REPLY7,REPLY8
%STRING(10) REPLY9,REPLY10,REPLY11
%INTEGERARRAY ORDER(1:100,1:2),SHORTAGE(0:3),SHORTAGE2(0:8)
%INTEGERARRAY ORDER2(1:100,1:2)
%BYTEINTEGERARRAY ALLOCATION(1:280),WARD(1:8),WARD2(1:8),WEIGHT1(1:8)
%BYTEINTEGERARRAY HALF(1:280),TOTAL3(1:8),TOTAL4(1:8),WEIGHT2(1:8)
%BYTEINTEGERARRAY ROSTER(1:280,0:110),EXP(1:280,1:25),AUG(1:280)
%BYTEINTEGERARRAY CHOICE(1:280,0:8),TOTAL(1:8),TOTAL2(1:8)
%BYTEINTEGERARRAY PLAN(1:280,-1:110),URGENCY(1:280,0:110)
%INTEGER COUNT,Z,COMP,COMP2,COMP3,COMP4,FLOAT,RESIDUE,A
%BYTEINTEGER FORTYNINE,LW,SC,OP,FIFTYFOUR,FN0,FN1,FN2,FN3,FN4,FN5,FN6
%BYTEINTEGER NIGHT,NEWVALUE,HOL,FN7,FN8,FN9,FN10
%INTEGER NO,WEEK,X,Y,CODE,SECT,I,J,K,L,M,DAY1
%REALARRAY TTT(1:16),TTTT(1:11)
%EXTERNALROUTINESPEC OPENDA(%INTEGER CHANNEL)
%EXTERNALROUTINESPEC READDA(%INTEGER CHANNEL,%INTEGERNAME SECT%C
,%NAME BEGIN,END)
%EXTERNALROUTINESPEC WRITEDA(%INTEGER CHANNEL,%INTEGERNAME SECT%C
,%NAME BEGIN,END)
%EXTERNALROUTINESPEC CLOSEDA(%INTEGER CHANNEL)
%EXTERNALLONGREALFNSPEC CPUTIME
NEWVALUE=0;NIGHT=0;K=0;RESIDUE=0
FN0=0;FN1=0;FN2=0;FN3=0;FN4=0;FN5=0;FN6=0;FN7=0;FN8=0;HOL=0


PRINT STRING('DO YOU WANT PRE-SELECTED PRINTOUT?');NEWLINE
READ STRING(REPLY11)
%IF REPLY11='YES' %THEN %START
              READ STRING(REPLY1)
              READ STRING(REPLY2)
              READ STRING(REPLY6)
              READ STRING(REPLY7)
              READ STRING(REPLY3)
              READ STRING(REPLY8)
              READ STRING(REPLY9)
              READ STRING(REPLY4)
                        %FINISH

PRINT STRING('DO YOU WANT RESULTS OUTPUT ON LINE PRINTER?')
NEWLINE
READ STRING(REPLY10)
%IF REPLY10='YES' %THEN %START
                           SELECT OUTPUT(40)
                        %FINISH



%ROUTINE QUICKSORT(%INTEGERARRAYNAME X,%INTEGER A,B)
         %INTEGER L,U,D,E
         %RETURN %IF A>=B
         L=A;U=B;D=X(U,2);E=X(U,1)
         ->FIND
  UP:    L=L+1
         ->FOUND %IF L=U
  FIND:  ->UP %UNLESS X(L,2)>D
         X(U,2)=X(L,2);X(U,1)=X(L,1)
  DOWN:  U=U-1
         ->FOUND %IF L=U
         ->DOWN %UNLESS X(U,2)<D
         X(L,2)=X(U,2);X(L,1)=X(U,1)
         ->UP
  FOUND:X(U,2)=D;X(U,1)=E
  QUICKSORT(X,A,L-1)
  QUICKSORT(X,U+1,B)
%END
```

```
 %ROUTINE FIND HALF(%INTEGER X)
 TTT(1)=CPUTIME
 COUNT=0
 %CYCLE I=X.1,X+27
 FNO=ROSTER(CODE,I)
    %IF FNO=5 %THEN ->35
    %IF FNO=10 %THEN ->40
    COUNT=COUNT+1
 %REPEAT
 35: HALF(CODE)=1 %UNLESS COUNT>27
 40: HALF(CODE)=2 %UNLESS HALF(CODE)=1 %OR COUNT>27
 HALF(CODE)=3 %UNLESS HALF(CODE)=1 %OR HALF(CODE)=2
 TTT(2)=CPUTIME
 %END


 %ROUTINE CALCULATE FLOAT(%INTEGER X)
 TTT(3)=CPUTIME
 %IF HALF(CODE)=1 %THEN%C
     %START
        LW=4;SC=4;OP=2;FORTYNINE=4;RESIDUE=4;FIFTYFOUR=0
        %CYCLE I=1.1.25
           J=EXP(CODE,I)
           %IF J=0 %THEN ->80
           %IF J=1 %THEN FORTYNINE=0 %AND RESIDUE=RESIDUE-1 %AND ->70
           %IF J=6 %THEN LW=0 %AND RESIDUE=RESIDUE-1 %AND ->70
           %IF J=7 %THEN SC=0 %AND RESIDUE=RESIDUF-1 %AND ->70
           %IF J=8 %THEN OP=0 %AND RESIDUE=RESIDUE-1 %AND ->70
           %IF J=5 %THEN FIFTYFOUR=1
70:AUG(CODE)=I
        %REPEAT
80:     FLOAT=COUNT-NIGHT-1-FORTYNINE-LW-SC-OP
      %FINISH
%IF HALF(CODE)=2 %THEN%C
     %START
        LW=4;SC=2;OP=1;FORTYNINE=3;RESIDUE=4;FIFTYFOUR=0
        %CYCLE I=8.1.15
        %IF EXP(CODE,I)=9 %THEN %START
        %CYCLE J=I+1,1,25
           FN7=EXP(CODE,J)
           %IF FN7=0 %THEN ->85
%IF FN7=1 %THEN FORTYNINE=0 %AND RESIDUE=RESIDUE-1 %AND->50
           %IF FN7=5 %THEN FIFTYFOUR=1 %AND ->50
           %IF FN7=6 %THEN %START
%IF LW=4 %THEN LW=2 %AND->50
%IF LW=2 %THEN LW=0 %AND RESIDUE=RESIDUE-1 %AND->50
                       %FINISH
%IF FN7=7 %THEN SC=0 %AND RESIDUE=RESIDUE-1 %AND->50
%IF FN7=8 %THEN OP=0 %AND RESIDUE=RESIDUE-1
50:AUG(CODE)=J
       %REPEAT
                                %FINISH
       %REPEAT
85:    FLOAT=COUNT-FORTYNINE-LW-SC-OP
     %FINISH
TTT(4)=CPUTIME
%END




%ROUTINE FIND URGENCY(%INTEGER A)

TTT(5)=CPUTIME
%IF RESIDUE=0 %THEN %START
%IF 1<PLAN(CODE,X-1)<5 %THEN URGENCY(CODE,X)=36+FIFTYFOUR%C
                             %ELSE URGENCY(CODE,X)=34+FIFTYFOUP
                    %FINISH %ELSE %START
                            URGENCY(CODE,X)=A-(RESIDUE*8)
                                  %FINISH
TTT(6)=CPUTIME
%END
```

```
%ROUTINE CALCULATE URGENCY(%INTEGER X)
TTT(7)=CPUTIME
FIFTYFOUR=0;RESIDUE=0;FLOAT=0;FORTYNINE=0;LW=0;SC=0;OP=0
%IF PLAN(CODE,X)#0 %THEN URGENCY(CODE,X)=38 %AND HALF(CODE)=1 %AND ->115
%IF EXP(CODE,1)=0 %THEN URGENCY(CODE,X)=33 %AND HALF(CODE)=1 %AND ->115
%IF EXP(CODE,1)=1 %AND EXP(CODE,2)=0 %THEN URGENCY(CODE,X)=35%C
%AND HALF(CODE)=1 %AND->115
%IF 1<EXP(CODE,1)<5 %AND EXP(CODE,2)=0 %THEN URGENCY(CODE,X)=35%C
%AND HALF(CODE)=1 %AND ->115
FIND HALF(X)
%IF HALF(CODE)=1 %THEN%C
   %START
     NIGHT=0
     %CYCLE I=X,1,X+COUNT
       %IF ROSTER(CODE,I)=4 %THEN NIGHT=NIGHT+1
     %REPEAT
     %IF COUNT-NIGHT-1>0 %THEN CALCULATE FLOAT(X) %AND ->45
     %IF COUNT-NIGHT-1=0 %THEN CALCULATE FLOAT(X) %AND%C
     URGENCY(CODE,X)=35-FIFTYFOUR
     %IF ROSTER(CODE,X)=4 %THEN%C
       %START
          CALCULATE FLOAT(X)
          URGENCY(CODE,X)=37-FIFTYFOUR
       %FINISH
->45
   %FINISH
     CALCULATE FLOAT(X)
45:
%IF FLOAT=-1 %AND COUNT-NIGHT-1=3 %AND OP=0%C
%THEN URGENCY(CODE,X)=37
%IF FLOAT<0 %THEN %START
     %IF HALF(CODE)=1 %AND COUNT-NIGHT-1>1 %THEN URGENCY(CODE,X)=99
     %IF HALF(CODE)=2 %THEN URGENCY(CODE,X)=99
%FINISH
%IF FLOAT=0 %THEN %START
                   %IF RESIDUE>0 %THEN URGENCY(CODE,X)=38-RESIDUE%C
                                 %ELSE FIND URGENCY(99)
                  %FINISH
%IF 0<FLOAT<9 %THEN FIND URGENCY(42-FLOAT-FIFTYFOUR-FIFTYFOUR)
%IF FLOAT=28 %THEN URGENCY(CODE,X)=0
%IF FLOAT>8 %THEN %START
                   PRINT STRING('FLOAT=')
                   WRITE(FLOAT,4)
                   NEWLINE
                  %FINISH
115:TTT(8)=CPUTIME
%END
```

```
TTT(9)=CPUTIME
%CYCLE J=1,1,280
   ALLOCATION(J)=0
   HALF(J)=0
   AUG(J)=0
   %CYCLE I=1,1,25
      EXP(J,I)=0
   %REPEAT
   %CYCLE I=0,1,110
      PLAN(J,I)=0
      URGENCY(J,I)=0
   %REPEAT
%REPEAT
OPENDA(20)
SECT=1
READDA(20,SECT,ROSTER(1,0),ROSTER(280,110))
CLOSEDA(20)
OPENDA(30)
SECT=1
READDA(30,SECT,PLAN(1,-1),PLAN(280,110))
CLOSEDA(30)
OPENDA(50)
SECT=1
READDA(50,SECT,EXP(1,1),EXP(280,25))
CLOSEDA(50)
%CYCLE I=1,1,280
   %CYCLE J=1,1,110
      %IF ROSTER(I,J)=5 %THEN PLAN(I,J)=10
      %IF ROSTER(I,J)=6 %THEN PLAN(I,J)=9
      %IF ROSTER(I,J)=10 %THEN PLAN(I,J)=11
   %REPEAT
%REPEAT
```

```
WEIGHT1(1)=5
WEIGHT1(2)=7
WEIGHT1(3)=7
WEIGHT1(4)=7
WEIGHT1(5)=15
WEIGHT1(6)=4
WEIGHT1(7)=6
WEIGHT1(8)=8

WEIGHT2(1)=12
WEIGHT2(2)=12
WEIGHT2(3)=12
WEIGHT2(4)=12
WEIGHT2(5)=30
WEIGHT2(6)=10
WEIGHT2(7)=10
WEIGHT2(8)=0


TTT(10)=CPUTIME

%CYCLE WEEK=49,1,61
    X=WEEK
NEWLINES(5);SPACES(25)
PRINT STRING('WEEK ');WRITE(WEEK,3);NEWLINE
SPACES(25);PRINT STRING('----------');NEWLINES(8)
    %CYCLE I=1,1,280
        ALLOCATION(I)=0
        HALF(I)=0
        %CYCLE J=0,1,8
            CHOICE(I,J)=0
        %REPEAT
    %REPEAT

%CYCLE I=0,1,8
    SHORTAGE(I)=0
    SHORTAGE2(I)=0
%REPEAT

%CYCLE I=1,1,8
    WARD(I)=0
    WARD2(I)=0
    TOTAL(I)=0
    TOTAL2(I)=0
    TOTAL3(I)=0
    TOTAL4(I)=0
%REPEAT


%CYCLE I=1,1,100
    %CYCLE J=1,1,2
        ORDER(I,J)=0
        ORDER2(I,J)=0
    %REPEAT
%REPEAT

WEIGHT1(5)=15
WEIGHT2(5)=30



%CYCLE CODE=1,1,280
    %IF PLAN(CODE,-1)=111 %THEN ->100
    %IF PLAN(CODE,WEEK)=11 %THEN PLAN(CODE,-1)=112 %AND ->100


TTT(11)=CPUTIME

CALCULATE URGENCY(WEEK)


TTT(12)=CPUTIME
```

```
%CYCLE I=0,1,8
     CHOICE(CODE,I)=0
%REPEAT
%IF 0<PLAN(CODE,X)<9 %THEN CHOICE(CODE,PLAN(CODE,X))=PLAN(CODE,X)%C
%AND ->15
FN3=ROSTER(CODE,WEEK)
%IF 4<FN3<7 %THEN CHOICE(CODE,1)=15-FN3 %AND->15

%IF HALF(CODE)=1 %THEN %START


%IF EXP(CODE,1)=0 %THEN %START
                     %CYCLE I=1,1,4
                       CHOICE(CODE,I)=I
                     %REPEAT
                      ->15
                   %FINISH
%IF EXP(CODE,1)=1 %THEN%C
%START
%IF EXP(CODE,2)=0 %THEN CHOICE(CODE,1)=2 %AND%C
                   CHOICE(CODE,2)=3 %AND%C
                   CHOICE(CODE,3)=4 %AND->15
%IF EXP(CODE,3)=0 %THEN CHOICE(CODE,6)=6 %AND%C
                   CHOICE(CODE,7)=7 %AND->15
%FINISH %ELSE %START
               %IF EXP(CODE,2)=0 %THEN CHOICE(CODE,1)=1 %AND%C
                                       CHOICE(CODE,6)=6 %AND%C
                                       CHOICE(CODE,7)=7 %AND->15
               %IF EXP(CODE,3)=0 %OR(EXP(CODE,4)=0 %AND%C
                                     FN3=2)%C
               %THEN %START
                        %CYCLE I=1,1,7
                          CHOICE(CODE,I)=I
                        %REPEAT
                        CHOICE(CODE,EXP(CODE,2))=0
                        ->15
                     %FINISH
            %FINISH
%IF FN3=ROSTER(CODE,WEEK+1)=3 %THEN %C
     %START
        %CYCLE I=1,1,8
          CHOICE(CODE,I)=I
        %REPEAT
        %CYCLE I=1,1,15
          %IF EXP(CODE,I)=1 %THEN CHOICE(CODE,1)=0
          %CYCLE J=5,1,8
            %IF EXP(CODE,I)=J %THEN CHOICE(CODE,J)=0
          %REPEAT
        %REPEAT
        %UNLESS FLOAT>0 %THEN %START
                                %CYCLE I=2,1,4
                                  CHOICE(CODE,I)=0
                                %REPEAT
                              %FINISH
        %IF FLOAT<2 %AND RESIDUE>0 %THEN CHOICE(CODE,5)=0
                                ->15
     %FINISH
FIFTYFOUR=0
%CYCLE I=1,1,15
  %IF EXP(CODE,I)=5 %THEN FIFTYFOUR=1
%REPEAT
%IF FIFTYFOUR=1 %THEN %START
                       %CYCLE I=2,1,4
                         CHOICE(CODE,I)=I
                      %REPEAT
                     %FINISH %ELSE%C
                     %START
                                       %CYCLE I=2,1,5
                                         CHOICE(CODE,I)=I
                                       %REPEAT
                     %IF NIGHT<2 %THEN CHOICE(CODE,5)=0
                     %FINISH
%FINISH %ELSE %START
%CYCLE I=1,1,15
  HOL=I
  %IF EXP(CODE,I)=9 %THEN->90
%REPEAT
```

```
90:
%CYCLE I=1,1,8
    CHOICE(CODE,I)=I
%REPEAT
%CYCLE I=HOL+1,1,25
    FN8=EXP(CODE,I)
    CHOICE(CODE,FN8)=0 %UNLESS 1<FN8<5
%REPEAT

LW=0
%CYCLE I=1,1,25
    %IF EXP(CODE,I)=6 %THEN LW=LW+1
%REPEAT
%IF LW<3 %THEN CHOICE(CODE,6)=6
%IF FN3=8 %THEN CHOICE(CODE,3)=0
%UNLESS FLOAT>0 %THEN %START
    %CYCLE I=2,1,4
        CHOICE(CODE,I)=0
    %REPEAT
%FINISH
%UNLESS FLOAT>1 %THEN CHOICE(CODE,5)=0
%FINISH



15:

TTT(13)=CPUTIME

100:
%REPEAT



TTT(14)=CPUTIME
PRINT STRING('DO YOU WANT COMPLETE URGENCY LISTINGS?');NEWLINE
READ STRING(REPLY1) %UNLESS REPLY11='YES'
%UNLESS REPLY1='YES' %THEN ->120


%CYCLE I=1,1,280
    %IF 110<PLAN(I,-1)<113 %THEN ->110
    NEWLINE
    WRITE(I,3);SPACES(5);WRITE(URGENCY(I,WEEK),3);SPACES(5)
    %CYCLE J=1,1,8
       %UNLESS CHOICE(I,J)=0 %THEN WRITE (CHOICE(I,J),2)%C
                             %ELSE %START
                                     SPACES(3)
                                   %FINISH
    %REPEAT
110:%REPEAT
NEWLINE
120:PRINT STRING('DO YOU WANT DAY DUTY LISTINGS?');NEWLINE
READ STRING(REPLY2) %UNLESS REPLY11='YES'


%CYCLE I=1,1,8
    TOTAL(I)=0
    TOTAL2(I)=0
%REPEAT
COMP=0
COMP2=0

NEWLINES(10)

SPACES(25);PRINT STRING('**DAY DUTY**')
NEWLINES(4)
SPACES(2)
PRINT STRING('CODE');SPACES(4);PRINT STRING('URGENCY');SPACES(4)
PRINT STRING('49  51  52  53  54  LW  SC  OP')
NEWLINES(2)
%CYCLE I=1,1,280
  %IF 110<PLAN(I,-1)<113 %OR URGENCY(I,WEEK)=38%C
  %OR ROSTER(I,WEEK)-(ROSTER(I,WEEK)//2)*2=0 %THEN ->125
  %IF REPLY2='YES' %THEN %START
  WRITE(I,3);SPACES(7);WRITE(URGENCY(I,WEEK),2);SPACES(8)
                         %FINISH
```

```
  COMP=COMP+1
  ORDER(COMP,1)=I
  ORDER(COMP,2)=URGENCY(I,WEEK)
  %IF REPLY2='YES' %THEN %START
  %CYCLE J=1,1,8
    %UNLESS CHOICE(I,J)=0 %THEN PRINT STRING('*   ')%C
    %AND TOTAL(J)=TOTAL(J)+1 %ELSE %START
                                       SPACES(4)
                                    %FINISH
  %REPEAT
  NEWLINE
  %FINISH %ELSE %START
     %CYCLE J=1,1,8
       %UNLESS CHOICE(I,J)=0 %THEN TOTAL(J)=TOTAL(J)+1
     %REPEAT
                 %FINISH
125: %IF URGENCY(I,WEEK)=38 %AND PLAN(I,WEEK)#10%C
     %AND ROSTER(I,WEEK)-(ROSTER(I,WEEK)//2)*2#0 %THEN %START
           %CYCLE J=1,1,8
              %IF 0<CHOICE(I,J)<9 %THEN TOTAL2(J)=TOTAL2(J)+1
           %REPEAT
           COMP2=COMP2+1
                                                         %FINISH
%REPEAT

%IF REPLY2='YES' %THEN %START
SPACES(1)
PRINT STRING('---         --         -   -   -   -   -   -   -   -')
NEWLINES(4)
%FINISH
WRITE(COMP,3);SPACES(15);%CYCLE I=1,1,8
                            WRITE(TOTAL(I),3)
                          %REPEAT
SPACES(2)
PRINT STRING(':MAX AVAILABLE')
NEWLINES(4)
WRITE(COMP2,3);SPACES(15);%CYCLE I=1,1,8
                               WRITE(TOTAL2(I),3)
                             %REPEAT
SPACES(2)
PRINT STRING(':PRE-ALLOCATED')
NEWLINES(4)
PRINT STRING('  73');SPACES(17)
PRINT STRING('12   8   8   8   4  15  10   8  :OPTIMUM')
NEWLINES(4)
PRINT STRING('  66');SPACES(17)
PRINT STRING('11   7   7   7   4  14   9   7  :MINIMUM')
NEWLINES(4)
SHORTAGE(0)=73-COMP2
SHORTAGE(1)=12-TOTAL2(1)
SHORTAGE(2)=8-TOTAL2(2)
SHORTAGE(3)=8-TOTAL2(3)
SHORTAGE(4)=8-TOTAL2(4)
SHORTAGE(5)=4-TOTAL2(5)
SHORTAGE(6)=15-TOTAL2(6)
SHORTAGE(7)=10-TOTAL2(7)
SHORTAGE(8)=8-TOTAL2(8)
WRITE(SHORTAGE(0),3);SPACES(15)
%CYCLE I=1,1,8
   WRITE(SHORTAGE(I),3)
%REPEAT
PRINT STRING('  :SHORTAGE')
NEWLINES(10)



QUICKSORT(ORDER,1,COMP)

PRINT STRING('DO YOU WANT DAY LISTING IN ORDER OF URGENCY?')
NEWLINE;READ STRING(REPLY6) %UNLESS REPLY11='YES'
%IF REPLY6='YES' %THEN %START
%CYCLE I=1,1,COMP
   WRITE(ORDER(I,1),3);SPACES(7)
   WRITE(ORDER(I,2),2);SPACES(3)
   %CYCLE J=1,1,8
      %UNLESS CHOICE(ORDER(I,1),J)=0 %THEN PRINT STRING('*   ')%C
      %ELSE %START
               SPACES(4)
            %FINISH
   %REPEAT
   NEWLINE
```

```
%REPEAT
NEWLINES(10)
%FINISH


%CYCLE I=1,1,8
   WARD(I)=0
%REPEAT

%CYCLE M=COMP,-1,1
   J=0
   %CYCLE I=1,1,8
      %UNLESS CHOICE(ORDER(M,1),I)=0 %THEN J=J+1 %C
      %AND WARD(J)=CHOICE(ORDER(M,1),I)
   %REPEAT
   %IF SHORTAGE(5)=0 %THEN WEIGHT1(5)=0
   %CYCLE I=1,1,J
      L=0
      %CYCLE K=1,1,J
         %IF SHORTAGE(WARD(I))*WEIGHT1(WARD(I))>=SHORTAGE(WARD(K))%C
         *WEIGHT1(WARD(K)) %THEN L=L+1
         %IF L=J %THEN ALLOCATION(ORDER(M,1))=WARD(I)%C
         %AND SHORTAGE(WARD(I))=SHORTAGE(WARD(I))-1 %AND->150
      %REPEAT
   %REPEAT
150:%REPEAT

PRINT STRING('DO YOU WANT DAY ALLOCATIONS?');NEWLINE
READ STRING(REPLY7) %UNLESS REPLY11='YES'
%CYCLE CODE=1,1,280
   FN9=ALLOCATION(CODE);FN10=HALF(CODE)
   %IF FN9=0 %THEN->155
%IF REPLY7='YES' %THEN %START
   WRITE(CODE,3);SPACES(7);WRITE(FN9,2);NEWLINE
%FINISH

%IF 0<FN9<5 %AND ROSTER(CODE,X)=1 %AND ROSTER(CODE,X+1)=1%C
%THEN %START
        DAY1=0
        %CYCLE I=0,1,8
           %IF ROSTER(CODE,X+1)=1 %THEN DAY1=DAY1+1 %ELSE->190
        %REPEAT
        190:%CYCLE I=0,1,DAY1-2
               PLAN(CODE,X+1)=FN9
            %REPEAT
        EXP(CODE,AUG(CODE)+1)=FN9
         ->155
      %FINISH
%IF 1<FN9<5 %AND ROSTER(CODE,X)=1 %AND ROSTER(CODE,X+1)=2%C
%THEN %START
        %CYCLE I=0,1,3
           PLAN(CODE,X+1)=FN9
        %REPEAT
        EXP(CODE,AUG(CODE)+1)=FN9
        ->155
      %FINISH
%IF 1<FN9<5 %OR (FN9=8 %AND FN10=2) %THEN PLAN(CODE,WEEK)=FN9%C
%AND EXP(CODE,AUG(CODE)+1)=FN9 %AND->155
%IF (FN9=1 %OR 5<FN9<8) %AND FN10=1 %THEN %START
                                          %CYCLE J=0,1,3
                                             PLAN(CODE,WEEK+J)=FN9
                                          %REPEAT
                                          EXP(CODE,AUG(CODE)+1)=FN9
                                          ->155
                                          %FINISH
%IF FN9=5 %OR (5<FN9<8 %AND FN10=2) %OR%C
(FN9=8 %AND FN10=1) %THEN %START
                          %CYCLE J=0,1,1
                             PLAN(CODE,WEEK+J)=FN9
                          %REPEAT
                          EXP(CODE,AUG(CODE)+1)=FN9
                          ->155
                          %FINISH
%IF FN9=1 %AND FN10=2 %THEN %START
                            %CYCLE J=0,1,2
                            PLAN(CODE,WEEK+J)=FN9
                            %REPEAT
                            EXP(CODE,AUG(CODE)+1)=FN9
                            %FINISH
155:%REPEAT
```

```
NEWLINES(5)
SPACES(19)
%CYCLE I=1,1,8
   WRITE(SHORTAGE(I),3)

%REPEAT
PRINT STRING('  :NEW SHORTAGE')
NEWLINES(10)


140:PRINT STRING('DO YOU WANT NIGHT DUTY LISTINGS?');NEWLINE
READ STRING(REPLY3) %UNLESS REPLY11='YES'
SPACES(25);PRINT STRING('**NIGHT DUTY**')


%CYCLE I=1,1,8
   TOTAL3(I)=0
   TOTAL4(I)=0
%REPEAT
COMP3=0
COMP4=0

NEWLINES(4);SPACES(2)
PRINT STRING('CODE');SPACES(4);PRINT STRING('URGENCY');SPACES(4)
PRINT STRING('49  51  52  53  54 LW  SC  OP')
NEWLINES(2)
%CYCLE I=1,1,280
  %IF 110<PLAN(I,-1)<113 %OR URGENCY(I,WEEK)=38%C
  %OR ROSTER(I,WEEK)-(ROSTER(I,WEEK)//2)*2#0 %THEN ->130
%IF REPLY3='YES' %THEN %START
  WRITE(I,3);SPACES(7);WRITE(URGENCY(I,WEEK),2);SPACES(8)
%FINISH
  COMP3=COMP3+1
  ORDER2(COMP3,1)=I
  ORDER2(COMP3,2)=URGENCY(I,WEEK)
     %IF REPLY3='YES' %THEN %START
  %CYCLE J=1,1,8
    %UNLESS CHOICE(I,J)=0 %THEN PRINT STRING('*   ')%C
    %AND TOTAL3(J)=TOTAL3(J)+1 %ELSE %START
                                  SPACES(4)
                                %FINISH
  %REPEAT
  NEWLINE
  %FINISH %ELSE %START
    %CYCLE J=1,1,8
      %UNLESS CHOICE(I,J)=0 %THEN TOTAL3(J)=TOTAL3(J)+1
    %REPEAT
              %FINISH
130: %IF URGENCY(I,WEEK)=38 %AND PLAN(I,WEEK)#9%C
     %AND ROSTER(I,WEEK)-(ROSTER(I,WEEK)//2)*2=0 %THEN %START
        %CYCLE J=1,1,8
           %IF 0<CHOICE(I,J)<9 %THEN TOTAL4(J)=TOTAL4(J)+1
        %REPEAT
        COMP4=COMP4+1
                                                  %FINISH
%REPEAT

%IF REPLY3='YES' %THEN %START
SPACES(1)
PRINT STRING('---               --         -   -   -   -   -   -   -   -')
NEWLINES(4)
%FINISH
WRITE(COMP3,3);SPACES(15);%CYCLE I=1,1,8
                         WRITE(TOTAL(I),3)
                        %REPEAT
SPACES(2)
PRINT STRING(':MAX AVAILABLE')
NEWLINES(4)
WRITE(COMP4,3);SPACES(15);%CYCLE I=1,1,8
                            WRITE(TOTAL4(I),3)
                          %REPEAT
SPACES(2)
PRINT STRING(':PRE-ALLOCATED')
NEWLINES(4)
PRINT STRING('  34');SPACES(17)
PRINT STRING(' 5   5   5   5   2   6   6   0  :OPTIMUM')
NEWLINES(4)
PRINT STRING('  28');SPACES(17)
PRINT STRING(' 4   4   4   4   2   5   5   0  :MINIMUM')
```

```
NEWLINES(4)
SHORTAGE2(0)=34-COMP4
SHORTAGE2(1)=5-TOTAL4(1)
SHORTAGE2(2)=5-TOTAL4(2)
SHORTAGE2(3)=5-TOTAL4(3)
SHORTAGE2(4)=5-TOTAL4(4)
SHORTAGE2(5)=2-TOTAL4(5)
SHORTAGE2(6)=6-TOTAL4(6)
SHORTAGE2(7)=6-TOTAL4(7)
SHORTAGE2(8)=0-TOTAL4(8)
WRITE(SHORTAGE2(0),3);SPACES(15)
%CYCLE I=1,1,8
   WRITE(SHORTAGE2(I),3)
%REPEAT
PRINT STRING('  :SHORTAGE')
NEWLINES(6)



QUICKSORT(ORDER2,1,COMP3)

PRINT STRING('DO YOU WANT NIGHT LISTING IN ORDER OF URGENCY?')
NEWLINE
READ STRING(REPLY8) %UNLESS REPLY11='YES'
 %IF REPLY8='YES' %THEN %START
%CYCLE I=1,1,COMP3
   WRITE(ORDER2(I,1),3);SPACES(7)
   WRITE(ORDER2(I,2),2);SPACES(8)
   %CYCLE J=1,1,8
      %UNLESS CHOICE(ORDER2(I,1),J)=0 %THEN PRINT STRING('*   ')%C
      %ELSE %START
                SPACES(4)
             %FINISH
   %REPEAT
   NEWLINE
%REPEAT
NEWLINES(10)
%FINISH


%CYCLE I=1,1,8
   WARD2(I)=0
%REPEAT

%CYCLE M=COMP3,-1,1
   J=0
   %CYCLE I=1,1,8
      %UNLESS CHOICE(ORDER2(M,1),I)=0 %THEN J=J+1 %C
      %AND WARD2(J)=CHOICE(ORDER2(M,1),I)
   %REPEAT
   %IF SHORTAGE2(5)=0 %THEN WEIGHT2(5)=0
   %CYCLE I=1,1,J
    L=0
    %CYCLE K=1,1,J
    %IF SHORTAGE2(WARD2(I))+WEIGHT2(WARD2(I))>=SHORTAGE2(WARD2(K))%C
    +WEIGHT2(WARD2(K)) %THEN L=L+1
    %IF L=J %THEN ALLOCATION(ORDER2(M,1))=WARD2(I)%C
    %AND SHORTAGE2(WARD2(I))=SHORTAGE2(WARD2(I))-1 %AND->130
    %REPEAT
   %REPEAT
130:%REPEAT

PRINT STRING('DO YOU WANT ALL ALLOCATIONS?');NEWLINE
READ STRING(REPLY9) %UNLESS REPLY11='YES'
%CYCLE I=1,1,280
   FN9=ALLOCATION(I);FN10=HALF(I)
   %IF FN9=0 %THEN->185
%IF REPLY9='YES' %THEN %START
   WRITE(I,3);SPACES(7);WRITE(FN9,2);NEWLINE
%FINISH

%IF 1<FN9<5 %OR (FN9=8 %AND FN10=2) %THEN PLAN(I,WEEK)=FN9%C
%AND EXP(CODE,AUG(CODE)+1)=FN9 %AND->185
%IF (FN9=1 %OR 5<FN9<8) %AND FN10=1 %THEN %START
                                           %CYCLE J=0,1,3
                                              PLAN(I,WEEK+J)=FN9
                                           %REPEAT
                                           EXP(CODE,AUG(CODE)+1)=FN9
                                           ->185
                                           %FINISH
```

```
%IF FN9=5 %OR (5<FN9<8 %AND FN10=2) %OR%C
(FN9=8 %AND FN10=1) %THEN %START
                          %CYCLE J=0,1,1
                             PLAN(I,WEEK+J)=FN9
                          %REPEAT
                          EXP(CODE,AUG(CODE)+1)=FN9
                          ->185
                          %FINISH
%IF FN9=1 %AND FN10=2 %THEN %START
                            %CYCLE J=0,1,2
                               PLAN(I,WEEK+J)=FN9
                            %REPEAT
                            EXP(CODE,AUG(CODE)+1)=FN9
                            %FINISH
185:%REPEAT

NEWLINES(5)
SPACES(19)
%CYCLE I=1,1,8
   WRITE(SHORTAGE2(I),3)
%REPEAT
PRINT STRING('  :NEW SHORTAGE')
NEWLINES(10)



145:PRINT STRING('DO YOU WANT UPDATED PLAN?');NEWLINE
READ STRING(REPLY4) %UNLESS REPLY11='YES'
%UNLESS REPLY4='YES' %THEN->160
SPACES(20);PRINT STRING('**UPDATED PLAN**');NEWLINES(4)

%CYCLE I=1,1,280
   %IF PLAN(I,-1)=111 %THEN->165
   NEWLINE
   %CYCLE J=25,1,60
      %UNLESS PLAN(I,J)=0 %THEN WRITE(PLAN(I,J),2)%C
                          %ELSE %START
                                SPACES(3)
                                %FINISH

   %REPEAT
165:%REPEAT

160:
TTT(15)=CPUTIME
TTT(16)=CPUTIME
%CYCLE I=1,1,5
   TTTT(I)=TTT(2*I)-TTT(2*I-1)
%REPEAT
%CYCLE I=6,1,10
   TTTT(I)=TTT(I+6)-TTT(I+5)
%REPEAT
TTTT(8)=TTT(14)-TTT(10)
NEWLINES(2)
%CYCLE I=1,1,10
   WRITE(I,3);SPACES(5);PRINT(TTTT(I),1,3);NEWLINE
%REPEAT
%REPEAT
%ENDOFPROGRAM
```

X280 - Sample printout (Started at week 26)

```
                         WEEK   52
                         ---------


                         **DAY DUTY**


CODE    URGENCY    49   51   52   53   54   LW   SC   OP

  22     30              *    *    *                   *
  24     36              *    *    *    *
  25     34              *    *    *    *
  26     36              *    *    *    *
  32     34              *    *    *    *
  34     33              *    *    *    *         *
  35      0         *                             *
  52     13              *    *    *         *    *    *
  61     13              *    *    *    *    *    *    *
  62      6         *    *    *    *    *    *    *    *
  63     13              *    *    *    *    *    *    *
  64     14         *    *    *    *    *    *         *
  65      6         *    *    *    *    *    *    *    *
  66     14         *    *    *    *    *    *         *
 141     34              *    *    *    *
 142     34              *    *    *    *
 144     36              *    *    *    *
 145     34              *    *    *    *
 146     37              *    *    *
 147     36              *    *    *    *
 151     36              *    *    *    *
 152     36              *    *    *    *
 154     35              *    *    *
 155     36              *    *    *    *
 163     37                                       *
 164     37                                  *
 171     33              *    *    *              *
 173     99         *                                  *
 174     31              *    *    *    *              *
 175     31              *    *    *         *
 183     23              *    *    *    *         *    *
 188     23              *    *    *    *         *    *
 191     22              *    *    *         *    *
 193     22              *    *    *    *    *         *
 194     35         *                             *    *
 222      7         *                        *    *
 223      7         *                        *    *
 224      7         *                        *    *
 225      7         *                        *    *
 226      7         *                        *    *
 ---     --         -    -    -    -    -    -    -    -


  40               12   30   30   30   23   16   18   14   :MAX AVAILABLE


  40                7    2    2    2    1   12    6    3   :PRE-ALLOCATED


  73               12    8    8    8    4   15   10    8   :OPTIMUM


  66               11    7    7    7    4   14    9    7   :MINIMUM


  33                5    6    6    6    3    3    4    5   :SHORTAGE
```

```
DO YOU WANT DAY LISTING IN ORDER OF URGENCY?
   35         0       *                       *
   62         6       *   *   *   *   *   *   *   *
   65         6       *   *   *   *   *   *   *   *
  226         7       *                   *   *
  225         7       *                   *   *
  224         7       *                   *   *
  223         7       *                   *   *
  222         7       *                   *   *
   61        13           *   *   *   *   *   *   *
   52        13           *   *   *       *   *   *
   63        13           *   *   *   *   *   *   *
   64        14       *   *   *   *   *   *       *
   66        14       *   *   *   *   *   *       *
  193        22           *   *   *   *   *       *
  191        22           *   *   *       *   *
  188        23           *   *   *   *       *   *
  183        23           *   *   *   *       *   *
   22        30           *   *   *               *
  174        31           *   *   *   *           *
  175        31           *   *   *       *
   34        33           *   *   *   *       *
  171        33           *   *   *           *
   32        34           *   *   *   *
  141        34           *   *   *   *
   25        34           *   *   *   *
  142        34           *   *   *   *
  145        34           *   *   *   *
  154        35           *   *   *
  194        35       *                       *   *
  151        36           *   *   *   *
  152        36           *   *   *   *
  144        36           *   *   *   *
  147        36           *   *   *   *
  155        36           *   *   *   *
   26        36           *   *   *   *
   24        36           *   *   *   *
  164        37                           *
  163        37                               *
  146        37           *   *   *
  173        99       *                           *




                      -1   0   0   0   0   0  -1   0 :NEW SHORTAGE
```

```
                          **NIGHT DUTY**

CODE    URGENCY     49    51    52    53    54    LW    SC    OP

  71       13                 *     *     *     *     *     *
  72       14        *     *     *     *     *     *
  75        8        *     *     *     *     *     *     *
  76        6        *     *     *     *     *     *     *
  81        4        *     *     *     *     *     *     *
  82        4        *     *     *     *     *     *     *
  83        4        *     *     *     *     *     *     *
  84        4        *     *     *     *     *     *     *
 121       37              *     *     *
 122       37              *     *     *
 125       37              *     *     *
 134       37              *     *     *
 135       37              *     *     *
 201       15              *     *     *     *     *     *
 203       15        *     *     *     *     *           *
 204       15              *     *     *     *     *     *
 205       15        *     *     *     *     *     *
 ---       --        -     -     -     -     -     -     -     -


  17                 9    17    17    17    12    11    10     0  :MAX AVAILABLE


  25                 3     0     0     0     6     2     4     0  :PRE-ALLOCATED


  34                 5     5     5     5     2     6     6     0  :OPTIMUM


  28                 4     4     4     4     2     5     5     0  :MINIMUM


   9                 2     5     5     5    -4     4     2     0  :SHORTAGE



DO YOU WANT NIGHT LISTING IN ORDER OF URGENCY?
  84        4        *     *     *     *     *     *     *
  83        4        *     *     *     *     *     *     *
  82        4        *     *     *     *     *     *     *
  81        4        *     *     *     *     *     *     *
  76        6        *     *     *     *     *     *     *
  75        8        *     *     *     *     *     *     *
  71       13              *     *     *     *     *     *
  72       14        *     *     *     *     *     *
 205       15        *     *     *     *     *     *
 204       15              *     *     *     *     *     *
 203       15        *     *     *     *     *           *
 201       15              *     *     *     *     *     *
 135       37              *     *     *
 134       37              *     *     *
 125       37              *     *     *
 122       37              *     *     *
 121       37              *     *     *



                     1     1     1     1    -4     1     1     0  :NEW SHORTAGE
```

A negative shortage is overstaffing - inevitable in this case

```
                          **UPDATED PLAN**

  4  4 10 10 10  9  9  9  9  8  7  7  4  4  6  6  5  5  4  1  1  1  4  6  6  4 11 11
  2  2 10 10 10  9  9  9  9  7  1  1  1  6  6  8  8  5  5  6  6  7  7  4  2  2 11 11
  5  3 10 10 10  9  9  9  9  6  6  6  6  7  7  2  2  2  8  8  1  1  1  2  3  4 11 11
  2  2  2 10 10 10  9  9  9  9  6  6  6  6  7  7  8  8  5  5  4  4  1  1  1  3 11 11
  3  3  3 10 10 10  9  9  9  9  8  7  7  7  6  6  1  1  1  3  3  6  6  5  5  2 11 11
  3  5  5 10 10 10  9  9  9  9  1  1  1  1  6  6  6  6  7  7  5  5  8  8  2  3 11 11
  4  4  4 10 10 10  9  9  9  9  2  7  7  1  1  1  6  6  6  6  8  8  5  5  4  2 11 11
  8  2  2  2  5  5 10 10 10  9  9  9  9  9  6  6  7  7  3  3  3  5  5  8  8  1  1  1       11
  6  6  3  3  3  3 10 10 10  9  9  9  9  9  1  1  1  6  6  5  5  7  7  4  6  6  2  8       11
  7  7  5  5  2  2 10 10 10  9  9  9  9  9  7  7  2  2  2  2  2  6  6  1  1  1  6  6       11
  7  7  4  4  4  4 10 10 10  9  9  9  9  9  7  7  3  3  1  1  1  6  6  6  6  8  4  3       11
  3  3  3  3  3  3 10 10 10  9  9  9  9  9  4  4  4  7  7  8  6  6  6  6  1  1  1  4       11
  4  4  4  4  4  4 10 10 10  9  9  9  9  9  3  3  6  6  7  7  1  1  1  8  6  6  3  4       11
  8  8  3  3  3  3  3 10 10 10  9  9  9  9  9  6  6  1  1  1  8  7  7  5  5  2  6  6       11
  7  2  2  2  2  2  2 10 10 10  9  9  9  9  9  1  1  1  4  4  7  7  6  6  6  6  8  2       11
  1  1  1  5  5  4  4 10 10 10  9  9  9  9  9  7  7  4  4  4  1  1  1  8  6  6  6  6       11
  8  3  3  4  4  5  5 10 10 10  9  9  9  9  9  2  2  2  6  6  8  1  1  1  6  6  4  3       11
  5  5  4  4  4  4  4 10 10 10  9  9  9  9  9  4  4  4  4  4  6  6  6  6  8  5  5  7  7    11
  1  1  1  1  2  2  2  2  2  2  2 10 10 10  9  9  9  9  6  6  7  7  7  5  5  3  1  1  1
  8  8  5  5  3  3  3  3  3  3  3 10 10 10  9  9  9  9  3  1  1  1  6  6  2  2  6  6
  1  1  1  1  8  8  5  5  4  4  4 10 10 10  9  9  9  9  4  7  7  1  1  1  2  3  6  6
  6  6  8  8  5  5  4  4  4  4  4  4 10 10 10  9  9  9  9  3  6  6  7  7  6  6  1  1  1
  1  5  5  8  8  3  3  3  3  3  3  3 10 10 10  9  9  9  9  3  6  6  1  1  1  5  5  3
  8  3  3  1  1  1  1  5  5  2  2  2 10 10 10  9  9  9  9  4  7  7  6  6  8  1  1  1
  3  8  8  6  6  6  6  5  5  3  3  3  3  3  3 10 10 10  9  9  9  9  1  1  1  2  2  4
  6  6  8  8  3  3  3  1  1  1  1  5  5  3  3 10 10 10  9  9  9  9  6  6  7  7  2  8
  8  8  7  7  7  7  5  5  2  2  2  2  2  2  2 10 10 10  9  9  9  9  2  1  1  1  4  2
  4  4  6  6  6  6  8  8  2  2  2  2  4  4  4 10 10 10  9  9  9  9  7  7  4  4  4  1  1  1
  7  7  7  7  8  8  4  4  5  5  4  4  4  4  4 10 10 10  9  9  9  9  6  6  6  6  3  5  5
  6  6  6  6  3  3  7  7  7  7  3  3  3  5  5 10 10 10  9  9  9  9  3  7  7  5  5  1  1  1
  6  5  5  8  8  7  7  7  7  2  2  2  2  2  2  2 10 10 10  9  9  9  9  1  1  1  3  6  6
  7  6  6  6  6  5  5  8  8  3  3  3  3  3  3  3 10 10 10  9  9  9  9  7  7  6  6  4
  5  1  1  1  1  8  8  7  7  7  7  3  3  3  3  3 10 10 10  9  9  9  9  2  3  3  7  7
  1  8  8  6  6  6  6  4  4  4  4  4  4  4  4  4 10 10 10  9  9  9  9  3  2  4  1  1  1
  7  7  7  8  8  4  4  4  1  1  1  1  5  5  4  4 10 10 10  9  9  9  9  4  4  5  5  1  1  1
  6  1  1  1  1  2  2  2  8  8  4  4  4  4  5  5 10 10 10  9  9  9  9  6  6  6  6  2
  2  7  7  7  7  6  6  6  6  8  8  5  5  2  2  2  2  2  2 10 10 10  9  9  9  9  9  3
  4  6  6  6  6  5  5  8  8  7  7  7  7  3  3  3  3  3  3 10 10 10  9  9  9  9  9  4
  6  1  1  1  1  7  7  7  7  8  8  3  3  4  4  5  5  4  4 10 10 10  9  9  9  9  9  6  6
  1  6  6  6  6  7  7  7  7  4  4  8  8  4  4  4  4  5  5 10 10 10  9  9  9  9  9  7  7
  7  2  1  1  1  1  6  6  6  6  3  5  5  8  8  3  3  3  3  3 10 10 10  9  9  9  9  9
  6  6  2  2  2  2  1  1  1  1  8  8  7  7  7  7  5  5  4  4 10 10 10  9  9  9  9  9
  1  1  7  7  7  7  6  6  6  6  8  8  5  5  2  2  2  2  2  2 10 10 10  9  9  9  9  9
  7  7  1  1  1  1  8  8  5  5  6  6  6  6  3  3  3  3  3  3 10 10 10  9  9  9  9  9
  6  4  4  4  8  8  1  1  1  1  7  7  7  7  4  4  4  4  5  5 10 10 10  9  9  9  9  9
  1  2  2  2  2  6  6  6  6  7  7  7  7  8  8  2  5  5  2  2  2  2  2  2 10 10 10  9  9  9  9
  2  1  1  1  1  2  2  6  6  6  6  8  8  7  7  7  7  2  2  2  2  2  2  2 10 10 10  9  9  9  9
  3  7  7  7  7  3  3  3  3  8  8  7  7  7  7  1  1  1  1  5  5  3  3  3 10 10 10  9  9  9  9
  4  7  7  7  7  1  1  1  1  4  4  4  8  8  6  6  6  6  3  3  3  3  3  3 10 10 10  9  9  9  9
  3  1  1  1  1  7  7  7  7  6  6  6  6  5  5  8  8  4  4  4  4  4  4  4 10 10 10  9  9  9  9
  1  1  3  3  3  7  7  7  7  8  8  6  6  6  6  5  5  3  3  3  3  3  3  3  2 10 10 10  9  9  9
  2  2  1  1  1  1  7  7  7  7  5  5  8  8  6  6  6  6  2  2  2  2  2  2  3 10 10 10  9  9  9
  2  2  6  6  6  6  1  1  1  1  7  7  7  7  5  5  8  8  3  3  3  3  3  3  2 10 10 10  9  9  9
  3  3  7  7  7  7  1  1  1  1  4  4  4  6  6  6  6  8  8  4  4  4  4  4  5  5 10 10  9  9  9
  4  4  6  6  6  6  4  4  4  7  7  7  7  1  1  1  1  8  8  2  5  5  2  2  4 10 10 10  9  9  9
        3  3  3  6  6  6  6  3  3  1  1  1  1  7  7  7  7  8  8  5  5  2  3  4  4  2 10 10 10
        2  2  2  6  6  6  6  7  7  7  7  8  8  1  1  1  1  5  5  3  3  3  2  2  2  3 10 10 10
        2  2  2  7  7  7  7  2  2  1  1  1  1  8  8  6  6  6  6  2  2  2  3  4  5  5 10 10 10
        4  4  4  7  7  7  7  6  6  6  6  4  4  4  1  1  1  1  8  8  4  4  4  3  5  5 10 10 10
        4  4  4  1  1  1  1  7  7  7  7  6  6  6  6  5  5  8  8  3  3  3  4  3  3  4 10 10 10
        1  1  1  2  2  2  2  6  6  6  6  7  7  7  7  8  8  1  2  2  2  2  2  4  5  5 10 10 10
        2  2  2  2  1  1  1  1  1  4  4  6  6  6  6  7  7  7  7  4  4  8  8  2  4  5  5 10 10
        1  1  1  1  3  3  3  3  3  7  7  7  7  8  8  6  6  6  6  4  4  4  4  2  2  5  5 10 10
        2  2  2  2  2  2  2  1  1  1  1  7  7  7  7  6  6  6  6  8  8  2  2  3  3  5  5 10 10
        3  3  3  3  6  6  6  6  4  4  5  5  1  1  1  1  8  8  7  7  7  7  3  3  3  2    10 10
        3  3  3  3  7  7  7  7  5  5  6  6  6  6  3  3  1  1  1  1  8  8  3  2  2  3    10 10
                 1  1  1  1  2  2  2  2  4  4  4  8  8  6  6  6  6  7  7  7  7  5  5
                 2  2  2  2  1  1  1  1  2  2  2  6  6  6  6  8  8  7  7  7  7  3
                 2  2  2  2  6  6  6  6  2  2  2  2  1  1  1  1  8  8  7  7  7  7
                 1  1  1  1  2  2  2  2  6  6  6  6  8  8  7  7  7  7  2  3  2  4
                 3  3  3  3  1  1  1  1  3  3  3  7  7  7  7  6  6  6  6  8  8  2
                 3  3  3  3  6  6  6  6  7  7  7  7  5  5  8  8  1  1  1  1  3  2
                 4  4  4  4  7  7  7  7  4  4  1  1  1  1  6  6  6  6  8  8  2  3
                 4  4  4  4  4  4  4  4  7  7  7  7  1  1  1  1  6  6  6  6  8  8
                 1  1  1  1  3  3  3  3  6  6  6  6  8  8  7  7  7  7  4  2  2  3
```

Final holiday

Week just allocated

9 = District

10 = Mid-year holiday

Each step is a new intake arriving

Continued overleaf

2	2	2	2	2	7	7	7	7	6	6	6	6	1	1	1	1	4	8	8	4	2			
3	3	3	3	1	1	1	1	3	3	3	3	6	6	6	6	8	8	7	7	7	7			
3	3	3	3	3	6	6	6	6	7	7	7	7	8	8	5	5	1	1	1	1	4			
4	4	4	4	7	7	7	7	1	1	1	1	6	6	6	6	3	3	8	8	2	2			
					1	1	1	2	2	2	2	7	7	7	7	8	8	6	6	6	6			
					1	1	1	3	3	3	3	6	6	6	6	5	5	7	7	7	7			
					2	2	2	1	1	1	1	2	2	2	6	6	6	6	8	8	7	7	7	7
					3	3	3	1	1	1	1	3	3	3	8	8	7	7	7	7	6	6	6	6
					3	3	3	6	6	6	6	1	1	1	1	7	7	7	7	5	5			
					4	4	4	7	7	7	7	1	1	1	1	6	6	6	6	8	8			
					1	1	1	1	4	4	4	4	6	6	6	6	8	8	4	3	7	7	7	7
					2	2	2	2	1	1	1	1	2	6	6	6	6	7	7	7	7			
					2	2	2	2	7	7	7	7	2	2	2	2	6	6	6	6	1	1	1	1
					3	3	3	3	6	6	6	6	7	7	7	7	1	1	1	1	8	8		
					4	4	4	4	7	7	7	7	5	5	1	1	1	1	8	8	4			
					4	4	4	4	6	6	6	6	4	4	4	1	1	1	1	8	8			
									2	2	2	1	1	1	1	2	2	3	6	6	6	6		
									3	3	3	1	1	1	1	3	3	5	5	6	6	6	6	
									2	2	2	6	6	6	6	1	1	1	1	4	5	5		
									1	1	1	2	2	2	2	7	7	7	7	6	6	6	6	
									3	3	3	6	6	6	6	3	3	7	7	7	7			
									3	3	3	3	3	3	3	1	1	1	1	6	6	6	6	
									4	4	4	7	7	7	7	4	4	4	4	6	6	6	6	
									1	1	1	3	3	3	3	6	6	6	6	3	7	7	7	7
									1	1	1	1	4	4	4	5	5	3	8	8	6	6	6	6
									4	4	4	4	1	1	1	1	4	7	7	7	7			
									3	3	3	3	7	7	7	7	1	1	1	1	8	8		
									4	4	4	4	4	4	5	5	6	6	6	6	8	8		
									2	2	2	2	6	6	6	6	5	5	1	1	1	1		
									2	2	2	2	2	2	2	6	6	6	6	1	1	1	1	
									1	1	1	1	2	2	2	2	2	6	6	6	6			
													3	3	3	3	1	1	1	1	4			
													1	1	1	1	3	7	7	7	7			
													4	4	4	4	6	6	6	6	2			
													2	2	2	2	1	1	1	1	6	6	6	6
													2	2	2	2	7	7	7	7	3			
													3	3	3	3	3	1	1	1	1			
													1	1	1	1	4	6	6	6	6			
													2	2	2	2	2	7	7	7	7			
													3	3	3	3	3	1	1	1	1			
													4	4	4	4	4	6	6	6	6			
													4	4	4	4	4	7	7	7	7			
																		1	1	1	1			
																		3	3	3	1	1	1	1
																		4	4	4	1	1	1	1
																		2	2	2	6	6	6	6
																		3	3	3	7	7	7	7
																		4	4	4	1	1	1	1
																		2	2	2	2			
																		3	3	3	3			
																		2	2	2	2			
																		3	3	3	3			
																		4	4	4	4			
																		4	4	4	4			

Note wards allocated ahead for up to three weeks

FILE IDENTIFIER: X108M

X108 - Program

```
%BEGIN
PRINT STRING('OFF WE GO!');NEWLINE
%STRING(10) REPLY1,REPLY2,REPLY3,REPLY4,REPLY5,REPLY6,REPLY7,REPLY8
%STRING(10) REPLY9,REPLY10,REPLY11,REPLY12,REPLY13,REPLY14
%INTEGERARRAY ORDER(1:36,1:2),SHORTAGE(0:6),SHORTAGE2(0:6)
%INTEGERARRAY ORDER2(1:36,1:2)
%SHORTINTEGERARRAY ALLOCATION(1:108),WARD(1:6),WARD2(1:6),WEIGHT1(1:6)
%SHORTINTEGERARRAY TOTAL3(1:6),TOTAL4(1:6),WEIGHT2(1:6)
%SHORTINTEGERARRAY ROSTER(1:108,0:72),EXP(1:108,1:6),GEN(1:108)
%SHORTINTEGERARRAY EXPT(1:108)
%SHORTINTEGERARRAY CHOICE(1:108,0:6),TOTAL(1:6),TOTAL2(1:6)
%SHORTINTEGERARRAY PLAN(1:108,0:72),URGENCY(1:108,0:72)
%INTEGER COUNT,Z,COMP,COMP2,COMP3,COMP4,FLOAT,RESIDUE,A,B
%SHORTINTEGER FORTYNINE,LW,SC,OP,FIFTYFOUR,FN0,FN1,FN2,FN3,FN4,FN5,FN6
%SHORTINTEGER NIGHT,NEWVALUE,HOL,FN7,FN8,FN9,FN10,FN11,QUARTER,SLACKINV
%INTEGER NO,WEEK,X,Y,CODE,SECT,I,J,K,L,M,N,DAY1,MALLOCT
%REAL ROOTSUMT,RWEEK
%REALARRAY TTT(1:16),TTTT(1:11),DIVISOR(1:6)
%EXTERNALROUTINESPEC OPENDA(%INTEGER CHANNEL)
%EXTERNALROUTINESPEC READDA(%INTEGER CHANNEL,%INTEGERNAME SECT%C
,%NAME BEGIN,END)
%EXTERNALROUTINESPEC WRITEDA(%INTEGER CHANNEL,%INTEGERNAME SECT%C
,%NAME BEGIN,END)
%EXTERNALROUTINESPEC CLOSEDA(%INTEGER CHANNEL)
%EXTERNALLONGREALFNSPEC CPUTIME
NEWVALUE=0;NIGHT=0;K=0;RESIDUE=0
FN0=0;FN1=0;FN2=0;FN3=0;FN4=0;FN5=0;FN6=0;FN7=0;FN8=0;HOL=0


PRINT STRING('DO YOU WANT PRE-SELECTED PRINTOUT?');NEWLINE
READ STRING(REPLY11)
%IF REPLY11='YES' %THEN %START
                READ STRING(REPLY1)
                READ STRING(REPLY2)
                READ STRING(REPLY6)
                READ STRING(REPLY7)
                READ STRING(REPLY3)
                READ STRING(REPLY8)
                READ STRING(REPLY9)
                READ STRING(REPLY4)
                             %FINISH

PRINT STRING('DO YOU WANT NORMAL OUTPUT?')
NEWLINE
READ STRING(REPLY12)
PRINT STRING('DO YOU WANT WEIGHTS?');NEWLINE
READ STRING(REPLY14)

PRINT STRING('DO YOU WANT RESULTS OUTPUT ON LINE PRINTER?')
NEWLINE
READ STRING(REPLY10)
%IF REPLY10='YES' %THEN %START
                               SELECT OUTPUT(40)
                             %FINISH


PRINT STRING('                  VERSION 11')

NEWLINES(5)


%ROUTINE QUICKSORT(%INTEGERARRAYNAME X,%INTEGER A,B)
         %INTEGER L,U,D,E
         %RETURN %IF A>=B
         L=A;U=B;D=X(U,2);E=X(U,1)
         ->FIND
  UP:    L=L+1
         ->FOUND %IF L=U
  FIND: ->UP %UNLESS X(L,2)>D
         X(U,2)=X(L,2);X(U,1)=X(L,1)
  DOWN: U=U-1
```

```
        ->FOUND %IF L=U
        ->DOWN %UNLESS X(U,2)<D
        X(L,2)=X(U,2);X(L,1)=X(U,1)
        ->UP
  FOUND:X(U,2)=D;X(U,1)=E
  QUICKSORT(X,A,L-1)
  QUICKSORT(X,U+1,B)
%END


%ROUTINE FIND CHOICE
TTT(2)=CPUTIME
%CYCLE I=0,1,6
   CHOICE(CODE,I)=0
%REPEAT
%IF 0<PLAN(CODE,WEEK)<7 %THEN CHOICE(CODE,PLAN(CODE,WEEK))%C
                      =PLAN(CODE,WEEK) %AND ->15
%IF GEN(CODE)=0 %AND EXP(CODE,1)=0 %THEN CHOICE(CODE,1)=1%C
%AND CHOICE(CODE,2)=2%C
%AND->15
%IF EXP(CODE,1)=0 %THEN CHOICE(CODE,1)=1
%CYCLE I=3,1,6
   %IF EXP(CODE,I)=0 %THEN CHOICE(CODE,I)=I
%REPEAT
%IF GEN(CODE)<5 %THEN CHOICE(CODE,2)=2
15:

TTT(3)=CPUTIME
%END



%ROUTINE CALCULATE URGENCY

%IF PLAN(CODE,WEEK)#0 %THEN URGENCY(CODE,WEEK)=99 %AND ->115
EXPT(CODE)=0
%CYCLE I=1,1,6
   EXPT(CODE)=EXPT(CODE)+EXP(CODE,I)
%REPEAT
%IF ROSTER(CODE,WEEK)=2 %THEN PLAN(CODE,0)=111 %AND ->115
%IF EXPT(CODE)=5 %THEN URGENCY(CODE,WEEK)=26+GEN(CODE)%C
                 %ELSE URGENCY(CODE,WEEK)=6*EXPT(CODE)+1%C
                                          +GEN(CODE)
%IF ROSTER(CODE,WEEK)=1 %AND ROSTER(CODE,WEEK-1)=0 %THEN %C
URGENCY(CODE,WEEK)=28
SLACKINV=EXP(CODE,1)*3+EXP(CODE,3)*2+EXP(CODE,4)*4%C
+EXP(CODE,5)*4+EXP(CODE,6)*2+GEN(CODE)
115:
%END




TTT(9)=CPUTIME
PRINT STRING('WRITE NUMBER OF QUARTER');NEWLINE
READ(QUARTER)
FN11=(QUARTER-1)*48
%CYCLE J=1,1,108
   ALLOCATION(J)=0
   GEN(J)=0
   %CYCLE I=1,1,6
      EXP(J,I)=0
   %REPEAT
   %CYCLE I=0,1,72
      PLAN(J,I)=0
      URGENCY(J,I)=0
      %IF I>0 %THEN ROSTER(J,I)=0
   %REPEAT
%REPEAT
%SHORTINTEGERARRAY MALLOC(1:68+FN11)
%IF QUARTER=1 %THEN %START
%CYCLE I=1,1,68+FN11
   MALLOC(I)=0
```

```
%REPEAT
MALLOCT=0
ROOTSUMT=0
%FINISH %ELSE %START
%CYCLE I=21+FN11,1,68+FN11
   MALLOC(I)=0
%REPEAT
%FINISH



%REALARRAY MEANMAL(21:68+FN11),ROOTSUM(21:68+FN11)


COUNT=0;A=1;B=20
%CYCLE I=1,1,108
   COUNT=COUNT+1
   %IF COUNT>6 %THEN A=A+4 %AND B=B+4 %AND COUNT=1
%IF B<73 %THEN FN7=B %ELSE FN7=72
%IF A>72 %THEN ->180
   %CYCLE J=A,1,FN7
      ROSTER(I,J)=1
   %REPEAT
%IF B+1>72 %THEN ->180
%IF B+4<73 %THEN FN8=B+4 %ELSE FN8=72
   %CYCLE J=B+1,1,FN8
      ROSTER(I,J)=2
   %REPEAT
180:
%REPEAT
SELECT INPUT(QUARTER+2)
%CYCLE I=1,1,30
   %CYCLE J=0,1,24
      READ(PLAN(I,J))
   %REPEAT
%REPEAT
%CYCLE I=1,1,30
   %CYCLE J=1,1,6
      READ(EXP(I,J))
   %REPEAT
%REPEAT
%CYCLE I=1,1,30
   READ(GEN(I))
%REPEAT
%CYCLE I=31,1,108
   PLAN(I,0)=112
%REPEAT



%IF QUARTER>1 %THEN %START
   READ(MALLOCT);READ(ROOTSUMT)
   %CYCLE I=21,1,20+FN11
      READ(MEANMAL(I))
      READ(ROOTSUM(I))
      READ(MALLOC(I))
   %REPEAT
%FINISH
SELECT INPUT(98)




%CYCLE I=1,1,108
   %CYCLE J=1,1,72
      %IF ROSTER(I,J)=2 %THEN PLAN(I,J)=10
   %REPEAT
%REPEAT


WEIGHT1(1)=4
WEIGHT1(2)=4
WEIGHT1(3)=4
WEIGHT1(4)=4
WEIGHT1(5)=4
WEIGHT1(6)=4
```

```
WEIGHT2(1)=4
WEIGHT2(2)=4
WEIGHT2(3)=4
WEIGHT2(4)=4
WEIGHT2(5)=4
WEIGHT2(6)=0


TTT(10)=CPUTIME

%CYCLE WEEK=21,1,68
   X=WEEK


%IF REPLY12='YES' %THEN %START
SELECT OUTPUT(50)
WRITE(WEEK,3);NEWLINE
%FINISH
%IF REPLY10='YES' %THEN %START
SELECT OUTPUT(40)
%FINISH
%IF REPLY12='YES' %THEN %START
NEWLINES(5);SPACES(25)
%FINISH %ELSE %START
   NEWLINE
%FINISH

PRINT STRING('WEEK ');WRITE(WEEK+(QUARTER-1)*48,3)
%IF REPLY12='NO' %THEN %START
   PRINT STRING(':')
%FINISH %ELSE %START
   NEWLINE
SPACES(25);PRINT STRING('----------');NEWLINES(8)
%FINISH
   %CYCLE I=1,1,108
      ALLOCATION(I)=0
      %CYCLE J=0,1,6
         CHOICE(I,J)=0
      %REPEAT
   %REPEAT

%CYCLE I=0,1,6
   SHORTAGE(I)=0
   SHORTAGE2(I)=0
%REPEAT

%CYCLE I=1,1,6
   WARD(I)=0
   WARD2(I)=0
   TOTAL(I)=0
   TOTAL2(I)=0
   TOTAL3(I)=0
   TOTAL4(I)=0
%REPEAT


%CYCLE I=1,1,36
   %CYCLE J=1,1,2
      ORDER(I,J)=0
      ORDER2(I,J)=0
   %REPEAT
%REPEAT




%CYCLE CODE=1,1,103
   %IF ROSTER(CODE,WEEK)=1 %AND PLAN(CODE,0)=112 %THEN %C
   PLAN(CODE,0)=0
   %IF 110<PLAN(CODE,0)<113 %THEN ->100
   %IF ROSTER(CODE,WEEK)=2 %THEN PLAN(CODE,0)=111 %AND ->100


TTT(11)=CPUTIME

CALCULATE URGENCY
```

```
TTT(1)=CPUTIME

FIND CHOICE

TTT(12)=CPUTIME






TTT(13)=CPUTIME

100:
%REPEAT



TTT(14)=CPUTIME
%IF REPLY12='YES' %THEN %START
PRINT STRING('DO YOU WANT COMPLETE URGENCY LISTINGS?');NEWLINE
READ STRING(REPLY1) %UNLESS REPLY11='YES'
%UNLESS REPLY1='YES' %THEN ->120


%CYCLE I=1,1,108
   %IF 110<PLAN(I,0)<113 %THEN ->72
   NEWLINE
   WRITE(I,3);SPACES(5);WRITE(URGENCY(I,WEEK),3);SPACES(5)
   %CYCLE J=1,1,6
      %UNLESS CHOICE(I,J)=0 %THEN WRITE (CHOICE(I,J),2)%C
                             %ELSE %START
                                     SPACES(3)
                                   %FINISH

   %REPEAT
72:%REPEAT
NEWLINE
120:PRINT STRING('DO YOU WANT AVAILABILITY LISTINGS?');NEWLINE
READ STRING(REPLY2) %UNLESS REPLY11='YES'
%FINISH



%CYCLE I=1,1,6
   TOTAL(I)=0
   TOTAL2(I)=0
%REPEAT
COMP=0
COMP2=0


%IF REPLY12='YES' %THEN %START
NEWLINES(10)
SPACES(25);PRINT STRING('**AVAILABILITY**')
NEWLINES(4)
SPACES(2)
PRINT STRING('CODE');SPACES(4);PRINT STRING('URGENCY');SPACES(4)
PRINT STRING('49  G  54  LW   SC  OP')
NEWLINES(2)
%FINISH

%CYCLE I=1,1,108
  %IF 110<PLAN(I,0)<113 %OR URGENCY(I,WEEK)=99%C
  %OR ROSTER(I,WEEK)-(ROSTER(I,WEEK)//2)*2=0 %THEN ->125
  %IF REPLY2='YES' %AND REPLY12='YES' %THEN %START
  WRITE(I,3);SPACES(7);WRITE(URGENCY(I,WEEK),2);SPACES(8)
                         %FINISH
  COMP=COMP+1
  ORDER(COMP,1)=I
  ORDER(COMP,2)=URGENCY(I,WEEK)
  %IF REPLY2='YES' %AND REPLY12='YES' %THEN %START
  %CYCLE J=1,1,6
    %UNLESS CHOICE(I,J)=0 %THEN PRINT STRING('*   ')%C
    %AND TOTAL(J)=TOTAL(J)+1 %ELSE %START
                                     SPACES(4)
                                   %FINISH
```

```
  %REPEAT
  NEWLINE
  %FINISH %ELSE %START
     %CYCLE J=1,1,6
       %UNLESS CHOICE(I,J)=0 %THEN TOTAL(J)=TOTAL(J)+1
     %REPEAT
               %FINISH
125: %IF URGENCY(I,WEEK)=99 %AND PLAN(I,WEEK)#10%C
     %AND ROSTER(I,WEEK)-(ROSTER(I,WEEK)//2)*2#0 %THEN %START
           %CYCLE J=1,1,6
              %IF 0<CHOICE(I,J)<7 %THEN TOTAL2(J)=TOTAL2(J)+1
           %REPEAT
           COMP2=COMP2+1
                                                     %FINISH
%REPEAT

%IF REPLY2='YES' %AND REPLY12='YES' %THEN %START
SPACES(1)
PRINT STRING('---       --       -   -   -   -   -   -')
NEWLINES(4)
%FINISH
%IF REPLY12='YES' %THEN %START
WRITE(COMP,3);SPACES(15);%CYCLE I=1,1,6
                          WRITE(TOTAL(I),3)
                        %REPEAT
SPACES(2)
PRINT STRING(':MAX AVAILABLE')
NEWLINES(4)
WRITE(COMP2,3);SPACES(15);%CYCLE I=1,1,6
                             WRITE(TOTAL2(I),3)
                           %REPEAT
SPACES(2)
PRINT STRING(':PRE-ALLOCATED')
NEWLINES(4)
PRINT STRING('  31');SPACES(17)
PRINT STRING(' 5   8   3   6   6   3  :OPTIMUM')
NEWLINES(4)
PRINT STRING('  29');SPACES(17)
PRINT STRING(' 4   7   3   6   6   3  :MINIMUM')
NEWLINES(4)
%FINISH
SHORTAGE(0)=31-COMP2
SHORTAGE(1)=5-TOTAL2(1)
SHORTAGE(2)=8-TOTAL2(2)
SHORTAGE(3)=3-TOTAL2(3)
SHORTAGE(4)=6-TOTAL2(4)
SHORTAGE(5)=6-TOTAL2(5)
SHORTAGE(6)=3-TOTAL2(6)
%IF REPLY12='YES' %THEN %START
WRITE(SHORTAGE(0),3);SPACES(15)
%CYCLE I=1,1,6
   WRITE(SHORTAGE(I),3)
%REPEAT
PRINT STRING('  :SHORTAGE')
NEWLINES(10)
%FINISH



QUICKSORT(ORDER,1,COMP)

%IF REPLY12='YES' %THEN %START
PRINT STRING('DO YOU WANT DAY LISTING IN ORDER OF URGENCY?')
NEWLINE;READ STRING(REPLY6) %UNLESS REPLY11='YES'
%IF REPLY6='YES' %THEN %START
%CYCLE I=1,1,COMP
   WRITE(ORDER(I,1),3);SPACES(7)
   WRITE(ORDER(I,2),2);SPACES(8)
   %CYCLE J=1,1,6
      %UNLESS CHOICE(ORDER(I,1),J)=0 %THEN PRINT STRING('*     ')%C
      %ELSE %START
              SPACES(4)
           %FINISH
   %REPEAT
   NEWLINE
%REPEAT
NEWLINES(10)
%FINISH
%FINISH
```

```
%CYCLE I=1,1,6
   WARD(I)=0
   DIVISOR(I)=TOTAL(I)+1
%REPEAT

%CYCLE M=COMP,-1,1
   J=0
   %CYCLE I=1,1,6
      %UNLESS CHOICE(ORDER(M,1),I)=0 %THEN J=J+1 %C
      %AND WARD(J)=CHOICE(ORDER(M,1),I)
   %REPEAT
   %IF SHORTAGE(1)=1 %THEN WEIGHT1(1)=1
   %IF SHORTAGE(2)=1 %THEN WEIGHT1(2)=1
   %IF SHORTAGE(3)=0 %THEN SHORTAGE(3)=-1 %AND WEIGHT1(3)=50
%IF REPLY14='YES' %THEN %START
NEWLINE
%CYCLE I=1,1,J
   PRINT (SHORTAGE(WARD(I))*(1/DIVISOR(WARD(I)))%C
*WEIGHT1(WARD(I)),3,3)
   SPACES(2)
%REPEAT
%FINISH
   %CYCLE I=1,1,J
      L=0
      %CYCLE K=1,1,J
%IF SHORTAGE(WARD(I))*(1/(DIVISOR(WARD(I))))%C
*WEIGHT1(WARD(I))>=%C
    SHORTAGE(WARD(K))*(1/(DIVISOR(WARD(K))))%C
*WEIGHT1(WARD(K)) %C
%THEN L=L+1
         %IF L=J %THEN %START
ALLOCATION(ORDER(M,1))=WARD(I)
SHORTAGE(WARD(I))=SHORTAGE(WARD(I))-1
%CYCLE N=1,1,6
   %UNLESS CHOICE(ORDER(M,1),N)=0 %THEN %START
   %IF DIVISOR(N)>1 %THEN DIVISOR(N)=DIVISOR(N)-1
   %IF 0<DIVISOR(N)<1 %THEN DIVISOR(N)=1/(1/DIVISOR(N)-1)
                                        %FINISH
%REPEAT
->150
%FINISH


      %REPEAT
   %REPEAT
150:%REPEAT

%IF REPLY12='YES' %THEN %START
PRINT STRING('DO YOU WANT DAY ALLOCATIONS?');NEWLINE
READ STRING(REPLY7) %UNLESS REPLY11='YES'
%FINISH
%CYCLE CODE=1,1,108
   FN9=ALLOCATION(CODE)
   %IF FN9=0 %THEN->155
%IF REPLY7='YES' %AND REPLY12='YES' %THEN %START
   WRITE(CODE,3);SPACES(7);WRITE(FN9,2);NEWLINE
%FINISH

%IF 0<FN9<3 %AND ROSTER(CODE,X)=1 %AND ROSTER(CODE,X-1)=0%C
%THEN %START
        %CYCLE I=0,1,2
           PLAN(CODE,X+I)=FN9
        %REPEAT
        %IF FN9=1 %THEN EXP(CODE,FN9)=1 %ELSE GEN(CODE)=%C
                                              GEN(CODE)+3

         ->155
      %FINISH
%IF FN9=2 %THEN PLAN(CODE,WEEK)=FN9 %AND GEN(CODE)=GEN(CODE)+1%C
%AND ->155

%IF 3<FN9<6 %THEN %START
                 %CYCLE J=0,1,3
                     PLAN(CODE,WEEK+J)=FN9
                 %REPEAT
                 EXP(CODE,FN9)=1
                 ->155
                 %FINISH
%IF FN9=3 %OR FN9=6 %THEN %START
```

```
                           %CYCLE J=0,1,1
                              PLAN(CODE,WEEK+J)=FN9
                           %REPEAT
                           EXP(CODE,FN9)=1
                           ->155
                           %FINISH
%IF FN9=1 %THEN %START
                 %CYCLE J=0,1,2
                    PLAN(CODE,WEEK+J)=FN9
                 %REPEAT
                 EXP(CODE,FN9)=1
                 %FINISH
155:%REPEAT
%IF SHORTAGE(1)>0 %THEN SHORTAGE(1)=SHORTAGE(1)-1
%IF SHORTAGE(2)>0 %THEN SHORTAGE(2)=SHORTAGE(2)-1
%IF SHORTAGE(3)<0 %THEN SHORTAGE(3)=SHORTAGE(3)+1
%CYCLE I=1,1,6
%IF SHORTAGE(I)<0 %THEN MALLOC(WEEK+FN11)=MALLOC(WEEK+FN11)%C
-SHORTAGE(I) %ELSE MALLOC(WEEK+FN11)=MALLOC(WEEK+FN11)+SHORTAGE(I)
%REPEAT

NEWLINES(5) %UNLESS REPLY12='NO'
SPACES(19)
%CYCLE I=1,1,6
   WRITE(SHORTAGE(I),3)
%REPEAT
%IF REPLY12='YES' %THEN %START
PRINT STRING('   ;NEW SHORTAGE')
%FINISH %ELSE %START
   NEWLINE
%FINISH
NEWLINES(10) %UNLESS REPLY12='NO'




145:%IF REPLY12='YES' %THEN %START
PRINT STRING('DO YOU WANT UPDATED PLAN?');NEWLINE
READ STRING(REPLY4) %UNLESS REPLY11='YES'
%UNLESS REPLY4='YES' %THEN->160
SPACES(20);PRINT STRING('**UPDATED PLAN**');NEWLINES(4)

%CYCLE I=1,1,108
   %IF PLAN(I,0)=112 %THEN->165
   NEWLINE
   %IF WEEK<31 %THEN %START
                      %CYCLE J=1,1,WEEK+3
                        %UNLESS PLAN(I,J)=0 %THEN WRITE%C
                        (PLAN(I,J),1) %ELSE %START
                                            SPACES(2)
                                            %FINISH
                      %REPEAT
                      %FINISH %ELSE %START
   %CYCLE J=WEEK-30,1,WEEK+3
      %UNLESS PLAN(I,J)=0 %THEN WRITE(PLAN(I,J),1)%C
                          %ELSE %START
                                SPACES(2)
                                %FINISH

   %REPEAT
   %FINISH
165:%REPEAT


160:%FINISH

TTT(15)=CPUTIME
TTT(16)=CPUTIME
   TTTT(5)=TTT(10)-TTT(9)
%CYCLE I=6,1,10
   TTTT(I)=TTT(I+6)-TTT(I+5)
%REPEAT
TTTT(8)=TTT(14)-TTT(10)
TTTT(4)=TTT(3)-TTT(2)
TTTT(3)=TTT(12)-TTT(1)
%IF REPLY12='YES' %THEN %START
NEWLINES(2)
%CYCLE I=3,1,10
   WRITE(I,3);SPACES(5);PRINT(TTTT(I),1,3);NEWLINE
%REPEAT
%FINISH
```

```
%IF REPLY12='YES' %THEN %START
NEWLINES(5)
%CYCLE I=21,1,WEEK
   WRITE(I,3)
%REPEAT
PRINT STRING(':WEEK')
NEWLINES(2)
%CYCLE I=21,1,WEEK
   WRITE(MALLOC(I),3)
%REPEAT
PRINT STRING(':DEVIATION')

NEWLINES(2)
%CYCLE I=21,1,WEEK
   WRITE(MALLOC(I)*MALLOC(I),3)
%REPEAT
PRINT STRING(':SQUARED DEVIATION')

MALLOCT=MALLOCT+MALLOC(WEEK)
RWEEK=WEEK-20
MEANMAL(WEEK)=MALLOCT/RWEEK
NEWLINES(5)
PRINT STRING('MEAN WEEKLY DEVIATION:      ')
PRINT(MEANMAL(WEEK),3,1)



ROOTSUMT=ROOTSUMT+(MALLOC(WEEK)*MALLOC(WEEK))
ROOTSUM(WEEK)=SQRT(ROOTSUMT/RWEEK)
NEWLINES(5)
PRINT STRING('ROOT MEAN SQUARED DEVIATIONS:      ')
PRINT(ROOTSUM(WEEK),3,2)

%FINISH %ELSE %START

I=WEEK+FN11
MALLOCT=MALLOCT+MALLOC(I)
RWEEK=I-20
MEANMAL(I)=MALLOCT/RWEEK
ROOTSUMT=ROOTSUMT+(MALLOC(I)*MALLOC(I))
ROOTSUM(I)=SQRT(ROOTSUMT/RWEEK)


%IF WEEK=68 %THEN %START

PRINT STRING('DO YOU WANT DEVIATIONS TO DATE?');NEWLINE
READ STRING(REPLY13)
%IF REPLY13='YES' %THEN %START
NEWLINES(5)
SPACES(30);PRINT STRING('ROOT');NEWLINE
SPACES(20);PRINT STRING('MEAN');SPACES(6);PRINT STRING('MEAN')
NEWLINE
SPACES(20);PRINT STRING('WEEKLY');SPACES(4);PRINT STRING('SQUARED')
NEWLINE
PRINT STRING('WEEK');SPACES(6);PRINT STRING('DEVIATION');SPACE
PRINT STRING('DEVIATION');SPACE;PRINT STRING('DEVIATIONS')
NEWLINES(3)


%CYCLE I=21,1,WEEK+FN11
   WRITE(I,3);SPACES(6)
   WRITE(MALLOC(I),3);SPACES(6)
   PRINT (MEANMAL(I),3,2);SPACES(4)
   PRINT (ROOTSUM(I),3,2)
   NEWLINE
%REPEAT
%FINISH


%FINISH
%FINISH
```

```
%IF WEEK=68 %AND REPLY12='NO' %THEN %START
   SELECT OUTPUT(QUARTER+3)

%CYCLE I=73,1,102
   WRITE(PLAN(I,0),2)
   %CYCLE J=49,1,72
      WRITE(PLAN(I,J),2)
   %REPEAT
%REPEAT

%CYCLE I=73,1,102
   %CYCLE J=1,1,6
      WRITE(EXP(I,J),2)
   %REPEAT
%REPEAT


%CYCLE I=73,1,102
   WRITE(GEN(I),2)
%REPEAT

WRITE(MALLOCT,2)
PRINT(ROOTSUMT,4,1)
%CYCLE I=21,1,WEEK+FN11
   PRINT (MEANMAL(I),3,2)
   PRINT (ROOTSUM(I),3,2)
   WRITE (MALLOC(I),3)
   NEWLINE
%REPEAT

%FINISH


%REPEAT
%ENDOFPROGRAM
```

```
WEEK  36
--------
```

X108 - Sample printout

```
**AVAILABILITY**

CODE    URGENCY    49   G  54  LW  SC  OP

15                   6  15  15  11   9  14  :MAX AVAILABLE


15                   2   0   3   4   4   2  :PRE-ALLOCATED


73                   5   8   3   6   6   3  :OPTIMUM


66                   4   7   3   6   6   3  :MINIMUM


16                   3   8   0   2   2   1  :SHORTAGE
```

```
DO YOU WANT DAY LISTING IN ORDER OF URGENCY?
  53           4        *   *   *   *   *   *
  52           4        *   *   *   *   *   *
  51           4        *   *   *   *   *   *
  50           4        *   *   *   *   *   *
  49           4        *   *   *   *   *   *
  54           7            *   *   *   *   *
  44          10        *   *   *   *       *
  46          11            *   *   *   *   *
  48          13            *   *       *   *
  37          17            *   *   *       *
  41          17            *   *   *       *
  39          17            *   *   *       *
  42          21            *   *       *
  34          23            *   *           *
  32          23            *   *           *
```

```
DO YOU WANT DAY ALLOCATIONS?
  32         2
  34         2
  37         4
  39         2
  41         2
  42         2
  44         1
  46         2
  48         2
  49         1
  50         5
  51         6
  52         1
  53         2
  54         4
```

```
                                    0    0    0    0    1    0   :NEW SHORTAGE



DO YOU WANT UPDATED PLAN?
                         **UPDATED PLAN**


5  5  4  4  4  4  1  1  1  6  6  3  3  2  2  10  10  10  10
4  4  3  3  5  5  5  5  1  1  1  2  2  6  6  10  10  10  10
1  2  2  6  6  3  3  4  4  4  4  5  5  5  5  10  10  10  10
2  1  1  1  6  6  2  5  5  5  5  4  4  4  4  10  10  10  10
2  2  5  5  5  5  4  4  4  4  6  6  3  3  2  10  10  10  10
3  3  2  6  6  2  5  5  5  5  4  4  4  4  2  10  10  10  10
2  2  5  5  5  5  4  4  4  4  1  1  1  6  6  2  3  3  2  10  10  10  10
2  2  4  4  4  4  3  3  5  5  5  5  1  1  1  2  2  6  6  10  10  10  10
2  2  1  1  1  2  2  6  6  3  3  4  4  4  4  5  5  5  5  10  10  10  10
2  2  3  3  2  1  1  1  6  6  2  5  5  5  5  4  4  4  4  10  10  10  10
1  1  2  2  2  2  5  5  5  5  4  4  4  4  6  6  2  3  3  10  10  10  10
1  1  2  2  3  3  2  6  6  2  5  5  5  5  4  4  4  4  2  10  10  10  10
         2  2  2  5  5  5  5  4  4  4  4  1  1  1  2  2  6  6  3  3  10  10  10  10
         2  2  2  4  4  4  4  3  3  5  5  5  5  6  6  1  1  1  2  2  10  10  10  10
         2  2  2  1  1  1  2  2  6  6  3  3  4  4  4  4  5  5  5  5  10  10  10  10
         2  2  2  3  3  2  1  1  1  6  6  2  5  5  5  5  4  4  4  4  10  10  10  10
         1  1  1  2  2  2  2  5  5  5  5  4  4  4  4  2  3  3  6  6  10  10  10  10
         1  1  1  2  2  3  3  2  6  6  2  5  5  5  5  2  4  4  4  4  10  10  10  10
                     2  2  2  5  5  5  5  4  4  4  4  6  6  2  1  1  1  2  3  3  10  10  10  10
                     2  2  2  4  4  4  4  3  3  6  6  1  1  1  2  2  5  5  5  5  10  10  10  10
                     2  2  2  1  1  1  2  2  6  6  3  3  5  5  5  5  4  4  4  4  10  10  10  10
                     2  2  2  3  3  2  1  1  1  2  5  5  5  5  4  4  4  4  6  6  10  10  10  10
                     1  1  1  2  2  2  2  5  5  5  5  4  4  4  4  2  6  6  3  3  10  10  10  10
                     1  1  1  2  2  3  3  2  2  2  4  4  4  4  5  5  5  5  6  6  10  10  10  10
                                 2  2  2  5  5  5  5  2  2  6  6  1  1  1  4  4  4  4  3  3  10  10  10
                                 2  2  2  4  4  4  4  1  1  1  2  6  6  2  3  3  5  5  5  5  10  10  10
                                 2  2  2  1  1  1  2  5  5  5  5  4  4  4  4  2  6  6  3  3  10  10  10
                                 2  2  2  3  3  2  2  1  1  1  4  4  4  4  5  5  5  5  6  6  10  10  10
                                 1  1  1  2  2  3  3  4  4  4  4  2  2  6  6  2  5  5  5  5  10  10  10
                                 1  1  1  2  2  3  3  2  2  6  6  2  4  4  4  4  5  5  5  5  10  10  10
                                             2  2  2  1  1  1  3  3  2  6  6  2  4  4  4  4
                                             1  1  1  4  4  4  4  5  5  5  5  2  2  2  2  2
                                             2  2  2  2  2  3  3  1  1  1  4  4  4  4  6  6
                                             2  2  2  5  5  5  5  1  1  1  2  4  4  4  4  2
                                             1  1  1  5  5  5  5  2  2  2  2  2  6  6  4  4  4  4
                                             1  1  1  6  6  2  2  2  2  2  3  3  4  4  4  4
                                                         2  2  2  5  5  5  5  1  1  1  2  4  4  4  4
                                                         2  2  2  6  6  5  5  5  5  2  1  1  1
                                                         2  2  2  1  1  1  2  5  5  5  5  2
                                                         2  2  2  2  2  3  3  1  1  1  4  4  4  4
                                                         2  2  2  5  5  5  5  1  1  1  2  2
                                                         1  1  1  4  4  4  4  2  2  6  6  2
                                                                     2  2  2  1  1  1  5  5  5  5
                                                                     2  2  2  5  5  5  5  1  1  1
                                                                     2  2  2  6  6  2  1  1  1
                                                                     1  1  1  2  2  2  2  2
                                                                     2  2  2  1  1  1  3  3
                                                                     1  1  1  4  4  4  4  2
                                                                                 2  2  2  1  1  1
                                                                                 2  2  2  5  5  5  5
                                                                                 2  2  2  6  6
                                                                                 2  2  2  1  1  1
                                                                                 2  2  2  2
                                                                                 1  1  1  4  4  4  4
              --------------------------------------------------------------------------------------

21   22   23   24   25   26   27   28   29   30   31   32   33   34   35   36:WEEK

 3    1    5    1    3    2    4    1    7    3    5    2    5    5    2    1:DEVIATION
```

X108 - Abbreviated output.

```
                VERSION 26

WRITE NUMBER OF QUARTER
DATA:1

WEEK    21:               0    0    0    0    0    0

WEEK    22:               0    0    0    0    0    0

WEEK    23:               1    0    0   -1    0    0

WEEK    24:               0    0    0    0    0    0

WEEK    25:               0   -1    0    0    0    0

WEEK    26:               0   -1    0    0    0    0

WEEK    27:               2   -1    0    0   -1    0

WEEK    28:               0    0    0    0    0    0

WEEK    29:               0    0    0    0    0    0

WEEK    30:               0   -1    0    0    0    0

WEEK    31:               1    0    0   -1    1   -1

WEEK    32:               0    0    0    0    0    0

WEEK    33:               0    0    0   -1    0    1

WEEK    34:               1   -1    0   -1    0    0

WEEK    35:               1    0    0    0    0   -1

WEEK    36:               0    0    0    0    0    0

WEEK    37:               0    0    0   -1    0    1

WEEK    38:               1   -1    1   -2    1   -1

WEEK    39:               1    0    0   -1    0    0

WEEK    40:               0    0    0    0    0    0

WEEK    41:               0    0    0    0    0    0

WEEK    42:               0   -1    0    1    0    0

WEEK    43:               0    0    0    0    0    0

WEEK    44:               0    0    0    0    0    0

WEEK    45:               0    0    0    1   -1    0

WEEK    46:               0    0    0    0   -1    1

WEEK    47:               0    0    0   -1    0    0

WEEK    48:               0    0    0    0    0    0

WEEK    49:               0    0    0    0    0    0

WEEK    50:              -1    0    0    1    0    0

WEEK    51:               1    0    0    1   -1   -1

WEEK    52:               0    0    0    0    0    0

WEEK    53:               0    0    0    0    0    0

WEEK    54:              -1    2    0    0   -1    0

WEEK    55:              -1    0    0    1    0    0

WEEK    56:               0    0    0    0    0    0

WEEK    57:               0    0    0    0    0    0
```

```
WEEK   58:                          0  -1   0   0   1   0

WEEK   59:                          0   0   0   0   0   0

WEEK   60:                          0   0   0   0   0   0

WEEK   61:                          0   0   0   0   0   0

WEEK   62:                         -1   1   0   0   0   0

WEEK   63:                          0   0   0   0   0   0

WEEK   64:                          0   0   0   0   0   0

WEEK   65:                          0   0   0   0   0   0

WEEK   66:                         -1   1   0   0   0   0

WEEK   67:                          0   0   0   0   0   0

WEEK   68:                          0   0   0   0   0   0
DO YOU WANT DEVIATIONS TO DATE?
DATA:'YES'



                                  ROOT
                       MEAN       MEAN
                       WEEKLY     SQUARED
WEEK         DEVIATION DEVIATION  DEVIATIONS


 21            0         0.00       0.00
 22            0         0.00       0.00
 23            2         0.67       1.15
 24            0         0.50       1.00
 25            1         0.60       1.00
 26            1         0.67       1.00
 27            4         1.14       1.77
 28            0         1.00       1.66
 29            0         0.89       1.56
 30            1         0.90       1.52
 31            4         1.18       1.88
 32            0         1.08       1.80
 33            2         1.15       1.82
 34            3         1.29       1.93
 35            2         1.33       1.93
 36            0         1.25       1.87
 37            2         1.29       1.88
 38            7         1.61       2.46
 39            2         1.63       2.44
 40            0         1.55       2.38
 41            0         1.48       2.32
 42            2         1.50       2.31
 43            0         1.43       2.26
 44            0         1.37       2.21
 45            2         1.40       2.20
 46            2         1.42       2.19
 47            1         1.41       2.16
 48            0         1.36       2.12
 49            0         1.31       2.08
 50            2         1.33       2.08
 51            4         1.42       2.17
 52            0         1.37       2.14
 53            0         1.33       2.10
 54            4         1.41       2.18
 55            2         1.43       2.16
 56            0         1.39       2.15
 57            0         1.35       2.12
 58            2         1.37       2.12
 59            0         1.33       2.09
 60            0         1.30       2.06
 61            0         1.27       2.04
 62            2         1.29       2.04
 63            0         1.26       2.01
 64            0         1.23       1.99
 65            0         1.20       1.97
 66            2         1.22       1.97
 67            0         1.19       1.95
 68            0         1.17       1.93
```

FILE IDENTIFIER : FASTNEW7 **HEURISTIC METHOD** - Program

```
%BEGIN
%REAL TEMP,ESTREDUCT
NEWLINES(2)
%STRING(20) REPLY1,REPLY2
%INTEGER COUNT,QQ,CMIN1,CMIN2,CHANGMIN,JSTORE,TSTORE
%INTEGER RECTRAN1,RECTRAN2,REPLY3
%INTEGER I,J,T,Q,IT,G,DS,GRADMAX,JGRAD,TGRAD,Z,HALFINTERVAL
%INTEGER TINT,IM,JM,TM,QM,PRICS,ECHANGE,EV,EVM,DEF,QMAX,QMIN
  SELECT INPUT(05)
%EXTERNALROUTINESPEC PROMPT(%STRING(20) S)
%EXTERNALLONGREALFNSPEC CPUTIME


PROMPT('GIRLS?');READ(IM)
PROMPT('WARDS?');READ(JM)
PROMPT('WEEKS?');READ(TM)
PROMPT('SCHEDULES?');READ(QM)
PROMPT('Z');READ(Z)
PROMPT('HALFINTERVAL:');READ(HALFINTERVAL)
%INTEGERARRAY AX(1:QM,1:TM),X,XX,XSTORE,TITE,LIMIT(1:QM)
%INTEGERARRAY QS,QSTORE(1:IM),CHANGE(1:QM-1,2:QM)
%INTEGERARRAY GRAD,GRADSTORE,D(1:JM,1:TM)
%REALARRAY C(1:QM), U(1:JM,1:TM),USTORE(1:JM,1:TM)
%REALARRAY V,SCHED(1:QM),TTT(1:3),TTTT(1:2),SUBMAX(1:IM)
%REAL MULT,M,STEP,N,ZIT,H,HI,HSTORE,PRIMALCOST,MULTMULT
%REAL SCHEDMIN,SCHEDMAX


%INTEGERFN A(%INTEGER J,T,Q)
   %IF AX(Q,T)=J %THEN %RESULT=1 %ELSE %RESULT=0
%END



EV=0;IT=0;MULT=2;ZIT=0;M=0;PRICS=0;PRIMALCOST=0;ECHANGE=0
H=0;RECTRAN1=-1;RECTRAN2=0

PROMPT('COSTS')
   %CYCLE Q=1,1,QM
      READ(C(Q))
      X(Q)=0
      SCHED(Q)=0
      LIMIT(Q)=IM
   %REPEAT

PROMPT('DEMAND')
%CYCLE J=1,1,JM
   %CYCLE T=1,1,TM
      READ(D(J,T))
      U(J,T)=0
      USTORE(J,T)=0
      GRADSTORE(J,T)=0
   %REPEAT
%REPEAT


PROMPT('SCHEDULES')
%CYCLE Q=1,1,QM
   %CYCLE T=1,1,TM
      READ(AX(Q,T))
   %REPEAT
%REPEAT



PROMPT('ESTIMATE')
READ(HI)
```

```
    PROMPT('PROBLEM SIZE')
READ(N)

PRINT STRING('DO YOU WANT EACH ITERATION?');NEWLINE
PROMPT('')
READSTRING(REPLY1)



PROMPT('EVALUATIONS:     ')
READ(EVM)



PRINT STRING('DO YOU WANT TO START AT THE BEGINNING?')
PROMPT('')
NEWLINE
READSTRING(REPLY2)
%IF REPLY2='NO' %THEN %START
PROMPT('NEW DATA?')
%CYCLE Q=1,1,QM
   READ(XX(Q))
%REPEAT
%FINISH

PRINT STRING('HOW MANY SCHEDULE LIMITS?');NEWLINE
READ (REPLY3)
%IF REPLY3>0 %THEN %START
%CYCLE I=1,1,REPLY3
   PROMPT('SCHEDULE:');READ(Q)
   PROMPT('LIMIT:');READ(LIMIT(Q))
%REPEAT
                    %FINISH


TTT(1)=CPUTIME


%CYCLE T=1,1,TM
   %CYCLE J=1,1,JM
      TITE(J)=0
   %REPEAT
   %CYCLE Q=1,1,QM
      TINT=AX(Q,T)
      TITE(TINT)=TITE(TINT)+1
   %REPEAT

   %CYCLE J=1,1,JM
      %IF TITE(J)=0 %AND D(J,T)>0 %THEN->600
      %IF TITE(J)=1 %THEN %START
%CYCLE Q=1,1,QM
   %IF AX(Q,T)=J %AND X(Q)<D(J,T) %THEN X(Q)=D(J,T)
%REPEAT
                           %FINISH

   %REPEAT
%REPEAT


TINT=0
%CYCLE Q=1,1,QM
   TINT=TINT+X(Q)
%REPEAT

%IF TINT>IM %THEN->700
IM=IM-TINT


%CYCLE J=1,1,JM
   %CYCLE T=1,1,TM
      G=0
      %CYCLE Q=1,1,QM
         G=G+A(J,T,Q)*X(Q)
      %REPEAT
      D(J,T)=D(J,T)-G
   %IF IM=0 %AND D(J,T)>0 %THEN ->700
   %REPEAT
%REPEAT
```

```
PRINT STRING('TIGHT ALLOCATIONS:');NEWLINE
TEMP=0
%CYCLE Q=1,1,QM
   TEMP=TEMP+1
   WRITE(X(Q),6)
   %IF TEMP=10 %THEN NEWLINE %AND TEMP=0
   XSTORE(Q)=X(Q)
   %IF REPLY2='YES' %THEN X(Q)=0 %ELSE X(Q)=XX(Q)
%REPEAT
NEWLINES(5)
PRINT STRING('ITERATION     DEF');SPACES(10)
TTT(2)=CPUTIME
TTTT(1)=TTT(2)-TTT(1)
PRINT STRING('SET-UP TIME:')
PRINT(TTTT(1),1,3)
NEWLINE


100:%IF EV=EVM %THEN ->200
EV=EV+1
IT=IT+1
TTT(2)=CPUTIME


%IF EV=1 %THEN %START
%IF REPLY2='YES' %THEN %START
   TEMP=C(1)
   TINT=1
   %CYCLE Q=2,1,QM
      %IF C(Q)<TEMP %THEN TEMP=C(Q) %AND TINT=Q
  %REPEAT
   X(TINT)=IM
   H=C(TINT)*IM
                         %FINISH
                  %FINISH %ELSE %START
H=0

%CYCLE J=1,1,JM
   %CYCLE T=1,1,TM
      H=H+D(J,T)*U(J,T)
   %REPEAT
%REPEAT



COUNT=0


%CYCLE Q=1,1,QM-1
   %CYCLE QQ=Q+1,1,QM
CHANGE(Q,QQ)=0

!%UNLESS X(Q)>0 %THEN ->300
%IF RECTRAN2+(100*Q+QQ)=0 %THEN ->300
%UNLESS ((-A(JSTORE,TSTORE,Q)+A(JSTORE,TSTORE,QQ))>0 %C
%AND X(Q)>0 %AND X(QQ)<LIMIT(QQ)) %THEN ->300
      %CYCLE J=1,1,JM
         %CYCLE T=1,1,TM
TINT=GRAD(J,T)+A(J,T,Q)-A(J,T,QQ)
%IF TINT>0 %OR GRAD(J,T)>0 %THEN CHANGE(Q,QQ)=%C
CHANGE(Q,QQ)+A(J,T,Q)-A(J,T,QQ)

         %REPEAT
      %REPEAT
COUNT=COUNT+1
%IF CCUNT=1 %THEN CHANGMIN=CHANGE(Q,QQ) %AND CMIN1=Q %AND CMIN2=QQ
%IF CHANGE(Q,QQ)<CHANGMIN %THEN CHANGMIN=CHANGE(Q,QQ)%C
%AND CMIN1=Q %AND CMIN2=QQ
300:
   %REPEAT
%REPEAT

%CYCLE Q=1,1,QM-1
   %CYCLE QQ=Q+1,1,QM
CHANGE(Q,QQ)=0
```

```
%IF RECTRAN2-(100*Q+QQ)=0 %THEN ->400
%UNLESS ((-A(JSTORE,TSTORE,Q)+A(JSTORE,TSTORE,QQ))<0 %C
%AND X(QQ)>0 %AND X(Q)<LIMIT(Q)) %THEN->400

     %CYCLE J=1,1,JM
        %CYCLE T=1,1,TM
TINT=GRAD(J,T)-A(J,T,Q)+A(J,T,QQ)
%IF TINT>0 %OR GRAD(J,T)>0 %THEN CHANGE(Q,QQ)=%C
CHANGE(Q,QQ)-A(J,T,Q)+A(J,T,QQ)

         %REPEAT
      %REPEAT
COUNT=COUNT+1
%IF COUNT=1 %THEN CHANGMIN=CHANGE(Q,QQ) %AND CMIN1=QQ %AND CMIN2=Q
%IF CHANGE(Q,QQ)<CHANGMIN %THEN CHANGMIN=CHANGE(Q,QQ)%C
%AND CMIN1=QQ %AND CMIN2=Q
400:
  %REPEAT
%REPEAT

%IF GRADMAX<X(CMIN1) %THEN TINT=GRADMAX %ELSE TINT=1
%IF GRADMAX<4 %THEN TINT=1
%IF RECTRAN1+RECTRAN2=0 %THEN TINT=X(CMIN1)//2
RECTRAN1=RECTRAN2
%IF CMIN1>CMIN2 %THEN %START
      RECTRAN2=-(100*CMIN2+CMIN1)
%FINISH %ELSE %START
  RECTRAN2=100*CMIN1+CMIN2
%FINISH

TINT=1

X(CMIN2)=X(CMIN2)+TINT
X(CMIN1)=X(CMIN1)-TINT


                        %FINISH


DEF=0;DS=0;GRADMAX=0

%CYCLE J=1,1,JM
   %CYCLE T=1,1,TM
      G=0
      %CYCLE Q=1,1,QM
        G=G+A(J,T,Q)*X(Q)
      %REPEAT
      GRAD(J,T)=D(J,T)-G
      %IF GRAD(J,T)>0 %THEN DEF=DEF+GRAD(J,T)
      DS=DS+GRAD(J,T)*GRAD(J,T)
      %IF GRAD(J,T)>GRADMAX %THEN GRADMAX=GRAD(J,T)%C
%AND JSTORE=J %AND TSTORE=T
  %REPEAT
%REPEAT

%IF GRADMAX<=0 %THEN ->200
WRITE(GRADMAX,5);WRITE(JSTORE,5);WRITE(TSTORE,5);NEWLINE

 %IF REPLY1='YES' %THEN %START
WRITE(IT,4);SPACES(5)
WRITE(DEF,4)




NEWLINE
TEMP=0
%CYCLE Q=1,1 ,QM
  TEMP=TEMP+1
  WRITE(X(Q),6)
  %IF TEMP=10 %THEN NEWLINE %AND TEMP=0
%REPEAT
NEWLINE

                      %FINISH
```

```
 TTT(3)=CPUTIME
 TTTT(1)=TTT(3)-TTT(2)
 TTTT(2)=TTT(3)-TTT(1)
 PRINT STRING('ITERATION TIME:');PRINT(TTTT(1),1,3)
 PRINT STRING('TOTAL TIME:');PRINT(TTTT(2),1,3)


 ->100
 200:%IF REPLY1 ='NO' %THEN %START
 PRINT STRING('ITERATIONS:  ');WRITE(IT,6);NEWLINE
 PRINT STRING('EVALUATIONS: ');WRITE(EV,6);NEWLINE
 %FINISH
 PRINT STRING('SOLUTION= ');WRITE(DEF,4);SPACES(10)
 NEWLINE
 TEMP=0
 %CYCLE Q=1,1,QM
       TEMP=TEMP+1
       WRITE(X(Q),6)
       %IF TEMP=10 %THEN NEWLINE %AND TEMP=0
   %REPEAT


 TEMP=0
 %CYCLE Q=1,1,QM
    TEMP=TEMP+1
    PRINT(SCHED(Q),4,1)
    %IF TEMP=10 %THEN NEWLINE %AND TEMP=0
 %REPEAT



NEWLINE;PRINT STRING(' DEFICITS:');NEWLINE

%CYCLE J=1,1,JM
   %CYCLE T=1,1,TM
      WRITE(GRAD(J,T),2)
      %IF T=TM %THEN NEWLINE
   %REPEAT
%REPEAT
NEWLINE


%IF EV=EVM %THEN %START
   PROMPT('')
   PRINT STRING('HOW MANY MORE EVALUATIONS?');NEWLINE
   READ (TINT)
   %IF TINT=0 %THEN ->500
   EVM=EVM+TINT
   ->100
                       %FINISH
PRINT STRING('TOTAL ALLOCATIONS:  ');NEWLINE
TEMP=0
%CYCLE Q=1,1,QM
   TEMP=TEMP+1
   WRITE(X(Q)+XSTORE(Q),6)
   %IF TEMP=10 %THEN NEWLINE %AND TEMP=0
%REPEAT


%IF GRADMAX<=0 %THEN->500
600:
PRINT STRING('PROBLEM IS INFEASIBLE');NEWLINES(2)
PRINT STRING('THERE IS NO SCHEDULE WHICH PUTS A GIRL ON WARD')
WRITE(J,1);PRINT STRING(' IN WEEK')
WRITE(T,2);NEWLINE
->500
700:
PRINT STRING('PROBLEM IS INFEASIBLE');NEWLINES(2)
PRINT STRING('THERE ARE TOO FEW GIRLS TO SATISFY THE TIGHT CONSTRAINTS')
NEWLINE


500:

%ENDOFPROGRAM
```

```
RUN(FNEW80)
```

Data set: REAL 13 (I=60,J=5,Q=20,T=30)

```
DO YOU WANT EACH ITERATION?
'YES'
EVALUATIONS:    20
DO YOU WANT TO START AT THE BEGINNING?
'YES'
HOW MANY SCHEDULE LIMITS?
0
TIGHT ALLOCATIONS:
  0  0  0  0  0  0  0  0  0  0
  0  0  0  0  0  0  0  0  0  0
  0  0  0  0  6  6  6  0  0  0
```

HEURISTIC METHOD

Sample output

```
SET-UP TIME: 1.369



ITERATION:    1
SHORTAGES: 271
SHORTAGE OF 17 ON WARD  4 IN WEEK  9

 16  0  9  0  7 10  0  0  0  0
  0  0  0  0  0  0  0  0  0  0
  0  0  0  0  0  0  0  0  0  0

ITERATION TIME: 0.411      TOTAL TIME: 1.827

ITERATION:    2
SHORTAGES: 227
SHORTAGE OF 13 ON WARD  4 IN WEEK  9

 16  0  5  0  7 10  0  0  0  0
  0  0  0  0  0  0  0  0  0  0
  0  0  0  0  0  0  0  0  0  4

ITERATION TIME: 1.178      TOTAL TIME: 3.040

ITERATION:    3
SHORTAGES: 242
SHORTAGE OF 19 ON WARD  2 IN WEEK  5

  8  0  5  0  7 10  0  8  0  0
  0  0  0  0  0  0  0  0  0  0
  0  0  0  0  0  0  0  0  0  4

ITERATION TIME: 1.183      TOTAL TIME: 4.258

ITERATION:    4
SHORTAGES: 213
SHORTAGE OF 17 ON WARD  2 IN WEEK  5

  8  0  3  0  7 10  0  8  2  0
  0  0  0  0  0  0  0  0  0  0
  0  0  0  0  0  0  0  0  0  4

ITERATION TIME: 1.307      TOTAL TIME: 5.601

ITERATION:    5
SHORTAGES: 198
SHORTAGE OF 16 ON WARD  2 IN WEEK  5

  8  0  2  0  7 10  0  8  2  0
  0  0  0  0  0  0  0  0  0  0
  1  0  0  0  0  0  0  0  0  4

ITERATION TIME: 1.317      TOTAL TIME: 6.953
```

```
ITERATION:    6
SHORTAGES: 149
SHORTAGE OF 12 ON WARD  2 IN WEEK  5

  8  0  2  0  7 10  0  4  2  0
  0  0  0  0  0  0  0  0  0  0
  5  0  0  0  0  0  0  0  0  4

ITERATION TIME: 1.336      TOTAL TIME: 8.324

ITERATION:    7
SHORTAGES: 136
SHORTAGE OF 10 ON WARD  4 IN WEEK  7

  8  0  2  0  4 10  0  4  2  0
  0  0  0  0  0  0  0  0  0  0
  8  0  0  0  0  0  0  0  0  4

ITERATION TIME: 1.330      TOTAL TIME: 9.689

ITERATION:    8
SHORTAGES: 100
SHORTAGE OF  7 ON WARD  1 IN WEEK 16

  8  0  2  0  4  5  0  4  2  0
  0  5  0  0  0  0  0  0  0  0
  8  0  0  0  0  0  0  0  0  4

ITERATION TIME: 1.222      TOTAL TIME: 10.946

ITERATION:    9
SHORTAGES:  80
SHORTAGE OF  5 ON WARD  1 IN WEEK 16

  8  0  2  0  4  3  0  4  2  0
  0  5  0  2  0  0  0  0  0  0
  8  0  0  0  0  0  0  0  0  4

ITERATION TIME: 1.221      TOTAL TIME: 12.203

ITERATION:  10
SHORTAGES:  65
SHORTAGE OF  5 ON WARD  4 IN WEEK  7

  8  0  2  0  4  3  2  2  2  0
  0  5  0  2  0  0  0  0  0  0
  8  0  0  0  0  0  0  0  0  4

ITERATION TIME: 1.246      TOTAL TIME: 13.484

ITERATION:  11
SHORTAGES:  54
SHORTAGE OF  4 ON WARD  4 IN WEEK  9

  8  0  2  0  2  3  2  2  2  0
  0  5  0  2  0  0  0  0  0  0
  8  0  0  0  0  0  0  0  2  4

ITERATION TIME: 1.389      TOTAL TIME: 14.909

ITERATION:  12
SHORTAGES:  71
SHORTAGE OF  6 ON WARD  2 IN WEEK  5

  4  0  2  0  2  3  6  2  2  0
  0  5  0  2  0  0  0  0  0  0
  8  0  0  0  0  0  0  0  2  4

ITERATION TIME: 1.736      TOTAL TIME: 16.680
```

```
ITERATION:  13
SHORTAGES:  52
SHORTAGE OF  5 ON WARD  2 IN WEEK 12

  4  0  2  0  2  3  3  2  5  0
  0  5  0  2  0  0  0  0  0  0
  8  0  0  0  0  0  0  0  2  4

ITERATION TIME: 1.682      TOTAL TIME: 18.397

ITERATION:  14
SHORTAGES:  44
SHORTAGE OF  3 ON WARD  2 IN WEEK  5

  6  0  2  0  2  3  3  2  3  0
  0  5  0  2  0  0  0  0  0  0
  8  0  0  0  0  0  0  0  2  4

ITERATION TIME: 2.366      TOTAL TIME: 20.799

ITERATION:  15
SHORTAGES:  38
SHORTAGE OF  2 ON WARD  2 IN WEEK  5

  6  0  2  0  2  2  3  2  3  0
  0  6  0  2  0  0  0  0  0  0
  8  0  0  0  0  0  0  0  2  4

ITERATION TIME: 1.689      TOTAL TIME: 22.523

ITERATION:  16
SHORTAGES:  32
SHORTAGE OF  2 ON WARD  2 IN WEEK 12

  6  0  2  0  2  2  3  1  3  0
  0  6  0  2  0  0  0  0  0  0
  8  0  0  0  0  0  0  0  2  5

ITERATION TIME: 1.702      TOTAL TIME: 24.260

ITERATION:  17
SHORTAGES:  23
SHORTAGE OF  1 ON WARD  1 IN WEEK  8

  6  0  2  0  2  1  3  1  3  0
  0  6  0  2  0  0  0  0  0  0
  8  1  0  0  0  0  0  0  2  5

ITERATION TIME: 2.340      TOTAL TIME: 26.641

ITERATION:  18
SHORTAGES:  19
SHORTAGE OF  1 ON WARD  1 IN WEEK 14

  6  0  1  0  2  1  3  1  3  0
  0  6  0  2  0  0  0  1  0  0
  8  1  0  0  0  0  0  0  2  5

ITERATION TIME: 2.428      TOTAL TIME: 29.104

ITERATION:  19
SHORTAGES:  15
SHORTAGE OF  1 ON WARD  2 IN WEEK  2

  6  0  1  0  1  1  4  1  3  0
  0  6  0  2  0  0  0  1  0  0
  8  1  0  0  0  0  0  0  2  5

ITERATION TIME: 1.604      TOTAL TIME: 30.743
```

```
ITERATION:  20
SHORTAGES:  11
SHORTAGE OF  1 ON WARD  1 IN WEEK  1

  6  0  1  0  1  0  4  1  3  0
  0  6  0  2  0  0  1  1  0  0
  8  1  0  0  0  0  0  0  2  5

ITERATION TIME: 2.933      TOTAL TIME: 33.712
SOLUTION=     11
  6  0  1  0  1  0  4  1  3  0
  0  6  0  2  0  0  1  1  0  0
  8  1  0  0  0  0  0  0  2  5

 DEFICITS:
  1  1  1  0  0  0  0  0  0  0  0  0  0  0  0  0  0 -1 -1 -1
 -1  0  0  0  1  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0
  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0
 -1 -1 -1  0  0  1  1  1  0  0  0  0  0  0  0  0  0  0  0  0
  1  0  0  0 -1 -1 -1 -1  0  0  0  0  0  0  0  0  0  1  1  1

HOW MANY MORE EVALUATIONS?
10


ITERATION:  21
SHORTAGES:   9
SHORTAGE OF  1 ON WARD  2 IN WEEK  2

  6  0  1  0  1  0  4  0  3  0
  0  7  0  2  0  0  1  1  0  0
  8  1  0  0  0  0  0  2  5

ITERATION TIME: 1.384      TOTAL TIME: 35.436


ITERATION:  22
SHORTAGES:   6
SHORTAGE OF  1 ON WARD  1 IN WEEK  1

  6  0  1  0  1  0  4  0  3  0
  0  6  0  2  0  0  1  1  0  0
  8  1  0  0  0  0  0  0  3  5

ITERATION TIME: 2.612      TOTAL TIME: 38.080
SOLUTION=      0
  5  0  1  0  1  0  4  0  3  0
  0  6  0  2  0  0  1  1  0  0
  8  1  0  0  0  0  0  0  3  6

 DEFICITS:
  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0
  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0
  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0
  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0
  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0

TOTAL ALLOCATIONS:
      5      0      1      0      1      0      4      0      3      0
      0      6      0      2      0      0      1      1      0      0
      8      1      0      0      6      6      6      0      3      6
```

NOTES

Introduction

1. Simpson Memorial Maternity Pavilion, Nurses Information Booklet. Internal circulation, undated.

2. Central Midwives' Board for Scotland, "Aims and Objectives". Undated. See Appendix A.

3. Ackoff, R.L. and M.W. Sasieni, "Fundamentals of Operations Research", John Wiley and Sons Inc. (1968) p278.

4. Mooney, G.H., Health Economics Research Unit, University of Aberdeen. Discussion papers Nos. 01-04/77, and Scottish Health Service Study No. 9, "Nursing workload per patient as a basis for staffing", Scottish Home and Health Dept. (1969).

5. Baligh, H.H., and D.J. Laughhunn, "An economic and linear model of the hospital", Health Service Research 4 (1969) 293-303, and Lee, S.M., "An aggregate resource allocation model for hospital administration", Socio-Econ. Plan. Sci. 7 (1973) 471-487, and Abernathy, W., N. Baloff, J. Hershey and S. Wandel, "A three stage manpower planning and scheduling model - a service-sector example", Opns. Res. 21 (1973) 693-711.

6. Davis, M. and R. Saunders, "Allocating student nurses by computer", N. Times 62 (1966) 467-9.

7. Gupta, I., J. Zareda and N. Dramer, "Hospital manpower planning by use of queueing theory", Health Service Res. 6 (1971) 76-82.

8. Williams, W.J., R.P. Covert and J.D. Steele, "Simulation modelling of a teaching hospital outpatient clinic", Hospitals JAHA41,128 (1967) 71-75.

9. Haussman, R.K.D., "Waiting time as an index of quality of nursing care", Health Service Res. 5 (Summer 1970) p92.

10. Maier-Rothe, C., and H.B. Wolfe, "Cyclical scheduling and allocation of nursing staff", Socio. Econ. Plan. Sci. 7 (1973) 471-487.

11. Francis, M.A., "Implementing a program of cyclical scheduling of nursing personnel", Hospitals, JAHA 40 (July 16, 1966) 108-125.

12. Howell, J.P., "Cyclical scheduling of nursing personnel", Hospitals, JAHA 40 (1966) 77-85.

13. Banks, I., "Nurse allocation", M.Sc. Thesis, University of Aston (1969).

14. Rothstein, M., "Hospital manpower shift scheduling by mathematical programming", Health Service Res. 8 (1973) 60-66.

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16. Warner, D.M. and J. Prawda, "A mathematical programming model for scheduling nursing personnel in a hospital", Management Science 19 (1972) 411-422.

17. Warner D.M., "Scheduling nursing personnel according to nursing preference: a mathematical programming approach", Operations Research 24 (1976) 842-856.

Chapter 1

1. As yet there have been no formal directives laid down by the E.E.C. Miss Phyllis Friend is Chief Negotiator to the E.E.C. on behalf of the Central Midwives Board (C.M.B.) for Scotland. Through her, Draft Council Directives have been received, and although the information they contain is sometimes speculative, the Simpson tends to bear them in mind in order to anticipate future moves, in order to lessen any possible disruption when they are forced to adopt certain measures. Formal rulings are expected to be agreed by 1980.

2. Francis (1966) op. cit.

3. Most member countries of the E.E.C. require two years training for midwives, but the C.M.B. are resisting attempts to make Britain confirm to this. At present the Simpson are considering a change to an eighteen month course.

4. C.M.B. for Scotland - Revision of Training (Oct. 1975). See Appendix B.

5. Personal communication with Senior Nursing Officer, 1974.

Chapter 2

1. C.M.B. Rules (1965) - Framed by the C.M.B. under the Midwives (Scotland) Act, 1951.

2. Ibid.

3. Ibid.

4. Ibid.

5. These are still only in draft form, but the implications of the anticipated changes are being examined. See Chapter 1, Note 1.

6. Black, R., "South Lothian College of Nursing and Midwifery - Midwifery training program", Lothian Health Board, Submission for implementation 1st December 1976, See Appendix C.

7. See Chapter 1, Note 1.

8. Black, R. (1976) *op. cit.*

9. John McFarlane, Director - Health Services O.R. Unit, University of Strathclyde, who described his work in a talk given to the Manpower Society Scottish Group entitled "Quantitative speculation on nursing roles" (1978).

10. Scottish Health Service Study No. 9 (1969) *op. cit.*

11. Personal communication with Miss Jamieson, Principal Nursing Officer, 1977.

Chapter 3

1. C.M.B. rules (1965) *op. cit.*

2. *Ibid.*

Chapter 4

1. Personal communication with Miss Tomlin, Senior Nursing Officer, 1974.

2. Ackoff and Sasieni (1968) *op. cit.*

3. *Ibid.*

Chapter 5

1. Haldi, J., "25 integer programming test problems", *Rept. of Graduate School of Business*, Stanford University, Calif. (1964), and
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Chapter 6

1. Fisher, M.L., "Optimal solution of scheduling problems using Lagrange multipliers: Part 1", *Operations Research* 21, No. 5 (1973) 1114-1127.

2. Held, M., and R.M. Karp, "The travelling salesman problem and minimum spanning trees", *Operations Research* 18 (1970) 1138-1162,
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3. Edinburgh Regional Computing Centre IMP Library: IMP DLINPR, "Modified Simplex method for linear programming", 19.330.501/ERCC.

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5. Dakin, R.J., "A tree search algorithm for mixed integ programming problems", *Computer Journal*, 8 (1965) 250-255,
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Taha (1971) *op. cit.* 326-342,
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Balas, E., "An additive algorithm for solving linear programs with zero-one variables", *Operations Research* 13 (1965) 517-546,
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Geoffrion, A.M., "An improved implicit enumeration approach for integer programming", *Operations Research* 17 No. 3 (1969) 437-454,
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Healey, W.C., "Multiple choice programming", *Operations Research* 12 (1964) 122-138.

6. Held, M., P. Wolfe and H.P. Crowder, "Validation of subgradient optimisation", *Mathematical Programming* 6 (1974) 62-88,
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Held and Karp (1970) *op. cit.*,
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Held and Karp (1971) *op. cit.*,
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Geoffrion, A.M., "Lagrangean relaxation and its uses in integer programming", *Mathematical Programming Study 2* (1974) 82-114,
and
Fisher (1973) *op. cit.*,
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Wolfe, P., "A method of conjugate subgradients for minimising nondifferentiable functions", *Mathematical Programming Study 3* (1975) 145-173.

7. Fisher (1973) *op. cit.*

8. *Ibid.*

9. *Ibid.*

10. Held and Karp (1970) *op. cit.*

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12. Motzkin, T. and I.J. Scheonberg, "The relaxation method for linear inequalities", Canadian Journal of Mathematics 6 (1954) 393-404.

13. Held and Karp (1970) op. cit.
and
Held and Karp (1971) op. cit.

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Chapter 8

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