DISTRIBUTION OF SCATTERED X-RAYS.

by

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INTRODUCTION.

It is known that many of the properties of the scattered X-rays can be explained on the basis of the classical theory as put forward by Sir J.J. Thomson and extended by Barkla. But many of the recent experiments with hard X-rays and γ-rays have revealed the inadequacy of the classical theory. Barkla, in his early work had pointed out that very soft X-radiations must be employed in order to obtain results such as could be explained on the simple classical theory. The discovery of partial polarisation of the primary radiation, the almost complete polarisation of the scattered radiation and the distribution of the scattered rays all confirmed the transverse wave theory. In particular the accuracy of Barkla's determination from the scattering experiments of the number of electrons in an atom, and Laue's discovery of the diffraction and interference of X-rays lent strong support to the classical theory of radiation and of X-ray scattering. Within the last few years, however, a new scattering phenomenon has been observed which is contrary to the usual electrodynamics. To account for this it is necessary to assume that X-rays are scattered as definitely directed quanta of radiant energy. This quantum theory of scattering, proposed by A.H. Compton,/

Compton, suggests that in the case of light elements when an X-ray quantum is scattered it spends all of its energy upon some particular electron. This electron in turn scatters the ray in some definite direction. The change in momentum of the X-ray quantum due to impact with the electron results in a recoil of the electron. The corresponding increase in the wave-length of the scattered beam is

$\lambda_\theta - \lambda_0 = \frac{2h}{mc} \sin^2\frac{\theta}{2} = .0484 \sin^2\frac{\theta}{2}$ where h is Planck's constant, m is the mass of the scattering electron, c is the velocity of light, and θ is the angle between the incident and the scattered ray. According to this theory the scattered radiation in general may consist of both modified (changed wave-length) and unmodified (unchanged wave-length) radiation. In special cases the radiation may be all modified, as in the case of γ -rays, or at the other extreme all unmodified, as in the case of ordinary light. According to Compton the modified radiation is scattered by loosely bound electrons which are free to recoil, while the unmodified radiation is scattered by electrons held too firmly to recoil in this manner.

The earliest experiments of Barkla, and later on by Compton, showed that the penetrating power of the secondary X-radiation is less than that of the primary radiation. Compton attributes this softening of the secondary/

secondary radiation to the change in wave-length of the scattered radiation, or what is the same thing, as due to modified radiation. Barkla associated the softening of the secondary X-radiation in certain cases, with a much more general phenomenon - the J-phenomenon. Nothing definite has yet been established regarding the fundamental nature of this J-phenomenon, and the investigation regarding this change in penetrating power is still in progress.

The successes and failures of the classical and quantum theories of radiation demonstrated the need of further investigation of the properties of these radiations and particularly of X-rays. Of these one of the most significant is the distribution of scattered radiation emitted by a substance traversed by an unpolarised primary beam.

Let us first consider the development of theories regarding the distribution of scattered X-rays. Barkla first showed, as a simple deduction from Thomson's theory of scattering, that for an unpolarised primary radiation the intensity of scattered radiation in any direction should be given by the equation

$$I_\varphi = I_{\frac{\pi}{2}} (1 + \cos^2 \varphi).$$

J.J. Thomson, (Conduction of Electricity through Gases, 2nd Edition) - on the basis of electromagnetic theory expressed the intensity scattering function for a free electron, as:-

$$I_\varphi = \frac{I_0 e^4}{2 m^2 r^2 c^4} (1 + \cos^2 \varphi)$$

where/

where I_ϕ represents the intensity of the scattered beam at an angle ϕ with the direction of the primary beam, at a distance r from the scattering electron, the mass of which is m and charge e in e.s.u. The primary beam is assumed to be unpolarised and its intensity is I_0; c stands for the velocity of light. If there are Z orbital electrons, and if the distances between these electrons are so small as to be negligible in comparison with the wave-length of the incident X-rays, then all the electrons act as a unit in the scattering process and the intensity of the rays scattered by a single atom is:-

$$I_{a'} = \frac{I_0 (Ze)^4}{2r^2(Zm)^2c^4}\left(1+\cos^2\phi\right)$$

$$= \frac{I_0 Z^2 e^4}{2r^2m^2c^4}\left(1+\cos^2\phi\right) = I_\phi Z^2$$

i.e. the intensity scattered by a single atom is Z^2 times the intensity due to a single free electron. If, on the other hand, the electrons in the atom are separated by distances large compared with the wave-length of X-rays, the electrons scatter independently and the intensity of the radiation scattered by a single atom is:-

$$I_a = I_0 \frac{Ze^4}{2m^2r^2c^4}\left(1+\cos^2\phi\right) = I_\phi Z$$

i.e. the intensity scattered by a single atom is Z times the intensity due to a single free electron.

Thus/

Thus the intensity of the scattered X-rays may vary by a factor of z, according to the degree of the concentration of the electrons within the atom. If the electrons are at distances comparable with the wave-length of X-rays, interference, constructive as well as destructive, will take place between the X-rays scattered by different electrons of the atom, and as a result the intensity scattered by the atom as a whole will then be between Z^2 and Z times that due to a single free electron.

The quantum theories predict a scattering function different from Thomson's formula. They are, however, based upon the assumptom that for the limiting case of long wave-lengths, where the motion imparted to the scattering electron is negligible, the intensity of the scattered rays should approach that assigned to it by Thomson's classical formula.

The older forms of the quantum theory were not successful in giving a unique solution for the scattering function. The newer forms, however, have been found to give such a solution and it is

$$I_\phi = Z I_e (1 + \alpha \operatorname{vers} \phi)^{-3}$$

$$\text{where } I_e = \frac{I_0 e^4}{2 m^2 r^2 c^4} (1 + \cos^2 \phi)$$

$\alpha = \frac{h\nu}{mc^2}$, h being Planck's constant, ν the frequency of X-rays and the other symbols have the same significance as before.

Breit/

Breit (Phys. Rev. 27, 362, 1926) was the first to suggest this formula on empirical grounds and reasoning from the correspondence principle. Dirac (Proc. Roy. Soc. A. CXI page 405, 1926) derived it theoretically from the quantum dynamics of Heisenberg; Born and Waller (Phil. Mag. 4, 1228, 1927) as well as Gordon (Zeits. f. Physik 39, 117, 1926) did the same on the principle of the de Broglie-Schroedinger wave theory. Klein and Nishina (Zeits. f. Physik 52, 582 1928) have derived a scattering function on the hypothesis of the spinning electron, using Dirac's relativistic quantum dynamics, which differs somewhat from the above. However, the deviations of the two formulae are of the order of $\left(\frac{h\nu}{mc^2}\right)^2$, while the Breit-Dirac expression differs from the classical Thomson formula by quantities of the order of $\frac{h\nu}{mc^2}$.
Compton's (Phys. Rev. 35, 925, 1930) theoretical expression is given by the equation:-

$$I_\phi = I_e\left\{F^2 + \left(Z - \frac{F^2}{Z}\right)(1 + \alpha \operatorname{vers}\phi)^{-3}\right\}$$

$$\text{Where } F = \int_0^\infty U(r)\,\frac{\sin kr}{kr}\,.\,dr, \qquad k = \frac{4\pi}{\lambda}\sin\left(\frac{\phi}{2}\right),$$

U(r) is the radial charge density of the atom measured in electrons per unit distance, and the other symbols have the same significance as before. His calculation, based on the classical electromagnetic theory is made of the intensity of the X-rays scattered by an atom in which the electrons are arranged with random/

random orientation, and with arbitrary radial distribution. Conversely, an expression is derived for the radial distribution of the electrons in an atom, assuming they have random orientation. This expression has the form of a Fourier integral which can be evaluated from observed intensities of scattering of X-rays for different wave-lengths and angles. A comparison of this calculation with Wentzel's quantum theory of X-ray scattering suggests the introduction of a certain correction factor to express more nearly the intensity of the modified rays. It is also noted that the interpretation $\psi \overline{\psi}$ as a probability of the occurrence of an electron leads to the correct value for the total intensity of the scattered X-rays.

Woo's (Phys. Rev. 38, 6, 1931) expression is given by the equation:-

$$I_\phi = I_e\left\{F^2(1-e^{-2M}) + \left(Z - \frac{F^2}{Z}\right)(1+\alpha \operatorname{vers}\phi)^{-3}\right\}$$

where e^{-2M} is the temperature factor and the other symbols have the same significance as before. The mathematical formulation of this theory is based on the theoretical investigation by Raman (Indian J. Physics 3, 357, 1928) and A.H. Compton (Phys. Rev. 35, 925, 1930) on the scattering of X-rays by a dynamic atom.

Barkla and Ayres (Phil. Mag. Feb.1911) were the first to study experimentally the distribution of scattered/

Barkla
&
Ayres

Fig.(1a).

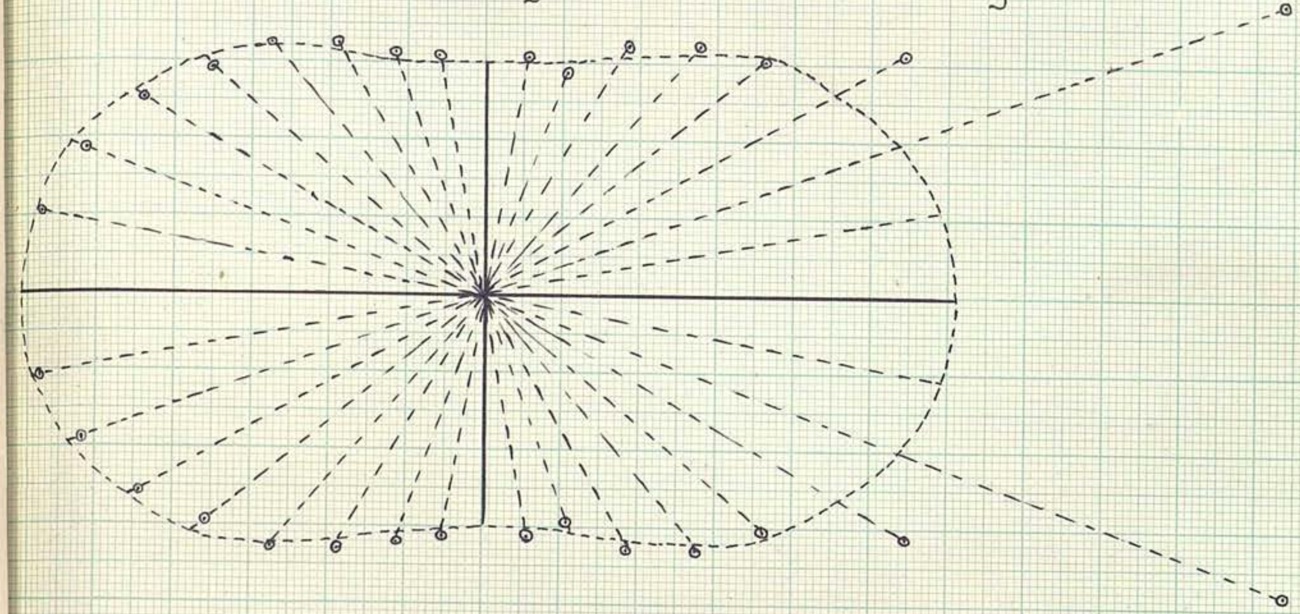
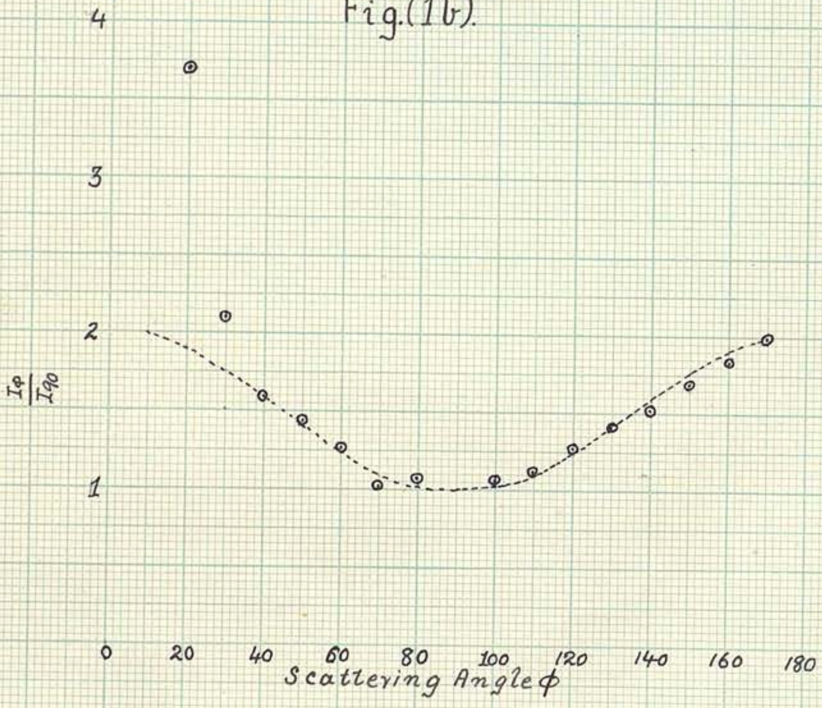


Fig.(1b).



scattered X-rays. They found that by using a very soft primary beam and carbon radiator, the ratio of the intensity $I\phi$ at any angle ϕ to the intensity I_{90} (suffix denoting the angle between the direction of scattering and that of propagation of the primary radiation) agrees remarkably well, with that given by the classical theory except for values of ϕ less than 30°. Figures 1 (a) and 1 (b) show the relation between intensity and scattering angle, obtained by them. The intensities are of course relative, that at 90° being taken as unity. In figures (1a and 1b) the curves show the theoretical distribution of the scattered radiation, while observations are indicated by small circles. The excess for small angles has since then been known as 'excess scattering'. It had previously been shown by Barkla (Phil. Mag. Feb. 1908) that the ratio $\frac{I\phi}{I_{90}}$ at $\phi = 170^{\circ}$ drops from nearly 2 to about 1.5 by increasing the penetrating power of the primary beam, showing the necessity for the use of long waves to obtain the classical result. This was confirmed in the experiments of Barkla and Ayres. In their experiments no correction was applied for variation in absorbability of scattered rays in different directions. For these long waves, however, the correction would of course by very small.

Owen (Camb. Phil. Soc. Proc. 16, 1911) investigated the distribution of the scattered radiation with particular/

particular regard to (1) hardness of the primary beam and (2) the thickness of the radiator. Using a thin radiator of filter paper he found the distribution on the incident side of the radiator in all cases to agree closely with the theoretical distribution given by the relation $I_\phi = I_{\frac{\pi}{2}}(1 + \cos^2\phi)$. In the case of the hardest rays examined (equivalent spark gap 7 cms.) he also obtained the same theoretical distribution on the emergent side - a result which does not appear to have been obtained by any other observer. A dissymmetry, however, appeared when soft rays were used and this increased with the softness of the rays and also slightly increased with the thickness of the radiator. In these experiments no correction was applied for polarisation or for variation in wave-length in different directions.

Crowther, too, about the same time (Proc. Roy. Soc. A. Vol. 85, 1911) in an investigation of the distribution of scattered X-radiation from aluminium, found a marked preponderance in the forward direction and a deficiency in the backward direction. The method of his experiment did not admit of sufficient accuracy for a detailed comparison with theory.

Some years later C.W. Hewlett, (Phy. Rev. 20 pp. 688 Dec. 1922) investigated the scattering of X-rays by mesitylene ($C_6 H_3 (CH_3)_3$). The agreement between the classical theory and experimental results was/

was again very satisfactory for angles greater than 30°. It is worthy of note, however, that these experiments were made with soft rays, the wave-length being about .7 A° and comparatively homogeneous. Between zero and 30° the intensity was greater than that given by $I_{\phi} = I_{\frac{\pi}{2}}(1+\cos^2\phi)$. This excess radiation for small angles of scattering is accepted on the classical theory as due to the superposition of secondary waves agreeing in phase.

All of the above investigations were conducted before the quantum theory of scattering was proposed by Compton.

Much more recently S. Chylinski (Phys. Rev. 42 pp. 153 Oct. 1932) has studied the distribution of the intensity of hard X-rays scattered by thin paraffin wax, at angles from 10° to 105°. He has made the necessary corrections for the variation in ionizing power of the rays scattered in different directions, due to Compton change in wave-length, assuming all the scattered radiation to be modified. His results show that the experimental value of $\frac{I_{\phi}}{I_{90}}$ is markedly in excess of the value to be expected on the Breit-Dirac theory.

As yet no systematic investigation had been made of the effects of (1) variation of wave-length of the primary radiation, (2) the nature of the scattering substance, (3) the thickness of the scattering substance and/

and (4) the method of excitation of the X-rays employed. The present work was undertaken with a view to testing the scattering functions as given by classical and quantum theories, and studying the influence of the variations referred to above. This seemed desirable in making an attempt to reconcile the conflicting results of various experimenters, and in order to obtain more information regarding the conditions necessary for the observation of the modification by scattering. (See papers on the J-phenomenon). In particular in this laboratory it has many times been found that when the radiator has been very thin, the difference between primary and scattered radiation has entirely disappeared, also the difference between the radiations scattered in different directions has under certain conditions vanished or become evident only by a discontinuity. The conditions most favourable for these results agreeing with classical theory, seemed to be (1) a thin scatterer, (2) a soft radiation and (3) possibly feeble intensity.

During the progress of this research Ivor Backhurst (Phil. Mag. Feb. 1934) has published the results of an investigation on the distribution of scattered energy, by using homogeneous X-rays of wavelengths .31 Å and .39 Å. He finds:-

(1)/

Backhurst

Fig.(2a).

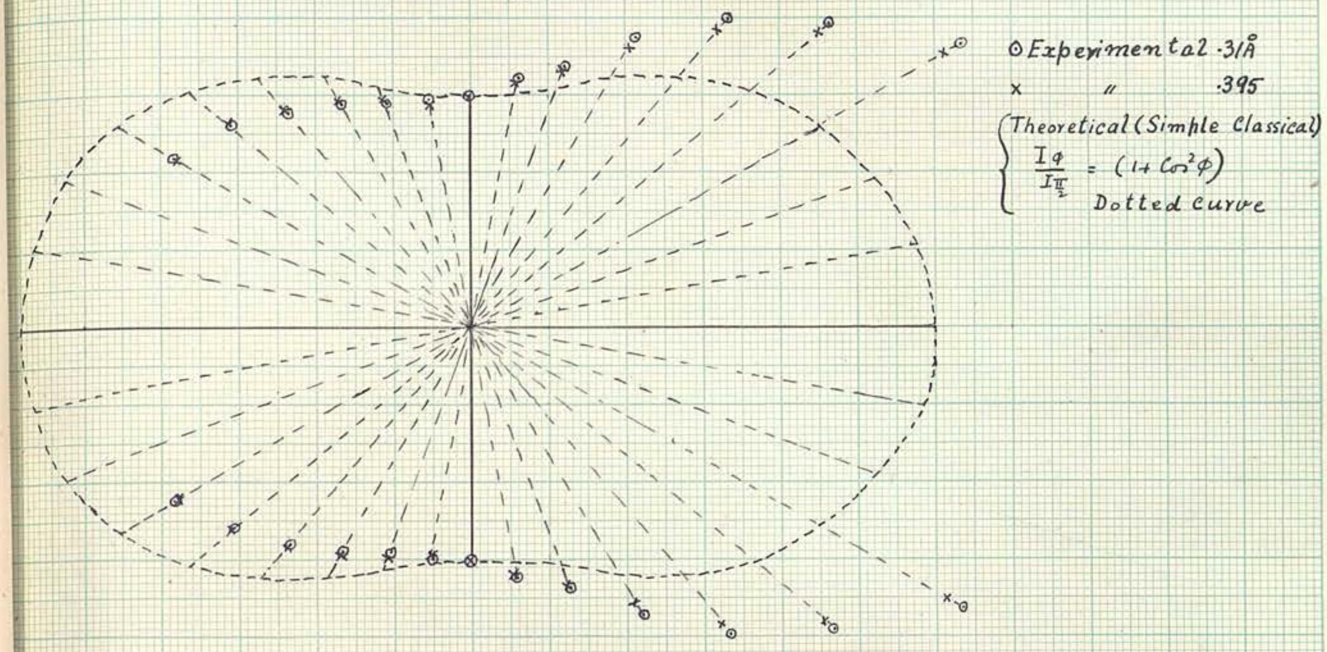
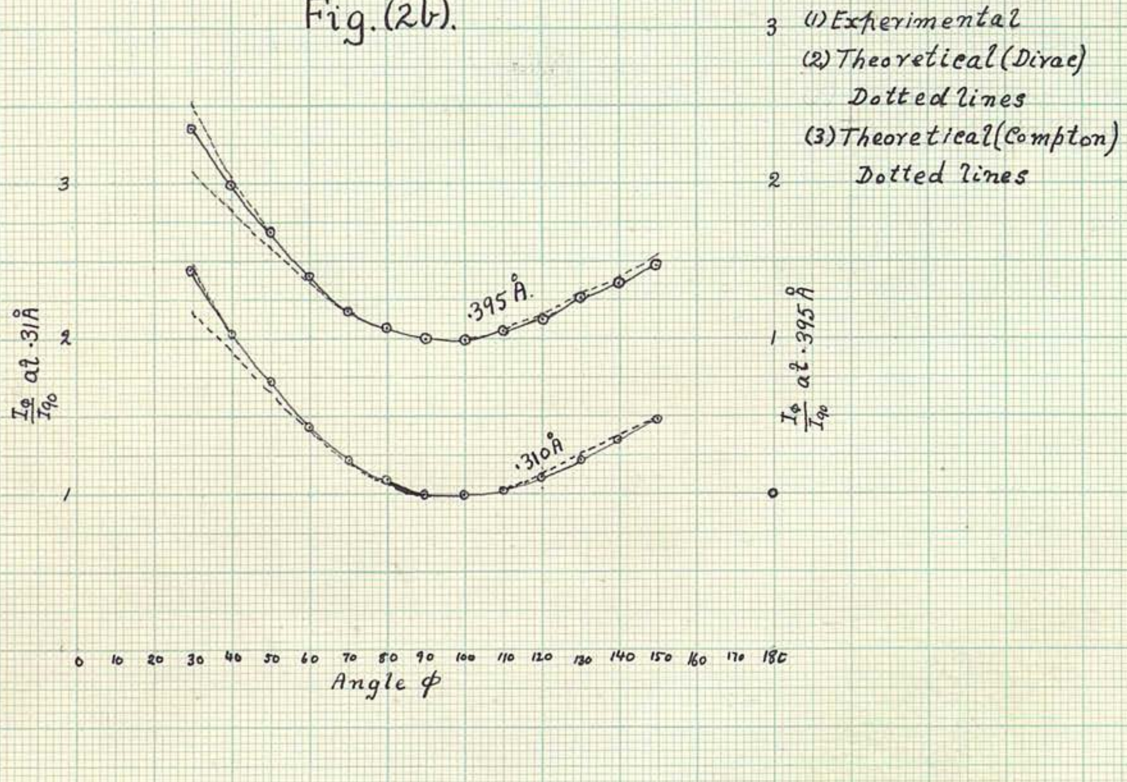


Fig.(2b).



(i) that the distribution with the angle of total scattered radiation (modified and unmodified together) from powdered beryllium at angles greater than 30° is in agreement - within a few per cent - with theoretical expressions based on Wave-Mechanics. The assumption is that the distribution is as from a gas i.e. only atomic scattering need be considered. Figure (2b) shows the relation between intensity and the scattering angle obtained by him using a beryllium scatterer in the form of powder contained in a cylindrical celluphane cell 2 mms. in diameter. The full curve (1) is experimental, the dotted curve (2) is that calculated from Dirac's equation and the dotted curve (3) is obtained from Compton's equation. (Fig.(2a) is drawn from Backhurst's experimental results for comparison with those of Barkla and Ayres fig.(1a) who used radiation of considerably greater wave-length).

(ii) The angular distribution of scattering at large angles is the same for all solids or liquids composed of atoms of low atomic numbers and is given by Dirac's equation. The relative intensities from paraffin wax, water, turpentine, benzene, alcohol and benzo-phenone are in agreement, within about 2 per cent, with those for beryllium. In his experiments corrections have been applied both for polarisation and for the variation in ionizing power in different directions.

The apparatus used and the method of investigation is described in the following pages.

EXPERIMENTAL.

The apparatus used in these experiments consisted essentially of:-

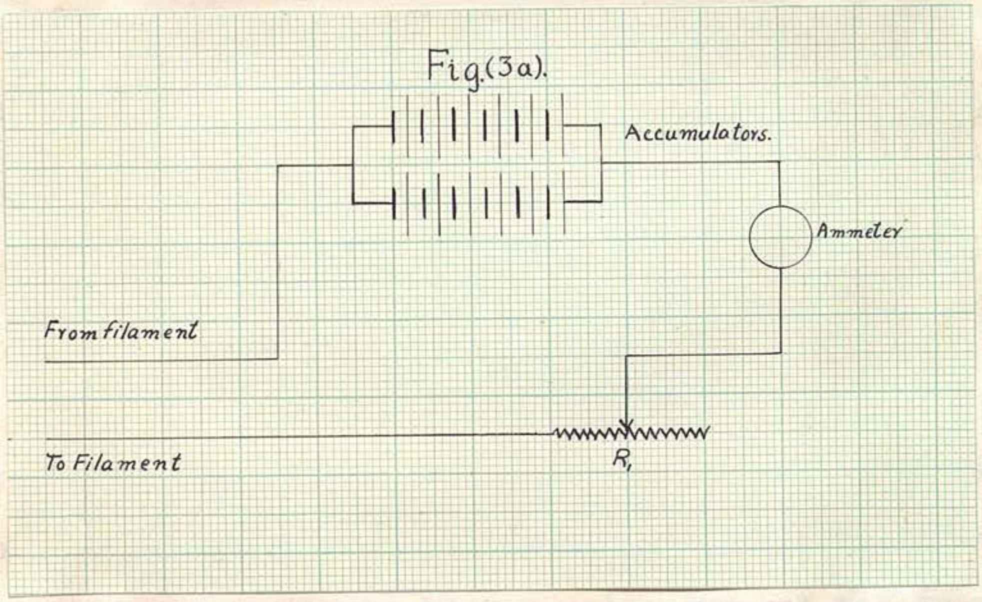
(i) X-ray tube and filament circuit;

(ii) High tension circuit;

(iii) Ionization chamber and electroscopes for comparing the intensity in different directions, and for standardising the intensity of the primary radiation.

(i) In all the experiments performed the source of X-rays was a Muller X-ray tube with tungsten anticathode. The arrangement for the control of the tube is shown in Fig. (3a).

Fig.(3a).

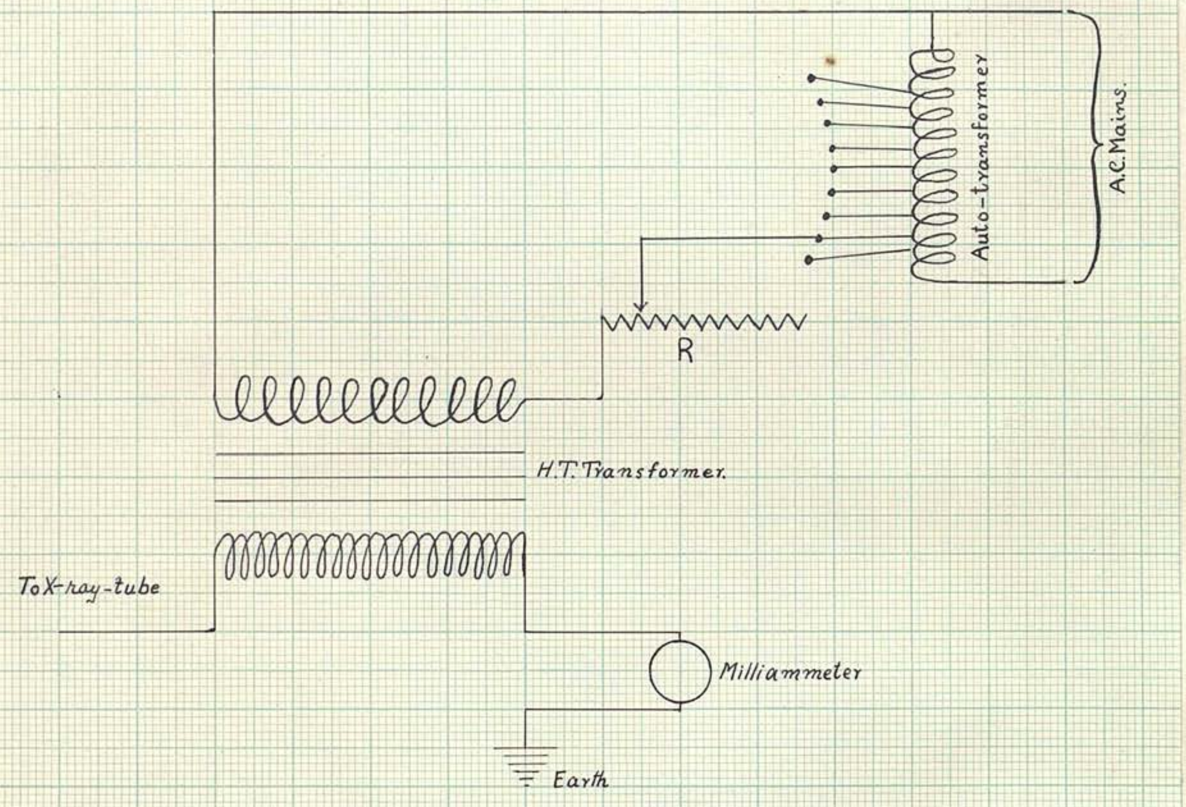


The rhoestat R_1 was used to adjust the filament current which was supplied from accumulators.

(ii)/

(ii) High tension circuit.

The alternating current from the 230-volt mains was passed through an auto-transformer fig (3b) tapped suitably to operate the high tension transformer, so that voltages varying by steps of 10 KV, from 10 KV to 90 KV (Peak Voltage) could be obtained. One end of the secondary of the high tension transformer was earthed and there the high tension current was passed through a milliammeter, as shown in fig.(3b), so that the current through the tube was always known. The current/


Fig(3b).

current through the tube could be adjusted by the rheostat R_1 Fig.(3a) and was kept constant throughout an experiment.

(iii) The ionization chamber Fig.(3c.) was cylindrical in shape 7.4 cms. long and 3.2 cms. in diameter, with an axial electrode, one end of which was bent and passed through an insulating plug, into the gold leaf electroscope. The window of the ionization chamber was of thin aluminium and at this end a narrow cylindrical lead tube 2 cms. in diameter, projected beyond the ionization chamber itself, so as to limit the rays entering the ionization chamber. The chamber was provided with inlet and outlet tubes (not shown) and was filled with sulphur-dioxide as the gas to be ionized.

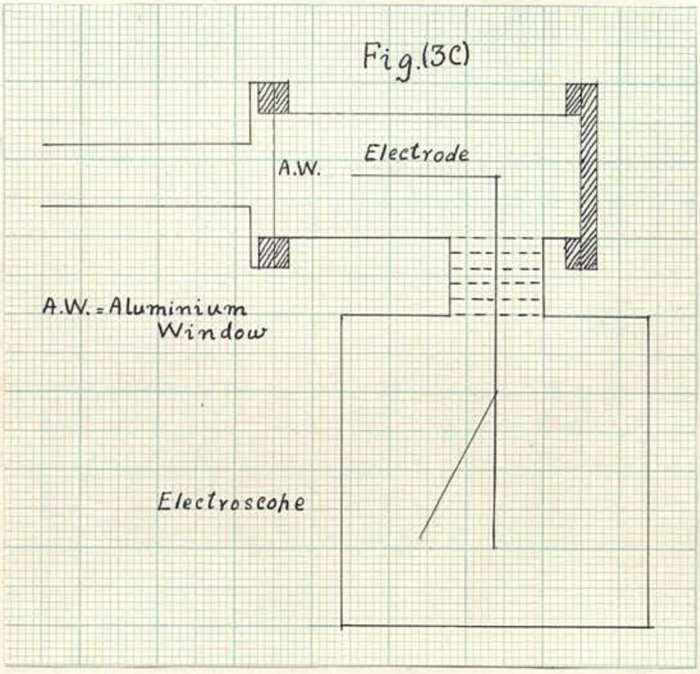


Fig.(3C)

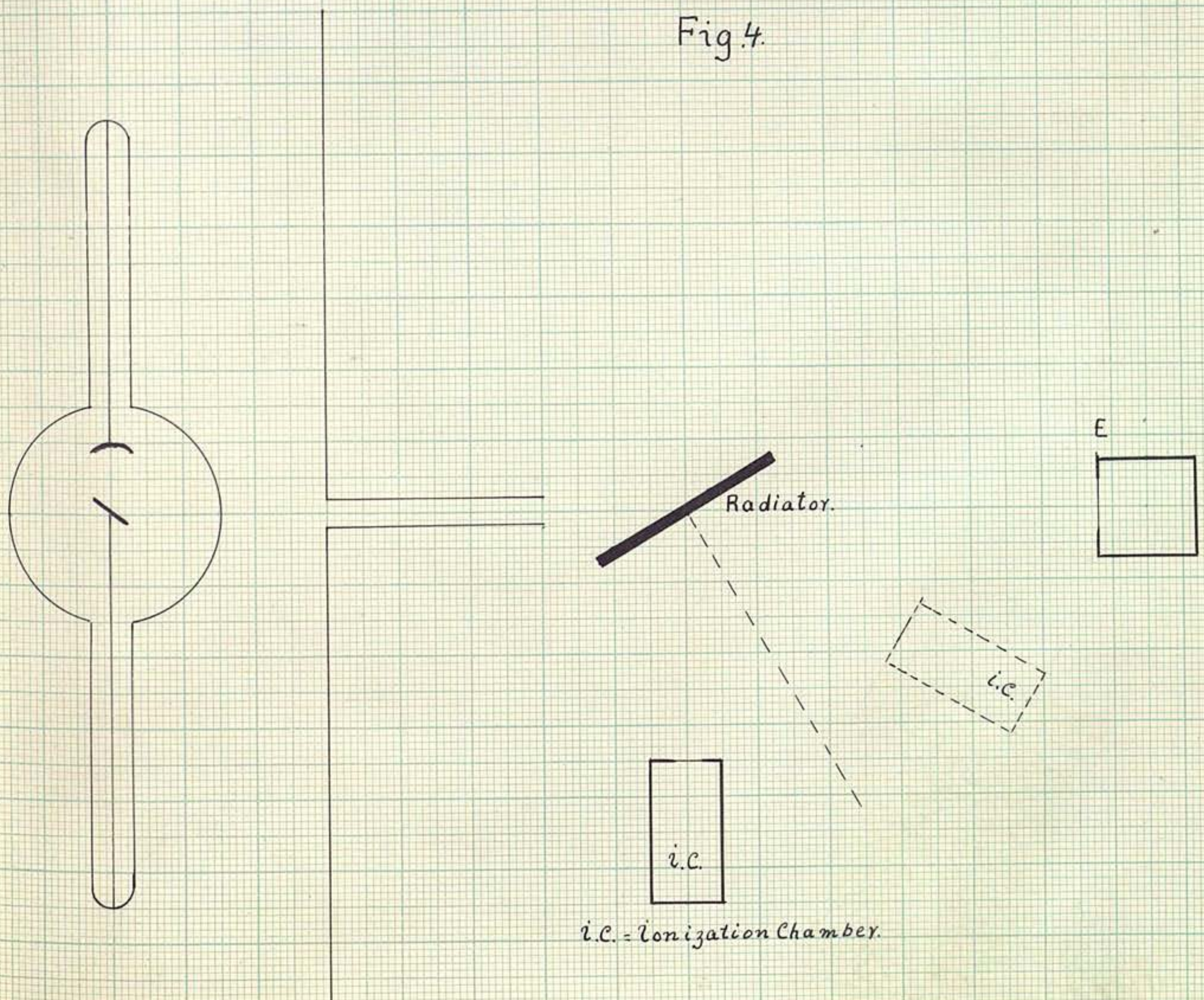
The electrode was always initially charged to a potential/

potential of 240 volts. The particular connections need not be described.

The ionization chamber, electroscope, and observing microscope were placed on a small wooden table at the end of an arm which was capable of rotating round a vertical axis. They were supported on a smooth horizontal circular graduated table. The position of the ionization chamber could be read on a circular scale. The axis of the ionization chamber was horizontal and radial.

Fig.4 shows the general arrangement of the apparatus used, to measure the intensity of scattered radiation in different directions.

Fig 4.



A second fixed electroscope E Fig.(2) was used to standardise the intensity of the primary radiation.

A beam of X-rays was directed horizontally on to a rectangular slab of the radiator, held in a vertical plane, with its centre on the axis of rotation of the ionization chamber. The radiator was capable of rotation round the same vertical axis. The intensities of secondary radiation proceeding in two directions making equal angles with the normal to the plate, were compared by observing the rates of deflection of the gold leaf when the axis of the ionization chamber was in the corresponding positions as S and S′ in Fig.5.

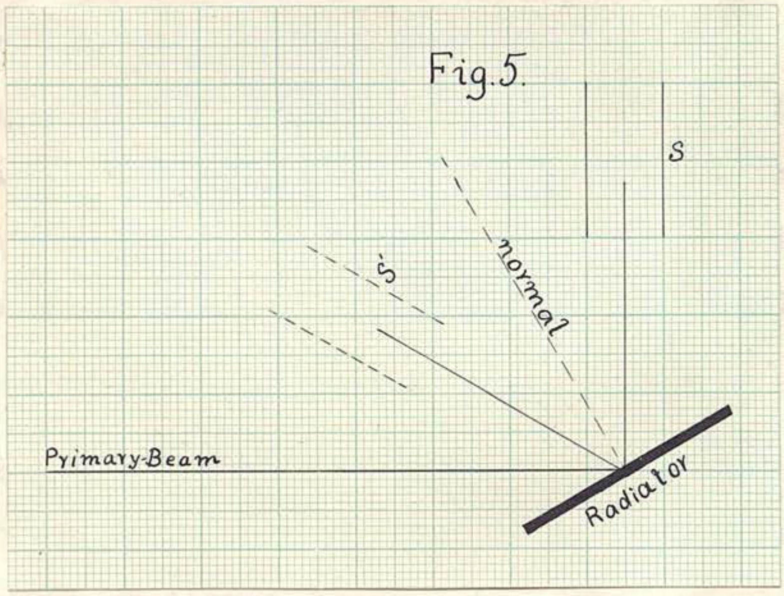


Fig.5.

As the distances in both carbon and air through which the measured secondary radiations travelled were the same in the two positions, the ionizations (ʒ) observed were proportional to the intensities of the radiation/

radiation proceeding from the atoms themselves, - neglecting any difference in absorbability in the two directions.

Taking the directions in pairs, results were obtained for the ratio $\frac{J_{\phi}}{J_{90}}$, ϕ being the angle between the direction of the central primary ray and central secondary ray in each case.

As the object of these experiments was primarily to study the variations of these ratios with the experimental conditions, and the number of these experimental conditions was so great, it was obviously impossible to observe the intensity for many values of ϕ. The three directions given by $\phi = 30^{\circ}, 90^{\circ}$ and 150° were considered sufficient for the purpose in view.

The effect of radiation scattered from air and of any other stray radiation was measured by direct experiment, and correction was made for this in determining the intensities of radiation from the solid scatterer.

Experiments were performed, with different thicknesses of paraffin-wax, carbon, and filter-paper as the scattering substances.

The radiations used were:

(a)/

(a) Soft rays, obtained by applying 30 K.V. (peak) to the X-ray tube; giving an average $\left(\frac{\overline{\mu}}{\rho}\right)_{Al}$ = 5.60 from '50% absorption'*;

(b) Harder rays by applying 80 K.V. to the X-ray tube; giving average $\left(\frac{\overline{\mu}}{\rho}\right)_{Al}$ = 1.88 from '50% absorption';

(c) Still harder rays obtained by filtering the primary beam at 80 K.V. with .5 cms. of aluminium to increase the average hardness of the beam by eliminating the softer components; giving average $\left(\frac{\mu}{\rho}\right)_{Al}$ = 0.71 from '50% absorption'.

The procedure adapted was to place a given scatterer in the position previously indicated (shown in fig.5) and by swinging the ionization chamber round the axis to the two positions (ø and 90) to obtain the ratio of ionizations $\frac{I_{\phi}}{I_{90}}$ for a certain primary radiation. The scattering substance was then turned through 180° about its axis and the ratio $\frac{I_{\phi}}{I_{90}}$ was again obtained. This was done in order to minimise the error due to lack of symmetry of the scatterer about the axis of rotation.

Similar/

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* The value of $\frac{\mu}{\rho}$ in aluminium was found from the equation $\frac{I}{I_0} = e^{-\mu x}$ as though the radiation was homogeneous, and using a thickness of aluminium which diminished the ionization by 50 percent.

Similar observations were made with each of the radiations a, b and c in turn (pages 19 and 20). The results thus gave the variation of $\frac{\mathfrak{I}_\phi}{\mathfrak{I}_{90}}$ with the character of the radiation scattered.

In other experiments, the thickness of the scattering substance was varied while the other conditions remained constant.

In other experiments again while the experimental conditions were kept constant, the material of the scattering substance was changed.

Before making a comparison of the experimental results with theory, it is essential that a number of corrections should be applied for:-

(i) Polarization in the incident radiation;

(ii) Variation in ionizing power of the radiation scattered in different directions, owing to dependence of wave-length on ϕ;

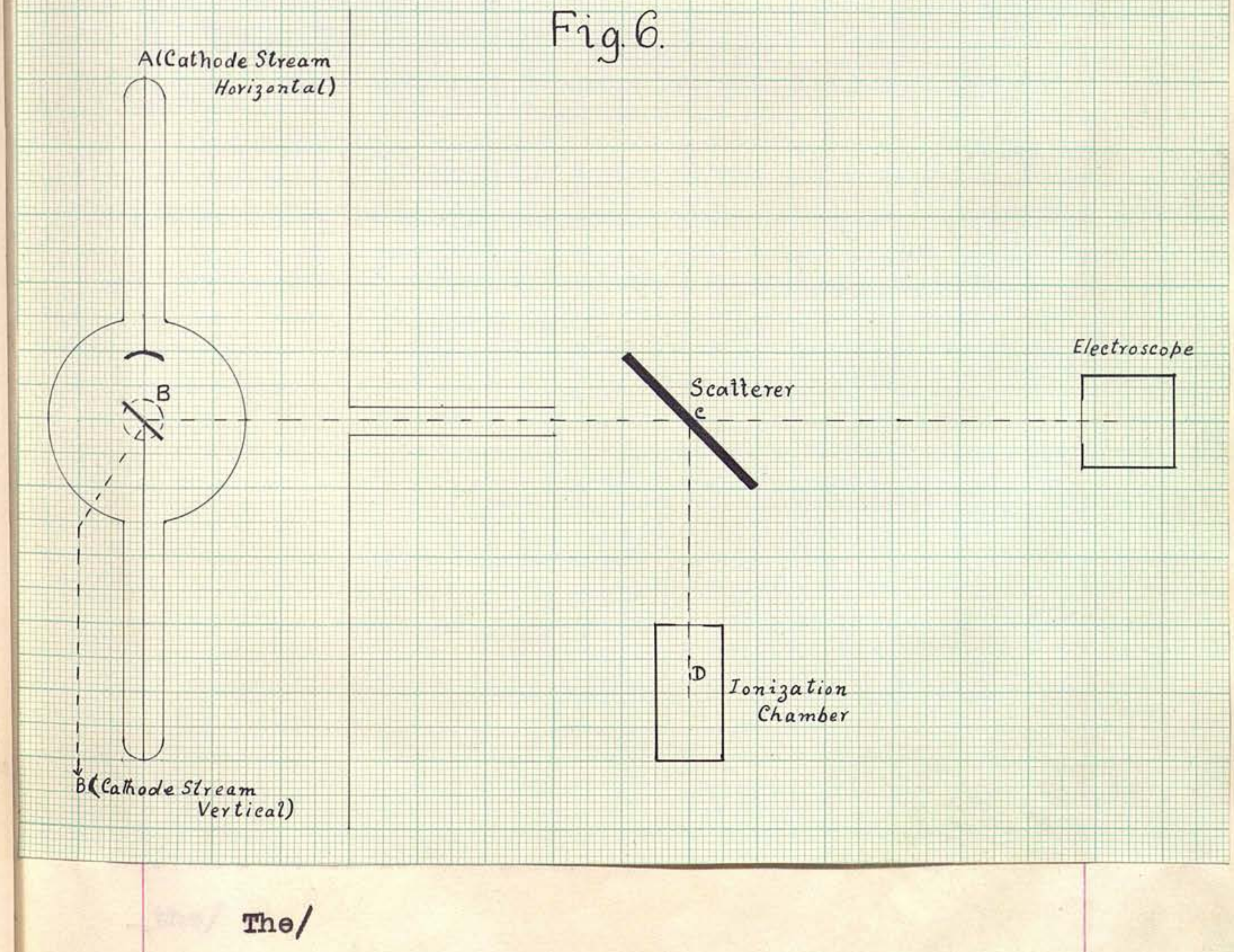
(iii) Obliquity of some of the radiation entering the ionization chamber.

The methods of correction are described below:-

POLARIZATION.

The amount of polarization present in the incident beam was obtained by measuring the scattering at $\phi = 90^{\circ}$ in the horizontal plane, when the cathode stream in the X-ray tube was horizontal, and when it was vertical. In order to do this a special stand with rotating device was made. The axis of rotation was made to coincide with the axis of the primary beam which was at right angles to the cathode stream. Fig.6 gives the diagram of the apparatus.

Fig.6.



The/

The ionization current was measured when the tube was in position A (cathode stream horizontal). The tube was then rotated through 90° and brought into position B (cathode stream vertical) and the ionization current was again measured. It was found that this current was greater when the tube was in position B. The percentage of polarization was calculated as follows.

We shall assume the partially polarized radiation consists of two parts, - (1) an unpolarized radiation of intensity U, and (2) a plane polarized radiation of intensity P. When the tube is in position A (horizontal) the electric vector of the polarized beam will be in the direction C D, and according to classical theory, the electrons of the scattering material will be accelerated in that direction due to this polarized beam. The intensity in this direction C D, and therefore the current in the ionization chamber, will be due only to the vertical component of the electric vector of the unpolarized beam. As the intensity of radiation proceeding in any direction θ with the electric vector is proportional to $\sin^2\theta$, the intensity due to the unpolarized primary may be written $\frac{KU}{2}$ since $\frac{1}{2}$ is the average value of $\sin^2\theta$; and the intensity due to the plane polarized primary varies from P to kP as the X-ray tube is turned about the/

the axis. When the tube is in position A this intensity of scattered radiation is $K(\frac{1}{2}u + 0)$ whereas in position B it is $K(\frac{1}{2}u + P)$.

The ionization current being proportional to the intensity of the ionizing radiation, we get:-

$$\frac{i_1}{i_2} = \frac{K\frac{u}{2}}{K(\frac{u}{2} + P)}$$

where i_1, is the ionization current for position A,

$$\therefore \frac{i_2}{i_1} = 1 + \frac{2P}{u}$$

and where i_2, is the ionization current for position B.

$$\text{i.e. } \frac{i_2 - i_1}{2i_1} = \frac{P}{u}$$

Thus from $\frac{i_2}{i_1}$ which is got by experiment, the percentage polarization $(\frac{P}{u+P} \times 100)$ can be determined.

Experiments were performed with paraffin wax and carbon as scattering materials and the amount of polarization was calculated as above.

Each observation was made twice and the mean of the two only slightly differing values was taken. The results obtained are shown in Table I.

Scattering Material.	Thickness of Scattering Material in cms.	K.V. on X-ray tube.	Average(λ) for '50% absorption'.	Deflection of Electroscope receiving primary beam.	Deflection of Secondary Electroscope at $\phi = 90^\circ$ when cathode stream horizontal.	Deflection of Secondary Electroscope at $\phi = 90^\circ$ when cathode stream vertical.	Percent Polarization.	Mean.
Paraffin Wax.	.62	30	5.6	40	11.4	13.4	8.1	8.1
"	"	"	"	40	11.6	13.6	7.9	
"	"	"	"	40	11.3	13.35	8.3	
Paraffin Wax.	.62	80	1.88	40	13.85	14.4	1.95	1.91
"	"	"	"	40	13.85	14.4	1.95	
"	"	"	"	40	13.4	13.9	1.85	
Carbon	.9	30	5.6	25	9.75	11.75	9.3	8.8
"	"	"	"	27	10.5	12.4	8.3	
"	"	"	"	27	10.4	12.4	8.8	
Carbon	.9	80	1.88	30	20	20.85	2.1	2.1
"	"	"	"	30	19.9	20.75	2.1	
Carbon	"	80 (beam filtered by .5 cms. of Al.)	.71	26	12	12.75	3.03	3.03
"	"	"	"	26	12	12.75	3.03	

From the above table it can be seen that the polarization in the primary radiation varies according to the different experimental conditions, as stated below:-

(i) The amount of polarization is affected by the Potential difference on the X-ray tube and increases from about 2 to 8.5 per cent with decrease of voltage from 80 K.V. to 30 K.V. This is an agreement with the earliest results of Barkla (Jahrbuch der Radioaktivitat und Elektronik V. Band Heft 3, 1908) who found that the amount of polarization varied from about 9 to 2.5 per cent by increasing the voltage on the X-ray tube. The decrease in the amount of polarization may be due to a greater proportion of secondary rays (unpolarized) from the anti-cathode at high voltage, thus diminishing the proportion of the polarized part.

(ii) The interposition of sheets of aluminium in the primary beam causes an increase in polarization. This at first sight seems contrary to the above - giving more polarization by hardening the primary radiation. This is also in agreement with the earliest work of Barkla (loc. cit.) and Ham (Phys. Rev. 30, 96, 1910); and can be explained as due to the softer secondary rays being cut off by aluminium, and thus leaving a greater proportion of polarized primary radiation.

(iii)/

(iii) The amount of polarization exhibited by different scattering materials - paraffin wax and carbon - shows very little difference (8.1% to 8.8% at 30 K.V. and 1.9% to 2.1% at 80 K.V.) This difference is so small that it is impossible to say with certainty whether it is due to a real difference in the perfection of scattering or to some slight experimental error.

The intensity of scattering from filter paper was too feeble to permit of accurate direct measurements of polarization, but any possible variation of the polarization correction must have been very small, since the nature of the scatterer does not to a great extent affect the amount of polarization exhibited. As a consequence the polarization experiments with filter paper were abandoned and the amount of polarization was assumed to be that found with paraffin wax as the scatterer.

To correct the ratio $\frac{J_{\phi}}{J_{90}}$ for the amount of polarization present in the incident radiation, - suppose x is the ratio of the ionizations due to scattered rays in directions ϕ and 90 degrees, set up by an unpolarized primary radiation,

Then/

Then the observed ratio $\frac{J_\phi}{J_{90}} = \frac{K x \frac{u}{2} + K P \cos^2\phi}{K \frac{u}{2}}$ (to a first approximatic
(assuming classical thec
(for the polarized part.

i.e. $$= x + \frac{2 P \cos^2\phi}{u}$$

$$= x + \frac{1.5 P}{u} \text{ since } \phi = 30^\circ \text{ or } 150^\circ$$

that is the observed ratio $\frac{J_\phi}{J_{90}}$ is greater by $1.5\,\frac{P}{U}$ due to polarized radiation. This must be subtracted from it to get the ratio for unpolarized radiation. The values of $\frac{1.5P}{U}$ for different cases were calculated from table I.

CORRECTION DUE TO VARIATION IN WAVE-LENGTH WITH DIRECTION OF SCATTERING.

According to the quantum theory of X-ray scattering, the wave-length of the modified radiation increases with increasing ϕ, hence when we are comparing I_ϕ with I_{90} there will be comparatively greater or less absorption in the ionization chamber according as ϕ is greater or less than 90° (both absorptions being compared with that of the rays scattered in direction $\phi = 90^\circ$) for the effective part of the absorption coefficient and hence the ionization is proportional to λ^3. Hence in order to compare the intensities in different directions a correction must be applied for the variation of the ionizing power of the radiation with direction. Hitherto the correction for the variation of wave-length has been neglected or has not been treated with sufficient accuracy to permit of close comparison with theory.

A correction might of course be calculated on the Compton theory if we knew the proportion of modified and unmodified radiations, as well as the approximate wave-lengths of the radiations. This, however, assumes a knowledge which we really do not possess; for in view of the experimental work done in this laboratory, we cannot accept the hypothesis of independent action of/

of the constituent radiations of a hetrogenous beam. Direct experiments on the absorbabilities of the radiations scattered in different directions have in some cases shown absolutely no difference (Barkla and Khastgir Phil. Mag. Sept. 1926) and (Barkla and Mackenzie Phil. Mag. Feb. 1926). Sometimes the difference - quite a marked one - suddenly appeared as a J- discontinuity. Owing thus to the uncertainties of the calculation and of the fundamental conditions affecting the phenomenon, it was decided to make a direct measurement of the difference of absorbabilities of the two scattered radiations compared. The superiority of this direct method is obvious.

This difference in absorbability of the secondary radiations in different directions cannot be found accurately by directly measuring the absorption-coefficients in these directions. In the present investigation this correction was treated as follows:-

Let I_ϕ and I_{90} be the true intensities of the two beams;

x_ϕ and x_{90} the percentage absorptions in the ionization chamber,

and z_ϕ and z_{90} the ionizations due to the two beams,

x then $\frac{z_\phi}{z_{90}} = \frac{KI_\phi x_\phi}{KI_{90} z_{90}}$ assuming ionizations are proportional to energies absorbed.

x_{90}

In order to get the true value of $\frac{I_\phi}{I_{90}}$ we must therefore/

therefore multiply the observed ratio $\frac{z_\phi}{z_{90}}$ by the inverse ratio of the absorptions i.e. $\frac{x_{90}}{x_\phi}$.

As over this range of wave-lengths, absorptions in sulphur-dioxide and in aluminium are proportional, it is sufficient to determine $\frac{x_{90}}{x_\phi}$ in a very thin layer of aluminium.

This was done by first finding the relation between ionization in sulphur-dioxide and thickness of intercepting aluminium, for the beam scattered at 90° - thus getting an absorption curve for the radiation scattered at 90°; and secondly by finding the relation between

$$\frac{\text{ionization by radiation scattered at } \phi^\circ}{\text{ionization by radiation scattered at } 90^\circ} = \frac{z_\phi}{z_{90}} = y$$

and the thickness of aluminium intercepting both beams - thus getting a relative absorption curve for the radiations scattered at ø and 90°.

By plotting the curves for these two we can get from the first the value of $\frac{1}{z_{90}} \cdot \frac{dz_{90}}{dt}$

Similarly from the second we can get the value of $\frac{1}{y}\frac{dy}{dt}$.

But $\frac{1}{y}\frac{dy}{dt} = \frac{1}{z_\phi}\frac{dz_\phi}{dt} - \frac{1}{z_{90}}\frac{dz_{90}}{dt}$ - - - - - (ii)

Thus knowing $\frac{1}{y}\frac{dy}{dt}$ and $\frac{1}{z_{90}}\frac{dz_{90}}{dt}$

$\frac{1}{z_\phi}\frac{dz_\phi}{dt}$ can be found from equation (ii)

The ratio $\frac{\frac{1}{z_{90}}\frac{dz_{90}}{dt}}{\frac{1}{z_\phi}\cdot\frac{dz_\phi}{dt}}$ however when t is small = $\frac{x_{90}}{x_\phi}$

because/

because the absorption by the SO_2 in the ionization chamber corresponds to a small thickness t of aluminium.

The following tables give the experimental results; few are also shown in the form of graphs.

It was found difficult in some cases with filter paper as the scattering material, to intercept the secondaries with as great a thickness of aluminium as in the case of paraffin wax and carbon, as the intensity of the scattered rays was very small, and the absorptions of the secondaries could not be measured accurately except for a small thickness of aluminium. Hence these experiments were performed with one small thickness only, and the ratio $\frac{x_{90}}{x_\phi}$ was calculated.

TABLE II. /

TABLE II.

PARAFFIN WAX.

K.V. 30 Average $\left(\frac{\overline{\mu}}{\rho}\right)_{Al.}$ = 5.6 from '50% absorption'.

Thickness of intercepting Al. in cms.	Corrected deflection of secondary Electroscope at 90° (z_{90})	Expt. $\frac{z_{150^\circ}}{\gamma_{90^\circ}} = y$	$\frac{x_{90}}{x_{150}}$
0	28.1	1.773	
.01	23.7	1.768	.967
.02	20.4	1.726	
.04	14.8	1.68	

K.V. 80 Average $\left(\frac{\overline{\mu}}{\rho}\right)_{Al.}$ = 1.88 from '50% absorption'

0	21.45	1.70	
.048	15.1	1.66	.936
.096	11.2	1.625	
.144	8.8	1.60	

K.V. 80 Beam filtered with .5 cms. of Al.

Average $\left(\frac{\overline{\mu}}{\rho}\right)_{Al.}$ = 0.71 from '50% absorption'.

0	42.1	1.725	
.048	36.4	1.69	.925
.06	32.1	1.69	
.144	28.2	1.70	

TABLE III.

PARAFFIN WAX.

K.V. 30 Average $\left(\frac{\overline{\mu}}{\rho}\right)_{Al.}$ = 5.6 from '50% absorption'.

Thickness of intercepting Al. in cms.	Corrected deflection of secondary Electroscope at 90° (z_{90})	Expt. $\frac{z_{30}}{z_{90}} = y$	$\frac{x_{90}}{x_{30}}$
0	19.7	2.462	
.01	16.6	2.470	
.02	14.3	2.461	1
.04	10.5	2.462	

K.V. 80 Average $\left(\frac{\overline{\mu}}{\rho}\right)_{Al.}$ = 1.88 from '50% absorption'.

0	29.8	2.04	
.048	21.1	2.06	1.014
.096	16.1	2.05	
.144	13	2.07	

K.V. ~~7~~80 Beam filtered with .5 cms. of Al.

Average $\left(\frac{\overline{\mu}}{\rho}\right)_{Al.}$ = 0.71 from '50% absorption'.

0	24.6	1.92	
.048	21.65	1.935	
.096	19.3	1.96	1.07
.144	17.05	1.99	

TABLE IV.

CARBON.

K.V. 30 Average $\left(\frac{\overline{\mu}}{\rho}\right)_{Al.}$ = 5.6 from '50% absorption'.

Thickness of Al. in cms.	Corrected deflection in secondary Electroscope at 90° (γ_{90})	Expt. Rate $\frac{\gamma_{150^\circ}}{\gamma_{90^\circ}} = y$	$\frac{x_{90}}{x_{150}}$
0	16.75	1.74	
.01	13.95	1.75	1
.02	11.8	1.73	
.04	8.8	1.75	

K.V. 80 Average $\left(\frac{\overline{\mu}}{\rho}\right)_{Al.}$ = 1.88 from '50% absorption'.

0	21.08	1.70	
.02	18.47	1.68	
.07	16.5	1.70	.962
.06	14.9	1.68	
.08	13.7	1.66	
.1	12.65	1.64	

K.V. 80 Beam filtered with .5 cms. of Al.

Average $\left(\frac{\overline{\mu}}{\rho}\right)_{Al.}$ = 0.71 from '50% absorption'.

0	16.4	1.70	
.048	14.35	1.70	.962
.096	12.85	1.68	
.144	11.6	1.65	

TABLE V.

CARBON.

K.V. 30 Average $\left(\frac{\overline{\mu}}{\rho}\right)_{Al.} =$ 5.6 from '50% absorption'.

Thickness of intercepting Al. in cms.	Corrected deflection of secondary Electroscope at 90°. (z_{90})	Expt. $\frac{z_{30^\circ}}{z_{90^\circ}} = y$	$\frac{x_{90}}{x_{30}}$
0	20.75	2.49	
.01	18.05	2.53	1.138
.02	15.55	2.57	

K.V. 80 Average $\left(\frac{\mu}{\rho}\right)_{Al.} =$ 1.88 from '50% absorption'.

0	21.5	2.063	
.048	16.95	2.097	1.103
.096	13.9	2.15	
.144	11.7	2.15	

K.V. 80 Beam filtered with .5 cms. of Al.

Average $\left(\frac{\mu}{\rho}\right)_{Al.} =$ 0.71 from '50% absorption'.

0	21.9	1.99	
.048	18.7	2.04	1.117
.096	16.8	2.04	
.144	14.95	2.06	

TABLE VI.

FILTER PAPER.

K.V. 30 Average $\left(\frac{\overline{\mu}}{\rho}\right)_{Al.}$ = 5.6 from '50% absorption'.

Thickness of intercepting Al. in cms.	Corrected deflection of secondary Electroscope at 90° (z_{90})	Expt. $\frac{z_{150^\circ}}{z_{90^\circ}} = y$	$\frac{x_{90}}{x_{150}}$
0	19.95	1.78	
.01	16.45	1.765	.948
.02	14.1	1.73	
.04	10.4	1.70	
K.V. 80 Average $\left(\frac{\overline{\mu}}{\rho}\right)_{Al.}$ = 1.88 from '50% absorption'.			
0	22.95	1.71	
.02	19.2	1.71	
.06	14.5	1.70	.98
.1	11.5	1.68	
K.V. 80 Beam filtered with .5 cms. of Al. Average $\left(\frac{\overline{\mu}}{\rho}\right)_{Al.}$ = 0.71 from '50% absorption'.			
0	15.2	1.80	
.04	13.3	1.80	1
0	15.2	1.79	
.04	13.3	1.79	

TABLE VII.

FILTER PAPER.

K.V. 30 Average $\overline{\left(\frac{\mu}{\rho}\right)}_{Al.} = 5.6$ from '50% absorption'.

Thickness of intercepting Al. in cms.	Corrected deflection of secondary Electroscope at 90° (z_{90})	Expt. $\frac{z_{30^\circ}}{z_{90^\circ}} = y$	$\frac{x_{90}}{x_{30}}$
0	10.4	3.21	
.01	8.8	3.21	1
0	10.4	3.20	
.01	8.8	3.21	

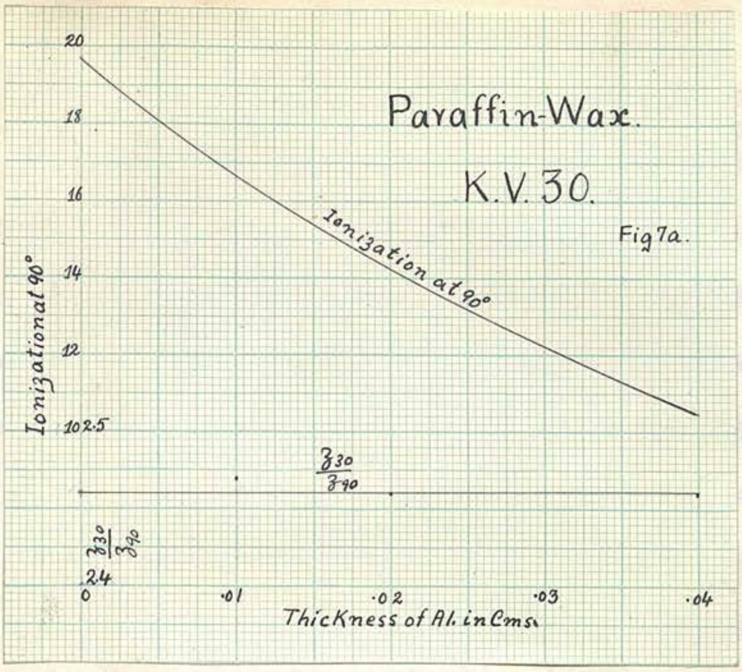
K.V. 80 Average $\overline{\left(\frac{\mu}{\rho}\right)}_{Al.} = 1.88$ from '50% absorption'.

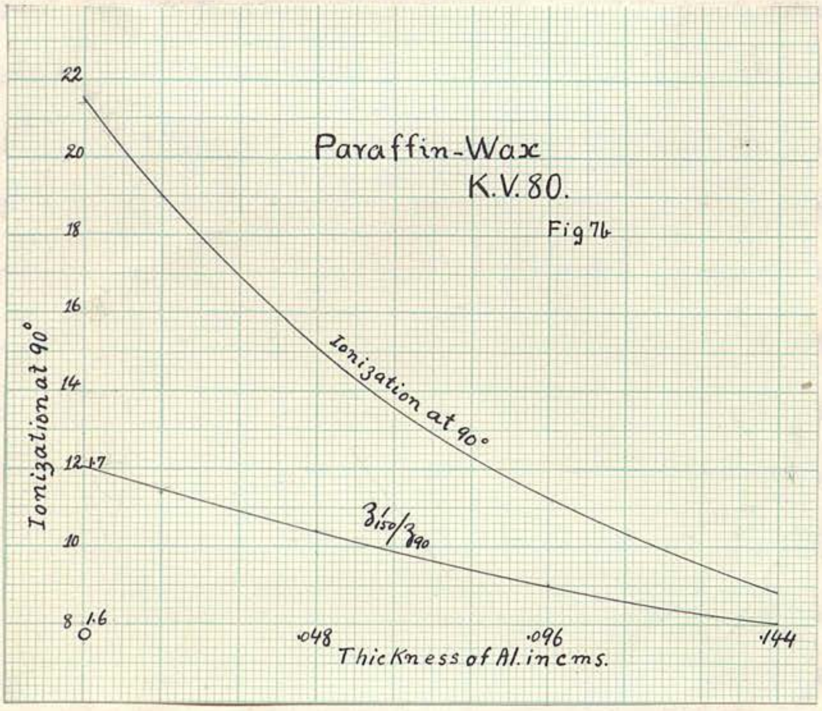
0	20.7	2.48	
.02	17.85	2.45	1
.06	13.5	2.46	
.1	10.6	2.44, 2.48	

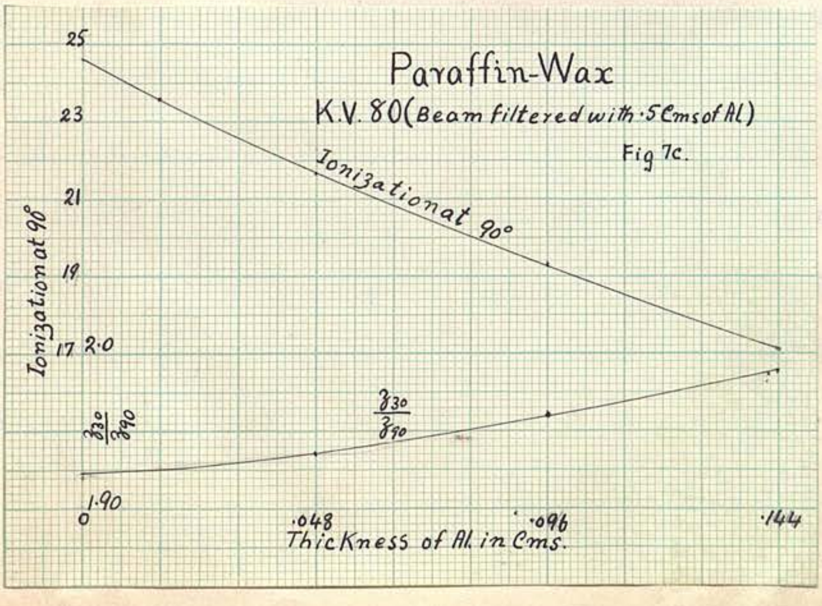
K.V. Beam filtered with .5 cms. of Al.

Average $\left(\frac{\mu}{\rho}\right)_{Al.} = 0.71$ from '50% absorption'.

0	12.9	2.086	
.04	11.6	2.085	1


Paraffin-Wax.
K.V. 30.
Fig 7a.
Ionization at 90°
Ionization at 90°
J30 / J90
J30 / J90
20
18
16
14
12
10 2.5
2.4
0
·01
·02
·03
·04
Thickness of Al. in Cms.


Paraffin-Wax
K.V. 80.
Fig 7b
Ionization at 90°
Ionization at 90°
J150/J90
22
20
18
16
14
12 1.7
10
8 1.6
0
·048
·096
·144
Thickness of Al. in cms.


Paraffin-Wax
K.V. 80 (Beam filtered with ·5 Cms of Al)
Fig 7c.
Ionization at 90°
Ionization at 90°
J30 / J90
J30 / J90
25
23
21
19
17 2.0
1.90
0
·048
·096
·144
Thickness of Al. in Cms.

From the above results it can be seen that in general, the ratio $\frac{J_{150}}{J_{90}}$ decreases, whereas $\frac{J_{30}}{J_{90}}$ increases with increasing thickness of absorbing aluminium. This of course, is what is to be expected owing to variation of wave-length of the modified radiation with the scattering angle. In a few cases, however, the ratio $\frac{J_{\phi}}{J_{90}}$ remained remarkably constant - within ½ to 1% - with increasing thickness of aluminium, thus showing equality of absorptions of the radiations scattered in different directions - within a small experimental error. In particular this is the case with filter paper as the scattering material, the radiation scattered at 30° having the same absorbability as that scattered at 90° with all radiations used. This confirms the experimental observations of Barkla and Khastgir (Phil. Mag. Sept. 1926) on the equality of absorptions of the two radiations scattered at 30° and 90° from filter-paper, observed in association with the J-phenomenon. Barkla and Mackenzie also obtained similar results when using paraffix wax and aluminium as scattering materials - the angles of scattering being 60° and 120°. But there was this striking difference between the results of this investigation and those of the experimenters named - they observed the expected difference in the form of discontinuities; whereas no 'absorption discontinuity' was observed in the present investigation.

OBLIQUITY.

As the scattered rays entering the ionization chamber did not coincide exactly with the direction of the axis of the beam experimented upon, it was necessary to make a small correction for obliquity. For the angles ϕ equal to 30 and 150 degrees the effect of obliquity on opposite sides of the axis approximately cancels out; it was neglected. For $\phi = 90^\circ$ the average value of $\cos^2\phi$ for the rays as estimated from the geometry of the apparatus would be about .01. Consequently the observed intensity was not I_{90} but 1.01 I_{90}.

Experiments performed as mentioned on pages (18,19,20 and 21) were corrected for polarization, change in wave-length in different directions and obliquity, as described in previous pages. The following tables give the corrected and uncorrected values of $\frac{J_\phi}{J_{90}}$ under different experimental conditions. The figures in brackets give the ratios $\frac{J_\phi}{J_{90}}$ obtained from the two faces of the scattering material.

SCATTERING SUBSTANCE. PARAFFIN WAX.

Angle ϕ	Thickness of Paraffin Wax in cms.	Expt. Ratio 30 K.V. without correction. $\left(\frac{J_\phi}{J_{90}}\right)$ a.	Expt. Ratio 80 K.V. without correction. $\left(\frac{J_\phi}{J_{90}}\right)$ b.	Expt. Ratio 80 K.V. Beam filtered with .5 cms. of Al. without correction. $\left(\frac{J_\phi}{J_{90}}\right)$ c.	a. (corrected)	b. (corrected)	c. (corrected)
150°	.62	1.78) 1.79) 1.785			1.62		
"	"		1.72) 1.72) 1.72			1.60	
				1.80) 1.77) 1.785			1.62
"	"	1.81) 1.82) 1.815			1.65		
"	"		1.72) 1.72) 1.72			1.60	
				1.76) 1.79) 1.775			1.61
"	"	1.78) 1.76) 1.77			1.60		
			1.71) 1.69) 1.70			1.58	
				1.75) 1.77) 1.76			1.60
"	.234	1.77) 1.82) 1.795			1.63		
"	"		1.73) 1.73) 1.73			1.61	
"	"			1.78) 1.82) 1.80			1.64

TABLE VIII (CONTD.)

Angle	Thickness of Paraffin Wax in cms.	Expt. Ratio 30 K.V. without correction. $\left(\frac{\mathcal{J}_\phi}{\mathcal{J}_{90}}\right)$ a.	Expt. Ratio 80 K.V. without correction. $\left(\frac{\mathcal{J}_\phi}{\mathcal{J}_{90}}\right)$ b.	Expt. Ratio 80 K.V. Beam filtered with .5 cms. of Al. without correction. $\left(\frac{\mathcal{J}_\phi}{\mathcal{J}_{90}}\right)$ c.	a. (corrected)	b. (corrected)	c. (corrected)
150°	.234	1.77) 1.82) 1.795			1.63		
			1.73) 1.73) 1.73			1.61	
				1.78) 1.82) 1.80			1.64
"	"	1.83) 1.84) 1.835			1.665		
			1.76) 1.77) 1.765			1.64	
				1.80) 1.80) 1.80			1.64
30	.62	2.38) 2.40) 2.39			2.28		
			2.07) 2.05) 2.06			2.08	
				1.91) 1.88) 1.895			2.00
"	"	2,33) 2.46) 2.395			2.29		
			2.04) 2.07) 2.055			2.07	
"	"			1.89) 1.92) 1.905			2.01

TABLE VIII (CONTD.)

Angle	Thickness of Paraffin Wax in cms.	Expt. Ratio 30 K.V. without correction. $\left(\frac{\mathfrak{z}_\phi}{\mathfrak{z}_{90}}\right)$ a.	Expt. Ratio 80 K.V. without correction. $\left(\frac{\mathfrak{z}_\phi}{\mathfrak{z}_{90}}\right)$ b.	Expt. Ratio 80 K.V. Beam filtered with .5 cms. of Al. without correction. $\left(\frac{\mathfrak{z}_\phi}{\mathfrak{z}_{90}}\right)$ c.	a. (corrected)	b. (corrected)	c. (corrected)
30	.234	2.31) 2.31) 2.31			2.20		
"	"		2.12) 2.08) 2.10			2.12	
				1.87) 1.86) 1.865			1.96

SCATTERING MATERIAL CARBON.

Angle	Thickness of carbon in cms.	Expt. Ratio 30 K.V. without correct. $\left(\frac{\mathcal{J}_\phi}{\mathcal{J}_{90}}\right)$ a.	Expt. Ratio 80 K.V. without correct. $\left(\frac{\mathcal{J}_\phi}{\mathcal{J}_{90}}\right)$ b.	Expt. Ratio 80 K.V.B.F. ·5 cms. of Al. without correct. $\left(\frac{\mathcal{J}_\phi}{\mathcal{J}_{90}}\right)$ c.	a. (corrected)	b. (corrected)	c. (correc
150°	.9	1.67) 1.72) 1.695			1.570		
"	"		1.67) 1.63) 1.65			1.57	
"	"			1.70) 1.70) 1.70			1.61
"	"	1.69) 1.71) 1.70			1.575		
"	"		1.66) 1.65) 1.655			1.58	
"	"			1.71) 1.71) 1.71			1.615
"	"	1.72			1.60		
"	"		1.68			1.60	
"	"			1.72			1.63
30	.9	2.53) 2.63) 2.58			2.80		
"	"		2.13) 2.09) 2.11			2.32	
"	"			1.99) 1.99) 1.99			2.19
"	"	2.49	2.06		2.70	2.26	
"	"			1.97			2.17

TABLE X.

SCATTERING MATERIAL. FILTER PAPER.

Angle	No. of filter papers.	Expt. Ratio 30 K.V. without correction. $\left(\frac{J_\phi}{J_{90}}\right)$ a.	Expt. Ratio 80 K.V. without correction. $\left(\frac{J_\phi}{J_{90}}\right)$ b.	Expt. Ratio 80 K.V.B.F. ·5 cms. without correction. $\left(\frac{J_\phi}{J_{90}}\right)$ c.	a. (corrected)	b. (corrected)	c. (corrected)
150°	24 (superficial density .1526 gms. per sq. cm.	1.78) 1.77) 1.775			1.575		
"	"		1.69) 1.69) 1.69			1.64	
"	"			1.79) 1.77) 1.78			1.750
"	"	1.77) 1.73) 1.75			1.55		
"	"		1.66) 1.71) 1.685			1.64	
"	"			1.77) 1.74) 1.755			1.725
"	"	1.79) 1.79) 1.79			1.59		
"	"		1.71) 1.71) 1.71			1.66	
"	"			1.80) 1.79) 1.795			1.765

Angle	No. of filter papers.	Expt. Ratio 30 K.V. without correction. $\left(\frac{J_\phi}{J_{90}}\right)$ a.	Expt. Ratio 80 K.V. without correction. $\left(\frac{J_\phi}{J_{90}}\right)$ b.	Expt. Ratio 80 K.V. B.F. 5 cms. without correction. $\left(\frac{J_\phi}{J_{90}}\right)$ c.	a. (corrected)	b. (corrected)	c. (corrected)
30°	24 (superficial density .1526 gms. per sq. cm.)	3.1) 3.12) 3.11			3.01		
"	"		2.38) 2.35) 2.365			2.36	
"	"			2.00) 2.06) 2.03			2.00
"	"	3.11) 3.11) 3.11			3.01		
"	"		2.37) 2.43) 2.40			2.39	
"	"			2.01) 2.05) 2.03			2.00
"	"	3.07) 3.12) 3.095			2.995		
"	"		2.41) 2.40) 2.405			2.40	
"	"			2.08) 2.03) 2.055			2.03

Experiments were performed to test the effect of thickness on the ratio $\frac{I_\phi}{I_{90}}$. The results of the experiments performed with paraffin wax as the scattering material are given in table XI.

Angle	Thickness of Paraffin Wax in cms.	Voltage on X-ray tube in K.V.	Expt. $\frac{J_\phi}{J_{90}}$		Corrected Ratio $\frac{I_\phi}{I_{90}}$	Remarks
150	.62	30	1.78 1.79	1.785	1.617	All these experiments performed with thick and thin paraffin wax show that the ratio is slightly greater in case of thin scatterer.
"	.234	"	1.77 1.82	1.795	1.627	
"	.62	"	1.81 1.82	1.815	1.646	
"	.234	"	1.83 1.84	1.835	1.665	
"	.973	80	1.73 1.69	1.71	1.59	The same as above.
"	.234	"	1.77 1.73	1.75	1.63	
"	.62	"	1.67 1.67	1.67	1.55	
"	.234	"	1.73 1.71	1.72	1.60	
"	.62	"	1.72 1.72	1.72	1.60	
"	.234	"	1.77 1.76	1.765	1.64	
"	.62	80 (B.F.)	1.80 1.77	1.785	1.624	
"	.234	"	1.78 1.82	1.80	1.637	
"	.62	"	1.76 1.79	1.775	1.614	
"	.234	"	1.80 1.80	1.80	1.637	

Angle	Thickness of Paraffin Wax in cms.	Voltage on X-ray tube in K.V.	Expt. $\frac{J_\phi}{J_{90}}$	Corrected Ratio. $\frac{I_\phi}{I_{90}}$	Remarks
30 30°	.62	30	2.38)2.39 2.40)	2.283	Shows that the ratio is slightly less in case of thin scatterer.
'	.234	"	2.31)2.31 2.31)	2.202	
'	.973	80	2.06)2.045 2.03)	2.064	Shows that the ratio is slightly greater in case of thin scatterer.
'	.234	"	2.02)2.10 2.18)	2.119	
'	.973	"	2.08)2.07 2.06)	2.088	
'	.234	"	2.15)2.075 2.0)	2.094	
'	.62	"	2.07)2.06 2.05)	2.08	
'	.234	"	2.12)2.10 2.08)	2.12	
'	.62	80 (B.F.)	1.91)1.895 1.88)	1.997	Shows that the ratio is slightly less in case of thin scatterer.
'	.234	"	1.87)1.865 1.86)	1.964	

The average values of the ratios of intensities at angles 150° and 30° as compared with 90° are given in tables XII and XIII respectively. They are also exhibited in figure 8.

TABLE XII. ANGLE 150°.

Substance	Thickness in cms.	K. V.	Expt. Ratio without correction. $\frac{\mathcal{J}_{150}}{\mathcal{J}_{90}}$	Expt. Ratio with correction. $\frac{I_{150}}{I_{90}}$
Paraffin Wax,	.62	30	1.79	1.62
"	.62	80	1.71	1.59
"	.62	80 (B.F.)	1.77	1.61
"	.234	30	1.81	1.64
"	.234	80	1.74	1.62
"	.234	80 (B.F.)	1.80	1.64
Carbon	.9	30	1.705	1.58
"	.9	80	1.66	1.58
"	.9	80 (B.F.)	1.71	1.62
Filter Paper	24 sheets (superficial density .1526 gms. per sq. cm.)	30	1.77	1.57
"	"	80	1.695	1.65
"	"	80 (B.F.)	1.78	1.75

TABLE XIII. ANGLE 30°

Substance	Thickness in cms.	K. V.	Expt. Ratio without correction. $\frac{J_{30}}{J_{90}}$	Expt. Ratio with correction $\frac{I_{30}}{I_{90}}$
Paraffin Wax.	.62	30	2.39	2.285
"	.62	80	2.06	2.075
"	.62	80 (B.F.)	1.90	2.005
"	.234	30	2.31	2.20
"	.234	80	2.10	2.12
"	.234	80 (B.F.)	1.865	1.96
Carbon	.9	30	2.535	2.75
"	.9	80	2.085	2.29
"	.9	80 (B.F.)	1.98	2.18
Filter Paper	24 sheets (superficial density .1526 gms. per sq. cm.	30	3.105	3.005
"	"	80	2.390	2.38
"	"	80 (B.F.)	2.04	2.01

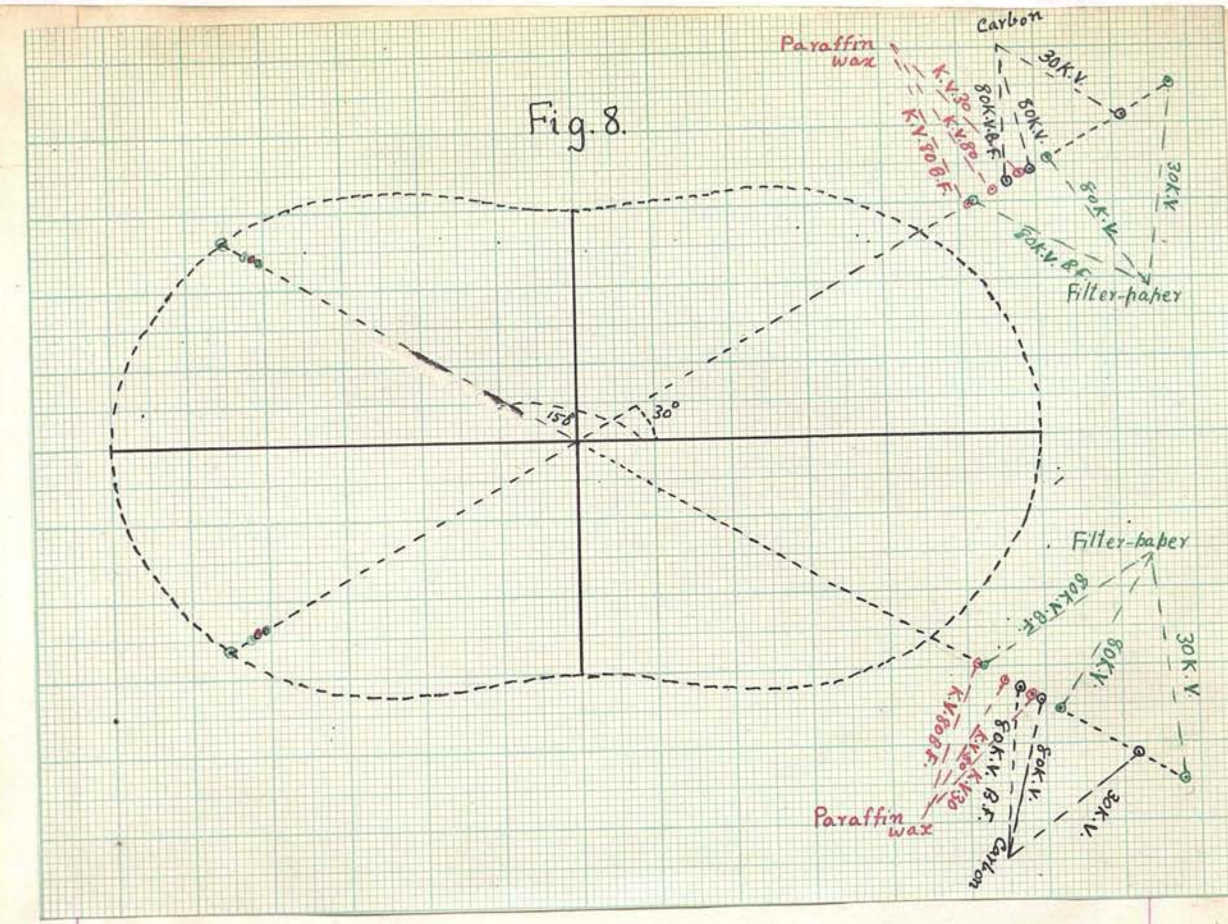


Fig. 8.

As the results of scattering from carbon did not show the variation in the ratio $\frac{z_{150}}{z_{90}}$ of anything like magnitude obtained by Barkla, and by Barkla and Ayres, it seemed desirable to determine whether this was due to the different method of excitation of the X-ray tube.

Experiments were therefore performed with an Induction coil as a high potential generator, and it was found that the uncorrected $\frac{z_{150}}{z_{90}}$ for Carbon as a scattering material, decreased from about 1.63 to 1.63 by increasing the voltage on the X-ray tube from about 45 K.V. to 100 K.V. on the X-ray tube. ~~as~~ the intensity of the rays obtained at 30 K.V. was very feeble. Experiments performed with transformer at 45 K.V. gave also an uncorrected ratio of $\frac{z_{150}}{z_{90}}$ = 1.67, thus showing no difference between the ratios obtained with an Induction coil and a transformer.

DISCUSSION.

From the above results it can be seen that:-

(1) With paraffin-wax and carbon as the scattering materials, the ratio $\frac{I_{150}}{I_{90}}$ does not change appreciably by varying the hardness of the primary radiation. The average value of $\frac{I_{150}}{I_{90}}$

at $\left(\frac{\mu}{\rho}\right)_{Al.} = 5.6$ is about 1.61
" $\left(\frac{\mu}{\rho}\right)_{Al.} = 1.88$ " " 1.59
" $\left(\frac{\mu}{\rho}\right)_{Al.} = 0.71$ " " 1.61

These values are somewhat less than the values to be expected on the simple classical theory, $(1+\cos^2\phi)$ being 1.75. They are lower than the value 1.69 found by Barkla and Ayres when using the softest possible radiations. The <u>uncorrected</u> values, however, are almost identical with theirs for carbon with soft rays. The difference is due to our larger correction for polarization.

Barkla in his earliest experiments, and later Barkla and Ayres, observed a considerable diminution in the ratio $\frac{I_{170}}{I_{90}}$ - from about 2 to 1.5 - with increasing frequency. Such a change we have been able to confirm to only a very slight extent.

It may be interesting to compare the above experimental results with the intensities predicted by/

by quantum theories. According to Dirac's theory, assuming all the radiation to be modified at a wave-length of .7 Å, the ratio $\frac{I_{150}}{I_{90}}$ will be about 1.60. Our value of $\left(\frac{\mu}{\rho}\right)_{Al.}$ = 5.6 corresponds to a wave-length of about .7 Å, and this gives an experimental ratio 1.61. The ratio should be slightly greater - of the order 1.64 - because of the presence of unmodified radiation.

Similiarly $\left(\frac{\mu}{\rho}\right)_{Al.}$ = 1.88 corresponds to an effective wave-length of about .49 Å. According to Dirac's theory at a wave-length of .49 Å, the ratio $\frac{I_{150}}{I_{90}}$ will be about 1.55, assuming all the radiation to be modified. The experimental ratio $\frac{I_{150}}{I_{90}}$ is about 1.59. This small difference may also be due to the presence of unmodified radiation. This agreement is quite satisfactory.

On the other hand filtering the primary beam at 80 K.V. and increasing the average hardness of the beam to $\left(\frac{\mu}{\rho}\right)_{Al.}$ = 0.71 does not decrease the ratio as would be expected according to Dirac's theory. In this region of wave-lengths the scattered radiation is nearly all modified and the ratio $\frac{I_{150}}{I_{90}}$ should be about 1.50 instead of 1.61 (experimental). Thus this theory proves satisfactory for some experimental results, but is evidently not so for others.

(2)/

(2) With filter-paper as the scattering material the ratio $\frac{I_{150}}{I_{90}}$ increases from about 1.57 to 1.65 by increasing the hardness of the beam from $\left(\frac{\mu}{\rho}\right)_{Al.} =$ 5.6 to $\left(\frac{\mu}{\rho}\right)_{Al.} =$ 1.88 and increases still further to about 1.75 by increasing the hardness of the beam to $\left(\frac{\mu}{\rho}\right)_{Al.} = 0.71$. This again is quite contrary to what would be expected on Dirac's or on Compton's theory; but it suggests that the ratio $\frac{I_{150}}{I_{90}}$ approaches the value to be expected on the simple classical theory by increasing the frequency of the incident radiation.

It may be noted that Owen's results (loc. cit.) with filter-paper give approximately equal values of $\frac{I_{150}}{I_{90}}$ for three different frequencies of the primary radiation, but if these results were corrected for polarization and change in wavelength they would confirm the above results with filter-paper - that is they would agree with the value $(1 + \cos^2\phi)$ only for the shorter waves. This confirmation is of some importance.

(3) The ratio $\frac{I_{30}}{I_{90}}$ with all the three substances is distinctly in excess over the values given by the equation $\frac{I_\phi}{I_{\frac{\pi}{2}}} = (1 + \cos^2\phi)$ and this 'excess scattering' at 30° decreases by hardening (diminishing the wave-length of) the primary radiation. This excess scattering if, as generally/

generally held, is due to the superposition of waves in the same phase in the forward direction will be less marked for shorter waves, and should ultimately vanish for very short waves.

(4) Considering now the effect of material. With paraffin wax and carbon the ratio $\frac{I_{150^\circ}}{I_{90}}$ is almost identical for both substances and for all the radiations employed. The results with filter paper as the scattering material agree with those of paraffin wax and carbon only for $(\frac{\mu}{\rho})_{Al.} = 5.6$ that is for long waves, but the ratio increases by increasing the hardness of the beam - a result tending to agreement with classical theory.

(5) The ratio $\frac{I_{30}}{I_{90}}$ is less, consequently the excess radiation is less with paraffin-wax than with carbon, as might perhaps be expected on the theory of electron distribution in an atom. The average distance between the electrons in paraffin-wax is greater than in carbon, hence the effect of superposition will be greater with carbon than with paraffin-wax. With filter-paper if we consider the scattering only due to carbon and oxygen, the average distances between the electrons will be less and there should be greater 'excess scattering' for filter paper than for carbon - which agrees with our results obtained/

obtained at $\left(\frac{\mu}{\rho}\right)_{Al.} = 5.6$ and $\left(\frac{\mu}{\rho}\right)_{Al.} = 1.88$; but at $\left(\frac{\mu}{\rho}\right)_{Al.} = 0.71$ the ratio $\frac{I_{30}}{I_{90}}$ is less for filter paper than for carbon, that is, the decrease with filter paper is greater than with other substances. Owen also found this rapid decrease in the ratio $\frac{I_{30}}{I_{90}}$, in his experiments the ratio falling from about 3.1 to 1.75 by increasing the voltage on the X-ray tube.

(6) The thickness of the scattering material as far as these experiments go does not markedly influence the ratio of intensities, though it was always observed that $\frac{I_{150}}{I_{90}}$ was slightly greater with a thin scatterer - tending towards the simple classical result. It was, however, not possible to measure the radiation from very thin layers such as have given such remarkable results in experiments associated with the J-phenomenon.

The values of $\frac{I_{30}}{I_{90}}$ with Paraffin Wax as the scattering material agree with those of Coven (Phy. Rev. 38, 1424, 1931) who, using the same scattering material obtained the ratio $\frac{I_{30}}{I_{90}} = 2.1$ at an effective wave-length of .32 Å. Chylinsiki using the same scattering material obtained the value $\frac{I_{30}}{I_{90}}$ to be equal to 3.07 at an effective wave-length of .23 Å, - a result which does not at all agree with the above.

It/

It is interesting to note the remarkable behaviour of filter paper when used as a scattering material. It is quite different from the behaviour of paraffin wax and carbon. With filter paper as the scattering material:-

(i) The ratio $\frac{I_{150}}{I_{90}}$ increases (within the limits of these experiments) by increasing the hardness or average frequency of the primary radiation and approaches the classical result.

(ii) The ratio $\frac{I_{30}}{I_{90}}$ decreases by increasing the hardness and approaches the classical result. (Confirmed by Owen's experiments).
Thus the scattering from paper has proved to be peculiarly sensitive to changes in wave-length of the primary radiation, and to be the only substance for which the classical distribution (or a very close approximation to it) has been obtained in all directions.

(iii) In addition, filter paper is the substance for which the scattered radiation has been found to be equally absorbable in different directions, say 30° and 90°, for all the primary radiations used.

(iv) Further, in very extensive series of experiments in this laboratory on the relation between the intensity/

intensity of radiation scattered at 90° from filter-paper and the wave-length of the primary radiation, a constancy has been obtained which strongly suggests a complete departure from quantum theory.

Finally, the simple relation $I\phi = I_{\frac{\pi}{2}} (1 + \cos^2\phi)$ does appear to hold very closely as a limiting case. When $\phi \nless \frac{\pi}{2}$ the values of $I\phi$ are low but under certain conditions approach and possibly reach $I_{\frac{\pi}{2}} (1 + \cos^2\phi)$. On the other hand when $\phi \ngtr \frac{\pi}{2}$ the values of $I\phi$ are high, but under certain conditions approach and possibly reach $I_{\frac{\pi}{2}} (1 + \cos^2\phi)$. We have no knowledge of any experiment showing too high a value when $\phi \nless \frac{\pi}{2}$; or too low a value when $\phi \ngtr \frac{\pi}{2}$. This is indicated in figure 8, showing the corrected ratios for the three scattering substances and for three radiations.

With regard to quantum theories, it cannot be said, that the experimental evidence of this paper on energy distribution of the scattered heterogeneous radiation, gives any strong support to these theories. While there are satisfactory agreements there are also quite serious discrepancies; and if we consider the magnitude of the deviations from the $(1 + \cos^2\phi)$ law - usually quite small except for small angles ϕ - the quantum theories put forward do not appear at all adequate. These and many other experiments suggest the/

AN X-RAY INVESTIGATION OF THE CRYSTALS OF DIPHENYL NITROSOAMINE

BY

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AN X-RAY INVESTIGATION OF THE CRYSTALS OF DIPHENYL NITROSOAMINE.

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Diphenyl nitrosoamine was crystallised from a mixture of alcohol and benzene. The crystals have been examined crystallographically and have been found to evelop c (001) and m (110) faces. q (011) and o $(10\bar{1})$ also appear but not in a well eveloped form. The crystals belong to the monoclinic prismatic class and the xial ratio is[1]

$$a : b : c = 0{\cdot}9635 : 1 : 1{\cdot}5637$$
$$\beta = 90^\circ\ 58'$$

The crystals were examined by the rotating crystal method. The rotation otographs were taken by means of Shearer X-ray tube fitted with copper ticathode: those obtained about a and b axes are shown in figures 1 and 2 late I). The lengths of the axes were determined from

$$l = \frac{n\lambda}{2z}\sqrt{(2r)^2 + (2D)^2}$$

where l = the length of the axis,
n = the order of the hyperbola,
λ = the wavelength of the incident X-rays,
z = the distance of a spot from the zero line,
r = the distance of the spot from the centre,
D = the distance of the plate from the crystal.

e mean values of a, b and c were thus found to be

$$a = 17{\cdot}08\ \text{Å};\quad b = 8{\cdot}8675\ \text{Å};\quad c = 28{\cdot}07\ \text{Å}.$$

The axial ratio is

$$a : b : c = 1{\cdot}926 : 1 : 3{\cdot}166$$

The value of β was assumed to be 90° 58′, the same as the value given by Groth. It can be seen from the above that the ratios of a & b and of c & b are exactly twice of that found by the crystallographic measurements.

Oscillation photographs about the a and b axes were taken at an interval of 15° and the indices of the reflecting planes corresponding to the spots on the oscillation photographs were worked out by Bernal's method of analysis.[2]

The planes observed, together with an approximate idea of their relative intensities, are given in tables 1 and 2. The method adopted in estimating the intensity of the spot was that used by Robertson.[3] The symbols used have the following meaning:

v. s. = very strong
s. = strong
m. s. = medium strong
m = medium
w. m. = weak medium
w = weak
v. w. = very weak.

Table 1

Axial Planes		(hol) Planes		(ho$\bar{l}$) Planes		(okl) Planes		(hko) Planes	
004	v. s.	202	m. s.	$20\bar{2}$	m. s.	012	s.	210	v. s
008	s.	206	v. s.	$20\bar{6}$	v. s.	016	v. s.	230	s.
0012	v. w.	2010	m.	$20\overline{10}$	m.	0110	m.	420	m.
020	v. s.	2014	m. s.	$20\overline{14}$	m.	0114	w. m.	610	w. m.
400	v. s.	404	s.	$40\bar{4}$	s.	024	s.	620	v. w.
600	w.	406	v. w.	...		028	s.	820	w.
800	w.	408	w. m.	...		0212	w. m.		
		...		$60\bar{2}$	m.	0214	m. s.		
		...		$60\bar{4}$	m.	032	s.		
		606	s.	$60\bar{6}$	w.	036	m. s.		
		6010	m. s.	$60\overline{10}$	w. m.	0310	m. s.		
		802	w.	...		048	m.		
		804	w. m.	$80\bar{4}$	w.				
		...		$80\bar{6}$	w.				

Table 2.—General Planes.

111	s.	$11\bar{1}$	s.	214	s.	$21\bar{4}$	s.	311	w.	$31\bar{1}$	m.
113	s.	…		218	m.	$21\bar{8}$	m.	313	v. s.	$31\bar{3}$	v. s.
115	v. s.	$11\bar{5}$	v. s.	…		$21\overline{10}$	m. s.	315	v. s.	$31\bar{5}$	s.
119	m. s.	$11\bar{9}$	m. s.	2112	w.	$21\overline{12}$	v. w.	317	m. s.	…	
113	w.	$11\overline{13}$	v. w.	222	s.	$22\bar{2}$	s.	319	m. s.	$31\bar{9}$	v. w.
115	w.	$11\overline{15}$	v. w.	226	s.	$22\bar{6}$	s.	3111	w. m.	$31\overline{11}$	v. w.
121	v. s.	121	v. s.	…		$22\bar{8}$	m. s.	3113	m.	…	
125	s.	$12\bar{5}$	v. s.	2210	m. s.	$22\overline{10}$	m. s.	321	v. s.	…	
127	m. s.	…		2214	w. m.	…		…		$32\bar{3}$	v. w.
129	m. s.	$12\bar{9}$	m. s.	232	m. s.	$23\bar{2}$	w. m.	325	m. s.	$32\bar{5}$	s.
131	s.	…		234	m.	$23\bar{4}$	m. s.	327	w. m.	$32\bar{7}$	w. m.
33	v. w.	$13\bar{3}$	w. m.	…		$23\bar{6}$	v. w.	3213	m.	…	
35	m.	$13\bar{5}$	m.	238	w. m.	$23\bar{8}$	w. m.	331	m.	$33\bar{1}$	m.
..		$13\bar{7}$	m.	…		$23\overline{10}$	m.	333	w.	$33\bar{3}$	v. w.
39	m.	$13\bar{9}$	w. m.	242	w. m.	…		337	m. s.	$33\bar{7}$	m. s.
1	w. m.	…		…		$24\bar{4}$	w. m.	…		$34\bar{3}$	w.
3	w. m.	…		246	w. m.	$24\bar{6}$	m.	345	w. m.	$34\bar{5}$	m.
5	m. s.	$14\bar{5}$	m. s.								
7	w.	$14\bar{7}$	m.								
2	m. s.	$41\bar{2}$	s.	511	m. s.	$51\bar{1}$	s.	612	m. s.	$61\bar{2}$	v. w.
6	w.	$41\bar{6}$	w. m.	513	v. s.	$51\bar{3}$	v. s.	614	w. m.	$61\bar{4}$	w.
8	v. w.	418		515	w.	$51\bar{5}$	w.	…		$61\bar{6}$	m. s.

F 4

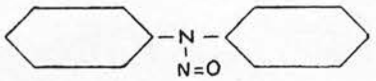
Table 2 (*continued*)

4110	m. s.	...		517	s.	$51\bar{7}$	m.	618	m. s.	$61\bar{8}$	m. s.
...		$411\bar{4}$	w.	519	m.	$51\bar{9}$	v. w.	6112	m. s.	...	
424	m.	$42\bar{4}$	w. m.	521	s.	$52\bar{1}$	s.	622	w. m.	$62\bar{2}$	m. s.
428	v. w.	$42\bar{8}$	m. s.	...		$52\bar{3}$	m. s.	626	w. m.	$62\bar{6}$	m.
4212	w. m.	$42\overline{12}$	v. w.	525	m. s.	$52\bar{5}$	m. s.	628	w. m.	$62\bar{8}$	m.
432	w. m.	...		527	m.	527	m.	6210	w.	$621\bar{0}$	v. w.
...		$43\bar{4}$	m. s.	...		$52\bar{9}$	v. w.	632	v. w.	$63\bar{2}$	m.
...		$43\bar{6}$	m.	5211	w.	$521\bar{1}$	v. w.	634	v. w.	$63\bar{4}$	v. w.
438	m. s.	...		5213	v. w.	...		638	w.	...	
4310	v. w.	$431\bar{0}$	v. m.	535	w.	$53\bar{5}$	w.	...		$631\bar{0}$	w.
444	m.	$44\bar{4}$	w. m.	...		$53\bar{3}$	w. m.	...		$64\bar{2}$	v. w.
...		$44\bar{8}$	w.	537	m. s.	$53\bar{7}$	m. s.	..		$64\bar{6}$	m.
				539	w. m.	$53\bar{9}$	w.				
				5311	w.	$531\bar{1}$	v. w.				
		711	w. m.	...		812	m. s.	...			
		713	m. s.	$71\bar{3}$	m.	816	w.	..			
		715	w.	...		...		824	w. m.		
		717	w.	$71\bar{7}$	w.						
		...		$71\bar{9}$	w.						
		723	v. w.	$72\bar{3}$	m.						
		725	m. s.	$72\bar{5}$	w.						
		731	m. s.	...							
		733	v. w.	...							

It will be seen from the above list that the planes (001) are quartered and (100) and (010) are halved and (hkl) planes are halved when (h+l) is odd. These halvings correspond to the space group C_{2h}^3 with Γ_m' Bravais Lattice.[4]

The number of molecules in the unit cell required by the space group C_{2h} is sixteen. The number of molecules in the unit cell calculated from the dimensions of the cell and the specific gravity of the crystals which was found to be 1·251, also comes out to be nearly sixteen. This indicates that the molecules of diphenyl nitrosoamine in the cell are asymmetric.

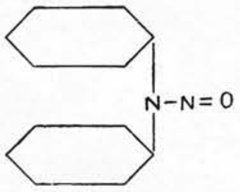
The chemical structure of diphenyl nitrosoamine is represented as



If the benzene rings are assumed to be plane rings of carbon atoms having liameter 1·42 Å and the centres of the carbon atoms and that of the nitrogen tom lie on the same line, the length of the molecule comes out to be nearly 2 Å.

It will be seen from the list of the planes that (hkl) and $(hk\bar{l})$ have in the najority of cases nearly the same intensity. The unit cell, therefore, behaves like n orthogonal cell: this, however, is expected as the angle β is nearly 90°.

From the quartering of the (001) planes it appears that the molecules of iphenyl nitrosoamine in the unit cell lie with theirlength parallel to the *c* axis. ut as the length provided for the purpose is only about 7 Å, it appears that the olecules in the cell are situated not in the manner indicated above but probably as



he considerations of the dimensions of this arrangement of the rings and those the unit cell further indicate that the two rings are situated nearer to the ac face an to the bc face and they are slightly inclined to each other and to the ac e. The oxygen atom is probably situated along the direction of the b axis. is arrangement of two rings is different from the one which has been observed in the unit cells of diphenyl or other substances of similar structure studied in this laboratory.

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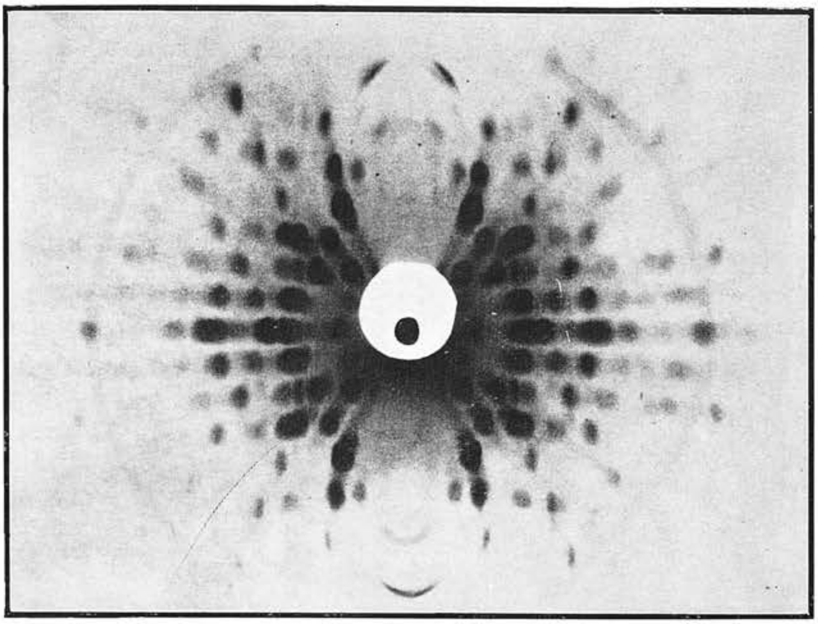
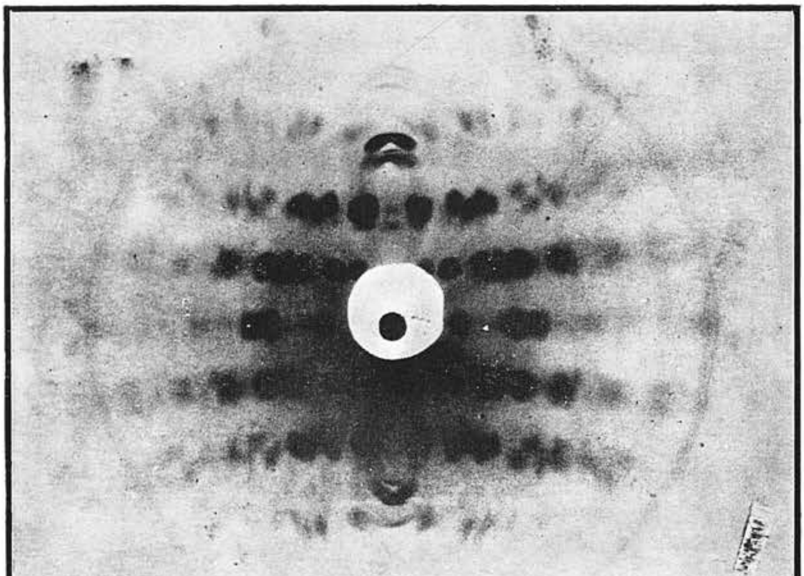


Rotation Photograph about the *a*-axis



Rotation Photograph about the *b*-axis

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