SOME ASPECTS OF THE QUARK MODEL OF ELEMENTARY PARTICLES

A Thesis

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by

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PREFACE

I am indebted to Dr. K.C. Bowler for suggesting the field of inquiry, and for his invaluable guidance, encouragement and help throughout the work leading to the preparation of this manuscript. It is also a pleasure to express my grateful appreciation of the hospitality extended to me at the Tait Institute by Professor N. Kemmer, F.R.S., and other members of the Department.

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The material presented in this dissertation is asserted to be original except where some reference has been cited.

M. Ayub

ABSTRACT

In the first two chapters the quark model of elementary particles is reviewed. The harmonic oscillator quark model is then used in the next two chapters as the basis for dynamical calculations of the energetically allowed radiative decays (some of these widths await experimental measurement) of a number of mesons. These include the radiative transitions of charmonium (Chapter 3), as well as those of all the "old" L=0 and "old" L=1 mesons (Chapter 4). In order to be able to calculate the decays

$$\psi \rightarrow \gamma\eta,\ \gamma\eta',\ \gamma X(2.8),\ \gamma\pi^{o},\ \gamma f(1270)$$

$$\psi' \rightarrow \gamma\eta,\ \gamma\eta\ ,\ \gamma X(2.8)$$

in the framework of the harmonic oscillator quark model, a broken SU(4) scheme is used to estimate the $(p\bar{p} + n\bar{n})$, $\lambda\bar{\lambda}$ and $c\bar{c}$ contents of the particles involved in the above decays (Note that X(2.8) is here identified with η_c). These particles belong naturally to three different (15+1)-plets of SU(4), namely the pseudoscalars with $J^{PC} = 0^{-+}$ (containing η, η' and X(2.8)), the vectors with $J^{PC} = 1^{--}$ (containing ω, ϕ and ψ) and the tensors with $J^{PC} = 2^{++}$ (containing f(1270), f'(1514) and χ(3552)). These results are compared with those available for some related models.

It is possible to derive some amplitude relations for the radiative decays of the lowest mass mesons on the basis of symmetry-breaking alone (i.e. without assumptions about quark dynamics). By classifying the vectors and the pseudoscalars according to two different (15+1)-plets of SU(4) and introducing a scheme for symmetry breaking, the matrix elements of the electromagnetic current may be calculated with SU(4) Wigner-Eckart theorem. The attempts of some authors to extend this approach to the calculation of the radiative widths are also discussed. This forms the substance of Chapter 5.

Finally, in Chapter 6, the applications of the MIT bag model to the calculation of hadron masses are surveyed critically and then employed to predict the masses of the excitations of the recently discovered heavy meson (here interpreted as $b\bar{b}$ systems - mesons with hidden beauty) at 9.4 GeV and also the masses of the ρ-like resonances.

C O N T E N T S

CONTENTS (Contd.)

C O N T E N T S (Contd.)

CHAPTER 1

THE QUARK MODEL OF ELEMENTARY PARTICLES

1.0 Introduction

The work in this thesis deals with the quark model of elementary particles and is divided into six chapters. The bulk of this work is about the investigation of the ability of the harmonic oscillator quark model to describe the radiative widths of mesons. The first chapter (from Sec. 1.1 onwards) is a review of the application of the quark model to the decay processes of hadrons, especially their radiative transitions. In the second chapter we review another aspect of the quark model, i.e. different techniques employed for the predictions (and relations) of (among) the hadron masses.

In Chapter 3 we describe our model and apply it to the radiative transitions of Charmonium. The transitions considered in this chapter are: $\psi' \to \gamma\chi_J$'s, χ_J's $\to \gamma\psi$ and $\psi \to \gamma X(2.8)$. All the decay width calculations have been performed with three sets of parameters (with charmed quark mass m_c between 1.3 - 2 GeV, the harmonic oscillator parameter α'^2 between 0.1 - 0.27 GeV^2 and m_u = 0.336 GeV). The results are compared with the bounds set on these radiative widths by the use of dipole sum rules and with the results obtained by the use of a dipole approximation formula for the decay width.

In Chapter 4, we consider the masses of the (15+1)-plets of SU(4) of the ground state vector, pseudoscalar and tensor mesons in a broken SU(4) model. We solve the mixing problems for the isoscalars contained in these multiplets and then apply our harmonic oscillator quark model (described in Chapter 3) to a host of mesonic radiative transitions. The decays considered are: $V(L=0) \to \gamma P(L=0)$, $P(L=0) \to \gamma V(L=0)$ (where

V(L=0) and P(L=0) stand for the "old" ground state vector and pseudoscalar mesons), the two body radiative decays of ψ (i.e. $\psi \rightarrow \gamma\eta, \eta'$, X(2.8), π, f(1270)), the radiative decays of ψ' (i.e. $\psi' \rightarrow \gamma\eta, \eta'$, X(2.8), X(3455)), the radiative decays of the charmed mesons (i.e. $D^* \rightarrow \gamma D$ and $F^* \rightarrow \gamma F$) and the radiative decays of the "old" L=1 mesons. The results are discussed and compared with available calculations of some other related models.

In Chapter 5, we get some relations between the amplitudes of the transitions of the ground state mesons, simply by considering the transformation properties of electromagnetic current in SU(4), using the eigenvectors for vector and pseudoscalar mesons found in Sec. 4.4 of Chapter 4 and employing the Wigner-Eckart theorem for SU(4). In the remainder of this chapter the work of some authors to get radiative widths for these transitions in certain broken SU(4) models (the approach being related to the one described in the first portion of this chapter) is discussed.

Chapter 6 is about the MIT bag quark model. There, we apply this model to predict the masses of the radial (and orbital) excitations of the recently discovered heavy meson at 9.41 GeV (the Υ-family analogous to the ψ-family) and the masses of the ρ-like resonances.

In Appendix A and Appendix B we give respectively, the wavefunctions and the integrals involved in the calculations of Chapter 3. References and footnotes are given at the end.

In the remainder of this chapter we review the application of the harmonic oscillator quark model to the decay processes of hadrons with an emphasis on their radiative decays.

1.1 Unitary Symmetry and the Quark Model

Because of the large number of strongly interacting particles there has always been an attempt to classify these particles in some scheme and correlate their properties. Notable among these models are: the Fermi-Yang model [1], Sakata model [2] and finally the quark model [3].

From an application of a certain model one cannot expect 100% exact results. One accepts the approximate results and is encouraged to put the model to more and more severe tests and find out the deficiencies of the model concerned. The important aspects of any model are:

1) To state clearly the rules of the game (as Lipkin calls it) while taking care not to change the rules while applying that model to explain a certain phenomenon.

2) To explore the consequences of the model in order to discover its limitations.

In this chapter, first we describe briefly the models which were introduced before the quark models and then we take the harmonic oscillator quark model for hadrons and discuss some of its successful applications and drawbacks [4]. Our main emphasis will be on the application of the model to the calculation of decay rates of hadrons. Another application is discussed in Chapter 2.

Fermi and Yang [1] took the nucleon and anti-nucleon as the basic building blocks and then in a sense, they could look upon all the other non-strange hadrons as composites of different numbers of the nucleons/ and anti-nucleons. As an example, if we consider just the isospin of the particle then the nucleon and anti-nucleon can be looked upon as eigenstates corresponding to the fundamental representation $\underline{2}$ of SU(2) and can be represented by an iso-spinor (or a tensor of rank one in a two-dimensional space) as follows:

$$A_i = \begin{pmatrix} p \\ n \end{pmatrix} \sim \underline{2} \; ; \qquad A^j = \begin{pmatrix} \bar{p} \\ \bar{n} \end{pmatrix} \sim \underline{\bar{2}} \equiv \underline{2} \tag{1.1}$$

($\sim$ means transforms as).

The direct product of the two fundamental representations of SU(2)

$$\underline{2} \otimes \underline{2} = \underline{1} \oplus \underline{3} \tag{1.2}$$

yields representations $\underline{1}$ and $\underline{3}$ of SU(2). The explicit isospin states in terms of the constituent nucleon and anti-nucleon can be easily found out and are given as follows:

$$\frac{1}{\sqrt{2}} (p\bar{p} + n\bar{n}); \quad I = 0, \quad I_3 = 0$$

$$\bar{p}n, \quad \bar{n}p, \quad \frac{1}{\sqrt{2}} (p\bar{p} - n\bar{n}); \quad I = 1, \quad I_3 = \pm 1, 0 \; . \tag{1.3}$$

These results can be interpreted in two ways. Either, one can say that the states with $I = 1$, $I_3 = \pm 1, 0$ tell us about the transformation properties of a pion triplet in isospin space or one can go a bit further and say that a pion triplet is a bound state of a nucleon and an anti-nucleon and thus get a model (Fermi-Yang model in this case). The state with $I = 0$ could not be identified with any physical particle at that time but now it can be identified with the η-meson. The difficulty which confronted this model is the fact that it could not incorporate the strange hadrons which were discovered in those days.

Sakata [2] extended the Fermi-Yang model by starting with a triplet of particles (adding a strange particle to the Fermi-Yang doublet) and an anti-triplet of these particles such that these could be looked upon as the eigenstates of the fundamental representations $\underline{3}$ and $\underline{\bar{3}}$ of SU(3) as follows:

$$A_i = \begin{pmatrix} p \\ n \\ \Lambda \end{pmatrix} \sim 3; \quad A^j = \begin{pmatrix} \bar{p} \\ \bar{n} \\ \bar{\Lambda} \end{pmatrix} \sim \bar{3} \, . \tag{1.4}$$

The direct product of the two fundamental representations of SU(3)

$$\underline{3} \otimes \underline{\bar{3}} = \underline{1} \oplus \underline{8} \qquad (1.5)$$

yields representations $\underline{1}$ and $\underline{8}$ of SU(3). The octet of states could be identified with the eight known pseudoscalar mesons if one supposes that the particles and antiparticles are in the spin anti-parallel states and in relative motion with $L = 0$ and could be identified with eight known vector mesons if the particles and anti-particles are assumed to be in the spin parallel states.

We run into difficulties if we try to apply the Sakata scheme to baryons. For example, combinations such as $pn\Lambda$ give $B = 3$ (p, n and Λ have $B = 1$ each), and such states are not observed. We can restrict ourselves to combinations like $p\bar{n}\Lambda$ (with B = 1) but then we cannot obviously exclude the combination $pn\bar{\Lambda}$ (with strangeness S = 1), which also is not observed in nature. One can find a way out of this difficulty by assuming that the basic triplet is not p, n, Λ (which are physical particles) but other particles with fractional baryon number. If one considers combinations of up to three such particles, the simplest assumption is that each has $B = \frac{1}{3}$. Such objects have been called quarks by Gell-Mann [5] and Zweig [6]. They showed that the "Eightfold Way" of Gell-Mann and Ne'eman (The Eightfold Way is based on SU(3) and a basic octet of particles without the mention of its origin) could be thought of in terms of combinations of a basic triplet of quarks. This model (the so-called quark model) can logically incorporate all the observed low-lying hadrons (also see Sections 1.2 amd 1.5). In the remainder of this chapter we shall review the development of the quark model of hadrons.

In the conventional quark model for hadrons [5,6], we have a triplet of quarks (denoted variously by p, n, λ; u, d, s or q_1, q_2, q_3) and a triplet of anti-quarks [7]. These so far hypothetical particles are assigned certain curious quantum numbers which are displayed in

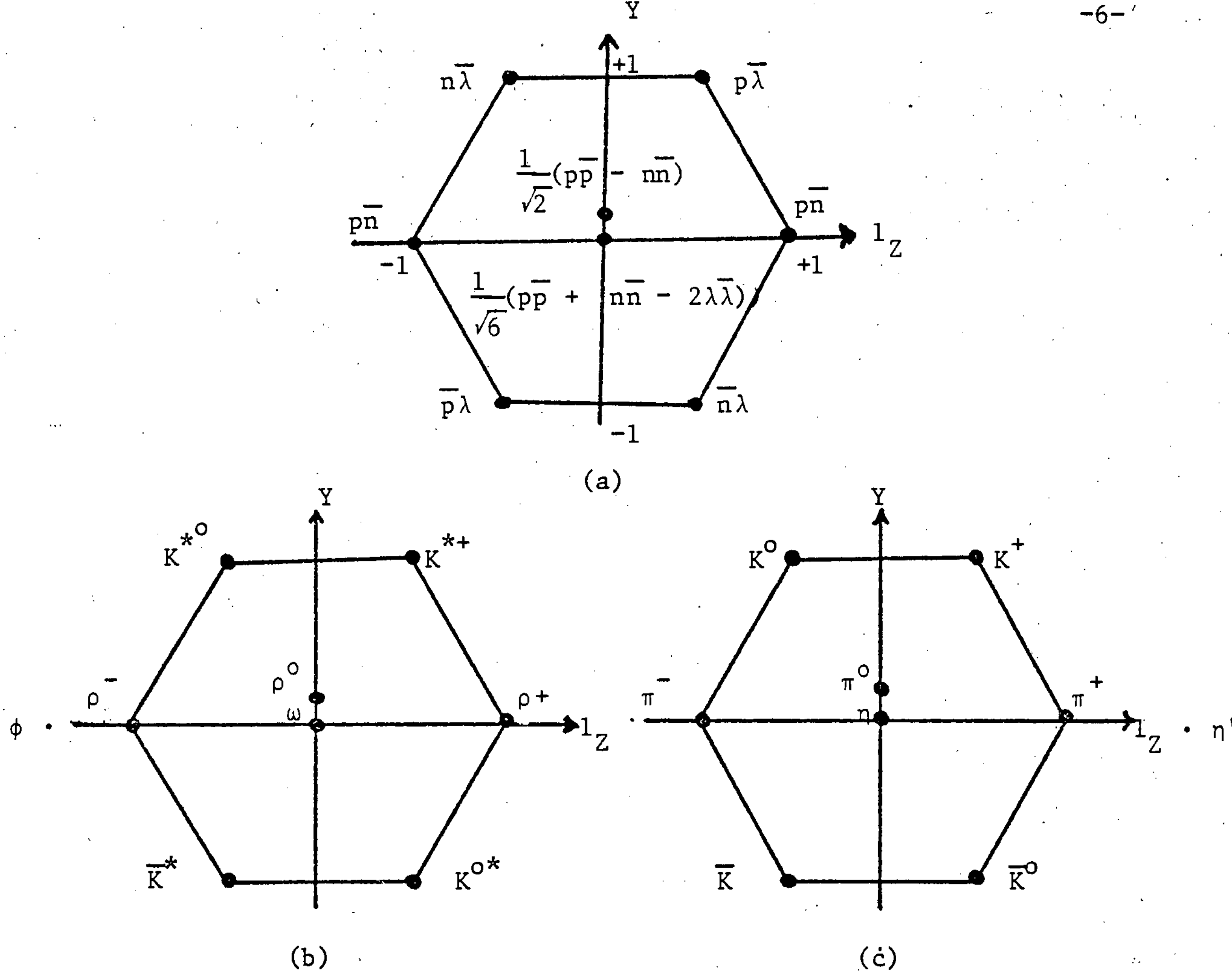


Fig. 1.1

a) Unitary spin states of an octet made of a quark and anti-quark.

b) The octet and singlet of vector mesons.

c) The octet and singlet of pseudoscalar mesons.

Table 1.1

Quantum Numbers of the Quarks

Quark	B	I	Y	S	Q
p	$\frac{1}{3}$	$\frac{1}{2}$	$\frac{1}{3}$	0	$\frac{2}{3}e$
n	$\frac{1}{3}$	$\frac{1}{2}$	$\frac{1}{3}$	0	$-\frac{1}{3}e$
λ	$\frac{1}{3}$	0	$-\frac{2}{3}$	-1	$-\frac{1}{3}e$

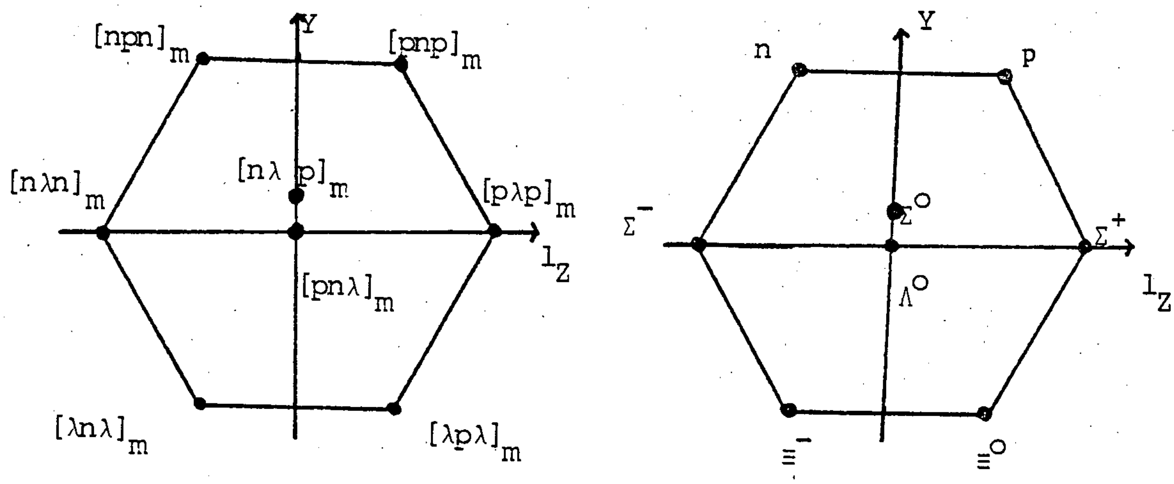


Fig. 1.2

a) Unitary spin states of an octet made of three quarks (the symbol $[\ ]_m$ means mixed symmetry w.r.t. the exchange of any two quarks).

b) Octet of lowest lying baryon states.

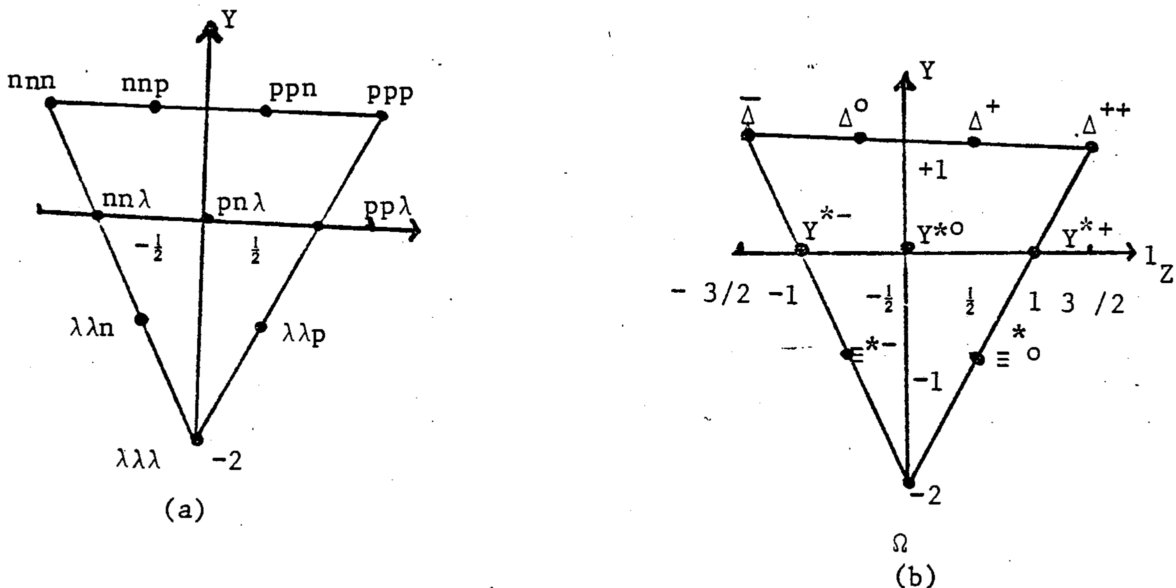


Fig. 1.3

a) Unitary spin states of a decuplet.

b) Decuplet of lowest lying baryon states.

Table 1.1. For example, they are fractionally charged and have fractional baryon numbers.

These quarks and antiquarks can be represented by the fundamental representations $\underline{3}$ and $\underline{\bar{3}}$ of SU(3) respectively. We are not yet considering the spins of the quarks and hence can represent them by a tensor of rank one in a 3-dimensional space as follows:

$$A_i = \begin{pmatrix} p \\ n \\ \lambda \end{pmatrix} \quad ; \qquad A^j = \begin{pmatrix} \bar{p} \\ \bar{n} \\ \bar{\lambda} \end{pmatrix} \tag{1.6}$$

As is clear from the table of the quantum numbers, mesons can be identified with states in the reduction $\underline{3} \otimes \underline{\bar{3}}$ and baryons with the states in the reduction $\underline{3} \otimes \underline{3} \otimes \underline{3}$ where

$$\underline{3} \otimes \underline{\bar{3}} = \underline{1} \oplus \underline{8} \tag{1.7}$$

$$\underline{3} \otimes \underline{3} \otimes \underline{3} = \underline{1} \oplus \underline{8} \oplus \underline{8} \oplus \underline{10} \tag{1.8}$$

The octet of baryons can be identified with $\underline{8}$ and the well known $J^P = {}^3/2^+$ decuplet of baryon resonances with $\underline{10}$. One can find the unitary-spin states for the octets of mesons, octet of baryons and the decuplet in a straightforward way [8]. The unitary spin states and the assignment of physical particles to the irreducible representations of SU(3) are shown in Fig. 1.1 to 1.3. The mesons η, η' and ω, ϕ are taken as mixtures of the octet $\underline{8}$ and singlet $\underline{1}$ of SU(3) (such matters are discussed in detail in Chapter 2) as follows:

$$\eta = \eta_8 \cos\theta_{PS} + \eta_1 \sin\theta_{PS}; \quad \eta' = \eta_8 \sin\theta_{PS} \simeq \eta_1 \cos\theta_{PS}; \ \theta_{PS} = -11^\circ$$

$$\omega = \omega_8 \sin\theta_V + \omega_1 \cos\theta_V \ ; \quad \phi = \omega_8 \cos\theta_V - \omega_1 \sin\theta_V; \ \theta_V = 35^\circ \tag{1.9}$$

The angles θ_{PS} and θ_V can be found out by diagonalizing a mass matrix or from the experimental decay widths.

If we include the spins of the quarks (and antiquarks) then we need a larger group which is SU(6) or a subgroup of SU(6) . Here the basic states can be represented by the components of an irreducible tensor (of one index) in the 6-dimensional space and one can show that

$$\underline{6} \otimes \underline{\bar{6}} = \underline{1} \oplus \underline{35} \tag{1.10}$$

One can also find the SU(2) and SU(3) contents in the irreducible representations of SU(6), i.e.

$$\begin{aligned} \underline{1} &= [\{1\}, 0] \\ \underline{35} &= [\{1\}, 1] \oplus [\{8\}, 0] \oplus [\{8\}, 1] \end{aligned} \tag{1.11}$$

where in the notation [{A}, B], A stands for SU(3) representation and B stands for SU(2) representation. We get the octets of 0^--mesons and 1^--mesons in the same representation of SU(6).

Also

$$\underline{6} \otimes \underline{6} \otimes \underline{6} = [20]_a + [56]_s + [70]_m + [70]_m \tag{1.12}$$

where

$$\begin{aligned} [56]_s &= [\{8\}, \tfrac{1}{2}] \oplus [\{10\}, \tfrac{3}{2}] \\ [70]_m &= [\{1\}, \tfrac{1}{2}] \oplus [\{8\}, \tfrac{1}{2}] \oplus [\{10\}, \tfrac{1}{2}] \oplus [\{8\}, \tfrac{3}{2}] \\ [20]_a &= [\{1\}, \tfrac{3}{2}] \oplus [\{8\}, \tfrac{1}{2}] \quad . \end{aligned} \tag{1.13}$$

a, s and m stand for anti-symmetric, symmetric and mixed with respect to the exchange of two of the quarks. The states of [56] are identified with the ordinary $\frac{1}{2}^+$ baryons and $\frac{3}{2}^+$ baryon resonances.

A little digression at this stage is perhaps in order, to emphasize that there does seem to be an SU(3) strong interaction (broken) symmetry regardless of whether or not explicit quark models are correct [9]. SU(3) symmetry group has led to remarkable classification of elementary particles, has predicted new particles, and given rise to many measurable consequences which are mostly in agreement with experiments. SU(3) can be presently looked upon as something "classic" in elementary particle physics, and it can be said that any future symmetry group, mixing internal and external quantum numbers (see next paragraph), must contain in some way the relevant aspects of SU(3).

Attempts have also been made to treat SU(6) as an approximate strong interaction symmetry - spin and SU(3) invariance of strong interaction, H_{strong}. The origin of this treatment of SU(6) goes back to Wigner's supermultiplet model of nuclear forces. Wigner postulated that these forces are invariant under rotations in spin space (the symmetry group - $SU(2)_s$) and isospin space (the symmetry group - $SU(2)_I$), and the combined transformations such that the overall symmetry group becomes SU(4). The main point to recognize here, is that SU(4) combines spin which is related to space-time symmetries (such as the Lorentz group and the translation group) and the internal symmetry group of isospin. This means that particles with different spin and isospin orientations can be treated on the same footing and thus making a supermultiplet. Another important point to note is that such a model is essentially a non-relativistic one.

These ideas were extended to elementary particle physics by Gursey, Radicati and Sakita [10]. The isospin group $SU(2)_I$ was replaced by the unitary symmetry group SU(3) and the overall invariance group (an approximate one, of course) after incorporation of the spin group $SU(2)_s$ becomes SU(6). The main successes of this model are

(a) The classification of the low-lying mesons and baryons in the 35-plet and 56-plet of SU(6) respectively.

(b) A simple group theoretical derivation of the ratio of the total magnetic moments of the proton and the neutron.

There have been attempts to modify the original SU(6) model, obviously for at least the following two reasons:

(c) Spin is not conserved in many elementary particle reactions.

(d) SU(6) model is not relativistic.

Now the question is - can one find a model which preserves the good features (a) and (b) of the SU(6) model and at the same time avoid the bad features (c) and (d). The answer given to this question is based on intuitive arguments that go as follows:

It is well known that invariance of a system under ordinary space rotations implies the conservation of the total angular momentum $\underline{J}$ ($\underline{J} = \underline{L} + \underline{S}$). In general, $\underline{L}$ and $\underline{S}$ are not separately conserved but if we suppose that all the particles in the system are at rest with respect to each other, then $L = 0$ and the conservation of J is equivalent to that of S. Alternatively, we can consider a more general situation where all the particles are in relatively parallel motion (collinear motion). In such a situation, the projection of $\underline{L}$ on the common direction of motion is zero and thus the component of $\underline{S}$ along the direction of motion is conserved.

The first example shows that for particles at rest, one can use the full group SU(6) without conflict with (c) but for collinear motion, only a subgroup can be used. This subgroup will not contain all the generators of the spin group but only one of them (say σ_3) which is conserved in this particular kinematical situation. For details of such considerations the reader is referred to a review article by Hey [11].

Whatever the virtues of the SU(6) model are, it has influenced the development of ideas in a positive way and beyond the group theoretical concepts, for example, those contained in the quark model (see the following sections) or in current algebra [11].

1.2 L-Excitation Model for Baryons

The successes of the quark model (quarks looked upon as mathematical objects) in classifying the lowest mass hadrons suggest that one should try to extend this device to the classification of higher mass hadrons. These states may be generated in two distinct ways. One is the addition of more quarks, so that, for example, a meson might be composed as a $\bar{q}\bar{q}qq$ object and a baryon as a $qqqq\bar{q}$ one. However, this method involves higher SU(3) multiplets such as $\underline{27}$ and still very large SU(6) multiplets. Such complications seem unattractive just as long as such higher multiplets remain unobserved. Alternatively, one can interpret the higher mass hadron states (for L-excited meson states, see Sec. 1.5) in terms of relative angular momentum (having the characteristic of an orbital angular momentum) arising due to the relative motion of the constituent quarks.

For the interpretation of the spectroscopy of the low-lying baryon states in terms of the excitation of their internal motion it is convenient to describe the three quark state in terms of two internal coordinates (see Sec. 1.3)

$$\underline{\rho} = \frac{1}{\sqrt{2}}(\underline{r}_1 - \underline{r}_2) \qquad (1.14)$$

and

$$\underline{\lambda} = (2\underline{r}_3 - \underline{r}_1 - \underline{r}_2)\sqrt{6} \quad . \qquad (1.15)$$

These two vectors transform according to the mixed representation (2-dimensional) of the three object permutation group S_3. ρ refers to the two body subsystem of quarks 1 and 2 whilst λ describes the position of the third quark. The orbital angular momentum $\underline{L}_\rho$ and $\underline{L}_\lambda$ associated with these vectors add vectorially to give the total orbital angular momentum, $\underline{L} = \underline{L}_\rho + \underline{L}_\lambda$. The total angular momentum $\underline{J}$ and the net parity P for the system is then given by

$$\underline{J} = \underline{L} + \underline{S} \qquad (1.16)$$

$$P = (-1)^{L_\rho + L_\lambda} \qquad (1.17)$$

where $\underline{S}$ denotes the total Pauli spin for the three quarks.

For handling L-excitations of three quarks (rather than two), it is convenient to introduce a specific model for the interaction. qq interactions of harmonic form are often adopted and are the simplest ones for a number of reasons:

(i) The Hamiltonian is separable in the ρ and λ coordinates which is a great simplification [20].

(ii) Wavefunctions of proper symmetry can be obtained in a straightforward way by using the ground state (symmetric) and creation operators [36]. These wavefunctions have simple analytic forms and are convenient to work with.

iii) Harmonic forces provide quark confinement, as is desired by experiment up to date.

Baryon spectroscopy in a shell-model based on harmonic oscillator forces is reviewed in the next section.

1.3 Symmetric Quark Model for Baryons

Up till now we have mainly considered the lowest-lying hadronic states. As mentioned in Sec. 1.2, the higher mass states need the consideration of excitations of the quarks which constitute a hadron (baryon for the present). As a baryon is a three quark system, it is desirable to postulate a certain potential through which the quarks interact. The one which is the simplest and has met with considerable success is the simple harmonic oscillator potential.

For a single particle moving in a central three-dimensional oscillator potential, the Hamiltonian can be written in obvious notation [20] as follows:

$$H = p^2/2M + \tfrac{1}{2}M\omega^2 r^2 \quad . \tag{1.18}$$

The Schrodinger equation [13] can be solved for the eigenvalues of this operator which are given by

$$E = (\ell + 2k' + \frac{3}{2})\omega \qquad (1.19)$$

where ℓ is the orbital angular momentum quantum number and $k' = 0, 1, 2, \ldots$ gives the nodes in the radial part of the wavefunction. The energy spectrum of the one body harmonic oscillator is shown in Fig. 1.4.

The eigenstates of the above Hamiltonian are given as follows [12,13]

$$\psi_{n\ell m}(r) = N(\alpha r)^{\ell} L_{k'}^{\ell+\frac{1}{2}}(\alpha^2 r^2)\exp(-\frac{\alpha^2}{2} r^2)Y_{\ell}^{m}(\sigma,\phi) \qquad (1.20)$$

where $n = \ell + 2k'$, $\alpha^2 = M\omega$ and L is a Laguerre polynomial. N is the normalization constant given by

$$|N|^2 = \frac{2\alpha^3 k'!}{\sqrt{\pi}(k'+\ell+\frac{1}{2})(k'+\ell-\frac{1}{2}) \ldots \frac{3}{2}\cdot\frac{1}{2}} .$$

If the quarks interact via harmonic oscillator potentials then the baryons (three quark system) are eigenstates of the Hamiltonian

$$H = \sum_j \frac{p_j^{\,2}}{2M_Q} + \frac{1}{2} M_Q\omega^2 \sum_{i<j} (r_i - r_j)^2 . \qquad (1.21)$$

For the purpose of labelling the states, it is both customary and more convenient to use the related <u>shell model</u> Hamiltonian given by

$$H_{sm} = \sum_{j=1}^{3} \frac{p_j^{\,2}}{2M_Q} + \frac{1}{2}M_Q\omega^2 \sum_j r_j^{\,2} . \qquad (1.22)$$

Its eigenstates are products of three one body harmonic oscillator wavefunctions. The shell model Hamiltonian contains more states (than the harmonic oscillator Hamiltonian) corresponding to higher oscillations of the centre of mass (the three body harmonic oscillator Hamiltonian can be reduced to the centre of mass motion and the relative motion of the quarks). These higher oscillations are referred to as <u>spurious</u> and removed by enumerating the states with the centre of mass held in its ground state, (1s).

The complete wavefunctions are determined by combining the spatial wavefunctions with SU(6) states such that the total wavefunction is symmetric. And thus symmetric, anti-symmetric and mixed symmetric harmonic oscillator states are to be combined with $\underline{56}$, $\underline{20}$ and $\underline{70}$ of SU(6) respectively.

In calculations such as the evaluation of decay widths, one has to remember to use the eigenstates of H (1.21) in which case a baryon is equivalent to two harmonic oscillators' system, the energy levels and spins of baryons are obtained by taking the direct product of the two oscillators. Trace of the transformation properties (with respect to the permutation group S_3) of wavefunctions corresponding to the excited levels can be easily kept by using creation operators and their properties under permutations [36]. Orbital angular momentum (L) and parity of these states are also determined in this procedure. The physical spin J of a multiplet is then obtained by combining the orbital angular momentum L (of the L-excitation level with proper symmetry) with intrinsic quark spin S (i.e. J = L + S).

The spectrum obtained (with the shell model notation for the energy levels) is the following:

First level : $(1s)^3 \to \underline{56}$ $(L = 0^+)$

Second level : $(1s)^2(^1p) \to \underline{70}$ $(L = \bar{1})$

Third level :

$(1s)^2(2s), (1s)^2(1d), (1s)(1p)^2$ $\to \underline{56}$ $(L = 0^+)$

$\to \underline{70}$ $(L = 0^+)$

$\to \underline{56}$ $(L = 2^+)$

$\to \underline{70}$ $(L = 2^+)$

$\to \underline{20}$ $(L = 1^+)$

TABLE 1.2

Classification of the Pion-Nucleon resonances in the L-excitation quark model with parafermi statistics (from Ref. 37).

$(1s)^3$	$(1s)^2(1p)$	$(1s)^2(2s),(1s)^2(1d),\ (1s)(1p)^2$	
56 $L = 0^+$	70 $L = 1^-$	56 $L = 0^+$	56 $L = 2^+$
$\{8,\frac{1}{2}\}$ $P_{11}(939)$			
$\{10,\frac{3}{2}\}$ $P_{33}(1236)$		$\{8,\frac{1}{2}\}$ $P_{11}(1450)$	
	$\{8,\frac{1}{2}\}$ $D_{13}(1512)$		
	$S_{11}(1540)$		
	$\{10,\frac{3}{2}\}$ $S_{31}(1630)$		
	$S_{33}(1670)$		
		$\{10,\frac{3}{2}\}$ $P_{33}(1690)$	
	$\{8,\frac{3}{2}\}$ $D_{15}(1680)$		$\{8,\frac{1}{2}\}$ $F_{15}(1690)$
	$S_{11}(1710)$		$P_{13}(1850)$
	$D_{13}(1730)$		
			$\{10,\frac{3}{2}\}$ $F_{35}(1880)$
			$P_{31}(1960)$
			$P_{3}(2160)$
			$F_{37}(1940)$

TABLE 1.3

Existing octets. There is one more Ξ state at 1635 MeV whose quantum numbers are not confirmed. The singlets $\Lambda(1405)(J^P = \frac{1}{2}^-)$ and $\Lambda(1520)(J^P = \frac{3}{2}^-)$ are ascribed to the [70], L = $\overline{1}$ octets. (From Ref. 37).

J^P	N	Λ	Σ	Ξ
$\frac{1}{2}^+$	939	1115	1190	1320
$\frac{1}{2}^+$	1420	1745	1610	-
$\frac{3}{2}^-$	1512	1700	1660	1820
$\frac{1}{2}^-$	1540	1670?	1650	-
$\frac{5}{2}^-$	1680	1830	1765	-
$\frac{5}{2}^+$	1690	1815	1915?	2030
$\frac{1}{2}^-$	1710	1750??	1760	-
$\frac{3}{2}^-$	1730	2020??	1920?	-
$\frac{3}{2}^+$	1850	1860	-	-

TABLE 1.4

Existing Decuplets. (From Ref. 37)

J^P	Δ	Σ	Ξ	Ω
$\frac{3}{2}^+$	1238	1385	1530	1675
$\frac{1}{2}^-$	1630	1760	-	-
$\frac{3}{2}^-$	1670	-	-	-
$\frac{3}{2}^+$	1690	1700	-	-
$\frac{5}{2}^+$	1880	-	-	-
$\frac{7}{2}^+$	1940	2030	2250?	-

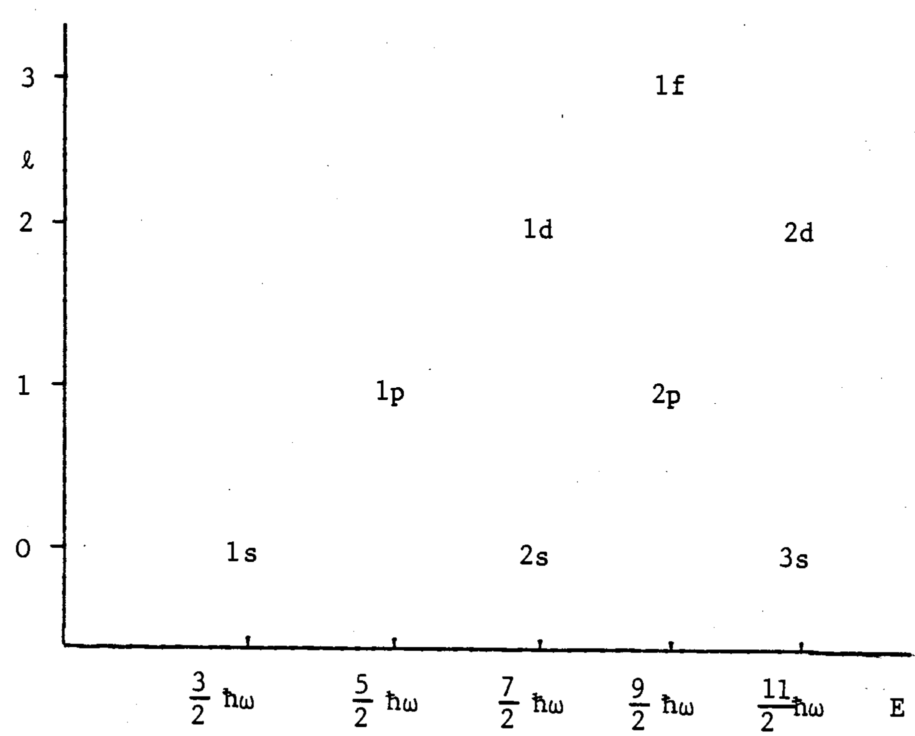


Fig. 1.4.

The energy spectrum of a three-dimensional harmonic oscillator potential.

As is apparent from Table 1.2, the third level yields an embarrassing profusion of states all of which are not observed. $\underline{56}$(L = 0^+) and $\underline{70}$ (L = 1^-), however, arise in a natural way as the ground and first excited states. These are the configurations that do seem to occur experimentally and all the required N^* resonances have been observed as shown in Table 1.2 [37]. The strange baryons also fit quite well and the position of the octets and decuplets is shown in Table 1.3 and Table 1.4 respectively [37].

1.4 Para-Fermi Statistics for Quarks

As can be noted, we have imposed a rule that the total states [SU(6) × O(3)] of baryons should be symmetric and thus obtain the so-called "symmetric quark model" [20] for baryons. It is because we obtain a nice description of the baryon spectrum by the imposition of this rule. Yet quarks are fermions (obey Pauli exclusion principle) and the total wavefunction of a system of fermions should be totally anti-symmetric with respect to the interchange of one another. Consequently, it seems as if quarks are curious objects. They are symmetrized in sets of three but otherwise anti-symmetric. One can consider para Fermi statistics [14] and impose the demand that physical particles are fermions or bosons - the quarks are then always bound - and so all physical three quark systems are totally symmetric.

Alternatively, one can make the three quark system totally anti-symmetric by introducing a new degree of freedom for the quarks. They can be painted red, yellow or blue. This RYB degree of freedom generates another SU(3) group called $[SU(3)]_{colour}$. Baryons can then be represented by

$$[SU(6) \otimes O(3) \otimes SU(3)_{colour}]\ \text{Anti-symmetric}$$

which demands that they should be colour singlets because a singlet of SU(3) is totally anti-symmetric. This is nice particularly because it does not affect the successes with the baryon spectrum. It is also useful in explaining the decay $\pi^o \to 2\gamma$ [15] and the experimental ratio of $R = \sigma_T(e^+e^- \to \text{hadrons})/\ \sigma(e^+e^- \to \mu^+\mu^-)$ [16].

1.5 The L-Excitation Model for Mesons

If we assume that the mesons have the simplest structure [17], namely $q\bar{q}$, the nine possible states can be displayed on a weight diagram (e.g. Fig. 1.1). Clearly there is an SU(3) octet and a singlet. The logical extension of this model is to interpret the higher meson states as rotational and/or vibrational excitations of the $q\bar{q}$ system. One can look upon the quark and anti-quark in a state of relative angular momentum L, giving rise to four nonets of parity $(-1)^{L+1}$, three of which have $S = 1$, $C = (-1)^{L+1}$ and $J = L+1, L, L-1$ and one which will have $S = 0$, $C = (-1)^L$ and $J = L$ (J is the total angular momentum which represents the physical spin of the particle). These states are usually denoted by the symbol ${}^{2s+1}L_J$ (borrowed from atomic spectroscopy), so that we have the nonets ${}^3L_{L+1}$, ${}^3L_L$, ${}^3L_{L-1}$ and ${}^1L_L$ each consisting of an SU(3) singlet and octet. Obviously, only certain combinations of J, P, and C are allowed. Thus the states

$$J^{PC} = 0^{--},\quad (\text{odd})^{-+},\quad (\text{even})^{+-} \tag{1.23}$$

are excluded and termed as exotic states. The assignment of mesons to the $L = 0$ multiplets (vector and pseudoscalar) is established by now. Possible assignments for $L = 1$ and $L = 2$ are given in Table 1.5 and Table 1.6 [17].

In Chapter 3 and Chapter 4, we assume that the harmonic oscillator model holds for mesons (that is, the quark anti-quark system interact

TABLE 1.5

Present status of N = 0 and 1 bands of the naive quark model (From Ref. 17).

Oscillator Quantum Numbers N	L	S	J^{PC}	Candidates I = 1	I = ½	I = 0	I = 0	Mixing (predominantly unmixed (U) or magically mixed (M))
0	0	0	0^{-+}	π(140)	K(494)	η(549)	η'(958)	U
0	0	1	1^{--}	ρ(770)	K^*(892)	ω(783)	ϕ(1020	M
1	1	0	1^{+-}	B(1235)	Q_B?	$[B_\eta]$	$[B'_\eta]$	?
1	1	1	2^{++}	A_2(1310)	K^*(1420)	f(1270)	f'(1514)	M
1	1	1	1^{++}	A_1?	Q_A?	D(1285)	E(1416)	?
1	1	1	0^{++}	δ(976)	κ(1250)	S^*(1000)	ε(∼1200)?	?

TABLE 1.6

Outline of the N = 2 (and higher) bands with some candidates (From Ref. 17).

Oscillator Quantum Numbers N	L	S	J^{PC}	Candidates I = 1	I = ½	I = 0	I = 0
2	0	0	0^{-+}	$[\pi^*]$	K^*?	$[\eta^*]$?	$[\eta'^*]$?
2	0	1	1^{--}	ρ^*(1600)	$[K^*_\rho?]$	$[\omega^*]$?	$[\phi^*]$?
2	2	0	2^{-+}	A_3?	L?	$[\eta_{A_3}]$?	$[\eta'_{A_3}]$?
2	2	1	3^{--}	g(1688)	K^*(1800)	ω_g(1670)	$[\phi_g]$?
2	2	1	2^{--}	[X]?	$[K_X]$?	$[\omega_X]$?	$[\phi_X]$?
2	2	1	1^{--}	$[\rho^{**}]$?	$[K^{**}]$?	$[\omega^{**}]$?	$[\phi^{**}]$?
	⋮						
3	3	1	4^{++}	-	-	h(2050)	-

via the harmonic oscillator potential) and apply it to the radiative decay widths of mesons. In order to accommodate the newly discovered mesons (ψ's and χ's etc.) we replace SU(3) by SU(4) for the unitary spin of particles.

1.6 Applications of Non-Relativistic Quark Model

The Quark Model [19] has been applied with considerable success to a large variety of processes, particularly those concerned with the electromagnetic properties of hadrons. The following review is meant to illustrate the application of the model to the estimation of decay rates and to understand the formula used for such processes (we shall use this formula for our calculations in Chapter 3 and Chapter 4).

The pioneering work in this topic (as far as we know) is due to Faiman et al. [20] and Becchi et al. [21]. Faiman et al. applied the model to the pionic and photonic decays of baryon resonances, while Becchi et al. employed it for the radiative decays of mesons.

1.7 Pionic Decays of Baryon Resonances

The basic tools which this particular model provides are the wavefunctions for baryons. Using these wavefunctions Faiman et al. [20] calculated the decay rates for the processes $N^* \to N\pi$ and $N^* \to \Delta\pi$. The main assumption of their calculations is that the emission of a pion takes place via a single quark de-excitation. The interaction which they use is the non-relativistic limit of the pion-nucleon interaction [22] and is given as follows:

$$\mathcal{H}_i^+ = \sqrt{2}\ \frac{f_q}{m} \sum_i \ (\underline{\sigma}_i \cdot \underline{k})\ \tau_i^+ \ \frac{1}{\sqrt{2E_\pi}}\ \exp(i\underline{k}\cdot\underline{x}_i) \qquad (1.24)$$

where summation is over the constituent quarks. f_q is the quark-pion coupling constant, m is pion-mass. k and E_π are the c.m.s. momentum and energy of the outgoing pion. Exponential $\exp(ik.x_i)$ comes from the eigenstate of the pion. $\frac{1}{\sqrt{2E_\pi}}$ is for normalization purposes. Since the wavefunctions for N^* and N (or Δ) are known, the amplitude M_{fi} for a decay process can be obtained to the lowest order in f_q. The decay rates can then be obtained by applying the Fermi Golden Rule [23]

$$\begin{aligned} d\omega &= 2\pi |M_{fi}|^2 \rho_f \\ &= 2\pi |M_{fi}|^2 \frac{dk_f}{(2\pi)^3 dE_f} \end{aligned} \tag{1.25}$$

for two body decays.

$\rho_f = \frac{dk_f}{(2\pi)^3 dE_f}$ is the phase space factor which can be obtained from the consideration of conservation of energy.

All the integrations involved can be done exactly by using the identity [24]

$$\int dr\, r^{\mu-1} J_\nu(k'r)\exp(-\alpha^2 r^2)$$

$$= \frac{\Gamma(\frac{1}{2}\nu + \frac{1}{2}\mu)(k'/2\alpha)^\nu}{2\alpha^\mu \Gamma(\nu + 1)} \times \exp(-\frac{k'^2}{4\alpha^2})\ {}^1F_1(\tfrac{1}{2}\nu - \tfrac{1}{2}\mu+1,\ \nu + 1;\ \frac{k'^2}{4\alpha^2}) \tag{1.26}$$

where the Bessel functions originate from the usual angular momentum expansion for $\exp(+ik.x_i)$ and ${}^1F_1(a, b; z)$ is a confluent hypergeometric function. The series expansion for ${}^1F_1(a, b; z)$ terminates if $\frac{1}{2}(\nu-\mu)$ is a negative integer.

For the unmixed states (i.e. N, Δ, D_{13}, D_{32}, S_{31}, F_{37} and F_{17}) there are two unknown parameters f_q^2 and α^2 (the harmonic oscillator constant). These can be estimated from the observed decay rates of any two of the unmixed states. With these values of f_q^2 and α^2, the decay rates of the remaining unmixed states can be found out. Their calculations yield

$$\alpha^2 = 0.1\ (\text{GeV})^2 \quad \text{and} \quad \frac{f_q^2}{4\pi} = 0.055 \tag{1.27}$$

All the calculated widths are close to the widths estimated by a phase-shift analysis. Considering that some of the wavefunctions that describe the resonances are completely different, the results are quite satisfactory.

There are two pairs of mixed states in $[70, L = \bar{1}]$ namely the two S_{11}'s and the two D_{13}'s. The physical states are defined as follows:

$$
\begin{aligned}
S_{11}(1710) &= \cos\theta_S\, S_{11}\{8, \tfrac{3}{2}\} + \sin\theta_S\, S_{11}\{8, \tfrac{1}{2}\} \\
S_{11}(1540) &= -\sin\theta_S\, S_{11}\{8, \tfrac{3}{2}\} + \cos\theta_S\, S_{11}\{8, \tfrac{1}{2}\}
\end{aligned} \qquad (1.28)
$$

and

$$
\begin{aligned}
D_{13}(1730) &= \cos\theta_D\, D_{13}\{8, \tfrac{3}{2}\} + \sin\theta_D\, D_{13}\{8, \tfrac{1}{2}\} \\
D_{13}(1512) &= -\sin\theta_D\, D_{13}\{8, \tfrac{3}{2}\} + \cos\theta_D\, D_{13}\{8, \tfrac{1}{2}\}\ .
\end{aligned} \qquad (1.29)
$$

Knowing f_q^2 and α^2, θ_S and θ_D can be estimated from the observed πN widths of $S_{11}(1540)$ and $D_{13}(1512)$. And then the decay rates of the remaining two states can be found. One gets two values for θ_S (i.e. $\theta_S = 35^o$ and $\theta_S = 90^o$). If we suppose that $\theta_S = 90^o$, the $S_{11}(1710)$ is mainly $\{8,\frac{1}{2}\}$ and $S_{11}(1540)$ is mainly $\{8, \frac{3}{2}\}$. This is the solution of Mitra and Rose [25] but it contradicts a selection rule deduced by Moorhouse [26] who showed that the N^* states originating from $\{8, \frac{3}{2}\}$ cannot be photoexcited from a photon. Experimentally, $S_{11}(1540)$ is strongly photoproduced, implying that it is not primarily $\{8, \frac{3}{2}\}$. There is no such contradiction with $\theta_S = 35^o$ so they used $\theta_S = 35^o$. Similarly one gets two values for θ_D ($\theta_D = 35^o$ and $\theta_D = 127^o$). They could use $\theta_D = 35^o$ on a similar type of reasoning. The results obtained with these values for the remaining two mixed states are not too bad.

1.8 Radiative Decays of Baryon Resonances

Faiman et al. [27] have used the harmonic oscillator model for baryons (described above) to estimate the radiative widths of the observed N^* resonances. Here the procedure of calculation is the same as above with the main assumption now being that the photon emission takes place via one quark de-excitation. They use the following interaction to calculate the matrix elements

$$\mathcal{H}_j = \sum_{j=1}^{3} \underline{J}_j \cdot \underline{A}(\underline{r}_j) = \sum_{j=1}^{3} q^{(j)} [-2ig\underline{S}^{(j)} \cdot (\underline{k} \times \underline{A}) + 2\underline{p}^{(j)} \cdot \underline{A}] \frac{e}{2M_Q} \quad (1.30)$$

where $eq^{(j)}$, $S^{(j)}$ and $p^{(j)}$ are the charge, spin and momentum of the j-th quark respectively. M_Q is the mass and g is the quark gyromagnetic ratio defined by $\mu = \frac{eg}{2M_Q}$, the quark scale magnetic moment which is taken equal to the proton magnetic moment. $\underline{A}$ is the electromagnetic field of the photon given by

$$\underline{A}(r_j) = \sqrt{4\pi} \sum_j \frac{\varepsilon}{\sqrt{2k_o}} \{a^+_{\underline{k}} \exp(-i\underline{k}\cdot\underline{r}_j) + a_{\underline{k}} \exp(i\underline{k}\cdot\underline{r}_j)\} \quad . \quad (1.31)$$

If we use the photon momentum k as the quantization axis and restrict ourselves to the righthanded photons for which $\underline{\varepsilon} = -\frac{1}{\sqrt{2}}(1, i, 0)$ then the above Hamiltonian can be simplified

$$\mathcal{H}_j = \sum_j a^{(j)} \mu_j \sqrt{4\pi} \frac{1}{\sqrt{k_o}} \exp(ikz^{(j)}) \left[kS_+^{(j)} - \frac{1}{g}(P_x^{(j)} + iP_y^{(j)}\right] . \quad (1.32)$$

And the radiative decay rates can be obtained (as for pionic decay) by applying the Fermi Golden Rule

$$d\omega = 2\pi |M_{fi}|^2 \rho_f \quad .$$

The only radiative decay width known with some degree of reliability at that time was for the process

$$\Delta^+(1236) \rightarrow p\gamma$$

with $\Gamma_\gamma = 0.65$ MeV. Using $\alpha^2 = 0.1(\text{GeV})^2$ (from pionic-decay of baryon resonances) and $T_\gamma = 0.65$ MeV, they calculated the parameter μ (i.e. $\mu = 0.18\ \text{GeV}^{-1}$). Many other decay rates have been estimated but there is very little data to compare those results with. One can also take the matrix elements of the interaction operator (considered above) between nucleonic states. This reproduces the well known ratio of the magnetic moments of the proton and neutron, i.e. $\mu^P/\mu^N = -\frac{3}{2}$.

1.9 Radiative Decays of Mesons

Becchi and Morpurgo [21] have considered the calculation of the decay widths for the processes $V \to \Pi + \gamma$ where V is a vector meson (lowest lying) and Π is a pseudoscalar meson (lowest-lying). In this section we shall review the main points of their calculations but before that it is desirable to describe the magnetic moments of hadrons (quarks) in the framework of the quark model (one needs the value of the quark magnetic moment for such applications).

1.10 Magnetic Moments of Quarks

Here, the main assumption is the one frequently employed in quark models, i.e. the assumption of additivity of certain properties of the constituents corresponding to an observable of a hadron. For example, the magnetic moment operator M_A of a hadron A can be written as

$$M_A = \sum_q M_q \qquad (1.33)$$

where the summation is over the constituent quarks. We suppose that the magnetic moment of the quark is proportional to its charge and write it as

$$M_q = \mu\left(\frac{e_q}{e}\right) \sigma_q \qquad (1.34)$$

where μ is a scale-parameter, adjusted to give the correct values of the proton and neutron magnetic moments. e_q is the quark charge and σ_q is the spin operator. As an example, we consider a proton which is constituted of two p-type and one n-type quarks. Its SU(6) state is given as follows:

$$|P, S_Z = \tfrac{1}{2}\rangle = \frac{1}{\sqrt{18}} [2|p\!\uparrow\! n\!\downarrow\! p\!\uparrow\rangle + 2|p\!\uparrow\! p\!\uparrow\! n\!\downarrow\rangle + 2|n\!\downarrow\! p\!\uparrow\! p\!\uparrow\rangle - |p\!\uparrow\! p\!\downarrow\! n\!\uparrow\rangle$$
$$- |p\!\uparrow\! n\!\uparrow\! p\!\downarrow\rangle - |p\!\downarrow\! n\!\uparrow\! p\!\uparrow\rangle - |n\!\uparrow\! p\!\downarrow\! p\!\uparrow\rangle - |n\!\uparrow\! p\!\uparrow\! p\!\downarrow\rangle - |p\!\downarrow\! p\!\uparrow\! n\!\uparrow\rangle]. \tag{1.35}$$

One can easily find out the expectation values of the above operator for such states. In particular one finds that

$$\mu_P = \mu, \qquad \mu_N = -\frac{2}{3}\mu, \qquad \mu_\Lambda = -\frac{1}{3}\mu \tag{1.36}$$

and the famous ratios:

$$\mu_P/\mu_N = -\frac{3}{2} \quad \text{and} \quad \mu_\Lambda/\mu_P = -\frac{1}{3} \tag{1.37}$$

which are in excellent agreement with experiments. For the decuplet one finds a general formula

$$\mu_A = Q_A\mu \tag{1.38}$$

where Q_A is the charge of the hadron A. Similarly one can find the magnetic moments of the [illegible] vector mesons.

From this discussion one can estimate the g-value for a quark [3]. As is known, the quark magnetic moment can also be written as

$$M_q = g\frac{e_q}{2em_q} S \quad . \tag{1.39}$$

Also $M_q = \mu(\frac{e_q}{e})\,\sigma_q$ (Eq. 1.34)

so that $g = (2.79)(\frac{m_q}{m_P})$

where $\mu = \mu_P = 2.79\,\frac{e}{2m_P}$ has been used.

So to know about the g-value of the quark, we need the value of its mass m_q. And it, in turn, demands the nature of the potential between the quarks [29] constituting a hadron. If we suppose that the potential between the quarks is of the type of the fourth component of a four-potential then the quark behaves just like a free particle of mass $m_q \geqslant 4$ GeV which gives $g \geqslant 12$. It shows a highly anomalous magnetic moment for the quarks. But if we suppose that the potential is a Lorentz scalar type and of depth U_o, then it can be shown that the effective mass of the quark reduces, i.e. $m_q \rightarrow m_q^* - U_o$. If we also require that the depth of the potential is such that it produces the observed masses of the bound system then

$$m_q^* \simeq \frac{m_M}{2} \simeq \frac{m_B}{3} \simeq 400 \text{ MeV} \qquad (1.40)$$

where m_M and m_B are the average masses of the 35-plet of mesons and 56-plet of the baryons respectively. And in this case we can choose the length of the potential quite large and thus the non-relativistic motion of the quarks is also assured. Moreover, the value of g becomes

$$g \leqslant 1 .$$

The value $g = 1$ as we will see later is favoured by electromagnetic decays of hadrons and also by the photoproduction of pions off the nucleons. In the calculations of the radiative decay rates of charmonium (Chapter 3) and the radiative decays of ordinary mesons (containing $c\bar{c}$ content, i.e., ω, ϕ, η, η' etc.), we take $g = 1$ for all the quarks but use a smaller magnetic moment for the charmed quark (i.e. $\mu_c = \frac{m_u}{m_c} \mu_P$) where m_u and m_c are the masses of the ordinary quark (u-type quark) and the charmed quark respectively (see below).

The magnetic moments of the proton and neutron determine the magnetic moments of the p-type and n-type quarks. They do not, of course, give any information on the magnetic moment of the strange quark. But in the applications to follow, the assumption is made that the third quark, too, has a magnetic moment proportional to its charge with the same constant of proportionality as for p-type and n-type quarks. Thus we have

$$M_q \equiv \mu_q = \sum_q \mu_p \left(\frac{e_q}{e}\right) \sigma_q \qquad (1.41)$$

with $q = 1, 2, 3$.

Now it is straightforward to calculate the radiative widths [28] of the vector mesons, i.e. the rates for the processes

$$V \rightarrow \Pi + \gamma \quad ,$$

Becchi and Morpurgo consider the following transitions

1) $\omega \rightarrow \pi^o + \gamma$ 2) $\omega \rightarrow \eta + \gamma$

3) $\rho \rightarrow \pi + \gamma$ 4) $\rho^o \rightarrow \eta + \gamma$

5) $K^{*+} \rightarrow K^+ + \gamma$ 6) $K^{*o} \rightarrow K^o + \gamma$

1.11 Method of Calculations

The non-relativistic form of interaction which they use is the one which arises from the interaction of orbital magnetic moment and spin magnetic moment with the electromagnetic field and can be written (equivalently) as given by (1.30)

$$\mathcal{H}_j = \sum_{j=1}^{2} q^{(j)} [-2ig\underline{S}^{(j)} . (\underline{k} \times \underline{A}) + 2\,\underline{p}^{(j)} . \underline{A}] \frac{e}{2m_q} \quad . \qquad (1.42)$$

For $L = 0 \rightarrow L = 0$ processes it takes the following form (from 1.32)

$$\mathcal{H}_j = \sum_j q^{(j)} \sqrt{4\pi}\, \mu_p\, S_+^{(j)} k \exp(ikZ^{(j)}) \frac{1}{\sqrt{k_o}} \quad . \qquad (1.43)$$

In (1.43) the direction of k is taken as Z-axis and right-handed ($\underline{\varepsilon} = -\frac{1}{\sqrt{2}}$ (1, i, 0)) photon is used. Becchi and Morpurgo neglect the symmetry-breaking and take the same spatial wavefunctions for the V-mesons and P-mesons (the vector nonet and the octet of pseudoscalars belong to the same $\underline{35}$-plet of SU(6)). Thus the states they use look like, for example,

$$\pi^0 = \tfrac{1}{4}[\bar{p}\uparrow p\downarrow - \bar{p}\downarrow p\uparrow - \bar{n}\uparrow n\downarrow + \bar{n}\downarrow n\uparrow]f(r)$$

$$\eta = \frac{1}{\sqrt{12}}[2\bar{\lambda}\uparrow\lambda\downarrow - 2\bar{\lambda}\downarrow\lambda\uparrow - \bar{p}\uparrow p\downarrow + \bar{p}\downarrow p\uparrow - \bar{n}\uparrow n\downarrow + \bar{n}\downarrow n\uparrow]f(r) \text{ etc. etc.} \quad (1.44)$$

In the long-wavelength approximation, i.e. $kr << 1$ (r is of the order of the dimension of the bound system) one can (presumably) take

$$\int f^{*A}(r)\ f^{A'}(r)\ \exp(i\underline{k}.\underline{r})dr = 1 \quad (1.45)$$

And the calculation of the matrix elements reduces to the matrix elements of $\mathcal{H}_j$ between the SU(6) states (spin and unitary spin states). Applying the usual <u>Fermi Golden Rule</u>

$$\omega = 2\pi|M_{if}|^2\ \rho_f$$

one can find the decay rates by averaging over the initial spin states and summing over the two states of photon polarization and multiplying by a proper phase-space factor [30]. They find in particular

$$\Gamma(\omega\to\gamma\pi^0) = 1.17 \text{ MeV}, \quad \Gamma(K^{*o} \to K^o\gamma) = 2.8 \times 10^{-1} \text{ MeV}$$

$$\Gamma(\phi\to\gamma\eta) = 3.04 \times 10^{-1} \text{ MeV}, \quad \Gamma(\phi \to \gamma\pi^0) \simeq 0$$

$$\text{and} \quad \Gamma(\rho\to\gamma\pi^0) = 1.2 \times 10^{-1} \text{ MeV}$$

It was only the first decay $(\omega \to \gamma\pi^0)$ which was known experimentally at that time and has quite good agreement with the calculated value, $\Gamma(\omega \to \gamma\pi^0) = 1.17$ MeV. But the remaining values for decay widths quoted above are very large and in contradiction with recent results.

In Chapter 4, we recalculate these and many other decays (some involve both the new and old mesons) within the framework of a harmonic oscillator model for mesons. The inclusion of the harmonic oscillator wavefunctions for V-mesons and P-mesons (without any approximation with $\exp(i\underline{k}.\underline{r}_i)$ and of course, with an added assumption about the magnetic moment of the charmed quark) improve the situation.

1.12 Some Ambiguities

1) The iso-singlets ω, ϕ, η and η' are generally taken as mixtures of the singlet $\underline{1}$ and octet $\underline{8}$ of SU(3). Thus ω, for example, comes out as [31]

$$\omega = \frac{1}{\sqrt{2}} [p\bar{p} + n\bar{n}]$$

and

$$\phi \simeq \lambda\bar{\lambda} \quad . \tag{1.46}$$

This is obtained by diagonalizing the mass-matrix so as to fit the masses within a multiplet. Similarly the Gell-Mann-Okubo mass formula demands that η should be taken as a pure octet state

$$\eta = \frac{1}{\sqrt{6}} [p\bar{p} + n\bar{n} - 2\lambda\bar{\lambda}] \tag{1.47}$$

Becchi and Morpurgo suggest that as there is no clear justification for the quadratic mass formula for bosons the above structures are probable but not definite and instead takes the structure

$$\omega = (2 + \lambda_\omega^2)^{-\frac{1}{2}} [p\bar{p} + n\bar{n} - \lambda_\omega \lambda\bar{\lambda}]$$

$$\eta = (2 + 2\lambda_\eta^2)^{-\frac{1}{2}} [p\bar{p} + n\bar{n} - \lambda_\eta \lambda\bar{\lambda}] \tag{1.48}$$

so the choice $\lambda_\omega = 0$, $\lambda_\eta = 2$ may be probable but not definite. They calculate the matrix elements with these structures but have to use the standard values $\lambda_\omega = 0$, $\lambda_\eta = 2$ in absence of anything better.

2) Another ambiguity which arises in these calculations concerns the phase-space factor. Some of the outgoing particles (other than the photon) are quite relativistic. Becchi and Morpurgo suggest that one should either take the relativistic interaction term or alternatively use the relativistic phase space factor. The phase space factor $2\pi\rho_f$ in its relativistic form is given by

$$2\pi\rho_f = \frac{k^2}{\pi} \frac{\omega_k}{m_V} \quad (1.49)$$

where k is the energy of the photon, ω_k the energy of the pseudoscalar meson and m_V the mass of the decaying vector meson. In a non-relativistic case $\frac{\omega_k}{m_V} \simeq 1$ but physical values give $\frac{\omega_k}{m_V} \simeq \frac{1}{2}$, showing that k is not at all small. In relativistic vertex calculations, the factor $\frac{\omega_k}{m_V}$ does not appear because one does not know the dependence of the vertex on the masses of the particles involved. Thus, they take $\frac{\omega_k}{m_V} = 1$, more or less as a prescription, with an appeal to the relativistic calculations where this factor does not appear. Thus the above calculations are relativistically true, provided one puts the factor $\frac{\omega_k}{m_V} = 1$ and are termed in the literature Quasi-Relativistic calculations. We find this prescription favourable for our calculations in Chapter 3 (the radiative decay widths of charmonium states) and Chapter 4 (the radiative decay widths of the ordinary mesons). The factor $\frac{\omega_k}{m_V}$ has not considerable effect on decay rates of charmonium states (except $\psi \to \gamma X(2.8)$) but the decay widths of the old mesons are decreased by a factor of almost half (because $\frac{\omega_k}{m_V} \simeq \frac{1}{2}$).

1.13 Single Pion-Photoproduction in the Quark Model

Copley et al. [32] have employed the quark model (described in the previous pages) to calculate the resonance contribution to the total and backward differential cross-sections for single pion photoproduction off nucleons

$$\gamma N \to N^* \to \pi N \ .$$

The basic assumption here is the usual one, that is, that the resonant state is obtained by a photo-excitation of a single quark in the nucleon. The interaction responsible for the excitation (in obvious notation) is the following

$$\mathcal{H}_j = \sum_j \underline{J}_j \cdot \underline{A}(\underline{r}_j) = \sum_{j=1}^{3} q^j [-2ig\, \underline{S}^j \cdot (\underline{k} \times \underline{A}) + 2\underline{p}^{(j)} \cdot \underline{A}] \frac{e}{2m_q} \quad (1.50)$$

$\mu = {}^{eg}/2m_q$ is the quark scale magnetic moment which is put equal to the proton magnetic moment, i.e. $\mu = 0.13\ (\mathrm{GeV})^{-1}$. $\underline{A}$ is the electromagnetic field of the photon which is given by

$$\underline{A}(\underline{r}_j) = \sqrt{4\pi} \frac{1}{\sqrt{2k_o}} \underline{\varepsilon} [a^+_{\underline{k}} \exp(i\underline{k}\cdot\underline{r}^{(j)}) + a_{\underline{k}} \exp(-i\underline{k}\cdot\underline{r}^{(j)}] . \quad (1.51)$$

If we take the direction of the photon as the Z-axis and restrict ourselves to the right-handed photons $(\underline{\varepsilon} = -\frac{1}{\sqrt{2}}(1,i,0))$, then the above Hamiltonian can be simplified as

$$\mathcal{H}_j = \sum_j q^{(j)} \mu_j \sqrt{4\pi} \frac{1}{\sqrt{k_o}} \exp(ikZ^{(j)}) [k\, S_+^{(j)} - \frac{1}{g}(p_x^{(j)} + ip_y^{(j)})] \quad (1.52)$$

As right-handed photons have helicity +1, the photo-excitation of a given resonance can be completely specified by defining the matrix elements of the above interaction between

a) the initial nucleon in a state with spin projection $m = -\frac{1}{2}$ and final resonance with $m = +\frac{1}{2}$.

b) the initial nucleon with $m = +\frac{1}{2}$ and the resonance with $m = \frac{3}{2}$ (for resonance with $J \geqslant \frac{3}{2}$). These matrix elements can be conveniently thought of as helicity amplitudes for the excitations of the nucleon into resonance states with helicity $\frac{1}{2}$ and $\frac{3}{2}$ respectively and are denoted by $A_{\frac{1}{2}}$ and $A_{3/2}$. And the contribution of the resonance to the total cross-section σ_T for a single pion production (as an example $\gamma p \to \pi^o p$) can be written as

$$\sigma_T = \begin{pmatrix} \frac{1}{3} \\ \frac{2}{3} \end{pmatrix} \frac{m_N}{m_R} \frac{x}{\Gamma} \{ |A_{\frac{1}{2}}|^2 + |A_{3/2}|^2 \} \quad (1.53)$$

where x and Γ are the elasticity and total decay width of the resonance and the factors $\frac{1}{3}$ and $\frac{2}{3}$ correspond to the resonances with isospin $I = \frac{1}{2}$ and $I = \frac{3}{2}$ respectively. Similarly for π^o-production in the backward direction

$$[\frac{d\sigma}{d\Omega}]_{\theta=\pi} = \begin{pmatrix} \frac{1}{3} \\ \frac{2}{3} \end{pmatrix} \frac{2J_R+1}{4\pi} \quad \frac{x}{T} \quad \frac{m_N}{m_R} |A_{\frac{1}{2}}|^2 \tag{1.54}$$

where J is the spin of the resonance.

From the expressions of $A_{\frac{1}{2}}$ for different resonance one can notice that cancellation between the contributions from the spin and the orbital parts of interaction can occur for a number of them. In particular, one finds that for photo-excitation from protons, the possibility of cancellation occurs for $D_{13}(1520)$, $S_{11}(1550)$, $F_{15}(1688)$, $P_{13}(1855)$, $D_{33}(1690)$ and $S_{31}(1640)$. The unknown parameters can be found by taking any two of these resonances. It is known experimentally that the contributions of $D_{13}(1520)$ and $F_{15}(1680)$ to backward photoproduction of π^o off protons is negligibly small. Copley et al. exploit this fact and put the corresponding matrix elements equal to zero, i.e. the amplitudes

$$A_{\frac{1}{2}}(D_{13}) = i\,\frac{2}{3}\,\frac{\sqrt{\pi}}{\sqrt{k}}\,\alpha\mu\,\exp\left(\frac{-k^2}{6\alpha^2}\right)\left[\frac{k^2}{\alpha^2} - \frac{1}{g}\right]$$

and

$$A_{\frac{1}{2}}(F_{15}) = -\,\frac{2}{5}\,\sqrt{\pi k}\,\mu\,\exp\left(\frac{-k^2}{6\alpha^2}\right)\left[\frac{k^2}{2\alpha^2} - \frac{1}{g}\right] \tag{1.55}$$

vanish if

$$k^2(D_{13}) = \tfrac{1}{2}k^2(F_{15}) \quad . \tag{1.56}$$

Experimentally, $k^2(D_{13}) = 0.22\ (\text{GeV})^2$

$k^2(F_{15}) = 0.34\ (\text{GeV})^2$

which are sufficient to make both of the amplitudes small for suitable values of α and g. Using $g = 1$ and the data for $F_{15}(1690)$, they obtain $\alpha^2 = 0.17\ (\text{GeV})^2$. These values can now be used to determine completely the resonant contributions to the total cross-section in the backward direction.

The agreement with experiment is good, particularly for $I = \frac{1}{2}$ resonances. For example, the quark model result for $D_{13}(1520)$ is $4.9 \times 10^{-2}\,\mu\text{b Sr}^{-1}$ while Walker's phase shift analysis [33] gives the value 8×10^{-2} μb Sr^{-1}. The agreement with experiment for total cross-section is also quite good, e.g. quark model result for $\sigma_T(\gamma p \rightarrow \pi^o p)$ ($D_{13}(1520)$ being the intermediate state) is 12.2 μb while Walker's analysis gives the value $\sigma_T = 16.6$ μb.

The only exception where the agreement with experiment is not good is the resonance $S_{11}(1550)$. This can be attributed to the fact that they assume no mixing between the quark spin $\frac{1}{2}$ and $\frac{3}{2}$ resonances. The result is improved if such mixing is considered. They have also calculated the contributions of $I = \frac{3}{2}$ resonances to the backward differential cross-section $(\gamma p \rightarrow \pi^o p)$. There too, the quark model predictions are of correct order of magnitude.

The <u>Naive Quark Model</u> (SU(6) quark model) has also been applied to weak processes [34], collision problems [3] , electromagnetic form factors of hadrons [35], and predictions and relations among the masses of hadrons [3].

In the next chapter we review briefly some of the techniques employed for prediction of masses and the mass-mixing problem (among particles of the same unitary spin) which arises due to the symmetry-breaking.

CHAPTER 2

MASSES OF PARTICLES IN THE QUARK MODEL

2.1 Introduction

Another successful application of the quark model and unitary symmetry is to obtain mass formulae expressing relations among the masses of particles which can be tested experimentally. In this chapter we review the different techniques which can be employed to get these relations. For the sake of simplicity we take SU(3) (for the unitary spin of particles) and its extension to SU(4) (the maximum group we use in this thesis is SU(4)). In principle the technique can be extended to SU(n) (n > 4). As we shall see in the following sections, it is the symmetry-breaking (not the exact symmetry) which enables us to make such predictions.

2.2 Symmetry-Breaking

The large difference between the masses of the particles within a supermultiplet (of SU(3)) shows that SU(3) is not an exact symmetry. Thus the strong interactions can be thought of as made up of two parts: a "very strong" interaction which is invariant under SU(3) and a "medium strong" interaction which breaks SU(3). In the absence of symmetry-breaking all the masses within a multiplet should be the same in the same way as the members of an iso-spin multiplet are assumed to be degenerate in the absence of the electromagnetic interaction. The strong interaction Hamiltonian H_{st} can be expressed as

$$H_{st} = H_{vs} + H_{ms} \qquad (2.1)$$

where H_{vs} (very strong interaction Hamiltonian) commutes with all the SU(3) generators

$$[H_{vs}, F_i] = 0 \qquad (2.2)$$

but

$$[H_{ms}, F_i] \neq 0 \qquad (2.3)$$

Gell-Mann [9c] postulated that H_{ms} (medium strong interaction Hamiltonian) transforms very simply under SU(3) transformations. This situation is analogous to the Normal Zeeman Effect in atomic Physics. If we neglect the spin of an electron (consider H atom for simplicity), the energy level E with orbital angular momentum L is (2L+1) - degenerate as a consequence of perfect spherical symmetry. The application of a weak magnetic field $\underline{B}$ in the Z-direction introduces an additional interaction energy H', given by

$$H' = \frac{e\hbar}{2mc} BL_Z = KL_Z . \qquad (2.4)$$

Obviously, H' is not invariant under the full rotation group O(3) but it is invariant under the rotations about the Z-axis. In first order perturbation theory the expectation value of H' is given as

$$\delta E_{Lm} = \langle\psi_{Lm}|H'|\psi_{Lm}\rangle = Km \qquad (2.5)$$

The perturbation is diagonal in the basis ψ_{Lm}, so degenerate perturbation theory is not required.

In strong interactions isospin and hypercharge (the two diagonal generators of SU(3)) are conserved so that

$$[Y, H_{ms}] = 0$$

$$[I_3, H_{ms}] = 0 \quad \text{and} \quad [I_\pm, H_{ms}] = 0. \qquad (2.6)$$

Gellmann suggested that H_{ms} should transform like one of the generators of SU(3) (c.f. Normal Zeeman Effect where L_Z is one of the generators of O(3)). One finds at once that the only generator out of SU(3) eight generators which commutes with Y, I_3 and $I_\pm$ is Y itself. This led to the postulate that H_{ms} transforms like the hypercharge operator Y.

2.3 U-Spin

We start by reviewing the U-spin technique to find relations among the masses of particles classified according to SU(3) multiplets. This technique was initiated and developed by Lipkin [38]. Like the isospin submultiplets one can also classify the particles of a supermultiplet in U-spin submultiplets. The U-spin generators U_3 and $U_\pm$ obey the same algebra as the I-spin generators. Hence all the apparatus of isospin can be carried over to U-spin. The particles of a U-spin multiplet have the same charge just as the particles of an isospin multiplet have the same hypercharge. The two labelling schemes (U, U_3, Q) and (I, I_3, Y) can be easily related. We give below the U-spin submultiplets of the octet (baryons or mesons) and the decuplet of baryons. Thus in the baryon octet p and Σ^+ have $(Q, U_3) = (+1, \frac{1}{2})$ and $(+1, -\frac{1}{2})$ respectively forming a U-spin doublet. n and Σ^o correspond to (0, +1) and (0, -1). The states at the origin (Σ^o, Λ^o) have definite total U and are superpositions of the states Σ^o and Λ^o of definite I. These superpositions are denoted by Σ^o_U and Λ^o_U according as U = 1 or 0 and are given as

$$\Sigma^o_U = \tfrac{1}{2}|\Sigma^o\rangle + \frac{\sqrt{3}}{2}|\Lambda^o\rangle \qquad (2.7a)$$

$$\Lambda^o_U = \frac{\sqrt{3}}{2}|\Sigma^o\rangle - \tfrac{1}{2}|\Lambda^o\rangle \qquad (2.7b)$$

In (7b) the position of the negative sign is arbitrary. These results are true for any octet. For a triangular supermultiplet such as the decuplet 10, there is no multiple occupancy of weights and the states can be regrouped into U-spin multiplets directly as follows:

$$\begin{array}{lll} U = 0 & Q = 2 & \Delta^{++} \\ U = \frac{1}{2} & Q = 1 & \Delta^{+}, \Sigma^{*+} \\ U = 1 & Q = 0 & \Delta^{o}, \Sigma^{*o}, \Xi^{*o} \\ U = \frac{3}{2} & Q = -1 & \Delta^{-}, \Sigma^{*-}, \Xi^{*-}, \Omega^{-}. \end{array} \qquad (2.8)$$

The generators of I-spin and U-spin groups are related as follows:

$$U_3 = -\tfrac{1}{2}I_3 + \tfrac{3}{4}Y$$

$$Q = I_3 + \tfrac{1}{2}Y$$

$$\therefore \quad Y = U_3 + \tfrac{1}{2}Q \qquad (2.9)$$

As it is postulated that H_{ms} transforms like Y and hence like a superposition of U_3, a component of a $U = 1$ vector in the U-spin space and of Q which commutes with the generators of U-spin group (U-spin scalar); it can be written as

$$H_{ms} = H_{ms}^{(s)} + H_{ms}^{(v)} \quad . \qquad (2.10)$$

Now the first order perturbation theory can be applied in a straightforward way to find the contribution to the masses due to H_{ms}. The expectation value of a U-spin scalar $H_{ms}^{(s)}$ does not depend on U_3 and gives

$$\langle U, U_3 \mid H_{ms}^{(s)} \mid U, U_3 \rangle = a \quad . \qquad (2.11)$$

The constant a varies with U, of course. The expectation value of $H_{ms}^{(v)}$ is proportional to U_3 and is given as

$$\langle U, U_3 \mid H_{ms}^{(v)} \mid U, U_3 \rangle = bU_3 \quad . \qquad (2.12)$$

The constant b varies with U (like a) and both a and b characterize a supermultiplet considered. With these preliminaries one easily obtains for the octet (say baryons) the relation

$$\tfrac{1}{2}(N + \Xi) = \tfrac{1}{4}(\Sigma + 3\Lambda) \qquad (2.13)$$

The charge labels can be omitted in the absence of electromagnetic interactions which split the masses within an iso-multiplet.

Relation (2.13) is the well-known Gellman-Okubo (G.M.O.) mass formula and should hold for any octet. For the case of $\tfrac{1}{2}^+$-baryons, the agreement with experiment is astonishingly (bearing in mind the use of the first order perturbation theory) good. In the case of 0^--mesons the agreement is not so good. Feynman noted that it can be improved if one uses squared masses in place of linear ones. Then one has

$$m_K^2 = \tfrac{1}{4}(m_\pi^2 + 3m_\eta^2) \quad , \tag{2.14}$$

since $m_{K^-} = m_K$ by CPT invariance and $\eta(549)$ is assigned to the iso-singlet contained in the octet. The mass formula for the decuplet is obtained in a straightforward way and one obtains the well-known equal spacing rule for decuplets

$$\Delta - \Sigma^* = \Sigma^* - \Xi^* = \Xi^* - \Omega \tag{2.15}$$

Ω^- was not known to exist when this relation was first obtained. The masses of Δ, Σ^* and Ξ^* were in good agreement with this formula. The mass of Ω^- was predicted to be 1680 MeV. The subsequent discovery [39] of a hypercharge minus two hyperon with a mass of 1686 ± 12 MeV provided convincing evidence in favour of SU(3) symmetry with the broken SU(3) scheme in which the symmetry breaking Hamiltonian transforms like the hypercharge operator. In seeking the assignment of 1^--mesons in the mass range 700-900 MeV to an octet one finds the $\rho(770)$ $(I = 1, Y=0)$ K^*, K^{*-} $(I = \frac{1}{2}, Y = \pm 1)$ but two candidates $\phi(1019)$ and $\omega(784)$ for the $I = Y = 0$ state. There are no other 1^--mesons in this mass range, so the simplest hypothesis is that we have an octet and a singlet. To see which of ω and ϕ is the octet member one can use the G.M.O. formula to predict the mass of the $I = 0$, $Y = 0$ state. One finds a value of 930 MeV which does not agree well with either ω or ϕ and this indicates a mixing between the two iso-singlets.

An explanation of this discrepancy can be given by considering the mixing of particle states with the same quantum numbers. As we think this to be a very useful exercise, we consider it in detail in the following section. In Chapter 4, we consider the mixing among the isoscalars belonging to the (15+1)-plets of $L = 0$ vectors, $L = 0$ pseudoscalars and $L = 1$ $(J = 2)$ mesons.

2.4 Mass-Mixing of Particles

Mass-mixing is another interesting application of symmetry breaking. We are familiar with a similar problem in elementary Quantum Mechanics where one considers the Stark effect to find the perturbation corrections to the levels of hydrogen energy spectrum. Such corrections are introduced through the symmetry breaking interaction $H' = eEZ$ (where e and E stand for the electronic charge and electric field respectively) which breaks the spherical symmetry of the hydrogen atom. As H' is invariant under rotations about the Z-axis, the states with the same m-values (corresponding to the principle quantum number n) get mixed and the degeneracy is decreased.

SU(3) symmetry breaking can be treated in a similar way. In the absence of SU(3)-breaking forces the nine vector mesons $(\underline{3} \otimes \underline{\bar{3}} = \underline{8} + \underline{1})$ are degenerate or nearly degenerate (the octet-singlet mixing is also suggested by their existence within a $\underline{35}$ of SU(6)).

In the absence of electromagnetic interactions we have got the following states (1^--mesons) to consider

$$\rho \equiv \phi^{(1)}, \quad K^* \equiv \phi^{(2)}, \quad K^{*-} \equiv \phi^{(3)}$$

$$\omega^o \equiv \phi^{(4)}, \quad \phi = \phi^{(5)}$$

The labelling for ϕ and ω can be interchanged. As H_{ms} transforms like Y for which

$$[Y, I^2] = 0, \quad [Y, I_3] = 0, \quad [Y, Y] = 0, \qquad (2.16)$$

the states with the same I^2, I_3 and Y values will get mixed in the presence of this interaction. So that ϕ and ω will be mixed and we have to think about diagonalizing a 3×3 matrix in the first inspection.

Applying the formula of degenerate perturbation theory [23]

$$\sum_i \alpha_i \langle\phi_n^{\,j} \mid H_{ms} \mid \phi_n^{(i)}\rangle = E_n^{(1)} \alpha_j \qquad (2.17)$$

where i and j stands for ρ, ϕ and ω. One gets the following equations:

$$\alpha_1 <\phi^{(1)}|H_{ms}|\phi^{(1)}> + o + \qquad o = \alpha_1 E^{(1)}$$

$$o \qquad + \alpha_2 <\phi^{(4)}|H_{ms}|\phi^{(4)}> + \alpha_3 <\phi^{(4)}|H_{ms}|\phi^{(5)}> = \alpha_2 E^{(1)}$$

$$o \qquad + \alpha_2 <\phi^{(5)}|H_{ms}|\phi^{(4)}> + \alpha_3 <\phi^{(5)}|H_{ms}|\phi^{(5)}> = \alpha_3 E^{(1)} \qquad (2.18)$$

The diagonalization of the 3×3 matrix is actually reduced to that of a 2×2 matrix. In terms of the squared matrix elements one has to diagonalize the following matrix:

$$\begin{pmatrix} m_{88}^2 & m_{81}^2 \\ m_{18}^2 & m_{11}^2 \end{pmatrix} \qquad (2.19)$$

a and b in m_{ab}^2 refer to the SU(3) representations, the octet and the singlet in the case under consideration. This matrix is hermitian and by the choice of phases of the states can be made real and hence symmetric, so that $m_{81}^2 = m_{18}^2$. The eigenvalues of the above matrix can be found out easily ($\because$ $m_\omega < m_\phi$) as follows:

$$\left.\begin{matrix} m_\phi^2 \\ m_\omega^2 \end{matrix}\right\} = \tfrac{1}{2}\{(m_{88}^2 + m_{11}^2) \pm [(m_{88}^2 - m_{11}^2)^2 + 4m_{81}^4]^{\frac{1}{2}}\} \quad . \qquad (2.20)$$

Let the corresponding orthogonal and normalized vectors be

$$\begin{pmatrix} \cos\theta \\ \sin\theta \end{pmatrix} \quad \text{and} \quad \begin{pmatrix} -\sin\theta \\ \cos\theta \end{pmatrix}$$

Then

$$\tan\theta = \frac{m_\phi^2 - m_{88}^2}{m_{81}^2} = \frac{m_{81}^2}{m_{88}^2 - m_\omega^2} \qquad (2.21)$$

and the perturbed physical states are then given by

$$|\phi> = \cos\theta|8> + \sin\theta|1> \qquad (2.22)$$

$$|\omega> = -\sin\theta|8> + \cos\theta|1> \qquad (2.23)$$

θ is called the mixing angle.

To apply these results to the 1^--mesons we suppose that the mass splitting in the octet is taken into account first so that m_{88}^2 is related to m_ρ^2 and m_K^2 by the G.M.O. mass formula

$$m_{88}^2 = \frac{1}{3}(4m_K^2 - m_\rho^2)$$

so that

$$m_{88} = 930 \text{ MeV}$$

m_{11}^2 and m_{81}^2 (or θ) is found from the observed ω- and ϕ-meson masses and are found as

$$m_{11} = 888 \text{ MeV}, \quad m_{81} = 456 \text{ MeV}, \quad \cos\theta = 0.7685.$$

$$\theta = 39.78^{o} .$$

Hence

$$|\phi> = 0.76|8> + 0.63|1>$$

$$|\omega> = -0.63|8> + 0.76|1> .$$

Thus, ϕ-meson has nearly 60% probability of being found in octet and 40% probability of being found in the singlet.

The mass diagonalization problem for the pseudoscalar particles $(\eta, \eta'(X^{o}))$ can be tackled on similar lines and one finds $\theta \simeq -11^{o}$ [37]. So to a good approximation, η and η' can be considered as a pure octet and a pure singlet respectively.

The U-spin technique which is familiar in the context of SU(3) symmetry has been extended by Kazi et al. [40,43] to SU(4). They have derived mass formulas for SU(4) multiplets which are the generalization of the mass formulas for SU(3) multiplets (like the well-known Gell-Mann-Okubo mass formula etc.).

2.5 Irreducible Tensor Operators - Wigner-Eckart Theorem

The results obtained by U-spin techniques (in the previous sections) can be reproduced exactly by postulating the transformation properties of the medium strong interaction with respect to a special unitary group and application of the Wigner-Eckart theorem for the group. We fill in the steps in the following for $\frac{1}{2}^{+}$ - baryons in order to illustrate the use of Wigner-Eckart theorem for SU(3) [44] and later for SU(4) [45].

For exact SU(3) symmetry the strong interaction should be an irreducible tensor (operator) of rank zero (i.e. a scalar). But obviously (as discussed earlier in this chapter) there is some perturbation which splits the multiplets (into isomultiplets) in such a way that I, I_3 and Y are still conserved. So the symmetry breaking operator (the medium strong interaction Hamiltonian) should transform like $I = 0$, $I_3 = 0$ and $Y = 0$ components of the different irreducible representations (denoted by μ) of SU(3). Denote an SU(3) tensor T belonging to the irreducible representation "μ" by

$$T^{(\mu)}_{II_3Y}$$

(compare with O(3) irreducible tensor operator $T^{(L)}_M$). So the symmetry-breaking operator responsible for the splitting of a supermultiplet (into isomultiplets) should look like

$$O_1 = \sum_\mu T^{(\mu)}_{000} = T^{(1)}_{000} + T^{(8)}_{000} + T^{(27)}_{000} + --- \qquad (2.24)$$

$\underline{1}$, $\underline{8}$, $\underline{27}$, ... contain the state with $I = 0$, $Y = 0$ (the electromagnetic interactions are being neglected which can be treated on similar lines).

The Wigner-Eckart theorem for an SU(3) irreducible tensor operator $T^{(\mu)}_\nu$ (where ν stands for the "magnetic" quantum numbers , I, I_Z and Y while μ for the irreducible representation of SU(3)) can be written as follows:

$$<\phi^{(\mu_3)}_{\nu_3} | T^{(\mu_2)}_{\nu_2} | \phi^{(\mu_1)}_{\nu_1}> = \sum_\gamma \begin{pmatrix} \mu_1 & \mu_2 & \mu_{3\gamma} \\ \nu_1 & \nu_2 & \nu_2 \end{pmatrix} <\mu_3 || T^{(\mu_2)} || \mu_1>_\gamma \qquad (2.25)$$

γ gives the summation over those irreducible representations which occur repeatedly in the direct product $\mu_1 \otimes \mu_2$. The SU(3) Clebsch-Gordon (C.G.) coefficients $\begin{pmatrix} \mu_1 & \mu_2 & \mu_{3\gamma} \\ \nu_1 & \nu_2 & \nu_3 \end{pmatrix}$ are defined by

$$\psi\begin{pmatrix} \mu_1 & \mu_2 & \mu_\gamma \\ & & \nu \end{pmatrix} = \sum_{\mu_1, \mu_2} \begin{pmatrix} \mu_1 & \mu_2 & \mu_\gamma \\ \nu_1 & \nu_2 & \nu_3 \end{pmatrix} \phi^{(\mu_1)}_{\nu_1} \phi^{(\mu_2)}_{\nu_2} \qquad (2.26)$$

The SU(3) C.G. coefficients are related to SU(3) isoscalar factors through SU(2) C.G. coefficients as

$$\begin{pmatrix} \mu_1 & \mu_2 & \mu_\gamma \\ \nu_1 & \nu_2 & \nu \end{pmatrix} = C(I_1 I_2 I;\ I_{1Z}\ I_{2Z}\ I_Z) \left(\begin{matrix} \mu_1 & \mu_2 \\ I_1 Y_1 & I_2 Y_2 \end{matrix} \middle| \begin{matrix} \mu_\gamma \\ \nu \end{matrix}\right) \qquad (2.27)$$

where $C(I_1 I_2 I;\ I_{1Z} I_{2Z} I_Z)$ is the SU(2) C.G. coefficient [46] and

$\left(\begin{matrix} \mu_1 & \mu_2 \\ I_1 Y_1 & I_2 Y_2 \end{matrix} \middle| \begin{matrix} \mu_\gamma \\ \nu \end{matrix}\right)$ is the SU(3) isoscalar factor [44,46]. For the medium strong interaction operator one can write

$$M \ (\text{or } m^2) = T^{(1)}_{(I=0,Y=0} + T^{(8)}_{(I=0,Y=0)} + T^{(27)}_{(I=0,Y=0)} \qquad (2.28)$$

where $M(m^2)$ stands for the mass of the baryons(mesons) [57]. For example, for nucleons, one gets

$$M_N = \langle\phi^{(8)}_{\nu_1}|M_{\nu_2}|\phi^{(8)}_{\nu_1}\rangle = \langle\phi^{(8)}_{\nu_1}|T^{(1)}_{\nu_2}|\phi^{(8)}_{\nu_1}\rangle + \langle\phi^{(8)}_{\nu_1}|T^{(8)}_{\nu_2}|\phi^{(8)}_{\nu_1}\rangle$$
$$+ \langle\phi^{(8)}_{\nu_1}|T^{(27)}_{\nu_2}|\phi^{(8)}_{\nu_1}\rangle \qquad (2.29)$$

where ν_1 stands for $I = \frac{1}{2}$ and $Y = 1$ and ν_2 stands for $I = 0$ and $Y = 0$. Noting that

$$\underline{8} \otimes \underline{1} = \underline{8}$$
$$\underline{8} \otimes \underline{8} = \underline{8}_1 + \underline{8}_2 + \cdots$$
$$\underline{8} \otimes \underline{27} = \underline{8} + \ldots$$

one gets after using SU(3) scalar factors [46]

$$M_N = 1\langle 8\|T^{(1)}\|8\rangle - \frac{\sqrt{5}}{10}\langle 8\|T^{(8)}\|8\rangle_1 + \tfrac{1}{2}\langle 8\|T^{(8)}\|8\rangle_2 + \frac{1}{3\sqrt{5}}\langle 8\|T^{(27)}\|8\rangle$$
$$= a_1 - \frac{\sqrt{5}}{10}\, a_{8_2} + \tfrac{1}{2} a_{8_2} + \frac{1}{3\sqrt{5}}\, a_{27} \qquad (2.30)$$

where $\mu_1 \to 8$ and the notation $\langle 8\|T^{(\mu)}\|8\rangle_\nu = a_{\mu_\nu}$ for the reduced matrix elements have been used. Similarly

$$m_{\Xi} = a_1 - \frac{\sqrt{5}}{10} a_{8_1} - \tfrac{1}{2} a_{8_2} + \frac{1}{3\sqrt{5}} a_{27}$$

$$m_{\Lambda} = a_1 - \frac{\sqrt{5}}{5} a_{8_1} - \frac{1}{\sqrt{5}} a_{27}$$

$$m_{\Sigma} = a_1 + \frac{\sqrt{5}}{5} a_{8_1} \qquad (2.31)$$

There are four constants and the four masses and hence the constants can be found out in terms of these masses as follows:

$$a_1 = \frac{1}{8}[2m_N + 2m_{\Xi} + m_{\Lambda} + 3m_{\Sigma}]$$

$$a_{8_1} = \frac{1}{\sqrt{5}}[3m_{\Sigma} - m_{\Lambda} - m_N - m_{\Xi}]$$

$$a_{8_2} = m_N - m_{\Xi}$$

$$a_{27} = -\left(\frac{9}{8\sqrt{5}}\right)[3m_{\Lambda} + m_{\Sigma} - 2m_N - 2m_{\Xi}] \qquad (2.32)$$

The Gell-Mann-Okubo (G.M.O.) mass relation

$$m_N + m_{\Xi} = \frac{3}{2} m_{\Lambda} + \tfrac{1}{2} m_{\Sigma}$$

is based on the assumption that the mass operator transforms as $T^{(8)}_{o,o,o}$ i.e. $a_{27} = 0$. In practice, this condition seems to be satisfied reasonably since $a_1 = 115.8 \frac{MeV}{C^2}$, $a_{8_1} = 91.34 \frac{MeV}{C2}$, $a_{8_2} = -379.54 \frac{MeV}{C^2}$ and $a_{27} = 11.9 \frac{MeV}{C^2}$.

The mass formula for the electromagnetic mass differences can be obtained by introducing a symmetry breaking operator (which splits the masses within an iso-multiplet) of the form

$$O_2 = \sum_{\mu I} T^{(\mu)}_{I,o,o} \qquad (2.33)$$

and use the SU(2) C.G. coefficients along with the SU(3) iso-scalar factors in that order. This form for O_2 is suggested by the fact that in electromagnetic interactions I_Z and Y are conserved but not I.

The equal spacing rule for the decuplet and the Gell-Mann-Okubo mass formula for the octets of pseudoscalar and vector mesons can be obtained similarly and are discussed in ref. [44].

The Wigner-Eckart theorem for SU(4) reproduces the Gell-Mann-Okubo formula for the old particles of SU(3) if one supposes that the medium strong interaction transforms as $\underline{8} + \underline{15}$ (where $\underline{15}$ is the adjoint representation of SU(4)) and that it conserves the quantum numbers, I, I_3, Y and C where C is the additional quantum number of SU(4) called Charm. A lot more relations can be obtained between the particles of a multiplet by assuming different transformation properties (under SU(4)) for the mass operator. SU(4) iso-scalar factors are given by Haacke et al. [47] and Rabl et al. [48].

In the following we make use of the SU(4) Wigner-Eckart theorem and obtain certain results which compare with those in Chapter 4 obtained by another approach. This also serves as a check of the accuracy of the SU(4) iso-scalar factors we have used.

2.6 Wigner-Eckart Theorem for SU(4)

In SU(4) the mass operator [41,42] generally used is given by

$$M\,(m^2) \quad = \quad T_o + T_8 + a' T_{15} \qquad (2.34)$$

where T_o is a SU(4)-scalar while T_8 and T_{15} belong to the same 15-dimensional representation of SU(4). We illustrate the use of SU(4) Wigner-Eckart theorem for the 15-plet of vector mesons containing ψ and D's (for new particles see Chapters 3 and 4). The ψ is assigned together with the usual ρ, K^*, ω and ϕ to the representations

$$\underline{4} \otimes \underline{4}^* \quad = \quad \underline{15} + \underline{1} \ . \qquad (2.35)$$

The pseudoscalar mesons are assigned to another $\underline{15} + \underline{1}$ representation. The SU(3) decomposition of $\underline{15}$ is

$$\underline{15} = \underline{1} + \underline{3} + \underline{3}^* + \underline{8} \tag{2.36}$$

where the representation $\underline{3}$ contains an SU(2) doublet $D(D^*)$ and singlet $F(F^*)$ of pseudoscalar(vector) mesons carrying non-zero charm C. If R denotes the SU(4) representation, μ the SU(3) representation, C the charm associated with the irreducible representation and ν the SU(3) quantum numbers I and Y, the matrix element of the operator m^2 can be written as [45]

$$\begin{pmatrix} R_1 \\ \mu_1\nu_1 \end{pmatrix} m^2 \begin{pmatrix} R_1 \\ \mu_1\nu_1 \end{pmatrix} = \sum_{\mu,\gamma,\gamma'} \begin{pmatrix} R_1 & R_2 & \Big| & R_{1\gamma} \\ \mu_1 C_1 & \mu_2 C_2 & \Big| & \mu_{1\gamma'} C_1 \end{pmatrix}$$

$$\times \sum_{\nu_1} \begin{pmatrix} \mu_1 & \mu_2 & \Big| & \mu_{1\gamma'} \\ \nu_1 & \nu_2 & \Big| & \nu_1 \end{pmatrix} \langle R_1 || R_2 || R_{1\gamma} \rangle \tag{2.37}$$

where $\begin{pmatrix} R_1 & R_2 & \Big| & R_{1\gamma} \\ \mu_1 C_1 & \mu_2 C_2 & \Big| & \mu_{1\gamma'} C_1 \end{pmatrix}$ is the SU(4) factor [47,48] and

$\begin{pmatrix} \mu_1 & \mu_2 & \Big| & \mu_1 \\ \nu_1 & \nu_2 & \Big| & \nu_1 \end{pmatrix}$ is the SU(3) isoscalar factor [46], $\gamma(\gamma')$ stands for SU(4) (SU(3)) representations $R_1(\mu_1)$ which occur repeatedly in $R_1 \otimes R_2$ $[\mu_1 \otimes \mu_2]$.

Using quadratic mass formula for mesons and the notation

$\begin{pmatrix} R_1 \\ \mu_1\nu_1 \end{pmatrix} T_o \begin{pmatrix} R_1 \\ \mu_1\nu_1 \end{pmatrix} = \overline{m}^2$ and $\langle 15 || 15 || 15_D \rangle = X_D$ it is easy to establish that [58]

$$(m^2)_\rho = \overline{m}^2 + \left(\frac{1}{3} - \frac{a'}{3\sqrt{2}}\right) X_D$$

$$(m^2)_{K^{*o}} = \overline{m}^2 + \left(-\frac{1}{6} - \frac{a'}{3\sqrt{2}}\right) X_D$$

$$(m^2)_{88} = \overline{m}^2 + \left(-\frac{1}{3} - \frac{a'}{3\sqrt{2}}\right) X_D$$

$$(m^2)_{D^*} = \bar{m}^2 + \left(\frac{1}{6} + \frac{a'}{3\sqrt{2}}\right)X_D$$

$$(m^2)_{F^*} = \bar{m}^2 + \left(-\frac{1}{3} + \frac{a'}{3\sqrt{2}}\right)X_D$$

$$(m^2)_{D^{-*}} = \bar{m}^2 + \left(\frac{1}{6} + \frac{a'}{3\sqrt{2}}\right)X_D$$

$$(m^2)_{F^{-*}} = \bar{m}^2 + \left(-\frac{1}{3} + \frac{a'}{3\sqrt{2}}\right)X_D \qquad (2.38)$$

If we put

$$\frac{1}{\sqrt{3}} X_D = A \quad \text{and} \quad -a' = a, \quad \text{we get}$$

$$(m^2)_{\rho} = \bar{m}^2 + \left(\frac{1}{\sqrt{3}} + \frac{a}{\sqrt{6}}\right)A$$

$$(m^2)_{K^{0*}} \quad \bar{m}^2 + \left(\frac{a}{\sqrt{6}} - \frac{1}{2\sqrt{3}}\right)A$$

$$(m^2)_{D^*} = \bar{m}^2 + \left(\frac{1}{2\sqrt{3}} - \frac{a}{\sqrt{6}}\right)A$$

$$(m^2)_{F^{+*}} = \bar{m}^2 + \left(-\frac{1}{\sqrt{3}} - \frac{a}{\sqrt{6}}\right)A = (m^2)_{F^{-*}}$$

$$(m^2)_{88} = \bar{m}^2 + \left(-\frac{1}{\sqrt{3}} + \frac{a}{\sqrt{6}}\right)A \qquad (2.39)$$

The mixed matrix elements can be written as

$$(m^2)_{00} = \begin{pmatrix} \underline{1} & \underline{1} & \Big| & \underline{1} \\ 10 & 10 & \Big| & 10 \end{pmatrix} < 1||1||1>$$

$$(m^2)_{08} = \begin{pmatrix} \underline{1} & \underline{15} & \Big| & \underline{15} \\ 10 & 80 & \Big| & 80 \end{pmatrix} \begin{pmatrix} \underline{1} & \underline{8} & \Big| & \underline{8} \\ I=0 & I=0 & \Big| & I=0 \\ Y=0 & Y=0 & \Big| & Y=0 \end{pmatrix} <\underline{1}||\underline{15}||15>$$

$$(m^2)_{015} = \begin{pmatrix} 1 & 15 & \Big| & 15 \\ 10 & 10 & \Big| & 10 \end{pmatrix} \begin{pmatrix} 1 & 1 & \Big| & 1 \\ I=0 & I=0 & \Big| & I=0 \\ Y=0 & Y=0 & \Big| & Y=0 \end{pmatrix} <\underline{1}||\underline{15}||15>$$

(2.40)

Each isoscalar factor is equal to one and hence

$$(m^2)_{00} = \bar{m}_o^{\,2} \quad \text{(say)}$$

$$(m^2)_{08} = B \quad \text{(say)}$$

$$(m^2)_{015} = B \quad . \qquad (2.41)$$

The notation $(m^2)_{00}$, $(m^2)_{08}$ and $(m^2)_{015}$ will become clearer in Chapter 4, where we consider the mass diagonalization problem for the $\underline{1} \oplus \underline{15}$ of vector- and pseudo-scalar mesons via a different approach (no explicit use of the Wigner-Eckart theorem) but get the same results as given above. We regard this agreement as a sort of check on the accuracy of the SU(4) iso-scalar factors [47,48].

2.7 Tensor Method

Here, we illustrate yet another method (i.e. the use of tensors [49]) to get mass relations among the particles of a multiplet (like G.M.O. mass formula for the octets and the equal spacing rule for the decuplets).

As we have noted in the previous sections, it is the medium strong interaction which produces mass-splitting in (SU(3)) multiplets, and hence it cannot be a scalar under SU(3) transformations. If we neglect the electromagnetic mass differences, and also use the fact that hypercharge is a good quantum number in strong interactions, then the three generators λ_1, λ_2 and λ_3 (of isospin) must commute with the mass operator. Thus the matrix elements of the medium strong interaction operator between states of different isospin or hypercharge should be zero. It follows that it (medium strong interaction operator) should be a $I = 0$, $Y = 0$ component of the irreducible SU(3) tensor operator. Now, there are a number of SU(3) representations (irreducible) which have $I = 0$, $Y = 0$ components but the simplest approximation would be to retain only the first two, i.e. the singlet and the octet. The singlet is excluded because it is an SU(3) scalar and hence the medium-strong interaction operator (M) must transform as the octet element given by

$$M = \frac{1}{\sqrt{6}} (A_1^{\ 1} + A_2^{\ 2} - 2A_3^{\ 3}) \quad . \tag{2.42}$$

In the matrix formalism of (SU(3)), it can be written as

$$M = \frac{c}{\sqrt{6}} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix} \tag{2.43}$$

where c is a real constant. This form for the mass operator [50] also results if one supposes that the quarks belonging to the $\underline{3}$ of SU(3) have distinct masses. Since q_1 and q_2 belong to an isospin doublet it is natural to assume that

$$m_1 = m_2 = m$$

and to suppose that the SU(3)-symmetry is broken by the third quark q_3 (i.e. $m_3 \neq m$). Let $\bar{m}$ be the average quark mass given by

$$\bar{m} = \frac{2m + m_3}{3} \tag{2.44}$$

The mass term in the quark Lagrangian can be written in the form

$$\begin{aligned} \mathcal{L}_m &= m(q_1^{\ 1} + q_2^{\ 2}) + m_3\, q_3^{\ 3} \\ &= m\, q_\alpha^{\ \alpha} + (m_3 - m) q_3^{\ 3} \\ &= \bar{m}\, q_\alpha^{\ \alpha} + \frac{3}{2}(m_3 - \bar{m})(q_3^{\ 3} - \frac{1}{3} q_\alpha^{\ \alpha}) \quad . \end{aligned} \tag{2.45}$$

If the part of the Lagrangian that accounts for the strong quark couplings and quark binding is SU(3) - invariant - then the symmetry-breaking part transforms like $A_3^{\ 3}$ component of the traceless tensor. $A^\alpha_{\ \beta}$ ($A^\alpha_{\ \beta} = q^\alpha_{\ \beta} - \frac{1}{3}\delta^\alpha_{\ \beta}\, q_\alpha^{\ \alpha}$). Now it is easy to see that this tensor transforms like λ_8 under SU(3) transformations. In terms of SU(3) matrix $A_3^{\ 3}$ ($A_3^{\ 3} = M$) can be written as

$$M = C \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix} \tag{2.46}$$

which is the same matrix as given by (2.43). M commites with the isospin generators $(\lambda_1, \lambda_2, \lambda_3)$ and hypercharge Y $(Y = \sqrt{(2/3)}\ \lambda_8)$.

In terms of the tensor formalism, we do not know the exact form of M but its matrix elements between two states (its expectation value) should be a scalar. Such a scalar can be made in two ways, namely

$$M_1 \equiv A^{*\ \alpha q \ldots t}_{\ \ jk \ldots m}\ M^{\alpha'}_{\alpha}\ A^{jk \ldots m}_{\alpha' q \ldots t} \tag{2.47a}$$

and

$$M_2 \equiv A^{*pq \ldots t}_{\alpha' k \ldots m}\ M^{\alpha'}_{\alpha}\ A^{\alpha k \ldots m}_{pq \ldots t} \tag{2.47b}$$

As an example, we give the mass relation obtained for the octet (Gell-Mann-Okubo mass formula).

The octet states in terms of the components of a tensor $A_i^{\ j}$ (i,j = 1,2,3) are given as follows:

$$A_1^{\ 3},\ A_2^{\ 3}\ ;\ A_1^{\ 2},\ \frac{1}{\sqrt{2}}(A_1^{\ 1} - A_2^{\ 2}),\ A^1_{\ 2}\ ;\ \frac{1}{\sqrt{6}}(A_1^{\ 1} + A_2^{\ 2} - 2A_3^{\ 3});\ A_3^{\ 2},\ A_3^{\ 1} \tag{2.48}$$

Their (I,Y)-values, respectively, are $(\frac{1}{2}, 1)$, $(1, 0)$, $(0, 0)$ and $(\frac{1}{2}, -1)$. One can easily find out the values of M_1 and M_2 for each isomultiplet and get the well-known G.M.O. mass formula (for baryons, say)

$$2M_{(\frac{1}{2},1)} + 2M_{(\frac{1}{2},-1)} = 3M_{(0,0)} + M_{(1,0)}$$

or in terms of particles

$$2M_N + 2M_{\Xi} = 3M_{\Lambda} + M_{\Sigma} \quad . \tag{2.49}$$

The singlet S, of (SU(3) is given as

$$S = \frac{1}{\sqrt{3}}(q_1^{\ 1} + q_2^{\ 2} + q_3^{\ 3}) \equiv \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \tag{2.50}$$

and one notes that both M_1 and M_2 are zero as it should be because there is no splitting in the singlet. The equal spacing rule for the decuplet of baryons can be obtained similarly in a straightforward way.

The tensor method can be extended to higher groups (SU(4) for example) but becomes cumbersome (perhaps not for all people).

2.8 Another Approach

In the previous sections we exploited the symmetry properties of the states and the mass operator to get mass relations. In this section we explain another method where one makes certain dynamical assumptions in addition to the underlying symmetry properties. For simplicity, we consider the conventional quark model in which baryons are looked upon as three-quark systems and mesons as quark and an anti-quark system.

Let us assume that the mass operator M of a hadron is given as [51,52]

$$M = W + M_s + \sum_{i<j} U_{ij} \tag{2.51}$$

where W is the kinetic energy (total) of the system, M_s is the sum of the quark masses and $\sum_{i<j} U_{ij}$ is the contribution from the quark interactions (for simplicity, only two body interactions are considered). If one assumes that the kinetic energy W is small, the SU(3)-breaking terms in W can be neglected. W will then be a constant for a multiplet and can be neglected for the mass differences. M_s, for the system can be written as

$$M_s = \sum_{i=1}^{n} m(i) \tag{2.52}$$

U will consist of SU(3)-invariant and SU(3)-breaking part. The SU(3)-invariant part, being a constant for a multiplet can be neglected for we consider the mass differences. The SU(3)-breaking interaction will contain an isospin-conserving part which depends on the total isospin and total hypercharge and electromagnetic interactions which split the isomultiplet. We can then write for U,

$$\sum_{i<j} U_{ij} = \sum_{i<j} [U_{IY}(i,j) + U_{e.m.}(i,j)] \tag{2.53}$$

where $U_{IY}(i,j)$ is the SU(2)-invariant interactions and $U_{e.m.}(i,j)$ is the SU(2)-breaking (electromagnetic) interactions between the i-th and the j-th quark.

The masses within an isomultiplet are degenerate if the electromagnetic interactions are neglected or in other words, u and d quarks (q_1 and q_2) have the same mass.

The mass of a baryon (for example) B is given by

$$M_B = <B|M|B> = M_o + <B|M_s|B> + \sum_{i<j} <B|U_{ij}|B> \quad (2.54)$$

where M_o is the contribution from SU(3)-invariant part of the mass operator and characterizes the supermultiplet. The terms $<B|M_s|B>$ and $\sum_{i<j} <B|U_{ij}|B>$ can be evaluated by using the baryon wavefunctions [3b].

As $U_{IY}(i,j)$ depends upon the isospin and hypercharge, it is convenient to introduce the notation for the expectation values of $U_{IY}(i,j)$ for different I- and Y-values. They are given as follows:

$$\begin{aligned} U_{IY} &= U_1 \quad \text{for } I = 1, \quad Y = \tfrac{2}{3} \\ &= U_2 \quad \text{for } I = 0, \quad Y = \tfrac{2}{3} \\ &= U_3 \quad \text{for } I = \tfrac{1}{2}, \quad Y = -\tfrac{1}{3} \\ &= U_4 \quad \text{for } I = 0, \quad Y = -\tfrac{4}{3} \end{aligned} \quad (2.55)$$

The expectation value $< q_j| M_s| q_j >$ will be given by (neglecting the mass difference between u- and d-type quarks)

$$\begin{aligned} &m \quad \text{for} \quad j = 1,2 \quad (\text{for u and d quarks}) \\ &m+\Delta \quad \text{for} \quad j = 3 \quad (\text{for the } \lambda \text{ quark}) \end{aligned} \quad (2.56)$$

where Δ is the mass difference between the strange and non-strange quark.

As an example, we consider a relation between the decuplet masses.

For $|\Delta> = q_1q_1q_1$

$$<\Delta|U_{IY}|\Delta> = 3U_1$$

$$<\Delta|M_s|\Delta> = 3m$$

$$M_\Delta = M_o + 3m + 3U_1 \quad .$$

Similarly, $M_{\Sigma}^* = (q_1q_1q_3 + q_1q_3q_1 + q_3q_1q_1)/\sqrt{3}$

$$<\Sigma^*|U_{IY}|\Sigma^*> \quad = \quad U_1 + 2U_3$$

$$<\Sigma^*|M_s\ |\Sigma^*> \quad = \quad 3m + \Delta$$

$$M_\Sigma^* \quad = \quad M_o + U_1 + 2U_3 + 3m + \Delta \quad .$$

Similarly

$$M_\Xi^* \quad = \quad M_o + 3m + 2\Delta + 2U_3 + U_4$$
$$m_\Omega \quad = \quad M_o + 3m + 3\Delta + 3U_4 \qquad (2.57)$$

We can now easily verify that the masses of the decuplet satisfy the relation

$$m_\Omega - m_\Sigma^* + m_\Sigma^* - m_\Delta \quad = \quad 2(m_\Xi^* - m_\Sigma^*) \quad . \qquad (2.58)$$

This relation is independent of any assumption of how the two-body force depends on I and Y. One can note from (2.57) that the decuplet masses satisfy the <u>equal spacing rule</u> if

$$U_1 + U_4 \quad = \quad 2U_3 \ . \qquad (2.59)$$

The Gell-Mann-Okubo mass formula for the octet is also reproduced if one uses the relation (2.59). One can proceed similarly to obtain relations among the masses of other multiplets.

For electromagnetic mass splitting one needs a form for the electromagnetic interactions. It is plausible to assume that it consists of the Coulomb interaction $U^{Coul}_{e.m.}(i,j)$,

$$U^{Coul}_{e.m.} \quad = \quad \frac{q_i\ q_j}{r_{ij}} \qquad (2.60)$$

where q_i is the charge of the i-th quark and r_{ij} is the relative distance of the quarks ($\frac{1}{r_{ij}}$ is assumed to be a constant) plus a spin-dependent magnetic interaction $U^{mag.}_{e.m.}(i,j)$

$$U^{mag.}_{e.m.}(i,j) \quad = \quad -\ (\frac{8\pi}{3})\ \mu_q(i)\cdot\mu_q(j)\ \delta(r_{ij}) \qquad (2.61)$$

where $\mu_q(i)$ is the magnetic moment of the i-th quark and $\delta(r_{ij})$ is the Dirac delta function. The expectation value for a symmetric S-wave spatial function ψ, for example, is given by

$$E_{ij} = \langle\psi|U^{\text{mag.}}_{\text{e.m.}}(i,j)|\psi\rangle = -C\mu_q(i) \quad \mu_q(j) \qquad (2.62)$$

where the positive parameter C is given by

$$C = \left(\frac{8\pi}{3}\right)\langle\psi|\delta(r_{ij})|\psi\rangle \quad .$$

For the details of such calculations the reader is referred to Lichtenberg [51,52], Gilman [53] and Perl [53].

In this chapter we have discussed one of the important aspects of symmetries, i.e. the exploitation of the symmetry-breaking effects for establishing relations among the masses of a multiplet of a group. As examples, we have considered SU(3) and SU(4) which are of immediate interest to us but the techniques can be extended to any SU(n) $(n > 4)$. We considered different approaches (yielding the same results) because we realize that a particular approach is sometimes more useful in a specific problem. The preference for the use of a particular type of approach depends upon the nature of the problem to be tackled and, of course, on personal liking.

Besides mass relations, these techniques find applications in extracting a lot more other properties of particles, like relations between matrix elements [54], prediction of decay widths [55] and breaking of coupling constants (in the context of different groups) [56] etc., etc.

In Chapter 6 we consider the methods for prediction of masses of hadrons within the framework of a different type of quark model called the MIT bag model.

CHAPTER 3

ELECTROMAGNETIC DECAYS OF NEW PARTICLES

3.1 Introduction to Charm

In the early sixties many authors, including Bjorken and Glashow [59] suggested that it would be attractive to have four types of quarks rather than the three then postulated, so as to exhibit some sort of symmetry with the four known leptons.

In 1970, Glashow, Iliopoulos and Maiani [60] argued that a fourth flavour of quark was not only desirable but necessary to explain the rareness of strangeness changing neutral weak processes, such as $K_L^o \to \mu^+\mu^-$ (see Chapter 4). They called the quantum number, associated with such a new quark Charm (see Sec. 3.3).

In a classic paper entitled "Search for Charm", Gaillard et al. [61] gave the theoretical predictions, based on the Charm Scheme (in the context of SU(4) symmetry), of a narrow vector particle ϕ_c (which is now identified with ψ/J), a pseudoscalar meson η_c, charmed mesons D's and F's (see Sec. 3.2 and Sec. 4.2) and a host of new (charmed) baryons (to be discovered). The notation for these particles, first initiated in this paper [61] is adopted now (more or less) universally in the literature.

3.2 Discovery of New Particles

Until late 1974 the only supporting evidence for the far reaching hypothesis of charm was the large e^+e^- annihilation cross-section at high energies at CEA and SPEAR. But then heavy narrow meson states, coupling to e^+e^-, were found at BNL [62] and SPEAR [63] in accordance with the charm hypothesis.

Since then new and dramatic experimental discoveries [65,66] have accumulated which make the case for charm quite a strong one. In addition to the charmed hadrons D^0 and $D^{\pm}$ [64] in e^+e^- annihilation at least nine new mesons have been discovered [64,66]. These are not charmed, but are thought to be made out of the $c\bar{c}$ system to contain hidden charm. They are the following:

(i)	$\psi(3.1)/J$	Narrow resonances seen in $e^+e^- \to$ hadrons
(ii)	$\psi'(3.684$	
(iii)	$\psi''(4.1)$	Broader resonances seen in $e^+e^- \to$ hadrons
(iv)	$\psi'''(4.4)$	(may be more than two in this region)
(v)	$\chi(3.414)$	Seen in
(vi)	$\chi(3.508)$	$\psi' \to \gamma\chi$, $\chi \to$ hadrons and $\psi' \to \gamma\chi$, $\chi \to \gamma\psi/J$
(vii)	$\chi(3.552)$	
(viii)	$\chi(3.455)$	-
(ix)	$X(2.8)$	Seen in $\psi/J \to X(2.8)$, $X(2.8) \to \gamma\gamma$

$\psi(3.1)$ was seen independently at BNL [62] and SPEAR [63]. At BNL it was seen in the production of e^+e^- by hadrons and called J. At SPEAR it was seen in the production of hadrons by e^+e^- with a c.m. energy of 3.1 GeV and called ψ (henceforth, we shall call the particle by the name ψ). Its mass and total width are known to be 3.095 GeV and (69 ± 7) keV respectively. Its spin and parity are found to be $J^P = 1^-$. It has hadron decays into states of odd G-parity (odd number of pions) and so is presumably an isoscalar like the ϕ. It is obviously a good candidate for $\phi_c(c\bar{c})$ (see Sec. 3.3). ψ' with mass 3.684 GeV has been seen decaying into ψ by emission of two pions or two photons [65]. Presumably this is an excited state of the $c\bar{c}$ system. Of the four intermediate states χ_J's (see Sec. 3.3 for assignment of quantum numbers), the first three have been seen in radiative transitions from ψ' (i.e. $\psi' \to \gamma\chi_J$'s) and all four have been seen in the electromagnetic cascades from ψ' (i.e. $\psi' \to \gamma\chi_J \to \gamma\gamma\psi$). The evidence for $X(2.8)$ comes from DESY [66], the mass being estimated as (2.83 ± 0.03)GeV. This evidence is, however, not fully firm.

Charmed particles (D's etc.) should be experimentally easy to detect through the analysis of their decays via the weak interactions. It is because charm is presumably conserved in strong and electromagnetic interactions but can change in weak processes and thus charmed particles should be dominantly decaying through weak interactions. The charm quantum number has been incorporated in the theory of weak decays (see Sec. 4.1) which provides certain selection rules for these decays (i.e. the so-called Cabibbo-forbidden and Cabibbo-allowed decays). The eventual discovery of charmed mesons has been, of course, guided to some extent by these rules. Narrow invariant mass peaks with an average mass of 1.865 GeV were found in the neutral states $K\pi$ and $K3\pi$ [64], observed in e^+e^--annihilation. The narrow widths and the presence of K's are good arguments for believing these to be the decay modes of the D^0(neutral) meson. Also, the lower limit on the observed masses of the recoil system (recoiling against $K\pi$ and $K3\pi$) is at least as large as that of the resonant peak combinations themselves. And thus the need for pair production of such objects (to conserve the new quantum number charm) is also satisfied. By now, a doublet of charmed pseudoscalar mesons (D) and a doublet of charmed vector mesons (D^*) have been detected in e^+e^--annihilation and their quantum numbers established. The masses of these mesons are such that the transition can occur only through pion or γ-ray emission.

3.3 Charmonium

It is now well established that the particle ψ is a bound state of the charmed quark and its anti-particle - that is, the object called charmonium of net charm quantum number zero [65,66]. This explanation received additional credence by the discovery, shortly thereafter, of a second resonance called the ψ', that can be interpreted as the first radially excited state of the ψ. In addition to ψ and ψ' peaks, there is a resonance [67] discovered recently at a centre of mass energy of 3.77 GeV, followed by a complex region which appears to exhibit peaks at 4.03 GeV and 4.4 GeV and perhaps also other possible structures. All of these states constitute radial excitation, of the charm-anticharm combination, assuming that they have the same intrinsic quantum numbers ($J^{PC} = 1^{--}$). Quantitatively, this suggestion holds up under critical examination but quantitative attempts to fit the energies of these peaks with a simple potential model have not been fully successful [65,72]. In our calculations, we have not concerned ourselves with the states above 3.684 GeV.

The next question is, can all the other states (mentioned above) be looked upon as bound states of charm and anticharm (i.e. states with hidden charm)? If we assume that the charm and anticharm from a system with orbital angular momentum L and spin S (see below for the quantum numbers of the charmed quark), then the parity P and charge conjugation number C of the state $^{2S+1}L_J$ (in the notation of atomic spectroscopy) are respectively given as (see Fig. 3.1) follows:

$$P = (-1)^{L+1}, \qquad C = (-1)^{L+S} \quad .$$

Thus the lowest lying states are

^1S_o	with	J^{PC}	$= 0^{-+}$
$^3S_1$	"		$= 1^{--}$
$^1P_1$	"		$= 1^{+-}$
$^3P_{0,1,2}$	"		$= 0^{++}, 1^{++}, 2^{++}$.

The determination of the quantum numbers of the intermediate states χ_J's rests on the observation of various decay modes. The state $\chi(3.414)$ cannot be $J^P = 0^-$ and $J^P = 1^+$ because it is observed to decay into $\pi^+\pi^-$, and K^+K^- states which would be forbidden for this assignment. Hence $\chi(3.414)$ must either be $J^P = 0^+$ or $J^P = 2^+$ state. The angular distribution of the photon from the radiative decay appears to be consistent with the $J^P = 0^+$

but not the $J^P = 2^+$ assignment. Thus $\chi(3.414)$ is a $J^P = 0^+$ state, the lowest member of the ^3P triplet. That $\chi(3.508)$ and (3.552) are the two higher members of the triplet is an assumption (a logical one). The angular distribution of the photons in the radiative decays is consistent with the assignment $J^P = 1^+$ and $J^P = 2^+$ for $\chi(3.508)$ and $\chi(3.552)$ respectively. The possibility of the state $\chi(3.455)$ being a member of the ^3P triplet is excluded due to the very small branching ratio for the transition from ψ' (to the state $\chi(3.455)$). It is, in fact, so small that it is not evident in the inclusive γ-ray spectra. It might be the pseudoscalar partner (2^1S_o) of ψ' (2^3S_1). In Chapter 4 we calculate the decay $\Gamma(\psi' \to \gamma\chi(3.455)$ with this assignment. For a complete review of the experimental situation concerning the new quark spectroscopy, the reader is referred to articles by Feldman et al. [66] and Schopper [65b].

Theoretically, with a phenomenological Hamiltonian [69] linear in the spin orbit force $\underline{L}\cdot\underline{S}$ the tensor force $S_{12} = (3\underline{\sigma}_1\cdot\hat{\underline{r}}\,\underline{\sigma}_2\cdot\hat{\underline{r}} - \underline{\sigma}_1\cdot\underline{\sigma}_2)$ and a spin-spin term $\underline{\sigma}_1\cdot\underline{\sigma}_2$, one can account for all known charmonium levels below 4 GeV (shown in Fig. 3.1 and Table 3.1) and predict the location of the rest of the spectrum. The states $\psi'(3.684)$ and $\psi''(3.772)$ can then be interpreted as representing a mixed $^3S_1 - {}^3D_1$ system. Theoretically expected states of charmonium are shown in Fig. 3.1 and the actual experimental situation (including decays of charmonium) is depicted in Fig. 3.2 [66,72].

The fourth quark c (which is responsible for these new states) additional to the SU(3) quark triplet (u, d and s) carries charm $C = +1$, just as the third quark carries strangeness $S = -1$. For definiteness, we give here the quantum numbers of these quarks (for a particular choice of quark charges).

Quantum Number	u	d	s	c
I_3	$+\frac{1}{2}$	$-\frac{1}{2}$	0	0
S	0	0	-1	0
C	0	0	0	+1
Q	$\frac{2}{3}$	$-\frac{1}{3}$	$-\frac{1}{3}$	$\frac{2}{3}$

In the SU(4) scheme the charmed quark charge can differ from the above value by an integer. For example, one can consider $Q_c = -\frac{1}{3}, -\frac{4}{3}$ etc. The choice $Q_c = \frac{2}{3}$ is the one adopted by Glashow, Iliopoulos and Maiani [60] for the construction of their weak interaction model in which the pairs (u,d) and (c,s) should have the same charge relationship, so that $Q_c = \frac{2}{3}$ was the obvious choice. $Q_c = -\frac{1}{3}$ is the possibility considered by Moffat [68]. All the quarks have baryon number $B = \frac{1}{3}$. The corresponding antiquarks $(\bar{u}, \bar{d}, \bar{s}, \bar{c})$ have the reversed values for the above quantum numbers. It is clear from the quantum numbers that charm C bears close analogy with the "strangeness" quantum number S. The two quantum numbers (for a hadronic system) are given by

$$S = (\text{number of } \bar{s} \text{ antiquarks}) - (\text{number of } s \text{ quarks})$$
$$C = (\text{number of } c \text{ quarks}) - (\text{number of } \bar{c} \text{ quarks}).$$

The charge for any hadronic system would be given by the extended Gell-Mann-Nishijima formula

$$Q = I_3 + \tfrac{1}{2}(C + S + B) \quad . \tag{3.1}$$

C and S are both additive quantum numbers, conserved in strong and electromagnetic interactions but not in weak interactions, in general.

In the following sections we shall calculate the radiative decay rates for the transitions $\psi' \to \gamma\chi_J's$, $\chi_J's \to \gamma\psi$ and $\psi \to \gamma X(2.8)$ with the identification shown in Fig. 3.1 and Table 3.1.

3.4 Radiative Transitions

The radiative transitions which we shall consider are shown diagrammatically in Fig. 3.1. All the states and transitions (except $\psi \to \gamma\eta_c(X(2.8))$ are rather well established. But the decay rates of the transitions χ_J's $\to \gamma\psi$ are not yet measured. And their measurement will be a very good test of our model which is described in the following sections of this chapter.

The lowest states in charmonium are the 1^3S_1 and 1^1S_o which are identified with $\psi(3.095)$ and $\eta_c(X(2.8))$ respectively. It is quite natural in a potential model (harmonic-oscillator potential in our case) to identify ψ' with $J^{PC} = 1^{--}$ as the 2^3S_1 state. The intermediate states denoted by χ_J's ($\chi(3.414)$, $\chi(3.508)$ and $\chi(3.552)$) can be identified with the second level of states $2^3P_{0,1,2}$ [65,69]. All the transitions except $\psi \to \gamma X(2.8)$ which we consider in this chapter are E1 transitions and depend rather heavily on the detailed knowledge of the potential.

In the following we assume an harmonic oscillator potential for charmonium. The spectrum shown in Table 3.1 seems roughly as expected from such a potential with a χ_o state at least about half way between the states ψ and ψ'. We have two unknown parameters, namely the harmonic oscillator constant α'^2 (α'^2 for the $c\bar{c}$ system to distinguish it from the corresponding constant α^2 for the ordinary $q\bar{q}$ system) and the magnetic moment of the charmed quark. We also assume (as usual) that the magnetic moment of the quark is inversely proportional to its mass and thus take the magnetic moment of the charmed quark to be smaller than those of the ordinary quarks by a factor of $\frac{m_u}{m_c}$ [3b,28] where m_u and m_c are the masses of the u- and c-type quarks respectively. The gyromagnetic ratios (g) for all the quarks are assumed as unity.

We have performed our calculations with three sets of parameters: i) $\alpha'^2 = 0.1$ GeV^2, $m_u = 0.336$ GeV, $m_c = 1.3$ GeV, (ii) $\alpha'^2 = 0.27$ GeV^2, $m_u = 0.336$ GeV, $m_c = 1.65$ GeV, (iii) $\alpha'^2 = 0.27$ GeV^2, $m_c = 2$ GeV,

m_u = 0.336 GeV (the non-relativistic treatments of charmonium give the charmed quark mass in the range 1.3 – 2 GeV . Hopefully, this will provide the reader with some idea of the variation of numerical values to be expected as one varies the underlying assumptions and input parameters. To keep the decay rates $\Gamma(\psi' \to \gamma\chi_J$'s) within the experimental limits [66,70], we have to use a bigger value of m_c (fixed m_u), as α'^2 is increased. With $\alpha'^2 = 0.1\ \text{GeV}^2$, m_c = 1.3 GeV (or $\alpha'^2 = 0.27\ \text{GeV}^2$, m_c = 2 GeV), we get sensible results for $\Gamma(\psi' \to \gamma\chi_J$'s). With increasing α'^2 (fixed m_u and m_c) the decay widths increase (except $\chi(3552) \to \gamma\psi$). We give a few more details of the calculations in the next section.

3.5 Detail of Calculations

The quark-antiquark system ($c\bar{c}$ in this case) is assumed to be described by the following Hamiltonian

$$H = \frac{1}{2M}(P_1^2 + P_2^2) + \frac{\kappa}{2}(\underline{r}_1 - \underline{r}_2)^2 \qquad (3.2)$$

where the suffices are quark indices, M is the quark mass and κ is the Hooke constant. Separating the centre of mass motion by putting

$$\begin{aligned} \underline{R} &= \tfrac{1}{2}(\underline{r}_1 + \underline{r}_2) \\ \underline{r} &= \frac{1}{\sqrt{2}}(\underline{r}_1 - \underline{r}_2) \\ \underline{P} &= \underline{P}_1 + \underline{P}_2 \\ \underline{P}_r &= \frac{1}{\sqrt{2}}(\underline{P}_1 - \underline{P}_2) \end{aligned}$$

one gets

$$H_{\text{internal}} = \frac{P_r^2}{2M} + \kappa r^2 \qquad (3.3)$$

Energy levels of this Hamiltonian are found to be

$$E = (n + \tfrac{3}{2})\omega = (\ell + 2k' + \tfrac{3}{2})\omega \qquad (3.4)$$

where ℓ is the orbital angular momentum quantum number, $k' = 0,1,2, \ldots$ is associated with the number of nodes in the radial wavefunction and $\omega = \sqrt{\frac{2\kappa}{M}}$. The corresponding wavefunctions can be written as [20,71]

$$\psi_{n\ell m} = N(\alpha r)^{\ell} L_{k'}^{\ell+\frac{1}{2}} (\alpha^2 r^2) \exp(-\frac{\alpha^2 r^2}{2}) Y_{\ell}^{m} (\theta,\phi) \tag{3.5}$$

where $\alpha^2 = M\omega$ and L is a Laguerre polynomial. The normalisation constant N is given by

$$|N|^2 = \frac{2\alpha^3 k'!}{\sqrt{\pi}(k' + \ell + \frac{1}{2})(k' + \ell - \frac{1}{2}) \ldots \frac{3}{2} \times \frac{1}{2}} .$$

The state of total angular momentum J (physical spin of the resonance) is obtained by coupling the spin state $|S, S_Z>$ with the spatial wavefunction as follows

$$|n,J,J_Z> = \sum_{m+S_Z=J_Z} (\ell m; SS_Z| JJ_Z) \psi_{n\ell m} |SS_Z> . \tag{3.6}$$

The total state is obtained by taking the product of the unitary spin wavefunction (the SU(4) content) with $|n,J,J_Z>$.

We use the following interaction Hamiltonian in our calculations [32]

$$\mathcal{H}_j = \sum_j q^{(j)} g \frac{e}{2M} [-2i \underline{S}^{(j)} \cdot \underline{k} \times \underline{A} + \frac{2\underline{p}^{(j)} \cdot \underline{A}}{g}] \tag{3.7}$$

where $eq^{(j)}$, $s^{(j)}$ and $p^{(j)}$ are the charge, spin and momentum operators of the j-th quark, M is the quark mass and g is the quark gyromagnetic ratio. For ordinary quarks (p, n, λ) g is defined by $\mu_j = \frac{eg}{2M} q^{(j)} = \mu_p(q^{(j)}/e)$ where μ_p is the proton magnetic moment. For the charmed quark we define $\mu_c = \frac{m_u}{m_c} \mu_p$. Thus the above Hamiltonian can be written

$$\mathcal{H}_j = \sum_j q^{(j)} \mu_c [-2i \underline{S}^{(j)} \cdot (\underline{k} \times \underline{A}) + \frac{2}{g} (\underline{p}^{(j)} \cdot \underline{A}] . \tag{3.8}$$

μ_j is replaced by μ_c because we are dealing with only the c-type quark (antiquark) in this chapter. $g = 1$ fits the baryon magnetic moments and we continue to take $g = 1$ even for the charmed quark. $\alpha'^2 = 0.1$ (GeV)^2 ($m_c = 1.3$ GeV, $m_u = 0.336$ GeV) which we use in these calculations is, incidentally, the value used by Faiman et al. [20] to fit $N^* \to N\pi$ and $N^* \to N\gamma$ decays. The electromagnetic field $\underline{A}$ of a photon of momentum k

and polarisation $\underline{\varepsilon}$ is defined to have the usual expansion [32]

$$\underline{A}(\underline{r}^j) = \sqrt{4\pi}\,\sqrt{2k_o}\,\underline{\varepsilon}\,[a^+_{\underline{k}} \exp(i\underline{k}.\underline{r}^j) + a_{\underline{k}} \exp(-i\underline{k}.\underline{r}^{(j)})]. \quad (3.9)$$

A simplified expression for the above interaction can be obtained by taking the photon momentum k along the Z-axis and considering the photons with right-handed polarisation $[\varepsilon = -\frac{1}{\sqrt{2}}(1, i, 0)]$. Then the Hamiltonian reduces to

$$\mathcal{H}_j = \sum_j q^{(j)} \mu_c \sqrt{4\pi} \frac{1}{\sqrt{k_o}} [k\, S_+^{(j)} - (p_x^{(j)} + ip_y^{(j)})] \exp(ikZ)^{(j)} \quad (3.10)$$

To determine the radiative emission at some angle θ to the Z-axis (the quantization axis) we recall that if a specified initial state has spin projection m along the Z-axis (the old Z-axis), it is a superposition of spin states m' along the new axis defined by θ. Thus if the resonance under consideration has spin J (the physical spin of the resonance) and projection m, the intensity of emission at this angle is given by the sum

$$\sum_{m'} \left| A_{|m'|}\, d^J_{m'm}(\theta) \right|^2 .$$

$A_{|m'|}$ denotes the amplitude where m' refers to the decaying particle, e.g. $A_{-1} = \langle \chi, J=0 | \mathcal{H}_j | \psi', J_Z = -1 \rangle$ etc. etc. The decay width Γ is then given by multiplying by the phase-space factor averaging over the initial spin states and summing over the two states of photon polarisation

$$\Gamma = \frac{1}{(2\pi)^2} \frac{2}{2J+1} \left(\frac{E_1 E_\gamma}{E_c}\right) k \int \sum_{m'} \left| A_{m'}\, d^J_{m'm}(\theta) \right|^2 d\Omega^\gamma(\theta,\phi) \quad (3.11)$$

In (3.11) k is the C.M. momentum of the photon and $d\Omega^\gamma(\theta,\phi)$ is the solid angle for the photon. E_c, E_1 and E_γ are the C.M. energy of the decaying resonance, final state hadron and photon respectively. The inclusion or exclusion of the factor $\frac{E_1}{E_c}$ (In the literature $\frac{E_1}{E_c} = 1$ is taken more or less as a prescription by following Becchi et al. [21]) does not make any appreciable change in the calculations of this chapter.

3.6 The Transitions: $\psi' \to \gamma\chi_J$'s, χ_J's $\to \gamma\psi$ and $\psi \to \gamma X(2.8)$

In the spectroscopy of new particle states observed in e^+e^- annihilation, the states $\psi(3095)$ and $\psi'(3684)$ are now clearly established in all their quantum numbers. The existence of the three states $\chi(3414)$, $\chi(3508)$ and $\chi(3552)$ is beyond doubt; the assignments $J^{PC} = 0^{++}$, 1^{++} and 2^{++} are consistent with all observations and are more or less strongly implied. However, we also consider the case when their quantum numbers are in the order of $J^{PC} = 2^{++}$, 1^{++} and 0^{++}. The remaining two states, $X(2.8)$ and $\chi(3455)$ have been more firmly established recently, but so far have no quantum numbers assigned. In the following, we shall tentatively look upon them as 0^{-+} states and the pseudoscalar partners of $\psi(3095)$ and $\psi(3684)$ respectively. The identifications (tentative) of these states in the harmonic oscillator potential (which we use in our calculations) are as shown in Fig. 3.1 and Table 3.1. The rates for the transitions $\psi' \to \gamma\chi_J$'s (χ_J's $\to \gamma\psi$) for the two assignments of quantum numbers are given in Tables 3.2 and 3.3 (Tables 3.4 and 3.5). The transition $\psi \to \gamma X(2.8)$ is considered with $X(2.8)$ as a pure $c\bar{c}$ state (like ψ's and χ_J's) and the pseudoscalar partner of $\psi(3095)$ (in Chapter 4, we consider a probable mass mixing in 16-plets (of SU(4)) of vector particles (containing ψ) and pseudoscalar particles (containing X(2.8)). The wave-functions and the integrals involved in the calculations are given in the Appendix A and Appendix B at the end.

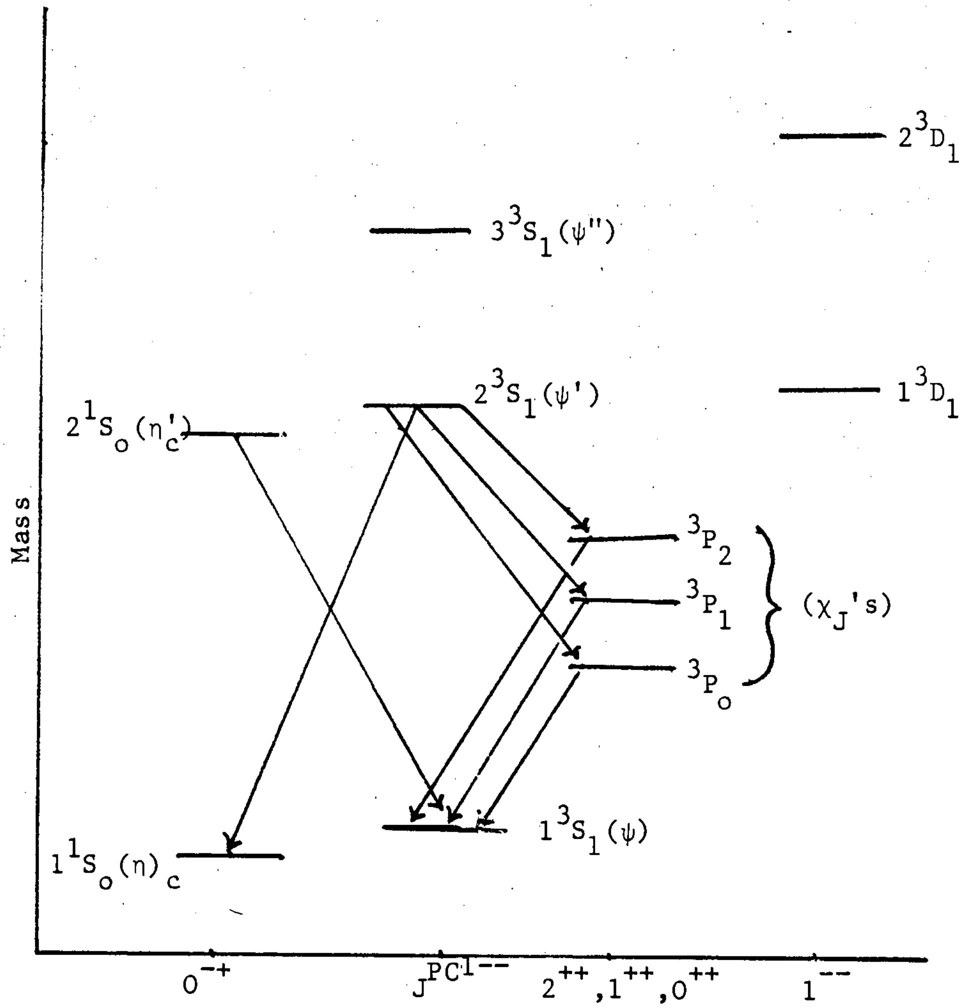


Fig. 3.1. The Spectrum of Charmonium (Ref. 66).

Table 3.1. Identification of Charmonium States

n = 0, L = 0			n = 1, L = 1			n = 2, L = 2, 0		
$^{2S+1}L_J$	J^{PC}	$c\bar{c}$(mass)	$^{2S+1}L_J$	J^{PC}	$c\bar{c}$(mass)	$^{2S+1}L_J$	J^{PC}	$c\bar{c}$(mass)
^1S_o	0^{-+}	$\sim$X(2.8)?	$3P_o$	0^{++}	$\chi(3.414)$			
			$3P_1$	1^{++}	$\chi(3.508)$	$^3S_1$	1^{--}	$\psi(3.68)$
$^3S_1$	1^{--}	$\psi(3,095)$	$3P_2$	2^{++}	$\chi(3.552)$			

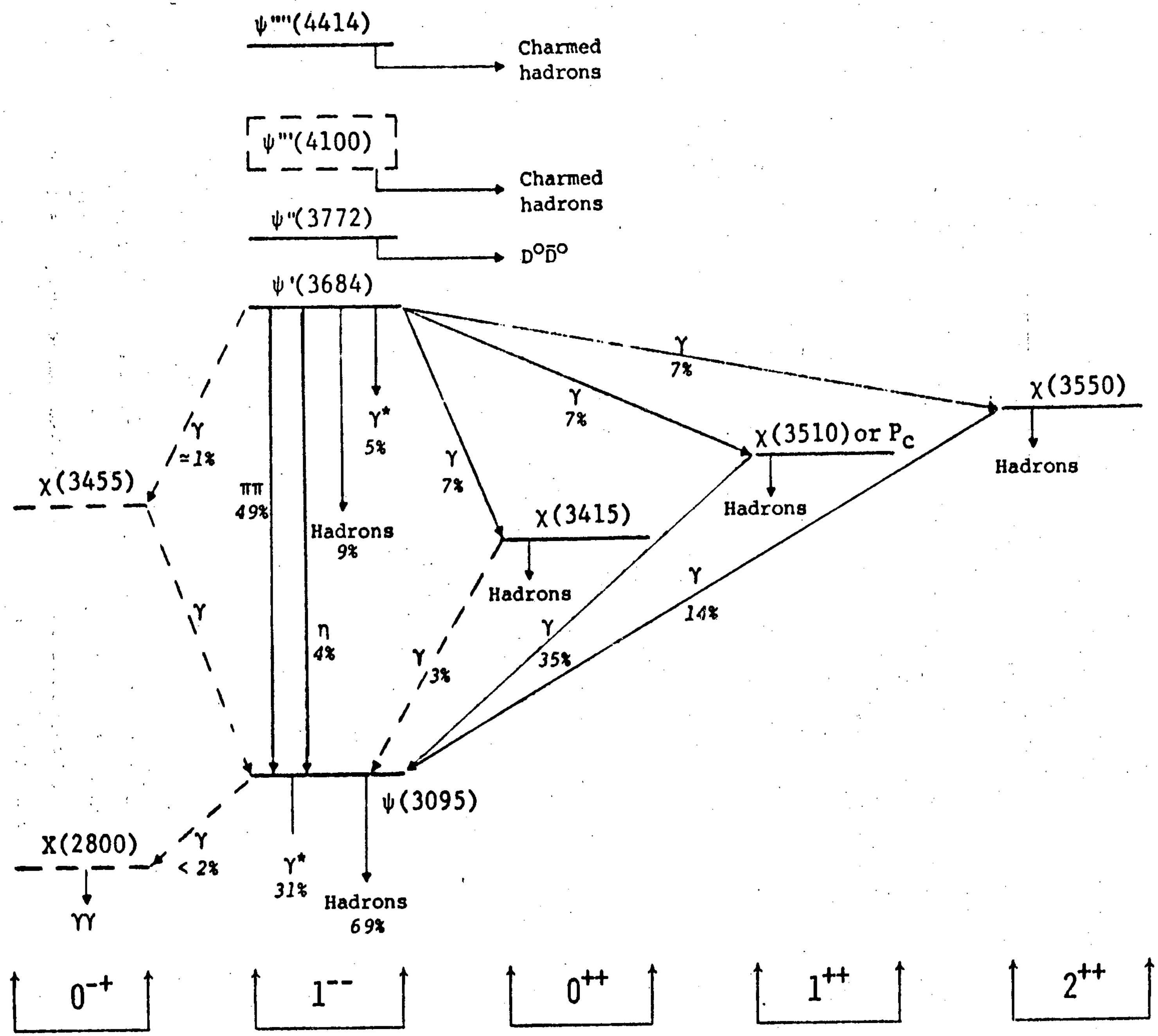


Fig. 3.2. The Spectrum of Charmonium based on experimental results (Refs. 66, 72).

TABLE 3.2 and TABLE 3.3

The rates of the decays $\psi'(3684) \rightarrow \gamma\chi_J$'s and $\psi \rightarrow \gamma X(2.8)$, calculated in the harmonic oscillator model as described in the text. Results in Table 3.2 correspond to the assignment $J^{PC} = 0^{++}$, 1^{++} and 2^{++} for $\chi(3414)$, $\chi(3508)$ and $\chi(3552)$ while the ones in Table 3.3 correspond to the assignment $J^{PC} = 2^{++}$, 1^{++} and 0^{++} for $\chi(3414)$, $\chi(3508)$ and $\chi(3552)$ in those orders. Col. 1: process; col. 2: the momentum of the γ; col. 3: the calculated width with the quasi-relativistic prescription of Becchi and Morpurgo [21]; col. 4: the experimental width (in keV).

TABLE 3.2

Process	k (GeV)	Γ(keV)			Γ(keV)
		$\alpha'^2 = 0.1$ GeV^2 $m_u = 0.336$ GeV $m_c = 1.3$ GeV	$\alpha'^2 = 0.27$ GeV^2 $m_u = 0.336$ GeV $m_c = 1.65$ GeV	$\alpha'^2 = 0.27$ GeV^2 $m_u = 0.336$ GeV $m_c = 2$ GeV	
$\psi' \rightarrow \gamma\chi(3414)$	0.2601	24	30.9	21	23 ± 8 [70]
$\psi' \rightarrow \gamma\chi(3508)$	0.1717	33	51.8	35	21 ± 8 [70]
$\psi' \rightarrow \gamma\chi(3552)$	0.1296	33	58.7	39.7	18 ± 7 [70]
$\psi \rightarrow \gamma X(2.82)$	0.2627	35	26.8	18	<2 keV [70]

TABLE 3.3

$\psi' \rightarrow \gamma\chi(3414)$	0.2601	56	111	75.7
$\psi' \rightarrow \gamma\chi(3508)$	0.1717	33	51.8	35
$\psi' \rightarrow \gamma\chi(3552)$	0.1296	8	13	8.9

TABLE 3.4 and TABLE 3.5

The rates of the decays χ_J's $\to \gamma\psi(3095)$, calculated in the harmonic oscillator model as described in that text. Results in Table 3.4 correspond to the assignment $J^{PC} = 0^{++}$, 1^{++} and 2^{++} for $\chi(3414)$, $\chi(3508)$ and $\chi(3552)$ while those in Table 3.5 correspond to the assignment $J^{PC} = 2^{++}$, 1^{++} and 0^{++} for $\chi(3414)$, $\chi(3508)$ and $\chi(3552)$ in those orders. Col. 1: process; col. 2: the momentum of the γ; col. 3: calculated width with the quasi-relativistic prescription of Becchi and Morpurgo [21]:

TABLE 3.4

Process	k(GeV)	Γ(keV)		
		$\alpha'^2 = 0.1$ GeV^2 $m_u = 0.336$ GeV $m_c = 1.3$ GeV	$\alpha'^2 = 0.27$ GeV^2 $m_u = 0.336$ GeV $m_c = 1.65$ GeV	$\alpha'^2 = 0.27$ GeV^2 $m_u = 0.336$ GeV $m_c = 2$ GeV
$\chi(3414) \to \gamma\psi$	0.304	35	28.9	19.6
$\chi(3508) \to \gamma\psi$	0.388	38	32.2	21.9
$\chi(3552) \to \gamma\psi$	0.427	28	10.9	7.47

TABLE 3.5

$\chi(3414) \to \gamma\psi$	0.304	6	8	55
$\chi(3508) \to \gamma\psi$	0.388	38	32.8	21.9
$\chi(3552) \to \gamma\psi$	0.427	96	68	46

TABLE 3.6

Comparison of the rates of the decays $\psi' \to \gamma\chi_J$'s and χ_J's $\to \gamma\psi$ with various theoretical sssumptions. (The states $\chi(3414)$, $\chi(3508)$ and $\chi(3552)$ are respectively taken as $J^{PC} = 0^{++}$, 1^{++} and 2^{++}). Col. 1: Process; col. 2: the partial widths calculated by Eichten et al. [95]; col. 3: the partial widths calculated by Jackson [94]; col. 4: "absolute" lower bounds on the decay widths of transitions χ_J's $\to \gamma\psi$ calculated by Jackson [94], using dipole sum rules; col. 5: our model results; col. 6: the experimental width.

Process	Γ(keV) with $m_c = 1.6$ GeV, $\alpha_s = 0.2$, $a = 0.2$ fm	Γ(keV) with $m_c = 1.65$ GeV, $\alpha'^2 = 0.27$ GeV^2	Γ(keV)	Γ(keV) with $m_u = 0.336$ GeV, $m_c = 1.65$ GeV, $\alpha'^2 = 0.27$ GeV^2	Γ(keV) experimental
$\psi' \to \gamma\chi(3414)$	42	31	-	30.9	23 ± 8
$\psi' \to \gamma\chi(3508)$	35	27	-	51.8	21 ± 8
$\psi' \to \gamma\chi(3552)$	24	19	-	58.7	18 ± 7
$\chi(3414) \to \gamma\psi$	160 → 90	225	94	28.9	-
$\chi(3508) \to \gamma\psi$	340 → 230	470	110	32.2	-
$\chi(3552) \to \gamma\psi$	485 → 320	625	120	10.9	-

The second values in the second col. are those given by Eichten et al. [95b] which correspond to the correction when the charmed meson continuum is taken into consideration in the spectrum of charmonium.

3.7 Discussion of the Results and Conclusions

As mentioned earlier, our results are dependent upon our assumptions about the charges, masses and magnetic moments of the quarks. For the charmed quark, we have assumed that its magnetic moment is smaller than those of the ordinary quarks by a factor of $\frac{m_u}{m_c}$ (magnetic moments of the ordinary quarks are usually taken as $\mu(\frac{q_i}{e})$ (i = 1,2,3) where $\mu = \mu_P = 2.79 \frac{e}{2m_P}$). This latter assumption itself is consequent upon the supposition that the magnetic moment of the quark is inversely proportional to its mass. We also assume that $Q_c = \frac{2}{3}e$ (the non-relativistic treatment of the new particles (the ψ-sector) generally takes $Q_c = \frac{2}{3}e$ and $m_c = 1.3 - 2$ GeV [65]). We have performed our calculations with three sets of parameters with m_c in the range of 1.3 - 2 GeV. Alternatively, m_u can be varied but that will disturb the conventional picture of the "old" hadrons where $\mu = \mu_P$ seems to be desirable. As can be noted by inspection of Table 3.2, we get reasonably good decay rates for the transitions $\psi' \to \gamma\chi_J$'s with the first and third set of parameters. Taking the simplicity of our model into consideration, even the results with the second set of parameters cannot be discarded altogether. As the level spacing of charmonium (and masses) demand a bigger value for α'^2 (certainly bigger than 0.1 GeV^2), $\alpha'^2 = 0.27$ GeV^2 seems to be a reasonably (still crude) good approximation [94]. In general, a bigger α'^2-value demands a bigger mass for the charmed quark (or alternatively a smaller mass for the ordinary u-type quark) in order to keep the decays for the transitions $\psi' \to \gamma\chi_J$'s within the experimental limits [70] or as good as the quark model results for the radiative decay rates of the "old" mesons (see Chapter 4).

We have considered two possibilities for the quantum numbers of the states $\chi(3414)$, $\chi(3508)$ and $\chi(3552)$. Firstly, we assume that they are $J^{PC} = 0^{++}$, 1^{++} and 2^{++} respectively and secondly that they are $J^{PC} = 2^{++}$, 1^{++} and 0^{++} respectively. The spin orbit interaction of the gluon theory [94] implies an ordering of the P states in energy with increasing

J while the second possibility for the quantum numbers is favoured by the non-relativistic spin-orbit potential [69c]. Also, the fact that $\chi(3414)$ is observed to decay to $\pi\pi$ or $K\bar{K}$ requires that it should have $J = 0$ or 2. Inspection of the Tables 3.2 and 3.3 shows that the present experimental data for $\Gamma(\psi' \to \gamma\chi_J$'s [70] favour the choice $J^{PC} = 0^{++}, 1^{++}, 2^{++}$ for $\chi(3414)$, $\chi(3508)$, $\chi(3552)$ in that order. For the decay rates of the transitions χ_J's $\to \gamma\psi$, we have no experimental data available and their measurement will throw more light on such considerations. It will also be a good test of our model.

The transitions $\psi' \to \gamma\chi_J$'s and χ_J's $\to \gamma\psi$ have also been considered by Eichten et al. [95]. They simulate the $c\bar{c}$ interaction by a simple potential that incorporates both the Coulomb and confinement forces

$$V(r) = -(\alpha_s/r)\ [1 - (^r/a)^2] \quad , \qquad (3.12)$$

The first term (the Coulomb term) reflects the behaviour of massless gluon exchange between quarks at small distances while the second term is meant to confine the quarks inside a hadron. One has to know three parameters in order to reproduce the features of the psion spectrum (they are, the quark mass m_c, the strong coupling constant α_s and the parameter a). This has been done by many people [65,95]. The transition rates in ref. 95 were calculated for an assumed mass of 3.45 GeV for all the three states χ_J's and the rates $\Gamma(\psi' \to \gamma\chi_J$'s) are proportional to (2J+1). Now the rates are to be corrected by the factor $(\frac{k}{\bar{k}})^3$ where k is the actual momentum of the photon and $\bar{k}$ is the mean value (i.e. $\bar{k}$ = 0.23 GeV) used in ref. 95. Similarly, the decay widths of the transitions χ_J's $\to \gamma\psi$ are to be corrected by the factor $(^q/\bar{q})^3$ where q is the actual momentum of the photon and $\bar{q}$ is the mean value (i.e. $\bar{q}$ = 0.36 GeV) as used in ref. 95. These results are compared with those of ours in Table 3.6.

Jackson [94] too, has calculated the widths of the transitions $\psi' \to \gamma\chi_J$'s and χ_J's $\to \gamma\psi$ with the use of the harmonic oscillator wavefunctions and the E1 (electric dipole) approximation formula

$$\Gamma(\mathrm{E1}) = \frac{4\alpha}{27} Q_c^{\,2} (2\,J_f + 1) k^3 \left|\langle f|r|i\rangle\right|^2 \tag{3.13}$$

where $Q_c = \frac{2}{3}e$, J_f is the spin of the final state particle, k is the momentum of the photon in the rest frame of the decaying particle and $\alpha = {}^1/137$. The decay widths of the transitions $\psi' \to \gamma\chi_J$'s and χ_J's $\to \gamma\psi$ (with $m_c = 1.65$ GeV and $\alpha'^2 = 0.27$ GeV^2) are shown in Table 3.6. The results in column 5 are our results (see Tables 3.2 - 3.5) with the same values of m_c and α'^2 (i.e. $m_c = 1.65$ GeV and $\alpha'^2 = 0.27$ GeV^2) and m_u = 0.336 GeV. The decay rates for both the transitions $\psi' \to \gamma\chi_J$'s and χ_J's $\to \gamma\psi$ are very sensitive (according to our calculations) to the change in the value of α'^2 (see Appendix B). The difference between our results and those of Eichten et al. and Jackson is mainly due to the use of different formulas (compare 3.11 and 3.13) and also because we have considered all the relevant matrix elements separately and without any approximation such as dipole approximation as all the algebra can be done without such a procedure. Moreover, for the meson states, we follow the usual procedure of combining the orbital angular momentum (L) with the intrinsic quark spin (S) to get the physical spin J. And thus we have only one (non-vanishing) matrix element for $\psi' \to \gamma\chi(3414)$ (and $\chi(3414) \to \gamma\psi$), two (non-vanishing) matrix elements for $\psi' \to \gamma\chi(3508)$ (and $\chi(3508) \to \gamma\psi$) and three (non-vanishing) matrix elements for $\psi' \to \gamma\chi(3552)$ (and $\chi(3552) \to \gamma\psi$), when $\chi(3414)$, $\chi(3508)$, $\chi(3552)$ are taken as $J^{PC} = 0^{++}\ 1^{++}\ 2^{++}$ states in that order. These matrix elements (in our case) are very complicated functions of k and α'^2 (see Appendix B) as compared to the ones involved in formula (3.13) (given by $\langle\chi_J\text{'s}|\gamma|\psi'\rangle = {}^1/\alpha$ and $\langle\psi|\gamma|\chi_J\text{'s}\rangle = \sqrt{\frac{3}{2\alpha}}$).

Jackson has also used dipole sum rules [94] to estimate the upper and lower bounds for the radiative and total decay widths of $\chi(3414)$, $\chi(3508)$ and $\chi(3552)$. Our results for χ_J's $\to \gamma\psi$ are far below even from his lower bounds on these transitions (see Table 3.6). The two dipole sum rules explicitly read as follows

$$2\mu \sum_j \omega_{ji} |\langle j|\underline{r}|i\rangle|^2 = 3 \qquad (3.14)$$

and

$$2\mu \sum_n \omega_{ns,2p} |\langle ns|\underline{r}|2p\rangle|^2 = -1 \qquad (3.15)$$

where μ is the reduced mass, $\omega_{ji} = E_j - E_i$ and the matrix element is that of the vector $\underline{r}$. If one assumes that the transitions $\psi \leftrightarrow \gamma\chi_J$'s saturates the sum rule then from (3.14)

$$|\langle 2p|r|1s\rangle|^2 \leq \frac{3}{2\mu k} \quad . \qquad (3.16)$$

In terms of the radiative states for χ_J's $\to \gamma\psi$ (from 3.13), this is equivalent to an upper bound

$$\Gamma(\chi_J \to \gamma\psi) < \frac{2\alpha k^2}{3\mu} Q_c^2 \quad . \qquad (3.17)$$

Similarly, from (3.15) one gets a lower bound of the form

$$\Gamma(\chi_J \to \gamma\psi) \geq \frac{2\alpha k_2^2}{q\mu} Q_c^2 + \frac{3}{2J+1} \left(\frac{k_2}{k_1}\right)^2 \Gamma(\psi' \to \gamma\chi_J) \qquad (3.18)$$

where k_1 and k_2 are the momenta of the photon in the processes $\psi' \to \gamma\chi_J$'s and χ_J's $\to \gamma\psi$ respectively. "Absolute" lower bounds on the decays are shown in column 4 of Table 3.6 for comparison.

The branching ratios for the three transitions $\psi' \to \gamma\chi_J$'s appear as $(7 \pm 2)\%$ in the particle data note book [46] and our results are in good agreement with these values. Our results also agree with the ones given by Badke et al. [96] and Luth et al. [97]. The branching ratios for $\psi' \to \gamma\chi_J$'s given there can be converted into radiative widths by using $\Gamma_t = 228$ keV [97]; one gets $\Gamma(\psi' \to \gamma\chi_J) = 17.5 \pm 6$ keV, 20 ± 7 keV and 18 ± 7 keV for the $J^{PC} = 0^{++}$, 1^{++} and 2^{++} states respectively. So on the whole we find that $\chi(3414)$, $\chi(3508)$ and $\chi(3552)$ taken as $J^{PC} = 0^{++}$, 1^{++} and 2^{++} states yield good results for the transitions $\psi'(3684) \to \gamma\chi_J$'s within the framework of the model we have used. The situation with the decays χ_J's $\to \gamma\psi(3095)$ is rather unclear. We get extremely small results for these rates as compared to the bounds set on these rates by Jackson [94].

We have also considered $\psi(3095) \rightarrow \gamma X(2.8)$ within the framework of the model used for other decays. It is an M1 decay and like the E1 decays we have not used the magnetic dipole approximation, i.e. we have not put $\exp(i\underset{\sim}{k}.\underset{\sim}{r}_i) = 1$. As in all the other decay calculations we have taken the terms in the expansion of $\exp(i\underset{\sim}{k}.\underset{\sim}{r}_i)$ which are compatible with the conservation of angular momentum (otherwise the overlap integral vanishes because of a property of the spherical harmonics). As for other models, the transition $\psi(3095) \rightarrow \gamma X(2.8)$ cannot be described reasonably if $X(2.8)$ is taken as the pseudoscalar partner of the vector $\psi(3095)$. We get a very large value of $\Gamma(\psi \rightarrow \gamma X(2.8)) = 18.35$ keV ($\Gamma_{exp} < 2$ keV). In Chapter 4 we consider broken SU(4) symmetry to estimate the mixture of $(p\bar{p} + n\bar{n})$ in the states $\psi(3095)$ and $\eta_c[X(2.8)]$. But as we shall find there, the situation is not improved by such considerations. There we also find that the particles η, η' and $X(2.8)$ taken together (in one multiplet) is a problem for the naive quark model. We cannot get consistent values (according to experiment to date) for the transitions $\psi(3095) \rightarrow \gamma P$ and $\psi'(3684) \rightarrow \gamma P$ (where P stands for η, η' and $X(2.8)$) with any tolerably acceptable assumptions. We discuss these problems in some more detail in Chapter 4.

We conclude this chapter with the remark that the experimental measurement of the decay rates of the transitions χ_J's $\rightarrow \gamma\psi(3095)$ seems very important for our model, particularly because the transitions $\psi'(3684) \rightarrow \gamma\psi_J$'s are so beautifully described by it.

CHAPTER 4

MASS MIXING IN BROKEN SU(4) AND THE RADIATIVE WIDTHS OF MESONS IN THE HARMONIC OSCILLATOR QUARK MODEL

4.1 Introduction

The renewed interest in the group SU(4) appears to suggest a recurring theme in High Energy Physics, which started with the introduction of SU(2) in the thirties to describe the charge states of the known particles at the time, namely, the proton, the neutron and the pion. This led to the concept of the conservation of the third component of isotopic spin I_3. The discovery of strange particles was incorporated in the higher symmetry group SU(3) and introduced a new conserved quantum number S, corresponding to the conservation of the strangeness number of hadrons in strong interactions. With the discoveries of the $\psi(3095)$ at SPEAR [63] and BNL [62] and the $\psi'(3684)$ at SLAC (and a number of intermediate states), we may be seeing the theme recurring again [73]. Thus, we may already have seen the first pieces of the broken higher symmetry group SU(4), which is the natural extension of SU(2) and SU(3). We are then led to consider a new additive quantum number called charm [60], which has a zero eigenvalue for the well known particles of SU(3), like the ρ, K^*, ω, ϕ etc. That SU(4) is indeed the correct extension that describes the ψ's seems to be verified because resonances (thought to be) carrying non-zero charm number, namely $D(D^*)$ have been discovered (and found places in the particle data notebook).

The charm schemes were first designed for use in theories of weak interactions, for they remove the strangeness changing contributions in the weak neutral currents [60]. In the Cabibbo theory of weak interactions the charge positive currents can be formally written [65d] at quark level, as

$$J^+ \sim \bar{p}(n \cos\theta_c + \lambda \sin\theta_c) \tag{4.1}$$

using $$n_c = n\cos\theta_c + \lambda\sin\theta_c \tag{4.2}$$

$$\lambda_c = n\sin\theta_c - \lambda\cos\theta_c \tag{4.3}$$

one gets

$$J^+ \sim \bar{p}\,\hat{J}^+ n_c = (\bar{p}\ \bar{n}_c)\begin{pmatrix}0 & 1\\ 0 & 0\end{pmatrix}\begin{pmatrix}p\\ n_c\end{pmatrix} \tag{4 4}$$

and

$$J^- \sim \bar{n}_c\,\hat{J}^- p = (\bar{p}\ \bar{n}_c)\begin{pmatrix}0 & 0\\ 1 & 0\end{pmatrix}\begin{pmatrix}p\\ n_c\end{pmatrix} \tag{4.5}$$

The V-A structure is neglected because we are considering only the symmetry properties of the currents.

In a gauge theory of weak interactions [65d], one expects a neutral current for which

$$\hat{J}_Z = [\hat{J}^+, \hat{J}^-] = \begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix}$$

so that $$J_Z \sim (\bar{p}\ \bar{n}_c)\begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix}\begin{pmatrix}p\\ n_c\end{pmatrix} = \bar{p}p - \bar{n}_c n_c \tag{4.6}$$

Substituting for n_c, one gets

$$J_Z \sim \bar{p}p - \bar{n}n\cos^2\theta_c - \bar{\lambda}\lambda\sin^2\theta_c - \sin\theta_c\cos\theta_c(\bar{n}\lambda + \bar{\lambda}n) \tag{4.7}$$

The unpleasant feature of this neutral current is the presence of strangeness changing neutral current with the strength of the same order as the strangeness changing charged currents. Experimentally, a large number of neutral current effects have been observed in the scattering of ν and $\bar{\nu}$ on nucleons and electrons but strong limits apply to the strangeness changing neutral current processes (like $K \to \mu\bar{\mu}$ and $K^- \to \pi^-\nu\bar{\nu}$ etc.). In order to know the working of such strong limits at quark level, it is necessary to look at strangeness changing neutral currents such as the decay $K^- \to \pi^-\nu\bar{\nu}$, for example. In the quark model of hadrons the K^- is made of $\bar{p}$ and λ-quarks and this decay $(K^- \to \bar{\pi}\nu\bar{\nu})$ would require the λ-quark to change into a n-quark by the neutral current (hadronic) while the leptonic neutral current decays into ν and $\bar{\nu}$. That is, however, not seen and therefore Cabibbo theory combined with the weak interaction gauge theories cannot be right.

In order to suppress the $\Delta S \neq 0$ neutral current, Glashow, Iliopoulos and Maiani [60] exploited the idea of a charmed quark (c) that had been introduced almost a decade ago by Hara [74] and also by Bjorken and Glashow [59]. The charged weak currents will be postulated to be, as an immediate extension of the previous case as follows:

$$J^{+} \sim \bar{p}n_c + \bar{c}\lambda_c = (\bar{p}\ \bar{n}_c;\ \bar{c}\ \bar{\lambda}_c) \begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} p \\ n_c \\ c \\ \lambda_c \end{pmatrix} \tag{4.8}$$

$$J^{-} \sim (\bar{p}\ \bar{n}_c;\ \bar{c}\ \bar{\lambda}_c) \begin{pmatrix} 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} p \\ n_c \\ c \\ \lambda_c \end{pmatrix} \tag{4.9}$$

and thus

$$\begin{aligned} J_z &\sim (\bar{p}\ \bar{n}_c;\ \bar{c}\ \bar{\lambda}_c) \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} \begin{pmatrix} p \\ n_c \\ c \\ \lambda_c \end{pmatrix} \\ &\sim \bar{p}p - \bar{n}_c n_c + \bar{c}c - \bar{\lambda}_c\lambda_c \\ &\sim \bar{p}p - \bar{n}n - \bar{\lambda}\lambda + \bar{c}c \end{aligned} \tag{4.10}$$

We note that the piece $(\bar{n}\lambda + \bar{\lambda}n)\sin\theta\cos\theta$ that arose from $\bar{n}_c n_c$ has been exactly cancelled by a similar piece in $\bar{\lambda}_c\lambda_c$, thus eliminating the $\Delta S \neq 0$ neutral currents as required by experiment.

4.2 The Group SU(4)

This introduction to SU(4) will serve to provide us with a brief review of the classification of hadronic states (particularly mesons with which we shall concern ourselves in the rest of this chapter) and will give the notations and conventions used in this thesis.

We assume that the wavefunctions for hadrons can be constructed from tensor products of the fundamental quartets $\underline{4}$ (denoted variously by p, n, λ, c or u, d, s, c or q_1, q_2, q_3, q_4) and conjugate quartets $\bar{\underline{4}}$ of SU(4) [61]. Baryons, constructed from three such quartets carry the irreducible representations of SU(4) [42]

$$\underline{4} \otimes \underline{4} \otimes \underline{4} = \underline{4} + 2(\underline{20'}) + \underline{20} . \qquad (4.11)$$

The $J^P = \frac{1}{2}^+$ baryons are assigned to the representation $\underline{20}'$. The $J^P = {}^3/2^+$ baryons are placed in $\underline{20}$ which contains a decuplet with $C = 0$.

Mesons presumably are products

$$\underline{4} \otimes \overline{\underline{4}} = \underline{1} + \underline{15} . \qquad (4.12)$$

The SU(3) decomposition of SU(4) representations are given as follows

$$\begin{aligned} \underline{4} &= \underline{1} + \underline{3} \\ \underline{20'} &= \underline{3} + \overline{\underline{3}} + \underline{6} + \underline{8} \\ \underline{20} &= \underline{1} + \underline{3} + \underline{6} + \underline{10} \\ \underline{15} &= \underline{1} + \underline{3} + \overline{\underline{3}} + \underline{8} . \end{aligned} \qquad (4.13)$$

We shall mainly be concerned with the $\underline{15} \oplus \underline{1}$ of SU(4).

4.3 Classification of Mesons in SU(4)

To apply our model later, we shall need the SU(2) decomposition of $\underline{1}$, $\underline{3}$, $\overline{\underline{3}}$ and $\underline{8}$ (of SU(3)) and also the quark-antiquark contents of the different states. It can be easily shown that the octet and the singlet are the usual octet and singlet of SU(3) with $C = 0$; $\underline{3}$ and $\overline{\underline{3}}$ have $C = -1$ amd $C = +1$ respectively. The mixing of the three isosinglets (contained in $\underline{15} + \underline{1}$) depends on the nature of SU(4)-breaking [61] which we take up in the next section.

In Table 4.1 we give the quark contents of the lowest lying meson states assigned to $\underline{15} + \underline{1}$ of SU(4) when there is no mixing (the ideal case of exact SU(4)). For completeness the identification is given for both the old and the new mesons.

Except for the isosinglet states, the wavefunctions of Table 4.1 hold for the tensor mesons as well (see section 4.5). In SU(3), the two isospin zero members of the vector or tensor nonets are mixed and are given (usually) as follows

$$\phi = \bar{s}\, s \qquad (4.14a)$$

$$\omega = (\bar{u}u + \bar{d}d)/\sqrt{2}\ . \qquad (4.14b)$$

Similarly with the same mixing angle ($\theta \simeq 35^{\circ}$) the tensor mesons would be given by

$$f' = \bar{s}\, s \qquad (4.15a)$$

$$f = (\bar{u}u + \bar{d}d)/\sqrt{2}\ . \qquad (4.15b)$$

Experimentally $\theta = 40^{\circ}$ for the vector mesons and $\theta = 30^{\circ}$ for the tensor mesons [46]. Thus to a fairly good approximation the physical states can be represented by the wavefunctions as given above.

4.4 Diagonalization of the Mass Matrix

Borchardt, Mathur and Okubo [75] have made predictions for baryon and meson masses based on the SU(4) symmetry-breaking interactions

$$H' = T_8 + a\, T_{15} \qquad (4.16)$$

where T_8 and T_{15} belong to the same 15-dimensional representation of SU(4).

The ψ(3095) is assigned together with the usual ρ, K^*, ω and ϕ to the representation

$$\underline{4} \otimes \underline{\bar{4}} = \underline{15} + \underline{1}\ .$$

The pseudoscalar mesons are assigned to another $\underline{15} + \underline{1}$ representation of SU(4). The SU(3) decomposition of $\underline{15}$ is

$$\underline{15} = \underline{1} + \underline{3} + \underline{\bar{3}} + \underline{8}\ .$$

TABLE 4.1 The Classification of Mesons in SU(4)

Label Pseudoscalar (Vector)	Representation SU(4) (SU(3))	Quark Content	Charm Quantum No.	Strangeness	Isospin I (I_3)
K^+ (K^{+*})	$\underline{15}$ ($\underline{8}$)	$\bar{s}u$	0	1	$\frac{1}{2}$ ($\frac{1}{2}$)
K^o (K^{o*})	"	$\bar{s}d$	"	1	$\frac{1}{2}$ ($-\frac{1}{2}$)
$\pi^+(\rho^+)$	"	$\bar{d}u$	"	0	1 (1)
$\rho^o(\rho^o)$	"	$(-\bar{u}u + \bar{d}d)/\sqrt{2}$	"	0	1 (0)
$\pi^-(\rho^-)$	"	$\bar{u}d$	"	0	1 (-1)
$\bar{K}^{o}(\bar{K}^{o*})$	"	$\bar{d}s$	"	-1	$\frac{1}{2}$ ($\frac{1}{2}$)
$K^-(K^{-*})$	"	$\bar{u}s$	"	-1	$\frac{1}{2}$ ($-\frac{1}{2}$)
η	15 ($\underline{8}$)	$\frac{1}{\sqrt{6}}(\bar{u}u + \bar{d}d - 2\bar{\lambda}\lambda)$	"	0	(0)
η'	1 ($\underline{1}$)	$\frac{1}{2}(\bar{u}u + \bar{d}d + \bar{\lambda}\lambda + \bar{c}c)$	"	0	(0)
$\eta_c(\psi)$	$\underline{15}$ ($\underline{1}$)	$\frac{1}{\sqrt{12}}(\bar{u}u + \bar{d}d + \bar{\lambda}\lambda - 3\bar{c}c)$	"	0	0

TABLE 4.1 (Contd.)

Label Pseudoscalar (Vector)	Representation SU(4) (SU(3))	Quark Content	Charm Quantum No.	Strangeness	Isospin I (I_3)
D^+ (D^{+*})	$\underline{15}$ ($\underline{\bar{3}}$)	$c\,\bar{d}$	1	0	$\frac{1}{2}$ ($\frac{1}{2}$)
D^o (D^{o*})	$\underline{15}$ ($\underline{\bar{3}}$)	$c\,\bar{u}$	1	0	$\frac{1}{2}$ ($-\frac{1}{2}$)
F^+ (F^{+*})	$\underline{15}$ ($\underline{\bar{3}}$)	$c\,\bar{s}$	1	+1	0 (0)
$\bar{D}^o$ ($\bar{D}^{o*}$)	$\underline{15}$ ($\underline{3}$)	$\bar{c}\,u$	-1	0	$\frac{1}{2}$ ($\frac{1}{2}$)
D^- (D^{-*})	$\underline{15}$ ($\underline{3}$)	$\bar{c}\,d$	-1	0	$\frac{1}{2}$ ($-\frac{1}{2}$)
F^- (F^{-*})	$\underline{15}$ ($\underline{3}$)	$\bar{c}\,s$	-1	-1	0

For the vector mesons $(i = 0,1, \ldots 15)$ the squared mass matrix for the $\underline{1} + \underline{15}$ representation can be written as follows.

$$(m^2)_{ij} = \overline{m}^2 \delta_{ij} + A(d_{i8j} + a\, d_{i15j})$$

$$(m^2)_{oi} = B(\delta_{8i} + a\, \delta_{15i})$$

$$(m^2)_{oo} = \overline{m}_o{}^2 \qquad (4.17)$$

where $\overline{m}^2$ and $\overline{m}_o{}^2$ are the SU(4) invariant squared masses of the regular representation $\underline{15}$ and the singlet representation, respectively and A and B are the reduced matrix elements. d_{ijk} (f_{ijk}) are the SU(4)-coefficients.

In the following, we have independently considered the mass diagonalization problem both for the $J^{PC} = 1^{--}$ (15+1)-plet of vector mesons and $J^{PC} = 0^{-+}$ (15+1)-plet of pseudoscalar mesons [76].

The mass-matrix to be diagonalized is a 3×3 matrix given by

$$m^2 = \begin{pmatrix} (m^2)_{00} & (m^2)_{08} & (m^2)_{015} \\ (m^2)_{08} & (m^2)_{88} & (m^2)_{815} \\ (m^2)_{015} & (m^2)_{815} & (m^2)_{1515} \end{pmatrix} \qquad (4.18)$$

In the case of the (15+1)-plet of vector mesons, the five unknown constants can be found out by using the masses of ρ, K^*, ω, ϕ and ψ. We obtain the following values

$$\overline{m}^2 = 2.78\ (\text{GeV})^2, \qquad A = -0.234\ \text{GeV})^2,$$

$$a = 21.363\ , \qquad \overline{m}_o{}^2 = 3.507\ (\text{GeV})^2,$$

$$B = -0.191\ (\text{GeV})^2 \quad . \qquad (4.19)$$

The analysis yields the following eigenvectors (spin unitary states) for the vector mesons.

$$\rho^{o} = \frac{1}{\sqrt{2}}(p\bar{p} - n\bar{n})$$

$$K^{*o} = n\,\bar{\lambda}$$

$$\omega = 0.705\ (p\bar{p} + n\bar{n}) - 0.054\ \lambda\bar{\lambda} - 0.051\ c\bar{c}$$

$$\phi = 0.037\ (p\bar{p} + n\bar{n}) + 0.998\ \lambda\bar{\lambda} - 0.044\ c\bar{c}$$

$$D^{*o} = c\,\bar{p}$$

$$F^{*} = c\,\bar{\lambda}$$

$$\psi = 0.037\ (p\bar{p} + n\bar{n}) + 0.04\lambda\bar{\lambda} + 0.998\ c\bar{c} \qquad (4.20)$$

Knowing the value of the parameter a in (4.17), we can also solve the mixing problem for the pseudoscalar mesons and the mass of the pseudoscalar partner of ψ(i.e. η_c) can be predicted. Our calculations give

$$m_{\eta_c} = 2.74\ \text{GeV}$$

when $\eta(549)$ and $\eta'(958)$ are taken as partners.

The mass squared matrix for the pseudoscalar mesons takes the same form as for the vector mesons, but the parameters are now

$$\overline{m^2} = 2.464\ (\text{GeV})^2\ , \qquad A = -0.263\ (\text{GeV})^2$$

$$a = 21.363 \qquad \bar{m}_o^{\,2} = 1.381\ (\text{GeV})^2$$

$$B = -0.081\ (\text{GeV})^2\ . \qquad (4.21)$$

The physical eigenstates for the $J^P = 0^-$ mesons are then given as the following

$$\pi^{o} = \frac{1}{\sqrt{2}}(p\bar{p} - n\bar{n})$$

$$K^{o} = n\,\bar{\lambda}$$

$$\eta = 0.502(p\bar{p} + n\bar{n}) - 0.704\ \lambda\bar{\lambda} + 0.033\ c\bar{c}$$

$$\eta' = 0.477(p\bar{p} + n\bar{n}) + 0.695\ \lambda\bar{\lambda} + 0.246\ c\bar{c}$$

$$D = c\,\bar{p}$$

$$F = c\,\bar{\lambda}$$

$$\eta_c = 0.138(p\bar{p} + n\bar{n}) + 0.153\lambda\bar{\lambda} + 0.968\ c\bar{c} \qquad (4.22)$$

In Table 4.2 we show the masses obtained from the diagonalization procedure. The significant feature lies in the fact that it gives the masses of D and F (pseudoscalar mesons) lower than those of the corresponding vector mesons D^* and F^*. This is in agreement with the calculations of Borchardt et al. [75] but in contradiction with those of D.H. Boal et al. [77]. Another remarkable point is the fact that the coefficients of $(p\bar{p} + n\bar{n})$ and $\lambda\bar{\lambda}$ are very sensitive functions of the input masses. In particular, using $m_\rho = 760$ MeV (in the particle data notebook $m_\rho = (770 \pm 10$ MeV) we get the following eigenstates for ω, ϕ, and ψ (to be compared with the previous ones)

$$\begin{aligned} \omega &= 0.706(p\bar{p} + n\bar{n}) - 0.044\, \lambda\bar{\lambda} + 0.013\, c\bar{c} \\ \phi &= 0.032(p\bar{p} + n\bar{n}) + 0.999\, \lambda\bar{\lambda} + 0.011\, c\bar{c} \\ \psi &= 0.01(p\bar{p} + n\bar{n}) + 0.011\, \lambda\bar{\lambda} - 0.999\, c\bar{c} \end{aligned} \tag{4.23}$$

We have also considered the case when E(1420) is taken as the partner of $\eta(549)$. Using the same value of a ($a = 21.363$), the mass of the pseudoscalar partmer of ψ (i.e. η_c) comes out to be

$$m_{\eta_c} = 3.026 \text{ GeV}.$$

So the intermediate charmonium state $\chi(3.455)$ (which is thought to be a $J^{PC} = 0^{-+}$) cannot be a good candidate for partnership with $\eta(549)$ and E(1420) and there is no other 0^{-+} state in this region of mass. In the following we shall look upon $\chi(3.455)$ as the pseudoscalar partner of $\psi'(3684)$ and a pure $c\bar{c}$ state (like $\psi'(3684)$ and the other χ_J's). The mass of the charmed mesons (D's and F's) are not affected by this choice (i.e. when E(1420) is taken as the partner of $\eta(549)$. It is because their masses are determined by the masses of π and K according to our calculations. The parameters in this case are given as follows:

$$\begin{aligned} \bar{m}^2 &= 2.464 \text{ GeV}^2, \quad & A &= -0.263 \text{ GeV}^2 \\ a &= 21.363 & \bar{m}_o^{\,2} &= 4.101 \text{ GeV}^2, \\ B &= -0.153 \text{ GeV}^2 \end{aligned} \tag{4.24}$$

TABLE 4.2

Predicted masses obtained from the diagonalization of the mass squared matrix

Particle		Predicted Mass (MeV)	Experimental Mass (MeV)
$J^P = 1^-$	ρ^o	770†	770 ± 10[a]
	K^{*o}	892†	892.2 ± 0.5[a]
	ω	783†	782.7 ± 0.3[a]
	ϕ	1020†	1019.7 ± 0.3[a]
	D^{*o}	2180	2010[a]
	F^*	2226	-
	ψ	3095†	3098 ± 0.3[a]
$J^P = 0^-$	π^o	135†	135.96[a]
	K	496†	495.7[a]
	η	549†	548.8 ± 0.6[a]
	η'	958†	957.6 ± 0.3[a]
	D^o	2164	1876 ± 15[a]
	F	2216	-
	η_c	2.745	-

(a) See Ref. 46.

†These masses used as input

The physical eigenstates for the $J^P = 0^-$ mesons now are given as follows (only the eigenstates of η, E and η_c are affected).

$$\eta = 0.454(p\bar{p} + n\bar{n}) - 0.766\ \lambda\bar{\lambda} + 0.0037\ c\bar{c}$$

$$E = 0.528(p\bar{p} + n\bar{n}) + 0.663\ \lambda\bar{\lambda} - 0.05\ c\bar{c}$$

$$\eta_c = -0.027(p\bar{p} + n\bar{n}) - 0.027\ \lambda\bar{\lambda} - 0.999\ c\bar{c} \qquad (4.25)$$

In Sec. 4.8, we use the states given by (4.20) for vector mesons and the states given by (4.22) for the pseudoscalar mesons.

4.5 Mass Diagonalization for Tensor Mesons

The branching ratio of the transition $\psi \to \gamma f(1270)$ has recently been measured experimentally [78]. It is with a view to determining the decay rate of this transition within the framework of our model, that we have tackled the mass diagonalization problem of $J^{PC} = 2^{++}$ mesons classified according to the (15+1)-plet of SU(4). f(1270) is a tensor meson with quantum numbers $J^{PC} = 2^{++}$. Now we have a sufficient number of particles belonging to the $J^{PC} = 2^{++}$ (15+1)-plet of SU(4), to enable us to carry out our diagonalization procedure and to find the $c\bar{c}$ contents of the particles f(1270) and f(1514) (the particles $A_2(1310)$, $K^*(1420)$, f(1270), f'(1514) and $\chi(3552)$ are assumed to be belonging to the (15+1)-plet of SU(4) and these are sufficient to provide us with the eigenvectors which we need for the calculation of the decay rate of the transition $\psi(3095) \to \gamma f(1270)$).

There are two ways to determine the eigenvectors of the states f(1270), f'(1514) and $\chi(3552)$ (J = 2?) which can mix in the broken SU(4). Firstly, one can use the value of a, determined from the masses $J^P = 1^-$ vector mesons (a = 21.363 in our case). In this case one predicts the mass of the χ which is mainly a $c\bar{c}$ state. With linear mass formula

Mathur et al. [75] find $m(\chi(c\bar{c})) = 3.414$ GeV and with quadratic mass formula, $m(\chi(c\bar{c})) = 3.8$ GeV. None of these two values give credence to the wider speculations that $\chi(3552)$ is a tensor meson (there is one state $X(3414)$ which has recently been confirmed to be a $J^{PC} = 0^{++}$ state - see Chapter 3 on new particles). Secondly, one can use the masses of the three tensors $f(1270)$, $f'(1514)$ and $\chi(3552)$ (along with those of $A_2(1310)$ and $K^*(1420)$, determine a and calculate the eigenvectors of $f(1270)$, $f'(1514)$ and $\chi(3552)$. It is this approach which we have adopted in this section.

We obtain the following values for the five unknown parameters

$$\begin{aligned} \overline{m}^2 &= 4.524(\text{GeV})^2, \qquad & A &= -0.347\ (\text{GeV})^2 \\ a &= 18.432 & \overline{m}_o^{\,2} &= 4.665\ (\text{GeV})^2 \\ B &= -0.259\ (\text{GeV})^2\ . \end{aligned} \tag{4.26}$$

The analysis yields the following eigenvectors for the mixed tensor mesons.

$$\begin{aligned} f(1270) &= 0.7046(p\bar{p} + n\bar{n}) + 0.0824\ \lambda\bar{\lambda} - 0.0146\ c\bar{c} \\ f'(1514) &= 0.0588(p\bar{p} + n\bar{n}) - 0.9964\ \lambda\bar{\lambda} + 0.01\ c\bar{c} \\ \chi(3552) &= 0.0099(p\bar{p} + n\bar{n}) + 0.0113\ \lambda\bar{\lambda} + .999\ c\bar{c} \end{aligned} \tag{4.27}$$

The decay rate of the transition $\psi \rightarrow \gamma f(1270)$ can now be easily calculated (Section 4.8).

4.6 Photon Transitions of Mesons

In Chapter 3 we calculated the radiative decay widths of the so-called new particles (charmonium) in the framework of the harmonic oscillator quark model with certain assumptions about the masses, charges and magnetic moments of the quarks. Both the initial and final states were taken as pure $c\bar{c}$ states. There, we considered the following three sets of decays.

$$\psi'(3684) \rightarrow \begin{cases} \gamma\chi(3414) \\ \gamma\chi(3508) \\ \gamma\chi(3552) \end{cases} \qquad \text{(E1 transitions)}$$

$$\left.\begin{matrix} \chi(3414) \\ \chi(3508) \\ \chi(3552) \end{matrix}\right\} \rightarrow \gamma\psi(3095) \qquad \text{(E1 transitions)}$$

$$\psi(3095) \rightarrow \gamma\eta_c(X(2.8)) \qquad \text{(M1 transition)}$$

These results, as mentioned earlier, are highly dependent on the assumptions about the masses, charges and magnetic moments of the quarks. We have good agreement with experiment in the case of the first set of transitions, for the second set of decays, no experimental data is available to compare our results with, and the decay width of the transition $\psi(3095) \rightarrow \gamma X(2.8)$ comes out to be much larger as compared to the experiment.

In the following we shall extend our model to the calculations of the radiative widths of the lowest lying mesons (Section 4.8) and the "old" $L = 1$ mesons (Section 4.9). For the latter meson (old) decays, we have no experimental data and our results are to be tested against future experiments [79].

4.7 Procedure and Assumptions

The procedure and assumptions are almost the same as adopted in Chapter 3 for calculations of the decay widths for the process $c\bar{c} \rightarrow \gamma + c\bar{c}$ except that now both the c-type quark and ordinary quarks are involved. As in Chapter 3 the c-type quark is taken to be heavier than the ordinary quarks which themselves are taken as degenerate. The magnetic moment (as before) of the c-type quark is taken to be smaller than those of the usual quarks. Following Becchi et al. [21], we take

$$\mu_i = \mu_p \left(\frac{q_i}{e}\right) \sigma_i \qquad (4.28)$$

where μ_i is the magnetic moment of the i-th quark, $i = 1, 2$ specifying the kind of quark (respectively, the p-type quark and the n-type quark) and $(q_1, q_2) = (\frac{2}{3}, -\frac{1}{3})$ are the charges of the p-type and n-type quark respectively. In (4.28) μ_p is the magnetic moment of the proton, $\mu_p = 2.79 \frac{e}{2m_p}$ where m_p is the proton mass ($m_p = 0.938$ GeV). As emphasized in Ref. 21, the underlying values of the magnetic moments are proportional to the charges. Beg and Pais [80] have observed that this is the only assumption which gives rise to the ratio $\mu_p/\mu_N = -\frac{3}{2}$ in the 56 of SU(6). It is also assumed that the magnetic moment of the λ-type quark is also proportional to its charge and (4.28) is replaced by

$$\mu_i = \mu_p \left(\frac{q_i}{e}\right) \sigma_i \tag{4.29}$$

where $i = 1, 2, 3$.

The wavefunctions used are the outer product states of the unitary-spin, spin and the harmonic oscillator wavefunctions. As usual for hadrons L-S coupling is used to obtain the physical spin of the particles. All the states (except f(1270)) considered in Section (4.8) are $L = 0$ states and all the initial states considered in Section 4.9 are $L = 1$ states.

For $L = 0 \rightarrow L = 0$ process the electromagnetic interaction operator $\mathcal{H}_j$ (Eq. 3.10) takes the form

$$\mathcal{H}_j = \sum_j q^{(j)} \mu_j \sqrt{4\pi} \frac{1}{\sqrt{k_o}} k \, S_+^{(j)} \exp(ikz^{(j)}) \tag{4.30}$$

where $\mu_i = \mu_p$ for the ordinary quarks and $\mu_j = \mu_c = \frac{m_u}{m_c}\mu_p$ for the c-type quarks. The formula used in the calculation of the decay width is given by Eq. 3.11.

4.8 The Transitions $V(L=0) \to \gamma P(L=0)$, $P(L=0) \to \gamma V(L=0)$, $\psi'(3684) \to \gamma\eta, \eta', X(2.82)$ and $\psi(3095) \to \gamma f(1270)$.

The transitions considered in this section can be classified into the following sets according to the type of calculations.

(I) Decays which involve only "old" L = 0 mesons (the results are shown in Table 4.3 and Table 4.4)

(II)(a) Two body radiative decays of ψ i.e. $\psi \to \gamma P$ (where $P = \eta, \eta'$ and X(2.8)) and $\psi \to \gamma\pi^o$

(b) $\psi \to \gamma f(1270)$.

The results are shown in Table 4.5.

(III)(a) Two body radiative decays of ψ', i.e. $\psi' \to \gamma P$ (where $P = \eta, \eta'$ and X(2.8)).

(b) $\psi' \to \gamma\chi(3.455)$. $\chi(3.455)$ is taken to be the pseudoscalar partner of ψ' and a pure $c\bar{c}$ state. The results are shown in Table 4.6.

(IV) Two body radiative decays of charmed particles, i.e. $D^* \to \gamma D$ and $F^* \to \gamma F$ (Table 4.7).

The following three sets of parameters have been used in these calculations [81].

(a) $\alpha^2 = 0.055\ GeV^2$, $\alpha'^2 = 0.27\ GeV^2$, $m_u = 0.336$ GeV, $m_c = 1.65$ GeV
(b) $\alpha^2 = 0.055\ GeV^2$, $\alpha'^2 = 0.27\ GeV^2$, $m_u = 0.336$ GeV, $m_c = 2$ GeV
(c) $\alpha^2 = 0.055\ GeV^2$, $\alpha'^2 = 0.1\ GeV^2$, $m_u = 0.336$ GeV, $m_c = 1.3$ GeV .

As mentioned earlier, α^2 and α'^2 are the harmonic oscillator parameters for the ordinary $q\bar{q}$- and $c\bar{c}$ system respectively. The decay widths of the transitions in Set I are insensitive to the values of α'^2 (and m_c value). It is because of zero or negligible $c\bar{c}$ content in both the initial and final particles. $\alpha^2 = 0.055\ GeV^2$ gives reasonable decay rates for the transitions in Set I and we continue to use this value for the rest of the transitions considered in this chapter. $\alpha^2 = 0.1\ \text{GeV}^2$ (for transitions in Set I) is almost equivalent to putting the overlap integral equal to unity which is, however, not desirable (see Sec. 4.9).

The decays of the charmed mesons (Set IV) are also unaffected by (acceptably large) variations in the value of α''^2 (Table 4.7).

In each of these transitions (except $\psi \to \gamma f(1270)$), there is only one non-zero matrix element (i.e. $\underline{A}_1 = \langle f, J{=}0|\mathcal{H}_j|i, J_z = -1\rangle$ or $A_1 = \langle f, J_z = 1|\mathcal{H}_j|i, J{=}0\rangle$ for initial vector- or pseudoscalar particle respectively, in the notation of Sec. 3.5). In the case of $\psi \to \gamma f(1270)$ there are three non-zero matrix elements. Here, too, we have considered two possibilities for the harmonic oscillator parameter α'^2 (for the $c\bar{c}$ system) and $\alpha^2 = 0.055\ \text{GeV}^2$ in both cases (we have not considered $\alpha^2 = 0.1\ \text{GeV}^2$ because this value is not favoured by transitions in the Set I).

4.9 Radiative Widths of "Old" $q\bar{q}$ L = 1 Mesons

In one of their papers Rosner et al. [88] express their hope that in the near future, it will be possible experimentally to measure the radiative decays of the low-lying positive parity mesons with $J^{PC} = 2^{++}$, 1^{++}, 0^{++} and 1^{+-}. They have made some predictions of these decay widths, using the Melosh-type approach with the additional help of the vector dominance hypothesis. Their strongest predictions deal with the electromagnetic decays of the new candidates for $q\bar{q}$, L = 1 mesons, however, they have also done calculations for the electromagnetic transitions of the "old" $q\bar{q}$, L = 1 mesons: the f_o, A_2, A_1, B, δ and others below 1.5 GeV. As these mesons tend to have large hadronic widths, their electromagnetic branching ratios should be small [89].

In the following we shall present our results obtained according to

TABLE 4.3

The rates of the decays $V(L=0) \rightarrow \gamma P(L=0)$ and $P(L=0) \rightarrow \gamma V(L=0)$ (V stands for the vector particle and P stands for the Pseudoscalar Particle), calculated in the harmonic oscillator model as explained in the text. Col. 1: Process; col. 2: the momentum of the γ; col. 3: the calculated width (in keV) with quasi-relativistic prescription of Becchi and Morpurgo [21]; col. 4: the calculated width without the prescription of Becchi and Morpurgo [21]; col. 5: the experimental decay width (in keV).

Process	k(GeV)	Γ(keV)	Γ(keV)	Γ(keV)
$K^{*o} \rightarrow \gamma K^o$	0.308	220	144	75 ± 35 (a)
$\omega \rightarrow \gamma \pi^o$	0.379	817	419.9	870 ± 86 (b)
$\rho^o \rightarrow \gamma \pi^o$	0.373	87	45	35 ± 10 (a)
$\phi \rightarrow \gamma \pi^o$	0.501	3.9	1.98	5.9 ± 2.1 (a)
$K^{*+} \rightarrow \gamma K^+$	0.308	55	36	<80 (b)
$\eta' \rightarrow \gamma \rho^o$	0.169	133	109	<304 (b)
$\phi \rightarrow \gamma \eta$	0.362	168	108	65 ± 15 (a)
$\omega \rightarrow \gamma \eta$	0.199	6.7	5	$[3^{+2.5}_{-1.8}]$ (c)
$\rho^o \rightarrow \gamma \eta$	0.184	61.8	46	50 ± 13 (c)
$\phi \rightarrow \gamma \eta'$	0.06	0.96	0.9	-
$\eta' \rightarrow \gamma \omega$	0.159	14.6	12	<80 (d)

$\alpha^2 = 0.1\ GeV^2$ $\alpha'^2 = 0.27\ GeV^2$, $m_u = 0.336$ GeV, $m_c = 1.65$ GeV

(a) See Refs. 82 and 46.

(b) See Ref. 46.

(c) See Refs. 83 and 46.

(d) See Ref. 84.

TABLE 4.4

The rates of the decays $V(L=0) \rightarrow \gamma P(L=0)$ and $P(L=0) \rightarrow \gamma V(L=0)$, calculated in the harmonic oscillator model as explained in the text. Col. 1: Process; col. 2: the momentum of the γ; col. 3: the calculated width (in keV) with quasi-relativistic prescription of Becchi and Morpurgo [21]; col. 4: the calculated width without the prescription of Becchi and Morpurgo [21]; col. 5: the experimental decay width (in keV).

Process	k(GeV)	Γ(keV)	Γ(keV)	Γ(keV)
$K^{*o} \rightarrow \gamma K^o$	0.308	181	118.6	75 ± 35 (a)
$\omega \rightarrow \gamma\pi^o$	0.379	609	313	870 ± 86 (b)
$\rho^o \rightarrow \gamma\pi^o$	0.373	65.8	33.9	35 ± 10 (a)
$\phi \rightarrow \gamma\pi^o$	0.501	2.33	1.2	5.9 ± 2.1 (a)
$K^{*+} \rightarrow \gamma K^+$	0.308	45	29.6	< 80 (b)
$\eta \rightarrow \gamma\rho^o$	0.169	125.5	103	< 304 (b)
$\phi \rightarrow \gamma\eta$	0.362	122.5	78.9	65 ± 15 (a), (82 ± 17) (b)
$\omega \rightarrow \gamma\eta$	0.199	5.7	4	$[3 {}^{+2.5}_{-1.8}]$ (c)
$\rho^o \rightarrow \gamma\eta$	0.184	57.6	43	50 ± 13 (c)
$\phi \rightarrow \gamma\eta'$	0.06	0.92	0.86	-
$\eta' \rightarrow \gamma\omega$	0.159	14	11.8	<80 (d)

$\alpha^2 = 0.055$ GeV^2, $\alpha'^2 = 0.1$ GeV^2, $m_u = 0.336$ GeV, $m_c = 1.3$ GeV

(a) See Refs. 82 and 46.

(b) See Ref. 46.

(c) See Refs. 83 and 46.

(d) See Ref. 84.

TABLE 4.5

The rates of the decays $\psi(3095) \to \gamma P(P = \eta, \eta', X(2.8))$, $\psi(3095 \to \gamma\pi^o$ and $\psi(3095) \to \gamma f(1270)$, calculated in the harmonic oscillator model as described in the text.

Col. 1: Process; col. 2: the momentum of the γ; col. 3: the calculated width (in eV) with quasi-relativistic prescription of Becchi and Morpurgo [21]; col. 4: the calculated width without the prescription of Becchi and Morpurgo [21]; col. 5: the experimental width (in eV).

Process	k(GeV)	Γ(eV)	Γ(eV)	Γ(eV)
$\psi \to \gamma\eta$	1.498	820	420	$94\pm30^{(a)}, 55\pm12^{(b)}$
$\psi \to \gamma\eta'$	1.399	43×10^3	23×10^3	$\leq 450eV^{(a)}, 152\pm117^{(b)}$
$\psi \to \gamma X(2.8)$	0.262	39×10^3	35×10^3	$8.3 \pm 3.5^{(b)}$
$\psi \to \gamma\pi^o$	1.544	4.2	2	$<350eV^{(a)}, 5\pm3.2^{(b)}$
$\psi \to \gamma f(1270)$	1.287	197.7	115.5	$(71 - 213)^{(c)}$
$\alpha^2 = 0.055$ GeV^2,	$\alpha'^2 = 0.27$ GeV^2,	$m_u = 0.336$ GeV,	$m_c = 1.65$ GeV	
$\psi \to \gamma\eta$	1.498	573	295	
$\psi \to \gamma\eta'$	1.399	20×10^3	10.9×10^3	
$\psi \to \gamma X(2.8)$	0.262	29×10^3	26×10^3	
$\psi \to \gamma\pi^o$	1.544	4.2	2	
$\psi \to \gamma f(1270)$	1.287	107	62	
$\alpha^2 = 0.055$ GeV^2,	$\alpha'^2 = 0.27$ GeV^2,	$m_u = 0.336$ GeV,	$m_c = 2$ GeV	
$\psi \to \gamma\eta$	1.498	56	29	
$\psi \to \gamma\eta'$	1.399	3	1.6	
$\psi \to \gamma X(2.8)$	0.262	53	48	
$\psi \to \gamma\pi^o$	1.544	4.2	2	
$\psi \to \gamma f(1270)$	1.287	8.6	5	
$\alpha^2=0.055$ GeV^2,	$\alpha'^2 = 0.1$ GeV^2,	$m_u = 0.336$ GeV,	$m_c = 1.3$ GeV	

(a) See Ref. 85. (b) See Ref. 86. (c) See Ref. 78.

TABLE 4.6

The rates of the decays $\psi'(3684) \to \gamma P$ ($P = \eta, \eta', X(2.8)$) and $\psi'(3684) \to \gamma\chi(3.455)$, calculated in the harmonic oscillator model as described in the text.

Col. 1: Process; col. 2: the momentum of the γ; col. 3: the calculated width with the quasi-relativistic prescription of Becchi and Morpurgo [21]; col. 4: the vector dominance model result [87]; col. 5: the experimental width available.

Process	k(GeV)	Γ(keV)	Γ(keV)	Γ(keV)
$\psi' \to \gamma\eta$	1.801	0.749	5.1 ± 1.6	< 0.3[(a)], < 5[(b)]
$\psi' \to \gamma\eta'$	1.717	39	20 ± 10	< 3.2[(a)], < 100[(b)]
$\psi' \to \gamma X(2.8)$	0.778	20.8	5^{+7}_{-3} MeV	-
$\psi' \to \gamma\chi(3.455)$	0.222	15	-	-
$\alpha^2 = 0.055$ GeV^2, $\alpha'^2 = 0.27$ GeV^2, $m_u = 0.336$ GeV, $m_c = 1.65$ GeV				
$\psi' \to \gamma\eta$	1.801	0.509		
$\psi' \to \gamma\eta'$	1.717	26		
$\psi \to \gamma X(2.8)$	0.778	14		
$\psi \to \gamma\chi(3.455)$	0.222	10.5		
$\alpha^2 = 0.055$ GeV^2, $\alpha'^2 = 0.27$ GeV^2, $m_u = 0.336$ GeV, $m_c = 2$ GeV				
$\psi \to \gamma\eta$	1.801	0.53×10^{-1}		
$\psi \to \gamma\eta'$	1.717	4		
$\psi \to \gamma X(2.8)$	0.778	94		
$\psi \to \gamma\chi(3.455)$	0.222	20.8		
$\alpha^2 = 0.055$ GeV^2, $\alpha'^2 = 0.1$ GeV^2, $m_u = 0.336$ GeV, $m_c = 1.3$ GeV				

(a) These results come from DASP (See Ref. 87).

(b) These results come from SPEAR (See Ref. 87).

TABLE 4.7

The rates of the decays $D^* \to \gamma D$ and $F^* \to \gamma F$, calculated in the harmonic oscillator model as described nn the text.

Col. 1: Process; col. 2: the momentum of the γ; col. 3: the calculated width (keV) with the quasi-relativistic prescription of Becchi and Morpurgo [21]; col. 4: the calculated width without such a prescription; col. 5: the experimental decay width (keV) available.

Process	k(GeV)	Γ(keV)	Γ(KeV)	Γ(KeV)
$D^{*+} \to \gamma D^+$	0.135	1.24	1.15	-
$D^{*o} \to \gamma D^o$	0.135	34	31.8	-
$F^* \to \gamma F$	0.107	0.625	0.593	-
α''^2 = 0.068 GeV^2, m_u = 0.336 GeV, m_c = 1.3 GeV				
$D^{*+} \to \gamma D$	0.135	1.295	1.208	-
$D^{*o} \to \gamma D^o$	0.135	35.6	33	-
α''^2 = 0.1 GeV^2, m_u = 0.336 GeV, m_c = 1.3 GeV				

TABLE 4.8

The rates of the decays of "old" $q\bar{q}$ L=1 mesons, calculated in the harmonic oscillator model as described in the text.

Col. 1: Process; col. 2: the momentum of the γ; col. 3: the calculated width (keV) with the quasi-relativistic prescription of Becchi and Morpurgo [21]; col. 4: the result obtained by Rosner et al. [88]; col. 5: the experimental decay width (keV) available.

Process	k(GeV)	Γ(keV)	Γ(keV)	Γ(keV)
$A_2 \to \gamma\pi^+$	0.647	495	348	-
$K^{**+} \to \gamma K^+$	0.624	471	312	-
$K^{**o} \to \gamma K^o$	0.624	0	0	-
$A_1 \to \gamma\pi^+$	0.541	597.8	338 - 3600	-
$B^+ \to \gamma\pi^+$	0.602	59 10^{-3}	108 - 490	
$f_o \to \gamma\rho$	0.401	278	750 - 4000	-
$\delta^+ \to \gamma\rho^+$	0.171	6.6	-	-
$B^+ \to \gamma\rho^+$	0.367	352.7	-	-
$A_1 \to \gamma\rho^+$	0.28	13.8	-	-
$A_2 \to \gamma\rho^+$	0.428	39.8	-	-

our harmonic oscillator model and compare with those of Rosener et al. [88]. The procedure of calculations is similar to that adopted in Chapter 3 for the decays of $L = 1$ new mesons, the χ_J's. The only difference is that now we have only ordinary quarks, p, n and λ and the harmonic oscillator constant α^2 ($\alpha^2 = 0.055$ (GeV)^2) for the ordinary quarks system. Moreover, now we have μ_p (in place of μ_c) to be taken as the scale magnetic moment of each quark. The structure of the spatial wavefunctions is the same as that of the χ_J's (listed in Appendix A) and the integrals involved have also been evaluated and listed in Appendix B. The tensors f, A_1, A_2, B and δ are assumed to have quantum numbers shown in Table 1.5 (of Chapter 1). As usual f(1270) (like ω) is taken as containing only non-strange quarks. This belief is also strengthened (is it?) by our diagonalization program (Sec. 4.5) where f(1270) comes out to be made of almost 100% non-strange quarks. The results are shown in Table 4.8.

4.10 Discussion of the Results and Conclusions

For the radiative decays in this chapter, we have used the harmonic oscillator quark model with SU(4) symmetry broken through a scheme which is the generalization of the symmetry breaking in SU(3) (Secs. 4.4 and 4.5). As mentioned earlier, our results depend upon our assumptions about the harmonic oscillator parameters (α^2 and α'^2), masses and charges of quarks and also on the belief that the magnetic moment of the charmed quark is smaller than those of the ordinary quarks.

As shown in Chapter 3, the parameters $\alpha'^2 = 0.1$ GeV^2, $m_u = 0.336$ GeV and $m_c = 1.3$ GeV (or $\alpha'^2 = 0.27$ GeV^2, $m_u = 0.336$ GeV and $m_c = 2$ GeV) fit the decays $\psi'(3684) \to \chi_J$'s quite reasonably well; we have performed calculations in this chapter with these two values of α'^2 (the non-relativistic treatment of the new particles (the ψ sector) gives the mass of the charmed quark in the range of $m_c = 1.3 - 2$ GeV).

With $\alpha^2 = 0.055\ GeV^2$, we are able to get acceptable widths for all the old lowest-lying L=0 mesons ($\alpha^2 = 0.1\ GeV^2$ is equivalent (in most of the transitions), to putting the spatial overlap integral of the harmonic oscillator wavefunctions equal to almost unity). As is obvious, α''^2 (the constant for a system of an ordinary quark and charmed quark - e.g. D^*'s and F^*) should lie between α^2 and α'^2 [81]. With $\alpha'^2 = 0.1\ GeV^2$, $m_u = 0.336$ GeV and $M_c = 1.3$ GeV we estimate $\alpha''^2 = 0.068\ GeV^2$. The decays of D^*'s and F^*, however, are not very sensitive to the values of α''^2 (because of small momentum k in the factor $\exp\left(\frac{-k^2}{4\alpha''^2}\right)$ - see Table 4.8).

The value of $\alpha^2 = 0.055\ GeV^2$ (for the ordinary quark antiquark system) seems to be very small but not unbelievable if we look through the literature for the range of α^2-values used to explain the different properties of hadrons. It ranges roughly between 0.06 - 0.18 GeV^2. Thornber [35] considered point quarks without a form factor and used $\alpha^2 = 0.06\ GeV^2$ equal to the electromagnetic proton radius $R^2 = 16\ (GeV)^{-2}$. Faiman et al. [20,27] used $\alpha^2 = 0.1\ GeV^2$ to fit the baryon decays $N^* \rightarrow N\pi$ and $N^* \rightarrow \gamma N$. Copley et al. [32], considering backward photoproduction of π^o, and trying to explain its vanishing for $D_{13}(1508)$ and $F_{15}(1688)$ found a value of $\alpha^2 = 0.17\ GeV^2$, taking $\mu = \mu_p$. The masses of hadrons favour even a larger α^2 value. For example, Minamikawa et al. [90] have explained electromagnetic mass differences of baryons with $\alpha^2 = 0.18\ GeV^2$. It seems as if level spacing of multiplets (and masses of hadrons), according to harmonic oscillator potential, favour a larger α^2-value as compared with decays and cross-sections.

The quark model approach to meson radiative decays is plausible at least in the following sense: experimentally large or small widths are predicted to be large or small respectively. There is, however, an ambiguity in applying the quark model non-relativistically, particularly to the processes in which the final particles are by necessity highly relativistic.

The inspection of the tables of results shows that the introduction of the harmonic oscillator wavefunctions improves the overall agreement with the experiment. Especially, in view of the new experimental values [82,46], $\Gamma(\rho^o \to \gamma\pi^o) = 35 \pm 10$ keV, $(K^{*o} \to \gamma K^o) = 75 \pm 35$ keV, $\Gamma(\phi \to \gamma\eta) = 65 \pm 15$ keV and $\Gamma(\phi \to \gamma\pi^o) = 5.9 \pm 2.1$ keV, the approximation $\exp ik.r_i = 1$ (used by Becchi et al. [21]) does not seem to be desirable. Calculations with $\alpha^2 = 0.055$ GeV^2 bring all the decay widths (differing otherwise) of old L= 0 mesons (Set I) within a factor of less than two of the recent experimental data.

In Set II, the transitions $\psi \to \gamma P$ (P = η, η' and X(2.8)) are equally sensitive to α^2 and α'^2 (and the m_c value) but the decay widths $\Gamma(\psi \to \gamma\eta)$ and $\Gamma(\psi \to \gamma\eta')$ can not be made consistent (according to the presently available experimental data) with a reasonable set of parameters. The decay width $\Gamma(\psi \to \gamma X(2.8)$ comes out to be very large. $\psi \to \gamma\pi^o$ is independent of α'^2 and favours $\alpha^2 < 0.1$ GeV^2. The transitions of the charmed mesons, too, are less sensitive to the α'^2-value (Table 4.7). The transition $F^* \to \gamma F$ is calculated with $m_F* = 2.065$ GeV and $m_F = 1.925$ GeV [91].

The experimental situation with $\psi' \to \gamma P$ (P = η, η', X(2.8), $\chi(3.455)$) is not quite clear. In the literature, there are widely different values for these decay rates and hence it is impossible to derive any firm conclusion except for the fact that our quark model results are considerably lower than those obtained by some other models, notably by the vector dominance model [87].

Another notable point is that at this stage of the experimental situation (especially when the radiative widths of the old and new L=1 mesons are not known), it is difficult to test the quasi-relativistic prescription of Becchi and Morpurgo [21]. In the case of most of the old meson decays, the transitions $\psi \to \gamma P$ (P = η, η'), $\psi' \to \gamma P$ (P = η, η') and $\psi \to \gamma\pi^o$, the use of the quasi-relativistic prescription of Becchi and Morpurgo almost doubles the decay rates; the decay rates of the charmed mesons are not affected appreciably. With the use of the quasi-relativistic prescription of Becchi and Morpurgo, the overall agreement of the transitions considered in this chapter demands $\alpha^2 < 0.1$ GeV^2 for the ordinary quark antiquark system. The only decay which can afford $\alpha^2 = 0.1$ GeV^2 is the transition $\omega \to \gamma\pi^o$. With $\alpha^2 = 0.055$ GeV^2, we get a reasonable decay rate for $\psi \to \gamma\pi^o$. The transition $\psi \to \gamma f(1270)$ involves both α^2 and α'^2 and the two pieces of the matrix elements (from the ordinary $q\bar{q}$ and $c\bar{c}$ contents) compensates each other effects and with $\alpha^2 = 0.055$ GeV^2 and $\alpha'^2 = 0.27$ GeV^2 ($m_c = 1.65 - 2$ GeV), we get as good a value as for the old meson rates. The transitions $\psi \to \gamma\eta$, η, X(2.8) (and similarly $\psi' \to \gamma\eta$, η', X(2.8)) cannot be reconciled with one another and indicate that the mixing problem of the pseudoscalars (η, η', X(2.8)) has to be treated differently from that of the vectors or that it is even premature to include X(2.8) in the established particles [72] and carry out such a program.

Our results for the radiative widths of the low-lying positive parity mesons can be tested against future experiments and can play an important role in settling the value for the harmonic oscillator parameter, α^2. The results in Table 4.8 (for $\alpha^2 = 0.055$ GeV^2), are very sensitive to the α^2-value; for example with $\alpha^2 = 0.1$ GeV^2, we get

$$\Gamma(K^{**+} \to \gamma K^+) = 575 \text{ keV}$$
$$\Gamma(A_2^+ \to \gamma\pi^+) = 642 \text{ keV}$$
$$\Gamma(A_1^+ \to \gamma\pi^+) = 599 \text{ keV}$$
$$\Gamma(B^+ \to \gamma\pi^+) = 8.7 \text{ keV}$$

Our results for $A_2^+ \to \gamma\pi^+$, $K^{**+} \to \gamma K^+$, $K^{**o} \to \gamma K^o$ and $A_1^+ \to \gamma\pi^+$ are not entirely different from those of Rosner et al. (Table 4.8) and can be brought (together) to an agreement level by adjusting (slightly) the value of α^2. And this can be done with α^2 certainly less than 0.1 GeV^2.

We conclude this chapter with the remark that our results are quite flexible in the sense that the parameters in our calculations are adjustable and maximum experimental information is desirable to settle these parameters once and for all. With the presently available experimental information, it is difficult to argue insistently in favour of specific values for constants like quark masses, quark magnetic moments or, for that matter, any quantity related to quarks, and our case is no exception. Anyway, it is encouraging to note that the naive quark model gives predictions for quite a large number of meson radiative transitions which are in acceptably good agreement with experiment. The overall fitting of the experimental data on radiative widths of mesons seems to favour the following set of parameters:

$$\alpha^2 = 0.055 \text{ GeV}^2, \quad \alpha'^2 = 0.27 \text{ GeV}^2, \quad m_u = 0.336 \text{ GeV}, \quad m_c = 1.65 - 2 \text{ GeV}.$$

That the harmonic oscillator potential gives such apparently reasonable results is also encouraging from the point of view of recent developments in the quark model which seek the permanent confinement of quarks inside a hadron. Such a confinement can be achieved by a choice of static $q\bar{q}$ interaction of the form $V(r) \sim r^{\varepsilon}$ and the most favoured one seems to be the one which rises linearly. But as noted in ref. 65a, the order of appearance of successive levels with increasing energy is insensitive to the details of the interaction, provided it is confining. It has been shown explicitly by Gromes et al. [92] that the linear potential can be approximated by an appropriate oscillator potential for the energy levels for the S-states (the Schrödinger equation with a linear potential can be solved analytically only for L = 0, i.e. S-states). The sum of linear and Coulomb potential (which is in fashion nowadays) can still be approximated by an appropriate harmonic oscillator potential, as long as the spectrum is essentially determined by the linear part (that this is true is shown in ref. 92). Another point which goes in favour of this approximation (of the linear and harmonic oscillator potential) is in fact that the contribution (from the overlap integral of the wavefunctions) comes from the region where the linear and oscillator potential do not differ too much [92] (especially for the low-lying levels). For hadronic spectroscopy of a linear confining potential, the reader is referred to a paper by Gunion et al. [93].

CHAPTER 5

APPLICATION OF SU(4) TO RADIATIVE TRANSITIONS

5.1 Introduction

In this chapter, we first discuss the consequences of SU(4)-symmetry for radiative transitions, $V(L=0) \rightarrow \gamma P(L=0)$ (where V and P represent the (15+1)-plets of vectors and pseudoscalars respectively), derived simply by placing the particles in irreducible representations, assuming an appropriate behaviour (under SU(4)) of the electromagnetic current, and applying the Wigner-Eckart theorem for the matrix elements of the transitions. It is simply a generalization of the well-known techniques of isospin symmetry and SU(3) symmetry [38]. Once we know the transformation properties of the interaction under a group (SU(4) in our case), then the Wigner-Eckart theorem gives a set of relations between the physical transition amplitudes and the reduced transition amplitudes from which one can obtain relations between physical amplitudes to be tested experimentally.

Thus, the problem of obtaining experimental consequences of SU(4) is a question of coupling between irreducible representations which can be solved by different methods. One has the pedestrian method of carrying out the Clebsch-Gordon composition [38] or applying the Wigner-Eckart theorem [31], but many other methods are employed to estimate the physical quantities like decay widths etc.

In the remainder of the Chapter (Sec. 5.6) we shall discuss briefly the work of some authors on the application of SU(4) to the radiative transitions of the low-lying mesons (i.e. $V(L=0) \rightarrow \gamma P(L=0)$ and $P(L=0) \rightarrow \gamma V(L=0)$) and compare the situation with our harmonic oscillator model results (Chapter 4), particularly with regards to the transitions $\psi \rightarrow \gamma P$ ($P = \eta, \eta', X(2.8)$), $\psi \rightarrow \gamma\pi^o$, $\phi \rightarrow \gamma\pi^o$, $\rho^o \rightarrow \gamma\pi^o$ and $K^{*o} \rightarrow \gamma K^o$.

5.2 Electromagnetic Current in SU(4)

In SU(4). we have at our disposal three additive quantum numbers I_3, Y and Z; they correspond to the three generators of (SU(4)) which can be simultaneously diagonalized [42,47].

The quartet has the following quantum numbers

$$q \equiv \begin{pmatrix} A \\ A \\ B \\ C \end{pmatrix} \quad \text{with} \quad \left\{ \begin{array}{llll} A: & I = \frac{1}{2}, & Y = \frac{1}{3}, & Z = \frac{1}{4} \\ B: & I = 0, & Y = -\frac{2}{3}, & Z = \frac{1}{4} \\ C: & I = 0, & Y = 0, & Z = -\frac{3}{4} \end{array} \right. \tag{5.1}$$

In order to have integral charges for hadrons, it is convenient to introduce in the definition of the charge, the baryon number N in addition to I_3, hypercharge Y and Z. The three conserved quantum numbers (in strong interactions) I_3, Y and Z are then related to the charge Q by the extended Gell-Mann - Nishijima relation,

$$Q = I_3 + \tfrac{1}{2}Y + aZ + bN \quad . \tag{5.2}$$

The new quantum number (new with respect to I and Y) charm is defined by

$$C = aZ + bN \tag{5.3}$$

where a and b are constants which depend on the choice of a specific model for the quark charges. For the choice of fractional charges of the four quarks in the Moffat model [68] we have $a = \frac{1}{3}$ and $b = -\frac{1}{4}$ while in the Glashow-Illiopoulos and Maiani (GIM) model [60] (which we adopted to take $Q_c = \frac{2}{3}$ in Chapter 3 and Chapter 4), $a = -1$ and $b = \frac{3}{4}$. With this choice the usual particles characterized by $aZ + bN = 0$ satisfy the eightfoldway charge formula.

Charge Q (and hence the electromagnetic current J_μ) can be related to the generators of SU(4) and can be written in the GIM model as

$$Q = F_3 + \frac{1}{\sqrt{3}} F_8 - \sqrt{\frac{2}{3}} F_{15} + \frac{\sqrt{2}}{3} F_o \tag{5.4}$$

where

$$F_n = \frac{\lambda_n}{2} \quad n = 0,1, \ldots 15 \quad \text{and} \quad \lambda_o = \frac{I}{\sqrt{2}} \qquad (5.5)$$

We assume in the following that the electromagnetic current (J_μ) has the same SU(4) structure and write

$$J_\mu = V_\mu^{(3)} + \frac{1}{\sqrt{3}} V_\mu^{(8)} - \sqrt{\frac{2}{3}} V_\mu^{(15)} + \frac{\sqrt{2}}{3} V_\mu^{(0)} \quad . \qquad (5.6)$$

In our treatment, we are not concerned with the Lorentz index μ; we need only the SU(4) transformation properties which for the four terms read as follows:

Term	SU(4) Representation	Subgroup SU(3) Representation	Subgroup SU(2) Representation
$V_\mu^{(3)}$	$\underline{15}$	$\underline{8}$ (I=1, Y=0)	$\underline{3}$ (I=1)
$V_\mu^{(8)}$	$\underline{15}$	$\underline{8}$ (I=0, Y=0)	$\underline{1}$ (I=0)
$V_\mu^{(15)}$	$\underline{15}$	$\underline{1}$ (I=0, Y=0)	$\underline{1}$ (I=0)
$V_\mu^{(0)}$	$\underline{1}$	$\underline{1}$ (I=0, Y=0)	$\underline{1}$ (I=0)

Jointly, the transformation properties of a term will be denoted by $[\underline{A}\ \{\underline{B}(\underline{C})\}]$ where $\underline{A}$, $\underline{B}$ and $\underline{C}$ stand for the SU(4)-. SU(3)- and SU(2)-representation respectively. For example, in this notation $V_\mu^{(3)}$ transforms as $[\underline{15}\{\underline{8}(\underline{3})\}]$ etc.

5.3 Assignment of Particles

Over and above the well-known nonets of vector and pseudoscalar particles, there is a place for four new particles in each of the 15-plet of vectors and pseudoscalars (two doublets and two singlets). An additional complexity is introduced because of symmetry-breaking in which case the isoscalars (ω, ϕ, ψ; and η, η', η_c(X(2.8)?)) can mix to generate the physical masses.

In the following we shall use the states obtained by the consideration of broken SU(4) [given by Eqs. 4.20 and 4.22]. Expressed in terms of the representations of SU(4), they read as follows:

For vectors:

$$\psi = -0.832[\underline{15}\{\underline{1}\}] - 0.003[\underline{15}\{\underline{8}\}] + 0.556[\underline{1}\{\underline{1}\}]$$

$$\omega = 0.435[\underline{15}\{\underline{1}\}] + 0.619[\underline{15}\{\underline{8}\}] + 0.653[\underline{1}\{\underline{1}\}]$$

$$\phi = 0.347[\underline{15}\{\underline{1}\}] - 0.784[\underline{15}\{\underline{8}\}] + 0.514[\underline{1}\{\underline{1}\}] \quad (5.7)$$

For scalars:

$$\eta_c(X(2.8)) = 0.963[\underline{15}\{\underline{1}\}] - 0.012[\underline{15}\{\underline{8}\}] - 0.27[\underline{1}\{\underline{1}\}]$$

$$\eta = 0.058[\underline{15}\{\underline{1}\}] + 0.984[\underline{15}\{\underline{8}\}] + 0.166[\underline{1}\{\underline{1}\}]$$

$$\eta' = 0.263[\underline{15}\{\underline{1}\}] - 0.178[\underline{15}\{\underline{8}\}] + 0.947[\underline{1}\{\underline{1}\}] \quad (5.8)$$

The SU(2) representation $\underline{C}$ in $[\underline{A}\{\underline{B}(\underline{C})\}]$ is $\underline{1}$ (i.e. I=0, Y=0) in each of the above terms and is omitted.

The quantum numbers of the new mesons classified according to the two 15-plets of SU(4) in the GIM model are shown in Table 5.1.

5.4 Wigner-Eckart Theorem for SU(4)

The Wigner-Eckart theorem for SU(4) has been discussed in detail in Chapter 2 and the references therein. If R denotes the SU(4) representation, μ the SU(3) representation, C the charm associated with the irreducible representation and ν the SU(3) quantum numbers (I and Y), then the matrix element of the current J_μ can be written as

$$\begin{pmatrix} R_1 & R_2 & R_3 \\ \mu_1\nu_1 & \mu_2\nu_2 & \mu_3\nu_3 \end{pmatrix} = \sum_{\mu,\gamma,\gamma'} \left(\begin{matrix} R_1 & R_2 \\ \mu_1 C_1 & \mu_2 C_2 \end{matrix} \middle| \begin{matrix} R_3\ \gamma \\ \mu_{3\gamma'}\quad C_3 \end{matrix}\right) \times$$

$$\times \sum_{\nu_3} \begin{pmatrix} \mu_1 & \mu_2 & \mu_{3\gamma'} \\ \nu_1 & \nu_2 & \nu_3 \end{pmatrix} \times (I_1, I_{1Z}; I_2\ I_{2Z} | I_3\ I_{3Z}) \times \langle R_1 || R_2 || R_{3\gamma} \rangle \quad (5.9)$$

for a process of the type $R_3 \to R_1 + R_2$ (the subscript 2 stands for the electromagnetic current J_μ).

TABLE 5.1

The quantum numbers of new mesons in the 15-plet of SU(4) according to the GIM model.

SU(3)	Label	Isospin I	I_Z	Hypercharge	Charm
{3}	$\bar{D}^{*o}$ ($\bar{D}^{o}$)		$\frac{1}{2}$	0	-1
	$\bar{D}^{*}$ ($\bar{D}$)	$\frac{1}{2}$	$-\frac{1}{2}$	0	-1
	$\bar{F}^{*}$ ($\bar{F}$)	0	0	-1	-1
{3*}	D^{+*} (D^{+})	$\frac{1}{2}$	$\frac{1}{2}$	0	1
	D^{o*} (D^{o})	$\frac{1}{2}$	$-\frac{1}{2}$	0	1
	F^{+*} (F^{+})	0	0	1	1
{1}	$\psi(\eta_c)$	0	0	0	0

In (5.9) $\begin{pmatrix} R_1 & R_2 & | & R_{3\gamma} & \\ \mu_1 C_1 & \mu_2 C_2 & | & \mu_{3\gamma'} & C_3 \end{pmatrix}$ is the SU(3)-singlet for

$R_1 \otimes R_2$ [47,48], $\begin{pmatrix} \mu_1 & \mu_2 & \mu_{3\gamma'} \\ \nu_1 & \nu_2 & \nu_3 \end{pmatrix}$ is the SU(3) isoscalar

factor [47,48] for $\mu_1 \otimes \mu_2$ and $(I, I_Z; I_2, I_{2Z} \mid I_3\ I_{3Z})$ is the SU(2) Clebsch-Gordon coefficient [46] to take account of the charge states in an isospin multiplet. Now it is straightforward (but laborious) with the transformation properties of the current J_μ (given by 5.6) and the assignment of the mesons to the (15+1)-plet of SU(4) as discussed in the previous section to get the transition amplitudes.

5.5 Matrix Elements of the Electromagnetic Current in SU(4)

In this section we give the relations among the transition amplitudes of the processes $V(L=0) \to \gamma P(L=0)$. Particularly, the processes with the photon momenta not widely different are related. This is meant to minimize the effect of mass splitting in SU(4) multiplets. The relations among the amplitudes of the following sets of processes have been considered. All these relations are satisfied with the available experimental data (the branching ratios for $D^{*+} \to \gamma D^+$ and $D^{o*} \to \gamma D^o$ are not known).

a) $\rho^o \to \gamma\pi^o$, $K^{*+} \to \gamma K^+$, $K^{*o} \to \gamma K^o$.

b) $\rho \to \gamma\eta$, $\phi \to \gamma\pi^o$, $\omega \to \gamma\pi^o$,

c) $K^{o*} \to \gamma K^o$, $K^{+*} \to \gamma K^+$, $\rho \to \gamma\eta$.

d) $\psi \to \gamma\pi$, $\psi \to \gamma\eta$, $\psi \to \gamma\eta'$, $\psi \to \gamma\eta_c$, $D^{*+} \to \gamma D^+$, $D^{o*} \to \gamma D^o$.

The five reduced matrix elements involved in the relations among the physical amplitudes and the reduced amplitudes are the following:

$X_A = \langle \underline{15} || \underline{1} || \underline{15} \rangle$, $X_B = \langle \underline{1} || \underline{15} || \underline{15} \rangle$

$X_C = \langle \underline{1} || \underline{1} || \underline{1} \rangle$, $X_D = \langle \underline{15} || \underline{15} || \underline{15}_D \rangle$

and

$X_E = \langle \underline{15} || \underline{15} || \underline{1} \rangle$

Elimination of the reduced matrix elements gives the following relations among the processes quoted above.

(a) $M(\rho \to \gamma\pi^o) = \frac{2}{3} M(K^{*+} \to \gamma K^+) + \frac{1}{3} M(K^{*o} \to \gamma K^o)$

(b) $M(\omega \to \gamma\pi^o) = 1.275M(\phi \to \gamma\pi^o) - 2.85M(\rho \to \gamma\eta)$

(c) $M(K^* \to \gamma K^o) = M(K^{*+} \to \gamma K^+) + 1.76M(\rho \to \gamma\eta)$

(d) $M(\psi \to \gamma\pi^o) = 1.602M(D^{o*} \to \gamma D^o) - 6.39M(D^{+*} \to \gamma D^+) + 0.448M(\psi \to \gamma\eta')$
$- 11.254M(\psi \to \gamma\eta) - 5.4M(\psi \to \gamma\eta_c)$

These matrix elements can also be used to calculate the mesonic radiative widths but the number of parameters become too large to extract anything reliable. Moreover, the incorporation of the phase space factor becomes more dubious (than SU(3)) because SU(4) is obviously a worse symmetry (than SU(3)). But there are, however, artifacts by many authors in this field, which we shall describe briefly in the following section. It must, however, be pointed out that no consistent picture of the mesonic decays (including the problematic decays mentioned at the end of section 5.1) has emerged so far (as far as we know).

5.6 Two-Body Radiative Decays of Mesons in Broken SU(4) Schemes

A number of papers have been contributed to this subject, of which we shall discuss only a few. This will, hopefully, demonstrate the method of extension of the above procedure to calculate radiative widths.

We begin with the work of Kazi et al. [98]. Using the U- and W-spin classification of mesons [40] and the U - and W-spin invariance of the electromagnetic current J, they are able to express all the matrix elements $\langle P|J|V\rangle$ in terms of $\langle \pi^o|J|\rho^o\rangle$. All the decay widths are normalized to $\Gamma(\omega \to \gamma\pi^o) = 870$ keV. They have also considered the possibility of E(1416) as being the partner of η but the overall predictions are far from satisfactory when compared with experiments.

In another paper Kazi et al. [56a] consider the SU(4) breaking of meson coupling constants (in addition to breaking in masses and wave-functions) by introducing a Hamiltonian of the form

$$\mathcal{H} = a_o T_o + a_8 T_8 + a_{15} T_{15} \tag{5.10}$$

which has similar properties to the one used for mass-breaking in SU(4) (see Chapter 4). With the additional help of vector dominance hypothesis, they consider a host of decays including strong decays $(V \to PP,\ V \to VP)$ and radiative decays $(V \to \gamma P,\ P \to 2\gamma)$. They use the following inputs to find the unknown parameters: $\Gamma(\rho^o \to e^+e^-) = 6.45$ keV, $\Gamma(\omega \to e^+e^-) = 0.76$ keV, $\Gamma(\phi \to e^+e^-) = 1.34$ keV, $\Gamma(\psi \to e^+e^-) = 4.8$ keV; $\Gamma(\rho^o \to \pi^+\pi^-) = 150.4$ MeV, $\Gamma(K^{*+} \to K^+\pi^o) = 16.6$ MeV; $\Gamma(\omega \to \gamma\pi^o) = 870$ keV, $\Gamma(K^{*o} \to \gamma K^o) = 110$ keV, $\Gamma(\psi \to \gamma\eta) = 94$ eV.

As far as the strong decays are concerned, their predictions (according to the scarce experimental data, to date) are consistent. But for mesonic radiative decays, they find that in their framework of coupling constant breaking the rates of the transitions $K^{*o} \to \gamma K^o$ and $\phi \to \gamma\eta$ can not be made consistent. A similar problem is noted in the relation between the decay widths of the transitions $\rho^o \to \gamma\pi^o$ and $\omega \to \gamma\pi^o$. The ~~result~~ for $\eta \to 2\gamma$ is also not satisfactory. Similarly, $\Gamma(\psi \to \gamma\pi^o)$ comes out too small (i.e. 0.61 eV) while $\Gamma(\psi \to \gamma X(2.8))$ comes out too large, (i.e. 27.8 keV), in contradiction with experiment.

Another approach to get a consistent picture of the mesonic radiative widths is the investigation of the dependence of the width on the masses (other than the phase space factor) of the particles involved. This has led to the study of the so-called spectrum generating group $SU(n)_E$ [91,99]

(distinguished from the so-called approximate symmetry group). It is the group whose generators commute with the generators (generalized) of the Poincaré group, i.e.

$$[\hat{P}_\mu, SU(n)_E] = 0, \qquad [L_{\mu\nu}, SU(n)_E] = 0 \qquad (5.11)$$

where $\hat{P}_\mu = P_\mu/M$ (generalized momenta), P_μ are the momentum operators and M is the mass operator. $L_{\mu\nu}$ are the generators of the homogeneous Lorentz group. In such a scheme, the connections between the transition operator H_μ (say) and the electromagnetic current V_μ^{el} $(= V_\mu^{\pi^o} + \frac{1}{\sqrt{3}} V_\mu^{\eta} - \sqrt{\frac{2}{3}} V_\mu^{\eta_c} + \frac{\sqrt{2}}{3} V_\mu^{o}$, in SU(4)) are made in such a manner that in the symmetry limit, they lead to the usual expression of the hadronic current (e.g. $H_\mu = g\{V_\mu^{el}, M^p\}$ where $p = 0$ will give the usual expression $H_\mu = g\, V_\mu^{el}$ and in this case, as is well known, the symmetry breaking is taken into account by using physical masses in the phase space factor). The net effect of this procedure (the use of generalized momenta in place of ordinary momenta) is the introduction of a phenomenological suppression factor depending on the masses of the decaying and product particles in the decay width formula. With such considerations, Bohm et al. [91,99], for example, find the suppression factor G_{VP} (for $V \to \gamma P$), given by

$$G_{VP} = (m_V^p + m_P^p)/m_V m_P, \quad p = \tfrac{1}{2}, 1, 3/2 \qquad (5.12)$$

where m_V (m_P) stand for the vector (pseudoscalar) meson masses. Unfortunately, this approach too involves too many parameters and approximations to extract any useful information. Moreover, the overall consistency cannot be checked if the mass mixing of the isoscalars is taken into consideration (which must be) because of a further increase in the number of parameters.

Concluding this chapter, we note that so far the decays $\psi \to \gamma P$ ($P = \eta, \eta', X(2.8)$), $\psi \to \gamma\pi^o$, $\phi \to \gamma\pi^o$, $\rho^o \to \gamma\pi^o$, $K^{*o} \to \gamma K^o$ and $\phi \to \gamma\eta$ have not been described consistently with a host of the rest of the observed mesonic radiative transitions within the framework of SU(4) (broken) symmetry or some other model. We also note retrospectively that our results (Chapter 4) for the transitions $\psi \to \gamma\pi^o$, $\phi \to \gamma\pi^o$, $\rho^o \to \gamma\pi^o$, $K^{*o} \to \gamma K^o$ and $\phi \to \gamma\eta$ are not unacceptably bad but the problem with the inconsistency of the transitions $\psi \to \gamma P$ ($P = \eta, \eta'$, X(2.8)) is yet to be tackled. As the results of the transitions $\psi \to \gamma P$ ($P = \eta, \eta'$X(2.8) are independent (according to our calculations of Chapter 4) of the rest of the radiative mesonic decays, we think that the solution of this problem may lie in a different treatment (different from that of the corresponding vector particles) of the mixing problem of the pseudoscalar particles (i.e. η, η' and η_c (X(2.8)?)).

CHAPTER 6

6.1 The MIT Bag Model

It has become widely accepted that hadrons are constituted of quarks with fractional charges and this is the basic theme on which the quark models are based. Hadron spectroscopy and many other related experimental observations are very nicely explained by this approach [3,4].

Nevertheless, quarks as free particles have never been detected experimentally. This negative aspect of the experiment motivates the idea of the quark confinement and accordingly, quarks are thought to be permanently bound inside hadrons. A relativistic quark model of hadrons based on this idea was proposed by Chodos et al. [100] and hence after called by the name of their institute namely, the MIT bag model.

The MIT bag model is a description of hadrons in which confinement of the hadronic constituents (such as quarks and gluons) is allowed in a Lorentz invariant manner. The confinement is obtained by assuming that the bag possesses a constant positive energy for unit volume, B. Its effect is to add a term to the usual stress energy tensor:

$$T^{\mu\nu} = T^{\mu\nu}_{field} - g^{\mu\nu}B \tag{6.1}$$

inside the bag while outside the bag $T^{\mu\nu}$ vanishes. Requirement of energy momentum conservation, and confinement lead to boundary conditions on the fields at the surface of the bag. The quarks are assumed to be interacting among themselves relatively weakly by the exchange of an octet of massless, coloured gluons coupled in the manner of Yang-Mills to their colour indices.

The equation for the massive quark mode functions is the Dirac equation (neglecting the coupling to the colour variables)

$$(- i \overline{\gamma}\cdot\overline{\nabla} + \gamma^o\omega + m)q(x) = 0 \tag{6.2}$$

characterized by two boundary conditions

$$-i\gamma^{\mu} n_{\mu} q(x) = q(x) \qquad (6.3)$$

$$n^{\mu} \frac{\partial}{\partial x^{\mu}} [\bar{q}(x) q(x)] = -2B \qquad (6.4)$$

where n_{μ} is a space-like normal to the surface, B is the outside pressure confining the quarks and $q(x)$ is the quark field. (6.3) implies $\bar{q}q = 0$ on the surface while (6.4) is responsible for the stability of the system (bag). It is this latter condition which keeps the surface of the bag spherical. It is the main approximation of the bag model, called the "spherical cavity (or fixed sphere)" approximation.

In the spherical cavity approximation the second boundary condition strongly restricts the possible quark modes. As it turns out, only two solutions are allowed. The first one is given as

$$q(r,t) = \frac{N(x)}{\sqrt{4\pi}} \begin{pmatrix} \sqrt{\frac{\omega+m}{\omega}}\, i\, j_o\, (x\,\frac{r}{R})\, U_m \\ -\sqrt{\frac{\omega-m}{\omega}}\, j_1\, (x\,\frac{r}{R})\, \underline{\sigma}\cdot\hat{\underline{r}}\, U_m \end{pmatrix} \qquad (6.5)$$

In (6.5) m is the mass of the quark, R is the radius of the spherical bag, U_m is a two component Pauli spinor, and j_i are spherical Bessell functions. This solution is also denoted by $S_{\frac{1}{2}}$. The second solution is given by

$$q(r,t) = \frac{N(x)}{\sqrt{4\pi}} \begin{pmatrix} \sqrt{\frac{\omega+m}{\omega}}\, i\, j_1\, (x\,\frac{r}{R})\, \underline{\sigma}\cdot\hat{\underline{r}}\, U_m \\ \sqrt{\frac{\omega-m}{\omega}}\, j_o\, (x\,\frac{r}{R})\, U_m \end{pmatrix} \qquad (6.6)$$

It corresponds to L = 1 in the non-relativistic limit and is denoted by $P_{\frac{1}{2}}$. The normalization constant N(x) is taken so that

$$\int d^3x\, q^{+}(x)\, q(x) = 1 \qquad (6.7)$$

and is given as

$$N^2 = \frac{\omega(\omega - m)}{R^3\, j_o^{\,2}(x)\{2\omega(\omega - {}^1/R) + \frac{m}{R}\}} \qquad (6.8)$$

with ω defined as

$$\omega = \frac{1}{R} [x^2 + (mR)^2]^{\frac{1}{2}} \quad . \tag{6.9}$$

The first boundary condition (given by 6.3) yields an eigenvalue condition for the mode frequencies x. For $S_{\frac{1}{2}}$, one has

$$\tan x = \frac{x}{1 - \sqrt{x^2 + (mR)^2} - mR} \tag{6.10a}$$

For $P_{\frac{1}{2}}$ the eigenvalue condition is given by

$$\tan x = \frac{x}{1 + \sqrt{x^2 + mR)^2} - mR} \tag{6.10b}$$

The first condition is familiar from the study of the masses of the ground state hadrons [101] while the second condition has been employed for the study of baryon excitations by Donoghue et al. [102] and mesons made of $P_{\frac{1}{2}}$ - quark quantum modes have been studied by Cleymans [103]. The solutions of the Eq. (6.10a) and Eq. (6.10b) can be read from graphs shown explicitly in these two references. The successive eigenmodes in the two cases are respectively denoted by $1S_{\frac{1}{2}}(x_{1-1})$, $2S_{\frac{1}{2}}(x_{2-1})$, ... and $1P_{\frac{1}{2}}(x_{11})$, $2P_{\frac{1}{2}}(x_{21})$, ...

The general solution to Eqs. 6.2 - 6.4 can be defined in terms of the complete system of cavity eigenstates

$$q_\alpha(x,t) = \sum_{n\kappa jm} N(\omega_{n\kappa j}) a_\alpha(n\kappa jm)\, q_{n\kappa jm}(x,t) \tag{6.11}$$

where there is an infinite sum over the integral values of n for each j, κ and m; n labels the radial excitations of quarks for given angular momentum quantum numbers. κ is the Dirac quantum number [100], $\kappa = \pm(j+\frac{1}{2})$ which differentiates the two states of opposite parity for each value of j (the quadratic boundary condition (6.4) restricts the modes which may be excited and allows only $j = \frac{1}{2}$ solutions to the Dirac equation and corresponds to $j = \frac{1}{2}$, $\kappa = -1$ and $j = \frac{1}{2}$, $\kappa = 1$ respectively).

Like the ordinary field theory, the quark annihilation operators

$a_\alpha(n,\kappa,\ j=\frac{1}{2},\ m)$ for negative n are defined as antiquark creation operators with positive energy:

$$a_\alpha(n\ \kappa\ j=\tfrac{1}{2}\ m) \equiv b_\alpha(n\ \kappa\ m), \quad n > 0$$

$$a_\alpha(n\ \kappa\ j=\tfrac{1}{2}\ m) \equiv d^+_\alpha(-n,\ -\kappa\ \ m) \quad n < 0 \tag{6.12}$$

These operators are assumed to satisfy the well-known anti-commutation relations for fermions,

$$\{b_\alpha(n\ \kappa\ m),\ b^+_\alpha(n\ \kappa\ m)\} = \{d_\alpha(n\ \kappa\ m), d^+_\alpha(n\ \kappa\ m)\} = 1 \tag{6.13}$$

and all other anti-commutators are zero.

Thus the Dirac field operator may also be written as

$$q_\alpha(\overline{x},t) = \sum_{n>0,\kappa=\pm1,m=\pm\frac{1}{2}} N(n,\kappa)\ \{b_\alpha(n\ \kappa\ j=\tfrac{1}{2}\ m)q_{n\kappa j=\frac{1}{2}m}(\overline{x},t)$$

$$+ d^+_\alpha(n\ \kappa\ j=\tfrac{1}{2}\ m)q_{-n\ -\kappa\ j=\frac{1}{2}\ m}(\overline{x},t)\} \tag{6.14}$$

where b^+_α and d^+_α create quark and antiquark excitations with the functions $q_{n\ \kappa\ j\ m}(\vec{x},t)$ in the bag. The vacuum state or empty cavity is defined as a state $|0>$ such that

$$b_\alpha(n\ \kappa\ m)\,|0> = 0 = d_\alpha(n\ \kappa\ m)\,|0>. \tag{6.15}$$

6.2 Applications

After the introduction of the MIT bag model, there has been a good deal of effort to explore its phenomenological content. Various studies have been carried out on the mass-spectrum of hadrons and other static properties of the ground state baryons. With the same theoretical framework, the model has also been applied to strong, electromagnetic and weak processes [104-108].

In the following sections we first briefly review the salient features of the application of the bag model to the masses of hadrons and then apply the model to the masses of the recently discovered resonances of the Υ-family and those of the ρ-like resonances. At the end, we comment on the inadequacy of the spherical cavity approximation for the masses of the excited states and the likely approach for improvement.

6.3 Hadron Masses

The MIT bag model was first successfully applied to the masses of the light hadrons by Chodos et al. [100] and De Grand et al. [101]. The formula which they used for the masses of hadrons has now become widely accepted and is taken as a prescription. The formula consists of the following terms:

a) volume energy term $E_V = \frac{4}{3}\pi BR^3$ and the zero-point energy term $E_0 = \frac{-Z_0}{R}$, both of which depend only on the radius of the hadron and are assumed to originate from the quantum fluctuations of the system. As discussed in ref. 101, the theoretical significance of the second term is somewhat unclear and is treated only phenomenologically.

b) Each quark contributes its rest and kinetic energy to the hadron mass. This energy contribution is denoted by E_Q and given by

$$E_Q = N_0\, \omega(m_0, R) + N_s \omega(m_s, R)$$

where N_0, N_s, m_0 and m_s are the respective numbers and masses of the non-strange and strange quarks and $\omega(m,R) = \frac{1}{R}\,[x^2 + (mR)^2]^{\frac{1}{2}}$.

c) The contributions which come from the colour magnetic exchange and colour electric parts of the gluon interaction. They are respectively denoted by ΔE_m and ΔE_E.

As mentioned earlier, quarks are not the only objects residing inside the bag. They are coupled to eight massless vector particles called gluons, which are the mediators of interactions between the quarks.

Like quarks, gluons too must be confined inside hadrons (bags). Gluons as vector particles are described by Maxwell equations (similarly to the photons). The mass spectrum of the ground state hadrons have been calculated in the lowest order of the quark-gluon coupling α_c.

In the lowest order of the quark-gluon coupling $\alpha_c = \frac{g^2}{4\pi}$ the gluon exchange graphs are shown in Fig. 6.1. Also, to the lowest order in α_c the non-Abelian gluon self-coupling does not contribute and the gluons act as eight independent Abelian fields without self-interaction. The problem reduces to ordinary electro- (magneto-) statics and the boundary conditions for the gauge fields on the surface of the bag may be written as

$$\hat{r}\cdot\vec{E}^a = 0 \tag{6.16}$$

$$\hat{r}\cdot\vec{B}^a = 0 \quad . \tag{6.17}$$

The index "a" denotes colour and runs from 1 to 8. E^a and B^a are the gluon electric and magnetic field vectors. They are defined as the time-space and the space-space components of the field tensor $F^a_{\mu\nu}$, respectively.

As shown in ref. 101, $\Delta E_E = 0$ if all of the quarks in a given hadron have the same mass. Even when the quarks have different masses, ΔE_E is very small, so long as the masses are not too different. For massless u- and d-type quarks and $m_s < 300$ MeV, $\Delta E_E < 5$ MeV. Thus ΔE_E can be neglected for charmonium, mesons with beauty (Sec. 6.4) and ρ-like resonances (Sec. 6.5).

The colour magnetostatic interaction energy in the final form [101] can be written as

$$\Delta E_m = 8\alpha_c \lambda \sum_{i>j} (\vec{\sigma}_i\cdot\vec{\sigma}_j) \frac{\mu(x,m_jR)\,\mu(x,\ m_jR)}{R^3} \; I(m_iR,\ m_jR) \quad \text{where} \tag{6.18}$$

$\lambda = 1$ for baryon and 2 for meson, σ are Pauli matrices.

$\mu(x,\ mR)$ and $I(m_iR,\ m_jR)$ are given by the following expressions:

$$\mu(x,\ mR) \quad = \quad \frac{R}{6}\ \frac{4\sqrt{x^2 + (mR)^2} + 2m_i\ R - 3}{2\sqrt{x^2 + (mR)^2}(\sqrt{x^2 + (mR)^2} - 1) + mR} \tag{6.19}$$

for $S_{\frac{1}{2}}$ states and

$$\mu(x,\ mR) \quad = \quad \frac{R}{6}\ \frac{4\sqrt{x^2 + mR)^2} - 2mR + 3}{2\sqrt{x^2 + mR)^2}\ (\sqrt{x^2 + (mR)^2} + 1) + mR} \tag{6.20}$$

for $P_{\frac{1}{2}}$ quantum modes.

$$I(m_i R, m_j\ R) \quad = \quad 1 + (x_i \sin^2 x_i - \tfrac{3}{2} y_i)^{-1} (x_j \sin^2 x_j - \tfrac{3}{2} y_j)^{-1}$$

$$\times\{-\tfrac{3}{2} y_i y_j - 2x_i x_j \sin^2 x_i \sin^2 x_j + \tfrac{1}{2} x_i x_j [2x_i \mathrm{Si}(2x_i)$$

$$+ 2x_j \mathrm{Si}(2x_j) - (x_i + x_j)\mathrm{Si}(2(x_i + x_j) - (x_i - x_j)\mathrm{Si}(2(x_i - x_j))]\} \tag{6.21}$$

where $y_i = x_i - \sin x_i$ and $\mathrm{Si}(x) = \int_0^x \frac{\sin t}{t}\, dt$.

The mass of the hadron of radius R is then given by

$$M(R) \quad = \quad E_V + E_o + E_Q + \Delta E_m + \Delta E_E \quad . \tag{6.22}$$

The quadratic boundary condition requires that the quark and gluon field pressure balance the external pressure B locally on the bag-surface. It can be shown that in the static spherical approximation, this equilibrium is obtained if $M(R)$ is minimum. Thus, the true radius of the hadron R_o is obtained by $\frac{\partial M}{\partial R} = 0$ and the mass of the hadron is given by $M(R_o)$.

The SU(3) symmetry breaking is introduced by assigning a different mass to the strange quark. The Σ and Λ would remain degenerate if the only effect of the SU(3) breaking were in the quark mass-kinetic term. However, the presence of the strange quark mass also modifies the wavefunction of the strange quark and therefore causes a secondary SU(3) breaking through the gluon magnetic interaction. This splits the Σ and Λ in the right direction. The detailed calculations and Tables of results for ground state hadrons are given in ref. 101.

The masses of the orbitally excited hadrons (made of u-, d- and s-type quarks) i.e. configurations like $(1S_{\frac{1}{2}})^2(1P_{\frac{1}{2}})$ and $(1S_{\frac{1}{2}})(1P_{\frac{1}{2}})$ have also been estimated within the framework of the "fixed sphere" bag model [109]. With the parameters from the fit of the ground state hadrons and gluon hyperfine interactions to lowest order, the calculated masses of these excited states are generally found to be lighter than the observed masses.

Bowler et al. [110] have considered the radially excited baryon states in the framework of the MIT bag model. The authors find the mixing of the non-strange states belonging to the $[56, 0^+]$ and $[70, 0^+]$ multiplets of the bag model. In the absence of the gluon interactions all these states are degenerate with a mass around 1600 MeV. Assuming that the dominant effect on the degeneracy (lifting of the degeneracy) is only due to the direct magnetic gluon interaction ΔE_m, they find that the Roper resonance (NP11(1470)) consists of two nearly degenerate P11 states. They also find that only one of them (the lower mass state) is coupled strongly to photons [111].

The "fixed sphere" bag model has also been applied to the charm spectroscopy [112] but with less success. The charmed meson mass is estimated by using the average mass of $\psi(3095)$ and X(2.8) i.e. $\overline{M} = \frac{1}{4}[3M(J=1) + M(J=0)] = 3025$ MeV (see Section 6.4). Varying the values of B (between 115 - 145 MeV) and Z_0 (between 0 - 2), Donoghue et al. [112] find $m_c = 1300 - 1500$ MeV. All the other parameters are known from the fit to the ground state hadrons [101]. From among the gluon interactions, they consider only the dominant direct magnetic gluon interaction. The masses of the charmed hadrons have also been calculated but there is little data to compare the theoretical results with. A candidate for a charmed baryon $(C = 1, S = 0)$ of mass 2426 MeV [113] is not much different from the bag model result (i.e. 2458 - 2536 MeV). The bag model result for the lightest charmed meson (averaged over the spin value) is less than the corresponding experimental value (which is available now) by about 109 MeV. The bag model predictions in the $c\bar{c}$ sector are even worse. The bag model, for example, can not reproduce enough excitation energy to interpret $\psi'(3684)$ as the first radial excitation of $\psi(3095)$. Moreover, if X(2.8) is interpreted as the pseudoscalar partner of $\psi(3095)$, the bag model does not predict the correct ortho-para splitting for any reasonable values of the bag parameters.

One can not locate the fault (for disagreement) clearly. It may be that the spherical cavity approximation is not a realistic one and that a more reasonable picture for hadronic systems is a long thin bag rather than a spherical one. Excited mesons (when quark and antiquark are in different modes) have another general problem of leaving the quadratic boundary condition time-dependent (i.e. B changes with time). Experimentally, very little is known about the state X(2.8) and the theoretical predictions based on the available data can not be conclusive.

In the following we shall apply the "fixed sphere" bag model to the newly discovered mesons with beauty (hidden beauty) at 9.41 GeV and 10.06 GeV etc. The semi-classical nature of the bag model suggests that it should work better in predicting masses of heavier mesons. At the end, we take another exercise and calculate the masses of the ρ-like mesons and comment on the inadequacy of the "fixed sphere" bag model.

6.4 Masses of Mesons with Beauty (Hidden) in The Bag Model

Recent experiments [114] with the reaction

$$P + (Cu, Pt) \rightarrow \mu^+ + \mu^- + \text{anything}$$

show structures in the differential cross-section which have been interpreted as resonances with masses of 9.41, 10.06, 10.058 and 10.92 GeV.

The width of these states is less than experimental resolution, suggesting that they may be new narrow resonances (Υ-family) much like the ψ-family. Most analyses put the mass of the new quark (beauty quark, denoted by b) at 4 - 5 GeV, which means that the Υ particle can be interpreted as a $b\bar{b}$ state. An SU(5) mass formula [115] can be used to predict the masses of other particles in the 24-plets of vector particles and pseudoscalar particles. In particular, the mass of the pseudoscalar counterpart of Υ (i.e. η_b) comes out to be $M(\eta_b) = 9.15$ GeV.

In the following, we shall assume that the interpretation of Υ and the mass $M(\eta_b)$(= 9.15 GeV) as predicted are valid; the latter assumption, however, as we shall see below, does not affect our results significantly. We further assume that Υ and η_b are $(1S_{\frac{1}{2}})^2$states in the bag model. From this we can determine the mass m_b of a beauty quark and then calculate the mass spectrum of hadrons with such a pair of quarks. We calculate M[(1S)(2S)], M[(1S)(3S)], M[(1S)(4S)] and $M[(1P)^2]$.

In our estimation of the mass m_b, we can neglect the effect of the ortho-para splitting produced by the magnetic coupling of gluons to quarks on the following grounds.

The contribution of the gluon magnetic interaction to the mass operator appears as follows (for $S_{\frac{1}{2}}$ states, for example)

$$M = \bar{M} + \Delta\vec{\sigma}_1\cdot\sigma_2 \qquad (6.23)$$

where $\bar{M}$ is the contribution from effects other than the gluon magnetic interaction and independent of the spin of a particle. Δ is given by the expression (in the notation of the previous sections)

$$\Delta = \frac{16\alpha_c\ \mu^2(x,\ mR)}{R^3}\ (1 + J) \qquad (6.24)$$

where

$$\mu(x,\ mR) = \frac{R}{6}\ \frac{4\sqrt{x^2+(mR)^2} + 2mR - 3}{2\sqrt{x^2+(mR)^2}(\sqrt{x^2+(mR)^2}-1) + mR} \qquad (6.25)$$

$$J = [x\sin^2 x - \frac{3}{2}(x - \sin x\cos x)^{-2}]\{-2x^2\sin^4 x -$$

$$- \frac{3}{2}(x - \sin x\cos x)^2 + \tfrac{1}{2}x^2[4x\,Si(2x) - 2x Si(4x)]\} \quad (6.26)$$

It is easy to see that

$$\overline{M} = \tfrac{1}{4}[3M(J=1) + M(J=0)] \quad (6.27)$$

which for Υ and η_b equals [116]

$$\overline{M} = \tfrac{1}{4}[3 \times 9.41 + 9.15] = 9.345 \text{ GeV} \quad (6.28)$$

so we can avoid the complicated magnetic interaction in the calculation of m_b, provided we use the average mass of Υ and its pseudoscalar counterpart in the above sense. We find $m_b = 4.935$ GeV.

The mass spectrum of hadrons composed of $S_{\frac{1}{2}}$ quantum modes (specifically, (1S)(2S), (1S)(3S), (1S)(4S) and $P_{\frac{1}{2}}$ modes (i.e. $(1P_{\frac{1}{2}})^2$ can now be calculated in a straightforward manner in accordance with the prescription of De Grand et al. [101]. The dominant contribution for a given quark is always $E_Q = (m_q{}^2 + \frac{x^2}{R^2})^{\frac{1}{2}}$, which is the energy of a non-interacting particle confined to a sphere of radius R. The volume energy $E_V = \frac{4}{3}\pi B R^3$ is associateted with the pressure parameter B, which ensures the stability of the bag.

The theoretical significance of the zero point energy $E_o = -\frac{Z_o}{R}$ is somewhat unclear but was used in ref. 101 in finding the overall fit to hadron masses. The other contribution we shall deal with arises from the gluon magnetic interaction. However, this latter contribution, as shown below, is negligible when the quark is very heavy and is in an excited mode. For a very large quark mass the magnetostatic energy decreases like the product of non-relativistic magnetic moments, $E_{mag} \sim (\frac{1}{2m})^2$ [117]. The gluon magnetostatic energy of the $b\bar{b}$ pair in the Υ and Υ' can be compared with those of $u\bar{u}$ $(d\bar{d})$ in the ρ^o, $c\bar{u}$ pair in the D^{o*} and $c\bar{c}$ in ψ as follows:

$$E_{mag}((1S)^2)_{b\bar{b}} \sim 26 \text{ MeV}$$

$$E_{mag}[(1S)(2S)]_{b\bar{b}} \sim 4 \text{ MeV}$$

$$E_{mag}[(1S)^2]_{c\bar{c}} \sim 28 \text{ MeV}$$

$$E_{mag}[(1S)^2]_{c\bar{u}} \sim 47 \text{ MeV}$$

$$E_{mag}[(1S)^2]_{u\bar{u}} \sim 109 \text{ MeV}$$

so we can safely neglect the effect of gluon magnetic interaction for the radially excited states of $b\bar{b}$ pairs.

6.5 The Mass Spectrum

The mass spectrum of mesons composed of $S_{\frac{1}{2}}$ (i.e. (1S)(2S), (1S)(3S) and (1S)(4S) and $P_{\frac{1}{2}}$ quantum modes (i.e. $(1P_{\frac{1}{2}})^2$ can now be calculated in a straightforward manner in accordance with the prescription of De Grand et al. [101]. All the parameters involved have been fixed by fitting the masses of the light hadrons. They have the following values:

$$\alpha_c = 0.55, \quad (B)^{\frac{1}{4}} = 0.145 \text{ GeV}, \quad Z_o = 1.84.$$

In the special case of Υ', this leads to the following expression:

$$M_{\Upsilon'}(R) = \frac{4}{3}\pi BR^3 + \frac{\sqrt{x_{1-1}^2 + (mR)^2}}{R} + \frac{\sqrt{x_{2-1}^2 + (mR)^2}}{R} - \frac{Z_o}{R}$$

$$+ 16\,\alpha_c \frac{\mu(x_{1-1}, mR)\mu(x_{2-1}, mR)}{R^3} (1 + J_{1S,2S}) \qquad (6.29)$$

where

$$J_{1S,2S} = (x_{1-1}\sin^2 x_{1-1} - \tfrac{3}{2}y_1)^{-1}(x_{2-1}\sin^2 x_{2-1} - \tfrac{3}{2}y_2)^{-1}$$

$$\times\{-\tfrac{3}{2}y_1 y_2 - 2x_{1-1}\, x_{2-1}\sin^2 x_{1-1}\sin^2 x_{2-1}$$

$$+\tfrac{1}{2}x_{1-1}x_{2-1}[2x_{1-1}\mathrm{Si}(2x_{1-1}) + 2x_{2-1}\mathrm{Si}(2x_{2-1}) - (x_{1-1}+x_{2-1}) \times$$

$$\mathrm{Si}(2(x_{1-1}+x_{2-1})) - (x_{1-1}-x_{2-1})\mathrm{Si}(2(x_{1-1}-x_{2-1}))]\} \qquad (6.30)$$

with

$$y_i = x_{i-1} - \sin x_{i-1} \cos x_{i-1}, \quad i = 1, 2 \tag{6.31}$$

and

$$Si(x) = \int_0^x \frac{\sin t}{t} dt \quad . \tag{6.32}$$

Minimization with respect to the radius of the bag gives:

$$\frac{\partial M_T(R)}{\partial R} = 4\pi BR^2 - \frac{A+B-1.84}{R^2} + m^2(\frac{1}{A} + \frac{1}{B}) - (16\alpha_c/36R^2)CDJ_{1S,2S}$$
$$+ \frac{16\alpha_c J_{1S,2S}D}{36R}\left[\frac{(4m^2R/A) + 2m}{2A(A-1) + mR} - C'\{2m^2R\frac{(A-1)}{A} + m\}\right]$$
$$+ \frac{16\alpha_c J_{1S,2S}}{36R}\left[\frac{(4m^2R/B) + 2m}{2B(B-1) + mR} - D'\{2m^2R\frac{(B-1)}{B}\}\right] \tag{6.33}$$

where

$$A = \sqrt{x_{1-1}^2 + (mR)^2}$$

$$B = \sqrt{x_{2-1}^2 + (mR)^2}$$

$$C = \frac{4\sqrt{x_{1-1}^2 + (mR)^2} + 2mR - 3}{2\sqrt{x_{1-1}^2 + (mR)^2}\{\sqrt{x_{1-1}^2 + (mR)^2} - 1\} + mR}$$

$$C' = \frac{C}{2\sqrt{x_{1-1}^2 + mR)^2}\{\sqrt{x_{1-1}^2 + (mR)^2} - 1\} + mR}$$

$$D = C[x_{1-1} \xrightarrow{\text{replaced by}} x_{2-1}]$$

$$D' = C'[x_{1-1} \xrightarrow{\text{replaced by}} x_{2-1}] \ . \tag{6.34}$$

The energy of the state in configuration (1S)(2S) can now be determined from the simultaneous solution of two linear boundary conditions of the type given by (6.10a)(one each for x_{1-1} and x_{2-1}) and one non-linear boundary condition which is equivalent to putting (Sec. 6.4):

$$\frac{\partial M_{T'}(R)}{\partial R} = 0$$

we find

$$x_{1-1} = 3.05, \quad x_{2-1} = 6.105, \quad mR = 16.971$$

so that

$$R = 3.438\ (\text{GeV})^{-1} \quad (m_b \text{ is known from the ground state, } \Upsilon).$$

For this value of the radius the mass is:

$$M_{\Upsilon'} = 9.8 \text{ GeV.}$$

The gluon magnetic interaction contribution, ΔE_{mag} is:

$$\Delta E_{mag} \simeq 4 \text{ MeV.}$$

Without gluon magnetic interaction, we get

$$M_{\Upsilon'} = 9.78 \text{ GeV.}$$

Thus we note that the contribution of magnetic gluon interaction to the masses of excited states of the $b\bar{b}$ system is negligible. The masses of the remaining excited states (1S)(3S), (1S)(4S) and $(1P)^2$ are calculated without the magnetic gluon interaction, whose contribution in these configurations is expected to be less than 4 MeV. The results are shown in Table 6.1.

(1S)(2S) and $(1P)^2$ with nearly equal mass can be candidates for Υ'. There are other excited states in the bag model, the (1S)(1P) configurations (χ_b's), whose energies lie between the $(1S)^2$ ground state and its (1S)(2S) excitation. Up to date, there is no experimental evidence for such states; these states are the equivalents of the χ's states in between $\psi(3095)$ and $\psi(3684)$ in the Charm Sector. In the absence of spin dependent forces (which are expected not to make much difference) the bag model predicts

$$M[(1S)(1P)]_{b\bar{b}} \simeq 9.64 \text{ GeV.}$$

Inspection of the Table 6.1 shows that the masses predicted for the excited states are too low, a situation somewhat similar to the Charm sector, where the static sphere bag model can not sustain excitation energies of more than 350 MeV [112].

6.6 Masses of ρ-like Resonances in the Bag Model

In this section we calculate the masses of the meson states associated with the quark modes, (1S)(2S), (1S)(3S), ... $(2S)^2$ and $(1P)^2$, the quarks involved being massless. These masses can be calculated quite easily by the now familiar prescription [101]. The masslessness of the quarks involved makes the calculations much simpler. The parameters required are already known from the fit of the masses of light hadrons. The values of the parameters are:

$$\alpha_c = 0.55, \quad B^{\frac{1}{4}} = 0.145 \text{ GeV}, \quad Z_o = 1.84.$$

In the special case of ρ' (interpreted as (1S)(2S)), its mass $M_{\rho'}(R)$ is given by

$$M_{\rho'}(R) = \frac{4}{3}\pi BR^3 + \frac{2.04}{R} + \frac{5.4}{R} - \frac{1.84}{R} + \Delta E_m \tag{6.35}$$

where $2.043(= x_{1-1})$ and $5.4(= x_{2-1})$ are the eigenvalues corresponding to the eigenmodes (1S) and (2S). The magnetic gluon interaction (for massless quarks) ΔE_m is given by

$$\Delta E_m = 16\alpha_c \frac{\mu(x_{1-1}, R)_{1S}\, \mu(x_{2-1},R)_{2S}}{R^3} (1 + J_{1S,2S}) \tag{6.36}$$

The expressions for $\mu(x,R)$ and $J_{1S,2S}$ are the same as in Sections (6.3) and (6.4) except that the quarks are massless now.

The three equations to be solved are:

1) $$\frac{\partial M_{\rho'}(R)}{\partial R} = 0$$

2) $$mR = \frac{x^2(\tan^2 x - 1) + \tan x(2x - \tan x)}{2 \tan x(x - \tan x)}$$

TABLE 6.1

Masses of heavy hadrons ($b\bar{b}$ states). All masses are quoted in GeV, R in $(GeV)^{-1}$. The contributions to the hadron mass E_V, E_o and E_Q, defined in the text are listed.

Configuration	Particle	M_{exp}	M_{bag}	R_o	E_o	E_V	E_Q
$(1S)^2$	$\frac{1}{4}[3M(\Upsilon)+M(\eta_b)]$	9.345	9.345	1.786	- 1.03	0.010	10.416
(1S)(1P)	χ_b		9.639	2.56	- 0.719	0.031	10.327
(1S)(2S)	Υ'		9.785	3.438	- 0.549	0.075	10.259
$(1P)^2$	Υ'		9.726	3.13	- 0.587	0.057	10.256
(1S)(3S)	Υ''		10.086	4.416	- 0.416	0.159	10.343
(1S)(4S)	Υ'''		10.424	5.084	- 0.362	0.243	10.542

$B^{\frac{1}{4}} = 0.145$ GeV, $Z_o = 1.84$, $m_b = 4.935$

one for each x_{1-1} and x_{2-1}.

It can easily be found out for $m_u = m_d = 0$, that

$$x_{1-1} = 2.043, \quad x_{2-1} = 5.4 \quad \text{and} \quad R = 5.7\ (\text{GeV})^{-1}.$$

For this value of the radius the mass is:

$$M_{\rho'} = \frac{4}{3}\pi B R^3 + \frac{2.043}{R} + \frac{5.4}{R} - \frac{1.84}{R} + \frac{0.24}{R}$$

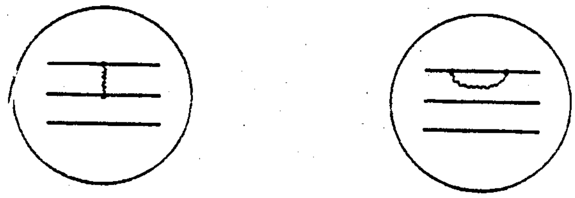
$$= 1.368\ \text{GeV}.$$

It is slightly less than the mass associated with the $(1P)^2$ configuration $(M(1p)^2 = 1.43\ \text{GeV})$[103] as expected. The predicted masses associated with other configurations are shown in Table 6.2.

It seems as if the observed ($\rho(1200)$?) and $\rho(1600)$ have no place in the static sphere bag model predictions for the likely excitations.

6.7 The Inadequacy of the Static Sphere Bag Model

From the review in Sec. 6.3 and our calculations in Sections 6.4 and 6.5, we note that the static sphere bag model successfully predicts the masses and other static properties of light hadrons but does rather badly when the masses of the excited states of hadrons (both radially excited and orbitally excited states) and processes such as decays are considered. We saw this explicitly in Section 6.4 for the excited states in the beauty sector (Υ', Υ'', ...) and in Section 6.5 for the resonances with ρ-like quantum numbers. For the first excitation (radial) in the beauty sector, the bag model predicts $M[1S(2S)]_{b\bar{b}} =$ 9.78 GeV while the experimental candidate seems to be at about 10.05 GeV; for the ρ', the bag model predicts $M_{\rho'}[(1S)(2S)]_{u\bar{u}} = 1.36$ GeV, which is about 200 MeV short of the $\rho'(1600)$. It is similar for the case in the charm sector where the bag model falls short of reproducing



(a) (b)

Fig. 6.1 Gluon Interaction diagrams for the three quark systems (baryons) in the lowest order of α_c. There are similar diagrams for the quark-antiquark systems (mesons). (a) Gluon exchange; (b) gluon self-energy.

TABLE 6.2

Masses of ρ-like resonances. All masses are quoted in GeV, R in GeV^{-1}.

Configuration	Particle	R_o	M_{bag}	M_{exp}
(1S)(2S)	ρ'	5.695	1.368	1.6
$(1P)^2$	ρ'	5.72	1.4	
$(2S)^2$		6.445	1.982	
(1S)(3S)		6.33	1.878	

the mass of ψ' (i.e. (3684 MeV) by 350 MeV [112]. The static sphere bag model has been applied to the orbitally excited states of baryons [109] where, too, it is found to be in poor agreement with experiment. The consistent failure of the static sphere bag model in the case of excitations is presumably because of the quadratic boundary condition which is not satisfied for excited quark modes and the static spherical approximation breaks down. The excited eigenmodes do not generate a spherically symmetric, classical pressure and correspond to fluctuating, non-spherical bags.

In a more general formulation of the bag model, one should take account of the motion of the bag boundary which is coupled to the motion of the quarks. Such a program was initiated by Rebbi [118] but so far, the complexities involved in the application to the realistic models of hadrons have not been mastered satisfactorily.

APPENDICES

Appendix A

Here we give the total states which have been used in the text. In each case the state with the highest J_Z-value is given.

$$|\psi'(3684), J_Z=1> \;=\; \left(\frac{\alpha'^3}{24\sqrt{\pi}}\right)^{\frac{1}{2}} \frac{4}{\sqrt{4\pi}} \exp\left(\frac{-\alpha'^2 r^2}{2}\right)(3 - 2\alpha'^2 r^2)c\bar{c}\;\alpha(1)\alpha(2)$$

$$|\chi(3414),\; J=0> \;=\; A\, r \exp\left(\frac{-\alpha'^2 r^2}{2}\right) \; c\bar{c} \times \frac{1}{\sqrt{6}}[\sqrt{2}\;\alpha(1)\alpha(12)Y_1^{-1} + \sqrt{2}\beta(1)\beta(2)Y_1^1 - \alpha(1)\beta(2)Y_1^o - \beta(1)\alpha(2)Y_1^o]$$

$$|\chi(3508), J_Z=1> \;=\; A\, r \exp\left(\frac{-\alpha'^2 r^2}{2}\right) c\bar{c} \times \tfrac{1}{2}[\sqrt{2}\alpha(1)\alpha(2)Y_1^o - \alpha(1)\beta(2)Y_1^1 + \beta(1)\alpha(2)Y_1^1]$$

$$|\chi(3552), J_Z=2> \;=\; A\, r \exp\left(\frac{-\alpha'^2 r^2}{2}\right) \; c\bar{c}\;\alpha(1)\alpha(2)Y_1^1$$

where

$$A \;=\; \sqrt{\frac{2}{3}}\left(\frac{4\alpha'^3}{\sqrt{\pi}}\right)^{\frac{1}{2}} \alpha'$$

$$|\psi(3095), J_Z=1> \;=\; \left(\frac{4\alpha'^3}{\sqrt{\pi}}\right) \times \frac{1}{\sqrt{4\pi}} \exp\left(\frac{-\alpha'^2 r^2}{2}\right) c\bar{c}\;\alpha(1)\alpha(2)$$

$$|X(2.8),\; J=0> \;=\; \left(\frac{4\alpha'^3}{\sqrt{\pi}}\right)^{\frac{1}{2}} \frac{1}{\sqrt{4\pi}} \exp\left(\frac{-\alpha'^2 r^2}{2}\right) c\bar{c}\;\frac{1}{\sqrt{2}}[\alpha(1)\beta(2) - \beta(1)\alpha(2)]$$

Appendix B

Here we give the evaluated indefinite integrals which occur in the matrix elements considered in the text:

i) The integrals involved in the transitions $\psi' \to \gamma\chi_J$'s come out in the following forms.

$$I^{(0)} \;=\; 7(I_1^{(0)} + I_2^{(0)}) - 2\alpha'^2(I_3^{(0)} + I_4^{(0)})$$

where

$$I_1^{(0)} \;=\; \frac{0.66467}{\alpha'^5}\exp\left(\frac{-k^2}{8\alpha'^2}\right)\left[1 - \frac{1}{1.5}\left(\frac{k^2}{8\alpha'^2}\right)\right]$$

$$I_2^{(0)} \;=\; 0.0554\,\frac{k^2}{\alpha'^7}\exp\left(-\frac{k^2}{8\alpha'^2}\right)$$

$$I_3^{(0)} = \frac{1.66167}{\alpha'^7} \exp\left(-\frac{k^2}{8\alpha'^2}\right) \left[1 - \frac{2}{1.5}\left(\frac{k^2}{8\alpha'^2}\right)\right]$$

$$I_4^{(0)} = 0.1938 \frac{k^2}{\alpha'^9} \exp\left(-\frac{k^2}{8\alpha'^2}\right) \left[1 - \frac{1}{3.5} \frac{k^2}{8\alpha'^2}\right)]$$

And

$$I^{(S)} = 3\, I_1^{(S)} - 2\, \alpha'^2\, I_2^{(S)}$$

where

$$I_1^{(S)} = \frac{\sqrt{\pi} \times k}{8 \times \sqrt{2}\, \alpha'^5} \exp\left(-\frac{k^2}{8\alpha'^2}\right)$$

$$I_2^{(S)} = \sqrt{\frac{\pi}{2}}\ \frac{2.5}{8}\ \frac{k}{\alpha'^7} \exp\left(-\frac{k^2}{8\alpha'^2}\right) \left[1 - \frac{1}{2.5}\left(\frac{k^2}{8\alpha'^2}\right)\right]$$

$I^{(0)}$ and $I^{(S)}$ are respectively the contributions from the orbital and spin part of the interaction. α'^2 is the well known harmonic oscillator constant (for a $c\bar{c}$ system) and k is the momentum of the emitted photon in the rest frame of the decaying particle.

(ii) The integrals involved in the transitions χ_J's $\to \gamma\psi(3095)$ come in the following forms.

$$I^{(0)} = \sqrt{\frac{8\pi}{3}}\ \left[-\alpha'^2(I_1^{(0)} + I_2^{(0)}) + I_3^{(0)} + I_4^{(0)}\right]$$

where $I_1^{(0)}$ and $I_2^{(0)}$ are given as above while $I_3^{(0)}$ and $I_4^{(0)}$ are given as

$$I_3^{(0)} = \frac{0.443}{\alpha'^3} \exp\left(-\frac{k^2}{8\alpha'^2}\right) = \left(\frac{\sqrt{\pi}}{4\alpha'^3} \exp\left(-\frac{k'^2}{4\alpha^2}\right)\right.$$

$$I_4^{(0)} = 0.02215 \frac{k^2}{\alpha,^5} \exp\left(-\frac{k^2}{8\alpha'^2}\right) \left[1 + \frac{1}{3.5}\left(\frac{k^2}{8\alpha'^2}\right)\right]$$

And

$$I^{(S)} = \sqrt{\frac{\pi}{2}}\ \frac{k}{8\alpha'^5} \exp\left(-\frac{k^2}{8\alpha'^2}\right) .$$

$I^{(0)}$ and $I^{(S)}$, as before, are the contributions from the orbital and spin part of the interaction respectively.

(iii) The only integral involved in the decay $\Psi(3095) \to \gamma X(2.8)$ comes from the spin part of the interaction and is given by

$$I^{(S)} = \int \exp(-\alpha'^2 r^2) j_o(k'r) r^2 dr = \frac{\sqrt{\pi}}{4\alpha'^3} \exp\left(-\frac{k'^2}{4\alpha'^2}\right) = I_3^{(0)}$$

$$\left(\because \quad k' = \frac{k}{\sqrt{2}}\right) .$$

The simplified forms for $I^{(0)}$ and $I^{(S)}$ have been obtained by employing the master formula,

$$\int_0^\infty dr\ r^{\mu-1} J_\nu(k'r) \exp(-\alpha'^2 r^2) = \frac{\Gamma(\frac{1}{2}\nu + \frac{1}{2}\mu)\left(\frac{k'^2}{2\alpha'}\right)^\nu}{2\alpha^\mu\ \Gamma(\nu + 1)}$$

$$\times \exp\left(-\frac{k'^2}{4\alpha'^2}\right) {}^1F_1\left(\tfrac{1}{2}\nu - \tfrac{1}{2}\mu + 1,\ \nu + 1;\ \frac{k'^2}{4\alpha'^2}\right) .$$

The relation between the ordinary Bessel function $J_\ell(k'r)$ and the spherical Bessel function $j_\ell(k'r)$ is given by

$$j_\ell(k'r) = \left(\frac{\pi}{2k'r}\right)^{\frac{1}{2}} J_{\ell+\frac{1}{2}}(k'r) .$$

${}^1F_1$ (a, b; x) is the confluent hypergeometric function and can be approximated as ${}^1F_1 \simeq (1 + \frac{a}{b} x)$ for the cases we have considered in the text (Chapter 3). The approximated ${}^1F_1$'s have been checked with the tabulated ones and found identical (up to fourth decimal point at least). Confluent hypergeometric functions involved in the calculations in Chapter 4, however, can not be approximated as above; (i.e. ${}^1F_1 \simeq (1 + \frac{a}{b}\ x)$ and are calculated by use of well-known recurrence relations in these functions.

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