THE MATHEMATICAL AND ECONOMIC FOUNDATIONS OF ACCOUNTING MEASUREMENT

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I declare that I am the author of this work, that unless otherwise stated all work was carried out by myself and that this work is original and has not been submitted in part or in full for any other degree.

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ABSTRACT

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ABSTRACT

In many respects, the present state of the theory of accounting measurement resembles that of probability theory before the path breaking analysis of A.N. Komolgorov. In accounting, as in "pre-Komolgorov" probability theory, there have been numerous attempts at providing a set of axioms for accounting measurement, all of which have either been ignored or subjected to varying degrees of criticism. By building on these prior attempts, the present thesis proposes an alternative set of axioms and then investigates its implications for accounting measurement in general.

The unifying conception has been alluded to already. The thesis endeavours to show that the theory of accounting measurement is, in fact, grounded upon three axioms, and it is the specification of the information assumed given by these axioms, which is the source of many (if not all) of accounting's problems. The remainder of the thesis deals with the more important of these problems. Thus, chapter three concerns itself with the statistical estimation and identifiability of accounting measurement rules; chapter four, with the commonly encountered models (or interpretations) of the axiom system alluded to above; chapter five, with some numerical methods for estimating the replacement cost of asset disposals (a necessary piece of datum if we are to provide the axiom system with a replacement cost interpretation), whilst chapter six, relying on the capital theory of Irving Fisher, deals with the economic foundations of accounting measurement.

There are two major conclusions which emerge from the study.

Firstly, by summarizing the antecedent conditions which must be satisfied before it is possible to generate accounting measurements, the "axiomatic method" provides a useful framework from which to determine (and organize) the relative importance of measurement problems in accounting. However, much remains to be done if the method is to achieve its "ideal" function as a watershed or "clearing house" for measurement problems in accounting. Secondly, Irving Fisher's "capital theory" possesses far greater potential for accounting theory than has hither to been realized. Specifically, by deriving Fisher's "investment opportunity locus" from first principles, as distinct from assuming it to be exogeneously specified, it is possible to provide an economic rationale for each of the measurement systems alluded to in chapter four.

CHAPTER ONE

AN INTRODUCTORY NOTE

1.0 Introduction

Recent years have witnessed the emergence of a bewildering volume of books and articles each concerned with some aspect of measurement in accounting.[1] This topic, which Professor Ijiri aptly dubbed the theory of accounting measurement,[2] arose from two principal considerations. First and foremost of these, was the realization that the antecedent conditions which need to be satisfied, before it is possible to construct accounting measurements, have nowhere been adequately specified or documented.[3] Yet, without a thorough understanding of the ingredients which go to make up accounting measurements, it is

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1. See, for example, any of the following

   Bierman, H.J. "Measurement in Accounting", The Accounting Review, XXXVIII, 3 (July 1963), pp.501-507.
   Mattessich, R. Accounting and Analytical Methods. Homewood, Illinois: Richard D. Irwin, Inc., 1964.
   Chambers, R.J. "Measurement in Accounting", Journal of Accounting Research, 3, 1 (Spring 1965), pp.17-25.
   Chambers, R.J. Accounting, Evaluation and Economic Behavior. Englewood Cliffs, New Jersey: Prentice-Hall, Inc., 1966.
   Ijiri, Y. The Foundations of Accounting Measurement. Englewood Cliffs, New Jersey: Prentice-Hall, Inc., 1967.
   Larson, K.D. "Descriptive Validity of Accounting Calculations", The Accounting Review, XLIV, 1 (January 1969), pp.38-47.
   Moonitz, M. "Price Level Accounting and Scales of Measurement", The Accounting Review, XLV, 3 (July 1970), pp.465-475.
   Vickery, D.W. "Is Accounting a Measurement Discipline?" The Accounting Review, XLV, 4 (October 1970), pp.731-742.
   Sterling, R.R. Theory of the Measurement of Enterprise Income. Lawrence, Kansas: The University of Kansas Press, 1970.
   Ijiri, Y. Theory of Accounting Measurement. American Accounting Association, 1975.

2. Ibid.

3. Ijiri, Foundations, p.X.

doubtful if there can be any systematic advance in accounting as a scientific discipline.[4] That such improvement is warranted is demonstrated by the fact that accounting practice is an uncomfortable compromise of rules and procedures, some of which possess no basis in logic, others being contradictory in nature.[5] A second consideration, however, stems from the oft-made assumption that accounting measurements are the product of some exact scientific procedure.[6] More often than not, of course, accounting measurements are the outcome of a compromise between the three competing objectives of accuracy, economy and versatility.[7] As a consequence, the reliability of some accounting measurements may be open to question. Yet, accountants continue to operate in a vacuum of reliability which fails to provide any form of error measurement.

The present volume documents our contribution to the theory of accounting measurement and is predicated on two assumptions. Firstly, we claim that the precept embodied in the approach which treats accounting as a measurement discipline, possesses both practical and theoretical utility; that is, by specifying the essential ingredients of accounting measurement, it enables us to differentiate between the important and peripheral areas of accounting theory.[8]

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4. Ibid.

5. Tilley, I. "A Critique of Historical Cost Accounting", Accounting and Business Research, 5, 19, (Summer 1975), pp.185-197.

6. Mattessich, op.cit., p.12.

7. Ibid.

8. Ijiri, op.cit., p.X.

Obviously, effective response to the multitude of criticisms and challenges currently confronting the accounting discipline requires an effective base from which to determine the relative significance of each. For similar reasons, the approach is significant from a pedagogical point of view.[9] In this respect, a student equipped with a thorough understanding of the basic ingredients which go to make up accounting measurements, is better placed to comprehend the complex fabric of rules and procedures embodied in accounting practice. Finally, by understanding accounting in its simplest form, we can compare it with measurement systems in other fields of science. Such comparisons enable us to integrate into accounting the desirable features of these other disciplines.[10]

A second and more important consideration, however, derives from the fact that there is, as yet, no generally accepted theory of accounting measurement. Indeed, the works of Mattessich and Ijiri who, collectively, have undoubtedly been the most influential and prolific writers on this aspect of accounting theory, have both been subject to a welter of criticism and debate. Since this is a topic to which we devote considerable attention in the text, it suffices here to note that Ijiri's work has been criticized on the grounds that it is not, in fact, a deductive theory of accounting measurement,[11] whilst Mattessich's system has been variously attacked for its preoccupation with the double entry bookkeeping system[12] and also for its unnecess-

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9. Ibid., p.XI.

10. Ibid.

11. Chambers, R.J. "Measurement in Current Accounting Practice", The Accounting Review, XLVII, 3 (July 1972), p.504.

12. Most, K.S. "The Planning Hypothesis as a Basis for Accounting Theory", Abacus, 9, 2 (December, 1973), p.131.

arily complicated nature.[13]

Taken together, these considerations suggest the existence of a prima facie case for yet another research project whose objective is to probe into the foundations of accounting measurement. It is the purpose of the present thesis to undertake such an analysis. In the next section, therefore, we provide an outline of the content of the present volume.

13. Chambers, R.J. "Accounting and Analytical Methods: A Review Article", Journal of Accounting Research, 4, 1 (Spring 1966), pp.106-107.

1.1 Scope and Content

Recall that the principal objective of the present work is to set forth an analytical structure as a base from which to build a unifying theme for the theory of accounting measurement. Such a structure is, in fact, derived and analyzed in chapter two of the text. We shall there argue that the theory of accounting measurement is grounded upon three axioms and it is these axioms which summarize a sufficient set of conditions for generating accounting measurements. The axioms, in turn, assume the existence of the "accounting measurement space" $(P_t, \mathcal{G}_t, L_t)$, where P_t is a "property set", $\mathcal{G}_t$ is an algebra of "resource sets" generated by the "property set" P_t, and L_t is a real valued measurement rule defined on the algebra $\mathcal{G}_t$. We shall see that it is the specification of the configuration $(P_t, \mathcal{G}_t, L_t)$ which is the source of many (if not all) of accounting's problem areas.

Having introduced the concept of an "accounting measurement space", we turn, in chapter two, to a more detailed analysis of the nature of accounting measurement. We commence the chapter with an analysis of the Stevens measurement scheme; the usual point of departure for discussions focussing on accounting measurement.[14] Contrary to "popular belief", we shall find Stevens' work to possess very little <u>direct</u> significance to the theory of accounting measurement. Indeed, its principal function seems to be as a device for vetting the "meaningfulness" of the "numerical procedures" applied to measurements when there is a choice in the unit (of measurement) in which the results of measurement are expressed. We shall conclude the chapter

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14. See footnote 1.

by investigating a variety of techniques for estimating the bias and objectivity of accounting measurements. Specifically, by imposing the assumption that the measurements analyzed represent a random sample from a normal frequency function, we shall demonstrate how the sample's mean and variance may be used as a base from which to construct point and interval estimates of the sample's bias and objectivity.

In chapter four, we shall complete our analysis of the accounting measurement systems by investigating the properties of a general "valuation" model; that is, a model which can meaningfully accommodate the replacement cost, net realizable value[15] and C.P.P. measurement systems. The model, in fact, was first proposed by Edwards and Bell in the context of replacement cost accounting, but its properties were not fully investigated by its authors. As a consequence, the model's generality has not been fully appreciated. We shall see that the system is based on two "fundamental" theorems, both of which shall be stated, proved and illustrated in the context of the measurement systems alluded to above. The first, and more important of these theorems provides a means for computing the (potentially) realizable "holding gains" accruing during an interval of time. When the model is provided with a replacement cost interpretation, the theorem requires (as an input) the accumulated replacement cost of disposals during the time interval. This has proved to be one of the most intractable problems confronting the adherents of the replacement cost measurement system. In chapter five, therefore,

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15. We shall henceforth take the terms "net realizable value" and "market value" to be synonymous.

we shall examine several methods for estimating the replacement cost of disposals over a specified time interval. The first three of these are polynomial based numerical methods abstracted from the discipline of "numerical mathematics". The relevance of such methods to the problem at hand has not been investigated, and yet, on the surface, they would seem to possess considerable potential. Having achieved this, we shall then examine two methods (the Edwards and Bell technique and the modified midpoint rule) which have been hinted at by accountants, but whose properties have not been fully investigated.

In the final and somewhat lengthy chapter, we shall examine the economic foundations of accounting measurement. Basing our work on the capital theory of Irving Fisher,we shall provide an economic rationale for each of the measurement systems alluded to in chapter four. Specifically, we shall show that the ratio of a firm's current operating profit to the replacement cost of goods sold during some productive interval T, can be utilized to bound the firm's market value at the end of the next productive interval (T+1). The realizable operating profit (of the market value system) will be shown to measure the contribution of a firm's productive activities (as against purely holding operations) to the variation in the firm's market value over the productive interval covered by the income statement. Finally, we shall demonstrate that the real realized income (of the C.P.P. system) measures the increased command of a firm's resources over a composite of consumptive services as a result of the firm's prior productive investments. In words, each of the measurement systems will be shown to possess some degree of utility to the owners of productive resources.

We shall now turn our attention to the first of these topics; namely the axiomatic foundations of accounting measurement.

CHAPTER TWO

THE AXIOMS OF ACCOUNTING MEASUREMENT*

*This chapter, with minor modifications, is to appear in a forthcoming number of Accounting and Business Research.

2.0 Introduction

In many respects, the present state of the theory of accounting measurement resembles that of probability theory before the publication of Kolmogorov's famous paper.[1] Like probability, there have been numerous attempts at providing a set of axioms for accounting measurement all of which have either been ignored, or attracted varying degrees of criticism.[2] To some extent this is understandable, since accounting is essentially a pragmatic discipline, and, therefore, attempts at axiomatizing its basic constructs may appear as alien and unnecessarily esoteric. But probability theory is designed to model a pragmatic discipline; a discipline which owes its origins to Blaise Pascal (1623-1662) and the "gambling houses" of France.[3] Consequently, the pragmatic nature of a discipline is of little significance to the decision of whether to axiomatize its basic constructs. Indeed, the effort to axiomatize the theory of accounting measurement is the "logical" outcome of the recent tendency of accountants to subject their "dogma" to more rigorous analysis.

> "The mathematical development of any science culminates in the axiomatic formulation of its contents ... The axiomatic method is simply a superb technique for summarizing our knowledge in a given field and for finding further knowledge deductively. This involves inevitably logico-mathematical operations, sometimes of great complexity. If the state of axiomatization of an empirical field has been reached, which is a state of some perfection, mathematics is indispensable

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1. Kolmogorov, A.N. <u>Grundbegriffe der Wahrscheinlichkeitsrechnung</u>. Berlin, 1933.

2. Mattessich, R. <u>Accounting and Analytical Methods</u>. Homewood, Illinois: Richard D. Irwin, Inc., 1964, p.447.

3. Eisen, M. <u>Introduction to Mathematical Probability Theory</u>. Englewood Cliffs, New Jersey: Prentice-Hall, Inc., 1969, p.2.

> "... Axiomatics does not burst upon the scene unprepared. There will have been a vast amount of preparatory exploration and thinking, much of it tentative and in parts. Some will have been in mathematical form, some not."[4]

The most notable attempts at axiomatizing the theory of accounting measurement are those provided by Mattessich[5] and Ijiri.[6] Mattessich's system is the earlier and more obscure of the two attempts. It has been variously criticized for its preoccupation with the double entry bookkeeping system[7] and for its unnecessarily complicated nature.[8] Yet Mattessich was the first to admit that his system

> "... is not a finished structure, but a foundation hopefully stable enough to serve others as a basis for further ventures."[9]

Further, he expressed the opinion that the system would eventually be simplified.[10] In this respect, since Ijiri's system is

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4. Morgenstern, O. quoted in Mattessich, op. cit., p.448.

5. Ibid., pp.32-45 and pp.448-465.

6. Ijiri, Y. The Foundations of Accounting Measurement. Englewood Cliffs, New Jersey: Prentice-Hall, Inc., 1967, pp.87-99.

7. Most, K.S. "The Planning Hypothesis", Abacus. 9, 2 (December 1973), pp.130-131.
   Chambers, R.J. "Accounting and Analytical Methods: A Review Article", Journal of Accounting Research. 4, 1 (Spring 1966), pp.106-107.
   For an example of multidimensional bookkeeping see Ijiri, op. cit., Chapter 5.

8. Chambers, op. cit.

9. Mattessich, op. cit., p.447.

10. Ibid., p.32, p.291.

composed of three[11] "axioms" (as compared with Mattessich's eighteen)[12] and is not based upon the double entry bookkeeping system[13] (Mattessich's duality axiom[14]) it may, at first sight, appear to provide the simplified system predicted by Mattessich. There are two reasons, however, why this is not the case. Firstly, Ijiri's axiom system is stated for historic cost accounting measurement only,[15] whereas of course, Mattessich's system is stated for accounting measurement in general.[16] Thus, Ijiri's system is not capable of modelling the non-historical cost accounting measurement systems.[17] Secondly, in Ijiri's system, the valuation rules are designed to "complement" the axioms rather than being the deductive consequences of them.[18] As such, Ijiri's axiom system is not a deductive theory of accounting measurement and any pretence to rigour within his system is purely superficial.[19]

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11. Ijiri, op cit., p.90.

12. Mattessich, op cit., pp.32-45.

13. Ijiri, Y. "Axioms and Structures of Conventional Accounting Measurement," The Accounting Review, XL, 1 (January 1965) p.36.

14. Mattessich, op cit., pp.33-34.

15. Ijiri, loc cit.

16. Mattessich, op cit., p.32.

17. Ijiri, Foundations, p.98.

18. This view was expressed to the writer in correspondence from Ijiri dated February 16, 1976. See also Chambers, R.J. "Measurement in Current Accounting Practice: A Critique", The Accounting Review, 47, 3 (July 1972), p.504.
Ijiri, Y. "Measurement in Current Accounting Practice: A Reply", The Accounting Review, 47, 3 (July 1972), pp.520-521.

19. This point receives more consideration below.

It is our view, however, that these are problematic limitations which are easily overcome. Specifically, it is our view that Ijiri's system can be modified so as to provide a set of axioms for accounting measurement which whilst being perfectly general, also retains the simplicity of the original system.

It is the purpose of this chapter to expand upon the issues isolated above. To this end, the chapter is divided into four sections. In the first section, we shall elucidate the significant features of an axiom system. This section is included for the dual purpose of providing the "uninitiated" with some "feel" for the workings of an axiom system, and, at the same time, to facilitate evaluation of certain comparisons made by Ijiri with the axiom system of Euclidean geometry. In the second section, we shall examine the mathematical propriety of Ijiri's system in some detail. Needless to say we shall find it to contain several deficiencies. In the third section, we shall propose a method by which these deficiencies may be overcome without at the same time detracting from the simplicity of Ijiri's system. Finally, in the fourth section we shall compare Ijiri's system with the modified version proposed in section three.

We now turn to a consideration of the first of these topics, namely a consideration of the significant features of the axiomatic method.

2.1 Axiomatics and Euclidean Geometry

A deductive system T may be characterized as a collection of statements (theorems, lemmas and corollaries) which may be derived from a set of "basic" statements called axioms.[20] The axioms are viewed as assumptions which are entertained purely because of the theorems they imply.[21] There is no consideration of their truth value.

> "Many propositions formerly regarded as self-evident ... are now known to be false. Indeed contradictory propositions about every variety of subject matter ... have ... at different times, been declared as fundamental intuitions and therefore self-evidently true. But whether a proposition is obvious or not depends on cultural conditions and individual training, so that a proposition which is 'self-evidently true' to one person or group is not to another."[22]

If the set of axioms from which the statements in T are derived is finite, then T is said to be finitely axiomatizable.[23] Thus the "propositions" contained in Euclid's Elements are finitely axiomatizable because they have been variously proved by employing a finite set of axioms.[24]

Every set of axioms contains a collection of primitive or undefined terms.[25] The function of the axioms is to specify the relations

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20. Beth, E.W., The Foundations of Mathematics. Amsterdam: North-Holland Publishing Company, 1965, p.81.

21. Cohen, M.R. and E. Nagel, An Introduction to Logic and Scientific Method. New York: Harcourt, Brace and World Inc., 1934, p.133.

22. Ibid., p.131.

23. Enderton, H.B., A Mathematical Introduction to Logic. New York: Academic Press, Inc., 1972, p.146.

24. Beth, op. cit., p.139.

25. Cohen and Nagel, op. cit., p.239.

which must or are considered to hold between the undefined terms.[26] The necessity for such primitive terms arises for the following reasons:

> "... when questioned of the truth or the reason for believing the truth of an assertion, we usually justify our belief by indicating that it ... can be deduced from certain other assertions which we accept. If somebody ... continues to ask for definitions or deductions, it is obvious that one of two things will happen. Either we find ourselves travelling in a circle, making use, in our answers, of concepts and assertions whose meaning and justification we originally set out to explain; or, at some stage, we refuse to supply any more definitions and deductions and reply bluntly that the concepts and assertions we employ in our answer are already the most basic which we take for granted."[27]

In Hilbert's axiomatization of Euclidean geometry for example, the primitive terms[28] are "point", "straight line", "order" (a point lies between the points x and y), "congruence" (congruence of line segments and of angles) and "incidence" (a point lies on a line, a line lies on a plane, a point lies in a plane). Other "concepts" are defined in terms of the primitives.[29]

> "... if A and B are points on a straight line a, the segment AB or BA can be defined as the set of points on a and between A and B."

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26. Ibid., p.135.
27. Wang, H., A Survey of Mathematical Logic. Peking: Science Press, 1962, p.1.
28. Weyl, H., Philosophy of Mathematics and Natural Science. Princeton: Princeton University Press, 1949, p.1.
29. Beth, op. cit., p.139.

"If a is a straight line and if B and C are points not on a, we shall say that B and C have similar position with respect to a if and only if the segment BC does not contain a point on a."

All definitions can be reduced to statements containing only the primitive terms. In the above examples, even though "similar position" is defined in terms of "segment" it can be reduced to a definition purely in terms of the primitives by merely replacing "segment" by its definition in the text. Thus in mathematics "definitions are implicit, the subject being defined in terms of the axioms which it must satisfy."[30] It is partly because of this that Euclid's Elements fail to provide a satisfactory answer to the problem of axiomatizing geometry. Euclid's "axioms"[31] consist of five "common notions" and five "postulates" and are reproduced in Table 2.1.[32] "Point" and "line" are obvious primitives,[33] yet Euclid defines them as "that which has no part" and "breadthless length" respectively.[34] In words, explicit definitions are provided. This caused Weyl to remark that Euclid

"... begins with opov definitions; but they are only in part definitions ... the most important among them are descriptions, indications of what is intuitively given. Nothing else, in fact, is possible after all for the basic geometri-

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30. Cohen and Nagel, op.cit., p.238.

31. Wang, loc.cit.

32. Heath, T.L., The Thirteen Books of Euclid's Elements, Volume 1, Cambridge: Cambridge University Press, 1908, pp.154-155.

33. Wang, op.cit., p.2.

34. Heath, op.cit., p.153.

TABLE 2.1

AXIOMS OF EUCLIDEAN GEOMETRY

POSTULATES

Let the following be postulated:

1. To draw a straight line from any point to any point.

2. To produce a finite straight line continuously in a straight line.

3. To describe a circle with any centre and distance.

4. That all right angles are equal to one another.

5. That, if a straight line falling on two straight lines make the interior angles on the same side less than two right angles, the two straight lines, if produced indefinitely, meet on that side on which are the angles less than the two right angles.

COMMON NOTIONS

1. Things which are equal to the same thing are also equal to one another.

2. If equals be added to equals, the remainders are equal.

3. If equals be subtracted from equals, the remainders are equal.

4. Things which coincide with one another are equal to one another.

5. The whole is greater than the part.

> cal concepts such as 'point', 'between', etc.; but as far as the deductive construction of geometry is concerned, descriptions of this kind are evidently irrelevant."[35]

Of course, the purpose of Euclid's system was to facilitate the provision of proofs of geometrical propositions. The method used was to argue deductively from the axioms and definitions to the desired proposition. As an example of this, Euclid's first proposition of Book I concerning the existence of equilateral triangles and its "proof" are reproduced in Table 2.2.[36] Note that it involves a statement of the proposition to be proved followed by a sequence of assertions in terms of the axioms and definitions, culminating in what was to be proved - the proposition itself.[37] Thus, the axioms imply the proposition.

Although Euclid's work attracted criticism practically from the time of its completion,[38] it was not until the end of the nineteenth century that Hilbert, amongst others, proved that Euclid appealed to a number of tacit "presuppositions" besides the axioms explicitly laid down, in proving several propositions.[39] They are the so-called "order" axioms which concern the "betweenness" properties of points and lines.[40]

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35. Weyl, op.cit., p.19.

36. Heath, op.cit., pp.241-242.

37. Cohen and Nagel, op.cit., p.136.

38. Beth, op.cit., p.139.

39. Ibid.

40. Ibid.

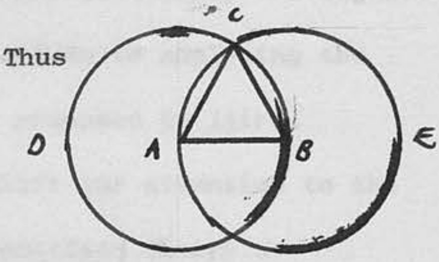
TABLE 2.2

EUCLID'S PROPOSITION I, BOOK I

On a given finite straight line to construct an equilateral triangle.

Proof

Let AB be the given finite straight line. Thus it is required to construct an equilateral triangle on the straight line AB.



With centre A and distance AB let the circle BCD be described (Post 3); again, with centre B and distance BA let the circle ACE be described (Post 3); and from the point C, in which the circles cut one another, to the points A, B let the straight lines CA, CB be joined (Post 1). Now, since the point A is the centre of the circle CDB, AC is equal to AB (Def. 15). Again, since the point B is the centre of the circle CAE, BC is equal to BA (Def. 15). But CA was also proved to equal AB; therefore each of the straight lines CA, CB is equal to AB. And things which are equal to the same thing are also equal to one another (C.N. 1); therefore CA is also equal to CB. Therefore, the three straight lines CA, AB, BC are equal to one another. Therefore the triangle ABC is equilateral; and it has been constructed on the given finite straight line AB. (Being) what it was required to do.

Thus, in "recent" years two major criticisms of Euclid's axiomatization of geometry have emerged. Firstly, he endeavoured to make explicit definitions of the primitive terms; secondly, he made implicit assumptions in "proving" propositions involving order.

Recall that the purpose of this section was to isolate the significant features of the axiomatic method as a prelude to analyzing the axiomatized method of accounting measurement proposed by Ijiri. Having accomplished the former task we now shift our attention to the latter, namely an examination of Ijiri's axiomatized theory of "conventional" accounting measurement.

2.2 Ijiri's System Criticized

Our inquiry into Ijiri's system shall endeavour to reveal two things. Firstly, we shall argue that Ijiri's system is not a deductive theory of accounting measurement and that any pretence to rigour within his system is purely superficial. Secondly, it will be argued that Ijiri was in error in eschewing a set theoretic foundation for accounting measurement.

2.2.1 Euclidean Geometry

The stated purpose of Ijiri's Foundations of Accounting Measurement was to approximate

> "... conventional accounting by devising a relatively simple set of axioms and valuation rules in the same manner that scientists in other fields have tried to develop a relatively simple set of concepts in order to explain complicated phenomena to a satisfactory degree."[41]

The resulting system devised by Ijiri is reproduced in Table 2.3[42,43] and is claimed to have the following properties:

> "The set of axioms and the set of valuation rules ... correspond to the set of axioms and the set of theorems in Euclidean geometry in the sense that if the set of

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41. Ijiri, op.cit., p.88.

42. Ibid, pp.90-95.

43. The "basic class" referred to is some numeraire, usually money. Ijiri has gone to great lengths to show that accounting measurements can be formulated by using some other numeraire such as wheat.
Ijiri, Y., "Physical Measures and Multi-Dimensional Accounting", in R.K. Jaedicke, Y. Ijiri, and O. Nielsen (editors), Research in Accounting Measurement, New York: American Accounting Association, 1966, pp.150-164.

TABLE 2.3

IJIRI'S HISTORIC COST "AXIOM SYSTEM"

Axioms

Control

There exists a method by which resources under the control (present or future, positive or negative) of a given entity at any time t are uniquely determined at that time or later.

Quantities

There exists a method by which all resources are uniquely partitioned into a collection of classes so that for each class a non-negative and additive quantity measure is defined and so that we are indifferent to any two sets of resources in the same class if and only if their quantities are the same.

Exchanges

There exists a method by which all changes in the resources controlled by a given entity up to any time t are identified at that time or later and are partitioned uniquely into an ordered set of pairs of an increment and a decrement, where the increment belongs to one and only one class.

Valuation Rules

Basic Rule 1

The value of any set of (present and future) resources in the basic class is defined to be equal to its quantity as determined by the quantity measure for the class.

Basic Rule 2

The value of an empty set is defined to be equal to zero.

TABLE 2.3.

Value Allocation Rule

Allocate the value of all resources in each class before the exchange to outgoing resources in the class and remaining resources in the class in proportion to their quantities. The sum of values allocated to outgoing resources in each class is the value of the decrement. Decrease the value of resources in each class by the value allocated to outgoing resources in the class.

Valuation Imputation Rule

If the resources in the increment belong to a non basic class, set the value of the increment equal to the value of the decrement. Increase the value of resources of the class by the value of the increment.

Value Comparison Rule

If the resources in the increment belong to the basic class, calculate a value gain or loss by subtracting the value of the decrement from the value of the increment.

axioms is granted the valuation rules can be applied in a purely mathematical way without making any empirical judgement ... (The axioms) are not a mere listing of concepts ... but are tied logically and mathematically to the set of valuation rules ..."[44]

However, in an axiom system there are the axioms themselves, definitions made in terms of the axioms, and theorems, lemmas and corollaries derived from the axioms; there are no valuation rules.[45] Thus

"... it is not clear just how ... the (valuation) rules are related to or derived from the axioms."[46]

The confusion is aggravated by the fact that at different times Ijiri has described the valuation rules as both definitions and theorems. Thus, having formulated the axioms of control, quantities and exchanges Ijiri declares

"Our task now is to define a method by which these heterogeneous quantity measures are converted into a homogeneous measure called a value measure."[47]

Yet in a later publication the following assertion appears

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44. Ijiri, Foundations, p.88.

45. See the previous section on "Axiomatics and Euclidean Geometry".

46. Dyckman, T.R., "The Foundations of Accounting Measurement", The Accounting Review, XLIII, 1 (January 1968), p.200.

47. Ijiri, op.cit., p.91.

"The set of valuation rules (listed in Table 2.3) is not the only set of such rules that can be derived from the three axioms, just as numerous theorems can be derived from Kolmogorov's axioms of probability or from the axioms of Euclidean geometry."[48]

Despite this latter and similar assertions it is our view that the valuation rules are purely definitional in nature. The Basic Rules one and two are explicitly stated definitions,[49] whilst no evidence has been provided by Ijiri to substantiate the view that the axioms imply the valuation rules as theorems. Further, conventional accounting is viewed

"as though it consisted of a set of axioms on the one hand and a set of valuation rules on the other. These are extracted from conventional accounting ..."[50]

In other places Ijiri describes the axioms as "empirical judgements" or "abilities" which when satisfied leave only the "computational procedure" of applying the valuation rules.[51] It would thus seem that the "logical" connection between the axioms and the valuation rules is an "empirical one" in that either the axioms contain sufficient information to operationalize the valuation rules or they do not. The "mathematical connection" is merely the computational one of "applying"[52] the valuation rules using

48. Ijiri Y., "Axioms for Historical Cost Valuation: A Reply", Journal of Accounting Research, IX, 1 (Spring 1971), p.184.

49. Basic Rule One is also poorly formulated. In none of the axioms is "basic class" mentioned. Thus, the definition is not (directly or indirectly) stated in terms of the primitives.

50. Ijiri, Foundations, p.88 (emphasis added).

51. Ibid., pp.84-85.

52. Ibid., p.88.

the information provided by the axioms.

We may state the "logical" connection between the axioms and the valuation rules in the following terms. Let P denote the statement "the information assumed by the axioms is known" and Q denote "we can operationalize any set of historic cost valuation rules". Then the connection is given thus[53]

P if and only if Q.

Note that the axioms do not imply the valuation rules in the sense that the axioms of Euclidean geometry imply the corresponding theorems. The theorems of Euclidean geometry are obtained by deductive argument; the valuation rules are defined.[54] It is misleading, therefore, for Ijiri to compare his "axiom system" with that of Euclidean geometry.

The mathematical connection between the axioms and the valuation rules is illustrated as follows.[55] Suppose an entity's property set at time t consists of 2,000 bushels of wheat with an historical cost of £2,000. In the interval [t, t+1] 1,000 bushels of the wheat are sold for £1,500. At time (t+1) the axiom of control is satisfied by noting that the entity owns some wheat. The axiom of quantities specifies that there are

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53. "... the set of axioms is necessary and sufficient to support the set of valuation rules." Ibid., p.88 (emphasis added).

54. See section 2.1.

55. Ijiri, Foundations, pp.92-95.

1,000 bushels of this commodity remaining, whilst the axiom of exchanges identifies that in order to obtain the £1,500 cash, 1,000 bushels of wheat was sacrificed. Given this information we can "apply"[56] the valuation rules. The value allocation rule allocates a value of £1,000 to the wheat sold whilst the value comparison rule recognizes a profit of £500 on the transaction. Note that once we know the information implied by the axioms there remains only the computational procedure of "applying" the valuation rules.

In some sense the theorems of Euclidean geometry are "applied" in the same way. Thus, for example, once the coordinates of Zurich ($\underset{\sim}{x}$) and London ($\underset{\sim}{y}$) are determined, the calculation of the distance between them $\|\underset{\sim}{x}-\underset{\sim}{y}\|$[57] is indeed a purely "mathematical" exercise involving no "empirical judgement". The subtle difference of course is that the axioms of Euclidean geometry imply the theorem that the linear distance between two points is $\|\underset{\sim}{x}-\underset{\sim}{y}\|$ whilst Ijiri's axioms do not imply the valuation rules as theorems.

As noted above, this is not the only source of contention in Ijiri's treatment of "conventional" accounting measurement.

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56. Ibid., p.88.

57. The distance can also be computed by defining the positive definite inner product $\langle \underset{\sim}{u},\underset{\sim}{v}\rangle = \sqrt{\underset{\sim}{u}^T.\underset{\sim}{v}}$ where $\underset{\sim}{u}$ and $\underset{\sim}{v}$ are real vectors. Thus

$$\langle \underset{\sim}{x}-\underset{\sim}{y},\ \underset{\sim}{x}-\underset{\sim}{y}\rangle = \sqrt{(\underset{\sim}{x}-\underset{\sim}{y})^T.(\underset{\sim}{x}-\underset{\sim}{y})}$$
$$= \|\underset{\sim}{x}-\underset{\sim}{y}\|$$

Specifically, Ijiri's testimony that it is not possible to construct a set theoretic based axiom system of accounting measurement is questionable. We now proceed to expand upon this proposition.

2.2.2 Set Theory

In developing his "axiomatic" theory of historical cost measurement Ijiri discarded a set theoretic foundation on the grounds that

> "... the mathematical notions of set, field, etc. are all based on two-valued logic where an element either belongs to or does not belong to the set or field. However, assets on the balance sheet may be shown as belonging to the entity either positively or negatively. Thus, a resource can take any of three states with respect to the entity. It belongs to the entity positively, it belongs to the entity negatively or it does not belong to the entity."[58]

The axiom of exchanges was introduced to overcome this problem.

> "It was not until I separated control criteria and recognition criteria that I felt completely comfortable about the set of resources as the starting point for constructing the axiomatic system."[59]

However, if the quantification of assets and liabilities is separated from their valuation it is a relatively simple matter to construct a set theoretic based axiom system. Suppose

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58. Ijiri, "Axioms for Historic Cost Valuation", p.183.

59. Ibid., p.184.

for example, that an entity purchases on credit 10,000 widgets at £2 each thus incurring a debt of £20,000. Its property set consists of 10,000 widgets and an account payable. This process involves the binary operation of partitioning assets and liabilities into two sets - those belonging to the entity positively and those not belonging to the entity. A measurement rule can then be defined which appropriates a "value" of £20,000 to the widgets and - £20,000 to the accounts payable. In the next section one such system is specified.

Despite these criticisms Ijiri's contribution to the theory of accounting measurement is original and unique.[60] He rid the theory of accounting measurement of the shackles of double entry bookkeeping, realizing that accounting measurement encompasses more than just a formal recording function.[61] When an accountant determines the unit cost of stock an accounting measurement has occurred and this may or may not be recorded in a set of books. In words, it is no more necessary to have a set of formal rules governing the way measurements shall be written down on paper in accounting than it is in Euclidean geometry or statistical inference. However, by far his greatest contribution are the axioms of historical cost measurement. It is these which are the foundations of the generalized theory of accounting measurement exhibited in the next section.

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60. We echo the following remark

"... Ijiri's work ranks with Edwards and Bell's classic ... as a must for serious scholars of accounting thought."

Dyckman, op.cit., pp.199-200.

61. Ijiri, "Axioms and Structures of Conventional Accounting Measurement", op.cit., p.36.

We have now demonstrated that Ijiri's "axiomatic model" of "conventional" accounting measurement is not, in fact, a deductive theory and that he was incorrect in eschewing a set theoretic foundation for accounting measurement. In the next section we shall develop upon this theme by providing a set theoretic based deductive theory of accounting measurement.

2.3 Axioms of Accounting Measurement

In the previous sections it was claimed that Ijiri's system could be modified so as to provide a set theoretic based deductive theory of accounting measurement. In this section we shall demonstrate how this may be achieved. After stating the modified system and illustrating its practical implementation in the context of an historical cost accounting example, we shall state some formal consequences of the axioms and demonstrate how various accounting concepts such as "asset", "liability" and "profit" may be defined within the system.

2.3.1 Axioms and Resource Sets

Accounting measurement is concerned with the monetary expression of resources belonging to a designated entity. Thus for a mathematical theory of accounting measurement the essential ingredients are the existence of a non-empty set of resources belonging to a clearly defined accounting entity upon which can be imposed in a consistent and comprehensive fashion, a measurement rule which associates a real number with each element of the resource set.

Given an accounting entity the set of resources belonging to that entity at time t is called a property set and will be denoted P_t. The individual resources generically denoted by p comprise the elements of the property set and satisfy the following conditions[62]

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62. The union of sets $p_{j,t}$ and $p_{k,t}$ denoted $p_{j,t} \cup p_{k,t}$ is the set all elements which belong to $p_{j,t}$ or $p_{k,t}$ or to both. The intersection of sets $p_{j,t}$ and $p_{k,t}$ denoted $p_{j,t} \cap p_{k,t}$ is the set of elements which belong to $p_{j,t}$ and also to $p_{k,t}$. If j = k then $p_{j,t} \cap p_{k,t} = p_{j,t} \cap p_{j,t} = p_{j,t}$ which is not empty.

(i) $P_t = \bigcup_{j=1}^{n} p_{j,t}$ $\qquad j = 1, 2, \_\_\_\_, n$

(ii) $p_{j,t} \cap p_{k,t} = \emptyset$ $\qquad j \neq k$

Hence, the individual resources form a partition[63] of the property set P_t and will be called simple resources or alternatively simple resource sets. Unions of simple resource sets are called compound resources or compound resource sets. Simple and compound resource sets shall be collectively referred to as "resource sets". Let $\mathcal{F}_t$ be the family of subsets of P_t which are generated by the simple resource sets. The elements of $\mathcal{F}_t$ have the following properties[64]

(i) If $A, B \in \mathcal{F}_t$ then $A \cup B \in \mathcal{F}_t$

(ii) If $A \in \mathcal{F}_t$ then $A^c \in \mathcal{F}_t$

where the complement in (ii) is with respect to P_t. A collection of sets having these properties is called an algebra or field.[65]

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63. The family of non-empty sets $B_j \quad j \in I$ is said to form a partition of the set A if and only if
    (i) $\bigcup_{j \in I} B_j = A$
    (ii) For any $i \neq j$ $B_i \cap B_j = \emptyset$
    $\emptyset$ is the empty or null set; the set having no elements.

64. The complement of a set A denoted A^c is the set of elements in P_t which are not in A.

65. Beth, op.cit., pp.163-164.

The conditions mean that $\mathcal{F}_t$ is closed under the formation of unions and complements. In addition, it can be shown that $\mathcal{F}_t$ is closed under the formation of intersections and that $\mathcal{F}_t$ contains the empty set.[66] These closure properties ensure that we will never need to consider a resource set which does not belong to the entity under consideration because it is not possible to manipulate any collection of sets using only the permissible set operations of union, intersection and complementation and so obtain a resource set which does not belong to the entity; that is, is not in $\mathcal{F}_t$. Further, for any A, B and C in $\mathcal{F}_t$ the following "conditions" also hold[67]

(i)	$A \cup B$	=	$B \cup A$
(ii)	$A \cap B$	=	$B \cap A$
(iii)	$A \cup (B \cap C)$	=	$(A \cup B) \cap (A \cup C)$
(iv)	$A \cap (B \cup C)$	=	$(A \cap B) \cap (A \cap C)$

(v) There is in $\mathcal{F}_t$ an element X such that, for any Y in $\mathcal{F}_t$

$$Y \cup (A \cap X) = Y$$

and

$$Y \cap (A \cup X) = Y$$

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66. By (i) and (ii) $A^c \cup B^c \in \mathcal{F}_t$

By (ii) $(A^c \cup B^c)^c \in \mathcal{F}_t$

But $(A^c \cup B^c)^c = A \cap B \in \mathcal{F}_t$

Proving that $\mathcal{F}_t$ is closed under the formation of intersections.

By axiom $P_t \in \mathcal{F}_t$

By (ii) $P_t^c \in \mathcal{F}_t$

But $P_t^c = \emptyset \in \mathcal{F}_t$

Thus, proving that the empty set is in $\mathcal{F}_t$.

67. Ibid., p.164.

The triple $(\mathcal{F}_t, \cup, \cap)$ is called a Boolean algebra.[68] Equipped with this knowledge we exhibit in Table 2.4[69] a set of axioms which are essential to any theory of accounting measurement. In words, historical cost, market value, price level adjusted and replacement cost measurement may serve as models of the axiom system.

Having furnished the modified set of axioms for accounting measurement we now illustrate their use in the context of an historical cost accounting example.

2.3.2 <u>An "Historic cost" Example</u>

The balance sheet of the Dyer Company Limited as of January 1, 1909 and the transactions for the year ending December 31, 1909 are exhibited in Table 2.5. We define the simple resource sets in the following terms since they exhibit the properties demanded by the axiom of quantities.[70]

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68. *Ibid.*

69. The reader versed in probability theory will see that this axiom system is based on Komolgorov's axiomatization of a finite probability space. It was Littleton who first emphasized the "statistical nature" of accounting measurement

"... the subject matter of accounting is inescapably economic and its basic methodology is unquestionably statistical in character."

Littleton A.C. *Structure of Accounting Theory*, New York: American Accounting Association, 1953, p.8.

However, no one has taken up the obvious implication of this for an axiomatized theory of accounting measurement.

70. In general, simple resource sets are not unique. For example, if it suited our purpose we could specify the simple resource sets to be equity, assets and liabilities. This is done, in fact, in the section on "Profit, Assets and Liabilities".

TABLE 2.4

AXIOMS OF ACCOUNTING MEASUREMENT

1. Axiom of Control

There exists a "property set" P_t which is uniquely defined for all non-negative real t.

2. Axiom of Quantities

There exists an algebra $\wp_t$ generated by the "simple resource sets" $p_{j,t}$, j = 1, 2, ___, n and having the following properties

(a) $\bigcup_{j=1}^{n} p_{j,t} = P_t$

(b) $p_{j,t} \cap p_{k,t} = \emptyset \qquad j \neq k$

for some positive integer n.

3. Axiom of Measurement

There exists a mapping called a "measurement rule"

$L_t : \wp_t \longrightarrow IR$

with the property

$L_t(A_{j,t} \cup A_{k,t}) = L_t(A_{j,t}) + L_t(A_{k,t})$

for any pair of disjoint sets $A_{j,t}$ and $A_{k,t} \in \wp_t$.

TABLE 2.5

DYER COMPANY LIMITED

Balance Sheet - January 1, 1909

Shareholders' Funds	£	Fixed Asset		£
Capital	50,000	Building		80,000
Profit unappropriated	60,000	Less Aggregate Depreciation		20,000
	110,000			60,000
Current Liability		**Current Assets**		
Trade creditors	5,000	Cash	25,000	
		Trade debtors	10,000	
		Securities	10,000	
		Stock	10,000	55,000
	£115,000			£115,000

Stock: Recorded using "perpetual lifo"; 1,000 units at £10 (per unit).

Building: Purchased January 1, 1904. Straight line depreciation is used where the life estimation is 20 years (no salvage value).

Transactions in the year ending December 31, 1909

1909

Jan. 30 : Purchased (on credit) 500 units at £11 (per unit).

Feb. 28 : Sold 800 units (on credit) at £20 (per unit).

Mar. 31 : Received £10,000 from debtors (no discounts).

April 30 : Paid £8,000 to creditors (no discounts).

Aug. 31 : Sold 500 units (on credit) at £21 (per unit).

Nov. 30 : Purchased 300 units (on credit) at £13 (per unit).

$p_{1,5}$ = "building"

$p_{2,5}$ = "cash"

$p_{3,5}$ = "trade debtors"

$p_{4,5}$ = "securities"

$p_{5,5}$ = "stock"

$p_{6,5}$ = "trade creditors"

Note that the simple resources form a partition of the property set P_5. The 64 subsets[71] which may be formed from the property set P_5 determine the algebra $\mathcal{E}_5$. The algebra $\mathcal{E}_5$ is the domain of the following measurement rule

$$L_5(p_{j,5}) = \begin{cases} 60{,}000 & \text{if } j = 1 \\ 25{,}000 & \text{if } j = 2 \\ 10{,}000 & \text{if } j = 3 \\ 10{,}000 & \text{if } j = 4 \\ 10{,}000 & \text{if } j = 5 \\ -\,5{,}000 & \text{if } j = 6 \end{cases}$$

The measurement rule L_5 depreciates the building using a straight line allowance of 5% p.a. Note that once the measures of the simple resources are given, the measure of every other set in the algebra $\mathcal{E}_5$ can be determined because such sets are merely unions of simple resource sets.[72]

Recall that the axiom system exhibited in Table 2.4 repre-

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71. See Theorem 1, below: $2^6 = 64$.

72. In reaching this conclusion we use Theorem 4 below.

sents a deductive theory of accounting measurement. As such the axioms imply certain statements about accounting measurement which can be obtained by "deductive reasoning" alone. In the next section, therefore, we isolate some consequences of the axioms.

2.3.3 The System Developed

In this section we state some formal consequences of the axioms particularized above. All proofs are relegated to the appendix so that we may concentrate on the more important task of interpreting the significance of the results.

Theorem 1

If P_t is the union of n (a positive integer) simple resource sets then $\mathcal{P}_t$ has 2^n elements.

In the example of the previous section P_5 was the union of six simple resources. The algebra $\mathcal{P}_5$ formed from this set has as its elements, the empty set, six simple resources, fifteen sets containing two simple resources, twenty sets containing 3 simple resources, fifteen sets containing four simple resources, six sets containing five simple resources and the property set P_5, itself.[73]

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73. One "easy" method of determining the elements of the algebra is to use "Pascal's Triangle".

Elements in P	Triangle	Elements in $\mathcal{P}_t$
0	1	1
1	1 1	2
2	1 2 1	4
3	1 3 3 1	8
4	1 4 6 4 1	16
5	1 5 10 10 5 1	32
6	1 6 15 20 15 6 1	64

Each element in the triangle is obtained by adding the elements to the right and left in the preceding row.

Theorem 2

$$L_t(\emptyset) = 0$$

The significance of this result is that it implies that Ijiri's system is in some sense redundant,[74] because by Basic Rule 2 this result is defined and therefore is not a consequence of the axioms. Thus, if a set is empty, its measure is zero, irrespective of what type of measurement rule is used.

Definition 1

The triple $(P_t, \mathcal{E}_t, L_t)$ is called an accounting measurement space.

Definition 2[75]

Suppose $A_{j,t} \in \mathcal{E}_t$; then $L_t(A_{j,t})$ is called the measure of $A_{j,t}$.

Definition 3

Suppose $A_{j,t} \in \mathcal{E}_t$ is not a simple resource set; then $A_{j,t}$

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74. Ijiri's system may not be redundant in the mathematical sense of course; that is it may not be possible to prove the Basic Rule Two from Ijiri's axioms. For a discussion of redundancy see Cohen and Nagel, <u>op.cit.</u>, pp.143-147.

75. The measure appropriated to each resource in $\mathcal{E}_t$ is unique by virtue of the fact that L_t is a mapping.
See Giles J.R., <u>Real Analysis</u>, Sydney: John Wiley and Sons, Australasia Pty. Ltd., 1972, p.13.

is called a compound resource set.[76]

The importance of these definitions stems from the following result

Theorem 3

The accounting measurement space $(P_t, \mathcal{F}_t, L_t)$ is completely described by its simple resource sets and their measures.

The importance of this theorem is that it implies that once the simple resources and their measures are known, P_t is known, and the measure of every set in $\mathcal{F}_t$ can be determined. Thus, in the case of the Dyer Company Limited, knowledge of the simple resources and their measures is sufficient to determine the measure of "current assets".

Theorem 4

If $A_{j,t} \in \mathcal{F}_t$ $j = 1, 2, \text{———}, n$ is a disjoint sequence of resource sets then

$$L_t\left(\bigcup_{j=1}^{n} A_{j,t}\right) = \sum_{j=1}^{n} L_t(A_{j,t})$$

76. That compound resource sets exist follows from Theorem 1. It was stated that $\mathcal{F}_t$ has 2^n elements where n is the number of simple resources. Hence, if we consider the empty set to be a compound resource, there are $2^n - n$ compound resource sets.

Theorem 5

If $A, B \in \mathcal{E}_t$ then

$$L_t(A \cup B) = L_t(A) + L_t(B) - L_t(A \cap B)$$

Theorems 4 and 5 together imply that in general, accounting measurements are not additive. Thus, for example, the measure of current assets and non-monetary assets is not necessarily the sum of their separate measures.

Theorem 6

Suppose $B = \bigcup_{j=1}^{n} A_{j,t} \in \mathcal{E}_t$

is a disjoint sequence of resource sets with the properties

(i) $L_t(A_{1,t}) = L_t(A_{2,t}) = \text{———} = L_t(A_{n,t})$

(ii) $L_t(B) = T$

then $L_t(A_{j,t}) = n^{-1}T \qquad j = 1, 2, \text{———}, n$

In effect, Theorem 6 states a set of assumptions which justify appropriating the same measure to every element of a resource set. There are numerous instances of this practice in accounting.[77]

The importance of these theorems is that they are true in

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77. Horngren C.T., Cost Accounting: A Managerial Emphasis, Englewood Cliffs, New Jersey: Prentice-Hall, Inc., 1972, Chapters 4 and 17.

every accounting measurement system which satisfies the axioms. They are not the only theorems which can be deduced from the axioms but merely a sample of the more obvious and useful.

Besides serving as a means through which the basic properties of accounting measurement rules can be derived, however, the axioms also enable us to define key accounting concepts in a clear and unequivocal manner. In the next section, therefore, we demonstrate the procedure by which definitions of accounting concepts may be made in terms of the axioms.

2.3.4 Profit, Assets and Liabilities

If the axiom system is to serve as a model of accounting measurement we need to define a profit measure.

Definition 4

The mapping $\pi : \mathbb{R}^2 \longrightarrow \mathbb{R}$ defined by

$$\pi(t, t-n) = L_t(P_t) - L_{t-n}(P_{t-n})$$

for all real $t > n$ is called the "profit measure" of the interval $[t-n, t]$.[78]

This is the "usual" definition of profit

"The income figure for a period is the difference between the value of assets at the end of the period and the

78. The profit measure is a real valued function with domain the Cartesian space $\mathbb{R} \times \mathbb{R} = \mathbb{R}^2$. This is so because in order to operationalize it requires two real numbers; one each for $L_t(P_t)$ and $L_{t-n}(P_{t-n})$ respectively.

value of assets at the beginning."[79]

Note that our profit measure is "defective" in that dividends, prior period adjustments, capital contributions and similar items are treated as income or expense of the interval $[t-n, t]$. This could be avoided by the addition of extra axioms. However, whilst this would improve the predictive ability of our axiom system it would do very little to enhance the analytical exposition.

Unlike Ijiri's algorithm for computing income[80] our profit measure is "balance sheet" oriented. This does not imply that our system is incapable of providing an analysis of the "economic phenomena" (transactions in the historic cost system) connecting any two balance sheets. Thus, for the historic cost system, define a reporting rule under which financial statements are prepared after each transaction. The increment and decrement of each transaction can then be specified by comparing the latest balance sheet with its immediate predecessor. Suppose we analyze the transactions of the Dyer Company Limited in terms of their effect on equity, assets and liabilities. We construct the "transactions matrix" $A_{5,6}$ for the year ending December 31, 1909.

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79. Ijiri, Foundations, p.97.

80. Ijiri's method for computing profit is stated as follows:

"An exchange involves two sets of resources, an increment d^+ and a decrement d^- ... . All changes in the assets are partitioned into a set of pairs (d^+, d^-); when all increments d^+'s and all decrements d^-'s in the set of pairs are added together to derive I^+ and I^-, respectively, we obtain the income (I^+, I^-) for the period ..." Ibid., p.89.

		CR	
	Equity	Assets	Liabilities
Equity	a_{11}	a_{12}	a_{13}
DR Assets	a_{21}	a_{22}	a_{23}
Liabilities	a_{31}	a_{32}	a_{33}

The element a_{21} of the transactions matrix, for example, represents a debit to assets and credit to equity. An example of such an entry is the Feb. 28 sale of stock.

Dr.	Trade debtors	16,000	
Cr.	Sales		16,000

The complete transactions matrix of the Dyer Company Limited is exhibited below:

$$A_{5,6} = \begin{bmatrix} 0 & 17,500 & 0 \\ 26,500 & 10,000 & 9,400 \\ 0 & 8,000 & 0 \end{bmatrix}$$

Suppose we let $\underset{\sim}{x}_5$ be the vector whose elements are equity, assets and liabilities respectively of the Dyer Company Limited as of January 1, 1909.

$$\underset{\sim}{x}_5 = \begin{bmatrix} -110,000 \\ 115,000 \\ -\ \ 5,000 \end{bmatrix}$$

If we let $\underset{\sim}{x}_6$ be the equivalent vector as of December 31, 1909

$$\underset{\sim}{x}_6 = \begin{bmatrix} -119{,}000 \\ 125{,}400 \\ -\ \ 6{,}400 \end{bmatrix}$$

then the connection between $\underset{\sim}{x}_6$ and $\underset{\sim}{x}_5$ may be described by the equation

$$\underset{\sim}{x}_6 = \underset{\sim}{x}_5 + (A_{5,6} - A_{5,6}{}^T)(\underset{\sim}{y} + \underset{\sim}{k})$$

where $\underset{\sim}{y}^T = [1,1,1]$ and $\underset{\sim}{k}$ is any vector in the kernel or null space of $(A-A^T)$.[81] Using the first elements of $\underset{\sim}{x}_6$ and $\underset{\sim}{x}_5$ we now compute the income of the Dyer Company Limited for the year ending 31, 1909.

$$\pi(\ _6,\ \ _5) = L_6(P_6) - L_5(P_5)$$
$$\pi(\ _6,\ \ _5) = 9{,}000$$

Perhaps the most contentious parts of our axiom system are the "implied" definitions of asset and liability which we now make explicit.

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81. The Kernel or Null space of a homomorphism $\emptyset : 1R^n \longrightarrow 1R^m$ represented by the matrix $(A-A^T)$ is the set of vectors K such that for any $\underset{\sim}{k} \in K$

$$(A - A^T)\underset{\sim}{k} = \underset{\sim}{0}$$

That is, the kernel is the set of vectors mapped to the zero vector.

Definition 5

Suppose $p_{j,t} \in \mathcal{P}_t$ is a simple resource set. If $L_t(p_{j,t}) > 0$ then $p_{j,t}$ is called an asset. If $L_t(p_{j,t}) < 0$ then $p_{j,t}$ is called a liability.

At first sight this definition may appear to contain several deficiencies. For example, since accumulated depreciation has a negative measure, our system appears to classify it as a liability. However, we can exclude accumulated depreciation from an entity's property set, on the grounds that the set being measured is the fixed asset and the measure afforded this asset is time dependent. That is, the measurement rule

$$L(A_{j,t}) = \begin{cases} Y \dfrac{t^* - t}{t^* - t^o} & \text{if } t^* \geqslant t \\ 0 & \text{if } t^* < t \end{cases}$$

appropriates a measure to the fixed asset $A_{j,t}$ by netting accumulated depreciation $Y \dfrac{t - t^o}{t^* - t^o}$ against the cost Y.[82] Hence, there is no need for "accumulated depreciation" to appear as a simple resource set in the entity's property set and thus the need

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82. $(t^* - t^o)$ is the anticipated productive life (in years) of the asset. t^o is the date the asset is put into service and t is the anticipated date of withdrawal from service. The net book value of the asset at time t, assuming zero scrap value, is computed thus:

$$Y - Y \frac{t - t^o}{t^* - t^o}$$

which by factoring $t^* - t^o$ may be shown to yield the measurement rule for $t^* \geqslant t$.

for a separate measure is avoided. Similar treatments can be afforded prepayments, provision for doubtful debts and deferred income and expense.

A second line of argument is that the definition appears to classify shareholders' equity as a liability. This criticism is avoided by excluding shareholders' equity from the property set P_t.[83] This does not detract from the validity of the axiom system because the property set P_t is then composed of the assets and liabilities making up shareholders' equity.

We have now stated and illustrated a modified version of Ijiri's "axiom system" which has the property that it represents a set theoretic based deductive theory of accounting measurement. Further, the basic properties of accounting measurement rules were derived using the axioms and some key accounting concepts were defined in terms of them. To conclude this chapter we shall compare and contrast Ijiri's "axiom system" with the modified version developed above.

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83. Obviously if the "profit measure" of definition 4 is not to be identically zero, it is essential that we give P_t this interpretation.

2.4 A Comparison of Systems

In this section we particularize the connection between Ijiri's "axiom system" and that developed above. In this respect, perhaps the most basic and important difference between the two systems is that ours is an axiom system of accounting measurement in general, whilst Ijiri's system is stated for historical cost accounting measurement only. Ijiri's aim was to construct an axiom system for which "conventional" accounting served as a model. As such it was

> "... based upon such principles as historical cost, realization and accrual."[84]

This meant that

> "... such concepts as current market values, replacement costs and net realizable values."[85]

were, of necessity, neglected. Our analysis, however, is based upon the assumption that there are certain procedures which are common to all accounting measurement systems. Thus, our system is as relevant to replacement cost and market value measurement as it is to historical cost measurement.

Secondly, whilst in each system the axiom of control partitions resources into two sets - those which belong to an accounting entity and those which do not - there is a fundamental difference in the axiom

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84. Ijiri, op.cit., p.98.

85. Ibid.

of quantities in the respective systems. Ijiri's system requires unit quantification as a basis for valuation through the axiom of exchanges and the valuation rules. In our system, the axiom of quantities defines an algebra which forms the domain of a real valued measurement rule. Thus, in our system unit quantification is not strictly necessary.[86]

Finally, Ijiri's valuation rules taken in conjunction with the axiom of exchanges states one historic cost measurement rule. In principle, this measurement rule satisfies our third axiom though strictly from a mathematical point of view it is impossible for it to do so. By the axiom of exchanges a change in the property set can be represented by the unique ordered pair (d^+,d^-) where d^+ is an "increment" and d^- is a "decrement". In our system the measurement rule provided by the axiom of measurement is defined on the algebra $\mathcal{F}_t$ generated by property set P_t. But since d^- is not in general a resource set in $\mathcal{F}_t$, its measure $L_t(d^-)$ is in general undefined. However, it is entirely "permissible" to state a measurement rule L_t where the measure afforded a resource set is derived by using Ijiri's valuation rules. Thus suppose an entity exchanges £2,000 cash for 2,000 bushels of wheat. Whilst the value imputation rule does not satisfy the axiom of measurement because its domain includes d^- which in general is not in the algebra upon which L_t is defined, the measurement rule $L_t(p_{j,t}) = 2{,}000$ where $p_{j,t}$ is the simple resource set containing 2,000 bushels of wheat, does satisfy the axiom. Hence while the valuation rule does not satisfy the axiom, the measure derived from its use does.

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86. This conclusion does not imply that our axiom system cannot accommodate unit quantification. By redefining the axiom of measurement so that its domain is the set of subsets of P_t, it is possible to partition each simple resource set so as to define a quantity measure of the set.

2.5 Summary

The purpose of this chapter was to construct an axiomatic theory of accounting measurement. In order to accomplish this task it was necessary to specify the properties possessed by an axiom system. Undoubtedly, the best known axiom scheme is the geometrical system formulated by Euclid. Consequently, a brief review of his system was undertaken.

Ijiri's "axiom system" was then analyzed and found to lack certain of the properties possessed by Euclid's system. This was so despite Ijiri's claim that Euclid's and his system are "analogous" in principle. However, it was shown that Ijiri's system contained the germ of an acceptable axiomatic theory of accounting measurement.

Finally, a set of three axioms was defined and several theorems were derived. It was found that this system could accommodate many commonly encountered concepts such as income, asset and expense.

APPENDIX 2A

Theorem 1

If P_t is the union of n (a positive integer) simple resource sets then $\mathfrak{P}_t$ has 2^n elements.

Proof

There are $C_o^n = 1$ empty sets in $\mathfrak{P}_t$; $C_1^n = n$ simple resource sets; C_2^n compound resource sets containing two simple resources; and so on. There are thus

$$\sum_{j=o}^{n} C_j^n \text{ elements in } \mathfrak{P}_t. \quad \text{But } \sum_{j=o}^{n} C_j^n a^j b^{n-j} = (a+b)^n$$

Letting $a = b = 1$ proves the result.

Remark

For the data of the Dyer Company Limited as of January 1, 1909, six simple resource sets were defined. Theorem 1 implies that there are $2^6 = 64$ resource sets in the algebra generated by the simple resources. Letting a denote the simple resource "building", b the simple resource "cash" and so on, the elements of the algebra can be depicted as follows:

Resource sets containing zero elements

Ø. 1

Resource sets containing one element

a, b, c, d, e, f. 6

Resource sets containing two elements

ab, ac, ad, ae, af, bc, bd, be, bf, cd, ce, cf, de, df, ef. 15

Resource sets containing three elements

abc, abd, abe, abf, acd, ace, acf, ade, adf, aef,

bcd, bce, bcf, bde, bdf, bef,

cde, cdf, cef,

def. 20

Resource sets containing four elements

abcd, abce, abcf, abde, abdf, abef, acde, acdf, acef, adef,

bcde, bcdf, bcef, bdef,

cdef. 15

Resource sets containing five elements

abcde, abcdf, abcef, abdef, acdef,

bcdef. 6

Resource sets containing six elements

abcdef 1

64

Note that the compound resource containing six elements is the property set P_5 and the resource sets containing one element are, in fact, the simple resource sets.

Theorem 2

$$L_t(\emptyset) = 0$$

Proof

By the "Laws of the Albegra of Sets", for any $A \in \mathcal{B}_t$

(i) $A \cup \emptyset = A$

(ii) $A \cap \emptyset = \emptyset$

From (ii) A and $\emptyset$ are disjoint. Thus, applying the axiom of measurement

$$L_t(A \cup \emptyset) = L_t(A) + L_t(\emptyset)$$

$$L_t(A) = L_t(A) + L_t(\emptyset)$$

$$L_t(\emptyset) = 0$$

Remark

Suppose for any resource set $A_{j,t} \in \mathcal{F}_t$ (trade debtors, land, accrued charges, etc.) we have $A_{j,t} = \emptyset$. Then Theorem 2 implies

$$L_t(A_{j,t}) = 0$$

Theorem 3

The accounting measurement space $(P_t, \mathcal{F}_t, L_t)$ is completely described by its simple resource sets and their measures.

Proof

Since P_t is the union of the simple resource sets which by hypothesis are known, we know P_t. Since we know the simple resource sets, we can construct the algebra $\mathcal{F}_t$, since it consists of all possible unions of the simple resource sets. By applying the axiom of measurement and Theorem 4 to all possible unions of simple resources we can compute the measures of the compound resources.

Theorem 4

If $A_{j,t} \in \mathcal{F}_t$, j = 1, 2, ———, n is a disjoint sequence of resource sets then

$$L_t\left(\bigcup_{j=1}^{n} A_{j,t}\right) = \sum_{j=1}^{n} L_t(A_{j,t})$$

Proof

Define $B_2 = \bigcup_{j=2}^{n} A_{j,t}$

It then follows from the axiom of measurement

$$L_t\left(\bigcup_{j=1}^{n} A_{j,t}\right) = L_t(A_{1,t} \cup B_2)$$

$$L_t\left(\bigcup_{j=1}^{n} A_{j,t}\right) = L_t(A_{1,t}) + L_t(B_2)$$

Define $B_3 = \bigcup_{j=3}^{n} A_{j,t}$

It then follows from the axiom of measurement

$$L_t\left(\bigcup_{j=1}^{n} A_{j,t}\right) = L_t(A_{1,t}) + L_t(A_{2,t} \cup B_3)$$

$$L_t\left(\bigcup_{j=1}^{n} A_{j,t}\right) = L_t(A_{1,t}) + L_t(A_{2,t}) + L_t(B_3)$$

Continuing this process proves the result.

Remark

Theorem 3 effectively says that once the simple resource sets and their associated measurements are known, we can compute the measurement of any compound resource set in the algebra generated by the simple resources. Suppose, for example, we desire to determine the measure of current assets for the Dyer Company Limited as of January 1, 1909. By Theorem 4 we have

$$L_5\left(\bigcup_{j=2}^{5} p_{j,5}\right) = \sum_{j=2}^{5} L_5(p_{j,5})$$

$$= 25{,}000 + 2 \times 10{,}000$$

$= 55{,}000$

By similar procedures we may compute the measure of any collection of resource sets in the algebra $\mathcal{F}_5$.

Theorem 5

If $A, B \in \mathcal{F}_t$ then $L_t(A \cup B) = L_t(A) + L_t(B) - L_t(A \cap B)$

Proof

By the "Laws of the Algebra of Sets" the following may be proved

$$A \cup (B \cap A^c) = (A \cup B) \cap (A \cup A^c)$$

$$= A \cup B$$

As the sets A and $(B \cap A^c)$ are disjoint we may apply the axiom of measurement

$$L_t(A) + L_t(A^c \cap B) = L_t(A \cup B) \qquad \text{(i)}$$

$$B = B \cap (A \cup A^c)$$

$$= (B \cap A) \cup (B \cap A^c)$$

As the sets $(B \cap A)$ and $(B \cap A^c)$ are disjoint we apply the axiom of measurement

$$L_t(B) - L_t(A \cap B) = L_t(A^c \cap B) \qquad \text{(ii)}$$

substituting (ii) into (i) gives the result.

Remark

Define $N = p_{1,5} \cup p_{4,5} \cup p_{5,5}$ to be the non-monetary assets of the

Dyer Company Limited as of January 1, 1909. Similarly, define $C = \bigcup_{j=2}^{5} p_{j,5}$ to be the current assets as of the same date. Note that $N \cap C = p_{4,5} \cup p_{5,5}$. From Theorem 5 we have

$$
\begin{aligned}
L_5(N \cup C) &= L_5(N) + L_5(C) - L_5(N \cap C) \\
&= 80{,}000 + 55{,}000 - 20{,}000 \\
L_5(N \cup C) &= 115{,}000
\end{aligned}
$$

Since the simple resource sets "securities" and "inventory" are both current assets and non-monetary assets, the measure of the union of current assets and non-monetary assets, is not the sum of their separate measures. To avoid double counting, we must subtract the measure of this common element.

Theorem 6

Suppose $B = \bigcup_{j=1}^{n} A_{j,t} \in \mathcal{F}_t$ is a disjoint sequence of resource sets with the properties

(i) $L_t(A_{1,t}) = L_t(A_{2,t}) = \text{———} = L_t(A_{n,t})$

(ii) $L_t(B) = T$

then $L_t(A_{j,t}) = n^{-1}T \qquad j = 1, 2, \text{———}, n$

Proof

By hypothesis we have

$$nL_t(A_{j,t}) = T$$

for some arbitrarily chosen j, proving the result.

Remark

For the Dyer Company Limited as of January 1, 1909 define each unit of stock to be a simple resource. Suppose it is "known" that the measure of each unit of stock (each element of the compound resource "stock") has the same measure. It follows from Theorem 6 that stock has a "value" of £1 (per unit).

CHAPTER THREE

MEASUREMENT IN ACCOUNTING

3.0 Introduction

In the previous chapter the mathematical foundations of accounting measurement were examined in some detail. Using a set of three axioms, some properties common to all accounting measurement systems were derived and their implications examined. Absent, however, was a discussion of the factors which influence the specification of accounting measurements; that is, the allocation of numbers to the resource sets composing the algebra $\mathcal{E}_t$. That accountants are prone to disagreement on this aspect of the accounting function is well documented.[1] Yet, despite this, there has been no attempt at formalizing a statistical theory of accounting measurement. In our view, one reason for this is that efforts at providing a logical framework for accounting measurement have adopted methods which, essentially, are alien to accounting. A prime example is the repeated reference one finds in the accounting literature to the work of Stevens.[2] Whilst Stevens' work would appear to bear some significance for accounting measurement, it is our view that its implications for measurement, in general, have not been fully appreciated by accountants.

For these reasons, the purpose of this chapter is to examine measurement in accounting at two levels. Firstly, we undertake to analyze Stevens' measurement scheme. After introducing the concept of a measurement rule, the "scales" of measurement are defined and

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1. Sterling, R.R. "Cost Versus Values: An Empirical Test", The Australian Accountant, 41, 5 (June 1971), pp.218-21.

2. Stevens, S.S. "On the Theory of the Scales of Measurement", Science, C111 (June 7, 1946), pp.677-80.

illustrated. We then examine the "meaningfulness" of the statistical manipulationsapplied to each of these scales. This permits us to analyze the propriety of some recent empirical research in accounting.

In the second part of the chapter we undertake a statistical analysis of accounting measurement. Using Stevens' scales of measurement as a basis, the likeness ratio is introduced as a means of quantifying the correlation between imperfectly related measurement rules. We conclude the chapter with some discussion of the estimation techniques which may be employed when there is disagreement between accountants concerning the measurement to be associated with a specific resource set. We now focus on the first of these topics; namely the Stevens measurement scheme.

3.1 Stevens' Measurement Scheme

That contemporary writers on accounting measurement attribute some importance to Stevens' work is illustrated by the fact that few recent publications in accounting measurement fail to mention it in some way.[3] Seldom, however, does the discussion advance past the descriptive stage. Stevens' measurement scales are "defined" in some vague sense, usually by a series of examples, but the implications of this classification scheme for measurement in general, let alone accounting measurement, are rarely entertained.[4] For this reason, this section has as its purpose the illustration of the important features of Stevens' work and their implications for measurement in general. In some later sections, we will examine its implications for accounting measurement.

3.1.1 Mappings and measurement

Given two sets X and Y, a mapping from X into Y or a function from X into Y associates with each element in X one and only

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3. See, for example, any of the following

   Bierman, H.J. "Measurement and Accounting", The Accounting Review, 38, 3 (July 1963) pp.501-507.

   Chambers, R.J. "Measurement in Accounting", Journal of Accounting Research, 3, 1 (Spring 1965), pp.32-62.

   Bierman, H.J. Financial Accounting Theory, New York: The MacMillan Company, 1965, p.333.

   Chambers, R.J. Accounting, Evaluation and Economic Behavior. Englewood Cliffs, New Jersey: Prentice-Hall, Inc., 1966, pp.84-89.

   Larson, K.O. "Descriptive Validity of Accounting Calculations", The Accounting Review, 44, 1 (January 1969), pp.38-47.

   Moonitz, M. "Price Level Accounting and Scales of Measurement", The Accounting Review, 45, 3 (July 1970), pp.465-475.

   Sterling, R.R. Theory of the Measurement of Enterprise Income. Lawrence, Kansas: The University of Kansas Press, 1970, pp.66-71.

4. A notable exception is provided by

   Mattessich, R. Accounting and Analytical Methods. Homewood, Illinois: Richard D. Irwin, Inc., 1964, Chapter 3.

one element f(x) in Y.[5] We say f maps or transforms X into Y and write $f: X \longrightarrow Y$. The set X is called the domain of f and f(X) is called the range or image of f.[6] Thus $f: \mathbb{R} \longrightarrow \mathbb{R}$ defined by $f(x) = x^2$ is a mapping since each real x[7] has one and only one real square.[8] However, the "relation" $g: \mathbb{R}^+ \longrightarrow \mathbb{R}$ defined by $g(x) = \sqrt{x}$ is not a mapping since each positive real x has two square roots in $\mathbb{R}$.[9]

Suppose we have the mapping $f: X \longrightarrow \mathbb{R}$. Then, we say the ordered pair (f;X) forms a measurement rule. We refer to the image of f (or subsets thereof), that is f (X), as a measurement series. We consider some examples of "measurement rules"

1. The measurement of intelligence is described in the following terms

   $I \; : \; P \longrightarrow \mathbb{R}$

   where P is a set of people and I associates with each element in P (i.e. each person) an "intelligence score" in $\mathbb{R}$.

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5. Giles, J.R. Real Analysis. Sydney: John Wiley and Sons Australasia Pty Ltd., 1972, p.13.

6. Note that f(X) need not be identical to Y. Functions possessing this property are called onto. Functions possessing the property that for each y in f(X) there exists only one x in X such that y = f(x) are called one-to-one. Ibid., pp.13-14.

7. For the properties of the real number system, see

   Ibid., pp.1-7.

8. Ibid., pp.13-14.

9. Ibid.

2. The measurement of temperature is described in the following terms

$$F : T \longrightarrow \mathbb{R}$$

where T is a set of points in time. F associates with each element in T, a "measurement" (farenheit, centigrade based) in $\mathbb{R}$. Note that T is not restricted to points in time. We may, for example, define T to be the set of points on a surface, so that F measures the temperature at each point of the surface, at some point in time.[10]

3. An accounting measurement rule is described in the following terms

$$L_t : \mathcal{P}_t \longrightarrow \mathbb{R}$$

where $\mathcal{P}_t$ is the algebra generated by the property set P_t. L_t associates with each set in $\mathcal{P}_t$ (accounts payable, cash, securities, etc.) a "measurement" in $\mathbb{R}$. This may be a "replacement cost" measure, an "historic cost" measure and so on.

We are justified, therefore, in describing the process of measurement in the following terms

> "... measurement ... is defined as the assignment of numerals to objects or events according to rules."[11]

---

10. Measurement rules may be derived from the conditions which it is known they must satisfy. For example, the "heat equation" $U_t = U_{xx}$ with initial conditions $U(x,o) = \sin 2x$, $o \leqslant x \leqslant \pi$ and $U(o,t) = U(\pi,t) = o$ has the solution $U(x,t) = e^{-4t}\sin 2x$. Here $U(x,t)$ is the temperature of a point x on a rod at time t.

11. Stevens, <u>op.cit.</u>, p.677.

It is noted, however, that different measurement rules produce different measurements. Temperature, for example, may be measured in a variety of ways, the most common, of course, being the farenheit and centigrade systems. Thus, for any given empirical situation, the metrician is likely to be confronted with a choice of measurement rule or, more precisely, a choice in the unit of measurement. We are then faced with three specification problems[12]

1. Identifying the measurement rules (admissible measurement rules) appropriate to a given empirical situation.

2. Determining the group affiliation of the collection of measurement rules obtained from (1).

Having satisfied (1) and (2) the third specification problem is stated in the following terms

3. Determining the "numerical procedures" which may "meaningfully" be applied to the chosen measurement series.

There is, of course, some inadequacy in purely verbal descriptions of this kind. What, for example, is meant by such obscure expressions as "meaningful" and "numerical procedures", for their significance is not clear from context. It is vital, however, that we eliminate any confusion inherent in these state-

12. Ibid., p.678.

ments for they are the hub of the Stevens measurement scheme. In the next section, therefore, we shall provide a more refined interpretation of the measurement concepts implied by these statements.

3.1.2 Measurement Scales

In this section, our objective is to fix exactly the meaning of the three specification problems isolated in the previous section. Having achieved this, we shall then be in a better position to understand the significance of Stevens' measurement scheme to the theory of accounting measurement, a topic deferred to some later sections. For the moment, however, we concentrate on the problem at hand, namely the provision of a more refined interpretation of the measurement concepts introduced above.

The first two specification problems resolve themselves in what Stevens termed the measurement scales. Since the factors determining the set of "admissible measurement rules" vary according to the empirical situation being analyzed and tend therefore to be somewhat fluid, it is not possible to provide in any substantive sense a definition of the measurement scales, a point acknowledged by Stevens.[13] Once, however, the set of "admissible measurement rules" is known, rigour may be compromized, a fact which is illustrated by the following definition of the measurement scales

> The measurement rules f and g are J scaled if there exists a function π in J such that

13. Stevens, S.S. "Measurement, Statistics and the Schemapiric View", Science, CLXI (August 30, 1968), p.850.

$$g = \pi . f$$

A J measurement scale C_J is a collection of measurement rules $X \longrightarrow \mathbb{R}$ which are mutually J scaled.

The precise form of the measurement scale is determined, of course, by the "group structure" of the set J. Stevens, for example, introduced the four measurement scales displayed in Table 3.1.[14] Note that each of these scales is defined by the group affiliation of the collection of mappings relating each pair of "admissible measurement rules." Thus in the Stevens scheme, measurement occurs on a nominal, ordinal, interval or ratio scale according to whether each pair of admissible measurement rules is related by a one-to-one mapping (permutation group), monotonic (increasing) mapping (isotonic group), linear (increasing) mapping (general linear group) or similarity (increasing) mapping (similarity group). Whilst these are undoubtedly the best known measurement scales, other lesser known scales have been (and may be) introduced as circumstances dictate.[15]

Taken by themselves, the measurement scales serve merely as a convenient receptacle for classifying measurements according to the transformations which may be applied to each. Whilst this

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14. Stevens, S.S. "Mathematics, Measurement and Psychophysics", in Stevens, S.S. (ed.) Handbook of Experimental Psychology. New York: John Wiley & Sons, Inc., 1951, p.25.

15. Suppes, P. and J.L. Zinnes "Basic Measurement Theory", in R.D. Luce, R.R. Bush and E. Galanter (eds.) Handbook of Mathematical Psychology, Volume 1. New York: John Wiley and Sons, Inc., 1963, pp.1-76.

TABLE 3.1

THE STEVENS MEASUREMENT SCALES

Measurement Scale	Group Structure	Examples	Permissible Statistics
Nominal	Permutation group	Classification of people by sex, religion, occupation, etc. Numbering of football players	Number of cases Mode Contingency correlation
Ordinal	Isotonic group	Moh's scale of hardness Military ranking Intelligence tests	Median Percentiles Rank order correlation
Interval	General linear group	Temperature Calendar time Potential energy	Arithmetic mean Standard deviation product moment correlation
Ratio	Similarity group	Weight Density Length	Geometric mean Harmonic mean Coefficient of variation

is a function of some import,[16] their main role derives from the fact that they provide a means of solving the third specification problem; namely, determining the "numerical procedures" which may meaningfully be applied to each scale. This concept may be more precisely defined as follows

> A "numerical procedure" ϕ is $J(\sim, f)$ scale meaningful if when
>
> $\phi(f^*) \sim \phi(f')$
>
> then
>
> $\phi(\pi \cdot f^*) \sim \phi(\pi \cdot f')$
>
> for all f^*, $f' \subset f$ in C_J and π in J.

where it will be recalled that the J measurement scale C_J is a collection of measurement rules which are mutually J scaled. The tilde ($\sim$) is one of the arithmetic operations "greater than" ($>$), "equivalence" ($=$) or combinations thereof ($\leqslant$ or $\geqslant$).

As an example of the implementation of the above definition we prove that the arithmetic mean may be "meaningfully" applied to the interval measurement scale. We thus suppose the intervally scaled measurement rule f to possess the property $\overline{f}^* > \overline{f}'$ where $\phi(f^*) = \overline{f}^* = \sum_{j=1}^{n} f^*_j$ and $\phi(f') = \overline{f}' = \sum_{k=1}^{m} f'_k$ are the arithmetic means of the two f measurement series. Transforming each measurement series by the general (increasing) linear group $\pi \cdot f = a + b^2 f$ implies $\phi(\pi \cdot f^*) = a + b^2 \overline{f}^* >$ $\phi(\pi \cdot f') = a + b^2 \overline{f}'$ or that comparison of the arithmetic means of

16. Chambers, loc.cit.

intervally scaled measurement series is a "meaningful" procedure. In words, for an interval measurement scale, the relationship between arithmetic means is independent of the measurement rule employed.

In Table 3.1 we list the statistical procedures which may meaningfully be applied to each of the Stevens measurement scales. Several factors deserve emphasizing. The permissible statistics are cumulative. Since, for example, the similarity group is a subset of the general linear group, a statistic permissible to the interval scale is also permissible to the ratio scale. Similarly, a statistic permissible to the ordinal scale is also permissable to the interval and ratio scales. Should, however, a statistic be applied to a measurement scale for which it is not permissible (for example, the geometric mean applied to intervally scaled measurements), conclusions concerning the statistic become dependent on the measurement rule utilized.[17]

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17. A numerical example may help to clarify this point. Consider the two temperature series $A_c = (2,10)$ and $B_c = (4,6)$ where the measurements are on the centigrade scale. The comparable measurements on the farenheit scale are $A_F = (35.6,50)$ and $B_F = (39.2, 42.8)$ where these figures are obtained by applying the familiar formula $F = 32+\frac{9}{5}C$ to the centigrade measurements. Denote by G, the geometric mean of each series, in which case we have

$$G(A_c) = \sqrt{2 \times 10} = 4.47 < G(B_c) = \sqrt{4 \times 6} = 4.90$$

for the centigrade scale measurements. For the farenheit scale measurements we have

$$G(A_F) = \sqrt{35.6 \times 50} = 42.19 > G(B_F) = \sqrt{39.2 \times 42.8} = 40.96$$

Thus, using the geometric mean as a criterion, the centigrade scale measurements indicate that the "A" temperature series is "hotter" than the "B" temperature series. If, however, we employ the farenheit scale, precisely the opposite result is obtained. The "B" temperature series is "hotter" than the "A" temperature series.

This completes our analysis of the Stevens measurement scheme. In summary we note that the Stevens measurement scales are merely a device for classifying measurements according to the transformations which may be applied to each but that in so doing it provides a means for determining the "meaningfulness" of the numerical procedures applied to such measurements. We now focus our attention on the more important task of investigating the significance of Stevens' work to the theory of accounting measurement.

3.2 The Accounting Implications

In this section our objective is to examine the accounting implications of the Stevens measurement scheme. These are at least two in number. Firstly, Stevens' work has been used to cast doubt on the verity of some recent empirical research in accounting. That it is important to resolve this issue derives from the fact that the methods concerned are widely used in practice and if they be invalid they may be the source of some invalid empirical generalizations. Secondly, in many aspects of his measurement function, the accountant must choose one of several admissable measurement rules. Recall that in the Stevens scheme, measurement scales are defined in terms of the relationship which exists between such rules. Although it is unlikely that alternative accounting measurement rules will have any deterministic relationship, the Stevens scheme does provide a rationale for utilizing one accounting measurement rule as a means of estimating another.

In the present section we shall examine each of these topics in some detail. We commence with the implications of Stevens' work for empirical research in accounting.

3.2.1 The Meaningfulness of Some Recent Empirical Accounting Research

A significant feature of Stevens' work, at least as far as accounting is concerned, is that it has been utilized to cast doubts[18] upon the validity of some recent empirical research

18. Peasnell, K.V. "The Objectives of Published Accounting Reports: A Comment", Accounting and Business Research, 4, 17 (Winter 1974) pp.71-76.

conducted by Carsberg, Hope and Scapens.[19] Since a similar statistical methodology was adopted in the empirical investigations conducted by Fisher,[20] Lee and Tweedie[21] and Baker and Haslem,[22] it is important that we evaluate the criticism's authenticity. This we proceed to do.

Each of the research projects noted above reports results obtained from requesting questionnaire respondents to rank financial information in some preferred order. Thus, for example, in the Carsberg et al study

> "The nub of our enquiry was expressed in a question which asked respondents to rank, on a seven point scale, the importance they thought should be attached to a number of possible objectives for published accounts."[23]

As a basis for comparisons the ranks were summed over all respondents and the mean and standard deviation of each objective was computed. It was this procedure which attracted the attention of Professor Peasnell.

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19. Carsberg, B., A. Hope and R.W. Scapens. "The Objectives of Published Accounting Reports", Accounting and Business Research, 5, 15 (Summer 1974), pp.162-173.
20. Fisher, J. "Financial Information and the Accounting Standards Steering Committee", Accounting and Business Research, 5, 16 (Autumn 1974), pp.275-285.
21. Lee, T.A. and D.P. Tweedie. "Accounting Information: An Investigation of Private Shareholder Usage", Accounting and Business Research, 5, 20 (Autumn 1975), pp.280-297.
22. Baker, H.K. and J.A. Haslem. "Information Needs of Individual Investors", The Journal of Accountancy, 136, 11 (November 1973), pp.64-69.
23. Carsberg, et al., op.cit., p.170.

'... one cannot agree with Carsberg et al that "averages seem to be a reasonable way of summarising the replies" ... because the averaging procedure is based on the ... erroneous assumption that the measurements are in the interval scale.'[24]

The criticism may be more fully appreciated by reference to the following matrix

$$
\begin{array}{c|c|cccc}
 & & & \text{Objective} & & \\
 & \underline{\text{Individual}} & \underline{1} & \underline{2} & \underline{3} & \underline{n} \\
 & \underline{1} & X_{11} & X_{12} & X_{13} \sim\!\!\sim\!\!-\!\!- & X_{1n} \\
\text{Rank Allotted} & \underline{2} & X_{21} & X_{22} & X_{23} \sim\!\!\sim\!\!-\!\!- & X_{2n} \\
 & \underline{3} & X_{31} & X_{32} & X_{33} \sim\!\!\sim\!\!-\!\!- & X_{3n} \\
 & \vdots & \vdots & \vdots & \vdots & \vdots \\
 & \underline{m} & X_{m1} & X_{m2} & X_{m3} \sim\!\!\sim\!\!-\!\!- & X_{mn}
\end{array}
$$

The elements x_{ij} of this matrix define a measurement rule, f: $Z_1^n x Z_1^m \longrightarrow Z_1^{n'}$ where $Z_1^{n'}$ is the set of integers contained in the interval $[1,n]$ representing the ranks allotted to each of n objectives, Z_1^m is the set of integers contained in the interval $[1,m]$ representing the m respondents and $Z_1^n = Z_1^{n'}$ represents each of the n objectives. Thus, for example, the entry X_{23} represents the rank allotted to the third objective by the second respondent, or in functional form, $f(2,3) = X_{23}$ where X_{23} is, of course, a positive integer.

To substantiate the view expressed by Professor Peasnell we must show that there exist at least two admissable measurement rules and that these rules are ordinally scaled; that is, linearly independent. Recall, from Table 3.1, that the mean and standard

24. Peasnell, op.cit., p.75.

deviation are not statistics permissible to the ordinal scale (that is, are not meaningful numerical procedures to the ordinal scale). It follows that on the ordinal scale the relationship between means and standard deviations is (in general) dependent upon which of the admissible measurement rules is utilized to express the results of measurement. The question, turns, therefore, on whether there exists a pair of linearly independent methods (measurement rules) for denoting ranks.

Our examination of the literature indicates that there is but one numerical procedure (that is, one admissible measurement rule) available for denoting ranks; namely, denotation of ranks by the positive integers 1, 2, ———, n, where n is the number of objects ranked. As such, the numbers denoting ranks (z) may only be transformed by the identity mapping $\pi(z) = z$. Measurement rules possessing this property are said to be absolutely scaled,[25] and have the additional feature that each of the statistics listed in Table 3.1 is a meaningful numerical procedure with respect to the measurement rule.[26] The consequence of this, of course, is that Professor Peasnell's criticism of the Carsberg et al paper is unsubstantiated. In words, computing the mean and standard deviation of a set of ranks is a "meaningful" numerical procedure.

We conclude, therefore, that estimating the "true" rank-

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25. Suppes and Zinnes, op.cit., p.25.

26. This follows from the fact that the identity mapping is obtained from the similarity mapping $\pi(z) = \alpha z$, by setting $\alpha = 1$.

ings by the procedure utilized in each of the above noted studies (the arithmetic mean of the respondents' ranks) is a meaningful operation. It may not, however, be the most "efficient" means of doing so. This is a topic we devote some time to in the ensuing section.

3.2.2 The Problem of m Rankings

Carsberg *et al* estimate the "true" ranking of n objects on the basis of the (arithmetic) mean rank taken over the m questionnaire respondents.[27] This procedure, however, is deficient in two respects

1. It presupposes the existence of consensus amongst the m respondents.
2. There is no criterion by which to judge the "efficiency" of the estimated rankings.

To overcome the first objection the coefficient of concordance has been proposed.[28]

$$W = \frac{12S}{m^2(n^3-n)}$$

where m is the number of respondents and n is the number of

27. Carsberg *et al*, *loc. cit.* See also
    Lee and Tweedie, *op.cit.*, p.282.
    Baker and Haslem, *op.cit.*, p.65.
    Fisher, *op.cit.*, p.280.

28. Kendall, *op.cit.*, p.95.

objects being ranked. S is the sum of the squared differences between the total of the ranks attributed to each object and the average attributed to all objects. Define the following

$$U = \frac{(m-1)W}{1-W}$$

$$V_1 = (n-1) - \frac{2}{m}$$

$$V_2 = (m-1)V_1$$

$$V = m(n-1)W$$

On the presumption that there is no consensus between the m respondents it can be shown for $n > 5$ and $m > 3$ that U is distributed as an $F(V_1, V_2)$ variate.[29] When $n > 7$, however, a more convenient test is provided by the fact that V has an approximate X^2 frequency function with (n-1) degrees of freedom.[30] When neither of these conditions is satisfied we resort to the use of specially prepared Tables.[31]

For the second problem, it may be shown that to rank objects according to the sum of ranks allotted to each provides a "best" estimate in a "least squares" sense.[32] The ranking

29. See Appendix 3A. See also Ibid., pp.107-111.

30. Ibid., p.98.

31. Ibid., pp.184-188.

32. Ibid., p.101.

thus obtained remains invariant when the rank sums are transformed to their arithmetic means.[33] It would seem, therefore, that the research reported above provides rankings consistent with the "least squares" rankings. Unfortunately, this is not necessarily the case.

In the Carsberg et al study, questionnaire respondents were requested to rank ten potential uses of financial statements on a scale from one to seven.[34] Thus, the rank allotted to each use was not necessarily unique. Similar procedures were adopted by Lee and Tweedie[35] and Baker and Haslem.[36] The consequence of this is that the estimated rankings reported in each of these papers are not necessarily consistent with the "least squares" estimates because under the "least squares" criterion each object must receive a unique rank from each respondent.[37]

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33. Obviously if $\sum_{i=1}^{m} x_{i,j} \sim \sum_{i=1}^{m} x_{i,k}$ then $\frac{1}{m}\sum_{i=1}^{m} x_{i,j} \sim \frac{1}{m}\sum_{i=1}^{m} x_{i,k}$.

34. Carsberg et al, loc.cit.

35. Lee and Tweedie, loc.cit.

36. Baker and Haslem, loc.cit.

37. Kendall, op.cit., p.101 and p.114.

Note that the studies referred to in the test requested questionnaire respondents to map various kinds of financial information into a set of mutually exclusive "importance" ranks. Under this scheme, it is possible for each type of financial datum to receive the same importance rank.

This completes our analysis of the implications of Stevens' work for empirical accounting research. In summary, we note that his work is of vital importance to any research project involving some species of measurement. Recall, however, that his work does bear a more direct relationship to accounting measurement in that it can be used as a base from which to rationalize the statistical estimation of one accounting measurement rule by recourse to another. In the next section we shall develop this theme in more detail.

3.2.3 The Likeness Ratio

In the Stevens system, scales are defined in terms of relations amongst admissible measurement rules. Thus, the measurements obtained under one measurement rule can be transformed into their equivalent measurements under another rule by merely applying the transformation which defines the scale type. In accounting measurement, however, whilst there are usually several potentially useful measurement rules, it is doubtful if there is any deterministic relation between them. Since financial statements must, of necessity, limit the number of valuation bases reported, users may be denied some potentially useful information. A partial solution to this problem was provided by Ijiri in the form of the linear aggregation coefficient.[38] The square of the linear aggregation coefficient, which Ijiri dubbed the aggregation effectiveness coefficient, is a summary measure designed to reveal the degree of identifiability

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38. Ijiri, Y. The Foundations of Accounting Measurement. Englewood Cliffs: New Hersey: Prentice-Hall, Inc., 1967, p.130.

between any two accounting aggregations.[39] Unfortunately, our axiom system is not stated in a form which facilitates use of the aggregation effectiveness coefficient due to the absence of quantification in the sense implied by Ijiri's axiom of quantities. In this section, therefore, we shall define and investigate the properties of the likeness ratio which is designed as the analogue of Ijiri's aggregation effectiveness coefficient.

In this respect, suppose a set of financial statements to be prepared under the valuation basis implied by the measurement rule $(L_t; \zeta_t)$. Suppose, however, a user of these statements "prefers" the valuation basis implied by the unknown measurement rule (L'_t, ζ_t). Since L_t is known, it may be possible to decrease the user's uncertainty by estimating L'_t by the following method

$$y_j = \beta x_j + e_j$$

where $y_j = L'_t(S_j)$ and $x_j = L_t(S_j)$ for all S_j in ζ_t, e_j is the error from estimating y_j by βx_j and β is a parameter. In order to determine a "best" value for β we must choose an optimality criterion. In this respect, the quantity $1 - \frac{\sum (y_j - \beta x_j)^2}{\sum y_j^2}$ is the fraction of the squared values of the

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39. Ibid., p.126.

y's that is eliminated as a result of estimating y by x. For "exact" fits the ratio assumes a value of unity. When the fit is not exact the ratio decreases in value as the fit deteriorates. Consequently, we employ this ratio as a criterion for gauging how useful one measurement rule is in estimating another. Specifically, define the following function as an optimality criterion

$$\lambda^2(\beta) = 1 - \frac{\sum_{j=1}^{N} (y_j - \beta x_j)^2}{\sum_{j=1}^{N} y_j^2}$$

where each summation is taken over the $N = 2^n$ measurements obtained by respectively applying L'_t and L_t to ξ_t. Differentiating $\lambda^2(\beta)$ with respect to β implies $\lambda^2(\beta)$ attains its maximal value when β assumes the following figure[40]

$$\hat{\beta} = \frac{\sum xy}{\sum x^2}$$

where the summation subscripts have been dropped for convenience. This result implies that $\lambda^2(\beta)$ attains its maximal value at the point $\left[\hat{\beta}, \lambda^2(\hat{\beta})\right]$ where[41]

$$\lambda^2(\hat{\beta}) = \frac{(\sum xy)^2}{\sum x^2 \sum y^2}$$

40. See Appendix 3B.

41. See Appendix 3B.

We thus define $\lambda^2(\hat{\beta})$ as the likeness ratio.

The likeness ratio possesses all the properties one would expect of a "determination coefficient". The range of $\lambda^2(\hat{\beta})$ is defined by[42]

$$0 \leqslant \lambda^2(\hat{\beta}) \leqslant 1$$

whilst it can be shown that $\lambda^2(\hat{\beta}) = 0$ if and only if $\hat{\beta} = 0$.[43] Further, if the fit is exact in the sense that $e_j = 0$ for all j, then $\lambda^2(\hat{\beta}) = 1$.[44]

The likeness ratio may be used as a means of choosing the simple resources to be reported in financial statements. We may, for example, define a standard such as $\lambda^2(\hat{\beta}) = 0.95$ to be "satisfactory" and then where possible, choose the simple resources so that "satisfactory" approximations can be made to the measurements of other valuation bases. As an example of this, we compute the likeness ratio for the historic cost and replacement cost measurement rules of the Dyer Company Limited as of December 31, 1909. The data upon which the computations are based are contained in Appendix 3C to this chapter and Table 4.9 of Chapter 4. Applying the equation for $\lambda^2(\hat{\beta})$ we have

42. See Appendix 3B.

43. See Appendix 3B.

44. See Appendix 3B.

$$\lambda^2(\hat{\beta}) = \frac{(\sum xy)^2}{\sum x^2 \sum y^2}$$

$$= \frac{(335,\ 402.2)^2}{(302,\ 465.5)(379,\ 576.7)}$$

$$\lambda^2(\hat{\beta}) = 0.9798$$

A likeness ratio as "significant" as this implies that the historic cost measurement rule is "likely" to be quite useful in estimating the replacement cost measurements of the Dyer Company's resources. Had the fit been less precise, it may have been possible to improve the value of the likeness ratio by aggregating some simple resources into compound resources whilst disaggregating others. In any event, the likeness ratio seems not only to provide a means for determining the content of financial statements, but also a summary measure of the identifiability of accounting measurement rules.

This completes our consideration of Stevens' work as it affects accounting measurement. In summary, we note that the main import of Stevens' work seems to be to empirical research in accounting, although it can be used as a base from which to rationalize the estimation of one accounting measurement rule by recourse to another rule. We now entertain a topic which has received surprisingly little attention in the accounting literature; namely a probabilistic analysis of accounting measurement rules.

3.3 A Probabilistic Analysis of Accounting Measurement

In the analysis to date, there has been little discussion of the factors influencing the appropriation of numbers to the resource sets composing the algebra $\mathscr{E}_t$. These numbers have been taken as somehow determined outside the axiom system. That accountants are prone to disagreement over this procedure is well documented.[45] Indeed, the American Accounting Association went so far as to suggest the incorporation of interval estimates of accounting measurements into financial statements.

> "... pressure exists for an expansion of the scope of accounting. The Committee believes that initially this expansion will be reflected in accounting reports with multiple valuations ... . (An) aspect of multiple valuations involves the use of non-deterministic measures or quantum ranges ... ."[46]

In this section we shall examine two methods for implementing this procedure. The first method imposes the strong assumption that the measurement series under examination possesses a normal frequency function, whilst the second relaxes this assumption and imposes the alternative condition that the measurement series merely possesses mean and variance. We now turn to the first of these methods.

3.3.1 "Normal" Measurement[47]

Suppose the measurement series $x_1, x_2, \_\_\_\_, x_n$ to consist of n

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45. Sterling, "Cost versus Values: An Empirical Test", *op.cit.*

46. American Accounting Association. *A Statement of Basic Accounting Theory.* Evanston, Illinois: American Accounting Association, 1966, p.65.

47. The analysis of this section may, at first sight, appear to be similar to the work conducted by Ijiri and Jaedicke. See Ijiri, Y. and R.K. Jaedicke, "Reliability and Objectivity of

(Contd.)

metricians' estimates of the measure of a resource set using the rules of a particular measurement system, such as, for example, the method of replacement cost. Define the sample objectivity measure of the measurement series as

$$V = \frac{1}{n}\sum_{j=1}^{n}(x_j - \alpha)^2$$

where α is determined so as to minimize V. Differentiating with respect to α and setting the result to zero gives[48]

$$\frac{dV}{d\alpha} = 0 = \frac{-2}{n}\sum_{j=1}^{n}(x_j - \alpha)$$

so that

$$\alpha = \frac{1}{n}\sum_{j=1}^{n} x_j$$

---

47. (Contd.)

Accounting Measurements", The Accounting Review, XLI, 3 (July 1966), pp.474-83. There are, however, several differences. Firstly, Ijiri and Jaedicke's "alleged value" is not, in general, equivalent to our "true value". The "true value" is the first moment (about the origon) of the metricians' frequency function. Ijiri and Jaedicke have little to say on how the "alleged value" is derived. Secondly, Ijiri and Jaedicke did not discuss how estimates of the bias and objectivity measurements implied by their system could be obtained. See also,

Ijiri, op.cit., Chapter 7 for some more discussion on this.

48. Since $\frac{d^2V}{d\alpha^2} = 2 > 0$ we are assured of a minimum. See Hancock,

H. Theory of Maxima and Minima. Boston: Ginn and Company, 1917, p.4 or any elementary calculus text.

is merely the arithmetic mean of the measurement series.

Suppose the elements of the measurement series are random elements from a frequency function possessing the following parameters

1. $\mu = E(x)$
2. $\sigma^2 = E\left[x - E(x)\right]^2$

We call μ the "true value"[49] (or consensus value) of the resource and σ^2 the "objectivity measure" of the resource. This permits the sample reliability measure to be defined in the following terms

$$R = \frac{1}{n}\sum_{j=1}^{n} (x_j - \mu)^2$$

Since α was determined so as to minimize the sample objectivity measure we have that $R \geqslant V$. The precise relationship between these quantities, however, can be derived in the following manner

$$nV = \sum (x_j - \bar{x})^2$$

where the summation subscripts have been dropped for convenience. Continuing we have

$$\begin{aligned} nV &= \sum \left[(x_j - \mu) - (\bar{x} - \mu)\right]^2 \\ &= \sum \left[(x_j - \mu)^2 - 2(x_j - \mu)(\bar{x} - \mu) + (\bar{x} - \mu)^2\right] \end{aligned}$$

---

49. This terminology was introduced by Morgenstern. See Morgenstern, O. On the Accuracy of Economic Observations. Princeton: Princeton University Press, 1963, p.76.

$$nV = \sum (x_j - \mu)^2 - 2(\bar{x} - \mu)\sum (x_j - \mu) + n(\bar{x} - \mu)^2$$

But since

$$\sum (x_j - \mu) = n(\bar{x} - \mu)$$

we may restate the above expression in the following terms

$$nV = \sum (x_j - \mu)^2 - n(\bar{x} - \mu)^2$$

Define $B = (\bar{x} - \mu)^2$ as the sample bias measure thus implying

$$n(V + B) = nR$$

so that the non-negative sample bias measure is the exact difference between R and V. Suppose we impose the following assumption

> The measurement series x_1, x_2, ———, x_n is a random sample from a normal frequency function.

and divide nV by the objectivity measure σ^2 thus giving

$$\frac{nV}{\sigma^2} = \frac{nR}{\sigma^2} - \frac{nB}{\sigma^2}$$

It can then be shown

1. The variate $\bar{x} = \frac{1}{n}\sum x_j$ possesses a normal frequency

function with mean μ and variance $\frac{\sigma^2}{n}$.[50]

2. The variate $\frac{nV}{\sigma^2}$ possesses a /Chi squared frequency function with (n - 1) degrees of freedom.[51]

3. The variate $\frac{nR}{\sigma^2}$ possesses a /Chi squared frequency function with n degrees of freedom.[52]

4. The variate $\frac{nB}{\sigma^2}$ possesses a /Chi squared frequency function with one degree of freedom.[53]

5. The variate $\sqrt{\frac{nB}{\sigma^2}}$ possesses a normal frequency function with zero mean and unit variance.[54]

and

6. The variate $\sqrt{\frac{(n-1)B}{V}}$ possesses a t frequency function with (n-1) degrees of freedom.[55]

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50. Mood, A.M. and F.A. Graybill. Introduction to the Theory of Statistics. New York: McGraw-Hill Book Company, Inc., 1963, p. 146.
51. *Ibid.*, p.230.
52. *Ibid.*, p.227.
53. *Ibid.*, p.230.
54. *Ibid.*
55. Suppose y to be a normal variate with zero mean and unit variance. Let u be a x^2 variate with k degrees of freedom and suppose u and y to be independently distributed. Then the random variable

$$t = \frac{y\sqrt{k}}{\sqrt{u}}$$

has a student's t frequency function with k degrees of freedom. Recall that the variate
(Contd.)

We are thus provided with a powerful set of tests by which to determine the consistency of alternative hypotheses concerning the variates B, R and σ^2. We illustrate their use by recourse to the following example

> An accountant is required to estimate a building's replacement cost and objectivity measures. A random sample of 25 metricians produces the following statistics
>
> (i) $\bar{x} = 10$
>
> (ii) $V = 100$
>
> Construct 95 per cent confidence intervals for the variates B, σ^2 and the ordered pair (B, σ^2).

From result 6 above we know that the variate $\sqrt{\frac{(n-1)B}{V}}$ has a t frequency function with (n-1) degrees of freedom. Sub-

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55. Contd.

$$y = \sqrt{\frac{nB}{\sigma^2}}$$

has a normal frequency function with zero mean and unit variance, whilst the variate

$$u = \frac{nV}{\sigma^2}$$

has a x^2 frequency function with (n-1) degrees of freedom. Since the frequency functions of $\bar{x}$ and V are, in fact, independently distributed we may use the above result. Substituting we have

$$t = \sqrt{\frac{nB}{\sigma^2}} \sqrt{\frac{\sigma^2}{nV}} \cdot \sqrt{n-1}$$

$$t = \sqrt{\frac{(n-1)B}{V}}$$

has a t frequency function with (n-1) degrees of freedom. See <u>Ibid</u>., p.233 and p.255.

stituting the given values of n and V implies

$$p\left[\left|\sqrt{0.24B}\right| \leqslant 2.064\right] \quad = \quad 0.95$$

This may be restated as

$$p\left[B \leqslant 17.750\right] \quad = \quad 0.95$$

This result implies that the probability of the sample bias measure exceeding 17.750 is 0.05. A similar procedure may be applied in estimating the objectivity measure σ^2. By virtue of result 2, the variate $\frac{nV}{\sigma^2}$ possesses a /Chi squared frequency function with (n-1) degrees of freedom. Substituting the given values of n and V implies

$$p\left[\frac{nV}{\sigma^2} \geqslant 13.8\right] \quad = \quad 0.95$$

which may be restated as

$$p\left[\sigma^2 \leqslant \frac{nV}{13.8}\right] \quad = \quad 0.95$$

$$p\left[\sigma^2 \leqslant 181.59\right] \quad = \quad 0.95$$

Thus, the probability of the objectivity measure exceeding 181.59 is 0.05.

The procedures specified above provide separate interval estimates of the sample bias measure B and the objectivity measure σ^2. In addition, however, it is possible to construct a confidence region in (B, σ^2) parameter space. This procedure provides some insight into the possible values which B and σ^2 may jointly assume. Since the frequency functions of $\bar{x}$ and V are independently distributed,[56] we employ results 2 and 5 above, in which case it follows[57]

$$p\left[\left|\sqrt{\frac{nB}{\sigma^2}}\right| \leqslant 2.24\right] = 0.975$$

which may be restated as

$$p\left[\frac{nB}{\sigma^2} \leqslant 5.02\right] = 0.975$$

and

$$p\left[\frac{nV}{\sigma^2} \geqslant 12.4\right] = 0.975$$

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56. Ibid., p.255.

57. The confidence region determined here is but one of an infinite number of possibilities. To be more precise, we must choose real numbers h and k so that

$$p\left[\left|\sqrt{\frac{nB}{\sigma^2}}\right| \leqslant 2.24\right] = h$$

and

$$p\left[\frac{nV}{\sigma^2} \geqslant 12.4\right] = k$$

where $h.k = 0.95$. We have chosen $h = k = \sqrt{0.95} \approx 0.975$ but there are obviously an infinite number of other possibilities.

The two inequalities obtained from the above equations taken in conjunction with the fact that both B and σ^2 must assume non-negative values,[58] determines the required confidence region. Substituting the observed values of n and V into the above expressions, we have

$$B \leqslant 0.20\ \sigma^2$$

$$\sigma^2 \leqslant 201.61$$

$$B,\ \sigma^2 \geqslant 0$$

The confidence region implied by these equations is graphed in Figure 3.1. The probability of both B and σ^2 being interior points of this confidence region is 0.95.

As the justification of these procedures is grounded in their practical utility, we illustrate their application by recourse to the example pursued above. Suppose the following alternate criteria are provided by "management" as necessary conditions for the inclusion of $\bar{x}$ in the financial statements.

1. The probability of the sample bias measure exceeding 20 (twenty) must not be greater than 0.05.

2. The probability of the sample objectivity measure exceeding 150 must not be greater than 0.05.

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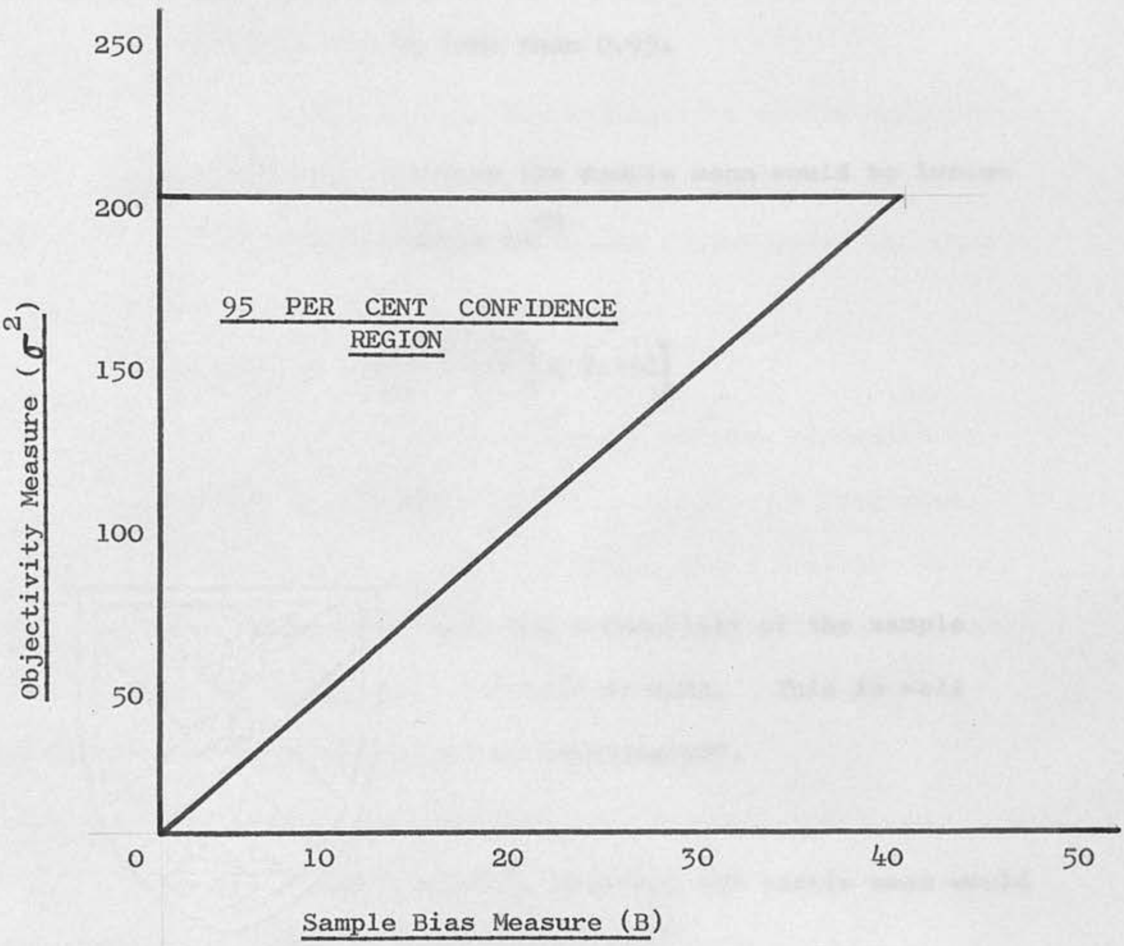
58. Since B and σ^2 are squared real numbers, this must be the case.

FIGURE 3.1

CONFIDENCE REGION IN (B, σ^2) PARAMETER SPACE



3. The probability of the ordered pair (B, σ^2) being contained in the region defined by the upper half of the rectangle formed from the origon and the point (30,150) must not be less than 0.95.

Under the first criterion the sample mean would be included in the financial statements as[59]

$$p\left[\ B \leqslant 20\right] \quad = \quad p\left[\left|\sqrt{\frac{(n-1)B}{V}}\right| \leqslant 2.191\right]$$

$$p\left[\ B \leqslant 20\right] \quad = \quad 0.980$$

This result implies that the probability of the sample bias measure exceeding 20 is (1-0.98) or 0.02. This is well within the tolerance specified by "management".

Using the second criterion, however, the sample mean would be excluded from the financial statements[60]

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59. Recall that n = 25 and V = 100. It follows, therefore, that

$$\frac{(n-1)B}{V} = \frac{24B}{100} \leqslant \frac{24}{100} \ \text{x}\ 20$$

$$\left|\sqrt{\frac{(n-1)B}{V}}\right| \leqslant 2.191$$

The probability reported in the text may then be obtained by interpolating on the t distribution function tables.

60. $$\frac{\sigma^2}{nV} = \frac{\sigma^2}{2500} \leqslant \frac{150}{2500}$$

$$16.67 \leqslant \frac{nV}{\sigma^2}$$

The probability reported in the text may then be obtained by interpolating on the distribution function tables.
/Chi squared

$$p\left[\sigma^2 \leqslant 150\right] = p\left[\frac{nV}{2} \geqslant 16.67\right]$$

$$p\left[\sigma^2 \leqslant 150\right] = 0.86$$

This result implies that the probability of the objectivity measure exceeding 150 is (1-0.86) or 0.14. This is outwith the probability specified by "management" and consequently the sample mean is excluded from the financial statements.

Utilization of the third criterion effects exclusion of the sample mean from the financial statements. To determine the probability of the ordered pair (B, σ^2) being interior points of this region we must determine the real numbers d and g where

$$p\left[\left|\sqrt{\frac{nB}{\sigma^2}}\right| \leqslant d\right] = h$$

and

$$p\left[\frac{nV}{\sigma^2} \geqslant g\right] = k$$

so that the "vertex" of the confidence region occurs at the point (30,150). Using this condition, the second of the above equations and recalling that n = 25 and V = 100, we have

$$g = \frac{nV}{\sigma^2}$$

$$= \frac{25 \times 100}{150}$$

$$g = 16.67$$

Similarly, from the first equation we have

$$d = \left|\sqrt{\frac{nB}{\sigma^2}}\right|$$

$$= \left|\sqrt{\frac{25 \times 30}{150}}\right|$$

$$d = 2.24$$

Using tables and the fact that $\sqrt{\frac{nB}{\sigma^2}}$ possesses a normal frequency function with zero mean and unit variance whilst $\frac{nV}{\sigma^2}$ possesses a x^2 frequency function with (n-1) degrees of freedom, we have

$$p\left[\left|\sqrt{\frac{nB}{2}}\right| \leqslant 2.24\right] = 0.975$$

and

$$p\left[\frac{nV}{2} \geqslant 16.67\right] = 0.86$$

or, h = 0.975 and k = 0.86. Since the frequency functions of $\bar{x}$ and V are independently distributed,[61] these results imply that the probability of the ordered pair (B, σ^2) being an interior point of the region specified by "management" is

$$p\left[\left\{\left|\sqrt{\frac{nB}{\sigma^2}}\right| \leqslant 2.24\right\} \cap \left\{\frac{nV}{\sigma^2} \geqslant 16.67\right\}\right]$$

$$= p\left[\left|\sqrt{\frac{nB}{\sigma^2}}\right| \leqslant 2.24\right] . p\left[\frac{nV}{\sigma^2} \geqslant 16.67\right]$$

$$= h \times k$$

$$= 0.84$$

Thus, the probability of the ordered pair (B, σ^2) being an

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61. Mood and Graybill, op.cit., p.255.

exterior point of the region is (1-0.84) or 0.16. Since this is outwith the probability specified by "management" the sample mean is excluded from the financial statements.

The normal frequency function occupies a conspicuous position in statistical theory if only because it has been found to be a "good" approximation to many empirical frequency functions.[62] Misleading conclusions may be derived, however, when the normal assumptions are erroneously employed.[63] In the next section, therefore, we shall investigate some methods which may be utilized when the normal assumptions are violated.

3.3.2 "Non-Normal" Measurement

In cases when there is evidence of "non-normality"[64] but we are assured that μ and σ^2 exist, there are two results of some significance. The first of these may be stated as

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62. Ibid., p.156.

63. Lusk, E. "Normal Assumptions in Decision Making", Accounting and Business Research, 3, 10 (Spring 1973), pp.133-144.

64. For some "goodness of fit tests", see

Kendall, M.G. and A. Stuart. The Advanced Theory of Statistics, Volume 2. London: Charles Griffin and Company Limited, 1973, Chapter 30.

On the "sensitivity" of normal tests to departures from normality; that is, the robustness of these tests, see

Ibid., Chapter 31, especially pp.483-484.

follows[65]

1. Let $\bar{x}$ be the mean of a random sample of size n.

Then the variate

$$\sqrt{\frac{nB}{\sigma^2}}$$

has an asymptotic normal frequency function with zero mean and unit variance.

This result is known as the central limit theorem. The rate of convergence to "normality" and, therefore, the degree of approximation, depend on the sample size and the frequency function being sampled. As an example of the theorem's application, suppose the normal approximation to be reasonable and that σ^2 is known. It then follows that

$$p\left[\left|\sqrt{\frac{nB}{2}}\right| \leqslant z_{\frac{1}{2}\alpha}\right] = (1-\alpha)$$

represents a $(1-\alpha)$ confidence region for the sample bias measure. In words, the probability of the sample bias measure (B) being an interior point of the interval $\left[0, \frac{z^2_{\frac{1}{2}\alpha}\sigma^2}{n}\right]$ is $(1-\alpha)$. A similar procedure may be adopted for the objectivity measure (σ^2) when the sample bias measure is known. It is unlikely, of course, that the precise values of B or σ^2 (or both) will be

65. For a proof of this theorem, making the redundant assumption that the frequency function's moment generating function exists, see Mood and Graybill, op.cit., pp.149-150. For proofs using more general conditions, see
Freund, J.E. Mathematical Statistics. Englewood Cliffs, New Jersey: Prentice-Hall, Inc., 1971, p.208.
Keeping, E.S. Introduction to Statistical Inference. New York: D. Van Nostrand Company, Inc., 1962, pp.90-92.

known, and as a consequence, this result is of little use in the form in which it is stated.

A far more significant result, however, is provided by the following theorem[66]

If y is random variable with finite second moment then

$$p\left[\,|y| \geqslant t\,\right] \leqslant E(y^2)t^{-2}$$

for all non-negative real t.

This result is known as Chebyshev's inequality, so named after the Russian mathematician who discovered it. A proof of the theorem (in this form) is provided in Appendix 3D.

Chebyshev's theorem can be used to prove some so called "ergodic" theorems. To illustrate, make the substitutions $y = \sqrt{B}$ and $t = k\sigma$ in which case we have

$$p\left[\,|\sqrt{B}| \geqslant k\sigma\,\right] \leqslant \frac{E(B)}{k^2\sigma^2}$$

$$= \frac{E\left[(\bar{x} - \mu)^2\right]}{k^2\sigma^2}$$

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66. For some other forms of this inequality, see
Mood and Graybill, op.cit., pp.148-149.
Freund, op.cit., pp.149-151.
Keeping, op.cit., pp.45-46.

When μ is known to exist, but we are not assured that σ^2 exists, then the Markov Inequality may be of some assistance. See Ibid., p.45.

$$p\left[B \geqslant k^2\sigma^2 \right] \leqslant \frac{1}{nk^2}$$ 67

Taking limits across this inequality we have

$$\lim_{n \to \infty} p\left[B \geqslant k^2\sigma^2 \right] = 0$$

Thus, for "large" samples, it is likely that the sample bias will be negligible. A similar result holds for the sample objectivity measure (V) in its relation to the objectivity measure (σ^2). To illustrate, let $s = \sqrt{V}$ $y = (s - \sigma)$ and $t = k\sigma$, in which case we have

$$p\left[\left| s - \sigma \right| \geqslant k\sigma \right] \leqslant \frac{E\left[(s - \sigma)^2\right]}{k^2\sigma^2}$$

$$= \frac{E(s^2) - 2\sigma E(s) + \sigma^2}{k^2\sigma^2}$$

But since

$$E(s^2) = \frac{n-1}{n}\sigma^2$$ 68

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67. This result follows from the fact that

$$\sigma^2_{\bar{x}} = E\left[(\bar{x} - \mu)^2\right]$$

$$\sigma^2_{\bar{x}} = \frac{\sigma^2}{n}$$

A proof of this result is provided in Mood and Graybill, op.cit., p.146.

68. Freund, op.cit., p.257.

and

$$E(s) = \sigma\left(1 - \tfrac{1}{4}n + O\tfrac{1}{n}2\right)^{69}$$

It follows that

$$p\left[\left|s - \sigma\right| \geqslant k\sigma\right] \leqslant \frac{1}{k^2\sigma^2}\left[\frac{n-1}{n}\sigma^2 - 2\sigma^2\left(1 - \tfrac{1}{4}n + O\tfrac{1}{n}2\right) + \sigma^2\right]$$

$$= \frac{1}{k}2\left[\frac{2n-1}{n} - 2\left(1 - \tfrac{1}{4}n + O\tfrac{1}{n}2\right)\right]$$

Taking limits across this inequality we have

$$\lim_{n \to \infty} p\left[\left|s - \sigma\right| \geqslant k\sigma\right] = 0$$

or that for "large" samples $s = \sqrt{V}$ is likely to be a reasonable approximation for σ. This, of course, implies that for "large" n, the sample objectivity measure (V) is likely to be "good" approximation to the objectivity measure (σ^2).

The "ergodic" theorems (concerning B and σ^2) derived above are of considerable practical significance since they imply for large samples taken from frequency functions possessing second moment about the mean (σ^2), that the measurement bias (B) is likely to be negligible whilst the sample objectivity measure (V) is likely to be a reasonable approximation for

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69. Keeping, op.cit., p.209.

the actual objectivity measure (σ^2). When the sample size is small,[70] however, knowledge of the underlying frequency function is a necessity if we are to make substantive conclusions.

This completes our analysis of the probabilistic foundations of accounting measurement. We now summarize the contents of the present chapter.

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70. Note that Sterling found the normal frequency function to be a reasonable approximation to the actual frequency function in his study. See Sterling, "Cost versus Values: An Empirical Test", op.cit., p.220.

3.4. Summary

In this chapter we have endeavoured to analyze accounting measurement by building on the axiom system developed in the previous chapter. Since few recent publications in accounting measurement fail to devote some attention to the Stevens measurement scheme, we commenced the chapter with an examination of the important features of Stevens' Measurement scheme. Having achieved this, we proceeded to examine its potential to the theory of accounting measurement. In this respect, it was suggested that the main import of Stevens' work lies in the province of empirical research. It would seem to bear little significance to accounting measurement *per se* unless, of course, it were to be demonstrated that the several accounting measurement rules have some deterministic relationship.

Stevens' measurement scheme can be used, however, as a base from which to rationalize the estimation of one accounting measurement rule by recourse to another. The likeness ratio was thus defined as a means of measuring the identifiability of any two accounting measurement rules. The likeness ratio was also shown to possess all the properties one would expect of a "determination coefficient".

As a final exercise, we undertood an analysis of the "estimation" techniques which may be utilized when there is disagreement between accountants concerning the measurements to be accorded a specific resource set. Specifically, this part of our work developed upon a theme initially developed by Ijiri and Jaedicke and enabled us to specify a means of incorporating interval estimates into financial statements. An unsatisfactory feature of the analysis, however, is that it assumed that measurements were normally distributed.

APPENDIX 3A

THE COEFFICIENT OF CONCORDANCE

On the assumption that all (m) observers are independent in their judgements, Pitman has shown that the frequency function of W may be approximated by the Beta distribution

$$dF = \frac{\Gamma(p+q)}{\Gamma(p)\Gamma(q)} W^{p-1}(1-W)^{q-1} dW$$

where $\quad p = \frac{1}{2}(n-1) - \frac{1}{m}$

and $\quad q = (m-1)p$

provided $m > 3$ and $n > 5$. Recall that n is the number of objects being ranked. This frequency function may be more usefully restated as

$$dF = \frac{\Gamma(p+q)}{\Gamma(p)\Gamma(q)} \left[\frac{W}{1-W}\right]^{p} \left[1+\frac{W}{1-W}\right]^{-(p+q)} \frac{1}{W(1-W)} dW$$

Make the substitution

$$z = \tfrac{1}{2} \log \frac{(m-1)W}{1-W}$$

$$2dz = \frac{1}{W(1-W)} dW$$

Thus implying that the frequency function may be restated as

$$dF = \frac{2\Gamma(p+q)}{\Gamma(p)\Gamma(q)} (m-1)^{q} e^{2pz} \left[(m-1) + e^{2z}\right]^{-(p+q)} dz$$

Letting $V_1 = 2p$, $V_2 = 2q$ and eliminating m, the above frequency function may be restated as

$$dF = \frac{2\Gamma\left(\frac{V_1+V_2}{2}\right)}{\Gamma\left(\frac{V_1}{2}\right)\Gamma\left(\frac{V_2}{2}\right)} \left[\frac{V_1}{V_2}\right]^{\frac{V_1}{2}} \cdot e^{V_1 z} \left[1 + \frac{V_1}{V_2} e^{2z}\right]^{-\frac{1}{2}(V_1+V_2)} dz$$

which is the frequency function of Fisher's z variate with V_1 and V_2 degrees of freedom. In words, the variate $z = \frac{1}{2}\log_e \frac{(m-1)W}{1-W}$ has a z frequency function with $V_1 = (n-1) - \frac{2}{m}$ and $V_2 = (m-1)V_1$ degrees of freedom. Suppose now we make the substitution

$$f = e^{2z}$$

$$\tfrac{1}{2}f^{-1}df = dz$$

then the frequency function of Fisher's z variate may be re-expressed as

$$dF = \frac{\Gamma\left(\frac{V_1+V_2}{2}\right)}{\Gamma\left(\frac{V_1}{2}\right)\Gamma\left(\frac{V_2}{2}\right)} \left[\frac{V_1}{V_2}\right]^{\frac{V_1}{2}} f^{\frac{V_1}{2}-1} \left[1 + \frac{V_1}{V_2}f\right]^{-\frac{1}{2}(V_1+V_2)} df$$

which is the frequency function of Fisher's f variate with V_1 and V_2 degrees of freedom. Hence, the variate $f = \frac{(m-1)W}{1-W}$ has an f frequency function with $V_1 = (n-1) - \frac{2}{m}$ and $V_2 = (m-1)V_1$ degrees of freedom. On this topic generally see

Kendall, M.G. Rank Correlation Methods. (London: Charles Griffin & Company Limited, 1948), pp.107-111.

APPENDIX 3B

THE LIKENESS RATIO

For any resource set $S_j \in \zeta_t$, define $y_j = L_t'(S_j)$ and $x_j = L_t(S_j)$ for the $N = 2^n$ sets in ζ_t. Define the function

$$\lambda^2(\beta) = 1 - \frac{\sum_{j=1}^{N} (y_j - \beta x_j)^2}{\sum_{j=1}^{N} y_j^2}$$

The stationary points of $\lambda^2(\beta)$ occur where

$$\frac{d\lambda^2(\beta)}{d\beta} = 0 = \frac{2\sum xy - 2\beta\sum x^2}{\sum y^2}$$

where the summation subscripts have been dropped for convenience. This result implies $\lambda^2(\beta)$ has a stationary point when β assumes the following value

$$\hat{\beta} = \frac{\sum xy}{\sum x^2}$$

Since

$$\frac{d^2\lambda^2(\beta)}{d\beta^2} = \frac{-2\sum x^2}{\sum y^2} < 0$$

there is a maximum at the point $\left[\hat{\beta},\ \lambda^2(\hat{\beta})\right]$. Substituting the expression for $\hat{\beta}$ into the expression for $\lambda^2(\beta)$ implies

$$\lambda^2(\hat{\beta}) = \frac{(\Sigma_{xy})^2}{\Sigma_x{}^2\Sigma_y{}^2}$$

By the Cauchy-Schwartz Inequality we have

$$(\Sigma_{xy})^2 \leq \Sigma_x{}^2\Sigma_y{}^2$$

thus implying

$$0 \leq \lambda^2(\hat{\beta}) \leq 1$$

Substituting the expression for $\hat{\beta}$ into the expression for $\lambda^2(\hat{\beta})$ implies

$$\lambda^2(\hat{\beta}) = \frac{\hat{\beta}^2\Sigma_x{}^2}{\Sigma_y{}^2}$$

In this form, $\lambda^2(\hat{\beta}) = 0$ if and only if $\hat{\beta} = 0$.

Finally, when $y_j = \hat{\beta}\, x_j$ (that is, $e_j = 0$ for all j) then

$$\lambda^2(\hat{\beta}) = \frac{\hat{\beta}^2\Sigma_x{}^2}{\Sigma_y{}^2}$$

$$= \frac{\hat{\beta}^2\Sigma_x{}^2}{\Sigma(\hat{\beta}x)^2}$$

$$\lambda^2(\hat{\beta}) = 1$$

Thus, when the fit is exact, then $\lambda^2(\hat{\beta}) = 1$.

APPENDIX 3C

COMPUTATION OF THE LIKENESS RATIO

Components	y	x	xy	x^2	y^2
Empty Set					
0	0	0	0	0	0
	0	0	0	0	0
Simple Resource sets					
1	60.2	56.0	3,371.2	3,136.0	3,624.0
2	7.5	5.9	44.3	34.8	56.3
3	26.5	26.5	702.3	702.3	702.3
4	20.0	10.0	200.0	100.0	400.0
5	27.0	27.0	729.0	729.0	729.0
6	-6.4	-6.4	41.0	41.0	41.0
	134.8	119.0	5,087.8	4,743.1	5,552.6
Compound resources (two elements)					
1,2	67.7	61.9	4,190.6	3,831.6	4,583.3
1,3	86.7	82.5	7,152.8	6,806.3	7,516.9
1,4	80.2	66.0	5,293.2	4,356.0	6,432.0
1,5	87.2	83.0	7,237.6	6,889.0	7,603.8
1,6	53.8	49.6	2,668.5	2,460.2	2,894.4
2,3	34.0	32.4	1,101.6	1,049.8	1,156.0
2,4	27.5	15.9	437.3	252.8	756.3
2,5	34.5	32.9	1,135.0	1,082.4	1,190.3
2,6	1.1	-0.5	-0.5	0.2	1.2
3,4	46.5	36.5	1,697.2	1,332.3	2,162.3
3,5	53.5	53.5	2,862.3	2,862.3	2,862.3
3,6	20.1	20.1	404.0	404.0	404.0
4,5	47.0	37.0	1,739.0	1,369.0	2,209.0
4,6	13.6	3.6	49.0	13.0	185.0
5,6	20.6	20.6	424.4	424.4	424.4
	674.0	595.0	36,392.0	33,133.3	40,381.2

Compound resources (three elements)

Components	y	x	xy	x^2	y^2
1,2,3	94.2	88.4	8,327.3	7,814.6	8,873.6
1,2,4	87.7	71.9	6,305.6	5,169.6	7,691.3
1,2,5	94.7	88.9	8,418.8	7,903.2	8,968.0
1,2,6	61.3	55.5	3,402.2	3,080.3	3,757.7
1,3,4	106.7	92.5	9,869.8	8,556.3	11,384.9
1,3,5	113.7	109.5	12,450.2	11,990.3	12,927.7
1,3,6	80.3	76.1	6,110.8	5,791.2	6,448.1
1,4,5	107.2	93.0	9,969.6	8,649.0	11,491.8
1,4,6	73.8	59.6	4,398.5	3,552.2	5,446.4
1,5,6	80.8	76.6	6,189.3	5,867.6	6,528.6
2,3,4	54.0	42.4	2,289.6	1,797.8	2,916.0
2,3,5	61.0	59.4	3,623.4	3,528.4	3,721.0
2,3,6	27.6	26.0	717.6	676.0	761.8
2,4,5	54.5	42.9	2,338.1	1,840.4	2,970.3
2,4,6	21.1	9.5	200.5	90.3	445.2
2,5,6	28.1	26.5	744.7	702.3	789.6
3,4,5	73.5	63.5	4,667.3	4,032.3	5,402.3
3,4,6	40.1	30.1	1,207.0	906.0	1,608.0
3,5,6	47.1	47.1	2,218.4	2,218.4	2,218.4
4,5,6	40.6	30.6	1,242.4	936.4	1,648.4
	1,348.0	1,190.0	94,691.1	85,102.6	105,999.1

Compound resources (four elements)

Components	y	x	xy	x^2	y^2
1,2,3,4	114.2	98.4	11,237.3	9,682.6	13,041.6
1,2,3,5	121.2	115.4	13,986.5	13,317.2	14,689.4
1,2,3,6	87.8	82.0	7,199.6	6,724.0	7,708.8
1,2,4,5	114.7	98.9	8,683.4	9,781.2	13,156.1
1,2,4,6	81.3	65.5	5,325.2	4,290.3	6,609.7
1,2,5,6	88.3	82.5	7,284.8	6,806.3	7,796.9
1,3,4,5	133.7	119.5	15,977.2	14,280.3	17,875.7
1,3,4,6	100.3	86.1	8,635.8	7,413.2	10,060.1
1,3,5,6	107.3	103.1	11,062.6	10,629.6	11,513.3
1,4,5,6	100.8	86.6	8,729.3	7,499.6	10,160.6
2,3,4,5	81.0	69.4	5,621.4	4,816.4	6,561.0
2,3,4,6	47.6	36.0	1,713.6	1,296.0	2,265.8
2,3,5,6	54.6	53.0	2,893.8	2,809.0	2,981.2
2,4,5,6	48.1	36.5	1,755.7	1,332.3	2,313.6
3,4,5,6	67.1	57.1	3,831.4	3,260.4	4,502.4
	1,348.0	1,190.0	113,937.6	103,938.4	131,236.2

Components	y	x	xy	x^2	y^2
Compound resources (five elements)					
1,2,3,4,5	141.2	125.4	17,706.5	15,725.2	19,937.4
1,2,3,4,6	107.8	92.0	9,917.6	8,464.0	11,620.8
1,2,3,5,6	114.8	109.0	12,513.2	11,881.0	13,179.0
1,2,4,5,6	108.3	92.5	10,017.8	8,556.3	11,728.9
1,3,4,5,6	127.3	113.1	14,397.6	12,791.6	16,205.3
2,3,4,5,6	74.6	63.0	4,699.8	3,969.0	5,565.2
	674.0	595.0	69,252.5	61,387.1	78,236.6
Compound resources (six elements)					
1,2,3,4,5,6	134.8	119.0	16,041.2	14,161.0	18,171.0
	134.8	119.0	16,041.2	14,161.0	18,171.0
	4,313.6	3,808.0	335,402.2	302,465.5	379,576.7

Using this information we are enabled to compute the following parameter

$$\hat{\beta} = \frac{\Sigma xy}{\Sigma x^2}$$

$$= \frac{335,402.2}{302,465.5}$$

$$\hat{\beta} = 1.1089$$

The likeness ratio may be computed as

$$\lambda^2(\hat{\beta}) = \frac{\hat{\beta}^2 \Sigma x^2}{\Sigma y^2}$$

$$= \frac{(1.1089)^2(302,465.5)}{(379,576.7)}$$

$$\lambda^2(\hat{\beta}) = 0.9798$$

APPENDIX 3D

CHEBYSHEV'S INEQUALITY

Theorem

If y is a random variable with mean μ and variance σ^2, then for all $t > 0$

$$p\left[\,|y| \geqslant t\,\right] \leqslant E(y^2)t^{-2}$$

Proof

We prove the result for the simple random variable defined on the finite probability space $(\Omega, \mathcal{F}, p)$. The generalization to elementary random variables and random variables is straight forward. By theorem

$$y^2(w) = \sum_{j=1}^{N} y_j^2 \, IA_j(w)$$

$$y^2(w) = \Sigma' y_j^2 \, IA_j(w) + \Sigma'' y_j^2 \, IA_j(w)$$

where Σ' is the summation over the y_j possessing the property $|y_j| \geqslant t$ and Σ'' is the summation over the y_j possessing the property $|y_j| < t$. Taking expectations across this equality

$$E\left[y^2(w)\right] = \Sigma' y_j^2 P(A_j) + \Sigma'' y_j^2 P(A_j)$$

Since $y_j^2 \geqslant 0$ and $P(A_j) \geqslant 0$ for all j this implies $\Sigma'' y_j^2 P(A_j) \geqslant 0$. This, in turn, implies

$$E\left[y^2(w)\right] \geqslant \Sigma' y_j^2 P(A_j)$$

By hypothesis on Σ', $\left| y_j \right|$ t, thus implying

$$E\left[y^2(w)\right] \geqslant \Sigma' y_j^2 P(A_j)$$

$$\geqslant t^2 \Sigma' P(A_j)$$

$$\Sigma' P(A_j) \leqslant E\left[y^2(w)\right] t^{-2}$$

But since

$$\Sigma' P(A_j) = P\left[\left| y \right| \geqslant t\right]$$

Giving the result

$$P\left[\left| y \right| \geqslant t\right] \leqslant E\left[y^2(w)\right] t^{-2}$$

CHAPTER FOUR

MODELS OF ACCOUNTING MEASUREMENT

4.0 Introduction

In the second chapter of this work, we developed what was loosely described as an axiom system of accounting measurement. Recall that an axiom system may be characterized as a deductive theory whose constituent parts are a set of primitive notions or concepts, a set of axioms defined in terms of those concepts, and the theorems, lemmas and corollaries which may be derived from the axioms and definitions.[1] In this respect, it is well to remember that a deductive theory is purely formal in that neither its primitive concepts or its axioms have any connection to reality.[2] A deductive theory may, of course, be interpreted; that is, empirical meaning may be assigned to its primitive concepts.[3] When an accounting interpretation is provided, the deductive theory constitutes a formalized model of accounting measurement.[4] Since the model is based on a set of axioms, it is called an axiomatic model of accounting measurement.[5]

To date we have examined an "historic cost" interpretation of the axiom system developed in the second chapter of this work. In the present chapter, we shall complete our analysis of the accounting measurement systems by investigating and illustrating the properties

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1. Beth E.W. The Foundations of Mathematics. Amsterdam: North Holland Publishing Company, 1965, p.81.

2. Ibid.

   Barker, S.F., Philosophy of Mathematics. Englewood Cliffs, New Jersey: Prentice-Hall, Inc., 1964, pp.91-94.

3. Weddepohl, H.N. Axiomatic Choice Models. Rotterdam: Rotterdam University Press, 1970, p.7.

4. Ibid.

5. Ibid.

of a general "valuation" model; that is, a model which can meaningfully accommodate the replacement cost, net realizable value and C.P.P. measurement systems. This would seem to be of some significance because it implies that the numerical processes associated with "adjusting" a set of "historic cost" financial statements to an alternative "valuation" basis are analogous in principle.

The "valuation" model which we shall investigate was first proposed by Edwards and Bell in the context of replacement cost accounting.[6] Its properties, however, were not fully examined by its authors, and consequently, the generality of the system has not been fully appreciated.

The nucleus of the Edwards and Bell system is provided by two "fundamental" theorems. We will commence the present chapter by stating these theorems and illustrating their use by recourse to the Dyer Company example employed in Chapter 2. Proofs of the theorems are provided in Appendixes 4A and 4B. To illustrate the generality of the Edwards and Bell System, we shall then extend it into the realm of C.P.P. and net realizable value measurement. Finally, we shall examine the difficulties associated with incorporating the theorems into the axiom system developed in Chapter 2.

Before examining each of these topics, however, we reproduce in Table 4.1 the financial information pertaining to the Dyer Company which was first introduced as Table 2.5 of Chapter 2. Table 4.2 contains the movement in balance sheet items for the year ending December 31, 1909. Table 4.3 contains the historical cost financial

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6. Edwards, E.O. and P.W. Bell. The Theory and Measurement of Business Income. Berkley, California: University of California Press, 1961.

statements for the year ending December 31, 1909. Finally, Table 4.4 contains the Dyer Company's balance sheet as of January 1, 1909 where the replacement cost basis of measurement has been used. The Tables are introduced at this point to facilitate the reader's appreciation of some ensuing examples.

TABLE 4.1

THE DYER COMPANY LIMITED

(a) Balance Sheet - January 1, 1909	Amount
Shareholder's Funds	£
Capital	50,000
Profit unappropriated	60,000
	110,000
Current Liability	
Trade creditors	5,000
	£115,000
	Amount
Fixed Asset	£
Building	80,000
Less Aggregate depreciation	20,000
	60,000
Current Assets 6(a)	
Stock	10,000
Trade debtors	10,000
Securities	10,000
Cash	25,000
	55,000
	£115,000

Stock: Recorded on a perpetual LIFO basis - 1000 units at £10 (per unit)

Building: Purchased January 1, 1904. Straight line depreciation is used where the life estimation is twenty years (no scrap value).

6(a) It should be emphasized that we are here assuming application of the "pure" historic cost model. Thus, the fact that stock's historic cost (£10,000) exceeds its market value (£9,000, TABLE 4.16(a), P. 152) is not considered.

(b) Transactions in the year ending December 31, 1909

1909		Debtors	Creditors	Purchases	Sales
		£	£	£	£
Jan 30	Purchased on credit 500 units at £11 (per unit).			5,500	
Feb 28	Sold 800 units (on credit) at £20 (per unit).				16,000
Mar 31	Received £10,000 from debtors (no discounts).	10,000			
Apr 30	Paid £8000 on trade creditors (no discounts).		8,000		
Aug 31	Sold 500 units (on credit) at £21 (per unit).				10,500
Nov 30	Purchased 300 units (on credit) at £13 (per unit).			3,900	
		£10,000	£8,000	£9,400	£26,500

TABLE 4.2

MOVEMENT IN BALANCE SHEET FIGURES

(a) Trade Creditors

	Table	Amount
		£
Balance - January 1, 1909	4.1(a)	5,000
Purchases	4.1(b)	9,400
		14,400
Payments	4.1(b)	8,000
Balance - December 31, 1909		£6,400

(b) Aggregate Depreciation

	Table	Amount
		£
Balance - January 1, 1909	4.1(a)	20,000
Depreciation	4.1(a)	4,000
Balance - December 31, 1909		£24,000

(c) Stock

		← Quantity →			← £ →		
Date	Table	Dr	Cr	Balance	Dr	Cr	Balance
1909							
Jan 1	4.1(a)			1,000			10,000
30	4.1(b)	500		1,500	5,500		15,500
Feb 28	4.1(b)		500	1,000		5,500	10,000
	4.1(b)		300	700		3,000	7,000
Aug 31	4.1(b)		500	200		5,000	2,000
Nov 30	4.1(b)	300		500	3,900		5,900

(d) Trade Debtors

	Table	Amount
		£
Balance - January 1, 1909	4.1(a)	10,000
Sales	4.1(b)	26,500
		36,500
Receipts	4.1(b)	10,000
Balance - December 31, 1909		£26,500

(e) Cash

	Table	Amount
		£
Balance - January 1, 1909	4.1(a)	25,000
Receipts	4.1(b)	10,000
		35,000
Payments	4.1(b)	8,000
Balance - December 31, 1909		£27,000

TABLE 4.3

THE DYER COMPANY LIMITED

(a) (Historic Cost) Profit and Loss Statement for the year ending December 31, 1909

	Table	Amount
		£
Sales	4.1(b)	26,500
Less Cost of Goods Sold		
Beginning stock	4.1(a)	10,000
Purchases	4.1(b)	9,400
		19,400
Ending stock	4.2(a)	5,900
		13,500
		13,000
Depreciation	4.1(a)	4,000
		9,000
Profit unappropriated - January 1, 1909	4.1(a)	60,000
Profit unappropriated - December 31, 1909		£69,000

(b) (Historic Cost) Balance Sheet - December 31, 1909

	Table	Amount
		£
Shareholders' Funds		
Capital	4.1(a)	50,000
Profit unappropriated	4.3(a)	69,000
		119,000
Current Liability		
Trade creditor	4.2(e)	6,400
		£125,400

	Table	Amount
Fixed Asset		£
Building	4.1(a)	80,000
Less Aggregate depreciation	4.2(b)	24,000
		56,000
Current Assets		
Stock	4.2(a)	5,900
Trade debtors	4.2(c)	26,500
Securities	4.1(a)	10,000
Cash	4.2(d)	27,000
		69,400
		£125,400

TABLE 4.4

THE DYER COMPANY LIMITED

(a) (Replacement Cost) Balance Sheet - January 1, 1909

	Table	Amount
Shareholders' Funds		£
Capital	4.1(a)	50,000
Profit unappropriated	4.1(a)	60,000
Unrealized cost savings	4.4(b)	2,500
		112,500
Current Liability		
Trade creditors	4.1(a)	5,000
		£117,500

	Table	Amount
Fixed Asset		£
Building	4.4(a)	82,000
Less Aggregate depreciation	4.4(a)	20,500
		61,500
Current Assets		
Stock	4.4(a)	11,000
Trade debtors	4.1(a)	10,000
Securities	4.1(a)	10,000
Cash	4.1(a)	25,000
		56,000
		£117,500

Building: The replacement cost of the building in its original condition as of January 1, 1909 is £82,000. As of December 31, 1909 it is £86,000.

Stock: The replacement cost is £11 (per unit) as of January 1, 1909, £12 (per unit) as of February 28, 1909, £13 (per unit) as of August 31, 1909 and £15 (per unit) as of December 31, 1909.

Securities: The market value of securities was £10,000 as of January 1, 1909 and £20,000 as of December 31, 1909.

(b) Unrealized Cost Savings

	Table	Building	Stock	Total
		£	£	£
Replacement cost	4.4(a)	61,500	11,000	72,500
Historic Cost	4.1(a)	60,000	10,000	70,000
Unrealized cost savings		£ 1,500	£ 1,000	£ 2,500

4.1 The Mathematical Foundations of Edwards and Bell

Our objective in this section is to state and illustrate two theorems which are peculiar to the method of accounting advocated by Edwards and Bell. The first of these theorems provides a means of computing the potentially realizable "holding gains" accruing on a simple resource during some time interval $[0,T]$, whilst the second reconciles the resource's unrealized cost savings as of time T with its replacement cost as of that date. If we are to be more specific about these theorems, however, it is necessary to define and illustrate the various cost savings concepts which are employed within the Edwards and Bell model, since it is in terms of these that the theorems shall be stated. This we proceed to do.

4.1.1 Cost Savings Concepts

Of central importance to the Edwards and Bell method of accounting are the definitions of realizable cost savings, realized cost savings and unrealized cost savings. In this section we shall provide definitions of these concepts. As far as practicable, we shall retain the definitions and notation which were introduced in Chapter 2.

Suppose a simple resource is an element of every property set defined in the interval $[0,T]$. The increase in the replacement cost measure during the interval is called the realizable cost savings accruing on the simple resource in the interval $[0,T]$. Thus suppose ten bolts were purchased at a cost of £1 (per bolt) at time 0. At time T suppose the same bolts could be replaced for £1.50 (per unit). The realizable cost savings accruing on the bolts during $[0,T]$ amount to £5.00. If the bolts had a replacement cost measure of £1.25 (per bolt) at time t, $0<t<T$, then

the realizable cost savings accruing on the bolts in the interval [t,T] amount to £2.50. Suppose a simple resource is an element of every property set defined in the interval [0,T] but is not an element of the next succeeding property set. The increase in the replacement cost measure during [0,T] is called the realized cost savings accruing on the simple resource during [0,T]. Suppose the bolts referred to above were disposed of at time T. The realized cost savings accruing on the bolts amounts to £5.00 during [0,T] and £2.50 during [t,T].

The difference between the realizable cost savings and the realized cost savings accruing on a simple resource in the interval [0,T] is called the unrealized cost savings accruing on the simple resource during [0,T]. The difference between the replacement cost measure and the historic cost measure at time T is called the unrealized cost savings of the simple resource at time T.[7] Thus, if 5 of the bolts referred to above were disposed of at time t, the realizable cost savings accruing during [0,T] amount to £3.75, the realized cost savings amount to £1.25 and the unrealized cost savings as of T amount to £2.50.

In Table 4.5 we summarize the results of this section under the assumption that five of the bolts were disposed of at time t for £1.25 (per bolt) leaving five bolts on hand at time T with a replacement cost of £1.50 (per bolt). Recall that each bolt was purchased for £1.00. We are now in a position to state and illustrate the two fundamental theorems referred to above.

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7. <u>Ibid</u>., p.115.

TABLE 4.5

COST SAVINGS CONCEPTS

		£
(a)	Realizable Cost Savings	
	Gain accruing on 5 units held over the interval [0,T] 5 × (1.50 − 1.00)	2.50
	Gain accruing on 5 units disposed of at time t. 5 × (1.25 − 1.00)	1.25
		£3.75
(b)	Realized Cost Savings	
	Gain accruing on 5 units disposed of at time t . 5 × (1.25 − 1.00)	£1.25
(c)	Unrealized Cost Savings	
	Realizable Cost Savings	3.75
	Less Realized cost savings	1.25
		£2.50
Represented by:		
	Gain accruing on 5 units held over the interval [0,T] . 5 × (1.50 − 1.00)	£2.50

4.1.2 Two Fundamental Theorems

In this section our objective is to state and illustrate what we shall call the "fundamental" theorems of the Edwards and Bell system. The theorems are called "fundamental" because they form the foundations of the Edwards and Bell method of accounting and as such they are the vehicle through which we shall extend the Edwards and Bell system into the realm of the other measurement system.

We now state the first of these theorems

> Let ψ be the replacement cost measure of a simple resource at time 0, plus additions (at cost) in the interval $[0,T]$ less disposals (at the replacement cost measure at time of disposal) in the interval $[0,T]$. Let ξ be the replacement cost measure of the simple resource at time T. The difference $\xi - \psi$ is the realizable cost savings[8] accruing on the simple resource during $[0,T]$.

This result is proved in Appendix 4A.

In Table 4.6 we compute the realizable cost savings accruing on the Dyer Company's stock during the year ending December 31, 1909. We first compute the realizable cost savings directly, and then by using the above theorem. Note, however, that although the theorem is illustrated in relation to stock, it is stated as applying to any simple resource, such as, for example, the Dyer Company's building or securities.

The second theorem is stated in the following terms

> Let ${}^{A}T$ be the unrealized cost savings at time T and ${}^{B}T$ be the historic cost at time T, of a simple resource. Let ${}^{I}T$ be the "quantity measure" of the simple resource at time T. Then

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8. To what extend Edwards and Bell were aware of this theorem's existence is a point for conjecture. They almost certainly realized the theorem applied to stock, but judging from the procedures they applied in computing the realizable cost savings accruing on fixed assets, were unaware of theorem's relevance to other resources. At no stage, however, did they state, let alone prove the theorem in the form presented in this chapter. See, also, appendix 4C to this chapter and <u>Ibid</u> pp.146-47, pp.188-193.

TABLE 4.6

REALIZABLE COST SAVINGS ACCRUING ON STOCK IN THE YEAR ENDING DECEMBER 31, 1909

(a) Direct Calculation

	Table	Amount £
Gain accruing on 200 units held the entire year 200 x (15 - 11)	4.2(a),4.4(a)	800
Gain accruing on 800 units sold February 28. 800 x (12 - 11)	4.1(b),4.4(a)	800
Gain accruing on 500 units sold August 31. 500 x (13 - 11)	4.1(b), 4.4(a)	1,000
Gain accruing on 300 units of ending stock 300 x (15 - 13)	4.2(a),4.4(a)	600
Realizable cost savings		£3,200

(b) By Theorem

	Table	Amount £
Replacement cost - December 31, 1909 (500 x 15)	4.2(a),4.4(a)	7,500
Beginning stock (1000 x 11)	4.1(a),4.4(a)	11,000
Purchases	4.1(b)	9,400
		20,400
Replacement cost of goods sold (800 x 12 + 500 x 13)	4.2(a),4.4(a)	16,100
		4,300
Realizable cost savings		£3,200

$$\frac{A_T + B_T}{I_T} = R_T$$

where R_T is the replacement cost measure (per unit) of the simple resource at time T.[9]

This result is proven in Appendix 4B.

In Table 4.7 we apply this result to the Dyer Company's stock for the year ending December 31, 1909. Again, note that although we illustrate the theorem in relation to stock, it is stated for any simple resource and as such is also applicable to the Dyer Company's building or securities.

We now extend the above results to the Dyer Company's other assets for the year ending December 31, 1909.[10] Table 4.8 applies the theorems to the data contained in Tables 4.1, 4.2, 4.3, and 4.4 whilst Table 4.9 exhibits the corresponding replacement cost financial statements. Table 4.10 contains the journal entries required to effect the inclusion of the replacement cost measurements into the books of the Dyer Company Limited.[11] In preparing Table 4.10, we have assumed that the balance sheet exhibited in Table 4.4(a) plus the historical cost income and expense of the year ending December 31, 1909 have been recorded in the Dyer Company's books.

We have now accomplished the first objective of this chapter, which was to examine the mathematical foundationsof the Edwards and

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9. This theorem was neither stated or proved by Edwards and Bell. They were, however, aware of its existence. See <u>Ibid</u>, p 220.

10. For a discussion of the problemsencountered in applying the fundamental theorems to fixed assets see Appendix 4C to this chapter.

11. For a discussion of the theoretical underpinnings of the replacement cost interpretation of the model, see <u>Ibid</u>, pp 88-103, and chapter 6 <u>infra</u>.

TABLE 4.7

RECONCILIATION OF COST SAVINGS CONCEPTS FOR STOCK IN THE YEAR ENDING DECEMBER 31, 1909

		Table	Amount
(a)	Realized Cost Savings		£
	Replacement cost of goods sold (800 x 12 + 500 x 13)	4.2(a),4.4(a)	16,100
	Historic cost of goods sold (800 x 10 + 500 x 11)	4.3(a)	13,500
	Realized cost savings		£ 2,600
(b)	Unrealized Cost Savings	Table	Amount £
	Unrealized cost savings - January 1, 1909 1000 x (11 - 10)	4.4(b)	1,000
	Realizable cost savings	4.6(b)	3,200
			4,200
	Realized cost savings	4.7(a)	2,600
	Unrealized cost savings		£1,600
(c)	Replacement Cost Data	Table	Amount £
	Historic cost - December 31, 1909	4.2(a)	5,900
	Unrealized cost savings	4.7(b)	1,600
	Replacement cost		£7,500
	Units		500
	Replacement cost (per unit)		£15.00

TABLE 4.8

COMPUTATION OF COST SAVINGS

(a) Realizable Cost Savings for the year ending December 31, 1909

	Table	Securities	Building	Stock	Total
		£	£	£	£
Replacement cost - December 31, 1909	4.4(a)	20,000	60,200*	7,500	87,700
Replacement cost - January 1, 1909	4.4(a)	10,000	61,500*	11,000	82,500
Additions	4.1(b)	-	-	9,400	9,400
		10,000	61,500	20,400	91,900
Replacement cost of disposals	4.2(a),4.4(a)	-	4,200t	16,100	20,300
		10,000	57,300	4,300	71,600
Realizable cost savings		£10,000	£ 2,900	£ 3,200	£16,100

* $\frac{7}{10} \times 86,000 = 60,200, \quad \frac{3}{4} \times 82,000 = 61,500$

t Depreciation is computed on the average replacement cost of the year $\frac{1}{20} \times \frac{86,000 \times 82,000}{2}$ = 4,200. See also Appendix 4C.

(b) Realized Cost Savings for the yeareending December 31, 1909

	Table	Securities	Building	Stock	Total
		£	£	£	£
Replacement cost of disposals	4.2(a),4.4(a)	-	4,200	16,100	20,300
Historic cost of disposals	4. 3(a)	-	4,000	13,500	17,500
Realized cost savings		-	£ 200	£ 2,600	£ 2,800

(c) Unrealized Cost Savings for the year ending December 31, 1909

	Table	Securities	Building	Stock	Total
		£	£	£	£
Realizable cost savings	4.8(a)	10,000	2,900	3,200	16,100
Realized cost savings	4.8(b)	-	200	2,600	2,800
Unrealized cost savings		£10,000	£ 2,700	£ 600	£13,300

(d) Unrealized Cost Savings as of December 31, 1909

	Table	Securities	Building	Stock	Total
		£	£	£	£
Balance - January 1, 1909	4.4(b)	-	1,500	1,000	2,500
Additions	4.8(c)	10,000	2,700	600	13,300
Unrealized cost savings		£10,000	£ 4,200	£ 1,600	£15,800

(e) Replacement Cost Measurement as of December 31, 1909

	Table	Securities	Building	Stock	Total
		£	£	£	£
Historic cost - December 31, 1909	4.3(b)	10,000	56,000	5,900	71,900
Unrealized cost savings - December 31, 1909	4.8(d)	10,000	4,200	1,600	15,800
Replacement cost measurements		£20,000	£60,200	£ 7,500	£87,700

TABLE 4.9

FINANCIAL STATEMENTS

(a) Profit and Loss Statement for the year ending December 31, 1909

	Table	Amount
		£
Sales	4.1(b)	26,500
Less Replacement Cost of Goods Sold		
Beginning stock	4.4(a)	11,000
Purchases	4.1(b)	9,400
		20,400
Ending stock	4.8(e)	7,500
		12,900
Realizable cost savings	4.6(b)	3,200
		16,100
		10,400
Depreciation	4.8(a)	4,200
Current operating profit		6,200
Realizable cost savings	4.8(a)	16,100
Business profit		22,300
Unrealized cost savings	4.8(c)	13,300
Realized profit		9,000

(b) Balance Sheet - December 31, 1909

	Table	Amount
Shareholders' Funds		£
Capital	4.1(a)	50,000
Profit unappropriated	4.3(a)	69,000
Unrealized cost savings	4.8(d)	15,800
		134,800
Current Liability		
Trade creditors	4.2(a)	6,400
		141,200

Fixed Assets	Table	Amount
		£
Building	4.4(a)	86,000
Less Aggregate depreciation	4.4(a) 4.8(a)	25,800
		60,200
Current Assets		
Stock	4.8(e)	7,500
Trade debtors	4.2(d)	26,500
Securities	4.4(a)	20,000
Cash	4.2(e)	27,000
		81,000
		141,200

TABLE 4.10

REPLACEMENT COST JOURNAL ENTRIES

		Dr	Cr
1.	Cost of goods sold	11,000	
	Stock		10,000
	Stock valuation adjustment		1,000
	To transfer beginning stock to cost of goods sold		
2.	Stock	5,900	
	Stock valuation adjustment	1,600	
	Cost of goods sold		7,500
	To record ending stock		
3.	Cost of goods sold	9,400	
	Purchases		9,400
	To transfer purchaes to cost of goods sold		
4.	Current operating profit	16,100	
	Cost of goods sold		16,100
	To transfer the replacement cost of goods sold to current operating profit		
5.	Cost of goods sold	3,200	
	Realizable cost savings		3,200
	To record realizable cost savings accruing on stock		
6.	Building valuation adjustment	2,900	
	Realizable cost savings		2,900
	To record realizable cost savings accruing on Building		

		Dr	Cr
7.	Depreciation	200	
	Building valuation adjustment		200
	To record depreciation of building on a replacement cost basis		
8.	Current operating profit	4,200	
	Depreciation		4,200
	To transfer depreciation to current operating profit		
9.	Sales	26,500	
	Current operating profit		26,500
	To transfer sales to current operating profit		
10.	Securities valuation adjustment	10,000	
	Realizable cost savings		10,000
	To record realizable cost savings accruing on securities		
11.	Current operating profit	6,200	
	Realizable cost savings	16,100	
	Business profit		22,300
	To record Business profit		
12.	Business profit	22,300	
	Unrealized cost savings		13,300
	Realized profit		9,000
	To record (historical cost) realized profit		
13.	Realized profit	9,000	
	Profit unappropriated		9,000
	To transfer realized profit to profit unappropriated		

Bell system. We turn now to the more important task of extending the results into the realm of some other measurement systems.

4.2 Current Purchasing Power Accounting[12]

In this section our objective is to extend the Edwards and Bell method of accounting into the realm of current purchasing power accounting. In this respect, the fundamental theorems which were stated and illustrated in the previous section can be extended in a fairly straight forward manner to other accounting measurement systems, including the current purchasing power system. So as to avoid confusion with the Edwards and Bell replacement cost system, we shall prefix the C.P.P. cost savings concepts with the term "fictional".[13] We thus speak of fictional realizable cost savings, fictional realized cost savings and fictional unrealized cost savings. These descriptions seem advisable because they reflect the increased amounts necessary to maintain command over a composite of consumption goods.

The "traditional" C.P.P. financial statements of the Dyer Company Limited for the year ending December 31, 1909 are contained in Tables 4.11(a) and 4.12.[14]

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12. The C.P.P. interpretation of the model offered by Edwards and Bell is unnecessarily complex and there is little (we would venture so far as to say no) discussion of its underlying mathematical framework. Because of this, the rationale underlying many of their computations is vague, to say the least.

13. Ibid., pp 124-129.

14. We have employed an "averaging" technique in restating the Dyer Company's financial statements to a C.P.P. basis. By this we mean, for example, that sales are assumed to occur at the midpoint of the period under consideration, thus justifying use of the midpoint index value in their restatement. This procedure may, however, result in some inaccuracy. See the ensuing chapter 5 for some further discussion on this point.

TABLE 4.11

CURRENT PURCHASING POWER ACCOUNTING

(a) "Traditional" Balance Sheet - January 1, 1909

	Table	Raw	Multiplier	Adjusted
Shareholders' Funds		£		£
Capital	4.1(a)	50,000	$\frac{150}{100}$	75,000
Profit unappropriated	4.1(a)	60,000	*	65,000
		110,000		140,000
Current Liability				
Trade creditors	4.1(a)	5,000		5,000
		£115,000		£145,000

	Table	Raw	Multiplier	Adjusted
Fixed Asset		£	£	£
Building	4.1(a)	80,000	$\frac{150}{100}$	120,000
Less aggregate depreciation	4.1(a)	20,000	$\frac{150}{100}$	30,000
		60,000		90,000
Current Assets				14(a)
Stock	4.1(a)	10,000		10,000
Trade debtors	4.1(a)	10,000		10,000
Securities	4.1(a)	10,000		10,000
Cash	4.1(a)	25,000		25,000
		55,000		55,000
		£115,000		£145,000

* Balancing figure

Index: The index's value was 100 on January 1, 1904, 150 on January 1, 1909, 155 on June 30, 1909 and 160 on December 31, 1909.

14(a) We are here assuming stock was acquired on December 31, 1908.

(b) "Edwards and Bell" Balance Sheet - January 1, 1909

	Table	Amount
Shareholders' Funds		£
Capital	4.1(a)	50,000
Profit unappropriated	4.1(a)	60,000
Fictional unrealized cost savings	4.11(c)	30,000
		140,000
Current Liability		
Trade creditors	4.1(a)	5,000
		£145,000

	Table	Amount
Fixed Asset		£
Building	4.11(a)	120,000
Less Aggregate depreciation	4.11(a)	30,000
		90,000
Current Assets		
Stock	4.1(a)	10,000
Trade debtors	4.1(a)	10,000
Securities	4.1(a)	10,000
Cash	4.1(a)	25,000
		55,000
		£145,000

(c) Fictional Unrealized Cost Savings as of January 1, 1909

	Table	Building
C.P.P. value - January 1, 1909	4.11(a)	90,000
Historic cost - January 1, 1909	4.1(a)	60,000
		£ 30,000

TABLE 4.12

TRADITIONAL METHOD OF CURRENT PURCHASING POWER ACCOUNTING

(a) Net Monetary Items

	Table	January 1	Table	December 31
		£		£
Trade debtors	4.1(a)	10,000	4.3(b)	26,500
Cash	4.1(a)	25,000	4.3(b)	27,000
		35,000		53,500
Trade creditors	4.1(a)	5,000	4.3(b)	6,400
		£30,000		£47,100

(b) Loss from Holding Net Monetary Items in the year ending December 31, 1909

	Table	Raw	Multiplier	Adjusted
		£		£
Net monetary items - January 1, 1909	4.12(a)	30,000	$\frac{160}{150}$	32,000
Sales	4.1(b)	26,500	$\frac{160}{155}$	27,355
		56,500		59,355
Purchases	4.1(b)	9,400	$\frac{160}{155}$	9,703
		47,100		49,652
Net monetary items - December 31, 1909				47,100
Loss from holding net monetary items				£ 2,552

(c) <u>(C.P.P.) Profit and Loss Statement for the year ending December 31, 1909</u>

	Table	Raw	Multiplier	Adjusted
		£		£
Sales	4.1(b)	26,500	$\frac{160}{155}$	27,355
Less Cost of Goods Sold				
Beginning stock	4.1(a)	10,000	$\frac{160}{150}$	10,667
Purchases	4.1(b)	9,400	$\frac{160}{155}$	9,703
		19,400		20,370
Ending stock	4.2(c)	5,900	$\frac{160}{155}$	6,090
		13,500		14,280
		13,000		13,075
Depreciation	4.1(a)	4,000	$\frac{160}{100}$	6,400
		9,000		6,675
Loss from holding net monetary items	4.12(b)	-		2,552
Net profit		£ 9,000		£ 4,123

(d) <u>(C.P.P.) Appropriation Statement for the year ending December 31, 1909</u>

	Table	£
Profit unappropriated - January 1, 1909	4.11(a)	65,000
Index adjustment $(\frac{160}{150} - 1) \times 65,000$	4.11(a)	4,333
		69,333
Net profit	4.12(c)	4,123
Profit unappropriated - December 31, 1909		£73,456

(e) (C.P.P.) Balance Sheet – December 31, 1909

	Table	Raw	Multiplier	Adjusted
		£		£
Shareholders' Funds				
Capital	4.1(a)	50,000	$\frac{160}{100}$	80,000
Profit unnapropriated	4.12(d)	69,000		73,456
		119,000		153,456
Current Liability				
Trade creditors	4.2(a)	6,400		6,400
		£125,400		£159,856

	Table	Raw	Multiplier	Adjusted
		£		£
Fixed Asset				
Building	4.1(a)	80,000	$\frac{160}{100}$	128,000
Less Aggregate depreciation	4.2(b)	24,000	$\frac{160}{100}$	38,400
		56,000		89,600
Current Assets				
Stock	4.2(c)	5,900	$\frac{160}{155}$	6,090
Trade debtors	4.2(d)	26,500		26,500
Securities	4.1(a)	10,000	$\frac{160}{150}$	10,666
Cash	4.2(e)	27,000		27,000
		69,400		70,256
		£125,400		£159,856

TABLE 4.13

COMPUTATION OF FICTIONAL COST SAVINGS

(a) Fictional Realizable Cost Savings for the year ending December 31, 1909

	Table	Securities	Building	Stock	Total
		£	£	£	£
C.P.P. value - December 31, 1909	4.12(e)	10,667	89,600	6,090	106,357
C.P.P. value - January 1, 1909	4.11(a)	10,000	90,000	10,000	110,000
Additions (at cost)	4.1(b)	-	-	9,400	9,400
		10,000	90,000	19,400	119,400
C.P.P. value of disposals	4.12(c)	-	6,400	14,280	20,680
		10,000	83,600	5,120	98,720
Fictional realizable cost savings		£ 667	£ 6,000	£ 970	£ 7,637

(b) Fictional Realized Cost Savings for the year ending December 31, 1909

	Table	Securities	Building	Stock	Total
		£	£	£	£
C.P.P. value of disposals	4.12(c)	-	6,400	14,280	20,680
Historic cost of disposals	4.3(a)	-	4,000	13,500	17,500
Fictional realized cost savings		£ -	£ 2,400	£ 780	£ 3,180

(c) Fictional Unrealized Cost Savings for the year ending December 31, 1909

	Table	Securities	Building	Stock	Total
		£	£	£	£
Fictional realizable cost savings	4.13(a)	667	6,000	970	7,637
Fictional realized cost savings	4.13(b)	-	2,400	780	3,180
Fictional unrealized cost savings		£ 667	£ 3,600	£ 190	£4,457

(d) Fictional Unrealized Cost Savings as of December 31, 1909

	Table	Securities	Building	Stock	Total
		£	£	£	£
Balance - January 1, 1909	4.11(c)	-	30,000	-	30,000
Additions	4.13(c)	667	3,600	190	4,457
Unrealized cost savings		£ 667	£33,600	£ 190	£34,457

(e) C.P.P. Measurements as of December 31, 1909

	Table	Securities	Building	Stock	Total
		£	£	£	£
Historic cost - December 31, 1909	4.3(b)	10,000	56,000	5,900	71,900
Fictional unrealized cost savings December 31, 1909	4.13(d)	667	33,600	190	34,457
C.P.P. measurements		£10,667	£89,600	£ 6,090	£106,357

TABLE 4.14

EDWARDS AND BELL C.P.P. FINANCIAL STATEMENTS

(a) (C.P.P.) Profit and Loss Statement for the year ending December 31, 1909

	Table	Amount
		£
Sales	4.1(b)	26,500
Less Cost of Goods Sold		
Beginning stock	4.1(a)	10,000
Purchases	4.1(b)	9,400
		19,400
Ending stock	4.2(c)	5,900
		13,500 13,000
Depreciation	4.2(b)	4,000
Realized Profit		9,000
Fictional realizable cost savings	*	9,334
		(334)
Fictional unrealized cost savings	4.13(c)	4,457
C.P.P. Income		£4,123

$* \ 140{,}000 \times (\frac{160}{150} - 1) = 9{,}334$ (See Table 4.11(b))

(b) (C.P.P.) Profit and Loss Appropriation Statement for the year ending December 31, 1909

	Table	Amount
		£
Profit unappropriated - January 1, 1909	4.11(b)	60,000
Fictional realizable cost savings	4.14(a)	9,334
C.P.P. income	4.14(a)	4,123
		13,457
		73,457
Fictional unrealized cost savings	4.13(c)	4,457
Profit unappropriated - December 31, 1909		£69,000

(c) (C.P.P.) Balance Sheet - December 31, 1909

	Table	Amount
Shareholders' Funds		£
Capital	4.1(a)	50,000
Profit unappropriated	4.14(b)	69,000
Fictional unrealized cost savings	4.13(d)	34,457
		153,457
Current Liability		
Trade creditors	4.3(b)	6,400
		£159,857

	Table	Amount
Fixed Asset		£
Building	4.12(e)	128,000
Less Aggregate depreciation	4.12(e)	38,400
		89,600
Current Assets		
Stock	4.12(c)	6,090
Trade debtors	4.2(c)	26,500
Securities	4.12(e)	10,667
Cash	4.2(d)	27,000
		70,257
		£159,857

TABLE 4.15

C.P.P. ACCOUNTING JOURNAL ENTRIES

		Dr	Cr
1.	Profit and loss	9,334	
	Fictional realizable cost savings		9,334
	To record the fictional realizable cost savings for the year ending December 31, 1909		
2.	Building (C.P.P.) Adjustment	3,600	
	Stock (C.P.P.) adjustment	190	
	Securities (C.P.P.) adjustment	667	
	Profit and loss		4,457
	To record the C.P.P. adjustments for the year ending December 31, 1909		
3.	Profit and loss	4,123	
	C.P.P. income		4,123
	To record the C.P.P. income of the year ending December 31, 1909		
4.	Fictional realizable cost savings	9,334	
	C.P.P. income	4,123	
	Profit and loss appropriation		9,000
	Fictional unrealized cost savings		4,457
	To record the unrealized component of fictional realizable cost savings		

We call these statements "traditional" because they employ the "usual" methods of restating the historical cost financial statements to a C.P.P. basis.[15] Their Edwards and Bell counterparts are contained in Tables 4.11(b), 4.13 and 4.14.[16] Table 4.15 contains the journal entries required to effect the inclusion of the "Edwards and Bell" C.P.P. measurements into the books of the Dyer Company Limited. In preparing Table 4.15, we have assumed that the balance sheet exhibited in Table 4.11(b) plus the historical cost income and expense of the year ending December 31, 1909 have been recorded in the Dyer Company's books.

In restating the Dyer Company's historical cost financial statements to their "Edwards and Bell" C.P.P. equivalents, we have employed a third theorem of commensurate importance to those stated in the previous section. The theorem is stated as follows

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15. "Accounting for changes in the purchasing power of money," Provisional Statement of Standard Accounting Practice 7, May 1974.
American Institute of Certified Public Accountants. "Reporting the Financial Effects of Price Level Changes," Accounting Research Study 6, 1963.

16. In computing the fictional realizable cost savings accruing on a resource, Edwards and Bell assume that the beginning quantity (I_0) is held while the index varies from that prevailing at the beginning to that prevailing at the end. Assuming that the index is r_1 at the midpoint of the interval and r_2 at the end and presuming the index to be "normalized" so that its beginning value is unity implies that the fictional realizable cost savings on I_0 amount to $(r_1r_2-1)I_0$. The increment P, is assumed by Edwards and Bell to be acquired at the midpoint of the interval, thus implying fictional realizable cost savings of $(r_2-1)P_1$. The total fictional realizable cost savings are thus $(r_2-1)P_1 + (r_1r_2-1)I_0$. This result is literally "plucked out of the air" by Edwards and Bell. (Ibid., pp.235-239), but is easily rationalised in terms of the "fundamental" theorems proved above. See Appendix 4E for some further discussions on this.

Let F_j be the "historical cost" shareholders' equity at time j, plus the fictional unrealized cost savings at time j. Let r_j and r_k be the values of a specified index at times j and k respectively. The total of the fictional realizable cost savings during the interval $[j,k]$ amounts to

$$\left(\frac{r_k}{r_j} - 1 \right) F_j$$

where $k > j$.[17]

This result is proved in Appendix 4E.

The theorem implies that the total of the Dyer Company's fictional realizable cost savings in the year ending December 31, 1909 amount to

$$140{,}000 \times \left(\frac{160}{150} - 1\right) = \underline{\underline{£9{,}334}}$$

where from Table 4.11(b), the sum of the historical cost shareholders' funds and the fictional realized cost savings as of December 31, 1909 amount to £140,000, whilst from Table 4.11(a) the index's value is 160 and 150 as of December 31, 1909 and January 1, 1909 respectively.

At first sight, this result may appear to conflict with the contents of Table 4.13(a) where the fictional realizable cost savings are listed at £7,637. The discrepancy, however, is accounted for as the difference between the loss from holding net monetary items (£2,552) and the adjustment which is necessary to restate sales on a C.P.P. basis (£855). These figures may be obtained from Table 4.12(c). The fictional realizable cost savings in the year ending December 31, 1909 is then seen to be composed of the following items

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17. Edwards and Bell provide an intuitive and somewhat unsatisfactory proof of this result.
Ibid., p 250.

For a rigorous proof employing the "fundamental" theorems see Appendix 4E.

	£
Building	6,000
Stock	970
Securities	667
Sales	(855)
Monetary loss	2,552
	£9,334

Of this figure, the amounts listed in Table 4.13(c) and totalling £4,457 remain unrealized as of December 31, 1909. The fictional realized cost savings in the year ending December 31, 1909 are the sum of the £3,180 listed in Table 4.13(b), the loss from holding net monetary items (£2,552) less the adjustment to sales (£855).

We have now partially fulfilled the second objective of this chapter which was to illustrate how the Edwards and Bell method of accounting may be extended into the realm of other measurement systems.[18] To complete the analysis, we now extend the model into the realm of market value (net realizable value, current cash equivalent etc.) accounting.

4.3 Market Value Accounting

The point of departure for extending the Edwards and Bell technique into the province of market value measurement is provided by Table 4.16. This Table restates the Dyer Company's balance sheet as of January 1, 1909 on a market value basis, and also provides the necessary information for preparing the market value financial statements for the year to December 31, 1909. So as to avoid confusion

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18. For a discussion of the theoretical foundations of the C.P.P. interpretation of the model, see

Ibid., pp.121-131 and chapter 8. See also chapter 6 *infra*.

TABLE 4.16

MARKET VALUE ACCOUNTING

(a) Balance Sheet - January 1, 1909

	Table	Amount
Shareholders' Funds		£
Capital	4.1(a)	50,000
Profit unappropriated	4.1(a)	60,000
Unrealized capital gains	4.16(b)	4,000
		114,000
Current Liability		
Trade creditors	4.1(a)	5,000
		£119,000

	Table	Amount
Fixed Asset		£
Building	4.16(a)	65,000
Current Assets		
Stock	4.16(a)	9,000
Trade debtors	4.1(a)	10,000
Securities	4.1(a)	10,000
Cash	4.1(a)	25,000
		54,000
		£119,000

Building: Market value as of January 1, 1909 is £65,000. As of December 31, 1909, the building's market value is £63,000.

Stock: Market value is £9 (per unit) as of January 1, 1909, £10 (per unit) as of February 28, 1909, £11 (per unit) as of August 31, 1909 and £13 (per unit) as of December 31, 1909.

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Securities: The market value of securities was £10,000 as of January 1, 1909 and £20,000 as of December 31, 1909.

(b) Unrealized Capital Gains

	Table	Building	Stock	Total
		£	£	£
Market value	4.16(a)	65,000	9,000	74,000
Historic cost	4.1(a)	60,000	10,000	70,000
Unrealized Capital Gains		£ 5,000	£(1,000)	£4,000

with the Edwards and Bell replacement cost system, we shall refer to the "cost savings" concepts of market value measurement as "capital gains".[19] We thus speak of realizable capital gains, realized capital gains and unrealized capital gains. The realizable capital gains thus represent the increase in market value of a firm's resources during some interval of time. The realized capital gains represent the realizable capital gains which have been realized through use or sale during the period, whilst the unrealized capital gains represent the difference between the market value of a firm's resources and their historic cost at a point in time.

Tables 4.17[20] and 4.18 apply the Edwards and Bell model to the market value information contained in Table 4.16[21] In this respect

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19. Ibid., pp.80-88.

20. As in the case of the replacement cost system, direct calculation of the realizable holding gains is more cumbersome. This may be illustrated by computing the realizable capital gains accruing on stock

Gain accruing on 200 units held the entire year 200 x (13-9)	£ 800
Gain accruing on 800 units sold February 28 500 x (10-11) + 300 x (10-9)	(200)
Gain accruing on 500 units sold August 31 500 x (11-9)	1,000
Gain accruing on 300 units of ending stock 300 x (13-13)	-
Realizable capital gains	£1,600

The underlying these figures may be obtained from Tables 4.2(a) and 4.16(a). The figure of £1,600 corresponds with the realizable capital gains accruing on stock which appears in Table 4.17(a).

21. For a discussion of the theoretical foundations of the market value interpretation of the model, see Ibid., chapter 2 and chapter 6 infra.

TABLE 4.17

COMPUTATION OF CAPITAL GAINS

(a) Realizable Capital Gains for the year ending December 31, 1909

	Table	Securities	Building	Stock	Total
		£	£	£	£
Market value - December 31, 1909	4.16(a)	20,000	63,000	6,500	89,500
"Market" value - January 1, 1909	4.16(a)	10,000	65,000	9,000	84,000
Additions (at cost)	4.1(b)	-	-	9,400	9,400
		10,000	65,000	18,400	93,400
Market value of disposals	4.2(a)				
	4.16(a)	-	4,417	13,500	17,917
		10,000	60,583	4,900	75,483
Realizable capital gains		£10,000	£ 2,417	£ 1,600	£14,017

(b) Realized Capital Gains for the year ending December 31, 1909

	Table	Securities	Building	Stock	Total
		£	£	£	£
Market value of disposals	4.2(a)				
	4.16(a)	-	4,417	13,500	17,917
Historic cost of disposals	4.3(a)	-	4,000	13,500	17,500
Realized capital gains		-	£ 417	-	£ 417

(c) Unrealized Capital Gainsfor the year ending December 31, 1909

	Table	Securities	Building	Stock	Total
		£	£	£	£
Realizable capital gains	4.17(a)	10,000	2,417	1,600	14,017
Realized capital gains	4.17(b)	-	417	-	417
Unrealized capital gains		£10,000	£2,000	£1,600	£13,600

(d) Unrealized Capital Gains as of December 31, 1909

	Table	Securities	Building	Stock	Total
		£	£	£	£
Balance - January 1, 1909	4.16(b)	-	5,000	(1,000)	4,000
Additions	4.17(c)	10,000	2,000	1,600	13,600
Unrealized capital gains		£10,000	£ 7,000	£ 600	£17,600

(e) Market Value Measurements as of December 31, 1909

	Table	Securities	Building	Stock	Total
		£	£	£	£
Historic cost - December 31, 1909	4.3(b)	10,000	56,000	5,900	71,900
Unrealized capital gains December 31,1909	4.17(d)	10,000	7,000	600	17,600
Market value measurements		£20,000	£63,000	£6,500	£89,500

TABLE 4.18

FINANCIAL STATEMENTS

(a) Profit and Loss Statement for the year ending December 31, 1909

	Table	Amount
		£
Sales	4.1(b)	26,500
Less Cost of Goods Sold		
Beginning stock	4.16(a)	9,000
Purchases	4.1(b)	9,400
		18,400
Ending stock	4.17(e)	6,500
		11,900
Realizable capital gains	4.17(a)	1,600
		13,500
		13,000
Depreciation	4.17(a)	4,417
Realizable operating profit		8,583
Realizable capital gains	4.17(a)	14,017
Realizable profit		22,600
Unrealized capital gains	4.17(c)	13,600
Realized profit		£ 9,000

(b) Balance Sheet - December 31, 1909

	Table	Amount
Shareholders' Funds		£
Capital	4.1(a)	50,000
Profit unappropriated	4.3(a)	69,000
Unrealized capital gains	4.17(d)	17,600
		136,600
Current Liability		
Trade creditors	4.2(a)	6,400
		£143,000

	Table	Amount
Fixed Assets		£
Building	4.16(a)	63,000
Current Assets		
Stock	4.17(e)	6,500
Trade debtors	4.2(d)	26,500
Securities	4.17(e)	20,000
Cash	4.2(e)	27,000
		80,000
		£143,000

there is one point which requires emphasizing. The market value of building disposals (market value depreciation) is listed at £4,417 in Table 4.17(a) whereas the building's market value declined only by £2,000 in the year to December 31, 1909.[22] In computing the £4,417 appearing in Table 4.17(a) a distinction was drawn between changes in market value due to the consumption of the productive services embodied in the building and those attributable to the passage of time.[23] These latter gains are usually referred to as "holding gains".[24] Presuming the firm's depreciation policy to be an adequate reflection of market value in the absence of holding effects, then the building may be characterized as consisting of 15 equally valued units of unused service potential as of January 1, 1909 and 14 such units as of December 31, 1909. Each unit has a market value of £4,333[25] as of January 1, 1909 and £4,500[26] as of December 31, 1909. If we now suppose the market value of these units to increase linearly over the year to December 31, 1909, and assume that disposals occur in equal amounts at each of n equally spaced points of this year, then the market value of such disposals may be computed in the following manner.[27]

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22. See Table 4.16(a)
23. *Ibid*., pp.70-80.
24. *Ibid*.
25. From Table 4.16(a) the total market value is £65,000. Hence, the market value (per unit) is $\frac{65,000}{15}$ or £4,333. See,also, Appendix 4C.
26. From Table 4.16(a) the total market value is £63,000. Hence, the market value (per unit) is $\frac{63,000}{14}$ or £4,500. See,also, Appendix 4C.
27. Recall that the sum of the first n integers is $\frac{n}{2}(n+1)$. The figure may also be computed by taking the arithmetic mean on the interval $[0,1]$ of the function $n(t) = 4,333 + 167t.$ Thus we have

$$\int_0^1 n(t)dt = \int_0^1 (4,333 + 167t)dt$$
$$\int_0^1 n(t)dt = 4,417$$

This topic receives more consideration in the ensuring chapter.

$$\sum_{j=1}^{n} \frac{1}{n}\left[4,333 + \frac{167j}{n}\right] = 4,333 + \frac{167}{n^2}\sum_{j=1}^{n} j$$

$$= 4,333 + \frac{167}{2n^2}\, n(n+1)$$

$$\sum_{j=1}^{n} \frac{1}{n}\left[4,333 + \frac{167j}{n}\right] = 4,333 + 84\left(1+\frac{1}{n}\right)$$

Supposing n to be "large" it necessarily follows that the figure of £4,417 appearing in Table 4.17(d) is a reasonable approximation to the market value of the disposals. The realizable capital gains accruing on the building in the year to December 31, 1909 may likewise be rationalized.

The balance of the computations appearing in Table 4.17(a) are quite straight forward and are obtained by applying the fundamental theorems stated earlier in this chapter. The journal entries by which these figures are incorporated into the Dyer Company's books are contained in Table 4.19. In preparing this Table, we have assumed that the balance sheet exhibited in Table 4.4(a) plus the historical cost income and expense of the year ending December 31, 1909 have been recorded in the Dyer Company's books.

We have now achieved the second objective of this chapter which, it will be recalled, was to illustrate the generality of the Edwards and Bell model of accounting measurement. We now progress to the final topic of this chapter, namely consideration of the problems associated with incorporating the fundamental theorems, developed in section 4.1.2 of this chapter, into the axiom scheme proposed in chapter 2.

4.4 Axiomatic Treatment

In section 4.1.2 of this chapter, two "fundamental" theorems of the Edwards and Bell accounting model were stated. In so doing, "definitions" of the terms realizable cost savings, realized cost savings and unrealized cost savings were employed. Further, these definitions were shown to generalize to the other measurement systems. If these concepts could be defined within the axiom system developed in chapter 2, it would be

TABLE 4.19

MARKET VALUE JOURNAL ENTRIES

		Dr	Cr
1.	Cost of goods sold	9,000	
	Stock valuation adjustment	1,000	
	Stock		10,000
	To transfer beginning stock to cost of goods sold.		
2.	Stock	5,900	
	Stock valuation adjustment	600	
	Cost of goods sold		6,500
	To record ending stock.		
3.	Cost of goods sold	9,400	
	Purchases		9,400
	To transfer purchases to cost of goods sold.		
4.	Depreciation	417	
	Building valuation adjustment		417
	To record depreciation of building on a market value basis.		
5.	Realizable profit	17,917	
	Depreciation		4,417
	Cost of goods sold		13,500
	To transfer depreciation and cost of goods sold.		

		Dr	Cr
6.	Cost of Goods sold	1,600	
	Building valuation adjustment	2,417	
	Securities valuation adjustment	10,000	
	Realizable capital gains		14,017
	To record the realizable capital gains accruing on stock, building and securities.		
7.	Sales	26,500	
	Realizable capital gains	14,017	
	Realizable profit		40,517
	To transfer realizable capital gains and sales.		
8.	Realizable profit	22,600	
	Realized profit		9,000
	Unrealized capital gains		13,600
	To record realizable profit		
9.	Realized profit	9,000	
	Profit unappropriated		9,000
	To record (historic cost) realized profit		

possible to prove the theorems directly from the axioms. Unfortunately, this task is rather imposing. To illustrate the reason for this, consider the following definition of realizable cost savings which, at first sight, may appear to be reasonable

> Suppose the simple resource set $p_{j,t} \in \mathcal{G}t$ for all $t \in [T,T^1]$. The mapping $R: \mathbb{R}^2 \longrightarrow \mathbb{R}$ defined by
> $$R(T^1,T) = L_{T^1}(p_{j,t}) - L_T(p_{j,t})$$
> is called the realizable cost savings of $p_{j,t}$ in the interval $[T,T^1]$.

This definition certainly "works" in the case of "current" simple resources such as stock or securities. Thus, the realizable cost savings accruing on stock in the period from January 30, 1909 to February 28, 1909 amounts to £1,500.[28] Applying this definition to the simple resource "building", however, implies that the realizable cost savings in the year ending December 31, 1909 amount to -£1,300,[29] compared with the "correct" figure of £2,900. One method of overcoming this is to replace the above expression for R by the following

$$R(T^1,T) \quad = \quad \left[L_{T^1}(p_{j,t}) - L_T(p_{j,t})\right] \int_T^{T^1} f(t,T,T^1)dt$$

where $f(t,T,T^1)$ is a real valued integrable function defined on the interval $[T,T^1]$. In the case of the building, for example, the following definition is appropriate

$$R(T,T^1) \quad = \quad \left[L_{T^1}(p_{1,5}) - L_T(p_{1,5})\right] \int_T^{T^1} \left(\frac{20-t}{20}\right) dt$$

where $f(t) = \frac{20-t}{20}$, defined on the interval $[0,20]$, is an expression of the proportion of the building's productive life remaining at time t. Whilst, in the case of inventory and similar current items this definition "works" provided

$$\int_a^{\ell} f(t,a,\ell)dt \ = \ 1$$

for all intervals $[a,\ell]$ in $[T,T^1]$, the precise form of f is not

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28. See Tables 4.2(a) and 4.4(a). 1500 x (13-12) = 1500.

29. See Table 4.4(a) and 4.9(b) 60,200 - 61,500 = -1300.

endogenous to the axiom system, and consequently, the modified definition is of little use. Thus, we choose not to incorporate these concepts into the axiom system. Unfortunately, this reduces the number of propositions, specific to replacement cost measurement which can be proved from the axioms. The two "fundamental theorems" stated earlier, for example, are not theorems of the axiom system. This does not imply, however, that the axiom system cannot be applied to the measurement systems examined in this chapter. To prove this, Appendix 4D applies the axiom system exhibited in chapter 2 to each of the measurement systems examined in this chapter.

4.5 Summary

Our objective in this chapter has been to provide a general model of accounting valuation; that is, a model which can meaningfully accommodate the replacement cost, net realizable value and C.P.P. measurement systems. The import of this for accounting measurement is that it implies that the "numerical processes" associated with restating a set of historical cost financial statements to a C.P.P., net realizable value or replacement cost basis of measurement are the same in principle. That is, there is a single adjustment procedure applicable to all accounting measurement systems; not separate procedures for each.

The basis of these procedures is to be found in two "fundamental" theorems. These theorems provide respectively a means of computing the potentially realizable "holding gains" for some time interval, and the unrealized holding gains at a point in time. Although each theorem was proved in relation to replacement cost accounting, they were found to generalize to other measurement systems. In this respect, we found little difficulty in extending the theorems into the province of market value and C.P.P. accounting.

We concluded the chapter by examining the possibility of proving the theorems within the axiom scheme particularized in chapter 2. Given the simplicity of the scheme therein advanced, this proved to be rather imposing.

APPENDIX 4A

COMPUTATION OF REALIZABLE COST SAVINGS

Theorem

Let ψ be the replacement cost measure of a simple resource at time 0, plus additions (at cost) in the interval $[0,T]$ less disposals (at the replacement cost measure at the time of disposal) in the interval $[0,T]$. Let ξ be the replacement cost measure of the simple resource at time T. The difference $\xi - \psi$ is the realizable cost savings accruing on the simple resource during the interval $[0,T]$.

Proof

(i) Suppose an entity disposes of $\sum_i \gamma_i Q$ units during $[0,T]$ where (Q) is the quantity measure of the simple resource at time 0. Assume without loss of generality

(i) $\sum_i \gamma_i \leq 1$

(ii) there is a FIFO physical flow.

Suppose the entity acquires $\sum_j \beta_j Q$ units during $[0,T]$. By (i) and (ii) the units held at both time 0 and time (T) amount to $Q - \sum \gamma_i Q$ and thus have corresponding realizable cost savings of $(R_T - R_o)(1 - \sum \gamma_i)Q$ where R_T and R_o are the replacement cost measures (per unit) at time T and time 0 respectively. The realizable cost savings accruing on disposals during $[0,T]$ amount to $\sum (R_i - R_o)\gamma_i Q$ where R_i is the replacement cost measure of disposals at the time of disposal. The realizable cost savings accruing on purchases during $[0,T]$ amount to $\sum (R_T - R_j)\beta_j Q$ where R_j is the acquisition price (per unit) of purchases. Hence, the realizable cost savings accruing during $[0,T]$ amount to

$$R[0,T] = (R_T - R_o)(1 - \sum \gamma_i)Q + \sum (R_i - R_o)\gamma_i Q + \sum (R_T - R_j)\beta_j Q$$

$$R\,\boxed{0,T} = (R_T - R_o)Q + (\Sigma\beta_j - \Sigma\gamma_i)R_TQ - (\Sigma R_j\beta_j - \Sigma R_i\gamma_i)Q$$

To prove the proposition, we must show that the method stated in the conclusion gives the above result. Computing the quantities therein

$$\psi = R_oQ + \Sigma R_j\beta_j Q - \Sigma R_i\gamma_i Q$$
$$\xi = R_TQ \quad (1 - \Sigma\gamma_i) + \Sigma\beta_j$$

so that

$$\begin{aligned}\xi - \psi &= R_TQ \quad (1 - \Sigma\gamma_i) + \Sigma\beta_j \quad - R_oQ - \\ &\quad \Sigma R_j\beta_j Q + \Sigma R_i\gamma_i Q \\ &= (R_T - R_o)Q + (\Sigma\beta_j - \Sigma\gamma_i)R_TQ - \\ &\quad (\Sigma R_j\beta_j - \Sigma R_i\gamma_i)Q \\ \xi - \psi &= R\,\boxed{0,T}\end{aligned}$$

thus proving the result.

(ii) To show that assumption (i) ($\sum_i \gamma_i \leq 1$) does not affect the proposition's validity we prove the proposition under the assumption $\Sigma\gamma_i > 1$. Let $\sum_j \alpha_j Q + \sum_k \delta_k Q$ be acquisitions of the interval $\boxed{0,T}$. Suppose that $\sum_j \alpha_j Q$ are disposed of in the interval $\boxed{0,T}$. The realizable cost savings of the interval $\boxed{0,T}$ are computed as follows

$$R\,\boxed{0,T} = \sum_i (R_i - R_o)\gamma_i Q + \sum_j (R_j' - R_j)\alpha_j Q + \sum_k (R_T - R_k)\delta_k Q$$

where R_i and R_j' are replacement cost measures (per unit) at time of disposal and R_j and R_k are acquisition costs (per unit).

Applying the above proposition we have

$$\psi = R_oQ + (\sum_j R_j\alpha_j + \sum_k R_k\delta_k)Q - (\sum_i R_i\gamma_i + \sum_j R_j'\alpha_j)Q$$

$$\xi = \sum_k R_T\delta_k Q$$

so that

$$\begin{aligned}
\xi - \psi &= \sum R_T \delta_k Q - R_o Q - \left(\sum R_j \alpha_j + \sum R_k \delta_k\right) Q \\
&\quad + \left(\sum R_i \gamma_i + \sum R_j' \alpha_j\right) Q \\
&= \sum (R_i - R_o) \gamma_i Q + \sum (R_j' - R_j) \alpha_j Q \\
&\quad + \sum (R_T - R_k) \delta_k Q \\
\xi - \psi &= R\,[0,T]
\end{aligned}$$

By similar procedures, we may show the proposition holds for other physical flows.

APPENDIX 4B

REALIZABLE COST SAVINGS AND REPLACEMENT COST

Theorem

Let A_T be the unrealized cost savings and B_T be the historic cost measure of a simple resource both at time T. Let I_T be the "quantity measure" of the simple resource at time T. Then

$$\frac{A_T + B_T}{I_T} = R_T$$

where R_T is the replacement cost measure (per unit) of the simple resource at time T.

Proof

(i) We use the same notation as in part (i) of the theorem proved in Appendix 4A and assume without loss of generality

(i) there are no unrealized cost savings carried forward from previous periods,

(ii) there is a FIFO physical flow,

(iii) $\sum_i \gamma_i \leqslant 1.$

The realized cost savings of the interval $[0,T]$ amount to $\Sigma (R_i - R_o)\gamma_i Q$. It thus follows that

$$A_T = R[0,T] - \Sigma (R_i - R_o)\gamma_i Q$$

$$A_T = (R_T - R_o)(1 - \Sigma\gamma_i)Q + \Sigma (R_T - R\ )\beta_j Q$$

The historic cost measure of the simple resource at time T is

$$B_T = (1 - \Sigma\gamma_i)R_o Q + \Sigma R_j \beta_j Q$$

This implies

$$A_T + B_T = (R_T - R_o)(1 - \Sigma\gamma_i)Q + \Sigma (R_T - R_j)\beta_j Q + (1 - \Sigma\gamma_i)R_o Q + \Sigma R_j \beta_j Q$$

$$A_T + B_T = R_T Q(1 - \Sigma\gamma_i + \Sigma\beta_j)$$

It follows that

$$\frac{A_T + B_T}{I_T} = \frac{R_T Q(1 - \Sigma\gamma_i + \Sigma\beta_j)}{Q(1 - \Sigma\gamma_i + \Sigma\beta_j)}$$

$$\frac{A_T+B_T}{I_T} = R_T$$

thus proving the result.

(ii) To show that assumption (iii) has no effect on the proposition's validity we assume $\sum \gamma_i > 1$ and adopt the notation of part (ii) of the proof in Appendix 4A. It then follows that

$$A_T = R\,[0,T] - \sum (R_i - R_o)\gamma_i\, Q - \sum (R'_j - R_j)\,\alpha_j\, Q$$

$$A_T = \sum (R_T - R_k)\,\delta_k Q$$

and

$$B_T = \sum R_k\, \delta_k Q$$

so that

$$\frac{A_T+B_T}{I_T} = \frac{\sum (R_T-R_k)\,\delta_k Q + \sum R_k\,\delta_k Q}{\sum \delta_k Q}$$

$$= \frac{R_T Q \sum \delta_k}{Q \sum \delta_k}$$

$$\frac{A_T+B_T}{I_T} = R_T$$

which was to be proved. By similar procedures, we may show that the procedure holds for other physical flows.

APPENDIX 4C

REALIZABLE COST SAVINGS (CAPITAL GAINS) AND FIXED ASSETS

In computing the realizable cost savings accruing on fixed assets during the interval $[T,T+1]$, Edwards and Bell employ a method which they proved (under a redundant set of assumptions) as applying to stock. In applying the method, a necessary piece of information is the weighted average acquisition cost of the interval $[T,T+1]$. In the case of fixed assets this datum is unlikely to exist simply because fixed assets are, by nature, "wasting" resources. It is doubtful, therefore, if the method has a legitimate application to fixed assets. The problem can, however, be overcome by employing the following result.

Theorem

Suppose there are no acquisitions of a simple resource during the interval $[T,T+1]$ and the replacement cost of disposals amount to

$$\sum_{k=1}^{n} r(t_k) \cdot s(t_k) = r(z)\sum_{k=1}^{n} s(t_k)$$

where $r(t)$ is the replacement cost of a unit of the simple resource at time t and $s(t)$ is disposals (in units) of the simple resource at time t. The realizable cost savings accruing on the simple resource during $[T,T+1]$ can then be computed in either of the following ways

(a) Assume that the beginning quantity is held over the interval $[T,T+1]$ while the replacement cost varies from that prevailing at time T to that prevailing at time (T+1). The excess (or deficiency) of the ending quantity over the beginning quantity is assumed acquired (or disposed of) at the replacement cost prevailing at time z.

(b) Assume that the beginning quantity is held whilst its replacement cost varies from that prevailing at time T to that prevailing at time z. The ending quantity is assumed acquired at time z and held while its replacement cost varies to that prevailing at time (T+1).

Proof

We prove each of these results in turn.

(a) The quantity held at time (T+1) is Ie where by hypothesis

$$Ie = Ib - S$$

where $S = \sum_{k=1}^{n} s(t_k)$ is disposals during $[T,T+1]$ and Ib is quantity held at time T. From the first fundamental theorem we have

$$\xi = r(T+1)Ie$$

$$\xi = r(T+1)Ib - r(T+1)\sum_{k=1}^{n} s(t_k)$$

Also

$$\psi = r(T)Ib - \sum_{k=1}^{n} r(t_k)\, s(t_k)$$

which, by hypothesis, may be restated as

$$\psi = r(T)Ib - r(z)\sum_{k=1}^{n} s(t_k)$$

we thus have

$$\xi - \psi = \left[r(T+1) - r(T)\right] Ib - \left[r(T+1) - r(z)\right]\sum_{k=1}^{n} s(t_k)$$

But

$$-\sum_{k=1}^{n} s(t_k) = Ie - Ib$$

thus implying

$$\xi - \psi = \left[r(T+1)-r(T)\right]Ib + \left[r(T+1) - r(z)\right](Ie-Ib)$$

proving the result.

(b) To obtain the second result, we merely refactor the above expression

$$\xi - \psi = r(T+1)Ib-r(T)Ib + r(T+1)Ie - r(T+1)Ib-r(z)Ie + r(z)Ib$$

$$\xi - \psi = \left[r(T+1)-r(z)\right]Ie + \left[r(z)-r(T)\right]Ib$$

completing the proof.

Replacement Cost Measurement

In applying the above results to the Dyer Company's building we impose the following assumptions.

(i) The building consists of 15 equally valued units of unused service potential as of January 1, 1909 and 14 such units as of December 31, 1909. Each unit has a replacement cost of £4,100 as of January 1, 1909 and £4,300 as of December 31, 1909.

(ii) Disposals can be taken as occurring at time $z = \frac{2T+1}{2}$, the midpoint of the interval $[T,T+1]$.

(iii) The replacement cost (per unit) at time $z = \frac{2T+1}{2}$ is £4,200 (per unit).

Applying method (a) we have

$$\left[r(T+1)-r(T)\right]Ib + \left[r(T+1) - r\left(\frac{2T+1}{2}\right)\right](Ie-Ib)$$

$$= (4{,}300-4{,}100) \times 15 - (4{,}300-4{,}200)$$

$$= 2{,}900$$

Applying method (b) we have

$$\left[r(T+1)-r\left(\frac{2T+1}{2}\right)\right]Ib + \left[r\left(\frac{2T+1}{2}\right)-r(T)\right]Ie$$

$$= (4{,}300-4{,}200) \times 15 + (4{,}200-4{,}100) \times 14$$

$$= 2{,}900$$

Note that this result agrees with Table 4.8(a). For some further comment on this see Edwards and Bell, pp.144-148 and pp.188-193.

<u>Market Value Measurement</u>

The methods used in the text can be obtained by imposing assumptions similar to those employed for replacement cost measurement

(i) The building consists of 15 equally valued units of unused service potential as of January 1, 1909 and 14 such units as of December 31, 1909. Each unit has a market value of £4,333 as of January 1, 1909 and £4,500 as of December 31, 1909.

(ii) Disposals can be taken as occurring at time $z = \frac{2T+1}{2}$, the midpoint of the interval $[T,T+1]$.

(iii) The market value at time $z = \frac{2T+1}{2}$ is £4,417 (per unit).

Applying method (a), we have

$$\left[r(T+1)-r(T)\right]Ib + \left[r(T+1)-r\left(\frac{2T+1}{2}\right)\right](Ie-Ib)$$

$$= (4,500-4333) \times 15 - (4500-4,417)$$

$$= 2,422$$

Applying method (b) we have

$$\left[r(T+1)-r\left(\frac{2T+1}{2}\right)\right]Ib + \left[r\left(\frac{2T+1}{2}\right) - r(T)\right]Ie$$

$$= (4,500-4,417) \times 15 + (4,417- 4333) \times 14$$

$$= 2421$$

Note that this result agrees with Table 4.17(a), the slight discrepency being caused by rounding.

APPENDIX 4D

"VALUATION" MODELS OF THE AXIOMS OF ACCOUNTING MEASUREMENT

Each of the valuation models provide the same interpretation for the axiom of control and the axiom of quantities. Thus, we have

(a) Axiom of Control

January 1, 1909

$$P_5 = \bigcup_{j=1}^{6} p_{j,5}$$

where

$p_{1,5}$ = building

$p_{2,5}$ = cash

$p_{3,5}$ = trade debtors

$p_{4,5}$ = securities

$p_{5,5}$ = stock

$p_{6,5}$ = trade creditors

December 31, 1909

$$P_6 = \bigcup_{j=1}^{6} p_{j,6}$$

where

$p_{1,5}$ = building

$p_{2,5}$ = cash

$p_{3,5}$ = trade debtors

$p_{4,5}$ = securities

$p_{5,5}$ = stock

$p_{6,5}$ = trade creditors

(b) Axiom of Quantities

January 1, 1909

The algebra $\mathcal{G}_5$ consists of the 64 sets generated by the property set P_5.

December 31, 1909

The algebra $\mathcal{G}_6$ consists of the 64 sets generated by the property set P_6.

The measurement rules applicable to <u>replacement cost measurement</u> are then defined in the following terms

(c) Axiom of Measurement

January 1, 1909

$$L_5(p_{j,5}) = \begin{cases} 61{,}500 \text{ if } j = 1 \\ 25{,}000 \text{ if } j = 2 \\ 10{,}000 \text{ of } j = 3 \\ 10{,}000 \text{ if } j = 4 \\ 11{,}000 \text{ if } j = 5 \\ -5{,}000 \text{ if } j = 6 \end{cases}$$

December 31, 1909

$$L_6(p_{j,6}) = \begin{cases} 60{,}200 \text{ if } j = 1 \\ 27{,}000 \text{ if } j = 2 \\ 26{,}500 \text{ if } j = 3 \\ 20{,}000 \text{ if } j = 4 \\ 7{,}500 \text{ if } j = 5 \\ -6{,}400 \text{ if } j = 6 \end{cases}$$

The profit measure of the year ending December 31, 1909 is

$$\begin{aligned}\pi(6,5) &= L_6(P_6) - L_5(P_5)\\ &= 134{,}800 - 112{,}500\\ \pi(6,5) &= 22{,}300\end{aligned}$$

which is the Business profit for the year ending December 31, 1909. Building, cash, trade debtors, securities and cash satisfy the definition of an asset. Trade creditors satisfies the definition of a liability.

The measurement rules applicable to <u>market value measurement</u> are defined as follows.

<u>(c^i) Axiom of Measurement</u>

<u>January 1, 1909</u> — <u>December 31, 1909</u>

$$L'_5(p_{j,5}) = \begin{cases} 65{,}000 & \text{if } j = 1\\ 25{,}000 & \text{if } j = 2\\ 10{,}000 & \text{if } j = 3\\ 10{,}000 & \text{if } j = 4\\ 9{,}000 & \text{if } j = 5\\ -5{,}000 & \text{if } j = 6\end{cases} \qquad L'_6(p_{j,6}) = \begin{cases} 63{,}000 & \text{if } j = 1\\ 27{,}000 & \text{if } j = 2\\ 26{,}500 & \text{if } j = 3\\ 20{,}000 & \text{if } j = 4\\ 6{,}500 & \text{if } j = 5\\ -6{,}400 & \text{if } j = 6\end{cases}$$

The profit measure of the year ending December 31, 1909 is

$$\begin{aligned}\pi(6,5) &= L_6(P_6) - L_5(P_5)\\ &= 136{,}600 - 114{,}000\\ \pi(6,5) &= 22{,}600\end{aligned}$$

which is the Realizable profit for the year ending December 31, 1909. Building, cash, trade debtors, securities and cash satisfy the definition of an asset. Trade creditors satisfies the definition of a liability.

The measurement rules applicable to <u>C.P.P. measurement</u> are defined as follows

(c^{ii}) Axiom of Measurement

January 1, 1909

$$L''_5(p_{\dot{j},5}) = \begin{cases} 90,000 & \text{if } \dot{j} = 1 \\ 25,000 & \text{if } \dot{j} = 2 \\ 10,000 & \text{if } \dot{j} = 3 \\ 10,000 & \text{if } \dot{j} = 4 \\ 10,000 & \text{if } \dot{j} = 5 \\ -5,000 & \text{if } \dot{j} = 6 \end{cases}$$

December 31, 1909

$$L''_6(p_{\dot{j},6}) = \begin{cases} 89,600 & \text{if } \dot{j} = 1 \\ 27,000 & \text{if } \dot{j} = 2 \\ 26,500 & \text{if } \dot{j} = 3 \\ 10,667 & \text{if } \dot{j} = 4 \\ 6,090 & \text{if } \dot{j} = 5 \\ -6,400 & \text{if } \dot{j} = 6 \end{cases}$$

The profit measure afforded by Definition 4 of chapter 2 does not provide the C.P.P. income reported in Tables 4.12(c) and 4.14(a). The reason for this is that the Shareholders' Funds as of January 1, 1909 is multiplied by the ratio of the index as of December 31, 1909 and the index as of January 1, 1909 before the C.P.P. income is computed. To overcome this problem we define a fourth axiom as follows

Axiom of Indexing

There exists a real number $i_{T,t}$ called a "price level index" uniquely defined for all real $t > T \geqslant o$.

The profit measure may then be redefined as follows

Definition 4^i

The mapping $\pi : \mathbb{R}^2 \longrightarrow \mathbb{R}$ defined by

$$\pi(t,T) = L_t(P_t) - i_{T,t}\, L_T(P_T)$$

is called the "profit measure" of the interval $[T,t]$.

In the present example the axiom of indexing is satisfied by noting that $i_{5,6} = \frac{16}{15}$. Applying Definition 4^i implies that the C.P.P. income for the year ending December 31, 1909 is computed as follows

$$\pi(6,5) = L_6(P_6) - i_{5,6}\, L_5(P_5)$$

$$= 153,456 - \frac{16}{15} \times 140,000$$

$$\pi(6,5) = 4,123$$

The simple resources Building, cash, trade debtors, securities and cash satisfy the definition of an asset whilst the simple resource trade creditors satisfies the definition of a liability.

APPENDIX 4E

C.P.P. MEASUREMENT

Let the following be defined accordingly

F_t ≡ Shareholders' funds at time t.

M_t ≡ Net monetary items at time t.

N_t ≡ Net non-monetary items (excluding stock) at time t.

A_t ≡ Acquisitions of non-monetary items (excluding stock) at time t.

I_t ≡ Stock of inventory at time t.

S_t ≡ Sales of stock at time t.

P_t ≡ Purchases of stock at time t.

E_t ≡ Expenses (excluding cost of sales and depreciation) at time t.

D_t ≡ Depreciation at time t.

r_t ≡ A price index at time t.

Supposing all transactions to occur at discrete points in time denoted by t = 1, 2, ____, n, define the loss from holding net monetary items during the interval $[j,k]$ to be $(r_k -1)M_j$.
Consequently the "monetary loss" during the interval $[0,1]$ is $(r_1-1)Mo$, having a"price level adjusted value" of $r_2(r_1-1)Mo$ at t = 2.
The "monetary loss" over the interval $[0,2]$ amounts to $r_2(r_1-1)Mo + (r_2-1)M_1$ where $(r_2-1)M_1$ is the monetary loss during $[1,2]$. We may restate this as

$$
\begin{aligned}
r_2(r_1-1)Mo + (r_2-1)M_1 &= r_2r_1Mo - r_2Mo + r_2M_1 - M_1 \\
&= r_2r_1Mo - Mo + r_2M_1 - M_1 - r_2Mo + Mo \\
&= (r_2r_1-1)Mo + (r_2-1)M_1 - (r_2-1)Mo \\
r_2(r_1-1)Mo + (r_2-1)M_1 &= (r_2r_1-1)Mo + (r_2-1)(M_1-Mo)
\end{aligned}
$$

The "current purchasing power income" of the interval $[0,2]$ is defined in the following terms

$$(F_2-Fo) - (r_2r_1-1)Mo - (r_2-1)(M_1-Mo) + (r_2-1)(S_1+I_1-P_1-E_1) - (r_1r_2-1)(Io+Do)$$

where $(F_2-Fo) = S_1-(Io+P_1-I_1) - E_1-Do$ is the "historic cost" income of the interval $[0,2]$, $(r_2r_1-1)Mo + (r_2-1)(M_1-Mo)$ is the loss from holding net monetary items during $[0,2]$ and $(r_2-1)(S_1+I_1-P_1-E_1) + (r_1r_2-1)(Io+Do)$ is the adjustment to income and expense of the interval $[0,2]$. Adding and subtracting r_1r_2Fo from the above expression gives $F_2-r_1r_2Fo + (r_1r_2-1)(Fo-Mo-Do) + (r_2-1).(Mo+S_1-M_1-P_1-E_1) - (r_1r_2-1)Io + (r_2-1)I_1$

Noting that $Fo - Mo = No + Io$ and $M_1 = Mo+S_1-(P_1+A_1-E_1)$ allows the above expression to be restated as

$$F_2-r_1r_2Fo + (r_1r_2-1)(No+Io-Do) + (r_2-1)(A_1+I_1) - (r_1r_2-1)Io$$

Noting that $N_1 = No + A_1 - Do$ allows the above expression to be restated as

$$F_2 - r_1r_2Fo + (r_1r_2-1)(No-Do) + (r_2-1)(N_1-(No-Do)+I_1)$$

Adding and subtracting Fo gives

$$(F_2-Fo) + (r_1r_2-1)(No-Do) + (r_2-1)(N_1-(No-Do)+I_1) - (r_1r_2-1)F_o$$

It follows that

$$(F_2-Fo) - (r_1r_2-1)Mo-(r_2-1)(M_1-Mo) + (r_2-1)(S_1+I_1-P_1-E_1) - (r_1r_2-1).(Io+Do)$$

$$=$$

$$(F_2-Fo) - (r_1r_2-1)Fo + (r_1r_2-1).(No-Do) + (r_2-1)(N_1-(No-Do)+I_1)$$

From which it follows that $(r_1r_2-1)Fo$ is equivalent to

$$(r_1r_2-1)Mo + (r_2-1)(M_1-Mo) - (r_2-1)(S_1+I_1-P_1-E_1) + (r_1r_2-1)(Io+Do) + (r_1r_2-1)(No-Do) + (r_2-1)(N_1-(No-Do)+I_1)$$

This expression is in turn equivalent to

$$(r_1r_2-1)Mo + (r_2-1)(M_1-Mo) - (r_2-1)S_1+(r_2-1)E_1+(r_2-1)P_1 + (r_1r_2-1)Io + (r_1r_2-1)No + (r_2-1)(N_1-(No-Do))$$

Our objective is to show that this expression is the fictional realizable cost savings of the interval $[0,2]$. The fictional realizable cost savings accruing on stock is computed using the first fundamental theorem (Appendix 4A)

$$\Psi = Io + P_1 - (r_1 r_2 Io + r_2 P_1 - r_2 I_1)$$

$$\Psi = r_2 I_1 - \left[(r_2 - 1)P_1 + (r_1 r_2 - 1)Io\right]$$

$$\xi = r_2 I_1$$

$$\xi - \Psi = (r_2 - 1)P_1 + (r_1 r_2 - 1)Io$$

Similarly, the fictional realizable cost savings accruing on the non-monetary items (excluding stock) amount to

$$\Psi = No + A_1 - r_1 r_2 Do$$

$$\Psi = No + N_1 - (No - Do) - r_1 r_2 Do$$

$$\xi = r_1 r_2 (No - Do) + r_2 \left[N_1 - (No - Do)\right]$$

$$\xi - \Psi = (r_1 r_2 - 1)No + (r_2 - 1)\left[N_1 - (No - Do)\right]$$

Note that the expression for the fictional realizable cost savings on stock and non-monetary items (excluding stock) appear in the expression for $(r_1 r_2 - 1)Fo$. The other components of this quantity are the loss from holding net monetary items and the adjustments to sales and expense respectively. This completes the proof.

CHAPTER FIVE

A PROBLEM IN ACCOUNTING MEASUREMENT: ESTIMATION OF THE REPLACEMENT COST OF ASSET DISPOSALS

5.0 Introduction

In the previous chapter the Edwards and Bell method of accounting was presented as a general model of accounting measurement in the sense that it could be meaningfully adapted and applied to any of the several accounting measurement systems. It will be recalled that the model is grounded on two theorems, the first and more important of which provides a means for computing the (potentially) realizable "holding gains" accruing during some interval of time $[T,T+1]$. This theorem requires, as an input, the accumulated "value" of disposals during $[T,T+1]$, where the term "value" is to be interpreted in the context of the measurement system being utilized. With the exception of replacement cost measurement, this "problem" has proved to be of relatively minor importance.[1] In the case of replacement cost measurement, however, it has proved to be a major obstacle to implementation.[2] For these reasons, the purpose of the present chapter is to examine several methods for estimating the replacement cost of disposals during the interval $[T,T+1]$.

The present chapter, in fact, develops two variations on a theme. The first of these is concerned with the relevance of some polynomial

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1. Net realizable value accounting has at no time been advocated by the professional accounting bodies. In Statement of Standard Accounting Practice No.7, which dealt with C.P.P. adjustments to historic cost figures, the problem was virtually ignored. Only in the following publications was the problem recognised as being of some importance.

   "Current Cost Accounting," Exposure Draft 18, Accounting Standards Committee 1976, pp.85-90.

   Inflation Accounting, Cmnd 6225, HMSO, 1975, pp.179-186.

2. Exposure Draft, loc.cit.
   Inflation Accounting Committee, loc.cit.
   Hamilton, S. "Field Testing ED 18: The Practical Reality," The Accountant's Magazine, LXXXI (May 1977), p.195.

interpolation based numerical techniques to the problem of estimating the replacement cost of disposals. The relevance of these methods to the problem at hand has not been investigated, and yet, on the surface they would seem to hold considerable potential. Having achieved this, we shall then examine two numerical methods which have been hinted at by accountants but whose properties have not been fully investigated. The first, which utilizes the weighted average cost of acquisitions, was introduced by Edwards and Bell.[3] The second, which is basically a simple averaging technique, was alluded to by both Edwards and Bell and the Inflation Accounting Steering Group amongst others.[4]

We now focus our attention on the topic of polynomial interpolation since this provides the background material necessary for an understanding of the polynomial based numerical techniques to be examined in section 5.2.

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3. Edwards, E.O. and P.W. Bell. The Theory and Measurement of Business Income, Berkley, California: The University of California Press, 1961, pp.144-45.

4. Ibid. p.192.
Inflation Accounting Steering Committee. Guidance Manual on Current Cost Accounting. Institute of Chartered Accountants in England and Wales, 1976, pp.85-90.

5.1 Interpolation

Each of the numerical techniques particularized in section 5.2 utilizes an interpolating polynomial to estimate a function defined on an interval $[a, b]$. Suppose we have (n+1) elements denoted by x_i, $i = 0, 1, 2, \ldots, n$ from the domain $[a, b]$ of a real function f which we call quadrature points or nodes,[5] and whose corresponding functional values are $f(x_i)$. A polynomial $P_n(x)$ is said to interpolate $f(x)$ on the nodes x_i if and only if[6]

$$P_n(x_i) = f(x_i) \qquad i = 0, 1, 2, \ldots, n \tag{1}$$

It can be shown that the nth degree polynomial defined on the (n+1) nodes and their corresponding functional values is unique.[7] Further, there are various methods for determining the (n+1) coefficients of the polynomial.[8] One such method, attributed to Lagrange, proceeds by using the equations[9]

$$l_i(x) = \frac{\prod_{\substack{j=0 \\ i \neq j}}^{n} (x - x_j)}{\prod_{\substack{j=0 \\ i \neq j}}^{n} (x_i - x_j)} \tag{2a}$$

$$F_n(x) = \sum_{j=0}^{n} l_j(x) f(x_j) \tag{2b}$$

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5. Isaacson, E. and H.B. Keller, <u>Analysis of Numerical Methods</u>. New York: John Wiley and Sons, Inc., 1966, p.300.

6. Henrici, P. <u>Elements of Numerical Analysis</u>. New York: John Wiley and Sons, Inc., 1964, p.183.

7. <u>Ibid</u>.

8. Isaacson and Keller, <u>op.cit</u>., chapter 6.

9. Henrici, <u>op.cit</u>., p.184.

where

$$f(x) = P_n(x) + e_n(x) \tag{2c}$$

for all x in the interval $[a, b]$ and $e_n(x)$ is the error from approximating $f(x)$ by the interpolating polynomial $P_n(x)$. It can be shown, provided certain assumptions[10] are satisfied, that

$$e_n(x) = \frac{f^{(n+1)}[z(x)]}{(n+1)!} \prod_{j=0}^{n} (x-x_j) \tag{3}$$

where $z(x)$ is located in the smallest interval containing the points $x_0, x_1, \_\_\_, x_n$. However, since $z(x)$ is in general unknown,[11] we can bound the error by letting $M_{n+1} = \max \left| f^{(n+1)}(x) \right|$ for all x in the interval $[a, b]$ whence

$$\left| e_n(x) \right| \leqslant \frac{M_{n+1}}{(n+1)!} \prod_{j=0}^{n} \left| x-x_j \right| \tag{4}$$

Table 5.1 provides an example of the interpolating procedures particularized above.

10. Ibid., p.187.

11. Ibid.

TABLE 5.1

EXAMPLE OF POLYNOMIAL INTERPOLATION

(a) Interpolation

Given that $\log_e 1.0 = 0$, $\log_e 1.1 = 0.09531$ and $\log_e 1.3 = 0.26236$ we determine a second degree interpolating polynomial for the function $f(x) = \log_e x$ on the interval $[1.0, 1.3]$

Lagrangian Coefficients

$$\ell_o(x) = \frac{(x-1.1)(x-1.3)}{(1-1.1)(1-1.3)}$$

$$\ell_o(x) = \frac{x^2-2.4x+1.43}{0.03}$$

$$\ell_1(x) = \frac{(x-1)(x-1.3)}{(1.1-1)(1.1-1.3)}$$

$$\ell_1(x) = \frac{x^2-2.3x+1.3}{-0.02}$$

$$\ell_2(x) = \frac{(x-1)(x-1.1)}{(1.3-1)(1.3-1.1)}$$

$$\ell_2(x) = \frac{x^2-2.1x+1.1}{0.06}$$

Interpolating Polynomial

$$P_2(x) = \sum_{j=0}^{2} \ell_j(x) f(x_j)$$

$$= \frac{0.(x^2-2.4x+1.43)}{0.03} - \frac{0.09531(x^2-2.3x+1.3)}{0.02} + \frac{0.26236(x^2-2.1x+1.1)}{0.06}$$

$$P_2(x) = -0.3928x^2+1.7780x-1.3852$$

(b) Error Bound

Given that $f(x) = \log x$ then $f^{(3)}(x) = \frac{2}{x}3$ and $\max \left| f^3(x) \right|$ on the interval $[1.0, 1.3]$ is 2. We thus have

$$\left| e_2(x) \right| \leqslant \frac{1}{3} \left| x-1 \right| . \left| x-1.1 \right| . \left| x-1.3 \right|$$

where $x \epsilon [1.0, 1.3]$

Recall that the purpose of this section was to provide the background material necessary for an understanding of the numerical integration techniques which may be utilized in estimating the replacement cost of asset disposals. Having furnished this background, we now turn to a consideration of these numerical techniques.

5.2 Numerical Methods

Our objective in this section is to apply some of the commonly encountered numerical integration techniques to the problem of estimating the replacement cost of asset disposals during some interval of time. The relevance of these methods to the problem at hand can be explained in the following terms. Consider the composite function

$$u(t) = r(t).s(t)$$

where $r(t)$ is the function whose value is the replacement cost of a unit of resource at time t and $s(t)$ is the function whose value is the rate of change in accumulated disposals at time t. If $u(t)$ is integrable over the closed interval $[T,T+1]$ then our problem is to evaluate the integral[12]

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12. Define the function $S(t)$ whose value is accumulated disposals (in units) at time t. Let $S(t)$ be monotone increasing on $[T,T+1]$ and define the jth increment of $S(t)$ as t varies from t_{j-1} to t_j accordingly

$$S_j = S(t_j)-S(t_{j-1})$$

for t_j and t_{j-1} in $[T,T+1]$. Suppose the interval $[T,T+1]$ to be partitioned into n subintervals so that in general the replacement cost of disposals during $[t_{j-1},t_j]$ is $r(\xi_j)\Delta S_j$ for some ξ_j in $[t_{j-1},t_j]$. This implies that the replacement cost of disposals during $[T,T+1]$ is

$$c\,[T,T+1] = \sum_{j=1}^{n} r(\xi_j)\Delta S_j$$

If this sum tends to a finite limit as the lengths of the subintervals tend to zero we write

$$c\,[T,T+1] = \int_T^{T+1} r(t)dS(t)$$

and call such a limit the Stieltjes integral. If we further suppose $r(t)$ to be continuous and $S(t)$ to be differentiable on $[T,T+1]$ then it follows

$$c\,[T,T+1] = \int_T^{T+1} r(t)s(t)dt$$

where $\frac{dS(t)}{dt} = s(t)$ is the rate of change in accumulated disposals at time t. On this point, see
Ferrar, W.L. Integral Calculus. Oxford: Oxford University Press, 1958, pp.150-58.

$$c\left[T,T+1\right] = \int_{T}^{T+1} r(t).s(t)dt \tag{5}$$

Where $c\left[T,T+1\right]$ is the replacement cost of asset disposals during the interval $\left[T,T+1\right]$. When the integral in (5) cannot be evaluated analytically, we may resort to any of the several numerical integration techniques alluded to above. Each of these techniques estimates the integral $\int_a^b f(x)dx$ by using the following procedure

$$\int_a^b f(x)dx = \sum_{j=1}^{n} w_j f(x_j) + E$$

where the w_j are a set of weights and E is the error associated with the method. If the technique integrates polynomials of degree m or less exactly, but is not exact for polynomials of higher degree, then it is said to have m degree precision.[13]

We now turn to a consideration of three such methods, namely the midpoint rule, the trapezoidal rule and Simpson's rule.

5.2.1 Midpoint Rule

Suppose we estimate the function f defined on the interval $\left[a, b\right]$ by interpolating on the node $\frac{a+b}{2}$ and its corresponding functional value.[14] This implies

$$\int_a^b f(x)dx = \int_a^b P_0(x)dx + E \tag{6a}$$

$$= \int_a^b f\left[\frac{a+b}{2}\right]dx + E \tag{6b}$$

$$\int_a^b f(x)dx = (b-a)f\left[\frac{a+b}{2}\right] + E \tag{6c}$$

The error[15] from approximating the integral of f(x) by the integral of the constant interpolating polynomial is obtained from the equation

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13. Isaacson and Keller, op.cit., p.301.

14. Ibid., p.316

15. Ibid.

$$E = \frac{f^{(2)}[z(x)]}{24}(b-a)^3 \tag{7a}$$

for some z(x) in the interval $[a, b]$. As z(x) is in general unknown[16] we can bound the error by letting $M_2 = \max\left|f^{(2)}(x)\right|$ for all x in the interval $[a, b]$ whence

$$|E| \leqslant \frac{M_2(b-a)^3}{24} \tag{7b}$$

This rule has one degree precision and thus integrates polynomials of degree one and zero exactly. Further, the method can be used to estimate the replacement cost of disposals in the interval $[T, T+1]$ by replacing f(t) by the composite function u(t) = r(t).s(t). This implies

$$\int_T^{T+1} r(t).s(t)dt = \int_T^{T+1} P_0(t)dt + E \tag{8a}$$

$$= \int_T^{T+1} u\left[\frac{2T+1}{2}\right]d + E \tag{8b}$$

$$\int_T^{T+1} r(t).s(t)dt = r\left[\frac{2T+1}{2}\right]s\left[\frac{2T+1}{2}\right] + E \tag{8c}$$

where $E = \frac{u^{(2)}[z(t)]}{24}$ (9a)

for some z(t) in the interval $[T, T+1]$. A bound on the error is given by

$$|E| \leqslant \frac{M_2}{24} \tag{9b}$$

The above method of estimating the integral of f(x) and u(t) is called the midpoint rule.[17] However, the more commonly encountered numerical methods are the Trapezoidal rule and Simpson's rule.

5.2.2 Trapezoidal Rule

The Trapezoidal rule approximates the function f(x) by interpolating a linear polynomial $P_1(x)$ on the nodes a and b and their corresponding functional values.[18] This implies

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16. Ibid.

17. Ibid.

18. Ibid.

$$\int_a^b f(x)dx = \int_a^b P_1(x)dx + E \tag{10a}$$

$$= \int_a^b \sum_{j=0}^{1} l_j(x)f(x_j)dx + E \tag{10b}$$

$$\int_a^b f(x)dx = \frac{(b-a)}{2}\left[f(a) + f(b)\right] + E \tag{10c}$$

The error[19] from approximating f(x) by the integral of the linear interpolating polynomial is obtained from the equation

$$E = \frac{-f^{(2)}\left[z(x)\right]}{12}(b-a)^3 \tag{11a}$$

for some z(x) in the interval $\left[a, b\right]$. As z(x) is in general unknown[20] we can bound the error in the same fashion as the midpoint rule

$$\left|E\right| \leqslant \frac{M_2}{12}(b-a)^3 \tag{11b}$$

Like the midpoint rule the trapezoidal rule has one degree precision. Further, the trapezoidal rule can be used to estimate the replacement cost of disposals in the interval $\left[T,T+1\right]$ by replacing f(t) by u(t). This implies

$$\int_T^{T+1} r(t).s(t)dt = \int_T^{T+1} P_1(t)dt + E \tag{12a}$$

$$= \int_T^{T+1} \sum_{j=0}^{1} l_j(t)u(t_j)dt + E \tag{12b}$$

$$\int_T^{T+1} r(t).s(t)dt = \tfrac{1}{2}\left[u(T) + u(T+1)\right] + E \tag{12c}$$

where

$$E = \frac{-u^{(2)}\left[z(t)\right]}{12} \tag{13a}$$

for some z(t) in the interval $\left[T,T+1\right]$. A bound on the error is given by

$$\left|E\right| \leqslant \frac{M_2}{12} \tag{13b}$$

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19. Ibid.

20. Ibid.

5.2.3 Simpson's Rule

Simpson's rule approximates the function $f(x)$ by interpolating a quadratic polynomial $P_2(x)$ on the nodes a, $\frac{a+b}{2}$ and b and their corresponding functional values.[21] This implies

$$\int_a^b f(x)dx = \int_a^b P_2(x)dx + E \tag{14a}$$

$$= \int_a^b \sum_{j=0}^{2} l_j(x)f(x_j)dx + E \tag{14b}$$

$$\int_a^b f(x)dx = \frac{(b-a)}{6}f\left[(a) + 4f\left[\frac{a+b}{2}\right] + f(b)\right] + E \tag{14c}$$

The error[22] from approximating the integral is obtained from the equation

$$E = \frac{-f^{(4)}\left[z(x)\right]}{2880}(b-a)^5 \tag{15a}$$

for some unknown[23] $z(x)$ in the interval $\left[a, b\right]$. A bound on the error is given by

$$\left| E \right| \leqslant \frac{M_4}{2880}(b-a)^5 \tag{15b}$$

where $M_4 = \max \left| f^4(x) \right|$ for all x in the interval $\left[a, b\right]$. This rule has 3 degree precision since it integrates polynomials of degree three or less exactly. In addition, like the trapezoidal and midpoint rules, the method can be used to approximate the replacement cost of disposals in the interval $\left[T, T+1\right]$ by replacing $f(t)$ by $u(t)$. This implies

$$\int_T^{T+1} r(t).s(t)dt = \int_T^{T+1} P_2(t)dt + E \tag{16a}$$

$$= \int_T^{T+1} \sum_{j=0}^{2} l_j(t)u(t_j)dt + E \tag{16b}$$

$$\int_T^{T+1} r(t).s(t)dt = \frac{1}{6}\left[u(T) + 4u\left[\frac{2T+1}{2}\right] + u(T+1)\right] + E \tag{16c}$$

where

$$E = \frac{-u^{(4)}\left[z(t)\right]}{2880} \tag{17a}$$

for some $z(t)$ in the interval $\left[T, T+1\right]$. A bound on the error is given

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21. <u>Ibid</u>.

22. <u>Ibid</u>.

23. <u>Ibid</u>.

by

$$\left| E \right| \leq \frac{M_4}{2880} \tag{17b}$$

5.2.4 An Example

As an example of the implementation of the above procedures assume the functions

$$r(t) = le^{kt} \tag{18a}$$

and[24]

$$s(t) = m \tag{18b}$$

are defined on the interval $\left[0, 1\right]$. It then follows that the replacement cost of disposals in the interval $\left[0, 1\right]$ is computed thus

$$c\left[0, 1\right] = \int_0^1 r(t).s(t)dt \tag{19a}$$

$$= lm\int_0^1 e^{kt}dt \tag{19b}$$

$$c\left[0, 1\right] = \frac{lm}{k}\left[e^k - 1\right] \tag{19c}$$

We call the absolute error expressed as a fraction of the replacement cost of disposals c $\left[0, 1\right]$ the relative error R where

$$R = \frac{\left| E \right|}{c\left[0, 1\right]} \tag{20}$$

This expression gives a better basis for gauging the accuracy of our calculations than $\left| E \right|$ because it relates the error to the quantity being estimated.

In Tables 5.2 and 5.3 we apply each of the numerical methods particularized above to estimate the integral (19b) under the assumption l = m = 1 and for various positive values of k. Column 1 of each method in Table 5.3 contains the estimate of (19b) obtained from applying

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24. This assumption implies that the accumulated disposals at time t is described by the function $S(t) = mt$, since $\frac{dS(t)}{dt} = s(t) = m$. See footnote 12, above.

the rule, column 2 contains the actual error, column 3 contains the error bound, whilst column 4 contains the maximum relative error. The Fortran program from which the figures in Table 5.3 are generated is contained in Appendix 5A.[25]

5.3 Some Alternative Methods

A problem in implementing each of the methods specified above is that the precise form of the functions r(t) and s(t) are unknown.[26]

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25. The accuracy of the above methods can be increased by splitting the interval $[T,T+1]$ into n equally spaced subintervals and then applying the method to each subinterval. For the midpoint rule, for example, the error of the jth subinterval is

$$E_j = \frac{1}{24^3} u^{(2)}[z_j(t)]$$

where $z_j(t)$ is bounded in the interval $\left[T + \frac{j-1}{n}, T + \frac{j}{n}\right]$. It follows that the total error is given by

$$E_j = \frac{1}{24n^3} \sum_{j=1}^{n} u^{(2)}[z_j(t)]$$

$$E = \frac{1}{24n^2} u^{(2)}[\bar{z}(t)].$$

for some $\bar{z}(t)$ in the interval $[T,T+1]$. This result implies that a sufficient condition for convergence is that $u^{(2)}(t)$ is bounded on $[T,T+1]$. Similar procedures apply to the other rules. The error from applying the trapezoidal rule to the n equally spaced subintervals of $[T,T+1]$ is

$$E = \frac{-1}{12n^2} u^{(2)}[\bar{z}(t)]$$

whilst for Simpson's rule the error is

$$E = \frac{-1}{2880n^4} u^{(4)}[\bar{z}(t)]$$

26. In general it is not necessary to know the precise form of u(t) in order to operationalize the above procedures. Estimates of (5) can be obtained from a finite set of points $[t_j, u(t_j)]$ but knowledge of $u^{(n)}(t)$ is necessary to bound the error of the estimate.

TABLE 5.2

NUMERICAL METHODS APPLIED

Midpoint Rule

$$c\,[0,\,1] = e^{\frac{k}{2}} + E$$

where

$$|E| \leqslant \frac{k^2 e^k}{24}$$

and

$$R = \frac{k^3}{24} \cdot \frac{e^k}{e^k - 1}$$

Trapezoidal Rule

$$c[0,\,1] = \tfrac{1}{2}\left[1 + e^k\right] + E$$

where

$$|E| \leqslant \frac{k^2 e^k}{12}$$

and

$$R = \frac{k^3}{12} \cdot \frac{e^k}{e^k - 1}$$

Simpson's Rule

$$c[0,\,1] = \frac{1}{6}\left[1 + 4e^{\frac{k}{2}} + e^k\right] + E$$

where

$$|E| \leqslant \frac{k^4 e^k}{2880}$$

and

$$R = \frac{k^5}{2880} \cdot \frac{e^k}{e^k - 1}$$

TABLE 5.3

EXAMPLE OF NUMERICAL METHODS

		Midpoint				Trapezoidal				Simpson			
k	C [0,1]	Estimate	Actual Error	Error Bound	Relative Error	Estimate	Actual Error	Error Bound	Relative Error	Estimate	Actual Error	Error Bound	Relative Error
0.00	1.0000	1.0000	0.0000	0.0000	0.0000	1.0000	0.0000	0.0000	0.0000	1.0000	0.0000	0.0000	0.0000
0.05	1.0254	1.0253	0.0001	0.0001	0.0001	1.0256	0.0002	0.0002	0.0002	1.0254	0.0000	0.0000	0.0000
0.10	1.0517	1.0513	0.0004	0.0005	0.0004	1.0526	0.0009	0.0009	0.0009	1.0517	0.0000	0.0000	0.0000
0.15	1.0789	1.0779	0.0010	0.0011	0.0010	1.0809	0.0020	0.0022	0.0020	1.0789	0.0000	0.0000	0.0000
0.20	1.1070	1.1052	0.0018	0.0020	0.0018	1.1107	0.0037	0.0041	0.0037	1.1070	0.0000	0.0000	0.0000
0.25	1.1361	1.1331	0.0030	0.0035	0.0029	1.1420	0.0059	0.0067	0.0059	1.1361	0.0000	0.0000	0.0000
0.30	1.1662	1.1618	0.0044	0.0051	0.0043	1.1749	0.0087	0.0101	0.0087	1.1662	0.0000	0.0000	0.0000
0.35	1.1973	1.1912	0.0061	0.0072	0.0060	1.2095	0.0122	0.0145	0.0121	1.1974	0.0000	0.0000	0.0000
0.40	1.2296	1.2214	0.0082	0.0099	0.0081	1.2459	0.0164	0.0199	0.0162	1.2296	0.0000	0.0000	0.0000
0.45	1.2629	1.2523	0.0106	0.0132	0.0105	1.2842	0.0212	0.0265	0.0210	1.2630	0.0000	0.0000	0.0000
0.50	1.2974	1.2840	0.0134	0.0172	0.0132	1.3244	0.0269	0.0343	0.0265	1.2975	0.0001	0.0001	0.0000

However, in general for $r(t)$ we do know a set of nodes and their corresponding functional values whilst sales (in units) of the interval $[T,T+1]$ are also known. Hence, if a numerical method is to be of practical significance, it must be capable of providing estimates of $C[T,T+1]$ and its error using only this information set.

In this section we shall consider two such methods. The first of these was stated by Edwards and Bell but only proved under a redundant set of assumptions.[27] Further, its properties were not fully investigated by its authors. The second method is original and, so far as we are aware, has not been advanced in the form in which it is presented.

5.3.1 The Edwards and Bell Method

To derive the Edwards and Bell method, suppose disposals to occur at the points t_1, t_2, ____, t_m in $[T,T+1]$ and impose the following assumption

> There is a constant ratio of acquisitions to disposals at each "disposal point" in $[T,T+1]$.

Using this assumption we now prove the following result

> The replacement cost of disposals during $[T,T+1]$ is the weighted average acquisition cost during $[T,T+1]$ multiplied by unit disposals during $[T,T+1]$

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27. The method was proved under the following redundant assumptions

(i) Acquisitions are described by the function

$$B(t) = ke^{rt}$$

where k is the constant ratio of purchases to sales, and

(ii) Acquisition cost (per unit) is described by the function

$$R(t) = e^{pt}$$

On this point, see Edwards and Bell, op.cit., pp.144-145.

To prove this result, suppose $r(t_j)$ to be the replacement cost (per unit) of resource at time t_j and $s(t_j)$ to be disposals (in units) also at time t_j. It follows that the replacement cost of disposals at time t_j is given by

$$c(t_j) = r(t_j).s(t_j) \tag{21a}$$

whilst the replacement cost of disposals during $[T,T+1]$ amount to

$$c\,[T,T+1] = \sum_{j=1}^{m} c(t_j) \tag{21b}$$

$$c\,[T,T+1] = \sum_{j=1}^{m} r(t_j).s(t_j) \tag{21c}$$

To prove the proposition we must show that the method defined in the conclusion yields the above result. We thus define the weighted average purchase price during the interval $[T,T+1]$ in the following terms

$$w\,[T,T+1] = \frac{\sum_{j=1}^{m} r(t_j).a(t_j)}{\sum_{j=1}^{m} a(t_j)} \tag{22a}$$

where $a(t_j)$ is acquisitions (in units) at time t_j. By hypothesis we have

$$a(t_j) = \ell\, s(t_j) \tag{22b}$$

where ℓ is the (constant) ratio of acquisitions to disposals at time t_j. This assumption implies

$$w\,[T,T+1] = \frac{\sum_{j=1}^{m} r(t_j).\ell s(t_j)}{\sum_{j=1}^{m} \ell s(t_j)} \tag{22c}$$

$$w\,[T,T+1] = \frac{\sum_{j=1}^{m} r(t_j).s(t_j)}{\sum_{j=1}^{m} s(t_j)} \tag{22d}$$

Using equation (21c) the above expression may be restated as

$$c\left[T,T+1\right] = w\left[T,T+1\right] \sum_{j=1}^{m} s(t_j) \qquad (22e)$$

thus proving the result. In Appendix 5B to this chapter we compute the realizable cost savings implied by this result.[28]

Table 5.4 applies the above result to the data of the Best Company using the assumption that the ratio of acquisitions to disposals in the year ending December 31, 1909 is 0.8. The corresponding realizable cost savings are computed in Appendix 5B.

When the assumptions employed in deriving the above result are not satisfied, it can still be applied as a means of <u>estimating</u> the replacement cost of disposals, although we are then confronted with the problem of ascertaining the magnitude of the error involved. To obtain an expression for the error of this estimating procedure, expand r as a Taylor series about the point t=w in which case we have[29]

$$r(t_j) = r(w) + (t_j - w)r^1(\xi_j) \qquad (23)$$

where w is a point in the interval $\left[T,T+1\right]$[30] such that $r(w) = w\left[T,T+1\right]$

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28. The results proved in Appendix 5B were initially proved by Edwards and Bell using the redundant set of assumptions specified in footnote 27.
Edwards and Bell, <u>op cit</u>., pp.146-148.

29. In expanding r(t) as a Taylor series we impose the assumption that $r^{(1)}(t)$ is defined on the interval $\left[T,T+1\right]$
Giles, J.R. <u>Real Analysis</u>. Sydney: John Wiley and Sons, Australia Pty. Ltd., p.86.

30. Imposing the condition that r(t) is a monotone increasing mapping guarantees this result. See
<u>Ibid</u>., pp.62-63.

TABLE 5.4

STOCK EXAMPLE

(a) Acquisitions and Disposals

The Best Company Limited accounts for stock by the perpetual FIFO method. The following data relate to the year ending December 31, 1909.

1909	
Jan 1	Stock on hand 1,000 units with an historic cost of £10 (per unit). Replacement cost is £11 (per unit). Sold 500 units. Purchased 400 units at £11 (per unit).
Mar 1	Sold 800 units. Replacement cost is £12 (per unit). Purchased 640 units at £12 (per unit).
May 1	Sold 600 units. Replacement cost is £13 (per unit). Purchased 480 units at £13 (per unit).
July 1	Sold 200 units. Replacement cost is £14 (per unit). Purchased 160 units at £14 (per unit).
Sept 1	Sold 300 units. Replacement cost is £15 (per unit). Purchased 240 units at £15 (per unit).
Nov 1	Sold 500 units. Replacement cost is £16 (per unit). Purchased 400 units at £16 (per unit).
Dec 31	Replacement cost is £20 (per unit).

(b) Direct Calculation of Replacement Cost of Disposals

$$c\left[0,\ T\right] = \sum_{j=1}^{6} r(t_j)S(t_j)$$

$$= (500 \times 11)+(800 \times 12)+(600 \times 13)+(200 \times 14)$$

$$+(300 \times 15)+(500 \times 16)$$

$$c\left[0,\ T\right] = 38{,}200$$

(c) <u>Theorem Calculation of Replacement Cost of Disposals</u>

$$W\left[0,\ T\right] = \frac{\sum_{j=1}^{6} r(t_j)a(t_j)}{\sum_{j=1}^{6} a(t_j)}$$

$$= \frac{(400\times 11)+(640\times 12)+(480\times 13)+(160\times 14)+(240\times 15)+(400\times 16)}{(400 + 640 + 480 + 160 + 240 + 400)}$$

$$= \frac{30{,}560}{2{,}320}$$

$$W\left[0,\ T\right] = 13.1724$$

$$c\left[0,\ T\right] = W\left[0,\ T\right].\ \sum_{j=1}^{6} S(t_j)$$

$$= 13.1724 \times 2900$$

$$c\left[0,\ T\right] = 38{,}200$$

and ξ_j is an unknown number bounded in the interval $[T, T+1]$.

Substituting equation (23) into equation (21c) allows the expression for the replacement cost of disposals to be restated as

$$c\,[T,T+1] = \sum_{j=1}^{m}\left[r(w) + (t_j - w)r^1(\xi_j)\right] s(t_j) \tag{24a}$$

$$= r(w) \sum_{j=1}^{m} s(t_j) + \sum_{j=1}^{m}(t_j - w)r^1(\xi_j)s(t_j) \tag{24b}$$

$$c\,[T,T+1] = w\,[T,T+1] \sum_{j=1}^{m} s(t_j) + E \tag{24c}$$

It follows that the error associated with this estimation technique may be expressed as

$$E = \sum_{j=1}^{m}(t_j - w)r^1(\xi_j)s(t_j) \tag{25a}$$

Letting $M_1 = \max |r^1(t)|$ on the interval, $[T,T+1]$ and given that $|t_j - w| \leqslant 1$[31] it then follows that a bound for E is given by the following expression

$$|E| \leqslant M_1 \sum_{j=1}^{m} s(t_j) \tag{25b}$$

Empirical research has shown this formulation to yield poor approximations under a wide class of circumstances.[32] The accuracy of the method may be increased, however, by partitioning the interval $[T,T+1]$ into several subintervals and then applying the method to each. To illustrate, suppose the interval $[T,T+1]$ to be subdivided into $n \leqslant m$ subintervals $[t_{j(k-1)}, t_{j(k)}]$ k = 1, 2, ____, n where $t_{j(o)} = t_o = T$ and $t_{j(n)} = t_m = (T+1)$. The contribution of the interval $[t_{j(k-1)}, t_{j(k)}]$ to the replacement cost of disposals c $[T,T+1]$ is $r(w_k) \sum_{j=j(k-1)+1}^{j(k)} s(t_j) + E_k$ where w_k is a point in the interval $[t_{j(k-1)}, t_{j(k)}]$ such that $r(w_k) = w\,[t_{j(k-1)}, t_{j(k)}]$,

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31. This result follows from the assumption that r is monotone increasing.

32. See any of the following
Dickerson, P.J. Business Income - A Critical Analysis. Berkley, California: University of California Press, 1965, p.9.
Chambers, R.J. "Edwards and Bell on Business Income," The Accounting Review, XL, 4 (October 1965), pp.737-738.
Benjamin, J. "The Accuracy of the Period-End Method for Computing the Current Cost of Materials Used," Abacus, 9, 1 (June 1973), pp.73-80.

$\sum_{j=j(k-1)+1}^{j(k)} s(t_j)$ is accumulated disposals over the interval $[t_{j(k-1)}, t_{j(k)}]$ and E_k is the error from applying the technique to the interval $[t_{j(k-1)}, t_{j(k)}]$. It thus follows that the replacement cost of disposals during $[T, T+1]$ amounts to

$$c[T,T+1] = \sum_{k=1}^{n} \left[r(w_k) \sum_{j=j(k-1)+1}^{j(k)} s(t_j) \right] + E_k \tag{26a}$$

$$c[T,T+1] = \sum_{k=1}^{n} \sum_{j=j(k-1)+1}^{j(k)} r(w_k) s(t_j) + \sum_{k=1}^{n} E_k \tag{26b}$$

where $E = \sum_{k=1}^{n} E_k$ is the error from applying the method to the interval $[T, T+1]$. To bound this error, note that

$$E_k = \sum_{j=j(k-1)+1}^{j(k)} (t_j - w_k) r^1(\xi_j) s(t_j) \tag{27a}$$

is the error associated with applying the technique to the interval $[t_{j(k-1)}, t_{j(k)}]$. If we suppose the intervals to be of equal length, it follows that $|t_j - w_k| \leq \frac{1}{n}$ [33] in which case we have

$$|E_k| \leq \frac{M_1}{n} \sum_{j=j(k-1)+1}^{j(k)} s(t_j) \tag{27b}$$

where it will be recalled $M_1 = \max |r^1(t)|$ on the interval $[T, T+1]$. By virtue of this result and the triangle inequality we have[34]

$$|E| \leq \sum_{k=1}^{n} |E_k| \tag{27c}$$

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33. See footnotes 30 and 31 above.

34. Noting that

$$E = \sum_{k=1}^{n} E_k$$

it follows from the triangle inequality that

$$\left| \sum_{k=1}^{n} E_k \right| \leq \sum_{k=1}^{n} |E_k|$$

or that

$$|E| \leq \sum_{k=1}^{n} |E_k|$$

Giles, <u>op.cit.</u>, p.8

$$\leqslant \sum_{k=1}^{n} \frac{M_1}{n} \sum_{j=j(k-1)+1}^{j(k)} s(t_j) \tag{27d}$$

$$\left| E \right| \leqslant \frac{M_1}{n} \sum_{j=1}^{m} s(t_j) \tag{27e}$$

This result implies that the absolute error can be reduced to any desired level by merely increasing the number of intervals utilized. To illustrate, suppose E* is set as an "acceptable" absolute error. In words, we must have

$$\left| E \right| \leqslant E^* \tag{28a}$$

which is achieved when

$$E^* \geqslant \frac{M_1}{n} \sum_{j=1}^{m} s(t_j) \tag{28b}$$

$$n \geqslant \frac{M_1}{E^*} \sum_{j=1}^{m} s(t_j) \tag{28c}$$

where n is the number of "quadrature points" necessary to guarantee an absolute error of E* or less. Although M_1 will seldom be known, note that a sufficient condition for convergence is that the function r^1 be bounded on the interval $[T,T+1]$. In practical terms, this means that as n increases, the method converges to the actual replacement cost of disposals during $[T,T+1]$ if the rate of increase in price has an upper bound on the interval $[T,T+1]$.

We now focus on a second method which we have chosen to call the modified midpoint rule. The reason for this is that estimates of the replacement cost of disposals are obtained by evaluating r(t) at the midpoint of each interval analyzed.

5.3.2. A Modified Midpoint Rule

The Edwards and Bell technique for estimating the replacement cost of disposals during the interval $[T,T+1]$ is suggestive of a simpler procedure which has been alluded to by a variety of authors.[35] It may be broadly

35. Exposure Draft 18, loc.cit.
Inflation Accounting Committee, loc.cit.
Edwards and Bell, op.cit., p.192.

described as the "averaging technique" and is applied by costing disposals at the midyear replacement cost. In terms of the notation employed to date, it may be stated as follows

$$c\left[T,T+1\right] = r\left(\frac{2T+1}{2}\right)\sum_{j=1}^{m} s(t_j) \qquad (29)$$

where $r(\frac{2T+1}{2})$ is the replacement cost (per unit) at the midpoint of the interval $\left[T,T+1\right]$. To illustrate the mechanics of the method, consider the data of Table 5.4. The replacement cost (per unit) as of July 1, 1909 is £14, where July 1 is the midpoint (in time) of the year ending December 31, 1909. Since disposals amount to 2,900 units, we estimate their replacement cost at (14 x 2,900) or £40,600. It will be recalled from Table 5.4(b) that the actual replacement cost of disposals amounts to £38,200 and it would seem, therefore, that the method offers a simple but reasonably accurate means of estimating the replacement cost of disposals. We proceed, therefore, to investigate its properties in further detail.

To obtain an expression for the error of this estimating procedure, expand r as a Taylor series about the point $t = \frac{2T+1}{2}$ in which case we have[36]

$$r(t_j) = r\left(\frac{2T+1}{2}\right) + \left(t_j - \frac{2T+1}{2}\right)r^1(\xi_j) \qquad (30)$$

where ξ_j is an unknown real constant bounded in the interval $\left[T,T+1\right]$.

Substituting (30) into 21(c) allows the expression for the replacement cost of disposals to be restated as

$$c\left[T,T+1\right] = \sum_{j=1}^{m}\left[r\left(\frac{2T+1}{2}\right) + \left(t_j - \frac{2T+1}{2}\right)r^1(\xi_j)\right]s(t_j) \qquad (31a)$$

$$(31b)$$

$$= r\left(\frac{2T+1}{2}\right)\sum_{j=1}^{m} s(t_j) + \sum_{j=1}^{m}\left(t_j - \frac{2T+1}{2}\right)r^1(\xi_j)s(t_j)$$

$$c\left[T,T+1\right] = r\left(\frac{2T+1}{2}\right)\sum_{j=1}^{m} s(t_j) + E \qquad (31c)$$

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36. See footnote 29.

It follows that the error associated with this estimation technique may be expressed as

$$E = \sum_{j=1}^{m} (t_j - \frac{2T+1}{2}) r^1(\xi_j) s(t_j) \qquad (32a)$$

Recalling that $M_1 = \max |r^1(t)|$ on the interval $[T,T+1]$ and noting that $|t_j - \frac{2T+1}{2}| \leq \frac{1}{2}$ implies that the error is bounded as follows

$$|E| \leq \frac{M_1}{2} \sum_{j=1}^{m} s(t_j) \qquad (32b)$$

Like the Edwards and Bell technique, the accuracy of this method can be increased by partitioning the interval $[T,T+1]$ into several subintervals and then applying the method to each subinterval. Thus, suppose the interval $[T,T+1]$ to be subdivided into $n \leq m$ subintervals $[t_{j(k-1)}, t_{j(k)}]$ $k = 1, 2,$ ____, n where $t_{j(o)} = t_o = T$ and $t_{j(n)} = t_m = T + 1$. The contribution of the interval $[t_{j(k-1)}, t_{j(k)}]$ to the replacement cost of disposals $c\,[T,T+1]$ is $r(\frac{t_{j(k-1)} + t_{j(k)}}{2}) \sum_{j=j(k-1)+1}^{j(k)} s(t_j) + E_k$ where $(\frac{t_{j(k-1)} + t_{j(k)}}{2})$ is the midpoint of the interval $[t_{j(k-1)}, t_{j(k)}]$, $\sum_{j=j(k-1)+1}^{j(k)} s(t_j)$ is accumulated disposals over the interval $[t_{j(k-1)}, t_{j(k)}]$ and E_k is the error from applying the technique to the interval $[t_{j(k-1)}, t_{j(k)}]$. It thus follows that the replacement cost of disposals during $[T,T+1]$ amounts to

$$c\,[T,T+1] = \sum_{k=1}^{n}\left[r\left(\frac{t_{j(k-1)}+t_{j(k)}}{2}\right)\sum_{j=j(k-1)+1}^{j(k)} s(t_j) + E_k\right] \qquad (33a)$$

$$c\,[T,T+1] = \sum_{k=1}^{n}\ \sum_{j=j(k-1)+1}^{j(k)} r\left(\frac{t_{j(k-1)}+t_{j(k)}}{2}\right)s(t_j) + \sum_{k=1}^{n} E_k \qquad (33b)$$

where $E = \sum_{k=1}^{n} E_k$ is the error from applying the method to the interval $[T,T+1]$. To bound this error note that

$$E_k = \sum_{j=j(k-1)+1}^{j(k)} \left(t_j - \frac{t_{j(k-1)}+t_{j(k)}}{2}\right) r^1(\xi_j)\,s(t_j) \qquad (34a)$$

is the error associated with applying the technique to the interval $[t_{j(k-1)}, t_{j(k)}]$. If we suppose the intervals to be of equal length, it follows that $\left|t_j - \frac{t_{j(k-1)}+t_{j(k)}}{2}\right| = \left|t_j - \left(T + \frac{2k-1}{2n}\right)\right| \leqslant \frac{1}{2n}$ in which case we have

$$|E_k| \leqslant \frac{M_1}{2n} \sum_{j=j(k-1)+1}^{j(k)} s(t_j) \qquad (34b)$$

By virtue of this result and the triangle inequality we have[37]

$$|E| \leqslant \sum_{k=1}^{n} |E_k| \tag{34c}$$

$$\leqslant \sum_{k=1}^{n} \frac{M_1}{2n} \sum_{j=j(k-1)+1}^{j(k)} s(t_j) \tag{34d}$$

$$|E| \leqslant \frac{M_1}{2n} \sum_{j=1}^{m} s(t_j) \tag{34e}$$

This result implies that the absolute error can be reduced to any desired level by merely increasing the number of intervals utilized. Thus, supposing E* is set as an "acceptable" absolute error, we have

$$E^* \geqslant \frac{M_1}{2n} \sum_{j=1}^{m} s(t_j) \tag{35a}$$

$$n \geqslant \tfrac{1}{2} \frac{M_1}{E^*} \sum_{j=1}^{n} s(t_j) \tag{35b}$$

where n is the number of "quadrature points" necessary to guarantee an absolute error of E* or less. As in the case of the Edwards and Bell technique, a sufficient condition for convergence is that r^1 be bounded on the interval $[T,T+1]$. Unlike the Edwards and Bell technique, however, this method has not been subjected to empirical testing. In the next section we undertake to rectify this situation.

5.3.3. A Simulated Test

The analysis of the previous section indicates that the modified midpoint rule is likely to be accurate under a fairly wide class of circumstances. Specifically, if the rate of change in inventory price is bounded, then the error associated with the method is bounded and, indeed, can be reduced to any "acceptable" level by merely increasing the number of quadrature points. Since, however, $r^1(t)$ can only be estimated,[38] we are only enabled to "approximate" an upper bound

37. See footnote 34.

38. To operationalize the error bound implied by (25b), (27e), (32b) and (34e) requires a complete specification of the function $r^1(t)$ on the interval $[T,T+1]$. Since, in general, $r^1(t)$ is unknown on this interval, it is not possible to obtain such a bounding.

for the error, and there is no guarantee that the approximation will be accurate. Thus, in this section we undertake to test more rigorously the accuracy of the proposed method.

The method of testing was to simulate a daily inventory price and quantity series, assuming a 256 day year.[39] The daily price series was generated by the following process

$$r(t) = a + bt + e(t) \qquad (36)$$

where t is time and is bounded in the interval $[0, 1]$,[40] $r(t)$ is price at time t (the daily price) and a and b are parameters denoting respectively inventory price at $t = 0$ and the rate of change in inventory price; that is, $r^{1}(t) = b$ for all t in $[0, 1]$. $e(t)$ is a random variable having a normal frequency function with mean μ and variance σ^2. For testing purposes it was assumed $a = 10$ whilst b was allowed to vary in increments of 0.5 over the interval $[0, 10]$. The values attributed to b imply an inflation rate

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38. Continued
However, by estimating $r^{1}(t)$ at the nodes of the interval by some numerical technique we can obtain an "idea" of the error involved. One such numerical method is the centred difference approximation of the first derivative

$$r^{1}(t) = \frac{r(t+h) - r(t-h)}{2h} + E$$

where the error is computed from the equation

$$E = -\frac{1}{6}h^2 r^{(3)}\left[z(t)\right]$$

for some unknown $z(t)$ in the interval $[t-h, t+h]$. If we ignore this error term and approximate M_1 by the maximum of $r^{1}(t)$ on the set of nodes, it is possible to obtain a "crude" approximation of the maximum absolute error on the interval $[T, T+1]$. See Isaacson and Keller, op cit., p.293

39. A 256 day year was chosen because it facilitated the programming without, at the same time, abstracting from the generality of the results obtained.

40. Thus at day t in the year, a fraction $\frac{t}{256}$ of the year has elapsed.

which varies in multiples of 5%, from 5% to 100% per annum. Finally, it was assumed that $\mu = 0$ and $\sigma = \frac{1}{2}$.

Daily unit sales were generated by assuming the existence of a linear trend with a sinusoidal seasonal term. Specifically, unit sales were generated by each of the following processes

$$s(t) = \alpha + \beta t + e^1(t) \qquad (37a)$$

$$s(t) = \alpha + \beta t + \gamma \sin(2\pi t) + e^1(t) \qquad (37b)$$

$$s(t) = \alpha + \beta t + \gamma \sin(\pi t) + e^1(t) \qquad (37c)$$

$$s(t) = \alpha + \beta t - \gamma \sin(\pi t) + e^1(t) \qquad (37d)$$

The parameters α and β denote respectively unit sales at t = 0 and the rate of change in unit sales in the absence of seasonal elements; that is, $s^1(t) = \beta$ if $\gamma = 0$. The parameter γ is the maximum absolute value of the seasonal factors.[41] Finally, $e^1(t)$ is a random

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41. The trend of a time series is usually estimated by the method of "least squares". In our case, this can be achieved in either of two ways. One method is to define the function

$$L(a, b) = \int_0^1 \left[a + bt - s(t)\right]^2 dt$$

where s(t) is the sales generating function, and minimize L with respect to the parameters a and b. A more convenient method, however, is to define the positive definite inner product space

$$\langle f(t); g(t) \rangle = \int_0^1 f(t).g(t)\, dt$$

and note that $B = \left\{1, \sqrt{12}(t - \frac{1}{2})\right\}$ forms an orthonormal basis for the space of linear functions on the interval $[0, 1]$. The "closest" linear function to s(t) (in the "least squares" sense) is given by $\sum_{j=1}^{2} l_j B_j$ where the l_j are the "fourier co-efficients" with respect to B. As an example, let

$$s(t) = \alpha + \beta t + \gamma \sin(2\pi t)$$

we then have

$$l_1 = \langle 1; \alpha + \beta t + \gamma \sin(2\pi t) \rangle$$

$$= \int_0^1 \alpha + \beta t + \gamma \sin(2\pi t) \quad dt$$

$$l_1 = \alpha + \tfrac{1}{2}\beta$$

$$l_2 = \langle \sqrt{12}(t - \tfrac{1}{2}); \alpha + \beta t + \gamma \sin(2\pi t) \rangle$$

$$= \sqrt{12} \int_0^1 (t - \tfrac{1}{2})(\alpha + \beta t + \gamma \sin(2\pi t))dt$$

$$l_2 = 12\left(\frac{\beta}{12} - \frac{\gamma}{2\pi}\right)$$

We then compute the "least squares" estimate h(t) as follows variable having a normal frequency function with mean μ^1 and variance $\sigma^{1\,2}$. For testing purposes it was assumed $\alpha = 100$, $\beta = 2.5$, $\gamma = 25$, $\mu^1 = 0$ and $\sigma^1 = 5$.

The random variables $e(t)$ and $e^1(t)$ were generated as follows. Suppose r_1 and r_2 to be two uniformly distributed and independent random variables defined on the interval $[0, 1]$. Then, the random variable[42]

$$x = (\sqrt{-2 \log_e r_1}) \cos(2\pi r_2) \qquad (38)$$

has a normal frequency function with zero mean and unit variance. It follows that the random variable $z = x\sigma$ has zero mean and variance σ^2. Values for r_1 and r_2 were generated by calling the function G05AA(Y) from the Edinburgh Regional Computing Centre FORTRAN Compiler. This function generates a sequence of pseudo random numbers from the uniform frequency function defined on the interval $[0, 1]$.

The results of each simulation are contained in Tables 5.5 through 5.8. Each table contains the actual replacement cost of goods

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41. Continued

$$h(t) = \sum_{j=1}^{2} 1_j B_j$$

$$= (\alpha + \tfrac{1}{2}\beta) + 12\left(\frac{\beta}{12} - \frac{\gamma}{2\pi}\right)(t - \tfrac{1}{2})$$

$$h(t) = \left(\alpha + \frac{3\gamma}{\pi}\right) + \left(\beta - \frac{6\gamma}{\pi}\right) t$$

This estimate provides a negative trend when $\beta < \frac{6\gamma}{\pi}$. Since, in our example, $\beta = 2.5$ and $\gamma = 25$, the "least squares" trend is negative for every simulation, whereas, of course, the actual trend is positive. For each of the other sales generating functions, positive "least squares" trends are obtained. For the function $s(t) = \alpha + \beta t + \gamma \sin(\pi t)$ the "least squares" trend is $(\alpha + \frac{2\gamma}{\pi}) + \beta t$ whilst for the function $s(t) = \alpha + \beta t - \gamma \sin(\pi t)$ the "least squares" trend is $(\alpha - \frac{2\gamma}{\pi}) + \beta t$. On this topic generally see Yamane, T. Statistics., New York : Harper & Row, Publishers, Inc., 1973, Chapter 13.

42. Box, G.E.P. and M.E. Muller. "A Note on the Generation of Normal Deviates," Annals of Mathematical Statistics, XXIX (1958), pp.610-611.

sold corresponding to one of the sales generating functions and several estimates thereof to varying degrees of accuracy. The estimates were obtained by applying the following form of equation (33b) to the simulated data[43]

$$c\left[0,\,1\right] = \sum_{k=1}^{2^{(i-1)}} \sum_{j=1+(k-1)2^{(9-i)}}^{k2^{(9-i)}} r\left[\frac{2k-1}{2^{i}}\right] s\left[\frac{j}{256}\right] \tag{39}$$

The summation limit $n = 2^{(i-1)}$ was initially set to unity by letting $i = 1$ and then doubled by increasing i in unit increments until the following relative error condition was satisfied

$$\left|\frac{(E - A)}{A}\right| \leqslant d \tag{40}$$

where E is the estimate obtained from equation (39), A is the actual replacement cost of goods sold and d, the relative error, succesively assumes the values 0.10, 0.05, 0.025 and 0.01. The "days" column appearing to the right of each estimate represents the size of the n intervals (in days) over which equation (39) was applied. Thus, for example, if $n = 8$, the method was applied eight times during the year or equivalently, every $\frac{256}{8}$ = 32nd working day.

A brief inspection of Tables 5.5 through 5.8 should lead the reader to the conclusion that convergence to within 2½% of the actual replacement cost of disposals is virtually guaranteed if the modified midpoint rule is applied at the end of every thirty second working day.[43(a)] Further, in 75% of the cases, this same thirty two day "quadature period" results in the estimate of the replacement cost of disposals converging to within 1% of the actual figure. Since a thirty two day "quadrature period" means the method is applied eight times ($\frac{256}{32} = 8$) in a full year, or less than once a month, the method seems to guarantee a high degree of accuracy at the cost of very little time and effort.[44] This

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43. This equation is obtained by setting T = 0 in equation (33b).

44. In the case of stock compensating effects in the errors will, in all likelihood, increase the method's accuracy.

43(a) If the difference between the actual replacement cost and the estimate thereof is expressed as a fraction of the firm's net profit, the above results may not be as impressive.

ESTIMATE OF THE REPLACEMENT COST OF GOODS SOLD

		0.01		0.025		0.05		0.10	
Inflation Rate (%)	£ Actual	£ Estimate	Days	£ Estimate	Days	£ Estimate	Days	£ Estimate	Days
5	265,853.19	268,413.88	64	268,413.88	64	258,563.13	128	282,234.00	256
10	273,313.75	272,457.81	8	278,238.81	32	282,834.13	128	287,886.50	256
15	277,440.25	279,835.25	16	273,282.06	256	273,282.06	256	273,282.06	256
20	285,294.25	286,942.75	256	286,942.75	256	286,942.75	256	286,942.75	256
25	292,975.75	294,601.63	32	288,263.19	256	288,263.19	256	288,263.19	256
30	295,946.19	294,142.00	16	292,079.13	128	305,358.94	256	305,358.94	256
35	304,655.88	302,955.50	128	302,955.50	128	315,569.88	256	315,569.88	256
40	308,650.81	307,518.00	32	304,374.88	128	304,374.88	128	283,484.94	256
45	316,535.13	313,986.75	64	320,526.19	256	320,526.19	256	320,526.19	256
50	325,191.81	326,208.56	16	328,763.69	128	328,763.69	128	343,226.00	256
55	329,339.25	329,153.19	64	333,728.81	256	333,728.81	256	333,728.81	256
60	338,258.38	337,583.81	16	344,944.13	128	344,944.13	128	361,663.88	256
65	342,946.75	339,933.88	128	336,870.00	256	336,870.00	256	336,870.00	256
70	349,261.81	351,668.56	32	343,859.69	128	343,859.69	128	371,519.31	256
75	357,662.44	359,905.75	128	354,022.19	256	354,022.19	256	354,022.19	256
80	362,385.56	359,084.19	256	359,084.19	256	359,084.19	256	359,084.19	256
85	371,906.13	372,467.81	8	362,646.75	256	362,646.75	256	362,646.75	256
90	379,422.56	380,185.38	128	371,999.25	256	371,999.25	256	371,999.25	256
95	383,211.13	385,039.81	64	385,039.81	64	396,222.31	128	363,697.31	256
100	391,683.31	390,822.75	32	390,822.75	32	381,634.69	256	381,634.69	256

$$\underline{\underline{Q = \alpha + \beta t}}$$

TABLE 5.6

ESTIMATE OF THE REPLACEMENT COST OF GOODS SOLD

		0.01		0.025		0.05		0.10	
Inflation Rate (%)	£ Actual	£ Estimate	Days	£ Estimate	Days	£ Estimate	Days	£ Estimate	Days
5	265,385.19	267,905.69	64	267,905.69	64	255,833.19	128	282,234.31	256
10	272,326.31	271,005.88	8	276,493.06	32	281,528.31	128	287,886.38	256
15	275,986.69	273,281.75	256	273,281.75	256	273,281.75	256	273,281.75	256
20	283,176.38	284,787.13	64	286,942.69	256	286,942.69	256	286,942.69	256
25	290,414.44	288,262.88	256	288,262.88	256	288,262.88	256	288,262.88	256
30	293,139.50	290,245.56	32	287,773.31	128	305,358.94	256	305,358.94	256
35	301,135.94	301,275.63	128	301,275.63	128	315,569.38	256	315,569.38	256
40	304,373.13	302,212.94	32	300,602.25	128	300,602.25	128	283,485.19	256
45	311,972.88	310,257.88	64	307,229.31	128	320,525.94	256	320,525.94	256
50	319,936.75	320,897.50	16	323,560.31	128	323,560.31	128	343,225.81	256
55	323,708.63	323,195.44	64	330,662.13	128	333,729.31	256	333,729.31	256
60	332,173.94	333,853.38	64	338,548.38	128	338,548.38	128	361,663.88	256
65	336,337.38	336,869.75	256	336,869.75	256	336,869.75	256	336,869.75	256
70	342,099.50	344,057.31	32	335,807.44	128	335,807.44	128	371,518.81	256
75	350,091.56	351,252.63	128	354,022.31	256	354,022.31	256	354,022.31	256
80	354 447.25	353,320.06	64	359,084.44	256	359,084.44	256	359,084.44	256
85	363,578.69	362,646.69	256	362,646.69	256	362,646.69	256	362,646.69	256
90	370,241.56	371,999.13	256	371,999.13	256	371,999.13	256	371,999.13	256
95	373,451.69	374,594.56	64	374,594.56	64	363,696.56	256	363,696.56	256
100	381,539.19	381,634.56	256	381,634.56	256	381,634.56	256	381,634.56	256

$$Q = \alpha + \beta t + \gamma \sin(2\pi t)$$

TABLE 5.7

ESTIMATE OF THE REPLACEMENT COST OF GOODS SOLD

		0.01		0.025		0.05		0.10	
Inflation Rate (%)	£ Actual	£ Estimate	Days	£ Estimate	Days	£ Estimate	Days	£ Estimate	Days
5	307,508.19	306,287.50	32	311,063.06	64	299,139.94	128	326,547.75	256
10	316,300.31	315,221.81	8	321,228.00	32	327,325.94	128	333,196.25	256
15	321,174.06	323,917.63	16	316,392.00	256	316,392.00	256	316,392.00	256
20	330,089.38	332,061.69	256	332,061.69	256	332,061.69	256	332,061.69	256
25	338,858.75	340,748.38	32	333,463.69	256	333,463.69	256	333,463.69	256
30	342,536.50	340,670.06	16	338,048.81	128	353,442.88	256	353,442.88	256
35	352,471.81	350,463.31	128	350,463.31	128	365,094.13	256	365,094.13	256
40	357,284.75	355,659.44	32	352,324.19	128	352,324.19	128	328,192.81	256
45	366,419.88	362,871.50	64	371,144.69	256	371,144.69	256	371,144.69	256
50	376,075.00	377,264.06	16	380,176.75	128	380,176.75	128	397,000.31	256
55	381,242.19	381,379.31	64	386,397.50	256	386,397.50	256	386,397.50	256
60	391,259.13	390,416.25	8	399,061.75	128	399,061.75	128	418,502.50	256
65	396,851.00	393,413.13	128	389,906.69	256	389,906.69	256	389,906.69	256
70	404,113.25	406,911.25	32	397,935.81	128	397,935.81	128	430,035.56	256
75	413,588.94	409,648.13	256	409,648.13	256	409,648.13	256	409,648.13	256
80	419,398.81	415,625.25	256	415,625.25	256	415,625.25	256	415,625.25	256
85	430,131.00	430,773.94	8	419,732.06	256	419,732.06	256	419,732.06	256
90	438,536.81	439,470.63	128	430,132.00	256	430,132.00	256	430,132.00	256
95	443,162.19	444,932.06	64	444,932.06	64	458,253.63	128	420,776.25	256
100	453,025.38	452,657.50	32	441,730.25	128	441,534.50	256	441,534.50	256

$$Q = \alpha + \beta t + \gamma \sin(\pi t)$$

TABLE 5.8

ESTIMATE OF THE REPLACEMENT COST OF GOODS SOLD

Inflation Rate (%)	£ Actual	0.01 £ Estimate	0.01 Days	0.025 £ Estimate	0.025 Days	0.05 £ Estimate	0.05 Days	0.10 £ Estimate	0.10 Days
5	224,198.50	225,765.38	64	225,765.38	264	217,986.81	128	237,920.88	256
10	230,327.94	229,694.19	8	235,249.81	32	238,342.56	128	242,576.94	256
15	233,704.81	235,753.13	16	230,172.19	256	230,172.19	256	230,172.19	256
20	240,498.75	241,823.69	256	241,823.69	256	241,823.69	256	241,823.69	256
25	247,092.25	248,453.94	32	243,061.88	256	243,061.88	256	243,061.88	256
30	249,356.94	247,613.75	16	246,109.44	128	257,275.00	256	257,275.00	256
35	256,840.06	255,447.00	128	255,447.00	128	266,045.00	256	266,045.00	256
40	260,016.50	259,376.94	32	256,426.06	128	256,426.06	128	238,777.56	256
45	266,649.44	265,102.31	64	269,907.94	256	269,907.94	256	269,907.94	256
50	274,308.63	275,153.44	16	277,351.19	128	277,351.19	128	289,452.06	256
55	277,436.75	276,927.44	64	281,060.44	256	281,060.44	128	281,060.44	256
60	285,257.38	284,751.25	8	290,826.13	128	290,826.13	256	304,824.81	256
65	289,042.38	286,455.13	128	283,833.81	256	283,833.81	128	283,833.81	256
70	294,410.56	296,425.25	32	289,782.81	128	289,782.81	128	313,002.25	256
75	301,736.25	303,507.38	128	298,396.31	256	298,396.31	256	298,396.31	256
80	305,372.69	302,544.06	256	302,544.06	256	302,544.06	256	302,544.06	256
85	313,680.88	314,161.63	8	308,796.50	16	305,561.38	256	305,561.38	256
90	320,307.13	320,899.75	128	313,866.25	256	313,866.25	256	313,866.25	256
95	323,260.00	325,146.75	64	325,146.75	64	334,190.06	128	306,617.63	256
100	330,342.69	328,988.50	32	328,988.50	32	321,735.19	256	321,735.19	256

$$Q = \alpha + \beta t - \gamma \sin(\pi t)$$

conclusion, of course, is very much influenced by the assumptions underlying the simulation, but we see no reason to doubt the conclusion's generality, even when alternative assumptions are specified. The FORTRAN programs from which the figures in Tables 5.5 through 5.8 were generated are contained in Appendix 5C.

Having provided two methods for estimating the replacement cost of disposals over the interval $[T, T+1]$, we now focus our attention on the relative merits of each.

5.3.4 A Comparison of Methods

The choice of whether to utilize the Edwards and Bell technique or the modified midpoint rule depends to a certain extent on personal preference. Each has the desirable characteristic of converging to the actual replacement cost of disposals as the number of intervals over which the method is applied is increased. Several points, however, require noting.

In implementing the Edwards and Bell method, we assume that the weighted average acquisition cost is defined for each interval to which the method is applied. If, for example, there are no acquisitions in some interval, the Edwards and Bell technique cannot be applied to the interval. Thus, the method is of no significance to the problem of estimating the replacement cost depreciation accruing on a fixed asset. Note, however, that the replacement cost (per unit) at the midpoint of each interval will almost certainly be defined. Indeed, the modified midpoint rule is, in fact, employed by Edwards and Bell to estimate replacement cost depreciation.[45]

Secondly, the modified midpoint rule has certain computational advantages over the Edwards and Bell technique. Specifically, in

45. Edwards and Bell, loc.cit.

applying the modified midpoint rule to some interval $[t_{k-1}, t_k]$ we merely require the replacement cost (per unit), $r(\frac{t_{k-1}+t_k}{2})$, at the interval's midpoint. The Edwards and Bell technique, however, demands the more lengthy procedure of computing the weighted average acquisition cost of the interval $[t_{k-1}, t_k]$. Because of this, the Edwards and Bell technique is likely to require much more data and computing than the modified midpoint rule.

Finally, the modified midpoint rule seems to provide a high degree of accuracy when applied to disposals on a monthly basis.[46] We did not perform an analogous series of tests on the Edwards and Bell technique because, given any disposal pattern, the firm's acquisition policy is likely to be affected by a variety of factors such as the cost of capital, current investment levels, ordering costs etc.[47] In some "pilot simulations" designed to test the accuracy of the Edwards and Bell technique and for which these parameters were assumed to be exogenously determined, we were also confronted with the problem that some time intervals involved no acquisitions and thus the weighted average acquisition cost was not defined for the interval. This meant, of course, that any attempt at testing the Edwards and Bell technique which included such intervals were, of necessity, abandoned. Therefore, given the superficial nature of the assumptions which, of necessity, are imposed in testing the Edwards and Bell technique, plus the fact that the weighted average acquisition cost is not always defined, resulted in our abandoning any attempt at duplicating the simulation tests on the Edwards and Bell technique.

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46. See previous section.

47. Weston, J.F. and E.F. Brigham. Managerial Finance. New York : Dryden Press, 1975. See especially the sections on inventory management.

This completes our analysis of the numerical methods which may be applied to the problem of estimating the replacement cost of disposals during some interval of time. We now summarize the contents of the present chapter.

5.4 Summary

In the previous chapter we proposed a general model of accounting measurement. One vexing problem restricting the model's practicability concerns the computation of the accumulated "value" of disposals during the period covered by the financial statements where, it will be recalled, the term "value" is to be interpreted in the context of the measurement system being utilized. Although this problem is applicable to all the measurement systems, it seems to have caused most consternation to the advocates of the replacement cost measurement system. In the present chapter, therefore, we analyzed several methods for estimating the replacement cost of disposals during the interval $[T,T+1]$.

In introducing these methods we defined two functions, the first of which was denoted by $r(t)$ and represented the replacement cost (per unit) of a resource at time t. The second function, denoted by $s(t)$, represented the rate of change in accumulated disposals at time t. When the composite function $u(t) = r(t).s(t)$ is integrable, then our problem is to estimate the quantity

$$c\,[T,T+1] = \int_{T}^{T+1} r(t).s(t)dt \qquad (5)$$

When, however, the integral cannot be evaluated analytically, there exist an assortment of numerical approximating techniques of varying accuracy and computational ease. Three of the better known numerical integration techniques were examined, but their relevance was questioned due to the fact that $s(t)$ is likely to be unknown and thus the integrability of the composite function cannot be guaranteed.

For this reason, we analyzed two numerical methods which have been hinted at by accountants, but whose properties have not been fully investigated. The first of these methods, which utilizes the weighted average cost of acquisitions, was introduced by Edwards and Bell. In this respect, we proved that the method converges to the actual replacement cost of disposals as the number of intervals to which it is applied is increased. Computationally, however, the method is very cumbersome and has the disadvantage that the weighted average acquisition cost may not be defined for the interval to which it is applied. To overcome these problems, we defined a modified midpoint rule and investigated its properties. Like the Edwards and Bell technique, it has the property of converging to the actual replacement cost of disposals as the number of intervals to which it is applied is increased, but it is not burdened with the lengthy computational procedures of the Edwards and Bell method. Further the method seems to afford a high degree of accuracy when applied on a monthly basis.

```
      C                          APPENDIX 5A
      C                          -----------
      C
      C
      C
      C
      C      ***************************************
      C      * THIS PROGRAM COMPUTES THE REPLACEMENT *
      C      * COST OF ASSET DISPOSALS DURING THE    *
      C      * INTERVAL (T,T+1) USING EACH OF THE    *
      C      * FOLLOWING QUADRATURE TECHNIQUES       *
      C      ***************************************
            DIMENSION R(12)
            DO 300 I=1,50
            FI=I
            FI=FI*0.01
            E1=EXP(FI)
            E2=EXP(FI/2.)
            A=1./FI*(E1-1.)
      C      ********************
      C      * MIDPOINT RULE    *
      C      ********************
            R(1)=E2
            R(2)=ABS(A-R(1))
            R(3)=FI**2*E1/24.
            R(4)=R(3)/A
      C      ********************
      C      * TRAPEZOIDAL RULE *
      C      ********************
            R(5)=0.5*(1.+E1)
            R(6)=ABS(A-R(5))
            R(7)=2.*R(3)
            R(8)=2.*R(4)
      C      ********************
      C      *SIMPSONS RULE     *
      C      ********************
            R(9)=0.16667*(1.+4.*E2+E1)
            R(10)=ABS(A-R(9))
            R(11)=R(7)*FI**2/240.
            R(12)=R(11)/A
            WRITE(6,200)FI,A,(R(J),J=1,12)
        200 FORMAT(1X,F6.2,2X,F6.4,12(2X,F6.4))
        300 CONTINUE
            STOP
            END
```

APPENDIX 5B

REALIZABLE COST SAVINGS

Theorem

If there is a constant ratio of acquisitions to disposals at each "disposal point" t_j in $[T,T+1]$, then the realizable cost savings of the interval $[T,T+1]$ may be computed in either of the following ways

(a) Assume the beginning quantity is held over the interval $[T,T+1]$ whilst the replacement cost changes from that prevailing at time T to that prevailing at time (T+1). The excess (or deficiency) of the ending quantity over the beginning quantity is assumed to be held whilst its replacement cost varies from the weighted average acquisition price to that prevailing at the time (T+1).

(b) Assume that the beginning quantity is held while its replacement cost changes from that at time T to the weighted average acquisition cost of the interval $[T,T+1]$. The ending quantity is assumed acquired at the weighted average acquisition price and held while its replacement cost rises to that prevailing at time (T+1).

Proof

We prove each of these results in turn

(a) The quantity held at time (T+1) is I_e where

$$I_e = I_b + A - S$$

and $A = \sum_{k=1}^{n} a(t_k)$ is acquisitions during $[T,T+1]$ $S = \sum_{k=1}^{n} S(t_k)$ is disposals during $[T,T+1]$ and I_b is quantity held at time T. We can thus restate I_e in the following terms

$$I_e = I_b + \sum_{k=1}^{n} a(t_k) - \sum_{k=1}^{n} S(t_k)$$

The replacement cost of I_e at time (T+1) is thus

$$\mathcal{E} = r(T+1)\, I_e$$

$$\mathcal{E} = r(T+1)\, I_b + r(T+1) \sum_{k=1}^{n} a(t_k) - r(T+1) \sum_{k=1}^{n} S(t_k)$$

By hypothesis we have

$$a(t_k) = \ell\, S(t_k)$$

thus implying

$$\mathcal{E} = r(T+1)\, I_b + (\ell - 1) r(T+1) \sum_{k=1}^{n} S(t_k)$$

The replacement cost of I_b at time T is $r(t)\, I_b$. Acquisitions (at cost) during $[T,T+1]$ amount to $\sum_{k=1}^{n} r(t_k)a(t_k)$ whilst the replacement cost of disposals during $[T,T+1]$ amounts to $\sum_{k=1}^{n} r(t_k)S(t_k)$. Hence, from the first fundamental theorem, (Appendix 4A of Chapter 4) we have

$$\psi = r(T)\, I_b + \sum_{k=1}^{n} r(t_k)a(t_k) - \sum_{k=1}^{n} r(t_k)S(t_k)$$

which by virtue of the fact $a(t_k) = \ell\, S(t_k)$ may be restated as

$$\psi = r(T)\, I_b + (\ell - 1) \sum_{k=1}^{n} r(t_k)S(t_k)$$

Applying the first fundamental theorem of Chapter 4

$$\xi - \psi = \left[r(T+1) - r(T) \right] I_b + (\ell - 1) \left[r(T+1) \sum_{k=1}^{n} S(t_k) - \sum_{k=1}^{n} r(t_k)S(t_k) \right]$$

The first term of this expression is the beginning quantity multiplied by the difference between the replacement cost at time T+1 and the replacement price at time T. The second term can be reexpressed in the following form

$$(\ell-1)\left[r(T+1) \sum_{k=1}^{n} S(t_k) - \sum_{k=1}^{n} r(t_k)S(t_k) \right]$$

$$= (\ell-1)\left[r(T+1) \sum_{k=1}^{n} S(t_k) - \frac{\sum_{k=1}^{n} r(t_k)S(t_k)}{\sum_{k=1}^{n} S(t_k)} \sum_{k=1}^{n} S(t_k) \right]$$

$$= (\ell-1) \sum_{k=1}^{n} S(t_k) \left[r(T+1) - W(T,T+1) \right]$$

On the r.h.s. of this expression, the quantity $(\ell-1) \sum_{k=1}^{n} S(t_k)$ is the excess (or deficiency) of acquisitions over disposals whilst $[r(T+1) - W(T,T+1)]$ measures the increase in replacement cost at time (T+1) over the weighted average acquisition cost of the interval $[T,T+1]$. This proves the result.

(b) Before proving the second result, we compute the following quantity

$$W[T,T+1](I_b - I_e) = -W[T,T+1](\ell - 1)\sum_{k=1}^{n} S(t_k)$$

$$= \frac{-(\ell - 1)\sum_{k=1}^{n} r(t_k)S(t_k)}{\sum_{k=1}^{n} S(t_k)} \cdot \sum_{k=1}^{n} S(t_k)$$

$$W[T,T+1](I_b - I_e) = -(\ell - 1)\sum_{k=1}^{n} r(t_k)S(t_k)$$

Applying the first fundamental theorem of Chapter 4

$$\xi - \psi = r(T+1)\, I_e - \left[r(T)I_b + \sum_{k=1}^{n} r(t_k)a(t_k) - \sum_{k=1}^{n} r(t_k)S(t_k)\right]$$

$$= r(T+1)\, I_e - \left[r(T)I_b + (\ell - 1)\sum_{k=1}^{n} r(t_k)S(t_k)\right]$$

$$= r(T+1)\, I_e - \left[r(T)I_b - W(T+1)(I_b - I_e)\right]$$

$$- \quad = \left[W(T,T+1) - r(T)\right] I_b + \left[r(T+1) - W(T,T+1)\right] I_e$$

The first term in this expression is quantity at time T multiplied by the difference between the weighted average acquisition cost of the interval $[T,T+1]$ and the replacement cost at time T. The second term is the quantity at time (T+1) multiplied by the difference between the replacement cost at time (T+1) and the weighted average acquisition cost of the interval $[T,T+1]$. This proves the result.

In Table 5.9 we apply these results to the data of Table 5.4.

TABLE 5.9

STOCK EXAMPLE

(a) Direct Calculation of Realizable Cost Savings

	£
Gain accruing on 500 units sold January 1. 500 × (11-11)	-
Gain accruing on 800 units sold March 1. 800 × (12-11)	800
Gain accruing on 600 units sold May 1. 100 × (13-11) + 500(13-12)	700
Gain accruing on 200 units sold July 1. 140 × (14-12) + 60(14-13)	340
Gain accruing on 300 units sold Sept. 1. 300 × (15-13)	600
Gain accruing on 500 units sold Nov. 1. 120 × (16-13) + 160 × (16-14) + 220(16-15)	900
Gain accruing on 420 units of ending stock. 20 × (20-15) + 400 × (20-16)	1,700
	£5,040

(b) Theorem Calculation of Realizable Cost Savings

Applying the first method

$$\left[r(T) - r(0)\right] I_b + (\ell - 1) \sum_{j=1}^{6} s(t_k) \left[r(T) - W(0,T)\right]$$

$$= (20 - 11) \times 1000 - \frac{1}{5} \times 2900 \times (20 - 13.1724)$$

$$= 9000 - 3960$$

$$\left[r(T) - r(0)\right] I_b + (\ell - 1) \sum_{j=1}^{6} s(t_k) \left[r(T)\ W(0,T)\right]$$

$$= 5040$$

Applying the second method

$$\left[W(0,T) - r(0)\right] I_b + \left[r(T) - W(0,T)\right] I_e$$

$$= (13.1724 - 11) \times 1000 + (20 - 13.1724) \times 420$$

$$= 2172.41 + 2,867.59$$

$$\left[W(0,T) - r(0)\right] I_b + \left[r(T) - W(0,T)\right] I_e$$

$$= 5040.$$

(c) Stock Record

	Quantity			£		
Date	Dr	Cr	Bal	Dr	Cr	Bal
1909						
Jan 1	-	-	1,000	-	-	10,000
31	-	500	500	-	5,000	5,000
	400	-	900	4,400	-	9,400
Mar 1	-	500	400	-	5,000	4,400
	-	300	100	-	3,300	1,100
31	640	-	740	7,680		8,780
May 1	-	100	640	-	1,100	7,680
	-	500	140	-	6,000	1,680
31	480	-	620	6,240		7,920
July 1	-	140	480	-	1,680	6,240
	-	60	420	-	780	5,460
31	160	-	580	2,240	-	7,700
Sept 1	-	300	280	-	3,900	3,800
30	240	-	520	3,600	-	7,400
Nov 1	-	120	400	-	1,560	5,840
	-	160	240	-	2,240	3,600
	-	220	20	-	3,300	300
30	400	-	420	6,400	-	6,700

```
      C                    APPENDIX 5C
      C                    -----------
      C
      C
      C     ******************************************
      C     * THIS PROGRAM SIMULATES THE REPLACEMENT   *
      C     * COST OF ASSET DISPOSALS AND ITS ERROR    *
      C     * FOR THE INTERVAL (T,T+1),ASSUMING THE    *
      C     * EXISTENCE OF A LINEAR TREND IN QUANTITIES*
      C     * AND A NORMALLY DISTRIBUTED ERROR TERM    *
      C     * FOR BOTH QUANTITIES AND PRICE.           *
      C     ******************************************
            DIMENSION A(256,3),R(1024)
            DATA P,P1,A1,A2,B1,S1,S2 /6.28318,3.14159,100.,10.,2.5,5.,0.5/
            B2=0.
            WRITE(6,10)
         10 FORMAT(44H1RATE     ACTUAL      ESTIMATED   COMMENTS  )
            DO 1100 N=1,20
            B2=B2+0.5
       C    ******************************************
       C         GENERATION OF THE RANDOM NUMBERS
       C    ******************************************
            DO 20 J=1,1024
            R(J)=G05AAF(Y)
         20 CONTINUE
       C    ******************************************
       C      * GENERARTION OF THE PRICE QUANTITY SERIES *
       C    ******************************************
            DO 100 I=1,256
            FI=I
            T=FI/256.
            JJ=4*(I-1)
            J1=JJ+1
            J2=JJ+2
            J3=JJ+3
            J4=JJ+4
            X1=SQRT(-2.*ALOG(R(J1)))*COS(P*R(J2))
            X2=SQRT(-2.*ALOG(R(J3)))*COS(P*R(J4))
            E1=X1*S1
            E2=X2*S2
            QJ=A1+B1*T+25.*SIN(P1*T)+E1
            PJ=A2+B2*T+E2
            CJ=PJ*QJ
            II=I-1
            IF(II)40,40,60
         40 A(1,1)=PJ
            A(1,2)=QJ
            A(1,3)=CJ
            GO TO 100
         60 A(I,1)=PJ
            A(I,2)=QJ+A(II,2)
            A(I,3)=CJ+A(II,3)
        100 CONTINUE
       C
       C
       C
```

```
      C
      C
      C
      C
      C
      C
      C
      C
      C
      C
      C       ********************************************
      C       *           NUMERICAL TECHNIQUE             *
      C       ********************************************
              DO 500 I=1,8
              J=2**(I-1)
              K=128/J
              N3=0
              DO 400 M=1,J
              N1=N3
              N2=N1+K
              N3=N2+K
              IF(N1)200,200,300
          200 ES=A(N2,1)*A(N3,2)
              GO TO 400
          300 ES=ES+A(N2,1)*(A(N3,2)-A(N1,2))
          400 CONTINUE
              TEST=ABS((A(256,3)-ES)/A(256,3))
              IF(TEST-0.025)700,700,500
          500 CONTINUE
              WRITE(6,600)B2,A(256,3),ES
          600 FORMAT(F4.1,2X,2(F12.2,2X),15H NO CONVERGENCE)
              GO TO 1100
          700 WRITE(6,800)B2,A(256,3),ES,J
          800 FORMAT(F5.1,2(F12.2,2X),19H CONVERGENCE AFTER ,I3,11H ITERATIONS)
         1100 CONTINUE
              STOP
              END

*OPTIONS IN EFFECT*  NOID,EBCDIC,SOURCE,NOLIST,NODECK,LOAD,NOMAP
*OPTIONS IN EFFECT*  NAME = MAIN     , LINECNT =        58
*STATISTICS*    SOURCE STATEMENTS =        59,PROGRAM SIZE =      9050
*STATISTICS*  NO DIAGNOSTICS GENERATED
```

CHAPTER SIX

THE ECONOMIC FOUNDATIONS OF ACCOUNTING MEASUREMENT

6.0 Introduction

Our exposition to date has been concerned with an abstract analysis of accounting measurement to the almost total neglect of the more pragmatic issues which plague contemporary accounting practice. In the present chapter we endeavour to remedy this position. We should note at the outset, however, that the phrase "pragmatic issues" conceals a host of unresolved problem areas, and given the confines of the present work it would be an achievement indeed if we were to consider but a few of them. Many of accounting's problem areas, however, share a common origin in that they arise out of economic considerations, a point acknowledged by Chambers,[1] Ijiri[2] and Sterling[3] amongst others. For this reason, the objective of the present chapter is to examine the economic foundations of the theory of accounting measurement.

The logical framework of the present chapter follows the general equilibrium analysis of Irving Fisher[4] in both spirit and form. Since Fisher's work is but one of several competing economic specifications, we must explain why this choice is not to be seen as arbitrary. This we proceed to do.

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1. Chambers, R.J. Accounting, Evaluation and Economic Behaviour, Englewood Cliffs, New Jersey: Prentice-Hall, Inc., 1966, pp. 349-352.
2. Ijiri, Y. The Foundations of Accounting Measurement. Englewood Cliffs, New Jersey: Prentice-Hall, Inc., 1967, p.69.
3. Sterling, R.R. Theory of the Measurement of Enterprise Income. Lawrence, Kansas: The University of Kansas Press, 1970, pp. 193-245.
4. Fisher, Irving. The Theory of Interest. New York: The MacMillan Company, 1930.

Perhaps the most obvious reason is that the Sharpe-Lintner[5] asset pricing model which has had and continues to have such a profound influence on accounting theory,[6] is but a generalization of Fisher's theory of interest to a world of uncertainty.[7] Unfortunately, the asset pricing model deals with one period consumption-investment decisions and is not easily generalized to several periods.[8] Since many accounting propositions are concerned with the predictive properties of accounting measurements, it is of some importance that our analysis anticipates more than a single consumptive-productive interval. By "regressing" to the pioneering work of Irving Fisher and thus imposing conditions of "perfect knowledge",[9] we retain many of the asset pricing model's features without the restriction of a single productive-consumptive interval.

A second reason, however, is that the Fisherine system is a convenient device through which to examine both a priori and empirical propositions concerning the firm. This may seem to be somewhat surprising, especially as Fisher eschewed an analysis of the product and factor markets underlying the "investment opportunity locus".[10]

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5. Sharpe, William F. "Capital Asset Prices: A Theory of Market Equilibrium under conditions of Risk", Journal of Finance, XIX (September 1964), pp. 425-442.
   Lintner, John. "The Valuation of Risk Assets and the Selection of Risky Investments in Stock Portfolios and Capital Budgets", Review of Economics and Statistics, XLVII (February 1965), pp. 13-37.
6. Gonedes, N.J. and Dopuch, N. "Capital Market Equilibrium, Information Production and Selecting Accounting Techniques: Theoretical Framework and Review of Empirical Work", Supplement to Journal of Accounting Research (1974), pp. 48-129.
7. Jensen, Michael, C. "The Foundations and Current State of Capital Market Theory", in Jensen, Michael C. (ed.) Studies in the Theory of Capital Markets. New York: Praeger Publishers, Inc., 1972, p.2.
8. Ibid., pp. 16-17.
9. Fisher, op.cit., p. 99.
10. Fisher, op.cit., pp. 143-149. The "investment opportunity locus" is more commonly referred to nowadays as the "productive opportunity locus". See Hirshleifer, J. Investment, Interest and Capital. Englewood Cliffs, New Jersey: Prentice-Hall, Inc., 1970. p.13.

For as Fisher himself observed

> "Walras and Pareto determine the rate of interest simultaneously with all other unknowns of the problem - the quantities of the commodities exchanged and the other services used in their production and the prices of the commodities and the services while I try ... at the outset to get [these] interactions cancelled out, leaving only the income stream and (labour) sacrifice".[11]

If, however, we derive a firm's "investment opportunity locus" from first principles, it is a relatively simple matter to resurrect the theory of the firm implied by the Fisherine analysis. Indeed, this leads to a much clearer exposition of the theory of the firm than is usually encountered in price theory texts. Other reasons for analyzing the Fisherine system could be cited, but perhaps we have now established a <u>prima facie</u> case.

Our analysis of the Fisherine system and its accounting implications is divisible into three sections. In the first section we shall summarize the mathematical form of Fisher's "second approximation"[12] to the theory of interest. The "second approximation" is relevant because it assumes that each agent's[13] income stream may be modified by investments in productive facilities[14] thus permitting the introduction of the firm as a device through which consumptive resources

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11. Fisher, <u>op.cit.</u>, p.519.

12. <u>Ibid.</u>, pp. 302-315.

13. "... the analysis applies not only to individuals proper but also to groupings of individuals such as households ... The key assumption is that any such groupings can be treated as a unitary body making decisions analagous to those of a self interested individual". Hirshleifer, <u>op.cit.</u>, p.1.

14. Fisher, <u>op.cit.</u>, p.125.

as of one date are physically transformed into consumptive resources as of another date. This, of course, provides a base from which to examine the economic foundations of the accounting measurement systems introduced in Chapter 4. Having discussed and illustrated the mathematical foundations of Fisher's "second approximation" we then illustrate its implementation by means of a practical example. The example, in fact, is designed to serve a dual function. Its main task is to serve as a device through which to examine the economic foundations of the accounting measurement systems introduced in Chapter 4, but it is also designed to promote a better understanding of the purely mathematical analysis of the previous section. As a final exercise, we shall examine each of the accounting measurement systems introduced in Chapter 4 in the context of the Fisherine system. The emphasis is on replacement cost measurement if only because Fisher's work seems to bear most relevance to that measurement system, but the market value and C.P.P. systems are also examined. Needless to say, we shall find the Fisherine system an extremely valuable device through which to examine accounting propositions concerning the firm.

We now focus on the first of these topics, namely an examination of the mathematical foundations of Fisher's "second approximation" to the theory of interest.

6.1 Fisher's "Second Approximation"

In this section we summarize Fisher's second approximation to the theory of interest; namely, the optimal allocation of consumption expenditures over time assuming "that all available income streams can be definitely foreseen".[15] More precisely, we impose the

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15. Ibid.

following assumptions

1. The "economic agent" has a remaining life of (T-t) years, where t is the present and T is the date of "death". The interval (t,T) is divisible into (T-t) subintervals $[t,t+1]$, $[t+1, t+2]$, ____, $[T-1,T]$ with consumption occurring and income being received at the beginning of each interval.

2. The economic agent acts <u>as if</u>[16] it maximizes a utility function $U(c_t, c_{t+1}, \_\_\_\_, c_T)$ which relates "satisfaction" to the consumption series $c_t, c_{t+1}, \_\_\_\_ c_T$.

3. At the commencement of the interval $[j,j+1]$ in $[t,T]$; that is, at time j

   (a) the agent has an endowment of wealth W_j, which consists of loans made through the capital market. Loans are made, repaid and renewed at the beginning of each time interval.

   (b) the agent makes a net disinvestment of p_j in productive facilities. The set of potential disinvestment patterns is described by the function $K(p_t, p_{t+1}, \_\_\_\_, p_T) = 0$.

4. The agent has equal and costless access to information about ruling security prices. Buyers, sellers and issuers of securities take the prices of securities as given and there are no brokerage fees, transfer taxes, or other "transactions" costs incurred when securities are sold.[17]

5. The agent has an exogenously determined and known income stream of $y_t, y_{t+1}, \_\_\_\_, y_T$. Further, the one period interest rates ${}_jr_{j+1}$ for each interval $[j,j+1]$ in $[t,T]$ are also known.

Using these assumptions the agent's life-time consumption profile may be stated in the following terms

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16. This phrase is quite significant from a methodological stance. See Friedman, M. "The Methodology of Positive Economics" in Friedman, M. (ed.) <u>Essays in Positive Economics</u>. Chicago: University of Chicago Press, 1953, pp. 3-43.

17. These are the usual assumptions of a perfect capital market. See Fama, E.F. and M.H. Miller. <u>The Theory of Finance</u>. Hindsdale, Illinois, Dryden Press, Inc., 1972, p.22.

$$W_{t+1} = (W_t + y_t + p_t - c_t)(1 + {}_t r_{t+1}) \qquad (t)$$

$$W_{t+2} = (W_{t+1} + y_{t+1} + p_{t+1} - c_{t+1})(1 + {}_{t+1} r_{t+2}) \qquad (t+1$$

$$\vdots$$

$$W_T = (W_{T-1} + y_{T-1} + p_{T-1} - c_{T-1})(1 + {}_{T-1} r_T) \qquad (T-1)$$

$$c_T = W_T + p_T + y_T \qquad (T)$$

since $(y_j + p_j - c_j)$ measures the excess (or deficiency) of "income" over consumption expenditures at time j and W_j is the "endowment" carried forward from the interval $[j-1, j]$. Hence, $(W_j + y_j + p_j - c_j)$ is "invested" in (or borrowed from) the capital market at time j and, with interest, yields $(W_j + y_j + p_j - c_j)(1 + {}_j r_{j+1})$ at time $(j+1)$. Equation (T) imposes the condition that the agent leaves neither bequest nor debt at the time of death. Substituting (t) into (t+1) implies

$$W_{t+2} = \left[(W_t + y_t + p_t - c_t)(1 + {}_t r_{t+1}) + y_{t+1} + p_{t+1} - c_{t+1}\right](1 + {}_{t+1} r_{t+2}) \qquad (2)$$

continuing this process and dividing the end product by $\prod_{j=t}^{T-1} (1 + {}_j r_{j+1})$ implies

$$c_t + \frac{c_{t+1}}{(1 + {}_t r_{t+1})} + \frac{c_{t+2}}{\prod_{j=t}^{t+1} (1 + {}_j r_{j+1})} + \cdots + \frac{c_T}{\prod_{j=t}^{T-1} (1 + {}_j r_{j+1})}$$

$$=$$

$$W_t + y_t + \frac{y_{t+1}}{(1+{}_tr_{t+1})} + \frac{y_{t+2}}{\prod_{j=t}^{t+1}(1+{}_jr_{j+1})} + \text{———} + \frac{y_T}{\prod_{j=t}^{T-1}(1+{}_jr_{j+1})}$$

$$+ p_t + \frac{p_{t+1}}{(1+{}_tr_{t+1})} + \frac{p_{t+2}}{\prod_{j=t}^{t+1}(1+{}_jr_{j+1})} + \text{———} + \frac{p_T}{\prod_{j=t}^{T-1}(1+{}_jr_{j+1})} \tag{3}$$

or, the present value of the consumption series $c_t, c_{t+1},$ ———, c_T is equivalent to the present value of the income series $y_t, y_{t+1},$ ———, y_T and the "investment series" $p_t, p_{t+1},$ ———, p_T. Thus, the agent's "problem" may be stated as maximizing the satisfaction obtainable from the consumption series, subject to the following constraints

1. The agent must choose a permissible investment pattern in productive facilities. Specifically, the investment pattern must be one described by the function $K(p_t, p_{t+1},$ ———, $p_T) = 0$.

2. The agent must choose a consumption series so that its present value is equivalent to the sum of the present value of the income series and the present value of the investment series.

To state this in mathematical form, define the Lagrangian L, in the following terms

$$L = U(c_t, c_{t+1}, \text{———}, c_T) - \lambda_1 K(p_t, p_{t+1}, \text{———}, p_T)$$

$$-\lambda_2 \Bigg[c_t + \frac{c_{t+1}}{(1+{}_tr_{t+1})} + \frac{c_{t+2}}{\prod_{j=t}^{t+1}(1+{}_jr_{j+1})} + \text{———} + \frac{c_T}{\prod_{j=t}^{T-1}(1+{}_jr_{j+1})} -$$

$$W_t - y_t - \frac{y_{t+1}}{1+{}_tr_{t+1}} - \frac{y_{t+2}}{\prod_{j=t}^{t+1}(1+{}_jr_{j+1})} - \text{------} - \frac{y_T}{\prod_{j=t}^{T-1}(1+{}_jr_{j+1})} -$$

$$p_t - \frac{p_{t+1}}{(1+{}_tr_{t+1})} - \frac{p_{t+2}}{\prod_{j=t}^{t+1}(1+{}_jr_{j+1})} - \text{------} - \frac{p_T}{\prod_{j=t}^{T-1}(1+{}_jr_{j+1})} \Bigg]$$

$$(4)$$

Maximizing with respect to the consumption series, the investment series, λ_1 and λ_2 gives the following system of equations

$$\frac{\partial L}{\partial c_t} = \frac{\partial U}{\partial c_t} - \lambda_2 = 0$$

$$\frac{\partial L}{\partial c_{t+1}} = \frac{\partial U}{\partial c_{t+1}} - \frac{\lambda_2}{(1+{}_tr_{t+1})} = 0$$

$$\frac{\partial L}{\partial c_{t+2}} = \frac{\partial U}{\partial c_{t+2}} - \frac{\lambda_2}{\prod_{j=t}^{t+1}(1+{}_jr_{j+1})} = 0$$

$$\vdots$$

$$\frac{\partial L}{\partial c_T} = \frac{\partial U}{\partial c_T} - \frac{\lambda_2}{\prod_{j=t}^{T-1}(1+{}_jr_{j+1})} = 0$$

$$\frac{\partial L}{\partial p_t} = \lambda_2 - \lambda_1 \frac{\partial K}{\partial p_t} = 0 \qquad (5)$$

$$\frac{\partial L}{\partial p_{t+1}} = \frac{\lambda_2}{(1+{}_tr_{t+1})} - \lambda_1 \frac{\partial K}{\partial p_{t+1}} = 0$$

$$\frac{\partial L}{\partial p_{t+2}} = \frac{\lambda_2}{\prod_{j=t}^{t+1}(1+{}_jr_{j+1})} - \lambda_1 \frac{\partial K}{\partial p_{t+2}} = 0$$

$$\vdots$$

$$\frac{\partial L}{\partial p_T} = \frac{\lambda_2}{\prod_{j=t}^{T-1}(1+{}_jr_{j+1})} - \lambda_1 \frac{\partial K}{\partial p_T} = 0$$

$$\frac{\partial L}{\partial \lambda_1} = K(p_t, p_{t+1}, \_\_\_\_, p_T) = 0$$

plus equation (3). This system has as its solution the consumption series c_t^1, c_{t+1}^1, c_{t+2}^1, ____, c_T^1 and the investment series p_t^1, p_{t+1}^1, p_{t+2}^1, ____, p_T^1. for which

$$\frac{\partial U}{\partial c_t} = \frac{\partial U}{\partial c_{t+1}}(1+{}_tr_{t+1}) = \frac{\partial U}{\partial c_{t+2}}\prod_{j=t}^{t+1}(1+{}_jr_{j+1}) = \_\_\_\_ = \frac{\partial U}{\partial c_T}\prod_{j=t}^{T-1}(1+{}_jr_{j+1})$$

and (6)

$$\frac{\partial K}{\partial p_t} = \frac{\partial K}{\partial p_{t+1}}(1+{}_tr_{t+1}) = \frac{\partial K}{\partial p_{t+2}}\prod_{j=t}^{t+1}(1+{}_jr_{j+1}) = \_\_\_\_ = \frac{\partial K}{\partial p_T}\prod_{j=t}^{T-1}(1+{}_jr_{j+1})$$

The fact that the system of equations defined in (6) decomposes into one set of equations which is dependent for its solution only on the function $U(c_t, c_{t+1}, \_\_\_\_, c_T)$ and a second set of equations which, in turn, is dependent for its solution only on the function $K(p_t, p_{t+1}, \_\_\_\_, p_T)$ is known as the separation theorem. This theorem asserts, <u>inter alia</u>, that the choice of an optimal consumption series decomposes itself into two independent decisions.[18] The first of these involves choosing the investment series which maximizes wealth, where wealth is defined in the following terms

$$W(t) = W_t + y_t + p_t + \sum_{i=t+1}^{T} \frac{(y_i + p_i)}{\prod_{j=t}^{t-1} (1+{}_jr_{j+1})} \tag{7}$$

The second, the allocation of this greatest wealth to an optimal consumption series. Any excess (deficit) of consumption expenditures over "money income" is financed (invested) through the capital market at the prevailing one period rate of interest.

The above analysis also serves to determine the equilibrium return earned by each asset in the economy. To illustrate, suppose the "economy" to consist of I agents each with the same time horizon of (T-t) years.[19] Then market equilibrium establishes (T-t) interest rates which equate the productive and consumptive demands and supplies for "money" in each period. The equilibrium position is obtained from the following set of equations:

<u>"Impatience Principle A"</u>[20] <u>I(T-t) equations</u>

$$\frac{\partial U}{\partial c_{t+1}} \Big/ \frac{\partial U}{\partial c_t} = Y_{t+1}(c_t, c_{t+1}, \underline{\quad\quad}, c_T)$$

18. Hirshleifer, <u>op.cit</u>., p.14.

19. I, T and t must all assume integer values. We shall henceforth assume $W_t=0$.

20. Fisher, <u>op.cit</u>., p.148.

$$\begin{aligned}
\frac{\partial U}{\partial c_{t+2}} \Big/ \frac{\partial U}{\partial c_{t+1}} &= Y_{t+2}(c_t, c_{t+1}, ____, c_T) \\
&\vdots \\
\frac{\partial U}{\partial c_T} \Big/ \frac{\partial U}{\partial c_{T-1}} &= Y_T(c_t, c_{t+1}, ____, c_T)
\end{aligned} \tag{8}$$

These equations determine the marginal rates of substitution in consumption between funds at time j and time $j-1$ as a function of the consumption series defined on the interval $[t,T]$. Since there are I agents and (T-t+1) consumption dates there are I(T-t) such equations.

<u>"Impatience Principle B"</u>[21] <u>I(T-t) equations</u>

$$\begin{aligned}
\frac{\partial U}{\partial c_{t+1}} \Big/ \frac{\partial U}{\partial c_t} &= \frac{1}{(1+{}_t r_{t+1})} \\
\frac{\partial U}{\partial c_{t+2}} \Big/ \frac{\partial U}{\partial c_{t+1}} &= \frac{1}{(1+{}_{t+1} r_{t+2})} \\
&\vdots \\
\frac{\partial U}{\partial c_T} \Big/ \frac{\partial U}{\partial c_{T-1}} &= \frac{1}{(1+{}_{T-1} r_T)}
\end{aligned} \tag{9}$$

The necessary condition for the individual to have attained an optimal consumption series is that the marginal rate of substitution between successive consumption dates j-1 and j must be equivalent to $\frac{-1}{(1+{}_{j-1}r_j)}$ where, it will be recalled, ${}_{j-1}r_j$ is the one period rate of

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21. <u>Ibid</u>.

interest prevailing in the interval $[j-1,j]$. Since there are I agents and (T-t+1) consumption dates there are I(T-t) such equations.

"<u>Market Principle A</u>"[22] <u>(T-t+1) equations</u>

$$\left.\begin{aligned}
\sum_{i=1}^{I} c_{t,i} &= \sum_{i=1}^{I} y_{t,i} + \sum_{i=1}^{I} p_{t,i} \\
\sum_{i=1}^{I} c_{t+1,i} &= \sum_{i=1}^{I} y_{t+1,i} + \sum_{i=1}^{I} p_{t+1,i} \\
&\;\;\vdots \\
\sum_{i=1}^{I} c_{T,i} &= \sum_{i=1}^{I} y_{T,i} + \sum_{i=1}^{I} p_{T,i}
\end{aligned}\right\} \qquad (10)$$

These "conservation relations" express the condition that the total consumption occurring at time j, j=t, t+1,____,T must be equivalent to the exogenous endowments and productive investments as of the date. These are, in effect "market clearing" equations. Since there are (T-t+1) consumption and investment dates, there are (T-t+1) of these equations.

"<u>Market Principle B</u>"[23] <u>I equations</u>

$$c_{t,1} + \sum_{i=t+1}^{T} \frac{c_{t,1}}{\prod_{j=t}^{i-1}(1+{}_jr_{j+1})} = y_{t,1} + p_{t,1} + \sum_{i=t+1}^{T} \frac{y_{i,1}+p_{i,1}}{\prod_{j=t}^{i-1}(1+{}_jr_{j+1})}$$

22. <u>Ibid</u>, p.149.

23. <u>Ibid</u>.

$$\begin{aligned}
c_{t,2} + \sum_{i=t+1}^{T} \frac{c_{i,2}}{\prod_{j=t}^{i-1}(1+{}_j r_{j+1})} &= y_{t,2} + p_{t,2} + \sum_{i=t+1}^{T} \frac{y_{i,2}+p_{i,2}}{\prod_{j=t}^{i-1}(1+{}_j r_{j+1})} \\
&\vdots \\
c_{t,I} + \sum_{i=t+1}^{T} \frac{c_{i,I}}{\prod_{j=t}^{i-1}(1+{}_j r_{j+1})} &= y_{t,I} + p_{t,I} + \sum_{i=t+1}^{T} \frac{y_{i,I}+p_{i,I}}{\prod_{j=t}^{i-1}(1+{}_j r_{j+1})}
\end{aligned} \tag{11}$$

These are the "wealth constraints" and imply that for each of the I agents, loans must be repaid with interest so that each agent leaves neither debt nor bequests at time of "death".

"Investment Opportunity Principle A" 24 — I(T-t+1) equations

$$\begin{aligned}
K(p_t, p_{t+1}, ___, p_T) &= 0 \\
\frac{\partial K}{\partial p_{t+1}} \bigg/ \frac{\partial K}{\partial p_t} &= W_{t+1}(p_t, p_{t+1}, ___, p_T) \\
\frac{\partial K}{\partial p_{t+2}} \bigg/ \frac{\partial K}{\partial p_{t+1}} &= W_{t+2}(p_t, p_{t+1}, ___, p_T) \\
&\vdots \\
\frac{\partial K}{\partial p_T} \bigg/ \frac{\partial K}{\partial p_{T-1}} &= W_T(p_t, p_{t+1}, ___, p_T)
\end{aligned} \tag{12}$$

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24. Ibid., p.148.

These equations determine the marginal rates of substitution in production between funds at timej and time j-1 as a function of the investment series defined on the interval (t,T). These equations differ from those specified in "Impatience Principle A" because in the implicitly defined productive opportunity locus $K(p_t, p_{t+1}, \_\_\_\_, p_T) = 0$, specification of (T-t) disinvestment variables determines the remaining unknown disinvestment variable. Since there are I agents each with one productive opportunity locus and (T-t) marginal rates of substitution in production there are I(T-t+1) equations under this head.

"Investment Opportunity Principle B" 25 — I(T-t) equations

$$\frac{\partial K}{\partial p_{t+1}} \Bigg/ \frac{\partial K}{\partial p_t} = \frac{1}{(1+{}_t r_{t+1})}$$

$$\frac{\partial K}{\partial p_{t+2}} \Bigg/ \frac{\partial K}{\partial p_{t+1}} = \frac{1}{(1+{}_{t+1} r_{t+2})}$$

$$\vdots \qquad\qquad \vdots \tag{13}$$

$$\frac{\partial K}{\partial p_T} \Bigg/ \frac{\partial K}{\partial p_{T-1}} = \frac{1}{(1+{}_{T-1} r_T)}$$

The necessary condition for an agent to have attained an optimal production strategy is that the marginal rate of substitution between successive production dates j-1 and j must be equivalent to $\frac{-1}{(1+{}_{j-1} r_j)}$ where, it will be recalled, ${}_{j-1} r_j$ is the one period rate of

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25. Ibid.

interest prevailing over the interval $[j-1,j]$. Since there are (T-t+1) production dates, there are I(T-t) equations under this head.

The above analysis implies that there are a total of 4 I(T-t+1)-2I+(T-t+1) equations to determine the unknowns of the economy. Since there are I agents and (T-t+1) production and consumption dates, there are I(T-t+1) consumption expenditures and I(T-t+1) production expenditures to be determined. Similarly, there are I(T-t) marginal rates of substitution in consumption and I(T-t) marginal rates of substitution in production to be determined. Finally, though of the utmost importance in Fisher's analysis, there are (T-t) interest rates to be determined. This gives a total of 4I(T-t+1)-2I+(T-t) unknowns in the economy. Since one of the equations appearing under the "Market Principles" head is redundant, this implies that we have 4I(T-t+1)-2I+(T-t) equations and the same number of unknowns. As such the system is fully determined.

We have now completed our analysis of the mathematical underpinnings of the Fisherine system. Our next objective is to illustrate its implementation by recourse to a practical example. Recall that there is a twofold reason for doing so. Firstly, in the third section of this chapter, we shall employ the example as a device through which to examine accounting propositions concerning the firm. As a second and equally important reason, however, it is designed to sharpen the analysis of the previous section. To simplify the analysis we shall consider only a one period example, but it is important to remember that a multi-period example may also have been utilized.

6.2 An Example

In this section our purpose is to illustrate the Fisherine system in terms of a practical example. We shall assume that consumption occurs and productive investments are made at each of two consecutive dates. The analysis may be generalized to more than a single consumptive interval, of course, but this would result in considerable complication without corresponding analytical benefit.

The example shall be worked in three sections. In the first and second sections the equations describing consumptive and productive equilibrium respectively shall be set forth whilst in the third section we shall derive the equilibrium solution in terms of these equations. We thus turn to the first of these topics, namely derivation of the equations describing consumptive equilibrium.

6.2.1 The Agents

We suppose the economy to be composed of two equally numerous classes of agents, which we shall label type J and type U agents respectively. We shall assume that only type U agents possess productive opportunities, but that both sets of agents have identical utility functions for consumption expenditures. Further, we employ the "representative individual"[26] device to justify an analysis of the economy based on the assumption that it is composed of one type J agent and one type U agent. We thus proceed to define the economy in the manner utilized by Fisher.

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26. Hirshleifer, op.cit., p.107.

"Impatience Principle A" 2 equations

We shall suppose the agents' utility functions for consumption expenditures to be of the following form

$$J(b_o, b_1) = \sqrt{b_o b_1}$$

$$U(c_o, c_1) = \sqrt{c_o\, c_1}$$

where b_o and c_o are the consumption expenditures of agent J and agent U respectively at time zero, whilst b_1 and c_1 are the equivalent expenditures at time 1. These functions define the following marginal rates of substitution in consumption

$$\frac{\partial J}{\partial b_1} \Big/ \frac{\partial J}{\partial b_o} = \frac{b_o}{b_1} \tag{14a}$$

and

$$\frac{\partial U}{\partial c_1} \Big/ \frac{\partial U}{\partial c_o} = \frac{c_o}{c_1} \tag{14b}$$

These equations together satisfy "Impatience Principle A".

"Impatience Principle B" 2 equations

"Impatience Principle B" requires that the marginal rates of substitution (in consumption) between successive consumption dates j-1 and j must be equivalent to $\frac{-1}{(1+{}_{j-1}r_j)}$ where, it will be recalled, ${}_{j-1}r_j$, is the one period rate of interest prevailing during the interval $[j-1,j]$. As the marginal rates of substitution (in consumption) are defined by equations (14a) and (14b), this implies

$$b_o = \frac{b_1}{(1+{}_or_1)} \tag{15a}$$

and

$$c_o = \frac{c_1}{(1+{}_or_1)} \tag{15b}$$

These equations together satisfy "Impatience Principle B".

"Market Principle A" 2 equations

"Market Principle A" expresses the condition that the total of the agents' consumption expenditures at each consumption date are to be equivalent to the sum of the exogenous endowments and productive disinvestments as of the date. In terms of the present example this implies

$$c_o + b_o = 2y_o + p_o \tag{16a}$$

$$c_1 + b_1 = p_1 \tag{16b}$$

where we impose the condition that all agents possess the same initial endowment. Recall that only type U agents possess productive opportunities.

"Market Principle B" 2 equations

This principle expresses the condition that agents leave neither bequests nor debt at time of "death". In terms of the present example this implies

$$c_o + \frac{c_1}{(1+{}_or_1)} = y_o + p_o + \frac{p_1}{(1+{}_or_1)} \tag{17a}$$

and

$$b_o + \frac{b_1}{(1+{}_or_1)} = y_o \tag{17b}$$

These equations together satisfy "Market Principle B".

We now introduce the firm as the device through which production occurs.

6.2.2 The Firm

Although Fisher did not integrate the more "traditional" theory of the firm with his interest theory, he was fully aware that the connection could be made.[27] Fisher preferred to commence his analysis rather further out, taking the productive opportunity locus as somehow specified outside the system.[28] Whilst this simplified the analysis considerably, it restricts our capacity to examine the accounting implications of Fisher's work. In this section, therefore, we shall illustrate the method by which a firm's productive opportunity locus is derived from the more traditional principles of price theory and then use the resulting construct to satisfy Fisher's "Investment Opportunity Principles".

We thus impose the following assumptions on the firm

1. During the interval $[0,1]$, the firm produces a single non-storable commodity, the demand for which is given by

$$q = 500-20p \tag{18a}$$

   where q is the quantity produced and sold during $[0,1]$ and p is the price (per unit).

2. The firm's production function which relates output q to factor usage during $[0,1]$ is of the form

$$q = F(x,y,1) = 10\sqrt{xy} \tag{18b}$$

   where x and y are variable factors of production (in units) and z is a fixed factor of production exogenously specified at one unit.

3. Factors of production are purchased in perfectly competitive markets at time zero. Revenue income is received at time one.

4. To finance production during $[0,1]$ securities are issued to a single owner. Revenue income is distributed to the firm's owner at time one.

We thus suppose the firm to issue securities in the amount of

27. Fisher, op.cit., p.131.

28. Ibid., p.519.

I pounds. The optimal production strategy is then determined by maximizing

$$F(x,y,1) = 10\sqrt{xy}$$

subject to the constraint

$$p_x x + p_y y + p_z = I$$

where p_x, p_y and p_z are the factor prices prevailing at time zero. Define the Lagrangian L, as follows

$$L = 10\sqrt{xy} + \lambda(p_x x + p_y y + p_z - I)$$

and maximize with respect to the choice variables x, y and λ. We thus have

$$\frac{\partial L}{\partial x} = 5\sqrt{\frac{y}{x}} + \lambda p_x = 0 \qquad (19a)$$

$$\frac{\partial L}{\partial y} = 5\sqrt{\frac{x}{y}} + \lambda p_y = 0 \qquad (19b)$$

$$\frac{\partial L}{\partial \lambda} = p_x x + p_y y + p_z - I = 0 \qquad (19c)$$

Equations (19a) and (19b) together imply the following "expansion path"[29]

$$v_x = v_y \qquad (20)$$

where $v_x = p_x x$ and $v_y = p_y y$ are the sums expended on factors x and y respectively during $[0,1]$. Substituting (20) into (19c) implies

$$v_x = \tfrac{1}{2}v \qquad (21)$$

where $v = I - p_z$ is the sum expended on variable resources during $[0,1]$.

29. Liebhafsky, H.H. The Nature of Price Theory. Homewood, Illinois: The Dorsey Press, Inc., 1963, p.145.

To determine the cost function implied by the assumptions imposed on the firm, substitute $x = \frac{v_x}{p_x}$ and $y = \frac{v_y}{p_y}$ into the firm's production function, equation (18b), thus implying

$$\frac{1}{10}\sqrt{p_x p_y}\; q \quad = \sqrt{v_x v_y} \tag{22}$$

which by virtue of equations (20) and (21) may be restated in each of the following forms

$$v_x = \frac{1}{10}\sqrt{p_x p_y}\; q \tag{23a}$$

$$v = \frac{1}{5}\sqrt{p_x p_y}\; q \tag{23b}$$

Thus, the optimal variable cost of production may be obtained from the expression $\frac{1}{5}\sqrt{p_x p_y}\, q$. The total cost of production is obtained by adding the fixed cost (p_z) to the variable cost. This implies the following total cost of production for the firm

$$c(q) \;= p_z + \frac{1}{5}\sqrt{p_x p_y}\; q \tag{24}$$

which, as expected, is linear.[30]

In terms of the theory of production,[31] the "income constraint" during $[0,1]$ is $v = \frac{1}{5}\sqrt{p_x p_y}\; q$ and the maximum quantities of x and y which may therefore be used in production are $\frac{v}{p_x} = \frac{1}{5}\sqrt{\frac{p_y}{p_x}}\; q$ and $\frac{v}{p_y} = \frac{1}{5}\sqrt{\frac{p_x}{p_y}}\; q$. The "isocost line"[32] is obtained by interpolating on the points $\left[\frac{v}{p_x}, 0\right]$ and $\left[0, \frac{v}{p_y}\right]$ thus giving

30. This follows from the fact that F is homogeneous of degree one.

31. Ibid., p.140.

32. Ibid.

$$v = v_x + v_y \tag{25}$$

Isoquants[33] are obtained by letting q assume an assortment of non-negative real numbers in (18b) as follows

$$y = \left[\frac{K}{10}\right]^2 \frac{1}{x} \tag{26}$$

where K is some permissible value of q. Each isoquant specifies the most efficient "technical" combination of x and y which may be used to produce the output (in our case K) which is implied by the isoquant.[34]

Having determined the set of optimum production strategies, we now determine the maximum returns to be earned from each. To accomplish this task, restate the firm's demand function in the following equivalent form

$$p = \frac{500 - q}{20}$$

Multiplying through this expression by q implies that revenue income at time one amounts to

$$S = \frac{500q - q^2}{20}$$

where S = pq. Substituting equation (24) into the above expression implies

$$S = \frac{1}{20\,m^2}(c - p_z)(500m + p_z - c) \tag{27}$$

where from equation (24) $m = \frac{1}{5}\sqrt{p_x p_y}$ is the firm's marginal cost of production. Equation (27) specifies the maximum return to be obtained at time one from investing c in productive resources at time zero. This is the point at which Fisher commences his analysis of firms as they affect the rate of interest.[35] We are thus in a position to specify the equations satisfying Fisher's "Investment Opportunity Principles".

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33. Ibid., pp. 133-136.
34. Ibid.
35. Fisher, op.cit., chapter 6.

"Investment Opportunity Principle A" — 2 equations

It is a relatively simple matter to derive the productive opportunity locus from equation (27). Suppose we impose the assumption that the type U agent is bequeathed an endowment which is sufficient to finance productive investments at time zero. In terms of our earlier notation this implies that $y_o = c = -p_o$, since an investment in productive facilities at time zero represents a negative cash flow at that time. As an investment of $-p_o$, in productive facilities at time zero returns an amount $s = p_1$ at time one, equation (27) may be restated in the following implicit form

$$K(p_o,p_1) = \frac{-1}{80}(p_o+2000)(p_o+3000)-p_1 = 0 \qquad (28)$$

where we have assumed $p_z = 2000$ and $p_x = p_y = 10$. Equation (28) is the firm's productive opportunity locus and, as such, defines the productive opportunities for physically transforming time zero consumption into time one consumption.[36] From the productive opportunity locus we derive the marginal rate of substitution (in production) between funds at time zero and funds at time one.

$$\frac{\partial K}{\partial p_1} \bigg/ \frac{\partial K}{\partial p_o} = \frac{80}{5000 + 2p_o} \qquad (29)$$

Equations (28) and (29) together satisfy Fisher's "Investment Opportunity Principle A".

"Investment Opportunity Principle B" — 1 equation

"Investment Opportunity Principle B" requires that the marginal rate of substitution (in production) between the successive production

36. Ibid.

dates j-1 and j must be equivalent to $\frac{-1}{(1+{}_{j-1}r_j)}$ where it will be recalled, ${}_{j-1}r_j$, is the one period rate of interest prevailing during the interval $[j-1,j]$. In the present context, the marginal rate of substitution (in production) between the successive production dates zero and unity is defined by equation (29), thus implying

$$p_o = 40(1+{}_or_1) - 2500 \tag{30}$$

This equation satisfies "Investment Opportunity Principle B". Our progress to date is summarized in Table 6.1 where we provide a complete listing of the equations derived above. We now determine equilibrium values for each of the unknowns appearing in this system of equations.

6.2.3 The System Solved

From Table 6.1 it will be observed that we have eleven equations expressed in terms of the following ten unknown variables

$$\frac{\partial J}{\partial b_1}\bigg/\frac{\partial J}{\partial b_o},\ \frac{\partial U}{\partial c_1}\bigg/\frac{\partial U}{\partial c_o},\ \frac{\partial K}{\partial p_1}\bigg/\frac{\partial K}{\partial p_o},\ b_o,\ b_1,\ c_o,\ c_1,\ p_o,\ p_1 \text{ and } {}_or_1$$

Recall, however, that one of the equations appearing under the "Market Principles" head is redundant. To illustrate this, add equations (17a) and 17b) thus giving

$$c_o + b_o + \frac{c_1}{(1+{}_or_1)} + \frac{b_1}{(1+{}_or_1)} = 2y_o + p_o + \frac{p_1}{(1+{}_or_1)}$$

Substituting equation (16a) into the above expression and multiplying through by $(1+{}_or_1)$ gives

$$c_1 + b_1 \quad = \quad p_1 \tag{16b}$$

TABLE 6.1

THE FISHERINE EQUATIONS

Impatience Principle A — 2 equations

$$\frac{\partial J}{\partial b_1} \Big/ \frac{\partial J}{\partial b_o} = b_o b_1^{-1} \tag{14a}$$

$$\frac{\partial u}{\partial c_1} \Big/ \frac{\partial u}{\partial c_o} = c_o c_1^{-1} \tag{14b}$$

Impatience Principle B — 2 equations

$$b_o = \frac{b_1}{(1+{}_o r_1)} \tag{15a}$$

$$c_o = \frac{c_1}{(1+{}_o r_1)} \tag{15b}$$

Market Principle A — 2 equations

$$c_o + b_o = 2y_o + p_o \tag{16a}$$

$$c_1 + b_1 = p_1 \tag{16b}$$

Market Principle B — 2 equations

$$c_o + \frac{c_1}{(1+{}_o r_1)} = y_o + p_o + \frac{p_1}{(1+{}_o r_1)} \tag{17a}$$

$$b_o + \frac{b_1}{(1+{}_o r_1)} = y_o \tag{17b}$$

Investment Opportunity Principle A — 2 equations

$$K(p_o, p_1) = -\frac{1}{80}(p_o + 2000)(p_o + 3000) - p_1 = 0 \tag{28}$$

$$\frac{\partial K}{\partial p_1} \Big/ \frac{\partial K}{\partial p_o} = \frac{80}{5000+2p_o} \tag{29}$$

Investment Opportunity Principle B — 1 equation

$$p_o = 40(1+{}_o r_1) - 2500 \tag{30}$$

which is equation (16b). Hence, one of the "Market Principles" equations is redundant. We are thus left with ten independent equations to determine the ten unknowns of the system. We proceed, therefore, to determine each of these unknowns.

The available supply of consumptive services at time one is obtained by substituting equation (30) into equation (28), the firm's productive opportunity locus

$$p_1 = -\frac{1}{80}\left[40(1+{}_or_1) - 500\right].\left[40\ (1+{}_or_1) + 500\right]$$

which may be restated as

$$p_1 = 3125 - 20(1+{}_or_1)^2 \tag{31}$$

The type U agent's demand for these services may be obtained by substituting equation (15b) into (17a) as follows

$$\frac{2c_1}{(1+{}_or_1)} = y_o + p_o + \frac{p_1}{(1+{}_or_1)}$$

Recall that each agent was bequeathed an endowment of $y_o = -p_o$ which was sufficient to finance productive investments at time zero. Using this condition, it follows that

$$c_1 = \frac{1}{2}\,p_1 \tag{32}$$

represents the type U agent's "consumption function" for time one consumptive services.

To obtain the type J agent's demand for time one consumptive services, substitute equation (15a) into (17b) thus obtaining

$$b_1 = -\frac{1}{2}\,p_o(1+{}_or_1) \tag{33}$$

where we have used the condition that $y_o = -p_o$. Equation (33) is the type J agent's "consumption function" for time one consumptive services.

Adding equations (32) and (33) determines the agents' aggregate demand for time one consumptive services

$$c_1 + b_1 \quad = \quad \frac{1}{2}\left[p_1 - p_o(1+{}_or_1)\right]$$

By virtue of equations (30) and (31), the aggregate demand function may be restated as

$$c_1 + b_1 \quad = \quad \frac{1}{2}\left[3{,}125 + 2500\ (1+{}_or_1) - 60(1+{}_or_1)^2\right] \qquad (34)$$

As by "Market Principle A" we must have

$$c_1 + b_1 \quad = p_1 \qquad (16b)$$

or that the aggregate demand for time one consumptive services must be equivalent to the aggregate supply, it follows using equations (31) and (34) that

$$0 = -3125 + 2500\ (1+{}_or_1) - 20(1+{}_or_1)^2 \qquad (35)$$

which has as its solution ${}_or_1 = 0.2628$. The consequences of this interest rate are depicted in Table 6.2. The results set forth in Table 6.2 may be obtained by direct substitution in the various functions. Thus, for example, the initial endowment $y_o = 2{,}449.49$ may be obtained by substituting ${}_or_1 = 0.2628$ into equation (30) and recalling that $y_o = -p_o$. By similar use of equation (31) it follows that $p_1 = 3093.11$, whilst this result may be used in conjunction with equation (32) to obtain the type U agent's time one consumption of 1546.46.

By analogous procedures we may derive the price and factor usage data relating to the type U agent's productive activities. Since by hypothesis $p_z = 2000$ and $p_x = p_y = 10$ it follows from equation (24) that the firm's cost function is defined by

$$C(q) \quad = \quad 2{,}000 + 2q$$

TABLE 6.2

EQUILIBRIUM PRODUCTION AND CONSUMPTION

Individual	Endowment		Productive Solution		Consumptive Solution	
	y_o	y_1	p_o	p_1	c_o	c_1
U	2,449.49	-	-2,449.49	3,093.11	1,224.75	1,546.56
J	2,449.49	-	-	-	1,224.74	1,546.55
Totals	4,898.98	-	-2,449.49	3,093.11	2,449.49	3,093.11

Using the conditions $y_o = -p_o = c$, it follows that $q = 224.74$. Using this result and equations (20), (21) and (18a) we have $x = y = 22.47$ and $p = 13.76$. These results imply the income statement portrayed in Table 6.3.

There are several points about this Table which require emphasizing. Firstly, it will be observed that the firm's (maximum) return over cost for the interval $[0,1]$ is equivalent to the rate of interest prevailing during the interval. This position characterizes an economy in "long run equilibrium" in the sense that only "normal returns" are earned.[37] Secondly, the firm's net income during $[0,1]$ is in fact the excess of the firm's market value at time one over its market value at time zero.[38]

This completes our analysis of the Fisherine system. We now turn to the more important task of examining its significance to accounting theory.

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37. Liebhafsky, op.cit. p.290.

38. Fama and Miller, op.cit., p.74.

TABLE 6.3

INCOME STATEMENT FOR THE INTERVAL [0,1]

Sales (224.7449 x 13.7628)		3,093.11
Expenses		
Factor x (22.4745 x 10)	224.75	
Factor y (22.4745 x 10)	224.74	
Factor z (2000 x 1)	2,000.00	2,449.49
Realized profit		643.62
Rate of return (over cost)		0.2628

6.3 The Accounting Implications

Our interest in Fisher's work is restricted solely to its accounting implications. Unfortunately, the theory is developed under several restrictive assumptions, the consequence of which is to reduce its analytical power, at least for accountants. For example, the presumed existence of perfect knowledge, and hence the absence of risk, implies that there is but one rate of interest prevailing over each time interval,[39] and obviously false specification of reality.[40] Despite this, however, there are several respects in which the model does contain both *a priori* and empirical implications for accounting theory. Indeed, Fisher was probably first in utilizing the model for this purpose

> "Past cost does not affect present valuations except indirectly as it affects future expected income and cost... The only cases in which cost ... is equal to value is where this value is also equal to the estimate of worth on the basis of future expectation; when, in other words, cost is superfluous as a determinant of value".[41]

In this section we shall deploy Fisher's analysis to provide a rationale for each of the measurement models specified in Chapter 4. We commence the section with an analysis of the replacement cost measurement system, placing particular emphasis on the predictive properties attributed to current operating profit by Edwards and Bell.[42] In this respect, Fisher's work contains both *a priori* and

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39. Fisher, *op.cit.*, p.206.

40. *Ibid.*

41. *Ibid.*, p.467.

42. Edwards, E.O. and Bell, P.W. *The Theory and Measurement of Business Income*. Berkley, California: The University of California Press, 1961.

empirical implications for replacement cost measurement which only partially supports the position taken by Edwards and Bell. Specifically we shall prove under a set of restrictive assumptions, that the current operating profit (over cost) of one interval is a lower bound for the ratio of realized (operating) income to cost of the next interval. Whilst the empirical evidence is certainly consistent with this hypothesis it is also consistent with several competing hypotheses, so much so that using a purely empirical criterion, no one hypothesis distinguishes itself over the others.

Having analyzed replacement cost measurement in the context of the Fisherine system we then focus our attention on market value measurement. In this respect, we shall find Fisher's analysis to corroborate much of what Edwards and Bell have to say about the market value system of measurement. As a final exercise, we examine the C.P.P. system of accounting measurement. Unfortunately, we shall find Fisher's work to be of little assistance in providing a satisfactory rationale for this measurement scheme.[42(a)] Indeed, it would seem that the works of Walras,[43] Pareto[44] and Hicks[45] hold more potential in this regard since their analyses commence with the product and factor markets omitted from the Fisherine analysis.

We now turn our attention to the first of these topics, namely a consideration of the replacement cost scheme of accounting measurement.

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43. Hicks, J.R. "Léon Walras," Econometrica, 11 (1934), pp. 338-348.

44. Hicks, J.R. Value and Capital. Oxford: The Clarendon Press, 1946, pp. 13-18.

45. Ibid.

42(a) As Fisher was the first adherent of indexation, we are here, perhaps, being unduly harsh. see Fisher, Irving. The Purchasing Power of Money. New York: The McMillan Company, 1911.

6.3.1 The Predictive Ability of Current Operating Profit

Although Edwards and Bell[46] dubbed their book the "Theory and Measurement of Business Income", throughout most of the work "business income" occupies a fairly subordinate position to " current operating profit". One of the more important, if not the most important proposition contained in the book, for example, attributes a predictive property to current operating profit.

> "Current operating profit can be used for predictive purposes if the existing production process and the existing conditions under which that process is carried out are expected to continue in the future; current operating profit then indicates the amount the firm can expect to make in each period over the long run".[47]

The stability conditions implied by this statement - stable technology, demand and factor prices - are unrealistic to say the least. Indeed, they imply that current operating profit, business profit and realized profit are identically equal and constant through time.[48] After acknowledging the verity of this criticism,[49] Edwards and Bell substitute the following proposition in its place.

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46. Edwards and Bell, op.cit.

47. Ibid., p.99.

48. Revsine, L. Replacement Cost Accounting. Englewood Cliffs: New Jersey: Prentice-Hall, Inc., 1973, p.119.

49. Edwards and Bell, loc.cit.

> "If a particular production process promises a larger current operating profit in this period than that promised by any other production process, is it [not] reasonable to assume that the production process will also promise higher current operating profits in subsequent periods than alternative processes, even though conditions have changed in those periods?"[50]

But this proposition is not free of criticism. We may legitimately question the motive for providing such "predictions" to the owners of productive facilities. There is no suggestion by Edwards and Bell that current operating profit represents relevant information to the owner of such productive facilities, save for the purpose of predicting itself. The circularity of this contention is obvious and has been well documented elsewhere.[51] But unless we can provide some rationale for current operating profit, there can be no justification for investigating its empirical significance. Fortunately, however, Fisher's work does provide some insight into this problem.

The relevance of Fisher's work to the problem at hand may be stated in the following terms

> Consider the intervals $[i,j]$ and $[j,k]$ with $i < j < k$. Let the function $S_t(q)$ describe the "monetary value" of a firm's sales (at time t) in terms of its output (q) and suppose $S_k(q') \geqslant S_j(q')$ where q' is output produced at time i and sold at time j. Then the ratio of current operating profit to the replacement cost of goods sold during $[i,j]$ is a lower bound for the maximal ratio of realized (operating) profit to the cost of goods sold during $[j,k]$.

To prove this result, let $C_i(q) = \underset{\sim}{p}_i^T \cdot \underset{\sim}{Q}_i(q)$ be the function whose value is the minimum production cost of q at time i; where $\underset{\sim}{p}_i$ is the vector of (per unit) factor prices at time i and $\underset{\sim}{Q}_i(q)$ is the vector whose elements are the factors of production (in units)

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50. Ibid.

51. Lee, T.A.: "The Cash Flow Accounting Alternative for Corporate Financial Reporting" in Cees Van Dam (ed.) Trends in Managerial and Financial Accounting. Leiden/Boston: Martinus Nijhoff Social Sciences Division. 1978, p.68.

consumed in producing q. It then follows that $C_j\ (q') \leqslant \underset{\sim}{p}_j^T \underset{\sim}{Q}_i\ (q')$. Since by hypothesis $S_j(q') \leqslant S_k(q')$ we then have

$$S_j\ (q') + C_j\ (q') \quad \leqslant \quad S_k\ (q') + \underset{\sim}{p}_j^T \circ \underset{\sim}{Q}_i\ (q')$$

or, more conveniently

$$S_j\ (q') - \underset{\sim}{p}_j^T \circ \underset{\sim}{Q}_i\ (q') \quad \leqslant \quad S_k\ (q') - C_j\ (q')$$

Now since $C_j\ (q') \leqslant \underset{\sim}{p}_j^T \cdot \underset{\sim}{Q}_i\ (q')$, it necessarily follows that

$$C_j(q')\ \left[S_j(q') - \underset{\sim}{p}_j^T \cdot \underset{\sim}{Q}_i\ (q')\right] \leqslant \underset{\sim}{p}_j^T \cdot \underset{\sim}{Q}_i\ (q') \left[S_k(q') - C_j\ (q')\right]$$

From which it follows

$$\frac{S_j\ (q') - \underset{\sim}{p}_j^T \cdot \underset{\sim}{Q}_i\ (q')}{\underset{\sim}{p}_j^T \cdot \underset{\sim}{Q}_i\ (q')} \leqslant \frac{S_k\ (q') - C_j\ (q')}{C_j\ (q')} \tag{36}$$

The expression $\dfrac{S_j\ (q') - \underset{\sim}{p}_j^T \cdot \underset{\sim}{Q}_i\ (q')}{\underset{\sim}{p}_j^T \cdot \underset{\sim}{Q}_i\ (q')}$ is the current operating profit of the interval $\left[i,j\right]$ divided by the replacement cost (at time j) of the factors consumed (at time i) in producing the firm's output q'.

The expression $\dfrac{S_k\ (q') - C_j\ (q')}{C_j\ (q')}$ is the ratio of realized (operating) income to cost from producing q' during $\left[j,k\right]$. This ratio can never exceed the maximum ratio of realized (operating) income to cost during $\left[j,k\right]$, thus proving the result.

The significance of this result to the owners of productive facilities is that under certain circumstances it can be used to provide a lower bound on the firm's market value at time k. Specifically, suppose we impose the following "normal return" assumption.

> During the interval $\left[j,k\right]$ the firm's maximal ratio of realized (operating) income to cost is equivalent to the rate of interest prevailing during $\left[j,k\right]$.

Using this assumption, we now illustrate how the above result may be used to bound the firm's market value at time k. Suppose q* to be the output which maximizes the ratio of realized (operating) profit to cost during $[j,k]$. In terms of the above proposition, this implies

$$\frac{S_j(q') - p_j^T \cdot Q_i(q')}{p_j^T \cdot Q_i(q')} \leqslant \frac{S_k(q^*) - C_j(q^*)}{C_j(q^*)} \tag{37}$$

From this, it follows that the return $S_k(q^*)$ (at time k) to productive investments $C_j(q^*)$ (at time j) has the following lower bound

$$C_j(q^*) \left[1 + \frac{S_j(q') - p_j^T \cdot Q_i(q')}{p_j^T \cdot Q_i(q')}\right] \leqslant S_k(q^*) \tag{38}$$

The surplus of funds to productive requirements at time j amounts to $S_j(q') - C_j(q^*)$. Since, by hypothesis, the maximum return (over cost) during $[j,k]$ defines the rate of interest prevailing during the interval, these funds accumulate to $\left[S_j(q') - C_j(q^*)\right] \cdot \left[1 + \frac{S_j(q^*) - C_j(q^*)}{C_j(q^*)}\right]$ at time k. By virtue of equation (37) these funds have the following lower bound as of that date.

$$\left[S_j(q') - C_j(q^*)\right] \cdot \left[1 + \frac{S_j(q') - p_j^T \cdot Q_i(q')}{p_j^T \cdot Q_i(q')}\right] \leqslant \left[S_j(q') - C_j(q^*)\right] \cdot \left[1 + \frac{S_j(q^*) - C_j(q^*)}{C_j(q^*)}\right] \tag{39}$$

The firm's market value at time k is given by the sum of the productive returns, $S_k(q^*)$, and the accumulated value of surplus funds $\left[S_j(q') - C_j(q^*)\right] \cdot \left[1 + \frac{S_j(q^*) - C_j(q^*)}{C_j(q^*)}\right]$. Using equations (38) and (39), a lower bound for this sum is given by the following expression

$$S_j(q')\left[1+\frac{S_j(q')-\underset{\sim}{p}_j^T\cdot\underset{\sim}{Q}_i(q')}{\underset{\sim}{p}_j^T\cdot\underset{\sim}{Q}_i(q')}\right] \leqslant S_k(q^*)+\left[S_j(q')-C_j(q^*)\right].$$
$$\left[1+\frac{S_k(q^*)-C_j(q^*)}{C_j(q^*)}\right] \tag{40}$$

Note that knowledge of the firm's current operating profit ratio and productive returns at the prior production date is sufficient to operationalize this lower bound. For computational purposes, however, it may be more convenient to restate equation (40) in the following equivalent form

$$\frac{\left[S_j(q')\right]^2}{\underset{\sim}{p}_j^T\cdot\underset{\sim}{Q}_i(q')} \leqslant \frac{S_j(q')\cdot S_k(q^*)}{C_j(q^*)} \tag{41}$$

To illustrate the application of these results, suppose the firm whose income statement is exhibited in Table 6.3 to continue in existence over the interval $[1,2]$. Impose the assumption that demand for the firm's output remains stable during $[1,2]$ but that factor prices vary to 8,18 and 2400 for x, y and z respectively at time one. Suppose also that (at best) the firm earns a "normal return" over cost during $[1,2]$. Substituting the factor prices into equation (27) gives

$$S = \frac{5}{576}(C-2400)(3600-C) \tag{42}$$

Recall that equation (42) specifies the maximum return, S, which may be obtained at time two from investing C in productive facilities at time one. It follows that the optimum return (over cost) is obtained from the expression

$$\frac{d}{dC}\left[\frac{S-C}{C}\right] = \frac{d(S/C)}{dC} = \frac{5}{576}\left[\frac{8,640,000}{C^2}-1\right] = 0$$

which implies C = 2,939.39 as the optimum investment in productive facilities at time one. Using this result in conjunction with equation (42) implies S = 3,093.11 and $\frac{S}{C} = 1.0523$. These results yield the income statement exhibited in Table 6.4. Note that the productive strategy depicted in this income statement is "optimal" in the sense that it provides a higher return (over cost) than any alternative strategy.

In Table 6.5 the current operating profit of the interval $[0,1]$ is computed in conformity with the method proposed by Edwards and Bell and utilized in proving the above propositions. Note that the current operating profit ratio $\frac{108.77}{2984.34} = 0.0364$ is in fact a lower bound for the ratio of realized (operating) income (to cost) $\frac{153.72}{2939.39} = 0.0523$. Further, by using equation (40) a lower bound for the firm's market value at time two is $S_j(q')\left[1+\frac{S_j(q')-\underset{\sim}{p}_j^T \cdot \underset{\sim}{Q}_i(q')}{\underset{\sim}{p}_j^T \cdot \underset{\sim}{Q}_i(q')}\right] =$ (3,093.11 x 1.0364) or 3,205.70. The actual market value is of course

$$\frac{S_j(q').S_k(q^*)}{C_j(q^*)} = \frac{(3,093.11)^2}{2,939.39}$$ or 3,254.87 which may be verified from equation (41).

To be sure, the above analysis is founded upon an extremely simplistic view of the economy. Firms are viewed as consisting of a sequence of "cash" based ventures. Further, as productive resources are purchased at time of use, increased factor prices are instantaneously reflected in firms' realized incomes. Finally, we assume conditions of constant technology (represented by a time invariant production function), of non-decreasing demand and certainty with respect to competing productive opportunities, returns and interest rates, etc. These

TABLE 6.4

INCOME STATEMENT FOR THE INTERVAL [1,2]

Sales (224.7448 x 13.7628)		3,093.11
Expenses		
Factor x (33.7117 x 8)	269.69	
Factor y (14.9830 x 18)	269.70	
Factor z (2400 x 1)	2,400.00	
		2,939.39
Realized (operating) profit		153.72
Interest income*		8.04
Realized profit		161.76
Rate of return (over cost)		0.0523

* $(3093.11 - 2939.39) \times \dfrac{153.72}{2939.39}$

TABLE 6.5

CURRENT OPERATING PROFIT FOR THE INTERVAL [0,1]

Sales (224.7449 x 13.7628)		3,093.11
Expenses		
Factor x (22.4745 x 8)	179.80	
Factor y (22.4745 x 18)	404.54	
Factor z (2400 x 1)	2,400.00	
		2,984.34
Current operating profit		108.77
Realizable cost savings		
Factor x (22.4745 x-2)	(44.95)	
Factor y (22.4745 x 8)	179.80	
Factor z (2400-2000)	400.00	
		534.85
Business profit		643.62
Unrealized cost savings		-
Realized profit		643.62
Current operating profit (over cost)		0.0364

assumptions are, of course, a simplification of reality designed to facilitate the derivation and analysis of a priori propositions. But this does not imply that the model is devoid of empirical content, for the significance of an economic model lies not in the accuracy of its assumptions, but in the predictive ability of its conclusions.[52] In the next section, therefore, we consider the empirical implications of the above model.

6.3.2 The Empirical Significance of Current Operating Profit

In the previous section it was demonstrated under conditions of non-decreasing demand that the current operating profit ratio of an interval is a lower bound for the return (over cost) of the next succeeding interval. We observed that the assumptions utilized in reaching this conclusion were, in some instances, a false specification of reality and that a possible consequence of this is that the model may be a poor device through which to make empirical generalizations. In this section, therefore, we undertake to test the model against some available empirical evidence. We should like to emphasize from the very beginning, however, that the tests conducted herein are of a pilot nature only and that much more comprehensive testing procedures need to be adopted if the results reported herein are to be regarded as valid empirical generalizations.[53]

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52. Friedman, op.cit.
53. Several reasons may be cited for this conclusion. Firstly, the sample analyzed is small, consisting of nineteen observations drawn from the period 1930-1949. Data for the ensuing period was not available in the Edinburgh region and could not be obtained given the time constraint placed upon the present work. Secondly, the current operating profit of each year is an estimated figure. Should these estimates show a consistent bias (in one direction or the other), the estimates of the parameters α and β may not be minimum variance unbiased. Finally, we assume that the unknown error term possesses a normal frequency functio[n]. Should this assumption be unjustified, then the conclusions concerning β contained in the test may be in error. For a more comprehensive treatment of these points, see Mood, A.M. and Graybill, F.A. Introduction to the Theory of Statistics. New York: McGraw-Hill Book Company, Inc., 1963, Chapter 13.

Table 6.6 contains estimates of the return (over cost) and the current operating profit ratio of U.S. firms over the two decades 1930 through 1949. Most of the data underlying these ratios were initially reported by Edwards and Bell. Where necessary, however, we have complemented the material with information from other sources, these being noted in the Table itself. The data contained in this Table lend themselves to both regressive and non-parametric tests, the results of which we now summarize.

The first test applied to the data of Table 6.6 was the non-parametric "sign test".[54] Specifically, define the parameter $p = \frac{1}{2}$ to be the expected proportion of cases in which the current operating profit ratio of one interval exceeds the return (over cost) of the next. The specification of $p = \frac{1}{2}$ is based on the assumption that there is in fact no relationship between these two ratios. If, in addition, we suppose increased sales to reflect an increased demand for firms' output, then there are thirteen (13) instances[55] where the return (over cost) exceeds the current operating profit ratio and one (1) instance[56] where the current operating profit ratio exceeds the return (over cost). The probability of this event, given that $p = \frac{1}{2}$, is computed as follows

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54. Freund, J.E. Mathematical Statistics. Englewood Cliffs, New Jersey: Prentice-Hall, Inc., 1971, pp. 343-344.

55. The thirteen instances are 1933 through 1937 (inclusive), 1939 through 1943 (inclusive) and 1946 through 1948 (inclusive).

56. This being the year 1944.

TABLE 6.6

ESTIMATES OF THE RETURN (OVER COST)

(In millions of dollars)

Year	1930	1931	1932	1933	1934	1935	1936	1937	1938	1939
Realized income	3,185	-776	-2,983	153	1,656	2,986	5,636	6,113	3,053	6,219
Current operating profit	5,901	1,382	-1,821	-1,888	832	2,534	4,663	5,605	3,568	5,053
Cost of production	131,940	108,834	85,467	85,164	98,520	109,227	124,952	134,613	116,323	125,700
Current cost of production	129,224	106,676	84,305	87,205	99,344	109,679	125,925	135,121	115,808	126,866
Realized income (over cost)	0.0241	-0.0071	-0.0349	0.0018	0.0168	0.0273	0.0451	0.0454	0.0263	0.0495
Current operating profit ratio	0.0457	0.0130	-0.0216	-0.0217	0.0084	0.0231	0.0370	0.0415	0.0308	0.0398
Sales	136,588	108,057	81,638	84,234	101,490	114,650	132,723	142,443	120,454	132,878

TABLE 6.6 (Contd.)

Year	1940	1941	1942	1943	1944	1945	1946	1947	1948	1949
Realized income	9,086	16,751	20,657	24,316	23,027	18,749	22,126	28,836	32,164	25,538
Current operating profit	8,369	13,558	18,499	22,546	21,752	17,159	15,333	20,620	26,744	24,376
Cost of production	138,889	173,757	194,292	221,466	235,654	234,102	263,555	336,130	376,378	365,063
Current cost of production	139,606	176,950	196,450	223,236	236,929	235,692	270,348	344,706	381,798	366,225
Realized income (over cost)	0.0654	0.0964	0.1063	0.1098	0.0977	0.0801	0.0840	0.0858	0.0855	0.0700
Current operating profit ratio	0.0599	0.0766	0.0942	0.1010	0.0918	0.0728	0.0567	0.0598	0.0700	0.0666
Sales	148,237	190,432	217,861	249,592	262,201	255,408	288,954	367,746	410,966	393,450

Sources and Notes: The figures for realized income and current operating profit were obtained from Edwards and Bell The Theory and Measurement of Business Income, p.228. The figures for sales and production cost were obtained from U.S. Department of Commerce, Statistical Abstract of the United States (1931-1960). The figures for current cost of production were obtained by adding realized cost savings for fixed assets and inventory, as reported by Edwards and Bell, to production cost.

$$P\left[j \leqslant 1 \;\middle|\; p = \tfrac{1}{2} \right] = \sum_{j=0}^{1} {}^{14}C_j \left(\tfrac{1}{2}\right)^{14}$$

$$= \frac{15}{16384}$$

$$P\left[j \leqslant 1 \;\middle|\; p = \tfrac{1}{2} \right] = 0.0009$$

Given this result, it is unlikely in the extreme that the current operating profit ratio bears no relationship to the return (over cost).[57] We thus reject this contention and accept the alternative hypothesis that under conditions of non-decreasing demand, the current operating profit ratio of one interval is a "consistent" lower bound for the return (over cost) of the next.

The exact relationship between the current operating profit ratio and the return (over cost) is, of course, dependent on the nature of the demand function confronting the firm and also the available productive opportunities. If, however, we make the usual concession to convenience and assume the relationship to be linear, we would then have it that

$$i_{t+1} = \alpha + \beta r_t + \epsilon_t$$

57. Should the years of decreasing aggregate sales be included, the evidence is much less impressive. For we then have that the years 1931, 1932, 1938, 1944, 1945 and 1949 are "counter examples" to the theory advanced. We then have

$$P\left[j \leqslant 6 \;\middle|\; p = \tfrac{1}{2} \right] = \sum_{j=0}^{6} {}^{19}C_j \left(\tfrac{1}{2}\right)^{19}$$

$$= \frac{43,796}{524,288}$$

$$P\left[j \leqslant 6 \;\middle|\; p = \tfrac{1}{2} \right] = 0.0836$$

Given this result, we accept the null hypothesis $p = \frac{1}{2}$ at the 0.05 significance level.

where i_{t+1} is the return over cost of the interval $[t,t+1]$, r_t is the current operating profit ratio of the interval $[t-1,t]$, ϵ_t is an unobservable error term and α and β are unknown parameters. For the n = 14 observations analyzed above, the "least squares" estimates of α and β are a = 0.0269 and b = 0.8739 with the (unbiased) estimate of the R^2 statistic being 0.91. If we now suppose $\beta = 0$ and that the error term has a normal frequency function with mean zero, it follows that the variable[58] $F = \dfrac{(n-2)\, b^2 \sum (r_t-\bar{r})^2}{\sum e_t^2}$

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58. Suppose U to be a /Chi squared variate with m degrees of freedom and V to be Chi squared variate with n degrees of freedom. Further, suppose U and V to be independent. It can then be shown that the variate $F = \dfrac{nU}{mV}$ has an F(m,n) frequency function. By theorem, the variate $(b-\beta)\sqrt{\sum (r_t-\bar{r})^2} \big/ \sigma$ has a normal frequency function with zero mean and unit variance, thus implying that the variate $U = (b-\beta)^2 \sum (r_t-\bar{r})^2 \big/ \sigma^2$ has a X^2 frequency function with one degree of freedom. Also, the variate $V = \sum (i_{t+1}-a-br_t)^2 \big/ \sigma^2 = \sum e_t^2 \big/ \sigma^2$ possesses a /Chi squared frequency function with (n-2) degrees of freedom. Since U and V are independent, it follows that the variate

$$F = \frac{(n-2)(b-\beta)^2 \sum (r_t-\bar{r})^2 \big/ \sigma^2}{\sum e_t^2 \big/ \sigma^2}$$

$$F = \frac{(n-2)(b-\beta)^2 \sum (r_t-\bar{r})^2}{\sum e_t^2}$$

has an F(1,n-2) frequency function. Setting $\beta = 0$ yields the result displayed in the text. On this topic generally, see Mood and Graybill, op.cit., p.211, pp. 226-232 and p.333.

58(a) Note that although the apriori results underlying the above analysis relate only to individual firms, the empirical tests were conducted on aggregate data. The main reason for use of aggregated data was to control for risk.

where $e_t = i_{t+1} - a - br_t$, has an $F(1,n-2)$ frequency function. In our case, we have an observed value of $F = 137.73$ with 1 and twelve degrees of freedom. Since $P\left[F(1,12) \geqslant 9.33 \mid \beta = 0\right] = 0.01$, we reject the contention that $\beta = 0$ and accept the alternative hypothesis that under conditions of increasing demand i_{t+1} and r_t are in fact related.[58(a)]

At first sight, the above results may appear to provide conclusive evidence for the proposition advanced, namely that under conditions of increasing demand, the current operating profit ratio of an interval is a lower bound for the return (over cost) of the next succeeding interval. We should note, however, that the data exhibited in Table 6.6 are also consistent with several competing hypotheses, so much so that on purely empirical grounds, no one hypothesis distinguishes itself over the others. This point is pursued further in Appendix 6A to this chapter.

A second motive for the provision of replacement cost financial information derives from the desire of a firm's owners to maintain the firm's "productive capacity". Unfortunately, the usual means of implementing this requirement effects a consistent over-statement of the costs necessary to achieve the objective. This is a topic we pursue in the next section.

6.3.3 The Maintenance of Productive Capacity

The reputed predictive ability of current operating profit is but one of two major reasons cited in the literature for the provision of replacement cost data to the owners of productive facilities. A second and perhaps more familiar justification relates to the maintenance of productive capacity. The <u>Guidance Manual on Current</u>

Cost Accounting expresses the argument in the following terms

> "CCA [Current Cost Accounting] in computing profit attempts to deduct from ... revenue the amount needed to restore the productive capacity ... consumed [in generating that revenue]." 59

The usual method of implementing this procedure, and indeed the method illustrated in the Manual,[60] is to cost the factors consumed (in producing the output from which the firm's revenue is generated) at the replacement costs prevailing at the time of the output's sale. The mechanics of the method were, in fact, illustrated earlier.

Recall that the income statement exhibited in Table 6.3 represents the optimal productive strategy for a firm over the interval [0,1]. Table 6.5 computes the current operating profit for the firm over the interval [0,1] on the assumption that factor prices (per unit) have varied from 10, 10 and 2000 for factors x, y and z respectively at time zero to 8, 18 and 2400 respectively at time one. Thus, if we apply the "logic" of the manual, it would seem that the firm should retain 2,984.34 if it is to maintain its productive capacity at 224.7449 units.

That such is not the case is illustrated in Table 6.4 where this same output (224.7449 units) is produced at a cost of 2,939.39, marginally less than the 2,984.34 "predicted" by Table 6.5. The fallacy in computing the replacement cost of disposals by the method demonstrated in the Manual is that it assumes that the optimal

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59. Inflation Accounting Steering Group, Guidance Manual on Current Cost Accounting. Institute of Chartered Accountants in England and Wales, 1976, p.13.

60. Ibid., pp. 13-14.

productive strategy is not affected by variations in the prices of factor inputs. That the set of optimum productive strategies available to a firm is dependent on factor prices is intuitively obvious and for the firm under consideration is demonstrated by the fact that the prices of factors x, y and z appear as arguments in equation (24), the firm's cost function. Indeed, if the firm under consideration were to be availed with an "income constraint"[61] of 2984.34 it would be enabled to produce $\frac{2984.34 - 2400}{2.4} = 243.475$ units of output, as is evident from substituting the factor prices ($p_x = 8$, $p_y = 18$ and $p_z = 2400$) at time one into equation (24). This is "substantially" (8.3%) in excess of the 224.7449 units mooted in Table 6.5.

To isolate the existence of the above problem and to suggest the means by which it may be solved are, of course, completely different issues. For whilst it is a relatively simple matter to prove the problem's existence, its solution does not come so readily to mind. If, however, we are acquainted with the firm's production (or cost) function, the most efficient means of reproduction may be calculated therefrom. Experience has shown, however, that these are not easily obtained and that there is a labyrinth of statistical and conceptual barriers confronting any potential endeavour in this direction.[62] Obviously, research into the area is required, however, for the consequence of shunning it is the provision of potentially misleading financial "information".

This completes our analysis of the replacement cost scheme of accounting measurement. We now focus attention on the market value system of accounting measurement.

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61. Liebhafsky, op.cit., p.140.

62. Johnston, J. Statistical Cost Analysis. New York: Econometric Handbook Series, 1960.

6.3.4 Realizable (Operating) Profit

In the analysis to date, we have assumed that the prices prevailing in the factor markets at time t represent the replacement costs prevailing at that time. Since, by hypothesis, the factor markets are characterized by perfect competition and, therefore, unhindered entrance and perfectly homogeneous (interchangeable) products, it follows that the prices prevailing in the markets may also be viewed as realizable (market) values.[63] We are thus provided with the means for investigating the validity of certain propositions relating to "the" market value system of accounting measurement.

In Table 6.7 we exhibit the realizable operating profit for the interval [0,1] corresponding to the optimal productive strategy displayed in Table 6.3, under the assumption that the market values (per unit) of factors x, y and z have varied from 10,10 and 2000 respectively at time zero to 8, 18 and 2400 respectively at time one. The measurement model upon which this income statement is based was described in section 4.3 (of chapter 4), although at that stage we made only sketchy comment on the model's significance. We are now in a position to make more concrete assertions.

Realizable operating profit was vested with the following significance by Edwards and Bell

> "[Realizable] operating profit arises because at least some of the assets of the firm have changed their form (or place) during the production moment. Operating profit is attributable solely to this change in form".[64]

In the context of the present example, it is clear how Edwards

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63. Liebhafsky, op.cit., p.21.

64. Edwards and Bell, op.cit., p.88.

TABLE 6.7

REALIZABLE OPERATING PROFIT FOR THE INTERVAL [0,1]

Sales (224.7449 x 13.7628)		3,093.11
Expenses		
Factor x (22.4745 x 8)	179.80	
Factor y (22.4745 x 18)	404.54	
Factor z (2400 x 1)	2,400.00	
		2,984.34
Realizable operating profit		108.77
Realizable capital gains		
Factor x (22.4745 x -2)	(44.95)	
Factor y (22.4745 x 8)	179.80	
Factor z (2400-2000)	400.00	
		534.85
Realizable profit		643.62
Unrealized capital gains		-
Realized profit		643.62

and Bell arrived at this conclusion. For had the firm refrained from production and merely elected to hold the productive resources in their original (unused) form over the interval $[0,1]$, the firm's market value would have been (108.77) less than it otherwise is. In contrast, a negative realizable operating "profit" indicates that the firm is better in electing to hold the productive resources in their original (unused) form and disposing of them at the succeeding production date. Thus, the realizable operating profit of a productive interval represents the potential contribution of production (as against purely holding activities) to the firm's market value at the end of the productive interval. This, of course, is precisely the function attributed to it by Edwards and Bell.

Realizable profit, however, is a conglomerate in the sense that it represents the combined contribution of the firm's productive and holding activities over some productive interval, to the firm's market value at the end of that interval. It was accorded the following significance by Edwards and Bell

> "When realizable profit falls below interest on opportunity cost (and is not expected to exceed it in the future), the date of abandonment has arrived".[65]

In terms of the above example this implies that should the firm's maximum return (over cost) be less than the rate of interest, then the resources available for investment in productive facilities should be loaned through the capital market (at the rate of interest). In light of Fisher's work, this makes obvious sense, since to invest in productive facilities under these circumstances (whether for holding or productive reasons) implies subordination of the wealth

65. Ibid., p. 101.

maximization criterion and thus contradiction of the separation theorem.[66]

This completes our analysis of the market value scheme of accounting measurement. We now focus our attention on the C.P.P. system of accounting measurement.

6.3.5 Current Purchasing Power

Several approaches have been utilized by accountants in an endeavour to provide a satisfactory rationalization for the provision of "current purchasing power" financial statements to the owners of productive facilities.[67] Each attempt, however, has been the recipient of varying degrees of criticism, so much so that it is not unfair to say that, with the possible exception of historical cost measurement, the C.P.P. system has been endowed with the most fragile of conceptual foundations. Nor should this be surprising, for Professor Hicks, when confronted with the problem of defining an "appropriate index number of prices" opined

> "To this question there is, I believe, no completely satisfactory answer". 68

66. Hirshleifer, op.cit., p.14.

67. For an excellent summary of the attempts at rationalizing the C.P.P. measurement system, see Gynther, R. "Why Use General Purchasing Power", Accounting and Business Research, 5,14 (Spring 1974), pp. 141-156.

68. Hicks, op.cit., p.175.

This opinion was expressed as long ago as 1939, and there has been little in the intervening period to suggest that Hicks' opinion was unfounded.[69] To what extent Fisher's analysis can be utilized in providing a solution is a moot point, though it is our view that the full potential of his work will not be realized until the link between utility functions for consumption goods and utility functions for consumption expenditures is completely specified. Recall that the theory of price-index numbers as formulated by Allen et al[70] in the first half of this century was developed in terms of utility functions for consumption goods, whereas, of course, Fisher's analysis proceeds in terms of utility functions for consumption expenditures. Although some research has been conducted along these lines, it has been far from conclusive.[71] Whilst we would not pretend to have the answer to this important question, the reader is entitled to a statement of our opinion and of its relevance to the problem at hand, namely the "theory" of current purchasing power accounting. We thus proceed.

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69. Gynther, op.cit., p.145.

70. Allen, R.G.D. "On the Marginal Utility of Money and its Application", Economica, XIII (May 1933), pp. 186-209.
Staehle, H. "A Development of the Economic Theory of Price Index Numbers," Review of Economic Studies, 11 (1934-35), pp. 163-188.

71. Fama, E.F. "Multiperiod Consumption-Investment Decisions", American Economic Review, LX (March 1970), pp. 163-174.
Fama, E.F. "Ordinal and Measurable Utility", in Jensen, op.cit., pp. 125-145.

A utility function for consumption expenditures may be decomposed into a utility function for consumption goods by merely imposing the condition that the amount available for consumption at any time must be expended on some finite number of consumption goods. In terms of the notation employed above, this implies

$$U(C_t, C_{t+1}, \_\_\_\_, C_T) = U\left(\sum_{j=1}^{n} p_j^{(t)} q_j^{(t)}, \_\_\_\_, \sum_{j=1}^{n} p_j^{(T)}, q_j^{(T)}\right)$$

where $p_j^{(t)}$ is the price (per unit) of the jth consumption good at time t and $q_j^{(t)}$ is the quantity consumed. Two species of market equilibrium equations suggest themselves. The first, which we shall label "the Consumptive Equilibrium Equations", would have it that the optimal consumption series $(C_t', C_{t+1}', \_\_\_\_, C_T')$ must be completely expended on consumption goods. The second, which we shall call "the Goods Market Clearing Equations", would have it that the demand for each consumption good must be equivalent to its supply. Supposing for the moment, that the prices and available supply of consumption goods and the optimal consumption series of each agent to be known, we may approach the task of decomposing each agent's optimal consumption vector into a vector of consumption goods by a procedure analogous to that employed in section 6.1. We proceed, therefore, to illustrate the method.

Supposing there to be l consumptive dates, m economic agents and n consumption goods, then there are lmn consumptive expenditures to be determined. There are lm "consumptive equilibrium equations" and ln "goods market clearing equations" by which to determine the above expenditures. Since l of the "goods market clearing equations" may be derived from the other l(m+n)-1 equations, there are thus

$l(m+n-1)$ independent equations by which to determine the lmn consumptive expenditures. This represents a deficiency of $l[mn-(m+n)+1]$ of independent equations over the unknown consumptive expenditures. These equations, or "nearly all"[72] of them, would need to be specified if we are to build even the simplest goods market into the Fisherine system.

In Table 6.8 these principles are applied to the data of Table 6.2. Recall that there are $l = 2$ consumption dates and $m = 2$ economic agents in this example. If, in addition, we suppose there to be two consumption goods available at each consumptive date, it follows there $l(m+n-1)=6$ independent equations by which to determine $lmn = 8$ unknown consumptive expenditures. Two independent equations must, therefore, be exogenously specified. For illustrative purposes we shall take $q_{1},_{1} = 22.475$ and $q_{1},_{2} = 100.000$ thus rendering the system determinate. The reason for this choice will become evident as we proceed. The consumptive solution implied by these equations and those contained in Table 6.8 is displayed in Table 6.9.

The content of this Table may be employed to examine the utility of C.P.P. financial information to the owners of productive facilities. Observe that the weighted average price of consumption goods at time one is $\frac{3093.12}{283.536}$ or 10.909, whereas at time zero the equivalent figure is $\frac{2,449.50}{244.95}$ or 10. The ratio of these figures $\frac{10.9090}{10} = 1.0909$ is a "price index" of prices at time one in terms of prices at time zero. Using this index, we exhibit in Table 6.10 the C.P.P. income statement for the optimal productive strategy depicted in Table 6.3. The income figure portrayed therein may be dissected further, as follows

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72. Some of the equations may need to be exogenously specified. See Friedman, M. "A Theoretical Framework for Monetary Analysis", Journal of Political Economy, 78, 2 (March 1970), pp. 217-222.

TABLE 6.8

EQUILIBRIUM EQUATIONS

Consumptive Equilibrium — 4 Equations

$$10q_{1,1} + 10q_{2,1} = 1{,}224.75$$

$$10q_{1,2} + 10q_{2,2} = 1{,}224.75$$

$$10q'_{1,1} + 12q'_{2,1} = 1{,}546.56$$

$$10q'_{1,2} + 12q'_{2,2} = 1{,}546.56$$

Goods Market Clearing — 4 Equations

$$q_{1,1} + q_{1,2} = 122.475$$

$$q_{2,1} + q_{2,2} = 122.475 \quad *$$

$$q'_{1,1} + q'_{1,2} = 154.656$$

$$q'_{2,1} + q'_{2,2} = 128.880 \quad *$$

* Redundant equations

TABLE 6.9

CONSUMPTIVE EQUILIBRIUM

Good	Agent 1		Agent 2		Totals	
	Units	£	Units	£	Units	£
1	22.475	224.75	100.000	1,000.00	122.475	1,224.75
2	100.000	1,000.00	22.475	224.75	122.475	1,224.75
Totals	122.475	1,224.75	122.475	1,224.75	244.95	2,449.50
1'	34.656	346.56	120.000	1,200.00	154.656	1,546.56
2'	100.000	1,200.00	28.880	346.56	128.880	1,546.56
Totals	134.656	1,546.56	148.880	1,546.56	283.536	3,093.12

TABLE 6.10

REAL REALIZED INCOME FOR THE INTERVAL [0,1]

Sales (224.7449 x 13.7628)		3,093.11
Expenses		
Factor x (22.4745 x 10)	224.75	
Factor y (22.4745 x 10)	224.74	
Factor z (2000 x 1)	2,000.00	
		2,449.49
Realized income		643.62
Fictional realizable cost savings (2,449.49 x 0.0909)		222.68
		420.94
Fictional unrealized cost savings		-
Real realized income		420.94

	Units	Price	Total
Consumptive units available at time one	283.536	10.9090	3,093.11
Consumptive units invested at time zero	244.95	10.9090	2,672.17
Real realized income			420.94
Fictional realizable cost savings			222.68
Realized income			643.62

Recall that although only type U agents possess productive opportunities, each type J agent contributes half the amount required to finance the type U agents' productive investments at time zero. As such, each type U and type J agent combined sacrificed 122.475 units of goods one and two respectively at time zero.[73] At time one this "investment" returns an entitlement to 154.656 units of good one and 128.880 units of good two. Note that the C.P.P. income (real realized income) of the interval [0,1] is the excess of the consumptive units available for consumption at time one over the consumptive units "invested" at time zero "valued" at the weighted average price of units available for consumption at time one. In this sense the C.P.P. income records the "increased" command over goods one and two accruing to each type U and type J agent collectively as a consequence of the productive investments made at time zero.

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73. The consumptive units sacrificed at time zero are computed by dividing the amount invested at time zero (2,449.49) by the weighted average price of goods available for consumption at time zero (10.00). We then have $\frac{2,449.49}{10}$ = 244.95 units are sacrificed at time zero and this may readily be decomposed into goods one and two by observing the ratio in which these goods are produced is one to one.

Before concluding this section, we make a short comment on the place of the fictional realizable cost savings within the C.P.P. measurement scheme. From the computations conducted above, it should be clear that the fictional realizable cost savings represents that part of realized income which is necessary to maintain the agents' consumptive potential at time one at 244.95 units.[74] The obvious difficulty with this line of reasoning is that the ratio of good one to good two is different at each consumptive date and as a consequence, it does not seem legitimate to take their difference in striking the C.P.P. income of the interval.

This completes our treatment of the accounting measurement systems specified in chapter four. We now summarize the content of the present chapter.

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74. Note that the excess of the weighted average price of goods available for consumption at time one over the equivalent weighted average price at time zero multiplied by the consumptive potential at time zero yields the fictional realizable cost savings of the interval [0,1]. That is 244.95 x (10.9090 - 10) = 222.68.

6.4 Summary

The objective of this chapter was to provide the means for rationalizing each of the measurement systems specified in chapter four. In this respect, we chose as the point of departure, the general equilibrium theory of Irving Fisher. A sufficient reason for doing so was that Fisher's work is the foundation for the Sharpe-Lintner asset pricing model, but unlike that model, it is not restricted to a single productive-consumptive interval.

Having analyzed the mathematical foundations of the Fisherine system, we illustrated its implementation by means of a practical example. The example, in fact, was designed to fulfil a dual purpose. Its main function was to serve as a device through which to examine accounting propositions concerning the firm but, as a secondary objective, it was intended to "sharpen" the mathematical foundations of the Fisherine system presented in the previous section.

We then proceeded to examine each of the measurement systems introduced in chapter four in the context of the Fisherine system. We found that Fisher's work contained both a priori and empirical implications for replacement cost measurement and that these provided only partial support for the predictive properties attributed to current operating profit by Edwards and Bell. In contrast, Fisher's analysis seemed to corroborate much of what Edwards and Bell had to say about market value measurement. As a final exercise, we examined the C.P.P. system of accounting measurement. Fisher's work seemed to be of little assistance in providing a satisfactory rationale for this measurement scheme. We expressed the view that the full potential of Fisher's work to this measurement scheme would not be realized until the link between utility functions for consumption goods and utility functions for consumption expenditures was more completely specified.

APPENDIX 6A

INCOME NUMBER PREDICTIONS

Several empirical studies have investigated the predictive properties of accounting income numbers. Those most cited in the literature are the following

Frank, Werner. "A Study of the Predictive Significance of Two Income Measures", Journal of Accounting Research, 7, 1 (Spring, 1969), pp. 123-36.

Simmons, K. and Gray, J. "An Investigation of the Effect of Differing Accounting Frameworks on the Prediction of Net Income", The Accounting Review, 44, 4 (October 1969), pp. 757-76.

Buckmaster, D.A., Copeland, R.M. and Dascher, P.E. "The Relative Predictive Ability of Three Accounting Models", Accounting and Business Research, 7,27 (Summer, 1977), pp. 177-186.

Each of these studies either eschews the *a priori* foundations of income predictions or suggest rather ambiguously and without elaboration that knowledge of a prior income series facilitates prediction of future income numbers. That a theoretical foundation for such propositions is required is demonstrated by the fact that several hypotheses are consistent with the empirical evidence. This fact is amply illustrated by Table 6.11.

Table 6.11 provides purely empirical estimates of the relationship between various accounting income numbers based upon the method of simple linear regression. In words, the relation

$$y_{j+1} = \alpha + \beta x_j + \epsilon_j$$

was estimated from the data of Table 6.6. y_{j+1}, the independent variable, is regressed against the lagged dependent variable x_j

TABLE 6.11

ALTERNATIVE PREDICTIVE HYPOTHESES

(1) #	(2) Dependent Variable	(3) Independent Variable	(4) a	(5) b	(6) R^2	(7) F	(8) Sig	(9) D.W.	(10) Sig	(11) n	(12) Remarks
1	Return (over cost)	C. O. P. ratio	0.0269	0.8739	0.9132	137.73	0.01	1.42	-	14	-
2	Return (over cost)	C. O. P. ratio	0.0128	0.9184	0.5998	27.98	0.01	0.78	0.01	19	D.W. Score indicates positive serial correlation
3	Realized Income	Realized Income	2702.4	1.0296	0.9536	268.43	0.01	1.95	-	14	-
4	Realized Income	Realized Income	1615.4	0.9623	0.8806	133.75	0.01	1.28	-	19	D.W. Score inconclusive at 5% and 1% significance
5	Realized Income	C. O. P.	3102.0	1.2114	0.8930	109.45	0.01	1.30	-	14	D.W. Score inconclusive at 5% significance
6	Realized Income	C. O. P.	1676.8	1.1105	0.7847	66.60	0.01	1.10	0.05	19	D.W. Score indicates positive serial correlation
7	C. O. P.	C. O. P.	2052.4	1.0501	0.9270	166.08	0.01	1.29	-	14	D.W. Score inconclusive at 5% significance
8	C. O. P.	C. O. P.	1102.7	0.9870	0.8659	117.18	0.01	1.04	0.05	19	D.W. Score indicates positive serial correlation

NOTES: When n = 14 the regression includes only data for which demand is increasing. When n = 19 both increasing and decreasing demand data sets are included.

to obtain the "least squares" estimates a and b of the parameters α and β. Estimates of these parameters (for each of eight regressions) are listed in columns four and five of Table 6.11. If we now assume the error terms ϵ_j, j = 1,2,_____, n to possess independent normal frequency functions and that $\beta = 0$, it follows that the variable

$$F = \frac{(n-2)b^2 \sum (x_j - \bar{x})^2}{\sum e_j^2}$$

has an F(1,n-2) frequency function (see footnote 58 to this chapter). We are thus provided with a means of testing the null hypothesis, $H_o : \beta = 0$ against the alternative hypothesis, $H_1 : \beta \neq 0$. Column 7 of Table 6.11 contains the F statistic for each of the eight regressions listed therein. Column eight records the level of significance at which the null hypothesis can be rejected for each of the regressions. Note that in each case the null hypothesis is in fact rejected in favour of the alternative hypothesis and that for each regression the relationship is stronger if the regression is restricted to those years over which aggregate sales is increasing.

Before analyzing the above results in more detail a brief note on the Durbin-Watson statistic (denoted D.W. in Table 6.11) is warranted. A necessary condition for the "least squares" estimates (a and b) of the parameters α and β to be "minimum variance unbiased" is that the unobservable random variables ϵ_j do not exhibit autocorrelation of any order. Durbin and Watson proposed a test for the existence of autocorrelation in normally distributed errors, based on the statistic

$$= \frac{\sum_{j=2}^{n} (e_j - e_{j-1})^2}{\sum_{j=1}^{n} e_j^2}$$

where $e_j = y_{j+1} - a - bx_j$. Critical values for this statistic are tabulated in

Durbin, J. and Watson, G.S. "Testing for Serial Correlation in Least Squares Regression II", Biometrika, 38 (1951), pp. 173-175.

Yamane, T. Statistics. Harper and Row, Publishers, Inc., 1973 pp. 1096-1098.

A method for removing the effects of autocorrelation is provided in the latter text. For our purposes, however, it is sufficient to note that the existence of autocorrelation implies that the variance $\sigma^2(b)$ of b around β will in general be underestimated. This, in turn, implies that there is a tendency to reject the null hypothesis $H_o: \beta = 0$ when, in fact, it is true. In words, adjusting for the effects of autocorrelation can only make the "results" worse. For some further discussion on this see

Durbin, J. and Watson, G.S. "Testing for Serial Correlation in Least Squares Regression I," Biometrika, 37 (1950), pp. 409-428.

Yamane, T. Statistics. Harper and Row, Publishers, Inc., 1973, pp. 998-1009.

For the regressions reported in Table 6.11, the Durbin-Watson statistic was significant on three occasions and inconclusive for two others. Since the three regressions exhibiting a significant Durbin-Watson statistic provided the three worst empirical relationships (as measured by the R^2 statistic), we did not adjust for the effects of the autocorrelation but rather ignored the affected regressions in any further statistical manipulations.

R.A. Fisher has shown that for bivariate normal frequency functions, the statistic $z = \frac{1}{2} \log_e \frac{1 + r}{1 - r}$ where

$$r = \frac{\sum xy - n\bar{x}\bar{y}}{\sqrt{(\sum x^2 - n\bar{x}^2)(\sum y^2 - n\bar{y}^2)}}$$

has an approximate normal frequency function with mean $E(z) = \frac{1}{2}\log_e \frac{1+\rho}{1-\rho}$, where ρ is the "correlation" between x and y, and variance $\frac{1}{n-3}$. For some more discussion on this, see

Freund, J.E. Mathematical Statistics. Englewood Cliffs, New Jersey: Prentice-Hall, Inc., 1971, p.381.

It thus follows that the variate

$$q = \frac{\sqrt{n-3}}{2} \quad \log_e \frac{(1+r)(1-\rho)}{(1-r)(1+\rho)}$$

has a normal frequency function with zero mean and unit variance. The above expression may be converted to a $(1-\alpha)$ confidence interval for ρ by solving the double inequality $-z_{\frac{1}{2}\alpha} \leqslant q \leqslant z_{\frac{1}{2}\alpha}$ where $z_{\frac{1}{2}\alpha}$ is the normal deviate corresponding to a probability of $(1-\alpha)$ that there will occur on random sampling a deviation from the mean of $z_{\frac{1}{2}\alpha}$ times the standard deviation or greater (in absolute terms). For the regressions not exhibiting significant autocorrelation, the following 0.95 confidence intervals were obtained

#	ρ^2	r^2
1	$0.7425 \leqslant \rho \leqslant 0.9725$	0.9132
3	$0.8560 \leqslant \rho \leqslant 0.9855$	0.9536
4	$0.8100 \leqslant \rho \leqslant 0.9535$	0.8806
5	$0.6890 \leqslant \rho \leqslant 0.9660$	0.8930
7	$0.7880 \leqslant \rho \leqslant 0.9760$	0.9270

Note that each confidence interval overlaps to some extent and thus on purely empirical grounds it is not possible to prefer one regression over another. This point, of course, was made in the text.

CHAPTER SEVEN

A CONCLUDING NOTE

7.0 Summary

The theory of accounting measurement is and will remain an interminable and contentious area for accounting researchers. For if and when a set of basic postulates (or axioms) for accounting measurement is defined and agreed upon, it will always be possible to specialize the analysis in the interests of simplicity and concreteness on the one hand or to generalize it in the interests of wider applicability on the other. The ultimate objective of this exercise, of course, is to provide a structure of concepts and relationships which define a unique set of measurement procedures for each potential measurement problem. Thus, for example, the "choice" of depreciation method would then resolve itself as a deductive consequence of the postulates or axioms. The present volume documents our contribution to this objective. The foundation and unifying theme of the work is contained in the axiom system exhibited in Table 2.4 of chapter two. We there saw that the theory of accounting measurement is grounded upon three axioms and it is these axioms which summarize a set of sufficient conditions for generating accounting measurements. The axioms, in turn, assume the existence of the data set $(P_t, \mathcal{E}_t, L_t)$, where P_t is a "property set", $\mathcal{E}_t$ is an algebra of "resource sets" and L_t is a measurement rule. Recall that the triple $(P_t, \mathcal{E}_t, L_t)$ was, in fact, called an "accounting measurement space" and that it is the specification of this configuration which is the source of many (if not all) of accounting's problem areas.

In chapter three, therefore, we focused more particularly upon the nature of accounting measurement. Specifically, we suggested that the Stevens' measurement scheme, which is the usual point of departure for discussions focusing on accounting measurement, is a useful device for determining the significance of the "numerical procedures" applied to measurements when there is a choice in the unit (of measurement) in which the results of measurement are expressed. Yet, despite the importance which contemporary writers on accounting measurement associate with the Stevens' scheme, it is doubtful whether it bears any direct significance to the theory of accounting measurement if only because it is questionable as to whether the collection of admissible measurement rules in accounting (historic cost, replacement cost, C.P.P. etc.) is capable of being "scaled" (that is, share a common group allegiance) in the sense implied by Stevens' work. However, Stevens' scheme was suggestive of a simple procedure for measuring the degree of identifiability between any pair of accounting measurement rules (a contributing factor to the definition of the likeness ratio) and it was also demonstrated as being of considerable practical significance to any form of empirical research in accounting. In concluding chapter three, we investigated a variety of techniques for estimating the bias and objectivity associated with accounting measurements. Specifically, by imposing the assumption that the measurements analyzed represent a random sample from a normal frequency function, it was demonstrated how the sample's mean and variance (the sample objectivity measure) could be used as a basis for constructing interval estimates of the sample's bias and (actual) objectivity measurements.

In chapter four, we completed our analysis of the accounting measurement systems by examining a general model of accounting valuation, a scheme which was designed to satisfy a dual objective. Its main purpose was to demonstrate that the axiom system exhibited in Table 2.4 of chapter two can, in fact, be meaningfully applied to measurement problems involving some species of "valuation", but it was also designed to show that the "numerical procedures" associated with adjusting a set of historical cost financial statements to some alternative basis of valuation, are analogous in principle. Recall that the model is founded upon two theorems, the first and more important of which provides a means of determining the (potentially) realizable "holding gains" accruing on a firm's resources during some interval of time T. When the model is provided with a replacement cost interpretation, the theorem requires (as an input) the accumulated replacement cost of disposals during T. The sheer complexity associated with computing this figure has proved to be a major bugbear to the adherents of the replacement cost measurement model.

In chapter five, therefore, we examined several methods for estimating the replacement cost of disposals over the interval $[T,T+1]$. In fact, five such methods were examined. The first three of these, namely the midpoint rule, the trapezoidal rule and Simpson's rule, are drawn from the topic of numerical mathematics, a discipline which, on the surface, appears to hold considerable potential for the problem at hand. Unfortunately, each of these "quadrature techniques" assumes the existence of a function which describes the rate of change in the firm's accumulated disposals for all t in $[T,T+1]$, a quantity which would seldom be known. As such, the methods are somewhat impracticable. However, the remaining two methods, namely the Edwards and Bell technique and the modified midpoint rule, seem to provide a practicable and reasonably accurate means of estimation, with the modified midpoint rule being the more computationally efficient of the two.

In the final and somewhat lengthy chapter, we examined the economic foundation of accounting measurement. Basing our analysis on the capital theory developed by Irving Fisher, we were able to provide an economic rationale for each of the measurement systems investigated in chapter four. We established, under very general conditions, that the ratio of a firm's current operating profit to the replacement cost of goods sold during the productive interval T is a lower bound for its return (over cost) during the next succeeding productive interval (T+1). The realizable operating profit (of the market value system) was shown to measure the contribution of a firm's productive activities (as against purely holding operations) to the variation in the firm's market value over the productive interval covered by the income statement. Finally, we demonstrated that the real realized income (of the C.P.P. system) measures the increased command of a firm's resources over a composite of consumptive services as a result of the firm's prior productive investments. In words, each measurement system was found, for a given class of circumstances, to possess some degree of utility to the owners of productive facilities.

There is, of course, a multitude of topics which we have chosen either to ignore or furnish with the most superficial of treatments. Clearly, little else could be expected from a work of the present proportions. This does not deny, however, that a host of further generalizations and applications await development. In the next section, therefore, we shall endeavour to provide the reader with some insight into the likely direction of these investigations.

7.1 Conclusions and Prognosis

The line of advance most forcefully demanded by the analysis of previous chapters lies in the province of the axiomatic foundations of accounting measurement. For whilst the axiom scheme exhibited in Table 2.4 of chapter two is undoubtedly a reasonable abstraction of the procedures associated with generating accounting measurements, it is far too general to be of much practical utility. Indeed, as presently constituted, it possesses only the most trivial of deductive consequences and is far removed from its ideal function as a watershed or "clearing house" for measurement problems in accounting. Whether, in fact, it is possible for the theory of accounting measurement to achieve the level of sophistication implied by this objective is a moot point. Our own view is that some improvement upon the axiom scheme exhibited in Table 2.4 is inevitable, but that to achieve the ideal is akin to taking the "breeks off a hielanman".[1]

A second, and far more pressing practical consideration, concerns the estimation of the replacement cost of goods sold during a financial period. It was claimed in chapter five of the text that the Edwards and Bell technique and the modified midpoint rule provide a practicable and reasonably accurate means of overcoming this obstacle. For firms possessing a "small" number of inventory lines this assumption is undoubtedly justified, but as

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1. For an excellent discussion of the problems involved, see Morrison, A.M.C. "The Role of the Reporting Accountant Today - II", The Accountant's Magazine, LXXXIV (October 1970), p.468.

the number of stock items increases these methods too are likely to become increasingly cumbersome.[2] It would seem, therefore, that there is a need for a comprehensive research programme directed toward providing a more satisfactory solution to this problem. Our own opinion is that the solution will emerge from a synthesis of the methods of numerical mathematics and mathematical statistics.

Finally, there is the sine qua non of accounting measurement - namely, its economic foundations. Although the Fisherine system, which was analyzed in chapter six of the text, was found to be a useful device through which to examine accounting propositions concerning the firm, we would be practising self-delusion if we were to pretend that it does not possess "weaknesses". The first of these concerns the introduction of productive resources into the Fisherine scheme. Recall that the text of the present volume introduced the firm as a device through which contemporaneous consumptive resources are productively transformed into future consumptive services, and that for reasons of simplicity and clarity of exposition, this was achieved by considering the firm to consist of a sequence of cash based ventures. Although this aspect of the topic of capital theory is one of the most formidable and intractable spheres of economic science, there is a handful of standard (albeit conflicting) expositions which may be "productively" utilized by

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2. To illustrate, suppose a firm which carries m inventory lines, applies one or other of the above rules on n occasions during an accounting period. Since each of the m inventory lines requires n (per unit) replacement costs and n periodic sales figures, this implies that a total of 2mn pieces of datum must be supplied (per accounting period) if the rule is to be applied to inventory in its entirety. Thus, for example, a firm possessing m = 10,000 inventory lines and which utilizes the rule on a monthly basis (n = 12, as suggested in the text), requires 2mn = 2 x 12 x 10,000 or 240,000 pieces of datum (per annum), if the rule is to be comprehensively applied. Our view is that firms in possession of a computing machine would find this a rather trivial exercise, provided the data collection phase (unit replacement costs and periodic sales) of its operations were efficiently organized.

accounting theorists. We are here, of course, referring to the Knightian, Bohm-Bawerk and "durable goods" concepts of "real capital".[3] In our view, it is only by examining these more "realistic" approaches to "real capital" theory that the relevance of the "accrual accounting" measurement systems to the owners of productive facilities, will emerge. A second consideration concerns the incorporation of risk and uncertainty into the Fisherine system. Recall that in a world of uncertainty, firms may produce information about themselves as well as undertake physical production. Although some progress along these lines has been made, there still remains much to be done.[4]

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3. Hirshleifer, J. Investment, Interest and Capital. Englewood Cliffs, New Jersey: Prentice-Hall, Inc., 1970, Chapter 6.

4. Gonedes, N.J. and Dopuch, N. "Capital Market Equilibrium, Information Production, and Selecting Accounting Techniques: Theoretical Framework and Review of Empirical Work", Studies on Financial Accounting Objectives, 1974, pp. 48-129. Supplement to Journal of Accounting Research, 12 (1974).

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