ROTATION AND CHEMICAL COMPOSITION OF EARLY-TYPE STARS

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ABSTRACT

Some early-type stars with abnormal line strengths for their intrinsic colours are investigated in order to determine their structure and the chemical composition of their atmospheres. The abnormal line strengths in several B-type stars (α Scl, 36 Lyn, 20 Tau and HD 175156) are shown to be probably due to large ranges of surface temperature such as would be observed in rapidly rotating stars of normal chemical composition, if viewed pole-on. These stars differ from the peculiar A stars, which rotate slowly and have abnormal chemical composition. Members of the Mn group of peculiar A stars rotate more slowly than those of the Si-4200 group, although the mean intrinsic colours of the two groups are similar.

Previous work on abundance determination in peculiar A stars is reviewed. The chemical composition of the manganese star 53 Tau is studied by a refined differential curve-of-growth method with α Lyr as the comparison star. Groups of lines of similar mean excitation and ionisation potentials and wavelengths are used so that the results are independent of the structure of 53 Tau, which is shown to be abnormal. A new double-line peculiar A spectroscopic binary HR 4072 was discovered, and it was found that excesses of Sr and Y are common to both components. Spectral variations in some peculiar A stars are described.

Well established abundance abnormalities in peculiar A stars may be explained on a modified form of van den Heuvel's theory that peculiar A stars have been secondaries of binary systems with initial separations in the range 10 a.u. to 100 a.u. approximately. The primaries, initially earlier than spectral type B8, have evolved and exploded as type II supernovae. During the explosions material enriched in heavy elements by interior nuclear reactions was transferred to the surfaces of the secondaries, which are now observed as peculiar A stars and are still on the main sequence. Excesses of Be and Si and deficiencies of O are caused by surface nuclear reactions on the secondaries after the explosions of the primaries. Mn stars differ from other peculiar A stars in many respects; the differences may be due to the primaries expanding beyond the Roche limits before they explode. On the oblique rotator theory, spectrum variables result from irregular distributions of surface nuclear reactions and ion migration on the secondaries. Some other theories of peculiar A stars are reviewed and shown to be in conflict with observation.

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CHAPTER 1

INTRODUCTION

One of the most striking features of the spectra of many early-type stars is the considerable broadening of the spectral lines. An explanation in terms of the Doppler effect immediately suggests itself, since the line widths are proportional to wavelength. Differential motions in the line of sight could be caused by atmospheric turbulence or by the rotation of the whole star. Studies of the radial velocity curves of eclipsing binaries have shown that stellar rotation rather than atmospheric turbulence is the predominant broadening agent in most main sequence early-type stars. If a rotating star is partially eclipsed, the uneclipsed segment has rotational as well as orbital motion in the line of sight, and distortion of the radial velocity curve results; such distortions have been observed in several early-type eclipsing binaries.

From the line widths it is possible to deduce the projected rotational velocity vsini, where v is the equatorial rotational velocity and i is the inclination of the axis of rotation to the line of sight. Values of vsini have been determined by various authors for numerous stars, and their results have been collected in the catalogue by Boyarchuk and Kopylov (1964). Equatorial rotational velocities v are known for only a few stars, as there is no general method of determining inclinations of axes of rotation. However, the mean equatorial rotational velocities of various groups of stars may be compared, if one can assume that the distributions of the inclinations are similar. Thus for normal main sequence stars the mean equatorial rotational velocity is much greater for early-type stars than for late-type stars, since the mean projected rotational velocity vsini decreases towards later types, the decrease being most rapid near spectral type F2.

The mean value of vsini for early-type main sequence stars is about 150 km/sec, but values as high as 400 km/sec have been found. These values may actually be underestimated owing to the neglect of certain second-order effects such as limb darkening and rotational distortion of the stars (Stoeckley, 1967). At any rate, there is good reason to believe that some early-type stars have equatorial rotational velocities close to that required for rotational break-up ($\sim$500 km/sec). For such a star, the effective surface gravity will be much lower at the equator than at the pole. Since the surface brightness is proportional to the effective surface gravity (von Zeipel, 1924), the effective surface temperature will be lower at the equator than at the pole; the expected temperature range is several thousand degrees. If the star was observed pole-on, the lines would be sharp, and the effect of the large range in surface temperature on their strengths could be studied. This problem is important, because the strengths of lines are used to derive the chemical composition of stellar atmospheres. The assignment of a unique surface temperature to a rapidly rotating star could lead to the derivation of erroneous abundances.

Detailed investigations have been carried out on the chemical compositions of the atmospheres of several sharp-line stars. For example, τ Sco (MK type BOV), γ Peg (B2IV), and α Lyr (AOV) were studied by Scholz (1967), Aller and Jugaku (1959), and Hunger (1955, 1960) respectively. As these stars have normal line strengths, it is unlikely that they are rotating rapidly. Their atmospheres were found to have chemical compositions resembling that of the Sun (G2V), which is a typical population I star. Broadly speaking, the composition of the atmospheres of normal main sequence stars is determined by the composition of the interstellar medium out of which they were formed, since products of nuclear reactions in the stellar interiors contaminate the atmospheres substantially only at later stages in the evolution of the stars.

Of particular interest among early-type stars is the remarkable group known as peculiar A stars. They comprise about 10% of all stars with Henry Draper spectral types B8 to A5. Although they lie near the main sequence in the colour magnitude diagram and have normal Balmer line profiles, they are characterised by abnormally strong lines of certain elements as compared with normal main sequence stars of the same colour. Their spectral features show considerable diversity, but they may be broadly classified into several groups according to which elements have the most prominent enhanced lines. Table 1 lists the mean Henry Draper spectral type and B - V colour index for each group as given by Jaschek and Jaschek (1958).

Table 1

GROUPS OF PECULIAR A STARS

Elements with enhanced lines	Mean HD sp. type	Mean B - V
Mn	B8.5	-0.09
Si-4200	A0	-0.13
Si	A0	-0.08
Si-Eu-Cr	A0	-0.07
Eu-Cr	A1	+0.01
Eu-Cr-Sr	A2	+0.09
Sr	A3	+0.13

Magnetic fields have been measured by Babcock (1958a) in about 100 peculiar A stars. The largest field recorded so far is 34400 gauss in the Si star HD 215441 (Babcock, 1960). Many peculiar A stars show variations within a few days in their magnetic field strengths, line strengths, line widths, radial velocities, luminosities, and colours. In some stars the variations are periodic, but the variations of the different quantities do not always correlate.

The mean value of vsini for peculiar A stars is 41 km/sec, which is much less than the value of 139 km/sec for normal main sequence stars with HD types B8 to A2 (Slettebak, 1954). This difference must be due to a real lack of broad-line peculiar A stars, since peculiar A stars have frequently been discovered in the course of objective-prism surveys with low spectral resolution, and the enhanced lines of SiII and CrII would still be visible if vsini were as high as 400 km/sec (Walker, 1966). It has been suggested that peculiar A stars are viewed at low inclinations to their axes of rotation (e.g. Jaschek and Jaschek, 1958); this would explain the low mean value of vsini. However, it does not seem possible to account for the enhanced lines of the rare-earth elements and the periodic variations simply in terms of an inclination effect. The production of the peculiar lines is not confined to polar regions, since the enhanced lines in a peculiar A star are not generally sharper than other lines of similar strength; thus the peculiarities can be observed at high inclinations. Furthermore, peculiar A stars have similar space motions and a similar distribution about the galactic plane as population I stars, and it would therefore be expected that their distribution of inclinations would also be the same as for normal stars. These general arguments indicate that peculiar A stars have low equatorial rotational velocities and are intrinsically different from normal stars.

As the abnormal line strengths in peculiar A stars are not due to rapid rotation, some other cause must be sought. The spectral lines suffer Zeeman broadening because of the presence of magnetic fields, but this is insufficient to account for the enhancement of the lines of some elements (Boyarchuk and others, 1960). Selective excitation of certain lines seems to be ruled out, since no emission lines have been found and, allowing for Zeeman broadening, the strengths of the lines lie on normal curves-of-growth. There is no general correlation between the enhancement of the lines and ionisation potentials. Tidal distortion by close companion stars is also excluded as a general explanation, as the frequency of spectroscopic binaries is low among peculiar A stars (Jaschek and Jaschek, 1958). For some ions (e.g. MnII, EuII, and GdII) the line strengths in some peculiar A stars are not exceeded in any other type of star. It therefore seems that the enhancement is at least partly due to real abundance anomalies.

The aim of this thesis will be to determine the nature of the abundance anomalies in peculiar A stars and the origin of these stars. As a first step, a search will be made for rapidly rotating pole-on stars of normal chemical composition with surface temperatures in the same range as those of peculiar A stars. Certain difficulties are encountered at the lower end of this range of surface temperature. Metallic-line stars occur at this part of the main sequence, and it is not clear whether they are related to the peculiar A stars or to normal stars or form a separate group. The sharp decrease in the values of vsini near spectral type F2 also poses a difficulty in any discussion of the distribution of values of vsini. For these reasons, attention will be confined mainly to the higher end of the range of surface temperature covered by the peculiar A stars. The identification of normal rapidly rotating pole-on stars will enable them to be excluded from the peculiar A group and will indicate the effect of a large range in surface temperature on line strengths. The rotation of the hotter groups of peculiar A stars will then be studied. Previous work on the chemical composition of peculiar A stars will be reviewed, and reliable information on the abundance anomalies will be summarised. A refined differential curve-of-growth method for abundance determination will be developed in a detailed study of the Mn star 53 Tau and applied to the double-line peculiar A spectroscopic binary HR 4072. The variable peculiar A stars will then be described. Finally, an attempt will be made to establish the origin of peculiar A stars from the available information on their abundance anomalies and their other properties.

CHAPTER 2

RECOGNITION OF NORMAL RAPIDLY ROTATING POLE-ON STARS

The effect of rapid rotation on surface conditions

A simple representation of a rapidly rotating star is the Roche model in which the stellar mass M is concentrated in the central region. This model was investigated by Slettebak (1949) and Ireland (1965). Following Ireland's treatment, the mean radius R_a of the star is defined as $\frac{1}{2}(R_e + R_p)$, where R_e and R_p are the equatorial and polar radii respectively. The surface of the star is considered as an equipotential, and the combined potential of the gravitational and rotational forces at the equator is equated to that at the pole. Thus

$$\frac{GM}{R_e} + \tfrac{1}{2}v^2 = \frac{GM}{R_p}, \tag{1}$$

where G is the gravitational constant. Hence

$$R_e = R_a - \frac{2GM}{v^2} + \sqrt{R_a^2 + \left(\frac{2GM}{v^2}\right)^2}, \tag{2}$$

and

$$R_p = R_a + \frac{2GM}{v^2} - \sqrt{R_a^2 + \left(\frac{2GM}{v^2}\right)^2}, \tag{3}$$

The effective surface gravity is $g_e = \frac{GM}{R_e^2} - \frac{v^2}{R_e}$ at the equator and $g_p = \frac{GM}{R_p^2}$ at the pole.

As an example, consider a main sequence star of spectral type B5 for which $M = 7\ M_{\odot}$ and $R_a = 4\ R_{\odot}$ (Allen, 1963). If v is, say, 400 km/sec, then $R_e = 1.115\ R_a$, $R_p = 0.885\ R_a$, and $\frac{g_p}{g_e} = 3.35$. Now the surface brightness H is proportional to the effective surface gravity (von Zeipel, 1924), and $H = \sigma T_{eff}^{\ 4}$, where σ is Stefan's constant and T_{eff} is the effective surface temperature. Hence $T_{eff} \propto g^{1/4}$, and the effective temperature at the pole will be 1.35 times greater than at the equator. Since the mean effective temperature of a B5 star is 16,500°K, the temperature difference between the pole and the equator will be about 5000 degrees.

A possible method of recognising rapidly rotating pole-on B-type stars is suggested by the analysis by Su-Shu Huang and Struve (1956) of Maia (20 Tau), which is the only B-type star in the Pleiades with a small value of vsini (30 km/sec). They observed lines in its spectrum corresponding to a large range in temperature and concluded that it was probably a normal rapidly rotating star viewed pole-on. The HeI lines in 20 Tau are abnormally weak for the spectral type given by the Balmer lines. This suggests a method of finding stars resembling 20 Tau, since the strengths of HeI lines have been measured in a large number of B-type stars. Of course, some slowly rotating stars may also have weak HeI lines due to a deficiency of helium, and it is not certain that the weakness of the HeI lines in 20 Tau is directly related to the proposed large range in surface temperature.

Stars with weak HeI lines

Other population I stars reported as having weak HeI lines for their spectral types include ε Cas (Morgan, Keenan and Kellman, 1943), 3 CenA (Jugaku, Sargent and Greenstein, 1961), α Scl (Jugaku and Sargent, 1961), and ADS 4193B (Slettebak, 1963). McNamara and Larsson (1962) noted that three stars in the Orion association (HD 36629, HD 37058, and HD 37807) had weak HeI lines for their intrinsic colours. Searle and Sargent (1964) also found that the HeI 4471 line in 36 Lyn is weaker than in normal stars of the same colour. All these stars have small values of vsini. They may either be rapidly rotating pole-on stars or have abnormal chemical composition. In either case spectral classification will be difficult, and the resulting spectral type will depend on which line ratios are used. In the following discussion the quantity

$$Q = (U - B) - 0.72(B - V) \tag{4}$$

will be used instead of spectral type, since a unique value of Q may be assigned to every star. Moreover, Q will not be greatly affected by abundance anomalies, because hydrogen is by far the most abundant element. Q is approximately independent of interstellar reddening, and since all the stars to be discussed are only slightly reddened, the ratio $E_{U-B}/E_{B-V} = 0.72$ (Johnson and Morgan, 1953) will be taken.

Table 1

EQUIVALENT WIDTHS IN mA FOR

B-TYPE STARS

Star	HD	MK	vsini	Q	HeI 4471	CII 4267	MgII 4481	SiII 4128 + 4131	SiIII 4553
<u>Normal stars</u>									
τ Sco	149438	B0V	13	-0.84	947	82	90		132
γ Peg	886	B2IV	5	-0.70	1280	206	160	70	115
ι Her	160762	B3IV	0	-0.56	1240	208	223	147	
τ Her	147394	B5IV	20	-0.45	650	100	220	180	40
π Cet	17081	B7V	15	-0.40	540	90	320	260	30
21 Aql	179761	B7V	0	-0.35	390	70	240	180	absent
HR8348	207840	B7V		-0.32	440	70	360	260	absent
ν Cap	193432	B9V	5	-0.13	90	20	400	210	30
α Lyr	172167	A0V	0	-0.01	38	absent	339	134	absent
<u>Stars with weak HeI lines</u>									
3 CenA	120709	B4IVp	0	-0.68	576	110	141	138	53
ε Cas	11415	B3IIIp	28	-0.51	718	152	251	176	
α Scl	5737	B8IIIp	0	-0.43	310	151	226	200	50
36 Lyn	79158		27	-0.40	190	120	270	200	?
20 Tau	23408	B7III	30	-0.35	165	94	191	169	absent

Equivalent widths of the HeI 4471, CII 4267, MgII 4481, SiII 4128 + 4131, and SiIII 4553 lines for stars with weak HeI lines and for normal stars with small values of vsini are given in Table 1. It is expected that the rotation of most of the normal stars will be slow. The equivalent widths were measured on high-dispersion spectra by Scholz (1965) for τ Sco, Aller and Jugaku (1958) for γPeg, Wright and others (1964) for ι Her, Searle and Sargent (1964) for γ Her, π Cet, 21 Aql, HR 8348, ν Cap and 36 Lyn, Hunger (1955) for α Lyr, Jugaku, Sargent and Greenstein (1961) for 3 CenA, Jugaku and Sargent (1961) for α Scl, and Su-Shu Huang and Struve (1956) for 20 Tau. The equivalent widths for ε Cas were measured on a spectrum at a dispersion of 5.6 A/mm taken with the Edinburgh 36-inch telescope. The values of vsini are from the catalogue by Boyarchuk and Kopylov (1964). Q was calculated from the UBV photometry by Johnson (1955), Iriarte and others (1965), or that quoted by Searle and Sargent (1964). Q for 3 CenA was found from the value D = 0.17 of the Balmer discontinuity quoted by Jugaku, Sargent and Greenstein (1961) using the relation

$$D = 0.525 + 0.525Q$$

given by Becker (1963).

The equivalent widths of the HeI 4471 and CII 4267 lines are plotted against Q in Fig. 1 so that the stars with weak HeI lines may be compared with the normal stars. Similar diagrams were constructed for the MgII 4481, SiII 4128 + 4131, and SiIII 4553 lines. Information on other lines was obtained from the references already quoted and from the spectra listed in Table 2. The lines in a spectrum of HD 37058 were recently identified by Sargent and others (1967).

Table 2

HIGH-DISPERSION SPECTRA EXAMINED FOR ABNORMAL LINE STRENGTHS

Star	Plate	Date	Dispersion	Wavelength Range
α Scl	Mt. Wilson Ce 10055	1955, Sep. 8	3 A/mm	3800 - 4800
"	" 10092	1955, Sep. 30	"	3800 - 4800
"	" 10097	1955, Oct. 1	"	3700 - 4850
"	" 10104	1955, Oct. 2	"	3700 - 4950
"	" 10161	1955, Oct. 27	"	3800 - 4800
π Cet	" 3668	1944, Dec. 26	"	3600 - 4700
20 Tau	" 2919	1942, Nov. 24	"	3440 - 4650
"	" 4014	1945, Oct. 18	"	3650 - 4650
"	" 9424	1954, Sep. 20	"	3700 - 4800
HD 175156	" 3601	1944, Oct. 5	"	3750 - 4600
ε Cas	Edinburgh 82/64	1964, Oct. 31	5.6 A/mm	4000 - 4600
36 Lyn	" 85/64	1964, Nov. 1	"	4000 - 4600
"	" 94/64	1964, Nov. 10	"	4000 - 4600

The strengths of lines in the stars with weak HeI lines relative to the strengths in normal stars with the same values of Q are given in Table 3; in this table a blank indicates that no information is available.

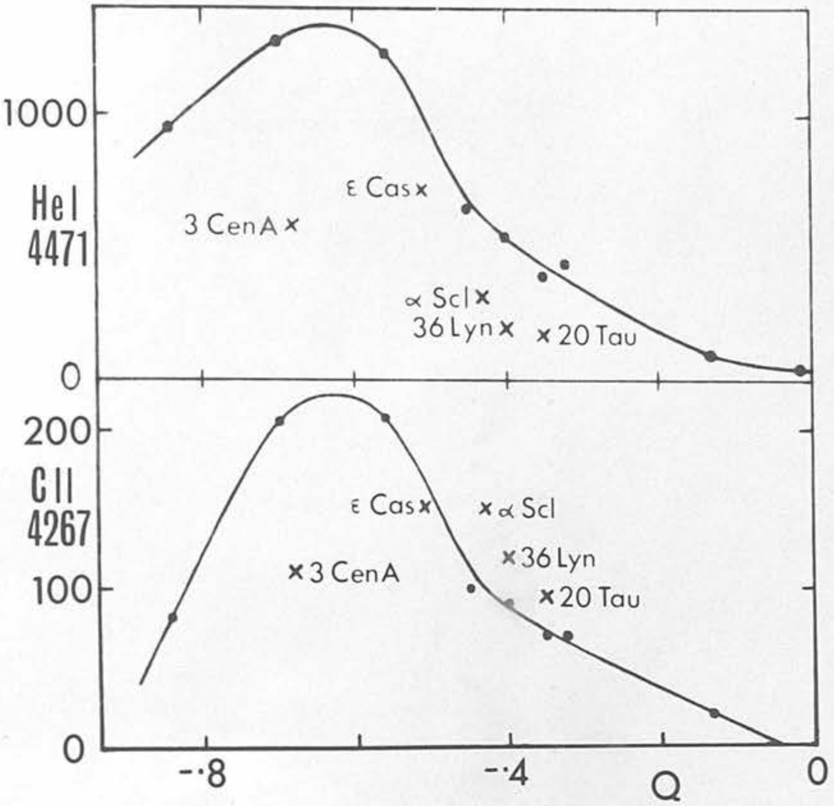

Figure 1

EQUIVALENT WIDTHS OF THE HeI 4471

AND CII 4267 LINES IN B-TYPE STARS

Dots denote normal stars and crosses

denote stars with weak HeI lines.

Table 3

STRENGTHS OF LINES RELATIVE TO NORMAL STARS

Star	3 CenA	HD 37058	ϵ Cas	α Scl	36 Lyn	20 Tau
Q	-0.68	-0.62	-0.51	-0.43	-0.40	-0.35
HeI 4471	weak	weak	rather weak	weak	weak	weak
CII 4267	weak		normal	rather strong	rather strong	rather strong
MgII 4481	normal		normal	normal	normal	rather weak
SiII 4128 4131	rather strong		normal	normal	normal	rather weak
SiIII 4553	weak		weak	normal	presence doubtful	absent
ScII 4247	absent		absent	strong	strong	absent
TiII lines	absent	strong	absent	strong	strong	strong
CrII lines	absent		absent	strong	strong	strong
FeII lines	strong		absent	strong	strong	strong
SrII 4215	absent	strong	absent	strong	strong	absent

Discussion

It must first be emphasised that of the stars with weak HeI lines only those whose other lines suggest a large range in surface temperature are likely to be normal rapidly rotating pole-on stars. The interpretation of the strengths of the HeI lines themselves will be difficult, because they are strongly influenced by the Stark effect. The abundance of helium in the atmosphere may also be affected by the extent to which mixing occurs between the stellar interior and the atmosphere.

Table 3 shows that the stars HD 37058, α Scl, 36 Lyn, and 20 Tau form a group with similar line strengths. The TiII, CrII, ScII and SrII lines are normally observed only in stars later than B8, but their presence in this group of stars would be explained if there is a large range in surface temperature in each star. The star HD 175156 (MK type B5III, vsini = 11 km/sec) may also belong to this group, since lines with a large range in excitation and ionisation potentials (HeI, CII, NII, MgII, SiII, SiIII, SII, CaII, TiII and FeII) are present on the Mount Wilson spectrum. The ScII and SrII lines have lower excitation and ionisation potentials than the TiII and CrII lines, and their absence in 20 Tau in spite of the large value of Q suggests that the range in surface temperature is smaller in 20 Tau than in α Scl and 36 Lyn.

An alternative explanation of the line strengths in HD 37058, α Scl, 36 Lyn, 20 Tau and HD 175156 might be that at least some of these stars belong to a previously unrecognised group of peculiar A stars. Many peculiar A stars have weak HeI lines for their intrinsic colours (Searle and Sargent, 1964). The low values of vsini are due to slow rotation rather rapid rotation viewed pole-on. Peculiar A stars may have fairly large ranges of surface temperature due to causes other than rapid rotation, but it is not possible to account for all their line strength anomalies in this way. On the other hand, the presence of all the lines so far identified in HD 37058, α Scl, 36 Lyn, 20 Tau and HD 175156 can be accounted for by large ranges in surface temperature.

Table 4

COMPARISON BETWEEN α SCL AND π CET

Ion	$\bar{\chi}$ in ev	χ_r in ev	$\bar{\chi} + \chi_r$	Relative line strengths
SiIII	19	33	52	weaker in π Cet
NII	18	29	47	absent in π Cet
CII	17	24	41	slightly weaker in π Cet
SII	16	23	39	similar
FeIII	8	30	38	absent in π Cet
SiII	10	16	26	similar
MgII	10	15	25	similar
CrII	4	17	21	absent in π Cet
FeII	3	16	19	weaker in π Cet
TiII	2	14	16	weaker in π Cet
ScII	0	13	13	absent in π Cet
CaII	0	12	12	partly interstellar?
SrII	0	11	11	absent in π Cet

A detailed comparison between the strengths of lines in α Scl and those in the normal star π Cet, which has a similar value of Q, is given in Table 4. All elements (except HI and HeI) with observed spectral lines in α Scl are listed in order of decreasing $\bar{\chi} + \chi_r$, where $\bar{\chi}$ is the mean excitation potential of the lower levels of the atomic transitions producing the lines, and χ_r is the ionisation potential of the ion. It will be noted that there is a large range in the values of $\bar{\chi} + \chi_r$ in α Scl; lines with extreme values are weaker or absent in π Cet, while lines with intermediate values have similar strengths in the two stars. Evidently, there is a large range in surface temperature in α Scl. The HeI lines are an exception, being stronger in π Cet; this may be related to the fact that they are affected to a greater extent by Stark broadening than the other lines. Sargent and Strittmatter (1966) found that the spectrum of ADS 4193B was similar to that of α Scl.

It thus appears that the abnormal line strengths in HD 37058, α Scl, 36 Lyn, 20 Tau, HD 175156 and ADS 4193B can be satisfactorily accounted for on the basis of normal chemical composition if there is a large range in surface temperature in each of these stars. The most likely cause of the large ranges in surface temperature is rapid rotation viewed pole-on. 3 CenA cannot be considered as a star of normal chemical composition, since the analysis by Jugaku, Sargent and Greenstein (1961) shows that the relative abundances of certain elements are abnormal by factors which are too large to be explained by errors including those due to structural abnormality. An examination of the spectrum of ϵ Cas failed to reveal any additional line strength anomalies.

A Roche model for a star with a mean temperature of 16,500°K and an equatorial rotational velocity of 350 km/sec was worked out by Ireland (1965). He found that when such a star is viewed pole-on, about 30% of the light comes from the polar regions with surface temperatures within 1000 degrees of that at the pole and about 10% comes from the equatorial regions with surface temperatures within 3000 degrees of that at the equator. The mean angular distance from the axis of rotation of these equatorial regions is about 80°. Table 5 gives approximate estimates of the temperatures at the pole and at 80° from the pole in α Scl, 36 Lyn, 20 Tau, and HD 175156. The estimates of the polar temperatures were made by comparing the lines of greatest $\chi + \chi_r$ in these stars with the same lines in normal main sequence stars and determining the temperatures of normal stars in which these lines are 10/3 times stronger than in α Scl, 36 Lyn, 20 Tau, and HD 175156; the polar temperatures are about 500 degrees higher than these temperatures. Corrections were made to the line strengths to allow for the curvature of the curve-of-growth. The temperatures at 80° from the axis of rotation in α Scl, 36 Lyn, 20 Tau, and HD 175156 were determined in a similar way from the lines of lowest $\chi + \chi_r$ by finding the temperatures of normal main sequence stars in which these lines are 10 times stronger. In this case corrections were made using Saha's ionisation equation, since the surface gravity and electron pressure are lower at the equator in rapidly rotating stars than the mean surface gravity and electron pressure in

Table 5

TEMPERATURES IN RAPIDLY ROTATING POLE-ON STARS

Star	Temperature at pole	Temperature 80° from pole	Difference
α Scl	20,000°K	9,000°K	11,000°
36 Lyn	18,000°K	8,000°K	10,000°
20 Tau	17,500°K	9,000°K	8,500°
HD 175156	19,000°K	9,500°K	9,500°

slowly rotating main sequence stars. The scale of effective temperatures of normal main sequence stars by Aller (1963) was used. The justification of the above procedure is that the lines of highest and lowest $\chi + \chi_r$ are formed in α Scl, 36 Lyn, 20 Tau, and HD 175156 only in the regions of their surfaces with extreme temperatures. The errors in the temperature estimates in Table 5 are probably less than ±2,000°. The temperature differences are similar to those predicted by Ireland (1965) for stars near rotational break-up. He considered Roche models with a mean temperature of 16,500°K. For an equatorial rotational velocity of 350 km/sec the temperature at the pole was 20,700°K and the temperature at 80° from the axis of rotation was 14,000°K. The corresponding temperatures at rotational break-up were 22,200°K and 14,000°K. His models neglected the effect of limb-darkening, but he pointed out that this would probably increase the observed temperature difference between pole and equator.

Another way of accounting for the observed ranges of temperature in α Scl, 36 Lyn, 20 Tau and HD 175156 might be to suppose that these stars are binaries. However, no periodic radial velocity variations have been noted in any of these stars. Also, since both the HeI and the CII lines decrease in strength in normal stars towards spectral type A0 (Figure 1), it would not be possible to explain the weakness of the HeI lines and the slight enhancement of the CII lines in α Scl, 36 Lyn and 20 Tau simply by having pairs of slowly rotating stars of different temperatures. These arguments do not, of course, entirely exclude the rather unlikely possibility of a very close binary viewed almost exactly perpendicular to the orbital plane.

For a rapidly rotating star viewed nearly pole-on it would be expected that the rotational broadening for lines with low values of $\chi + \chi_r$ formed near the equator would be greater than for polar lines with high values of $\chi + \chi_r$. Unfortunately, this effect will usually be masked by errors in the measurement of line-widths and possibly by large-scale turbulence associated with surface currents of matter caused by the large range in surface temperature. Equivalent widths and central depths of lines in 20 Tau were measured by Su-Shu Huang and Struve (1956). There is a slight indication that for a given equivalent width the central depth is greater for lines with high values of $\chi + \chi_r$ as would be expected if rotation is the main broadening agent; however, measurements of higher accuracy will be required before a conclusive result can be obtained.

Sargent and Searle (1966) and Sargent and Strittmatter (1966) have criticised the suggestion that α Scl, 36 Lyn, 20 Tau, and HD 37058 are normal rapidly rotating pole-on stars. They pointed out that the ratios of the strengths of the HeI and CII lines, which have similar excitation and ionisation potentials, are abnormal in α Scl, 36 Lyn and 20 Tau. This does not necessarily rule out the pole-on hypothesis, because the HeI lines suffer Stark broadening and are so strong that they will be formed much higher in the atmosphere than the CII lines. Their other main argument is that in the sharp-line Be stars, which are almost certainly rapidly rotating pole-on stars, the strengths of the HeI lines are normal for the intrinsic colours. This argument is also inconclusive, as any attempt to assess the effect of rotation on the strengths of the HeI lines would require a detailed knowledge of the atmospheric structure and would have to take account of Stark broadening and limb darkening. Moreover, there is the possibility that the Balmer lines in emission might affect (B - V) or the correction for interstellar reddening in the Be stars. It is interesting to note that Slettebak (1954) found no trace of Hα emission in the rapidly rotating stars α Leo (MK type B7V, vsini = 352 km/sec) and ζ Aql (MK type B9V, vsini = 365 km/sec). If these stars were viewed pole-on, their spectra would probably be similar to those of α Scl, 36 Lyn and 20 Tau.

Sargent and Strittmatter (1966) noted that HD 37058 lies on the lower edge of the main sequence for the nebula region of the Orion association and that such a position in the colour magnitude diagram would imply slow rotation. This type of argument is subject to the uncertainties of association membership, distance spread within the association, interstellar reddening corrections, and age differences among association members. 20 Tau does not lie near the lower edge of the Pleiades main sequence, but this should not be regarded as an argument for rapid rotation, since most of the brighter stars in the Pleiades are probably evolving off the main sequence. A more powerful argument for the slow rotation of HD 37058 may be the recent discovery of a magnetic field of about 2000 gauss by Sargent and others (1967). Magnetic fields of this strength are uncommon except in peculiar A stars, which are slow rotators; indeed, Sargent and his colleagues suggested that the Orion stars with weak HeI lines are an extension of the peculiar A sequence towards bluer colours. No magnetic field has been discovered in α Scl, although the lines are sufficiently sharp for a strong field to be detected (Chapter 3).

To sum up, the theoretical prediction of large ranges of surface temperature in rapidly rotating stars is confirmed by observation. All the available evidence supports the view that α Scl, 36 Lyn, 20 Tau and HD 175156 are examples of normal rapidly rotating stars viewed pole-on. On the other hand, the strong magnetic field in HD 37058 suggests that this star may belong to the peculiar A group and rotate slowly. The cause of the weakness of the HeI lines in α Scl, 36 Lyn and 20 Tau remains an unsolved problem awaiting the construction of detailed models of rapidly rotating stars. It is still an open question whether there might be a real deficiency in the atmospheric abundance of helium due to some unknown factor.

CHAPTER 3

ROTATION OF THE SILICON AND MANGANESE STARS

Line widths

There have been various suggestions that the types of peculiar A stars may be related to rotation (Sargent, 1964). The silicon and manganese stars have the highest colour temperatures among the peculiar A stars, and the rotation of these stars is now examined.

Lists of the Si-4200, Si, and Mn stars brighter than $m_v = 7.0$ were compiled from the catalogues by Jaschek and Jaschek (1958) and Bertaud (1959, 1960, and 1965). Stars with uncertain classifications were excluded, but in cases where the uncertainty was between the Si-4200 and Si groups the classification given by Jaschek and Jaschek (1958) was adopted. The line at 4200A has recently been identified by Bidelman (1962) as a high excitation line of SiII; the stars in which this line is present have therefore the highest excitation temperatures of the stars with abnormally strong lines of SiII. Tables 1 and 2 give the lists of Si-4200 and Mn stars; a similar list for the 44 Si stars was compiled. The mean values of Q were calculated for each star from the UBV photometry by Osawa (1959), Osawa and Hata (1960, 1961), Abt and Golson (1962), Cousins and Stoy (1963), Crawford (1963), Eggen (1963), and Iriarte and

Table 1

Si-4200 STARS BRIGHTER THAN $m_v = 7.0$

HD	Name	w	vsini	Q	No. of Sources
3580		1.4:			
12767	ν For	(2.4)	60	-0.37	2
14392	63 And	3:		-0.29	1
18296	21 Per	0.6:	0	-0.23	3
19832	56 Ari	(3)		-0.33	1
25267	τ^9 Eri	(0.5)	22	-0.32	4
25823	41 Tau	0.43	10	-0.38	3
27039	56 Tau	1.5	43	-0.30	2
29305	α Dor	(4.7)	175	-0.27	1
32549	11 Ori	1:	29	-0.04	3
34452	HR1732	(1.5)	57	-0.43	1
73340	HR3413			-0.44	1
74535	HR3466			-0.39	1
133652	HR5619	0.5:			
133880	HR5624	2:			
170000	φ Dra	1.5:	60	-0.28	2
175744	HR7147			-0.41	2
192913		0.2:		-0.19	1
196178	HR7870	1.5		-0.43	1
203585	HR8180			-0.04	1
221006	HR8919				
223640	108Aqr	0.8:	68	-0.31	2
224166					

Table 2

MN STARS BRIGHTER THAN $m_v = 7.0$

HD	Name	w	vsini	Q	No. of Sources
358	α And	(1.1)	50	-0.34	3
3322	HR 149			-0.33	2
27295	53 Tau	(0.1)	0	-0.21	2
33904	μ Lep	0.30	0	-0.32	5
53244	γ CMa	1:	19	-0.39	3
75333	14 Hya	(0.7)	13	-0.27	3
78316	κ Cnc	0.13	11	-0.36	3
110073	1 Cen	0.2:	25	-0.38	3
129174	π^1 Boo	0.4	0	-0.32	1
144206	ν Her	(0.1)	0	-0.25	3
145389	φ Her	(0.1)	0	-0.22	3
172044	HR6997	(0.9)	39	-0.42	3
174933	112Her	(0.1)	23	-0.37	3
178628	HR7268				
207857	HR8349			-0.41	2
221507	HR8937	0.3:	25	-0.29	3

Table 3

MEAN LINE WIDTHS

	Si-4200 stars	Si stars	Mn stars
Mean value of Q	-0.30	-0.18	-0.33
Mean value of w	1.56 ± 0.27A	1.46 ± 0.30A	0.42 ± 0.10A
Mean value of vsini	52 km/sec	70 km/sec	16 km/sec

others (1965). Q will not be greatly affected by abundance anomalies, since it depends mainly on the size of the Balmer discontinuity. Searle and Sargent (1964) found that Balmer lines Hγ and Hδ of the peculiar A stars have profiles which do not differ systematically from those of normal main sequence stars of the same colour. Fig. 1 shows that the distribution of the values of Q is approximately the same for the Si-4200 and the Mn stars, while the values of Q for the Si stars are generally larger. Tables 1 and 2 also give the spectral line widths w determined by Babcock (1958a) from coudé spectra at dispersions of 2.2, 4.5 and 10 A/mm and the projected rotational velocities vsini from the catalogue by Boyarchuk and Kopylov (1964). The values of w in brackets were found by transforming the widths of the MgII 4481 line determined by Su-Shu Huang (1953) from 3-prism spectra at dispersions of 10.4 and 13.3 A/mm to Babcock's system. The values of w for 56 Ari and HD 124224 were determined by Deutsch (1958a).

The mean values of Q, w, and vsini for each group and the standard deviations of the means are given in Table 3. The distribution of line widths w in each group is shown in Fig. 2. It will be seen that the line widths of the Si-4200 and Si stars appear to be generally larger than those of the Mn stars. The mean equivalent width of the SiII 4200 line for the 6 stars in Table 1 measured by Searle and Sargent (1964) is 163 mA, while the equivalent widths of the strongest

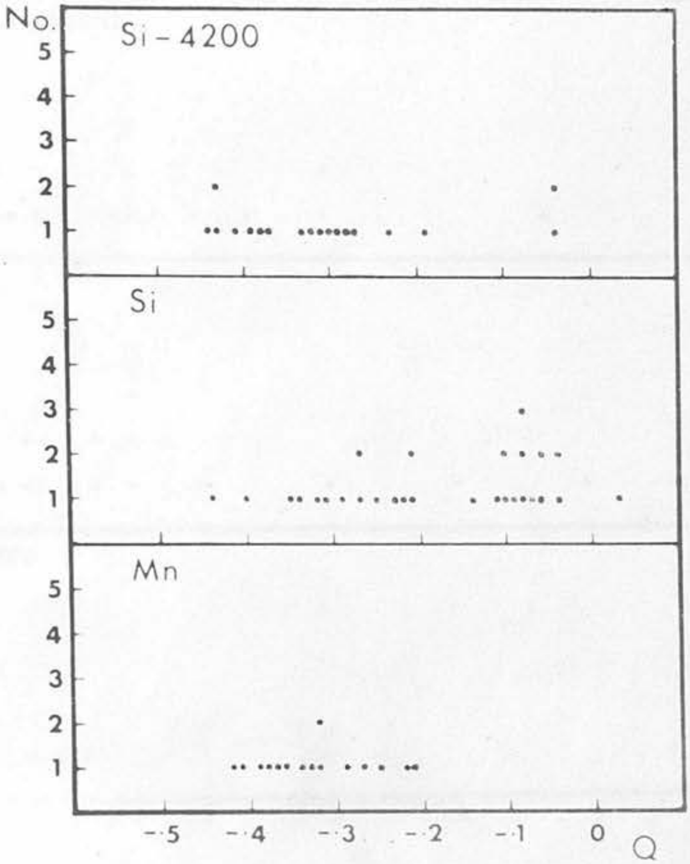

Figure 1

DISTRIBUTION OF THE VALUES OF Q FOR

Si-4200, Si, AND Mn STARS

The distributions are similar for the Si-4200 and Mn groups, but the values of Q for the Si stars are generally larger.

Mn lines in the Mn star 53 Tau are only about 80 mA (Aller and Bidelman, 1964). Broad-line Si-4200 stars will therefore be more easily detected than broad-line Mn stars. This may partly explain the lack of broad-line Mn stars in Fig. 2. However, there must be a real difference in the distribution of line widths between the Si-4200 and Mn groups, since the distributions are quite different even for stars with $w < 2$. In particular, the preponderance of stars with very sharp lines in the Mn group is notable. The distribution of line widths for the Si group is similar to that for the Si-4200 group.

Discussion

Other stars near the main sequence with values of Q similar to those of the Si-4200 and Mn stars may be divided into the following groups:-

(1) Sharp-line stars which are normal in the sense that their line strengths may be accounted for from a consideration of excitation and ionisation potentials assuming normal chemical composition. The rotation of these stars is slow. Unlike the sharp-line peculiar A stars, they have no measurable magnetic field (Babcock, 1958b).

(2) A group of stars represented by α Scl, 36 Lyn, and 20 Tau whose line strengths may also be accounted for on the basis of normal chemical composition, it if is assumed that they are rapidly

rotating and viewed pole-on so that there is a large range in effective surface temperature.

(3) The Si and Si-Eu-Cr groups of peculiar A stars, which are a continuation of the Si-4200 group as the colour temperature is decreased.

(4) Broad-line stars in which only a few lines can be seen. Most of these will be normal rotating stars viewed at high inclinations to the axis of rotation.

(5) Stars with Hα in emission. Most, and perhaps all, of these stars will be rotating rapidly.

The possibility that there is a connection between the peculiar A stars and the rapidly rotating pole-on stars (group (2)) seems to be eliminated. The excitation and ionisation potentials and Zeeman patterns of the MgII 4481 and SiII 4131 lines are similar, and the ratios of the equivalent widths of these lines are normal in α Scl, 36 Lyn, and 20 Tau. Searle and Sargent (1964) found a distinct gap in the ratios MgII 4481/SiII 4131 between the Si-strong (i.e. the Si-4200, Si, and Si-Eu-Cr groups) and other peculiar A and normal stars. They pointed out that this gap is not due to a selection effect, since they observed stars which had been classified as Si stars but which proved to be stars with normal silicon lines. The Si-strong stars are thus quite distinct from normal rapidly rotating pole-on stars.

Lines of manganese are absent in α Scl in the wavelength range 3600 to 8600A (Jugaku and Sargent, 1961), although Morgan (1931) classified it as a manganese star. Su-Shu Huang and Struve (1956) gave only a doubtful identification of the MnII line at 4253.02A in an examination of 2 coudé spectra of 20 Tau in the wavelength range 3710 to 4735A. A search was made for manganese lines on the Mount Wilson spectra of α Scl and 20 Tau and on the Edinburgh spectra of 36 Lyn. The only certain identification of manganese was the multiplet No.3 of MnII in one of the spectra of 20 Tau. Its presence could not be determined in the spectra of α Scl or 36 Lyn, since none of them extended sufficiently far into the ultraviolet (i.e. to $\lambda < 3500$A). The presence in 20 Tau of the lines of this multiplet, which have low excitation and ionisation potentials, can be accounted for by the range in surface temperature produced by rapid rotation viewed pole-on. The Mn stars are therefore also distinct from normal rapidly rotating pole-on stars.

Babcock (1958a) and Gollnow (1962) found that the lines in 20 Tau and α Scl were too broad for the measurement of magnetic fields. Table 4 gives the line widths for α Scl, 36 Lyn, and 20 Tau. On several of the Mount Wilson spectra of α Scl, w was estimated to be 0.3A; this agrees with the value of w deduced from Su-Shu Huang's measure of the width of the MgII 4481 line. α Scl therefore has lines which are sufficiently sharp for a strong magnetic field to be detected if present. The presence or absence of a magnetic field in α Scl would be of interest, since all sharp-line Si-strong and Mn stars have magnetic fields (Babcock, 1958b).

Table 4

LINE WIDTHS FOR RAPIDLY ROTATING POLE-ON STARS

HD	Name	w	vsini	Q
5737	α Scl	(0.3)	0	-0.43
79158	36 Lyn	(1.1)	27	-0.40
23408	20 Tau	0.8	30	-0.35

There are three possible ways of explaining the preponderance of sharp-line stars in the Mn group as compared with the Si-4200 group:-

(1) The equatorial rotational velocities of the Si-4200 stars may be larger than those of the Mn stars, and each group may include members viewed at all angles of inclination to the axis of rotation.

(2) The Si-4200 and Mn stars may be similar in abundance anomalies and equatorial rotational velocities, but the Mn stars may be viewed at smaller angles of inclination to the axes of rotation, an overabundance of manganese being formed in the polar region and an overabundance of silicon in the equatorial region in every peculiar star of either group.

(3) The third possibility is that the Mn stars have an overabundance of manganese only in the polar regions and an approximately normal abundance of silicon over the whole surface, while the Si-4200 stars have an approximately normal abundance of manganese over the whole surface and an overabundance of silicon except possibly at the pole. A polar "patch" of manganese in the Mn stars would be observed only in stars viewed at fairly low inclinations to the axis of rotation, and this could explain the preponderance of sharp-line stars in this group.

Spectra, taken with the Edinburgh 36-inch telescope, of the broader-lined Mn stars (α And, π^{1}Boo and HR 6997) were examined, but there is no indication that the Mn lines are sharper than the Si lines. This would appear to rule out the second and third explanations, which suggest that the enhanced Mn lines are formed only in polar regions. The first explanation, which states that the equatorial rotational velocities of the Si-4200 stars are larger than those of the Mn stars, is probably the correct one.

Further investigation of peculiar A stars which exhibit periodic spectrum variations would be desirable, since in many cases the period of rotation may be the same as the period of the spectrum variation. Deutsch (1956), Bonsack (1958), and Peterson (1966) have shown that the periodic variations of the strengths of the SiII lines in the Si-4200 star 56 Ari and the Si star HD 124224 are due to the silicon-rich regions being brought into view periodically as the stars rotate. The line widths and the periods (0.73 days for 56 Ari and 0.52 days for HD 124224) therefore indicate that these stars are viewed nearly equator-on. No conclusive results are available for the Mn stars α And and π'Boo, which may have periodic spectrum variations (Deutsch, 1947). Most peculiar A stars with regular spectrum variations have periods of several days. This may constitute additional evidence that equatorial rotational velocities are smaller for peculiar A stars than for normal main sequence stars of similar intrinsic colours.

CHAPTER 4

PREVIOUS WORK ON ABUNDANCES IN PECULIAR A STARS

Methods of determining abundances

The three main methods of determining the chemical composition of stellar atmospheres are the curve-of-growth method, the differential curve-of-growth method, and the model-atmosphere method. Caution must be exercised in applying these methods to peculiar A stars in view of the lack of knowledge on the structure of these stars.

The fundamental assumption in the curve-of-growth method is that the formation of lines may be adequately described by assigning a single temperature and electron pressure to the whole atmosphere. Consequently, stratification of the atmosphere and possible temperature differences over the surface (e.g. between pole and equator or in areas of magnetohydrodynamic activity) are neglected. The model-atmosphere approach specifies the variation of temperature and electron pressure with optical depth and takes account of the effects of stratification in the atmosphere. Nevertheless, it is usually assumed for simplicity that the star is spherical, and serious errors could arise if a spherical model were applied to a rotating star or to a close binary. Application of these methods to peculiar A stars is therefore not justified without further investigation except as a means of searching for structural abnormalities.

Abundances derived by the curve-of-growth and model-atmosphere methods are also subject to errors due to the uncertainty in laboratory gf-values. Considerable confusion has resulted, because authors tend to use different sources of gf-values. Comparison of one peculiar A star with another is therefore difficult. The most satisfactory procedure would be to select a normal star for which a reliable model-atmosphere may be derived and to determine abundances for this star. A refined differential curve-of-growth method would then give abundances for peculiar A stars without the need for additional gf-values. If the model-atmosphere or the gf-values used for the normal star were subsequently improved, the abundances in the peculiar A stars could be rapidly modified without a complete re-analysis of the data on each star. The chief problem is how to refine the differential curve-of-growth method so that the results will not be affected if the peculiar A stars have abnormal stratification or temperature differences over their surfaces.

Previous abundance determinations in peculiar A stars

There have been several attempts to derive abundances in individual peculiar A stars. Qualitatively, these attempts serve to indicate that the abundances of many elements are abnormal and that the rare earths are overabundant in Eu-Cr stars such as α^2 CVn.

Quantitatively, however, most of the results must be considered unreliable, because they would be affected by structural abnormalities in the stellar atmospheres.

Certain lines in peculiar A stars, for example those of OI, MgII and SiII, have excitation and ionisation potentials which make their strengths insensitive to temperature and electron pressure for the ranges of temperature and pressure being considered. Accordingly, abundances of the corresponding elements may be derived without a detailed knowledge of atmospheric structure. Reliable relative abundances may also be obtained by using groups of lines of similar excitation and ionisation potentials and wavelengths. These methods were employed by Sargent and Searle (1962), Searle and Sargent (1964), and Searle, Lungershausen and Sargent (1966) in determining abundances of O, Mg and Si and relative abundances of Ti, Cr, Mn, and Fe for peculiar A stars by differential curve-of-growth analyses. Their results are given in Table 1, where the notation

$$(X) = X_{star} - X_{normal}$$

is used, X being the logarithmic abundance ratio of two elements. Their values of Δ(Ti/Fe) for Mn and Si-4200 stars are omitted, because the ionisation corrections for titanium are rather large for these stars. Table 2 gives the mean abundance ratios for the various groups of stars. Oxygen has a normal abundance in the Mn group but is deficient in other groups. The abundance of magnesium is normal in all groups. Silicon is overabundant in the Si-4200, Si and Si-Eu-Cr groups and normal in the other groups. The relative abundances of chromium, manganese and iron are normal, except in the Mn group where they vary from star to star.

Although these results are the most reliable obtained so far, there are two sources of error which might be removed by using spectra at a higher dispersion (i.e. higher than 10 A/mm). Searle and Sargent (1964) assumed that the microturbulent velocities were zero in all the stars, whereas Searle, Lungershausen and Sargent (1966) found that the microturbulent velocities were smaller in the Mn stars than in normal stars. The other groups of peculiar A stars had apparently larger microturbulent velocities as compared with normal stars, but it should be pointed out that the Zeeman broadening of spectral lines will affect the curve-of-growth in the same way as an increase in the microturbulent velocity. Abundances derived from moderately strong lines would be affected by errors in the microturbulent velocity. A higher dispersion would enable a more accurate determination of the microturbulent velocity; alternatively, weaker lines on the linear part of the curve-of-growth could be used.

Table 1

ABUNDANCES IN PECULIAR A STARS

Star	HD	Type	Δ(O/H)	Δ(Mg/H)	Δ(Si/H)	Δ(Ti/Fe)	Δ(Cr/Fe)	Δ(Mn/Fe
53 Tau	27295	Mn		-0.3	+0.1		+0.5	+1.3
μ Lep	33904	Mn	-0.3	-0.5				
14 Hya	75333	Mn					+0.5	+1.3
κ Cnc	78316	Mn	+0.3	-0.8	+0.1		-0.4	+1.2
π' Boo	129174	Mn		+0.1			+0.4	+1.3
υ Her	144206	Mn	-0.3	0.0	+0.1			
φ Her	145389	Mn		-0.3	-0.2			
112Her	174933	Mn	-0.4	-0.3	-0.5		-1.6	+0.1
ν For	12767	Si-4200		-1.0	+0.8			
63 And	14392	Si-4200		-0.2	+1.7			
21 Per	18296	Si-4200	-1.0	-0.8	+2.0		0.0	+0.2
41 Tau	25823	Si-4200	-1.4	-0.5	+1.8		-0.4	-0.3
HR1732	34452	Si-4200		+0.5	+2.0			
15 Cnc	68351	Si-4200		-0.3	+1.7			
108Aqr	223640	Si-4200	-0.4	-0.1	+1.8			
ρ HerA	157779	Si		+0.3	+0.8:			
α^2CVn	112412	Si-Eu-Cr	-1.0	-0.5	+1.0			
HR6176	149822	Si-Eu-Cr		+0.3	+2.1			
ο Aur	38104	Eu-Cr		-0.2	< -0.8			
17ComA	108662	Eu-Cr		+0.4	+0.7	-0.1	+0.5	+0.2
78 Vir	118022	Eu-Cr		+0.2	-0.6:			
ω Her	148112	Eu-Cr		+0.2	-1.1:			
45 Her	151525	Eu-Cr	-1.0:	-0.1				
HR8216	204411	Eu-Cr		+0.6	+0.8	+0.2	+0.4	+0.3
κ Psc	220825	Eu-Cr	< -2.0	+0.7	-0.1	+0.1	+0.3	-0.1

Table 1 (continued)

ABUNDANCES IN PECULIAR A STARS

Star	HD	Type	Δ(O/H)	Δ(Mg/H)	Δ(Si/H)	Δ(Ti/Fe)	Δ(Cr/Fe)	Δ(Mn/
β CrB	137909	Eu-Cr-Sr	-1.8	0.0				
10 Aql	176232	Eu-Cr-Sr	-0.9	+0.2				
γ Equ	201601	Eu-Cr-Sr	-1.7	+0.6		+0.1	+0.1	-0.2
HR 410	15144	Sr		+0.5	+0.1	0.0	+0.3	+0.2
ω Oph	148898	Sr	-1.6	+0.5				

Table 2

ABUNDANCES FOR PECULIAR A GROUPS

Group	Mn	Si-4200 and Si	Si-Eu-Cr and Eu-Cr	Eu-Cr-Sr and Sr
Range in Δ (O/H)	-0.4 to +0.3	-1.4 to -0.4	-2.0 to -1.0	-1.8 to -0.9
No. of stars	4	3	3	4
Mean Δ (O/H)	-0.2	-0.9	-1.3	-1.5
Range in Δ (Mg/H)	-0.8 to +0.1	-1.0 to +0.5	-0.5 to +0.7	0.0 to +0.5
No. of stars	7	8	9	5
Mean Δ (Mg/H)	-0.3	-0.3	+0.2	+0.4
			Eu-Cr only	
Range in Δ (Si/H)	-0.5 to +0.1	+0.8 to +2.0	-1.1 to +0.8	+0.1
No. of stars	5	8	6	1
Mean Δ (Si/H)	-0.1	+1.6	-0.2	+0.1

The other source of error in the results in Table 1 is associated with the lack of knowledge of the physical conditions and structure of the atmospheres. Errors of this type depend mainly on the excitation and ionisation potentials of the lines used, and are small only because of the careful selection of lines. Searle and Sargent (1964) adopted values of ionisation temperatures and electron pressures which they derived from B - V colour indices and Balmer line profiles. It would be better to derive atmospheric parameters for making ionisation corrections and corrections for differences in the excitation potentials of the lines from an investigation of an element which produces lines with wide ranges of excitation and ionisation potentials. This would normally require spectra at a higher dispersion, because equivalent widths of many lines would be needed; otherwise, random errors might be larger than the systematic errors of the method used by Searle and Sargent (1964). Their method for making the small ionisation and excitation corrections is wrong in principle for stars which might have ranges of surface temperature.

Study of the light elements lithium, beryllium and boron is important, because they could be destroyed by thermonuclear reactions with protons at temperatures higher than about 3×10^6 degrees Kelvin such as occur in stellar interiors. Unfortunately, it has proved very difficult to find suitable lines of these elements in peculiar A stars. Two lines of BeII at 3130.4 and 3131.1A can be measured in the hotter peculiar A stars, but they are blended in cooler stars.

Fairly reliable abundances may be derived from these lines, since the ionisation corrections are small. Bonsack (1961) found that beryllium was overabundant in the Si-Eu-Cr star α^2 CVn by a factor of about 25. Sargent, Searle and Jugaku (1962) examined spectra of 25 peculiar A stars and noted that the BeII lines were abnormally strong in four out of ten Mn stars (corresponding to an overabundance by a factor of about 100); the lines were very weak or absent in the other six Mn stars and in five Si-4200 stars. The excess of beryllium in α^2 CVn and the four Mn stars is probably due to surface nuclear reactions involving ions accelerated by magnetic fields. Surface physical conditions, such as the configuration of magnetic fields, might be different in the other six Mn stars, so that different nuclear reactions would take place. An alternative explanation is that beryllium is destroyed by convective mixing with the interior in normal stars and in the six Mn stars with very weak BeII lines. It seems unlikely, however, that convective mixing would be an effective mechanism in six Mn stars and not in the other four. In other words, it is more plausible to suggest that only the surface conditions, rather than the entire structures of the stars, vary from star to star in the Mn group. Furthermore, the convection zones of slowly rotating early-type stars are believed to be shallow during their main sequence life-times (Rudkjöbing, 1942).

For some elements the enhancement of lines is so large that there is no doubt about the reality of the overabundances. Thus the rare-earth elements (but not barium) are overabundant by large factors in many stars of all groups except the Mn group (Sargent, 1964), and phosphorus and gallium are overabundant in some Mn stars (Jaschek and others, 1965). Strontium is probably overabundant in many peculiar A stars.

CHAPTER 5

THE CHEMICAL COMPOSITION OF THE MANGANESE STAR 53 TAU

Introduction

A suitable star for assessing the various methods of determining chemical composition is the manganese star 53 Tau, which has very sharp spectral lines. Aller and Bidelman (1964) studied its spectrum at high dispersion (2 A/mm) and obtained equivalent widths for lines in the wavelength range 3677 to 4756A. They performed an approximate abundance analysis by the normal curve-of-growth method, but pointed out certain discrepancies which they suggested might be resolved by a model-atmosphere approach. Warner (1965) repeated the curve-of-growth analysis for some elements using additional laboratory determinations of f-values. A model-atmosphere study has also been carried out by Auer and others (1966). The aim of the present differential curve-of-growth analysis is to try to derive the chemical composition of the atmosphere of 53 Tau without making any assumptions about its structure, which may be abnormal. Comparisons between metastable and non-metastable lines will be avoided in case the assumption of local thermodynamic equilibrium is not valid.

α Lyr has been chosen as the comparison star, since its UBV colours are similar to those of 53 Tau, and Hunger (1955, 1960) has carried out a model-atmosphere abundance analysis based on high-dispersion spectra. Table 1 gives the UBV colours for the two stars. The intrinsic colour index $(B - V)_0$ was obtained using the nomogram given by Johnson (1958).

Table 1

UBV COLOURS FOR 53 TAU AND α LYR

Star	HD	B - V	U - B	$(B - V)_0$	Source
53 Tau	27295	-0.05	-0.24	-0.07	Osawa and Hata (1960)
		-0.10	-0.28	-0.07	Crawford (1963)
α Lyr	172167	0.00	-0.01	0.00	Johnson (1955)

Wallerstein and Hunziker (1964) obtained abundances for several early A-type stars relative to α Lyr by the differential curve-of-growth method, and found that there was apparently a large range in the values of the metal to hydrogen abundance ratios. The value obtained for α Lyr was intermediate for stars with low space velocities; 53 Tau also has a low space velocity. Hunger found that the chemical composition of α Lyr was similar to that of the Sun.

Construction of the curve-of-growth

Since the equivalent width W is proportional to the number of atoms "above the photosphere" in the appropriate state of excitation and ionisation only for weak lines, the construction of a curve-of-growth is necessary for each star. This will give the corrected equivalent width W_{corr} for a strong line, i.e. the equivalent width which the line would have had if the curve-of-growth were linear.

The same form of the curve-of-growth as used by Aller and Bidelman (1964) will be adopted. The ordinate is $\log (Wc/\lambda v)$, where c is the velocity of light, and

$$v = \sqrt{\left(\frac{2kT}{M} + \xi_t^{\,2}\right)}. \qquad (1)$$

in equation (1), k is Boltzmann's constant, T the gas kinetic temperature, M the mass of the atom, and ξ_t the microturbulent velocity. The abscissa of the curve-of-growth is

$$\log X_o = \log N_r - \theta_{exc}\chi + \log gf\lambda + C(\lambda). \qquad (2)$$

where N_r is the number of atoms in the rth stage of ionisation, χ the excitation potential of the lower level of the atomic transition, g the statistical weight of the lower level, f the oscillator strength, and λ the wavelength of the line.

$$\theta_{exc} = \frac{5040}{T_{exc}}, \qquad (3)$$

where T_{exc} is the excitation temperature.

$$C(\lambda) = \log \frac{e^2\sqrt{\pi}}{mc} - \log v - \log u(T) + \log \frac{\kappa(\lambda_0)}{\kappa(\lambda)}, \qquad (4)$$

where e and m are the charge and mass of the electron respectively, u(T) the partition function, and the last term takes account of the variation of the continuous absorption coefficient $\kappa(\lambda)$ with wavelength.

v and θ_{exc} are not known initially, but the empirical curve-of-growth for the Sun given by Wright (1944) may be fitted to a plot of log (W/λ) against log gfλ for an element with a large number of measured lines (both weak and strong) with a small range in excitation potential and well-determined gf-values. Multiplets (18) to (61) of TiII satisfy these requirements. The spectral lines of 53 Tau are very sharp (Aller and Bidelman, 1964), and ξ_t may be taken as zero. The curve-of-growth for 53 Tau derived from the plot of log (W/λ) against log gfλ will therefore be applicable to all elements, if a correction term log $\sqrt{M}$ is added to log (W/λ) before Wright's curve-of-growth is fitted. Hunger (1955) derived upper limits for ξ_t in α Lyr from the curves-of-growth for TiII and FeI as 3 km/sec and 2 km/sec respectively. His adopted model atmosphere for α Lyr gives an effective temperature $T_{eff} = 9500^{\circ}$. Putting $T = T_{eff} = 9500^{\circ}$, $\sqrt{\frac{2kT}{M}} = 1.8$ km/sec for titanium. ξ_t^{2} is therefore of the same order as $\frac{2kT}{M}$, but its exact value is uncertain. Since the ratios of the atomic weights of the elements to be considered to that of titanium lie in the range 0.5 to 1.9 (i.e. near unity), a mathematical approximation will be made, and the correction term to be added to log (W/λ) will be taken as

$\frac{1}{2}\log \sqrt{M}$ for α Lyr, i.e. putting $\xi_t^{\,2} = \frac{2kT}{M}$. Errors in these corrections will not have a serious effect on the derived abundances, provided that very strong lines are avoided; this restriction is necessary in any case, since it cannot be assumed that the curves-of-growth for 53 Tau and α Lyr will have the same shape as that for the Sun for lines stronger than the strongest TiII lines of multiplets (18) to (61).

Table 2 lists all the unblended lines of TiII in 53 Tau with gf-values by Tatum (1961), $\lambda > 3800$A, and no depression of the continuum by the Balmer lines. The multiplet number, the wavelength λ, and the excitation potential χ are taken from the multiplet tables by Moore (1945). Some of the gf-values listed by Tatum were obtained from solar and stellar spectra, and these values are given in brackets; the other gf-values are his laboratory determinations. All the gf-values are on the same scale. Fig. 1 shows Wright's curve-of-growth fitted to plots of $\log (W/\lambda) + \log \sqrt{M}$ and $\log (W/\lambda) + \frac{1}{2}\log \sqrt{M}$ against $\log gf\lambda$ for 53 Tau and α Lyr respectively for multiplets (18) to (61) of TiII ($\chi = 1.08$ to $\chi = 1.24$). The lines with 45° slope were used for deducing $\log W_{\text{corr}}$.

Table 2

CURVE-OF-GROWTH FOR TiII LINES

Multiplet	λ	χ	log gfλ	log (W/λ) + log $\sqrt{M}$ 53 Tau	log (W/λ) + $\frac{1}{2}$log $\sqrt{M}$ α Lyr
11	4025.1	0.60	(-6.26)	-16.11	
	4012.4	0.57	-5.99	-15.94	
12	3813.4	0.60	-6.07	-16.06	
	3814.6	0.57	-5.85	-15.92	
18	4469.2	1.08	(-6.29)	-16.75	
19	4395.0	1.08	-4.66	-15.73	-10.23
	4443.8	1.08	-4.99	-15.76	-10.33
	4450.5	1.08	-5.81	-16.03	-10.68
20	4294.1	1.08	-5.29	-15.83	-10.30
	4287.9	1.08	(-5.99)	-16.07	-10.83
31	4468.5	1.13	-4.87	-15.72	-10.29
	4501.3	1.11	-5.14	-15.74	-10.30
	4444.6	1.11	(-6.37)	-16.38	
34	3913.5	1.11	-4.61	-15.67	-10.10
	3932.0	1.13	-5.94	-15.97	-10.68
40	4441.7	1.18	(-6.39)	-16.49	
	4470.9	1.16	(-6.17)	-16.42	
	4417.7	1.16	-5.50	-15.94	-10.56
	4464.5	1.16	(-5.72)	-16.08	-11.11

Table 2 (continued)

CURVE-OF-GROWTH FOR TiII LINES

Multiplet	λ	χ	log gfλ	log (W/λ) + log $\sqrt{}$ M 53 Tau	log W/λ) + $\frac{1}{2}$log$\sqrt{}$ M α Lyr
41	4300.1	1.18	(-4.87)	-15.81	-10.21
	4290.2	1.16	(-5.20)	-15.76	-10.35
	4301.9	1.16	(-5.31)	-15.90	-10.59
	4312.9	1.18	-5.51	-15.89	-10.57
	4307.9	1.16	-5.37	-15.94	
	4315.0	1.16	-5.32	-15.88	
	4321.0	1.16	(-6.07)	-16.18	
50	4534.0	1.23	(-4.74)	-15.74	
	4563.8	1.22	-5.18	-15.79	-10.39
	4590.0	1.23	-6.15	-16.26	-10.78
51	4399.8	1.23	-5.56	-15.93	-10.51
	4394.1	1.22	(-5.96)	-16.15	-10.88
	4418.3	1.23	(-6.17)	-16.24	
60	4544.0	1.24	(-6.90)	-16.75	
	4568.3	1.22	(-7.10)	-16.86	
61	4395.8	1.24	(-6.26)	-16.14	-11.08
	4409.5	1.23	(-6.69)	-16.49	
	4411.9	1.22	(-6.69)	-16.79	
82	4549.6	1.58	-4.48	-15.75	
	4572.0	1.56	-4.58	-15.79	-10.24
	4529.5	1.56	(-5.67)	-16.12	

Table 2 (continued)

CURVE-OF-GROWTH FOR TiII LINES

Multiplet	λ	χ	log $gf\lambda$	log (W/λ) + log $\sqrt{M}$ 53 Tau	log (W/λ) + $\frac{1}{2}$log $\sqrt{M}$ α Lyr
87	4028.3	1.88	-5.20	-15.95	-10.65
	4053.8	1.88	-5.37	-15.94	-10.60
93	4421.9	2.05	(-5.83)	-16.38	
104	4367.7	2.58	(-5.04)	-16.08	-10.82
	4386.9	2.59	(-5.09)	-16.16	-10.95
105	4163.6	2.58	-4.37	-15.78	-10.43
	4171.9	2.59	-4.53	-15.86	-10.46
	4174.1	2.59	(-5.18)	-16.39	-11.06
106	4064.3	2.59	(-5.63)	-16.54	
115	4488.3	3.11	-4.81	-15.93	-10.89
	4411.1	3.08	-4.96	-16.10	-10.84
	4456.7	3.11	(-4.97)	-16.75	

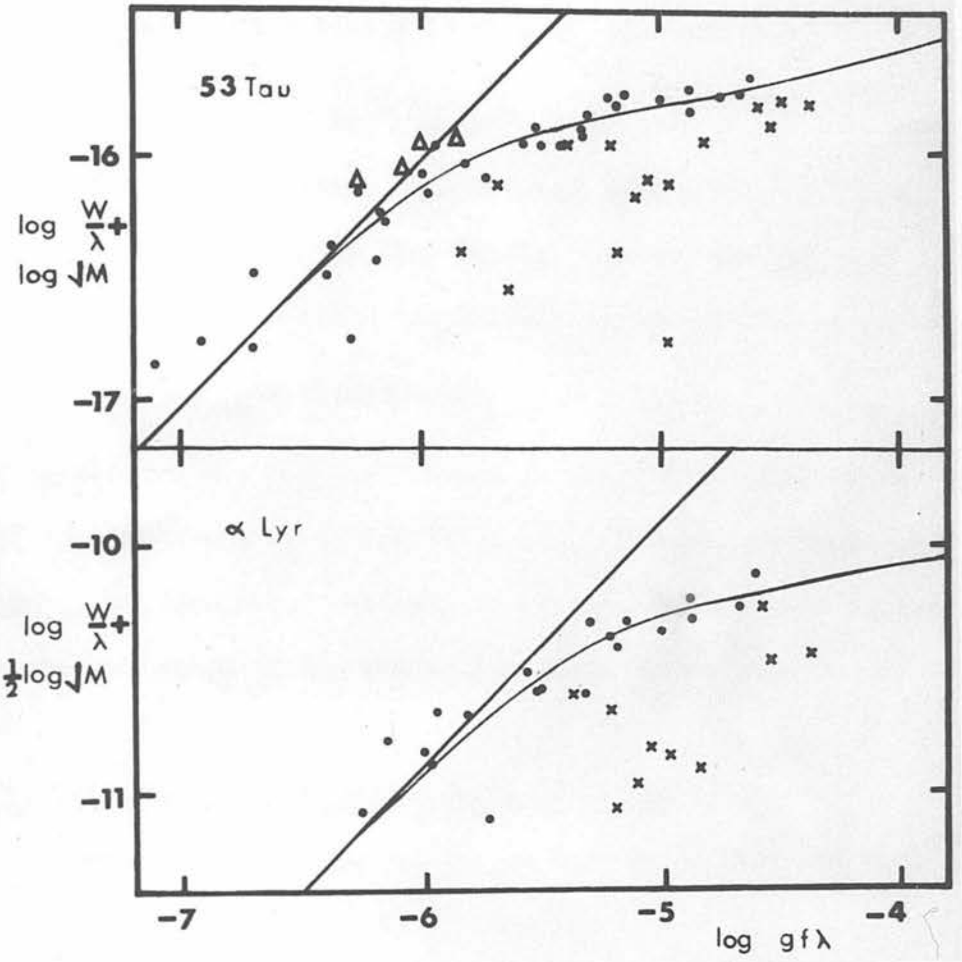

Figure 1

CURVES-OF-GROWTH FOR TiII LINES IN 53 TAU AND α LYR

Triangles denote lines of multiplets (11) and (12), dots multiplets (18) to (61), and crosses multiplets (82) to (115).

The apparent excitation temperature in 53 Tau

The difference $\Delta\delta\log X_o$ in the deviations $\delta\log X_o$ from the curve-of-growth between 53 Tau and α Lyr is plotted against the excitation potential χ in Fig. 2. The notation

$$\Delta X = X_{53\ \mathrm{Tau}} - X_{\alpha\ \mathrm{Lyr}}, \tag{5}$$

where X is any quantity, will be used throughout. A least squares solution of the plot in Fig. 2 gives the apparent difference in excitation temperature between 53 Tau and α Lyr as

$$\Delta\theta_{\mathrm{exc}} = -0.03 \pm 0.06. \tag{6}$$

This will be the real difference in excitation temperature only if the structure of 53 Tau is normal. The mean error ± 0.06 in $\Delta\theta_{\mathrm{exc}}$ is due almost entirely to random errors in the equivalent widths W, since errors in the gf-values cancel out.

The apparent electron pressure P_e in 53 Tau

From the equivalent widths of MgI, MgII, CaI, and CaII lines in α Lyr, Hunger (1955) found

$$\theta_{\mathrm{ion}} = 0.55 \pm 0.03, \qquad \log P_e = 2.98 \pm 0.3, \tag{7}$$

where

$$\theta_{\mathrm{ion}} = \frac{5040}{T_{\mathrm{ion}}} \tag{8}$$

and T_{ion} is the ionisation temperature. If $\Delta\theta_{\mathrm{ion}} = \Delta\theta_{\mathrm{exc}}$, then the apparent value of θ_{ion} is 0.52 for 53 Tau. Using these values, the Saha ionisation equation was applied to the

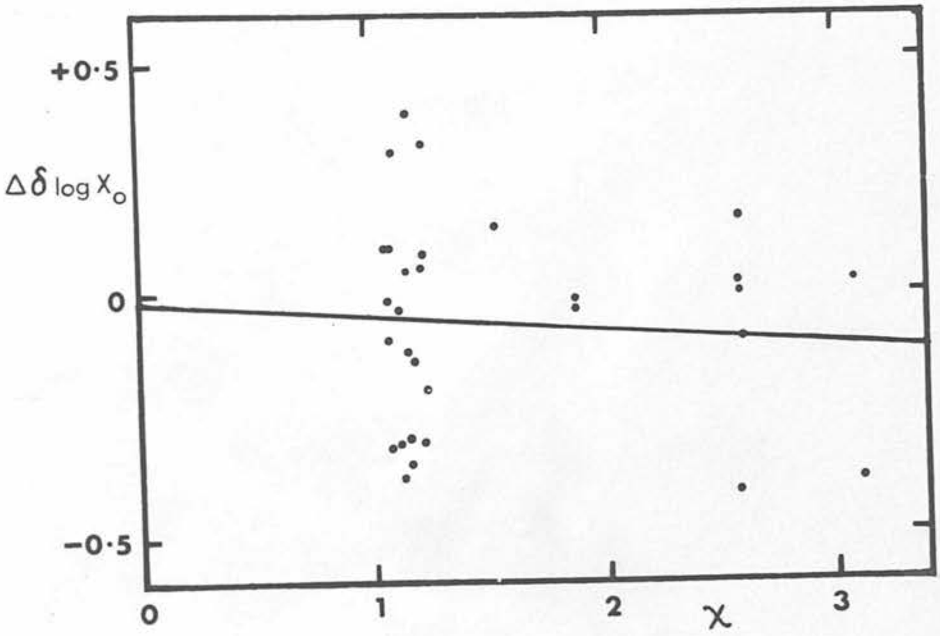

Figure 2

DETERMINATION OF THE APPARENT DIFFERENCE $\Delta\theta_{exc}$

IN THE VALUES OF θ_{exc} FOR 53 TAU AND α LYR

The least squares solution is shown; this gives $\Delta\theta_{exc} = -0.03 \pm 0.06$.

lines of multiplets (20), (41) and (43) of FeI and multiplets (27) and (28) of FeII in 53 Tau and α Lyr listed in Table 3, and it was found that

$$\log P_e = 2.50 \tag{9}$$

for 53 Tau. This deduction of $\log P_e$ for 53 Tau from a comparison with α Lyr avoids the need for gf-values. Lines of CrI were not found in 53 Tau by Aller and Bidelman (1964). Thus the strongest line of CrI in 53 Tau probably has $W < 0.010$A, since $W = 0.005$A for the weakest measured lines. The strongest line of CrI in α Lyr is CrI(1) at $\lambda = 4254.3$A. Application of Saha's equation to this line and the 4 lines of CrII(31) listed in Table 3 gives $\log P_e < 2.68$, which checks the value in equation (9). The value $\log P_e = 2.50$ is only an *apparent* value, since it has been deduced on the assumption that the structure of 53 Tau is normal.

Relative abundances in 53 Tau and α Lyr

Table 3 lists all the unblended lines common to 53 Tau and α Lyr with $\lambda > 3800$A, $W > 0.010$A in both stars, with corrections $\log W_{corr} - \log W < 0.40$, and with no depression of the continuum by the Balmer lines in 53 Tau. Some extra lines not satisfying all these conditions have had to be included to provide abundances of additional elements. The lines of CaII(3) and ScII(7) are blended with lines of FeI(5) and FeI(693) respectively in α Lyr. The equivalent widths of these blends have been deduced from the

Table 3

EQUIVALENT WIDTHS IN 53 TAU AND α LYR

			53 Tau		α Lyr		Δ log W_{corr}
Multiplet	λ	χ	log W	log W_{corr}	log W	log W_{corr}	-0.03χ
MgII(10)	4390.6	9.96	-1.51	-1.46	-1.39	-1.32	-0.44
	4384.6	9.95	-1.77	-1.76	-1.61	-1.58	-0.48
SiII(1)	3853.7	6.83	-1.21	-0.96	-1.34	-1.22	+0.06
CaII(3)	3736.9	3.14	-1.28	-0.99	-0.94	+0.32	-1.40
ScII(7)	4246.8	0.31			-1.39	-1.29	
(15)	4320.7	0.60			-1.47	-1.41	
TiII(19)	4450.5	1.08	-1.33	-1.15	-1.51	-1.45	+0.27
(20)	4287.9	1.08	-1.39	-1.25	-1.68	-1.65	+0.37
(34)	3932.0	1.13	-1.33	-1.09	-1.57	-1.51	+0.39
(40)	4417.7	1.16	-1.24	-0.95	-1.39	-1.29	+0.31
	4464.5	1.16	-1.38	-1.24	-1.94	-1.94	+0.67
(41)	4301.9	1.16	-1.21	-0.82	-1.44	-1.35	+0.50
(50)	4590.0	1.23	-1.55	-1.49	-1.60	-1.56	+0.03
(51)	4399.8	1.23	-1.24	-0.93	-1.35	-1.23	+0.26
	4394.1	1.22	-1.46	-1.36	-1.72	-1.69	+0.29
(61)	4395.8	1.24	-1.44	-1.33	-1.92	-1.92	+0.55
(87)	4028.3	1.88	-1.29	-1.03	-1.52	-1.45	+0.36
(104)	4367.7	2.58	-1.39	-1.25	-1.66	-1.62	+0.29
	4386.9	2.59	-1.47	-1.38	-1.79	-1.77	+0.31
(105)	4174.1	2.59	-1.72	-1.68	-1.92	-1.92	+0.16
(107)	3761.9	2.58	-1.36	-1.13	-1.46	-1.36	+0.15
(115)	4488.3	3.11	-1.23	-0.91	-1.72	-1.69	+0.69
	4411.1	3.08	-1.41	-1.29	-1.68	-1.95	+0.57

Table 3 (continued)

EQUIVALENT WIDTHS IN 53 TAU AND α LYR

Multiplet	λ	χ	53 Tau log W	 $\log W_{corr}$	α Lyr log W	 $\log W_{corr}$	$\Delta \log W_{corr}$ $-0.03\,\chi$
CrI(1)	4254.3	0.00			-1.60	-1.56	
CrII(31)	4261.9	3.85	-1.40	-1.25	-1.56	-1.51	+0.14
	4275.6	3.84	-1.52	-1.44	-1.90	-1.89	+0.33
	4284.2	3.84	-1.52	-1.44	-1.79	-1.77	+0.21
	4269.3	3.84	-1.85	-1.83	-1.74	-1.71	-0.24
(44)	4558.7	4.06	-1.21	-0.84	-1.12	-0.79	-0.17
	4588.2	4.05	-1.21	-0.82	-1.25	-1.09	+0.15
	4618.8	4.06	-1.28	-1.04	-1.32	-1.20	+0.04
	4634.1	4.05	-1.37	-1.21	-1.41	-1.32	-0.01
	4555.0	4.05	-1.46	-1.36	-1.66	-1.63	+0.15
	4592.1	4.06	-1.52	-1.45	-1.47	-1.40	-0.17
	4616.6	4.05	-1.42	-1.30	-1.54	-1.50	+0.08
(162)	4145.8	5.30	-1.64	-1.59	-1.79	-1.77	+0.02
MnI(2)	4030.8	0.00	-1.38	-1.18	-1.53	-1.47	+0.29
	4033.1	0.00	-1.38	-1.18	-1.71	-1.68	+0.50
FeI(20)	3820.4	0.86	-1.82	-1.78	-1.15	-0.65	-1.16
(41)	4383.5	1.48	-2.22	-2.22	-1.03	-0.32	-1.94
(43)	4045.8	1.48	-1.89	-1.87	-1.07	-0.38	-1.53
FeII(27)	4416.8	2.77	-1.96	-1.95	-1.31	-1.17	-0.86
	4303.2	2.69	-1.70	-1.65	-1.29	-1.13	-0.60
(28)	4178.9	2.57	-1.60	-1.54	-1.14	-0.73	-0.89

Table 3 (continued)

EQUIVALENT WIDTHS IN 53 TAU AND α LYR

			53 Tau		α Lyr		
Multiplet	λ	χ	log W	log W_{corr}	log W	log W_{corr}	Δ log W_{corr} -0.03χ
FeII(37)	4629.3	2.79	-1.62	-1.57	-1.17	-0.93	-0.72
	4555.9	2.82	-1.82	-1.80	-1.17	-0.93	-0.95
	4515.3	2.83	-1.92	-1.91	-1.21	-1.00	-0.99
	4491.4	2.84	-1.77	-1.74	-1.36	-1.24	-0.59
	4520.2	2.79	-1.64	-1.59	-1.24	-1.06	-0.61
	4489.2	2.82	-1.89	-1.87	-1.41	-1.32	-0.63
(38)	4522.6	2.83	-1.40	-1.25	-1.13	-0.78	-0.55
	4508.3	2.84	-1.57	-1.51	-1.12	-0.75	-0.85
(186)	4635.3	5.93	-1.72	-1.68	-1.60	-1.56	-0.30
(190)	3939.0	5.89	-2.29	-2.29	-1.65	-1.61	-0.86
SrII(1)	4077.7	0.00	-1.41	-1.12	-1.22	-0.87	-0.25
	4215.5	0.00	-1.62	-1.53	-1.37	-1.22	-0.31
ZrII(41)	4149.2	0.80	-1.58	-1.46	-1.67	-1.63	+0.15

equivalent widths of other lines in the same multiplets using the gf-values for FeI by Corliss and Warner (1964) and subtracted from the total equivalent widths measured by Hunger (1955).

Since

$$\begin{aligned} \log W_{corr} &= \log X_o + A \\ &= \log N_r - \theta_{exc}\chi + \log gf\lambda + C(\lambda) + A, \qquad (10) \end{aligned}$$

where A is a constant,

$$\Delta \log N_r = \Delta \log W_{corr} - 0.03\chi + \Delta \log v + \Delta \log u(T). \qquad (11)$$

The mean values of $\Delta \log W_{corr} - 0.03\chi$ for groups of lines with similar excitation potentials have been listed in Table 4, and equation (11) has been used to calculate $\Delta \log N_r$ for each group. $\Delta \log u(T)$ was found from the lists of partition functions by Aller (1960) and Allen (1963). Saha's ionisation equation has been applied to both stars using the nomogram by Unsöld (1955) to deduce $\Delta \log N$ from $\Delta \log N_r$.

The values of $\Delta \log N$ are only apparent values. In order to obtain the true values $\Delta' \log N$, the differences between the apparent values $\Delta \log N$ for groups of lines with similar mean excitation and ionisation potentials and wavelengths have been found and are shown in Fig. 3; the arrows point towards the element with the larger value of $\Delta \log N$. Fig. 3 gives the true relative values of $\Delta' \log N$, and the zero point of the $\Delta' \log N$ scale may be fixed by using lines which are insensitive to temperature and electron pressure, since for these lines $\Delta' \log N = \Delta \log N$. The lines in Table 3 which are least sensitive to temperature and electron pressure are those of SiII, CrII and FeII, and the mean values of $\Delta \log N$ for the lines of these ions are -0.04, -0.05, and -0.86 respectively. The final values of $\Delta' \log N$ are given in Table 5. The upper limit for $\Delta' \log N$ for scandium has been deduced assuming that the lines at

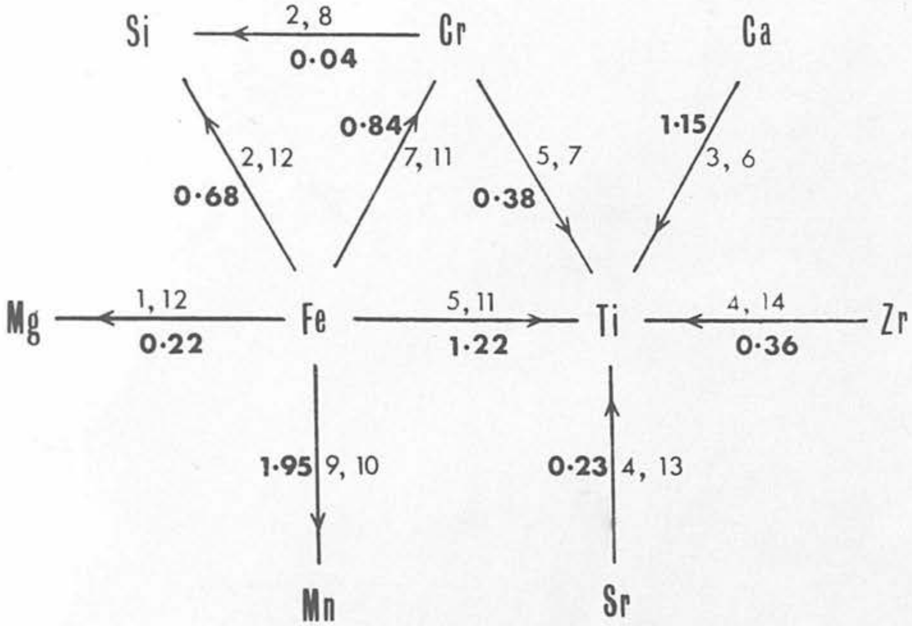

Figure 3

RELATIVE VALUES OF Δ'log N OBTAINED IN THE

REFINED DIFFERENTIAL CURVE-OF-GROWTH

COMPARISON OF 53 TAU WITH α LYR

The numbers of the groups of lines in Table 4 are indicated. The arrows point to the element with the larger value of Δ log N.

Table 4

VALUES OF Δ LOG N_r AND ΔLOG N

Group of lines	Multiplets	Mean $\Delta \log W_{corr}$ -0.03χ	$\Delta \log N_r$	$\Delta \log N$
1	MgII(10)	-0.46	-0.55	-0.50
2	SiII(1)	+0.06	-0.04	-0.04
3	CaII(3)	-1.40	-1.50	-1.01
4	TiII(20),(34),(41)	+0.42	+0.29	+0.41
5	TiII(104),(105),(115)	+0.34	+0.21	+0.33
6	TiII(107)	+0.15	+0.02	+0.14
7	CrII(31),(44)	+0.05	-0.07	-0.05
8	CrII(162)	+0.02	-0.10	-0.08
9	MnI(2)	+0.40	+0.27	+1.03
10	FeI(20),(41),(43)	-1.54	-1.68	-0.92
11	FeII(27),(28),(37),(38)	-0.75	-0.90	-0.89
12	FeII(186),(190)	-0.58	-0.73	-0.72
13	SrII(1)	-0.28	-0.48	+0.18
14	ZrII(41)	+0.15	-0.05	+0.05

Table 5

ABUNDANCES IN 53 TAU AND α LYR

Element	Δ 'log N	log N α Lyr	log N 53 Tau
H		12.0	(12.0)
Mg	-0.64	7.7	7.1
Si	-0.08	8.2	8.1
Ca	-0.79	6.3	5.5
Sc	< -0.5	3.4	< 2.9
Ti	+0.36	4.8	5.2
Cr	-0.02	5.6	5.6
Mn	+1.19	5.3	6.5
Fe	-0.86	6.5	5.6
Sr	+0.13	2.8	2.9
Y	+0.30	2.1	2.4
Zr	0.00	2.9	2.9

4246.8A and 4320.7A would have been noted by Aller and Bidelman (1964), if their equivalent widths were greater than 0.010A in 53 Tau. The value Δ 'log N = +1.09 for manganese given by Fig. 3 is uncertain, since the ionisation corrections for MnI and FeI are large. A more reliable value Δ 'log N = +1.24 was deduced from the new equivalent width measures by Auer and others (1966) of the additional lines at 3482.9, 3488.7, and 3495.8 of MnII(3) and 3504.9 and 3510.8 of TiII(88).

Giving this latter value double weight, a mean value Δ'log N = +1.19 was adopted for manganese. The value Δ'log N = +0.30 for yttrium was obtained from the new measures of the line of YII(14) at 4177.5A. The values of log N for α Lyr in Table 5 are those given by Hunger (1960), and they have been used to derive the values of log N for 53 Tau.

The main source of error in the relative values of Δ'log N will be random errors in the equivalent widths W, which amount to about ± 0.005A for 53 Tau (Auer and others, 1966) and ± 10% for α Lyr (Hunger, 1955). Errors in W will produce errors in $\Delta \log W_{corr}$ particularly for strong lines which almost lie on the "flat" part of the curve-of-growth. Large errors may also be expected for very weak lines which are difficult to measure and may be affected by blends. Errors in W for the titanium and iron lines will affect the accuracy of the apparent values of $\Delta\theta_{exc}$ and log P_e respectively. The FeI lines are weak in 53 Tau, and hence the probable error in the apparent value of log P_e will be about ± 0.3. However, the effect of errors in $\Delta\theta_{exc}$ and log P_e has been minimised by the use of groups of lines of similar excitation and ionisation potentials. It should also be noted that an error in the adopted value of $\Delta\theta_{ion}$ will produce a corresponding error in the derived value of log P_e, but the ionisation corrections will be only slightly affected. Thus, although the error of ± 0.06 in $\Delta\theta_{exc}$ implies an error of ± 0.06 in $\Delta\theta_{ion}$, the resulting errors will be small. For the same reason, it does not matter much if the assumption that

$\Delta\theta_{ion} = \Delta\theta_{exc}$ is not strictly correct. The apparent values of $\Delta\theta_{exc}$ and $\log P_e$ have been derived only for lines with low excitation potentials, and it has unfortunately not been possible to check whether the same values apply to lines of high excitation potential. An examination of Tables 4 and 5 shows that the values of $\Delta'\log N$ and $\Delta \log N$ are rather similar. This means that the full benefit of the refinement of the differential curve-of-growth method is not apparent for 53 Tau, because the range of χ for the lines of each type of ion is small in 53 Tau and α Lyr. gf-values have not been used in the analysis, except in the initial construction of the curves-of-growth for TiII where any errors in the gf-values cancel out in the determination of $\Delta\theta_{exc}$. Hunger (1955) used gf-values to derive a value of $T_{ion} = 9200^{\circ}$K for α Lyr, but this value is unlikely to be seriously in error, since Brown and others (1964) found that $T_{eff} = 9200^{\circ}$K from an interferometric measure of the stellar radius. The longitudinal component of the magnetic fields in manganese stars is less than 1000 gauss (Babcock, 1958a), and the effect of magnetic intensification of the lines on the derived abundances should be small (Babcock, 1949; Boyarchuk and others, 1960).

It must be emphasised that the values of $\Delta'\log N$ refer to the whole observed surface of 53 Tau, and they give no information about the possible non-uniformity of the surface distribution of the elements. If there is a range of temperature and surface gravity over the surface of 53 Tau, the relative abundances derived for non-uniformly distributed elements might be erroneous.

There also remains some uncertainty in the zero point of the Δ'log N scale. The removal of this uncertainty would require reliable model-atmosphere studies of both 53 Tau and α Lyr. Contributions to the uncertainty are the possibility of systematic errors in the equivalent widths for 53 Tau and α Lyr, and the fact that the change of opacity between 53 Tau and α Lyr has been neglected owing to the difficulty of assigning a unique temperature to the whole of the surface of 53 Tau. The latter contribution to the uncertainty in the zero point of the Δ'log N scale should not be large, as the UBV colours of α Lyr are roughly similar to those of 53 Tau. Moreover, any change in opacity between 53 Tau and α Lyr would be expected to show a wavelength dependence, and there is no wavelength dependence of the values of $\Delta \log W_{corr} - 0.03\lambda$ in Table 3 for TiII, CrII, or FeII. Assuming that the zero point of the Δ'log N scale is correct, manganese is overabundant in 53 Tau by a factor of about 12, while iron is deficient by a factor of about 7. Calcium, scandium, and possibly magnesium are also deficient. The other elements listed in Table 5 have approximately normal abundances. The presence of a line of GaI at 4172.1A indicates a large excess of gallium (Aller and Bidelman, 1964).

Comparison with other investigations

A comparison between the values of Δ'log N and the results of abundance analyses by other authors is given in Table 6. The values of log N for α Lyr by Hunger (1960) have been subtracted from the values of log N for 53 Tau (log N(H) = 12.0) by Aller and Bidelman (1964), Warner (1965), and Auer and others (1966) to give the values of Δ log N listed. The large discrepancies between the values of Δ'log N and those of Δ log N may be attributed to several factors:-

(1) The gf-values used by Hunger (1960) for α Lyr and those used by other authors for 53 Tau were from different sources. Discrepancies of ± 1.0 in log gf between various sources are quite common in the literature. The values of Δ'log N do not depend on the accuracy of gf values.

(2) Aller and Bidelman (1964) and Warner (1965) adjusted the scales of their values of log N to make the abundance of iron in 53 Tau the same as that in the solar system and the Sun respectively. An attempt has been made in the present analysis to fix the zero point of the Δ'log N scale by a less arbitrary procedure, but it must be admitted that some uncertainty about the zero point remains. At any rate, the difference in the procedures may partly account for the discrepancies between the values of Δ'log N and Δ log N.

(3) Aller and Bidelman (1964) and Warner (1965) adopted the values $\theta_{exc} = \theta_{ion} = 0.45$ and log $P_e = 2.75$, which differ from the <u>apparent</u> values $\theta_{exc} = \theta_{ion} = 0.52$ and log $P_e = 2.50$ derived in the present analysis and used for finding Δ'log N. The discussion in the next

Table 6

COMPARISON WITH OTHER INVESTIGATIONS

Element	$\Delta'\log N$	$\Delta\log N$ Aller and Bidelman	$\Delta\log N$ Warner	$\Delta\log N$ Auer and others	$\Delta\log N$ Searle and Sargent
Mg	-0.6	-0.9:	-0.6:	-0.5	-0.3
Si	-0.1	-1.2	-0.9	-0.9	+0.1
Ca	-0.8	-1.6	-1.3	0.0	
Sc	<-0.5				
Ti	+0.4	+0.7	+1.6	+1.4	
Cr	0.0	-1.1	+0.4	-0.4	
Mn	+1.2	+1.6	+2.2	+1.8	
Fe	-0.9	-0.2	+0.1	-0.1	
Sr	+0.1	+1.4	+1.7	+0.9	
Y	+0.3	+1.6	+1.9	+1.9	
Zr	0.0	+1.3	+1.6	+2.1	

chapter will show that these differences have a large effect on the resulting abundances relative to α Lyr.

(4) The curve-of-growth analyses by Aller and Bidelman (1964) and Warner (1965) would be affected by abnormalities in the structure of 53 Tau. Certain types of structural abnormality would also affect the model-atmosphere analysis by Auer and others (1966).

CHAPTER 6

THE STRUCTURE OF THE MANGANESE STAR 53 TAU

Introduction

The refined differential curve-of-growth analysis of 53 Tau in the preceding chapter enabled abundances in 53 Tau to be derived without assuming that its structure was normal. Other authors, assuming normal structure, obtained conflicting abundances, but it is difficult to investigate the discrepancies as they could be due to several factors. The structure of 53 Tau will therefore be studied by carrying out a normal differential curve-of-growth analysis assuming a normal structure for 53 Tau and comparing the results with those obtained in the preceding chapter by the refined differential curve-of-growth method. In the normal differential curve-of-growth method the differences in the temperature and electron pressure between the two stars are derived from their intrinsic colours and the profiles of the Balmer lines, and no restriction on the choice of lines is made on the basis of their excitation and ionisation potentials; thus it is assumed that the structures of the two stars are similar. To avoid confusion, the notation for the normal differential curve-of-growth analysis will be δX instead of ΔX, where X is any quantity.

Normal differential curve-of-growth analysis

The values $\theta_{ion} = 0.45$ and $\log P_e = 2.75$ were derived by Aller and Bidelman (1964) for 53 Tau from the Balmer lines. From the UBV photometry previously quoted for 53 Tau and α Lyr $\delta(B - V)_o = 0.07^m$, and hence $\delta\theta_{ion} = \delta\theta_{exc} = -0.10$, if the relationship between T_{exc} and T_{ion} and $(B - V)_o$ for normal stars found by Kopylov (1963) is used. Since $\theta_{ion} = 0.55$ for α Lyr, the value $\theta_{ion} = 0.45$ for 53 Tau agrees with that expected from $(B - V)_o$. The error in $\delta(B - V)_o$ due to errors in the UBV photometry is $\pm 0.02^m$. The normal differential curve-of-growth analysis will therefore be carried out adopting the values $\delta\theta_{exc} = -0.10$, $\theta_{ion} = 0.45$ and $\log P_e = 2.75$ for 53 Tau and $\theta_{ion} = 0.55$ and $\log P_e = 2.98$ for α Lyr (Hunger, 1955).

Auer and others (1966) measured additional spectra of 53 Tau and obtained equivalent widths which agreed well with those of Aller and Bidelman (1964). They averaged their measures with those of Aller and Bidelman, and the average equivalent widths will be used in the present analysis. As the absorption coefficient may vary with wavelength in a different manner in the two stars, the wavelength range will be restricted as far as possible without drastically reducing the number of ions with lines present in both stars; thus only lines in the range 3850 to 4420A will be used. Values of $\delta \log W_{corr}$ and mean values of $\delta \log W_{corr} - 0.10\chi$, $\delta \log N_r$ and $\delta \log N$ were deduced for the groups of lines of similar excitation potentials listed in Table 1.

The values of $\delta \log N$ differ considerably in some cases from the values of $\Delta' \log N$ deduced in the refined differential curve-of-growth analysis. When $\delta \log N$ is plotted against $\bar{\chi} + \chi_r$, as in Fig. 1, a strong correlation becomes obvious. The abundance deficiency of iron in 53 Tau relative to the other elements was well established in the previous refined analysis, and the arrow in Fig. 1 indicates that a correction $- \Delta' \log N = 0.86$ added to $\delta \log N$ for iron would place the point for iron on the relation between $\delta \log N$ and $\bar{\chi} + \chi_r$ for the other elements. All the values of $\delta \log N$ were derived from lines of singly ionised atoms. The correlation cannot be removed by any reasonable change in the value $\log P_e = 2.75$ for 53 Tau. The value $\theta_{ion} = 0.45$, which agrees with that expected from $(B - V)_o$, would therefore have to be increased to remove the correlation. The increase required corresponds to a change in $(B - V)_o$ of more than twice the error ($\pm 0.02^m$) in $\delta (B - V)_o$ due to errors in the UBV photometry. This suggests that the structure of 53 Tau is abnormal.

Table 1

DIFFERENTIAL CURVE-OF-GROWTH ANALYSIS

Multiplet	λ	χ	$\delta \log W_{corr}$	Mean $\log W_{corr}$ -0.10χ	$\delta \log N_r$	$\delta \log N \bar{\chi}$	$+\chi_r$
MgII(10)	4390.6	9.96	-0.10	-1.12	-1.21	-0.83	24.9
	4384.6	9.95	-0.14				
SiII(1)	3853.7	6.83	+0.15	-0.53	-0.62	-0.57	23.1
TiII(20)	4287.9	1.08	+0.36	+0.29	+0.21	+0.76	14.8
(34)	3932.0	1.13	+0.35				
(40)	4417.7	1.16	+0.38				
(41)	4301.9	1.16	+0.46				
(51)	4394.1	1.22	+0.37				
(61)	4395.8	1.24	+0.53				
(104)	4367.7	2.58	+0.46	+0.15	+0.07	+0.62	16.2
	4386.9	2.59	+0.47				
(105)	4174.1	2.59	+0.29				
CrII(31)	4261.9	3.85	+0.28	-0.19	-0.18	+0.03	20.4
	4275.6	3.84	+0.37				
	4284.2	3.84	+0.23				
	4269.3	3.84	+0.12				
FeII(27)	4416.8	2.77	-0.66	-0.95	-1.04	-0.91	18.8
	4303.2	2.68	-0.57				
(28)	4178.9	2.57	-0.81				
SrII(1)	4077.7	0.00	-0.21	-0.18	-0.32	+0.96	11.0
	4215.5	0.00	-0.15				
YII(14)	4177.5	0.40	+0.08	+0.04	-0.13	+0.99	12.7
ZrII(41)	4149.2	0.80	+0.11	+0.03	-0.08	+0.42	14.8

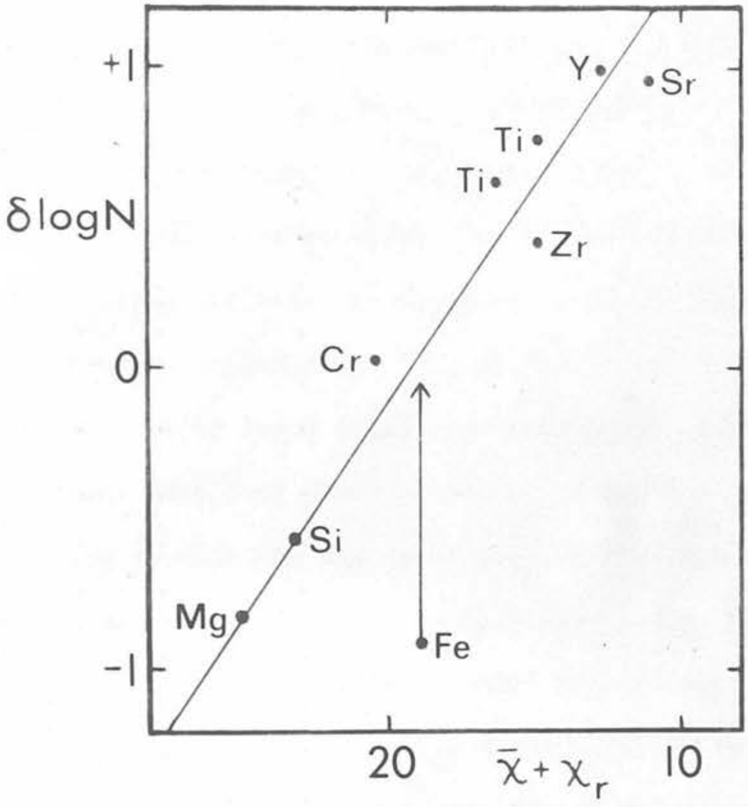

Figure 1

RELATION BETWEEN $\delta \log N$ AND $\bar{\chi} + \chi_r$

OBTAINED IN THE NORMAL DIFFERENTIAL

CURVE-OF-GROWTH COMPARISON OF

53 TAU WITH α LYR

The arrow indicates the effect of the correction $-\Delta' \log N = 0.86$ added to $\delta \log N$ for iron.

Model-atmosphere analysis

Before discussing possible causes of structural abnormality in 53 Tau, it is worth while enquiring whether the abundance anomalies derived from the spherical model of 53 Tau by Auer and others (1966) would correlate with $\chi + \chi_r$. To avoid errors due to uncertainties in gf-values, a comparison will be made line by line with the recent model-atmosphere analysis of α Lyr by Strom and others (1966), which is based on the mean equivalent widths given by Hunger (1955). The values of log N for 53 Tau were found from the values of log Nf by Auer and others using the same gf-values as Strom and others. Two models with effective temperatures T_{eff} of 9000°K and 9500°K and a surface gravity given by log g = 3.7 were calculated by Strom and others for α Lyr, and the resulting pairs of values of log N were averaged. Table 2 lists the results for the lines used in the normal differential curve-of-growth analysis. There is again a correlation between Δ log N and $\chi + \chi_r$ (Fig. 2) except for iron, which was shown in the previous chapter to be deficient relative to the other elements in 53 Tau. Microturbulence was neglected in the analysis of α Lyr by Strom and others. Of the lines listed in Table 2 the only two which are sufficiently strong to be seriously affected by microturbulence are FeII 4178.9 and SrII 4077.7. If these lines are omitted, the mean values of Δ log N change from -0.82 to -0.70 for Fe and from +1.35 to +1.58 for Sr;

Table 2

MODEL-ATMOSPHERE ANALYSIS

Multiplet	λ	χ	log N α Lyr	log Nf 53 Tau	log f	log N 53 Tau	Mean Δlog N	$\bar{\chi} + \chi_r$
MgII(10)	4390.6	9.96	7.50	6.01	-1.15	7.16	-0.35	24.9
	4384.6	9.95	7.36	5.89	-1.11	7.00		
SiII(1)	3853.7	6.83	7.47	5.03	-2.30	7.33	-0.14	23.1
TiII(20)	4287.9	1.08	3.97	3.84	-1.95	5.79	+1.84	14.8
(34)	3932.0	1.13	4.10	3.58	-2.20	5.78		
(40)	4417.7	1.16	4.36	4.48	-1.73	6.21		
(41)	4301.9	1.16	4.12	4.79	-1.26	6.05		
(51)	4394.1	1.22	4.04	4.16	-1.67	5.83		
(61)	4395.8	1.24	3.82	3.60	-2.21	5.81		
(104)	4367.7	2.58	3.95	4.49	-1.19	5.68	+1.66	16.2
	4386.9	2.59	3.80	4.42	-1.14	5.56		
(105)	4174.1	2.59	4.02	3.98	-1.52	5.50		
CrII(31)	4261.9	3.85	4.71	3.75	-1.81	5.56	+0.72	20.4
	4275.6	3.84	4.31	3.44	-1.82	5.16		
	4284.2	3.84	4.52	3.59	-1.71	5.30		
	4269.3	3.84	5.03	3.22	-2.11	5.33		
FeII(27)	4416.8	2.77	6.24	3.44	-2.08	5.52	-0.82	18.8
	4303.2	2.68	6.29	3.29	-2.32	5.61		
(28)	4178.9	2.57	6.53	3.23	-2.24	5.47		
SrII(1)	4077.7	0.00	3.53	3.58	-1.07	4.65	+1.35	11.0
	4215.5	0.00	2.93	3.21	-1.30	4.51		
YII(14)	4177.5	0.40	2.35	3.25	-0.94	4.19	+1.84	12.7
ZrII(41)	4149.2	0.79	2.64	3.65	-1.03	4.68	+2.04	14.8

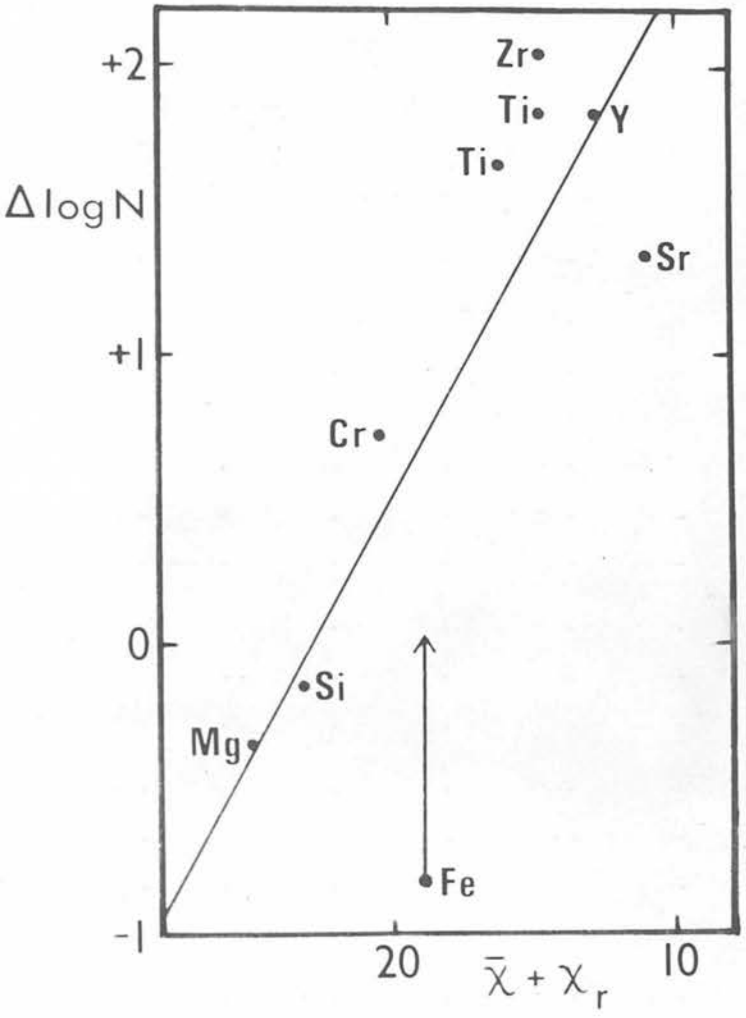

Figure 2

RELATION BETWEEN $\Delta \log N$ AND $\bar{\chi} + \chi_r$ OBTAINED IN THE COMPARISON OF MODEL-ATMOSPHERES OF 53 TAU AND α LYR

The arrow indicates the effect of the correction $-\Delta' \log N = 0.86$ added to $\delta \log N$ for iron.

the effect on Figure 2 is therefore small. For the other lines in Table 2 the error in log N due to the neglect of microturbulence is less than 0.20.

It is unlikely that the abundance anomalies in 53 Tau would be related to the ionisation potentials of the elements, and a correlation of Δ log N with excitation potential is apparent from the two groups of TiII lines. One of the models must therefore be faulty. The models of α Lyr appear satisfactory, since the values of T_{eff} and log g agree well with the independent values $T_{eff} = 9200 \pm 300^{\circ}K$ and $\log g = 3.85 \pm 0.2$ derived from the interferometric measure of the angular radius of the star by Brown and others (1964). Hence the most probable explanation of the correlation of Δ log N with $\chi + \chi_r$ is that the model-atmosphere of 53 Tau constructed by Auer and others fails to take account of some kind of structural abnormality.

Possible causes of structural abnormality in 53 Tau

The normal differential curve-of-growth comparison of 53 Tau with α Lyr is equivalent to a direct comparison of line strengths in 53 Tau with those in a sharp-line normal star with the same $(B - V)_0$ and Balmer line profiles. It was necessary to use α Lyr, because no equivalent widths for such a normal star were available. Moreover, many of the lines in 53 Tau would not be present in a normal star with the same $(B - V)_0$; 53 Tau and α Lyr have a large number of lines in common. A direct comparison of the line strengths in α Scl with those in a normal star of similar intrinsic colour (π Cet) was given in Chapter 2, and a correlation between the relative line strengths and $\chi + \chi_r$ was found. The ratios of the line strengths in α Scl to those in π Cet were smallest for lines with intermediate values of $\chi + \chi_r$. This was interpreted as an indication of a large range of temperature over the surface of α Scl. The normal differential curve-of-growth comparison of 53 Tau with α Lyr showed that the values of δ log N were smaller for lines with intermediate values of $\chi + \chi_r$ (the lines of MgII and SiII) than for lines with low values of $\chi + \chi_r$. Accordingly, there would seem to be a range of surface temperature in 53 Tau also. For α Scl, the idea of a large range in surface temperature removes all the apparent abundance anomalies (except a possible deficiency of helium for which only Stark-broadened lines are available). In the case of 53 Tau, however, the deficiency of iron and the excess of manganese relative to other elements remain as real abundance anomalies even when a range of surface temperature is admitted.

Assuming that the abundance of carbon is normal in 53 Tau, further support for the existence of a range of surface temperature may be given from a consideration of the CII 4267 line, which has high excitation and ionisation potentials ($\chi + \chi_r = 42$ eV). The strength of this line is normal in 53 Tau for the value of $(B - V)_0$, and therefore δ log N is approximately zero, which is larger than the values of δ log N for the MgII and SiII lines. Thus the lines with intermediate values of $\chi + \chi_r$ have the smallest values of $\delta \log N$, as would be expected if a range of surface temperature is present in 53 Tau. Variability or an error in $(B - V)_o$ could produce a correlation of $\delta \log N$ with $\chi + \chi_r$ but not a minimum value of $\delta \log N$ for lines with intermediate values of $\chi + \chi_r$. The validity of this argument is uncertain, because it is not known whether the abundance of carbon is normal in 53 Tau.

Various types of structural abnormality may be suggested for 53 Tau:-

(1) The stratification of the atmosphere may be abnormal, especially since magnetic fields have been observed in some manganese stars. However, the relative strengths of the metastable lines of SiII(1) and the non-metastable lines of SiII(3) are about the same as in normal stars, whereas in shell stars the relative strengths of the SiII(1) and SiII(3) lines are very different from normal stars. This shows that the abnormality in the stratification in 53 Tau, if present, is much less severe than in the case of shell stars.

(2) Rapid rotation viewed pole-on may be discounted, because the Mn stars rotate very slowly (Chapter 3).

(3) The structure of 53 Tau might be distorted by the tidal interaction of a close companion star of much lower luminosity. This is rather unlikely, since no periodic radial velocity variations have been found,

and the frequency of short-period spectroscopic binaries is low among peculiar A stars (Jaschek and Jaschek, 1958).

(4) The range in surface temperature found in 53 Tau might be explained by a phenomenon similar to sunspots but on a grander scale. The small variations in UBV magnitudes in some Mn stars with magnetic fields (Abt and Golson, 1962) might also be accounted for in this way.

CHAPTER 7

THE CHEMICAL COMPOSITION OF THE BINARY PECULIAR A STAR HR 4072

Introduction

The orbit of the spectroscopic binary star HR 4072 = HD 89822 ($\alpha_{1950} = 10^{h}\ 20^{m}$, $\delta_{1950} = +65^{\circ}\ 49'$, $m_{v} = 4.9$) was studied by Baker (1912) and Schlesinger (1912) who derived a period of 11.58 days and an eccentricity of 0.38. They were unable to find the secondary spectrum due to the fainter component. Babcock (1958a) observed strong spectral lines of MgII, SiII, CaII, TiII, CrII, and FeII and weak lines of CrI, FeI, and MnII, and found a magnetic field of about 300 gauss reversing in less than 90 days.

Two spectra of this star at a dispersion of 5.6 A/mm were taken with the Edinburgh 36-inch telescope on May 18, 1966. Several spectral lines which could not be identified were noticed. A third spectrum was obtained on May 23, 1966, and it was found that the unidentified lines had shifted by about 3A with respect to the other lines. This showed that the unidentified lines were due to the secondary component. It was also noted that the YII lines were abnormally strong in both components. Since few double-line peculiar A binaries are known and the lines in HR 4072 are very sharp, an abundance analysis of the two components was carried out to determine the chemical composition of the atmosphere of the primary component and to investigate whether the composition of the secondary is similar. This binary star will also be of interest with regard to the theory proposed by Renson (1963) that many of the peculiar A stars have a close companion star of lower luminosity with a magnetic field moving in an eccentric orbit.

Measurement of equivalent widths

The spectra were taken on Eastman-Kodak 103a0 emulsion and were uniformly widened 1 mm. The wavelength range suitable for measurement was from 4070 to 4510A. The plates were calibrated in a separate multiple-slit spectrograph a few hours after exposure at the telescope and brush developed in Kodak D19b developer for 4 minutes at about 20°C. The two spectra taken on May 18, 1966 were on the same plate, and the projected spectrograph slit-width on the plate was 30 microns. The third spectrum taken on May 23, 1966 was obtained with the projected slit-width reduced to 20 microns to improve the resolution of the spectral lines. The exposure times were 82, 64, and 162 minutes respectively.

The spectra were traced with a Joyce-Loebl microdensitometer using a magnification of 50 and a projected slit-width of 20 microns. Equivalent widths for unblended lines were measured on the third spectrum. Further measures were obtained by finding the products of the central depths and half-widths of the lines and transforming the products to the equivalent width scale. This was done for each of the three spectra to give three additional sets of measures. The four sets of measures were then averaged, and the mean equivalent widths W in mA with the number of measures n are listed in Table 1. For the broad lines of HeI(14), CII(6), and MgII(4) all the measures were direct measures of equivalent width.

Table 1

EQUIVALENT WIDTHS

Multiplet	λ	χ	Primary		Secondary	
			W	n	W	n
HeI(14)	4471.48 .69	20.87	58	1		
CII(6)	4267.02 .27	17.97	38	1		
MgII(4)	4481.13 .33	8.83	262	3	55	3
(9)	4433.99	9.96	43	4		
(10)	4390.59	9.96	46	4		
SiII(3)	4130.88	9.80	140	4		
	4128.05	9.79	135	4		
ScII(7)	4246.83	0.31	<10	4		
TiII(19)	4395.03	1.08	79	4	25	1
	4443.80	1.08	81	4	32	1
	4450.49	1.08	48	4		
(20)	4294.10	1.08	72	4	37	2
	4287.89	1.08	39	4		
(31)	4468.49	1.13	88	4	25	2

Multiplet	λ	χ	Primary		Secondary	
			W	n	W	n
	4501.27	1.11	85	4	36	2
	4444.56	1.11	22	2		
TiII(40)	4441.73	1.18	24	4		
	4417.72	1.16	70	2		
	4464.46	1.16	39	4		
(41)	4300.05	1.18	81	4		
	4290.22	1.16	89	2		
	4301.93	1.16	73	2		
	4312.86	1.18	65	4	27	2
	4320.97	1.16	36	2		
(51)	4399.77	1.23	53	2		
	4394.06	1.22	28	2		
	4418.34	1.23	25	2		
(61)	4395.85	1.24	34	2		
(94)	4316.81	2.04	34	2		
(104)	4367.66	2.58	44	4		
	4386.86	2.59	43	2		
(105)	4163.64	2.58	60	2		
	4171.90	2.59	73	2		
	4174.09	2.59	29	2		
(115)	4488.32	3.11	38	2		
	4411.08	3.08	40	4		
CrI(1)	4254.35	0.00	41	4		
CrII(26)	4179.43	3.81	32	2		
(31)	4242.38	3.85	71	4	28	2
	4261.92	3.85	57	2		
	4275.57	3.84	43	2		
	4284.21	3.84	41	2		
	4269.28	3.84	36	4		

Multiplet	λ	χ	Primary		Secondary	
			W	n	W	n
CrII(162)	4145.77	5.30	41	2		
MnII(6)	4326.76	5.37	44	4		
-	4136.91	6.14	38	2		
-	4478.63	6.64	23	2		
FeI(41)	4383.55	1.48	51	2	40	2
	4404.75	1.55	47	4	26	4
(42)	4271.76	1.48	47	2		
	4325.77	1.60	82	2	40	4
(152)	4187.80	2.41	38	2		
(800)	4219.36	3.56	31	2	23	2
FeII(27)	4351.76	2.69	66	4	26	2
	4303.17	2.69	67	4		
	4385.38	2.77	68	2		
	4128.74	2.57	32	2		
(28)	4178.86	2.57	75	4	22	2
	4296.57	2.69	56	2	31	2
	4122.64	2.57	36	4		
	4258.16	2.69	46	4		
(37)	4491.40	2.84	55	2	27	2
	4489.19	2.82	51	4		
	4472.92	2.83	25	4		
	4508.28	2.84	69	3	39	3
-	4451.55	-	29	4		
SrII(1)	4077.71	0.00	120	4	41	2
	4215.52	0.00	113	4	49	4
YII(1)	4204.69	0.00	49	2	8	1
(5)	4309.62	0.18	78	2	31	2
	4398.02	0.13	58	4	24	2
	4422.59	0.10	53	4		

Multiplet	λ	χ	Primary		Secondary	
			W	n	W	n
YII	4358.73	0.10	68	4		
(13)	4374.94	0.41	101	4	40	3
(14)	4124.91	0.41	54	4		
	4177.54	0.41	121	4	39	2
ZrII(41)	4149.22	0.80	39	4		

Analysis of the primary

The chemical composition of the atmosphere of the primary will be determined by a differential curve-of-growth analysis with α Lyr as the comparison star in the same way as for 53 Tau. Curves-of-growth are shown in Fig. 1 for multiplets (19) to (61) of TiII, which have a small range in excitation potential. At a dispersion of 4.5 A/mm the lines in HR 4072 are extremely sharp (Babcock, 1958a), and the microturbulent velocity ξ_t must therefore be low. Unfortunately no spectra at a higher dispersion are available. ξ_t will be taken as zero, and the correction term log $\sqrt{M}$ added to log (W/λ) will then make the curve-of-growth for HR 4072 applicable to elements with different atomic masses M. The correction term for α Lyr is approximately $\frac{1}{2}$ log $\sqrt{M}$. The lines with 45° slope were used for deducing the values of log W_{corr} in Table 2.

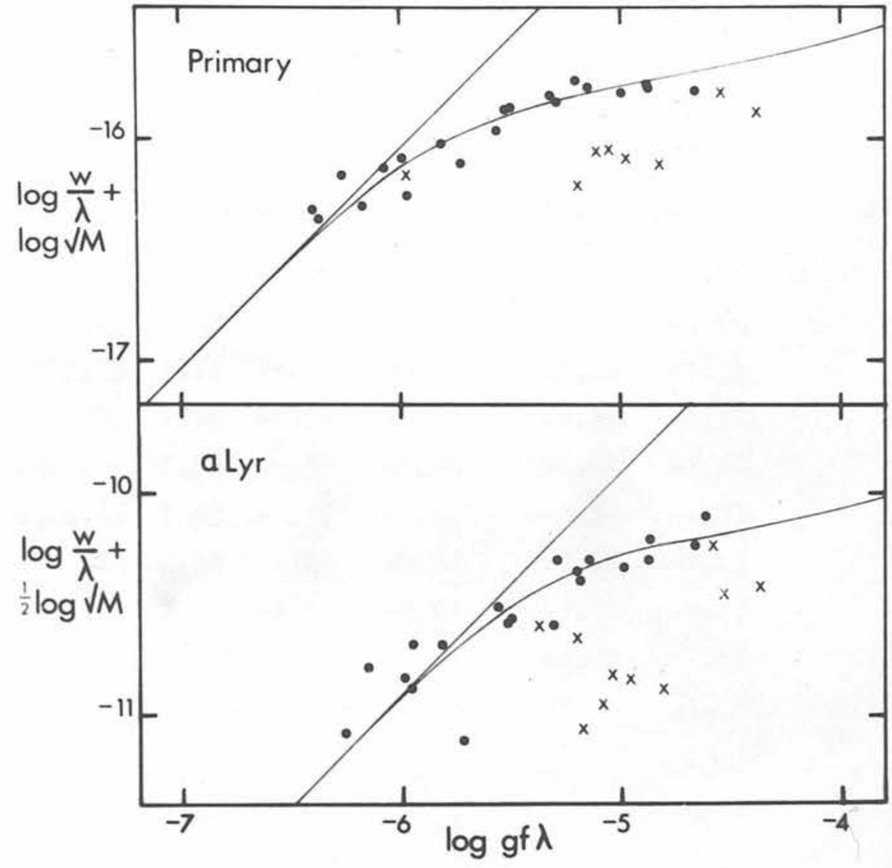

Figure 1

CURVES-OF-GROWTH FOR TiII LINES

IN HR 4072 (PRIMARY) AND α LYR

Dots denote lines of multiplets (19) to (61), and crosses multiplets (82) to (115).

Table 2

CORRECTED EQUIVALENT WIDTHS

			Primary		α Lyr		
Multiplet	λ	χ	log W	$\log W_{corr}$	log W	$\log W_{corr}$	$\Delta_1 \log W_{corr}$
MgII(9)	4433.99	9.96	-1.37	-1.31	-1.77	-1.75	+0.44
(10)	4390.59	9.96	-1.34	-1.27	-1.39	-1.32	+0.05
SiII(3)	4130.88	9.80	-0.85	+0.39	-1.18	-0.98	+1.37
	4128.05	9.79	-0.87	+0.29	-1.17	-0.96	+1.25
ScII(7)	4246.83	0.31	<-2.00	<-1.99	-1.39	-1.29	<-0.70
TiII(20)	4294.10	1.08	-1.14	-0.63	-1.15	-0.83	+0.20
	4287.89	1.08	-1.41	-1.30	-1.68	-1.65	+0.35
(41)	4301.93	1.16	-1.14	-0.63	-1.44	-1.35	+0.72
	4312.86	1.18	-1.19	-0.77	-1.42	-1.32	+0.55
(104)	4367.66	2.58	-1.36	-1.22	-1.66	-1.62	+0.40
	4386.86	2.59	-1.37	-1.24	-1.79	-1.77	+0.53
(105)	4163.64	2.58	-1.22	-0.87	-1.29	-1.13	+0.26
	4171.90	2.59	-1.14	-0.60	-1.32	-1.18	+0.58
(115)	4488.32	3.11	-1.42	-1.32	-1.72	-1.69	+0.37
	4411.08	3.08	-1.40	-1.29	-1.68	-1.65	+0.36
CrII(31)	4242.38	3.85	-1.15	-0.61	-1.48	-1.41	+0.80
	4261.92	3.85	-1.24	-0.92	-1.56	-1.51	+0.59
	4275.57	3.84	-1.37	-1.21	-1.90	-1.89	+0.68
	4284.21	3.84	-1.39	-1.25	-1.79	-1.77	+0.52
	4269.28	3.84	-1.44	-1.33	-1.74	-1.71	+0.38
(162)	4145.77	5.30	-1.39	-1.24	-1.79	-1.77	+0.53
FeI(41)	4383.55	1.48	-1.29	-1.04	-1.03	-0.32	-0.72
	4404.75	1.55	-1.33	-1.13	-1.24	-1.05	-0.08
(42)	4271.76	1.48	-1.33	-1.12	-1.24	-1.04	-0.08

Table 2 (continued)

CORRECTED EQUIVALENT WIDTHS

Multiplet	λ	χ	Primary		α Lyr		
			log W	log W_{corr}	log W	log W_{corr}	Δ_1 log W_{corr}
FeI(152)	4187.80	2.41	−1.42	−1.29	−1.46	−1.37	+0.08
(800)	4219.36	3.56	−1.51	−1.44	−1.50	−1.44	0.00
FeII(27)	4351.76	2.69	−1.18	−0.72	−1.16	−0.86	+0.14
	4303.17	2.69	−1.17	−0.66	−1.29	−1.13	+0.47
	4385.38	2.77	−1.17	−0.69	−1.31	−1.17	+0.48
(28)	4296.57	2.69	−1.25	−0.93	−1.39	−1.28	+0.35
	4122.64	2.57	−1.44	−1.32	−1.62	−1.58	+0.26
	4258.16	2.69	−1.34	−1.14	−1.73	−1.70	+0.56
SrII(1)	4077.71	0.00	−0.92	+0.79	−1.22	−0.87	+1.66
	4215.52	0.00	−0.95	+0.71	−1.37	−1.22	+1.93
YII(13)	4374.94	0.41	−1.00	+0.50	−1.79	−1.76	+2.26
(14)	4177.54	0.41	−0.92	+0.77	−1.60	−1.54	+2.31
ZrII(41)	4149.22	0.80	−1.41	−1.15	−1.67	−1.63	+0.48

The difference $\Delta\delta \log X_o$ in the deviations $\delta \log X_o$ from the curve-of-growth between HR 4072 and α Lyr is plotted against the excitation potential χ in Fig. 2, where $\log X_o$ is the abscissa of the curve-of-growth. The notation

$$X = X_{HR\ 4072} - X_{\alpha\,Lyr}, \tag{1}$$

where X is any quantity, will be used. Where there is likely to be confusion, a suffix "1" will be used for the primary and a suffix

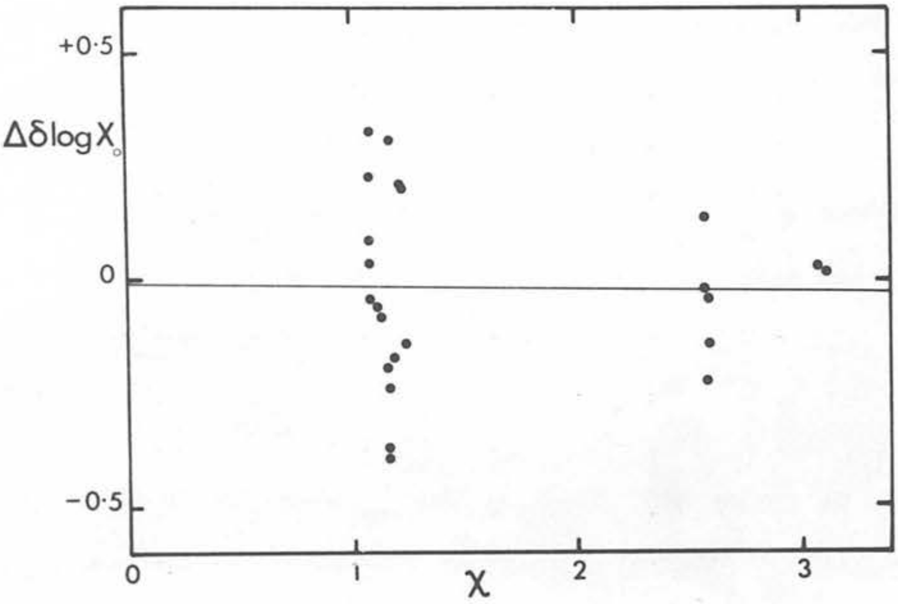

Figure 2

DETERMINATION OF THE APPARENT DIFFERENCE

$\Delta_1 \Theta_{exc}$ IN THE VALUES OF Θ_{exc} FOR

HR 4072 (PRIMARY) AND α LYR

The least squares solution is shown; this gives $\Delta_1 \theta_{exc} = -0.008 \pm 0.057$.

"2" for the secondary. A least squares solution of the plot in Fig. 2 gives the apparent difference in θ_{exc} between the primary and α Lyr as

$$\Delta_1 \theta_{exc} = -0.008 \pm 0.057. \qquad (2)$$

Since this difference is very small, it will be taken as zero to simplify the analysis. Using the values $\theta_{ion} = 0.55$ and $\log P_e = 2.98$ derived by Hunger (1955) for α Lyr and putting $\Delta_1 \theta_{ion} = 0$, Saha's ionisation equation was applied to the lines of FeI and FeII listed in Table 2, and it was found that the apparent value of $\log P_e$ for the primary was

$$\log P_e = 2.44. \qquad (3)$$

The mean values of $\Delta_1 \log W_{corr}$ and $\Delta_1 \log N_r$ for groups of lines of similar excitation potentials are listed in Table 3. Saha's ionisation equation was used to deduce the apparent values of $\Delta_1 \log N$ from the values of $\Delta_1 \log N_r$. These apparent values of $\Delta_1 \log N$ will be the same as the true values $\Delta_1' \log N$ only if the structure of the primary is normal and the secondary is much fainter than the primary.

Table 3

VALUES OF Δ_1LOG N_r AND Δ_1LOG N

Group of lines	Multiplets	Mean $\Delta_1 \log W_{corr}$	$\Delta_1 \log N_r$	$\Delta_1 \log N$
1	MgII(9),(10)	+0.24	+0.15	+0.16
2	SiII(3)	+1.31	+1.21	+1.21
3	ScII(7)	< -0.70	< -0.84	< -0.71
4	TiII(20),(41)	+0.45	+0.30	+0.35
5	TiII(104),(105),(115)	+0.42	+0.27	+0.32
6	CrII(31)	+0.59	+0.43	+0.44
7	CrII(162)	+0.53	+0.37	+0.38
8	FeI(41),(42),(152),(800)	-0.16	-0.33	+0.21
9	FeII(27),(28)	+0.38	+0.21	+0.21
10	SrII(1)	+1.79	+1.56	+1.96
11	YII(13),(14)	+2.28	+2.05	+2.29
12	ZrII(41)	+0.48	+0.25	+0.28

The differences between the apparent values of $\Delta_1$log N for groups of lines of similar mean excitation and ionisation potentials and wavelengths are shown in Fig. 3, where the arrows point to the element with the larger value of $\Delta_1$log N. Fig. 3 gives the true relative values of Δ_1'log N, and the zero point of the Δ_1'log N scale may be fixed by using lines which are insensitive to temperature and electron pressure; the lines of CrII are most suitable. A correction must be made for the "dilution" of the primary lines by the light from the secondary. For the MgII 4481 line the relative luminosities L_1 and L_2 of the primary and secondary components is approximately given by

$$L_1/L_2 = W_1/W_2 = 4.76, \qquad (4)$$

since this strong line is insensitive to differences in temperature, electron pressure, and the abundance of magnesium between the primary and the secondary. The value of Δ_1'log N for Cr is thus

$$\begin{aligned}\Delta_1'\log N &= \Delta_1\log N + \log 5.76 - \log 4.76\\ &= +0.51. \qquad (5)\end{aligned}$$

Fig. 3 may now be used to obtain the values of Δ_1'log N for Sc, Ti, Fe, Sr, Y, and Zr listed in Table 5. The lines of MgII(9), MgII(10), and SiII(3) are also insensitive to temperature and electron pressure, and the values of Δ_1'log N for Mg and Si may be found directly as in equation (5).

The lines of MnII in the primary are not present in α Lyr, but two of them have been measured by Auer and others (1966) in the manganese star 53 Tau. The previous comparison of 53 Tau with α Lyr gave a value Δ'log N = +1.19 for Mn for 53 Tau. Using the notation

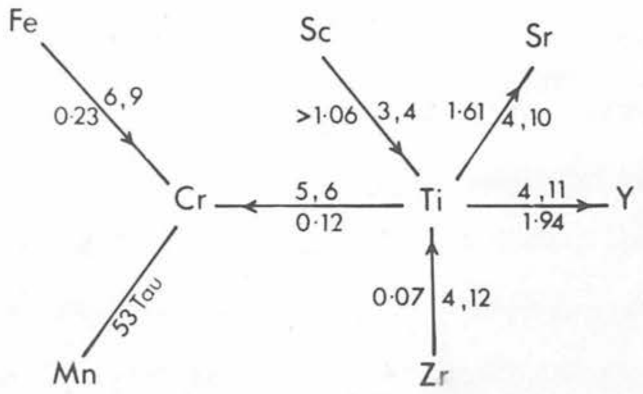

Figure 3

RELATIVE VALUES OF Δ_1'log N OBTAINED IN THE REFINED DIFFERENTIAL CURVE-OF-GROWTH COMPARISON OF HR 4072 (PRIMARY) WITH α LYR

The numbers of the groups of lines in Table 3 are indicated. The arrows point to the element with the larger value of Δ log N.

$$\delta_1 X = X_{HR\ 4072} - X_{53\ Tau}, \tag{6}$$

where X is any quantity, a comparison between the primary and 53 Tau is given in Table 4 for Mn and Cr. $\delta_1 \theta_{exc} = +0.03$, and the mean values of $\delta_1 \log N_r$ for the lines of MnII and CrII listed are -0.66 and +0.42 respectively. Since $\Delta_1{}'\log N = +0.51$ for Cr, the resulting value of $\Delta_1{}'\log N$ for Mn is +0.64.

The final values of $\Delta_1{}'\log N$ are listed in Table 5. The values of log N given by Hunger (1960) for α Lyr were used to derive the values of log N in this table for the primary. For elements with lines on the linear part of the curve-of-growth, the results should be more accurate than those for 53 Tau, since the ionisation corrections are smaller and based on five suitable lines of FeI instead of three weak lines of FeI. Although an error in the adopted value of $\Delta_1 \theta_{ion}$ will produce a corresponding error in the derived value of $\log P_e$, the resulting ionisation corrections will not be greatly affected. The chief uncertainty in the results is the zero point of the $\Delta_1{}'\log N$ scale.

Table 4

CORRECTED EQUIVALENT WIDTHS FOR MnII AND CrII

Multiplet	λ	χ	Primary log W	 log W_{corr}	53 Tau log W	 log W_{corr}	$\delta_1 \log W_{corr} + 0.03\chi$
MnII	4136.91	6.14	-1.42	-1.29	-1.12	-0.23	-0.88
	4478.63	6.64	-1.64	-1.60	-1.27	-0.97	-0.43
CrII(26)	4179.43	3.81	-1.49	-1.40	-1.66	-1.61	+0.32
(31)	4261.92	3.85	-1.24	-0.92	-1.39	-1.23	+0.43
	4275.57	3.84	-1.37	-1.21	-1.58	-1.52	+0.43
	4284.21	3.84	-1.39	-1.25	-1.60	-1.55	+0.42
	4269.28	3.84	-1.44	-1.33	-1.85	-1.83	+0.62
(162)	4145.77	5.30	-1.39	-1.22	-1.64	-1.59	+0.53

Table 5 shows that the chemical composition of the atmosphere of the primary component of HR 4072 is abnormal. There are large excesses of silicon, strontium and yttrium, and scandium is probably deficient. The excess of strontium is interesting, because the primary is much hotter than the Sr stars; it would suggest that excesses of strontium are not confined to the coolest peculiar A stars.

Table 5

ABUNDANCES IN THE PRIMARY

Element	Δ_1'log N	log N α Lyr	log N Primary
H		12.0	(12.0)
Mg	+0.24	7.7	7.9
Si	+1.29	8.2	9.5
Sc	$<$ -0.67	3.4	$<$ 2.7
Ti	+0.39	4.8	5.2
Cr	+0.51	5.6	6.1
Mn	+0.64	5.3	5.9
Fe	+0.28	6.5	6.8
Sr	+2.00	2.8	4.8
Y	+2.33	2.1	4.4
Zr	+0.32	2.9	3.2

Analysis of the secondary

The primary will be taken as the comparison star in the analysis of the secondary, and the notation

$$\Delta_2 X = X_{secondary} - X_{primary}, \tag{7}$$

where X is any quantity, will be employed. Unfortunately, it is not possible to determine $\Delta_2 \theta_{exc}$ because of the shortage of lines in the secondary. Adopting $\theta_{exc} = 0.7$ as an approximate estimate of the excitation temperature of the secondary, a curve-of-growth for the YII lines was constructed in Fig. 4 by plotting $\log (W/\lambda) + \log\sqrt{M}$ against $\log gf\lambda - 0.7\chi$ using the gf-values by Corliss and Bozman (1962). An error in the estimate of θ_{exc} will not be serious because of the small range in χ for the lines. Assuming that $\log P_e = 2.44$ as in the primary, a value $\Delta_2\theta_{ion} = +0.04$ was derived by applying Saha's ionisation equation to the lines of FeI and FeII listed in Table 6; $\Delta_2\theta_{exc}$ was also taken as +0.04. An error in the adopted value of $\log P_e$ will produce a corresponding error in the derived value of $\Delta_2\theta_{ion}$, but the ionisation corrections will be only slightly affected. The effect of an error in the value of $\Delta_2\theta_{exc}$ may be more serious, since the shortage of lines prevents the use of groups of lines of similar mean excitation potentials.

Values of $\Delta_2\log N_r$ and $\Delta_2\log N$ were deduced and listed in Table 7. It will be noted that the ionisation corrections $\Delta_2\log N - \Delta_2\log N_r$ are small. The values of $\Delta_2'\log N$ were found by adding log 5.76 to the values of $\Delta_2\log N$ to take account of the "dilution" of the secondary lines by the light of the primary.

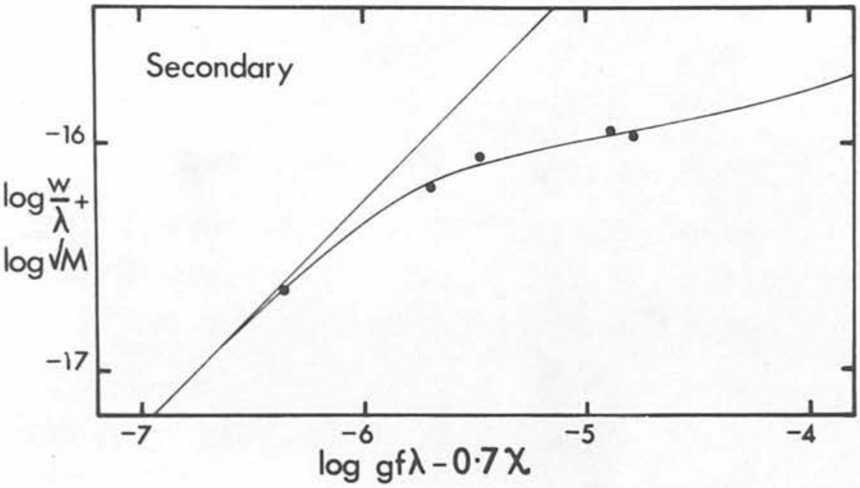

Figure 4

CURVE-OF-GROWTH FOR YII LINES IN HR 4072 (SECONDARY)

Table 6

CORRECTED EQUIVALENT WIDTHS

Multiplet	λ	χ	$\log W_2$	$\log W_{corr}$	$\log W_1$	$\log W_{corr}$	$\Delta_2 \log W_{corr} + 0.04\chi$
TiII(19)	4395.03	1.08	-1.60	-1.48	-1.10	-0.47	-0.97
	4443.80	1.08	-1.49	-1.31	-1.09	-0.46	-0.81
(20)	4294.10	1.08	-1.43	-1.12	-1.14	-0.63	-0.45
(31)	4468.49	1.13	-1.60	-1.49	-1.06	-0.30	-1.14
	4501.27	1.11	-1.44	-1.20	-1.07	-0.36	-0.80
(41)	4312.86	1.18	-1.57	-1.42	-1.19	-0.77	-0.60
CrII(31)	4242.38	3.85	-1.55	-1.37	-1.15	-0.61	-0.61
FeI(41)	4383.55	1.48	-1.40	-0.98	-1.29	-1.04	+0.12
	4404.75	1.55	-1.58	-1.43	-1.33	-1.13	-0.24
(42)	4325.77	1.60	-1.40	-0.98	-1.09	-0.28	-0.64
(800)	4219.36	3.56	-1.64	-1.52	-1.51	-1.44	+0.06
FeII(27)	4351.76	2.69	-1.58	-1.43	-1.18	-0.72	-0.60
(28)	4178.86	2.57	-1.66	-1.55	-1.12	-0.36	-1.09
	4296.57	2.69	-1.51	-1.28	-1.25	-0.93	-0.24
(37)	4491.40	2.84	-1.57	-1.42	-1.26	-0.99	-0.32
(38)	4508.28	2.84	-1.41	-1.03	-1.16	-0.68	-0.24
SrII(1)	4215.52	0.00	-1.31	-0.09	-0.95	+0.71	-0.80
	4077.71	0.00	-1.39	-0.44	-0.92	+0.79	-1.23
YII(5)	4309.62	0.18	-1.51	-1.09	-1.11	+0.06	-1.14
	4398.02	0.13	-1.62	-1.42	-1.24	+0.61	-2.02
(13)	4374.94	0.41	-1.40	-0.61	-1.00	+0.50	-1.09
(14)	4177.54	0.41	-1.41	-0.58	-0.92	+0.77	-1.33

Table 7

RESULTS FOR THE SECONDARY

Element	Multiplets	Mean $\Delta_2 \log W_{corr} + 0.04\chi$	$\Delta_2 \log N_r$	$\Delta_2 \log N$	$\Delta_2' \log$
Ti	TiII(19),(20),(31),(41)	-0.79	-0.81	-0.86	-0.10
Cr	CrII(31)	-0.61	-0.66	-0.67	+0.09
Fe	FeII(27),(28),(37),(38)	-0.50	-0.53	-0.53	+0.23
Sr	SrII(1)	-1.02	-1.05	-1.41	-0.65
Y	YII(5),(13),(14)	-1.40	-1.43	-1.65	-0.89

Table 7 shows that the relative abundances of titanium, chromium, iron, strontium, and yttrium are similar in the primary and secondary components, although the overabundance of strontium and yttrium relative to titanium, chromium, and iron may be slightly smaller in the secondary. The zero point of the $\Delta_2' \log N$ scale is uncertain because of the uncertainty in the ratio of the luminosities of the two components.

Discussion

Probably the chief limitation on the accuracy of the relative abundances in the primary and secondary components is the uncertainty in the microturbulent velocity ξ_t, as some lines on the "flat" part of the curves-of-growth have been used. On the basis of the extreme sharpness of the lines (Babcock, 1958a) and in the absence of another reliable source of information, ξ_t has been assumed to be zero in both the primary and secondary components. Allowing for the "dilution" of the primary lines by the light from the secondary, an attempt was made to determine ξ_t in the primary by comparing the curve-of-growth for TiII with the corresponding curves-of-growth for 53 Tau and α Lyr. Estimates of 1 or 2 km/sec were obtained for ξ_t, but these are unreliable because of the scatter about the curves-of-growth and the possibility of systematic errors in the equivalent widths between the stars. The uncertainty in ξ_t for HR 4072 is, however, unlikely to affect the general conclusion that excesses of strontium and yttrium are common to both components.

Not many other double-line peculiar A binaries are known. Searle and Sargent (1965) found that the two components of 41 Eri were sharp-line manganese stars with GaII lines and that both components had an approximately normal Mn/Fe abundance ratio with Mn and Fe in excess relative to Cr and Ti. They suggested that the two components had acquired these similar abundance anomalies in their circumstellar envelopes during their final stages of contraction to the main sequence. Abt and others (1966) found that HD 98088 was a double-line peculiar A binary with a mass ratio of 1.34. A weak secondary spectrum was also noted in the manganese star 112 Her by Aller (1966).

If the mass luminosity relation

$$\log L/L_{\odot} = 3.3 \log M/M_{\odot} \tag{8}$$

quoted by Allen (1963) holds for peculiar A stars, the ratio of the masses M_1/M_2 of the two components of HR 4072 is approximately 1.6. The orbital elements for the primary given by Schlesinger (1912) would then imply that the angle of inclination i_{orb} which the normal to the orbital plane makes with the line of sight is about 29°. However, if the period of rotation of the primary is the same as the orbital period (11.58 days), the extreme sharpness of the lines suggests that the angle of inclination i_{rot} which the axis of rotation makes with the line of sight is less than 10°. There thus seems to be a difference between the inclination angles i_{orb} and i_{rot} or a difference between the orbital and rotational periods. A more accurate mass ratio could, of course, be derived from a series of radial velocity measures for both components.

Since the lines in HR 4072 are very sharp, it may be expected that further investigations at higher resolution will be worth while. It would be particularly interesting to know whether the secondary component has a magnetic field with variations related to those in the primary.

CHAPTER 8

VARIABLE PECULIAR A STARS

Magnetic field variations

Extensive measurements of the longitudinal component of magnetic fields in peculiar A stars were made by Babcock (1958a). Effective fields of several hundred to several thousand gauss were found in sharp-line stars. The fields are variable, and Babcock (1958b) classified stars according to the type of variation. α-variables have fields which reverse polarity periodically; β-variables have reversing fields but apparently no periodicity, while γ-variables have irregular variations of constant polarity. These classifications describe existing observations but may be modified subsequently; for example, periods may be discovered later for some stars originally classified as β-variables. Variations in radial velocity were also observed by Babcock, but in some stars he found no obvious correlation between the radial velocity variations and the magnetic field variations. Abt and Golson (1962) found small variations in luminosity and colour in magnetic stars. These variations usually show periodicity in α-variables.

Spectrum variations

Stars which show periodic variations in their line intensities might be expected to offer information on the origin of abundance anomalies in peculiar A stars. However, the situation is far from simple, and observations of some spectrum variables are now described to illustrate the diversity of the problems encountered.

α^2 CVn is the best known peculiar A spectrum variable. Struve and Swings (1943) identified many lines of singly ionised rare-earth elements which varied in strength with a period of 5.5 days. Some other lines, such as those of CrII, varied out of phase, while a third group of lines, including those of MgII and SiII, did not show appreciable variations. The grouping of lines according to the type of radial velocity variation is similar. The rare-earth lines have minimum and maximum radial velocities 1.0 days before and 1.5 days after maximum line strength respectively. The CrII lines show a double wave in the radial velocity curve, and the MgII and SiII lines have constant radial velocities. The effective magnetic field varies periodically between -1400 gauss when the rare-earth lines are strongest and +1600 gauss when the chromium lines are strongest. No transverse field was detected when the longitudinal field was zero (Babcock, 1958a). All lines are sharper when the rare-earth lines are strongest. Böhm-Vitense (1966) has explained these observations assuming a period of rotation of 5.5 days and a cylindrically symmetric magnetic field inclined to the axis of rotation, overabundances of the rare earths and chromium being concentrated in the polar and equatorial regions respectively. Burbidge and Burbidge (1955) determined abundances in α^2CVn by a differential curve-of-growth comparison with γ Gem, but Sargent (1964) has pointed out that their adopted ionisation temperatures for both stars may be too low. Further observations on α^2CVn are now being made at the Mount Wilson and Palomar Observatories.

HD 125248 is an outstanding Eu-Cr variable with a period of 9.3 days. The grouping of lines according to their intensity variations is similar to that in α^2CVn. Analysis of the radial velocity variations is complicated by a secular variation in the mean velocity due to binary motion. The binary period of 1618 days given by Ledoux and Renson (1966) was found to be consistent with all the radial velocity measurements between 1947 and 1951 by Babcock (1951) and Deutsch (1958b) of the FeI lines which undergo only slight velocity variations in the 9.3-day period. Deutsch (1958b) noted a similarity in the grouping of lines for the radial velocity variations and equivalent width variations within the 9.3-day period. The effective magnetic field varies between +2100 gauss when the rare-earth lines are strongest and -1900 gauss when the chromium lines are strongest (Babcock 1951, 1958a). Attempts have been made to account for the observations by regarding the star as oscillating (Babcock, 1951), or by postulating a complicated asymmetric field configuration and distribution of elements with a period of rotation of 9.3 days (Deutsch, 1958b).

The "cross-over" effect was first discovered in HD 125248 by Babcock (1951). When the magnetic field is decreasing and crossing over the value zero, most lines in the spectrum formed by left-handed circularly polarised light become sharper than in the other spectrum. At the other cross-over point, when the magnetic field is increasing, the effect is reversed. A similar effect was later detected in other α-variables. The explanation of the effect is that different areas of the surface have different radial velocities and magnetic fields.

ϵ UMa is the brightest peculiar A star, but detailed analysis of it is difficult as its lines are rather broad. Guthnick (1931) established a period of 5.1 days for the variation of the strength of the CaII line at 3933A. All authors agree that the CrII lines vary out of phase with this line, but there is considerable disagreement about the phases of the variation of other lines. A series of spectra at a dispersion of 5.6 A/mm was taken on fine-grain Ilford R40 emulsion with the Edinburgh 36-inch telescope. Difficulty was experienced in identifying lines because of blending. The SiII 4131 line varies out of phase with the CrII lines. Most other lines are constant or vary in phase with the CrII lines. Lines of the rare earths are very weak or absent. Doubling of certain lines of TiII, VII, CrII, and FeII was reported by Struve and Hiltner (1943) at phases 1.36 to 1.59 days and 3.65 to 3.70 days after the minimum of the CaII 3933 line, but other lines of the same ions remained single. No doubling of lines could be detected on the Edinburgh spectra, which were taken at other phases.

Perhaps the most remarkable feature of ϵ UMa is the rapid changes in the Balmer line profiles (Wood, 1964). The equivalent widths of Hγ and Hδ vary 180° out of phase. There are rapid variations in the wings of Hβ, and a change of almost 4A in its equivalent width was found during a four-minute interval. Extraordinary changes in atmospheric structure must be occurring in short intervals of time.

73 Dra has abnormally strong lines of CrI, CrII, SrII, and EuII, and its CaII 3933 line is unusually weak (Morgan, 1933). The EuII 4205 line varies with a period of 20.3 days (Durham, 1943). The variation of other lines was investigated on low-dispersion spectra by Morgan (1933), Durham (1943) and Faraggiana and Hack (1962), but discordant results were obtained. Provin (1953) found that the visual magnitude varies with an amplitude of 0.04^m, maximum light occurring near the phase of maximum EuII line strength.

More recent observations made by Rakos (1963) enabled the times of maximum and minimum EuII line strength to be predicted, so that spectra could be taken near these phases with the Edinburgh 36-inch telescope. Two spectra were obtained at a dispersion of 5.6 A/mm on November 8, 1966 and November 20, 1966. Numerous sharp lines were visible on both spectra. The lines of SiII, CaI, ScII, TiII, MnI, MnII, SrII, YII, BaII, and EuII were much weaker on the second spectrum, whereas the lines of MgII, CrI, CrII, FeI, and FeII remained constant or were only slightly weaker on the second spectrum. A strong unidentified line at 4423.1A remained constant.

The behaviour of the CaII 3933 line was studied on six spectra at a dispersion of 4.5 A/mm by Wehlau (1960). A constant sharp component is superimposed on a broad component which varies in phase with the EuII lines. The broad component may be due to the wings of the line formed in small patches with a high CaII abundance, the remainder of the star's surface having a low CaII abundance. Rapid changes in the overall strength of the line may also occur (Honeycutt, 1966).

The magnetic field variations in 73 Dra have the same period as the spectrum variations (Preston, 1967), and the radial velocity variations are small (Babcock, 1958a). Wood (1964) reported rapid out of phase fluctuations in the profiles of the

Balmer lines similar to those found in ε UMa.

53 Cam, a Eu-Cr-Sr star, has a magnetic field which varies between -4350 and +3700 gauss with a period of 8.0 days. This is the largest range known among the α-variables. The field variations are accompanied by outstanding variations in the spectrum lines. All lines are much broader and the MgII and TiII lines are stronger when the field is of negative polarity (Babcock, 1958a).

HD 98088, another Eu-Cr-Sr star, is a double-line spectroscopic binary with a period of 5.9 days. The variations of the magnetic field between -1000 and +800 gauss and of the strength of the SrII lines in the primary also have this period. If the period of rotation of the primary is the same, the side carrying the positive magnetic field and producing the strongest SrII lines always faces the secondary component (Abt, 1953; Babcock, 1958a).

π'Boo is one of the manganese stars. Deutsch (1947) found from low dispersion spectra that some lines varied in strength, and he derived a provisional period of 2.24 days. Two spectra at a dispersion of 5.6 A/mm were obtained with the Edinburgh 36-inch telescope on May 9 and May 12, 1966 (73 hours apart), but no definite variations were detected. The relative intensities of the lines on both spectra agreed fairly well with those estimated by Jaschek and others (1965) on a Mount Wilson spectrum with a dispersion of

4.5 A/mm. Confirmation of the variations reported by Deutsch is therefore required.

Interpretation of the observations

It does not seem possible to explain all the periodic variations in line strengths without invoking the idea of periodic variations in abundances. For example, in α^2 CVn the rare-earth lines vary out of phase with the Cr lines. The amplitude of the variations does not correlate with excitation potential or Zeeman broadening; both weak and strong lines show large variations (Burbidge and Burbidge, 1955). Lines of an element in different stages of ionisation vary in phase. As these facts cannot be accounted for merely by a periodic variation in atmospheric structure, the main cause of the line variations is probably a periodic variation in the abundances of elements such as Cr and the rare-earths. The large periodic variations of the SiII 4128 and 4131 lines in 56 Ari and HD 124224 (Peterson, 1966) also indicate a variable abundance of Si, as these lines are insensitive to temperature and pressure; moreover, the MgII 4481 lines, which have similar strength, and similar excitation, ionisation and Zeeman parameters, remain constant.

Several theories have been proposed to explain the complex variations in peculiar A stars. The <u>oscillator</u> theory, put forward by Schwarzschild (1949), attributes the cause of the variations to mechanical oscillations of the star. Fluctuations in the strength of the magnetic field could be explained in this way, but this theory does not account for the reversal of polarity. It does not provide a satisfactory explanation for all the variations in line strength, and there is also the problem of how the oscillations are maintained.

The oblique rotator theory, proposed by Stibbs (1950) and Deutsch (1956), suggests that there is a dipole magnetic field inclined to the axis of rotation and that patches of different chemical composition exist on the surface of the star; the period of the variations is the same as the rotational period. Strong support for this theory is provided by the observations of the radial velocities and strengths of the SiII lines in 56 Ari and HD 124224 (Peterson, 1966) and by the fact that periodic spectrum variations usually correlate with the variations in magnetic field strength. Another powerful argument in favour of this theory is that, allowing for the errors in the observations, all the periodic spectrum variables have line widths less than or equal to the widths expected if they rotated with the period of the variations and were viewed equator-on (Ledoux and Renson, 1966). Additional evidence supporting the theory was recently summarised by Steinitz (1964). For some stars much more complex distributions of magnetic fields and surface abundance anomalies must be postulated; Deutsch (1958b) and Böhm-Vitense (1966) discussed the distribution required for HD 125248 and α^2CVn. Some form of magnetohydrodynamic activity would also have to be envisaged in order to account for the irregular magnetic field variations which are often observed.

Another theory is the solar cycle theory (Babcock, 1958b), which suggests that the variations in peculiar A stars are caused by a mechanism similar to that producing the sunspot cycle. Recent work by Jose (1965) shows that there is a correlation between the sunspot cycle and the motion of the Sun relative to the centre of mass of the solar system. As the period of rotation of the Sun is much shorter than the sunspot period, the solar cycle theory seems to conflict with the oblique rotator theory. Synchronisation between the rotational periods of peculiar A stars and the orbital periods of companions would only occur if the companions had stellar rather than planetary masses. In other words, peculiar A variables should be observed as binaries in which the magnetic and spectral variations correlate with the orbital motion, as suggested by Renson (1963). Such a correlation is actually observed for the double-line binary HD 98088, in which the orbital period is identical to the period of the magnetic and spectral variations (Babcock, 1958a). However, a similar correlation has not been found for other spectroscopic binaries, and other stars with short-period magnetic and spectral variations (e.g. 53 Cam, HD 71866, α^2CVn, and HD 125248) have not been observed to be short-period binaries. In most periodic spectrum variables the radial velocity variations are in agreement with the idea of a single rotating star with a complex surface distribution of abundance anomalies. The question of how the complex distributions form seems to be part of the more general problem of the origin of abundance anomalies in peculiar A stars.

CHAPTER 9

THE ORIGIN OF THE PECULIAR A STARS

Nuclear reactions in normal stars

In order to provide a framework for a discussion of the origin of the abundance anomalies in peculiar A stars, the various types of nuclear reactions, which are believed to occur in the interiors of normal early-type stars (Burbidge and others, (1957), are now listed.

1. Hydrogen burning converts hydrogen into helium by the carbon-nitrogen cycle at a temperature T of about 2 x 10^7 °K during the main sequence life-time.

2. Helium burning, at T $\sim 10^8$ °K forms C^{12} by the 3α-reaction and O^{16}, Ne^{20}, and perhaps Mg^{24} by further α-particle addition after the hydrogen burning zone has moved out from the centre of the star in the giant stage.

3. The α-process occurs at T $\sim 10^9$ °K, when the supply of helium at the centre of the star is exhausted, and the helium-burning zone moves out from the centre. α-particles released by (γ, α) reactions are captured by Ne^{20} to form Mg^{24}, Si^{28}, S^{32}, A^{36}, Ca^{40}, and probably Ca^{44} and Ti^{48} successively.

4. The e-process. At T $\sim$ 4 x 10^9 °K, nuclear reactions become so profuse that a statistical equilibrium can be set up in the time

available. The central temperature and density increase until the virial theorem is violated and the core implodes. Lighter elements fall inwards, heat up, and take part in nuclear reactions so rapidly that the star explodes as a type II supernova (Hoyle and Fowler, 1960). The relative abundances of the iron-peak elements injected into the interstellar medium are the same as those prior to the explosion, except that some Fe^{56} may be converted into helium by the energy released in the implosion.

5. The <u>s-process</u> is a slow neutron capture process with a time scale of $\sim 10^2$ to $\sim 10^5$ years. The neutron capture is followed by β-decay, and elements up to Bi^{209} are produced. When Bi^{209} captures a neutron, α-decay occurs before another neutron can be captured. Thus cycling between Pb^{206} and Bi^{209} results.

6. The <u>r-process</u> is a rapid neutron capture process (time scale ~ 1 sec) occurring during the supernova explosion. It builds up other heavy elements including the rare-earth elements Eu and Gd and elements heavier than Bi (Seeger and others, 1965).

7. The <u>p-process</u> is a proton capture process which produces some low-abundance heavy isotopes which are proton-rich. It is relatively unimportant.

Energy considerations

Silicon, which is one of the more abundant elements in normal stars (Si/H) $\sim 1/1000$ by mass), is overabundant by a mean factor of about 40 in Si stars (Searle and Sargent, 1964). This factor is reliable, since it is based on SiII lines which are insensitive to temperature and pressure. The mass of a normal AOV star is about 7×10^{33} gm (Allen, 1963). The luminosities of Si stars are the same as normal main sequence stars of the same colour index (this is known as some Si stars are members of galactic clusters), and the Balmer line profiles of Si stars resemble those of normal main sequence stars. Hence Si stars have normal surface gravities for their luminosities and therefore normal masses. Thus the mass of silicon in Si stars would be, if their composition were uniform throughout, about $7 \times 10^{33} \times 40/1000 \doteqdot 3 \times 10^{32}$ gm. Hence the number of atoms of silicon in a Si star would be about 6×10^{54}. The nuclear reactions producing the overabundance of silicon are not definitely known, but they are likely to involve energies of the order of 10 Mev per atom (Sargent and Searle, 1962). If the composition of Si stars were uniform, the total energy involved would be about 10^{50} ergs. A similar calculation shows that about 4×10^{49} ergs would be involved in producing the observed deficiency of oxygen throughout an AOV star.

For comparison, the energy radiated by a normal A0V star per second is about 3×10^{35} ergs (Allen, 1963). The main sequence life-time of such a star is about 3×10^{8} years, i.e. about 10^{16} sec (Von Hoerner, 1957). Hence the total energy radiated by a normal A0 star during its main sequence life-time is about 3×10^{51} ergs.

The energy involved in the nuclear reactions producing the abnormal abundances in peculiar A stars would therefore be of the order of a few percent of the total energy radiated by normal A-type stars, if the composition of the peculiar A stars were uniform throughout. The energy involved will be much less if the abundance anomalies are limited to the outer convection zone only. The depth of the outer convection zone of a normal main sequence A-type star is about 10^{3} km (Rudkjöbing, 1942). The presence of magnetic fields in peculiar A stars will tend to inhibit convection except in regions of great electromagnetic disturbance. Adopting to 10^{4} km as an overestimate of the depth of the outer convection zone in peculiar A stars, Fowler and others (1955) found that the outer convection zones comprised only about 3×10^{-6} of the total stellar mass. More extensive mixing occurs during the early stages of contraction to the main sequence (Hayashi, 1961) and after the star has evolved off the main sequence. It is therefore important to decide the evolutionary status of peculiar A stars.

Evolutionary status of peculiar A stars

The best method of determining the evolutionary status of a group of stars is to discuss the membership of the stars in clusters and associations. Jaschek and Jaschek (1962) found that peculiar A stars were members of galactic clusters with ages in the range 2×10^7 to 6×10^8 years. This range must now be extended, because Garrison (1967) recently found several peculiar A stars in the upper Scorpius complex, which is part of the Scorpius-Centaurus association and has an age of about 5×10^6 years. There is, however, no indication that peculiar A stars are associated with very young clusters or nebulosity, and no peculiar A stars have been found in clusters older than 6×10^8 years. The hotter types of peculiar A star appear only in the younger clusters. In all cases the peculiar A stars lie near the main sequence and appear to have normal luminosities for their masses. Sometimes they are two or three magnitudes fainter than the brightest main sequence stars in the cluster. There is therefore strong evidence that peculiar A stars have the same evolutionary status as normal main sequence stars.

Source of the abundance anomalies

Since peculiar A stars are found in clusters and visual binaries with normal members, it is difficult to see how they can have been formed out of interstellar matter of abnormal composition. Accretion of abnormal interstellar matter during the life-time of the star may be discounted for the same reason. In view of the energy considerations, it is rather unlikely that the abundance anomalies were produced during the early stages of contraction to the main sequence, assuming that convective mixing was present as in the contraction of normal stars (Hayashi, 1961). The abundance anomalies are therefore probably produced during the later stages of contraction or during the main sequence life-time. This conclusion, in conjunction with the energy considerations, indicates that preference should be given to theories of peculiar A stars which imply that their abundance anomalies are merely surface phenomena.

Fowler and others (1965) have suggested that peculiar A stars have returned to the vicinity of the main sequence after their giant phase with only a small loss of mass. This conflicts with the evolutionary status of peculiar A stars as determined from their membership of clusters and associations. It is also unlikely that the stars would return to the main sequence with normal luminosities for their masses and remain there after extensive nuclear transformations in the interior and mixing with the surface. According to their suggestion, these post-giant stars would remain on the main sequence for about 3×10^7 years, since peculiar A stars comprise about 10% of A-type stars and the main sequence life-time of a normal A0 star is about 3×10^8 years. This assumes that all stars of suitable mass pass through a peculiar A phase; if only some do, the post-giant period in the vicinity of the main sequence would have to be even longer.

As an alternative, they suggested that peculiar A stars are close binaries in which the companion star was initially more massive and is now highly evolved. The companion star and the peculiar A star will be designated as "primary" and "secondary" respectively. During the giant phase of the primary, material which had been processed by nuclear transformations in the interior was mixed with the surface, and the primary lost mass. In this way some of the processed material was lost into space, and some was transferred to the surface of secondary which is now observed as a peculiar A star. The primary may now be a white dwarf. This suggestion would explain why peculiar A stars are observed only on part of the main sequence. Since the primary has to be initially more massive, peculiar A stars of the earliest spectral types will not be observed. In late-type stars the convection zones are deep, and any abundance anomalies resulting from mass transfer from a companion star would be diluted. Descendants of the peculiar A stars with similar abundance anomalies have not been found, because the abundance anomalies are diluted by mixing with the interior after the peculiar A stars have evolved off the main sequence. Thus no red giants with Si, Mn, Eu, or Gd excesses are known.

This theory would mean that all peculiar A stars have, or previously had, close companion stars. Jaschek and Jaschek (1958) discussed the frequency of spectroscopic binaries among peculiar A stars. They pointed out that peculiar A stars have been more carefully studied than normal stars and that they have sharper spectral lines. Both these factors favour the discovery of more peculiar A spectroscopic binaries. Allowing for these selection factors by considering bright stars with vsini $\leqslant$75 km/sec, they found that only 13% of peculiar A stars are spectroscopic binaries as compared with 43% of normal stars. Although some spectroscopic binary peculiar A stars may have remained undiscovered because of non-periodic fluctuations in radial velocities, it is difficult to reconcile the small percentage of known binaries with the proposal, also put forward by Ledoux and Renson (1966), that *all* peculiar A stars are close binaries. Jaschek and Jaschek (1958) suggested that the lack of known spectroscopic binaries is simply due to most peculiar A stars being viewed pole-on and perpendicular to the orbital plane, but this conflicts with several lines of evidence that peculiar A stars are viewed at all angles of inclination. It may be concluded that at least some peculiar A stars are single or have only distant companions. Another difficulty is that, of the known spectroscopic binaries, some have to be excluded as far as the transfer of *processed* material is concerned, because their periods are so short (several days) that the primary would hardly have been able to expand at all after its main sequence life-time without reaching the Roche limit, and so only unprocessed envelope material could be transferred.

The mass transfer theory is attractive, because it provides a simple explanation for at least some of the abundance anomalies and for the position of peculiar A stars in the colour-magnitude diagram. Moreover, the energies involved present no difficulty. It is perhaps worth asking whether the low percentage of suitable spectroscopic binaries among the peculiar A stars is really a fundamental objection to the theory.

Van den Heuvel's theory

It seems that the only way to retain the mass transfer theory would be to suppose that peculiar A stars were previously secondaries in close binary systems. Van den Heuvel (1967) has pointed out that this is possible if the primary star lost more than half its mass in the post-giant phase by a supernova outburst during a time interval shorter than the orbital period. This would usually result in an increase in the major axis of the orbit or a complete separation of the two components. In the latter case the secondary would be ejected with a space velocity slightly less than its original orbital velocity. The condition for complete separation derived by Blaauw (1961) implies that the spectral type of the primary should have been earlier than B4 in most cases. Stothers (1963) showed that supernova outbursts occur only for stars earlier than spectral type B8 initially. These considerations led van den Heuvel to suggest an explanation for three groups of peculiar stars.

1. The "runaway" stars, which had primaries of spectral type earlier than B4 and were completely separated.

2. The peculiar A stars, which had primaries with spectral type between B4 and B8 and are now in enlarged orbits.

3. The metallic-line stars, which had primaries of spectral type later than B8 and are now observed as spectroscopic binaries (Abt, 1961), since no enlargement of the orbit takes place if the mass loss is slow.

If this theory were correct, it would account for the lack of binaries among the "runaway" stars (Blaauw, 1961) and the absence of metallic-line stars in clusters younger than 3×10^8 years (Jaschek and Jaschek, 1962).

It might be expected that the "runaway" stars would be an extension of the peculiar A stars towards earlier spectral types. Van den Heuvel suggested that spectral studies of the "runaway" stars would be desirable. Such a study was carried out by Wallerstein and Wolff (1965). They found that, unlike the peculiar A stars, the "runaway" stars had a distribution of values of vsini similar to that of normal main sequence stars. They also failed to detect any line strength anomalies in three sharp-line "runaway" stars. Another problem is how to explain the large space velocities after ejection (about 100 km/sec) without having the two components so close that the first

Lagrangian point limits the radius of the primary before it reaches the pre-supernova red giant stage. Blaauw (1961) proposed that the primaries were proto-stars of very large masses (about 250 $M_\odot$) which exploded before they reached the main sequence. This would explain the observed features of the "runaway" stars, in particular the absence of line strength anomalies. However, it must then be decided what has happened to stars which had main sequence primaries earlier than spectral type B4. Perhaps they are now peculiar A stars whose velocities of ejection have passed unnoticed because they are much less than 100 km/sec.

Van den Heuvel's explanation of metallic-line stars may also be called in question. Metallic-line stars are binaries, and most of them have short periods (Abt, 1961). With the present orbital dimensions, the primaries could not have expanded to the giant phase without reaching the Roche limit. One possibility is that mass transfer from a third component has taken place. However, metallic-line stars comprise about 20% of A-type stars, and it is doubtful whether the formation of suitable triple systems is sufficiently common to account for this percentage. Another possibility is that the orbital dimensions of the binaries were originally much larger and that slow mass loss from the primaries has decreased the sizes of the orbits (Su-Shu Huang, 1963). Some suitable mechanism for the slow mass loss would have to be found.

On the other hand, there seems to be no immediate objection to van den Heuvel's proposal for the peculiar A stars. Since no evidence has been found for the existence of a fourth group of peculiar stars which had primaries earlier than spectral type B4, it is proposed that such primaries could also give rise to the formation of peculiar A stars. With this modification, the implications of van den Heuvel's theory of the origin of peculiar A stars are now explored.

Formation of a typical peculiar A star

As a typical example, consider a binary with a primary of 16 $M_\odot$ (spectral type BOV) and a secondary of 3 $M_\odot$ (spectral type AOV). Since a red giant may expand to a radius of about 8 a.u., the initial separation of the two components should be greater than about 20 a.u. if the orbit is circular; otherwise the primary would expand beyond the Roche limit. The problem of what happens when the initial separation is less than this is discussed later in the sections on the Mn stars. For a separation of 20 a.u. the period is 20 years, and the orbital velocity of the secondary is 24 km/sec. The velocity of ejection of the secondary is usually between 50% and 100% of the orbital velocity (Blaauw, 1961), and the resulting peculiar A star will thus have an excess space velocity of between 12 and 24 km/sec, which will pass unnoticed in most cases. For an elliptical orbit the initial mean separation would have to be larger to avoid expansion of the primary beyond the Roche limit. On the other hand, if the mean separation is too large, the mass transferred will not be sufficient to produce noticeable abundance anomalies on the surface of the secondary. The mean separations probably lie in the range from 20 to 100 a.u., and 40 a.u. will be adopted as a typical value. Kuiper (1935) found that the most frequent separation for binaries was about 20 a.u. When the primary explodes to form a white dwarf of about 1 $M_\odot$, it loses about 15 $M_\odot$. The explosion will probably be a supernova of type II, since such supernovae have large masses and are associated with population I stars (Hoyle and Fowler, 1960); the space motions and galactic distribution of peculiar A stars indicate that they also belong to population I.

Van den Heuvel did not discuss the effects which a type II supernova explosion would produce on the surface of the secondary. The energy radiated in the explosion is of the order of 10^{49} ergs (Minkowski, 1964). If the explosion is isotropic, about 2×10^{41} ergs will reach an AOV secondary 40 a.u. away. Since this is the same as the energy radiated by an AOV star in about 8 days, the atmosphere of the secondary will heat up and expand. The velocity of ejection in the explosion ($\sim$5000 km/sec according to Minkowski, 1964) is much larger than the velocity of escape from the secondary ($\sim$700 km/sec). Thus the gravitational field of the secondary will not have a great influence on the motion of most of the ejected material. If all the ejected material (about 15 $M_\odot$) travelled at 5000 km/sec, the secondary might be expected to receive about $4 \times 10^{-7} M_\odot$ about 14 days after ejection. The material reaching the limb of the secondary will transfer momentum to its atmosphere, and some of the original atmospheric material will be removed. Material arriving at the central region of the surface will be mixed with the original atmosphere. The widths of the emission lines in a type II supernova spectrum indicate that the ejected material has a velocity dispersion of several thousand km/sec (Poveda, 1964). The velocity of escape from the primary in its giant phase before the explosion is much less than that of the AOV secondary. Consequently, material coming from the expanding shell and directly from the primary will also reach the neighbourhood of the secondary at low velocities and experience its gravitational attraction. Additional processed material could be accreted by the secondary in this way during the following months.

In general, a mass transfer theory cannot account for the large abundance deficiencies of certain elements (e.g. oxygen) in peculiar A stars, because this would mean that the original atmosphere of the secondary was almost entirely replaced by the material transferred and that practically no dilution of the transferred material by mixing with the outer layers of the secondary took place. If, in accordance with the previous energy and evolutionary considerations, the abundance anomalies in the peculiar A stars are regarded as merely surface phenomena, large abundance deficiencies must be the result of surface nuclear reactions. Further evidence for surface nuclear reactions is that beryllium, which would be destroyed by thermonuclear reactions with protons at the temperatures experienced in stellar interiors and supernovae, is overabundant by a factor of about 25 in the Si-Eu-Cr star α^2 CVn (Bonsack, 1961). One of the great advantages of the supernova theory of mass transfer is that it immediately suggests at least four ways in which surface nuclear reactions might occur.

1. γ-radiation from the supernova might induce nuclear reactions on the surface of the secondary.

2. Thermonuclear reactions might occur on the surface of the secondary, because the radiation from the supernova will heat up its atmosphere. Further heating will take place on the arrival of high-velocity material, since some of this material will leave the primary at temperatures higher than 10^9 degrees and will not have "cooled" appreciably in 14 days (Poveda, 1964; Shklovskii, 1962).

3. The kinetic energy of atoms with an atomic weight of 40 travelling at 5000 km/sec is 5 Mev, which is sufficient to induce nuclear reactions on arrival at the surface of the secondary.

4. The observed magnetic fields in peculiar A stars may have been transferred from the primary during its giant phase or the supernova

explosion, since magnetic fields have been found in red giants and supernova remnants (Van den Heuvel, 1967). Surface nuclear reactions involving ions accelerated in the magnetic field on the secondary might occur during the remainder of its main sequence life-time (Burbidge and others, 1958).

It is likely that the surface nuclear reactions involve only the lighter elements, since Coulomb barrier penetration is more difficult for heavy elements, and high neutron densities ($\sim 10^{24}/\text{cm}^3$ according to Burbidge and others, 1957) are required for the rapid neutron capture processes which are responsible for the formation of some of the rare earths such as Eu and Gd. It is interesting to note that nearly all the observed and suspected deficiencies in peculiar A stars are for elements with atomic weights less than 45, and that Fowler and others (1965) could not propose any interior nuclear reactions to account for the deficiency of oxygen and the suspected deficiencies of helium and carbon. The stable iron-peak elements (Cr, Mn, and Fe) are not likely to be affected by surface nuclear reactions. Surface spallation of Fe^{56} would require high-energy nautrons or protons ($\sim$20 Mev) and would greatly enhance the abundance of vanadium (Burbidge and Burbidge, 1958; Fowler and others, 1965). No great excess of vanadium has been found in the peculiar A stars.

There remains the possibility that (p,n) and (α,n) reactions involving the lighter elements might supply neutrons for slow neutron capture by the heavy elements on the surface of the secondary. Most of the neutrons will be captured by protons to form deuterium, but some might be captured by heavier elements (Burbidge and others, 1958). Since no great excess of Ba has been found in peculiar A stars, the supply of neutrons on the surfaces of secondaries is not sufficient to produce long neutron capture chains. With a limited supply of neutrons, it would be expected that only elements with atomic weights slightly higher than the most abundant elements would be enhanced. P, which is slightly heavier than Si, and Ga, which is slightly heavier than the iron-peak elements, are overabundant in some Mn stars and may be produced by surface neutron captures on the secondary. Alternatively, unless subsequent work shows that heavy s-process elements such as Sr and Ba are overabundant in Mn stars, it might be supposed that the supply of neutrons in the interiors of their primaries is abnormally low.

Distinctive properties of the Mn stars

The composition of the material transferred to the secondary in the supernova explosion is a reflection of the products of nuclear reactions occurring before and during the explosion in the interior of a normal star earlier than B8 initially. The only modification is that surface nuclear reactions involving the lighter elements take place on the secondary during and after the explosion. Although differences in surface composition might be expected from star to star among the peculiar A group, all peculiar A stars should show the same general surface composition in the sense that elements which are abundant in normal stars and groups of elements of similar atomic weights will behave in a similar way. The work of Searle and Sargent (1964) and Searle, Lungershausen and Sargent (1966) shows immediately that the surface composition of Mn stars differs radically from that of other peculiar A stars. The latter will be called the <u>main group</u> of peculiar A stars, simply because the Mn stars comprise only about 10% of peculiar A stars. The main group will be discussed first.

With regard to elements which are abundant in normal stars, reliable abundances are available for O, Mg and Si. O is deficient in all stars of the main group (Sargent and Searle, 1962), and the abundance of Mg is normal (Searle and Sargent, 1964). Overabundances of Si are much more common in the hotter sub-groups and may be due to surface nuclear reactions (Searle and Sargent, 1964). The relative abundances of the iron-peak elements are about normal (Searle, Lungershausen and Sargent, 1966). Rare-earth elements are overabundant in many stars of all sub-groups of the main group. According to the classification scheme of Jaschek and Jaschek (1958) the predominant feature of cooler peculiar A stars is a great enhancement of SrII lines. However, one of the hottest peculiar A stars (the Si-4200 star HD 34452) shows SrII lines which are not observed in normal stars of the same colour (Jaschek and Garcia, 1967) unless they have a large range of surface temperature due to rapid rotation viewed pole-on or to tidal distortion by a companion star. HD 34452 has slight spectral variations of period 2.47 days, a line width of about 1.5A, and a constant radial velocity. According to the oblique rotator theory, it would therefore be a single slowly rotating star viewed equator-on. The brighter component of HR 4072 is a Si star with a large excess of Sr (Chapter 7). Thus the overabundance of Sr seems to be common to all sub-groups of the main group. The surface compositions for members of the main group therefore appear to be roughly similar.

The Mn stars differ from members of the main group in the following respects.

1. They show a preponderance of very sharp-line stars which is not observed in the main group. This is due to the equatorial rotational velocities of most Mn stars being very low (Chapter 3).

2. When the magnetic field measurements by Babcock (1958a) are averaged, it is found that the Mn stars have generally lower observed components of the magnetic fields (400 gauss as compared with 800 gauss in the main group).

3. Curve-of-growth studies by Searle, Lungershausen and Sargent (1966) indicate that the Mn stars have lower microturbulent velocities

(2 km/sec as compared with 6 km/sec in the main group). Application of the theory of Zeeman broadening by Boyarchuk and others (1960) shows that this large difference is not a spurious result of the difference of 400 gauss in the mean observed components of the magnetic field between the Mn group and the main group.

4. No outstanding spectrum variables are known in the Mn group. The spectral variations of π^1Boo have still to be confirmed, and the period of spectral variation of the bright Mn star α And ($m_v = 2.1$) is unknown.

5. For no apparent reason, Mn stars are confined to the hotter end of the range of surface temperatures for peculiar A stars. The termination of the main group is naturally explained by the increasing depth of the outer convection zone towards later spectral types.

6. The abundances of O and Si are normal in the Mn group (Searle and Sargent, 1964).

7. The relative abundances of the iron-peak elements (Cr, Mn, and Fe) are abnormal in the Mn group and vary from star to star (Searle, Lungershausen and Sargent, 1966).

8. Rare-earth lines are absent in Mn stars, although they are present in some stars of the main group with similar colours.

9. P and Ga are greatly overabundant in some Mn stars but not in stars of the main group.

The origin of the Mn stars

The explanation of the Mn stars must have some similarity to that of the main group, since the Mn stars are also peculiar A stars by virtue of their galactic distribution, their position in the colour magnitude diagram, their slow rotation, their low binary frequency, and their magnetic fields. On the other hand, the modification to the supernova theory must be so drastic for the Mn stars that the course of nuclear reactions involving the iron-peak and rare-earth elements at the centre of the primary is changed. Perhaps one solution to this dilemma is that the structure of the primary is altered by expansion to its Roche limit before the supernova explosion.

Consider a primary of mass 16 $M_\odot$ separated by less than 20 a.u. from the secondary. When the primary expands beyond the Roche limit towards the end of its giant phase, it loses mass until the mass of its envelope becomes roughly equal to that of the core, the time interval involved being of the order of a day (Reddish, 1957). Since the mass of the core is about a third of the original mass of the star (Hayashi and others, 1962), the primary loses about 5 $M_\odot$. Some of the mass lost falls on the secondary which consequently increases in mass. This may explain why Mn stars are confined to masses greater than 4 $M_\odot$ corresponding to the hotter end of the range of surface temperatures for peculiar A stars. As the mass loss from the primary occurs in an interval which is much shorter than the orbital period of several years, the size of the orbit increases. Complete separation of the two components does not occur, because Blaauw's condition for separation implies a much greater mass loss. Since no mixing has occurred between the core and the envelope in the primary, the mass transferred to the secondary will have the same chemical composition as the interstellar matter out of which the binary system was formed. The secondary will still be in the early part of its main sequence life-time, and its chemical homogeneity will not be affected by the addition of some mass from the envelope of the primary. The secondary will therefore continue as a main sequence star. On the other hand, the structure of the primary has been drastically altered by the loss of about 5 $M_{\odot}$, and the ratio of the mass of the core to the total mass is quite different from that in a normal giant with a total mass of 11 $M_{\odot}$. The character of the subsequent nuclear reactions in the core is likely to be affected. Having equalised the masses of its envelope and its core, the primary will be stable for a time. However, if it is eventually to become degenerate, it must lose another 10 $M_{\odot}$ approximately in order to bring its mass below the Chandrasekkar limit of about 1.4 $M_{\odot}$. There will not be sufficient time during the remainder of its "giant" phase for this to occur by any normal process of continuous mass loss. The primary will therefore eject matter in some kind of violent process, possibly a supernova explosion, and some of the matter processed in the interior will reach the surface of the secondary, which will then be observed as a Mn star.

Consideration is now given to the problem of why the Mn stars comprise only about 10% of the total number of peculiar A stars. According to the above explanation, the initial distributions of the masses of both primaries and secondaries would be the same for the Mn group as for the main group. The critical factor is therefore the initial separation of the two components. If the separation is too large for the primary to expand beyond the Roche limit, a peculiar A star of the main group will be formed. If, on the other hand, the separation is sufficiently small for expansion beyond the Roche limit to take place, then the mass of the secondary will be increased, and the remainder of its main sequence life-time will be shortened. The mass of the primary is reduced, and its evolution may be retarded so that its explosion is postponed. Thus the life-time of a Mn star (the interval between the explosion of the primary and the end of the main sequence life-time of the secondary) will be short, and not many Mn stars will be observed. The evolution of binary systems with even smaller initial separations (of the order of 1 a.u.) is a complicated problem. Several expansions beyond the Roche limit may take place, and each may be accompanied by a large mass transfer. It is not clear whether the final result will be the formation of a Mn star.

Equatorial rotational velocities are low in the main group and very low in the Mn group. This cannot be due to any abnormal process of star formation, because the essence of the supernova theory is that a peculiar A star was originally the secondary of a normal binary. The slow rotation is therefore due to partial synchronisation of the rotation of the secondary to the orbital motion. Such synchronisation probably does not take place during the contraction to the main sequence, as there is no evidence that early B-type long-period binaries (the primaries of future peculiar A stars) have slow rotation. This argument is, of course, not conclusive, because the more massive primaries are less likely to become synchronised to the orbital motion than the secondaries. Another suggestive, but not conclusive, argument against synchronisation during the contraction stage is that "runaway" stars have the same distribution of values of vsini as normal stars (Wallerstein and Wolff, 1965). Synchronisation of the secondary during the main sequence life-time of the primary is even less likely, because the radii of the primary and secondary are too small and the separation between the two components is too large for tidal interaction. Synchronisation of the secondary does not take place after the supernova explosion of the primary, since the explosion enlarges the orbit. The synchronisation of the secondary should therefore occur during the giant phase of the primary and will be more complete for Mn stars, because the primary in its giant phase is sufficiently close to expand beyond the Roche limit. This is probably the main explanation of the preponderance of stars with very sharp lines in the Mn group. It is not, however, certain that partial synchronisation takes place in the formation of all stars of the main group; the absence of rapidly rotating peculiar A stars may also be partly due to dilution of the abundance anomalies by rotational mixing.

The abnormal relative abundances of the iron-peak elements in the Mn group imply that the equilibrium conditions under which these elements were formed differed from those in the primaries of the main group. Moreover, the conditions (temperature and neutron density) were different in the supernova explosion, since the r-process elements Eu and Gd are not observed in the Mn stars. The difference in the interior conditions between the primaries of the Mn and the main groups might be explained by the substantial loss of mass from the envelope of a Mn primary in its giant phase and the resulting change in the structure of the star. The mass loss depends on the initial separation between the primary and secondary, and this might explain why the relative abundances of the iron-peak elements vary from star to star in the Mn group. The mass loss will also affect the character of the supernova explosion and might be one factor contributing to the diversity of the light curves of type II supernovae which was noted by Minkowski (1964).

Changes in the composition of the interstellar medium

Type II supernovae ejecting several solar masses of processed material at the present frequency of about one per 60 years (Minkowski, 1964) during the history of the galaxy (about 1.5×10^{10} years) would have supplied about 10^9 $M_\odot$ of processed material (about 1% of the mass of the galaxy). This very rough estimate is sufficient to demonstrate that there should be some correlation between the changes in the composition of the interstellar medium and the abundance anomalies observed in peculiar A stars. The correlation would apply only to the elements which are not affected by surface nuclear reactions on the secondaries. Accordingly, the supply of iron-peak and r-process rare-earth elements to the interstellar medium will now be discussed.

The maximum rate of addition of heavy elements to the interstellar medium occurred in the early history of the galaxy (Clayton, 1964). During this stage the interstellar medium was well mixed. Subsequent mixing was only partial, and the composition of the interstellar medium became patchy as further stellar material was injected (Dixon, 1966). Apart from a few light elements (Li, Be and B), the surface composition of G-type dwarfs is the same as the composition of the interstellar medium at the time and place of formation. Differences in surface composition from star to star will therefore yield information on the composition of the material injected into the interstellar medium after the mixing became partial, provided that stars with extreme metal deficiencies are avoided.

Wallerstein (1962) and his colleagues determined the abundances of Na, Mg, Si, Ca, Sc, Ti, Cr, Mn, Fe, Ni, and Ba in 31 G-type dwarfs in the solar neighbourhood. For each element X the difference between the logarithm of the abundance ratio X/H in the star and that in the Sun, designated [X/H] by Wallerstein, was found. The mean values of [X/H] for the well observed metal-poor stars HD 22879, HD 114762, HD148 HD 157089, HD 165908, and HD 224930 and the metal-rich stars HD 10307, HD 34411, HD 86728, HD 102870, HD 114710, and HD 186408 are listed in Table 1. The maximum error in [X/Fe] for a star is less than ±0.20 in most cases. The antilogarithm of the difference between the metal-rich and the metal-poor mean values of [X/H] is the factor by which the element was enriched in the interstellar medium. Further abundance determinations were made by Helfer and others (1963) from spectra at a higher dispersion for the metal-poor stars 99 Her and 85 Peg and for the metal-rich stars ζ Her and β Com, and the enrichment factors for these stars were calculated in a similar way. Because of the different choice of stars, these enrichment factors are smaller than those derived from Wallerstein's results. The enrichment factors for the stars chosen by Helfer and others (1963) were transformed to the same scale as the enrichment factors derived from Wallerstein's results and averaged with the latter.

The predominant nuclear process believed to be responsible for the formation of each element (Burbidge and others, 1957; Seeger and others,

1965) is also given in Table 1. It is striking that Mn has a much higher enrichment factor than the other e-process elements; even if corrections are made for the hyperfine structure of the Mn lines (Wolff and Wallerstein, 1966), the mean enrichment factor for Mn would still be high (about 9.1). The enrichment factor for the r-process rare-earth element Eu is also high. The picture which emerges is that the enriching material supplied to the interstellar medium since mixing became partial has higher abundances of Mn and Eu relative to Cr and Fe than the metal-weak stars whose composition reflects that of the material ejected by the first generations of stars. It seems reasonable to suppose that the material enriching the interstellar medium originated in a variety of sources, two of which were the supernovae producing peculiar A stars of the Mn and the main groups. Supernovae producing peculiar A stars cannot be the only sources, because the observed abundances of Mn and Eu relative to Cr and Fe in the metal-rich stars would then be much greater. A third source might be the primaries of metallic-line stars, if a mass transfer process is involved in the formation of metallic-line stars. The chemical composition of metallic-line stars is still a matter of considerable debate (Hack, 1965), but the abundances of most elements relative to Cr and Fe seem to be roughly the same as those in metal-rich G-type dwarfs. Other sources supplying the interstellar medium might be type I supernovae, novae, red giants, Wolf-Rayet stars, planetary nebulae, and contact binaries; some of

Table 1

ENRICHMENT FACTORS

		Wallerstein (1962)			Helfer (1963)	
Element	Main Process	Mean metal-poor [X/H]	Mean metal-rich [X/H]	Enrichment factor	Enrichment factor	Mean Enrichment factor
Na	H burning	-0.56	+0.21	5.9	3.5	6.0
Mg	α	-0.31	+0.18	3.1	2.9	3.8
Si	α	-0.43	+0.20	4.3	3.0	4.5
Ca	α	-0.38	+0.20	3.8	2.7	3.9
Sc	s	-0.54	+0.26	6.3	3.3	5.9
Ti	α	-0.38	+0.20	3.8	2.5	3.7
V	e				2.7	4.0
Cr	e	-0.47	+0.22	4.9	3.1	4.9
Mn	e	-0.97	+0.15	13.2	7.0	14.0
Fe	e	-0.55	+0.25	6.3	3.8	6.5
Ni	e	-0.54	+0.24	6.0	2.9	5.3
Zn	s				5.5	11.1
Sr	s				4.4	8.3
Y	s				4.6	8.8
Zr	s				3.2	5.3
Ba	s	-0.62	+0.38	10.0	4.2	8.9
La	s				3.0	4.8
Ce	s				4.3	8.0
Nd	s				3.3	5.5
Sm	r, s				2.0	2.2
Eu	r				5.8	11.8

these sources may supply material which has been partially processed by nuclear reactions in their interiors. The relative importance of the various sources depends on the position in the galaxy and changes during the galactic history.

The foregoing discussion has a bearing on the problem of the origin of the elements in the solar system, since the Sun is a G-type dwarf which was formed about 5×10^9 years ago (Clayton, 1964) and is moderately rich in metals. Attempts have been made to deduce the physical conditions (temperature and density) under which the e-process elements in the solar system formed from the observed abundances of e-process isotopes (Burbidge and others, 1957). However, it appears that the e-process elements in the solar system originated in a variety of sources. Among these sources are the first generations of stars in the galaxy and the supernovae producing Mn stars; their interior conditions during the formation of e-process elements are reflected in the relative abundances of e-process elements in metal-weak G-type dwarfs and Mn stars respectively. As the abundance of Mn relative to Cr and Fe is about 100 times smaller in metal-weak G-type dwarfs than in Mn stars, the range of physical conditions under which the e-process isotopes in the solar system were formed is probably much greater than commonly realised.

Since the abundances of Mn and Eu relative to Cr and Fe in metal-weak G-type dwarfs are low, supernovae of the type producing peculiar A stars must have been less frequent in the early history of the galaxy.

It is probably not possible to investigate this for our own galaxy, but some other galaxies may still be in their early history. It is interesting to note that type II supernovae have been found only in spiral galaxies (Minkowski, 1964); our galaxy is now spiral, but its appearance was probably different in its early history.

Binary and variable peculiar A stars

In many cases it is difficult to decide, on the basis of nuclear properties alone, whether the abundance excess of a particular element in a peculiar A star is due to surface nuclear reactions or to material transferred from the primary. Observations of peculiar A binaries and spectrum variables are now discussed to see if they can throw any light on this problem and to find out whether their existence is consistent with the supernova theory.

With the aid of the bibliographies on peculiar A stars by Bertaud (1965) and Ledoux and Renson (1966), a list of spectroscopic binaries with known orbital periods was compiled (Table 2). The periods range from 3.0 to 3834 days.

Table 2

PECULIAR A SPECTROSCOPIC BINARIES

HD	Name	Type	Period	Eccentricity	
358	α And	Mn	97 days	0.5	
8441		Sr	106	$\neq 0$	
15144	HR 710	Sr	3.0	~ 0	
25267	τ^9 Eri	Si-4200	6.0	0.1	
27376	41 Eri	Mn	5.0		double-line (Searle and Sargent, 1965)
78316	κ Cnc	Mn	6.4	0.1	
89822	HR4072	Si-Sr	11.6	0.4	double-line (Chapter 7)
98088	HR4369	Eu-Cr, Sr	5.9	0.2	double-line (Abt, 1953)
125248	HR5355	Eu-Cr	1618		
137909	β Crb	Eu-Cr, Sr	3834	0.4	
174933	112Her	Mn	6.4	0.1	double-line (Aller, 1966)

Since it is uncertain whether all red giants with large masses expand to radii of about 8 a.u. before exploding and whether the explosions are isotropic, there is a possibility that the unobserved fainter components of the binaries with long periods of several years were initially primaries and are now white dwarfs. However, according to the supernova theory, the short period binaries must have been members of triple systems in which the most massive component (the primary) exploded and transferred material to the binary secondary star which is now observed. Thus the fainter components of the secondaries are still main sequence stars, and their radii are sufficiently large to produce observable eclipses in the case of a suitably orientated orbit. No peculiar A eclipsing binaries have yet been discovered, as the probability of obtaining a binary with a suitably orientated orbit is low and the number of known peculiar A binaries is still small. However, the main sequence character of the fainter components of the secondaries is also suggested by the existence of double-line binaries.

In order to form a binary peculiar A star, the original triple system must be dynamically stable during the evolution of the primary to the supernova stage. This means that the distance between the two components of the secondary before the supernova explosion has to be much less than the distance between the primary and the secondary. An interesting triple system is HD 157978-9 (McLaughlin, 1962); its dimensions are roughly similar to those required for the formation of a binary peculiar A star. Its brightest component is a G-type giant, and the other two stars are of spectral type A0. The orbital period of the A-type pair is about 3.76 days, while the G-type star revolves about the centre of mass of the system in 1170 days. Since the G-type star has a mass of about 5 $M_{\odot}$ and has reached the giant stage, the system has been dynamically stable for about 10^8 years. Thus the formation of peculiar A binaries with periods of several days is possible from the point of view of dynamical stability. A few peculiar A binaries (α And, HD 8441) have, however, periods of about 100 days. Their periods may have been increased to their present values during the supernova explosions of their primaries. It is difficult to estimate whether substantial increases in period have occurred, because the character of the explosions is not known in detail. For example, it is not known whether the assumption that the explosions are isotropic is valid.

Some peculiar A stars may retain their primaries as faint white dwarfs in enlarged orbits and might be observed as visual binaries. Two examples may be HD 98088 and γ Equ. HD 98088 = ADS 8115 is a double-line spectroscopic binary with a visual companion of absolute magnitude $M_v \sim +8$ at 1". The Eu-Cr-Sr star γ Equ = ADS 14702 is not a spectroscopic binary but has a faint visual companion ($M_v \sim +8$) at 2".

The double-line binaries provide a means of testing the idea that secondaries previously acquired processed material from their exploding primaries, because it would be expected that abundance anomalies due to the transfer of material would be present in both components of the secondaries. Accordingly, the relative abundances of the iron-peak elements Cr, Mn, and Fe should be about the same in both components. Both components of the double-line binary 41 Eri are Mn stars, and both have the uncommon feature that Fe as well as Mn is overabundant relative to Cr (Searle and Sargent, 1965). The relative abundances of Cr and Fe are similar in the two components of HR 4072 (Chapter 7).

The secondary spectra of the double-line binaries HD 98088 and 112 Her are too weak for accurate measurement. No r-process rare-earth elements have been observed in 41 Eri or HR 4072.

A search was made for elements which are overabundant in only one of the two components of HR 4072, because the overabundance of such elements would be due to surface nuclear reactions rather than to material transferred from the primary. The results of the previous differential curve-of-growth analysis of HR 4072 are summarised in Table 3, where the values of $\Delta'\log N = \log N_{\mathrm{HR\ 4072}} - \log N_{\alpha\,\mathrm{Lyr}}$ are given for each element. The SiII lines at 4128 and 4131A could not be found in the fainter component, and an approximate upper limit for the value of $\Delta'\log N$ for Si in this component was derived assuming that these lines have equivalent widths less than 15 mA. It will be observed that the large overabundances of Sr and Y are common to both components. The large overabundance of Si, however, is found only in the brighter component, and is therefore probably produced by surface nuclear reactions on this component. This conclusion is supported by the fact that overabundances of Si are more common in the hotter sub-groups of the main group (Searle and Sargent, 1964), whereas overabundances of the r-process rare-earths, which are due to mass transfer from the primary, occur in many stars of all sub-groups of the main group. The nuclear reactions occurring on the surface of the secondary are much more likely to be related to the surface temperature of the secondary than the nuclear reactions occurring in the interior of the primary.

Table 3

OVERABUNDANCES IN HR 4072

Element	Mg	Si	Ti	Cr	Mn	Fe	Sr	Y	Zr
Δ'log N (brighter component)	+0.2	+1.3	+0.4	+0.5	+0.6	+0.3	+2.0	+2.3	+0.3
Δ'log N (fainter component)		<+0.3	+0.3	+0.6		+0.5	+1.3	+1.4	

The conclusion of the review of the theories of peculiar A variable stars was that the oblique rotator theory would explain most of the observations, provided that complex surface distributions of abundance anomalies can be formed (Chapter 8). Since the strengths of lines of the r-process rare-earth elements Eu and Gd vary in many of the periodic spectrum spectrum variables, it might be supposed that material enriched in these elements was transferred from the exploding primary during part of the rotational period of the secondary and that a patch of material enriched in Eu and Gd was formed on the surface of the secondary. The patch might be prevented from spreading over the whole surface by the magnetic field. However, this idea encounters serious difficulties. If substantial accretion of additional material from the primary takes place for a few months after the supernova explosion, the distribution of Eu and Gd on the secondary will become much more uniform. Moreover, since about 10% of peculiar A stars are spectrum variables and the lifetime of a star of the main group is 10^8 to 10^9 years, the patches would have to remain for 10^7 to 10^8 years. This in turn implies that the distribution of the magnetic field of the secondary remains constant for a similar length of time. Magnetic fields are not found in normal sharp-line A-type stars (Babcock, 1958b), and the initiation of magnetic fields in peculiar A stars is therefore due to their possession of primaries until the supernova explosions. Since magnetic fields are not generally associated with binaries, the initiation of magnetic fields in peculiar A stars probably occurs on the arrival of high-velocity charged particles from the supernova explosions of the primaries. After the supernova explosion, the primary separates from the secondary, and in the absence of other effects, the magnetic field of the secondary will decay. The decay time for material at rest is $\tau = 4\pi\sigma l^2$, where σ is the electrical conductivity and l is the distance over which the magnetic field varies appreciably (Cowling, 1953). The magnetic field of the secondary should be limited to its surface, because it is probably due to the arrival of charged particles from the primary and because, if the observed field of 34,400 gauss in the Si star HD 215441 extends throughout the star, the magnetic energy would be at least 10^{41} ergs (Babcock, 1960). Thus σ is of the order of 3×10^{-8} emu at the surface (Cowling, 1953), and l is of the order of 10^{10} cm. Hence the decay time τ is of the order of 10^{6} years. The actual decay time will be much shorter, because the atmospheric material is not at rest. Microturbulent velocities of several km/sec are found in members of the main group (Searle, Lungershausen and Sargent, 1966), and the irregular fluctuations in the observed magnetic fields in some stars indicate magnetohydrodynamic activity (Babcock, 1958b). Thus the decay time will be several orders of magnitude too short for the maintenance of patches enriched in Eu and Gd for 10^7 to 10^8 years. Patches of Si-rich material formed by surface nuclear reactions on the secondary at the time of the supernova explosion of the primary will also disappear rapidly. Since the strengths of Si lines are frequently found to be variable, it must be supposed that new patches form when there is an irregular distribution of surface nuclear reactions during the remainder of the main sequence life-time of the secondary. Excesses of Si are not found in normal A-type stars which have no magnetic fields. It seems, therefore, that the supernova explosion of the primary initiates a magnetic field and surface nuclear reactions on the secondary, and that the magnetic field is maintained during the continuation of surface nuclear reactions during the remainder of its main sequence life-time. The distribution of the magnetic field and the patches probably varies on a short time scale ($\sim 10^3$ years perhaps). This still does not solve the problem of variable lines of Eu and Gd, because patches enriched in Eu and Gd cannot form by surface nuclear reactions.

Babcock (1963) proposed that irradiation by polarised light may preferentially align the atomic magnetic moments parallel or antiparallel to the magnetic field and that, if a large magnetic field gradient exists, atoms with large magnetic moments may migrate over the surface of the star. (Thiessen (1962) reported that the degree and direction of the polarisation of the Eu-Cr-Sr star HD 71866 vary with half the period of magnetic variation; various explanations, including synchrotron radiation, have been suggested (Steinitz, 1964)). Babcock's proposal would favour migration of Cr, Mn and Eu (but not Si). Atoms of Gd and other rare-earth elements also have large magnetic moments, but their flip resistance to disorientation is less than in the case of Eu. The migration theory is consistent with the observed variations in line intensities in α^2 CVn. The Eu lines show the largest variations, but fairly large variations in the lines of Cr and Gd are also found (Struve and Swings, 1943); although Si is overabundant by a factor of about 10 (Searle and Sargent, 1964), the SiII 4128 line remains almost constant. The magnetic field gradients may be smaller in Mn stars, since no outstanding variations in the intensities of Cr and Mn lines have been observed in Mn stars. Thus the complex surface distribution of abundance anomalies required by the oblique rotator theory is due to an irregular distribution of surface nuclear reactions together with ion migration, and may be accounted for on the basis of the supernova theory.

Since Si is one of the more abundant elements in normal stars (Si/H $\sim$ 1000 by mass), the excess of Si due to surface nuclear reactions in Si stars should be accompanied by deficiencies of other light elements which are even more abundant in normal stars (He, C, N, O, and Ne); large deficiencies of O have been established, and deficiencies of He and C are suspected in Si stars (Searle and Sargent, 1964). Variable stars may assist in the identification of the surface nuclear reactions, because light atoms have small magnetic moments and will not suffer migration to any great extent. Thus the abundances of the elements from which patches enriched in Si were formed should vary out of phase with the abundance of Si. In the variable Si stars 56 Ari and HD 124224, the strength of the HeI 4472 line varies out of phase with the SiII lines (Peterson, 1966), but it is very difficult to determine the abundance of He from strong HeI lines which are broadened by the Stark effect and are sensitive to changes in temperature. The variation of lines of other light elements has not yet been investigated.

The frequency of peculiar A stars

As a final check on the supernova theory, the frequency of peculiar A stars is discussed in order to decide whether the number of peculiar A stars predicted by the theory is of the same order as the number observed. For stars with initial spectral types earlier than B8, the mass loss during the giant phase is insufficient to bring the mass below the Chandrasekkar limit, and the star must explode if it is finally to become degenerate (Stothers, 1963). The numbers of stars per 1000 pc^3 of various spectral groups in the solar region were determined by Nassau and MacRae (1949), and the total number of O to B7 stars in the galaxy is about 10^6 (Stothers, 1963). Table 4 was compiled assuming that the relative numbers of stars of the various spectral groups is the same in the whole galaxy as in the solar region and that the relative numbers remain constant. The mean main sequence life-time τ_{ms} for each spectral group was estimated from the discussion of the turn-off points in colour-magnitude diagrams of galactic clusters given by von Hoerner (1957). The number of stars forming or exploding per year was found by dividing the observed number of stars by the main sequence life-time.

Table 4

FREQUENCIES OF VARIOUS GROUPS OF STARS

Spectral group	Mean τ_{ms}	No./1000 pc^3 in solar region	No. forming per year per 1000 pc^3 in solar region	No. in galaxy	No. explodin per year in galaxy
O to B3	10^7 years	5×10^{-3}	5×10^{-10}	2.5×10^5	2.5×10^{-2}
B4 to B7	10^8	1.5×10^{-2}	1.5×10^{-10}	7.5×10^5	7.5×10^{-3}
B8 to A0	2.3×10^8	1.7×10^{-1}	8×10^{-10}		
A1 to A5	7×10^8	5×10^{-1}	7×10^{-10}		

The frequency of type II supernovae in a galaxy (estimated by Minkowski (1964) to be about one per 60 years) is of the same order as the estimated frequency of explosions of O to B7 stars in our galaxy (about one per 30 years according to Table 4); this confirms Stothers' estimate of the spectral range of main sequence stars which eventually explode as type II supernovae.

In the solar region, the number of O to B7 stars exploding per year per 1000 pc^3 is about 6.5×10^{-10}. As the number of B8 to A5 stars forming per year per 1000 pc^3 is about 15×10^{-10}, and as about 10% of these stars become peculiar A stars during most of their main sequence life-times, the number of peculiar A stars forming per year per 1000 pc^3 is of the order of 2×10^{-10}. Thus, according to the supernova theory of the origin of peculiar A stars, about 30% of the O to B7 stars in the solar region have to be binaries with suitable orbital dimensions and mass ratios for the formation of peculiar A stars. Unfortunately, the detection of such binaries is difficult, as their mean separations probably lie in the range 10 to 100 a.u. They are too close to be detected as visual binaries, and their detection as spectroscopic binaries is not easy owing to their long periods and the small amplitudes of their radial velocity curves. There is, however, some <u>indirect</u> evidence that a large percentage of O to B7 stars are binaries which are suitable for the formation of peculiar A stars. Blaauw (1961) estimated that about 75% of O to B5 stars are binary or multiple; both visual and spectroscopic binaries are common in this spectral group. The average mass ratio M_2/M_1 for single-line spectroscopic binaries is about 0.62 for O to B2 stars and about 0.27 for B3 to B7 stars (Beer, 1956); these mass ratios are suitable for the formation of peculiar A stars. Kuiper (1935) estimated that about 10% of <u>all</u> stars brighter than absolute magnitude +6.5 are binaries with separations between 10 and 100 a.u. and magnitude differences less than 4^m. Thus the supernova theory of the origin of peculiar A stars is in agreement with the estimated frequency of type II supernovae and does not conflict with the estimated binary characteristics of O to B7 stars.

Sufficient information is available to permit a preliminary discussion of the frequency of peculiar A stars in galactic clusters. Table 5 lists the numbers of stars of spectral types B5 to A3 and luminosity classes IV and V on the MK system in five clusters. The numbers include peculiar A stars having similar intrinsic colours, and these peculiar A stars are named in the third column. Membership of the clusters was determined according to the normal criteria of positions, radial velocities, proper motions, magnitudes and colours. Only 3 peculiar A members have been found in a total of 131 B5 to A3 members. The detection of peculiar A stars is probably complete, because the classifications were made from spectra of moderately high dispersion. Other clusters do not have members with accurate spectral classifications in the whole of the range B5 to A3. Jaschek and Jaschek (1962) estimated that about 11% of all stars with Henry Draper spectral types B8 to A5 were peculiar A stars. Thus the percentage of peculiar A stars in clusters (about 2.3%) seems to be much lower than that among field stars.

Table 5

PECULIAR A STARS IN CLUSTERS

Cluster	Total no. of B8 to A5 members	Peculiar A members	Nearby peculiar A stars	Reference
α Persei	44	HD 22401	HD 14392	Kraft (1967)
Pleiades	29	none	41Tau, 53Tau, 56Tau	Abt and others (1965)
IC 2391	15	HD 74535	HD 73340	Buscombe (1965)
IC 2602	17	HD 92664	none	Whiteoak (1961)
IC 4665	26	none	none	Abt and Chaffee (1967
Total	131	3	5	

On the supernova theory, two explanations for the low percentage of peculiar A stars in clusters may be offered. Firstly, in the cases of complete separation of the binary components after the explosions of the primaries, the velocities of ejection of the secondaries (∼10 km/sec) are much larger than the velocities of escape from clusters (∼1 km/sec), and the clusters where some peculiar A stars were formed more than about 5 x 10^6 years ago are now difficult to identify. Some peculiar A stars lying near clusters (Jaschek and Jaschek, 1962) are listed in Table 5; these may have been ejected from the clusters less than 5 x 10^6 years ago. A second explanation of the low percentage of peculiar A stars in clusters may be that the clusters had no binaries with low mass ratios M_2/M_1 and that peculiar A stars at the cooler end of the range B5 to A3 have yet to form. On average, peculiar A stars are about 2 magnitudes fainter than the brightest main sequence star in the cluster, whereas for the clusters in Table 5, the A3V stars are 4 or 5 magnitudes fainter than the brightest main sequence star.

Concluding remarks

The foregoing discussion of a modified form of van den Heuvel's supernova theory of the origin of peculiar A stars has not revealed any fundamental objection to the theory. Although many details remain uncertain, the theory is at least consistent with a very wide range of observations of peculiar A stars. It explains why abundance abnormalities indicating late stages of evolution are found in the main sequence peculiar A group. Abundance deficiencies as well as excesses are accounted for. A plausible explanation can be given for some of the distinctive properties of the Mn stars, although it is perhaps too early yet to say whether Mn stars are the only peculiar stars that form as a result of explosions of the primaries after expansion beyond their Roche limits. The binary and variable characteristics of peculiar A stars are also explained in general terms. The theory has interesting implications for other branches of astrophysics, and further investigation of its various aspects would be desirable.

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