Minimal requirements for the cultural evolution of language

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To Özge and Mina, of course.

I confirm that I am the author of this thesis, and all work is my own except where indicated. The work has not been submitted for any other degree or professional qualification, and the included publication is my own work, except where indicated in the preface to that chapter.

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First up, the academic content of this thesis: I owe a massive debt to my supervisors Simon Kirby and Kenny Smith. Towards the end of my undergraduate degree, I fixated on two ideas. The first was that there must be some kind of evolutionary explanation for human language, and the second that we could use computer simulations of the emergence of language to inform linguistic theory. I put these ideas to a number of people in and around the University of London. The consensus was that maybe such a thing was possible, but there was nowhere in London to do it. Finally, one person (Lutz Marten, thanks!) suggested that I should take a look at the work coming out of the LEC group in Edinburgh. I read Simon's thesis while commuting to work over the following week and applied for a place at Edinburgh the next day. The MSc in Evolution of Language and Cognition run by Simon and Kenny took me from a position of near total ignorance to embarking on my PhD. As supervisors, Simon and Kenny allowed me pretty much free rein while providing an enormous amount of intellectual guidance and advice, in particular during our weekly group supervisions. Without their help and continued support, this thesis would not exist. So, huge thanks!

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Abstract

Human language is both a cognitive and a cultural phenomenon. Any evolutionary account of language, then, must address both biological and cultural evolution. In this thesis, I give a mainly cultural evolutionary answer to two main questions: firstly, how do working systems of learned communication arise in populations in the absence of external or internal guidance? Secondly, how do those communication systems take on the fundamental structural properties found in human languages, i.e. systematicity at both a meaningless and meaningful level? A large, multi-disciplinary literature exists for each question, full of apparently conflicting results and analyses. My aim in this thesis is to survey this work, so as to find any commonalities and bring this together in order to provide a *minimal account* of the cultural evolution of language.

The first chapter of this thesis takes a number of well-established models of the emergence of signalling systems. These are taken from several different fields: evolutionary linguistics, evolutionary game theory, philosophy, artificial life, and cognitive science. By using a common framework to directly compare these models, I show that three underlying commonalities determine the ability of any population of agents to reliably develop optimal signalling. The three requirements are that i) agents can create and transfer referential information, ii) there is a systemic bias against ambiguity, and iii) some mechanism leading to information loss exists.

Following this, I extend the model to determine the effects of including referential uncertainty. I show that, for the group of models to which this applies, this places certain extra restrictions on the three requirements stated above.

In the next chapter, I use an information-theoretic framework to construct a novel analysis of signalling games in general, and rephrase the three requirements in more formal terms. I then show that we can use these 3 criteria as a diagnostic

for determining whether any given signalling game will lead to optimal signalling, without the requirement for repeated simulations.

In the final, much longer, chapter, I address the topic of duality of patterning. This involves a lengthy review of the literature on duality of patterning, combinatoriality, and compositionality. I then argue that both levels of systematicity can be seen as a functional adaptation which maintains communicative accuracy in the face of noisy processes at different levels of analysis. I support this with results from a new, minimally-specified model, which also clarifies and informs a number of long-fought debates within the field.

Contents

Chapter 1

Introduction

This thesis consists of four chapters, two main topics, and one overarching theme: the cultural evolution of language. This is, of course, a vast area of ongoing research, and in this thesis I address only a small subset of the many pressing questions in the field. Nevertheless, the two main topics are (I think) central and important ones, namely: 1) In groups of agents capable of learned signalling, how do shared, functional signalling systems *self-organise*? 2) How do signalling systems take on the fundamental aspects of combinatorial and compositional structure found in human languages, i.e. *duality of patterning*?

Both of these topics are well-established. They have been the subject of many studies in several disciplines; they have used a variety of methodologies with multiple different theoretical underpinnings, and have attained results and drawn conclusions which sometimes appear to be completely at odds with each other. This underlines my main motivation in preparing this thesis: to investigate how much these seemingly contradictory accounts actually share in terms of both results and theory: to work out a *minimal account* for the cultural evolution of language.

The structure of this thesis is as follows: the first three chapters look at different aspects of the emergence of learned signalling, and the fourth (much longer) chapter is devoted to duality of patterning. In the first chapter, I directly compare and contrast a number of well-established computational models of signalling. I use a minimal framework which is capable of replicating all these models, and investigate the effects of incrementally adding mechanisms to discern their effects. My main finding here is that only three properties are required to ensure the development of optimal signalling: the creation and transfer of referential information, a systemic bias against ambiguity, and a mechanism leading to information loss. This chapter is presented in the form of the final edit of a journal article in Cognitive Science. The second chapter is an extension of the work in the first, where I investigate the effects of introducing referential uncertainty to the previously investigated models, which has implications for the three requirements for signalling. The third chapter draws this work together in the language of information theory. I show that when we use the correct information-theoretic description of any learned referential signalling model, we can re-cast the three requirements as a set of formal requirements, and that we can use these to diagnose whether any given model will reliably converge on a signalling system.

In the final chapter, I turn to duality of patterning. This commences with an extended review of the literature concerning the nature and evolution of duality of patterning and its component parts, combinatorial and compositional structure. I then argue that we can provide a simple, unified account of the emergence of duality which appeals to the role of functional adaptation for communicative success in the face of noisy processes at different levels of analysis. I present a simple, minimally-specified model as a proof-of-concept for these arguments, and show how this more abstract account can unify and dispel a number of apparent controversies found in various parts of the literature.

My main methodology is computational modelling, but I reference theoretical, comparative, and both field-based and lab-based empirical work throughout. I use basic mathematical notation in parts, but nothing in the way of formal proofs, and my intention is for any formalism to increase rather than decrease clarity.

Chapter 2

Minimal requirements for the emergence of learned signalling

The following is an exact reproduction of an article as accepted for publication in Cognitive Science. It represents my original work, as supervised by Simon Kirby and Kenny Smith, and discussed with Kevin Stadler. Simon Kirby, Kenny Smith, and Kevin Stadler all assisted in the preparation and editing of the article, but I take full responsibility for any and all mistakes.

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Minimal Requirements for the Emergence of Learned Signaling

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Abstract

The emergence of signaling systems has been observed in numerous experimental and real-world contexts, but there is no consensus on which (if any) shared mechanisms underlie such phenomena. A number of explanatory mechanisms have been proposed within several disciplines, all of which have been instantiated as credible working models. However, they are usually framed as being mutually incompatible. Using an exemplar-based framework, we replicate these models in a minimal configuration which allows us to directly compare them. This reveals that the development of optimal signaling is driven by similar mechanisms in each model, which leads us to propose three requirements for the emergence of conventional signaling. These are the creation and transmission of referential information, a systemic bias against ambiguity, and finally some form of information loss. Considering this, we then discuss some implications for theoretical and experimental approaches to the emergence of learned communication.

Keywords: Communication; Cultural evolution; Signaling games; Reinforcement learning; Feedback learning; Observational learning; Agent-based models; Exemplar theory

1. Introduction

Human language provides a uniquely flexible and expressive system of communication, but we are not the only species capable of communication. Signaling behavior is found

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throughout nature: Virtually every species has a means of communicating about, for example, the presence of food or predators, potential as a mate, or presence of a competitor. However, only human communication displays such *open-endedness* (Tomasello, 2010) in the number of signals learned, the contexts in which they are elicited, and the responses they effect. This flexibility arises because the basic building blocks of human language—words—are learned socially, by observing word use by others. In contrast, the articulatory form of signals in the vast majority of animal communication systems are, as far as we know, not socially learned. For instance, among our closest relatives, the form of alarm calls (distinctive calls used to warn conspecifics of the presence of particular types of predators) are thought to be largely genetically determined (Fedurek & Slocombe, 2011); even among the other apes, while the decision to employ a given call may be intentional (Slocombe et al., 2010), there is only limited evidence for any flexibility or group-level variation in the form of those calls (Crockford, Herbinger, Vigilant, & Boesch, 2004; Seyfarth & Cheney, 1986). There are of course obvious exceptions; for instance, many bird species are capable of both learning and innovating songs (Podos, Huber, & Taft, 2004). However, vocal learning in birds (and in other animals where it has been observed, such as cetaceans, elephants, and bats: Janik, 2014; Poole, Tyack, Stoeger-Horwath, & Watwood, 2005; Boughman, 1998) is most probably a case of convergent evolution, rather than reflecting some ancestral cognitive capacities shared by the extremely distant common ancestor of all vocal learning species.

Innately specified communication systems are presumably the product of natural selection (Maynard Smith & Harper, 2003). As such, the major questions concern the nature of the evolutionary route to signaling, as well as the selective pressures involved. The emergence of *learned* communication, on the other hand, is less well understood. First, we might ask when and why a learned system would replace an innately specified signaling system (Lachlan, Janik, & Slater, 2004; Ritchie & Kirby, 2006). Second, socially learned communication systems are potentially shaped by an entirely different set of pressures. In a learned communication system, unlike its innate equivalent, natural selection cannot directly tune the structure of the signaling system; rather, socially learned signaling systems are shaped by the processes through which they are learned and used (see literature review in next section). Their functional properties are then determined by the nature of the learning and usage mechanisms involved (which are themselves potential targets for biological evolution). Understanding the nature of these mechanisms is crucial to understanding when and how a learned communication system such as human language might evolve.

Our focus in this study is therefore: What are the necessary social and psychological adaptations which allow populations to develop, via processes of learning and use, functional learned communication systems? Semiotic experiments (such as surveyed by Galantucci & Garrod, 2011) demonstrate that human subjects can rapidly bootstrap communicative conventions across a range of modalities and interactive conditions. Moving beyond the laboratory, the recent emergence of indigenous sign languages (e.g., Nicaraguan Sign Language and Al-Sayyid Bedouin Sign Language: Senghas, Senghas, & Pyers, 2005; Sandler, Meir, Padden, & Aronoff, 2005) is a compelling reminder that functional communication systems are able to *self-organize* in human populations in the absence of any explicit, centralized coordination. Presumably, the same mechanisms also underlie the development of other human signaling conventions, including all other human languages. Identifying these mechanisms—which must be particular to human cognition and interaction—will shed light on what enables *Homo sapiens* to be such a fundamentally communicative species.

The emergence of functional learned communication has been studied across a number of seemingly loosely related disciplines, including Classical and Evolutionary Game Theory (e.g., Lewis, 1969; Nowak, Krakauer, & Kingdom, 1999; Skyrms, 2010), Artificial Life (e.g., Steels & Loetzsch, 2012), Cognitive Science (e.g., Barr, 2004), and Evolutionary Linguistics (e.g., Oliphant, 1996; Smith, 2002). The assumptions made and conclusions drawn in these various fields regarding the prerequisites for functional communication appear on the surface to be quite different, if not mutually incompatible. To cut through a rather confusing mesh of approaches, models, and results, we have created a framework to replicate a representative selection of the approaches outlined above. Having done this, we then identify a *basic framework*—an urn-model—which strips the individual models back to the simplest set of common underlying mechanics.

We then employ an *additive* approach to this framework: We first add the characteristic features of each model in terms of *interaction* and *learning*. None of these basic instantiations reliably lead to optimality; as such, we then investigate which particular *mechanisms* are responsible for doing so. By adding each mechanism in isolation, we are able to investigate exactly which are responsible for driving the behavior of each model. The subsequent direct comparison reveals that the apparent diversity of mechanisms driving the emergence of functional learned communication is overstated in the literature; in fact, the same fundamental processes underpin *all* of the current accounts.

The rest of this study is organized as follows. In Section 2 we discuss the issues of conventionality and optimality, and how these are tackled in models drawn from the various disciplines mentioned above. We first motivate and then describe the exemplar-style framework which we have used for our model replications in Section 3, before discussing each replication in more detail in Section 4, along with the adjustments we have made for comparison, and which aspects of each model are necessary for the development of optimal communication. In Section 5 we propose that three fundamental principles—the creation and propagation of referential information, a bias against ambiguity, and a mechanism leading to information loss—determine whether any system is able to bootstrap functional communication. Finally, this leads into a discussion in Section 6 regarding how this can help us interpret the various theories of the emergence of communication outlined above.

2. Past approaches to the emergence of functional learned signaling

For any form of communication to be functional, it must be *conventional*; in particular, there must be consensus within a population about how signals are produced and interpreted. Conventions are widespread in human populations and extend far beyond the communicative domain (we have, for instance, conventions about what to wear to work, what side of the road to drive on to get to work, what times one should work at, and appropriate language at work).

In his classic study, Lewis (1969) analyzes convention as a type of game-theoretic *coordination problem:* Two or more agents have a choice of behaviors, and coordinating those behaviors leads to mutual benefit. Even working under the assumption that such coordination provides a mutual benefit, the mechanisms leading to the establishment of conventions are not immediately obvious. Lewis proposed a critical role for *common knowledge* (Lewis, 1969; p. 56): All agents are aware of a set of propositions, each agent knows that every other agent also knows those propositions, and so on recursively *ad infinitum*. Populations of agents can employ this knowledge to create conventions by making rational choices targeting maximal individual payoffs. However, in the case of these *simple* conventions (where an atomic choice is made from an unordered set of alternative behaviors, such as the side of the road we drive on), Vylder (2008) shows that such sophisticated reasoning is unnecessary: Whenever agents *strongly amplify* observed behavior, population-wide agreement on a single convention is assured. (This is where agents sample from each other's behavior, and the suite of behaviors is represented as a ranked probability distribution. In strongly amplified copying, the ratio of likelihoods between any two subsequently ranked behaviors is strictly increased in favor of the more highly ranked one.)[1]

However, being conventional is not enough to ensure a functional communication system. Lewis's Signaling Game (1969) is the canonical problem in the emergence of learned communication. In its most basic form, the Signaling Game involves a single signaler and a single hearer. The signaler must communicate one of two possible world-states to the hearer with two available signals, and the hearer has a choice of two possible responses. Each world-state has a corresponding "matched" response which triggers a mutual payoff; mismatched responses provide no payoff. Lewis showed that even this simple game has several *Nash equilibria*, where a Nash equilibrium is any state where the best payoff for any given player is to continue with her current strategy, leading to a global stasis where no further change in play can occur. Only two of these Nash equilibria are *optimal* strategies, designated by Lewis as signaling systems, which guarantee that the hearer will select the appropriate response based only on the signaler's signal. The others—*pooling equilibria*—are stable but non-optimal strategies; for example, if the signaler sends the same uninformative signal for every world-state and the receiver always chooses the action with the greatest average payoff. This ambiguity of signal-to-response mapping will be referred to below as *homonymy*.

Moving beyond the simplest scenario of two world-states, two signals, and two responses results in a drastic increase in the number of possible system states, and the chance of multiple *partial* pooling equilibria: These are stable states that are a mixture of informative strategies and pooled, non-informative ones (see Fig. 1). As such, in addition to being conventional, a functional communication system must be at least somewhat informative; in Lewis's terms, it must allow the hearer to select the correct response with

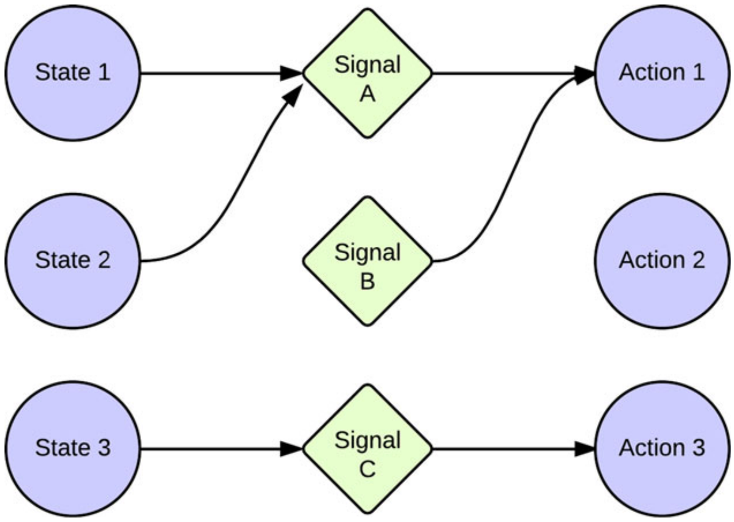

Fig. 1. An example of a *partial pooling equilibrium*. The first two states, signals and actions, are *pooled* and mutually uninformative: The speaker always produces the Signal A and the receiver always uses Action 1. State 3, however, leads to the informative Signal C and hence Action 3.

greater than chance frequency. The most functional systems are optimal; in Lewis's terms, such systems require that all world-states must map to at least one signal, and each of those signals must be unambiguously associated with a matched response. Identifying how conventional, optimal, learned signaling systems develop has therefore become the benchmark problem in this field; the existence of many non-optimal stable states in the face of Lewis's rational behavior suggests that the reliable development of optimal signaling occurs via some other means. In the sections that follow, we review the proposals made in various fields as to what that mechanism might be.

2.1. Payoff-based accounts

Game-theoretic accounts are driven by the idea of a *payoff*, which is instantiated through either increased evolutionary fitness (e.g., Nowak et al., 1999) or reinforcement learning (e.g., Skyrms, 2010), where individuals modify their behavior in response to the payoffs they receive.

Nowak et al.'s model involves the *natural selection of cultural variants* (Boyd & Richerson, 1985) and rests on two assumptions: First, the fitness of individuals (i.e., the number of their offspring) is determined by their communicative success within a population, and second, the resulting children learn from their parents (with some error), thereby inheriting their communication system via social learning. Numerical simulations show that, under these conditions, while some populations evolve optimal systems, many stabilize at partial pooling equilibria, some signals being associated with more than one meaning. However, these suboptimal states occur less as the chance of learning error increases: Error knocks the systems out of previously stable states. Note that the mechanism at play here—natural selection—is the same as that invoked to explain the evolution of signaling systems which are not socially learned, the only difference being that the mechanism through which behaviors are inherited is social learning, rather than genetic transmission.

Skyrms (2010) surveys a type of *reinforcement learning* devised by Roth and Erev (1995). Many forms of reinforcement learning (e.g., Bush & Mosteller, 1953) are essentially "memoryless": Each available behavior can be completely characterized by a single (probabilistic) value which describes its current state. The effect of any learning experience recalculates this value using a parameterized function. Roth–Erev reinforcement, on the other hand, models memory as a collection of tokens which gradually accumulate over a learner's lifetime; as such, calculating both behavior and the effects of learning must take into account not just the relative proportion of memory tokens, but their absolute count as well (experiences in later life contribute relatively little to the overall store of tokens, whereas early experiences have larger effects). Skyrms (2010) motivates the Roth–Erev model by showing that learners in a non-signaling scenario—one where they must learn to modify their behavior to maximize their expected returns when presented with an initially unknown distribution of world-states—are able to escape pooling equilibria by using Roth–Erev reinforcement learning, whereas parametric forms of reinforcement learning (Bush & Mosteller, 1953) are not. Incorporating this into the basic Lewis signaling game, Skyrms (2010) describes a simple iterative strategy: Each time a pair of agents successfully communicate, the associations involved are strengthened by both. A proof of convergence to optimality using Roth–Erev learning in the minimal signaling game (two world-states, signals and responses) is given in Beggs (2005). However, Barrett (2006) shows that including more states, signals, or responses immediately increases the possibility of non-optimal equilibria, and that simple reinforcement no longer leads to guaranteed convergence on optimal signaling. Barrett (2006) goes on to propose two solutions to this problem: the addition of negative reinforcement (also known as *punishment*, the term we shall use henceforth), where unsuccessful associations are decremented, and *forgetting* (which is further investigated in Barrett & Zollman, 2009). Both strategies greatly increase the likelihood that optimal signaling develops and guarantee it for certain parameter regimes.

Also using Roth–Erev reinforcement (but looking at pragmatic implicatures rather than signaling games), Franke and Jäger (2012) investigate the effects of *lateral inhibition:* After successful communication, competing associations are dampened. With this effect included, they show via simulation that optimal states are reached far more quickly. We draw attention to the role of lateral inhibition here, as it plays an important factor in several other models described below.

2.2. Interaction-based accounts

The program of artificial life research as set out in Steels (2012) and the neural network populations of Barr (2004) place a critical emphasis on the fundamental roles of *feedback* and *alignment*. Agents interact with each other multi-modally: Repeated attempts at local alignment ultimately lead to a globally functioning communication system.

As part of a larger program to investigate the evolution of language, Steels (2012) shows that multi-modal negotiations between embodied robotic agents situated in a complex environment lead to the development of multiple levels of language-like structures. The seminal *Naming Game* described in Steels and Loetzsch (2012) is a core element in this process. A population of agents is situated in an environment containing a number of objects. Pairs of randomly chosen agents are presented with a limited context of objects observable by both parties, with one particular target object chosen for a designated signaler to attempt to communicate to the hearer. The agents then execute a scripted series of actions. A signal is sent, the hearer's interpretation is checked, and the intended referent is indicated by the speaker in the event of failure. Both agents then potentially adjust their internal representations by strengthening and weakening associations, with these weight adjustments determined according to the particular scenario (success or failure) which has just occurred—these updates can be carried out by speaker, hearer, or both agents involved in the interaction. In addition, agents possess the ability to *innovate* terms for previously unseen objects, chosen from a very large signal space. This is a potentially critical difference with the other models discussed in this section, which assume limited signal spaces, and in which processes which eliminate homonymy are critical to establishing optimal signaling. In contrast, in a typical Naming Game simulation involves an initial stage in which the number of terms for any given object explodes, before a single term wins out for each, as the result of gradual *lateral inhibition* of competing terms. A consequence is that homonymy is very rare: "optimality" for these games tends to be defined not in terms of successful communication, but by when the lexicon is reduced to a minimal size. De Vylder and Tuyls (2006) show that, as shown with simple conventions in Vylder (2008), convergence on a minimal, unambiguous, conventional lexicon is guaranteed if agents utilize a *strongly amplifying* imitation function (as described in Section 2). Baronchelli (2010) further shows that hearer update (i.e., hearers updating their internal representations based on the success or failure of an interaction) is critical in the development of optimal lexicons, while speaker update plays a lesser role.

Barr (2004) looks at the role of common knowledge in the emergence of conventional communication. Employing populations of interacting agents (both neural network based and simpler association based), he showed that not only was common knowledge (about the signaling behavior of the population as a whole) unnecessary, but that population-wide convergence on a single system was significantly more likely when agents used *only* the information from individual interactions. The neural network model is rather sophisticated, but it includes a type of parametric reinforcement learning similar to Bush and Mosteller (1953) (outlined in Section 2.1). Also included is a form of lateral inhibition, (although described as a *mutual exclusivity bias*) which acts to promote one-to-one signal/meaning mappings. Barr's simulations reliably lead to states of optimal signaling: In a second set of results, Barr aims to counter a possible objection to his neural networks: as they sample over time from the whole population, they could be argued to be accruing a type of common knowledge. To this end, he uses a modified association-based model (based on Steels, 1997) which employs a "*stay/switch*" strategy—agents stick to successful strategies with some chance of switching to less successful ones. This model includes a type of memory where agents can be restricted to knowledge of their last n interactions.

In the end, both types of population, neural network and stay/switch, reliably arrived at global convergence on an optimal system.[2] A further observation was that stay/switch populations proved more efficient at developing globally optimal signaling when their memories were highly restricted, providing another strong counterexample to common knowledge-based explanations.

2.2.1. Reinforcement versus feedback learning: An aside

It is worth clarifying the differences between reinforcement and feedback accounts, as they actually share much in common. Another complicating point is that the models in Barr (2004) and, to a lesser extent, Steels and Loetzsch (2012) are described at different times in terms of both Reinforcement and Feedback learning.

One of the main factors distinguishing reinforcement (in its classic form) and feedback involves the availability of *referential information*. This describes how agents associate meanings with signals, both for when signals are sent and interpreted. In fact, in classic signaling games, referential information is irrelevant. Mutually available "meanings" are split into two: world-states perceivable by the speaker, and actions taken by the receiver. However, as every state has a single matched action which triggers a payoff event, we can (for the sake of direct comparison) temporarily overlook this distinction and see matched state/action pairs as directly equivalent to meanings in the other models. In any case, in reinforcement accounts the equivalent of referential information is only made available after a successful interaction, and it is provided by the *environment*: Signaler and receiver know that the intended and interpreted meaning have coincided, because they receive reinforcement from the environment; more subtly, in the event of failure the *absence* of positive reinforcement informs each party that his or her choice has been unsuccessful. In feedback learning, on the other hand, the environment cannot provide this information. Instead, the agents themselves must furnish it via "pointing" behavior: Simple social interactions, presumably via another modality, which are able to resolve reference. As such, although there is a near equivalent to the reinforcement described above, it is analyzed in terms of the interaction between the agents; the receiver must point at its interpreted referent, providing *Interpretation Feedback*, and the speaker must either indicate whether the receiver has selected the correct meaning (what we shall term *Yes/No Feedback*) or provide richer information by indicating its intended referent (henceforth *Referential Feedback*). With Yes/No Feedback, then, the situation resembles reinforcement learning in that full referential information is only made available after communicative success. The real difference between reinforcement learning and Yes/No Feedback learning is seen after failure. In Reinforcement learning, the speaker can only know that his or her intended signal/meaning association was unsuccessful. Similarly, the hearer is only aware that his or her interpreted association failed. This is also true for Referential Feedback, but due to the availability of Interpretation Feedback, extra information (about how the hearer interpreted the signal) is reliably available to the speaker. In Reinforcement learning this information is only available after successful communication. Referential Feedback plays a similar role, as it provides full information about the speaker's intended referent to the hearer; again, with reinforcement learning this is only available after success.

It is the availability of this extra information in feedback learning that allows for more subtle strategies than in reinforcement learning. With Reinforcement learning, agents must somehow promote successful associations and inhibit failed ones. This remains the case with Feedback learning, but speakers and hearers have reliable sources of referential meaning which are *independent* of communicative success. How this information is used, of course, depends on the particular model.

Interestingly, then, although Barr (2004) describes what must be a feedback model—in that alignment is verified through interaction—the interaction itself is placed in a black box. Because of this, the model uses an exact equivalent to the reinforcement dynamic: The extra information potentially available is not actually used. In models such as that of Steels and Loetzsch (2012), the interaction has a more fine-grained realization which is incorporated into the model, making the extra sources of information potentially usable. The question, then, is to determine what role those extra sources of information *do* play; this will be dealt with in Section 4.

2.3. Observational learning accounts

A third strand of work has focused on the evolution of communication via *iterated learning* (Kirby, 2001) — repeated cycles of production and observational learning, often but not always with population turnover. In generational turnover models, one generation of learners learns from behavior produced by the previous generation of learners and goes on to produce behavior which is observed and learned from by a subsequent generation of learners; alternatively, new agents acquire a signaling system by observing the existing population produce and/or interpret signals, then replace an older member of the population, implementing a gradual turnover of the population. This *observational learning* paradigm typically de-emphasizes the role of communicative interaction (see e.g., Oliphant, 1996; Smith, 2002): Agents are assumed to be unmodified by any further interaction after an initial phase of learning, and signaling conventions can therefore only develop during this initial stage of sampling and learning. For this reason, the models place a critical emphasis on the learning process itself. Furthermore, unlike the reinforcement and interaction-based models discussed above, these observational learning models typically do not include any referential uncertainty: Learners learn from observing meaning-signal pairs, rather than signals produced in some context which leaves its intended meaning unclear.

The models in Smith (2002) investigate how individual *learning biases* shape the evolution of signaling systems in populations through iterated learning. In this study, learners are modeled as simple associative networks, who adjust association weights after each learning exposure according to a particular learning rule: Smith varies these learning rules parametrically, to explore both the properties of learning at the individual level and the consequences of these individual-level processes for the signaling systems which develop in populations. Smith used three criteria to classify the effect of each learning rule: whether it produced agents capable of (a) *learning*, (b) *maintaining,* and (c) *constructing* optimal signaling systems; each criterion is a strict subset of the previous one. A property shared by all constructor-type rules is an implicit bias against homonymy in the form of lateral inhibition. Learners with such a bias are less likely to successfully learn homonymous meaning-signal mappings, and over many episodes of learning this bias eliminates homonymy entirely, leading to optimal signaling. Learning rules which are neutral to homonymy are sometimes capable of constructing functional signaling, but usually converge on suboptimal pooling equilibria. In contrast, biases against synonymy alone do not contribute toward the development of optimal systems, although they are required for the learning of optimal systems under certain assumptions about the relative size of the meaning and signal spaces (K. Smith, 2004).

Oliphant and Batali (1996) adopt an alternative approach within the observational learning framework, exploring a rational approach which they dub *obverter*. Their work starts from the observation that the rational approach to signaling is to maximize the chances of being correctly understood, while rational receivers will attempt to maximize the chance of correct interpretation. Obverter signalers leverage this fact by calculating which signal is most likely to be correctly interpreted as their intended meaning, based on the observed reception behavior of the population; similarly, obverter reception involves identifying which meaning is most likely to be signaled using the received signal, again based on observations of the population's production behavior. Oliphant and Batali first show that when agents have perfect information about the signaling behavior of the population (e.g., through unlimited observation of the production and reception behavior of that population), the communicative accuracy of a population will necessarily increase with every new generation of learners who apply the obverter approach to production and reception, eventually leading to convergence on an optimal system. Numerical simulations show that *approximating* this perfect knowledge, by estimating the population's signaling behavior from a limited number of observations of population behavior, is sufficient to guarantee optimal communication.

2.4. Summary

What conclusions can we draw from the above? First, there are clearly substantial differences between the various models (see Table 1 for a brief summary of their key features): What is striking is their heterogeneity, with each model having at least one unique feature, and no obvious universally shared property which might drive the evolution of functional signaling systems. Second, the models differ in the time-scales involved. Some accounts employ intergenerational learning and natural selection, and describe a process which takes place over multiple generations: Selective reproduction or language acquisition is the only mechanism of change. In contrast, in other models, signaling systems are negotiated between individuals over much shorter time periods. Finally, and most important from our perspective, the various models seem to require rather different cognitive capacities in individual agents. Skyrms' (and subsequently Barrett's) reinforcement learners have no social or cognitive capacity beyond the capacity to retain a set of signal/meaning association weights and the ability to recognize a payoff. Nowak's evolutionary model excludes even the requirement for individuals to recognize communicative success, leaving natural selection to do the work of tuning the population's communication

Table 1
A comparison of the major features of the models surveyed in Section 2

	Nowak	Steels	Barrett/Franke	Oliphant/Batali	Smith	Barr
Transmission	Vertical	Horizontal	Horizontal	Vertical	Vertical	Horizontal
Model Type	Association matrix	Associative	Numerical	Associative	Neural	Neural
Mod. Hearer/ Speaker?	H & S	H & S	H & S	H	H	H & S
Interaction	Mutual payoff	Feedback	Mutual payoff	Observation	Observation	Mutual payoff
Features	Natural selection	Inhibition	Frget/pun/ inhib	Obverter	Inhibition	Inhibition
Production/ Reception	Stochastic	Deterministic	Stochastic	Deterministic	Deterministic	Deterministic

system. In contrast, the cognitive apparatus required in the Naming Game and Observational Learning paradigms seems rather more demanding—they variously require mechanisms of speaker–hearer feedback (often glossed in these models as "pointing"), various processes of competition or lateral inhibition between signals and meanings, rational reasoning about the optimal signaling behavior, and possibly the ability to reliably infer a signaler's intended meaning. However, drawing strong conclusions as to the necessary cognitive prerequisites for the emergence of functional learned signaling seems premature, since the numerous implementational differences between the existing models potentially obscure a common underlying mechanism.

At this point some notions require clarification. In our simulations reported below, our key criterion will be whether a particular type of model develops *optimal signaling*—a completely unambiguous set of signals which cover all meanings—*reliably*, that is, 100% of the time. Human lexicons are not optimal but are supported by contextual cues to disambiguate words which would be ambiguous out of context (e.g., Piantadosi, Tily, & Gibson, 2012); Why, then, are we looking for reliable optimality when it does not appear in natural language? The first reason is largely historical: There is a long tradition in work looking at the evolution of signaling conventions to focus on the evolution of optimal signaling, with the development of signaling systems which are near-optimal in context being a more complex (and under-explored) question. Second, on a practical point, the work which follows demonstrates that models which do not *always* produce optimal systems actually very rarely do so, to an extent that would be highly dysfunctional in human language.

3. An exemplar-based framework

To cut through this diversity of models, we have replicated four of the six models above using a minimal framework.[3] The two which have been excluded are Nowak et al. (1999) and Barr (2004). The former is excluded simply because the mechanism driving the development of optimal signaling is the well-understood process of natural selection, simply operating on traits which are inherited culturally rather than genetically; as such, this work has relatively little to say about how the processes of learning and use might shape signaling systems. Barr (2004) is not replicated primarily because, unlike the other models reviewed above and presumably driven by his neural network implementation, meanings and signals are not atomic, but are represented as distributed patterns of activation across both input and output nodes. This feature is hard to reconcile with the other models presented here. However, as can be seen in Table 1, Barr employs a mixed design, incorporating reinforcement and lateral inhibition. Since these features can be seen independently in one or more of the other models, we hope to derive insights into these processes in isolation which will also apply to Barr's model.

We employ a simple *exemplar*-based "urn model" as a general framework for a number of theoretical and practical reasons. Although the replicated models involve several different forms of representation, all of them treat a signaling system as a set of associations between meanings and signals. The exemplar model captures this simply—meaning/signal pairs can be seen as "meaning" balls in "signal" urns (or vice versa): Mechanisms of learning and adjustment are equivalent to adding and removing the balls from the specified urn. This also allows for an unlimited number of novel signals in the same manner as Steels and Loetzsch (2012), unlike the fixed-size neural networks of, for example, Smith (2002). In addition, this representation is identical to the reinforcement learning of Roth and Erev (1995). Roth–Erev learning directly captures the first two of exemplar theory's "central notions of similarity, frequency, and recency" (Walsh, Möbius, Wade, and Schütze 2010), p.1. The third factor is more problematic, as all stored tokens in Roth–Erev learning are equally weighted. This factor means that Roth–Erev learning is a simplified exemplar model. With that said, recency effects are introduced by including *forgetting*, such as in Barrett and Zollman (2009). As such, the exemplar framework can directly replicate game-theoretic work such as Skyrms (2010) and Barrett (2006), as well as be easily extended to include the core mechanisms from feedback and observational learning accounts.

In the sections that follow, we describe the various components of our exemplar framework, before identifying a basic framework, a baseline instantiation of our model which includes the minimal ingredients to replicate the various results from the literature. In Section 4 we show these replications and explore various deviations from the minimal model which illuminate the fundamental mechanisms driving the evolution of optimal signaling.

3.1. Exemplars, agents, populations

Discrete meanings and signals m, s are drawn from unordered sets M, S. Each exemplar represents a simple bidirectional association between a single meaning m and signal s. Exemplars are atomic and unweighted. The population contains a set of agents A; each agent a consists of an unstructured set of N exemplars. The current number of exemplars of agent a associating meaning m with signal s is denoted by N_{ams}.

We assume that populations are fully connected: Each agent has an equal chance of interacting with any other. During an interaction, two agents are chosen (the particulars of this depend on the *population dynamic*, which is described in Section 3.1.5 below), and designated *speaker* and *hearer,* respectively. A *context* $C \subseteq M$ of c meanings is selected with uniform probability. Also with uniform probability, a single *topic* meaning t is selected from C. The speaker produces an *utterance* $u \in S$ associated with the topic which the hearer turns into an *interpretation* $i \in M$. Production and interpretation can be described as stochastic functions over probability distributions for signal production $p(u|t)$ and reception $r(i|u, C)$, where C is a context containing the topic t. The different ways in which exemplar storage gives rise to these probability distributions is specified below.

3.1.1. Production and reception

During production, given a target meaning t, an agent a must select a signal. In all the models we consider, each potential signal $s \in S$ has a weight proportional to the proportion of stored exemplars which feature topic t paired with s. As shown in Fig. 2, in the standard model the *production weight* of a signal s is simply:

$$P_{\mathrm{ats}} = \frac{N_{\mathrm{ats}}}{\sum_{s' \in S} N_{\mathrm{ats'}}} \tag{1}$$

We can similarly define the reception weighting of a potential interpretation $i \in M$, given an utterance u and a *context* C of potential meanings to which u could refer:

$$R_{\mathrm{amuC}} = \frac{N_{\mathrm{amu}}}{\sum_{m' \in C} N_{\mathrm{am's}}} \tag{2}$$

To select a signal based on P, or a meaning based on R, an agent could apply either a *stochastic* or *winner-take-all* procedure. A stochastic signaler (or receiver) simply samples a meaning (or signal) proportional to P (or R), that is, $p(u|t) \propto P_{\mathrm{atu}}$, $r(m|u, C) \propto R_{\mathrm{amuC}}$. A signaler applying a winner-take-all procedure selects with uniform probability among the set of meanings (or signals, during reception) with the maximum value of P (R for reception); that is,

$$p(s|m) = \begin{cases} 1/|S'| & \text{if } s \in S' \\ 0 & \text{otherwise} \end{cases} \tag{3}$$

where S' is the set of meanings for which P_{amu} is at a maximum, and $r(m|u, C)$ is similarly defined with respect to those signals for which R_{amuC} is at a maximum.

The *obverter* mechanic differs from these standard production and reception procedures in that the obverter weights for production are simply equivalent to the reception weights in the standard model, and vice versa, that is, $P_{\mathrm{obverter}} = R$, $R_{\mathrm{obverter}} = P$.

3.1.2. Communicative accuracy

Successful communication occurs when the hearer's interpretation matches the speaker's intended meaning, the topic (i.e., $t = i$). The communicative accuracy between a speaker a and hearer b, where T is the set of all contexts of size c is[4]:

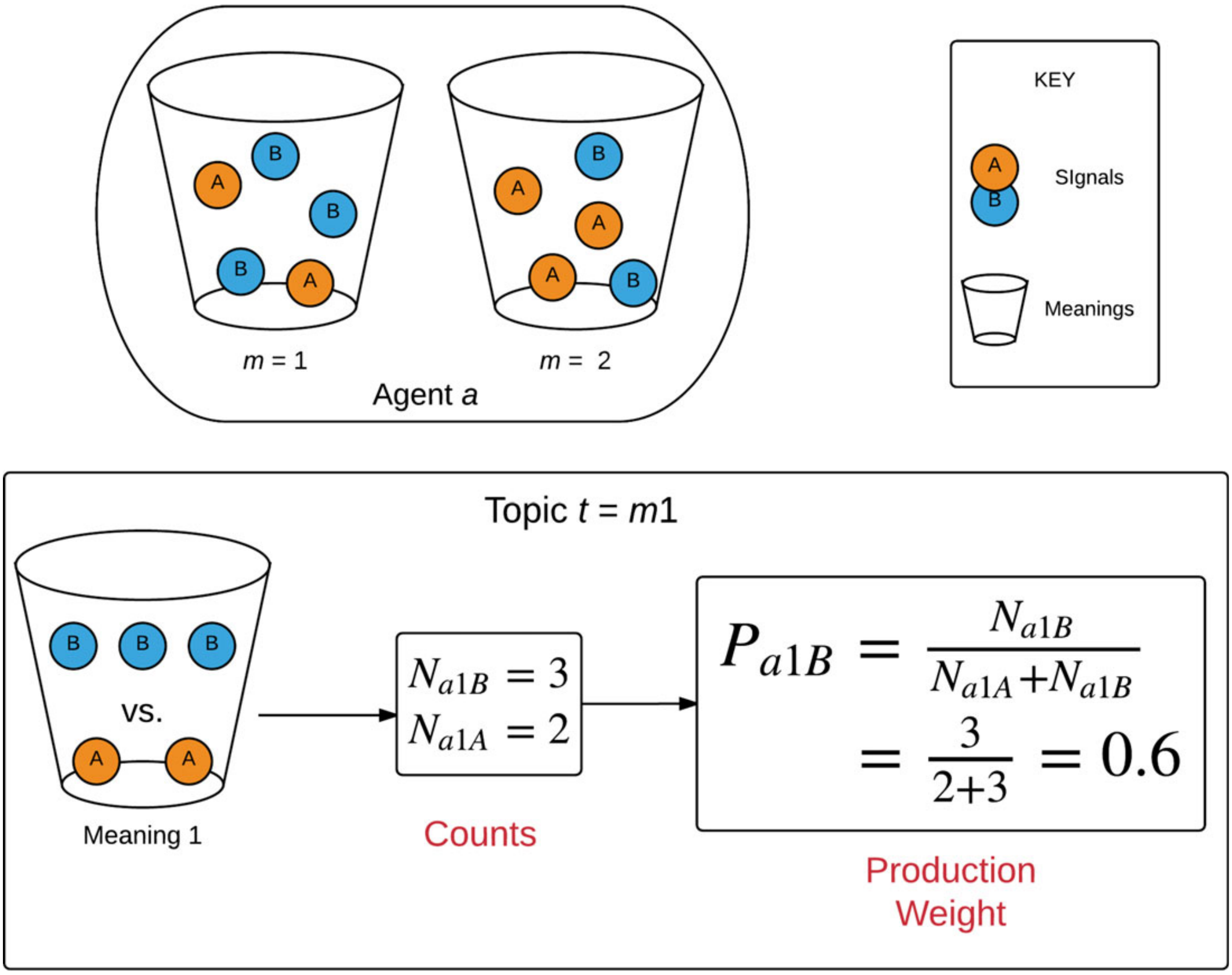

Fig. 2. An *urn-model* conceptualization of the notation for exemplar counts and production weights. Signal/meaning associations are represented as balls (signals) in urns (meanings). The weighting function simply counts the proportion of a given ball/signal in the urn/meaning chosen as topic, in this case meaning 1. For this example, WTA production would always produce signal B, whereas stochastic production would produce it 60% of the time. Although this diagram represents production weights P_{ams}, visualizing the reception weights R_{amsC} simply involves swapping the roles of signal and meaning to urns and balls, respectively, excluding any meanings which are not in the context C.

$$CA(a,b) = \frac{1}{C \cdot |T|} \sum_{C \in T} \sum_{m \in C} \sum_{s \in S} p(s|m,C) \times r(m|s,C) \tag{4}$$

The communicative accuracy of a population is the mean value after Eq. (4) is calculated for all possible pairs of agents in the population, with each member acting as both speaker and hearer.

3.1.3. Learning, deletion, inhibition, and memory

Learning always involves storing exemplars—after an interaction one or more meaning-signal pairs is added to an agent's memory. Memory limitations are modeled by enforcing a maximum allowed number of stored exemplars per agent. When this is surpassed, any newly stored exemplar results in the deletion of a randomly selected older exemplar.

Some models also involve deletion of exemplars, for example, in some reinforcement models to adjust the speaker and/or hearer's memories after an unsuccessful interaction. *Simple deletion* removes a single exemplar with a specific meaning-signal association. *Lateral inhibition* represents competition between newly introduced exemplars and already stored ones. This can be either *anti-homonymy*, which deletes exemplars which share the same signal as a given focal exemplar but a different meaning, or *anti-synonymy* for the converse process. Lateral inhibition can be either *minimal*, *broad*, or *maximal* inhibition. Minimal inhibition deletes only a single competing exemplar, selected with uniform probability from all competing exemplars. Broad inhibition affects all competing associations equally: A single exemplar is deleted for each competing type. In maximal inhibition, *all* competing tokens are removed. When this method is applied against homonyms and synonyms, it guarantees that all signal/meaning mappings will be one-to-one. This instantly removes the problem of how signals become unambiguous (although not the problem of how populations arrive upon a shared system), and as such we do not investigate this mechanism here.

In all the cases where lateral inhibition proves necessary, minimal inhibition is sufficient: As such, we use minimal inhibition in all the replications, which shows that stronger forms are not necessary for the evolution of optimal signaling.

3.1.4. Feedback and referential information

"Feedback" is not defined consistently across the literature, but it always describes a scenario in which referential information is transmitted *after* signaling (see Fig. 3). This may be about either the speaker's intended referent *(referential information)* or the hearer's interpretation *(interpretive information)*. Further to this, feedback can be either

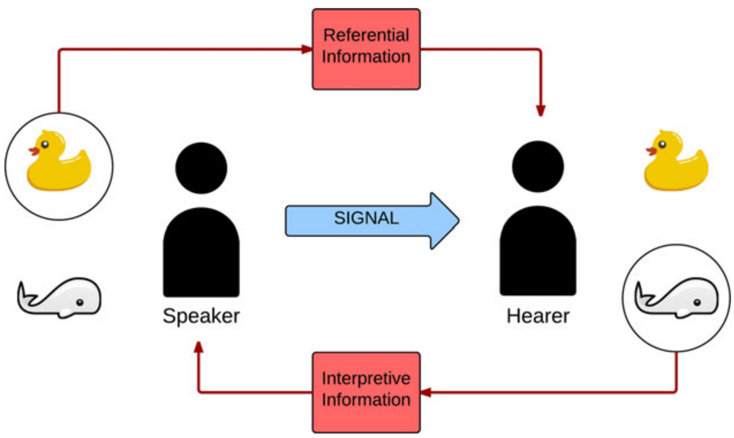

Fig. 3. *Feedback:* the referential elements that can make up the feedback found in the different models: Speakers select a meaning and produce a signal, which is interpreted by the hearer. Information about the intended and interpreted referent is, depending on the model, always, sometimes, or never transmitted.

full (the referent is unambiguously indicated) or *partial* (some information about the referent is supplied, by limiting the set of candidates). Depending on the particular model, neither, one, or both of the elements in may be involved.

In reinforcement learning, full referential and interpretive information is provided after successful communication (i.e., the environmental payoff indicates to the speaker that the hearer's interpretation was identical to the topic, and this indicates to the hearer that its interpretation was identical to the speaker's intended meaning); the absence of payoff from an unsuccessful communication provides partial referential and interpretive information (the signaler knows that the receiver's interpretation was not the topic; the receiver knows that the topic was some meaning $m \in C,\ m \neq i$). Observational Learning models provide full referential information: The hearer receives the topic plus the speaker's utterance.

Finally, several flavors of feedback are provided in the Naming Game, as discussed in Section 2.2.1. *Interpretation feedback* is when full interpretive information is provided (i.e., the hearer points at its interpretation). We again note that the interaction in the Naming Game requires that this information is always provided. With *Yes/No Feedback*, full referential information is provided after successful interpretation, but only partial referential information otherwise (i.e., after the hearer points, the speaker says yes or no). A richer alternative to this is *Referential Feedback:* Full referential information is provided (if the hearer is correct, the speaker confirms; if not, the speaker points at its intended referent).

3.1.5. Population dynamics

The population can be either *closed* or exhibit *gradual replacement*. Closed populations have no turnover: No agent leaves the population, no new individuals enter the population, and learning occurs after each interaction, such that all members of a population continue to learn over the entire run of the model. Under gradual replacement, old agents are continually replaced with new ones; a new agent is created and interacts with the established population a given number of times before joining the population by replacing the oldest agent. In this scenario, following the standard Observational Learning paradigm, agents only learn during their initial set of interactions, on entering the population, and do not learn in their subsequent interactions (when they are serving as language model for some other new individual).

In both cases, the populations are fully connected: In the closed dynamic, each interaction involves a new pair of agents selected at random from the population; with intergenerational turnover, the new learner is paired with a randomly selected member of the established population.

3.2. The basic framework

We first establish the *basic framework*. This represents what we have identified as the simplest abstraction of the models surveyed in Section 2. The purpose of this is twofold: First, by providing a common framework, we can meaningfully compare the different models. Second, and more important, we can apply the various mechanisms from those models to *build up* from the basic framework. This *additive* approach allows us to see, both individually and (if necessary) in combination, the effects of each mechanism and ultimately determine which factors are responsible for the development of optimal learned signaling.

The basic framework is a type of Roth–Erev "urn" model. Agents are represented as a collection of "meaning/signal" tokens. To produce a signal, for example, a ball is sampled at random from all the balls with the specified meaning. When a new exemplar is stored, a single new ball is added to the urn. Likewise, a deletion removes a single token. As such, this minimal framework employs stochastic production and reception. All other mechanisms are *modifications* of this base framework. As such, WTA production will choose from the most frequent tokens instead of random sampling; broad lateral inhibition deletes a single token of each competing type.

In addition to stochastic production and reception, the basic framework has the following properties which can be assumed to be true of all the following replications unless otherwise stated: The population consists of 10 agents, and we fix the number of meanings and signals at 5 each. There is no use of a restricted context ($C = M$). Agents have no memory limitations or any other form of exemplar deletion such as punishment or lateral inhibition. We employ a closed-group population dynamic unless otherwise stated. When the gradual replacement population dynamic is used, each new agent interacts with the existing population exactly 35 times.[5]

Before continuing, we must point out that we have made a number of necessarily arbitrary decisions about several parameters. For example, fixing the number of agents at 10 is problematic on at least two levels. The first is in terms of model comparison: In the studies we replicate here, populations range in size between two and ten thousand. Second, and perhaps more important, actual human populations are rarely limited to 10 individuals: We need to know whether the results we present here *scale up* in a reasonable way. Similar concerns can be raised over the choice to model populations as being fully connected (every agent can be called upon to communicate with every other agent) and to limit the number of both signals and meanings to five. The number of signals, for example, can be as low as two in the classic signaling Game of Lewis (1969) or be practically unbounded in a more accurate reflection of human communication (Steels & Loetzsch, 2012).

This being the case, we have opted to set these parameters at relatively small values. The main benefit of this trade-off is computational efficiency: We are able to run very large numbers of simulations, and this allows us to examine the *aggregate behavior* of any given configuration. That said, the question of scaling must be addressed. Due to the computational costs of running a large number of simulations, we are unable to provide exhaustive results for much larger numbers of agents, signals and meanings. However, results from smaller numbers of runs (see Section 5) indicate that the required number of interaction to lead to optimal signaling appears to increase linearly with the size of the population,[6] and quadratic growth with the number of signals and meanings. As such, the results presented here remain reasonable for, as an example, populations of 1,000 agents negotiating signaling systems with 100 meanings and signals. We will return to issues of how our framework can address these and other simplifications in the discussion in Section 5, in particular the fully connected network governing agent interactions and the uniform distribution of meanings.

4. Results: Exact and minimal replications

In each replication, our aim is twofold. The first is to confirm that our exemplar framework can replicate the original results. To this end, we include all mechanisms from the source paper. Second, and as our principal focus, we want to determine which of those mechanisms are responsible for leading to the reliable development of optimal signaling. This is the role of the basic framework: First, we add the base interaction of each model type—that is, the reinforcement dynamic, the feedback dynamic, or the observational learning dynamic as described in Section 3.1.4. We then test whether this alone leads to reliable optimality. If this is not the case, features from the original models are then added in, individually at first. For those individual features which do not individually lead to optimality, we then investigate whether combinations of those features do.

The purpose of this methodology is to chart out the full range of interactions between the different models and features. In the subsequent section, we provide an overview which argues that despite the initially bewildering range of design choices, they can all be seen as variations on three basic underlying themes. These three mechanisms are the creation and transfer of referential information, a form of information loss, and a bias against ambiguity.

4.1. Reinforcement models

We first replicate a basic reinforcement model using Roth–Erev learning as described in Skyrms (2010), before including mechanisms of (a) negative feedback and (b) forgetting (following Barrett & Zollman, 2009), and finally, as an exploratory measure, (c) gradual turnover. The basic reinforcement models require that one idiosyncratic feature is added to the basic framework, *initialization*: All agents begin with one exemplar of each possible association. This avoids a *lock-in* effect particular to reinforcement models, where agents will only ever produce the first successful signal associated with any given meaning (the result of this is that two agents who had initial success with different signal/meaning associations can never align with each other).

Our replication of Skyrms (2010), which we refer to as the *Pure reinforcement model*, uses the basic framework with the reinforcement feedback dynamic described in Section 3.1.4: After each successful interaction, both hearer and speaker update their exemplar store with the meaning-signal pair produced by the speaker. Following Barrett (2006), we then add memory limitations and punishment. Results of this set of simulations are shown in Figs. 4 and 5.[7]

Our replication reproduces the key features of the reinforcement models. Pure reinforcement learning as in Skyrms (2010) does not lead to optimality—no runs converge to

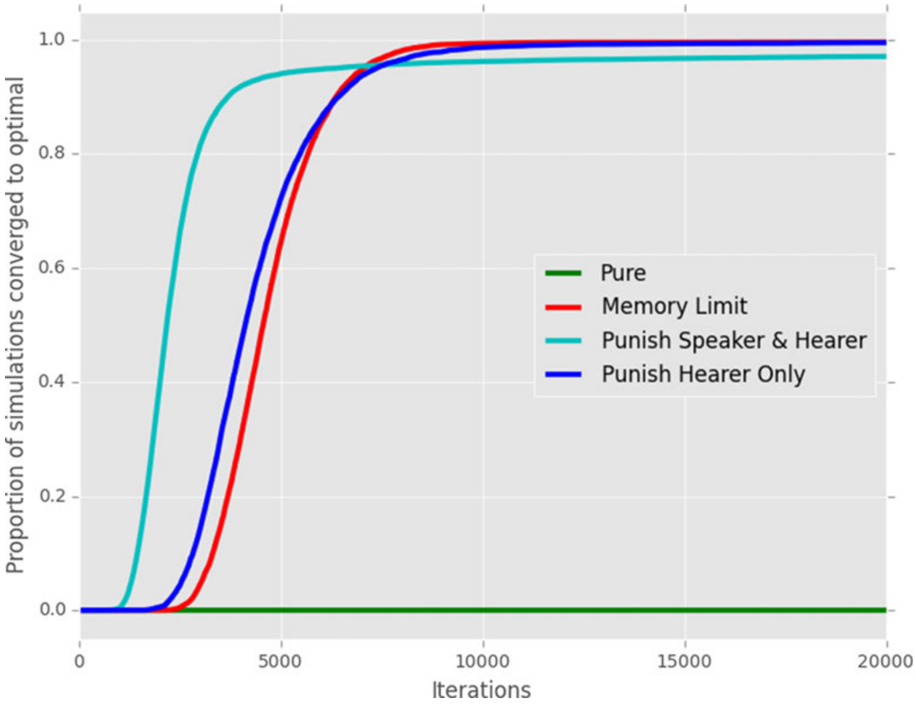

Fig. 4. Replication of classic reinforcement learning models. Here, we show the proportion of 10,000 simulations which had converged to an optimal communication system after a given number of iterations. Results are plotted for the pure reinforcement model ("Pure"), pure reinforcement with a memory limit of 35 exemplars ("Memory Limit"), pure reinforcement with punishment for both speaker and hearer ("Punish Hearer & Speaker"), or for the hearer only ("Punish Hearer Only").

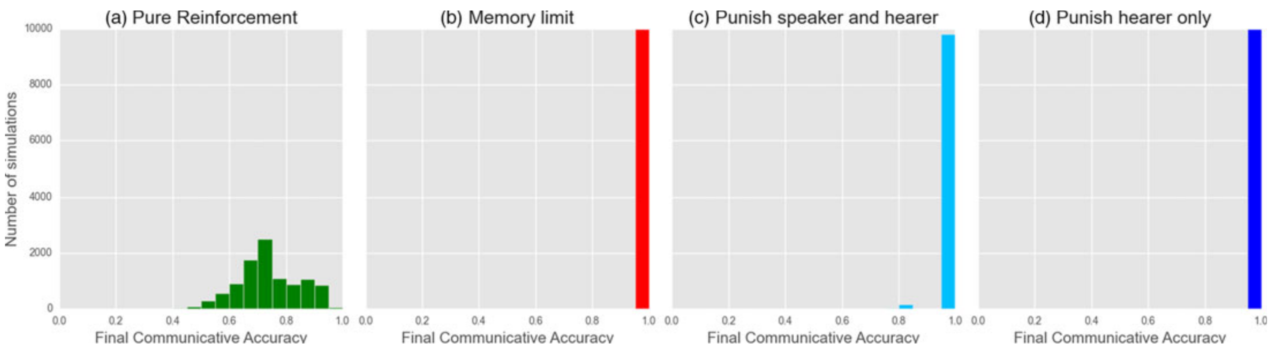

Fig. 5. Reinforcement models: comparing (a) pure reinforcement learning with the effects of (b) enforcing a memory limit of 35 exemplars or punishing failed associations for (c) both speaker and hearer or (d) only hearer. Figures indicate the final, stable distribution of communicative accuracy scores for the 10,000 simulations shown in Fig. 4 after 20,000 interactions.

optimal signaling. Fig. 4 indicates the proportion of 10,000 simulations that have converged after a given number of iterations. Similarly, Fig. 5a displays the final stable distribution of communicative accuracy scores for the same set of simulations. Our normal criterion for optimality is that CA = 1: even when a looser criterion is used (e.g., CA = 0.95), the number of populations which converge is small.

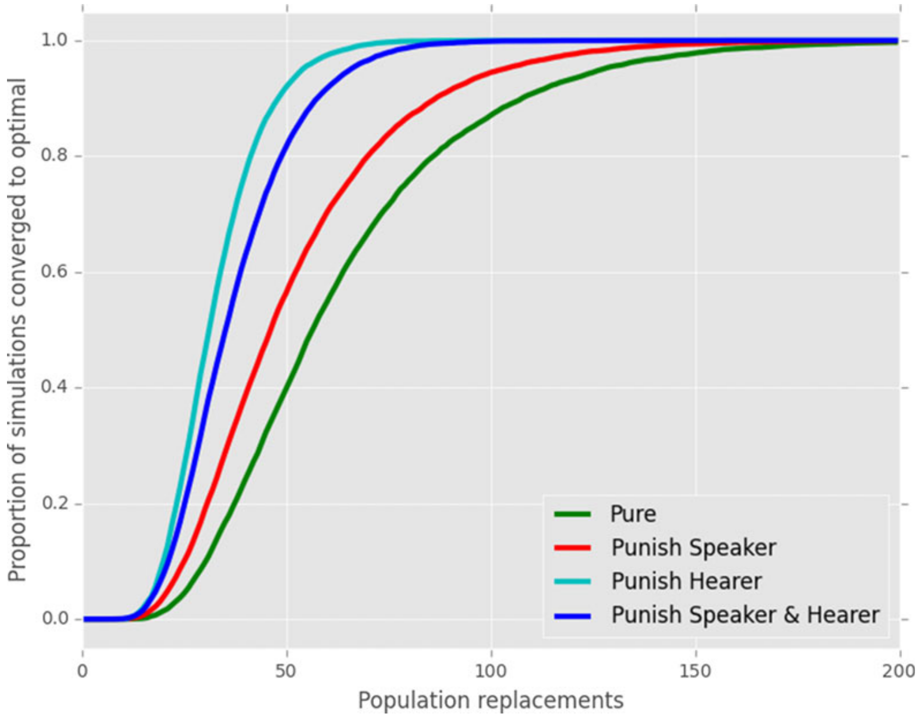

Fig. 6. (Modification of Barrett, 2006) Gradual population replacement using pure reinforcement, punishment of speaker only, hearer only, or both speaker and hearer.

Our first addition to the pure reinforcement model is a *memory limit* of 35 exemplars (Fig. 5b). We confirm that, as shown by Barrett (2006), this reliably leads to optimality; furthermore (not shown), the time taken to reach optimality reduces as the size of memory reduces unless the memory limit is too low to permit a stable set of stored exemplars (e.g., with 5 meanings and 5 signals, once the memory drops much below 25 exemplars). Moving to the second mechanism, *punishment*, we replicate the result from Barrett (2006). When punishment is employed by both speaker and hearer, this usually (but not always) leads to optimality (Fig. 5c). However, as can be seen in Figs. 4 and 5d, if only the hearer modifies his or her exemplar store after each interaction by reinforcement or punishment, optimal signaling reliably develops.

Finally, we provide the novel result (Fig. 6) that a third mechanism—gradual population turnover, rather than a closed group—also reliably leads to the emergence of optimal signaling: Under this population model, different types of feedback and updating merely impact on the time taken to convergence. This suggests that limited memory and negative reinforcement are not the only mechanisms that can lead to convergence in a reinforcement model—essentially any process that leads to the removal of stored exemplars (either targeted removal, as is the case of negative reinforcement; random removal, in the case of a memory limit; or wholesale removal, as effected by replacement of individuals) leads to the emergence of optimal signaling within the framework of the reinforcement paradigm.

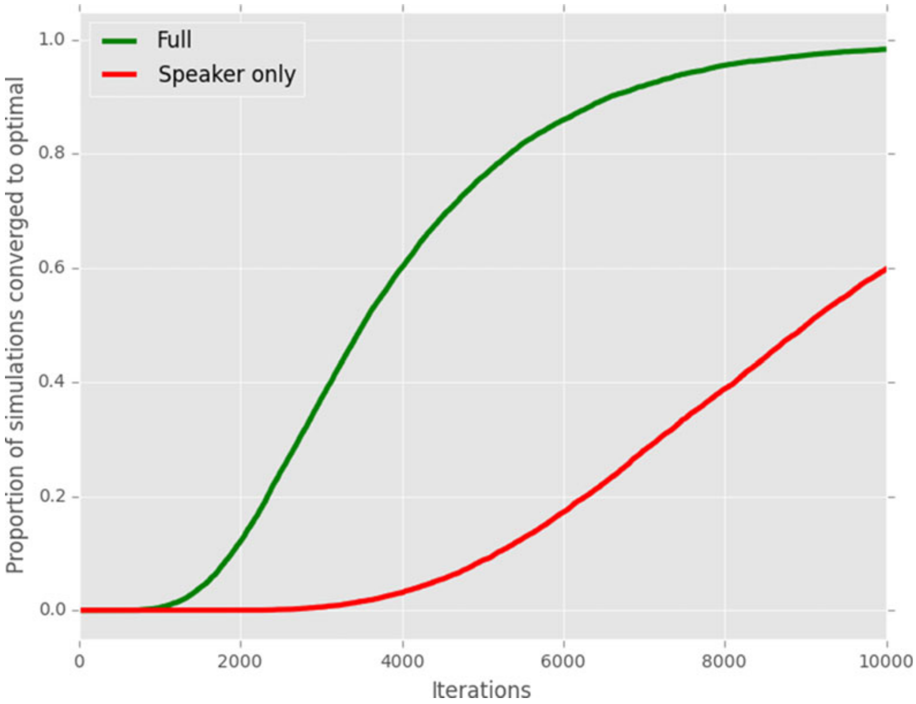

Fig. 7. Replication of the Naming Game results of Steels and Loetzsch (2012) and Baronchelli (2010), showing a replication of the full Naming Game ("Full"). We also replicate Baronchelli's results (with a model equivalent to the full Naming Game model, minus referential information), showing that when only the speaker learns from each interaction ("Speaker Only"), convergence is significantly slower than when both Speaker and Hearer learn from each interaction.

4.2. Naming game models

First, we replicate the Naming Game model of Steels and Loetzsch (2012) using the exemplar framework. The original model Naming Game model is essentially a reinforcement model with a number of added features. Core additions are as follows: (a) full interpretive information is provided after every interaction (i.e., the hearer "points" at its interpretation); (b) referential information is provided after every interaction, although the quality of this referential information depends on the success of the interaction (full referential information is provided after successful interpretation, but only partial referential information otherwise—that is, after the hearer "points" to its interpretation, the speaker says yes or no); (c) lateral inhibition of homonyms and synonyms after storing a new exemplar (see Section 3.1.3); (d) punishment of speaker and hearer after failed communication; (e) WTA rather than stochastic production and reception. Note that our replication differs from the original by limiting the number of signals to 5 and including no restricted context ($c = |M|$).

As can be seen in Fig. 7, we replicate the original Naming Game result, namely that optimal communication reliably emerges. Furthermore, we replicate the finding of Baronchelli (2010) that when only referential information is provided (i.e., the hearer does not identify their interpretation), whether speaker or hearer updates his or her exemplar store after interaction impacts on time to convergence: Hearer modification leads to rapid convergence, convergence under speaker-only modification is far slower. This result suggests that interpretative information (the hearer "pointing" to his interpretation) is not required for the emergence of optimal signaling in the Naming Game, assuming the presence of referential information—since referential information is provided in Observational Learning models, but interpretive information is not, this removes one source of difference between these models.

We are now in a position to explore the mechanisms of the model in isolation. As there are actually two feedback dynamics (Yes/No and Referential Feedback), both were investigated using the basic framework.

As discussed in Section 2.2.1, Yes/No Feedback is formally identical to reinforcement learning except for one factor: With Yes/No Feedback, the speaker has access to information about the hearer's interpretation. The original feedback model of Steels and Loetzsch (2012) uses a punishment mechanism which has two features: The first is the same as "negative reinforcement" in Barrett (2006)—the speaker punishes the association it used and failed with. The second, which is only available in this model, is that the speaker also punishes the *hearer's* failed association. In Section 4.1, we saw that punishing the hearer after failed communication leads to reliable optimality, while punishing the speaker does not always do so (although it usually does). In the case of Yes/No Feedback, however, we can see in Fig. 8a that when speakers *only* use interpretive information as the basis for update and punishment—that is, when they only punish the hearer's failed association—populations do not reliably develop signaling systems. As shown in Figs. 5b and c, restoring referential information does reliably leads to optimality. However, the importance of the punishment dynamic is illustrated by Fig. 5d, in which we can see that, for Yes/No Feedback *without* any punishment, lateral inhibition of homonyms and synonyms does not reliably lead to optimal signaling.

We can now look at the other feedback dynamic. While punishment is an effective strategy for Yes/No Feedback, it is not effective for Referential Feedback (not shown). Speaker-only learning and punishment only rarely leads to optimal signaling.[8]

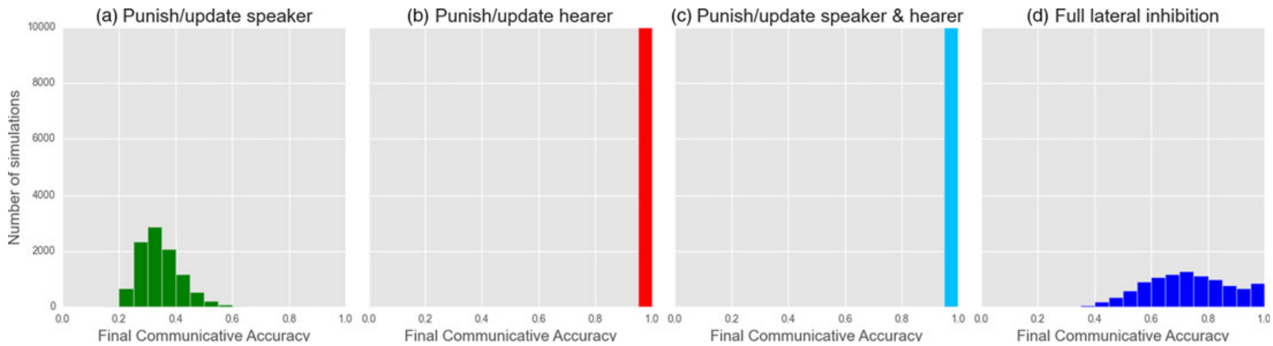

Fig. 8. Yes/No Feedback models: investigating the effects of different forms of punishment (a–c) and the application of lateral inhibition of both homonyms and synonyms for both speaker and hearer (d). Figures indicate the final, stable distribution of communicative accuracy scores for 10,000 simulations after 20,000 interactions.

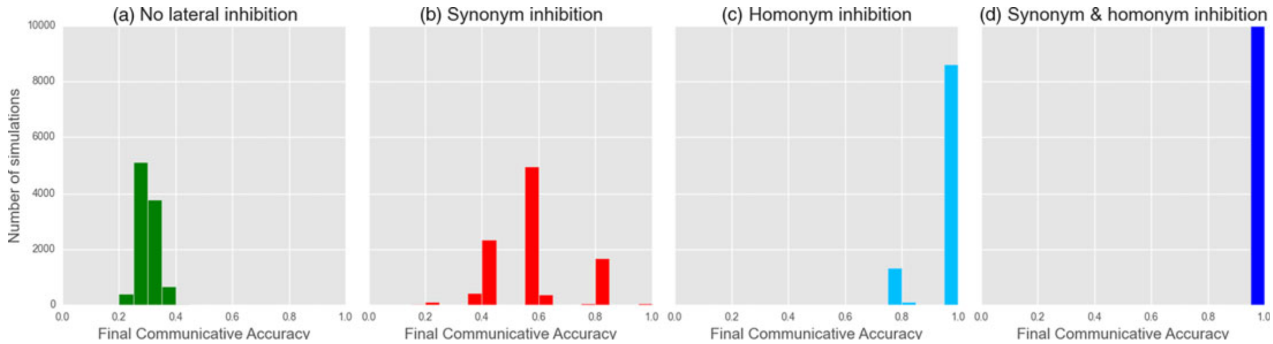

Fig. 9. Referential Feedback models: investigating the effects of no lateral inhibition (a), only synonyms (b), only homonyms (c) and both (d). Figures indicate the final, stable distribution of communicative accuracy scores for 10,000 simulations after 10,000 interactions.

Turning to Referential Feedback, the lateral inhibition mechanic (Fig. 9) proves to play a more significant role; specifically, synonyms and homonyms must always be inhibited in order for convergence to occur. A further observation (not shown here) is that the importance of inhibiting synonyms depends on who learns in an interaction. When both speaker and hearer are modified after an interaction, both homonyms and synonyms must also be inhibited; when only the hearer learns, only homonyms must be inhibited.

In summary, the different types of feedback—neither of which leads to optimality by themselves—require different additional mechanisms to do so. As Yes/No Feedback is a modified type of reinforcement learning, it is no surprise that punishment is a successful strategy. What is surprising, however, is that when *only* interpretive feedback (which is the only difference between this model and basic reinforcement with punishment) is used, it never produces a signaling system. Also unexpected is that lateral inhibition does not consistently lead to optimality for Yes/No feedback, while it is highly effective in conjunction with Referential Feedback. In contrast, we see the restricted circumstances in which punishment reliably leads to optimality for Referential Feedback (hearer-only learning, WTA production). In the final analysis, none of the mechanisms of the Naming Game are unnecessary (except interpretive information for the purposes of punishment). Instead, the model contains a suite of mechanisms, each most effective when applied to the two types of feedback dynamic incorporated into the original model.

4.3. Observational learning: Biased learning

Smith (2002) models agents as ($|M|\times|S|$) associative networks and explores the consequences of a range of weight-updating procedures on these networks. In doing so, he identifies two crucial features on which learning rules vary: presence or absence of biases against synonyms and homonyms. Our minimal replication allows us to manipulate these same biases, by enforcing different types of lateral inhibition. Employing a closed-group dynamic instead of the original gradual turnover, we compare the effects of these manipulations, as well as the role of WTA production/reception (as used in the original) and stochastic production/reception. As this is an observational learning model, only hearers

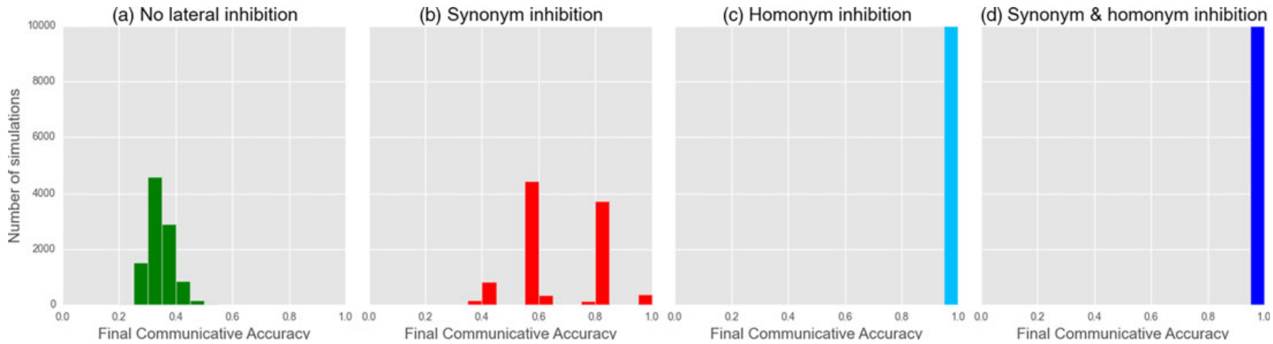

Fig. 10. Biased learner models: investigating the effects of no lateral inhibition (a), only synonyms (b), only homonyms (c) and both (d). Figures indicate the final, stable distribution of communicative accuracy scores for 10,000 simulations after 10,000 interactions.

are updated after any interaction, and full referential information is provided. Before proceeding, we point out the similarities between the Referential Feedback model of the previous section and the Biased learning model. The first difference is that the original biased learning model uses gradual turnover instead of closed groups. Second is the provision of information: Both Observational Learning and Referential Feedback guarantee the transmission of referential information, but the dynamic of referential feedback also includes interpretive information. However, we have shown in Section 4.2 that utilizing this information (by punishing failed associations) is not always an effective strategy when referential information is guaranteed.

Using the basic framework, we are able to replicate the results of the original (Fig. 10). The emergence of optimal signaling is dependent on learners employing the right type of lateral inhibition; critically, as seen for the Naming Game, agents must *inhibit homonyms*. Again, adding inhibition of synonyms has no additional effect, while inhibiting synonyms alone does not reliably lead to optimal signaling, and optimality never occurs when lateral inhibition is removed altogether. In further tests comparing WTA and stochastic production/reception, no difference was found apart from slightly faster convergence for WTA. This once again suggests that this difference between Smith's model and the Referential Feedback version of the Naming Game described in the previous section is superficial only.

4.4. Observational Learning models: Obverter models

Our replication of Oliphant and Batali (1996) uses the Observational Learning version of the basic framework, but also employs *obverter weighting* as described in Section 3.1.1. In line with the observational learning paradigm (Section 3.1.4), only hearers are updated after any interaction. Features of the original model which are not part of our basic framework are (a) WTA production/reception rather than stochastic and (b) gradual replacement population instead of a closed-group populations. We will examine the effects of these mechanisms, as well as the effects of adding a memory limit. As the

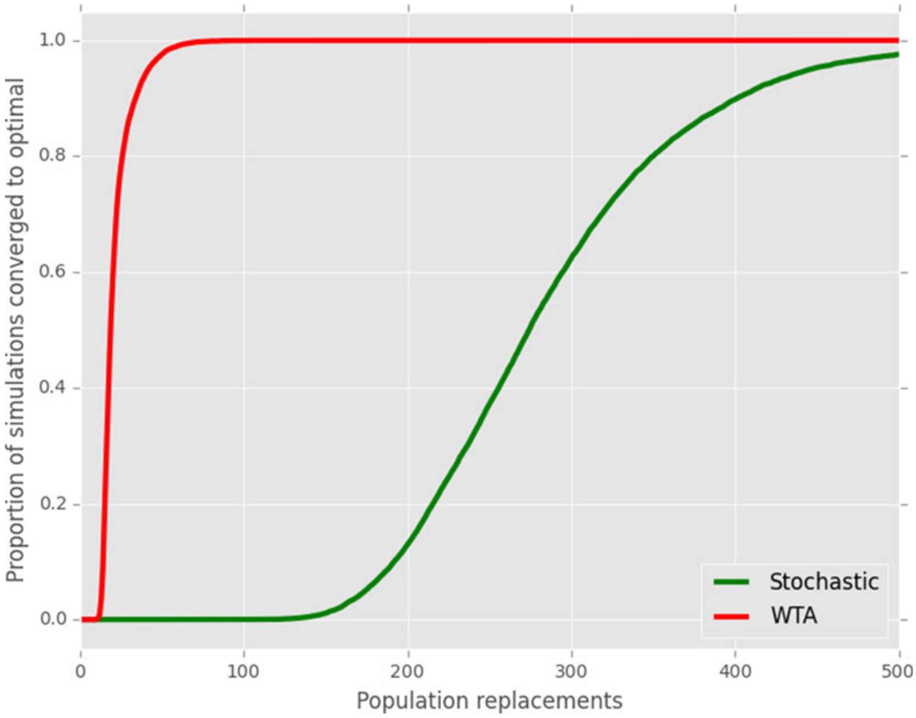

Fig. 11. Obverter production/reception, with gradual population replacement, and stochastic or WTA production/reception.

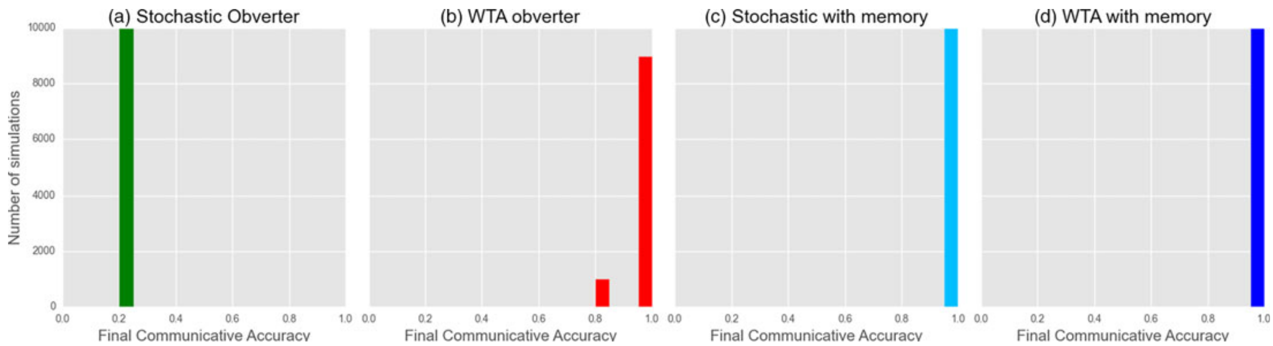

Fig. 12. Obverter learner models: investigating the effects of WTA production with and without a memory limited to 35 exemplars (a, b), and stochastic production with and without a memory limited to 35 exemplars (c, d). Figures indicate the final, stable distribution of communicative accuracy scores for 10,000 simulations after 20,000 interactions.

effects of lateral inhibition on observational learning models have been fully explored in Section 4.3, none of our replications include this mechanism.

We are able to replicate the original result of Oliphant and Batali (1996): A minimal replication employing gradual population turnover reliably leads to the emergence of optimal signaling when WTA production/reception is used. When stochastic production/reception is used, we still have guaranteed optimality, albeit over much longer time-scales (Fig. 11). Results for closed groups are strikingly different (Fig. 12). In particular, optimal systems do not reliably emerge unless a memory limit (here, a maximum of 35 tokens) is added to the model.

We see here an strong parallel with the results for Reinforcement Learning. Despite the presence of a *systemic* bias toward functional communication, both strategies are ineffective without the presence of some form of information loss, whether that is via intergenerational transmission or explicit memory loss.

5. Minimal requirements for optimal signaling

Using our minimal exemplar framework we were able to replicate the results of all the original models. As suspected, the internal representations of agents (network/associations/urn models, etc.) are not a factor behind the development of signaling; all that is necessary is a way to model agents who can capture associations between meanings and signals. Indeed, given the appropriate parameter settings, the replications using the basic framework produce results which are strikingly similar or identical (Fig. 13) when we compare a version of the Naming Game with hearer-only learning and only Referential Feedback with the Biased Learning model (both using lateral inhibition of homonyms).

Beyond this, what conclusions can we draw? First, passing *referential information* (as described in Section 3.1.4) from speakers to hearers is almost always essential. In the case of Observational Learning and classic variants of the Naming Game where referential information is provided, full referential information is always provided to the

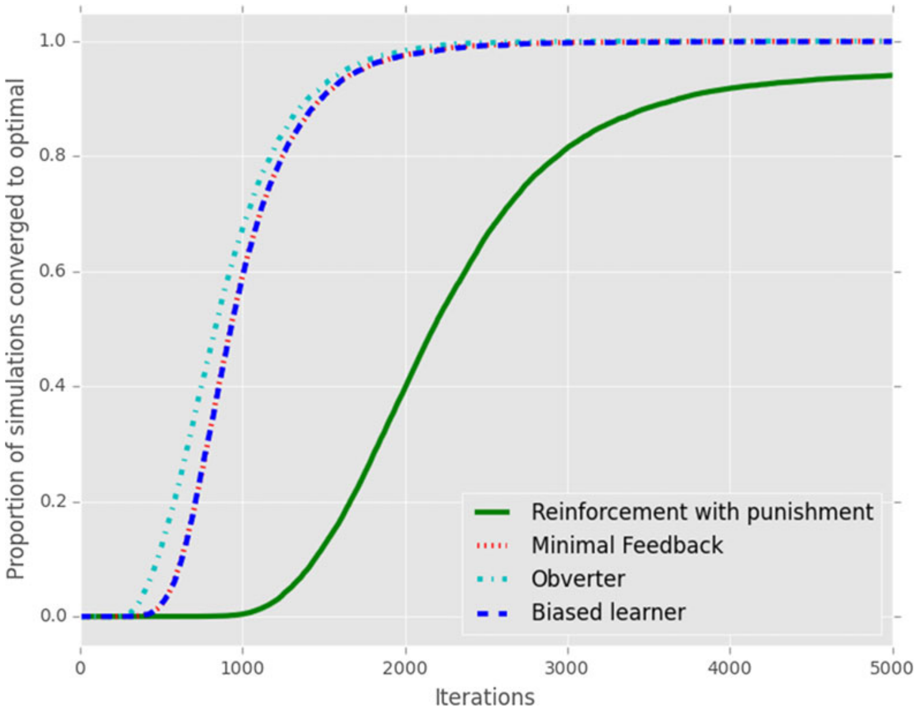

Fig. 13. *Model Comparison:* Comparison of the different models instantiated using the basic framework (stochastic production and reception, closed-group) of Reinforcement (with punishment, following [Barrett, 2006]), a minimal version of the Naming Game using only Referential Feedback, no punishment inhibition of homonyms and hearer-only update (Steels & Loetzsch, 2012), Obverters with a limited memory of 35 exemplars (Oliphant & Batali, 1996), and Biased learners with inhibition of homonyms (Smith, 2002).

hearer. This allows them to know with certainty which meaning the speaker intended to convey; the only difference between the referential feedback models and observational learning models is that, in the latter, the topic is immediately provided. In Naming Game models with referential feedback, it is provided after the signal has been sent. However, as evidenced by the reinforcement and Naming Game models with Yes/No Feedback only, it is enough for referential information to only be provided sometimes. More interestingly, reliable interpretive information (i.e., the hearer signaling its interpreted meaning to the speaker) does not appear to play a significant role, as can be seen in Naming Game models with Yes/No feedback where only the speaker learns from each interaction.

Second, *information loss* about previously stored associations must be present in some form. In reinforcement learning and the obverter instantiations of the observational learning model, this can be due to deletion, limited memory, or population turnover. In the case of Naming Game and Biased Learning models, this can be achieved via lateral inhibition.

In the Naming Game and Biased Learning models, lateral inhibition of homonyms also serves to provide a *bias against ambiguity*. This bias seems to be intrinsic to the reinforcement model and obverter models; we discuss this in greater detail below.

A summary of these findings can be seen in Fig. 14, which indicates how the different models relate to each other in terms of the basic framework, and which mechanisms are required for each model to reliably develop optimal signaling.

5.1. Referential information

One requirement for optimal communication is conventionality: A population agrees on how signals map to meanings. The problem is that while signals are overt, meanings are not. If only signals passed between agents, arriving at a convention for how meanings and signals are associated would be impossible; a mechanism for sharing this referential information therefore must exist. Fig. 3 in Section 3.1.4 shows the two types of information that can be made available in any interaction between agents. Referential information details how a speaker provides an intended referent; interpretive information pertains to how a hearer interprets a given signal. How does this apply in the models above?

In Observational Learning models, full referential information is always provided (guaranteeing transfer of meaning as well as signal between speaker and hearer), and interpretive information is never provided. In basic reinforcement learning, the environment provides full referential information and interpretive information after communicative success, and partial information of both types after failure (although in the latter case this information is only utilized when deletion of exemplars after unsuccessful communication is employed, i.e., punishment). The Naming Game model of Steels and Loetzsch (2012) involves two flavors of feedback. Referential Feedback ensures that, whether a given interaction has been a communicative success or a failure, full referential and interpretive information is always provided. With Yes/No Feedback, full interpretive information is always provided (as the hearer always indicates their interpretation), and the amount of referential information provided depends on the success of the interaction—full

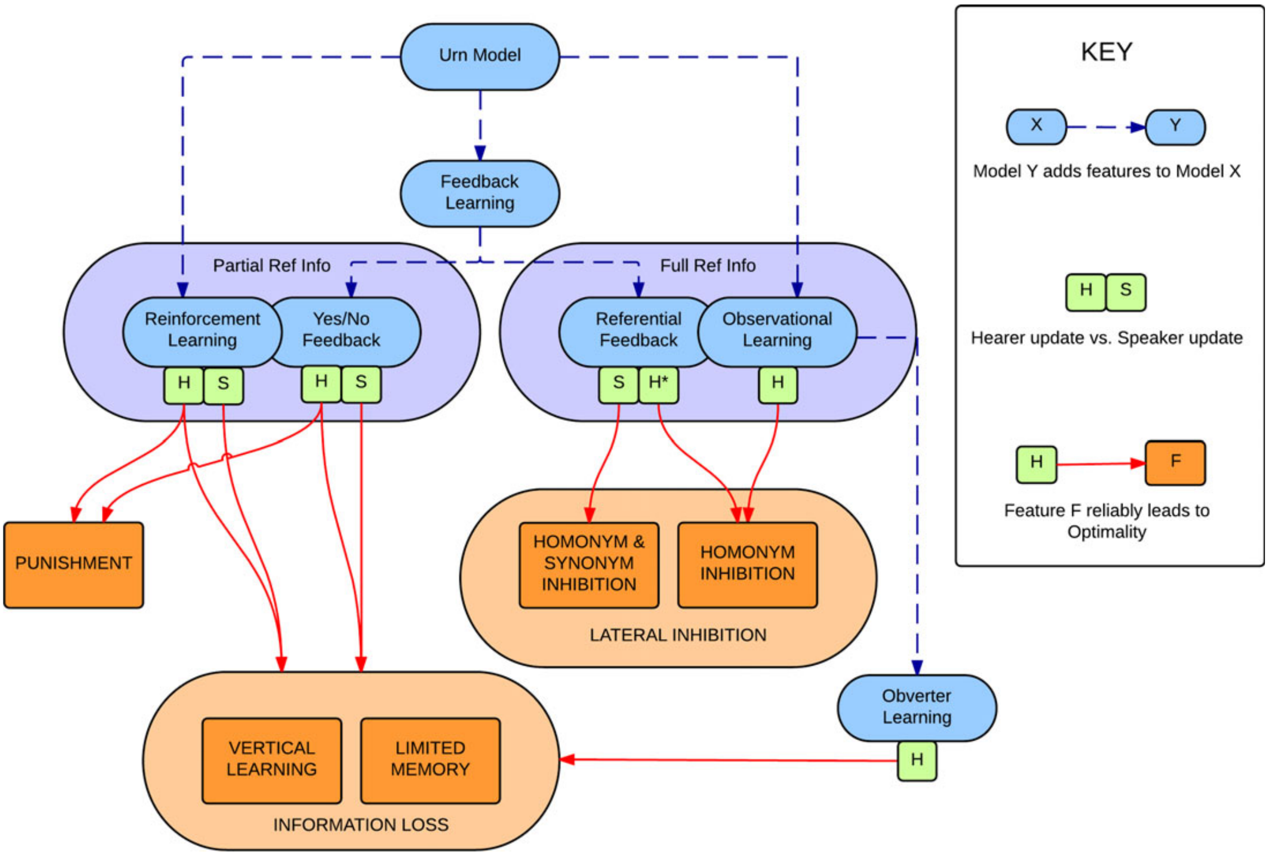

Fig. 14. An overview of how the different models (in blue) relate to each other in terms of the availability of referential information (guaranteeing either partial or full referential information), and the model mechanisms (in orange) which reliably lead those models to optimal signaling. Vertical learning and limited memory are grouped together as being types of information loss; the two forms of lateral inhibition are also grouped together. Green squares represent the impact of Speaker and Hearer update, respectively; for example, only hearer-directed punishment drives reinforcement to optimality, while both hearer and speaker-directed punishment are effective for Yes/No feedback models. For Referential Feedback, the * denotes the fact that a specific configuration (hearer-directed punishment with WTA production) does lead to optimality, but this is not otherwise indicated.

referential information is only provided after success (as the speaker confirms the hearer's interpretation), whereas partial referential information is provided after failure (some other member of the context must have been the intended meaning). Note again that this is almost equivalent to Reinforcement learning except for the fact that full interpretive information is always provided via the hearer's pointing action.

This analysis in terms of information flow rather than agent interaction clarifies a number of issues. First, the reason Reinforcement models and Yes/No-only variants of the Naming Game are slower to converge is simply that full referential information is not as frequently provided (i.e., only after successful communication, rather than after every interaction). Second, the *identical behavior* of the minimal instantiations of the Naming Game and biased Observational Learning models shown in Fig. 13 is no surprise; in both—despite the different sequencing of information transfer—hearers are always supplied with full referential information about the speaker's signaling behavior. Finally, while both Naming Game and Reinforcement models are often implemented so as to provide interpretive information, this is not a *necessary* component for the development of optimal signaling, as shown in Fig. 7 in Section 4.2, in agreement with the conclusions of Baronchelli (2010). The only exception to this is versions of Reinforcement and Feedback learning where only speakers learn; in this case, the guaranteed transfer of interpretive information proves to be critical in ensuring optimality.

Crucially, conventionality is established in a population whenever speakers propagate referential information to hearers. This is backed up by a mathematical result from Xue (2006). Xue employs populations of interacting Pólya urns; this is essentially a generalized version of our basic framework described in Section 3.2. Xue shows that whenever information is passed from speaker to hearer with positive probability, a state of conformity will always result. This, then, is the fundamental role of the various channels of information transfer between speaker and hearer: ensuring conformity primarily via the transfer of referential information about the topic from signalers to hearers.

5.2. *Bias against ambiguity*

With the spread of referential information, we can expect the development of conventionality in a population. Unless other pressures are at play, however, these conventions are unlikely to be optimal, as illustrated by the "imitation learner" of Oliphant and Batali (1996) and the "maintainers" of Smith (2002). A pressure toward optimality is created in various ways: when only successful associations are strengthened, when obverter weighting is used, or when there is inhibition of homonyms. What do these mechanisms have in common?

Our definition of optimality requires that an unambiguous signal exists for each meaning and that all signals are unambiguous. Ambiguity indicates the presence of some degree of homonymy: It is no surprise that the explicit deletion of homonyms leads to an unambiguous system. This *lateral inhibition* of homonyms is the pressure that drives optimality in biased Observational Learning (Smith, 2002) and Naming Game (Steels & Loetzsch, 2012) models.

In Reinforcement models, reinforcement strengthens successful associations, therefore preferentially increasing the weight of less ambiguous associations for the hearer. That association is then more likely to be used again, by the hearer when later acting as a speaker. This *rich-get-richer* process provides the necessary pressure against ambiguity. Obverters also have an inherent bias against ambiguity, manifested by picking the least ambiguous signals. However, in the absence of some form of memory loss, this bias is either never strong enough (with stochastic production) or not reliably so (WTA) to lead to guaranteed optimality.

We draw attention to the fact that the bias against ambiguity has multiple possible interpretations. In Reinforcement learning the bias in incorporated in both the environment (which provides a payoff when states and acts are matched), and the ability to recognize that payoff—or, when punishment is employed, the lack of a payoff. Similarly, in the nearly equivalent Naming Game model with Yes/No Feedback, information about success is established through *interaction*. In the Naming Game with referential feedback, as with observational learning models, the guarantee of referential information requires an internal bias in the form of lateral inhibition of homonyms. More sophisticated still (but mechanistically identical) is the “rational” approach of the obverter Observational Learning model.

As a final remark, we point out a parallel between this pressure against ambiguity and the “amplifying function” described in De Vylder and Tuyls (2006) (outlined here in Section 2.2), which works to eliminate synonyms and guarantees convergence in the homonymy-free naming game. The true function of the bias against ambiguity is seen in the way input meaning/signal distributions map to output meaning/signal distributions, that is, from what an agent learns, to what an agent produces. While De Vylder’s amplifying function acts to always privilege more common variants in the mapping from input to output, the required bias against ambiguity applies slightly differently. Instead, the bias ensures that the *least ambiguous* variants be amplified via repeated cycles of learning and production.

5.3. Information loss

Section 4.1 highlighted the necessity of *information loss* in the development of optimal communication. This can take several forms: simple forgetting through limited memory size, the sampling effects of gradual replacement of members of the population, or more targeted processes of deletion and lateral inhibition.

Why should information loss be beneficial, rather than a hindrance? In all models, the initial state of the population is highly disorganized: Individual agents have maximally ambiguous meaning/signal associations and are driven in many different, mutually incompatible directions as a result of their early interactions. If an optimal signaling system is to be established, the influence of these early disorganized states must be eliminated. In standard Roth–Erev learning (and hence also exemplar learning), learners have an effectively infinite memory and place an equal weight on all observations. Because the effect of new information is proportional to the count of previous observations, the *learning rate* steadily decreases over time. In the case of classic instantiations of the Reinforcement model—and also in obverter Observational Learning without population turnover or forgetting—we observe a slowing effect. In the long term, populations are trapped into non-optimal pooling equilibria. The only way to avoid this is some form of information loss. It provides a “plasticity” whereby non-optimal states can always be escaped.

As in the previous section, we note that the mechanisms which lead to information loss can have very different interpretations and apparent functions. However, they all share two key properties: (a) new information is privileged over old, and (b) the chance that a particular association “survives” (either within an individual or within the population as a whole) is proportional to its relative frequency, making possible the stability of frequent associations against noisy loss.

5.4. Comment: Simplifications and extensions

Our framework involves a number of simplifications. To expand on our comments in Section 3.2, increasing the population size appears to result in roughly linear growth in the number of interactions required for convergence, but quadratic growth when increasing the number of meanings and signals. To measure the effect of increasing the population, we took the average time to convergence over 10 simulations for the original population size. We then successively doubled that size to attain values for populations of 10, 20, 40, 80, 160, 320, 640, and 1,280. The time to convergence was found to roughly double for each doubling in population. A similar method was used for meanings and signals, with 5, 10, 20, 40, 80, and 160 tested. In this case, each doubling resulted in a roughly fourfold increase in time to convergence. However, due to the time constraints imposed by computational limitations, we are not able to make any strong claims on this basis, particularly in light of the fact that Barr (2004) observed something more like logarithmic growth when he increased the number of agents in his simulations. In any case, the linear or slower growth resulting from increased population sizes seems less problematic than the quadratic increase which occurs when more meanings and signals are used. The emergence of novel sign languages such as NSL involves large signal and meanings inventories. These results suggest that this process might necessarily be piecemeal, first establishing small number of conventional meaning-signal mappings and then expanding. It would be an interesting direction for future research to empirically verify whether this is indeed the case.

Moving away from matters of simple scaling, the fully connected populations used in the model are quite unlike the social structures found in actual human societies, which tend to exhibit the "small-world" property identified by Milgram (1967). This is a very rich area of study and impossible to treat thoroughly here, but we draw attention to the possibility of an interaction between the agent model and the network type; indeed, Barr (2004) shows that one very simple strategy (*stay/switch*, described in Section 2.2) is ineffective in fully connected populations but seemingly optimal in more sparsely connected networks. In some further preliminary work, we have investigated the effects of placing agents on small-world networks and also very sparsely connected lattice networks. For small-world networks, it appears that the short average path-length between any two agents leads to no great divergence from our observations above. For lattice networks, however, where any two agents can be separated by a significant number of intermediaries, we found that the global emergence of a single set of optimal signaling conventions was not guaranteed for the majority of simulation runs even under parameter setting which lead to convergence in fully connected populations; rather, populations converge on a series of local optimal conventions (as also seen in Smith, 2003). This certainly warrants further investigation, but we are encouraged by the fact that the more realistic network structures do not appear to conflict with our proposed general requirements.

Finally, as observed by Zipf (1936), words in natural languages tend to roughly follow a power law distribution; that is, the frequency $f(w)$ of a word scales more or less according to its frequency r rank r, so that $f(w) \propto \frac{1}{r}$. We amended our model so that the presentation of meanings had such a distribution, first for the case of 5 meanings and signals, and then for 50 meanings and signals, and compared the resulting number of iterations to convergence with those for uniform meaning distributions. These investigations found a distinct effect whereby the power-law distributions led to slower convergence, but within the same order of magnitude ($\approx$1.5 times as long for 5 signals and meanings, and $\approx$3 times as long for 50). As such, we are content that—at least in this case—our results are robust to manipulations of meaning frequency.

6. Conclusions

To reiterate the remarks of Section 5, we argue that the necessary requirements leading to the reliable development of optimal signaling conventions in populations of interacting agents are as follows:

1. A way of propagating *referential information*
2. The presence of a *bias* against ambiguity/homonymy
3. Some form of *information loss*

One way to look at these requirements is as a solution at Marr's *computational* level of analysis (Marr & Poggio, 1976). There are, however, multiple solutions at the *representational* level—presumably more than the four we have surveyed here. With this in mind, what can we say about which strategies are actually employed by humans, whether in naturalistic or experimental settings? At this point, it is worth re-examining two of the requirements—reference and bias—as there is a discernibly common pattern in how they are treated.

In contrast to Lewis's (1969) proposal that common ground must play a role in the establishment of optimal signaling, none of the theories here require global knowledge beyond agreement on a set of shared meanings and signals. Where they differ is in how reference is established through individual interactions. Reinforcement learning requires environmental cues to create reference where none existed before; models in the Naming Game framework assume the existence of referential meaning but concern themselves with how pairs create shared reference; in observational learning models, the salience of reference is assumed. In all cases, referential information is shared during interaction.[9] The different roles of reference in these models can be construed as involving increasing degrees of cognitive sophistication: first environmental stimulus, then explicitly negotiated, and finally implicitly available. Humans use all these strategies (Ashby, Maddox, & Bohil, 2002; Fay, Arbib, & Garrod, 2013; Scott-Phillips, Kirby, & Ritchie, 2009). Reinforcement learning is arguably the simplest account here in cognitive terms.

A similar trend can be seen with the bias against ambiguity. Reinforcement learning requires only recognition of variability in stimuli: *Operant conditioning* (surveyed in Staddon & Cerutti, 2003) has long been established as a common animal behavior. Inhibition of homonymy is a type of *mutual exclusivity bias* (Markman & Wachtel, 1988), and more complex pragmatic inference resembling obverter learning has been experimentally observed in language games by Frank and Goodman (2012). Again, reinforcement learning appears to present the simplest strategy, while not necessarily the most efficient one.

Modern humans can utilize all the cognitive abilities described above. Which, then, are involved in the self-organization of functional communication systems we see in the wild (e.g., in the emergence of homesign or indigenous sign languages such as NSL or ABSL and in the lab (e.g., in studies on experimental semiotics)? Presumably, there must be some interplay between the individual task demands of the particular communicative setting—whether naturalistic or experimental—and the cognitive expenditure which is required. It is very likely, for example, that certain communicative settings will favor a particular strategy where others favor a different one, perhaps selected depending on the type and quality of feedback available. We suggest that this may be a fruitful line of enquiry for future experimental work.

What we hope our comparative approach has shown is the *multiple realizability* of behavior that leads to the emergence of signaling conventions. Compelling evidence for the explanatory role of any particular mechanism (e.g., learning bias or feedback) should not be taken as evidence for that being the *only* explanation. Hopefully, we can instead *integrate* these numerous insights and gain a richer understanding of the tapestry of human communicative behavior.

Acknowledgments

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Notes

1. This is essentially a type of *conformist frequency-dependent bias* (Boyd & Richerson, 1985).
2. This is in the case of well-connected populations: Neural network populations did not arrive at global convergence when they were placed on “small neighborhood” network topologies.
3. The source code for the model can be found at https://github.com/matspike/Cogscisignaling
4. Formally, $T = \{C \subseteq \mathbb{P}(M) | \, |C| = c\}$.
5. This sufficiently reduces the probability that any given meaning is not sampled.
6. Note, however, that Barr (2004) observed non-linear, logarithmic growth when using expanded populations.
7. In subsequent figures, we will make use of time-course graphs when we feel some indication of relative speed of convergence is necessary. The final distribution of communicative accuracy scores will be shown otherwise. We point out that these distributions are *stable* and are not to be taken as intermediate stages which may yet develop optimality.

8. On closer inspection of individual simulation time-courses, hearer-only learning and punishment reliably tends toward optimality but almost never reaches it. This is due to the stochastic nature of the basic framework: Because minority meaning/signal associations are less likely to be used by the population, they are also increasingly less likely to be deleted (a type must be used to be unsuccessful). In the specific case of WTA hearer-only learning and punishment, this effect is avoided. This is the only time when WTA instead of stochastic processes prove to be a critical factor in the development of optimal signaling.
9. There is an issue we have not addressed here, which Smith (2005) refers to as the "signal redundancy paradox"; that is, why signal if meaning can be transferred otherwise, for example, via pointing? There are a number of approaches to this problem, ranging from the observation that signaling, once established, allows for both displaced reference and communication out of the line of sight, to the proposal by Smith (2005) that most meaning transfer must not be explicit but require sophisticated inference. This is very interesting work, but we follow the precedent in much of the earlier work on the evolution of signaling systems by deferring the study of this important question.

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Chapter 3

Signalling with referential uncertainty

3.1 Signalling games review

In the previous chapter, we looked at the problem of how learned signalling systems come into being. The main accounts seeking to explain this — reinforcement learning, feedback learning, biased observational learning and rational 'obverter' learning — appear at first sight to be mutually incompatible. However, by 1) using a simplified exemplar-style framework to directly compare the different theories, and 2) investigating the effect of incrementally adding individual features, we have seen that only 3 requirements underlie the ability of any system to reliably self-organise optimal signalling: the ability to create and transmit referential information, a systemic bias against ambiguity, and finally some form of information loss.

Of course, this is not the end of the story. As with any model, the framework presented in the previous chapter represents a significant simplification of reality, and there are a number of ways in which we could relax its assumptions. In this chapter, I address one of these: *referential uncertainty*. I investigate its implications on the previously investigated models, and show that it places further restrictions on the three requirements outlined above.

3.2 Signalling with referential uncertainty

Referential uncertainty is a feature which appears in some of the previous models, namely the signalling games with reinforcement and feedback learners but, notably, not models which involve observational learning. However, even in these cases, there is always some mechanism which allows reference to be *resolved*. In reinforcement learning (allowing for the fact that 'reference' is not defined in the relevant source literature) reference is only completely resolved after successful communication: for feedback learning, the various analogues of pointing behaviour provide the necessary referential information.

However, as famously pointed out by Quine (1960), for any utterance heard for the first time there are many possible references. Some mechanism must account for how language learners are able to solve this problem with such apparent ease, as is the case with children's ability to *fast-map* (Carey & Bartlett, 1978) new words with their referents after only a few exposures. One candidate mechanism (which has appeared in several guises) is that humans employ some cognitive or behavioural strategy which leads to a significant reduction in referential uncertainty. For example, phenomena such as a *shape bias* (Landau et al., 1988) and other biases for colour, size, and relevance may play some role in reducing uncertainty about the intended reference. Another candidate is *joint attention*— which Tomasello (2010) and others argue for being a prerequisite for the evolutionary emergence of human language — which, by directing the focus of both interlocutors towards the same topic, provides a natural reduction in uncertainty.

Even with such mechanisms in place, Quine argues that some meanings will always remain indistinguishable from each other as they always refer to the same stimuli.[1] What is necessary, then, is some principled way of acquiring words in the face of this uncertainty. While this is particularly pertinent for theories involving observational learning (where the transfer of referential information is simply assumed), it may also be of relevance to feedback and reinforcement learning accounts: in the former, for example, pointing behaviour may not be reliable, and even if it is Quine's essential problem remains (what exactly is being pointed at?) and in reinforcement learning it may be that different states of the world (i.e. different referents) both reward the same behaviour, leaving no way to distinguish between them.

Cross-situational learning strategies (e.g. Pinker, 1984; Siskind, 1996; Frank et al., 2009; Blythe et al., 2010; K. Smith, 2011) rely on an ability to track words along with some subset of the meanings with which they have co-occurred (depending on the strategy, this can be all co-occurring meanings, a memory-limited subset, or the current best guess). By doing this over many exposures, the correct mappings become statistically inferable (usually by simply choosing the most frequent meaning/label co-occurrences). As shown by Siskind (1996) and Blythe et al. (2010), strategies like this succeed with high probability when applied to the problem of learning a static set of associations, as long as there are enough exposures.

A. Smith (2001) provides an early investigation into how communication systems can be established without explicit meaning transfer. In a crucial difference from the models which we have already surveyed, Smith models the simultaneous development of an agent's internal representation of meaning alongside the lexi-

[1] See Blythe et al. (2014) for a discussion of what Quine intended to say and how it has been subsequently interpreted.

con. Using the obverter strategy from Oliphant & Batali (1996), Smith finds that some degree of communication does develop between agents in the absence of feedback. However, due to the fact that agents' internal representations of meaning categories are highly unlikely to ever be shared, stable and optimal communication is never attainable. In a simpler model in which agents share a categorial representation of the world, Vogt & Coumans (2003) investigate what they term the 'selfish game', a direct analogue of cross-situational learning in which agents never establish joint attention or provide any feedback. Vogt & Coumans employ populations of Bayesian learners which are also roughly equivalent to obverter learners. They conclude that these populations are able to bootstrap signalling conventions even when constrained by uncertain reference, but only when an iterated learning paradigm is employed.

Fontanari & Cangelosi (2009) use two-agent populations to investigate how two different types of learners negotiate a communication system in the face of referential uncertainty, what they refer to as 'unsupervised' and 'supervised' learners. Both supervised and unsupervised learners are simply represented: a single association matrix which determines their signal production and reception behaviour; a type of obverter reception[2]; and finally, a limited memory. Unsupervised learners use a learning algorithm which is roughly equivalent to that in Blythe et al. (2010), where a received signal is associated with every item in the provided context. Supervised learners, on the other hand, enact a type of Naming Game in which Yes/No feedback is provided and also punish failed associations. Interestingly, neither type of learner leads to the reliable development of signalling conventions: Fontanari & Cangelosi observe some differences between their rela-

[2] They descibe this using the term 'introverted obverter', coined by K. Smith (2003a). This refers to an obverter strategy which relies on personal observations instead of global knowledge, and is basically identical to the obverter learners described in the first chapter.

tive error rates, but shown them to behave identically in the limit.

At this point, I would like to make two key observations. Firstly, neither of Fontanari's learners instantiates a strategy which drives a pressure against homonyms. Lateral inhibition is used but only against *synonyms*, and as we have seen before, some pressure against ambiguous terms must exist for a population to converge on an optimal signalling system. Likewise, obverter reception alone does not lead to the elimination of ambiguity, as only production can lead to a change in signalling behaviour. And while the inhibition of homonyms is certainly not present in the unsupervised configuration, one might expect the punishment employed by the supervised learner to be sufficient, given that it also has a limited memory. As can be seen in the previous chapter, these mechanisms are sufficient for the Yes/No signalling game to reach an optimal state for both speakers and hearers. My suspicion here is that the way that lateral inhibition of synonyms in this model interacts with cross-situational learning interferes with the development of optimal systems. However there is a more substantial problem: the supervised learner is not engaging in cross-situational learning. If we analyse the interaction, it is identical to that of Steels & Loetzsch (2012) except for the fact that *only* Yes/No feedback is used. As soon as a mechanism is able to eliminate referential uncertainty, the essential problem of cross-situational learning evaporates. As an example, Steels' original model also included a restricted context, but is not described as requiring cross-situational learning. Because feedback is reliant on the hearer's interpretation of a signal, restricting the context can only serve to further decrease referential uncertainty, which is no longer necessary. For this reason, I will only investigate the behaviour of the unsupervised learner populations in the following section.

Finally, De Beule et al. (2006) observe that although the simulations of Vogt & Coumans (2003) do result in populations of optimal signallers, they never settle upon an efficient set of one-to-one signal/meaning conventions. Also using a type of 'introspective obverter' alongside a rather complex parametric update rule[3] which leads to the suppression of homonyms alongside the lateral inhibition of synonyms, they show that populations are able to reliably bootstrap optimal and efficient lexicons under referential uncertainty.

3.2.1 Notes on previous models

The models surveyed in the previous section are not a significant departure from the signalling games mapped out in Section 3.1, and all in fact use some form of obverter production or reception. Apart from the 'supervised' learner of Fontanari & Cangelosi (2009), they all restrict themselves to forms of observational learning. As stated before, this is because cross-situational learning ceases to have any relevance if there is a reliable mechanism for providing unambiguous referential information. As a result, no form of feedback learning can be considered, as both Yes/No feedback and Referential feedback result in the indication of a clear referent at least some of the time. Likewise, a modified form of reinforcement learning in which hearers are provided with a limited context along with every signal (as well as feedback on success) is still more informative than observational learning, in which only context and signal are ever received[4]. This being the case, we can restrict ourselves to the two main forms of observational learning, the biased learner and the Bayesian/obverter learners.

The models surveyed in the previous section don not explicitly refer to the

[3] One of the sources of this complexity is a method which ensures that agents remain responsive to new data which contradicts their current state but are unchanged by confirmatory data.

[4] In fact, for *any* model which includes the occasional provision of full referential information, reducing the context is *always* informative. It is only the case in observational learning that the *only* available referential information is via differential contexts.

idea of a 'biased learner'. However, both Fontanari & Cangelosi (2009) and De Beule et al. (2006) do feature lateral inhibition, although in both cases this affects synonyms instead of homonyms. All three of the models employ some form of obverter, on the other hand: in Vogt & Coumans and De Beule et al. an obverter function is used for production only, while in Fontanari & Cangelosi for reception only. This leads to to a prediction, based on the work of the previous chapter, that the inability of Fontanari's models to develop optimal systems can be likely be partially attributed to the fact that neither of the key mechanisms (i.e. inhibition of homonyms or obverter production with information loss) which lead to optimality for the standard observational learning models are present. The following section will investigate the effects of the main mechanisms which apply to observational learning, i.e. lateral inhibition, obverter learning, and information loss: this will be done in the context of the models we have just discussed, which requires the addition of a small number of extra features.

3.2.2 The exemplar framework extended

We will continue with the basic signalling game framework as outlined in Spike et al. (2016), with minimal adjustments.

The first decision we have to make is the choice of how to represent an observational event as an exemplar. In previous models, an unambiguous link between a specific meaning/topic $m = t$ and signal s created a single exemplar. In this new condition, only signals are presented unambiguously. Meanings, however, are only ever presented as a size c context $C \in M$, and the topic t is never explicitly communicated. As such, we are presented with a choice between:

1. Given the set of meanings $\{m_i, \ldots, m_j\} \subset C$, create c new exemplars, one for each m, each associated with signal s, or;

2. Create only a single new exemplar by using some procedure to select the best candidate meaning $m_{best} \in C$ and associate that with signal s. This can be done stochastically, proportionally to the weights of exemplars, or using winner-takes-all, which always takes the most heavily weighted candidate.

All previous models utilised the first approach, but the second approach is also a viable solution as will be seen in the following section.

We must also make a decision about how *lateral inhibition* can be instantiated. If we use the first procedure, creating a large amount of new exemplars with each observation, there is likely to be an interaction with the number of exemplars which are deleted. Previously, we distinguished between *minimal* inhibition — where a single competing exemplar is deleted — or *broad* inhibition — where one competing exemplar of each competitor type is deleted (e.g. for synonym inhibition, one exemplar with the same meaning but a different signal for all competing signals). In section 3.1, we demonstrated that there is no significant difference between the two: where inhibition is required, both reliably lead to optimality. But this is only when a single exemplar is added. In this case, we cannot assume that a single deletion will still be sufficient to drive towards optimality, and as such should look again at the difference between the two.

If we use the second procedure, on the other hand, and only add a single exemplar, we must consider how *homonym inhibition* might work. After being provided with a signal s and a context C (but *not* a topic t), and if we then wish to inhibit a competing *homonym* for m_{best} we need to determine if there is any difference between inhibiting *any* competing homonym, or only competing homonyms which are *in* the context, or only those which are *not in* the context. More formally, should we inhibit an exemplar $X = \langle m_k, s_k \rangle$ where:

1. $X = \langle m_k \neq m_{best} \ , \ s_k = s \rangle$ or

2. $X = \langle m_k \neq m_{best}$ & $m_k \in C$, $s_k = s \rangle$ or

3. $X = \langle m_k \neq m_{best}$ & $m_k \notin C$, $s_k = s \rangle$

Finally, we can look at the implications for the obverter strategy. As before, there is a distinction between obverter production and reception. We know from our previous work that obverter reception *alone* never leads to optimality. However, for obverter production, we must decide whether the context is taken into account during the process. That is, when selecting the most communicative signal, should the obverter disregard meanings which are not in the current context? Looking at Table 3.1, we can see how this can be an issue. When obverters pick a signal, they choose the one which is most likely to be interpreted correctly according to their observations. It is possible that a different signal could be more likely to be interpreted as the intended meaning if another meaning, not in the current context, were taken into account.[5]

3.2.3 Results

As in the previous section, the basic underlying framework is an urn-model. On top of this, we add individual mechanisms so that we can observe their effect, and in particular whether they reliably lead to optimal signalling. To recapitulate, as all models are necessarily observational learners, the main mechanisms are i) the Biased Learner which uses lateral inhibition of homonyms and/or synonyms, and ii) the Obverter learner. Additional mechanisms which can apply to both types of learner are the inclusion of a limited memory, closed group populations vs. vertical learners and WTA vs. stochastic production and reception.

[5]The corollary is trivial: obverter reception should only consider meanings present in the context (in this simplified model at least).

(A)

		S1	S2	S3
$t \rightarrow$	M1	0.4	0.3	0.1
	M2	0.3	0.4	0.0
	M3	0.3	0.3	0.9

(B)

			S1	S2	S3
	$t \rightarrow$	M1	1	0	0
C		M2	0	1	0
		M3	0	0	1

(C)

			S1	S2	S3
C	$t \rightarrow$	M1	0	0	1
		M2	0	1	0
		~~M3~~			

Table 3.1: Matrices demonstrating the process of obverter production where C is the context and t = M1 is the topic. (A) is a *reception* matrix and is used as the basis of obverter *production* in (B) and (C). If there an unrestricted context as in (B), the best signal is S1. If there is a restricted context as in (C), the best signal is now S3 (assuming WTA production/reception for simplicity).

For both biased learners and obverters, I investigate the difference between *context-recorders* which add an exemplar for each meaning in the context, and the *interpretation-recorders* which add only a single exemplar, representing the most likely interpretation. For context recorders, the strength of lateral inhibition (i.e. how many exemplars are removed) now needs to be taken into account. Interpretation-recorders only need to inhibit one competing homonym or synonym; for these, I look at whether competitors from only within or without the current context (or both) should be inhibited.

Applying only to obverter learners, I will look at whether it is important whether signallers consider only the meanings in the context, or all possible meanings.

All other model parameters are the same as the previous exemplar framework, with 10 agents and 5 signals and meanings. The context is now kept at three

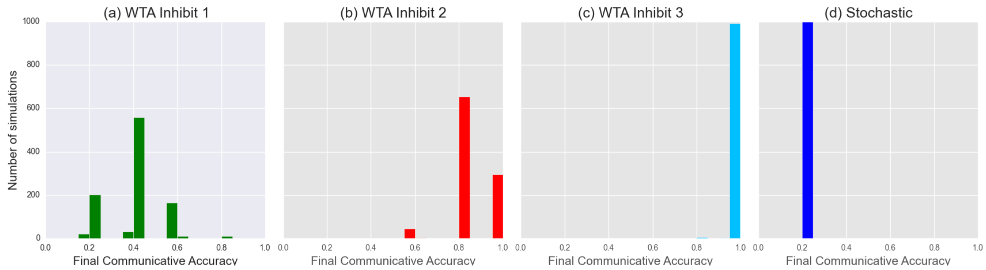


Figure 3.1: Context-recorder biased learners with lateral inhibition of homonyms. Images (a), (b), and (c) demonstrate WTA models, and the effects of increasing the lateral inhibition from 1 to 3 competing exemplars. Image (d) demonstrates that Stochastic models are incompatible with the context-recorder strategy.

meanings (including the topic) randomly selected with uniform probability at each signalling event.

Biased Learners

We can first separate the cross-situational biased learners into the *context-recorders* and the *interpretation-recorders*. While both types of learner are able to develop optimal signalling, the mechanisms which allow each to do so are not the same. To summarise the results: interpretation-recorders specifically require that both homonyms and synonyms are inhibited, and the inhibition of homonyms must include meanings which are present in the context. Context recorders, on the other hand, require only that homonyms are inhibited. However, lateral inhibition must ensure that more than one deletion is made for each competing token. Furthermore, while both stochastic and WTA production and reception are both effective for the interpretation-recorders, context-recorders necessitate WTA. As would be expected from the results from the models of simple signalling, the inclusion of a limited memory or vertical transmission has no effect: the information loss provided by lateral inhibition provides sufficient plasticity to the system.

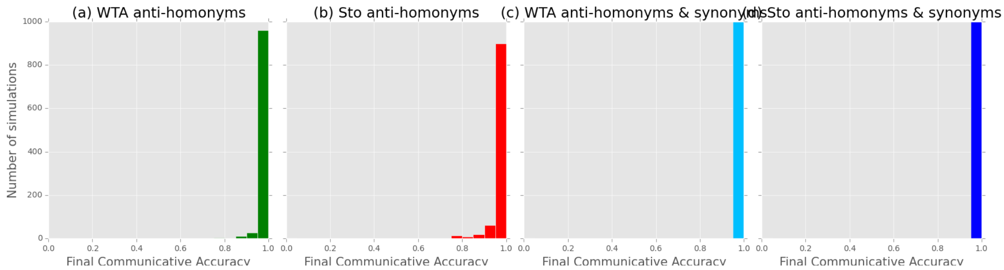


Figure 3.2: Interpretation recorder biased learners. For both WTA and Stochastic agents, lateral inhibition of both homonyms and synonyms is necessary to guarantee the emergence of optimal signalling, otherwise there is a small possibility that some meanings are left *uncovered* (see below).

Figure 3.1 shows WTA closed-group context-recorders and the effects of increasing the amount of lateral inhibition. When only one or two exemplars are deleted for each competitor, optimal systems do not develop, but increasing it to three exemplars tips the balance. The reason for this is simple: the probability of a competing homonym being inhibited P_i is smaller than P_r, the chance of being reinforced (in the case of these parameter settings, $P_i = 0.4$ and $P_r = 0.6$). This is simply the ratio between the size of the context and the number of meanings. If lateral inhibition is not strong enough, it appears that it populations are unable to overcome this imbalance and optimality never develops.

Similarly, the context-recorders in Figure 3.1(d) demonstrate that while WTA production and reception will always develop optimal signalling when enough lateral inhibition of homonyms is applied, stochastic agents are never able to do so. On reflection, this is also an obvious result: optimal systems for stochastic agents can only exist when each signal is associated with only exemplars of a single meaning. As context-recorders always associate multiple meanings with each received signal, optimal systems are necessarily impossible for this configuration.

Figure 3.2 shows interpretation-recorders, once again in closed groups with

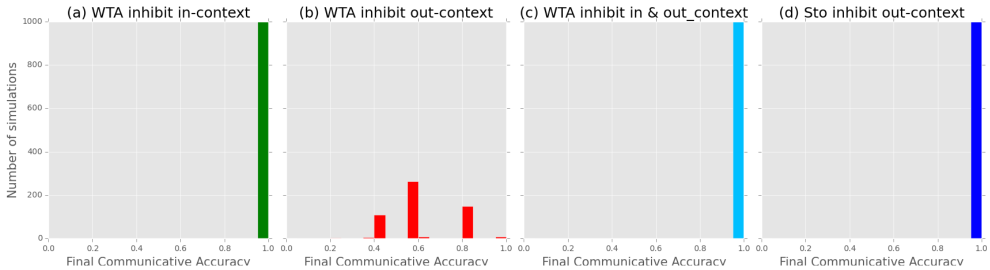


Figure 3.3: Interpretation recorder biased learners, demonstrating the importance of inhibiting meanings which are presented within the same context as the interpretation. This is the case for both WTA agents (a-c) and Stochastic agents (d).

minimal inhibition. For both stochastic and WTA production/reception, we can see that while inhibiting homonyms does not reliably lead to optimal signalling, inhibiting both homonyms and synonyms does. However, as we will see in more detail shortly, this is only as long as competing exemplars whose meaning is within the presented context are (at least some of the time) the target of inhibition.

The explanation for why synonyms must be inhibited is slightly difficult. In standard observational learning, every meaning is almost guaranteed to be associated with a signal. Because every meaning will be prompted at some stage, and due to the explicit presentation of the meaning/signal pair to the learner, each learner will reliably be presented with exemplars associated with all meanings. For interpretation-recorders, only *signals* are presented explicitly, whereas meanings are only available via context. The upshot of this is that biased learners are able to develop vocabularies which consist entirely of unambiguous signals, but because synonymy is permitted, certain meanings remain ‘uncovered’ with no associated exemplars. Table 3.2 provides an illustrated example of such a situation. As can be seen, an anti-synonymy bias is required to ‘free-up’ a signal for use with an uncovered meaning, as we see in the modelling work above.

Speaker	S1	S2	S3
M1	2	0	3
M2	0	1	0
M3	0	0	0

Hearer	S1	S2	S3
M1	4	0	1
M2	0	3	0
M3	0	0	0

Table 3.2: *Interpretation-recorders and uncovered meanings*. An example of a speaker-hearer pair where a sub-optimal state has been reached where one meaning is not *covered*. The speaker has no associations for M3, and will send any of the its repertoire of signals with uniform probability. The hearer will never interpret any signal as M3, and hence is incapable of recording a new exemplar associated with M3. This situation can only be remedied by including a bias against synonyms. Not that this situation would be the case if speaker and learner changed positions, and that no difference in behaviour would be expected between WTA and stochastic agents.

Finally, for interpretation-recorders, it is necessary for inhibition to apply to meanings which have been presented in the same context, as can be seen in Fig. 3.3. If within-context meanings are *not* inhibited, too much ambiguity is allowed to remain within the system for the development of optimal signalling: this may be due to the fact that meanings in this parameter set-up are more likely to appear within a context than without, hence the chance of reinforcement is always higher than the chance of being inhibited. Also, when inhibition does not occur within-context it may allow for pairs of homonyms to develop which remain stable over time.

Obverter Learners

Obverter learners can also be separated into context-recorders and interpretation-recorders. For obverters, however, the second strategy requires an extra consideration. When we looked at the models with explicit meaning transfer in Spike et al. (2016), we observed that only obverter *production* was a factor in the development of optimal signalling. As discussed earlier, this is because the gradual change in a signalling system towards optimality relies upon the production of new meaning/signal associations. When meaning transfer is explicit, it is the process of production which is entirely responsible for the creation of new associations. When there is indeterminacy of meaning, however, and interpretation is used as the basis of creating new exemplars, we must consider whether obverter reception might now play a role.

As such, the main parameters which I want to examine are whether obverter strategies lead to optimal signalling, and to re-confirm previous results that this also requires some form of information loss (either vertical learning or a limited memory). Additionally, we need to see whether it is necessary to confine obverter production to only those meanings which are present in the context.

Our results are as predicted: all obverter models which were able to create functional signalling systems required some form of information loss, whether this was a limited memory or vertical transmission. Also as expected, context-recorder obverters resemble the biased learners in that only WTA production leads to optimal signalling. Once again, this is a simple consequence of the fact that several exemplars are added with each exposure, meaning that stochastic production/reception will never lead to one-to-one mappings.

Staying with context-recorder obverters, another (perhaps surprising) result is that observational learning in which only the hearer is modified — as has been the case for all previous observational models — does *not* drive the development of optimal signalling. However, as can be seen in Fig. 3.4, if the speaker is also modified after every interaction to record an exemplar by associating the signal with its *intended meaning*, it does reliably drive the emergence of signalling sys-

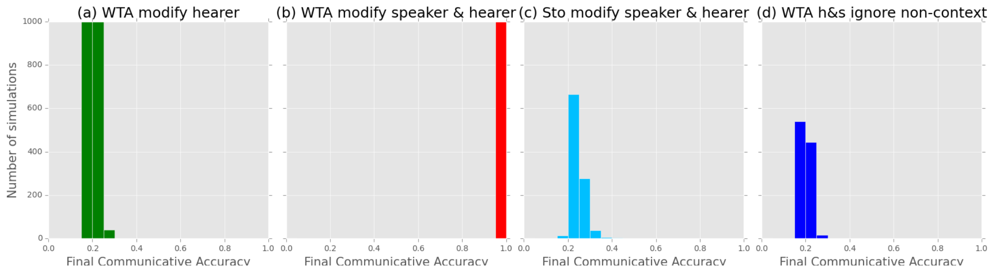


Figure 3.4: Context-recorder obverter learners with a memory limit of 150 exemplars (a larger memory is necessary because every meaning in the context is recorded), demonstrating for WTA agents the difference between modifying only the hearer after an interaction (a), or modifying both speaker and hearer (b). Stochastic agents (c) are once again incompatible with the context-recorder strategy, while (d) shows the necessity of using an obverter which considers possible meanings which are *not* within the provided context.

tems. This particular configuration is actually the same as the learners in Vogt & Coumans (2003). Exactly why this is the case is not clear, but it seems that the obverter bias against ambiguity is not strong enough to promote one-to-one mappings without the extra nudge provided by this type of speaker update. This intuition will be further explored in the next chapter.

This seems to be supported by the fact that for interpretation-recorders, only modifying the hearer is necessary. As seen in Fig. 3.5, as long as a type of information loss exists optimal signalling will reliably develop. This occurs over a longer time-scale than would be the case without referential uncertainty, but this is unsurprising given the extra noise and impoverished information transfer that characterises these models.

For both context-recorders and interpretation-recorders, a novel result (illustrated in Fig. 3.5) is that it is important for the obverter producer to *not* restrict themselves to only the meanings present in the context, but consider all possi-

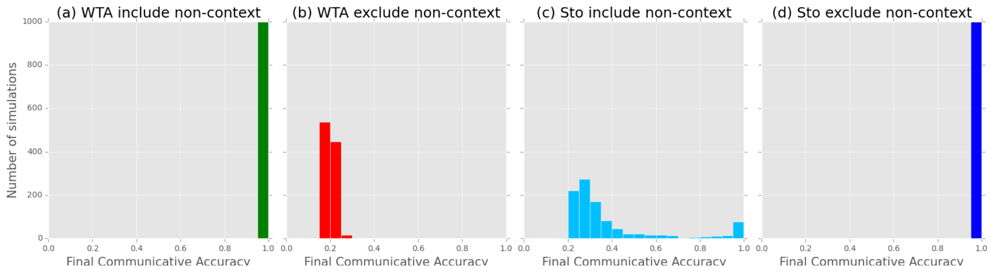


Figure 3.5: Interpretation-recorder obverter learners with a memory limit of 35 exemplars, showing that WTA obverters which do not consider meanings outside of the current context (a) are unable to reliably develop optimal signalling, but WTA obverters which do consider out-of-context meanings (b) are able to do so. Stochastic obverters (c) and (d) show the same pattern, but take significantly longer.

ble interpretations of their signal. In purely communicative terms this may seem highly unintuitive: as can be seen in Table 3.1, different contexts can make for crucial changes in interpretation. To understand why it is important to consider *all* possible meanings, we should keep in mind that an optimal system is defined across *all possible* contexts: while the obverter procedure which restricts itself to a particular context may be more effective at guaranteeing communicative success for that particular instance, it does not drive the *overall* signalling system towards optimality. To expand upon this: in standard referential signalling games, an obverter is not just trying to provide the most communicative signal for a given meaning. What obverter production really provides is information about the optimal signal for other agents to use to communicate the topic meaning to itself. In essence, the procedure is as much a form of *teaching* as it is *learning*. When reference is uncertain, the intended topic is now uncertain. If the obverter only takes into account the current context, it is equivalent to saying 'given *this particular* context, this is the best signal to communicate my intended meaning', as opposed to; 'given *any* context, here is the best way to communicate my intended meaning'. For this reason, ignoring the current context is actually an optimal strategy.

3.2.4 Discussion

Clearly, referential uncertainty does not require a radical rethink about which strategies lead to the development of optimal signalling. Besides the fact that it narrows the field to models of observational learning, the same requirements remain: the transfer of referential information, a bias against ambiguity, and a form of information loss. The role of information loss remains unchanged: this is provided by lateral inhibition in the case of biased learners, and a limited memory or vertical learning for obverters.

One place we see a slight difference is that a straightforward bias against ambiguity is, by itself, no longer a sufficient criterion of optimality. This manifests in different ways particular to each model. For context-recorders, whether the biased learners or obverters, this is because the act of adding multiple exemplars blocks the ability of the system to successfully settle on one-to-one mappings. Biased learners and obverters tackle this problem in necessarily different ways: biased learners must increase the effect of lateral inhibition to compensate. Obverters have no mechanism of deletion except for information loss, and so must compensate in the opposite way: the *speaker* simultaneously strengthens the one-to-one association between the signal and the intended meaning. In any case, both strategies lead towards the reliable development of optimal signalling.

For interpretation-recorders the pattern is slightly different. Biased learners can fall into a trap where certain meanings are left 'uncovered', i.e. they will never be the interpretation of any given signal. This is a case where, unlike in earlier models, synonymy is detrimental to communication. Because of this, the lateral inhibition of synonyms becomes an important factor for these models. By doing away with shared labels, they free words up which can be used for meanings which have been left uncovered. Interpretation-recorder obverters, on the other hand, are able to deal with this problem for the simple reason that information loss is already necessary for obverters: when multiple signals are spread out over a single meaning, there are less exemplars for each signal/meaning association than for meanings which have no synonymy. As such, they are a much more likely candidate for total loss at some point, freeing them up again for an uncovered meaning.

The most obvious difference between these models and those of Spike et al., of course, is that the amount of referential information available in these models is — by definition — less. Context-recorders and interpretation-recorders deal with this in seemingly different ways. In the former case, agents record all available information about the possible associations between meanings and signals, and then bootstrap this information by relying on mechanisms similar to the cross-situational learning algorithms presented in work such as Blythe et al. (2010). Although it might seem that interpretation-recorders jettison this same information when they record only their interpretation, which is quite likely to be incorrect. One way of understanding this is to first look at a simpler case: how can an *already* optimal shared signalling system remain stable? Agents will always use an unambiguous signal for the topic regardless of the context; that association will continue to be reinforced and the system will persist. How could a *new* agent learn this optimal system via interpretation-recording? Basically, the correct associations are more likely to be strengthened over time than incorrect ones: they will be in more of the presented contexts. Once we factor in information loss, there is surprisingly little to separate the two strategies. Context-recorders use all the available data to infer the most likely hypothesis, whereas interpretation-recorders use less memory to keep track of the best current hypothesis.

A final observation related to interpretation-recorders is that as long as the method of learning includes some bias against ambiguity — whether that is lateral inhibition or the obverter strategy — along with a way of ensuring that all meanings are covered (i.e. anti-synonymy or obverting) individual agents will be able to settle on *personal* vocabularies which are completely optimal even when no referential information is *ever* supplied, i.e. when the size of the context is equal to the number of meanings. The catch, of course, is that none of the personal systems are ever likely to be shared by other agents. This fits in well with the conclusions of the previous chapter: only two of the ingredients required for the development of optimality are present, namely the bias against ambiguity and information loss, while referential information transfer is not.

We can now look at the extra features which are required to allow observational learners to construct optimal signalling in the face of referential uncertainty. For biased learners and for obverters, we have seen that both the context-recording and the interpreter-recorder strategies will lead to reliable development of signalling, but with certain provisos. For context recorders using biased learning, inhibition must be strong enough to overcome the increase in the number of exemplars. Obverters using the same strategy cannot overcome the weight of the extra exemplars through inhibition: the only strategy left open is for *speakers* to solve the problem by strengthening their intended association. For interpretation recorders, on the other hand, the story is slightly more involved: biased learners must inhibit both homonyms *and* synonyms, and meanings within the presented context must be the target of this inhibition. The context plays another role for obverters, but in a seemingly counter-intuitive manner: optimal signalling only develops when obverters (during production) essentially behave as if there were no context.

How can we draw this together? The seemingly disparate features outlined above all have one thing in common: they counteract referential uncertainty and drive the emergence of *globally unambiguous* signals, and provide a solution to the problem of how one-to-one mappings can emerge when they are never explicitly available. As stated before, context-recorders and interpretation-recorders both rely on a simple inference from data, but just time this inferential step differently. As for the other necessary features, they all ensure that the inference is driven towards one-to-one mappings from signal to meaning.

Finally, as with the previous section, I'd like to stress that all of these accounts are feasible. Once again, it is the shared features which are worth emphasising. In fact, all of these are a modification of the bias against ambiguity. When reference is uncertain, we further require 1) a way of *inferring* unambiguous mappings from ambiguous data, and 2) a production mechanism which *minimises* the ambiguity of any given context. In other words, speakers need to create and select signals which are explicit as possible, and hearers need to be sensitive to the speaker's strategy.

Chapter 4

Information dynamics of learned signalling

4.1 Overview

My aim in this chapter is to cast the three requirements (referential information, bias against ambiguity, and information loss) presented in previous chapters in a more abstract, formal way. My motivation for this is *generality*.

The requirements are observations about the behaviour of computational models. A criticism often levelled against models like this (or modelling in general) is that there is always at least some element to their design which is an arbitrary choice. Despite the purpose of the exemplar/urn-model framework being to reduce this as much as possible, it is hard to prove that any or all of the requirements might be *specific* to some aspect of that model. For example, I have previously referred to the fact that urn-models use a type of Roth-Erev learning (Roth & Erev, 1995): Skyrms (2010) himself devotes some time to demonstrating their difference from another popular type of *parametric*[1] reinforcement learning,

[1]This is where learning directly affects the probability distributions which describe agent be-

namely that of Bush & Mosteller (1953). A very reasonable question to ask is whether the requirements still hold for a framework based around that type of learning. One response to this would be to repeat all of the previously detailed experiments around a new framework. This not only requires time, but wouldn't avoid the problem: any new proposed type of learning (or any other adjustment to the model) would again threaten to invalidate the observations. What is needed is a way to phrase the requirements in a general way which can avoid having to repeat the experiments ad infinitum.

In this chapter, I'll propose a simpler way of capturing the dynamics of *any* population of learning signallers, provided they are modelled as a single set of meaning/signal associations. My main tools in doing this are borrowed from *information theory*. Despite its relevant-sounding name, information theory has proven to be ill-suited for any theory of human communication, because it has no way of describing semantic/referential content (Shannon & Weaver, 1964), as we shall see later in this chapter. In this case, I am able to avoid this problem by enforcing a small number of extra constraints which apply specifically to signalling games. This allows me to construct a simplified state-space based on the idea of *conditional entropy*, and then show that we can represent the current state of any population of signalling agents as one point in this space. I then show how different types of learning and interaction determine the way in which the population navigates this space: in fact, the overall dynamic of any given model can be determined entirely by the *mean-field* effect of an interaction between two agents specified within that model. In this way, the *micro-level* — the basic *pairwise* interaction — can be used as a diagnostic of the global, *macro-level* dynamic. By doing this, I am able to cast the three requirements in more technical terms, namely:

haviour. In Roth-Erev learning, the probability distributions are produced as a function of other data, i.e. the relative numbers of exemplars.

1) agents must *imitate* with positive probability:, 2) pairwise interactions must reduce individual meaning/signal *conditional entropy*, and 3) individual learning rates must retain *plasticity*.

This chapter will proceed as follows: firstly, I will provide an overview of some dynamical analyses which have been applied to signalling games in the past, which has mainly (but not entirely) taken place within disciplines which use game-theoretic methodologies, both classical and evolutionary. In Section 4.2.1, I take a quick look at how the conventional way of measuring the optimality of a population's signalling system (*communicative accuracy*) does not lend itself to this kind of dynamical analysis. In Section 4.3, I provide an account of some basic information theoretic terms, and how they have been applied in the past to problems of human communication. After some preliminary definitions, I will then detail how we can construct the state-space, how we can situate the current state of a population as a point within that space, and how we can avoid the usual problems with information theory's indifference to reference. Next, in Section 4.5, I rephrase the first two requirements in terms of the dynamics of this space. Specifically, I will show that 1) referentially signalling agents which *imitate* each other in a specific way will always converge on a set of globally-shared signalling conventions, and 2) the mean-field pairwise interaction specified by any model will push the state towards certain parts of this space representing greater or lesser optimality. I will also tackle the *plasticity problem*, i.e. how information loss determines whether the state-space is actually navigable. Section 4.7 provides a visualisation of the space, the dynamics of processes like drift and bias, and a depiction of the non-uniform structure of that space. Finally, I show how the new criteria can be put into action in Section 4.8, and give an example of how we can analyse the global behaviour of *any* model in terms of the way it defines individual interaction and learning.

4.2 The dynamics of learned signalling

In game-theory, a typical concern is trying to discern whether optimal strategies exist for one or more players. For some types of game, for example *signalling games*, there is also interest in finding the most *mutually beneficial*[2] equilibria. However, as Huttegger et al. (2014, p.10874) point out, it is not enough to simply identify these optimal equilibria and then 'rely on faith' that they will be reached: we need to consider the dynamics of those games. What is true of the game-theoretic models is true of all models of the emergence of signalling: as we have seen in the previous chapter, even some configurations which superficially appear to contain the necessary requisites to ensure reliable optimal signalling do not. Similarly, we sometimes see models which do attain optimality, but only some of the time. It should be possible to determine the behaviour of models *prior* to running an exhaustive set of simulations: to do this, we need to have some understanding of the overall dynamics.

Huttegger et al. (2014) survey a number of dynamical analyses of game-theoretic signalling games which involve both the *replicator dynamic* and reinforcement learning. Despite the rich literature associated with the replicator dynamic, our focus on associative learning agents restricts our focus to the latter. Game theoretic treatments usually begin with analyses of the basic Lewis signalling game (one signaller and one receiver, two each of world states, signals and actions). Argiento et al. (2009) proves (for Roth-Erev reinforcement learners) that all games will converge on either a signalling system or a sub-optimal pooling equilibrium, and that only signalling systems are long-term stable: optimal signalling is guaranteed. Unfortunately, this neat result doesn't seem to hold for more agents, signals, meanings, states or actions: Huttegger et al. (2014, p.10879) state; "The case of $M = N = 2$ is very special". There is, as of yet, no dynamical analysis

[2] Also known as *Pareto Optimal*

which shows that pooling equilibria are not stable, and hence no way to ensure the emergence of optimality for reinforcement learning.

The Naming Game (Steels, 1997) has also been subjected to a number of dynamical analyses. As discussed in the previous chapter, however, the Naming Game focusses on how optimal vocabularies can be reached when there is an open signal space: the main consideration is how *synonymy* can be eliminated, and these analyses do not consider the possibility of homonyms. The proof in De Vylder & Tuyls (2006) shows that populations of interacting agents which employ an *amplifying function* will always settle on a minimal vocabulary, free of synonymy. Amplifying functions are defined as follows: if a given meaning m produces a set of signals according to probability vector $P_i = \langle p_1, \dots p_n \rangle$ ordered such that $p_k \geq p_{k+1}$ (i.e. in decreasing order of probability), then an amplifying function between the input vector P_i and the output vector P_o is such that ratio between any p_k / p_{k+1} is always larger in the output than the input. More succinctly, the ratio between the relative frequency of any signal and the next most frequent will always increase in the output, as can be seen in Table 4.1.

Input p_i	p_i / p_{i-1}	Output p_o	p_o / p_{o-1}
0.5	1.67	0.6	2
0.3	1.5	0.3	3
0.2	n/a	0.1	n/a

Table 4.1: An example of an amplifying function, where Input probabilities P_i are mapped to Output probabilities P_o. Note that individual probabilities can be increased, remain the same, or be decreased: amplification requires that the ratio between any probability and the one with the next lower rank increases between the input and output.

Other work investigating the dynamics of the Naming Game looks at the impact of different types of feedback on convergence rates, for example Baronchelli (2010), or the effects of different update strategies and network topologies (as surveyed in Loreto et al., 2011). As with De Vylder & Tuyls, the studies are also restricted to versions of the Naming Game without homonymy, and so do not directly address our topic, the issue of reliable optimality.

Beyond these studies, there has been no more general attempt to study the dynamics of populations of learning agents bootstrapping signalling systems. This is probably largely for historical reasons. For example, in game-theory, dynamical analysis tends to be the domain of either economists or evolutionary game theorists. As observed by Mühlenbernd (2013), these both take a *macro-level* perspective of populations employing *pure strategies*. For example, populations employing the *replicator dynamic* switch between a predefined set of signalling strategies and, critically, this requires perfect information about the current state of the whole population. The overall dynamic of such systems is directly driven by the macro-state of that system. Reinforcement learning, on the other hand (and all of the other models we have considered so far in this thesis), focuses on *micro-level* interactions which cause incremental adjustments to individual *behavioural strategies*. The numerous tools which are used to analyse the macro-level dynamics do not apply to the micro-level. Although there is evidence that some forms of reinforcement learning may converge to the replicator dynamic under certain conditions (Bergers & Sarin, 1995), this is likely to only hold for certain types of reinforcement learning (Beggs, 2005). For this reason, we are not able to employ these tools with any confidence. On the other hand, the statistical physicists who study Naming Game dynamics certainly are interested in the interaction between micro-level and macro-level processes. However, their focus (understandably) tends to be on things like convergence rates, evidence of symmetry-breaking and phase-transitions, and the effects of network topologies.

Our criterion is simpler than that of either the game-theoreticians or the statistical physicists: does a given model reliably converge upon a optimal signalling?

This means that our focus is simpler than in game-theory, which is concerned with various sub-optimal states, and the physically-inspired analyses of the Naming Game. This is the motivation for creating a space which is directly constructed around a micro-level interpretation of optimality. This allows us to chart a population's position within that space, and determine whether a model will *always* be drawn towards optimality. Before we move onto this, I need to address the issue of *communicative accuracy*.

4.2.1 Communicative accuracy

If we're looking for a measure of optimal signalling, why not use the ubiquitous *communicative accuracy*? It is the measure which is traditionally used in the field, and the one we have been using up until now. There are simple and computationally efficient ways to calculate it, even for large populations. However, *CA* is not a useful index for calculating the overall dynamic of a population of learning agents. This is because *CA* is a *macro-level* measure, instead of a *micro-level* one, as we shall see.

As an illustration, we can look at how *CA* would define an internally optimal language for a single agent. The definition of optimality from Chapter 1 is:

$$\forall\, m \in M\; \exists\, s \in S \;\; s.t.\;\; p(m|s) = 1.0 \tag{4.1}$$

i.e. all meanings are covered by at least one signal and all signals are unambiguous. We *visualise* optimality in terms of the formula for *CA* by using matrices for production $P(S|M)$ and reception $P(I|S)$ as illustrated in Fig. 4.1. The trivial definition of an optimal language, then, is any two production and reception matrices which lead to perfect interpretation, represented here by an identity matrix[3]

[3] An identity matrix is in which every entry on the leading diagonal is 1, and all other values are 0.

$$P(S|M) = \begin{array}{c c} & \begin{array}{ccc} s1 & s2 & s3 \end{array} \\ \begin{array}{c} m1 \\ m2 \end{array} & \begin{pmatrix} 0.5 & 0 & 0.5 \\ 0 & 1 & 0 \end{pmatrix} \end{array}, \quad P(I|S) = \begin{array}{c c} & \begin{array}{cc} i1 & i2 \end{array} \\ \begin{array}{c} s1 \\ s2 \\ s3 \end{array} & \begin{pmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 0 \end{pmatrix} \end{array}$$

$$P(I|M) = P(S|M) \cdot P(I|S) = \begin{array}{c c} & \begin{array}{cc} i1 & i2 \end{array} \\ \begin{array}{c} m1 \\ m2 \end{array} & \begin{pmatrix} \mathit{1} & 0 \\ 0 & \mathit{1} \end{pmatrix} \end{array} \longrightarrow \quad P(I = M) = 1.0$$

Figure 4.1: An example of an optimal signalling system (with some synonymy). Using a single production and a single reception matrix, we take the product of the two and then take the mean of all values on the diagonal, assuming a uniform distribution over $P(M)$.

on $P(I|M)$. Calculating CA is a multi-stage process which must consider both production and reception behaviour together, both of which are derived from an internal set of signal/meaning associations. The important point is that, even for a single agent, optimality defined in terms of *CA* is an *emergent* as opposed to an *internal* property: it requires a sequence of calculations, as opposed to being a lower level description, i.e. some clear property of the agent itself.

We can see another aspect of the problem when we consider an agent with an internally *sub-optimal* system. Looking at Fig. 4.2, consider the question of *how can this system become more optimal*. An answer of 'maximise $P(I = M)$' has no explanatory power: as already discussed, *CA* is a higher-level measure, and not itself an adjustable property of the system. Moving down a level, we run into more problems: 'adjust the production and reception matrices so that $P(I = M)$ is maximised' is just as uninformative: even this minimal production/reception system has four degrees of freedom[4] — which should be adjusted? Moreover,

[4]Because both the production and reception matrices are normalised over rows, adjusting one

$$\text{Learned associations} = \begin{array}{c c} & \begin{array}{cc} s1 & s2 \end{array} \\ \begin{array}{c} m1 \\ m2 \end{array} & \begin{pmatrix} 1 & 3 \\ 4 & 2 \end{pmatrix} \end{array}$$

$$P(S|M) = \begin{array}{c c} & \begin{array}{cc} s1 & s2 \end{array} \\ \begin{array}{c} m1 \\ m2 \end{array} & \begin{pmatrix} 0.25 & 0.75 \\ 0.67 & 0.33 \end{pmatrix} \end{array}, \quad P(I|S) = \begin{array}{c c} & \begin{array}{cc} i1 & i2 \end{array} \\ \begin{array}{c} s1 \\ s2 \end{array} & \begin{pmatrix} 0.2 & 0.8 \\ 0.6 & 0.4 \end{pmatrix} \end{array}$$

$$P(I|M) = P(S|M) \cdot P(I|S) = \begin{array}{c c} & \begin{array}{cc} i1 & i2 \end{array} \\ \begin{array}{c} m1 \\ m2 \end{array} & \begin{pmatrix} \mathit{0.5} & 0.5 \\ 0.33 & \mathit{0.67} \end{pmatrix} \end{array} \longrightarrow \quad P(I = M) = 0.58$$

Figure 4.2: An example of an internally *non-optimal* signalling system. Starting with the set of internal, learned associations, we derive (using a stochastic function) a production and reception matrix, take the product of the two and then take the mean of all values on the diagonal, assuming a uniform distribution over $P(M)$.

although we can calculate the effect of any adjustment on the *CA*, there is no immediate correspondence between the lower-level adjustments and the higher level *CA* measurement.

However, it *is* possible to describe optimality using agent-internal properties instead of the higher-level *CA* measure. As can be seen in Fig. 4.3, for a stochastic agent to have an internally-consistent optimal language, each signal must associate only with a single meaning, and each meaning must be associated with at least one signal. Winner-take-all agents are slightly more complex: each signal

value simultaneously adjusts the other value in that row, leaving two degrees of freedom per matrix.

$$
\text{(A)}\;\begin{array}{c}\\ m1\\ m2\\ m3\end{array}\!\!\begin{array}{c}\begin{array}{ccc} s1 & s2 & s3\end{array}\\ \begin{pmatrix} \boldsymbol{22} & 0 & 0\\ 0 & 0 & \boldsymbol{15}\\ 0 & \boldsymbol{19} & 0\end{pmatrix}\end{array}
\qquad
\text{(B)}\;\begin{array}{c}\\ m1\\ m2\\ m3\end{array}\!\!\begin{array}{c}\begin{array}{ccc} s1 & s2 & s3\end{array}\\ \begin{pmatrix} \boldsymbol{22} & 12 & 7\\ 11 & 9 & \boldsymbol{15}\\ 17 & \boldsymbol{19} & 13\end{pmatrix}\end{array}
\qquad
\text{(C)}\;\begin{array}{c}\\ m1\\ m2\\ m3\end{array}\!\!\begin{array}{c}\begin{array}{ccc} s1 & s2 & s3\end{array}\\ \begin{pmatrix} \boldsymbol{22} & \mathit{21} & 7\\ 11 & 9 & \mathbf{15}\\ 17 & \mathbf{19} & \mathit{18}\end{pmatrix}\end{array}
$$

Figure 4.3: Internal signal/meaning associations which map to (A) an optimal language for a stochastic agent, (B) an optimal language for a WTA agent, and (C) a non-optimal language for a WTA agent (or stochastic agent). Numbers in **bold** text are the maximum values for rows (meanings), numbers in *italics* are the maximum values for columns (signals). (C) is non-optimal because the reception system would not cover $m2$, despite the fact that there is no homonymy.

must associate with only a single maximum-value meaning, and that association must also be a maximum value for that meaning. In both cases, we're looking for matrices which will be converted by production and reception functions into stochastic matrices that have the *optimal characteristic* which has just been outlined.

The stochastic matrices we've been looking at, such as in Fig. 4.1, show the conditional probability of signals given meanings, or vice versa. The *optimal characteristic* is easy to recognise for reception matrices and can be seen in the matrix for $P(I|S)$: every signal-row must be populated by only zeroes and a single 1, and every meaning-column must contain at least one 1. Another way of saying this is that **every row of the matrix must be within at least one subset *permutation matrix***. A permutation matrix is any row or column-based shuffling of the identity matrix, as can be seen in Fig. 4.4. However, for reasons which will be explained in a subsequent section, we need to focus on a type of matrix which is derived from production, instead of reception, but all of the requirements for the optimal characteristic are the same as just described.

$$
\text{(A)}\quad
\begin{array}{c}
\\ s1 \\ s2 \\ s3
\end{array}
\begin{array}{c}
\begin{array}{ccc} i1 & i2 & i3 \end{array} \\
\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}
\end{array}
\qquad
\text{(B)}\quad
\begin{array}{c}
\\ s1 \\ s2 \\ s3
\end{array}
\begin{array}{c}
\begin{array}{ccc} i1 & i2 & i3 \end{array} \\
\begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}
\end{array}
\qquad
\text{(C)}\quad
\begin{array}{c}
\\ s1 \\ s2 \\ s3 \\ s4 \\ s5
\end{array}
\begin{array}{c}
\begin{array}{ccc} i1 & i2 & i3 \end{array} \\
\begin{pmatrix} 0 & 0 & \mathbf{1} \\ 0 & \mathbf{1} & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \\ \mathbf{1} & 0 & 0 \end{pmatrix}
\end{array}
$$

Figure 4.4: Stochastic matrices: (A) is the *identity matrix*, and (B) is a shuffled identity matrix, or *permutation matrix*. (C) is a matrix with the *optimal characteristic*, as every row can be incorporated into a subset which forms a permutation matrix: the characters in bold give an example of one of these subsets.

So, how do we move *towards* optimality? As summarised in the previous chapter, the key factor is that homonyms are suppressed while ensuring meanings are still covered. In reference to Fig. 4.3, internally optimal systems will look something like (A) or (B), mapping to matrices with the optimal characteristic. How do we describe this operation in terms of *CA*? We cannot: once again, communicative accuracy is an *emergent* measure, whose calculation requires all of the steps described above. This is again is the reason why describing the dynamics requires a different type of measure. A measure which relies on only agent-internal associations would give a direct, *micro-level* perspective, unlike the *macro-level* one given by *CA*.

This macro-level property of *CA* also renders it unsuitable for describing *population-level* dynamics. When a population has an optimal signalling system, it means not only that every agent has an internally optimal system, but also that all agents share that system. To chart the dynamics of how a population reaches consensus on one system, we need to be able to track how *similar* individual agents are to each other. Once again, this is not possible using *CA*. Not only is it a macro-level measure, but the two aspects of optimality — internal optimality and population consensus — are conflated within it. This is the second reason for using a micro-level, agent-focussed measure.

4.3 Information theory and human communication

Modern *information theory* (Shannon, 1948) is a description of the relationships between probabilistic data, channel capacities, compression, and noise. All of these are deeply implicated in human communication, and tools from information theory have been applied to problems connected to human language since they first became available. This has led to numerous advances such as efficient and robust transmission codes, new forms of encryption and code-breaking, and data-compression, amongst many others. However, as pointed out early on by Shannon & Weaver (1964), information theory is incapable of describing *meaningful communication*: it is blind to any type of *semantic content* or *reference*. To understand why, we first need to provide a brief sketch of basic information theory.

4.3.1 Basic information theory

Probability lies at the core of information theory. We can describe a system as a collection of states, and then assign a probability to each state. By doing this, we can then quantify the total amount of uncertainty associated with that system, or *entropy*. Despite its name, the fundamental measure in information theory is entropy, not information. Information is gained when entropy — uncertainty — is *lost*. As an illustration, picture a fair coin. Before flipping the coin, we know that the result will be one of two states, heads or tails. The probability of heads, $P_h = 0.5$, and of tails, $P_t = 0.5$. The entropy of a system, H, is traditionally measured in *bits*, using the base-2 logarithm:

$$H = -\sum_{p_i \in P} p_i \; log_2 \; p_i \tag{4.2}$$

So, a one-fair-coin system has an entropy $H = -(\frac{1}{2}\ log_2\ \frac{1}{2}) - (\frac{1}{2}\ log_2\ \frac{1}{2}) =$ 1 bit exactly. This understanding of information means that as the probabilities of individual states shift, so does the entropy of the system. For example, if a coin is 99% biased towards heads, there is very little uncertainty about the result of a flip, and the entropy is very low ($H = 0.014$ bits). Likewise, if there are more states, the entropy increases: in both the case of four independent coin flips and in the case of a fair 16-sided dice, the total entropy $H = 4$ bits: each system has sixteen equally likely resulting states.

Conditional entropy is related to the idea of conditional probability. For example, we'll use some data from a month of summer weather in Scotland. The weather in August, W, was recorded, with S denoting a sunny day, and R a rainy day:

$$W = \langle RRSRRRSRSRRRSRRRSSSRRRRSRRRSRSR \rangle$$

First of all, we can calculate the overall probability of a sunny day, $P(S) = \frac{1}{3}$, and a rainy day, $P(R) = \frac{2}{3}$ (ignoring the final day of the month for reasons which will become clear shortly). We can also use the data to create a matrix which tells us how many rainy days R_t and sunny days S_t were followed by rainy days R_{t+1} or sunny days S_{t+1}, and use this to build the conditional probability matrix showing the probability of tomorrow's weather given today's (Fig. 4.5).

Now for some entropy measures. The *overall* entropy of weather in August, $H(W) = -\frac{1}{3}\ log_2\ \frac{1}{3} - \frac{2}{3}\ log_2\ \frac{2}{3} = 0.92$ bits. But we can also use the table of conditional probabilities to calculate the entropy of weather tomorrow given a sunny day, $H(W_{t+1}|S_t) = -.2\ log_2\ .2 - .8\ log_2\ .8 = 0.72$ bits, or given a rainy

$$
\begin{array}{c c}
 & \begin{array}{cc} S_{t+1} & R_{t+1} \end{array} \\
\begin{array}{c} S_t \\ R_t \end{array} & \begin{pmatrix} 2 & 8 \\ 8 & 12 \end{pmatrix}
\end{array}
\longrightarrow
\begin{array}{c c}
 & \begin{array}{cc} P(S_{t+1}) & P(R_{t+1}) \end{array} \\
\begin{array}{c} P(S_t) \\ P(R_t) \end{array} & \begin{pmatrix} .2 & .8 \\ .4 & .6 \end{pmatrix}
\end{array}
$$

Figure 4.5: The conditional probability matrix showing the probability of tomorrow's weather given today's.

day, $H(W_{t+1}|R_t) = -.4\ log_2\ .4 - .6\ log_2\ .6 = 0.97$ bits.

Now, if we want to calculate the *overall* conditional entropy of the weather tomorrow given that we know today's weather, we need to combine the entropies given a sunny day and a rainy day. Now, if there were an equal probability of sunny or rainy days, we would just take the mean of the two values. But as rainy days are twice as likely as sunny ones, we multiply each entropy value by its overall probability (this is why we didn't use the final day of the month, as it is never W_t but only W_{t+1}, i.e. it is never a predictor of tomorrow's weather), and sum those values:

$$H(W_{t+1}|W_t) = (\frac{1}{3} \times 0.72) + (\frac{2}{3} \times 0.97) = 0.89 \text{ bits}$$

The example we've just worked through can be stated more formally:

$$H(X|Y) = -\sum_{y \in Y} p(y) \sum_{x \in X} p(x|y)\ log_2\ p(x|y) \tag{4.3}$$

Note that the conditional entropy, 0.89 bits, is less than the overall entropy, 0.92 bits. In fact, the conditional entropy is *never* larger than the overall entropy. When we add extra conditions, in this case what the weather was today, it can never increase uncertainty, only decrease it or not affect it. Information is strictly *additive*. Formally:

$$H(X|Y) \leq H(X) \tag{4.4}$$

4.3.2 Information theory and reference

How can we apply this to communication? Shannon (1948), was primarily concerned with how much information could be transferred over a *communication channel*. Consider the communication system in Fig. 4.6. There are two equally likely messages, hence $H(M) = 1$ bit. An optimal communication system implies that when I select a message and send a signal, it picks out a interpretation with no uncertainty, i.e. $H(M'|M) = 0$.

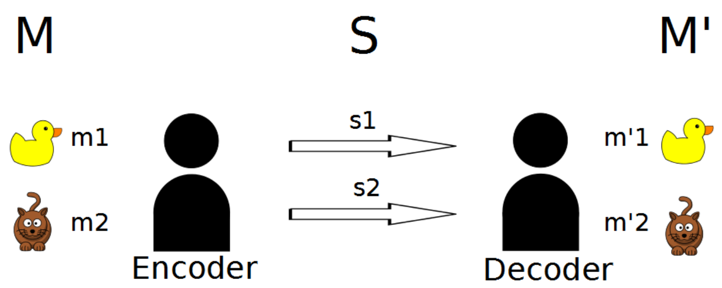


Figure 4.6: A simplified communication system. There are two messages, $m1$ and $m2$, which must be encoded and sent via one of two possible signals, $s1$ and $s2$, to be decoded into one of two interpretations, $m'1$ and $m'2$.

We now see the fundamental problem with using information theory to describe referential communication: *there is no connection between the initial message and its interpretation*. For example, while it is certainly true that the following communication system preserves reference, i.e.

$$m1 \rightarrow s1 \rightarrow m'1 \quad \& \quad m2 \rightarrow s2 \rightarrow m'2, \quad H(M'|M) = 0,$$

the same zero conditional entropy measure is *also* true of communication systems which are incorrectly interpreted 100% of the time!

$$m1 \rightarrow s1 \rightarrow m'2 \quad \& \quad m2 \rightarrow s2 \rightarrow m'1, \quad H(M'|M) = 0$$

In fact, there is no requirement for the messages and interpretations to have the same identity at all. Fig. 4.7 displays an information theoretically optimal system, but a nonsensical one in terms of referential human communication.

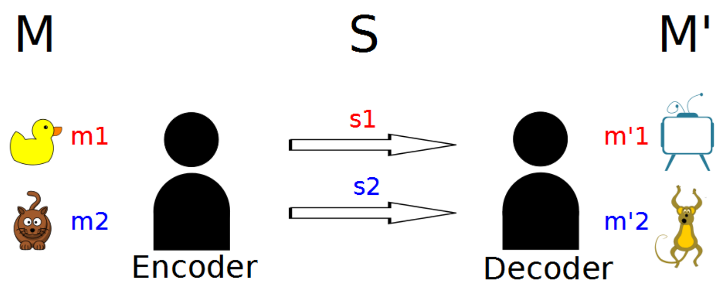


Figure 4.7: An 'optimal' communication system without any referential transfer. Although the interpretations are completely specified by the intended message, e.g. transmitting the message 'cat' will always be interpreted as 'angry monkey', the intended referent is never transmitted.

In fact, the semantic properties of communication were never part of the mathematical theory of communication as laid out by Shannon (1948). Weaver (1964) defines three levels of communication problems: (a) the technical problem, (b) the semantic problem, and (c) the effectiveness problem. Weaver stresses that Shannon's theory, by design, applies only to the first, technical level: in a final section, however, he discusses his hopes that a similarly sophisticated theory of meaning would soon be added to the framework. This did not prove to be the case.

There have been some more recent forays into incorporating meaning into information theory. Plotkin & Nowak (2000) consider the emergence of meaning in an evolutionary context, however they concentrate on how robust error-correcting codes can emerge under a particular fitness function. The methodology of the paper does not employ entropy or any related terms, and so does not address the problem of reference within information theory. Corominas-Murtra et al. (2014), on the other hand, focus exactly on this issue, and adapt standard measures of entropy and information to encompass referential meaning. Their paper demonstrates some interesting consequences of considering what they term *consistent information*, i.e. information which preserves reference. For example, whereas the entropy and information of a system are symmetric around their maximally uniform distribution, this is not the case for consistent information. However, while their exploration of the implications for reference of information theory is thorough, the methodology Corominas-Murtra et al. employ to measure referential transfer does not go as far as incorporating reference in a mathematically precise way: the crucial term they use is a direct equivalent of CA, the same external measure that doesn't provide us with an ability to track the micro-level dynamics of the system. The aim of the next section is to outline a measure which is able to do this.

4.4 Dynamics

We are interested in whether models reliably develop optimal signalling, which corresponds to $CA_{pop} = 1$. As discussed in Section 4.2.1, there is no immediately obvious way of tracking the dynamics of this process using CA.

To illustrate, we start by defining the change in communicative accuracy between the speaker and hearer in their current states sp, hr and their states after their interaction;

$$\Delta CA(sp,hr) = CA(sp',hr') - CA(sp,hr)$$

What we want to show is that the *mean-field expected change* in CA is positive, i.e. that we can expect the communicative accuracy to steadily increase between any two agents:

$$E[\Delta CA(sp,hr)] > 0 \tag{4.5}$$

But more than this, we also want to guarantee that this micro-level process leads to a macro-level increase in the communicative accuracy over the whole population:

$$E[\Delta CA(sp,hr)] > 0 \rightarrow E(\Delta CA_{pop} > 0) \tag{4.6}$$

The difficulty of doing this can illustrated by using an example population of two agents. Agents $a1$ and $a2$ have identical, maximally ambiguous signalling systems with two shared signals and meanings, ($CA(a1,a2) = \frac{1}{2}$ and $CA_{pop} = \frac{1}{2}$). If we assume that there is no *systemic* bias towards any particular system state, we need to show that repeated interaction between the two agents will inevitably be drawn towards an 'optimal' equilibrium state where $CA_{pop} = 1$).

Assuming that the only stable states of the system are *pure strategies* (where every meaning elicits only a single signal, and where each signal is interpreted as only a single meaning), and if we also permanently hold one agent as speaker and the other as hearer, only two out of the 16 possible pure strategies are optimal. How can we uniquely define these strategies in terms of (4.1)?

$$\forall m, sp, hr \; \exists s \;\; s.t. \;\; P(s \mid pro(sp,\, m)) \times P(m \mid rec(hr,\, s,)) = 1 \tag{4.7}$$

If individual signals only ever mapped to a single meaning—no homonymy—the condition in (4.7) is trivially true: no signals are ambiguous. Otherwise, the problem is that optimal signalling requires that both the 'production term' and the 'reception term' be maximised for some m, s and minimised for all others. Again, this is a system-level property, not the micro-level description we are looking for. Moreover, the state-space of signalling games is necessarily vast if we consider macro-states, as outlined below.

State-space of signalling games

Even the simplest two-agent pure-strategy signalling game has 16 states, of which only two are optimal. When we increase the number of meanings and signals, the total number of pure-strategies increases more than exponentially: the number of pure strategies is $|M|^{|S|} \cdot |S|^{|M|}$, $|S|!$ of which are optimal. The proportion of pure strategies which are optimal, P_{opt}, is: [5]

$$P_{opt} = \frac{|S|!}{|M|^{|S|} \cdot |S|^{|M|}} \quad \text{for } S \geq M \tag{4.8}$$

As $n \to \infty$, $\frac{n!}{n^n} \to 0$, and so (4.8) will approach zero much faster than this even. So, as we increase the number of signals and meanings an increasingly small

[5]This is calculated as follows: the number of pure strategies is the number of $|M| \times |S|$ production matrices which have one populated cell per row multiplied by the equivalent number of $|S| \times |M|$ reception matrices.

The number of pure strategies which are optimal is the number of $|M| \times |M|$ permutation matrices, $|M|!$, then taking into account $\frac{|S|!}{|M|!}$ ways of including redundant rows (with all zero-value cells - otherwise they would not be pure strategies), giving $|S|!$. For the latter, there is no need to multiply for production and reception matrices: each optimal reception matrix identifies a unique optimal production matrix.

A further point is that the total number of *non-pure* optimal strategies is infinite for $S > M$ due to the possibility of synonymy, however these will all correspond to one of the $|S|!$ optimal reception matrices, assuming every signal is used. This is a slightly counter-intuitive result which relies on the fact that the number of pure production strategies accounting for unused signals is the same as the number of pure reception strategies accounting for multiply-used signals.

proportion of pure-strategy states will be optimal. In any case, enumeration of all possible macro-states, determining which states are optimal, and then charting the transitions between these states is a laborious process — and this is only for the pure strategies, let alone more complex mixed or intermediate strategies. The following section describes a different way of charting the dynamics which avoids these problems.

4.4.1 Introducing the entropy state space

There is a way of reducing the size of the space described above and still tracking the degree of 'optimality' of a signalling system. Instead of directly measuring the communicative accuracy of a population as given in (4.1), we can use a measure of *conditional entropy* between signals and meanings. We derive this from the population average signalling behaviour, which is calculated by taking the mean values of the meaning/signal probability from the production and reception matrices of the whole population.

We know from Fig. (4.4) that matrices with the 'optimal characteristic', for example, PRO_{opt} might potentially allow synonymy but never homonymy: every signal must unambiguously map to either one or zero meanings. A concise way to describe this exists in information theory. From the production matrix $P(M|S)$, we can first calculate the distribution of signals $P(S)$. Then, assuming a uniform distribution of meanings for $P(M)$ (this is the case in most of the models we have surveyed, but is easily incorporated if not), we can use Bayes' rule to calculate the distribution of meanings conditional on signals, $P(M|S)$:

$$P(M|S) = \frac{P(S|M)\ P(M)}{P(S)} \tag{4.9}$$

From these, we can now derive $H(M|S)$, the conditional entropy of meanings given signals, Eq. (4.10).

$$H(M|S) = \sum_{s \in S} p(s) \sum_{m \in M} p(m|s)\ log_2\ p(m|s) \tag{4.10}$$

This is a direct measure of *recoverability*: when $H(M|S) = 0$, we know that every produced signal can be mapped back to a single meaning, i.e. there is no ambiguity. When we consider only signal production, this satisfies the requirement that all signals produced are unambiguous as to which meaning triggered them. However, this is not yet sufficient to guarantee optimality.

$H(M|S) = 0$ by itself does not necessarily satisfy the requirement that all meanings are *covered*, i.e. that every meaning has at least one corresponding signal. It could equally well be the case that every signal maps back unambiguously to the same meaning, which is far from optimal. The only way to avoid this is by making a strong assumption: every meaning will always produce some signal. Fortunately, every signalling model we have discussed in the previous chapter has made this assumption. This is one of the key factors which allows us to use the conditional entropy measure as a proxy for the optimality of the whole system.

What this gives us, then, is a measure that lets us track the optimality of production behaviour: when the conditional entropy $H(M|S) = 0$, we know that the population as a whole has coordinated on a signal production which has the 'optimal property'. Production, however, is only half the story: signals must also be interpreted.

4.4.2 Optimal reception and preserving reference

A functional signalling system has similar requirements for the optimality of both signal production and reception. For production, all signals must map back to a single meaning each, and all meanings must produce at least one signal. Likewise, for reception to be optimal, all signals should map to only a single interpretation with at least one signal for each interpreted meaning. On top of these, however, there is a third, critically important factor: reference must be *preserved*. The intended meaning and the interpreted meaning must be the same for each used signal.

The measure of conditional entropy for production which has been outlined above only tells us about the optimality of production behaviour. What we need is a way to extrapolate this measure, and show that optimal production implies both optimal reception, and that reference is preserved. The first apparent setback is that we can not simply use an equivalent conditional entropy measure for signal reception. This is because, in the case of reception, there is no reasonable constraint that every interpreted meaning is covered by a signal. This means that $H(I|S) = 0$ can be true of highly non-optimal systems, for example where all signals map unambiguously to the same meaning.

Similarly, conditional entropy measures of the mapping between intended and interpreted meanings are of no use: $H(I|M) = 0$ can be true of trivially non-optimal systems (e.g. all intended meanings lead to the same interpretation). Reversing the conditionality and using $H(M|I) = 0$ as a criterion, we can again rely on the fact that every meaning produces at least one signal. Unfortunately, this again falls prey to the fact that information theory is blind to reference, as described in Section 4.3.2. As an illustration, consider the situation where the speaker always sends signal a for State 1, and signal b for State 2; If the hearer always interprets signal a as State 2, and signal b as State 1, the correct reference is never communicated. In terms of Shannon's conditional entropy, however, the system is perfectly informative: there is never any uncertainty about how any signal is interpreted. Despite this apparent setback, the subsequent section presents a way of ensuring that reference is preserved.

Signal production as a proxy for the system

To summarise: we can't use the conditional entropy measures for the population reception matrix, or to map between intended and interpreted meanings. However, we can make a second assumption which allows us to avoid the problems involved with preserving reference. As outlined in Section 4.2.1, production and reception behaviour are both derived from the same set of observed signal/meaning pairings. This does not, however, imply that production and reception behaviour are equivalent: in fact, for any random set of observations, the behaviours will often translate to quite different signal/meaning mappings, as seen for example in Fig. 4.3 (C).

However, the situation is different when we consider 'optimal' production and reception. It turns out that when production and reception are a *monotonic* function of the same set of observed pairings, and if both production and reception functions have the *optimal characteristic*, then those optimal matrices must coincide exactly with each other. This means that for any meaning/signal association, the corresponding signal/interpretation association is such that the meaning and interpretation are the same. In this way, reference is preserved between optimal production and reception.

Functions are monotonic when they preserve the relative order of points in a dataset between input and output. Stochastic and WTA functions both have this property: while stochastic functions are directly proportional to the observed data, WTA preserves only the peaks of that distribution and sets all other values at zero[6].

[6]Actually, both of these functions are just two possible points along a continuous spectrum of more or less *regularising* functions. One way of describing this cline mathematically is by using a beta-binomial function such as in Reali & Griffiths (2010). Within this model, stochastic functions correspond to $\frac{\alpha}{2} = 1$, and WTA functions to $\frac{\alpha}{2} = 0$. Another possibility, not seen in signalling game models, is for $\frac{\alpha}{2} = \infty$, which would correspond to an anti-regularisation bias: all

To define the *optimal characteristic* for production matrices we first use the $P(S|M)$ matrix to construct the $P(M|S)$ matrix. We then require that every row of this matrix can form part of a subset which is a permutation matrix. As discussed in Section 4.2.1, this guarantees both that all signals are *unambiguous*, and that every meaning is *covered*. We can represent this as an $(|S| \times |M|)$ matrix.

We can describe three matrices: 1) the matrix of observed meaning/signal pairs $DATA$, 2) the matrix $PRO' = P(M|S)$ and 3) the matrix $REC = P(I|S)$. To guarantee that reference is preserved, we must first specify that PRO' and REC have the optimal property as described above. Before doing this, however, I should re-clarify the relationship between PRO and PRO': the former is an agent's production system, while the latter is a measure of *meaning recoverability* for that production system.

In the first case, with stochastic production, it is trivial that if both are optimal then $PRO' = REC$: the $DATA$ matrix which produces an optimal PRO' must have zero values everywhere except for where PRO' has 1 values. This in turn means that $DATA$ must specify a unique matrix REC which is identical to PRO', as seen in Fig. 4.8.

Showing that reference is preserved between production and reception for WTA agents is more involved. Komarova & Niyogi (2004, p. 14) prove that $PRO' = REC$ if both are optimal for any finite measure (i.e. monotonic function) on $DATA$ which has unique maxima.[7]

variants are produced according to a uniform distribution regardless of the stored frequencies.

[7] As a brief sketch of this proof: if optimal PRO' and REC are different, there must be a 1 in PRO' which represents a row maximum r_1 in DATA, but not a column maximum because it is not a 1 in REC. This maximum of that column, c_1 in DATA, in turn, must not be a row maximum. Again, the corresponding maximum r_2 in that row must have a corresponding larger value in the respective column, and so on. This leads to a chain of inequalities, but since there are a finite number of cells in the matrix, we eventually end up with $r_1 > r_1$, causing a contradiction, i.e. PRO and REC must be the same. Also, notice that we are discussing PRO rather than PRO' here: this

$$
\begin{array}{c|ccc} \text{PRO'} & m1 & m2 & m3 \\ \hline s1 & 0 & 0 & 1 \\ s2 & 0 & 1 & 0 \\ s3 & 0 & 0 & 1 \\ s4 & 0 & 1 & 0 \\ s5 & 1 & 0 & 0 \end{array}
\longleftarrow
\begin{array}{c|ccc} \text{DATA} & m1 & m2 & m3 \\ \hline s1 & 0 & 0 & 23 \\ s2 & 0 & 16 & 0 \\ s3 & 0 & 0 & 31 \\ s4 & 0 & 4 & 0 \\ s5 & 11 & 0 & 0 \end{array}
\longrightarrow
\begin{array}{c|ccc} \text{REC} & i1 & i2 & i3 \\ \hline s1 & 0 & 0 & 1 \\ s2 & 0 & 1 & 0 \\ s3 & 0 & 0 & 1 \\ s4 & 0 & 1 & 0 \\ s5 & 1 & 0 & 0 \end{array}
$$

Figure 4.8: For stochastic production/reception, if DATA maps to an optimal REC matrix, then it must necessarily map to an identical optimal PRO' matrix. This is because any given signals can only map to a single meaning, which is the only populated cell in that row.

This leaves two problematic cases for WTA: i) when *PRO* is optimal but *REC* is not, and ii) when either *PRO* or *REC* have more than one maximum, e.g. when two values in a row or column of *DATA* are the same. Later, in Section 4.7.3, we will demonstrate that states like this are only temporary, at least for the systems we are interested in.

For most other order-preserving functions on *DATA*, the same argument applies as for basic stochastic production: if optimal *PRO* and *REC* exist, they must be the same as all non-maximal values in *DATA* must be zero. This is true even for *floor functions*. If, for example, data points under a certain value were mapped to zero, with the rest to non-zero probabilities, we can simply regard those values as being equivalent to zero. In any case, none of the particular models we looked at so far have included such a feature.

The result of this is that, under the assumptions outlined above, we can mea-

is because optimal stochastic matrices can exhibit synonymy, but WTA will not (as long as there are unique maxima). This means that we need to use PRO' to evaluate the optimal characteristic for stochastic matrices, but this is not so for WTA.

sure the optimality of a signalling system in terms of $H(M|S)$ instead of the usual Communicative Accuracy. This allows us to significantly simplify the overall dynamics of signalling games, both in terms of a single interaction, and as a result also for populations of interacting agents. If we can show that the *population average* PRO'_{pop} satisfies:

$$H_{pop}(M|S) \to 0 \tag{4.11}$$

That is, that it will eventually reach a zero value. For any sufficiently well-defined signalling game, that is sufficient to guarantee that it will reliably develop optimal signalling.

4.5 Population entropy: two sources

As previously discussed in Section 4.2.1 in reference to communicative accuracy measures, there are two main components to the optimality of a system. The first is whether agents have *internally optimal* signalling systems, i.e. the optimal characteristic. The second is the degree to which signalling systems are *shared* between agents in the population. Communicative accuracy, being a macro-level measure, has no simple way of separating these two factors. Conditional entropy, on the other hand, is a micro-level feature which allows us to measure both the internal optimality and the difference between agents in the population.

To clarify, we can calculate the *population* production and reception matrices, PRO_{pop} and REC_{pop} respectively, by taking the mean value of each cell over the population: from the former we can then calculate PRO'_{pop} using Bayes rule. As laid out in Eq. (4.11), we now know that if $H_{pop}(M|S) = 0$, the population has converged on an optimal signal production behaviour.

Now, as we have just discussed, the population entropy $H_{pop}(M|S)$ is actually comprised of two distinct sources of referential uncertainty, which we term *individual* and *alignment* entropy, H_{ind} and H_{align}. As an illustration, see the 'populations' of 2 biased coins shown in Table 4.2.

Each coin has a chance of landing on heads $p_i(h)$, leading to a population average of landing on heads $P_{pop}(h)$. We can use this to calculate the population entropy H_{pop}. However, we can also calculate the *individual* entropies for each coin, H_1, H_2, and hence the mean individual entropy H_{ind}. The important thing to note is that $H_{pop} \geq H_{ind}$. The missing entropy comes from the *misalignment* of coins H_{align}. In Table 4.2, we can see that in Population 1, the individual entropy of each coin is maximised, but there is no entropy from misalignment. For Population 2, however, individual coins have no entropy, but the misalignment term is maximised. Population 3 and 4 show intermediate situations, where the system entropy is shared between the individual and alignment terms.

Population	Coin 1 $p_1(h)$	Coin 2 $p_2(h)$	$P_{pop}(h)$	H_{pop}	H_1, H_2	H_{ind}	H_{align}
1	0.5	0.5	0.5	1.0	1.0, 1.0	1.0	0.0
2	1.0	0.0	0.5	1.0	0.0, 0.0	0.0	1.0
3	0.7	0.3	0.5	1.0	0.88, 0.88	0.88	0.12
4	1.0	0.5	0.75	0.81	0.0, 1.0	0.5	0.31

Table 4.2: Coin 'populations' of two coins each: each coin has a chance of landing on heads $p_a(h)$ and population $P_{pop}(h)$. The population entropy is H_{pop}. *Individual* entropies for each coin, H_1, H_2, provide the mean individual entropy H_{ind}. The entropy from misalignment of coins is H_{align}. Note that $H_{pop} \geq H_{ind}$.

The population average probability of heads, P_h, is simply:

$$P_h = P_{pop}(h) = \frac{1}{|A|} \sum_{a \in A} p_a(h) \tag{4.12}$$

We calculate the population entropy using the probabilities for heads and tails, P_h and P_t:

$$H_{pop} = \sum_{k \in \{h,t\}} -P_k \; log \; P_k \tag{4.13}$$

The average individual entropy, on the other hand is:

$$H_{ind} = \frac{1}{|A|} \sum_{a \in A} \sum_{k \in \{h,t\}} p_{ak} \; log \; p_{ak} \tag{4.14}$$

As stated, the average of the individual entropies H_{ind} is always equal to or less than the entropy of the population average matrix, H_{pop}. This extra entropy is the uncertainty caused by misalignment between members of the population, H_{align}. The easiest way to calculate this is simply:

$$H_{align} = H_{pop} - H_{ind} \tag{4.15}$$

This definition seems somewhat arbitrary, but it is possible to specify its character more precisely. The entropy from misalignment is actually the average *Kullback-Leibler divergence* from P_{pop} to the individual probabilities p_a:

$$H_{align} = Avg.\; D_{KL}(P_{pop}||p_a) = \frac{1}{|A|} \sum_{a \in A} \sum_{k \in \{h,t\}} p_{ak} \; log \; \frac{p_{ak}}{P_{pop}(k)} \tag{4.16}$$

The KL-distance is an unusual measure: it is asymmetric, so $D_{KL}(A||B) \neq D_{KL}(B||A)$. One way of understanding it is as follows: the population average probability distribution P_{pop} can be regarded as a 'model' of the actual data. The KL-distance of any other distribution from the average tells us how well or badly the average models the data: it measures how much *extra information* we need to provide to describe the new distribution using the average/model as a baseline. As such, if the new distribution is the same as the model, the KL-distance $D_{KL} = 0$. If, as with Population 2 in Table 4.2, the new distribution is maximally different, the KL-distance reflects that.

In any case, the key point is that we now have a way to divide the overall population entropy into separate components for individual entropy H_{ind} and the population alignment entropy H_{align}. To restate Eq. 4.5 differently:

$$H_{pop} = H_{ind} + H_{align} \tag{4.17}$$

As we will see in the next sections, an important property of H_{align} is that it always approaches zero under certain conditions. There are a number of facts we can state about the terms which have been isolated on the right hand of this equation. This essential step is what lets us describe the overall dynamics of signalling games by analysing only pairwise interactions.

4.6 Separating entropy dynamics

In all of the models of the emergence of signalling that we have surveyed until now, agents learn from the behaviour of other agents and accommodate that information into their own behaviour. Despite apparent differences between the different models, central to all the instantiations of learning is some element of *imitation*. This is always shaped by other processes, for example learning biases or rational optimisation, but this is always a modification of learned as opposed to purely novel behaviour.

It turns out that whenever agents learn in this way, *consensus* in that population becomes inevitable. This is not to say, however, that there will be consensus on an *optimal* set of conventions. However, what it does mean is that the term representing alignment entropy in Eq. 4.17, H_{align}, will always be reduced to zero in an interacting population of imitating learners. Ultimately, what this allows us to do is concentrate on the term for optimality, H_{ind}. Before doing this, we will take a look at how the minimisation of alignment entropy happens.

Xue (2006) analysed a model of interacting Pólya urns. The urns were placed on a directed ring-graph in which interaction took place between successive urns over many cycles of the graph. Pólya urns are an even more simplified version of the urn models used in the basic framework for signalling games. An urn is filled with a number of differently coloured balls in some proportion. When a ball is sampled from each urn (and returned), a ball of the same colour is added to the next urn in the sequence, and so on. Xue's main result is that, after sufficient time, the proportion of ball colours will be the same in all of the interacting urns. Moreover, this is true "irrespective of the initial conditions, and the imitating probabilities... as long as groups imitate each other with positive probabilities" (Xue, 2006, p.1). This result is echoed by Pra et al. (2014), who investigate a similar scenario in a fully-connected population of urns, but where a parameter α is set over the likelihood of imitating the rest of the population, where $0 \leq \alpha \leq 1$, and $\alpha = 0$ defines a population of non-interacting urns. They show that for any $\alpha > 0$, populations will almost surely converge on the same composition. In both cases, then, the convergence on a set of shared conventions relies on the existence of imitating behaviour, but that imitating behaviour does not need to be strictly applied.

We can now take a look at this in practice. Fig. 4.9 demonstrates the time-course for the three conditional entropy measures from a population of simple imitation learners. The total entropy remains very stable at a high value throughout, but this is not the case for the other measures. The individual entropy has an initial spike: this is when an agent has only sampled a small number of times from other agents' behaviour, their own behaviour will be reasonably determinis-

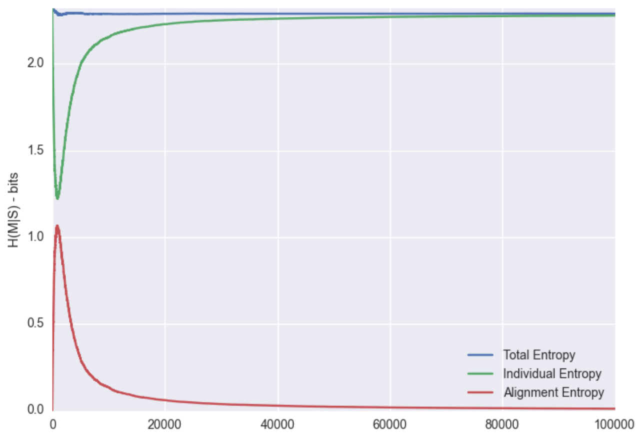


Figure 4.9: The entropy measures for a population of 100 imitator learners with stochastic production. The *total entropy* measures the conditional entropy $H(M|S)$ for the *population average* production matrix. The *individual entropy* is the average entropy $H(M|S)$ across all agents in the population. Finally, the *alignment entropy* is the difference between the former two measures, equivalent to the *mean Kulback-Leibler divergence* from the population average production matrix to the individual production matrices.

tic. However, as each agent has more learning experiences, they begin to resemble each other. As there is no systemic bias at work, the behaviour of individual agents becomes similar to the overall, highly random behaviour across the whole population. In this way, all agents become extremely similar (causing alignment entropy to almost disappear) but very random (hence the high levels of total and individual entropy).

Similarly, we can look at what happens in a population of slightly more advanced agents in Fig. 4.10. The WTA-production of the agents does force much more deterministic behaviour, and we do see a significant drop in overall entropy. However, because the main effect of the WTA mechanic is to eliminate synonyms but has no direct effect upon homonyms[8] , this does not result in a reliable mech-

[8]Although it will normally reduce homonymy due to the limited number of signals available:

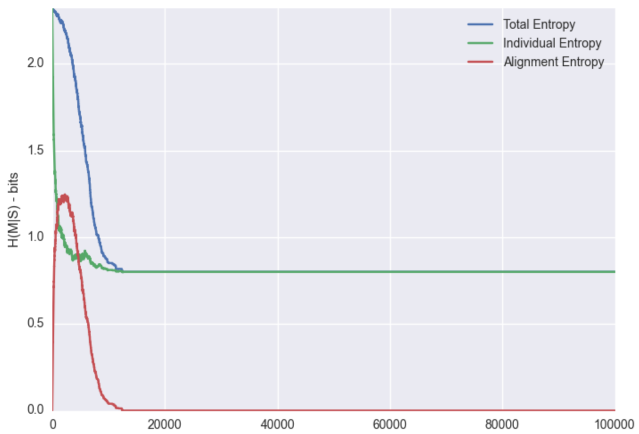


Figure 4.10: Entropy measures for a population of 100 imitator learners with WTA production.

anism for reducing the level of individual entropy to zero. Agents do however become highly similar to each other, sending the alignment entropy to zero. One way to think of this is that while signal *production* becomes completely deterministic, this does not result in a signalling system which has a deterministic *decoding* strategy.

Still looking at imitator learners, a further effect can be seen when we limit memory to 35 exemplars for a population of stochastic agents. Fig. 4.11 shows that this extra source of stochasticity prevents agents from becoming too self-similar, in effect acting as a floor value for alignment entropy. This value remains relatively stable. Manipulating the size of the exemplar memory to 70 exemplars (not shown here) had the effect of lowering this floor value. This is worth noting, because we know that the stochastic effect of information loss plays an important role: it provides the necessary plasticity for the anti-ambiguity bias to take effect. On the other hand, we can see here that noise actually counteracts the emergence of consensus/homogeneous behaviour.

of all of the possible states without synonymy, most have a reduced level of homonymy.

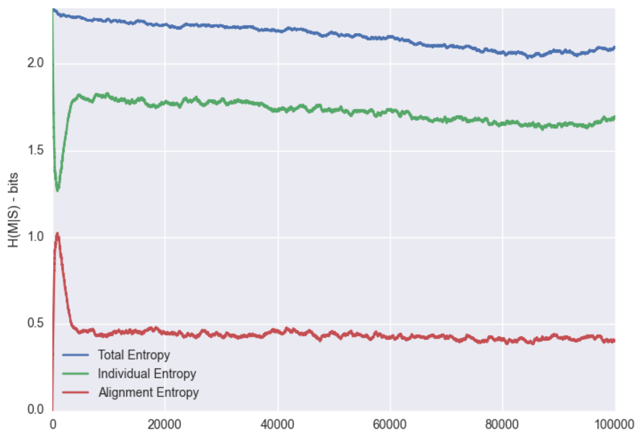


Figure 4.11: Entropy measures for a population of 100 imitator learners with stochastic production and a memory limited to 35 exemplars.

In Fig. 4.12, we can look at populations with the necessary bias against ambiguity. The reinforcement learners on the left become highly self-similar, but the lack of an information loss mechanism means that the learning rate slows down too much, and the individual entropy levels out. The anti-homonymy observational learners in the middle have both of the extra biases needed, and both individual and alignment entropy slowly decrease over time to zero. The figure on the right hand side is an example of when WTA obverter learners sometimes fail to converge on a signalling system ($\sim 10\%$ of the time). Obverter production provides a bias against ambiguity, but is not always strong enough to drive the individual entropy to zero. As we know, including any form of information loss would provide the necessary plasticity for both the reinforcement and obverter models to reliably develop signalling.

Finally, we can see what happens when agents do *not* imitate each other. In Fig. 4.13, a population of observational learners have the necessary bias against homonyms. However, in this population only *speakers* learn after an interaction,

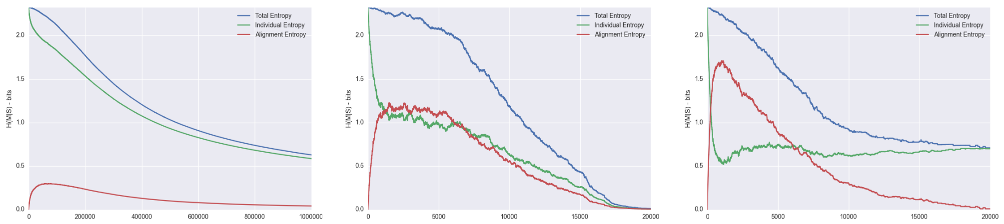


Figure 4.12: Entropy measures for populations of 100 agents with an anti-ambiguity bias. On the left, a basic reinforcement learning model; in the middle, observational learners with an anti-homonymy bias; on the right, WTA obverter learners with no information loss.

i.e. people are entirely solipsistic learners who only ever listen to themselves. The effect of this is that individual agents develop internal systems which are internally optimal, but not shared with others. As a result, we see individual entropy drop to zero: without any imitation the alignment entropy never decreases, until it is the only source of entropy in the population. The extremely defective self-copying learners on the right have no learning bias at all: agents are neither similar to each other nor internally optimal.

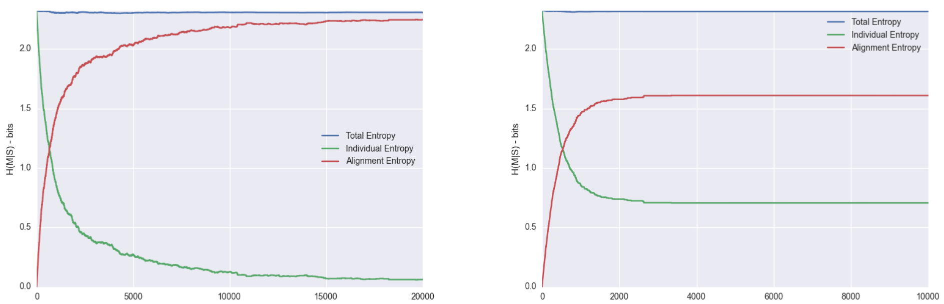


Figure 4.13: Entropy measures for a population of 100 observational learners where *only speakers* update after an interaction. On the left, agents have a bias against homonymy, while on the self-imitators on the right have no learning bias.

4.7 The entropy state space visualised

The previous section demonstrated that as long as agents *imitate* each other to some degree, alignment entropy will inevitably be reduced to zero (as long as no stochastic source is allowed to dominate, as in Fig. 4.11). The important factor, then, is the presence of a bias against ambiguity. With this in mind, we can track the dynamics of populations using their population average production matrix. Two measures from this, $H(M|S)$ and $H(S|M)$, corresponding to the degree of homonymy and synonymy, allow us to plot the current state of a population as a point within this 2-dimensional space.

Populations which are initialised as blank agents will have uniformly-distributed, maximally non-deterministic behaviour, and begin in the top right corner of the space laid out in Fig. 4.14. As signalling systems develop, the population navigates the space towards lower levels of homonymy and synonymy. When homonymy is eliminated, the population will reside on the baseline where $H(M|S) = 0$. Similarly, populations on the y-axis denote a lack of synonymy, and when the population is at the origin neither homonymy nor synonymy will remain.

One point which should be clarified is that each point in entropy space maps to multiple equivalent states of the population. One reason for this is that the identity of signal/meaning mappings are irrelevant, only the degree of ambiguity they represent. Secondly, because the point in space represents the *population average* production matrix, there are a huge number of ways that this can map to individuals within that population. What is certain, however, is that $H(M|S) = 0$ represents a population with absolute consensus on an optimal signalling system.

This section will show how the three properties which we identified earlier drive the dynamics of populations within the entropy space. Populations are represented as a point in a three-dimensional space, the third dimension being time. Each simulation plot below represents a typical transition through the space seen

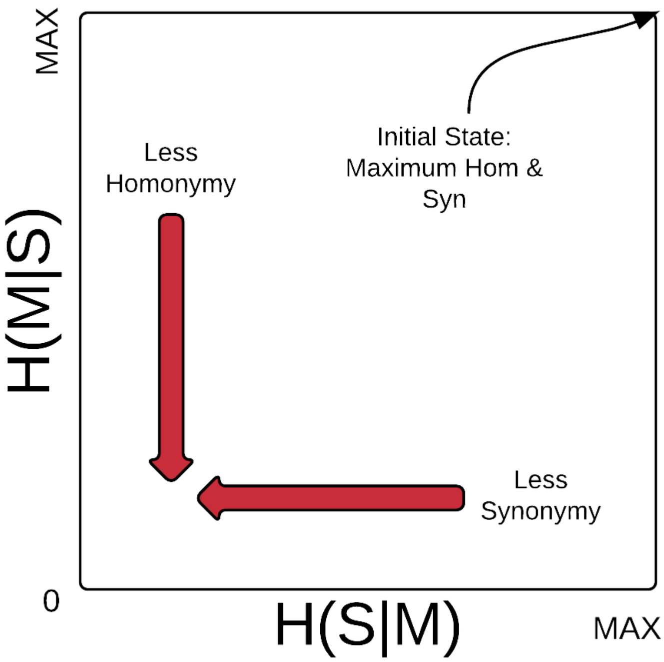


Figure 4.14: The entropy state-space of signalling games. The y-axis measures the overall level of homonymy, and the x-axis the level of synonymy. Any population which as developed optimal signalling will reside on the baseline of the space, where $H(M|S) = 0$. A population which has eliminated synonyms will be represented by a point on the left axis. When there is neither homonymy or synonymy, the population will be represented by a point in the lower left corner.

from 3 perspectives: homonymy vs. synonymy, homonymy vs. time, and synonymy vs. time. This perspective provides several advantages. For example, the time dimension displays the effects of *learning rate* and *slowdown*, and the pure entropy space provides information about correlations between homonymy and synonymy.

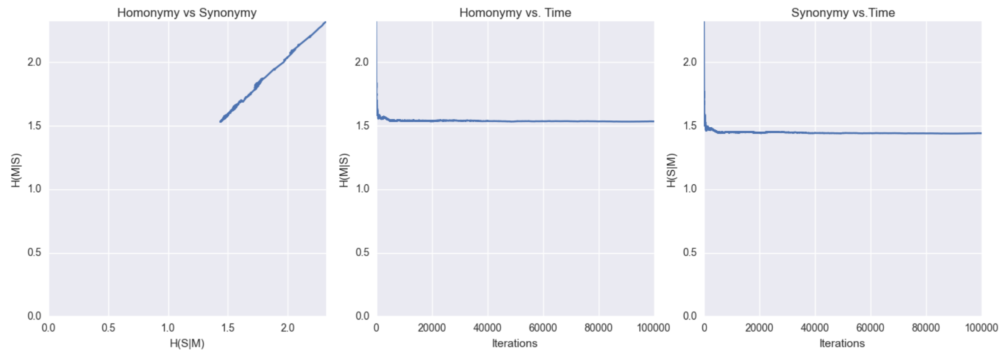


Figure 4.15: Entropy dynamics for a population of 100 imitatation learners. The left figure shows the passage of the population through entropy-space: the initial state of a population is at the top right-hand corner of the first graph, representing maximal entropy for both $H(M|S)$ and $H(S|M)$, and proceeds to move towards the centre of the graph as the entropies are reduced. The middle and right hand figures show how the population travels through the homonym and synonym dimensions over time.

4.7.1 Information loss is drift and eliminates synonymy

The population of imitation learners in Fig. 4.15 shows two important effects. The first is that, even without any systemic bias, the population is drawn towards less entropic states. Any chance irregularities (for example, some signals happen by chance to be less ambiguous than others) which are retained in the population will cause this to happen. Secondly, and more importantly, we can see the effect of *learning slowdown*. This is a natural consequence of the urn-model learner when there is no information loss: the initial ability to move through the space disappears, and the population is locked into a sub-optimal state.

In contrast, the population of imitator learners in Fig. 4.16 shows us that *with* information loss, the population is able to navigate the whole space: learning slowdown does not occur. This figure is a particularly clear demonstration that the dynamics of information loss in signalling populations are directly equivalent

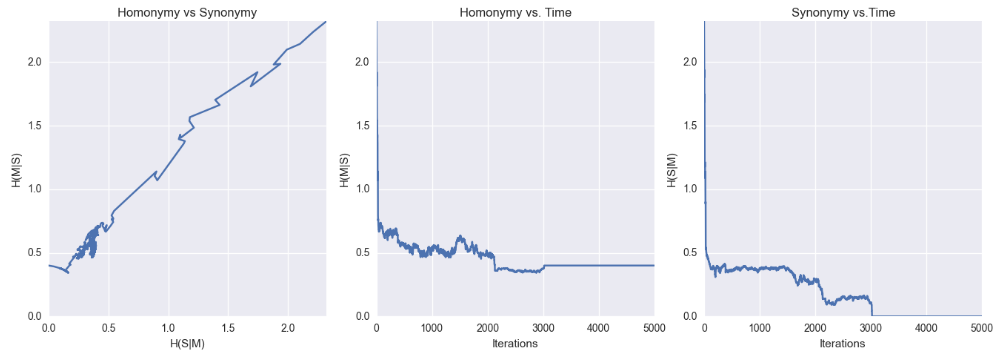


Figure 4.16: Entropy dynamics for a population of 2 imitation learners with a memory of 80 exemplars.

to *genetic drift*[9]. Both information loss in these models and genetic drift are processes which sample from populations to create their next stage. In both cases, they lead to *fixation*: variation is lost. Because of the nature of (most) signalling games, the loss of variation is in terms of *synonymy* rather than homonymy. New signals are produced in some relation to their proportional association with each meaning. These proportions will *drift* randomly, but once a particular association has been lost in a population, any synonyms will be eliminated. Just as with genetic drift, this process is inevitable, unless there is some mechanism to *reintroduce variation*.

In Fig. 4.17, also a population of imitator learners but this time with a very small and unstable memory of 25 exemplars, we again see the way that information loss acts against synonymy. In this case, however, the limitations placed on memory lead to the frequent loss of signal/meaning associations. In this case, a new signal association is innovated at random, reintroducing variation. This allows 'fixated' populations to re-enter the space, before fixating elsewhere, and so on — even sometimes temporarily eliminating homonymy. The subsequent dy-

[9]In this case, because there is a hard limit on the number of exemplars stored by any agent, it is analogous to the symmetric birth/death process modelled by Moran (1962)

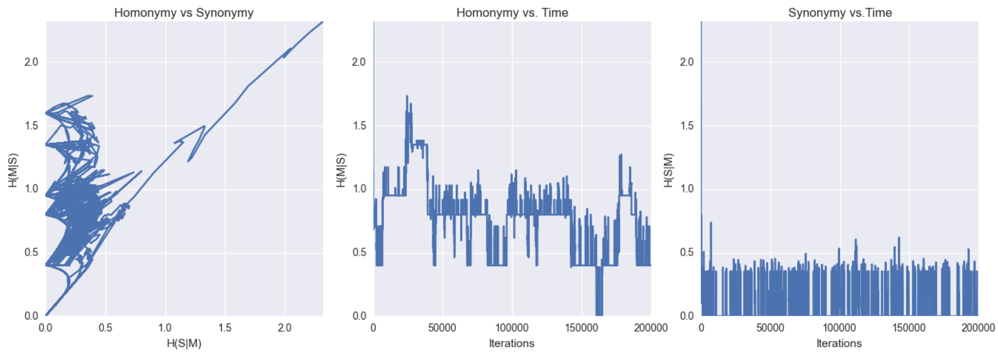


Figure 4.17: Entropy dynamics of a population of 2 imitation learners with a memory of 25 exemplars.

namics suggests that the possible positions of the population are not uniformly distributed within the space: this is in fact true, as will be shown later.

As a final point, it should be pointed out that information loss is not the only process which leads to the loss of synonymy. WTA production has a similar effect, as does (unsurprisingly) the explicit inhibition of synonyms. Similar observations have been made in the literature addressing *linguistic regularisation*, where some proponents argue for a stronger role for drift-like processes (e.g. Reali & Griffiths, 2010) and others for cognitive biases (e.g. Ferdinand et al., 2013). This discussion is not immediately relevant here, but it is important to recognise that information loss is not a completely neutral force: it implicitly acts against synonymy by driving out variation.

4.7.2 The bias against homonymy

Fig. 4.18 is another example of how, even with the necessary bias, a lack of information loss slows down learning. Although convergence is quick at first, it slows dramatically, not even within 10% of optimality after 1 million interactions. As seen before, the bias against ambiguity is not in itself sufficient.

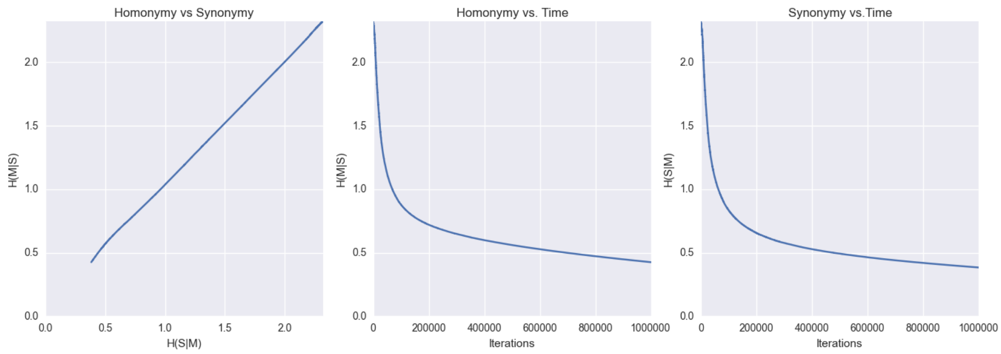


Figure 4.18: Entropy dynamics of population of 10 reinforcement learners without information loss.

Fig. 4.19 (reinforcement learners with a limited memory) and Fig. 4.20 (observational learners with a bias against homonyms) show the importance of information loss in the presence of the necessary bias. A further observation is that the trajectory through entropy-space reveals that, when a bias against homonymy exists, the amount of synonymy and homonymy are highly correlated, to the point of equality. This is *only* the case when there is a bias against homonymy: the biases against synonymy leave the population relatively unconstrained in their passage through the space.

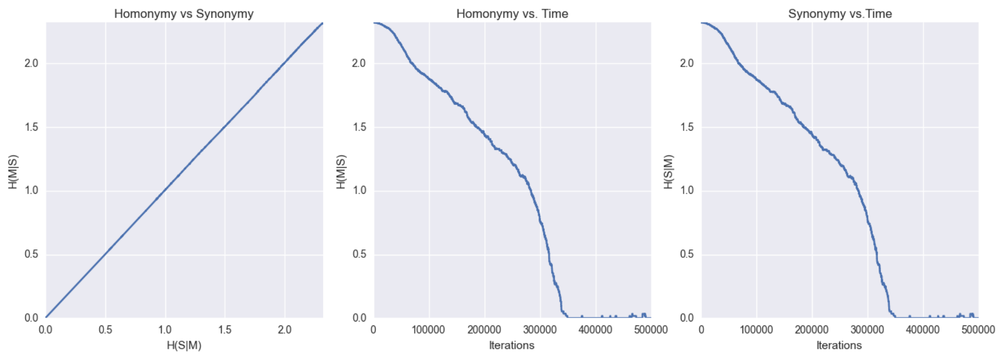


Figure 4.19: Entropy dynamics of 100 reinforcement learners with a 40 exemplar memory.

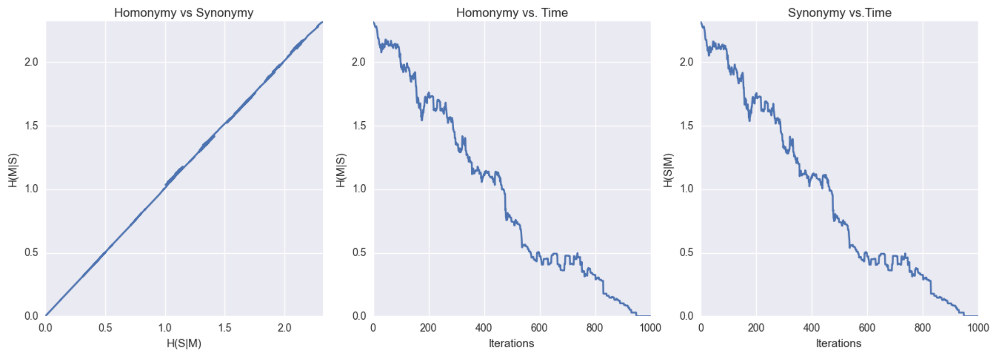
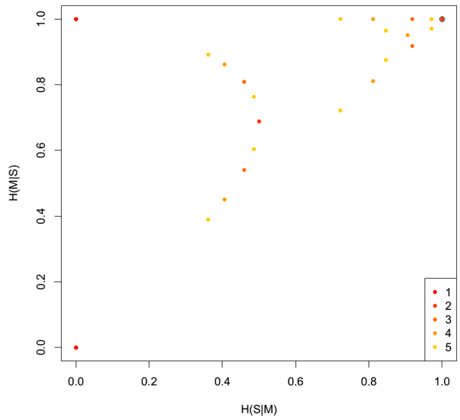


Figure 4.20: Entropy dynamics of population of 100 observational learners with an anti-homonymy bias.

What is this imbalance between synonymy and homonymy? It comes down to the fact that every meaning is always associated with some signal, but the reverse is not true. This has two implications: i) the population has many more states with a given level of synonymy than the equivalent level of homonymy, and ii) maximising the homonymy results in maximised synonymy, but not vice versa. To illustrate this using the models we have been using (where there are an equal number of meanings and signals), take the state where $H(M|S) = 0$. In this case, the lack of homonyms guarantees that no synonyms exist: there are no signals left free to associate with more than a single meaning. The alternative, when $H(S|M) = 0$ can exist for numerous levels of homonymy, because it is perfectly possible for meanings to associate with the same signal, leaving some signals unused. This is why, when homonymy is maximised by individual agents, this leads to the maximum possible synonymy. This has the effects of restricting the trajectories of populations to the linear path we observe them make through the entropy-space.

Fig. 4.21 shows the structure of the entropy space. Starting with only 2 signals and meanings, the graph displays the possible states for populations of increasing size. Firstly, it is clear that the space is not uniformly distributed. Secondly, the



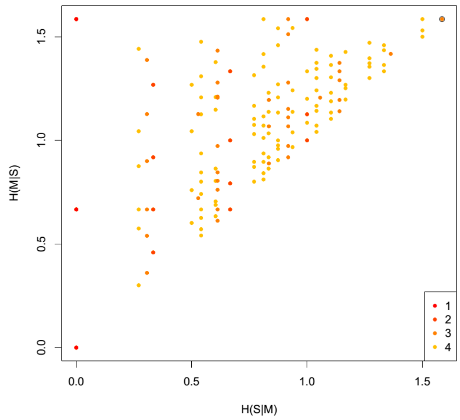


Figure 4.21: Possible states for populations with WTA production. The left hand figure shows the possible states for 2 signals and meanings, the right hand side for 3 signals and meanings. The dot colour represents possible states for populations with the corresponding number of agents: for example i the left figure, one agent populations can only have totally optimal systems (bottom left corner) or defunct systems (top right). Introducing more agents increases the number of states and possible positions within the space. *Image courtesy of Kevin Stadler*

effect of guaranteeing that all meanings produce a signal can be clearly seen: there are no possible states below the diagonal. This shows again why the bias against ambiguity acts to simultaneously minimise synonymy, restricting populations to a trajectory along the diagonal.

4.7.3 Optimal production leads to optimal reception

To summarise the results so far: as long as agents imitate each other with positive probability, there is some bias against ambiguous mappings, and information loss is able to provide the necessary plasticity, then a situation will arise where the population converges on an identical optimal signal production system. However, as pointed out in Section 4.4.2, it is possible to have optimal production systems which do not correspond to an optimal reception system. This directly implies a non-optimal system. A second problem is that is there are *non-unique maxima*

$$\begin{array}{cc} \text{(A)} & \begin{array}{ccc} s1 & s2 & s3 \end{array} \\ \begin{array}{c} m1 \\ m2 \end{array} & \begin{pmatrix} \mathbf{3} & 1 & \mathbf{3} \\ 1 & 2 & 1 \end{pmatrix} \end{array} \qquad \begin{array}{cc} \text{(B)} & \begin{array}{cc} s1 & s2 \end{array} \\ \begin{array}{c} m1 \\ m2 \end{array} & \begin{pmatrix} 1 & \mathbf{2} \\ \boldsymbol{\mathit{4}} & \mathit{3} \end{pmatrix} \end{array}$$

Figure 4.22: Two problematic association matrices: (A) has multiple maxima (shown in bold), and (B) maps to an optimal matrix for production, but not reception: production maxima are shown in bold and reception maxima in italics.

in the underlying association matrix, so the proof given in Komarova & Niyogi (2004) which guarantees that reference is preserved no longer holds for WTA learners. Fig. 4.22 gives an example for each of these defective cases.

At this point, it helps to take a micro-level view of the three requirements, as illustrated in Fig. 4.23. Firstly, although the various models implement the bias against ambiguity differently, they all lead to the same effect internally: a single meaning is maximised for each signal, while competing meanings are weakened either by direct inhibition or information loss. At the same time, every meaning will continue to produce new signals, and hence new associations. With WTA production, its implicit bias against synonymy ensures that each meaning maximises exactly one signal, while stochastic production reinforces multiple signals in proportion to their weights. Finally, information loss continually weakens association which are not being strengthened.

As we know, the net effect is to produce matrices with the optimal characteristic. Although stochastic production can result in multiple optima, this is not a problem: as outlined earlier in Section 4.4.2, the optimal PRO' matrices derived from production matrices must always map to an identical optimal reception matrix. For WTA production, on the other hand, the three pressures guarantee that each meaning will develop a single, unambiguous association with a signal, and that all other associations will be decremented over time. Because it is production

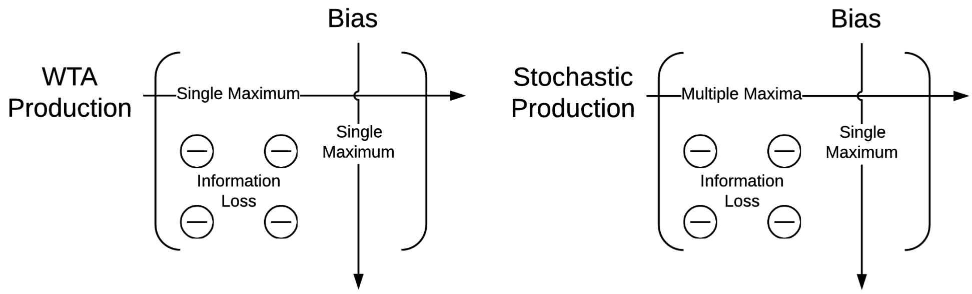


Figure 4.23: The internal forces on an association matrix under WTA and stochastic production: WTA production strengthens a single maximum per meaning, and stochastic production multiple maxima. The bias against ambiguity strengthens a single maximum per signal, while information loss weakens all associations which are not being strengthened.

and not reception which drives the development of agents over time, the suboptimal situations displayed in Fig. 4.22 are guaranteed to be transitional for any learners with the three necessary properties.

4.8 Diagnosing the optimality of a system

In this chapter I have outlined a way of describing the dynamics of populations of learning signallers by using micro-level, as opposed to the usual macro-level dynamics. The dynamics employ information-theoretic measures. The measures are able to avoid the normal problems with information theory and reference. They can do this thanks to two observations: first, by ensuring that agents will always produce a signal for a prompted meaning, and secondly by showing that the average production matrix of the population can be used as a proxy for the whole system. This measures the optimality of interacting populations using conditional entropy. We then saw that the population average entropy is actually comprised of two separate sources, individual entropy and alignment entropy. In any interacting population of agents which imitate each other with positive probability, alignment entropy will always be reduced to zero. In order to guarantee that optimality develops reliably, we need to show that individual entropy also reduces to zero. Critically, however, some form of information loss must also be present in order to provide the necessary plasticity to the system.

We are now in a position to restate the three requirements more precisely. To recap, we previously observed that reliable optimality required all systems to 1) create and transfer referential information, 2) have a bias against ambiguity, and 3) provide some form of information loss. We have now shown:

1. Agents must imitate each other's referential signalling with non-negative probability, as this drives alignment entropy to zero.

2. We can therefore expect agents to become very similar. Given this, the conditional entropy $P(M|S)$ of a pair of similar, interacting agents must decrease on average.

3. Information loss is necessary to prevent the learning rate from slowing down.

More formally:

1. Agents imitate with positive probability:
   $D_{KL}(P_{individual}||P_{population}) \rightarrow 0$

2. Interactions reduce individual entropy:
   $E[\Delta H(M|S)] < 0$

3. The learning rate must allow convergence on optimality:
   $\Delta H(M|S) \not\rightarrow 0$ if $H(M|S) > \varepsilon \;\; \forall \;\; \varepsilon > 0$

How can we apply this to a particular model? Simply by using these criteria to evaluate the course of a typical pairwise interaction. For example, if we consider a classic Roth-Erev reinforcement learner such as described by Skyrms (2010):

- **Q**: In an interaction, if a speaker produces a signal s for meaning m at time t, is the hearer more likely to imitate that action at time $t+1$?

  $$p_{t+1}(m|s) > p_t(m|s)?$$

  A: After success, both hearer and speaker reinforcement learners strengthen that association, so it is certainly true. After failure, no associations are reinforced. On average, then, agents do imitate with positive probability.

- **Q**: If two agents associate a meaning with signals in the same ranked order (which is guaranteed to happen if they imitate each other), will an interaction reduce $H(M|S)$ overall for the pair?
  A: Reinforcement learners strengthen successful associations, i.e. those which are de facto most likely to be the least ambiguous. Because the agents in the interaction are highly similar, the likelihood is that the strongest association for that meaning is also the strongest association for that particular signal, i.e. the signal with the lowest $H(m|s)$. Strengthening that association will reduce the overall $H(M|S)$.

- **Q**: Does the learning rate slow down?

  A: Yes, in the case of classic Roth-Erev reinforcement learning. However, applying a memory limit removes the problem, and we can expect convergence.

It is possible (and easier) to perform this procedure with the other models, and indeed any signalling game: as long as the details of interaction and learning are fully specified, the micro-level interaction will diagnose whether the system as a whole is able to self-organise an optimal signalling system. This also holds for models with referential uncertainty: as observed in Chapter 2, the important thing is that the model of learning is able to extract reliable referential information across contexts in a way which doesn't *increase entropy*, both in terms of $H(M|S)$ and $H(S|M)$. For example, consider the investigations of cross-situational learning in the last chapter: models which relied upon adding multiple exemplars after every learning experience necessarily increase the internal entropy with every step, and hence require another mechanism in order to reduce this.

4.9 Conclusion

This chapter has established the entropy state-space for analysing the dynamics of learned signalling games, and in particular has isolated two sources of entropy, individual and alignment entropy. This simplification gives us a way of determining the long-range dynamics of any such system without needing to resort to exhaustive simulations.

However, there are still some drawbacks. Firstly, this dynamical analysis is strictly tied to models which are based on agents which have a unified set of meaning/signal associations. Some models have historically used multiple, separate representations for signal production and reception: the arguments used in the previous sections do not necessarily apply to such models. On the other hand, the description should not be restricted to Roth-Erev style non-parametric models: Bush-Mosteller reinforcement learning, for example, still represents a set of meaning/signal associations which we can interpret in terms of conditional entropy. In fact, we can see that Bush-Mosteller learners will only reliably develop signalling systems if they also have the necessary bias against ambiguity and a mechanism of information loss, for example a noise parameter, but this warrants further investigation.

Another issue is that what is presented here is not a formal mathematical *proof*, although it incorporates a number of them. The main source of this is the problem of learning slowdown: it is not clear whether — in the limit — slowdown is eventually overcome, or whether the learning rate eventually does effectively stop. For all practical purposes, however, we can see that learning slows dramatically over short time-scales and does not recover over very long time-scales.

On the other hand, the basic associative model underlies many more complex representations of signalling behaviour, for example neural networks and Bayesian statistical models. Whenever the issue of optimality comes up, any system which can be stripped down to its associative centre means that we can use these tools to analyse it and determine whether it will develop consensus on a functional signalling system. It is hoped that this ability — to predict aspects of model behaviour in advance without recourse to lengthy numerical work — should help clarify certain aspects of the various fields which investigate the emergence of learned signalling systems.

Chapter 5

Duality of patterning

5.1 Introduction

Duality of patterning is the design feature that Hockett (1955, 1959) proposed to be unique to human language. I will depart from Hockett's original definition (more discussion later), and take duality of patterning to refer to the two levels of systematicity — one involving meaningful units and one meaningless — which are a characteristic of almost all human languages. The study of how these levels are structured (e.g. phonology vs. morphosyntax) constitutes much of modern linguistic theory. However, there has been much less of a focus on the study of duality as a property in and of itself. Recently, however, the resurgence of work in language evolution has brought with it a renewed interest in explaining abstract features of language like this. In particular, there is a growing body of work attempting to explain the emergence of *combinatoriality* and *compositionality*. This has in turn brought duality itself into the spotlight.

The main argument of this chapter is that we can define duality as the presence of both combinatoriality and compositionality, and that both are a functional adaptation which maintains *learnability* and *expressivity* in the face of noise. This draws heavily on work such as K. Smith et al. (2013) and Kirby et al. (2015), who show that *compositional* structure emerges under learnability and expressivity pressures. I extend these ideas to look at combinatoriality as well, and argue that noise plays a crucial determining role in the emergence of structure, demonstrating this with a computational model of the emergence of duality.

Before doing this, I will need to provide an overview of the three central notions involved in this chapter. All three — duality of patterning (henceforth DoP), compositionality and combinatoriality — are topical issues in language evolution, linguistics, and the philosophy of language. Because of this, there is no absolute consensus on even their definitions. My focus will be on evolutionary explanations for their emergence, for which there are a similar range of proposals: I aim to show that, for the most part, the different accounts are reconcilable.

As such, Section 5.2 is devoted to looking at past theoretical, empirical and modelling work on DoP, compositionality and combinatoriality, with a section taking each in turn. Section 5.3 is devoted to new modelling work. This includes a description of the model in Section 5.3.1, after which I outline the results in Section 5.3.3. Finally, Section 5.4 discusses these results: In Section 5.4.2 I look at implications for other related work, for example theories of proto-language, the role of the information bottleneck, and the difference between superficial and productive structure, and Section 5.4.3 makes some more model-specific comments.

5.2 Review

There is a broad literature dedicated to the fields of combinatoriality, compositionality and duality of patterning in language. Not all of this is restricted to studies of language evolution, although that will be my focus. I should also point out that the order of presentation differs from their historical appearance: I have done this mainly for reasons of theoretical coherence.

I begin with a look at duality of patterning in Section 5.2.1, including Hockett's original observation and some more recent discussion of how that applies to modern linguistic and philosophical thought. There is only one integrated model (Tria et al., 2012) which expressly focusses on the emergence of duality of patterning: most discussion of the relevant parts of that model (devoted to combinatoriality and compositionality) appear in the relevant sections. Likewise, I look at several pieces of empirical and experimental work focussing on duality, but will examine their analyses of combinatoriality and compositionality in those sections. My main aim here is to clarify and motivate the definition of duality of patterning I will be working with, as this will affect my approach to the individual levels of patterning. In particular, I will take the position of Ladd (2012) that duality of patterning — contrary to Hockett (1960) — describes two levels of systematicity, one of which (combinatoriality) is completely embedded in the other (compositionality).

Section 5.2.2 takes in the previous work on *combinatoriality*. There is less to review here, simply because this is the most recent of the three areas to appear in the literature. However, there has been a good deal of recent empirical and modelling work in this area which is of immediate relevance to this study, both in terms of what combinatoriality *is*, and of how it *appeared*. Although there is some discussion of the adaptivity of combinatorial signalling in terms of natural selection (e.g. Scott-Phillips & Blythe, 2013), I will mainly look at learning-based accounts of its emergence. Zuidema & de Boer (2009) provide an insightful analysis of different aspects of combinatoriality which I will use as the basis for the modelling work in later sections.

I address *compositionality* in Section 5.2.3. This represents a large body of work spanning a number of disciplines, much falling within the philosophy of language. There are numerous computational models and experimental studies of the cultural evolution of compositionality: these are my main focus. Several topics within this field intersect with larger debates in evolutionary linguistics, for example the respective roles of interaction, learning, and cognitive bias. Importantly, one group of models (N. Smith et al., 2013; Kirby et al., 2015) is of crucial importance in defining the key pressures of expressivity and learnability, although I argue that these pressures can be seen in almost all of the theories of the emergence of compositionality reviewed here.

Finally, both the sections on combinatoriality and compositionality outline some more formal approaches used to describe them, and how this applies to their definitions, measures, and explanations.

5.2.1 Duality of patterning - history

In his comprehensive survey of duality of patterning, Ladd (2012) points out that Hjelmslev (1961, 1st English edition) was the first to describe human language as consisting of a meaningful 'content plane' and a meaningless 'expression plane', and that these two levels of structure appeared to operate separately according to their own rules. This was likely the influence for both Hockett (1959) and Martinet (1984) when they formulated their closely related ideas of *duality of patterning* and *double articulation*. Although there are a number of distinctions between these two ideas, they both represent an attempt to capture an abstract property which appears to exist in human language but not in any other communication system found in nature. This leads to a number of questions: firstly, how exactly do we define these two levels, and how can we distinguish between them? Secondly, given that this duality appears to be a *design feature* of almost all human languages, is there some functional or evolutionary explanation which accounts for its presence? Stated differently, does the fact that duality exists tell us something about the nature of language? If some functional aspect of duality in and of itself can be identified, then it might be the result of genetic or cultural evolution. On the other hand, it is possible that each level of systematicity arose independently, and human language just happens to feature both. Before addressing these concerns, we should first take a look at the differences between the accounts of Hockett and Martinet.

Hockett (1959, p.33) defines duality of patterning as "a set of conventions in terms of smallest meaningful elements ... and also a set of conventions in terms of minimum meaningless but differentiating ingredients." This requires some clarification. Why is it necessary to identify the *smallest/minimal* elements, and what are they in any case? As Blevins (2014) remarks, the systematicity found in both meaningful and meaningless components of language is spread across several levels of analysis. Both morphemes and words are uncontroversially recognised as distinct meaningful units, and a thorough analysis of language considers both. Likewise, meaningless aspects of language show organisation at different levels from phonetic features and phonological categories to suprasegmental features such as tone, stress and pitch. This does not appear to pose a problem for Hockett who, as Ladd (2012) says, is focussed on the issue of *how* human communication is unique. From this perspective, it is enough to say that both meaningful and meaningless elements exist, regardless of whether they exist at different levels of analysis.

This contrasts with Martinet (1984) who draws a distinction between *primary articulation*, the system governing the organisation of meaningful elements, and *secondary articulation*, the system which mediates meaningless elements. Crucially, Martinet sees the secondary system as being completely embedded within the first. This is quite far from Hockett's position, in which the two levels of systematicity could in theory exist independently of each other, possibly even without interaction. Martinet describes a serial process in which language is comprised of discrete meaningful elements, and those elements are made up of meaningless elements. To reiterate, the key distinction is that, for Hockett, all language is made up of meaningful and meaningless elements, whereas for Martinet all language is made up of meaningful elements and all meaningful elements are made up of meaningless elements. The distinction might seem slight, but has a number of implications. The first regards how language is perceived as a process: for example a situation where some phonological mechanism exists which operates over strings of meaningless elements and outputting the result to a separate morpho-syntactic mechanism process (the Martinet view) versus two mechanisms which apply *simultaneously* to linguistic representations (the Hockett view).

Further to this, and as clearly aid out by Ladd (2012), there is an interesting interaction here with the second of Hockett's proposed design features (1960), *productivity*. This is the capacity of human language to produce and understand novel utterances by compositional combination of meaningful elements. Hockett's explanation of duality seems to focus only on the meaning-free aspect of language alone, with productivity explaining the systematicity of meaningful elements. His example for a non-productive system which displays duality is in line with this interpretation: he describes a system of five lamps, each of which can be set to one of three colours. This is a system capable of communicating $3^5 = 243$ different messages, but which is not productive beyond this hard limit. This is a system with 3 meaningless elements (the lamp colours) and potentially 243 meaningful elements (the configurations), which satisfies Hockett's interpretation of duality. However, while some (minimal) systematicity exists in the way meaningless elements are combined, no such thing exists for the meaningful elements. This is another key point of divergence from Martinet's double articulation, which makes specific reference to the *process* by which meaningful and meaningless elements are recombined.

A further implication of Martinet's perspective is that the emergence of meaningful systems preceded that of meaningless elements, while this does not follow from a strict interpretation of Hockett's concept of duality. However, Hockett (1960) also argues that combinatorial systematicity was likely the last human-specific design feature to emerge. In both cases, this is closely connected to their proposals for a *functional* role of combinatorial communication. Hockett (1959) proposed that it was a 'great convenience' for communication systems with large signal inventories to be composed of a limited number of meaningless elements, and that these large vocabularies were probably the most recent development. Martinet's explanation is similar, picking out the advantages of having a 'synchronically stable' set of meaningless elements available to play the key role of disambiguating between meaningful elements, while stressing the functionality of productive combination of meaningful elements. For both Hockett and Martinet, then, the idea is that productivity is an initial stage: this leads to an ever-increasing number of meaningful, re-combinable signal elements, necessitating a final stage in which systematic recombination of discrete, meaningless elements emerges.

At this point, it is worth pointing out that duality of patterning is not an absolute universal within human language. Blevins (2014) focuses on this problem, and observes that some sign languages feature no corollary to phonology, for example Al-Sayyid Bedouin Sign Language (Sandler et al., 2005). In this case, the language is expressive and compositionally productive, but there appears to be little internal structure to the signs themselves, which also display a high degree of iconicity. This is despite the fact that a large sign inventory exists, contradicting Hockett's proposal that limiting the number of meaningless elements is a functional response to a large vocabulary. In a different apparent contradiction to duality, Blevins lists a number of languages in which minimal distinguishing units are also meaningful, for example /ʔ/ in Kabardian ('say') and a more extreme case where grammatical inflection for 3rd person in Isthmus Mixe is expressed through palatalisation of the stem-initial consonant. Although these are rare phenomena, they do invalidate a very strong interpretation of Martinet's claim that *all* meaningful elements are composed of meaningless ones. Furthermore, sometimes the distinction between meaningful and meaningless is hard to clearly distinguish. For example, *phonesthemes* such as the word-initial /gl/ cluster in English are strongly associated with semantic properties of light and vision, but it is not clear where they lie on the dividing line between meaningful and meaningless. At the other end of the scale, there are meaningful elements of spoken languages which appear to be holistic, such as the positive and negative interjections [ʔʌ̃ˈhʌ̃] and [ʔʌ̃ˈʔʌ̃] in English. Both feature unconditioned nasalised vowels and a contrastive glottal stop, neither of which appear in other English words. This proves to be a common phenomenon cross-linguistically, particularly within interjections. As for how these phenomena impact on duality of patterning, Blevins (in parallel with both Martinet and Hockett) argues that ABSL's lack of duality is likely due to its age, and that sub-lexical structure is a secondary development which should be expected to emerge over time. Similarly, we should not be surprised by the presence of many apparent violations of duality found throughout more established languages: while duality of patterning appears to play a strong functional role towards creating large, productive lexicons, there is no reason why it should be expected to be absolutely ubiquitous.

It is impossible to continue much further without addressing exactly what the levels in duality of patterning are, and how they have been treated in the literature. Up to this point, I have only been referring to 'meaningful' and 'meaningless' elements: henceforth, I will follow de Boer et al. (2012) in referring to the meaningless level as *combinatoriality* and the meaningful level as *compositionality*. I will also be working with a definition of duality of patterning which is intermediate between Hockett and Martinet's: unlike Hockett, I will focus on the emergence of systematicity at both combinatorial and compositional levels of analysis. Unlike Martinet, however, I will not assume that combinatorial systematicity is entirely encapsulated within discrete compositional elements. Finally, unlike both scholars, I will not assume that either feature appeared before the other.

Aside from Tria et al. (2012), all of the studies reviewed below focus on the emergence of combinatoriality and compositionality in isolation. However, most make some implicit assumption about the nature of the other level, although this often boils down to whether the other level is present in the model. The nature of what entails combinatorial and compositional structure shows quite a lot of variation in the literature, along with the various proposals which have been put forward for the emergence of each. Because of this, I will review the two fields separately.

5.2.2 Combinatoriality

A broad definition for *combinatorial signalling* is simple: signals are composed of two or more concatenated discrete forms, chosen from a closed set of forms. We will first look at how there have been a number of different approaches to defining combinatoriality. The following section will outline how combinatorial communication is rare within the natural kingdom. After this I will outline several explanations which have been proposed for the emergence of combinatorial communication. Many of these are phrased in terms of optimality for learning or perception/production, but there are also a number of natural-selection based accounts. This leads into a more formal description of combinatoriality, how it emerged, and how to measure it. We will then take a look at how the different approaches we have surveyed interface with the idea of duality of patterning, and the existence of compositional mapping between the signal and meaning spaces. This leads into a more formal description of combinatoriality, how it emerged, and how to measure it.

Defining combinatorial communication

As we have seen in Section 5.2.1, Hockett (1959, 1960) and Martinet (1984) both assume a level of analysis which involves the recombination of discrete, meaningless elements, the development of which they see as both secondary to and subsequent to that of meaningful elements. As such, the process of *discretisation* would seem to be the fundamental step in the development of combinatoriality. This is not the whole story, however: while the emergence of discrete elements from a continuous form-space is an essential step, they are then combined. The nature of the process leading to combination needs to be considered.

Zuidema & de Boer (2009) take a comprehensive look at different aspects of combinatoriality, identifying three distinct stages. Firstly, a stage in which discrete elements form out of a continuous form space; secondly, a *superficially combinatorial* stage in which these discrete elements are reused over different signals; finally, a stage of *productive* re-use and perception, in which the elements are perceptually recognised and novel signals can be composed of these elements. I would add a fourth stage to these, in which the systems of recombination become "rule-based", but this will not be the focus of the study at hand.

The apparatus employed by humans to convey language (whether spoken or signed) has an inherently *analogue* nature: articulators move through a continuous, multi-dimensional space. The time dimension plays a particularly important role, not just in terms of allowing for an extra degree of freedom, but also because it complicates issues of perception: when there is no distinct boundary marker between elements (as there usually is not), the matter of where one segment ends and another begins is far from trivial. However, almost all human linguistic signalling behaviour is composed of discrete, re-combinable, non-semantic elements. At a first approximation, then, we need to account for how discrete elements resolved out of the continuous space. Some of the studies below (Oudeyer, 2006; Oudeyer & Kaplan, 2007; Studdert-Kennedy, 2005) focus solely on this, while in other studies it is a key, if not focal aspect (de Boer & Verhoef, 2012; Galantucci et al., 2010; Zuidema & de Boer, 2009; Verhoef, 2012; Verhoef et al., 2014).

The second property of combinatoriality is that these discrete elements propagate through the signalling system. As a very simple example of this, we can imagine a system in which signals can be composed from two different behaviours, A and B, and there is a closed set of signals $S = AA, BB, AB, BA$. In this case, because we are able to decompose all four signals into two component elements, we can describe it as being *superficially combinatorial*. However, there is as yet no real motivation for claiming the elements possess any psychological reality. This is, in fact, precisely the case with the minimally combinatorial call-system used by the putty-nosed money described in Section 5.2.2. There is no a priori reason to suspect that the monkey composes the combinatorial signal through some active cognitive process. The simplest assumption is that the monkeys have access to three holistic behaviours, one of which happens to be analysable into the others from an external perspective. For this reason, Zuidema & de Boer characterise this as a property of *E-language*[1]: that is, an external, observable property of language which does not represent an 'internal/generative' facility.

The next stage is *productive* combinatoriality. In contrast to its superficial counterpart, this is characterised as an *I-language* property. The discrete analytic components of signals are perceptually salient, optionally leading to the ability to construct novel signals out of the same building blocks. If, for example, the

[1]The I-language/E-language terminology is borrowed from Chomsky (1986)

putty-nosed monkeys began to create a succession of novel, meaningful signals constructed from 'pyow/hack' sequences, this would be compelling evidence that productive combinatoriality had arisen. However, as Verhoef (2012) observes, evidence of the psychological reality of segments is not necessarily derived from signal production alone: if evidence can be found of *perceptual* awareness of the segments, this would suffice as, but would likely be harder to prove.

Zuidema & de Boer (2009) stop at this point. Elsewhere, they and a number of other authors (Verhoef et al., 2014; Oudeyer, 2006; Berwick et al., 2011; Kirby, 2013) identify a fourth stage: the development of *phonotactic constraints* (which Oudeyer refers to as *strong combinatoriality*). This requires not just that the discrete elements can be recombined, but that the way in which this is done appears to be rule-governed in some sense: for example, restricting certain elements from appearing together sequentially within a meaningful unit, or in certain orders (e.g. in English, no morphemes contain an /ŋh/ or /hŋ/ sequence). However, just as with the distinction between superficial and productive combinatoriality, there must be some appreciation of the fact that apparently rule-based behaviour can have multiple explanations. The fact that there are constraints on which elements can occur together does not imply the existence of a generative faculty such as proposed by Berwick et al. (2011). Other possible explanations might be that some form of articulatory pressure disfavours certain sequences: Ohala (2005) goes as far as arguing that most phonological rules are the product of phonetic constraints. Likewise, even in the case that an internal rule-set exists but the signalling system has an inventory of limited size, it might simply be that what looks like a constraint is the result of a random gap. As is always the case when inferring internal properties from external evidence, the amount of data available must be taken into account before making any strong claims.

In the modelling work later in this chapter, I concentrate specifically on superficial combinatoriality. This is not meant in any way to discount previous work on the emergence of discrete elements from continuous forms, as this is an absolute prerequisite for the study here. However, I do show in Section 5.2.2 that mechanisms leading to combinatoriality (essentially, a response to a noisy channel) is explanatory for both the emergence of discreteness and increasing combinatoriality. Turning to the third and fourth stages, simple and rule-based productivity pose a different type of problem: as attention moves from properties of observed sequences towards properties of the mechanisms which produced those sequences, we are confronted with a multi-level *inferential* problem. Can we determine that a system exists, rather than random recombination? Similarly, assuming a system does exist, what is it? What is the best way of inferring and modelling the system? These are important issues, but I will set them aside for the time being.

Combinatorial communication in nature

In the vast majority of cases, combinatorial signalling is not a feature of animal signals, which are typically *holistic* and do not appear to be comprised of individual shared sub-elements. However, there are a number of notable exceptions. An example of this is the call system of the putty-nosed monkey, in which a 'pyow' call represents a warning about leopards, a 'hack' call warns about eagles, while a 'pyow/hack' sequence has an entirely different function, signalling to the group to relocate as food is scarce (Arnold & Zuberbühler, 2006, 2008). Importantly, the recombination of different forms does not lead to a composition of the respective meanings, and it is the ability to decompose the third signal into component parts shared by other signals which qualifies it as combinatorial. Aside from human communication, however, the canonical example of combinatorial signalling is found in *birdsong*. Many birds, for example the Bengalese finch, have songs which not only can be analysed into a number of discrete elements, where these individual elements do not themselves contribute to meaning, but that behaviour also appears to be productively rule-governed (Berwick et al., 2011; ten Cate & Okanoya, 2012). Tools developed by Kakishita et al. (2008) are able to first break the continuous signals down into discrete components and then infer finite-state grammars which produce them — we will return to some of these methods in the next chapter.

Remaining with birdsong, Collier et al. (2014) points out a difficulty in associating birdsong with human combinatorial communication: because none of this variation is associated with different signal/response behaviour, i.e. different 'meanings', it is not possible to make a comparison with phenomena such as minimal pairs in human natural languages. However, this may not ultimately pose a great problem. Firstly, it is more of an argument against the existence of duality of patterning than combinatoriality itself. Secondly, minimal pairs mainly serve as a useful inferential tool for identifying units, but other techniques are available. Finally, evidence such as presented by Engesser et al. (2015) points to the presence of exactly this kind of meaningful "phonemic" distinction in some birdsong: the chestnut-crowned babbler appears to distinguish between existing signals and novel ones where an extra signal element is inserted at some point within the original. The authors suggest that this is a possible intermediate stage prior to a phonological capacity with generative properties.

Previous explanations for the emergence of combinatorial signalling

Hockett (1959) proposed that the discrete combinatorial aspect of duality is well-designed for communication systems consisting of large numbers of messages: keeping the number of discriminating elements small is described as a "great convenience", but he does not clearly state exactly what the convenience is beyond "economy". This might seem to appeal to a more 'cognitive' perspective, i.e.

fewer elements need to be learned. However, in Hockett (1960) he is more explicit and motivates the economy as having a *physical* source: because the human articulatory/perceptual system has a finite resolution, any holistic signalling system of a sufficient size will inevitably run out of space for distinctive messages. Martinet (1984) also refers to its "obvious economical role": he argues that the discrete elements allow the maintenance of a *synchronically stable* system. He proposes that this would be impossible if meaningful elements were composed of "unanalysable grunts", as they would be subject to a form of iconicity-driven emotive modulation (i.e. the word for wind could vary in various qualities to express different strengths and types of wind), and the resulting variation would preclude the development of any shared signalling system.

Like Hockett, Nowak, Krakauer, & Dress (1999) and Nowak & Krakauer (1999) explore the idea that physical constraints interact with pressures for expressivity to produce combinatoriality. They outline an evolutionary game-theory analysis which proposes an *error-limit* explanation for the emergence of discrete forms. The authors define fitness in terms of the probability of successful communication and attain two main results. Firstly, discrete signalling strategies will always be more informative in the presence of any level of noise. Secondly, there is a 'maximally fit' number of discrete elements which is determined by the amount of noise, beyond which there are diminishing returns. As it turns out, this argument is actually a special case of *channel capacity* (Shannon, 1948) which we will look at in more detail in Section 5.2.2.

Another study relying on physical explanation is de Boer (2005), which focusses on vowel systems in a physiologically-inspired model. The study shows that a pressure for distinctiveness drives the emergence of plausibly human-like vowel inventories. Wedel (2006) employs more abstract exemplar models to illustrate a similar point: a pressure for distinctiveness combined with the physical limitations of an articulatory space results in the development of distinct vowel categories. Zuidema & de Boer (2009) and de Boer & Zuidema (2010) employ similarly abstract models including a time-dimension in which signals are plotted as *trajectories* instead of simple points. Once again, signals are optimised for expressivity, but the extra degree of freedom supplied by time has an interesting effect: to quote the original, "trajectories become far apart where possible and close together where necessary" (p.31). In the simulations, the corners of the two- and three-dimensional spaces become attractors: individual trajectories consist of periods of stasis in one of the corners interspersed by transitions between corners: in essence, this is a demonstration of how signal trajectories transition from being continuous to discrete, with each corner and transition representing a discrete element.

A number of accounts employ *information theory*. In particular, Fortuny & Corominas-Murtra (2013) proposes an account of combinatoriality (strictly speaking, they discuss duality of patterning, but in doing so they are referring to sub-lexical discrete coding) as optimised for *compressibility* and (similarly to Nowak, Krakauer, & Dress, 1999) *robust transmission*. The concept of robust transmission requires that signals are maximally distinct, in the same sense as the pressure for distinctiveness which is found in de Boer & Zuidema (2010). Ay et al. (2007) also propose that combinatorial coding allows for robust transmission, and that robustness provides a flexibility which can be co-opted for productive re-use. In their model of duality of patterning, Tria et al. (2012) show that the amount of noise in transmission determines the degree of robust, combinatorial coding taken on by the system in order to maintain expressivity: the more noise, the greater the degree of combination.

Other accounts, however, are not based on physical constraints upon expressivity. Oudeyer (2006) also uses vocal trajectories, the model consisting of coupled pairs of neural network agents. Oudeyer argues that his model demonstrates that physical constraints are unnecessary, and that a 'perceptual magnet' effect (e.g. Pierrehumbert, 2001) drives the formation of categories over repeated cycles of reception and production between agents. However, as a core element of this model is a Gaussian parameter which determines the likelihood of producing a target output value given an input value, the number of categories which emerge is a result of how large the parameter is: a large value means that categories will have widely-spread activation distributions, while small values result in narrow distributions: the former leads to few categories, while the latter leads to more. As such, the *number* of distinct categories which come into being is constrained by the size of the parameter in relation to the articulatory space. Since categories are an emergent property of groups of mappings, and those mappings are not 'tagged' in any way, there is no meaningful sense in which signals can be understood to be in competition — either they are grouped together or they aren't.[2] I would argue that this represents an implicit pressure for distinctiveness: if similar signals *cannot* be distinguished from each other, then multiple categories *must* be distinctive.

Verhoef (2012); Verhoef et al. (2014) also suggest another non-physical pres-

[2]There are a number of abstract structural similarities between Wedel (2006) and Oudeyer (2006), for the latter, particularly in the case of the two-dimensional simplifications found at the end of Chapter 6. A key difference between them is that in Wedel's model mappings are tagged with a categorical symbol, whereas categories are purely emergent in Oudeyer's model. This explains a curious difference between the two models: Wedel's models will collapse to a single category if there is no pressure for distinctiveness, while Oudeyer's models do not feature this problem. This is because of the *cue* which serves as the basis of producing a new signal in each model: in Wedel's model, the cue is a categorical tag, while in Oudeyer's model it is a random 'Go! signal' (p.80) which can presumably take on any input value in the perceptron range. Because of this, while categories are free to (and do) collapse into each other, new categories are guaranteed to emerge because they are not constrained to be near any existing cloud of mappings.

sure: *learnability*. In a series of experiments utilising an artificial articulator with a completely continuous characteristic (a slide-whistle), participants are asked to produce and repeat a series of initially randomly-produced slide-sequences (with no meaning attached to them). The sequences quickly take on discrete, combinatorial properties: distinguishable elements are repeated within and across individual signals. However, the number of sequences in Verhoef (2012), for example, is only 12 and does not threaten to overcrowd the articulatory space: they argue that "combinatorial structure is easier to learn and produce".

This view is echoed by Roberts & Galantucci (2014) and Roberts et al. (2015), who investigate the opposition between *iconicity* and *combinatoriality*. Iconic signals, resembling their intended meaning, are easily interpreted and thus highly expressive. On the other end of the spectrum, combinatorial signals display a high degree of *arbitrariness* and do not resemble their target. From a different perspective, however, because a set of combinatorial signals consists of a small amount of repeated elements, the assumption is that this would place less pressure on both memory and acquisition. They show that when the ability to produce iconic signals is removed, signals become increasingly combinatorial.

From an abstract perspective, we can see a number of common threads running through these accounts. A force for *expressivity* can be seen in the pressure to keep signals distinct in the physically-motivated models such as Nowak, Krakauer, & Dress (1999); Zuidema & de Boer (2009); de Boer & Zuidema (2010); Wedel (2014), but it is also present (more or less implicitly) in Oudeyer (the Go! signal) and Tria et al.'s simulations (when new words are invented in the event of communicative failure), and in the experimental design of Verhoef et al. (2014) and Roberts et al. (2015).

A force for *learnability* can be also be discerned, but often less obviously. Verhoef et al. (2014) and Roberts et al. (2015) are the most explicit, but Martinet's appeal to language *stability* is an appeal to learnability, as are *robustness* in Tria et al. (2012) and Ay et al. (2007), as is the *compressibility* constraint found in Fortuny & Corominas-Murtra (2013). In both cases, we see in most work that while neither pressure might be the *focus* of a study, it is still a component of the explanation.[3]

At this point, it's worth looking at what exactly a 'learnable, expressive' system is. Learnability is a property of both *signals* and *systems*: signals themselves must be perceivable and storable, while the system as a whole must also be reliably attained and maintained. In both cases, signals and systems must be *robust to acquisition and storage* in the face of information bottlenecks and noise. And so the presence of different degrees of noise itself determines what makes a learnable system: in the case of the noisy sampling effects of an transmission bottleneck it leads to a preference for compact, highly regular systems. Expressivity, also, has aspects relating to both signals and systems. Expressive signals (ignoring meaning for the time being) must be maximally *distinct* — optimally with no shared ele-

[3]There are two exception to this: Studdert-Kennedy (2005) proposes that the relevant unique adaptation in humans is that of the articulators, and that discreteness in spoken language is an inevitable result of this: I am not entirely convinced by this argument, as many continuous processes can be produced by discrete means: the orbit of the Moon around the Earth, for example, and the tides which that produces. Taking quite a different tack, Scott-Phillips & Blythe (2013) claim that combinatorial communication is unlikely to appear in species which do not engage in ostensive/inferential communication: because so much of human communication requires pragmatic inference, this is very likely to constantly produce novel signals in conjunction with each other which are then ritualised, a process which does not occur in other animals. While I fully agree with the fundamental importance of pragmatic inference and theory-of-mind in human communication, the rest of the models here focus more on the *process* by which populations transition from holistic to combinatorial signals, whereas Scott-Phillips & Blythe ask why such a thing might happen: the contrast is between a proximate and an ultimate explanation, and I am focussing here on the former.

ments between signals. We again see the role of noise, which determines what is *robust to transmission*. Likewise, expressivity requires that there must be enough signals to *cover* the the whole system of meanings.

As an example of how these forces might work at extremes, an optimally learnable system might be a single signal consisting of a single perceivable element. An optimally expressive system, on the other hand, would require that no elements are shared between signals (maximising distinctiveness), and that signals consist of a maximal number of characters (maximising robustness to transmission). Looking at it like this, we can see how a simple compromise between the two forces might lead to some form of combinatoriality: firstly, a small number of distinct elements gives us the property of discreteness; when these are spread across strings long enough to maintain sufficient distinctness, we see the emergence of superficial combinatoriality. However, as we shall see, what constitutes 'learnable' depends on the context defined by noise within the system.

I will return to this theme later in this chapter, but not before noting that combinatoriality increases in response to pressures affecting *only* the signalling space: either the number of *signals* overtakes the noise-determined capacity of the space to remain distinct, or the number of *elements* overtakes memory capacity, as defined by the number of tokens which are stable before noisy effects predominate. In either case, a signal-specific pressure is preventing them from remaining learnable and expressive. Looking at this from another direction, if perception allowed for infinitesimally different signals to be differentiated, and if memory was able to store them, a non-combinatorial system would be able to handle an infinity of distinct, learnable signals. Removing either of these capacities favours the development of a combinatorial system, although once again affected by the interaction between the number of unique signals needed and the amount of noise in the system.

Formal Descriptions of Combinatoriality

This section will look at combinatoriality from a more abstract perspective. Firstly, I will show that the presence of noise in any continuous space will place a limit on its information capacity, and hence on the maximum number of distinct signal elements possible. Because of this, whenever expressivity requires more than this capacity, the only option is to move towards some form of coding. I will then show that we can regard memory capacity as representing another form of noisy channel, which also places an effective limit on the number of viable signal elements. This leads into a description of learnability in terms of *robustness in acquisition and storage*, and unifies competing accounts of combinatoriality as being adaptations to noisy processes. Following this, I will have a look at different measures of combinatoriality.

One way of modelling a perfect communication channel is that it consists of an infinite set of distinguishable elements available for constructing signals. In the complete absence of noise, an infinite number of different single-character strings would remain learnable and expressive. As the perceptual resolution of the system is decreased, however, only a certain number of characters are available: the only way to remain expressive is by concatenating characters together. Reducing the capacity of memory has the *exact same effect*, limiting the feasible number of characters, again requiring that characters are concatenated to be expressive. The upshot of this is that the apparent tension between accounts based on channel capacity (e.g. Hockett) and learnability (e.g. Verhoef et al.) can be resolved at a certain level of abstraction: they both limit the practical resolution of the signal space, i.e. the number of 'characters' available, leading to combinatoriality as signals are extended.

The Shannon-Hartley Theorem (Shannon, 1948) guarantees that an analogue channel can never reliably retain an amount of information which exceeds C bits.[4] The only way to exceed this limit — in our case, the only way of increasing the number of distinct signals — is via *coding*, i.e. concatenating symbols together. Thus, the presence of noise (whatever the source) will always lead to the requirement that discrete elements are recombined once the channel capacity is exceeded for expressivity to be maintained.

As remarked earlier in Section 5.2.2, these more general principles underlie the game-theoretic approach of Nowak, Krakauer, & Dress (1999), who extend them to look at a number of more specifically relevant applications. In particular, instead of considering analogue channels, they look at what happens when transmission error is applied to signals across a number of different metric spaces. They first consider signals embedded in a one-dimensional space $x_i \in [0,1]$. Defining distance as d_{ij}, the similarity between any two points is $e^{-\alpha d_{ij}}$, where α is a noise parameter. On receiving a signal s_i, the distribution of interpretations for the signal is proportional to the ratio of their similarity to the sum of all similarities:

$$p(s_j|s_i) = \frac{s_j}{\sum_k s_k} \tag{5.1}$$

They show that, in this case, the maximally 'fit' number of signals F_{max} — corresponding to the most informative configuration — is:

$$F_{max} = 1 + \frac{\alpha}{2} \tag{5.2}$$

This relation will hold for any linear space affected by noise, both guaranteeing discreteness and determining the optimal number of discrete elements. However, this seems to contrast with the effects of noise in the simulations of Zuidema & de Boer (2009), in which there is no discretisation of the space and signals tend

[4] Related to this, Shannon's Noisy Coding Theorem tells us the maximum amount of information capacity given a certain tolerance for noise, which is always larger than C.

to cluster at the corners of the available space. The reason for this is that in that study distinctiveness is maximised within the 2-dimensional space but not in the extra time-dimension, which is why they extend through that and maximise spatial distance. Were this not the case, we would expect to see discreteness develop in two dimensions according to the optimal condition shown in Eq. 5.2.

Nowak, Krakauer, & Dress (1999) also consider a space where signals have a similarity of 1 to themselves and a similarity of some constant s to all other signals. This seemingly abstract idealisation is actually a rather compact model of *exemplar memory*, where representations compete with each other only on the basis of total storage. Nowak, Krakauer, & Dress attain another neat result: the maximally fit number of discrete signals within this space scales inversely with the similarity constant. That is, the more likely signals are to be confused, the smaller the number of maximally effective signals, and vice-versa.

If we look at this in terms of memory, this is equivalent to saying that whenever signals are assigned a constant weight, the system will have a corresponding optimal number of signal elements. This observation plays a key role in the models in Section 5.3. Unfortunately, we can't directly apply Eq. **??** to those models because they define noise probabilistically instead of using a similarity metric. However, we *can* use Shannon's information theoretic definition of channel capacity to demonstrate that this feature of 'maximal expressivity' is also true for the model here. The information capacity of a channel with distributions of sent signals S and interpreted signals R is defined as their mutual information $I(S;R)$ (see Fig. 5.3). This employs the joint entropy measure $H(S,R)$, which is the overall uncertainty associated with *both* S and R.

$$I(S;R) = H(S,R) - H(R|S) - H(S|R) \tag{5.3}$$

In a preview of the model described in 5.3.1, we can use this equation to

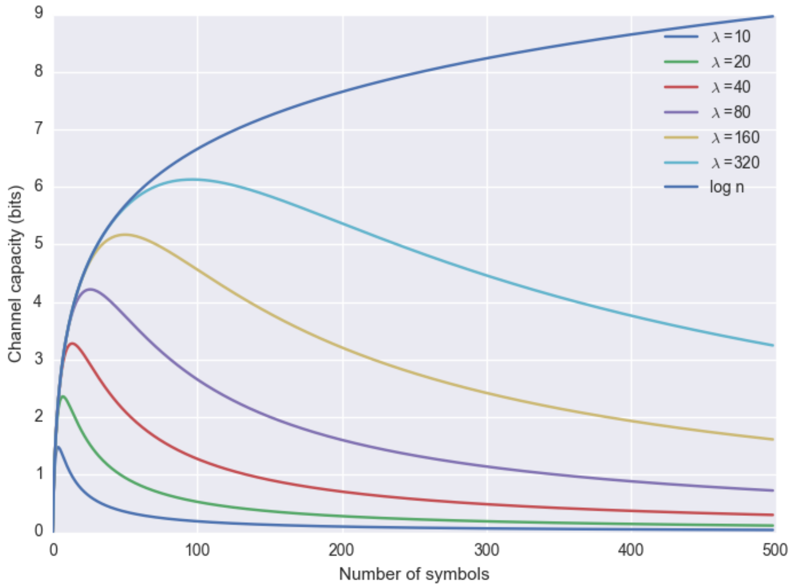


Figure 5.1: The information capacity of a 'discrete memory channel' where noise is determined by the parameter λ and the number of equiprobable symbols N such that $p(Flip) = e^{\frac{-\lambda}{N}}$

analyse the effects of noise in that model. Fig 5.1 describes a probabilistic version of the exemplar memory given in Eq. **??**, which includes a noise parameter λ. As this parameter increases (higher values corresponding to less probability of noise), we see the maximum information capacity defines a certain 'peak' number of symbols—literally the peaks of the individual curves here—beyond which noise takes over and the channel becomes increasingly less informative. In the absence of noise ($\lambda = \infty$), the information capacity of a channel converges to its standard information theoretic definition for N equiprobable signals, $log\ N$. Even in this more abstractly-defined memory space, noise determines the maximum number of characters.

Finally, Fortuny & Corominas-Murtra (2013) also point out that Hockett's argument has strong parallels with Shannon's noisy channel theorem, but they argue that channel capacity alone cannot account for the emergence of code-like discretisation. Fortuny states that coding allows for both data compression and robust transmission, and hence should be desirable as soon as any discreteness becomes available. For example, discrete coding means that more frequent meanings can be associated with shorter codes, an assumption which goes back to Zipf (1936). Fortuny's claim is a sound one, as long as we assume that signals are coded to be *productively optimal*, which relies (realistically) on the probabilities of meanings being different: short signals for frequent meanings, long signals for infrequent ones. However, in the absence of noise it is possible to have an infinite number of basic signals, all with the same minimal length: a completely optimal system. Because of this, noise must play an essential part in any explanation of combinatoriality.

Measures of Combinatoriality

As we have outlined previously, the concept of combinatoriality involves more than one concept. These are detailed below:

- The emergence of discrete elements from a formerly continuous space
- The appearance of these elements within and across different signals
- The productive recombination of these elements, possibly involving novel signals, entailing the mental 'reality' of the segments
- The emergence of rule-like behaviour governing the recombination of those strings

In the work surveyed previously, the focus was placed on one or both of the first two phenomena, because without investigating them first it is not possible to investigate the second pair. I will focus on the first two and look at the different ways in which they have been measured.

Detecting discrete elements

Determining whether continuous signals have developed discrete elements is much more challenging than might be expected. The fundamental issue is a problem of scale. Any regular, repeated features found in a continuous signal may themselves be composed of other regular, repeated features, and so on. To use English as an analogue, syllables, phonemes and features are all drawn from a finite set, but (most) analysis tends to regard the phoneme as the combinatorial unit. When we turn from language to the data from models, experiments, and animal studies, we have neither the amount of data nor any other linguistic cues available when working with natural languages, making the task highly non-trivial. Notwithstanding this, researchers have found a number of ways to detect discreteness. We will survey these now.

We can first look at methods used in comparative animal studies. Because the elements of the 'pyow-hack' monkey calls studied by Arnold & Zuberbühler (2006, 2008) were themselves produced in isolation, segmentation did not pose a problem. However, addressing the possibility that some undetected feature of the combined calls was triggering the different response, the researchers created their own synthetic calls by combining the isolated ones. When these were played to the monkeys it triggered a highly similar response, indicating the salience of the components. Although continuous birdsong presumably presents a more difficult problem, the techniques by which Kakishita et al. (2008) extract individual notes from waveform files is not reported, while Engesser et al. (2015) utilised software specifically designed for birdsong (Bioacoustics Research Program, 2011).

Moving to experimental work, Verhoef & Kirby (2010); Verhoef (2012); Verhoef et al. (2014) analyse their slide-whistle data using several techniques. They identify silence-separated segments, and then place them into categories using a different types of clustering models. Verhoef (2012) observed that participants were apparently paying more attention to the movement of the whistle-plunger than the sounds themselves. This inspired a measure mapping to this movement, which proved to be a significantly more reliable method of extracting combinatorial elements. In the visual modality, Roberts & Galantucci (2014); Roberts et al. (2015) also use separation as a criterion, in this case white-space between figures, before categorisation by a clustering algorithm. Giudice (2012) also uses a distance measure based on dynamic time-warping to calculate distances across all signals in a set, with higher similarity interpreted as greater combinatoriality.

In modelling work, Oudeyer (2006) uses Shannon entropy to analyse the activation levels across the agents' neural map outputs. As such, maximal entropy (uniform distributions) represent a complete lack of clustering, and decreasing entropy (more 'peaky' distributions) indicates increased discreteness. Zuidema & de Boer (2009) measure the distinctiveness of signal trajectories via *confusability* probabilities, in which parameterised Gaussian noise around any point of the trajectory is used to calculate the chance of being misinterpreted as its nearest neighbour. Straddling the boundary between the detection of individual elements and the measurement of combinatoriality itself, de Boer & Zuidema (2010) outline a measure of 'phonemicity' for the trajectories in their model. They observe that in combinatorial systems where trajectories are from point to point in a straight line, the endpoints of trajectories are more likely on average to be closer to each other than intermediate points are, and instantiate a composite measure for this.

Due to the difficulty of detecting discrete elements, most of the methods surveyed above either involve some human interaction (for example with the birdsong analysis software), miss out on possible further segmentation (when physical separation is used as the criterion), or rely on proxy measures in place of identifying actual elements. There is a philosophical issue at stake here: if we assume that the segments possess some psychological reality, the task is more than a simple mechanical one, as it requires inference about cognition. But if cognition is ignored, the problems of scale discussed at the beginning of this section become significant. In fact, identifying the fundamental units of speech is contentious even within mainstream linguistics: the phonetic realisation of phonemes are cloud-like and vary over multiple dimensions, overlapping significantly (e.g. Pierrehumbert, 2001), and the mental reality of the *phoneme* has been disputed since its inception (e.g. Twaddell, 1935).

Measuring superficial combinatoriality

Once a set of discrete elements has been established (whether this is through detection or if they are simply assumed), we need to establish a measure of combinatoriality. Again, this is not as simple as it might seem: combinatoriality is much easier to establish as a *property* than a scalar measure. To illustrate this, we can take a recent (de Boer & Zuidema, 2010) definition of combinatoriality: "...it combines a limited number of basic sounds into a potentially infinite set of complex utterances that all differ in meaning". Assuming the set of 'basic sounds', any measure must concentrate on the *degree* of combination, but what exactly this means is not immediately clear. Because there is no obvious measure, none of the various methods employed in the literature are exactly the same.

Verhoef (2012); Verhoef et al. (2014) define combinatoriality in terms of compression: the smaller the set of elements used to construct the entire signal space, the more combinatorially structured it is. Defined like this, Shannon entropy provides an ideal measure: they simply take the proportion of each element p_i across all signal strings and calculate $H = -\sum p_i \; log_2 \; p_i$. Lower entropy values are reached when there is a greater concentration on smaller sets of building blocks. There are some drawbacks with this particular measure, however: firstly, by taking the overall proportion of each element across all strings, there is no way of determining that elements are re-used in *different* strings. For example, the signal set $S_1 = \{AAA, BBB, CCC\}$ would render an identical measurement as the signal set $S_2 = \{ABC, BCA, CAB\}$. Secondly, a maximum value of combinatoriality is reached for $H = 0$, implying that everything is made of a single element. There are situations for which this does not pose a problem, for example $S_3 = \{A, AA, AAA\}$, but it would also hold true for $S_4 = \{A, A, A\}$ or even $S_5 = \{A\}$: at this point, maximally 'combinatorial' systems cease to appear combinatorial at all.

Zuidema & de Boer (2009) suggest an example measure for the 'degree of combination': "$\phi = \frac{N}{k}$, where ϕ is the measure of recombination (phonemicity), N is the number of words in the repertoire and k the number of building blocks." This would render high values for small sets of basic elements, and minimal values for larger sets. We're still faced with a problem, however: there are a number of confounding factors which will affect the value of ϕ. Firstly, just as with Verhoef et al.'s measure, it can't distinguish between elements being re-used within or across strings. Secondly, as another example, take $S_1 = \{ABC, CAB\}$ and $S_2 = \{ABC, CAB, BCA, ACB\}$. Although all strings maximally re-use the available elements, $\phi(S_1) = 0.66$ and $\phi(S_2) = 1.33$: the number of strings directly affects the score, and the nature of the measure means that there is no way to normalise for this. Similarly, both $S_3 = \{ABC, BAC, CBA\}$ and $S_4 = \{A, B, C\}$ produce $\phi(S_3) = \phi(S_4) = 1$, but S_4 bears the hallmark of holistic rather than combinatorial signalling.

In Galantucci (2005); Roberts & Galantucci (2014); Roberts et al. (2015), the *Form Recombination Index* (FRI) is used. After determining the set of elementary elements, the FRI provides a count of re-use *across signals*. This is done by first eliminating any repeated elements within signals, and then counting the number of times each unit features in a signal F alongside the number of times an element *could* have featured in a signal P, and then calculating $FRI = \frac{F}{P}$. This has some nice properties, in that it takes into account re-use across and within strings. There are some possible drawbacks, however: one is that its maximal value of 1 is reached when there is only one element, for example in the case of a one-word system composed of a single holistic character. A second is that, while the measure is more robust against string-length than others, it can still fall short: compare $S_1 = \{ABC, CAB, BCA\}$ and $S_2 = \{AB, BC, AC\}$. While elements are maximally re-used across strings in S_1, if we consider that (for some reason) string-length is constrained to 2 in S_2, then elements are being maximally re-used *for that string length*.

Tria et al. (2012) also define a measure of combinatoriality C, where there are F distinct word-forms, M meanings, and $m(f_i)$ is the number of distinct meanings whose signal contains the element f_i :

$$C = \frac{\sum_i (m(f_i) - 1)}{(M-1)F} \tag{5.4}$$

The term $(m(f_i) - 1)$ is used so that only elements which are used for two meanings are taken into account. One minor implication of the measure is that it suffers from the same problem as FRI, in that it can be affected by word length. However, it does a good job of registering combinatoriality across strings.

Finally, we could also define a measure of *optimal* combinatorial re-use C_x across strings. This is essentially a measure of compression, using the relationship between the optimal number of bits to store a lexicon expressing all meanings without using any coding/compression techniques $log\ |M|$ as a proportion of the number of bits encodable by the total number of signal elements $log\ |C|$ multiplied by the average word length $\frac{\sum |w|}{|W|}$.

$$C_x = \frac{\lceil log_2|M| \rceil \, |W|}{\lceil log_2|C| \rceil \sum_{w \in W} |w|} \tag{5.5}$$

This value for C_x could also be taken as an indicator of *redundancy*: the lower the value, the greater the degree of redundancy, and hence the more likely the signalling system is to be robust to noise.

In Section 5.3, both overall character entropy and Tria's combinatoriality measure will be used. Due to the fact that string length doesn't vary in the model, the measure for optimal combinatoriality is not used here, as it is better suited to systems which have more ability to adapt in this respect.

Implications for duality of patterning

Both Hockett (1959, 1960) and Martinet (1984) explicitly state that combinatorial structure in language would have developed as a separate step, secondary to productive compositionality. A recent animal study sharing this hypothesis is given a paper by Collier et al. (2014). They propose a re-interpretation of Campbell monkey's compound call systems (Arnold & Zuberbühler, 2006, 2008) as actually possessing a type of "syntactic blending", in which the semantics of eagle and snake signals are synthesised into a 'move on' signal. They argue that the human phonological capacity is likely a subsequent development to this kind of semantic compositionality, and the result of pressures of cultural transmission rather than genetics. In contrast to this, Engesser et al. (2015) claim that putty-nosed monkeys do possess a phonological capacity, and that they are even able to distinguish between established and artificial recombinations. Berwick et al. (2011) also concentrate on combinatoriality found in the natural world, suggesting that the main difference between the combinatorial ability found in birdsong, for example, and that of humans is that in humans it is wedded to compositional semantics. According to this view, both shared brain regions and convergent evolution probably play some role in the shared aspect of combinatorial ability. In placing emphasis on the combinatorial faculty, however, the focus is on the question of how it gained semanticity at all, whether productive or not. Ladd (2012) echoes this sentiment, putting forth the idea that duality of patterning is simply the combinatorial ability applied to different levels of structure. In contrast to this view, Kirby (2013) proposes the emergence of combinatorial and compositional structure as two independent major transitions in the evolution of language; he is keen to point out that neither change pre-supposes the other. In the theoretical literature at least, there is no consensus on whether compositionality preceded combinatoriality or vice versa, or indeed whether separate steps were necessary.

Looking at experimental and modelling work, there are few assumptions made about the existence of productive compositionality, which features only in the work of Tria et al. (2012). The main split lies between work which assumes Hockett's version of duality, i.e. the existence of a meaningful stratum, and that which doesn't. In the former camp, the experiments of Galantucci (2005); Roberts & Galantucci (2014); Roberts et al. (2015); de Boer & Verhoef (2012) all include the expression of meaning. Most modelling work lies in the latter camp, however: Oudeyer (2006); de Boer & Zuidema (2010); Zuidema & de Boer (2009) concentrate directly on the signal space. This is also the case in the experiments of Verhoef & Kirby (2010); Verhoef (2012); Verhoef et al. (2014). By distinguishing signals through form alone, no assumptions are made about the role of meaning, and hence duality. Returning to Tria et al. (2012), combinatorial and compositional structure are proposed to be the result of different pressures: either can emerge in isolation.

We can see, then, that literature on combinatoriality can be divided into three camps on the issue of duality: i) combinatorial systems pre-suppose compositional ones ii) the combinatorial *capacity* underlies both combinatorial and compositional systems, and iii) no commitment to duality by ignoring meaning. We will now turn to studies on the emergence of *compositionality*.

5.2.3 Compositionality

This section will proceed in a similar fashion to the one on combinatoriality: after a survey of semantic compositionality in nature, we will have a look at previous definitions, explanations and measures of compositionality, before looking at how work in this field interacts with theories of duality of patterning.

Defining compositionality

The first description of the compositional property of human language is often attributed to Frege (1884), but similar ideas can be traced back as far as Plato (Janssen, 2012). An informal definition of compositionality is:

> "The meaning of a complex expression is determined by its structure and the meanings of its constituents." (Szabó, 2013)

This definition of compositionality can be further analysed into:

1. There are multiple constituents.
2. Constituents can be combined.
3. Constituents are meaningful.
4. Combinations of constituents have meaning.
5. Constituents contribute towards the meaning of combinations.
6. The way constituents are combined also contributes meaning.

The first two aspects of compositionality describe its *combinatorial* properties. It is the meaningfulness of constituents, and their contribution towards a larger meaning which distinguishes compositionality from the combinatoriality discussed in the previous section. However, we do see in both cases that the manner of combination plays a key role (e.g. combinatorial *dog* vs. *god* and compositional 'man bites dog' vs.'dog bites man'). Seen like this, compositionality is what Hockett (1959, 1960) referred to as *productivity*, and what Martinet (1984) as described as *primary articulation*, and the compositional syntax which it entails is fundamental to human language.

More formally, Montague (1970) defined compositionality as a *homomorphism* between meaning and form. A homomorphism is a mapping from one structured space to another in which some aspect of the original structure is preserved. As an example from cartography, projecting maps from the surface of a sphere to a flat plane distorts many of the original features. By using different projections, certain properties of the original, such as direction, shape, or area can be preserved, usually at the expense of others. Each of these projections is a different homomorphism. Montague's formulation involves more technical apparatus than is needed here, but a simplified version (based on Szabó, 2013) defines expressions e, a meaning function m, a semantic operation G, and a syntactic operation F, and defines a compositional mapping as:

$$m(\ F(e_1,\ \ldots\ ,e_k)\ ) = G(\ m(e_1),\ \ldots\ ,m(e_k)\ ) \tag{5.6}$$

Working from the left, this says that the meaning of a particular syntactic combination of expressions is the same as a particular semantic combination of the meaning of the individual expressions. In a fully compositional language, then, the homomorphism implies that every semantic operation G necessarily has a matching syntactic operation F.

In this way, Montague is addressing more than just the *productivity* of language, but also its *systematicity*: the syntactic and semantic operations are of crucial importance. However, in line with much (but not all) previous work examining the emergence of compositional language, I will avoid complex operations for the time being and instead focus on a more restricted version of compositionality. This does not mean that I will ignore systematicity, just that my focus on a highly restricted version which assumes only *concatenation* of both form and meaning. As such, we can further simplify our definition of compositionality. Assuming a simple *additive* and *transitive* model of semantics[5], we can define a compositional mapping as:

$$m(e_1,...,e_k) = m(e_1) + \ldots + m(e_k) \tag{5.7}$$

A signalling system which is compositional in this way, then, needs to have an *analytic* property: every expression with a complex meaning must be *decomposable* into a set of smaller sub-meanings and sub-expressions, with each sub-meaning mapping to a distinct sub-element. This does not imply that there is only a single way to decompose either meanings or expressions: the compositional homomorphism requires only that one particular decomposition of each has this property. In practice, the majority of the models described below assume a *fixed* semantic structure, with most decomposition/recombination involving form elements: the model in the following section will also make this assumption.

We can use this simplified version of compositionality to develop a definition for compositional systems. Complex meanings $m \in M$ are composed of features $f \in F$, and expression-strings $s \in S$ are composed of sub-strings s_{ij}. A fully compositional system must satisfy:

[5] i.e. GREEN DOG = GREEN + DOG = DOG + GREEN

$$\forall f \in F \; \exists \, s_{ij} \; \text{ such that } \; p(f|s_{ij}) = 1 \;\; \wedge \;\; p(s_{ij}|f) = 1 \tag{5.8}$$

This says simply that every meaning feature is always associated with a particular sub-string, and that sub-string is always associated with the same meaning feature (note that this leaves open the possibility that some sub-strings are redundant, and do not form part of the compositional system). Referring back to the work on simple signalling games in Chapter 1, this basically says that an unambiguous, synonymy-free set of mappings exists on the level of individual meaning features. Note that this definition of compositionality implies completely optimality. Other optimal but non-compositional signalling systems are also possible, but they do not satisfy Eq. 5.8. This is because although it is true that every feature is associated with an unambiguous signal ($p(f|s_{ijk}) = 1$), the non-analyticity of holistic signals means that instances of that feature found in different complex meaning combinations will have no consistent associated signal ($p(s_{ijk}|f) \neq 1$). In Section 5.3.1, we will use this definition of compositionality to construct a relevant measure.

The issue of synonymy is worth addressing, as the simplified version of compositionality does not directly imply Eq. 5.8. Consider the following system:

{ ('bigdog':BIG+DOG),
('largefish': BIG+FISH),
('smalldog': SMALL+DOG),
('littlefish': SMALL+FISH) }

It is easy to infer meanings for 'dog' and 'fish', and hence by process of elimination the other terms. However, it is hard to tell exactly whether this is fully compositional: within the context of this small system, terms like 'small' and 'big' have the character of so-called *cranberry morphs*: they reliably indicate a particular property, but not in a systematically predictable way. A similar problem is seen with synonymy: introducing 'bigcat' and 'littlecat' would reveal a degree of systematicity, but not a system-wide one. Certainly, this would still not represent a fully compositional system, as Eq. 5.8 cannot distinguish between degrees of systematic synonymy. In fact, this 'argument from synonymy' has been used to attack the existence of a strict interpretation of the principle of compositionality: see (Pelletier, 1994), who argues that any good working definition of compositionality should be able to deal with only partially systematic relationships between meaning and form, as we see in all natural languages. Later, in section 5.3.1, the measure of compositionality we construct deals with this by measuring the degree to which form maps onto meaning probabilistically.

Pagin & Westerståhl (2010) build on the foundations laid by Frege (1884) and Montague (1970). They survey a number of discussions within philosophy on whether compositionality is a true feature of natural languages. Many of the arguments — especially the idea that compositionality is a requirement for language to be *learnable*, *productive*, and *systematic* — are well-established in other fields. One argument in particular, that compositionality minimises *complexity* has especially strong parallels with work such as Kirby et al. (2015). Assessing arguments against compositionality, they find the strongest case in a number of apparent natural language exceptions to the rule, including some tricky technical problems with the truth-conditionality of embedded clauses such as 'John believes X', the syntactic status of quotations, and the varying degrees to which idioms are analytic. On the whole, their assessment is that compositionality is not an unproblematic notion. Luckily, most of these problems do not affect the current study, apart from the issue of idioms. As discussed in Section 5.2.3, the appearance of systematicity (as is found in idioms) does not imply an active systematic process. We will revisit this distinction between internal and external systematicity (c.f. K. Smith, 2003b) in the model section later on.

Finally, up till this point we have been referring to abstract associations between meaning and signal. However, just as synonymous terms result in productive signalling which is not entirely systematic (i.e., more than one productive form for any given meaning), it is possible to describe systems which satisfy Eq. 5.8 which are systematic but not productive. For example, a completely systematic signalling system can be learned holistically, by rote. In this case, although the system itself bears all the hallmarks of compositionality, in cognitive terms it is not. K. Smith (2003b), recognising this fact, contrasts *I-compositionality* with *E-compositionality*, the former an internal cognitive property, and the latter an external systematic property. The model described in section 5.3.1 blurs the line somewhat between these two, but makes use of both.

Previous accounts for the emergence of compositionality

There is a large literature devoted to the emergence of compositionality in human language, spread over a number of disciplines. This encompasses a widely divergent set of perspectives, but discussion tends to centre around certain key issues, including i) the nature of proto-language, ii) which cognitive, social and environmental prerequisites are necessary, and iii) the role of compositionality in the emergence of linguistic systematicity. Particular attention has been focussed on the relative importance of cognitive biases and interaction, and along with this the roles of horizontal vs. vertical transmission and the effect of information bottlenecks. Although there has been a good deal of theoretical and comparative work on these issues, studies employing computational and mathematical modelling are particularly well-established in the field, and (partially inspired by this work) there is a fast-growing body of experimental work. The following section surveys this work, with particular emphasis on previous modelling and experimental accounts.

Comparative work

Compositional communication (as opposed to combinatorial) is a much less prominent topic of discussion in animal comparative studies. The primary reason for this is that there is scarce evidence that any non-human species employs compositional communication at all. Collier et al. (2014) put forward the strongest argument, claiming that Campbell monkeys utilise a 'lexical affix' which modifies their normal alarm calls to become more general. For example, instead of the normal 'eagle overhead', the affix would render 'something overhead', which would give the affix something akin to the semantics of a lexical modifier. In a related but more tentative claim, they offer up a compositional re-analysis of the putty-nosed monkey data (Arnold & Zuberbühler, 2008): the 'pyow-hack' signal, traditionally regarded as combinatorial could also be seen as compositional, where the two more specific warning calls merge into a more general one. The real problem with all such claims is the issue of internal/external systematicity just discussed: given the highly limited signalling systems, there simply isn't much data to work with due to the small size of the signalling repertoires, and any appeal to productive compositionality is on shaky ground. Berwick et al. (2013) argue along similar lines: the 'compositional creativity' of human language sets it apart from both highly combinatorial but non-meaningful birdsong and the meaningful but apparently non-creative signalling found elsewhere.

Protolanguage

The issue of *protolanguage* — i.e. the existence of an intermediate stage between an initial, presumably simple form of human communication and its full modern complexity — is highly contentious. There is strong disagreement between proponents of the idea that the language capacity is based on a single, transformative ability to manipulate mental symbols and hence must have appeared suddenly (e.g. Hauser et al., 2002, 2014), and those who suggest a more gradual emergence (e.g. Pinker, 1984; Jackendoff & Pinker, 2005; Hurford, 2011). This is a debate which has taken on a wider philosophical dimension, highly influenced by the model of language which is assumed. Because the work I am presenting here is an investigation into how linguistic structure can emerge as the result of processes of learning and cultural transmission, it is inevitably biased towards the second, gradualist perspective. This being the case, I will not take part in the debate, apart from pointing out my implicit position.

Within the gradualist tradition, another ongoing discussion regards the nature of protolanguage. The main point of contention revolves around its putative development, either i) an initial stage in which simple, atomic semantic units began to combine with each other (e.g. Tallerman, 2007, 2008; Hurford, 2011), or ii) a 'fractionating' process in which semantically complex *holistic* signals were gradually re-analysed into smaller *compositional* parts (e.g. Wray, 1998; Kirby, 2000; K. Smith, 2008; Arbib, 2011). While neither hypothesis is disprovable, much of the argument hinges on the emergence of the recombinatory process. In the atomic account, the ability to combine elements is all-or-nothing: in the case of holistic protolanguage, some degree of combination is also necessary, but allows for the possibility that the *appearance* of structure (i.e. similar to what Zuidema & de Boer (2009) describe as 'superficial' structure) can arise first. However, unlike the case of emergent combinatoriality, which can be driven by entirely random processes such as noise, any process leading to even the appearance of compositional structure requires *some* form of combinatorial re-use. The question, then, is how sophisticated that process is. Croft (2004) remarks that most 'holistic-first' models rely on a pre-existent 'representation of meaning', and that the generalisation process represents a movement towards an 'iconic mapping': hence, the process is not so much one of *transformation* but more one of *replication*. To illustrate this point, instead of invoking a complex set of partial internal representations which are subsequently manipulated, simply partial repetition of similar utterances should, by itself, be able to lead to compositionality. This is arguably the simplest form of recombination, and forms the basis of the model presented later in this chapter.

Models of the emergence of compositionality

Since several foundational works in the late 1990s, use of mathematical and computational modelling has been a mainstay of research into the evolution of compositionality. This encompasses a wide variety of approaches, working under seemingly quite different assumptions. Much of the work assumes either the Iterated Learning framework (Kirby, 2001, henceforth IL) or interaction-based accounts (surveyed in Steels, 2012). A number of studies cannot be easily classified into one of these camps: while I will address them, the division between IL and interaction-based will be my primary focus. My main purpose in this survey is to show that despite the apparently conflicting methodologies and theories, a number of common assumptions underlie most of the models. These have much in common with the philosophical arguments outlined in Section 5.2.3: compositional *systematicity* has the benefit of being *learnable* and *productive*. These notions are expressed in various different but related ways in the literature, often resulting in some nuance: take, for example, the terms ‘learnability’, ‘simplicity’, ‘compressibility’, and ‘low complexity’. Similarly, ‘productivity’ can be grouped together with ideas of ‘expressivity’, ’novelty’, ‘functionality’ and ‘a desire to communicate’. I will clarify how these terms relate to each other, but will first survey the previous models.

Nowak & Krakauer (1999) describe an evolutionary game-theoretic model of the emergence of ‘basic grammatical rules’ which boils down to an analysis of the relative stability of holistic vs. compositional languages in the face of noisy interference affecting the signal such that every word has an equal probability of being misunderstood. Their first finding is that 'grammatical' (compositional) languages are 'fittest' up to a certain noise threshold, after which holistic languages are better.[6] This is an interesting result, in that it runs contrary to arguments that syntactic communication is *more* robust to noisy processes. The reason for this is that noise in this model specifically affects *signals*, rather than associations: when noise levels are too high, using more words increases the likelihood of interference: I will discuss this further in section 5.4. Their second result is that for a compositional language to be preferable, the number of states which need to be described must be larger than the total number of individual signals. Although this may seem somewhat redundant, it does capture the idea that the benefits of compositionality are not simply that events must be described using multiple signals, but that by limiting the number of signals overall, the *system* is more robust to "mistakes in implementation and comprehension" (p.8031): it is more learnable. They further develop this result in Nowak et al. (2000), and derive a result showing that applying a learnability parameter to the model places a more complex limit on when compositional signalling is more adaptive than holistic strategies.

The IL framework (Kirby, 2001) is a development of earlier work looking at the evolution of linguistic syntax and compositionality (Kirby, 1996; Kirby & Hurford, 1997a,b; Kirby, 1998, 1999a,b, 2000). The key insight is that when asking where structure in language comes from, it is not enough to simply ascribe it to some mental faculty. Language is a cultural product, transmitted between people and down the generations. These repeated cycles of expression and infer-

[6]More specifically, the chance of mistaking any word is defined by an error rate ε. When $\varepsilon = 0$ then holistic and compositional languages are equally fit, and compositional languages are fitter up to a certain threshold $0 < \varepsilon < \varepsilon_{max}$.

ence from limited data shape the linguistic system as it is passed along. As well as simply being expressive, languages must be *learnable*. Although many studies (modelling and experimental) employ the metaphor of intergenerational transmission, this is not the essential aspect of the framework: work using IL attempts to determine how different cognitive, interactive and transmission pressures shape the structure of culturally transmitted communication systems.

One trend that can be seen in IL models is a gradual increase in abstraction. Early studies (such as Kirby, 2000, 2001) model learning in terms of *grammatical inference*. Chance correspondences between holistic signals with some shared meaning feature lead to a grammatical re-analysis using a chunking algorithm: this creates not just new 'lexemes', but also *rules* with intermediate categorical projections (e.g. the equivalent of noun phrases). The emphasis at this stage is on language as *adaptive system*: as it undergoes successive jumps between *I-language* and *E-language*, it adapts to survive this process, becoming more learnable over time.

Brighton & Kirby (2001); Brighton (2002, 2003) take a closer look at the relationship between learnability, the nature of induction, and a *transmission bottleneck*. This varies the amount of linguistic data which passes between generations, i.e. how many linguistic exposures a learning agent is provided with in order to infer their own signalling system. In particular, the idea that learning equates to *simplicity* is explored. Using an induction algorithm which generates finite-state transducers to describe the linguistic data, these grammars are then evaluated using a Minimum-Description-Length criterion, and the grammar with the smallest size is chosen at each generation. This bias towards simplicity drives the development of compositional grammars, in line with Kirby's earlier hypothesis that these are more learnable than holistic languages are. Moreover, they also investigate the *stability conditions* determined by the size of a transmission bottleneck, and show that compositional grammars remain stable under much smaller bottlenecks than do holistic ones.

Developing from these models, K. Smith (2003a,b) uses an associative network model to show that when agents are equipped with two *learning biases*, one in favour of one-to-one mappings between signals and meaning, and another bias to 'exploit regularities in the input data'.[7], He explores Kirby's insight that the emergence of compositional languages depends on the transmission bottleneck and shows that when it is large enough, languages will remain holistic, but that they become compositional under the pressure of a reduced bottleneck. Moreover, as this model is able to manipulate biases which were implicit in earlier work, it expands on the idea that compositionality favours learnability, but also that the biases reinforce *systematicity* and *expressivity*.

The bias for one-to-one mappings (further discussed in Brighton et al., 2005) is a direct analogue of Montague (1970), who defines compositionality as structure-preserving homomorphisms between form and meaning (see Section 5.2.3). This is explored in more abstract terms by Brighton & Kirby (2006), who model the cultural transmission of signals as topographical mappings between two continuous two-dimensional spaces. Initial mappings are completely random, and every generation a new set of signals must be transmitted for arbitrarily-chosen points in the meaning space. As the meaning space is continuous, every new meaning is novel and must be generated by analogy based on the three nearest signal/meaning vectors. Over the course of several generations, the topographical structure of both spaces become steadily more similar: moreover, the smaller the bottleneck, the faster this occurs.

[7]This is further explored in K. Smith et al. (2003), in which the authors show that the development of compositionality depends on the degree to which the agent perceives the world as being structured — the more structured, the more compositionality develops

This theme, that compositional structure is an adaptive response to maintain learnable, expressive systems, forms the basis of K. Smith et al. (2013); Kirby et al. (2015). A Bayesian model is used to directly control pressures for expressivity and learnability/compressibility, and shows three possible end-states depending on the pressures applied. When there is only a pressure for compressibility, degenerate non-communicative languages emerge whose high learnability stems from their consisting of only a single term. When there is only a pressure for expressivity, non-systematic holistic languages result. When both pressures apply, on the other hand, systematic compositional languages result.

Turning to work which focusses on the role of interaction, the stress is placed not so much on learnability but more on the idea that language structure facilitates optimal interpersonal communication. As a result, while these studies also use agent-based methodologies, they tend to use closed population structures and emphasise the role of *self-organisation*. The "Talking Heads" program of Steels (2012), features computational and embodied robotic agents which negotiate language bottom-up, all the way from a shared set of signals and categories to mutually-agreed syntactic and semantic systems. Due to the ambitious scale of this project, compositionality is not often the sole focus of this work, but often an implicit stepping-stone. However, some work within this paradigm has concentrated on compositionality alone.

Vogt (2005) directly compares intergenerational IL with closed population structures: using a version of the Naming Game (Steels, 1997) alongside a framework capable of fully recursive phrase-structure grammar, agents re-combine signals to form compositional mappings. A key difference between this model and others we have looked at (e.g. K. Smith, 2003a; K. Smith et al., 2013) is that agents will attempt to construct compositional signals whenever they are able to, even if they have access to a holistic signal for the target meaning. Because of this, compositional signalling happens both with and without iterated learning, although in the case of IL with multiple teachers a bottleneck is still needed to eliminate excess variation. In subsequent work, Vogt (2005) shows that semantic overextensions play a key role in the development of compositional signalling, echoing the 'systematicity bias' described by K. Smith (2003b).

De Beule & Bergen (2006) and De Beule (2008a) also employ interacting populations of agents who use a chunking model capable of processing hierarchical syntax. In De Beule & Bergen (2006), the model puts holistic grammars in competition with compositional ones. Both 'simple' and 'complex' topics are possible, the proportion determined by a ratio parameter between 0 and 1. As in other models, the language develops via gradual fractionation of utterances, but in this case only when a simple topic is compared with a complex one.[8] This being the case, they find that compositional languages will only develop when complex and simple topics are mixed, and furthermore that a high ratio of simple topics (what they describe as 'low task complexity') is required for compositionality to have any advantage.

De Beule (2008b) argues against the role of vertical transmission. When agents process input strings, they interact to evaluate their communicative success, and weight the resulting chunk accordingly. As this process of gradual negotiation succeeds in creating compositional language, the author proposes that this shows that only a 'desire to communicate' is required for the emergence of language structure. As such, the weight of the argument in this case is on *expressivity* as opposed to *learnability*. However, as with Vogt's models, the agent architecture of both models guarantees that compositional systems will be created whenever possible: chunking always takes place, no matter whether holistic sig-

[8] For example, if 'buba' means 'red fox', and 'bu' means red, an agent will infer that 'ba' means 'fox'. However, if an agent only sees 'buku' meaning 'red fish' and not 'bu', it will infer nothing.

nals are available. Moreover, chunks are not only weighted for success, but for *synonymy* as well: this, along with the chunking, is an explicit bias for the development of what will always be a minimally sized, and hence maximally learnable compositional system.

Gong (2010) does not focus specifically on the emergence of compositionality, but this forms part of his model comparing the effects of vertical, horizontal and *oblique* transmission (where vertical is strictly parent-child, oblique is any intergenerational transmission). Subsequently, in Gong (2011), he explicitly looks at the possibility that the co-evolution of compositionality and word-order lead to syntax. Gong shows that (as with all of the models surveyed here) the compositional aspect of the model develops via analogy with the semantic structure. Finally, although Gong stresses the role of 'socio-cultural factors' at work in his model, along with the role of interactivity, he ultimately points out that this is only one possible route to modern linguistic structure. In both of his papers, however, rather sophisticated mechanisms of syntactic manipulation and inference are involved.

Mechanism is a key concern of both Tria et al. (2012) and Franke (2014, 2016). Instead of the complex inference used in many of the other models, Tria et al. use a simple 'blending/repair' strategy. The semantic space is modelled as a random network, and agents initially send holistic strings to each other. Upon communicative failure, agents create new, compositional strings by blending terms for connecting nodes on the semantic network. This proceeds as follows: every node in the network represents a meaning, which can be associated with a string. If a particular meaning does not have an associated string, the model will create a new one by choosing two connected nodes which do have associated strings, and belding those together by taking a random sequence from the start of one and the end of the other. Tria et al. (2012) find that the degree of compositionality which develops depends on the degree of connectedness for the semantic network, similarly to K. Smith et al. (2003). However, there is an interesting divergence between this model and most of the others here in the relationship between compositionality and productivity. While compositional recombination is used to construct new terms, once they are established and shared throughout the population, they are used holistically. Because of this, signalling systems which appear highly compositional are only so *externally*: apparently compositional systematicity is not matched by an underlying compositionally productive system. This highlights the distinction between I-language and E-language raised by Kirby (2000), K. Smith (2003b) and others.

Franke (2014) raises an important question: all of the models surveyed here involve a productive, *creative* mechanism of various degrees of complexity. Franke asks what the *minimal* form of productive creativity might be, and in particular how observed pairings of meaning and form extend to unobserved ones. Franke settles on a model of reinforcement learning with 'spillover', a type of semantic overextension in which signals for seen complex meanings associate with unseen meanings on the basis of similarity. Arguing that this is an even less sophisticated mechanism than basic reinforcement learning, possibly due to "an inability to distinguish sharply", Franke shows that when *lateral inhibition* of competing synonyms and homonyms is applied sufficiently, there is a domain in which compositional signalling is the dominant strategy. This argument is actually very similar in both character and implementation to that of K. Smith (2003a,b): both over-extension to novel meanings and a bias for one-to-one signalling drives the emergent compositional mappings.

Finally, while not a model of compositionality per se, Johansson (2008) points out that the fractionation process involved in many of the models may not be empirically feasible. By randomly generating strings (of a fixed length) and complex meanings from a fixed pool of characters, he shows that the likelihood of coincidental form/meaning matches is increasingly unlikely as the number of terms increases. Johansson uses this to argue that counterexamples to compositionality would vastly outnumber positive ones since for any reasonably large vocabulary, statistically-influenced individuals would be swamped under negative data. This is a reasonable analysis, but there are several arguments against Johansson's position. An initial observation is that an initially small vocabulary and sound inventory are not unreasonable assumptions, and would allow for many possible overlaps, as would intergenerational cultural transmission over longer timescales. Perhaps more important, however, is the critique relating to inference: as Johansson states, there is a large body of work showing that humans are sensitive to low-level statistical correlations. It does not follow, however, that humans will abandon any correlations they experience because the majority of evidence is not correlated. In fact, the opposite is true: humans often perceive patterns where none exist (e.g. Whitson & Galinsky, 2008). It is reasonable to expect that earlier humans would exhibit similar tendencies, and have had a bias towards structural homomorphisms.

Experimental work

Empirical work has recently played an increasingly important role in investigating the cultural evolution of linguistic structure. This has been inspired by models such as those surveyed above, and also by work in both the experimental semiotics paradigm (surveyed in Galantucci & Garrod, 2011) and from cultural transmission experiments looking at human cultural evolution (surveyed in Mesoudi & Whiten, 2008). Using a variety of artificial population structures, human participants are typically presented with initially random artificial languages: over the course of multiple transmission events (either to new learners or interactively), compositionally structured languages begin to emerge.

The canonical IL experiment is Kirby et al. (2008): participants are presented with stimulae from a structured meaning space alongside random signals and are asked to reproduce those signals for transmission to the next generation (another participant). This process continues over the course of multiple generations. In an initial condition, the authors found that the signalling system tended to undergo *collapse* and become highly degenerate, with one shared label for most items. However, when an artificial pressure for expressivity was added (by removing homonyms and replacing them with new random signals), the languages quickly took on compositional structure. This proved to be the starting point for many more studies, a recent survey of which is provided by Tamariz & Kirby (2016). These manipulate a wide range of conditions, including the nature of the signal (e.g. linguistic vs.non-linguistic, gestural vs. written, the frequency distribution of terms), the task involved, the population dynamics (e.g. transmission chains vs. dyads vs. micro-societies, see Theisen-White et al., 2011), and the structure of the meaning space. These experiments found that cultural transmission resulted in compositionally structured systems under pressures of learnability and expressivity (Kirby et al., 2015), and moreover that this was influenced by the structure of the meaning space and the contextual relevance of certain meanings (Winters et al., 2015). As such, experimental work broadly holds up the over-arching hypothesis that signals take on compositional structure as a *functional* adaptation, and that this happens under pressures for expressivity and learnability.

Overview and issues

Having surveyed the field, there is much more overlap than might have been expected. In fact, most of the difference in opinion is down to the particular focus of individual studies, rather than any fundamental theoretical clash. Assuming the position that compositionality *does* exist in natural language (while holding on to certain caveats such as pointed out by Pagin & Westerståhl, 2010), we are now in a position to see exactly where opinion converges or divides.

A primary observation is that, from the original formulation of Montague (1970) onwards, the conceptualisation of compositionality as a homomorphic relation between structure in the meaning and signal spaces is pervasive throughout the literature. When this is explicit, this can be expressed in more or less concrete terms (compare Brighton, 2002 and Brighton & Kirby, 2006), but it is an assumption in all formal modelling work and experimental analysis. However, the 'homomorphism assumption' doesn't take a central position in many of the studies we have just looked at. The focus generally falls elsewhere, mainly because of different theoretical assumptions about both the protolanguage debate and the respective roles of interaction, transmission and learning in the cultural evolution of language. To a lesser degree, we also see larger issues intervene, such as the nature of semantic representation and the complexity of the cognitive abilities available. Whatever the approach, however, the fundamental question is *how* and *why* a homomorphic system arises.

Firstly, the protolanguage debate — and by extension, assumptions about the emergence of novel languages such as NSL and ABSL — is often more concerned with *how* compositional language emerged. The two opposing positions, regarding an initial stage that is atomic vs. holistic, require different processes. The holistic story requires a 'fractionation' stage, while the atomic-first narrative requires that some recombinatory mechanism comes into existence. Because of this, the homomorphism assumption is taken for granted in the atomic-first account, but is the optimal 'target' for the holistic-first one. However, this does not necessarily mean that the only difference between them is that the holistic account assumes a possibly superfluous initial stage before an equivalent atomic stage. Depending on the particulars of the fractionating mechanism, it is possible for signalling systems to develop which exhibit *superficial* but only limited *productive* compositionality. Because of this, the holistic-first path can be seen not so much as the emergence of atomic units, but as the gradual emergence of rule-like structure: compare this to the atomic-first account in which the recombination mechanism is either present or not. Because of this, theories which assume an initial presence of atomic units focus on the relative optimality of holistic vs. compositional coding, unlike holistic-first accounts which concentrate on processes of inferring simple units from larger ones.

In both cases, however, the implication is that some *functional* pressure must be responsible for the compositional homomorphism of modern human language. Holistic-first accounts tend to refer to simplicity in terms of either acquisition (typically IL work) or efficient storage (interaction-based work). Atomic-first accounts compare the relative efficiency of holistic and compositional systems, and tend to find that compositional systems are optimal for highly-structured semantic spaces. It is not surprising that a focus on vertical or horizontal transmission leads to different conclusions: in the case of vertical learning, the importance of learnability is inevitably stressed. When interactivity is available, productive, efficient coding is argued to be optimal for that purpose. Whenever we see this appeal for learnable/compressible systems, we can interpret it as a response to some noise/bottleneck process. The information bottleneck, then, is not simply a matter of intergenerational learning, but is equally a factor in horizontal acquisition, whether that relates to limitations of memory or interaction (e.g. smaller systems spread through populations more quickly). Certain views can be found in both camps, for example the role of structured meanings, and De Beule & Bergen (2006) stress the fact that learnability plays a critical role in the emergence of compositionality, albeit not necessarily as the result of an inter-generational process.

However, it is worth stressing that in *all* of these studies, we see both learnability and expressivity forming part of the argument that compositional one-to-one mappings are optimal, and that the type of system which emerges depends on the presence of some noisy process.

Finally, some mention must be made of *mechanism*. The theories and models here involve everything from rather powerful frameworks capable of complex hierarchical representations at one extreme down to basic reinforcement learning at the other. These mechanisms are not usually presented as being accurate cognitive models, but more because they serve to demonstrate certain theoretical ideas, for example compressible grammars. However, as pointed out by Franke (2013), there is a strong argument for identifying a minimal model. This is the motivation behind the model presented below.

5.2.4 Summary

There have been many evolutionary explanations for duality of patterning and its combinatorial and compositional components. I have focussed on cultural evolutionary explanations, but even here we see a wide range of apparently incompatible explanations. In particular, it is not clear whether combinatoriality is driven by physical or learning-based constraints on expressivity. In the case of compositionality, most accounts favour an appeal to learnability, compressibility, or economy, but there is no agreement about the role of transmission and/or interaction. My argue that we can abstract away from most of this by recognising the presence of **noise** in various guises. Structure, then, is an adaptive response to this noise. The model of the next section is a *minimal account* of the emergence of duality under these pressures.

5.3 Model

In Section 5.3.1, I provide a formal description of a model of the emergence of duality of patterning. The principle underpinning the model is simple: signalling systems are affected by noise at different levels. Minimal processes (of inhibition and recombination) act to preserve expressivity and learnability in the face of this noise, resulting in differently structured systems.

5.3.1 Model Description

In keeping with previous chapters, the fundamentals of the models are based on an exemplar-theoretic framework. A major difference, however, is that this model involves only a single agent which interacts with itself. The main reason for this decision is that, unlike previous sections, my focus is not on how agents establish a shared signalling system, but on internal properties of the signalling system itself: whether the system is combinatorial and/or compositional. The requirements established in the previous sections which ensure the development of optimal signalling systems (referential information, a bias against ambiguity and information loss) are all properties of the model presented here, which allows me to dispense with the agent-based methodology. Furthermore, models employing a self-interacting agent are established within the field, in particular the perception/production loop utilised in the exemplar models of Wedel (2003, 2006).

The fundamental basis of the model is that exemplars associate complex meanings and complex signals. There is no explicit representation of compositional mappings between between individual meaning features and sub-strings, and substring units are not stored separately. These intermediate aspects of the system appear only through analysis of the whole system. The reason for this design is, in part, theoretically motivated: exemplars typically represent observed events rather than internally generated representations. However, a more specific motivation is that restricting internal representation to observed complex exemplars allows for a tighter understanding of how information bottlenecks affect the development of structure. Previously, the information bottleneck was very clearly defined in the case of intergenerational transmission (typical of most iterated learning models), but much less so in other types of transmission, for example closed populations. In the iterated learning studies, the amount of information transmitted to the next generation is clearly specified, whereas in closed populations the main information bottleneck is individual agents' memory. This means that while internal representation has little effect on the bottleneck for intergenerational transmission, it is of crucial importance for memory bottlenecks. If we are able to avoid using internally generated representations (apart from those generated on the fly during production and interpretation), we can draw a direct analogy between the transmission bottlenecks and memory bottlenecks: in both cases, any structure will be inferred from the totality of the data, and the distinction between the two forms of bottleneck disappears. As such, the model can be seen as an abstraction of both processes. The collection of exemplars representing an agent can thus be seen as either the whole body of data passed between generations, or the memory of an individual agent.

Exemplars

The representation of an agent is simple: a set of exemplars E associating strings of characters with complex n-dimensional meanings M. All other features of the model are mechanisms which act over these exemplars, resulting in only three operations: the production, modification (via noise), and deletion of exemplars.

The signal aspect of each exemplar $e \in E$ consists of a string with a fixed length of k characters $c \in C$. C is unbounded in size: $|C| = \infty$. An individual string is an ordered set $s = \{c_1, ..., c_k\}$.

The meaning space consists of $d \in \{1,2\}$ dimensions, each of which is assigned a single feature, $f \in F$. The number of features per dimension $|F| = \phi$. Individual complex meanings $m \in M$ are ordered sets of features, $m = \{f_1, f_2\}$.

An exemplar is an association between a single complex meaning and a single string, represented as the set $e = \{m_i,\ s_i\}$.

Exemplar initialisation, string storage, and noise

It is first crucial to implement an *initialisation stage*. Population models are able to create a large amount of initial *variation*. Because there is no population in this model, if there is no random initialisation then processes leading to combinatoriality and compositionality would apply from the outset. As soon as any partially similar meaning is prompted, the model will attempt to construct a new compositional exemplar, composed of already present signal elements. Because of this, some degree of combinatoriality and compositionality is actually guaranteed without an initialisation stage.

The exemplar store is initialised by creating a number of exemplars equal to the maximum memory size. Complex meanings are produced randomly with uniform probability over all of the features of each dimension. Strings are concatenations of characters drawn from the infinite set C. Because of this, there is zero chance of any characters repeating either in or across strings: no combinatoriality or compositionality exists initially.

After the initialisation stage, the model starts to produce new exemplars. This process involves several stages, and so its description will be held back until the following section. When a new exemplar is created, strings are subject to *noise*. Noise represents an abstraction of both noisy *transmission* and noisy *memory*. The core concept of noise has been outlined previously in Section 5.2.2: limi-

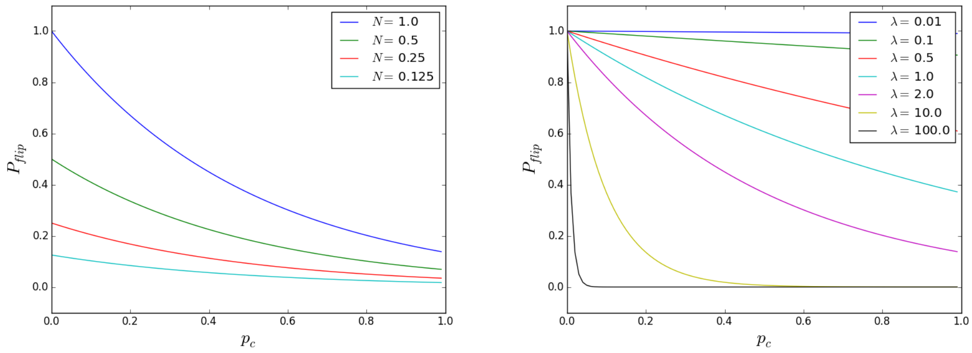


Figure 5.2: Demonstrating how the two parameters affect the relationship between the proportion of a character p_c and the probability of it flipping to another character P_{flip}. On the left, we see the effects of varying N, while keeping $\lambda = 2$. On the right, we hold $N = 1$ and plot for different values of λ.

tations of both production/perception and memory place a practical limit on the number of usable signal elements. A similar concept is used by Tria et al. (2012), who implement noise by modulating probability that a character will be flipped to another depending on the number of times which that character has been experienced. This model instantiates noise differently, along the lines of Nowak, Krakauer, & Dress (1999): when a new string is stored, there is a probability that each character will flip to another stored character. This probability P_{flip} that this happens is:

$$P_{flip} = N\, e^{-\lambda\, p_c} \tag{5.9}$$

where p_c is the overall proportion of character c across all stored exemplars in E. There are two parameters. As can be seen in Fig. 5.2, N limits the maximum probability of noise and takes values $0 < N < 1$. The λ parameter controls how much p_c affects P_{flip} and takes values $-\infty < \lambda < \infty$. When λ is large, higher values of p_c are affected much less than lower ones. When λ is small, higher values of p_c are affected similarly to lower ones.

If a character does flip, it is incorrectly stored as another previously-seen character. The probability $P(c|flip)$ of this is proportional to p_c:

$$P(c|flip) = \frac{p_c}{\sum_{i \in C} p_i} \tag{5.10}$$

Noise in this model is not implemented to affect the perception of meanings, only the associated signal. As with before, the reason for this idealisation is that we are focussing on the emergence of structures in the signal space, and are not (at this point) concerned with issues of reference or noisy perception of meanings.[9]

Signal production and interpretation

A basic principle of the model is that stored exemplars are compared with each other to create and interpret novel signals. When a prompted meaning does not perfectly match any exemplar in memory, the agent creates new signals by combining sub-strings. The sub-strings composing the new signal are those which, together, have the strongest association with the target meaning. Interpretation of novel signals is the inverse of this process: the agent finds the segmentation of the string which has the highest associations over all meaning dimensions and chooses that interpretation.

At each iteration of the model, a random meaning is provided as a prompt. The agent then examines its memory: if none of the features of any meaning dimension match any of the stored exemplars, the agent simply produces a new string comprised entirely of new characters drawn from C.

If, however, some features do partially match those of stored exemplars, the

[9]Note that structure and noise in the meaning space is a factor in several models, for example Brighton & Kirby (2006). These tend to involve a metric over signals and meanings which does not form a part of this model. In both cases, compositional mapping is the result of isomorphic mapping between structure in the signal and meaning spaces.

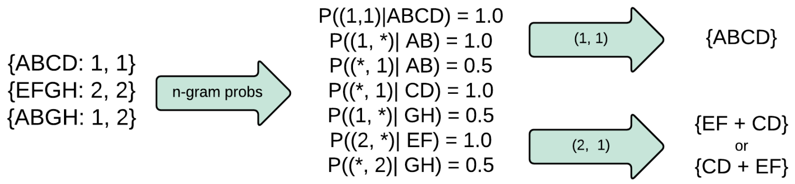


Figure 5.3: Demonstrating how conditional n-gram probabilities are used to produce new exemplars. The exemplars on the left produce the conditional probabilities $P(F|S)$. These are then used to produce new signals: when the prompted meaning is an already-stored exemplar, that will be used (this would be the case even if a compositional construction was available, as the model always selects sub-strings with maximal length for each meaning feature: in the case of holistic signals, both features map to a sub-string of length 4, while in compositional signals the length for each would be 2). The agent then chooses to combine exemplars which have the highest chance of correct interpretation. As shown in the model, there is no string ordering applied.

agent attempts to construct a new string which maximises the probability of correct interpretation. This probability-maximisation is equivalent to the *obverter* process described in previous chapters. The construction process creates new strings by trying to combine sub-strings from already-stored exemplars associated with the target features, and choosing the most expressive string. This proceeds as follows:

1. Assemble the set of candidate exemplars associated with at least one target feature.

2. Create a new set containing all possible decompositions of every string into d sub-strings (where d is the number of dimensions).

3. Use the previous set to create a new one which contains every possible combination of sub-strings which have length k. Thus, if $d = 2$ & $k = 4$, we could see all combinations of length 1 and 3, and 2 and 2.

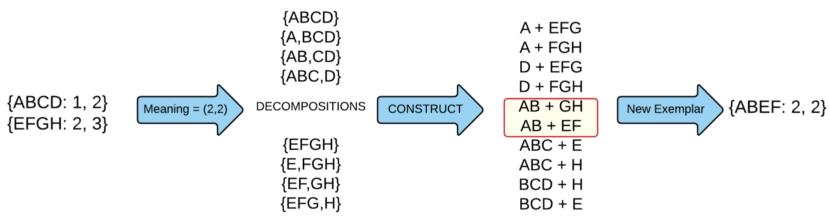


Figure 5.4: Demonstrating how exemplars are constructed, ignoring probabilities. Each exemplar is broken into d sub-strings. All of the possible re-combinations of sub-strings from the two different strings are created. Then, the combinations which maximise the average sub-string length for each meaning feature are selected, and a signal is chosen from them.

4. For each sub-string s of each new string combination, use the probability P_{ij} of each target feature f_{ij} to create a vector of probabilities $V_{sub} = \langle p_{1j}, \ldots, p_{dj} \rangle$. This represents the relative frequency of each target feature given that sub-string in the existing set of exemplars. Thus, when a sub-string is always associated with a particular feature, it registers 1; if never, then 0, 50 % of the time, then 0.5, and so on.

5. For each string combination, create a vector V_{best} which takes the highest value for each feature from the individual sub-string vectors, and calculate the mean value P_{avg} over this vector.

6. Assemble a set of candidate string combinations which have the maximum values for P_{avg}.

7. Transmit the most *robust* of these candidate string combinations: this maximises the length of sub-string for each target feature. As such, a holistic signal would always be selected if one is stored (as it maximises the sub-string length for all target features), but when choosing between a 3-1 length combination and a 2-2 length combination the latter will always be selected.

Figure 5.3 provides an illustration of how sub-strings probabilities contribute to the process, while Figure 5.4 gives an example of how sub-strings are re-combined to form new strings. We can now turn to signal interpretation, which involves the inverse process:

1. Generate all possible segmentations of the received string into d sub-strings.

2. For each sub-string, calculate the vector V_{sub} for the strongest association probability associated with that substring and $V_{feature}$ to record which meaning feature it relates to. If there is more than one, choose between them at random.

3. For each segmentation, create a vector V_{best} which takes the highest value for each feature from the individual sub-string vectors, and $V_{bestfeature}$ recording which meaning features these relate to.

4. Calculate the mean value P_{avg} for all V_{best}, and choose the segmentation with the highest value of P_{avg}, then interpret the signal as the corresponding $V_{bestfeature}$.

As can be seen, neither of these processes store structural information: all structure is inferred from the set of stored exemplars, but results only in a new signal/meaning association. At this stage, the process may not appear minimal: most of the structure has been shifted onto the mechanism of recombination. However, it represents two main operations: firstly, the simple fact that sub-strings are recombined, and secondly that the sub-strings with the strongest association with the target meanings are chosen. This is basically a recombinatory version of the obverter process described in the first chapter.

One design decision which warrants discussion is that all strings are of a specific length. This is certainly unrealistic: all natural languages feature variation in word length. Variation in word length is not the target of the present study, and is not directly informative about the emergence of duality of patterning. Another reason for limiting the length of words is simply that of computational complexity: if string lengths are allowed to vary, the number of possible combinations undergoes combinatorial explosion. In this model, a robustness constraint selects the longest segmentation for each meaning feature: if string length was allowed to vary, an opposing constraint (likely memory-based) would be necessary. All of these considerations are avoided by pre-determining the string length.

Finally, we should address two more design decisions: firstly, there is no assumption of *order*. Two sub-strings can be concatenated in any order without any impact on meaning. This is still consistent with a broad definition of compositionality, which states that meaning is a function of both individual parts and the rules of combination. It is just that, in this case, the rules of combination are very simple: two things placed together have the meaning of both. [10]

Secondly, unlike some previous models (and features of some natural languages), the model assumes that meaningful features are encoded by *contiguous* sub-strings. Meaningful units are cannot be distributed over non-contiguous parts of the string, patterned as $1-2-1$ for example. Computational complexity is again the main reason for the second decision, but in both cases there is theoretical support for the idea that simple concatenation preceded more complicated ordered or distributed structures (e.g. Jackendoff, 1999).

Inhibition and memory

Memory is instantiated by placing a limit on the number of exemplars which can be stored by the agent. When this limit is exceeded by adding a new exemplar, an

[10] In practice, however, this feature was actually relaxed to produce the illustrations in the results section: this is because Mantel tests were used for comparative purposes. These use Levenshtein edit distances which are unable to recognise (without parsing) that two swapped strings are equivalent. All results continue to hold when this is not the case, however.

old exemplar is selected at random and deleted.

A final feature of the model is a type of *synonymy inhibition*: when an exemplar is stored, it triggers the deletion of a competing synonym. This may seem unnecessary, but is used for the following reason. Take the following substrings and their strongest meaning associations: $\{AB : 1, BC : 1, EF : 2\}$. To create the compositional meaning $\{1\&2\}$, both $ABEF$ and $BCEF$ are viable options. This is technically a compositional system: meaning are a function of their parts. Typically, however, a requirement made in previous models and the theoretical literature is that signal-meaning relationships in optimally compositional systems are one-to-one: see Section 5.2.3 for more on this. As it stands, there is no explicit pressure in the model to drive this effect: meanings can be multiply expressed. In line with the principle of learnability, the simplest possible compositional system might require that synonymy is inhibited: this is tested in section 5.3.2 and found to be only partially true.

Similarly, there is a form of lateral inhibition against homonyms. Because sub-strings are not independently represented in the model, inhibition requires that whole exemplars are deleted. When a new exemplar is created, the best segmentation of the string may contain sub-strings which have the same probability of matching the target meaning. In this case, a single exemplar containing one of the competing sub-strings is then selected at random for deletion. As systematic compositionality emerges, only one sub-string will exist for any given complex meaning and inhibition will cease.

Diagnostics

The combinatoriality of a system employs two measures: the form recombination index (FRI) described by Tria et al. (2012) and in Section 5.2.2, and the overall character entropy. All of these measures are performed over both individual strings and over segments.

The compositionality of the system is measured in two ways. Firstly, in line with much work in the literature (e.g. Kirby et al., 2015), a Mantel test is used. This proceeds as follows: First the Levenshtein edit distance between all possible strings and the number of different meaning features is calculated, and the correlation between these two sets of values is calculated using Pearson's product moment (Pearson's r). This is then compared with similar correlations between multiple re-shufflings of the meaning features and the resulting Levenshtein edit distances. This creates a distribution of correlation measures, and the significance of the original correlation is measured using a z-score. In this way, the correlation tells us how similar the two spaces are, and the z-score tells us the level of significance at which to treat the correlation score.

However, it should be pointed out that there is a potential issue with the Mantel test when applied to combinatorial data which arises from *segmentation problems*. As an example, take the following set of exemplars:

$$S = \{\ \{AAAB : 1,1\}, \{AABA : 1,2\}, \{BBAB : 2,1\}, \{BBBA : 2,2\}\ \} \tag{5.11}$$

We can infer from this that the system is compositional, with the following elements:

$$AA = (1,*),\ \ BB = (2,*),\ \ AB = (*,1),\ \ BA = (*,2) \tag{5.12}$$

However, if we calculate the Levenshtein distances *without* taking this segmentation into account we see the problem shown in Table 5.1:

How can this problem be avoided? The problem lies with a mis-match between variable-length string edits and the discrete dimensionality of the meaning space, but disappears once we know the internal elements. The best solution, then,

Signal - Meaning	AAAB - 1,1	AABA - 1,2	BBAB - 2,1	BBBA - 2,2
AAAB - 1,1	0	1 vs. 1	2 vs.1	4 vs. 2
AABA - 1,2	**1 vs. 1**	0	4 vs. 2	2 vs. 1
BBAB - 2,1	**2 vs. 1**	4 vs. 2	0	2 vs. 1
BBBA - 2,2	4 vs. 2	2 vs. 1	2 vs. 1	0

Table 5.1: An example of how Levenshtein edit distance in the string space is not necessarily compatible with the edit distance in the meaning space: highlighted in bold is one case where — without segmentation — the same distance in the meaning space can map to different distances in the signal space.

is to infer the best set of elements which correspond to the dimensionality of the meaning space before performing the Mantel test.

An ‘internal’ measure of compositionality would be somewhat trivial in this model, only needing to register whether a holistic or compositional strategy is used by the agent for each possible meaning. Given the exemplar-based nature of the model, a fully compositional ‘internal’ strategy is impossible. Because exemplars represent mappings between complex meanings and strings, any complex meanings which are already stored are just produced holistically.

As an ‘external’ alternative to this, we can check the degree to which the definition of compositionality given in Eq. 5.8 is true. There are two ways to do this, one more strict than the other.

A compositional sub-mapping exists for a meaning feature it it can be unambiguously associated with some sub-string. That is to say:

$$\exists\, s_{ij} \in S \quad s.t. \quad p(f|s_{ij}) = 1 \;\wedge\; p(s_{ij}|f) = 1 \tag{5.13}$$

As such, a measurement of compositionality checks the degree to which mappings between sub-strings and meaning features are bijective. We need to specify a function $comp(f,S) \rightarrow [0,1]$, and calculate the compositionality of the system as:

$$C = \frac{1}{|M|} \sum_{f \in M} comp(f,S) \tag{5.14}$$

This leaves us to define the function $comp(f,S)$. A strict version of the measure checks for each feature and all sub-strings whether a bijective mapping exists or not, i.e. $comp_{strict}(f,S) = 1$ if Eq. 5.13 is true, otherwise $comp_{strict}(f,S) = 0$.

A more fine-grained measure, on the other hand is:

$$comp(f,S) = \forall s_{ij} \in S, \quad \max\left(p(f|s_{ij}) \cdot p(s_{ij}|f)\right) \tag{5.15}$$

One way of seeing Eq. 5.15 is that it scans through all sub-strings and their mappings to meaning features to find the most bijective set. Looking at it like this, it is actually very close to the measures of communicative accuracy employed in previous chapters, except for two things: firstly, it penalises synonymy, and secondly, it is based only on mappings to individual meaning features instead of complete meanings. As such, this measure is really showing the degree to which a synonymy-free signalling system exists at the level of features, rather than for complex meanings.

The information bottleneck

As outlined in Section 5.3.1, the information bottleneck is an abstraction of both intergenerational transmission and a limited memory: at any stage, the set of exemplars can be seen as the training data provided to a new generation, or as the internal representation of an agent. This has been done with the intention of showing that it is the existence of a bottleneck which is a driving force behind structure, rather than a particular instantiation of it.

The total number of possible complex meanings $k = F^d$. We can then analyse the information bottleneck by working out the expected proportion of meanings

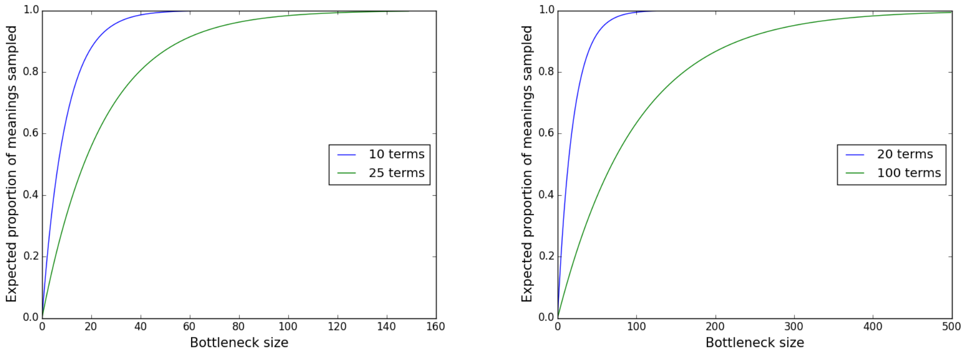


Figure 5.5: Demonstrating how the information bottleneck affects holistic and compositional systems. On the left, for a system with 2 meaning dimensions with 5 features each, a compositional system would require $5 \times 2 = 10$ terms, while a holistic system would require $5^2 = 25$ terms. The graph shows the expected proportion of all terms to be seen after a given number of samples. The graph on the right shows the expected proportions for a 2×10 meaning system, with 20 compositional and 100 holistic terms required.

to be sampled given a certain bottleneck size n. If there are k meanings, the probability that a given meaning m_i has *not* been seen after n samples is:

$$p(\neg m_i) = \left(\frac{k-1}{k}\right)^n \tag{5.16}$$

Because all meanings have an equal chance of being sampled, we can see that Eq. 5.16 also gives the overall expected proportion of unseen meanings after n samples.[11] So, the expected proportion of meanings $E[M]$ which *have* been seen is:[12]

$$E[M] = 1 - \left(\frac{k-1}{k}\right)^n \tag{5.17}$$

Fig. 5.5 gives a numerical example of the bottleneck in action. As can be seen, the bottleneck becomes increasingly important as the size of the meaning

[11] The proportion $R = p(\neg m_i) \times \frac{|M|}{|M|} = p(\neg m_i)$

[12] A very similar result can be found in Brighton (2003)

space increases. This is also valuable information about what type of system to expect from different parameters: for example, given a 2×10 meaning space, we would expect bottlenecks of around 100 exemplars to preserve stable compositional systems, and bottleneck sizes of over 500 exemplars to preserve stable holistic systems.

5.3.2 Model Results

The primary aim of this section is to show that different levels of structure — compostional and combinatorial — emerge when learnable, expressive signalling is put under pressures from noisy processes. These pressures can be seen as affecting either acquisition and/or storage. Pressures in the signal space alone drive combinatorial structure, while pressures related to the signal/meaning association space — the 'system' space — drive compositional structure. Because of this, we can see the development of four distinct classes of signalling system. This includes i) systems which are entirely holistic, i.e. non-compositional and non-combinatorial, like most animal communication, ii) combinatorial but non-compositional systems, similar to that of putty-nosed monkeys, iii) non-combinatorial but compositional systems, similar to ABSL, iv) combinatorial, compositional systems as found in human languages, and finally v) degenerate languages when the noise is such that no learnable languages are expressive and no expressive languages are learnable. Models which lead to these five types of system will be demonstrated in turn over the following sections, including an analysis of how particular model mechanisms (e.g. inhibition) affect this process, and how certain parameters (e.g. noise) determine stable configurations of the model. A final note: in the following sections, illustrations refer to individual results, but all broad statements refer to robust properties of the model over many runs. Aggregate results are presented when meaningful, and omitted when they add nothing

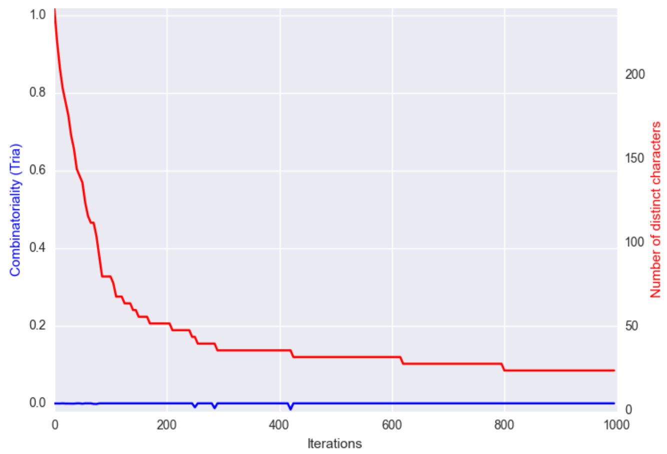


Figure 5.6: In the absence of any noise ($N = 0$) or complex meanings ($d = 1$, $|F| = 5$ with a 60 exemplar memory), there are no shared characters between any strings, and the blue line shows that combinatoriality (as defined by Tria et al., 2012) never develops. The right hand axis (red) demonstrates how the number of individual strings gradually reduces through memory loss, eventually leading to a single string per non-complex meaning. This also serves as a demonstration of how the combinatoriality measure is not affected by the number of strings or by synonymy.

to the analysis.

Combinatoriality alone

The prediction is that, in the absence of noise affecting the signal space or the system space, neither combinatoriality nor compositionality will result. Pressure in the signal space is applied via noise parameters, N and λ: We will see that the greater the amount of noise, the greater the degree of combinatoriality in the system, up until a point at which noise takes over and the system becomes degenerate.

The simplest possible system which we can use to illustrate maximally non-combinatorial signalling is when we set $N = 0$, removing the effects of noise from signalling entirely, and set $d = 1$, $|F| = 5$, so only holistic signalling is possi-

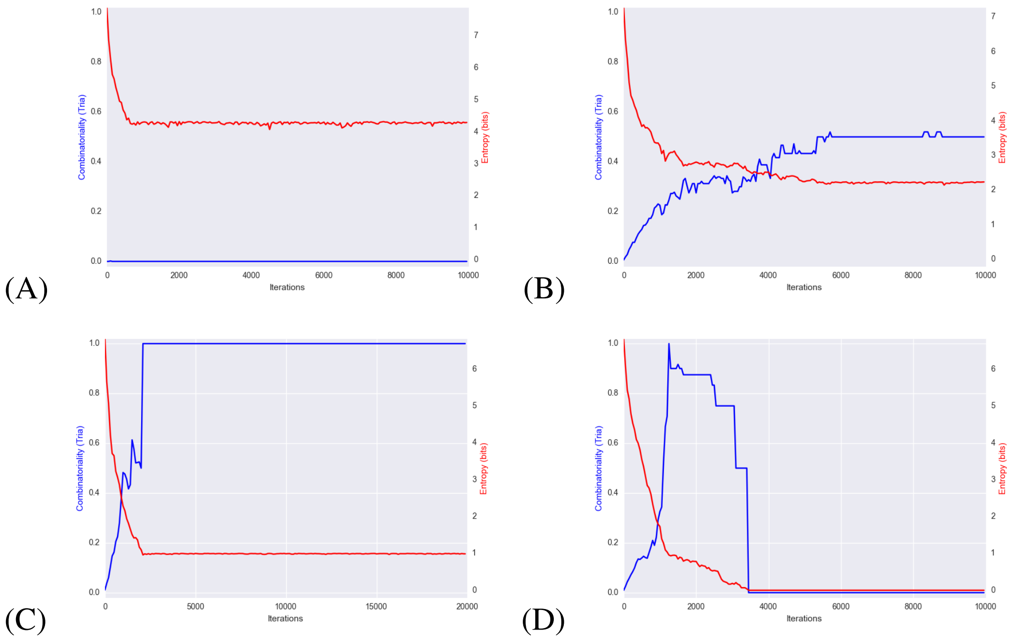


Figure 5.7: The effect of changing the λ noise parameter, keeping $N = 0.5$ and without complex meanings ($d = 1, |F| = 5, |E| = 60$). $\lambda = 200$ (A), 60 (B), 20 (C) and 10 (D). The blue line indicates the level of combinatoriality (Tria et al., 2012), while the red line shows the entropy over the character probability distribution. In (A) we see that when λ is high, the system is tolerant of many low probabilities and combinatoriality never develops. Increasing λ in (B) and (C) leads to gradually increasing levels of combinatoriality, but when λ is too high the system collapses to a single character and combinatoriality is lost.

ble. This is particularly enlightening example: the only scope for change in this configuration of the model is when a certain meaning/signal association is lost. However, the lack of complex meanings means that new signals cannot be formed by recombination, and will always consist of a new string of novel characters. This is demonstrated in Figure 5.6, where we can see that combinatoriality never develops. The only change that occurs in the model is the number of individual strings in the system, which gradually reduces via random memory deletion in a drift-like process.

We can now look at what happens when we allow a degree of noise ($N = 0.5$)

and manipulate the value of λ. As previously discussed, different values of λ directly determine the maximum number of stable elements that a combinatorial system can have, while N determines the overall level of noise. We can see this in Figure 5.7, which demonstrates the effects on combinatoriality for four different levels of λ. When λ is very high, elements are unaffected by noise (A). As λ is increased, the number of stable elements will also do so (B,C). If λ is too low, the system will not even tolerate elements with a proportion $p \approx 0.5$, meaning that not even binary coding remains stable (D).

It is important to understand that because λ operates over character probabilities, it directly determines the stable level of character entropy H_c, which is simply the classic entropy measure over the distribution of characters across all strings. The number of characters will vary somewhat, but will be in the region of $|C| \approx 2^{H_c}$. The level of combinatoriality, on the other hand, is more variable. Because there is no restriction on where characters appear across strings, it is a matter of chance whether they are evenly distributed. For example, four characters are sufficient to create a maximally combinatorial system across five strings of length four, but any repeated characters would result in a lower level of combinatoriality.[13]

The way λ constrains character entropy more than combinatoriality can be seen in Figure 5.8, which shows the aggregate behaviour of different values of λ over 100 runs each. There is a clear inverse relationship between the stable level of character entropy and the relative combinatoriality, but the random aspect of the latter leads to much wider and overlapping distributions. This is seen especially clearly in the case of when $\lambda = 20$, as the occasional collapse from binary coding

[13]The same would actually be the case for any maximally efficient system: take a binary coding across four strings of length 2, $S = \{AA, AB, BA, BB\}$, which would have a combinatoriality score of 0.75.

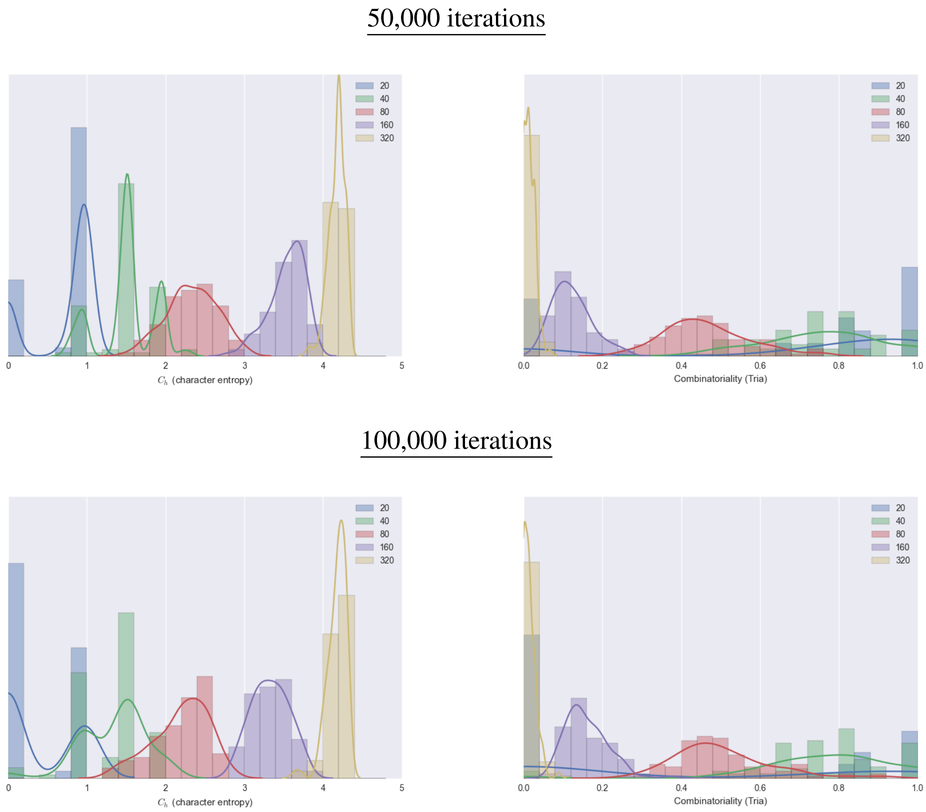


Figure 5.8: The distributions of final character entropy H_c and combinatoriality scores after 50,000 and 100,000 iterations for different values of the λ noise parameter, keeping $N = 0.5$ and without complex meanings ($d = 1, |F| = 5, |E| = 60$) with 100 simulation runs per condition. Note that while there is some drift towards lower entropy regimes, this has only a minimal effect on the amount of combinatoriality, and that both distributions are mostly stable over time.

to a single character means that the distribution is split between values of exactly 1 and 0.

To summarise: the system's tolerance to noise on the basis of individual character probabilities — i.e. the 'memory channel' — determines the levels of character entropy which remain stable over time. The character entropy roughly determines the number of characters, and this in turn has an effect on the combinatoriality of the signalling system, the exact relationship also depending on the number of strings overall and their length. There are two opposing regimes, one

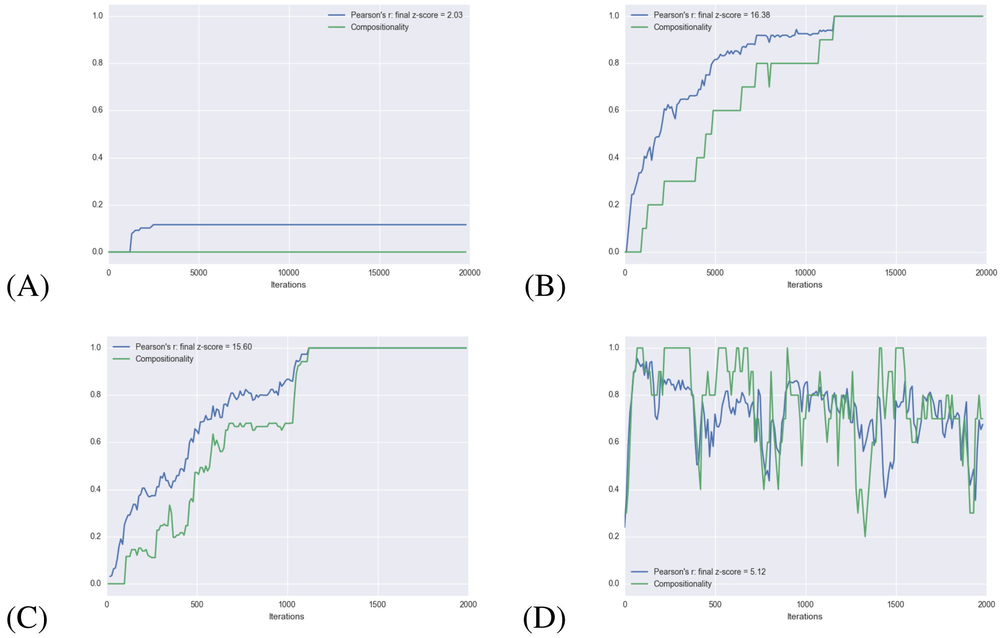


Figure 5.9: The effect of the bottleneck on the emergence of compositionality. ($d = 2$, $|F| = 5, N = 0$). The bottleneck sizes are 200 (A), 100 (B), 60(c) and 20 (D). Note how the two measures of compositionality track each other, and in (A) in particular, where the increased level of Pearson's r indicates that a very limited degree of compositional recombination has occurred, but this is only partial and not systematic. Also, while both (B) and (C) achieve compositionality, the time-scale at which this occurs differs by a factor of 10.

where the level of noise is too high to permit even two stable characters (degenerate), and one where noise is low enough that a completely non-analysable non-combinatorial system is stable. Between these regimes, lower levels of stable character entropy lead to higher combinatoriality.

Compositionality alone

We can now turn to compositional systematicity, which we expect to be driven by another noisy process, the information bottleneck. As previously discussed, the size of the bottleneck should determine whether the system that develops is holistic, compositional, or whether no stable system is possible. To investigate this, the overall noise parameter N is set at zero. The number of meaning dimensions d is increased to 2: we will compare values of $|F| = 5$ and 10, and look at how these interact with bottleneck sizes to produce more or less compositional systems, using the information shown in Figure 5.5 to guide us. Finally, we will have a look at the effect of removing the anti-synonymy bias.

Figure 5.9 demonstrates the effect of the information bottleneck. When the bottleneck is large enough, the initial holistic system remains stable. Beyond a certain threshold, however, holistic systems are not stable and compositional systems take over. Finally, there is another threshold, the 'expressive limit', where not even compositional systems remain stable (again, the only stable systems are degenerate). The bottleneck size corresponding to these thresholds directly depends on the size of the meaning dimensions, as previously illustrated in Fig. 5.5 (p.176).

One point that should be made here relates to the fact that the compositional recombinatory process only happens when a given complex meaning is not available (basically, when a 'memory gap' exists). Because the bottleneck is inherently noisy, even extremely large bottlenecks will occasionally experience a gap like this. This means that as long as a pressure for expressivity exists, the amount of compositionality in the system will increase, simply much more slowly as the bottleneck ratio increases.

The model includes anti-synonymy inhibition. This is designed to promote systematic one-to-one mappings between meanings and signals, and works by deleting a single exemplar which contains a sub-string which expresses a part-meaning equally well. However, there is reason to suspect that this inhibition is unnecessary. In the models of simple signalling in Chapter 1 and 2, a direct bias against synonymy was not required, as drift-like processes related to information

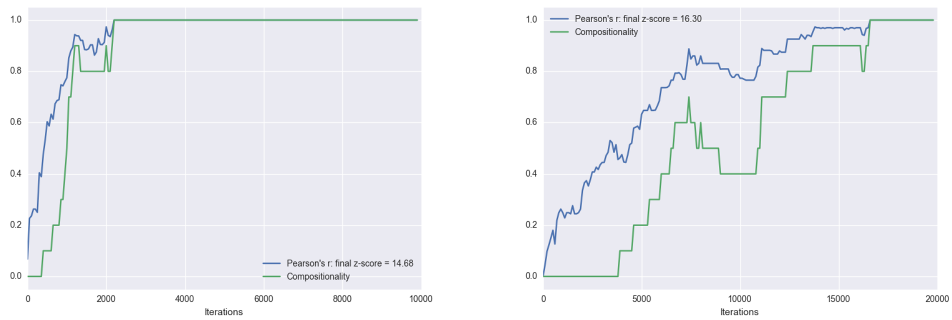


Figure 5.10: Showing the effects of removing anti-synonymy inhibition: under the same conditions as before, ($d = 2, |F| = 5, |E| = 60$ on the left and 100 on the right), stable compositionality still develops, just slightly more slowly than with inhibition.

loss inevitably lead to loss of synonymous variation. In Figure 5.10 we can see that this is also the case here: even without explicit inhibition, gradual information loss leads to the eventual elimination of synonyms, and one-to-one mappings reliably emerge.

To summarise, in the absence of character noise, the pressure for expressivity leads to the emergence of compositional systems when under sufficient pressure from noise due to the information bottleneck. When the information bottleneck is to small, no stable signalling system is possible. Beyond a certain size of bottleneck, only compositional signalling systems are stable, while holistic systems become more stable as the bottleneck size increases. However, as noisy processes always lead to the chance deletion of certain associations, compositional systems *always* develop eventually, albeit over increasingly large timescales. Finally, in the absence of character-based noise, anti-synonymy inhibition is not a necessary requirement for the development of compositionality.

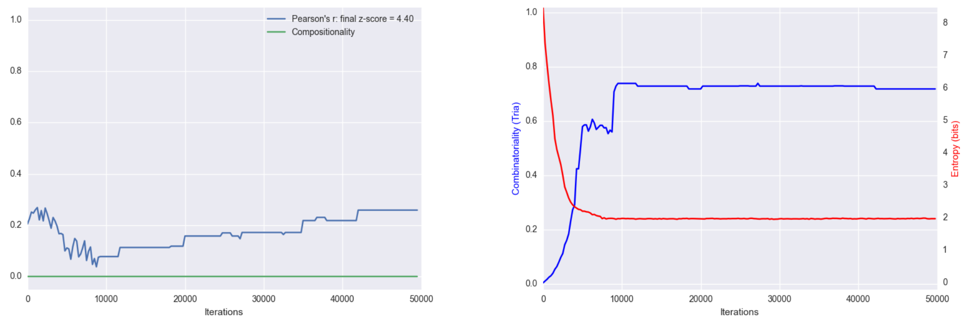


Figure 5.11: Compositionality (left) according to different measures for a completely holistic, combinatorial system (right). $d = 2$, $|F| = 5$, $|E| = 200$, $\lambda = 40$, $N = 0.5$. Note in particular the Pearson's r correlation which is supported by a z-score of 4.40, corresponding to significance at $p \ll 0.01$.

5.3.3 Duality of patterning

So far, we have looked at the emergence of combinatoriality and compositionality in isolation, and shown that combinatoriality and compositionality are both functional adaptations to noise in the signal-space and the signal-meaning association space respectively. As our working definition of duality of patterning is where both levels of structure exist, we will now take a look at what happens when both sources of noise are accounted for.

First of all, we need to look at some baseline cases: according to our measures, what degree of compositionality emerges in our combinatorial models, and vice versa? Figure 5.11 shows results for a combinatorial, completely holistic system. The measure for compositionality developed in section 5.3.1 registers nothing, but (perhaps unexpectedly) the correlation using Pearson's r appears to pick up a degree of compositional regularity which is supported by a Mantel test with a z-score of 4.40, which is significant at $p \ll 0.01$. This is the result of chance correspondences between patterns in the holistic signals and the structure of the meaning space. Importantly, the fact that the z-score is so high only serves to support a cor-

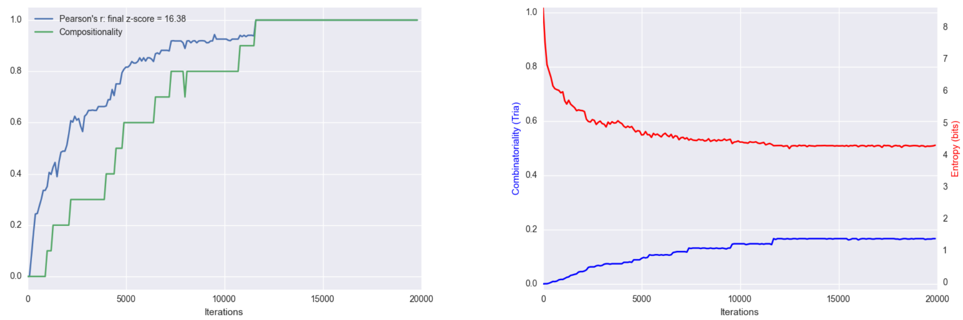


Figure 5.12: Combinatoriality (right) according to different measures for a compositional, non-combinatorial system (left). $d = 2$, $|F| = 5$, $|E| = 60$, $\lambda = 0.0$, $N = 0.0$. Note that 1) a compositional system where two characters are assigned to each feature requires $2 \times 10 = 20$ characters, and $log_2(20) = 4.3$ bits, which corresponds to the figure; 2) The combinatoriality measure registers the fact that compositional re-combination leads to some degree of combinatoriality.

relation which is actually rather low. Simply put, we can have high confidence in a low correlation. However, this result may highlight some issues for models and experiments which use the z-score as the primary measure of compositionality: I will return to this topic in Section 5.4.

Similarly, Figure 5.12 shows that the measures of combinatoriality we have been using are not completely independent of compositional structure. Firstly, the measure of combinatoriality developed by Tria et al. (2012) picks up on the fact that compositional re-use entails a degree of combinatoriality: this fact should be kept in mind when looking at the combinatoriality of any compositional system. The fact that the entropy measure roughly specifies the number of characters being used can also be taken into account: in the figure, the value of $H_c \approx 4.3$ bits corresponds to around 20 characters. This is how many characters would exist in a system which was non-combinatorial at the level of meaningful segments, but where some degree of combinatoriality exists at the complex word level by virtue of the shared meaningful elemnts across a compositional system. However, this is

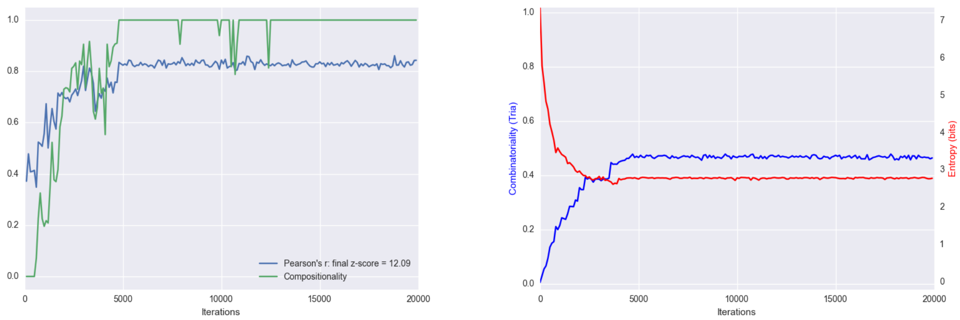


Figure 5.13: A system displaying duality of patterning. $d = 2$, $|F| = 5$, $|E| = 60$, $\lambda = 40$, $N = 0.5$. Compare with Figs. 5.11 and 5.12 which develop significantly more slowly. Note that although the system is perfectly compositional, the edit-distance problem discussed in Section 5.2.3 means that the Pearson's r measure never registers a perfect correlation.

not so say that these measures of combinatoriality and compositionality are fatally flawed, but just that some of them are not always independent of each other and that this should be kept in mind.

We can now look at what happens when we allow noisy processes to affect both the signal space and the signal-meaning space. Figure 5.13 shows that duality of patterning does indeed develop, with perfectly compositional, highly combinatorial systems arising. Another unexpected observation is that twin pressures which drive emergence of both levels of structure appear to feed off each other. Combinatoriality and compositionality both appear more quickly together than alone. On reflection, this is not so surprising: as shown in Figure 5.11, character based-noise inevitably brings about chance correlation between meanings and signals, while the gradual deletion of exemplars via noise contributes to drift-like processes of character loss.

Another example of the interaction between combinatoriality and compositionality can be seen in Figure 5.15. Because the system has stabilised on a character entropy of 2.3 bits, equivalent to 5 characters, this restricts the expressivity of

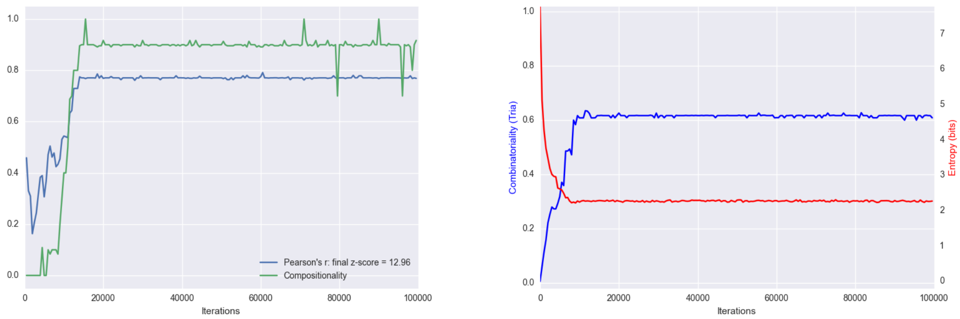


Figure 5.14: A system displaying duality of patterning. $d = 2$, $|F| = 5$, $|E| = 100$, $\lambda = 60$, $N = 0.5$. Unlike Fig. 5.13, the character entropy has stabilised at 2.3 bits $\approx$ 5 characters. This limits the expressivity of sub-strings to 25 possible expressive strings, and the system struggles to maintain complete compositionality.

any length-2 sub-string to 25 possible character permutations. This is only slightly more than the 20 distinct sub-strings which are required to maintain a compositional system for this dimensionality of meaning, and thus the system struggles to maintain absolute compositionality.

Finally, Figure 5.15 shows that, unlike non-combinatorial systems, anti-synonym inhibition is required for perfect compositionality to develop in the presence of combinatoriality. This is because the character-level noise which leads to combinatoriality constantly mutates sub-strings. As long as this process continues some level of synonymy will persist, which reduces compositionality under both of our measures.

5.4 Discussion

5.4.1 Overall results

The models of clearly demonstrate that both aspects of duality of patterning can be understood as a *functional adaptation* preserving expressivity and learnabil-

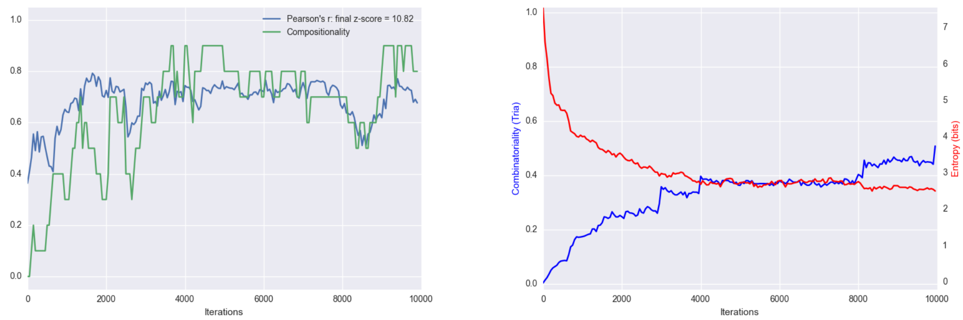


Figure 5.15: A system displaying duality of patterning without anti-synonymy inhibition. $d = 2$, $|F| = 5$, $|E| = 60$, $\lambda = 100$, $N = 0.5$. Unlike Fig. 5.13, the lack of inhibition means that synonymy reduces the level of compositionality.

ity in the face of noisy processes at different levels. At this point, it is worth having another look at exactly how we understand these two terms. My intention here is to rephrase them slightly, and see both in terms of how they relate to *noisy processes*. To do this, I am going to refer to the noise which applies directly to the character-level as *signal-directed* noise, and the noise due to the acquisition/storage bottlenecks will be called *system-directed* noise.

For a signalling system to be *expressive*, it requires both that every meaning has a corresponding signal, and that those signals are unambiguous. Signal-directed noise leads to a situation where signals which are more similar to each other, and hence more ambiguous. System-directed noise, whether memory or intergenerational learning, threatens the stability and permanence of signal-meaning associations. If either of these pressures are too strong, signalling systems are reduced to an incomplete set of ambiguous associations. In the models, we can see this for values of λ which are too high and for restrictively small bottlenecks. In fact, when noise is too high, no functional response is even possible, for example when not even two characters can be reliably distinguished, or when memory is smaller than the number of meanings. From this perspective, noise looks decidedly *anti-functional*, and antithetical to communication.

Noise also interacts with the *learnability* of a system, another term which we can separate into two components: learnability requires that signals can be both *acquired* and reliably *stored*. The story is less simple here: signal-directed noise can be seen as both acquisitional — physical limitations directly affecting the sensory-motor process — and related to memory/storage, seen for example in the experiments of Verhoef et al. (2014). In either case, the effect of signal-directed noise on learnability is to bring about discreteness, and then to provide a practical limit on the possible number of discrete elements. System-directed noise can relate to both acquisition (e.g. iterated learning bottlenecks) and storage (memory). Once again, both have the same effect: a practical limit to the size of system which can be stored.

We can now tie these ideas together. At both levels, noise acts to restrict the stable size of the system. This works directly against expressivity and provides a limit to what is learnable. When noise is too great, expressive and learnable systems are impossible. In the absence of noise, maximally expressive systems of arbitrary size can be learned. Structure emerges in the intermediate regimes, and the type of structure is determined by an interaction between the amount of noise and the size of the meaning space. If the number of signal elements determined by noise is at least as large as the number of meanings, there is no motivation for combinatorial signalling. If, on the other hand, this is not the case, combinatorial systems provide the only expressive, learnable solution. Similarly, if the number of associations robust to system-directed noise is larger than the number of complex meanings, compositional systematicity has no advantages. When system-directed noise is sufficiently large, only compositional systems are stable. To restate: linguistic structure is a functional adaptation to maintain learnable, expressive systems in the face of noisy processes.

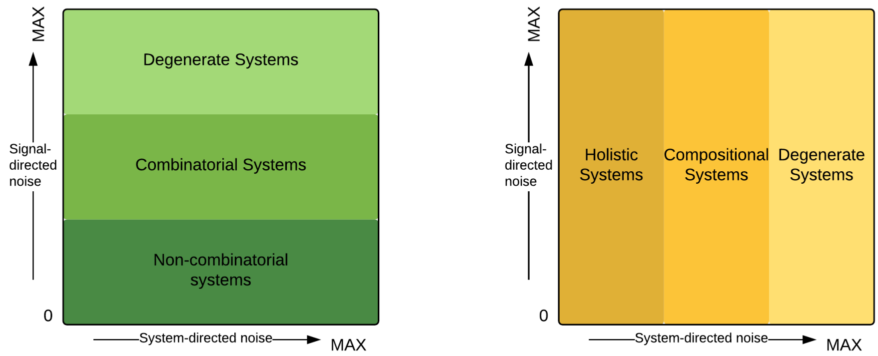


Figure 5.16: The hierarchy of robust systems as determined by noise. At low levels of noise, no combinatorial or compositional structure will develop, while at high levels of either type of noise, no expressive systems are possible. Between these extremes, the only stable systems which are expressive and learnable are combinatorially and compositionally structured.

Following from this, I would like to propose a slight modification to the position advanced by Kirby et al. (2015), that linguistic structure is an evolutionary trade-off between expressivity and learnability. I argue that *robustness to noise* also plays a critical role. Fig. 5.16 is an illustration of how different forms of noise determine the structure of learnable, expressive systems. Excepting when levels of noise are so high that they result in inexpressive degenerate systems, signalling which is both learnable and expressive is possible in every region of the graph.

5.4.2 Comparison with other theoreies

In Section 5.2, I argued that the apparently large diversity of opinion regarding the origins of duality of patterning, combinatoriality, and compositionality could be largely resolved by seeing the important role either learnability or expressivity plays in each. In the previous section I have shown that these factors, along with noise, are sufficient to explain the different levels of structure. However, it is worth briefly returning to some of the particular claims in the light of the previous work.

Firstly, I should point out that the work here has much in common with foundational studies carried out by Nowak and colleagues (e.g. Nowak, Plotkin, & Krakauer, 1999; Nowak & Krakauer, 1999; Nowak, Krakauer, & Dress, 1999; Nowak et al., 2000). In particular, the error-limit argument of Nowak, Krakauer, & Dress (1999) provides a very similar explanation for the emergence of what amounts to combinatorial structure, except phrased in terms of evolutionary game theory. Of course, both Nowak's analysis and the one here are really special cases of the Channel Capacity Theorem developed by Shannon (1948).

Also by Nowak et al. (2000) but somewhat less similar is an argument for 'syntax' (compositionality) which largely ignores noise and focuses on the increased expressivity afforded by compositional communication. This is argued to be an adaptive response to a expanding meaning space, and in particular where $d \geq 2$ and $|F| \geq 3$. This in perfect agreement with the model here, in whose terms it can be restated as saying that that the regions where compositional systems and holistic systems are robust completely overlap when there are less than three meanings. They also remark that the compositionality of language depends on the degree of structure in the meaning space, an argument which is echoed by Tria et al. (2012) and to some extent by work such as K. Smith et al. (2003). It is fair to say that my model takes the structured meanings for granted, but as I am looking at compositionality strictly in terms of homomorphisms between complex meaning and signals, I would argue that this does not present a real problem, as long as what structure there is within meanings is mirrored by structure within signals.

The role of *robustness to noise* as a determiner of structure allows a more abstract understanding of what previously appeared to be quite different hypotheses regarding the genesis of both combinatoriality and compositionality. From this perspective, there is no real dichotomy between the perception and learning-based accounts of de Boer & Zuidema (2010) and Verhoef et al. (2014), as both involve noisy processes, only differently situated (perception/production vs. memory). Similarly, there is no conflict between intergenerational accounts of compositionality such as Kirby (2001) and interaction-based ones such as De Beule (2008b), as in both cases compositional structure is motivated in terms of its compressibility, which entails robustness.

Finally, both Berwick et al. (2011) and Franke (2014) argue that the key innovation is not compositional structure itself, but the creative or generative process which underlies it. In particular, Franke remarks that models of compositionality to date had required rather sophisticated mental representations and their associated machinery, which simply moves the onus for explanation onto those faculties. The rather stripped-down reinforcement model Franke proposes as a response to this makes few assumptions beyond a form of semantic overextension and lateral inhibition. As the model presented here was also intended to be as simple as possible, now is a good time to evaluate whether this is so. Superficially at least, the exemplar representations are close to minimal: unlike many previous models, there are no *intermediate* or *internal* representations with causal effect in the model. To clarify this point, both the finite-state transducers of Brighton (2002) and Franke's association weights are internal properties of agents. In the exemplar model, compositional production of a novel complex signal occurs almost entirely on the basis of the set of signals which the agent has been exposed to. The sole exception to this is lateral inhibition, which is certainly an internal process. However, as seen in the first chapter, some form of anti-homonymy is required for any signalling system to reliably become optimal.

Also, the recombinatory operation involves what amounts to a inference over possible outputs. Certainly, the blending process modelled by Tria et al. (2012) appears less sophisticated, simply splicing random sections of strings with related meanings together. However, the inference process in this model may well be more complex in implementation than reality. When agents output sub-strings with the strongest associations to the target meaning, retrieving those associations might require minimal computation compared to the algorithmic descriptions used here. Much of the complexity is because strings are restricted to a certain length, a design choice which reduces complexity in other areas. The discreteness of characters is another possible issue, but one I hope to dealt via the channel capacity of any noise-affected continuous channel. As such, the representations and mechanisms of the model are arguably close to minimal.

5.4.3 Model-specific comments

Outside of the larger theoretical context, the model raises a number of issues of a more or less technical nature.

There are several manipulations of the model which could be suggested. I will look at four of these in particular:

1. Unlike work such as K. Smith et al. (2013); Kirby et al. (2015), there is no explicit manipulation of parameters for expressivity or learnability: expressivity is 'baked in', and the learnability of any system is simply a consequence of noise. On the other hand, removing expressivity would have a very predictable effect: without the relevant mechanisms, strings would remain ambiguous and systematicity would not emerge.

2. On a related point, expressivity in this model operates at the string level, calculated using the degree of association of sub-strings with target meanings.

However, a more fine-grained measure of ambiguity is possible, where the edit distance between different strings could be used to determine relative closeness, and hence chance of reproduction. In fact, previous versions of this model did include this feature. It was taken out for two reasons: firstly, the edit-distance measurement is an inherently combinatorial operation, and thus hard to justify on a platform of maximum simplicity. Secondly, it had a rather trivial effect, that of minimising combinatoriality: the less shared features between strings, the greater their mutual distance. Because of this, I chose not to include an extra dynamic affecting the development of combinatoriality which *itself* operates combinatorially.

3. The character noise mechanism does not include the possibility of interpreting as an entirely new character. This leads to a 'one-way' drift-like dynamic where characters can only ever be lost. However, I made the decision not to implement this feature for similar reasons as for omitting edit-distances: the feature would not change the region of stability for character entropy, but only serve to add more noise to the system and make results harder to interpret.

4. Signal-directed noise results in characters 'flipping'. In the model, the probability of the new character is determined by the relative proportion of characters in memory. Alternatively, this could have been implemented with a parameter affecting 'regularity bias': for example, more common characters could have a disproportionately large chance of being selected. The reason for excluding this was again one of simplicity: selecting a 'neutral model' of noise is simply less theoretically demanding. In any case, there is good deal of ongoing discussion regarding the respective roles of bias and drift in regularisation processes (e.g. Ferdinand et al., 2014; Reali & Griffiths,

2009), which is not the target of this particular study.

Less specific to the model is the observation that the Mantel score which is typically used to measure the compositionality of model and experimental data is potentially severely compromised when systems exhibit duality of patterning. As shown above, inevitable random correlations between form and meaning spaces result in highly significant, but reasonably small correlations even when systems are completely holistic. The main problem here is that recent studies have chosen to use not the correlation measure (Pearson's r), but the significance (z-score) as the structural metric. Because of this, it is important to minimize the chances that random correlations like these are driving apparent effects. In practice, this is unlikely to cause serious problems: the shortness of the strings in this model is likely to be a factor in the strength of the random correlations, and in any case the experimental data in question exhibits very clear structure. However, it is worth keeping this in mind in order to reduce the chance of false positives. The alternative, to adopt the measure of compositionality used here, has yet to be tested against experimental data.

A further issue is the distinction between I/productive structure and E/superficial structure, which is rather blurred in this model. All production is on the basis of actually observed exemplars, and if a complex meaning has been seen before the entire string is just produced holistically. As such — depending on the size of the information bottleneck — it often happens that a signalling system can exhibit complete compositionality, yet be produced almost completely holistically. This apparent paradox can be resolved in two ways: firstly, compositional structure for smaller systems has a strong advantage: it is highly robust to noise. This remains true even if the chance of noise is relatively small. Secondly, this observation is only really true for smaller dimensionalities, and the advantage of compositional production explodes as the number of meaning dimensions and features increases. However, this robustness feature for smaller meaning spaces can be argued as a feasible intermediate stage, partially independent of productivity arguments. Systems can attain structure piecemeal, only requiring minimal cognitive computation, moving over to greater degrees of computation as the meaning space expands. This *gradualist* account might be investigated in future theoretical and experimental work.

Finally, as has been commented before, this model takes the particular structure of meaning spaces for granted. Just as signal-directed noise results in combinatorial structure and system-directed noise leads to compositionality, it might well be the case that a parallel process of noise affecting *only* meanings might be explanatory of *categorical* structure. Silvey et al. (2014) showed that categorical structure developed during communication games is *not* optimised along categorically salient lines. One possible explanation for this is that task demand introduces representational noise which causes conceptual spaces to collapse into arbitrary non-functional structures. This hypothesis could easily be incorporated into a future expansion of this model which includes dynamic meaning spaces.

5.5 Conclusion

The broad thesis of this chapter is that duality of patterning is a functional response to maintain expressivity and learnability in the face of noisy processes affecting both signals and signal/meaning associations. Structure emerges at different levels depending on an interaction between the size/dimensionality of the meaning space and the amount of noise at a specific level. Structure at one level does not imply structure at the other, and either might be present or not. This gives us new insight into phenomena such as the apparent lack of combinatoriality in ABSL (Sandler et al., 2011), the presence of combinatoriality and the absence of compositionality in birdsong (Berwick et al., 2011), and the fully-fledged duality of human language.

Of course, it is not enough to just recognise that different theories have certain abstract properties in common, described here in terms of learning, expressivity, and robustness to noise. It is also important to identify the particular mechanisms in effect, for example memory vs. perception. However, as in the previous chapters, it is quite possible that multiple mechanisms are at play to similar effect. In this case, the more abstract understanding helps dispel apparent conflicts of analysis, methodology, and interpretation.

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