THE CREEP OF CONCRETE UNDER LOAD WITH SPECIAL REFERENCE TO THE USE OF PORTLAND-BLAST FURNACE CEMENT.

BY

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ILLUSTRATIONS

ILLUSTRATIONS (Contd.)

TABLES.

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INTRODUCTION

For many years now it has been realized that concrete is far from being a truly elastic material even within the range of stress allowed in good practice. The phenomenon of plastic yield, time yield, flow, or creep as it is variously termed has been recognized for some time but it is only recently that any adequate investigation of the subject has been made.

In this country, Faber[1] in 1927 gave the results of a series of time tests on reinforced concrete beams and suggested the use of a provisional "Plasticity Factor" to allow for the effects of creep in design. More recently, the Building Research Station published details[2] of a comprehensive series of experiments devised to arrive at an absolute value of creep, to determine the factors by which it is controlled, and to investigate the effects on the stress distribution in reinforced concrete members. Much work has also been undertaken by American experimenters, notably Davis[3] and it is proposed to give here a very brief resumé of the known facts regarding the creep of plain concrete before describing the experiments carried out by the Author.

Creep has been defined as "the non-elastic deformation which takes place with time when concrete is subjected to stress". Glanville[2]

(1) Figures refer to bibliography, Appendix No.4.

has shown that the curvature in the stress strain graph for concrete is due to creep and by an ingenious method for correcting back to zero time, has obtained a linear relationship; or in other words, the instantaneous deformation is proportional to the applied stress.

(See Authors' experiments under Modulus of Elasticity.)

The deformation of concrete subjected to stress for long periods at a constant temperature and humidity is affected by creep, changes in the elastic strain, and shrinkage, and by observing the two latter movements in similar control specimens a true creep-time curve may be arrived at.

A study of the existing literature on creep reveals the following facts.

(1) The rate at which creep takes place under constant conditions decreases with time[(1),(2),(3)etc] pointing to an eventual state of equilibrium. Specimens under stress for five years evidenced very slight but still perceptible creep.[(10)]

(2) Creep is approximately proportional to the applied stress and this proportionality becomes more marked with increase in time.[(1),(2)] Davis[(3)] found that the creep was proportionally greater for high stresses than for low ones but thought it reasonable to expect that the stress-creep ratio would become more nearly constant within the range of working stresses as the age increased.

(3) Creep decreases with the age at the time of loading[3] and this decrease is due to a smaller deformation in the early life of the concrete, the final rates of creep being comparable independant of the age at the time of loading.[2]

(4) Creep increases with the quantity of the aggregate used i.e. lean mixes show relatively greater creep than rich ones under similar conditions.[2][3] Glanville found that creep is approximately inversely proportional to the quantity of cement used.

(5) Creep increases with an increase in the quantity of the mixing water employed.[8][3]

(6) When tested in air the creep of concrete decreases with the rapidity of the hardening of the cement used.[2] Thus the creep is smaller with Rapid Hardening Portland Cement than with Normal Portland and still smaller with Aluminous cement.

(7) For Portland Cements creep is reduced with an increase in the humidity[3] and is about 50°/o less for a concrete completely immersed in water than for one stored in air with a relative humidity of 65°/o.[2] In general, the reverse holds for Aluminous Cement Concretes.

(8) A well graded aggregate results in less creep or, the higher the fineness modulus the smaller is the creep.[3]

(9) Creep is affected by the mineral character of the aggregate being least for limestone and increasing in the order quartz, granite, gravel, basalt, and sandstone.[3]

(10) The magnitudes of creep under compressive and tensile stresses are approximately equal.[5]

(11) In general, factors tending to increase the strength and elastic modulus tend to reduce the creep but no defined relationship has been discovered.

(12) Following the instantaneous elastic strain recovery on the release of load there is a time recovery which gives a general shape of curve similar to the creep-time curve but the magnitude is relatively small and equilibrium is reached in a comparatively short time.[3]

(13) Glanville[2] gives the following figures for the creep of 1:2:4 concretes tested in air and loaded at an age of 28 days. (In view of the approximate proportionality of creep to stress these may be quoted in inches per inch per unit stress.)

	Creep per unit length per lb. per sq. inch at age of one year.
Aluminous Cement	$0{\cdot}25 \times 10^{-6}$
Rapid Hardening Portland Cement	$0{\cdot}39 \times 10^{-6}$
Normal Portland Cement	$1{\cdot}02 \times 10^{-6}$

It will be seen from (6) and (13) above that the influence of the type of cement employed has been investigated; experiments have been performed with Normal Portland, Rapid Hardening Portland, and Aluminous Cements, three representative types. It was felt that a fourth class viz: Portland-Blast Furnace Cement merited an investigation which might give an indication of its behaviour under sustained stress, and which might yield also, further data regarding creep generally.

With this object in view Portland-Blast Furnace Cement was employed in the first series of tests and thereafter comparative tests with Normal Portland Cement were carried out under identical conditions.

DESCRIPTION OF APPARATUS.

The investigations described were carried out in the Strength of Materials Laboratory of the University of Edinburgh.

Throughout the tests the temperature was maintained at approximately 64°F by thermostats and electric heating elements. Apparatus for the control of humidity was not available but wet and dry bulb thermometer readings were taken simultaneously with all strain observations thus enabling local variations in the curves of movement with time to be explained and giving, also, an approximate mean relative humidity for the period of the tests.

The use of beams at once suggested itself as a ready method for obtaining the sustained stress for a determination of creep but was abandoned because of the uncertainty of the actual stresses which occur in a reinforced concrete beam. The number and the variety of the flexural theories for reinforced concrete were sufficient to prejudice the investigator against such a method of stress production. Axial loading was considered to be imperative and since the monopolisation of testing machines for long periods was impracticable, possible methods employing hydraulic and gravity loading were considered. These were abandoned on the grounds of cost or bulkiness and finally the spring loading apparatus shown in Figs. 1 and 2 was designed.

Essentially, the loading device consists of a rigid welded framework of rolled steel channels

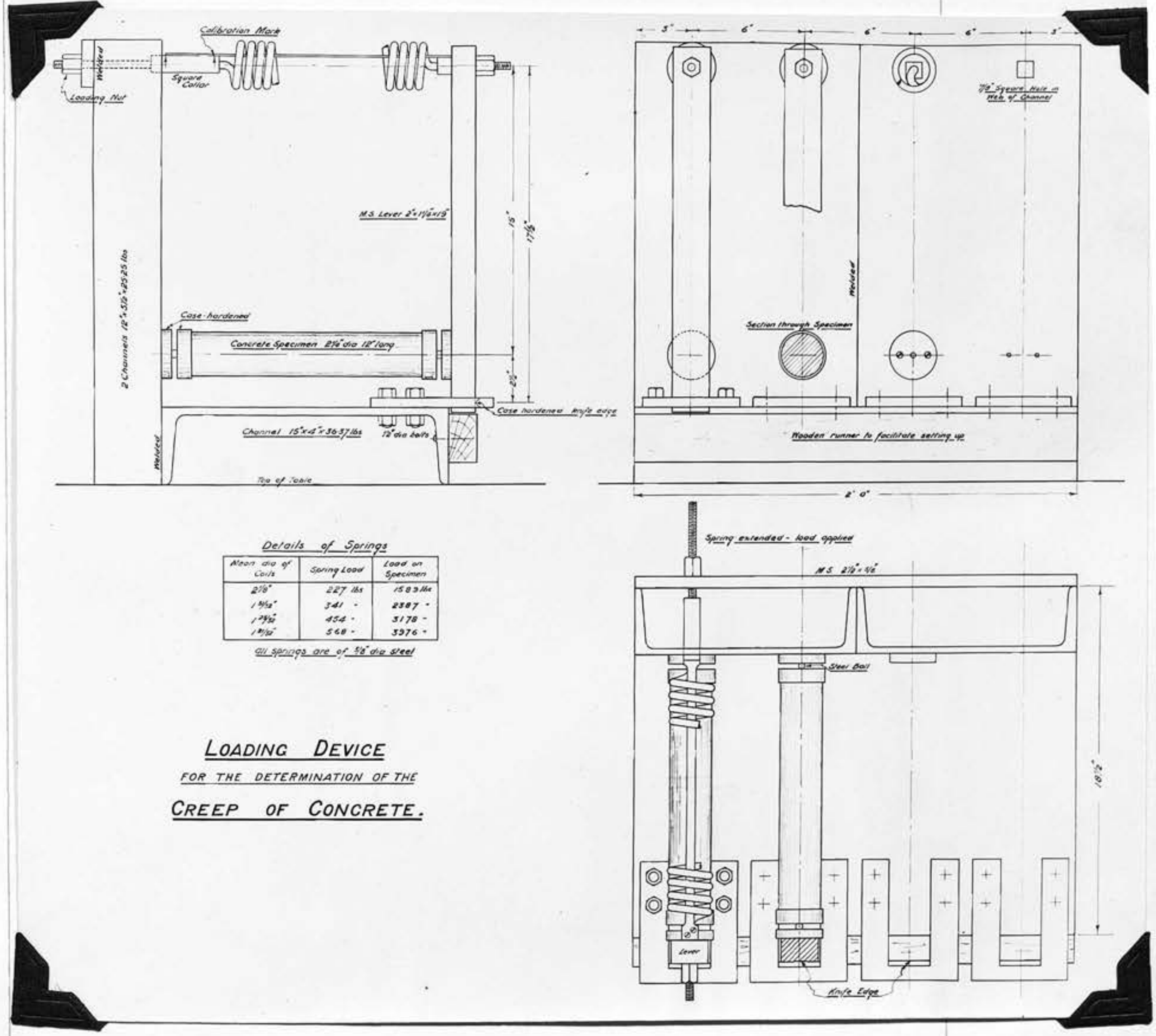


Fig. 1.

Diagram of Loading Device.

with four levers which react on case-hardened steel knife edges and which give a 7:1 magnification of the spring loads applied at their upper ends. This apparatus avoids the use of very heavy springs with little sacrifice to accuracy and enables four specimens to be stressed within a compact space. A square section brass collar fastened to the end of each spring by grub screws passes through a square hole in the web of the vertical channel thus securing a direct axial pull on the spring and obviating any tendency to torsion when the load is applied. A light brass strip attached to the lever end of the spring registers the deflection on the collar. Each spring was carefully calibrated by the manufacturers under gravity loading and the deflections corresponding to the desired loads were scribed on the brass collars; these deflections were checked in a low capacity testing machine in the laboratory prior to use.

The springs were designed to give a deflection of approximately $2\frac{1}{2}$ inches under the working load. It was found possible to control the deflections within about 0·02 in. of the desired amount, involving a maximum error in the concrete stress from this cause of only 0·8 $^{o}/_{o}$. With a lever device such as described, the creep of the concrete is of course amplified but periodic checks were made on the spring deflections, the loading nuts being given a fraction of a turn when it was found

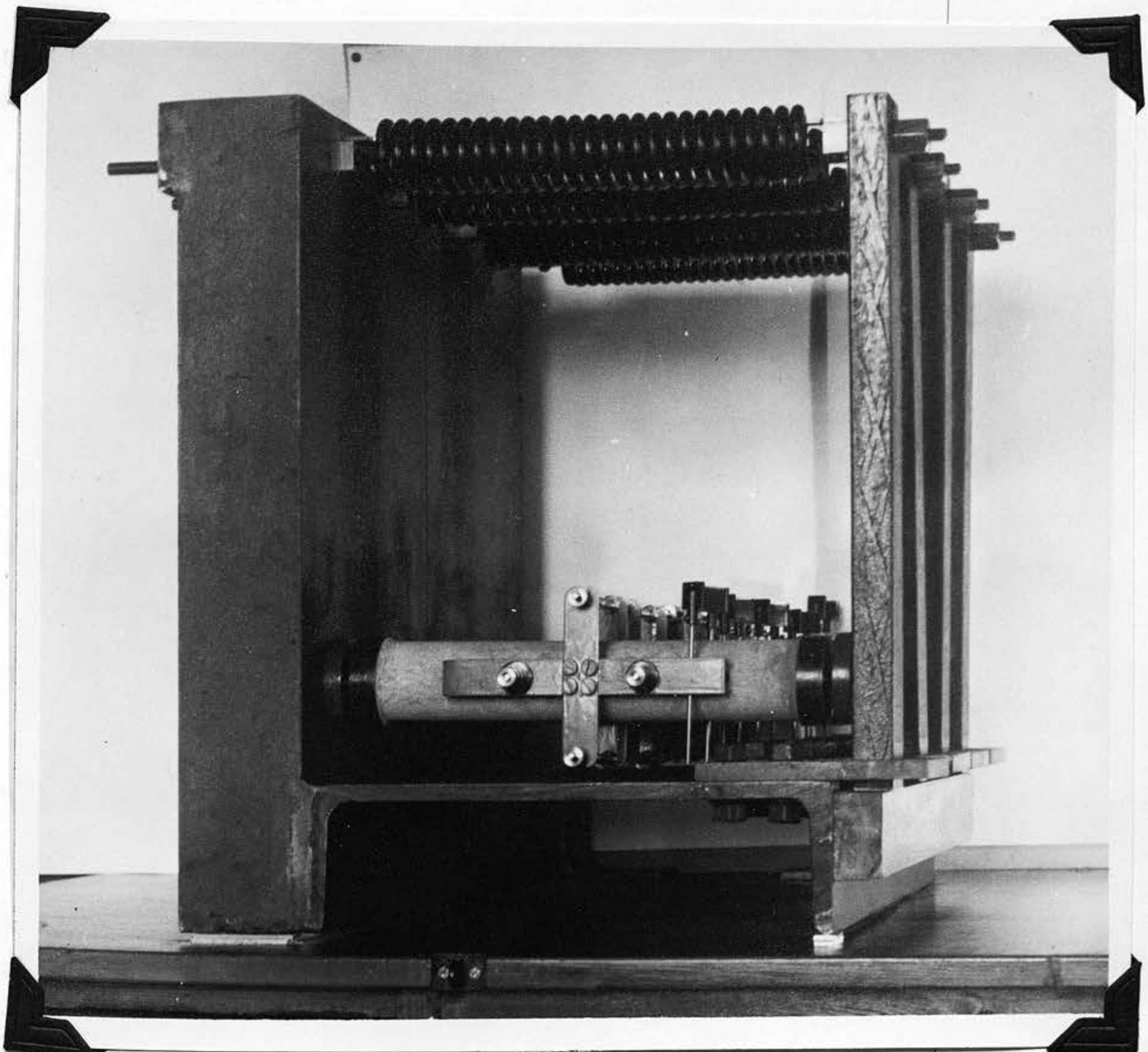


Fig. 2.

Loading Device.

necessary. The four springs were designed to give stresses of 400, 600, 800, and 1000 lb. per sq.in. in the specimens but the actual stresses were slightly higher owing to small differences in the lever distances.

At the conclusion of each time test the springs were carefully checked for creep in the steel. It was found that the reduction of load under the prescribed deflection was small, varying from 0·9 °/o to a maximum of 1·75 °/o. Thus there took place throughout the experiment a gradual reduction in the concrete stress due to the creep of the steel but this was not of such a magnitude as to be of great importance. Each spring was, of course, re-calibrated before being again employed for stress production.

The lever distances were carefully determined to within 0·01 in. giving rise to a possible error in the concrete stress from this cause of about 0·4 °/o. In order to avoid eccentricity and to insure a uniform stress across the section the load was applied through steel balls and case-hardened steel platens and, in addition, this feature provided a useful method of adjustment in that, by varying the diameter of one or both balls by 1/16 in. the lever was brought into a truly vertical position thus allowing latitude for slight variations in the length of the specimen.

The loading device was placed at one end of a table in the laboratory and beside it an unstressed specimen for observation of shrinkage was mounted in smooth V brackets which could offer no appreciable resistance to movement. The shrinkage specimen occupied a position of the same height and lateral spacing as the battery of four stressed specimens thus permitting the same treatment for observations of movement. Also, the unstressed specimen was thus subject to the identical conditions of temperature and humidity.

The whole apparatus, including the unstressed specimen, was covered by a light wooden cabinet which could be raised or lowered vertically by suitable lifting tackle attached to the roof of the laboratory. By this means, the extensometers in particular were protected from the deposition of dust inevitable with long time experiments. A hinged door was fitted to the side of the cabinet enabling observations to be made readily on all five specimens. The general layout is illustrated in Figs. 3, 4, and 5.

The weight of the massive welded frame prevented any movement and it was quite possible to replace any particular specimen without disturbing the remainder. Also, no risk of error was incurred due to changes in the scale distance of the extensometers.

After consideration of the relative

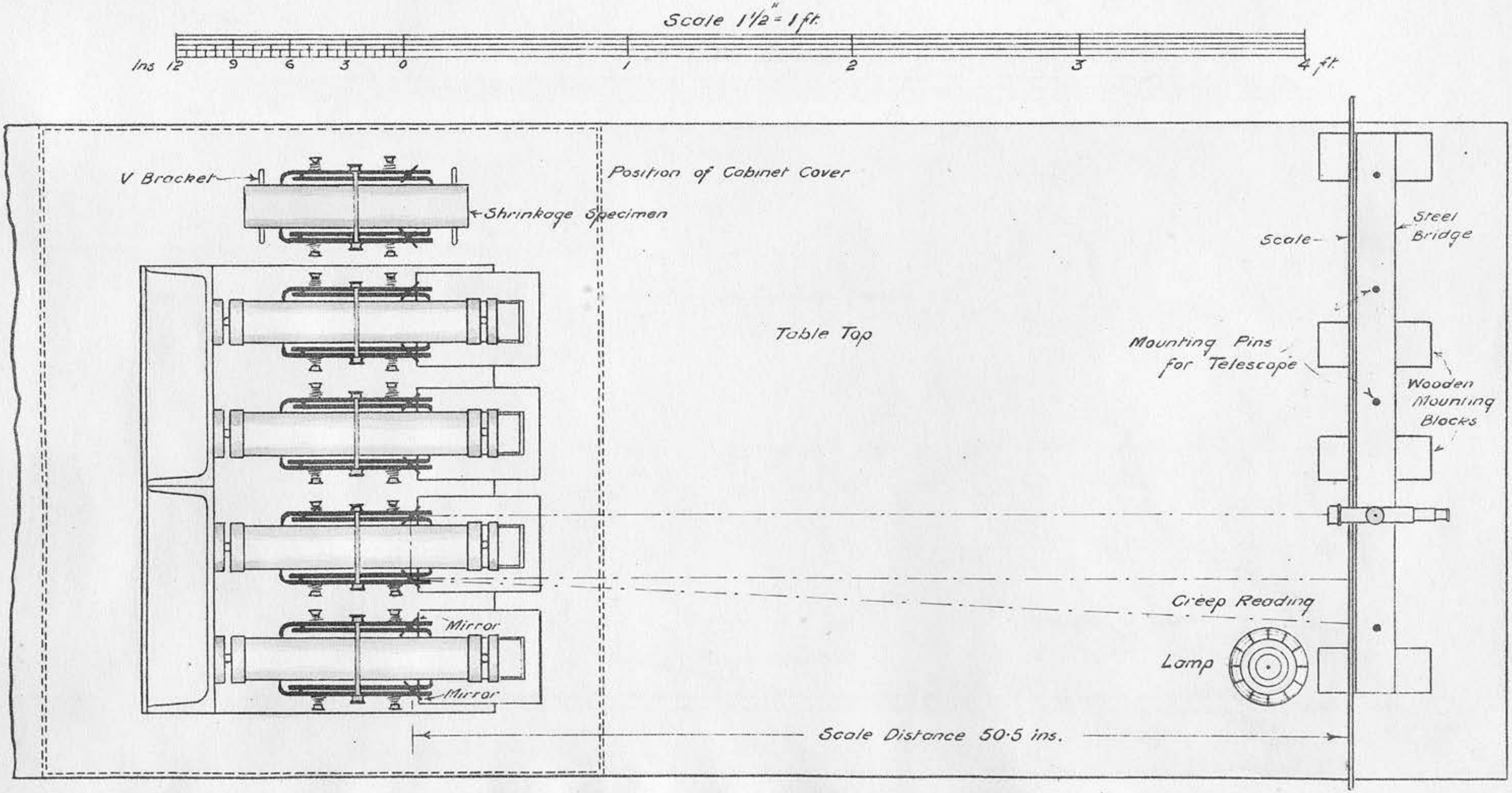


FIG. 3

PLAN OF CREEP APPARATUS.

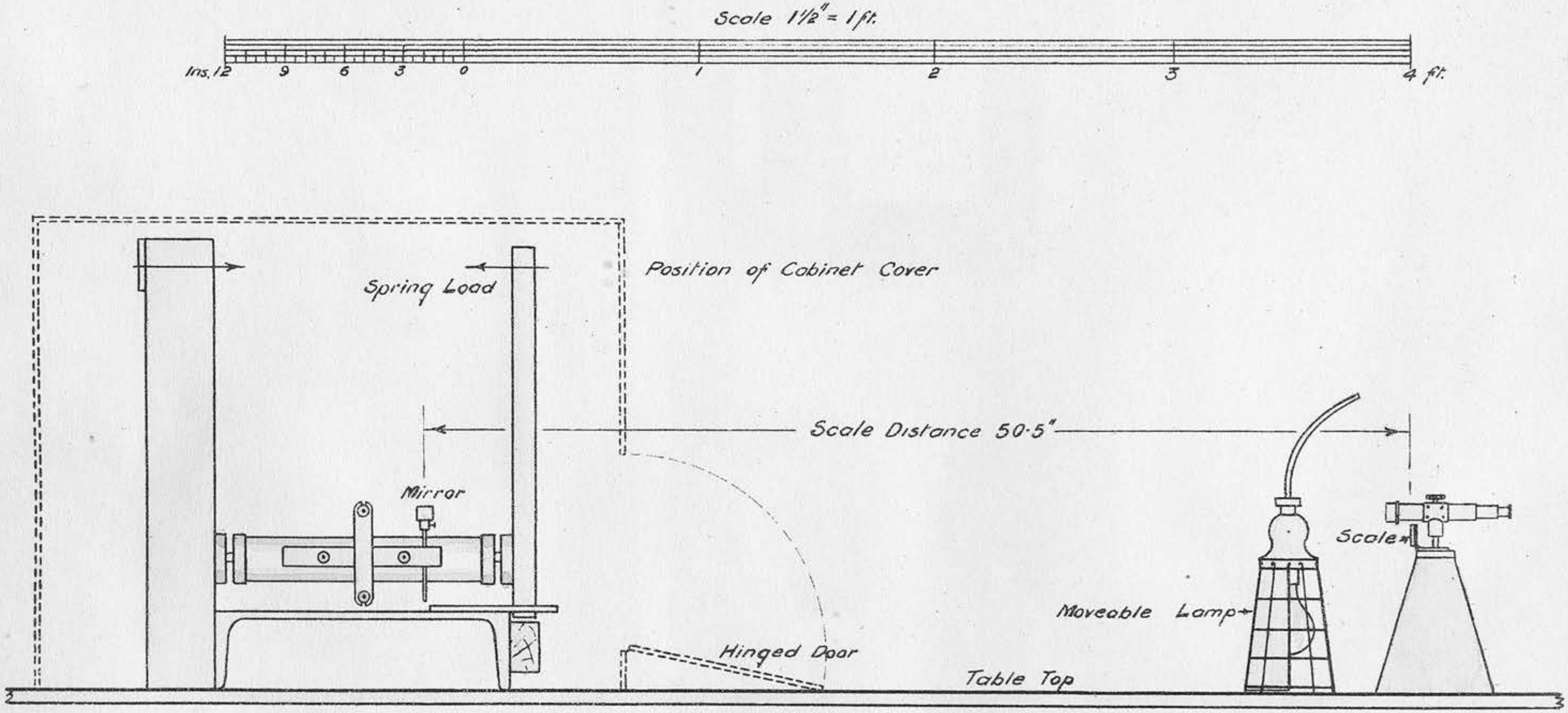


FIG. 4

ELEVATION OF CREEP APPARATUS

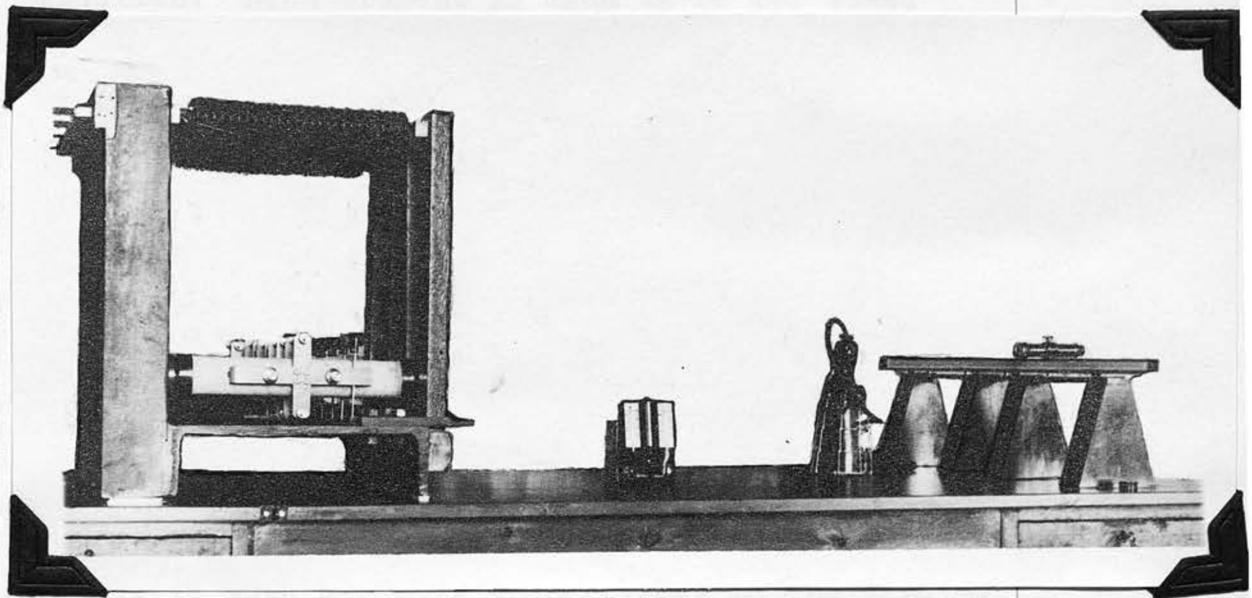
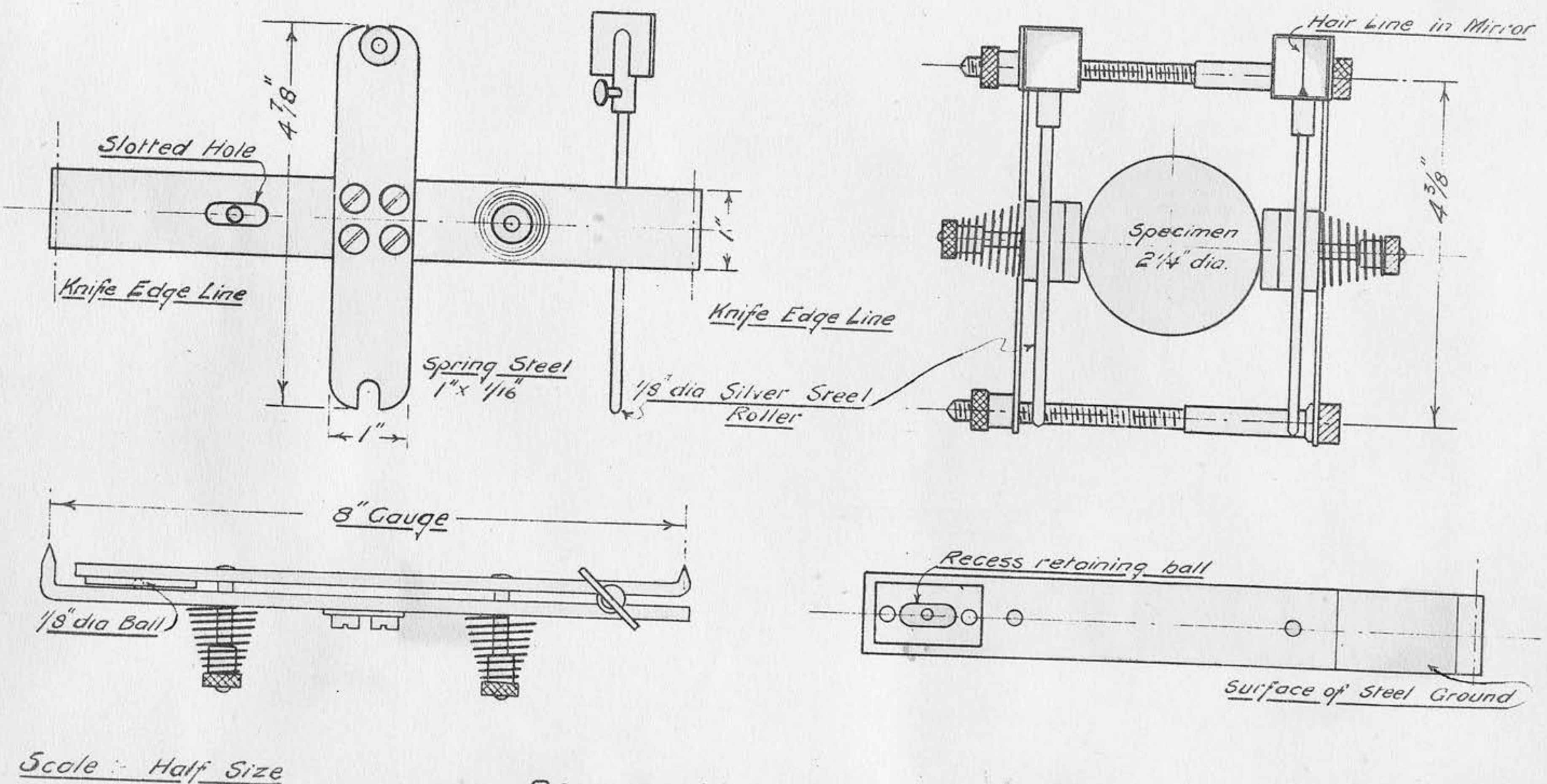


Fig. 5.

General Arrangement.

merits and costs of marketed strain gauges the optical instruments illustrated in Figs. 6 and 2 were designed and made up at a cost which represented a considerable saving on the commercial products.

The extensometer, an adaption of the Lamb roller mirror type, consists of two spring loaded elements which grip on opposite sides of the specimen. Each element is made up of two steel flats separated at one end by an 1/8 in. dia. ball and at the other end by an 1/8 in. dia. silver steel roller carrying a mirror. The contact surfaces are accurately ground and the pressure is controlled by springs. Strain in the specimen is imparted as a relative movement to the two flats which grip by means of knife edges at a gauge distance of 8 in. The angular displacement of the mirror is thus directly proportional to the strain. The dual advantages of increased magnification and particularly the automatic elimination of errors due to rotary or translatory movement of the specimen as a whole predisposed the Author to use the two elements conjointly instead of as two separate extensometers on opposite sides of the specimen. Certain modifications were introduced and the instruments were used in the manner indicated in Figs. 9 and 3. (When employing the instrument for the determination of the elastic modulus it was found more convenient to use the conventional layout indicated in Figs. 8 and 7.)



ROLLER MIRROR EXTENSOMETER.

FIG. 6

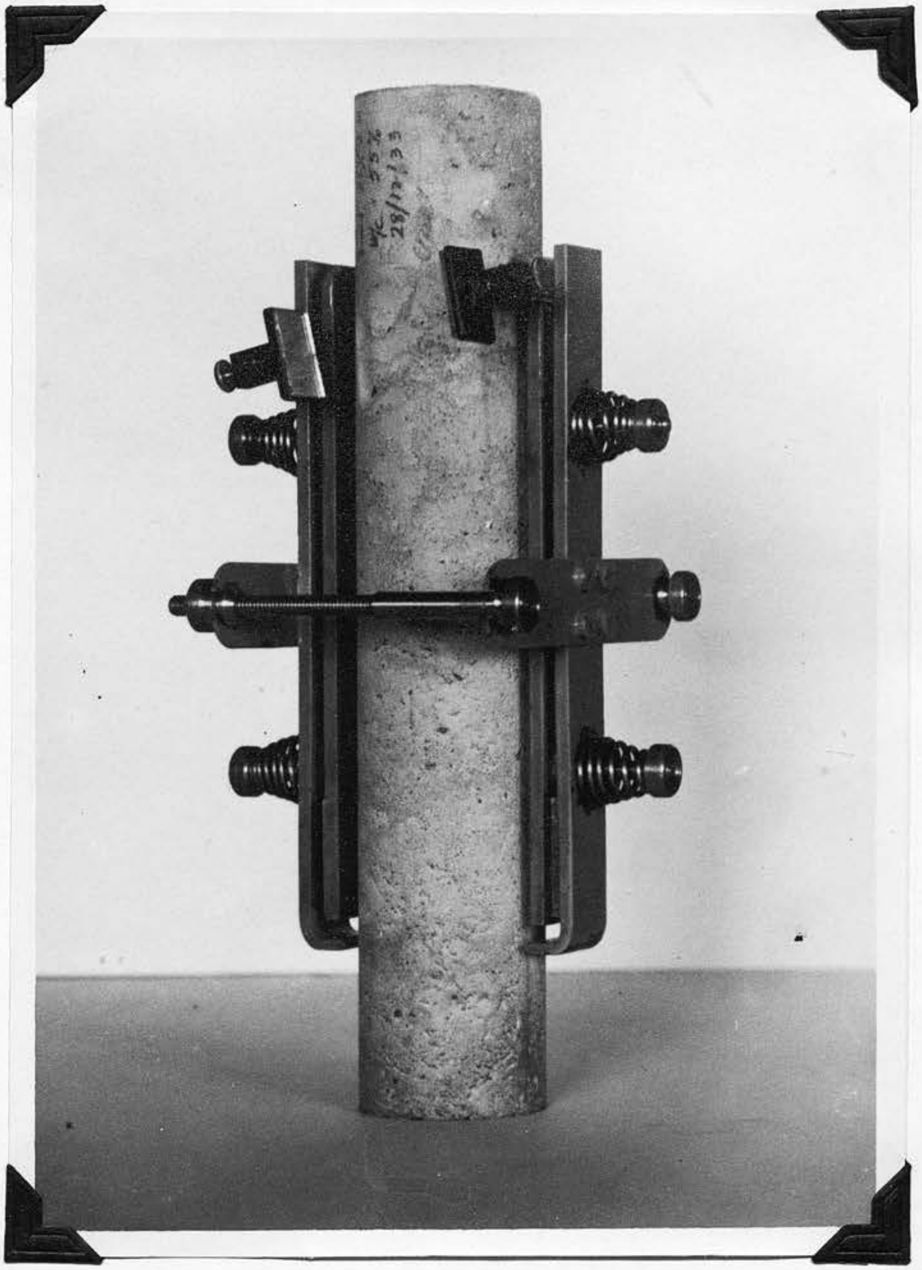


Fig. 7.

Extensometer fitted to Specimen for Elastic Test.

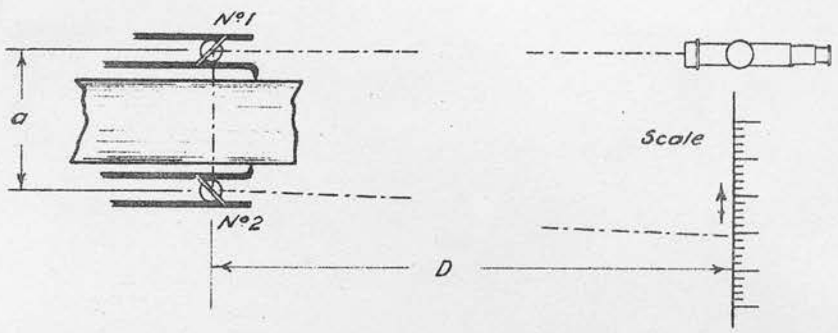


FIG. 9

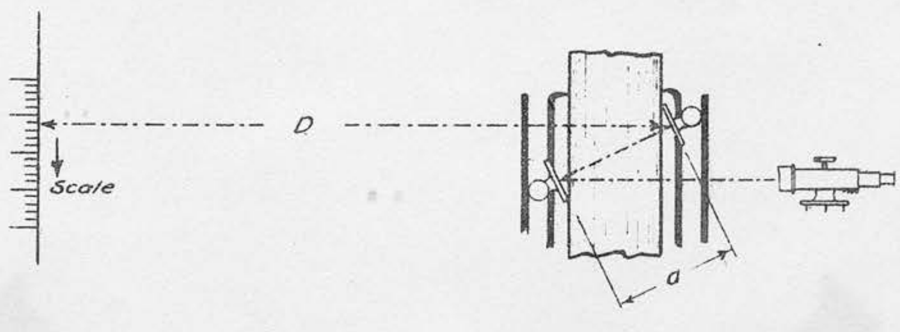


FIG. 8

Fig. 8. Conventional Layout of Extensometer. (Diagrammatic)

Fig. 9. Layout of Extensometer as Used for Creep Tests. (Diagrammatic)

A telescope with a vertical crosshair was employed and this could be mounted directly opposite the specimens on five vertical pins fitted to a steel bridge which, along with the horizontal scale, was secured to the table at the requisite height by means of wooden blocks. See Figs. 10, 3 and 4. The line of vision was reflected back by the two mirrors of the extensometer towards the source. The telescope was, of course, fitted temporarily only to the appropriate pin and to insure that the direction of the initial line of sight was not varied, a vertical line in the reflecting surface of No. 1 mirror was scribed directly in the roller axis; also, the mirrors were so mounted that their reflecting surfaces lay in the diametral planes of the rollers. The vertical line in No. 1 mirror was thus directly in the central axis of the roller.

In operation, the telescope was swung until the crosshair coincided with the mirror line, was locked in this position, and then the scale reflected through No. 2 mirror was brought into focus and the reading taken. In spite of the rotation of the roller, the vertical mirror line maintained its relative lateral position and, the telescope axis being fixed, the line of vision entering the instrument was directionally constant and thus the observed difference of scale reading was a measure of the angular deflection of the two mirrors which was directly proportional to the strain in the specimen. (See Appendix No. 1)

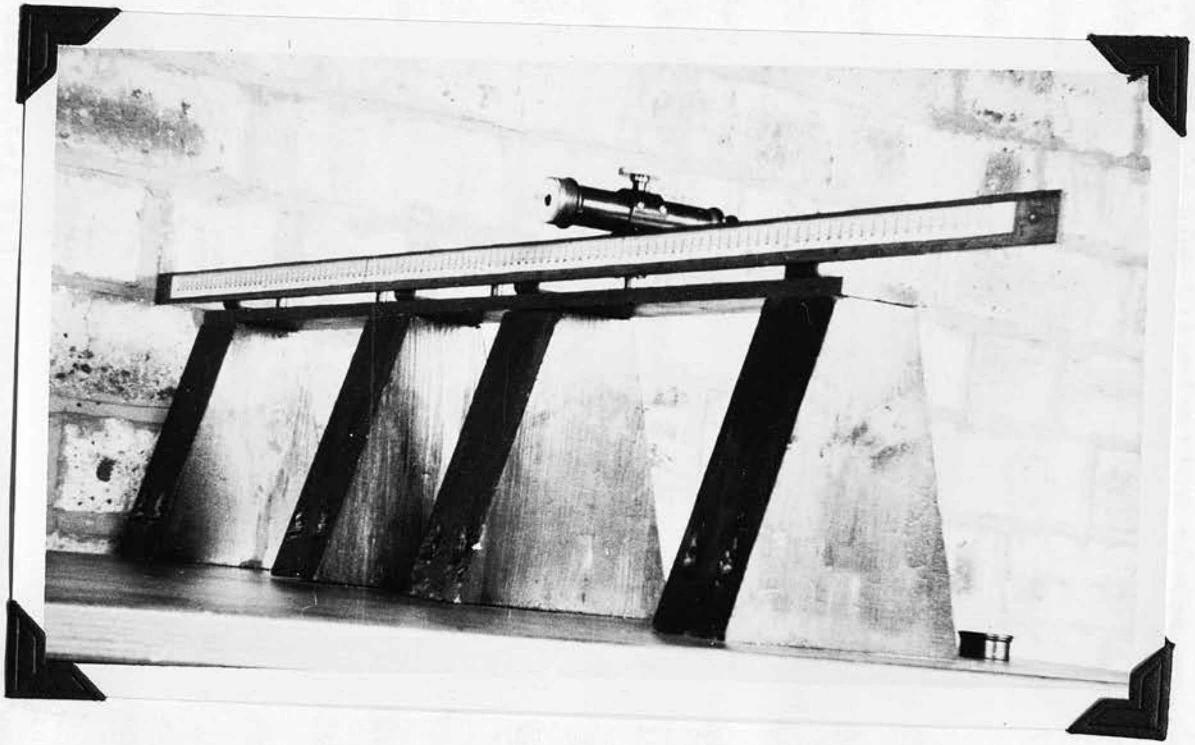


Fig. 10.

Mounting of Scale and Telescope.

Some optical difficulties were encountered in obtaining a clearly defined scale image; telescope apertures of varying diameter were experimented with until it was possible to get a clear, sharp reflection of the scale. A movable inspection lamp, with one side shielded, was placed on the table to provide for the illumination of the scale and the darkness within the cabinet insured a bright image thus enabling the mirror to be brought readily into the field of the telescope.

The scale was divided into 1/30ths of an inch and was placed at a distance of 50·5 in. from the extensometer rollers. With this arrangement, scale readings of 1/60th in. which were quite possible with the telescope used corresponded to 1×10^{-5} in. strain on an 8 in. gauge length and thus per unit length observations were made to $1{\cdot}25 \times 10^{-6}$ in. per in. Actually it was possible, with care, to obtain readings to $0{\cdot}63 \times 10^{-6}$ in. per in. by interpolating to quarter scale divisions but it was felt that such refinement was not justified in view of the other unavoidable experimental approximations.

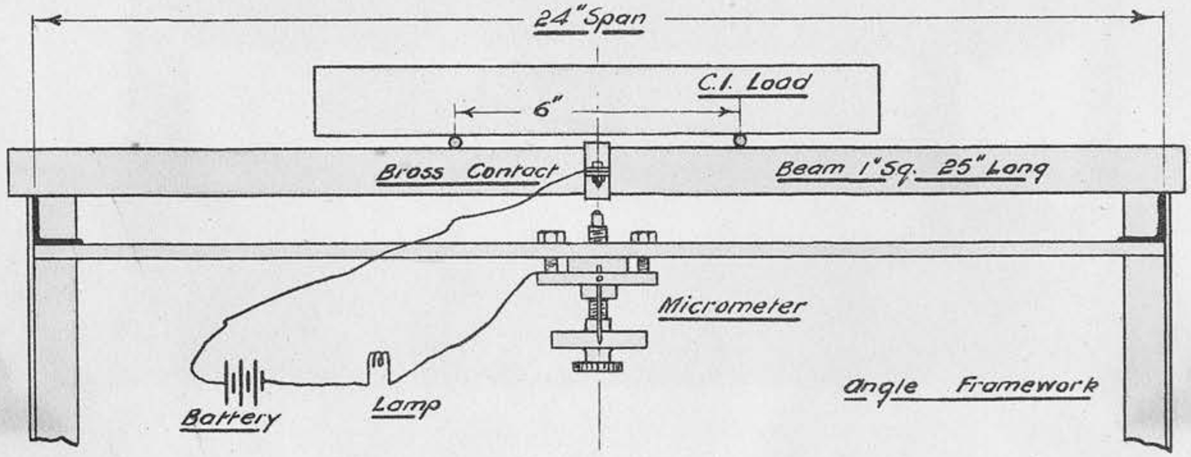
The magnitude of the maximum error arising from the fact that the scale was straight instead of seqmental was investigated; under the most unfavourable conditions i.e. with the largest possible rotation, the error was found to be of the order of 1 o/o and since normally it would be less than this figure, corrections were not considered to be essential.

With large deformations a tendency was noted for the line of vision to be deflected out of the field of No. 2 mirror owing to the rotation of No. 1, or to be screened by the vertical levers. This sometimes necessitated re-setting the mirrors; the readings immediately prior to and subsequent to the readjustment were noted (e.g. see Table 1) and since the operation required only a short period no appreciable loss of accuracy could result. It should be noted also, that this adjustment was required only after large deformations, i.e. when the rate of creep was so reduced that the movement occurring in a minute or two would be quite negligible.

The extensometers were lubricated with a light oil of low viscosity which insured free movement and avoided corrosion of the rollers and plates.

Throughout the experiments the whole of the apparatus functioned well and the Author feels that the layout described has much to recommend it. Strain observations were made simply and rapidly; the use of several telescopes and scales was avoided and measurements with the accuracy required for this type of work were made with quite a short scale distance and within a limited space.

To study further the effects of time loading on cements and mortars and to carry out comparative tests on Normal Portland and Portland-Blast Furnace cements the simple beam tester indicated in Figs. 11



Beam Testing Device

FIG. 11

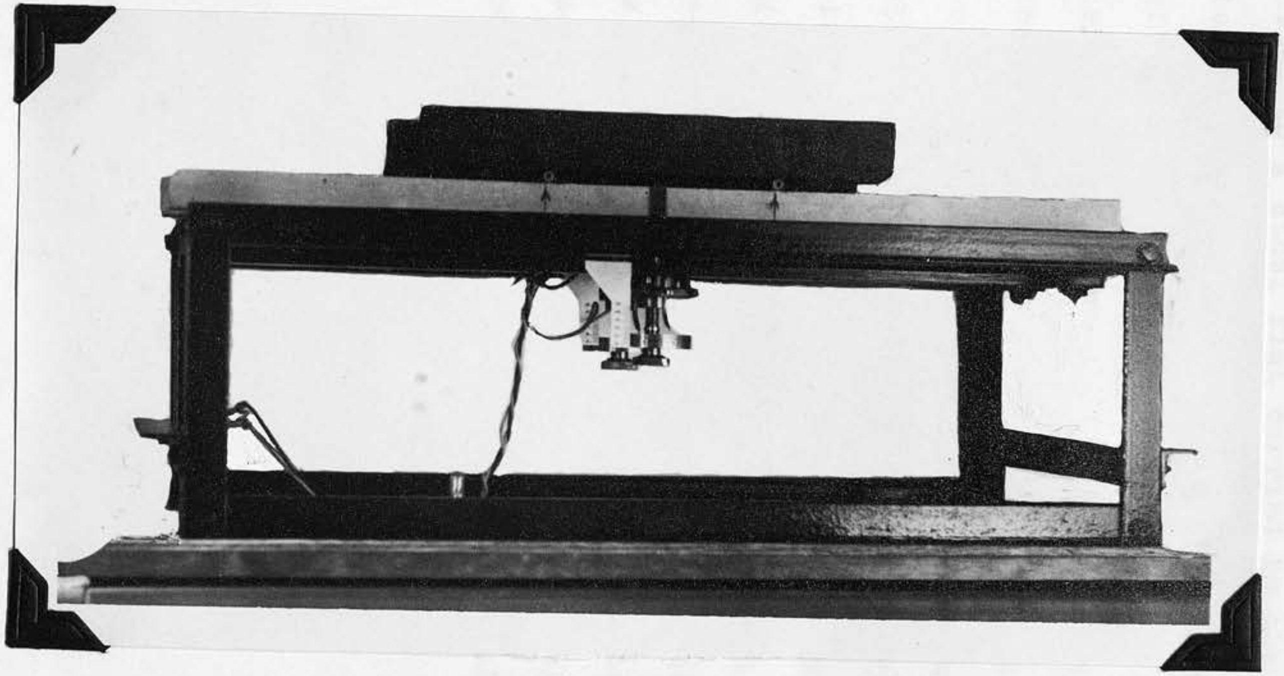


Fig. 12.

Beam Testing Device.

and 12 was designed. The device consisted of a framework or stool of light steel angles on which two beams could rest supported at 24 in. span. Centrally under each beam was placed a micrometer bolted to a steel flat carried by the frame. The micrometers read to 1×10^{-4} in. and an electrical tell-tale was arranged by securing a light brass strip to each beam; a small lamp was illuminated when contact took place between the ball in the micrometer head and the brass strip thus giving uniformity and accuracy in reading. The beams were 1 in. square in cross-section and loading was effected by a cast iron weight through rollers placed centrally 6 in. apart. The whole apparatus was placed in a cabinet in the laboratory and the specimens were thus subjected to the same atmospheric conditions as the cylinders under axial stress.

DESCRIPTION OF EXPERIMENTS.

For the first experiments hereafter referred to as Series I, Portland-Blast Furnace cement was employed. Six specimens, $2\frac{1}{4}$ in. in diameter and 12 in. long were cast in split cylindrical moulds. While $2\frac{1}{4}$ in. is small compared with normal dimensions in practical concrete construction a series of tests carried out at the Building Research Station[8] has shown that between the diameters of 6 and 3 in. the creep is independant of the size of the test piece. The mix was 1:2:4 by weight with crushed whinstone and river sand as aggregates. (See Appendix No. 2 for Mechanical Analysis). A water/cement ratio of 0·55 was employed resulting in a relatively dry but nevertheless a workable mix.

Six 3 in. cubes were cast from the same mix and these gave a mean crushing strength of 3735 lb. per sq.in. after 28 days water curing. The cylindrical specimens were placed in damp canvas for two days after removal from the moulds and thereafter were exposed to the air of the laboratory. At the age of 28 days four of the cylinders were placed in the testing machine and loaded, the remaining two being controls for the observation of shrinkage and elastic movement.

The ends of the specimens were bedded on the platens of the loading device with neat cement to avoid areas of concentrated stress. The test pieces were first merely held in position with no appreciable load while the extensometers were fitted and

	Date	Aug. 7	8	9	10	11	13	15	17	20	22	24	27
Specimen No.4 Series II	Temperature	64	65	66	65	65	64	66	65	64	63	64	65
	Relative Humidity	78	73	79	71	64	68	74	69	73	73	58	78
Normal Portland Cement	Days Old	28	29	30	31	32	34	36	38	41	43	45	48
1:2:4 Mix	Days under Load	0	1	2	3	4	6	8	10	13	15	17	20
65 o/o W/c Ratio	Scale Reading	On Loading 87·4-101·3	108·7	111·7	113·3	115·2	*** 106·6 118·6	109·9	111·6	114·4	115·4	117·0	120·4
Cast 10/7/34	Difference	* 13·9	7·4	3·0	1·6	1·9	3·4	3·0	2·0	2·8	1·0	1·6	3·4
400 lb/sq.in.	Total Movement after Loading on 8 inches in. x 10^{-4}	** 1.5	8·9	11·9	13·5	15·4	18·8	21·8	23·8	26·6	27·6	29·2	32·6
applied 7/8/34.	Creep plus Shrinkage ins.per in. x 10^{-6}	** 19	111	149	169	193	235	273	298	333	345	365	408

* True instantaneous elastic movement = 12·4 x 10^{-4} in.

** Creep during loading.

*** Mirrors re-set.

TABLE I. TYPICAL RECORD OF CREEP EXPERIMENT.

the mirrors adjusted.

After the initial reading had been taken the desired load was applied by rotating the loading nut on the spring. The application of the load required about 5 minutes and even in this short period some creep took place evidenced by the difference in the observed strain and that calculated on the true elastic modulus as determined by the control specimen. Observations were then taken every one or two days initially and at longer intervals thereafter for a period of 162 days.

The results are shown graphically in Fig. 13. It will be seen that the rate of creep, large initially, was reduced with increase in time; also that the creep and shrinkage were together more than four times the initial loading strain after a period of 162 days under stress. In Fig. 14 the elastic strain plus creep at different ages is plotted against stress; the diagram indicates clearly that creep is approximately proportional to the applied stress and that this proportionality becomes more pronounced with increase in age, the curve for 160 days being more nearly a straight line than that for shorter periods.

The specimen maintained at 840 lb. per sq. in. however, showed relatively less creep than the remaining three and this gave rise to much speculation. The cylinder was cast from the same mix and the curing and storage conditions were identical. A

Series I 1:2:4 Concrete Portland-Blast Furnace Cement Cured and Tested in Air Mean Rel. Hum. 60%

Loaded at 28 Days Creep during Loading included W/C Ratio 0·55 (by wt.) Cube Strength (28 Days) = 3735 lbs. per sq. in.

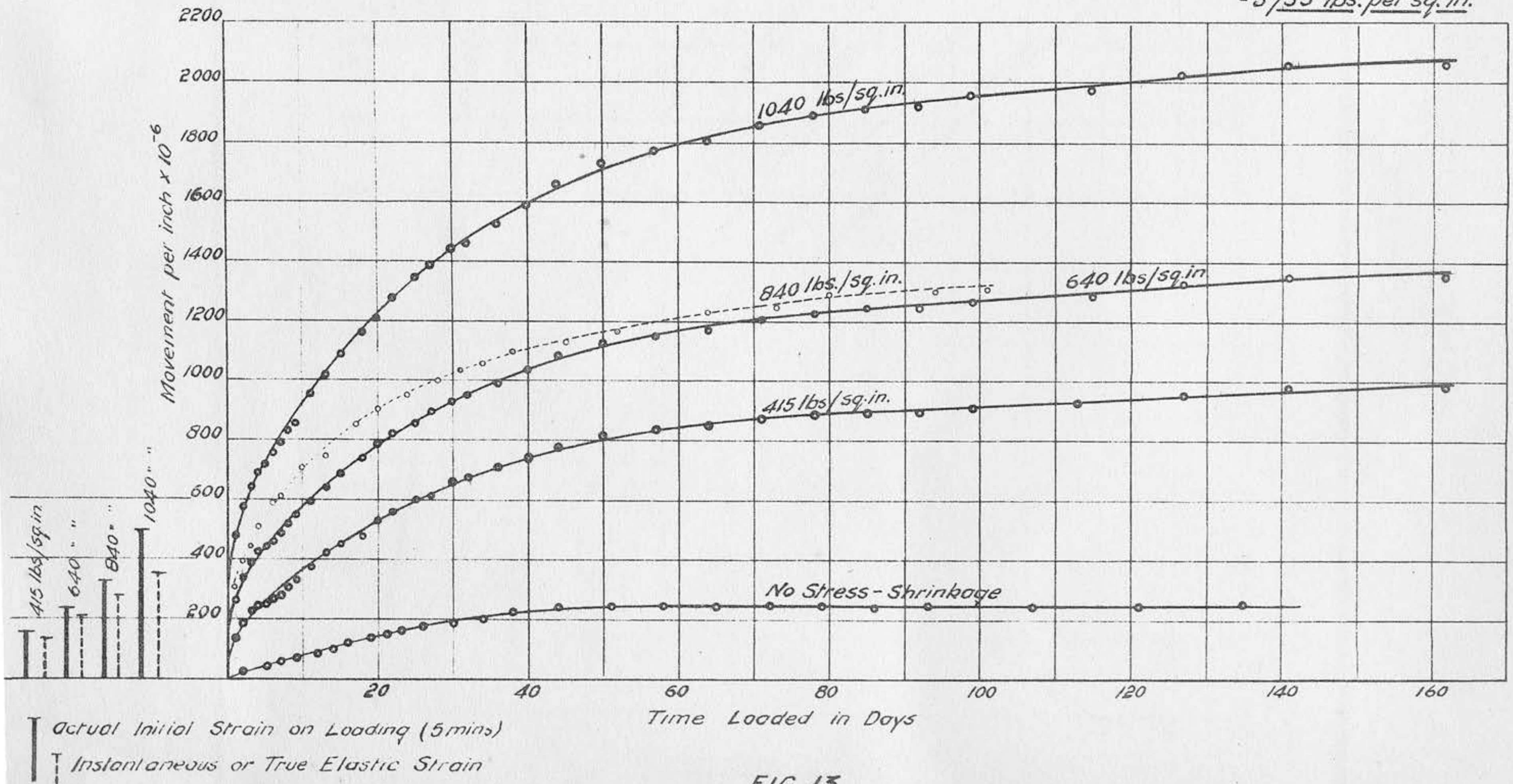


FIG. 13

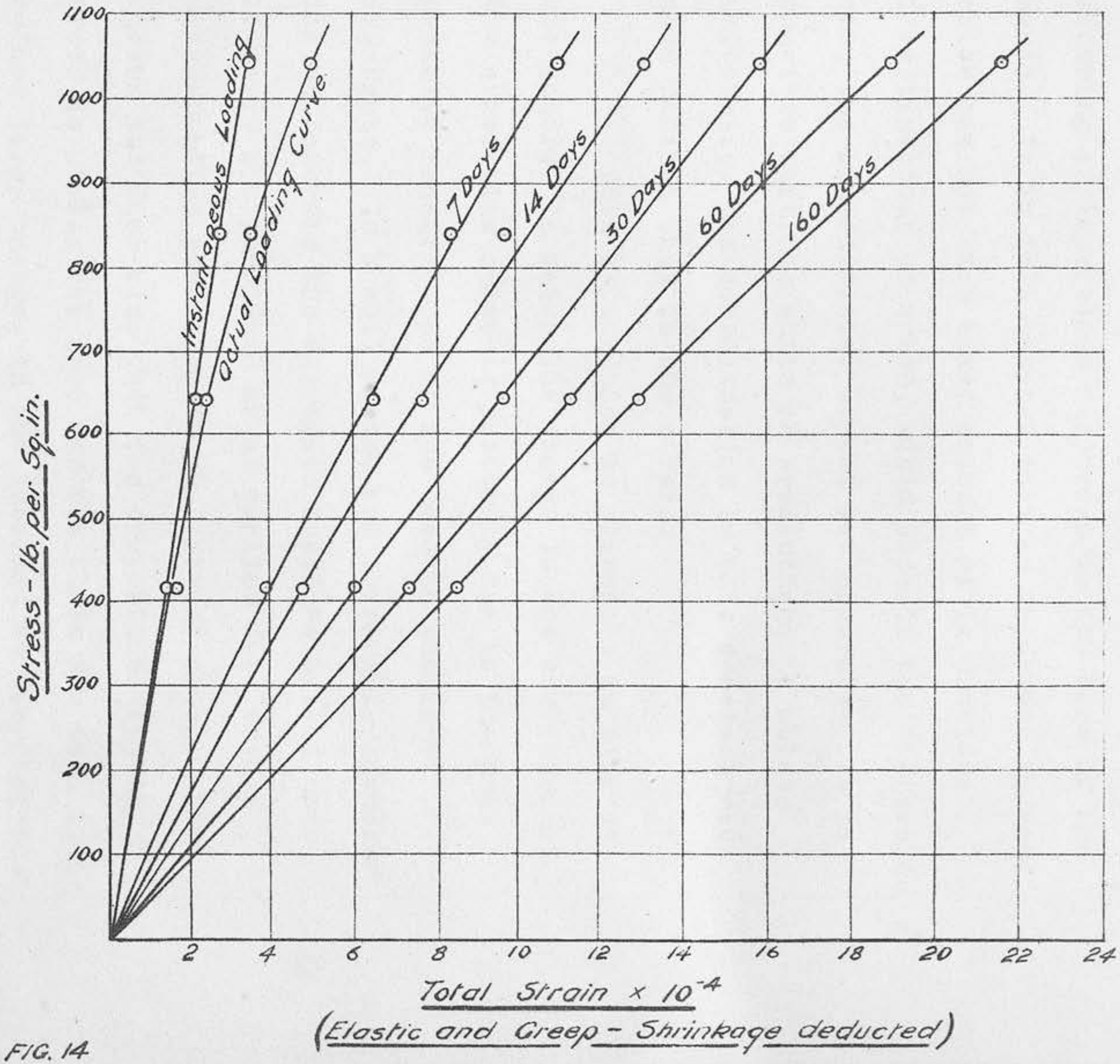

Instantaneous Loading
Actual Loading Curve
7 Days
14 Days
30 Days
60 Days
160 Days
Stress - lb. per Sq. in.
1100
1000
900
800
700
600
500
400
300
200
100
2
4
6
8
10
12
14
16
18
20
22
24
Total Strain × 10^{-4}
(Elastic and Creep - Shrinkage deducted)

FIG. 14

slight variation in the quantity of water present at casting however, may have taken place owing to a loss at the base of the mould thus giving a drier concrete which would be expected to evidence less creep. This was corrected in succeeding experiments by securing an impervious diaphragm to the base of the mould. It is felt however that an important factor, which has not been taken account of in previous investigations of creep, would explain the difference viz. the consolidation of the wet concrete. It is possible that, in spite of precautions to obtain uniformity, the consolidation in this specimen was more perfect than in the others.

This explanation is plausible in view of the theory[4] in which the change in the curvature of the creep-time curve is partially due to the progressive transference of the load to the inert aggregate. In a well rammed and thoroughly consolidated concrete the aggregate would be called upon to carry a higher load at an earlier date with a corresponding reduction in the rate of creep. The curves indicate also that the proportionality of creep to stress was good for the first few days i.e. before large changes in the internal stress distribution had taken place.

The remaining three specimens however, showed such similarity in the shape of the creep-time curves and such good proportionality of creep to stress that it was felt that undue weight should not

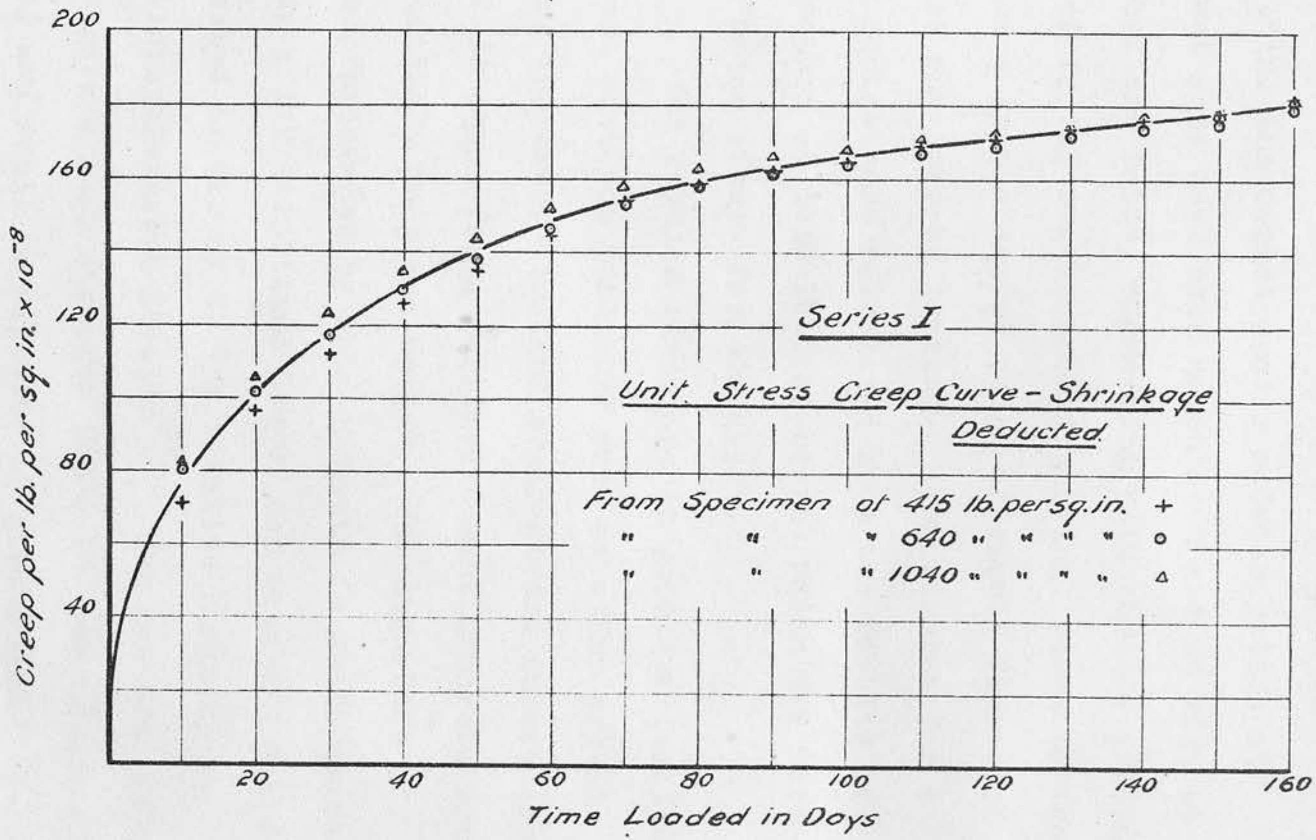

Series I
Unit Stress Creep Curve - Shrinkage Deducted
From Specimen of 415 lb. per sq. in. +
" " " 640 " " " " o
" " " 1040 " " " " △
Creep per lb. per sq. in. x 10^-8
200
160
120
80
40
20
40
60
80
100
120
140
160
Time Loaded in Days

FIG. 15

Table 2 shows strikingly the results of these tests in terms of an "effective" modulus or the ratio of stress to total strain, elastic plus creep (shrinkage is deducted). On the assumption that steel has an elastic modulus of 30×10^6 lb. per sq. in. the corresponding modular ratios at different ages have been calculated and it will be seen that the ratio, commencing with the low initial value of 10 for instantaneous loading, rises to the enormous - from a design point of view - figure of 70 after one year of loading. (The figures for 6 and 12 months were calculated from the results of the specimen at 1040 lb. per sq.in. which was maintained under stress for 400 days).

The results given are in good qualitative agreement with the published work of other experimenters; the creep-time and creep-stress curves are typical in shape for a Portland Cement Concrete but quantitatively they are unique. The magnitude of the creep evidenced by this concrete is about twice that for a 1:2:4 Portland Cement Concrete W/C Ratio 0.7, tested in air of 65 °/o relative humidity at the Building Research Station[2]. The low W/C ratio of 0.55 gave a high strength (3735 lb. per sq.in. at 28 days) and would be expected to give a small creep; an elastic modulus of 3.0×10^6 lb. per sq.in. was considered to be satisfactory; also, the mean density of the specimens was 147 lb. per cu.ft. - all evidence that the concrete was sound. The mean

TABLE 2.

Effective Modulus of Elasticity and Modular Ratio at Different Ages.

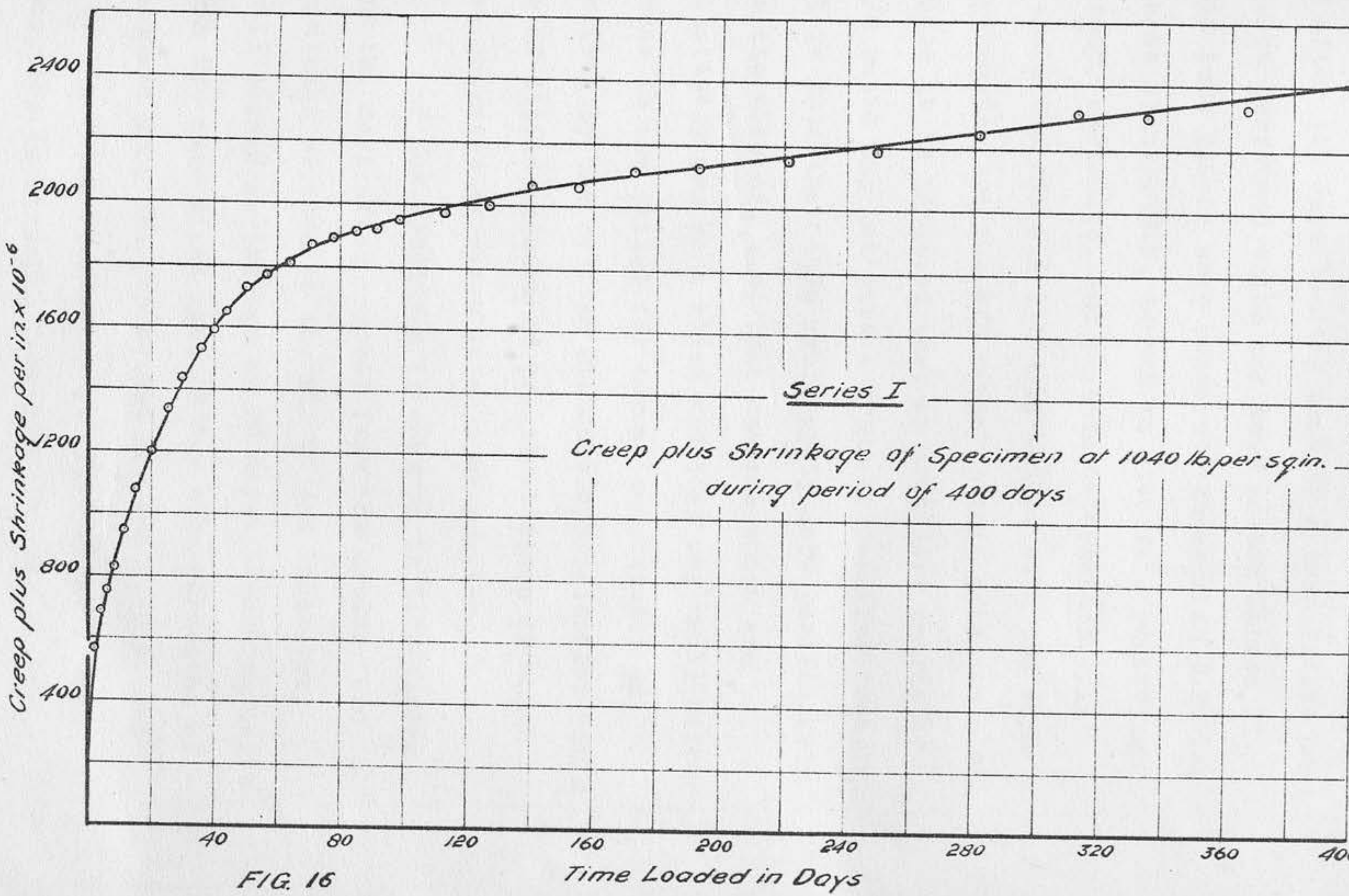
Portland-Blast Furnace Cement, 1:2:4 Mix (by weight) 0·55 Water/Cement Ratio tested in Air, Loaded at 28 Days.

Time	Effective Modulus $\times 10^6$ lb. per sq.in.	Modular Ratio
At Loading (Instantaneously	3·0	10
(Including Creep	2·12	14
After 7 Days	0·95	32
" 14 "	0·80	38
" 1 Month	0·65	46
" 2 Months	0·55	55
" 3 "	0·51	59
" 6 "	0·47	64
" 12 "	0·43	70

relative humidity during the period was 60 °/o, slightly lower than that in the test referred to, but it is improbable that this would account for the great difference in results.

That the large value of creep was due to the character of the cement at once suggested itself and further experiments were devised to confirm this. In view of the good proportionality of creep to stress shown in the first test it was decided that a single specimen would provide quantitative data for longer periods and would thus allow other tests to be carried out. The specimen at 1040 lb. per sq.in. therefore was maintained under load for 400 days and in Fig. 16 the total movement during this period is shown graphically. The almost linear creep-time relationship from 160 to 400 days is worthy of notice and would lead one to expect that equilibrium would only be attained after the elapse of a considerable time.

The longer test of this specimen enables a figure to be quoted for the unit stress creep at the age of one year, a quantity useful for comparison with the results of other investigations. After a period of 337 days i.e. one year from casting the total movement amounted to 2320×10^{-6} in. per in. which, after deduction of shrinkage gives a creep of $1{\cdot}99 \times 10^{-6}$ in. per in. per lb. per sq.in. This figure represents a movement slightly greater than that of a 1:3:6 N.P. cement concrete as found by


Series I
Creep plus Shrinkage of Specimen at 1040 lb. per sq. in. during period of 400 days
Creep plus Shrinkage per in. x 10^-6
2400
2000
1600
1200
800
400
40
80
120
160
200
240
280
320
360
400
Time Loaded in Days
FIG. 16

Glanville[2] and almost double that of a N.P. 1:2:4 concrete.

The removal of three specimens of Series I after 162 days loading enabled a comparative test to be performed which has been called Series II. Six test pieces were prepared, three with N.P. and three with P.-B.F. Cement to give data regarding creep, shrinkage, and elastic deformation. As before the mix was 1:2:4 by weight with the same aggregates but the W/C ratio in this case was 0.65. While it is admitted that an arbitrary choice of W/C ratio may not give a truly comparative test, it is felt that this would apply more particularly to the case of, say, neat cements which are relatively dry when mixed and that any small differences in the ratios for normal consistency as defined by the British Standard Specification would be of little account in practical mixes in which the quantity of water is so much greater.

The specimens were cured in damp canvas for two days after removal from the moulds, and were then exposed to the air of the laboratory until loaded at the age of 28 days. Crushing tests from the same mixes gave 28 day cube strengths of 2555 and 2905 lb. per sq.in. for the P.-B.F. and N.P. concretes respectively. Incidentally the reduction in strength from 3750 lb. per sq.in. (Series I 0.55 W/C Ratio) to 2555 lb. per sq.in. (Series II 0.65 W/C Ratio) gives a good example

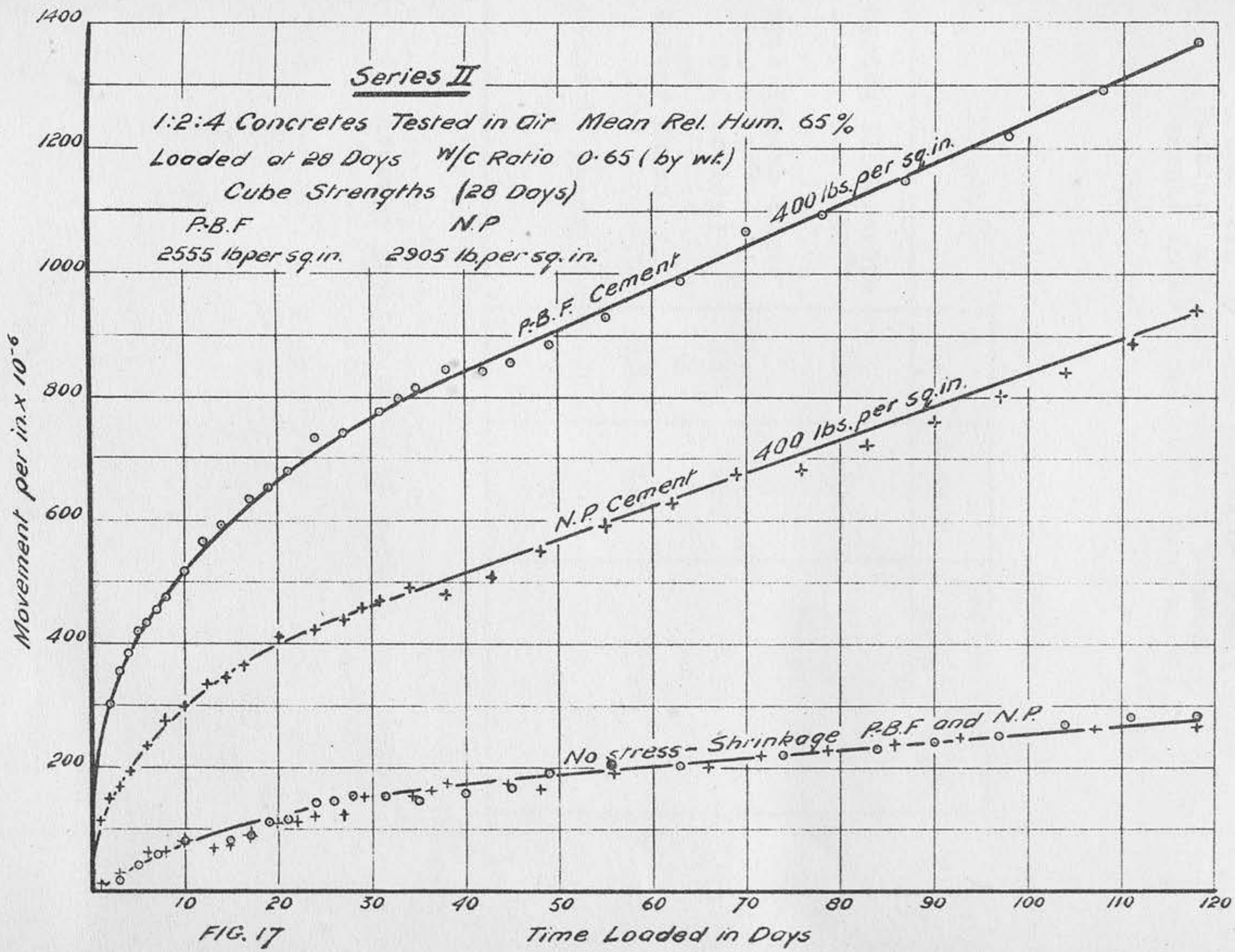

Series II
1:2:4 Concretes Tested in Air Mean Rel. Hum. 65%
Loaded at 28 Days W/C Ratio 0·65 (by wt.)
Cube Strengths (28 Days)
P.B.F N.P
2555 lb per sq.in. 2905 lb.per sq.in.
P.B.F. Cement
400 lbs. per sq.in.
N.P. Cement
400 lbs. per sq.in.
No stress - Shrinkage P.B.F and N.P.
Movement per in. x 10⁻⁶
1400
1200
1000
800
600
400
200
10
20
30
40
50
60
70
80
90
100
110
120
Time Loaded in Days

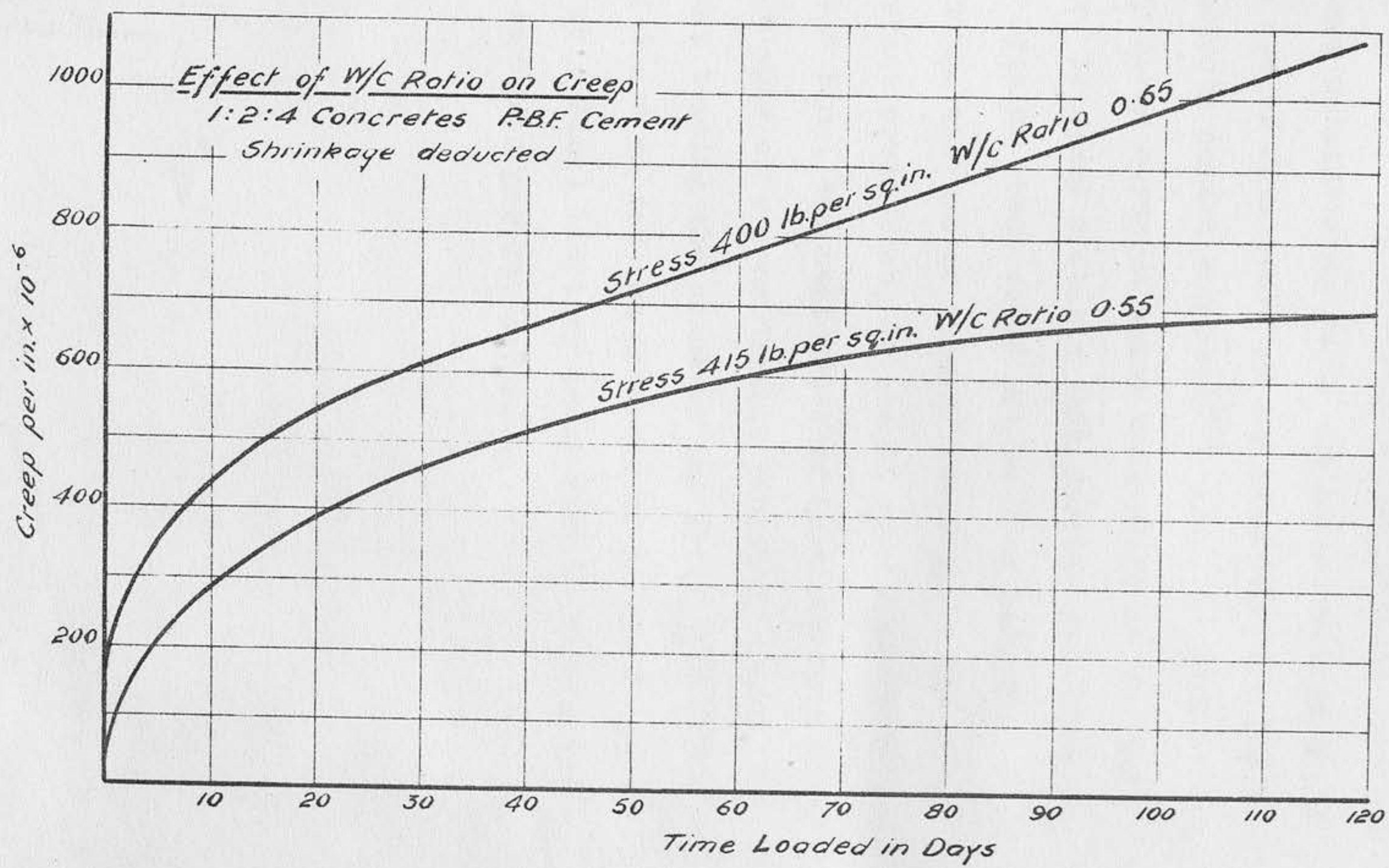
FIG. 17

TABLE 3

Comparison of Creep Values of Normal Portland and Portland-Blast Furnace Cements under Comparative Conditions.

	N.P.	P.-B.F.	Percentage Greater Creep with P.-B.F.
Series II Concrete in.per in. x 10^{-6}	660	1100	67
Neat Cement Beams (in. x 10^{-4} Deflection)	439	658	50
Mortar Beams (do)	160	315	97
Concrete Beams (do)	461	562	22

Average greater creep of P.-B.F. specimens 59 %.



Effect of W/c Ratio on Creep
1:2:4 Concretes P.B.F. Cement
Shrinkage deducted
Stress 400 lb. per sq. in. W/c Ratio 0·65
Stress 415 lb. per sq. in. W/c Ratio 0·55
Creep per in. × 10⁻⁶
1000
800
600
400
200
10
20
30
40
50
60
70
80
90
100
110
120
Time Loaded in Days

FIG. 18

above would apply to the concrete of Series I. However, although no definite forecast can be made, the Author feels, bearing in mind the different properties of the materials, that in this case the linear relationship would not be maintained up to the age of four years as has been reported for an Aluminous cement concrete.[10] Also, in view of the very large movement which took place in the 1st year, it seems probable that further reduction in the rate of creep could not be long delayed. Thus, although the use of Thomas' curve might not give an exact prediction, it should give some indication of the final eventual creep of this concrete after the elapse of a considerable period of time. For 28 day loading the value of the ratio $\frac{\text{Limiting Creep}}{\text{Creep in 1 Year}}$ is 1·3 and so for the concrete of Series I the probable final limiting creep will be $2 \cdot 59 \times 10^{-6}$ in. per inch per lb. per sq.in.

BEAM EXPERIMENTS.

A series of three tests was carried out with the beam testing device described. The beams were 25 in. long and one inch square in cross-section, and were cured in water after removal from the moulds until loaded at seven days old. Tests were carried out on neat cement, mortar, and concrete beams and in each case two beams were cast, one with Normal Portland Cement and the second with Portland-Blast Furnace Cement. The aggregates, mixes, W/C ratios, storage conditions and applied loads were identical and so the experiments were good comparative tests.

The following are the particulars of the beams tested.

Neat Cement:- W/C Ratio 0·24, 15·6 lb. applied at 7 days old and maintained for 41 days.

Mortar:- 1:3 (by weight) Leighton Buzzard Standard sand, W/C Ratio 0·32, 7·84 lb. applied at 7 days old and maintained for 71 days.

Concrete:- (In view of the necessarily small aggregate this might almost be considered a mortar.) 1:1:2 (by weight) river sand and crushed granite (up to 1/4 in.) W/C ratio 0·45, 7·84 lb. applied at 7 days old and maintained for 70 days.

In each case the initial deflection on the application of the load was observed and, since this required only a very short period of time, it is assumed that the results represent the instantaneous deflection without any deformation due to creep. Periodic readings were taken throughout the tests which were concluded by the observation of elastic recovery on the removal of the load and further observations of time recovery until equilibrium was established.

The results are shown graphically in Figs. 19, 20, and 21. In every case the large progressive increase in deflection with time is striking. This is particularly noticeable in the case of the concrete beams; with the Portland-Blast Furnace cement specimen the increase in deflection due to creep over a period of 70 days was $7\frac{1}{2}$ times the initial deflection on loading. The very large creep deformation in this test is explained by the high W/C ratio necessitated by the shape of the aggregate.

In view of the fact that creep is approximately proportional to stress and is the same for tension as for compression, it follows that at any point in the beam the total strain (elastic plus creep) is proportional to the stress, and thus deformation and deflection proceed proportionately. It is therefore possible to calculate the values of the "effective" modulus from the deflection data provided by these tests. It should be noted that

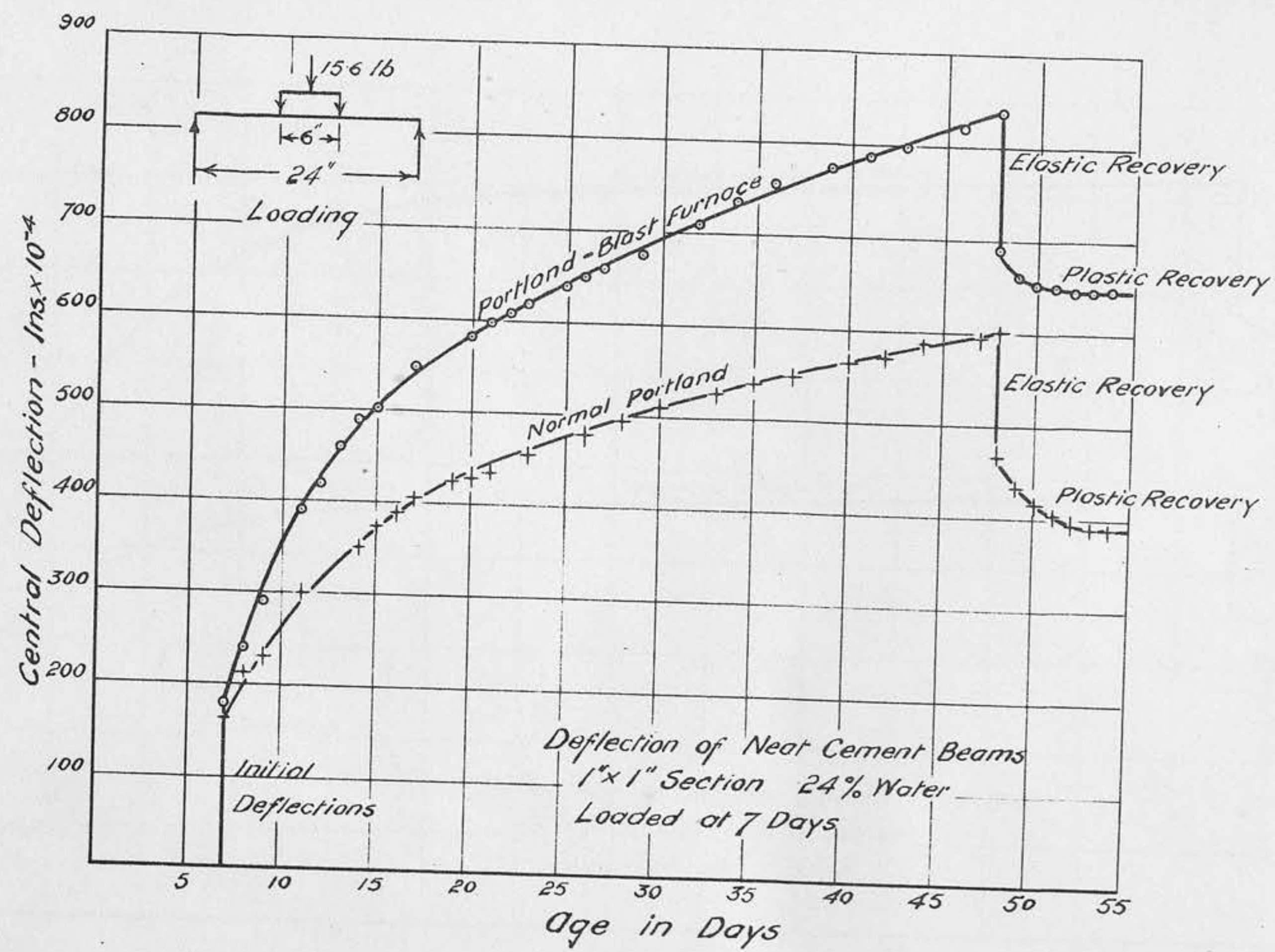
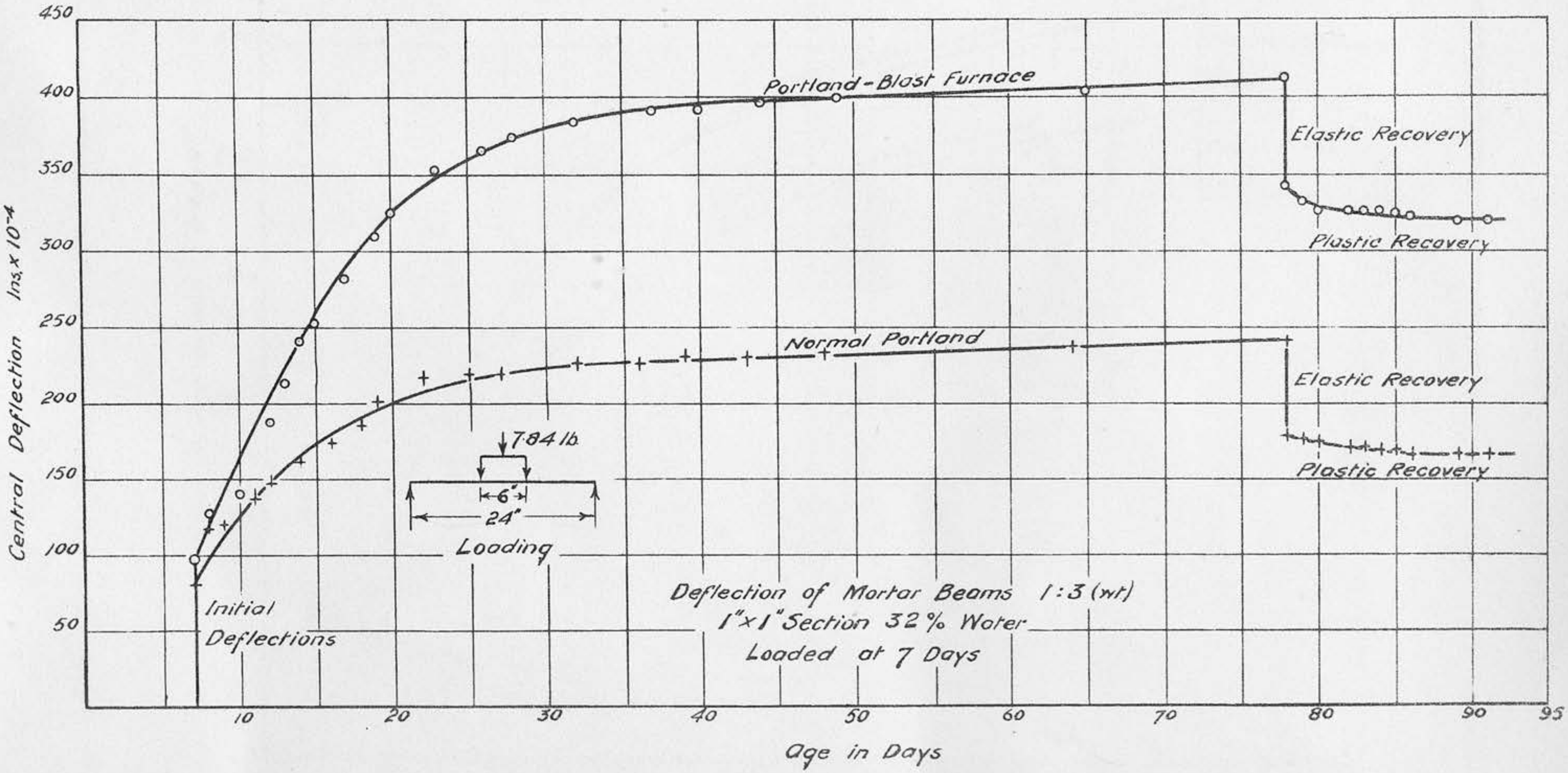
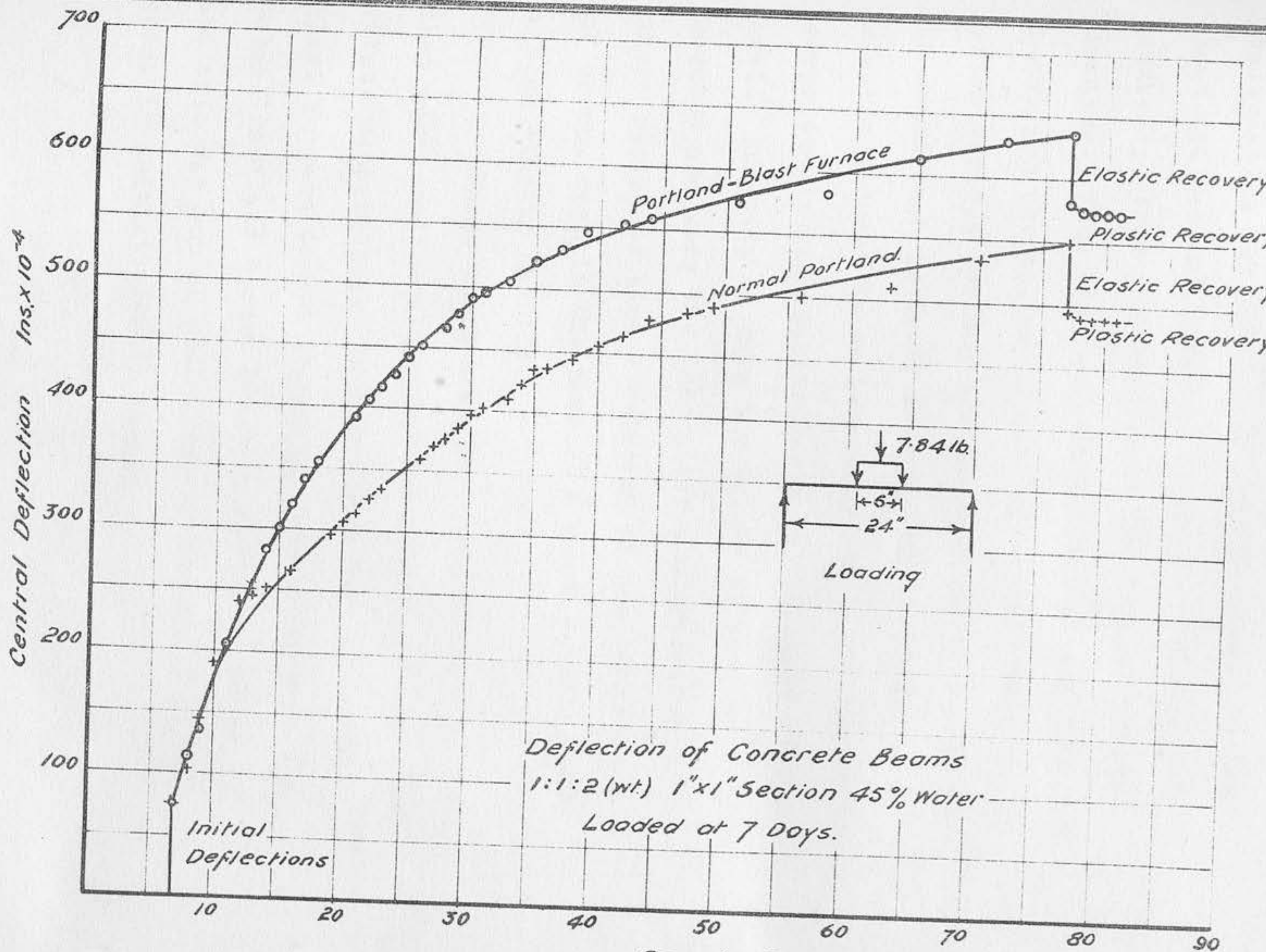

900
800
700
600
500
400
300
200
100
Central Deflection - Ins. x 10^{-4}
5
10
15
20
25
30
35
40
45
50
55
Age in Days
15.6 lb
6"
24"
Loading
Portland - Blast Furnace
Normal Portland
Elastic Recovery
Plastic Recovery
Elastic Recovery
Plastic Recovery
Initial
Deflections
Deflection of Neat Cement Beams
1" x 1" Section 24% Water
Loaded at 7 Days

FIG. 19



Central Deflection Ins. x 10-4
450
400
350
300
250
200
150
100
50
Portland - Blast Furnace
Elastic Recovery
Plastic Recovery
Normal Portland
Elastic Recovery
Plastic Recovery
7.84 lb
6"
24"
Loading
Initial
Deflections
Deflection of Mortar Beams 1:3 (wt)
1"x1" Section 32% Water
Loaded at 7 Days
10
20
30
40
50
60
70
80
90
95
Age in Days

FIG. 20



Portland-Blast Furnace
Normal Portland
Elastic Recovery
Plastic Recovery
Elastic Recovery
Plastic Recovery
7.84 lb.
5"
24"
Loading
Deflection of Concrete Beams
1:1:2 (wt.) 1"x1" Section 45% Water
Loaded at 7 Days.
Initial Deflections
Central Deflection Ins. x 10^-4
Age in Days
100
200
300
400
500
600
700
10
20
30
40
50
60
70
80
90

FIG. 21

the initial deflection is due solely to the applied load while during the time period the dead load is also operative. Due allowance however was made by calculating the initial deflection caused by dead load (it was not possible to measure this experimentally) from the results of the initial applied loading. Thus the total deflection at any time was known and the values of the "effective" modulus after various periods are shown in Table 4. It will be noted that in every case the value of the true elastic modulus is satisfactory and is in some cases surprisingly high particularly at the close of the test. In spite of this the values of the "effective" modulus are very low even after a comparatively short period of time - evidence of the large effects of creep, even in the case of a sound material with a high elastic modulus.

These tests were further justified in that they confirmed conclusively the results of the axial stress experiments. In every case the Portland-Blast Furnace cement showed greater creep than the Normal Portland cement under identical conditions, and this was out of proportion to the small differences in initial elastic strain.

TABLE 4.

BEAM TESTS.

Neat Cement.

Time	"Effective" Modulus x 10^6	
	Normal Portland	Portland-Blast Furnace
On Loading	3·02	2·74
After 1 Day	2·36	2·10
" 3 Days	1·90	1·49
" 7 "	1·47	1·06
" 14 "	1·19	0·88
" 28 "	0·97	0·71
" 41 "	0·87	0·63
* True Elastic Modulus after 41 days	3·68	3·33

Mortar.

On Loading	3·09	2·58
After 1 Day	2·60	2·12
" 3 Days	2·11	1·65
" 7 "	1·65	1·13
" 14 "	1·36	0·84
" 28 "	1·22	0·72
" 50 "	1·18	0·69
" 71 "	1·15	0·68
* True Elastic Modulus after 71 days	3·99	3·53

Concrete.

On Loading	3·3	3·3
After 1 Day	2·48	2·31
" 3 Days	1·47	1·43
" 7 "	1·10	0·99
" 14 "	0·89	0·72
" 28 "	0·66	0·54
" 50 "	0·56	0·48
" 70 "	0·52	0·447
* True Elastic Modulus after 70 days	4·58	4·42

* Computed from the elastic recovery on the removal of load.

TIME OR PLASTIC RECOVERY.

At the conclusion of each time experiment strain observations were taken for several days subsequent to the release of the load. In every case an "after effect" or time recovery was noted quite apart and distinct from the immediate elastic recovery, and results for the compression cylinders are shown graphically in Fig. 22. In the same way time recovery of deflection was observed for all the beam specimens and these results are recorded along with the creep curves in Figs. 19, 20, and 21.

In general shape the recovery and creep curves are very similar, but it will be noted that equilibrium is reached within about 10 days in all cases and that the magnitude of the total recovery is small compared with that of the creep or elastic deformation.

Since this phenomenon must affect to some extent the stress distribution in members subjected to live loads, the results were studied carefully with a view to discovering the factors which control the magnitude of the recovery. It seems probable that the period under load, the total creep, and possibly the elastic deformation may all affect the magnitude of the recovery but no relationship could be found for the results given. In Table 5 are shown the recoveries for the various specimens along with other relative data. It is noteworthy that the recovery in the case of the concrete beams - specimens of high W/C ratio and showing large creep - was

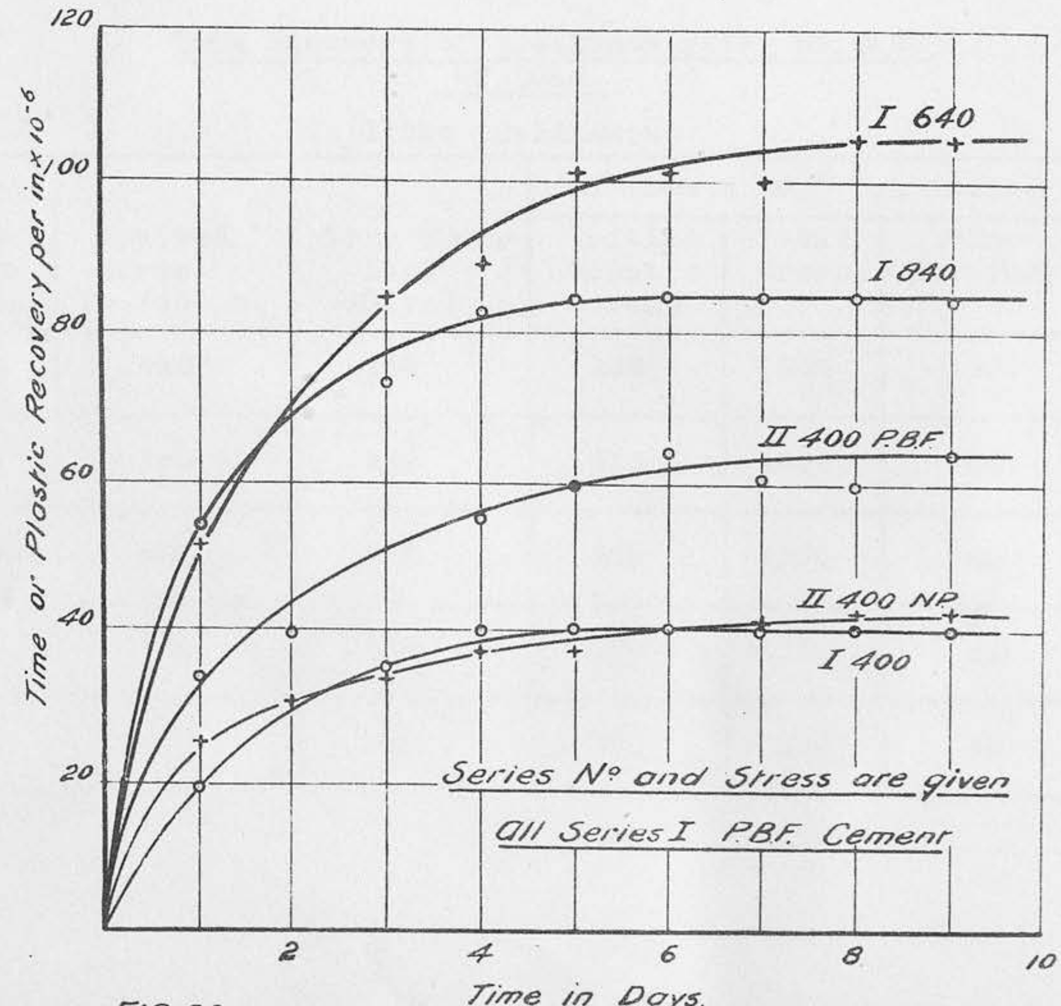

I 640
I 840
II 400 P.B.F.
II 400 N.P.
I 400
Series No and Stress are given
All Series I P.B.F. Cement
120
100
80
60
40
20
Time or Plastic Recovery per in. x 10^-6
2
4
6
8
10
Time in Days.

FIG. 22

TABLE 5.

Time Recovery of Specimens After Release of Load.

1:2:4 Cylinders.

Type of Cement	W/C Ratio	Applied Stress lb./sq.in.	Time Under Load (Days)	Inches x 10^{-6} per inch.			Time Recovery as a percentage of creep
				Initial Elastic Strain	Total Creep	Time Recovery	
P.-B.F.	0.55	415	162	138	710	40	5.6
P.-B.F.	0.55	640	162	213	1090	106	9.7
P.-B.F.	0.55	840	101	280	1300	85	6.5
P.-B.F.	0.65	400	118	225	1100	64	5.8
N.P.	0.65	400	118	155	660	44	6.7

TABLE 5 (Contd.)

24 in. Beams.

Beam	Type of Cement	Load (lb.)	Time Under Load (Days)	Inches x 10^{-4}			Time Recovery as a percentage of Creep
				Elastic Deflection	Creep Deflection	Time Recovery Deflection	
Neat Cement	N.P.	15·6	41	163	439	76	17·3
	P.-B.F.	15·6	41	180	658	44	6·7
Mortar	N.P.	7·84	71	80	160	12	7·5
	P.-B.F.	7·84	71	96	315	23	7·3
Con-crete	N.P.	7·84	70	75	461	7	1·5
	P.-B.F.	7·84	70	75	562	8	1·4

very small. Expressed as a percentage of the creep it will be noted that time recovery is a very variable quantity and it appears to be likely that a large number of tests under rigidly controlled conditions would be required before any forecast became possible. However, when it is borne in mind that the normal time of application of live load is short compared with the sustained periods in these tests and that in any case the recovery is such a small quantity, the whole question becomes one of academic interest rather than one of importance in practical design.

MODULUS OF ELASTICITY.

For the determination of the true elastic modulus an Olsen 10,000 lb. capacity lever testing machine was employed (Fig.23) which allowed the extensometer to be used in the conventional manner indicated diagrammatically in Fig. 9. In view of the greater space available 3/16 in. diameter rollers were employed with a vertical scale divided in 1/50 in. placed at a distance of 92·6 in. from the centre line of the specimen. A light theodolite provided a suitable optical instrument, and it was possible to read to half divisions on the scale which corresponded to $0{\cdot}5 \times 10^{-5}$ in. on a gauge length of 8 in. or $0{\cdot}6 \times 10^{-6}$ in. per inch.

With a view to obtaining a modulus as far as possible uninfluenced by creep, the dual precautions of a low applied stress and a treble application of the load were taken. The maximum stress employed was 302 lb. per sq.in. and the control specimen in Series I was tested on seven occasions throughout the duration of the experiment. Fig.24 shows typical stress-strain curves at 28 days and at 173 days after four intermediate tests.

In spite of the low stress, hysteresis loops were marked at first but were much reduced by repeated application of the load and increase in age. In the third loading of each test the ascending curve approximated to a straight line on the slope of which the modulus was computed. The later determinations of the modulus revealed very little

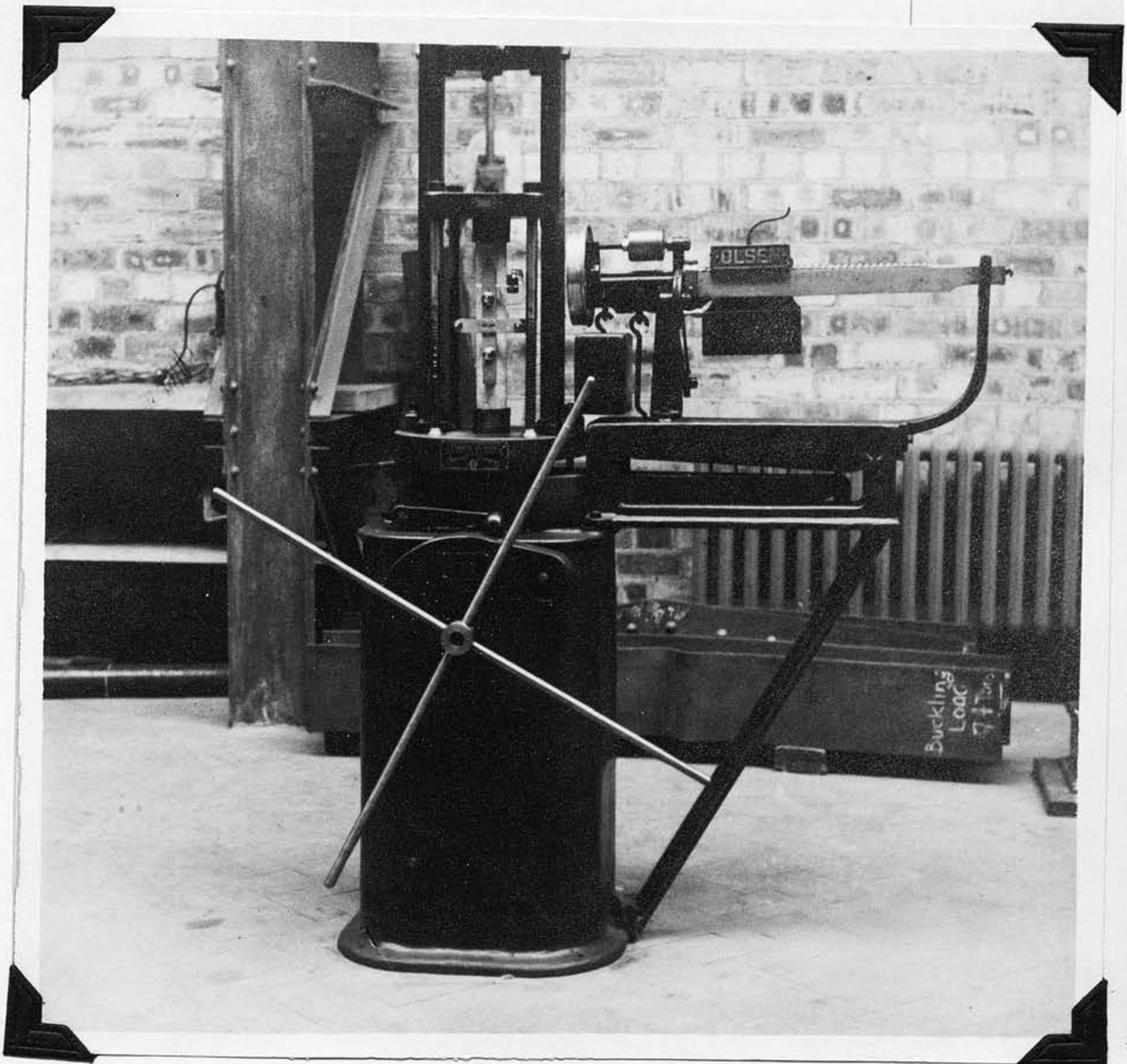
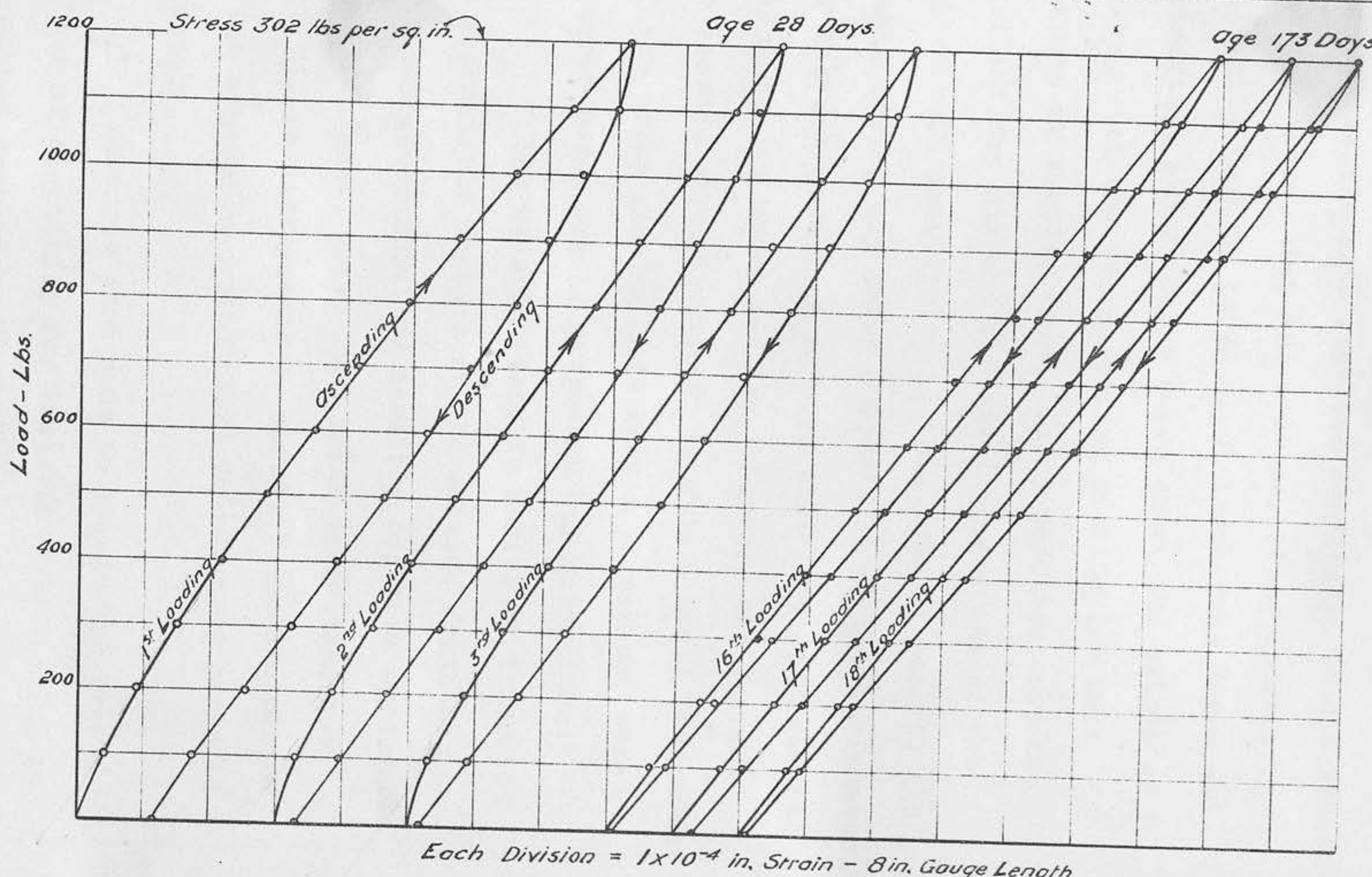


Fig. 23.

Testing Machine Employed for Elastic
Modulus Determinations.



ELASTIC TESTS OF CONCRETE.

FIG. 24

permanent set and gave an almost perfectly linear relationship, indicating in common with other experimental results[11] that concrete becomes more truly elastic with repeated loadings. It was noticeable however, that the modulus, after increasing slightly from 28 to 42 days, thereafter decreased steadily, and in Table 6 are shown the values of E along with the corresponding strains for unit stress at varying ages. It is possible that the initial dry mix and the small dimensions of the piece both contributed to this unusual result, the rapid drying out being prejudicial to the enrichment of the gel structure so essential to a high elastic modulus. As noted earlier, however, the changes in elastic strain amounted to only a small percentage of the creep and for this reason it was not considered necessary to introduce any correction.

The elastic tests in Series II indicated no appreciable changes in the modulus which for both concretes remained sensibly constant throughout the experiment, apart from small variations due probably to differences in moisture content caused by the atmospheric conditions immediately prior to loading.

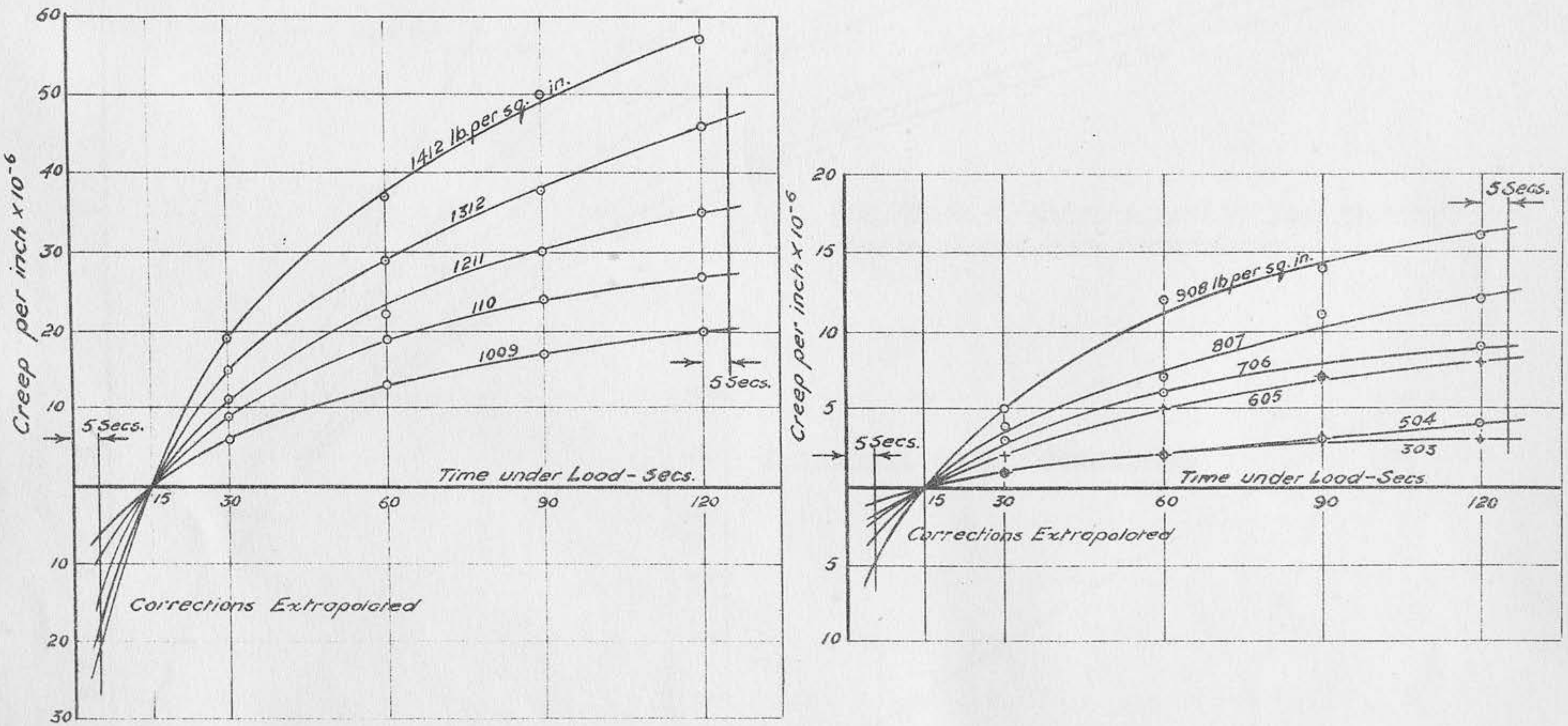
Attempts were made to obtain the true modulus of elasticity for instantaneous loading by the method devised by Glanville.[2] It was found impossible with the type of testing machine employed to apply the increment of load in the short period

TABLE 6.

Series I

Values of Elastic Modulus at Different Ages.

Age in Days.	Value of Modulus lb. per sq.in. x 10^6	Strain per lb. per sq. inch inches x 10^{-6}
28	3.0	0.333
35	3.04	0.33
42	3.04	0.33
56	2.95	0.34
97	2.81	0.356
173	2.70	0.37
360	2.53	0.395



Creep per inch x 10-6
1412 lb. per sq. in.
1312
1211
110
1009
5 Secs.
5 Secs.
Time under Load - Secs.
15
30
60
90
120
Corrections Extrapolated
Creep per inch x 10-6
908 lb per sq. in.
807
706
605
504
303
5 Secs.
5 Secs.
Time under Load-Secs
Corrections Extrapolated

FIG. 25

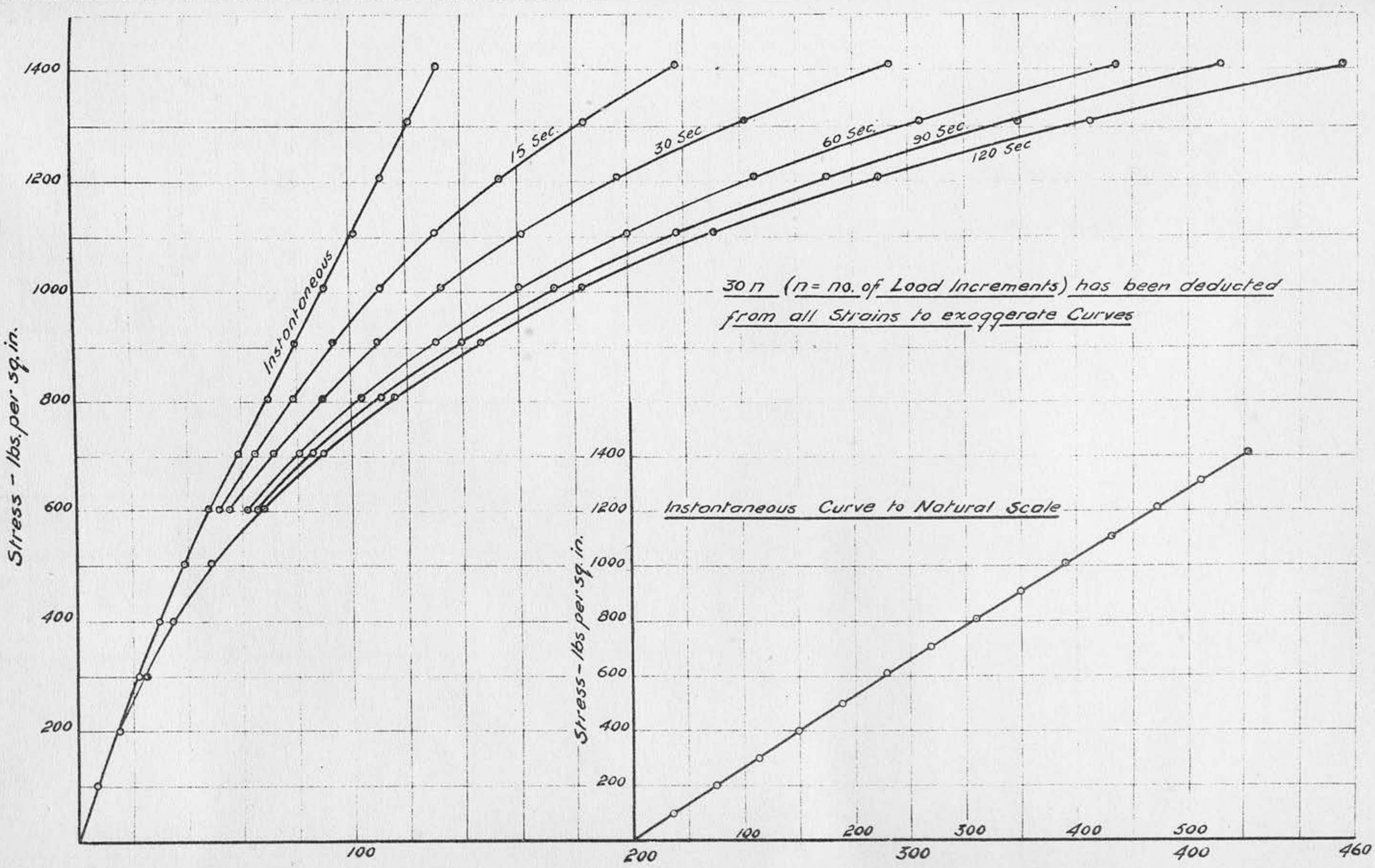

30 n (n = no. of Load Increments) has been deducted from all Strains to exaggerate Curves
Instantaneous
15 Sec.
30 Sec.
60 Sec.
90 Sec.
120 Sec.
Instantaneous Curve to Natural Scale
Stress - lbs. per sq. in.
Strain per in. x 10^-6

FIG. 26

TABLE 7.

1:2:4 Portland-Blast Furnace Cement Concrete, 0.55 W/C Ratio, Tested for the First Time at 316 Days.

Strain after periods of:- (With the exception of the scale readings all strains are x 10^{-6} in. per inch.)

Applied Load (lb.)	Stress (lb. per sq.in.)	15 Secs. Scale Reading	15 Secs. Total Strain	15 Secs. Increase during 15 secs.	15 Secs. Total during 15 secs. only	30 Secs. Scale Reading	30 Secs. Total Strain	30 Secs. Increase during 30 secs.	30 Secs. Total during 30 secs. only	30 Secs. Creep 15-30 secs.	60 Secs. Scale Reading	60 Secs. Total Strain	60 Secs. Increase during 60 secs.	60 Secs. Total during 60 secs. only	60 Secs. Creep 15-60 secs.	90 Secs. Scale Reading	90 Secs. Total Strain	90 Secs. Increase during 90 secs.	90 Secs. Total during 90 secs. only	90 Secs. Creep 15-90 secs.	120 Secs. Scale Reading	120 Secs. Total Strain	120 Secs. Increase during 120 secs.	120 Secs. Total during 120 secs. only	120 Secs. Creep 15-120 secs.	Creep on Lower Stress 120-125 secs. (By Extra-polation)	Creep 5-15 secs. (By Extra-polation)	Total creep correction	Increment for Instantaneous Loading.	Total Strain for Instantaneous Loading.
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
400	101	2·95	37	37	37	2·95	37	37	37	0	2·95	37	37	37	0	2·95	37	37	37	0	2·95	37	37	37	0	0	0	0	37	37
800	202	6·0	75	38	75	6·0	75	38	75	0	6·0	75	38	75	0	6·0	75	38	75	0	6·0	75	38	75	0	0	0	0	38	75
1200	303	9·02	113	38	113	9·04	113	38	113	0	9·08	114	39	114	1	9·11	114	39	114	1	9·12	114	39	114	1	0	0	0	38	113
1600	403	12·18	152	38	151	12·23	153	39	152	1	12·3	154	40	154	2	12·36	155	41	155	3	12·39	155	41	155	3	0	1	1	37	150
2000	504	15·6	195	40	191	15·68	196	41	193	1	15·78	197	42	196	2	15·84	198	43	198	3	15·92	199	44	199	4	0	1	1	39	189
2400	605	19·23	240	41	232	19·33	242	43	236	2	19·61	245	46	242	5	19·75	247	48	246	7	19·8	248	49	248	8	0	2	2	39	228
2800	706	23·3	291	43	275	23·5	294	46	282	3	23·75	297	49	291	6	23·85	298	50	296	7	23·98	300	52	300	9	0	2	2	41	269
3200	807	27·55	344	44	319	27·85	348	48	330	4	28·2	353	53	344	7	28·38	355	55	351	11	28·5	356	56	356	12	0	3	3	41	310
3600	908	32·1	401	45	364	32·5	406	50	380	5	33·0	413	57	401	12	33·2	415	59	410	14	33·35	417	61	417	16	0	5	5	40	350
4000	1009	37·15	464	47	411	37·6	470	53	433	6	38.15	477	60	461	13	38·5	481	64	474	17	38·75	484	67	484	20	0	6	6	41	391
4400	1110	42·75	534	50	461	43·45	543	59	492	9	44·24	553	69	530	19	44·65	558	74	548	24	44·85	561	77	561	27	0	9	9	41	432
4800	1211	49·2	615	54	515	50·08	626	65	557	11	50·96	637	76	606	22	51·6	645	84	632	30	52·0	650	89	650	35	0	14	14	40	472
5200	1312	56·8	710	60	575	58.0	725	75	632	15	59.1	739	89	695	28	59·8	748	98	730	38	60.5	756	106	756	46	1	19	20	40	512
5600	1412	65·5	819	63	638	67·0	838	82	714	19	68·5	856	100	795	37	69·5	869	113	843	50	70·1	876	120	876	57	2	21	23	40	552

so. This diagram also shows the instantaneous strains plotted to a natural scale. The figures for this test are given in Table 7 and in the penultimate column it will be noted that the strain increments for instantaneous loading are reasonably constant varying from 37 to 41 x 10^{-6} in. per inch with a mean of 39.4 x 10^{-6}.

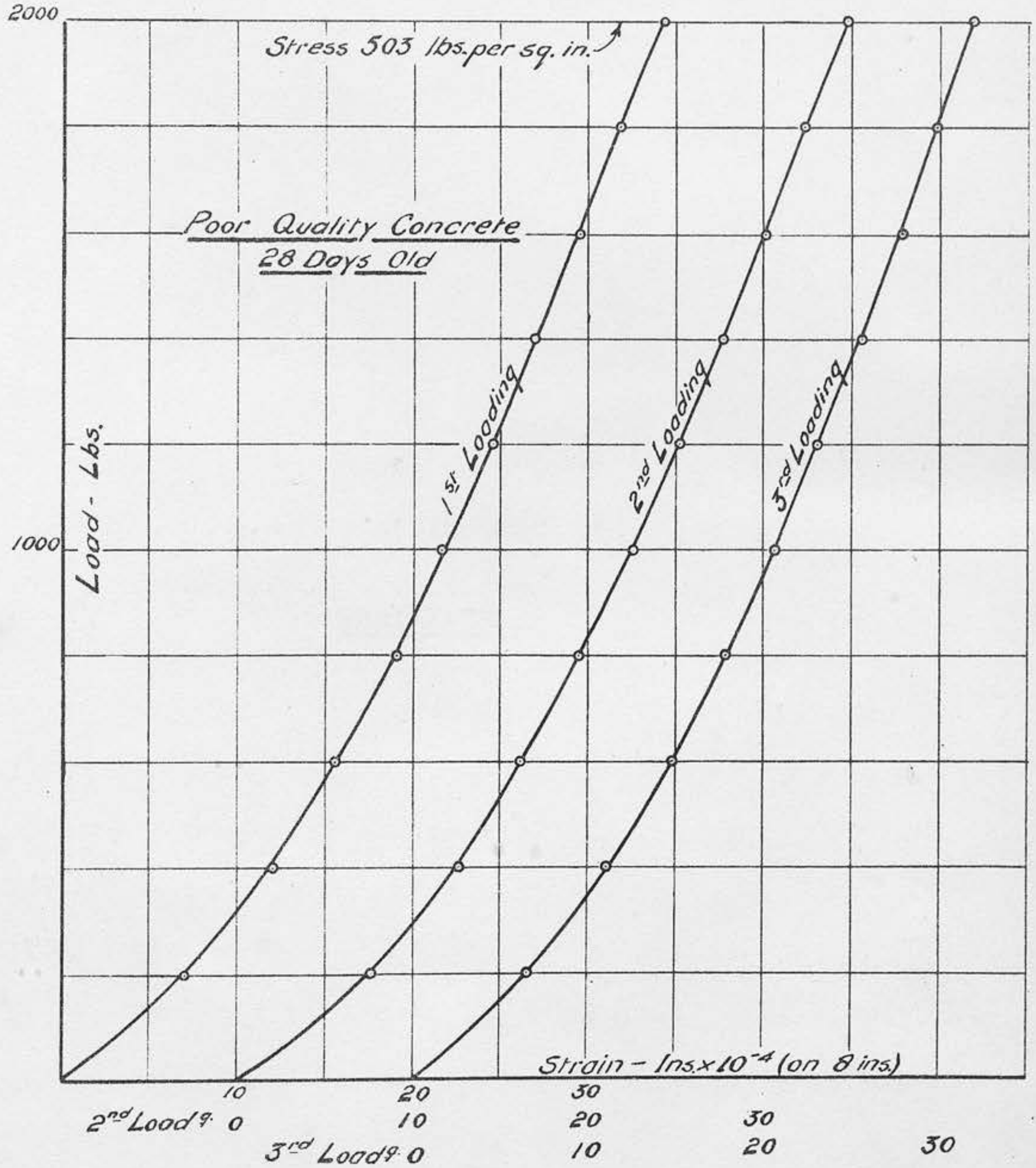
It is admitted that with the comparatively long time periods used in this test the corrections by extrapolation are somewhat problematical in that the extension to the curve is not very clearly defined, but in the example given it is improbable that the maximum error in the creep corrections exceeds 2 x 10^{-6} in. per inch.

At stresses higher than 1412 lb. per sq.in. the creep became so rapid that accurate Strain measurements were impossible. With a material showing less creep, e.g. Aluminous cement concrete, it is probable that the linear relationship would be more clearly defined and would be maintained up to higher stresses. However, the almost perfectly straight line in Fig. 26 justifies the conclusion that a direct proportionality exists between stress and strain for instantaneous loading and that the curvature in normal test results is caused by creep occurring during the comparatively short period of the experiment.

A study of the figures and curves given indicates that an elastic test up to about 500 lb.

per sq.in. will yield for this concrete, a true value of the modulus, provided that the time period for load increments does not exceed 120 sec. Normally the time would be much less than this and it is probable that the above statement would hold for all but the poorest of concretes.

Whilst acquiring a technique in the use of the optical extensometers, elastic tests on various concretes were performed. Some of the specimens were poor in quality and insufficiently consolidated and gave correspondingly low values for the elastic modulus. Several tests yielded curious curves of which those in Fig. 27 are typical. It will be seen that, contrary to the usual form, the strains for equal load increments are greater at lower stresses than at higher, being approximately constant at 500 lb. per sq.in. The Author feels that, in view of the poor consolidation, the movement at low stresses is largely due to the strain in the cementitious portion, and only as the stress rises does the aggregate receive a larger share of the load and so reduce the movement in view of its higher modulus of elasticity. On this assumption the initial slope of the curve would approximate to the modulus for the poor and rather porous cementitious material, while that at higher stresses would represent the modulus for the two portions resisting the load conjointly.



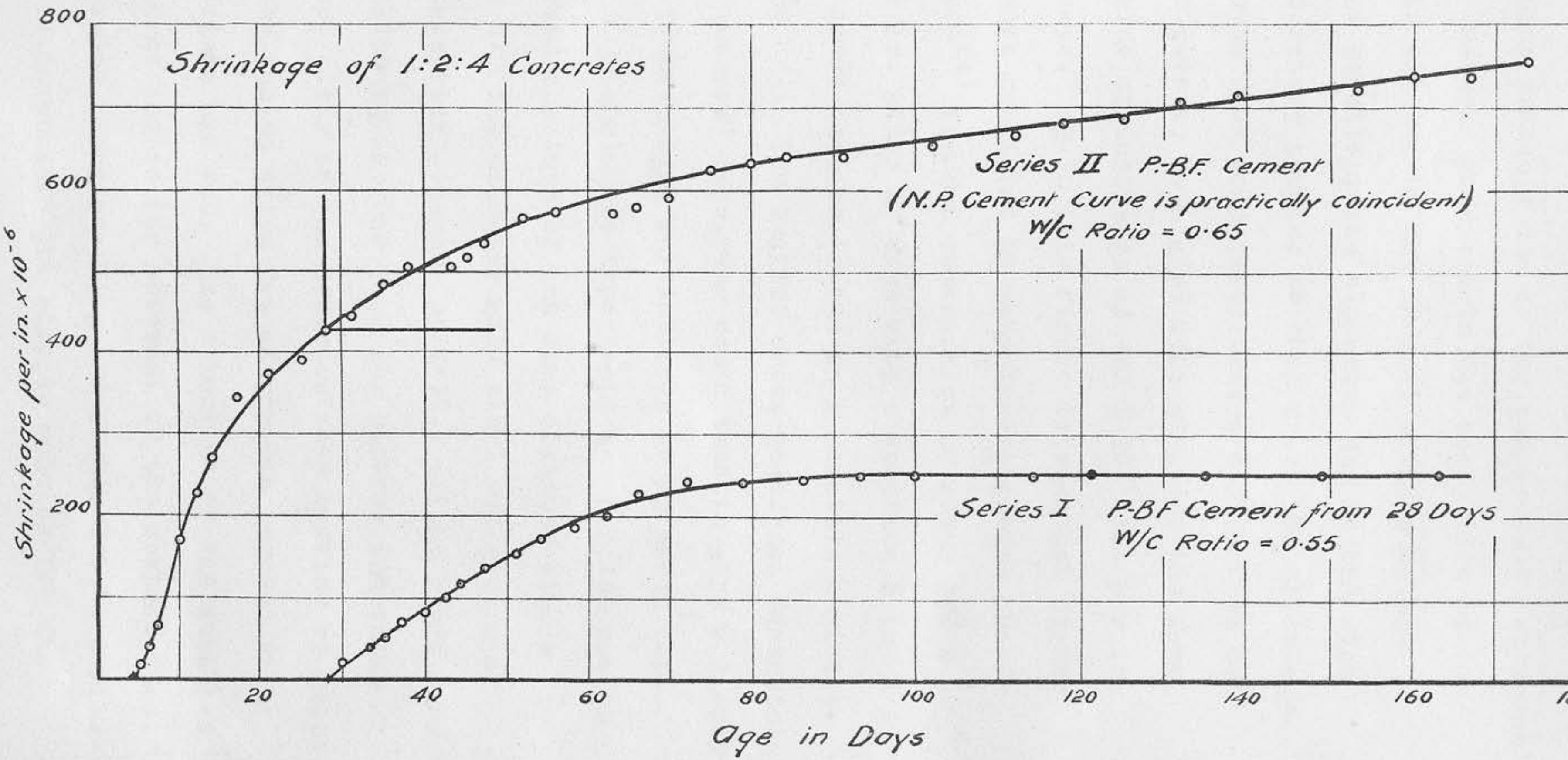
2000
1000
Stress 503 lbs. per sq. in.
Poor Quality Concrete
28 Days Old
Load - Lbs.
1st Loading
2nd Loading
3rd Loading
Strain - Ins. x 10^-4 (on 8 ins.)
10
20
30
2nd Load'g 0
10
20
30
3rd Load'g 0
10
20
30

FIG. 27

SHRINKAGE.

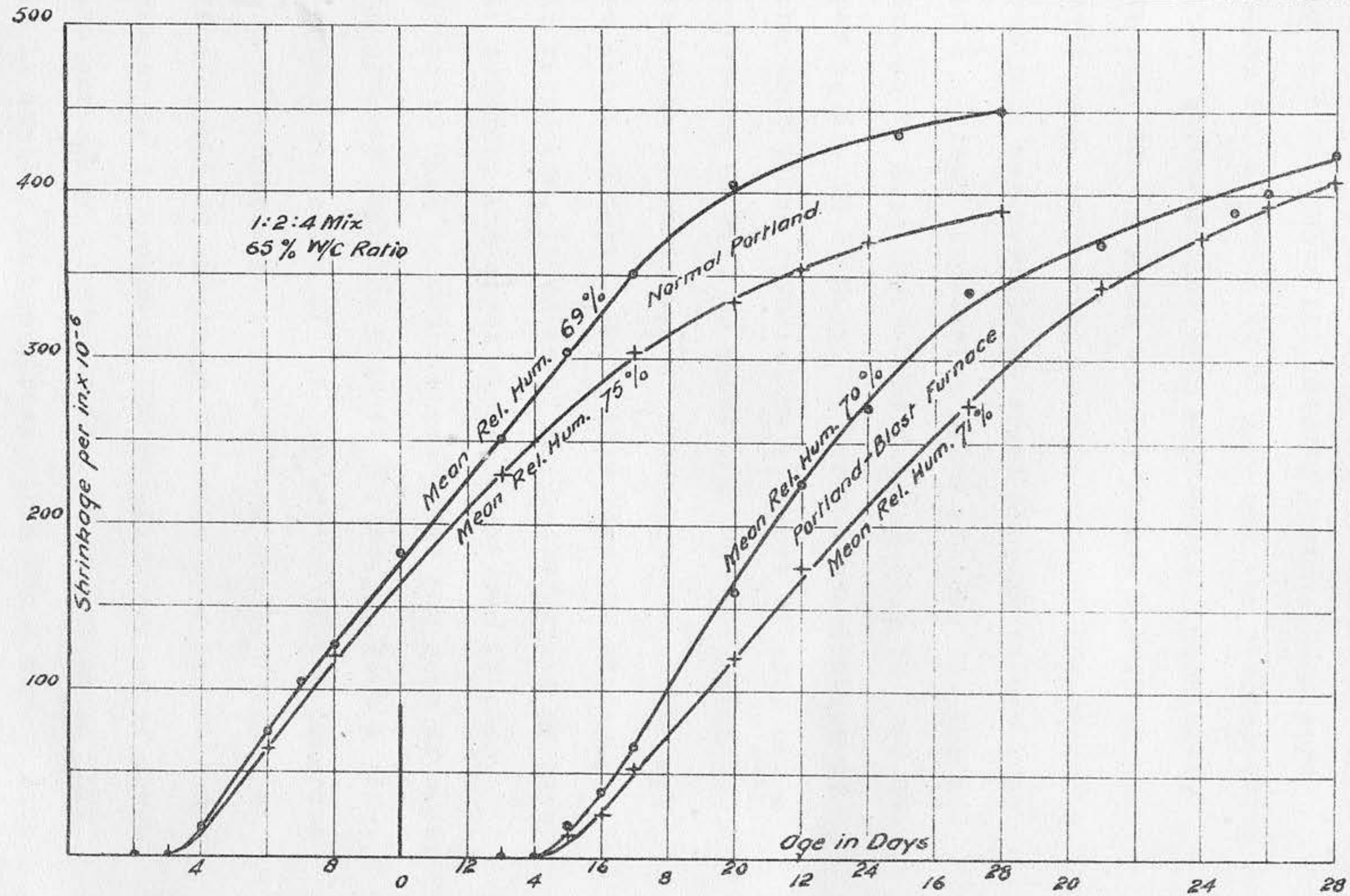
The investigation of shrinkage was not one of the objectives of this research but formed a necessary part of the work in order to arrive at creep by deduction. In Fig. 28 are shown the shrinkage curves for Series I and II and it is surprising to note that in the former case movement appears to have ceased at the age of about 100 days. It is felt that this cessation of shrinkage is probably due to the dryness of the mix and to the small dimensions of the specimen which would allow drying out to take place more rapidly. Faber[1] has given a curve drawn from a variety of sources representing the average shrinkage of concrete over a year and equilibrium is shown to be attained only after the elapse of this period. The value of 250×10^{-6} in. per in. from 4 weeks, however, compares favourably with the figure of 220×10^{-6} from Faber's curve. Series II curve is typical in shape but represents a shrinkage which is greater than the average, a result which the Author feels is caused by the large $\frac{\text{Surface}}{\text{Volume}}$ ratio. The approximate coincidence of the two Series II curves forms in itself some check on the experimental methods because large differences in the shrinkage movements of concretes made from N.P. and P.-B.F. cements would not be expected.

Fig. 29 shows shrinkage curves for four concrete specimens up to an age of 28 days. It is noteworthy that no movement was recorded until after


Shrinkage of 1:2:4 Concretes
Series II P-B.F. Cement
(N.P. Cement Curve is practically coincident)
W/C Ratio = 0·65
Series I P-B.F Cement from 28 Days
W/C Ratio = 0·55
Shrinkage per in. x 10⁻⁶
800
600
400
200
20
40
60
80
100
120
140
160
180
Age in Days
FIG. 28

the 3rd day with the N.P. and until after the 4th day in the case of the P.-B.F. specimen, this difference being probably due to the comparative "slowness" of the latter. Movement in the early life of concrete is cloaked by the thermal effects of the chemical reaction, the expansion due to the rise in temperature tending to counteract the shrinkage. The curves show some quantitative differences but the two cements both give approximately the same mean value of shrinkage of 420×10^{-6} in. per in. at 28 days. Again this figure is somewhat higher than those published by many investigators but Matsumoto[24] found a contraction of about 380×10^{-6} in. per in. after 28 days with a specimen 6 in. x 6 in. in cross section tested in a relative humidity of 40-80 $^{o}/_{o}$. The smaller cross-sectional dimensions of the Authors' specimens would tend to give a larger value of shrinkage, particularly at an early date.

A shrinkage experiment was carried out on a reinforced specimen of the same diameter with a central 3/8 in. diameter mild steel rod giving a percentage reinforcement of 2·78. Light rigid arms were welded to the bar at 8 in. centres and these were kept clear of the concrete when casting by means of rubber tubing which was afterwards removed when the specimen had set. The extensometer was attached to the arms and so the movement of the central steel was recorded, thus enabling the stress due to shrinkage to be computed. The elastic modulus for the



Shrinkage of 4 Concrete Specimens up to 28 Days

FIG. 29

steel was carefully determined by a test on a piece cut from the same bar. Portland-Blast Furnace Cement was employed and the concrete was identical to that of Series II. Strain observations were taken for a period of 58 days and the corresponding steel stresses have been represented graphically in Fig. 30. A small tensile stress was first evidenced, indicating an expansion of the concrete, but this was rapidly reversed and the stress rose to a maximum compressive value of 1800 lb. per sq. in. at 19 days thereafter falling away to 1200 lb. per sq.in. at 58 days. A careful examination of the test piece revealed no evidence of slip but a reference to Fig. 29 shows a marked reduction in the rate of shrinkage of this concrete at the age of about 19 days. Up to this time the shrinkage is rapid and, in spite of the relief given by creep in tension, the steel stress continues to rise. After 19 days it would appear that the rate of creep is greater than the rate of shrinkage, with a consequent reduction of tensile stress in the concrete and of compressive stress in the steel. Thus the development of shrinkage stresses in a reinforced member is dependant on the relative rates of creep and shrinkage. Probably Fig. 30 is unique in that it is similar to the stress curve for a member with a very high steel percentage or to the concrete stress curve in a completely restrained plain member. It should be borne in mind however, that

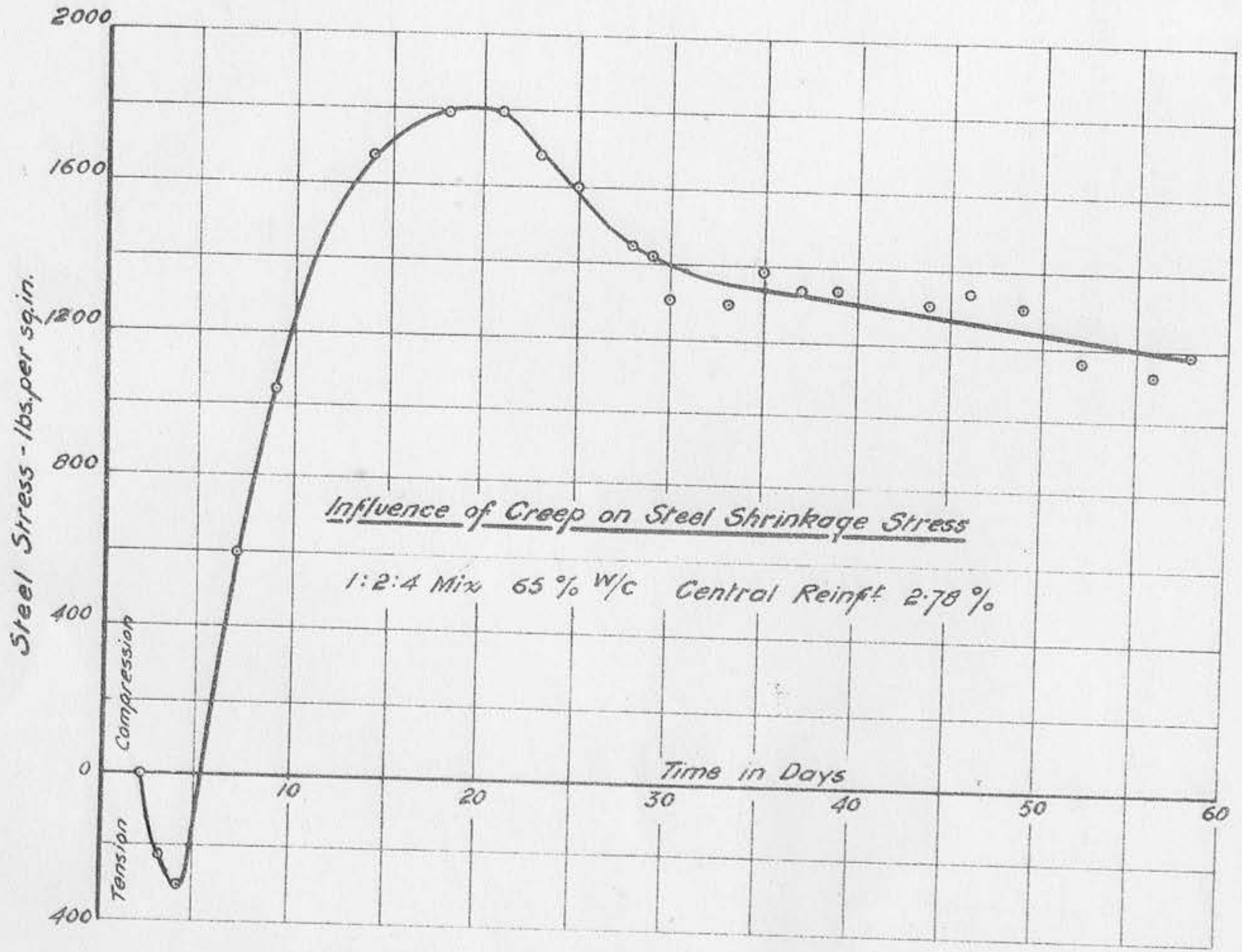

Influence of Creep on Steel Shrinkage Stress
1:2:4 Mix 65 % W/c Central Reinft 2·78 %
Steel Stress - lbs. per sq. in.
Time in Days
Compression
Tension
2000
1600
1200
800
400
0
400
10
20
30
40
50
60

FIG. 50

in the present case very large values of creep were recorded for this particular concrete (Series II. P.-B.F. cement) while the shrinkage of an unreinforced test piece was the same as that for a N.P. specimen. In other words the condition represented in Fig. 30 is more likely to occur with P.-B.F. than with N.P. cement concrete. This gives a good example of the manner in which creep operates to relieve stress.

CONCLUSIONS.

From the experiments described the Author concludes that :-

(1) Cements, mortars, and concretes yield with time under load.

(2) The rate of creep, greatest in the first two or three months, decreases with time and a final asymptotic or strain equilibrium state is indicated.

(3) Creep is approximately proportional to the applied stress.

(4) Creep increases with the quantity of the mixing water used.

(5) A time or plastic recovery follows the immediate elastic recovery on the release of load the rate of which decreases with time. Plastic recovery is small compared with the creep and equilibrium is reached within the comparatively short period of about 10 days.

(6) Under comparative conditions the creep of Portland-Blast Furnace cement concrete is markedly greater than that of Normal Portland cement concrete.

(7) The creep of Portland-Blast Furnace cement concrete, 1:2:4 mix, 0·55 W/C ratio, tested in air is of the order $1{\cdot}99 \times 10^{-6}$ in. per in. per lb. per sq.in. at the age of one year.

(8) In view of the large creep of Portland-Blast Furnace cement concrete, considerable relief is offered to the steel stress induced by shrinkage.

CRITICAL SURVEY OF CREEP.

The creep of concrete throws much light on the physical structure of the material, and in addition lends strong support to the "gel" theory as opposed to the "crystal" theory. Presuming that concrete consists of aggregate, cement gel, and water, creep is well explained by the loss of colloidal water. The loss of free water does not affect the volume of concrete but when colloidal water passes into the free state a reduction in dimensions is noted. Shrinkage is the result of this transformation under atmospheric conditions, and creep is really a shrinkage accelerated by stress i.e. the colloidal water is forced under pressure into the capillary channels as free water, finds its way to the surface and there evaporates. It should be noted that the shrinkage-time and creep-time curves are similar in form.

The elastic movement of concrete is, of course, made up of the deformations of both aggregate and gel. Davis,(3) however, has shown that longitudinal creep is not accompanied by appreciable permanent lateral deformation i.e. there is no plastic crystal flow and the creep occurs only in the gel. The conception of a gel structure is in agreement with the experimental data regarding the reduction of creep when concrete is maintained wet. Under these conditions the moisture gradient in the capillary channels is flat and the rate of seepage consequently slower. Also, concrete under stress at high temperature shows large permanent set, the loss of water being greatly facilitated by the gaseous state. It is probable, under constant humidity, that higher temperatures even within the ordinary atmospheric range will cause greater creep because of the freer movement of the water resulting from the reduced viscosity and the greater evaporation at the surface. Concrete mixes with high W/C ratios evidence large creep and this again satisfies the theory of a colloidal cementitious structure. The loss of water in a wet mix would perforce be greater than in the case of a dry mix with a corresponding increase in the magnitude of the creep. All experimental creep-time curves tend to a limiting asymptotic value after a considerable period of time; this would indicate that all the more loosely held water had been expelled by the applied stress.

With this theory in view it seems improbable that the behaviour of large masses of concrete can be predicted with certainty from experiments on comparatively small specimens. While tests[8] have shown that between diameters of 3 ins. and 6 ins. creep is independant of the size of the piece, it appears to be unlikely that the same deformation per unit stress will be shown by a member such as a large arch rib in which the loss of water must necessarily be retarded by the dimensions.

Staub[12] proposed an exponential equation to represent the time deformation of concrete based on the results of experiments by McMillan[13], Fuller and More[14], Hollister[15], Smith[16] and others. The equation was of the form

$$\rho = K \mu^p t^q$$

in which ρ is the creep, a function of the stress μ and the time t. K is a parameter depending on the properties of the material and the exponents p and q are constants determined by the properties of the material at the beginning of the stressed condition. The equation was applied to an elaborate arch theory and raised much criticism on the grounds that it represented that deformation must increase progressively for all time under stress. While the equation might represent creep for a limited time it is clear that the parameters would require periodic revision as the age increased.

Thomas[4] has evolved a mathematical expression for creep based on the assumption that concrete is composed of a cementitious material which behaves in a viscous manner when loaded, and an inert aggregate which does not creep. The rate of creep is reduced, firstly by the progressive transference of load to the inert aggregate, and secondly by the hardening of the cementitious material with age. With these factors as a basis for analysis the creep of concrete is expressed in the equation

$$c = Cf \left[1 - e^{-A\left\{ (t + a)^z - a^z \right\}} \right]$$

Where c is the creep, C, A, a, and z are constants for a particular concrete, e is the Napierian base, f is the applied stress, and t the time from loading. With this form of equation creep-time curves have been drawn which agree well with experimental results for Normal and Rapid Hardening Portland Cement Concretes. The value of this analysis is realized when it is seen that at infinite time the creep reaches a limiting calue of Cf, and thus the constant C represents the limiting value of creep under unit stress.

It is interesting with this theory in mind to study the approximate shape of the Authors' beam deflection curves for neat cement as compared with mortar. The rapid reduction in the rate of creep noticeable in the case of mortar at about 15 days under load, is absent in the curves for neat cements for which, after 40 days loading, the rate of creep is still considerable. This is probably due to the fact that in the case of neat cement the progressive reduction in the rate of creep is dependant solely on the increased viscosity of the material, (ignoring the small quantities of inert matter present) and the introduction of an inert aggregate tends to decelerate the rate.

It has been reported[8][10] that Aluminous Cement concrete yields almost a linear creep-time relationship from 6 months to 4 years, and the Author feels that an explanation might lie in the theory outlined above. An aluminous cement will attain a high resistance to creep at a comparatively early date, and in view of the fact that the overall creep is small, it is possible that the transference of stress to the aggregate is long delayed. Not only will the cementitious material rapidly attain a high resistance, but a resistance near to its maximum, and this fact would explain the almost linear relationship. It is easy to picture an Aluminous cement concrete under load for which the reduction in the rate of creep in its early life is due only to the increase in the viscosity of the cementitious portion, which, after about six months attains nearly to its maximum resistance, but still being not wholly elastic continues to yield steadily and proportionately with time. After an extended period, the aggregate may be called upon to carry a higher load with consequent reductions in the stress in the cementitious material, and in the rate of creep, thus finally leading to the asymptotic condition which would appear to be beyond doubt.

When a compressive load is removed from a concrete specimen, a small time recovery follows the instantaneous or elastic recovery. It is probable that after the release of the load the composite material is in a state of internal stress; the cementitious portion has deformed, but, owing to the considerable increase in viscosity it tends to resist complete recovery and thus induces stress in the aggregate. The strain in the aggregate due to this stress is gradually recovered by the yield of the cementitious material in the reverse direction to that which originally took place.

EFFECTS IN REINFORCED CONCRETE.

It is unnecessary to emphasise that the major significance of creep is found in its effects in reinforced concrete. While structures of this material have withstood the acid test of long service under arduous conditions, a better knowledge of the internal stress distribution is imperative before any real advance in design can be made. It has been reiterated that the redistribution of stress resulting from creep does not make reinforced concrete unsafe, and this is borne out by the rarity of failures in service structures. The desirability, however, of a more exact understanding of the actual stresses occurring cannot be denied.

Throughout the whole of the literature on concrete, references to the increasing deformation of the material with time under load are to be found. Test loads on concrete structures have yielded interesting results, considerable increase in deflection being noted even after a period of hours. In the discussion of Faber's (1) paper, Jackson cited practical cases of reinforced concrete structures where creep was evidenced by increase in deformation with time. The Author has in mind also, a large span reinforced concrete floor of recent date. The floor system shows no obvious cracks or signs of failure, but a light partition wall constructed longitudinally over a beam, has developed a large vertical crack near the floor level. It is felt that this is visible evidence of the results of creep and, of course, also of shrinkage but this latter probably to a lesser extent.

As a result of his experiments, Faber proposed the use of a provisional "Factor of Plasticity", or the ratio of eventual to original strain. If e is the elastic strain and c is the creep then the plasticity factor k is $\frac{e + c}{e}$, a constant independant of the stress and increasing with time. Experiments have revealed no definite relationship between elastic strain and creep, and in Table 8 are shown the results of the investigations described here from this point of view. It will be seen that the values of k vary considerably with the different specimens.

Glanville[2] has evolved a theory to evaluate the change in the stress distribution in a longitudinally reinforced concrete column. Assuming that the steel does not creep and that there is no plastic slip he finds

$$f_c = \frac{f_o}{e^{c/b}}$$

$$\text{and} \quad f_s^1 = f_a \cdot \frac{a_c}{a_s} \cdot \frac{1}{m} \cdot \left(1 - \frac{1}{e^{c/b}}\right)$$

where f_c is the concrete stress, f_o is the concrete stress on loading, e is the Napierian base, c is the creep of plain concrete under unit stress from the time of loading to the time under consideration, and b is a constant equal to $\frac{a_c}{a_s} \frac{1}{E_s} + \frac{1}{E_c}$. f_s^1 is the increase in steel stress due to

TABLE 8.

Factor of Plasticity (Faber).

Specimens		Value of k after periods of (days)					
		7	14	28	50	70	120
1:2:4 Concrete Loaded at 28 days.	(P.-B.F. Series I.	3.0	3.57	4.4	5.2	5.6	6.1
	(P.-B.F. Series II.	2.73	3.14	3.67	4.2	4.7	5.9
	(N.P. Series II.	2.22	2.52	2.94	3.45	4.00	5.25
Loaded at 7 days.	N.P. (Neat Cement.	2.05	2.54	3.11	-	-	-
	N.P. (Mortar.	1.87	2.27	2.53	2.62	2.69	-
	N.P. (Concrete.	3.0	3.7	5.0	5.9	6.35	-
	P.-B.F. (Neat Cement.	2.58	3.11	3.86	-	-	-
	P.-B.F. (Mortar.	2.29	3.08	3.58	3.74	3.80	-
	P.-B.F. (Concrete.	3.33	4.58	6.11	6.88	7.40	

creep, f_a is the steel stress immediately on loading, m is the true modular ratio $\frac{E_s}{E_c}$, and a_c and a_s are the areas of concrete and steel respectively.

It will be noted that both concrete and steel stresses are dependant on the value of c, the former being reduced and the latter increased as the magnitude of the creep increases. Direct experimental corroboration of the analysis was obtained by measuring the deformation of the longitudinal steel in loaded columns subjected to known constant conditions. Values of c were taken from creep tests on plain concrete identical to that used in the columns and gave calculated stresses which were in reasonable agreement with those obtained by direct measurement.

It is interesting in the light of this theory to calculate the probable stress in the vertical steel of a typical column of P.-B.F. cement concrete identical to that tested in Series I. Assume that a column 12 in. square with four 1 in. diameter vertical bars is loaded at 28 days with 137,800 lb. The true elastic concrete modulus was 3×10^6 lb. per sq.in. and with a steel modulus of 30×10^6 lb.per sq.in. the modular ratio is 10, giving initial stresses immediately on loading of 800 and 8,000 lb. per sq.in. in the concrete and steel respectively, i.e. $f_o = 800$ and $f_a = 8{,}000$ lb. per sq. in.

$$\frac{a_c}{a_s} = 44{\cdot}8, \quad m = 10,$$

$$b = \frac{a_c}{a_s}\frac{1}{E_s} + \frac{1}{E_c} = 1{\cdot}827 \times 10^{-6}$$

For the concrete tested, creep plus shrinkage after one year of loading at 1040 lb. per sq.in. was 2360 $\times 10^{-6}$, which, after deduction of shrinkage, gives a unit stress creep of $2{\cdot}03 \times 10^{-6}$ in. per in.

$$\frac{c}{b} = \frac{2{\cdot}03 \times 10^{-6}}{1{\cdot}827 \times 10^{-6}} = 1{\cdot}11$$

$$\text{Increase in Steel Stress} = f_s^1 = 8000\ \frac{44{\cdot}8}{10}\ \left(1 - \frac{1}{e^{1\cdot11}}\right)$$

$$= 24{,}000 \text{ lb.per sq.in.}$$

The shrinkage from 28 days was 250 $\times 10^{-6}$ in.per in. and the steel strain from this cause is given by (17)

$$e_s = \frac{s}{\frac{E_s}{E^1_c}\ \frac{a_s}{a_c} + 1}$$

where e_s = Steel strain due to shrinkage

s = Shrinkage during period considered

E^1_c = Effective concrete modulus.

$$\text{Thus } e_s = \frac{250 \times 10^{-6}}{\frac{30 \times 10^6}{0{\cdot}43 \times 10^6}\ \frac{3.14}{140{\cdot}8} + 1} = 99{\cdot}2 \times 10^{-6}$$

giving a steel stress due to shrinkage from 28 days of 3000 lb. per sq. in.

Thus after one year of loading the condition is :-

Initial Stress on loading	8000	lb./sq.in.
Increase in stress caused by creep	24000	"
" " " " " shrinkage	3000	"
	35,000	"

In addition there is the shrinkage stress up to the age of 28 days - another probable 2,000 lb. per sq.in., at least - making the total stress after one year of loading about 37,000 lb. per sq.in., a figure very near to, if not exceeding, the yield point of a typical mild steel. It should be noted that the steel percentage is 2·23, quite a typical figure, and that the calculation is for a drier mix (0·55 W/C ratio) than would normally be used in practice. It is more than probable, with a practical mix of a consistency suitable for reinforced work, that, in view of the larger creep, the yield point of the steel will be reached in the case of a Portland-Blast Furnace Cement Concrete column, even with normal calculated design stresses.

Extensive researches into the effects of creep in reinforced concrete columns have been carried out in the Universities of Illinois[18][19] and Lehigh[20] under the direction of a committee of the American Concrete Institute. Similar tests were carried out at both Universities, and while some quantitative differences appear in the results owing to discrepancies in humidity and differences in aggregates, the following conclusions were reached in both cases.

(1) The stress in the vertical steel increases with time under load as a result of creep and shrinkage, the stress at the age of one year being several times the initial stress on loading.

(2) The increase of stress is less for columns with a higher percentage reinforcement.

(3) The major effects of creep and shrinkage occur in the first 6-8 months of loading.

(4) The redistribution of stress due to shrinkage and creep have no effect on the ultimate strength of columns.

In the Lehigh tests the steel stress in a column with $1\frac{1}{2}$ % reinforcement increased from 6,000 to 37,000 lb. per sq.in., the corresponding figures for the Illinois experiments being 7000 and 24,000 lb. per sq.in. It is pointed out that the variation of stress distribution in service structures will probably be great when laboratory tests under practically parallel conditions yielded considerable differences.

It is clear that the combined effects of shrinkage and creep will induce high stresses in the longitudinal steel of columns. With low percentages of steel or with concrete of poor quality, the steel stress may rise to the elastic limit when further transference of load becomes impossible, and a state of stress equilibrium is reached. While stresses of this order may appear to be startling from the point of view of good practice, it does not follow that the factor of safety is reduced. The bars are supported laterally throughout their length by the surrounding concrete, and thus may be called upon to carry high stresses without a tendency to buckle.

These considerations however, emphasise a fact long recognized as a result of ultimate load tests, namely that the lateral binding in a column is of greater relative importance than the longitudinal reinforcement.

In beams, it follows that high stresses will be reached in the compression steel. Careful investigations have shown that the possibility of plastic slip in adhesion may be discounted, and thus, the deformations of the steel and the surrounding concrete being the same, the steel stress increases with time. By analogy from the case of a column, high stresses in the compressive steel are not necessarily dangerous, particularly when adequately bound laterally. As a result of creep, the neutral axis of a reinforced concrete beam is lowered, thus reducing the effective lever arm, and thus throwing an increased load on the tensile steel. This increase, however, must necessarily be relatively small, and in addition, there is the possible support of the concrete in tension.

EFFECTS IN DESIGN.

The recently published results of experiments on the creep of concrete might almost lead one to doubt whether the existing theory for reinforced concrete design is admissible. It is assumed that concrete is a truly elastic material within the ordinary range of stress with an elastic modulus of perhaps 2×10^6 lb. per sq.in., while actually, as a result of progressive deformation under load, the "effective" modulus may fall to about, $0{\cdot}6 \times 10^6$ lb. per sq. in., or to $0{\cdot}43 \times 10^6$ as in the Authors' experiments. The more recent text books on reinforced concrete design e.g.[21] merely mention the phenomenon of creep, and explain that the arbitrary choice of a modular ratio of 15 does in effect make some allowance for progressive deformation. It is pointed out that, with modern cements, the elastic modulus of a practical concrete may be $3\text{-}4 \times 10^6$ lb. per sq.in., while only 2×10^6 is assumed. Cements have improved and the knowledge of the fundamental properties of concrete has been extended, and thus the continued use of the low value of the elastic modulus gives an approximation to the true stresses under modern conditions.

The consideration of the effects of creep in design at once raises the question as to how the moments and shears in concrete structures may be influenced. In statically determinate beams the moments and shears are, of course, wholly dependant on the external forces, and are not controlled by the properties of the materials of construction. It would seem, however, prima facie, that in continuous structures, the progressive yield of the concrete would necessarily be an important factor in design. Nevertheless, in spite of the fact that the solution of problems in continuous structures is dependant on the theory of elasticity, it should be remembered that elastic strain plus creep always remains reasonably proportional to the stress, and that this proportionality is more marked with increase in time. In other words, the total deformation being approximately proportional to the stress, continuous concrete structures are still amenable to treatment by the elastic theory, the initial or true elastic modulus being replaced by the "effective" modulus.

An interesting paper by Dyson[22] appeared in 1922 in which methods of design were proposed without any reference to a modular ratio. This unorthodox suggestion was put forward in view of the non-elastic nature of concrete, although it would appear that creep was only known as permanent set and hysteresis, and the large deformations which occur under long periods of time had not been recognized. With regard to columns, however, Dyson was fully aware that high stresses might be reached in the longitudinal steel. He states "This contraction (shrinkage) induces a longitudinal shortening in the steel and may induce so much strain therein that the stress can actually reach the yield

point of the steel. That, however, can only apply in the case of a moderate quantity of steel."
While shrinkage alone may be unlikely to give rise to such high stresses in a typical column, it is certain that the combined effects of shrinkage and creep under sustained load may cause the steel stress to reach the yield point, particularly in the case of a poor quality concrete and a low steel percentage.

It is possible, when all the conditions are known, to forecast the behaviour of a plain concrete member by the use of an "effective" modulus of elasticity. With the introduction of reinforcing steel, however, the problem no longer admits of a simple solution, and it has been shown[(2)] that the use of an "effective" modulus for the calculation of the steel stress in a column may lead to a maximum error of 23 $^{o}/_{o}$. It is clear also, that attempts to limit the actual eventual stresses by the use of the formulae given would, for normal design purposes, be impracticable. In actual service structures the problem is further complicated by the irregularity in the application of dead load, variations in the magnitude and periods of application of live load, time recovery, the possible results of "scale effect", and variations in humidity and temperature in addition to all the long recognized factors in reinforced concrete design which are not amenable to mathematical treatment. Obviously all the conditions cannot be forecast, and a rational solution is impossible.

In the "Recommendations for a Code of Practice for the Use of Reinforced Concrete in Buildings" H.M. Stationery Office 1934, the following general assumptions are made.

(1) Both steel and concrete are perfectly elastic.

(2) Concrete carries no tensile stress.

(3) Plane sections remain plane after bending.

(4) The modular ratio has a value $\frac{40,000}{3x}$ (where 3x is the minimum cube strength at 28 days required for the works test, x being the permissible concrete stress in bending.)

Scott and Glanville[23] commenting on the above state, "In varying degrees all of them (the above assumptions) are inaccurate and it is known that their use does not lead to a true conception of the stresses actually developed. They should be regarded as a framework which, when used properly in conjunction with the appropriate permissible stresses enables design, the safety of which has been proved by practical experience and tests, to be made in a simple manner and with comparative ease". The quantity $\frac{40,000}{3x}$ is approximately the value of the "effective" modular ratio for typical concretes after one day of sustained loading. Thus the actual stresses on loading will be greater than the calculated for the concrete, and less for the steel, and this will be reversed as a result of creep.

With our present comparatively limited knowledge, this appears to be the only possible trend in design. A modular ratio is chosen giving calculated stresses which will be realized only at one period in the life of the structure, but stresses which, nevertheless, enable some justification to be shown for the choice of sections. The increase in knowledge of the physical properties of concrete has brought with it a train of difficulties for the designer, and it is evident that the results of his efforts must be a compromise, which, while not claiming to forecast the true stresses, may still result in an efficient and economical structure, the object of all structural design.

Appendix No. 1.

Proof of Extensometer.

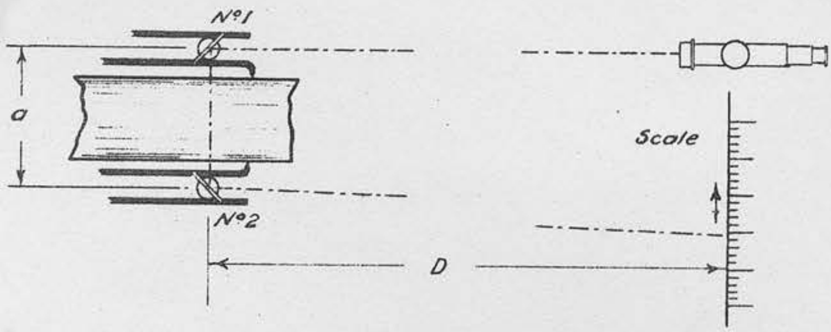


FIG. 9

Let the diameter of the rollers be d, and assume a strain of e in the specimen. 8/-

Then mirror No. 1 rotates $\frac{\cancel{2}e}{d}$ radians and causes a difference in scale reading of $\frac{2e}{d}$ (D + a)

No. 2 mirror rotates $\frac{\cancel{2}e}{d}$ and causes a difference in scale reading of $\frac{2e}{d}$ D. 8/-

$$\therefore \text{ Total Scale Difference } X = \frac{2e}{d}(2D + a)$$

$$\text{Whence } e = \frac{d}{2}\,\frac{X}{(2D + a)}$$

$$= \frac{d}{4}\,\frac{X}{(D + \frac{a}{2})}$$

Where d = diameter of rollers

D = scale distance

a = distance between rollers.

For the experiments described $d = \frac{1}{8}$ in.

$$\therefore \quad \frac{d}{4} = 0{\cdot}03125 \text{ in.}$$

It was decided that 1/60 in. on the scale should represent 0·00001 in. strain.

$$\text{Whence } 0{\cdot}00001 = 0{\cdot}03125 \ \frac{\frac{1}{60}}{(D + \frac{a}{2})}$$

$$(D + \frac{a}{2}) = 52{\cdot}08 \text{ ins.}$$

In this case $a = 3{\cdot}1$ ins.

$$\text{Whence } D = 50{\cdot}5 \text{ in.}$$

Appendix No. 2.

Grading of Aggregate.

Sieve.	Percentage Residue.	
	Sand.	Broken Stone.
1/2 in.	0	0
3/8 in.	0	44·8
No.4.	0·5	80·8
No.8.	34·1	96·5
No.18.	79·1	98·8
No.30.	86·2	100·0
No.50.	95·4	-
No.180.	100·0	-

Appendix No. 3.

Standard Tests on Cements.

	Normal Portland.	Portland-Blast Furnace.
Fineness (per cent)		
Residue on 72 Sieve	0·38	0·05
Residue on 170 Sieve	4·78	2·2
Setting Time		
o/o water for gauging	21·7	21·2
Initial (hrs.min.)	1 - 10	1 - 45
Final (hrs.min.)	2 - 55	3 - 15
Expansion. Le Chatelier		
after 3 hrs. boiling.	1·0 mm.	1·5 mm.
Tensile Strength. (lb. per sq. in.)		
1:3 mortar		
o/o water for gauging.	7·9	7·8
Age 3 days	400, 385, 365, 380, 360, 390,	370, 340, 350, 335, 350, 335.
Mean	380	347
Age 7 days	420, 440, 405, 440, 430, 415,	480, 490, 425, 435, 470, 400.
Mean	425	450

Appendix No. 4.

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