The Role of Structured Induction in Expert Systems

by

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DEDICATION

This thesis is dedicated to my wife Merryl who has stood by me through thick and thin, triumph and defeat, keeping me sane and showing me the meaning of love. Also to my parents Rene and Harvey for whose confidence and support I will always be grateful.

DECLARATION

I declare that this thesis has been composed by myself and the work is my own or performed under my direct supervision.

Alen D. Shapiro

Part of Chapter 6 has appeared in *Advances in Computer Chess 3* (Ed. M.R.B. Clarke, 1982) Oxford: Pergamon Press.

ABSTRACT

A "structured induction" technique was developed and tested using a rules-from-examples generator together with a chess-specific application package. A drawback of past experience with computer induction, reviewed in this thesis, has been the generation of machine-oriented rules opaque to the user. By use of the structured approach humanly understandable rules were synthesized from expert supplied examples. These rules correctly performed chess endgame classifications of sufficient complexity to be regarded as difficult by international master standard players. Using the "Interactive ID3" induction tools developed by the author, chess experts, with a little programming support, were able to generate rules which solve problems considered difficult or impossible by conventional programming techniques. Structured induction and associated programming tools were evaluated using the chess endgames King and Pawn vs. King (Black-to-move) and King and Pawn vs. King and Rook (White-to-move, White Pawn on a7) as trial problems of measurable complexity.

Structured solutions to both trial problems are presented, and implications of this work for the design of expert systems languages are assessed.

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CHAPTER 1

Introduction

1.1. Motivation

An "expert system" is a computer program that

(1) aims to emulate or outdo one or more human experts in a skilled diagnostic or other decision making task and
(2) explains its decisions to the user on demand.

The structure of an expert system can be split into three modules

- the "inference engine"
- the "knowledge-base" and
- the "knowledge acquisition module".

The knowledge-base contains a representation of expertise in the domain. There is also a "database" which contains transient information specific to the current state of the problem. The inference engine dictates how the rules in the knowledge-base are applied to the facts present from time to time in the "database". Database is placed in quotation marks because a) this usage is misleading: "situation model" would perhaps be better, and b) "database" is used later for something different. Use of the knowledge acquisition module usually requires a partnership between a computer scientist (knowledge engineer) and a specialist (domain expert) in the given field. Sometimes these are one and the same person.

To make an expert system one must choose (or develop) an inference engine and, consulting a domain expert, fill the knowledge-base with information of a type which can be called "prescriptive". This typically has the form of "if-then" rules each with associated degrees of confidence. e.g. IF (with some degree of certainty) the car battery is flat THEN conclude (with some measure of confidence) that the fan belt is loose. Some expert domains are such that a system with all confidence measures set to 0 or 1 (false or true) is adequate.

The choice of inference engine dictates the user-interface characteristics and defines some ordering over the information contained in the knowledge base. Designing an inference engine is now well understood. At first glance, knowledge gathering from the domain expert may also not seem to be

particularly hard. But it has become increasingly apparent that

> "the acquisition of domain knowledge (is) the bottleneck problem in the building of applications-oriented intelligent agents." (Feigenbaum, 1977).

Even with domain experts that are regularly available (by no means the normal situation since by their nature their time is in heavy demand) one rule per man-day debugged and installed in the knowledge base is reckoned adequate progress.

What is so difficult about getting correct rules out of an expert, since he is after all an expert? To answer this question it is important to realize that his expertise does not include the ability to explain the reasons for his professional decisions. When a chemical company hires a mass spectroscopist they are renting his ability to *interpret* spectra, not to explain how he makes the interpretations. Hence he is not to be regarded as necessarily expert in this second activity. Indeed in this activity he is not even in the normal sense a professional. Experts typically cannot describe their own reasoning processes. They have to a large extent forgotten how they learned their trade, which tends to be largely based on experience assimilated into a form of intuitive "know-how". Moreover domain experts are seldom computer scientists; hence they do not know how to install rules in a given software system nor do they know the form the rules should take for a particular inference engine. A direct interface between domain expert and expert system is needed. At present the interface is via the knowledge engineer. The knowledge engineer talks to the expert and extracts rules from the explanations he supplies, converting them to machine acceptable form and pointing out inconsistencies as they are discovered. This is the long, slow process of rule acquisition referred to before. The indications of the present work are that for moderately complex tasks complete success can never be achieved by this method alone, i.e. without use of rule-induction. It is significant that the largest operational rule-bases to be built without using induction have not yet much exceeded 2000 rules. Nievergelt (1977) showed that a grand-master's store of chess patterns amounts to some 50,000 in number. Although one pattern is not always equivalent to one rule, the implications for the construction of expert systems for problems of grand-master chess complexity are clear.

There is however one facet of the expert's skill that until recently has not been utilized: he is able to act as a skilled source of relevant examples to train an apprentice. If this skill could be tapped and fed into an expert system equipped with the power to generalize from examples it should alleviate the

knowledge-gathering bottleneck. Michalski (1980) has shown that it is possible by the use of mechanized inductive learning to build a complete expert system from a file of examples. Moreover the inductively built expert system was not only much cheaper to synthesize than a comparable system hand-built by conventional techniques: it showed strikingly superior accuracy of run-time decisions. The research, which was on diagnosing diseases in soy beans, showed that, at this level of problem complexity, the induced rules were understandable and mentally checkable by human experts in the test domain. These issues of cognitive compatibility are central and are more fully discussed in the last section of this introduction.

Another feature usually associated with expert systems is that of "knowledge refinement". The information content of an active knowledge base tends to increase as it is tuned and rules are added. It becomes an increasingly accurate store of expert chosen rules that with very little reformatting can be turned into a tutorial manual.

1.2. Historical background

The following selection from published contributions on machine learning over the past 25 years is focussed on just those which point to the possibility of incorporating learning in expert systems software. We omit work like that of A.L.Samuel (1957) based on the tuning of parameters of a pre-specified description as opposed to the structural modification of descriptions or the generation of new descriptions.

Hunt, Marin and Stone's (1966) CLS (Concept Learning System) was the first to generate rules automatically from examples. These generalizations were produced in the form of decision trees, functionally equivalent to compound conditional statements.

Michie and Chambers' (1968, 1969) real-time system BOXES "learned" to balance a pole on a moving cart. The system modified a set of 225 production rules on the basis of trial runs with a simulation displayed on a video monitor. The system could acquire expertise either in stand-alone mode from its own trial and error, or by observing the real-time decisions of an expert trained on the control task.

Winston (1970) and Barrow & Popplestone (1971) independently introduced relational graphs ("semantic nets") to describe visual scenes. Their programs modified these visual descriptions from example scenes.

In 1976 Michalski, together with Chilausky and Jacobsen showed cost-benefit advantages, both in the labour of rule-base construction and in run-time performance, of induction over traditional dialogue methods for building an expert rule-base. He later (1980) took his soy bean diagnosis a stage further with new material and multiple sources of expert knowledge for a more detailed comparison. The induced expert system again outperformed an expert system generated by conventional means of rule-acquisition from plant pathologists. User-transparency machine rules was reasonable, although not ideal.

Quinlan (1979) re-designed CLS on a more rational and efficient basis and adapted it to large example files by introducing a "refutation filter". The latter incrementally adds a selection of counter-examples to the working set, re-inducing an improved rule on each iteration. A full description of this algorithm is given later.

Quinlan (1983) produced machine-executable descriptions with 5 times greater run-time efficiency than the best human-produced descriptions. He also (1983) proved bounds on the run-time error of induced descriptions to be a) small and b) a function *only* of the size of training set.

1.3. Nature of reasoning

Two forms of logical reasoning are normally distinguished under the names *deductive* and *inductive*. Deductive reasoning is the process of reasoning from the general to the particular. The first recorded attempt at formalizing deduction was Aristotle's (see for example his *Analytica Posteriora* translated in McKeon, 1947). Following deductive rules one may deduce:

- facts from theories e.g. determine whether a particular bridge will stand by axiomatising and applying the laws of mechanics (an example of Aristotle's own is to determine whether a particular triangle has angles summing to two right angles by applying the laws of geometry).
- more specific theories from less specific theories e.g. to classify a class of bridges (or triangles) as capable of standing (or satisfying the two right angles condition, e.g. the class of plane triangles).

Rules for inductive reasoning were given by Francis Bacon in his *Novum Organum*. Induction is in a sense the reverse of deduction i.e. reasoning from the particular to the general. Following its rules one may induce:

- theories from facts e.g. in "scientific induction". Thus Galileo was able to develop theories of motion from observing metal spheres rolling down inclined planes. Similarly legal codifiers abstract over empirical accumulations of judicial precedents.

- more general from less general theories, as when Newton's scheme subsumed the partial formulations of Kepler concerning planetary motions.

Deduction guarantees that providing the theories used are correct, the deduced facts and more specific theories are also correct. Induction is a type of informed conjecture and no equivalent statement can be made, except in the special case that the training set of examples exhausts the entire universe of instances. Here the use of an induced theory is as a compact replacement for the facts rather than as a predictor of new facts. Thus induced descriptions are guaranteed valid only for the facts and sub-descriptions from which the induction was made. When a fact or class of facts is encountered which is misclassified by the description (a refutation example) a further cycle of induction is entered to construct or reconstruct a new description which fits the enlarged set of examples. Essential to the inductive cycle is the availability at every stage, of a source of information as to whether each given example does or does not satisfy the current descriptive role. We call such a source an "oracle". An oracle may be a tutor, or it may be (as in "scientific induction") the response of the real world to observation or experiment. Finally it may be a "fossil record" inscribed in machine memory by some exhaustive computation. This last form of oracle, important in the present work, is commonly (if misleadingly) referred to by the name "database".

The reason that it is not in general possible to provide an inverse to the deduction function is that unless *all* relevant facts are available, a collection of facts is not a replacement for a theory and a theory induced from an incomplete set of facts may always be refuted when further facts come to light. However the process of generating theories from possibly incomplete example sets is one of the mechanisms by which human concept learning takes place. When it is performed by machine the process is called *inductive inference* or *computer induction*.

1.4. The products of induction: friendly *versus* unfriendly

Human understandability is a key facet of expert system design. If a human professional cannot question and follow the reasoning processes of an automated system he cannot be expected to endorse its decisions and he will not use it even if he knows that statistically it may outperform him. If he does use it then he runs the risk of overriding its decisions at the wrong time through his lack of understanding of the machine's "game plan". D. Kopec (1982) illustrates this point with the following example:

In 1975 the productivity of the Hoogovens hot strip mill dropped abruptly and the question was, to what extent was this problem caused by a newly installed highly automated control system. Reporting on an 18-month study H. Voysey (1977) stated that the operators failed to fully understand the control theory of the programs used in the controlling computer, and this reinforced their attitude of "standing well back" from the operation - except when things were very clearly going awry. By intervening late, the operators let the productivity drop below that of plants using traditional control methods. So automation had led to lower productivity and operator alienation simultaneously.

Among the recommendations of the enquiry was the following:

> "Information displays should be designed to help the operator predict performance and to help him *understand* the decisions being taken by the automation, as opposed to the use of displays only to indicate the state of a process."

In this case the result was not too disastrous. However if the task environment were an air traffic control centre or nuclear power station mistakes like those illustrated above could then cost lives. On the other hand lives might be lost if advanced automation were *not* used.

One answer to this dilemma is not to make "unfriendly" man-machine systems in the first place. But it is already plain that the cost-effective production of expert systems themselves is a task that due to problem complexity needs to be performed (at least in part) by computer induction. This tool will then need to be constrained to produce only human-friendly (understandable) rules. This thesis reports a technique, structured induction, which enables this to be done.

1.5. Choice of chess as experimental test-bench

Newell, Shaw and Simon (1958) in their paper "Chess-playing programs and the problem of complexity" justified interest in chess from the Artificial Intelligence viewpoint in the following way:

> "Chess is the intellectual game *par excellence*. Without a chance device to obscure the contest, it pits two intellects against each other in a situation so complex that neither can hope to understand it completely, but sufficiently amenable to analysis that each can hope to out-think his opponent. The game is sufficiently deep and subtle in its implications to have supported the rise of professional players, and to have allowed a deepening analysis through 200 years of intensive study and play without becoming exhausted or barren. Such characteristics mark chess as a natural area for attempts at mechanization. If one could devise a successful chess machine, one would seem to have penetrated to the core of human intellectual endevour."

Michie (1982) identifies five key features that together make chess specially interesting when it comes to the study of the representation of human

knowledge in machines.

(1) the game constitutes a fully defined and well formalized domain
(2) the game challenges the highest levels of human intellectual capacity
(3) the challenge extends over a large range of cognitive functions such as logical calculation, rote-learning, concept-formation, analogical thinking, imagination, deductive and inductive reasoning
(4) a massive and detailed corpus of knowledge has been accumulated
(5) a generally accepted numerical scale of performance is available

Other problem domains exhibit some of the above features, all of which are important if quantitative scientific analysis is to be performed on a non-trivial, human-taxing task. Figure 1/1 puts some other "hard" tasks in this perspective.

Chess is so complex a test-bed that it would be unsuitable as a tool for measuring the success or failure of current inductive learning techniques were it not for the fact that by simple removal of pieces the problem may be reduced to less complex sub-problems. In the experiments detailed later, all but 3 or 4 pieces were removed to reduce problem complexity to tractable limits. At the 3-piece level (king and pawn against king) conventionally generated (human written) programs already existed that could assign a game value to any position in the problem space. There was also available a complete database of position values that could be used as an oracle. Not all these programs were 100% accurate (as the database showed). The programming problem was thus hard though not insoluble. At the 4-piece level (king and pawn against king and rook) the problem is thought to be too complex to be solved by conventional programming techniques short of complete enumeration of the space of positions. Such enumerations were performed in the present work to generate exhaustive lookup databases for reference and checking purposes.

1.6. The hypothesis to be tested

The expert tasks selected were (i) the classification of the King and Pawn versus King ending (KPK, 83622 legal positions with Black to move) into the classes "won" and "drawn"; (ii) the classification of a chess ending comprising 209,718 legal positions into the decision classes "won" and "not-won" for White, the ending in question being King and Pawn (the pawn situated on a7) against King and Rook, with White to move (KPa7KR with WTM).

TASK / FEATURE	school algebra	travelling salesman	preparing abstracts	analytical chemistry	IQ tests	chess
fully defined well formalized domain	yes	yes	no	no	no	yes
challenges human intellectual capacity	no	yes	no	yes	yes	yes
challenge across large range of cognitive functions	no	no	yes	no	?	yes
massive knowledge corpus available	no	no	no	yes	no	yes
numerical performance scale available	no	yes	no	no	yes	yes

Figure 1/1. Chess; a useful test domain. Some candidate tasks tabulated against features that would facilitate their use as test-bed problem domains. Note that only chess scores "yes" for *all* criteria. The symbol "?" indicates that the matter is controversial.

A question common to both tasks was:

For these domains is it possible to construct a complete and brain-compatible symbolic representation of the expert classificatory skill?

By "brain-compatible" we mean that a person can understand the representation and can also check it by mental application to any given example case. Note that some of today's computer programs fail the first criterion and almost all fail the second.

For the more complex domain (KPa7KR) it was known that;

(1) no such representation had ever been constructed by human agency.

(2) since the expertise is largely inaccessible to conscious review, no such representations, in the opinion of the chess-masters consulted, could ever be so constructed.

(3) no representations (brain-compatible or otherwise) could be constructed by known programming techniques other than by exhaustion of the domain.

(4) because of the inaccessibility of expertise to conscious review (2 above) the so-called "knowledge engineering" techniques of contemporary expert systems work would not be applicable.

(5) in the absence of information about the structural features that automatically generated rules should exhibit, conventional use of inductive learning techniques would be inadequate. Rules generated in this way might well be complete but not brain-compatible.

The last point concerns the need for constraints to be developed that will keep the products of induction within the bounds of brain-compatibility. The foregoing question can now be specialized as follows:

Can techniques for the use of induction be developed which, for the complex domain of KPa7KR WTM, make possible the generation of a complete and correct rule of solution which is both machine-executable and brain-compatible. When translated into English text can such a rule constitute a training aid?

It should be pointed out that the KPK problem, although almost trivial to a Master, is too complex for any but the most scrappy and vague rules to have been articulated in existing chess manuals. For the KPa7KR problem the author is not aware that any human codification has been achieved at all. A semi-automated means for codifying skills too complex for human articulation would plainly be of wide utility.

In the event a method was developed for hand-crafting hierarchical structures for the given problem domains. This was found to supply the needed framework for successful use of computer induction to meet the criteria italicised above.

CHAPTER 2

Programming tools

2.1. Conventions used

Throughout this thesis reference will be made to certain chess endgames and chess positions. The notation used to denote an endgame will specify white pieces in descending order of importance followed by the black counterparts (also in descending order of importance). Thus KPKR stands for the four piece endgame with white possessing king and a pawn against black's king and rook.

All references to chess positions will be made in algebraic form with files "A" to "H" from left to right along the bottom of the board and ranks "1" to "8" up the side. Pieces on particular squares will be denoted by colour (B or W) then piece type from the set [K, Q, R, B, N, P] (meaning king, queen, rook, bishop, knight, pawn) separated by a colon (":") from that piece's position. Thus BR:e4 stands for black rook on file 5 rank 4. The "wildcard" character "?" will be used to denote any value in the legal range and the square brackets "[ ]" will contain ranges of legal values. Thus BR:?[1-4] stands for black rook on any file and on ranks 1 to 4 inclusive. The symbol -> when encountered between a piece name X and square Y means "piece X moves to square Y", e.g. WK->a4 means the white king moves to a4.

The abbreviations BTM, WTM, WFW and WFB stand for Black-to-move, White-to-move, won-for-white and won-for-black respectively.

To designate complete endgames we reserve the freedom to drop brackets, colons etc. where this introduces no ambiguity. Thus KPa7KR stands for WK:??WP:a7BK:??BR:??.

2.2. Computing and programming environment

A major consideration before embarking on any substantial programming effort is the facility afforded by the available computing environments. For compute-bound tasks such as chess database generation a powerful processor is a prerequisite. For software development a versatile operating system is essential. The former need was filled by the Edinburgh DEC 10 (KL10) supported by SERC's Interactive Computing Facility. The latter need was filled by the UNIX

operating system running at first on a 6-user PDP 11/34 and later on an 8-user PDP 11/24. These machines were made available to the MIRU by Intelligent Terminals Limited. UNIX facilities like inter-process communication and makefile hierarchies were heavily utilized. The former was used for communicating between the inductive learning system (written in Pascal) and example transformation programs incorporating a CLIP emulator (written in C). The latter was used automatically to ensure that the executable portion of the software was always up to date with respect to the programs and data that described it.

2.3. ID3 induction program

ID3 (Iterative Dichotomiser Three) developed by J.R. Quinlan (1979), has at its heart an algorithm based on Hunt's Concept Learning System (1966) which, given a set of situations each described in terms of features (Quinlan calls these features *attributes)* and a class value, induces a situation classification rule (providing the set of classified situations contain no "clashes", see later). In principle, this rule is a program which can be run to determine the class values for a set of input situations. If the rule is generated from an exhaustive set of pre-classified situations then it is guaranteed to correctly classify *all* legal input situations. A classified situation is represented by a sequence of attribute values together with the corresponding class value and is known as a *decision-vector.*

An extension to this algorithm made by Quinlan in ID3 enables the CLS system iteratively to grow a small example set from an exhaustive file of pre-classified situations thereby allowing it to incrementally induce rules. This *working example set*, introduced by Quinlan, is made up of those examples used to induce a rule at any one stage of the inductive process. Quinlan calls this set the *window*. A brief outline of the ID3 algorithm is given in Figure 2/1.

The name of the attribute description file (mentioned earlier) is supplied as an argument to the ID3 program. The first line of this file contains the number of attributes to be used for the current induction run. Then for each attribute there is a line giving its name, number of values and symbol to be used in the decision rule for each value. Then there is a line containing the number of class values followed by a line containing the names of these class values to be used in the generated decision rule. For example given the attribute description file shown in Figure 2/2 and the data shown in Figure 2/3 ID3 produces the decision rule shown in Figure 2/4. An example of how such a decision rule should be interpreted is given in Figure 2/5.

(1) read in an attribute description file (described later) that describes the attribute characteristics to be used for this run.

(2) initialise the working set with a random working example set of situations of user-determined size ("start with" parameter).

(3) Induce a rule from the working set.

(4) Deal with clashes.

(5) Mark exceptions to the current rule that occur outside the working set.

(6) If no exceptions are found goto (9)

(7) Include a user determined number of exceptions in the working set (increasing working-set size). ("increase by" parameter).

(8) Print statistics for this iteration and goto (3).

(9) Print rule and exit.

A *clash* is said to occur if two or more decision-vectors have the same attribute values but different class values. This corresponds to examples that are described with identical features but for which different actions are recommended. Step (4) involves marking, with a keyword "clash" the point(s) in the incomplete rule where clashes arise. This informs the user that the current set of attributes is not adequate to classify the working set and another technique must be used here or the attribute set must be changed. A source of clashing decision-vectors is erroneous examples supplied by the expert. This aspect will be returned to.

Figure 2/1. Outline of the ID3 algorithm.

```
3
weather     3          wet   dry   blustery
sheltered   2          yes   no
soaked      2          yes   no
2
USE         DONT_USE
```

Figure 2/2. A sample ID3 attribute description file. The first line in the file (containing just a "3" indicates the number of attributes present in this problem.

WEATHER	SHELTERED	SOAKED	DECISION CLASS
wet	no	no	USE
dry	-	-	DONT_USE
blustery	-	-	DONT_USE
-	yes	-	DONT_USE
-	-	yes	DONT_USE

Figure 2/3. A sample ID3 example file. Note the "-" which means "don't care" is an extension to ID3 introduced by the author. An example with sheltered="-" is equivalent to two examples, one with sheltered="yes" and the other with sheltered="no". When both sheltered and soaked have the value "-" as in example 2, then this is equivalent to 2×2, i.e. 4, fully specified examples.

```
weather
        blustery: DONT_USE
        dry: DONT_USE
        wet: sheltered
                no: soaked
                        no: USE
                        yes: DONT_USE
                yes: DONT_USE
```

Figure 2/4. A sample ID3 decision rule. This decision rule would be produced by ID3 if presented with the data given in the two previous figures.

Test the weather. If it is blustery or dry then don't use your umbrella otherwise (if the weather is wet) test whether you are sheltered. If you are then don't use your umbrella. If you are not (sheltered) then test if you are already soaked. If you are then don't use your umbrella otherwise use it.

Figure 2/5. An English explanation for the "umbrella" decision tree. (Shown in Figure 2/4)

A more graphical representation for the same rule follows is given in Figure 2/6.

When a complete and correct rule has been generated, the working set contains example situations that embody all the relevant features for a solution to the domain. Given formal (machine executable) definitions of the attributes used, then this working set together with the induction system is a complete functional replacement for the decision-vector database for the domain as indeed is the induced rule. Likewise the decision-vector database is a replacement for the complete classification database. (It should be noted that none of these are replacements for any database giving more information than the simple class values that appear in the decision-vectors. A rule induced from examples whose class values denote only won or not won will not in general replace a

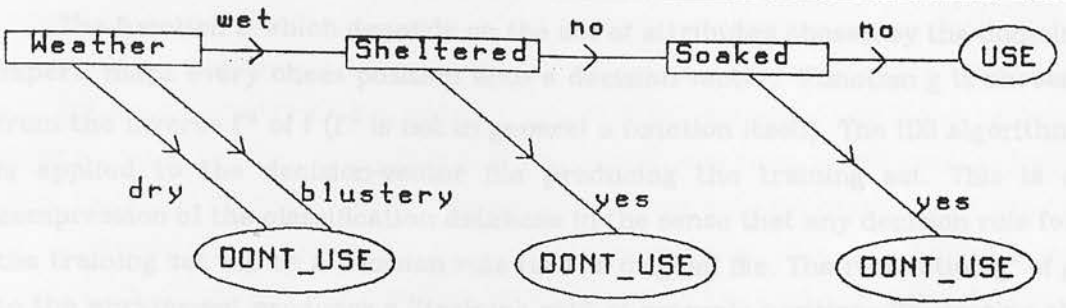


Figure 2/6. A "nodes and arcs" representation for the same rule.

database containing depth-of-win information.)

Resulting database compressions can be spectacular. In the case of two chess test domains (KPK and KPa7KR) the extent of this compression is shown in Figure 2/7.

Domain	Database size (space of legal positions)	Number of decision vectors classifying the total space	Size of training set required to induce a complete and correct unstructured classification rule
KPK	83622	2412	304
KPa7KR	209718	3196	175

Figure 2/7. KPK and KPa7KR database compression. Note that as we move from left to right in the above tabulation we traverse successive degrees of compression: from the initial universe of individuals to a smaller number of "primitive descriptions" in the form of conjuncts of primitive descriptor values ("decision-vectors"); then from the complete set of vectors to a subset discovered by the induction algorithm as sufficient for the purpose of inducing a descriptive rule. Later we shall see that by "structured" induction yet further compression can be achieved.

The process of database compression, and the corresponding generation of a training set is shown in Figure 2/8.

The function f, which depends on the set of attributes chosen by the domain expert, maps every chess position onto a decision-vector. Function g is chosen from the inverse f^{-1} of f (f^{-1} is not in general a function itself). The ID3 algorithm is applied to the decision-vector file producing the training set. This is a compression of the classification database in the sense that any decision rule for the training set will be a decision rule for the original file. The restriction g' of g to the working-set produces a "training set" of example positions illustrating all the features of relevance in the original universe.

2.4. Interactive ID3

Of the two problem domains to which the inductive learning technique was applied, only one (KPa7KR) was solved without using a database as oracle. In the case of KPK, since a set of databases was available, there was no problem in generating all legal decision-vectors and by applying Quinlan's ID3, producing a complete and correct decision tree. No database was initially available however

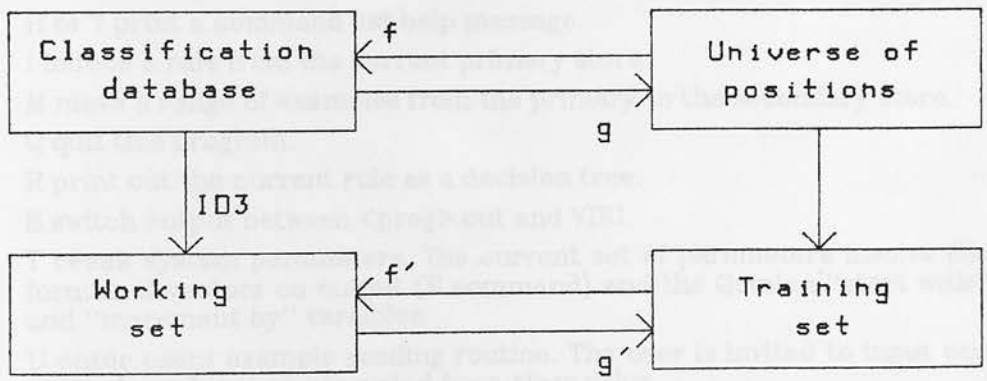


Figure 2/8. Relationship between data in ID3.

in the case of KPa7KR; the only source of class values were fallible human experts. In order to use Quinlan's ID3 it would have been necessary to ask these experts to supply (in advance) most if not all of the example positions (with their game values) that might be required. Experts are good at providing examples but not as a once-for-all training set supplied in advance. They function best when trying to portray single facets of the domain at a time in response to feedback from the inductive learner's mistakes. A new rule-generation tool had to be developed that allowed the interactive development of a rule by selective application of examples to a visibly developing rule. The tool was called Interactive ID3 (see Appendix A for a User Manual).

Interactive ID3 has been built as a shell around Quinlan's ID3 algorithm. An interactive command level allows the user to specify new examples, generate a new rule, move examples between stores ("primary" which is equivalent to Quinlan's "window" (working-set) store and "secondary" whose use will be explained later in this section). The available commands are:

- A add a range of examples to the primary store from the secondary store.
- C print out the current rule as a C function. The name of this function is supplied in the attribute description file and is referred to as <prog> from here.
- D delete a range of examples from either store.
- E print out a range of examples in decision-vector form.
- F read examples from the file <prog>.d.
- H or ? print a command list help message.
- I induce a rule from the current primary store.
- M move a range of examples from the primary to the secondary store.
- Q quit this program.
- R print out the current rule as a decision tree.
- S switch output between <prog>.out and VDU.
- T tweak system parameters. The current set of parameters involve the format of vectors on output (E command) and the Quinlan "start with" and "increment by" variables.
- U enter users example reading routine. The user is invited to input one example and is then prompted for a class value.
- X check the secondary store for any exceptions to the current rule. (This is done automatically after every induce command).
- !<command>, submit <command> to the operating system for immediate execution. Interactive ID3 is suspended until this command is finished.

Examples are processed depending on their status with respect to the most recently generated decision tree. Accordingly there are 4 types of examples:

(1) Examples that contradict the current rule.
(2) Examples that do not contradict the current rule but whose decision-vectors have not been encountered before.
(3) Examples that clash with examples previously encountered.
(4) Examples which add no new information about the space. Their decision-vectors have been encountered before.

Examples of the first type are added immediately to the working set.

Examples of the second type, though not adding any useful information at this stage, may be relevant later when the rule has changed and these examples now cause the rule to fail. This type of example is stored in the secondary store and all examples in this area are checked against subsequently generated rules. Any example in this store that fails to be classified correctly by a rule is reported to the user who has the choice as to whether it should be included in the working-set.

Examples of the third type are reported to the user and added to the secondary store.

Examples of the fourth type are ignored.

Since it would have been impractical for the expert to supply his examples in the form of decision-vectors, Interactive ID3 was given the capability to invoke and communicate with an attribute evaluation process. In chess this means that the expert supplies his examples as board-positions expressed in algebraic notation. Interactive ID3 passes the positions to an attribute evaluation process (described later) which calculates the relevant attribute values and passes them back to Interactive ID3. The expert is then prompted for a class value which is appended to the newly acquired attribute values to make an decision-vector. It is these vectors that are manipulated by Interactive ID3.

Like Quinlan's ID3, Interactive ID3 is invoked with the name of an attribute description file as its argument. In addition to the fields described in the previous section this file has a field for a comment appended to each attribute description line, a field for a question that is asked of the user as a prompt for his example class value, and a field containing the name of the relevant attribute evaluation program. This last field is also used to supply a name for the "C" function that may be generated. For example recall the "umbrella" attribute description file, Figure 2/9 shows this file suitably changed to reflect the conventions of Interactive ID3. The same rule would be produced but now using the "C" command the following "C" function shown in Figure 2/10 would be output. In

```
3
weather     3           wet   dry   blustery   What's the weather like?
sheltered   2           yes   no              Are you sheltered?
soaked      2           yes   no              Are you already soaked?
2
USE         DONT_USE
umbrella
Should you use your umbrella?
```

Figure 2/9. A sample Interactive ID3 attribute description file.

```
#include "umbrella.h"

umbrella() {
        switch(weather()) {
                case BLUSTERY:
                        return(DONT_USE);
                case DRY:
                        return(DONT_USE);
                case WET:
                        if(NOT sheltered()) {
                                if(NOT soaked()) {
                                        return(USE);
                                }
                                else {
                                        return(DONT_USE);
                                }
                        }
                        else {
                                return(DONT_USE);
                        }
        }
}
```

Figure 2/10. A "C" function generated by Interactive ID3.

this figure the first line (#include "umbrella.h") is a compiler directive to include a user defined file in the compilation. This file must specify values for all the constants in the function. For the "umbrella" problem the file would look as shown in Figure 2/11.

Since Interactive ID3 is driven solely by the user a technique of rule generation is needed. The best results have been obtained when the expert user is requested to select relevant examples that he thinks will be classified incorrectly by the current rule. In this way the rule is refined in a controlled and disciplined manner.

Interactive ID3 was developed for problems for which there was no classification database. But it was found useful as a tool for reducing the size of the set of examples necessary to produce a complete rule (training set). In the KPK "canrun" example described later, the size of the training set obtained using Quinlan's ID3 with a database oracle was reduced from 16 to 12 using Interactive ID3.

```
#define BLUSTERY  0
#define DRY       1
#define WET       2
#define USE       0
#define DONT_USE  1
#define NOT       !
```

Figure 2/11. Rule header file for the "umbrella" problem.

2.5. CLIP/C parallel array emulator

As mentioned previously, ID3 uses situations detailed in terms of attributes. In chess these attributes can be difficult to compute though easily understandable in terms of humanly perceived patterns. Often the "domain specialist" would express a concept that was powerful but expensive to program using conventional techniques. As with some computer vision problems, the trouble seems partly at least to arise from mismatch between the domain's intrinsically two-dimensional character and the one-dimensional architecture implicit in conventional languages. A specialised emulator was accordingly developed of an array-oriented machine for the specific purpose of coding primitive patterns. A brief description of this emulator follows. CLIP (Cellular Logic Image Processor) (Duff, 1978), is a hard-wired machine presenting the user with 32 planes of 96×96 bit arrays. The computations allowed are bitwise logical operations performed between planes. Since a CLIP machine was not available to us, a 100 plane 8×8 bit emulator was developed on the PDP 11/34. For this C-coded version, an earlier FORTRAN emulator (Zdrahal, Bratko and Shapiro, 1979) provided a useful design aid. The general form of a CLIP/C instruction is:

[c]op(plane1,plane2,plane3)

This means perform the logical operation "op" between plane1 and plane2 putting the result in plane3. The optional "c" in front of the operation means use the first argument complemented. Sometimes as in single-argument logical operations, (e.g. not) a different number of planes are used. There are also 2

special functions returning boolean values and 3 non-logical pattern transformation operators. The full set of library operations available with the current CLIP/C emulator are as follows.

1. Logical Operations

Three-Argument Operations

- and(P1, P2, P3) bit-and plane P1 with plane P2 leaving the result in plane P3.
- or(P1, P2, P3) bit-or plane P1 with plane P2 leaving the result in plane P3.
- xor(P1, P2, P3) bit-xor plane P1 with plane P2 leaving the result in plane P3.
- eqv(P1, P2, P3) set all the bits in P3 that have the same value in P1 and P2.
- cand(P1, P2, P3) bit-and plane $\overline{P1}$ with plane P2 leaving the result in plane P3.
- cor(P1, P2, P3) bit-or plane $\overline{P1}$ with plane P2 leaving the result in plane P3.
- cxor(P1, P2, P3) bit-xor plane $\overline{P1}$ with plane P2 leaving the result in plane P3.
- ceqv(P1, P2, P3) bit-eqv plane $\overline{P1}$ with plane P2 leaving the result in plane P3.

Two-Argument Operation

- not(P1, P2) bit complement plane P1 leaving the result in plane P2

2. Non-Logical Operations

Five-Argument Operation

- propagate(P1, P2, x ,y, P3) shift bit pattern in P1 x columns in the X direction and y rows in the Y direction repeatedly until the edge of the plane is reached or an equivalent bit in plane P3 is reached. The result is left in P2. Bits falling off the edge of plane P2 are lost.

Four-Argument Operation

- shift(x, y, P1, P2) shift bit pattern in P1 x columns in the X direction and y rows in the Y direction leaving the result in P2. Bits falling off the edge of plane P2 are lost.

Three-Argument Operations

- setbit(x, y, P1) set bit (x,y) (file and rank) in plane P1. Does not affect state of any other bits in the plane.

- bitset(x, y, P1) return true if bit (x, y) is set in plane P1, otherwise false.

Two-Argument Operations

- expand(P1, P2) expand the bit pattern in P1 leaving the result in P2. e.g. If P1 contains 2 bits set (as below) the result would be as shown in Figure 2/12.
- copy(P1, P2) copy plane P1 leaving the result in plane P2.
- same(P1, P2) return true if plane P1 is the same as plane P2, false otherwise.

One-Argument Operations

- init(P1) fill plane P1 with zeros.
- zero(P1) return true if plane P1 is all zero, else false.
- display(P1) print plane P1 on the console (in the same format as Figure 2/12).

The speed of the CLIP/C emulator was tested on the PDP 11/34 by evaluating EXPAND, the slowest operation. One expand requires about 250 microseconds, a very acceptable time. In chess terms each bit in an 8x8 CLIP/C plane corresponds to a square on a chess board. If a plane containing one non-zero square is EXPANDed then the resulting CLIP/C plane represents all the squares that a king attacks from the original square.

Let BKING, WKING, PAWN, BATTACK, WATTACK and RESULT be CLIP/C planes (P1

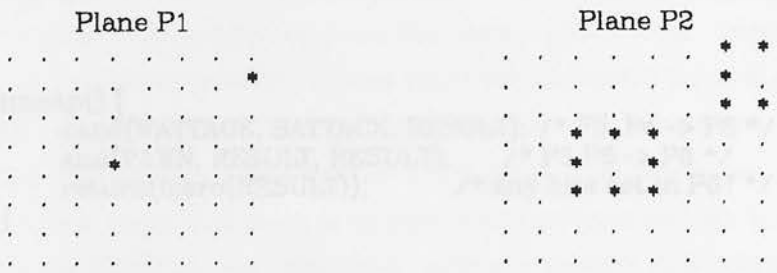


Figure 2/12. expand(P1, P2).

to P6 respectively) then if the initialisations shown in Figure 2/13 have been performed the CLIP/C function shown in Figure 2/14 returns the boolean value of whether the Black king can immediately capture the pawn.

On the PDP 11/34 imcap evaluates in under 440 micro-seconds.

2.6. Decision vector generators

Chess experts express their examples as chess positions. On the other hand Interactive ID3 receives its examples as decision-vectors. The conversion between these two example forms was accomplished by example transformation

```
init(BKING);
setbit(bkfile, bkrank, BKING);
init(WKING);
setbit(wkfile, wkrank, WKING);
init(PAWN);
setbit(pfile, prank, PAWN);

expand(BKING, BATTACK);
expand(WKING, WATTACK);
```

Figure 2/13. Initialisations for the "imcap" function.

```
imcap() {
    cand(WATTACK, BATTACK, RESULT); /* P5 .P4 -> P6 */
    and(PAWN, RESULT, RESULT);      /* P3.P6 -> P6 */
    return(!zero(RESULT));          /* any bits set in P6? */
}
```

Figure 2/14. imcap - implemented using CLIP/C.

programs (decision-vector generators, one for each induction task). These programs comprise a loop that reads chess positions. For each legal position a series of functions evaluated, each function corresponding to an attribute (like imcap) that the expert considers relevant to the current induction task.

In interactive mode the relevant transformation program is invoked by the inductive learning program (getting its name from the attribute description file). In non-interactive mode the relevant transformation program is compiled with (using a technique of conditional compilation) a legal move generator that generates all legal chess positions for the domain being tested. After adding the class value (from database lookup) the move generator program sorts and merges the replies to remove duplicates. The resulting decision-vector file is used as input to Quinlan's ID3.

Since the attribute transformation programs were separate UNIX processes with i/o along standard input/output channels they could be tested as separate modules independently of Interactive ID3.

2.7. Database generators

Database is used here to mean a pre-calculated collection of facts about a chess endgame. This information is ordered so that the address of any information about a position is a function of that position.

Chess databases differ from the more commonly encountered databases of facts (e.g. mathematical log tables) in one very important respect. There is generally no way to construct a particular entry in a chess database by a feasible function evaluation which does not involve reconstruction of a substantial part of the rest of the database. Exhaustive move-by-move evaluation by forward search becomes impractical at even the three piece level. Consider the KPK endgame; about seven possible moves must be evaluated from a given position on average. To determine the value of a position by exhaustive search would require (since the maximum depth of win is 38 ply) a search tree of 7^{38} nodes. There are many ways that such a search could be speeded up (e.g. illegal and repeated move pruning) but even these methods require unacceptable resource overheads when used to evaluate a single entry in isolation.

Current chess database generation techniques require that either all of an identifiable sub-part of the database be evaluated at once or none of it. This is because the developing database is used as a partial data store for other positions that may be evaluated as the algorithm progresses. Only upon completion

of the algorithm is it known that data contained in the database will not be subject to further change. So until the generation process is complete no one piece of data can be said to have the correct value. Only in some cases is it possible to split the generation of a database up into parts. Furthermore the order of evaluation of any of the sub-parts may be critical. For example, the generation of the KPKR WTM won-for-white / not-won-for-white database was split into 6 parts (one for each of the pawn ranks 2-7). Since positions with the Pawn on rank 2 might lead to positions with the Pawn on rank 3, the rank 3 generation had to take place before rank 2. Similarly for all the other possible Pawn ranks. The precise method of database generation is discussed in the next chapter. This section is concerned with the software tools that were required to facilitate database generation.

Chess endgame databases were used in two ways:

(1) provision of class values for decision-vectors in the KPK problem, - i.e. using the database rather than the expert as "oracle".
(2) checking the accuracy of induced rules in both the KPK and KPa7KR problems.

Use of databases in the second way ensured detection of any inaccuracy of the induced product. For the KPK experiment this was confirmatory only since the same database provided class values for the vectors used to induce this rule. For the KPa7KR experiment the use of a checking database provided an invaluable crosscheck since only expert supplied information had been used to form the rule.

There were two distinct database generator programs - one for each experiment. The principles are discussed more fully in the next section. In addition to these there were three intermediate databases generated in the process of creating a KPKR database for white to move with pawn on a7, for storing the values of "White-wins/White-does-not-win". (Note this KPa7KR database did not distinguish draws from Black-wins.) The problem in this case was the number of derived domains that can be encountered when either the pawn promotes or pieces are captured. These derived endgames are shown in Figure 2/15 (PX, RX etc. denote Pawn capture, Rook capture etc.).

In an attempt to cut down the work space some points were noted as follows:

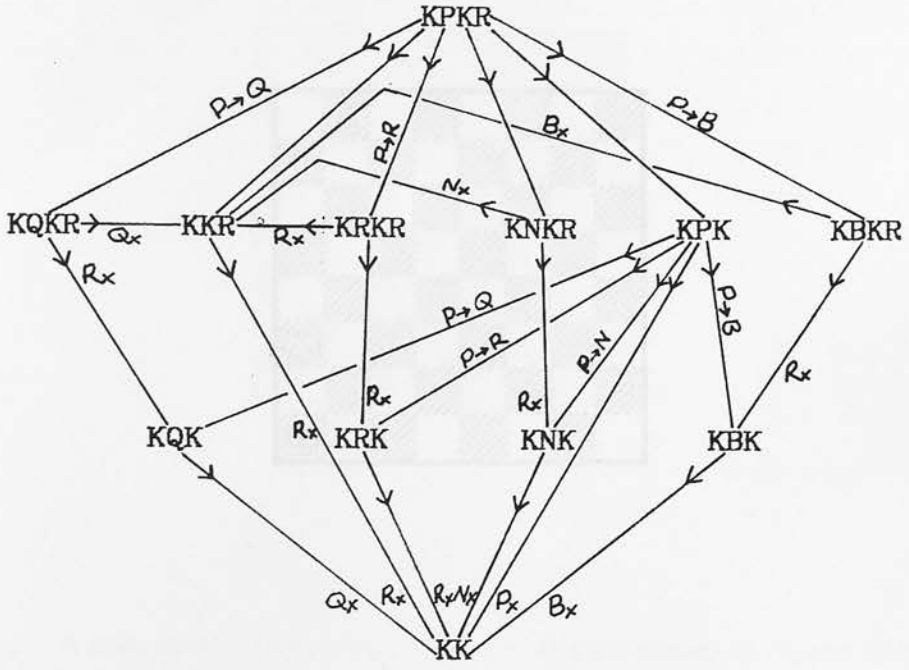


Figure 2/15. Derived endgames of KPKR.

(1) King against King (KK) is always a draw.

(2) No case where the pawn must promote to a knight is won for white (KNK and KNKR).

(3) There are no cases when the pawn is forced to promote to a bishop (KBK and KBKR).

(4) There was only one case when safe capture of the rook by the pawn could not be regarded as won-for-white (see Figure 2/16.)

(5) All cases where the white king captures the rook can be evaluated by the KPK subproblem "rank7" (described later).

(6) Except for the position shown in Figure 2/16 (which is treated as an exception) all captures of the rook by the white pawn can be dealt with very simply.

Thus the only derived databases that it was necessary to consider when building the KPKR database were KRKR and KQKR with "White-wins/White-does-not-win" information. These databases were kindly supplied by Ken Thompson of Bell Laboratories. Due to the nature of the database generation procedure, these

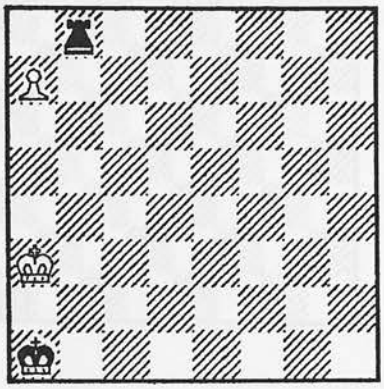


Figure 2/16. Can the BR be captured safely. When the WP captures the BR and promotes to either a queen or a rook the above position results in the stalemate position shown in Figure 2/17. Note that if the BR is not captured the position is won-for-black and if, on capturing the rook, the pawn promotes to anything other than a rook or queen the position is drawn.

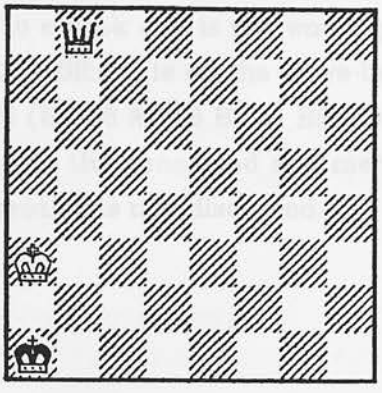


Figure 2/17. A stalemate. After white captures the BR shown in Figure 2/16 this stalemate occurs if white promotes the pawn to any piece that has a potential of winning (WR or WQ).

databases only needed to contain values for positions where the promoted white pawn is on rank 8. Unfortunately the same generation procedure requires that these databases report on black-to-move values and Ken Thompson's databases contained values for white-to-move only.

A program was written which converted Thompson's (WFW/not WFW, white-to-move KQKR) database to a WFW/not WFW, WQ on rank eight, black-to-move database (denoted KQ8KR), and generated every legal KQKR position with WQ on rank eight. From each of these positions every legal black move was generated and looked up in the KQKR database. The worst value for white looked up in this database for the set of legal black moves was put in the new (KQ8KR) database as the black-to-move game theoretic value. The same procedure was applied to KRKR giving a KR8KR database.

The merge of these 2 new databases then had to be performed (i.e. the worst white-value with black-to-move) resulting in the third and final derived

database namely a white-value K(R or Q)KR with (R or Q) on rank eight with black-to-move. Each of the lookahead programs took about a week to run on the PDP 11/34. The merge took about 2 hours. The results of the merge are summarized in Figure 2/18.

The merge results show that there is only one position that is won for white if the pawn is promoted to a rook and is not won if promoted to a queen. This position (WK:c2 WR:c8 BK:a1 BR:d4) is on the move-line from a position know as Saavedra's position (1895) (WK:b6 WP:c6 BK:a1 BR:d5) and its appearance in Figure 2/18 lends credibility to the generated and merged database. The general problem of database generation is now discussed.

KQ8KR \ KR8KR	not_won	won	illegal
not_won	218709	1	0
won	444753	6709	0
illegal	0	0	378404

Figure 2/18. KQ8KR and KR8KR BTM WFW/not WFW position tabulation. Note the presence of a position that is not WFW if the pawn is promoted to a queen but *is* WFW if the pawn is promoted to a rook.

CHAPTER 3

Database generation

3.1. The "database" problem

The last section touched on the need for an alternative to "dumb" search for generating chess databases. Two methods for cutting down the search space were suggested:

- throw away any illegal positions generated
- use remembered value for any position previously encountered

Throwing away illegal moves may reduce the average branching factor by a small amount. For KPKR the branching factor is between 8 and 10. The maximum depth of win is 38 ply, so a naive search would visit greater than 8^{38} nodes for every position to be evaluated (about 200,000 - 100,000 with Black-to-move and 100,000 with White-to-move) so small reductions in branching factor (of one or two) will make little impact on the search complexity. Far greater gains may be achieved by the sensible use of memory for partial result storage by exploiting the favourable space/time tradeoff, i.e. if the database generator is allowed to use memory, it will take less time to complete its task. More specifically, if the database generation algorithm has a storage slot allocated for every position in the KPK domain (about 200,000) it will not have to compute a value for any position more then once. Further gains can be made if the search space is limited to deal only with positions that lead to interesting positions (in our case these interesting positions were defined as those that are known to be won-for-white). This theme can be taken further: positions that lead to interesting positions are defined as interesting. Taken to its logical conclusion this theme (positions that eventually lead to positions that are won-for-white) defines a search strategy that is guaranteed to include all positions about which we would like to have some information. Ideally this information would be a number representing the depth of win. However for the purposes of the experiments performed, just the fact that these positions are won-for-white is sufficient.

In order to implement the above search strategy one needs to be able to obtain (or generate) all initially interesting positions (for KPK this means the set

of positions where White can *safely* promote to queen or rook (without creating stalemate)) these are known as "terminal" positions. One also needs a reverse legal move generator that can provide all legal predecessor positions to the one currently being investigated. Given these two resources the algorithm which is known as "standard backup" database generation is now described.

3.2. Principles of standard backup database generation

The standard backup needs to be able to store two values for every position in the space, called depth and counter. These can be thought of as functions applied to a position P. Depth(P) returns the current depth of win associated with position P and is initially set to undefined. Counter(P) is only applied to BTM positions and returns the number of as yet untested Black moves from position P. Counter(P) where P is a BTM position is initially set to the number of legal Black moves from P. Having defined these two functions the standard backup algorithm is shown in Figure 3/1 (from Niblett (1982)). Note the choice of BTM positions as the initially evaluated positions is arbitrary.

Steps (1) to (3) are initialisation and can be performed in one pass over the BTM and WTM positions. Steps (4) to (8) form the body of the algorithm and requires a pass through these positions for every ply depth evaluated. Step (4) requires a linear search through WTM positions to find any that are lost in depth i. If these are BTM positions their predecessors must have been WTM so White must have had a move that caused Black to lose. White would surely make this move (given perfect play) so the game theoretic value from the WTM predecessor positions (there may be more than one) is won in depth i + 1.

Looking at WTM positions that are won depth i is more complicated. Since the predecessor of a WTM position is BTM, given perfect play, Black would not play a move that resulted in a won-for-white position unless there were no choice. So before the BTM predecessors can be marked as won depth i + 1, we must make sure there are no *other* moves Black may make that are not-won-for-white. If there were any then given perfect play, Black would surely make one of these. This is where the "counter" function comes in. Every BTM position is marked with the number of legal moves that Black can make. If for a given BTM predecessor to a won-for-white depth i position, this number is one then this is the only move Black can make and it must be won-for-white depth i + 1. If this number is two then there is another move Black can make so decrement counter(this BTM predecessor) by one (giving one). If in the future this position is backed-up to again, counter(this BTM predecessor) will be found to be one and

(1) Set each BTM position P which is lost immediately as won for White depth 0 (depth(P) := 0). Set all other positions as undefined.

(2) For each BTM position in the endgame set a counter containing the number of legal moves from that position.

(3) Set i to 0.

(4) For each BTM position P lost at depth i (depth(P) = i), generate its legal predecessors $P_1,\ldots, P_i$. If any P_j has depth(P_j) = undefined then set depth(P_j) := i + 1.

(5) Set i := i + 1.

(6) For each WTM position P that is won in depth i generate its legal predecessors $P_1,\ldots, P_n$. If any P_j has counter(P_j) = 1 and is not marked as won, then mark it won depth i + 1 (depth(P_j) := i + 1). Otherwise decrement the counter by 1 (counter(P_j) := counter(P_j) - 1).

(7) Set i := i + 1.

(8) Repeat steps (4) to (7) until no new positions are marked as won for White. All positions P from which White can force a win will have been set with depth(P) equal to some positive integer.

Figure 3/1. The standard backup algorithm. Note the choice of BTM positions as the initially evaluated positions is arbitrary.

this position will then be marked won-for-white depth i + 1. Note this value for "i" is not necessarily the same as it was when the position was first encountered as a BTM predecessor to a won-for-white position - it is probably greater. This is all right since, as it was a Black move and given perfect play, the game-theoretic value is the depth along the longest path that Black can delay a loss. Extending this explanation, if the value of counter (at some BTM position) is found to be N then there are N - 1 other positions that have this BTM position as a predecessor. Only if all N - 1 positions are backed up to will a depth-of-win value be assigned to this BTM position. If not then depth(this BTM position) will remain undefined which means not-won-for-white.

Since for KPK the largest depth of win was found to be 38 ply the algorithm needs about 38×200,000 = 7,600,000 data accesses to complete the KPK database generation. Compared to the 8^{38}×200,000 accesses of a naive algorithm this represents a considerable saving and brings the task within the realms of feasi-

bility. There are further savings to be made by making a clever cache of positions and incorporating some forward look-ahead in the backup search. One such algorithm was used for the generation of the KPa7KR database. The algorithm is shown in Figure 3/2. This algorithm is very much less disk-bound during execution than the standard algorithm of Figure 3/1. Note that the output of the algorithm for WTM positions does not give us access to the depth of win for these positions, only whether they are won or not.

3.3. Difficulties and pitfalls

The main difficulties are concerned with a) finding all terminal positions (this process for KPa7KR has been described and was by no means easy) and b) ensuring that the reverse legal move generator works in strange cases e.g. it is

---

Let MAX_MOVES be the maximum number of legal moves (enumerated in some standard order) possible from a BTM position (e.g. MAX_MOVES is 22 for KPKR). Moveno(P) is a counter which takes values in the range 0 to MAX_MOVES. It is used in the algorithm to keep track of which moves have been considered from BTM positions. We assume that initially decided positions are with BTM.

(1) Set each BTM position P which is lost immediately as won for White depth 0 (depth(P) := 0). Set moveno(P) to be 0 for each BTM position P. Set all other positions as undefined.

(2) Set i to 0.

(3) For each BTM position P lost at depth i (depth(P) = i), generate its legal predecessors $P_1,\ldots, P_j$. If any P_j has depth(P_j) = undefined then set depth(P_j) := i + 1.

(4) Set i := i + 1.

(5) For each BTM position P not marked as lost - if the move represented by moveno(P) is illegal, impossible or leads to a WTM position increment moveno(P). If moveno(P) is incremented and moveno(P) < MAX_MOVES goto (5). If moveno(P) = MAX_MOVES all the possible moves from P are illegal or lead to lost positions so set depth(P) = i + 1.

(6) Set i to i + 1.

(7) If no new positions were marked as lost in steps (3) or (5) then STOP else goto (3).

Figure 3/2. The backup algorithm modified to be less disk-bound. Note that the output of the algorithm for WTM positions does not give us access to the depth of win for these positions, only whether they are won or not.

---

easy to forget that some positions are impossible to get to, bad data may be generated for positions backing-up to them. Since mistakes cost many CPU hours, trial and error is not a feasible approach. So modular design and testing is essential. Fortunately the algorithm is straight-forward.

3.4. The checking problem

Having generated a database the problem of how to test it remains to be solved. The testing method with mathematical tables might be to take some random sample (or all entries if the table is "small"), compute the tabulated function and check it against the corresponding database entry. However with these "complex" chess endgames there is in general *no* feasible computation of the tabulated function.

A very cheap check is to examine statistics like those obtained for the K(Q8 or R8)KR terminal position database. The rediscovery of Saavedra's position was a useful source of confidence (but hardly a foolproof test of database integrity).

A partial solution to this problem (making do) is to get someone else independently to attempt the database generation and then compare the databases "byte for byte". This is costly in resources and still does not guarantee that both generators have not made the same mistakes. Fortunately the generation algorithm is reasonably simple. But conceptual mistakes can and do creep in that would not be obvious to human designers of database generators.

The approach we took was as follows: The database was to be used as a checking device for other solutions (inductively derived from a chess-master as oracle) for the same problem (KPKR white-to-move with WP:a7). It would not be begging the question too much to use each to cross-check the other. The idea is that each may show up whole classes of errors in the other and the chance that each would contain exactly the same members of a given class of error as misclassified should be remote (since each uses a very different evaluation technique). The pitfall of this approach occurs if a detected error is treated in isolation from its class (as an exception) without finding its "class cause".

An error in the database was found in this way and it was indeed one that another database builder might well have overlooked. It was the stalemate position given in the previous chapter (WK:a3 WP:a7 BK:a1 BR:b8). This position was mistakenly included in the set of terminal positions as won-for-white. The consequences were not too disastrous for the rest of the database.

It may be wondered how a such a mistake could be made? The possibility of stalemate was considered and rejected since it was assumed that White could always avoid the draw by minimally underpromoting the pawn (i.e. to rook). In this case underpromotion does no good. If it is minimal, then stalemate occurs. If underpromotion is not minimal (i.e. to knight or bishop) the win fails from insufficient force.

3.5. Databases generated for the present work

Figure 3/3 shows a table of databases generated during the course of this research. Some further analysis yields the table shown in Figure 3/4. From these tables it can be seen that the deepest KPK win for white with BTM is a position within the main-pattern subproblem namely WK:h3 WP:b2 BK:h5 and is solved in 19 moves.

Endgame	Subset	Legal positions	Number of +ve outcome	Max depth to outcome (moves)	Database size (bytes)
KPK	WP:[a-d][2-7]	BTM 83622	48795 WFW	19	98304
"	pawn-can-run	BTM 83622	36958	5	98304
"	WP:a[2-7]	BTM 20919	12312 WFW	10	24576
"	main-pattern	BTM 31185	12796	19	49152
"	WP:[a-d][5-6]	BTM 21012	9765	14	32768
"	WP:[a-d]7	BTM 10506	237	2	16384
KQKR	WQ:?8	BTM 670172	451462 WFW	31	1892352
KRKR	WR:?8	BTM 670172	6710 WFW	3	1892352
KPKR	WP:a7	209718	129825 WFW	†	262144
KPKR	WP:b7	206424	124782 WFW	†	262144

Figure 3/3. Databases generated during the course of this research. Note: If won *versus* not-won information is all that is required, database sizes given in bytes can be regarded as their size in bits.

†Since the KPa7KR database was generated using the modified backup algorithm, this information is not available.

Database	Number of positions at maximum depth	Example position at maximum depth			
KPK (pawn-can-run)	4628 (5)	WK:a1	WP:a2	BK:d1	
KPK (WP:a[2-7])	21 (10)	WK:b2	WP:a2	BK:d2	
KPK (main-pattern)	1 (19)	WK:h3	WP:b2	BK:h5	
KPK (WP:[a-d][5-6])	41 (14)	WK:f4	WP:b2	BK:f8	
KPK (WP:[a-d]7)	237 (2)	WK:a6	WP:b7	BK:c6	
KQKR (BTM WQ:?8)	11 (31)	WK:a8	WQ:d8	BK:c3	BR:b4
KRKR (BTM WR:?8)	3 (3)	WK:a1	WR:a8	BK:a3	BR:c2

Figure 3/4. Some database statistics. The numbers in backets are the maximum depth of wins (in moves) as given in the previous figure.

CHAPTER 4

Techniques in the use of computer induction

4.1. Structured induction

Established programming techniques dictate that when faced with a complex programming task, the problem to be solved should be split into smaller more manageable subproblems. Each subproblem is usually programmed as a procedure, and procedures can typically be nested to any level. The precise choice and hierarchical order of subproblems is the product of top-down design (as in Structured Programming). We call the induction method developed from this idea by Shapiro and Niblett (1982) Structured Induction.

With computer induction the same human-generated top-down problem decomposition can be combined with bottom-up implementation of the bodies of the individual procedures, using example-value pairs obtained from an oracle, e.g. from an expert in the domain of the given procedure. The domain expert decides on the attributes that he knows to have most bearing on the problem to be solved. He takes each one of these in turn and in consultation with a programmer, they decide if these attributes are immediately codeable. If any are not then the decomposition process is repeated for each attribute that is not immediately codeable until none are left. This produces a hierarchical tree of subproblems whose leaf nodes are directly codeable attributes.

The next stage involves using inductive inference to solve each of the subproblems from the bottom of this hierarchy to the top. Interactive ID3 is applied to each subproblem in each hierarchical level in turn and a decision tree (in the form of a compilable function) for each is produced and tuned until the domain expert is satisfied that each deals with all the example situations that he intended they should when the problem decomposition was being performed.

This process of tuning involves the expert using the current rule to classify examples other than those so far used in the rule-training process. Failures to correctly classify a new example fall into two categories:

- examples that result in clashing decision-vectors (i.e. same attribute values but different class values), either caused by an incorrectly classified example or an incorrectly defined or missing attribute.

– examples that contradict the current rule.

A clash can be dealt with either by removing the incorrectly classified example (if any) or by redefining the faulty attribute. An example that contradicts the current rule indicates that the rule needs to be re-induced with this example included in the training set. The expert also has some control over the order with which attributes appear in the rule. In the case of a tie in the measure that is used to sequence attributes within a rule, their order of appearance in the attribute description file is used.

Each newly solved subproblem is then given a meaningful name which is used in the next level up as a simple codeable attribute. Each induced solution is now a directly executable piece of code that when run delivers a value that has the expert's seal of approval. This process is continued until there are no more subproblem hierarchy levels to ascend (the top of the decomposition tree has been reached).

At this stage a top level problem exists that when run, calls the lower level subproblems and attributes in an order determined by the inductive procedures applied at each level of the bottom-up implementation.

For example, consider the following problem "p" that an expert has decided is best split by attributes a_1 a_2 a_3. Since it is decided that both a_2 and a_3 are too complicated to be coded as attributes, they are treated as subproblems and the expert splits a_2 using attributes a_{21}, a_{22}, a_{23} and a_{24}, and a_3 is split using attributes a_{31} and a_{32}. The top-down problem decomposition is shown in Figure 4/1 (circled items are subproblems and boxed items are directly codeable attributes).

This completes the expert's hierarchical decomposition. (Note that the only ordering information contained in this decomposition tree is the depth at which a subproblem will be executed. There is no horizontal ordering implied.)

A programmer now codes the four procedures corresponding to attributes $a_{2[1\text{-}4]}$ and the expert uses Interactive ID3 to produce a rule that he likes (by providing relevant examples and suggesting attribute coding changes to the programmer). Suppose the expert approved rule is as shown in Figure 4/2 (the decision classes are called a_2D1 and a_2D2), and when the process is repeated for a_3, the rule produced is as shown in Figure 4/3.

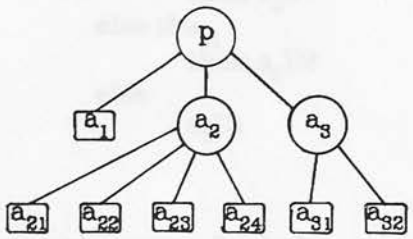


Figure 4/1. Top down decomposition of hypothetical problem "p". It should be noted that although attributes are shown as distinct entities there is no reason why they should not be functionally equivalent; for example in the above tree the nodes a_{21} and a_{32} might correspond to identically the same test. This is demonstrated for example, in the KPa7KR experiment (described later) where the attribute "r2ar8" appears several times in the structured solution.

```
if a24
        then a2D1
else if a21
        then a2D2
else
        a2D1
```

Figure 4/2. Hypothetical expert approved rule for a_2.

```
if a32
        then a3D1
else if a31
        then a3D2
else
        a3D1
```

Figure 4/3. Hypothetical expert approved rule for a_3.

These rules are then substituted for a_2 and a_3 respectively. Interactive ID3 is once again applied and the resulting rule (shown in Figure 4/4) is the solution to the original problem p.

We can now reproduce the problem decomposition diagram with irrelevant attributes removed and an implied left-to-right depth-first execution ordering as shown in Figure 4/5. Notice that if elements of the same decision tree are enclosed (as in this figure) in a separate envelope (dotted lines), then the whole structure maps in an obvious way onto a program in a hypothetical programming language, as shown in Figure 4/6. This correspondence is taken up in this thesis' final chapter, where a description may be found of the CDL-1 rule

```
if a2 = a2D1
        then pD1
else if a3 = a3D1
        then pD2
else if a1 = a1D1
        then pD1
else
        pD2
```

Figure 4/4. The top-level solution to the original problem "p".

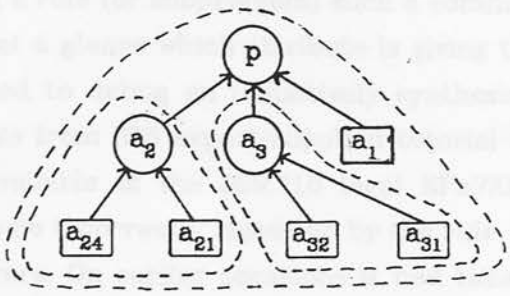


Figure 4/5. Decomposition diagram for hypothetical problem "p". A left-to-right depth-first execution ordering is implied.

p	-	program
a_2 then a_3 then a_1	-	body of p
a_2	-	procedure
a_3	-	procedure
a_1	-	primitive procedure
a_{24} then a_{21}	-	body of a_2
a_{32} then a_{31}	-	body of a_3
a_{24}	-	primitive procedure
a_{21}	-	primitive procedure
a_{32}	-	primitive procedure
a_{31}	-	primitive procedure

Figure 4/6. "p" mapped on to a hypothetical programming language.

language which took origin from the present work.

4.2. Self-commenting

Based on a proposal by D. Michie, self-commenting is a programming technique by which a humanly understandable commentary is generated whenever

the induced rule executes on a test example. This feature has two main uses: a) rule debugging and b) answering the "why did you decide that?" question at rule execution time.

When debugging a rule (or subproblem) such a commentary is helpful since it shows the expert at a glance which attribute is giving the wrong result. Self-commenting was used to debug an inductively synthesized KPa7KR rule (see later). After synthesis from 325 expert-supplied tutorial examples, a complete database became available of the 209,718 legal KPa7KR positions. Of these, fifteen were found to be incorrectly classified by the rule. It took only two hours to correct these errors. On earlier occasions it had taken about twenty times longer to correct errors of comparable subtlety.

The execution-time use has an impact on the problem of human understandability of automatically generated rules. It is not enough to inform someone that a particular rule has been vetted by an expert and is to be trusted. It is also not enough to present the rule as the definitive text on a problem and expect an inexperienced user to gain an expert's insight by inspection of this rule. He requires in addition the freedom to inspect the line of inference whereby the rule-base has arrived at its conclusion.

For self-commenting to work, the expert must insert a question text at the entry point of each problem, subproblem and primitive attribute, and also plant at each exit point a corresponding answer text (e.g. "YES", "NO"). At rule-execution time, the question text corresponding to each activated subproblem and attribute is output before the subproblem or attribute is executed. The question text is kept in the attribute description file where it also acts as a natural language description for the symbolic name given to attributes and subproblems listed in such files.

For example, consider the attribute "katri" from the KPa7KR experiment found in subproblem "dblat". "katri" is described in the "dblat" attribute description file the relevant line of which is shown in Figure 4/7. When Interactive ID3 forms a rule using this attribute it uses this information as shown in Figure 4/8.

This is only half of the commentary text available to the self-commenting system. The other half is the answers to the questions posed at subproblem and attribute entry. These answers are stored at the exit points of the coded attributes and subproblems. If self-commenting is enabled, the text corresponding to the relevant return point is output, forming a question-answer dialogue. For

katri 3 B W N Does any king control intersect point. If so, which

Figure 4/7. A fragment of the dblat attribute description file.

```
enter("katri", "Does any king control intersect point. If so, which", LEVEL);
switch(katri()) {
  case B:
       etc.
}
```

Figure 4/8. A fragment of the dblat decision rule. The function enter is a compile-time option that when enabled, causes its first 2 arguments to be printed and inserts a question mark. LEVEL is a compile-time constant that causes the question to be printed at an indent level corresponding to depth of the subproblem or attribute in the decomposition tree.

example, consider the "C" code for katri (shown in Figure 4/9) - the attribute used in the previous example.

The same commenting convention must be maintained when Interactive ID3 generates compilable code for a subproblem. This subproblem must call comment when it executes a return and the comment that is output must be displayed at a lower indent level since it answers the question that preceded the call to this subproblem. Part of Interactive ID3's output was shown in Figure 4/8, the rest of Interactive ID3's treatment of the call to the katri attribute is shown in Figure 4/10.

The self-commenting output from the induced KPa7KR rule for position WK:g8 WP:a7 BK:c7 BR:d4 (which is critically dependent on the value of katri) is shown in Figure 4/11. (The output that results from code given earlier as examples is shown in italics.)

```
katri() {
   if(bitset(wf, wr, IS_INTERSECT))
      return(comment(E, "%s", "", "", "NO  the WK is in check", LEVEL));
   or(BKING, BATT, 20);
   or(WKING, WATT, 22);
   and(IS_INTERSECT, 20, 21);
   and(IS_INTERSECT, 22, 23);
   if(!zero(21)) {
      if(zero(23))
         return(comment(B, "%s", "YES the BK does", "", "", LEVEL));
      return(comment(E, "%s", "", "", "NO  both do", LEVEL));
   }
   if(!zero(23))
      return(comment(W, "%s", "", "YES the WK does", "", LEVEL));
   return(comment(E, "%s", "", "", "NO  none do", LEVEL));
}
```

Figure 4/9. C code for attribute katri. The function "comment" is a compile-time option that is replaced by its first argument if comments are disabled. Otherwise it causes one of three strings to be output depending on the run-time value of its first argument. If these string slots are empty, the default action is to print out a simple YES/NO answer.

```
enter("katri", "Does any king control intersect point. If so, which", LEVEL);
switch(katri(LEVEL)) {
  case B:
   return(comment(TRUE, FALSE, "", "", "", LEVEL-1));
   break;
  case W:
   return(comment(FALSE, FALSE, "", "", "", LEVEL-1));
   break;
  default:
       etc.
}
```

Figure 4/10. Use of katri in the automatically generated C coded decision rule.

```
Is this position won for white? (pa7)
  Can the BR be captured safely? (rimmx)
  NO
  Does one or more B piece control the queening square? (bxqsq)
  NO
  Is the WK in stalemate? (stlmt)
  NO
  Is there a good delayed skewer threat? (ds)
    Is there a special opposition pattern present? (spcop)
    NO
    Is the WK one away from the relevant edge? (wtoeg)
    NO
  NO
  Is there a simple delay to white's queening the pawn? (dq)
    Is there a good delay because there is a mate threat? (thrmt)
      Does the BR attack a mating square safely? (rxmsq)
      NO because there is no mating square
    NO
    Is there a good delay because the WK is on square a8? (wka8d)
      Is the WK on square a8? (wkna8)
      NO
    NO
    Is there a good delay because the WK is in check? (wkchk)
      Is the WK in check? (wknck)
      NO
    NO
    Is there a good delay because of a double attack threat? (dblat)
      Is the WK on an edge and not on a8? (cntxt)
      YES
      Is the BK in the way? (bkblk)
      NO
      Is the BK attacked in some way by the promoted WP? (bkxbq)
      NO
      Does any king control intersect point. If so, which? (katri)
      YES the BK does
    YES
  YES
NO
```

Figure 4/11. Self-commenting output for position WK:g8 WP:a7 BK:c7 BR:d4. The output that results from code given earlier as examples is shown in italics.

4.3. Post-processing self-commentary texts

Let us now consider how this text might be re-organized (e.g. by a post-processor) so as to avoid the deep nesting structure and associated postponement of answers to questions which are high up in that structure. These features

have arisen because of the convention that explanatory text should appear in exactly the same order as that of their run-time invocation. Perhaps then, we should experiment with ways of relaxing this convention, so as to generate explanatory text with a more event-driven flavour.

A transformation which may be useful for this purpose is analogous to that used to convert ordinary bracketed (infix) algebraic/arithmetic functional expressions into "reverse Polish" (postfix) expressions. By this transformation an expression of the form:

$$(a + b) \times c$$

which need brackets because of the implied precedence of x over +, becomes

$$a\ b + c \times$$

which is executed in a strictly left-to-right order on a one-address machine (i.e. a machine with no memory except a stack) as shown in Figure 4/12.

The infix expression corresponding to the self-commented text shown above is shown in Figure 4/13. The same expression is shown in postfix form in Figure 4/14. The subproblems themselves are treated as operators. They are applied to the contents of the stack on which they leave their result.

This postfix expression could be used to generate an "after the event" explanation like the one shown in Figure 4/15.

A further refinement of the self-commenting idea suggested by Ivan Bratko would optionally condense the commentaries by omitting print-out of strings

---

STACK a
STACK b
ADD (the top 2 items on the stack, replacing them with the answer)
STACK c
MULTIPLY (the top 2 items on the stack replacing them with the answer)

Figure 4/12. Execution of the postfix expression "a b + c ×".

PA7(rimmx bxqsq stlmt DS(spcop wtoeg) DQ(thrmt(rxmsq) WKA8D(wkna8) WKCHK(wknck) DBLAT(cntxt bkblk bkxbq katri)))

Figure 4/13. Infix form for self-commented text shown in Figure 4/11. To avoid bulk only the attribute names not the explanation text, are used. The non-primitives are capitalized for ease of identification.

katri bkxbq bkblk cntxt DBLAT wknck WKCHK wkna8 WKA8D rxmsq THRMT DQ wtoeg spcop DS stlmt bxqsq rimmx PA7

Figure 4/14. Postfix form for self-commented text shown in Figure 4/13. The non-primitives (operators in the postfix expression) are capitalized for ease of identification.

seeing that the BK controls the intersect point
    when the BK is not attacked in some way by the promoted WP
    and the BK is not in the way
    and the WK is on an edge and not on a8
it follows that there is a good delay because of a double attack threat

seeing that the WK is not in check
it follows that there is not a good delay because the WK is in check

seeing that the WK is not on square A8
it follows that there is not a good delay because the WK is on square A8

seeing that the BR does not attack a mating square safely
it follows that there is not a good delay because there is a mate threat

seeing that there is a good delay because of a double attack threat
    when there is not a good delay because the WK is in check
    and there is not a good delay because the WK is on square A8
    and there is not a good delay because there is a mate threat
it follows that there is a simple delay to queening the pawn

seeing that the WK is not one away from the relevant edge
    when there is not a special opposition pattern present
it follows that there is not a good delayed skewer threat

seeing that there is a simple delay to white's queening the pawn
    when there is not a good delayed skewer threat
    and the WK is not in stalemate
    and one or more B piece does not control the queening square
    and the BR can not be captured safely
it follows that this position is not won for white

Figure 4/15. A postfix "after the event" explanation text.

associated with tests which are redundant to predicting the outcome *given the current rule*. Under this regime the postfix print-out given above could appear as shown in Figure 4/16.

This last form is not useful when the commentary is being used to debug the rule (due to the loss of information about other tests that were made). However providing the rule is an accepted orderly solution to the problem this terse form of reporting can be of use as a "to-the-point" explanation. If the reader has in his head a variant rule with reference to which not all the omitted tests are redundant, then in principle he might perceive the explanation as in some way lacking. Equally there could be tests in the text which according to his own rule

```
seeing that the BK controls the intersect point
     when the BK is not attacked in some way by the promoted WP
     and the BK is not in the way
     and the WK is on an edge and not on a8
it follows that there is a good delay because of a double attack threat
from this it follows that there is a simple delay to white's queening the pawn
from this it follows that this position is not won for white
```

Figure 4/16. "To the point" postfix self-commenting.

should have been omitted. The extent to which consumer dissatisfaction from this cause will be encountered is not yet known. The remedy is in any case immediate, since the user is free to switch to one of the unabridged options.

Comment was invited from US National Master Kopec on the three variant forms for the specimen worked example, (1) infix, (2) postfix and (3) abbreviated postfix, given in Figures 4/11, 4/15 and 4/16. His clearly expressed preference placed them in the order (3), (2), (1). Dr. Kopec felt in fact that the abbreviated postfix form promised to be fully adequate to the needs of the run-time expert user.

CHAPTER 5

Plan of the experiments

Two main experiments were performed:

- KPK (BTM)
- KPKR (WP:a7 WTM)

5.1. The first experiment

The first experiment (KPK) was partly to determine whether or not inductive learning techniques were applicable to reasonably complex problems. In this respect the work was largely confirmatory of that done by R. Quinlan (1979). New ground was, however, broken by the introduction of the "brain-compatibility" criterion and of the structured induction technique designed to meet it. It was felt that quantitative information on the cost of manufacturing a solution by induction was needed. Part of the KPK investigation concerned transparency of solutions. The experiment was split into two inductive learning experiments (structured and unstructured) and a database lookup program that performed the same function as the induced rules was added to give a comparison. Both structured and unstructured synthesis made full use of the available databases. No expert-directed learning took place (except in the choice of attributes and subproblems). The main aim of this experiment was to compare the costs associated with a) the synthesis and b) the execution of rules produced by the two styles of computer induction, unstructured (as in Quinlan) and structured.

The task of the expert (Dr T. Niblett) for this experiment was to partition the problem as he deemed necessary and then to aid in the choice of attributes. He also had to examine clashing examples as they arose and suggest the cause. Recall that a clash exists if a set of two or more decision-vectors agree in all values of the attributes but not in the decision-class value. Figure 5/1 shows a clash.

<long-legs, spotted, long-neck, herbivore, GIRAFFE>
<long-legs, spotted, long-neck, herbivore, NOT-GIRAFFE>.

Figure 5/1. A clash.

5.2. The second experiment

The aim of the second experiment (KPa7KR) was to solve a problem using inductive inference that was considered too complex to be hand coded, and furthermore to solve it without the use of a database. WTM was chosen because the previously solved KPK BTM problem is a derived endgame (WK capturing BR results in a BTM KPK game). It was decided to restrict the initial position of the WP to the most difficult single square on the board (a7) as an initial simplification (though still comprising over 200,000 legal positions) and to restrict the outcome values to be won/not won for white (as opposed to won/lost/drawn). This last simplification was to facilitate the generation of a checking database without which no quantitative results would be definitive. By subsequently adding a "lost-for-white" solution (Appendix C) a complete won-drawn-lost classifier was achieved.

Several experts were used for this experiment. Each provided a very useful cross-check on the other. Where possible they were kept from consulting each other so that their approaches were independent. In the U.S.A. Rob Gordon and Marcel Schoppers in a University of Illinois graduate class project (supervised by the author) provided the top-down problem decomposition and the attributes, debugging these until they could detect no mistakes. At this point the resulting decision trees were saved for future evaluation against a database (which was not yet available). These decision trees will be referred to as "stage one" in the results section. Appendix B contains the project report generated from this stage of the research. All three "experts" were far from expert in the unrestricted game of chess, being probably "B" players. KPa7KR is however a small enough sub-game for expertise to be acquirable, as experience showed.

The Gordon/Schoppers/Shapiro solution was brought back to the U.K. where Dr D. Kopec (a US National (FIDE) Master and former Scottish Chess

Champion) was asked to supply examples that he considered representative of the KPa7KR problem with a view to "breaking" the current decision rule. Dr. Kopec supplied examples in groups of about ninety thereby bringing to light a total of 23 "bugs". After the decision trees were debugged against each group of examples supplied by Dr. Kopec they were saved for future evaluation against a database (not available at this stage). Dr. Kopec supplied altogether 263 examples (of which 39 were duplicates) and the decision trees will be referred to as stages two through four in the results section.

The "lost for white" problem has subsequently been solved as part of a class project at the University of Illinois by R. Reinke (supervised by the author) but since the "value for black" database has not been completed at this time, the exact performance of this module although believed high is unknown. Using the tools described previously Reinke arrived at his solution in under two weeks of non-dedicated time. The won-for-white solution together with the lost-for-white solution provides a complete solution for the three valued KPKR (WP:a7, WTM) problem since; if not-won-for-white and not-lost-for-white then a position must be drawn. More recently he has solved WFW/not WFW WP:b7 (again without the use of a database). This solution was found to be 94.8% correct against MIRU's KPKR database. Appendix C contains his project report.

CHAPTER 6

KPK experiment

6.1. Introduction to the topic

The KPK endgame is one of the more elementary chess endgames, yet contains a surprising degree of complexity. It is very hard to program using a conventional tree search plus evaluation function since the maximum depth of win is 38 ply, leaving much domain-specific knowledge to be incorporated in the evaluation function. This allots to the programmer the task of dealing with the many exceptional positions that arise. In this environment it can take months to produce a correct program. Zuidema (1974) gives a good description of the programming problems that can arise from the "exceptions nuisance" with an even simpler ending (KRK) which he was (eventually) able to solve on a no-search basis.

Several attempts have been made to program KPK using a more modular and knowledge-based approach. We shall describe the play of KPK as given in the textbook "Pawn Endings" (Averbakh and Maizelis 1971) where a particularly lucid exposition is given. Following this we describe a top-level strategy developed by Dr. T. Niblett (1982). Although developed for WTM KPK positions this strategy formed the basis of the structured induction experiments performed on BTM KPK positions.

6.2. The play of KPK

The play of KPK is determined by two considerations;

- the square of the pawn, and
- the pawn's critical squares.

The square of the pawn is shown in Figure 6/1. When the pawn's rank is less than 5 the critical squares are as shown in Figure 6/2, otherwise as in Figure 6/3. If the White king can occupy one of the critical squares White wins whoever is to move.

Averbakh treats the rookpawn as a special case, with only one critical square, the location of which is independent of the pawn, as shown in Figure 6/4.

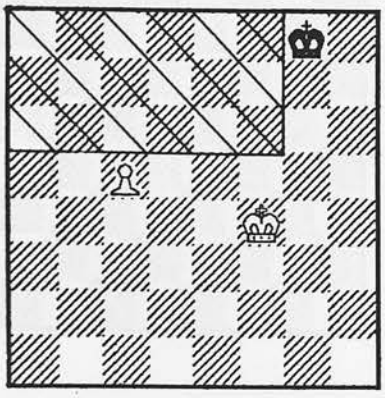


Figure 6/1. The square of the pawn. With White-to-move (WTM) the pawn advances, the square contracts and the Black King cannot move into the new square, allowing the White pawn to queen safely. With BTM the Black king can enter the square and so prevent the pawn queening.

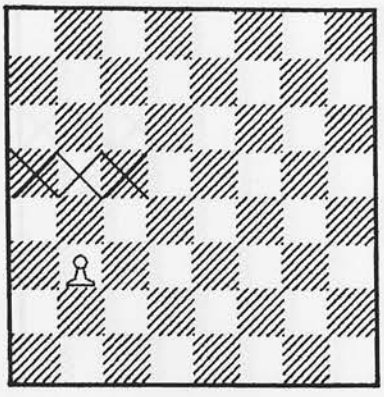


Figure 6/2. The critical squares of the pawn with the pawns rank < 5.

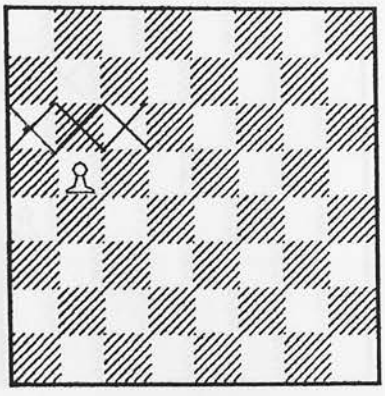


Figure 6/3. The critical squares of the pawn with the pawns rank >= 5.

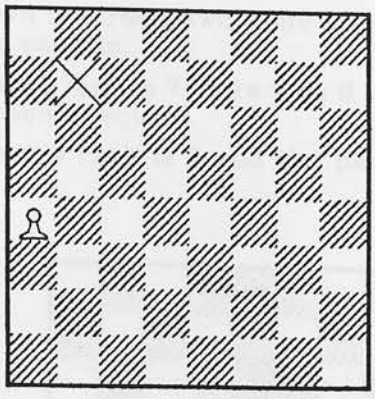


Figure 6/4. The rookpawn's critical square.

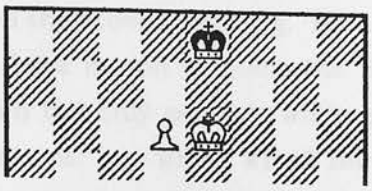
To accompany these general principles Averbakh gives about a dozen examples of play. Although his explanation is lucid, and to a human player complete, not enough information is given to specify correct play in the KPK ending. Bramer (1977) discusses this point. The task of the programmer is to fill in the details, producing a program that plays correctly.

An advice language strategy has been developed at the MIRU (Niblett 1982) which follows the broad outline of Averbakh's text. An English version of this strategy is given below. The KPK game is effectively split into a variety of subgames each of which is easier than the whole KPK game. This is the strategy on which we have based our experiments.

6.3. A top-level strategy for KPK

To determine whether White wins from any WTM position, consider the rules shown in Figure 6/5 in order.

- If the pawn can run White wins.
- Otherwise if the pawn is a rookpawn White wins iff a position is achievable where the pawn can run.
- Otherwise if the pawn's rank is 7 White wins if the White king can safely move next to the queening square.
- Otherwise if the pawn's rank is 6 and the pattern shown below holds White wins.



- Otherwise if the pawn's rank is less than or equal to 5 then White wins if he can achieve the pattern above.
- Otherwise if the position fits the pattern of Figure 6/2 or Figure 6/3 (with the White king on a marked square) White wins.
- Otherwise White wins iff the pattern of Figure 6/2 or Figure 6/3 (with the White king on a marked square) is achievable.

Figure 6/5. Rules for White to win from any WTM KPK position.

6.4. Previous computer work

The history of earlier work on this endgame has been reviewed by Kopec (1982). The main milestones which he discusses are the following.

1977 M.R.B. Clarke's complete lookup database for KPK. For each position the won-drawn status was given and the depth of win or draw (e.g. the depth at which Black captures the WP). A complete statistical tabulation was given.

1975 L. Harris' ALGOL 60 search-oriented program classified positions with approximately 98% accuracy (as later shown by Bitner and Hansche).

1977 D. Beal's exceptionally fast and compact won/drawn classifier took the form of a decision table comprising 48 rules. These proved to be completely opaque to human inspection, even though correct and highly effective. The algorithm did not include move-generation, nor depth information.

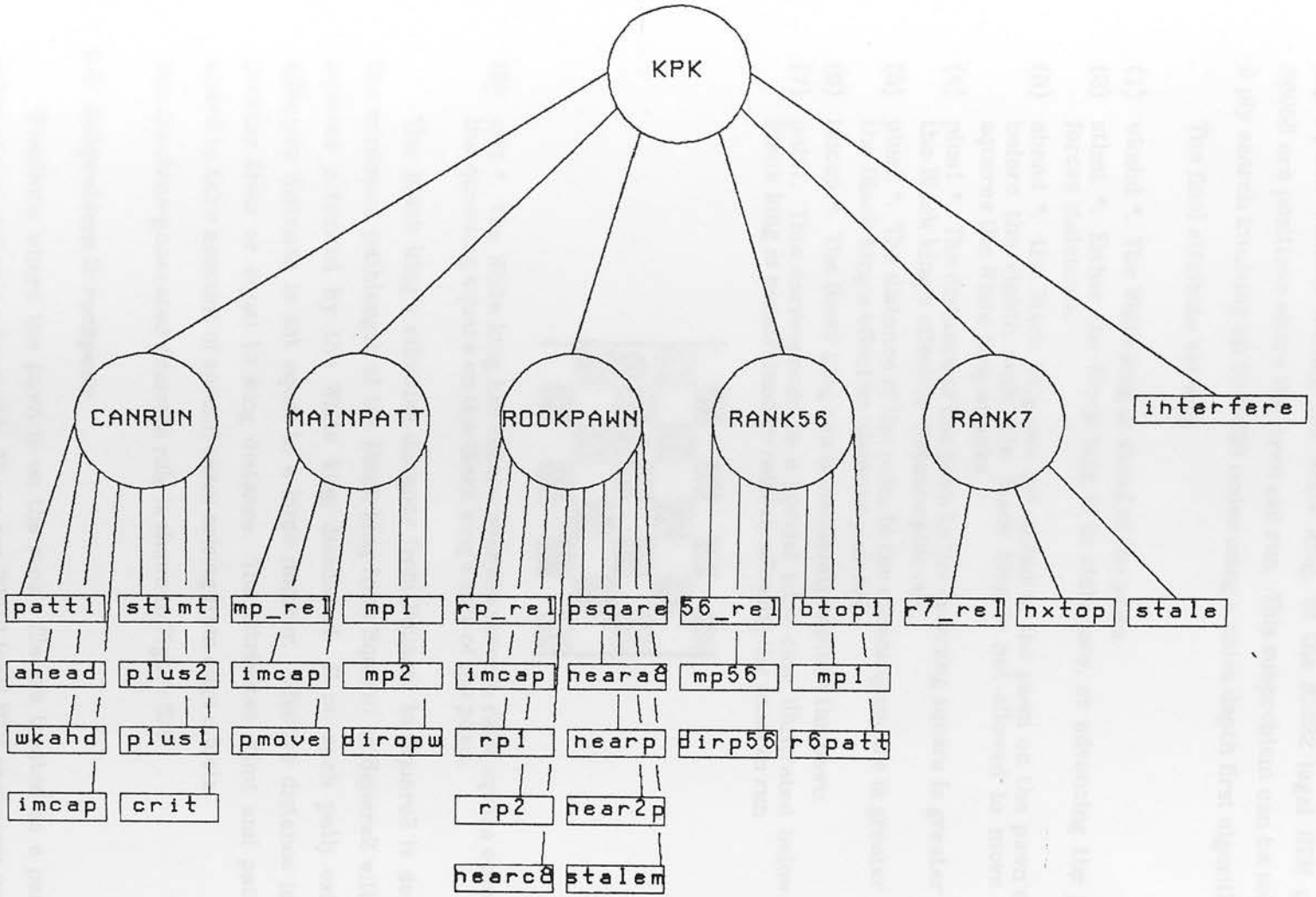
1977 M. Bramer wrote a pattern-based program comprising some 20 rules which made the best White move from any KPK WTM position. In the realm of KPK this miniature "expert system" is the example *par ex-*

cellence of the classical knowledge engineering approach of extracting rules from the expert for codification and testing. In this case extraction proceeded by introspection, since domain expert and programmer were the same person.

6.5. Structured induction of decision rules

The size of the KPK problem space must be discussed, partly because previous published statements on this subject have not been entirely unambiguous or free from error. The figure for the number of legal BTM positions that is usually quoted (97992, (Clarke 1977)) includes as legal 390 positions that have no legal white-to-move predecessor. There are 388 such positions where the black king is in check when the pawn is on the second rank e.g. WK:c1 WP:a2 BK:b3. The point is that the pawn could not have moved to place the black king in check. The other two positions are when the only possible white predecessor moves were king moves from illegal positions. e.g. WK:a1 WP:a2 BK:c1 (BK:c2 generates the other example). That brings the size of the space down to 97602. This figure includes 13980 pawn on rank 8 positions which according to our convention here are no longer within the KPK sub-domain. Thus the figure for the number of legal KPK positions with BTM that is used in this paper is 83622.

The aim was to produce fairly simple decision trees, reflecting the domain expert's intuitive feel for the structure of the various subproblems. We will discuss the individual cases, giving our conclusions at the end. All attributes marked with "*" are evaluated using the CLIP/C emulator. All attributes marked with "O" are subproblems.



6-8

6.6. KPK problem decomposition tree

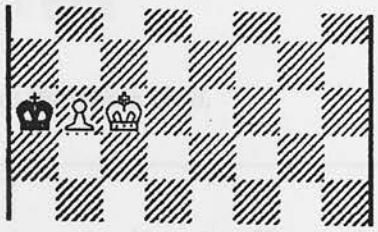
(subproblems are circled, primitive attributes are boxed)

6.7. Subproblem 1: pawn-can-run

The pawn can run in a position if the pawn can safely advance to the queening square without moving the White king. Of the 83622 legal BTM positions, 36958 are positions where the pawn can run. This subproblem can be solved by a 9 ply search involving up to 30000 nodes using a naive depth first algorithm.

The final attribute set was:

(1) wkahd *. The White king is ahead of the pawn.
(2) stlmt *. Either the Black king is in stalemate, or advancing the pawn forces stalemate.
(3) ahead *. the Black king can get ahead of the pawn on the pawn's file, before the eighth rank. The Black king is not allowed to move onto squares the White king attacks.
(4) plus1 *. The distance of the pawn to the queening square is greater than the Black king's effective distance plus one.
(5) plus2 *. The distance of the pawn to the queening square is greater than the Black king's effective distance plus two.
(6) imcap *. The Black king can immediately capture the pawn.
(7) patt1. This corresponds to a special edge case illustrated below. the Black king is constrained to retreat, allowing the pawn to run.



(8) crit *. The White king has control over the seventh rank square covering the queening square on the Black king's side of the pawn.

The Black king's effective distance from Square1 to Square2 is defined as the minimum pathlength of the Black king from Square1 to Square2 with all the squares attacked by the White king disallowed. If no such path exists the effective distance is set equal to a large number. Effective distance is always greater than or equal to king distance. The attributes stlmt and patt1 were added to take account of special cases arising from edge effects.

The machine-generated decision rule is shown in Figure 6/6.

6.8. Subproblem 2: rookpawn

Positions where the pawn is on the rook's file are treated as a particular subproblem, following Averbakh. There are 20919 legal BTM rookpawn positions, 12312 won for White. This was the most difficult problem, probably because

No of examples in final working-set = 16.
No of nodes in final tree = 17.
Time taken to generate rule = 0.60 CPU seconds.

```
patt1
    f: ahead
        f: wkahd
            f: imcap
                f: stlmt
                    f: plus2
                        t: plus1
                            f: crit
                                f: FALSE
                                t: TRUE
                            t: FALSE
                        f: TRUE
                    t: FALSE
                t: FALSE
            t: FALSE
        t: TRUE
```

Figure 6/6. pawn-can-run decision tree.

Averbakh's guidelines were inadequate. We needed 8 iterations to produce a clash-free attribute set. The initial attribute set had 4 elements, the final one 10.

The final attribute set was:

(1) rookpawn_relevant. Is pawn on rooks file and not able to run.
(2) nearc8 *. The Black king can reach c8 before the White king.
(3) neara8 *. The Black king can reach a8 before the White king.
(4) nearp. The Black king is nearer the pawn than the White king.
(5) psquare *. The Black king can move inside the "pawn's square".
(6) rp1. The pattern shown in Figure 6/7.
(7) rp2 *. The Black king can trap the White king on the edge of the board.
(8) imcap *. Black can immediately capture the pawn
(9) near2p *. Essentially the pattern shown in Figure 6/8.
(10) stalemate *. Positions which lead to stalemate.

The machine-generated decision rule is shown in Figure 6/9.

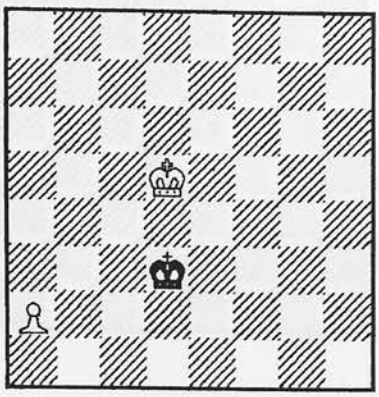


Figure 6/7. rookpawn pattern 1 (rp1). BK->c3 (to get near enough to the WP to capture it). If the WP->a4 then BK->b4 will win the WP. If instead of WP->a4, White plays WK->c5 then BK->b4 or BK->b3 are illegal (BK moves into check). The nearest the BK can get to the WP is b2 but then WP->a4 allows the WP to beat the BK to the "queening square".

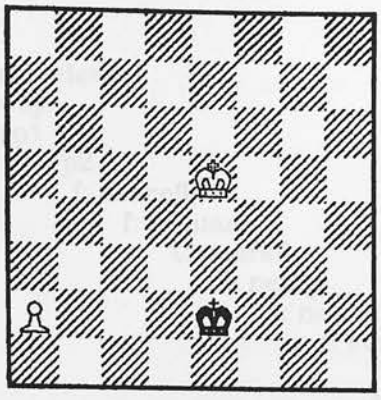


Figure 6/8. Can the BK get next to the WP? (near2p). BK->d3 (to get near enough to the WP to capture it). If the WP->a4 then BK->c4 will ensure that the WP doesn't escape. If instead of WP->a4, White plays WK->d5 then BK->c4 becomes illegal and to maintain a threat to the WP, Black must play BK->c3. We have now reached the position shown in Figure 6/7.

No of examples in final working-set = 20.
No of nodes in final tree = 20.
Time taken to generate rule = 0.84 CPU seconds.

```
rookpawn_relevant
   t: imcap
      f: rp1
         f: rp2
            f: nearc8
               f: psquare
                  t: neara8
                     f: nearp
                        f: near2p
                           f: stalemate
                              f: TRUE
                              t: FALSE
                           t: FALSE
                        t: FALSE
                     t: FALSE
                  f: TRUE
               t: FALSE
            t: FALSE
         t: TRUE
      t: FALSE
   f: FALSE
```

Figure 6/9. rookpawn decision tree.

6.9. Subproblem 3: get-to-main-pattern

This attribute deals with all positions where main-pattern holds or is achievable, the rank of the pawn is less than five, and the pawn is not a rookpawn. There are 31185 positions in this category, 12796 of which satisfy the advice. The vast majority of positions were easy to classify with a few hundred proving harder because the pawn is on the second rank, and/or both pieces have to move to achieve main-pattern.

The final attribute set was:

(1) mainpatt_relevant. Pawn is not on rooks file and pawn rank <= 5.
(2) imcap *. The Black king can take the pawn immediately.
(3) nearerx. The Black king can get ahead of the White pawn, disallowing mainpatt.
(4) diropb *. The Black king has the opposition and the White king is not more than 1 rank ahead of the pawn.

(5) diropw *. The White king is one rank ahead of the pawn and has the opposition.

(6) anyop *. White has the distant opposition and can get ahead of the pawn.

(7) mp1 *. Moving the White king alone White can get to mainpatt 1 rank ahead of the pawn.

(8) mp2 *. Moving the White king alone White can get to mainpatt 2 ranks ahead of the pawn.

(9) pmove. By first moving the pawn White can get to mainpatt.

The machine-generated decision rule is shown in Figure 6/10.

6.10. Subproblem 4: rank56

Rank56 holds in any position where the pawn's rank is 5 or 6, it is not a rookpawn, and either mainpatt or r6patt holds or is achievable.

This problem was easy compared with the previous examples since no special cases arose due to the edge of the board or the pawn's initial double move. There are 21012 legal positions of which 9768 satisfy rank56.

No of examples in final working-set = 7.
No of nodes in final tree = 13.
Time taken to generate rule = 0.28 CPU seconds.

```
mainpatt_relevant
    t: imcap
        f: pmove
            f: mp1
                f: mp2
                    f: diropw
                        f: FALSE
                        t: TRUE
                    t: TRUE
                t: TRUE
            t: TRUE
        t: FALSE
    f: FALSE
```

Figure 6/10. main-pattern decision tree.

The final attribute set was:

(1) rank56_relevant. Pawn is not on rooks file and is on rank five or six.
(2) dirop56. The 6th rank pattern holds or can be achieved.
(3) btop1 *. The Black king is or can get directly in front of the pawn.
(4) mp56 *. By moving the king alone White can get to main-pattern.
(5) mp1 *. Moving the king alone White can get to main-pattern 1 rank ahead of the pawn
(6) r6patt. Special pattern shown in Figure 6/11.

The machine-generated decision rule is shown in Figure 6/12.

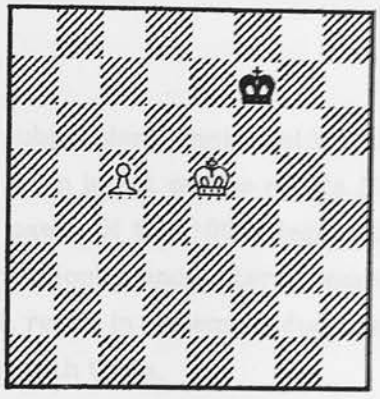


Figure 6/11. Can White take the opposition from Black? (r6patt). Here BK->e7 (opposition) is answered by WP->c6. Then after BK->d8 or BK->e8, White can take the opposition.

No of examples in final working-set = 7.
No of nodes in final tree = 13.
Time taken to generate rule = 0.38 CPU seconds.

```
rank56_relevant
    t: mp56
        f: dirop56
            f: btop1
                t: mp1
                    f: r6patt
                        f: FALSE
                        t: TRUE
                    t: TRUE
                f: TRUE
            t: TRUE
        t: TRUE
f: FALSE
```

Figure 6/12. rank56 decision tree.

6.11. Subproblem 5: rank7

This was the simplest subproblem. Rank7 holds if the pawn is on the seventh rank, canrun is false, the pawn is not on the rook's file and the white king can force its way next to the pawn. Of the 10506 legal rank7 positions 237 satisfy these conditions. The only difficulty encountered was that 3 positions that logically ought to satisfy rank7, result in stalemate due to edge conditions. An extra attribute was added to cope with them.

The final attribute set was:

(1) rank7_relevant. Pawn is not on rooks file and is on rank 7.
(2) nxtop *. The White king can force its way next to the pawn.
(3) stale *. Initial stalemate condition.

The machine-generated decision rule is shown in Figure 6/13.

6.12. Top level solution for KPK

At this point our use of Quinlan's ID3 moves to a higher level in which we treat the previously-synthesized decision trees as primitive attributes. Quinlan's ID3 can then be applied again to form a decision tree in terms of these *sub-trees*.

No of examples in final working-set = 3.
No of nodes in final tree = 7.
Time taken to generate rule = 0.20 CPU second.

```
rank7_relevant
        t: nxtop
            t: stale
                f: TRUE
                t: FALSE
            f: FALSE
        f: FALSE
```

Figure 6/13. rank7 decision tree.

Although it has been proved that a combination of these subproblems is sufficient to determine the value of any position for WTM, this turns out not to be the case for BTM positions. There is a small class of positions for which an extra attribute must be added. Consider the position shown in Figure 6/14. To deal with this an extra attribute called interfere was added to detect patterns like that shown in this figure.

The attribute set for the complete KPK Black-to-move problem was: O can-run, O rookpawn, O mainpatt, O rank56, O rank7 and interfere. The first five of these attributes represent the previously-synthesized decision trees.

The machine-generated decision rule is shown in Figure 6/15.

6.13. Unstructured induction of decision rule

The final problem we tackled was to produce a decision tree using attributes from all the subproblems, without regard for the imposed top-down structure of the problem. Our intention was to get some idea of the extra complexity introduced, in human terms, by removing our previous structuring of the problem. In addition we would be able to compare the absolute sizes of the two decision trees produced for the full KPK game.

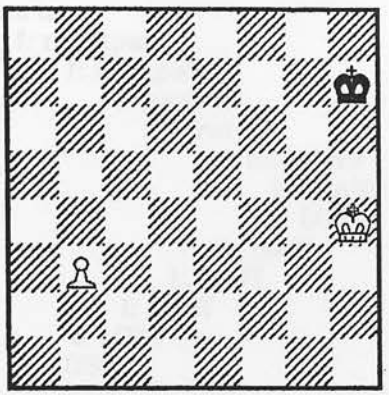


Figure 6/14. Interference between attributes - a case study. In this position with BTM Black can prevent White forcing main-pattern by BK:h6, and canrun by BK:g6, BK:g7 or BK:g8. Black cannot prevent *both* of these however, and this situation is precisely where the attributes given by the 6 decision rules for the subproblems are inadequate.

No of examples in final working-set = 9.
No of nodes in final rule = 13.
Time taken to generate rule = 0.30 CPU seconds.

```
canrun
    f: mainpatt
        f: rookpawn
            f: rank56
                f: rank7
                    f: interfere
                        f: DRAWN
                        t: LOST
                    t: LOST
                t: LOST
            t: LOST
        t: LOST
```

Figure 6/15. top-level kpk decision tree.

The 32 attributes used were:

- imcap wkahd stlmt ahead plus1 plus2 patt1 crit
  from the CANRUN set of attributes.
- rookpawn_relevant nearc8 neara8 nearp psquare rp1 rp2 near2p stale-mate
  from the ROOKPAWN set of attributes.
- mainpatt_relevant nearerx diropb diropw anyop mp1 mp2 pmove
  from the MAINPAT set of attributes.
- rank56_relevant dirop56 btop1 mp56 r6patt
  from the RANK56 set of attributes.
- rank7_relevant
  from the RANK7 set of attributes.

It was also necessary to include "interfere". The decision tree generated from these attributes can be found in Appendix D. It should be noted that six attributes (patt1 nearerx, mainpatt_relevant, nxtop and stale) were shown to be unnecessary for this particular unstructured solution. They were left out of the decision tree by Quinlan's ID3. By varying the user-supplied parameters to Quinlan's ID3, ie. size of the initial working-set, number of contradicting decision-vectors to be added to the working-set with each iteration and the pseudo-random number seed, it was possible to generate a whole series of unstructured decision trees each equivalent in function but omitting different

subsets of the above five attributes in the different cases. In all, 128 unstructured trees were generated. The particular unstructured tree detailed in this paper was one of the best (i.e. smallest number of nodes). The user-supplied parameters to Quinlan's ID3 which were discussed earlier did not appear to have any effect on the structured tree. Criteria for the choice of this tree are discussed in the next section.

6.14. Costs associated with decision rules

Inductive synthesis by machine of machine-executable descriptions can be seen as a *manufacturing process*, with the customary categories of cost, efficiency etc. The analogy can be taken further by likening the databases used to a factory building, the computer to a complex of machine tools and the system programs within the computer to the tool settings. The production side of this manufacturing process can be viewed as the development of attributes (components), the determination of legal decision-vectors (performance and design specification of product) and the synthesis of decision trees (assembly of production prototypes from components). Since both the structured and unstructured trees used the same attributes, the cost of developing attributes for these solutions (pilot R&D) was the same. However the other two components of this manufacturing cost (decision-vector generation and decision tree synthesis) merit some discussion. The first of these (the tooling up cost) is a cost that is independent of the number of decision trees finally produced (all 128 unstructured trees were generated from the one set of decision-vectors). Figure 6/16 shows the figures for decision-vector generation. The measure of computational cost chosen was that of store times time, in kilo-byte second (kbs for short).

To evaluate the second component of manufacturing cost (decision tree synthesis) it is necessary to know the size of Quinlan's ID3, the average CPU time to generate an unstructured tree and the total CPU time to generate the structured solution. Figure 6/17 shows the relevant figures.

The unstructured solution needed a larger version of Quinlan's ID3 to cope with the extra number of attributes present at one time. This alteration did not add to the tree synthesis time since the structured trees took the same time to generate on the larger Quinlan's ID3. It must be assumed that the discrepancy in tree synthesis cost is a true reflection of the added complexity introduced when generating large unstructured decision trees.

Problem	Time (CPU seconds)	Size (kbytes) program+database	No. of legal vectors	Cost (kbs)
Structured				
canrun	1760	9.07+12.29	51	37590
rookpawn	543	10.16+ 3.69	69	7521
mainpat	881	9.14+ 7.37	14	14545
rank56	665	10.98+ 4.91	11	10566
rank7	189	9.20+ 2.46	3	2204
kpk (top-level)	2116	9.27+30.72	12	84619
Total			160	157045
Unstructured	7252	15.14+12.29	1901	198922

Figure 6/16. Cost of decision-vector generation. Note the cost of generating decision-vectors for the structured solution is the sum of the costs for the six sub-problems.

Problem	Time (CPU seconds)	Size (kbytes)	Cost (kbs)
Structured			
canrun	0.60	35.27	21.16
rookpawn	0.84	35.27	29.63
mainpat	0.28	35.27	9.88
rank56	0.38	35.27	13.40
rank7	0.20	35.27	7.05
kpk (top-level)	0.30	35.27	10.60
Total			91.72
Unstructured	140.00	40.62	5686.80

Figure 6/17. Decision tree synthesis cost. Note the non-linear increase in the cost of generating "large" decision trees.

As with other manufacturing exercises the accuracy and efficiency of the manufactured products themselves is also important. To evaluate this factor the decision trees were run and checked for accuracy against a database (Clarke 1977). All the decision trees generated were 100% accurate. Because there were so many unstructured trees (128 were tested) the statistics relating to this group is split into three parts; best, average and worst. The same measure of cost (the product of store and processing time in kilo-byte seconds) was used. Only one test program was used to determine the execution cost of each decision tree. Many features of this program could be enabled or disabled using a technique of *conditional compilation*. By disabling both the database checking and the tree checking, a measure for the overheads of this program (legal move generation, statistics gathering and evaluation, variable initialisation etc.) could be determined. Subtraction of these values from the raw data yielded the adjusted or true costs. The raw figures are shown in Figure 6/18, Figure 6/19 and the average time to classify a KPK position with BTM is shown in Figure 6/20.

Overhead		
kbytes	seconds	kbs
7.234	15.7	114

Database (program + data) + Overhead		
kbytes	seconds	kbs
8.084 + 12.288 = 20.372	391.7	7979

Structured tree + Overhead + Database evaluation		
kbytes	seconds	kbs
14.884	1984.7	29540

Unstructured tree + Overhead + Database evaluation (average of 128 trees, working-set increments from 1-1900)		
kbytes	seconds	kbs
15.062	2389	35983

Unstructured tree + Overhead + Database evaluation (best, working-set increment 250)		
kbytes	seconds	kbs
15.012	1555	23344

Unstructured tree + Overhead + Database evaluation (worst, working-set increment 1600)		
kbytes	seconds	kbs
15.114	3144	47518

Figure 6/18. Raw run-time statistics. These statistics were collected on the run-time costs of decision trees and database when used to classify KPK with BTM.

Database		
kbytes	seconds	kbs
13.14	376	4941

Structured tree		
kbytes	seconds	kbs
6.8	1593	10832

Unstructured tree (average)		
kbytes	seconds	kbs
7.0	1997	13979

Unstructured tree (best)		
kbytes	seconds	kbs
6.9	1163	8025

Unstructured tree (worst)		
kbytes	seconds	kbs
7.0	2752	19264

Figure 6/19. Actual cost of classifying all KPK positions with BTM.

Technique	Time (msecs.)
Database	4.5
Structured tree	19.1
Unstructured tree (average)	23.9
Unstructured tree (best)	13.9
Unstructured tree (worst)	32.9

Figure 6/20. Average time to classify a KPK position with BTM.

6.15. Programmer costs

It took about 6 man weeks to complete this portion of the work described. At least half this time was spent considering special cases for the two subproblems rookpawn and get-to-mainpatt. This was due to considerable difficulty found with the iterative refinement of the attributes (steps 3 and 4 above). When there were only one or two classes of clash the choice of refinement was critical. Changing the definition of an attribute to eliminate one clash caused other clashes with different attributes. This arises when the attributes do not strictly delineate between drawn and won positions over the whole space, but only for the clash being considered. Thus, any change had to be considered for its global effect rather than just locally. It is not clear whether this was due to a bad choice of attributes for the two problems since it did not happen elsewhere, or whether the problems were inherently more difficult. Further experiments should clear this point up.

Despite these problems, total programming time was far faster than with conventional techniques, where the problem can consume several programmer months.

6.16. Structural features of rules

An interesting point that arose from our work on the subproblems is the uniform nature of the decision trees created. Each has one branch with depth one offshoots and this form *also* characterizes the tree synthesized in the final top-level pass (see earlier). This may correspond to the domain expert's conceptualisation of the problem. It is certainly very different from the result Quinlan

(1979) reports for the KRKN ending, and from the trees synthesized for KPK using unstructured induction.

Another point concerns the ubiquitous "special cases". If we eliminate the two attributes PATT1 and STLMT dealing with special cases in the pawn-can-run problem, we produce a 13-node rule (instead of 17) which correctly evaluates 99.998% of the pawn-can-run space. For more complex problems it may be more effective to produce a rule which leaves incorrectly classified a small class of such exceptions, thus keeping the rule within the "human window" (Michie 1983a). This could produce compact, humanly accessible rules which people could use successfully in the vast majority of cases, accepting the penalty of having to resolve the remaining cases by other means.

Looking at the manufacturing costs it seems that decision-vector generation for the structured solution is 20% cheaper than for the unstructured solution. For this particular problem this represents a saving of over 40,000 kilo-byte seconds. Furthermore, to be reasonably sure of generating an unstructured decision tree that is more cost effective than the structured tree, several unstructured trees must be sampled, the production cost of each one being over 5000 kbs. Since only 36 of the 128 unstructured solutions were cheaper to run than the structured solution, to achieve a cost effective result a number of unstructured trees would need to be generated. This makes the cost of manufacturing a good unstructured solution greater than 40,000 kbs. This is very much more expensive than the structured solution and adds emphasis to the point that adding a humanly understandable structure does not necessarily add to the cost of solving a problem.

CHAPTER 7

KPa7KR experiment

7.1. Introduction to the topic

In relation to its complexity the text book treatment of KPKR is even more sketchy than that of KPK. Indeed the particular subset of KPKR that was chosen for this experiment (WTM WP:a7) is not mentioned at all in any of the books (listed in Appendix E) consulted by the author. This is in line with the conclusion to which the author's investigations have driven him namely that *the fraction of expert skill which is explicitly accessible (and hence can be codified in books) is a decreasing function of problem-domain complexity.* This conclusion if generally valid would have a bearing on the "complexity barrier" by which further progress in expert systems work is now confronted.

The original problem decomposition was attempted according to the following rationale:

Since the white pawn is one square away from queening there is only one reason for White not winning i.e. White is delayed from safely promoting the pawn. Thus the top-down problem decomposition centres on what might be the existing delays to White's promoting the pawn and when such a delay exists can Black capitalize? Due to the good potential mobility of the black rook, Black will either capitalize quickly or not at all. The types of delay that are reflected in the decomposition structure are:

- Double attack

  Delays of this category are encountered when the white king is skewered against the promoted pawn by the black rook or when the black rook forks the white king and promoted pawn. Consider the positions shown in Figure 7/1.

- Delayed skewer

  This type of position can result in a double attack (described above). An example of a "delayed skewer" is given in Figure 7/2.

- White king in check

  Another potential delay arises when White is in check. Since the white king

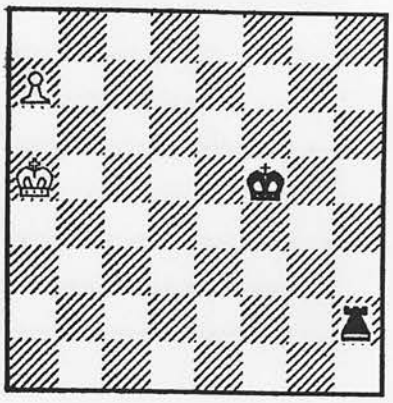


Figure 7/1. Example of a "double attack" position. If White pushes the pawn to a8 and promotes then Black can move its rook to a2 with check. White has to move his king out of check thereby allowing the black rook to capture the newly promoted white pawn. Complications arise when promoting the pawn puts the black king in check (thus preventing the rook skewer) e.g. with BK->d5. BR:g7 introduces another complication; as the pawn promotes then BR->a7 can be taken by the promoted WP (Q or R). Further problems arise when the white king is too close to the square on which the black rook is threatening to skewer e.g. with BR:g4, pawn promotes, BR->a4, white king can now take the rook. Taking this last variation further with the starting position WK:a5 WQ:a8 BK:b3 BR:g4, the white king is prevented from taking the rook because to do so would place the white king in check. In all of these cases if White does not promote the pawn then the black rook can at worst move to cover the 8th rank which is guaranteed to allow the rook to capture the pawn. This capture may not be safe for Black but it will always result in a not-won-for-white game value.

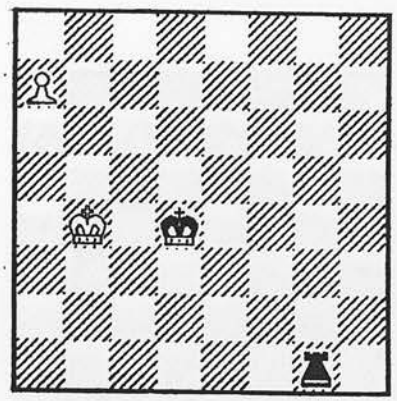


Figure 7/2. Example "delayed skewer". If White promotes the pawn then BR->b1 with check. The white king has to move out of check but is constrained to move leftward because moving to the right would be into check from the black king. On moving to the left (a3, a4 or a5), BR->a1 (check) results in an immediate skewer. The same type of complications set in as with the immediate skewer except now that two white king moves are involved, the white king distance to the black rook skewering square must be one greater than before e.g. BR:g2 will not work because WP->a8, BR->b2 (check), WK->a3, now BR->a2 can be captured by the white king.

must move out of check, the black rook may be able to move to the 8th rank or the "A" file thus preparing to capture the pawn which is threatening to promote. For example see Figure 7/3.

- White king on pawn promotion square

  When the white king is on the pawn promotion square the delay involved is due to White having to move this piece before promoting the pawn. If Black can safely attack the promoting square in one move then the value is not-won-for-white. An example of this type of position is given in Figure 7/4.

- Black is threatening checkmate

  The idea here is that if Black is threatening to checkmate White then White

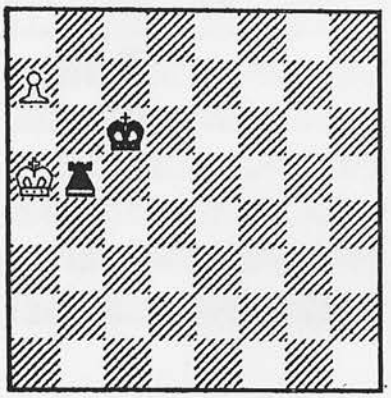


Figure 7/3. Example "white king in check" delay position. Here the white king is in check and must move out of the way, but is constrained to remain on the "A" file (a4 or a5) by the rook. The white king cannot take the rook because it is defended by the black king. After the white king has moved the black rook would normally be moved to rank 8 (to guarantee a not-won-for-white value) but in this case the 8th rank square is defended by the white pawn. The BR->a5 move is defended by the white king. In this case BR->b1 introduces a good skewer threat and results in not-won-for-white. Other complications involve multiple check where white king and black rook may walk from one end of the board to the other before a result is obtained.

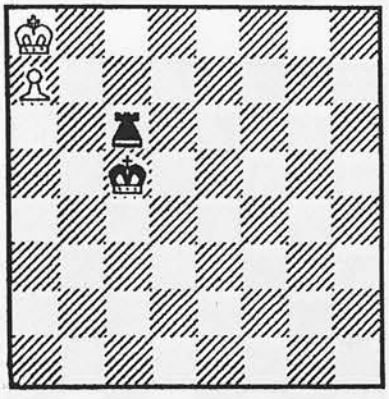


Figure 7/4. An example "white king on pawn promotion square" position. The white king must move to either b7 or b8. The black rook will be moved to a6 to ensure that the promoting pawn can be captured. However if the initial white king move is to b8, the WR to a6 move is blocked. Blacks answer to this is BR->b6 (check) this forces the white king to a8 or c8 or c7. Which ever of these moves White makes, Black answers with BR to a6 eventually resulting in the capture of the pawn and a not-won-for-white value. In order for this type of complication to work, the black king must defend the BR to b6 move.

must do something about this threat possibly resulting in a delay to promoting the pawn. Here is a possible delay of this type that actually does not result in a delay to White promoting the pawn. For example see Figure 7/5.

The complete (100% correct) decision structure comprised 9 trees at 4 different levels, making up 108 nodes in all. This structure was run 15 times on all the legal positions in the KPa7KR WTM domain. The timing results are shown in Figure 7/6.

7.2. Previous computer work

Apart from automatic construction of lookup databases, no computer construction of verifiably correct knowledge systems has previously been recorded on this endgame (with or without restrictions to any one piece), nor indeed on

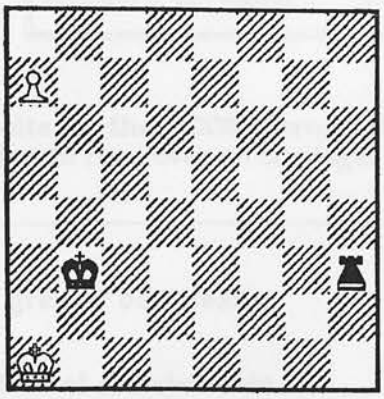


Figure 7/5. A "threatened checkmate" position. Here the threat is that Black may move his rook to h1 with checkmate. However if WP->a8 promotes to a queen, the h1 square is attacked by this piece. If the starting position was with BR:h4 then after WP->a8, BR->a4 (check) and then the newly promoted queen must take the BR (WQ->a4) and BK->a4 captures the queen, result not-won-for-white (draw).

Cost of classifying all legal (209718) positions (kbs)	
mean	29224.1
standard deviation	674.1

Figure 7/6. Timing results for the (100% correct) structured decision rule. Mean and standard deviation for 15 runs over all the legal KPa7KR WTM positions."

any other endgame of greater complexity.

7.3. Structured induction of decision rule

Since all the examples originated from domain experts it was essential to produce simple decision trees that could readily be understood so that the expert could improve them after inspection by incremental addition of refutation examples. This also meant that the attributes themselves had to be simple. The rules of thumb used to make sure this was the case were a) attributes should not be longer than a screenful of C code and b) subproblems should have solution trees invoking no more than seven attributes. The latter was *strictly* adhered to (none of the solution trees exceeded seven attributes). The former was treated mainly as a guideline. However when one of the attributes (btoqs in subproblem wkchk) grew to 55 lines of C code it was turned into a subproblem. The process of turning btoqs into a subproblem was semi-automatic as follows:

(1) Take all the examples so far supplied by the experts and evaluate the original function (in this case btoqs) storing the result for future use as position/class-value pairs.
(2) Inspect the original function code and draw a box around each return point and its associated code.
(3) Group the boxes into larger more meaningful boxes and give each of these a name.
(4) Invoke Interactive ID3 on these named groups of code using the position/class-value pairs in (1) as training examples.

The use of Interactive ID3 allows for the possibility of a more sensible ordering of code than was present in the original function. Appendix F shows the original btoqs attribute code and the name assignments made by the author. The

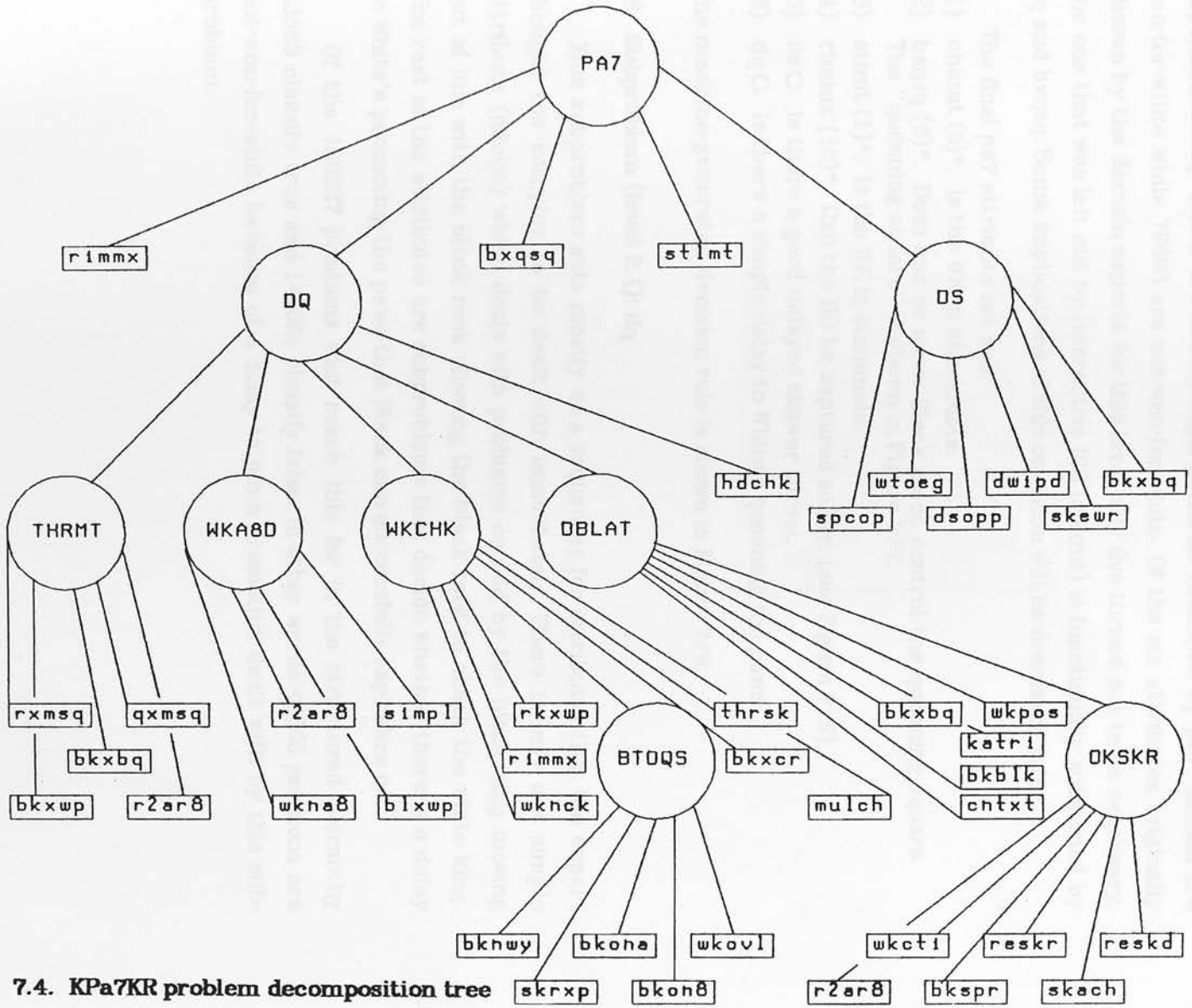
original 55 lines of C code became 61 lines but of the five attributes chosen, none exceeded 21 lines and the result was a more humanly understandable function.

The average length of hand-coded attributes was about fifteen statements of C. Once again, the problem decomposition reflected the domain expert's intuitive feel for the problem's structure. This intuitive feel caused a possible anomaly; the subproblem "ds" (delayed skewer) should probably be housed within "dq" (good delay to queening) but one of the experts (Dr. Kopec) was happier with it being a top-level subproblem.

A full description of the attributes and subproblems follows. Attributes that are marked with "*" are evaluated using the CLIP/C emulator. Attributes that are marked with "O" are subproblems. The number in brackets after every C-coded attribute is the number of C statements that defines that attribute. There are two exceptions to this measurement:

(1) The top-level attributes chkmt (5 lines) calls two other C functions (white_king_in_check (6 lines) and no_white_king_move (3 lines)).
(2) The top-level attribute stlmt (1 line) calls three C functions (white_king_in_check, no_white_king_move and no_white_pawn_move (7 lines).

These other C functions do not figure in the attribute size measurement as they are treated as primitive functions like those of the CLIP emulator (e.g. cand (5 lines)). One-line attributes are of the form "return(<expression>)".



7.4. KPa7KR problem decomposition tree

(subproblems are circled, primitive attributes are boxed)

7.5. Subproblem (level 1): pa7

This is the top-level tree for KPKR with WP:a7 and WTM. It was the last tree in the structured set to be generated (due to the requirements of bottom-up implementation). There are 209718 legal positions classified by pa7, 129825 are won-for-white while 79893 are not-won-for-white. Of the six attributes originally chosen by the domain experts for this level only five turned out to be necessary. The one that was left out by Interactive ID3 (chkmt) is functionally subsumed by dq and bxqsq. Some implications of this omission will be discussed later.

The final pa7 attribute set was:

(1) chkmt (5)*. Is the WK in checkmate.
(2) bxqsq (6)*. Does one or more Black piece control the queening square. The "queening square" is shown in Figure 7/7.
(3) stlmt (1)*. Is the WK in stalemate.
(4) rimmx (15)*. Can the BR be captured safely. (see Figure 7/8).
(5) ds O. Is there a good delayed skewer threat.
(6) dq O. Is there a simple delay to White's queening the pawn.

The machine-generated decision rule is shown in Figure 7/9.

7.6. Subproblem (level 2.1): dq

This subproblem acts mostly as a sequencer for attributes that the expert thought too complex to be dealt with immediately. There is only one simple attribute (hdchk) which deals with problems caused by the black king moving out of line with the black rook allowing the black rook to check the white king. The rest of the attributes are subproblems that decide whether there is a delay to White's promoting the pawn that Black can successfully capitalize on.

Of the 188327 positions that reach this far in the structured hierarchy 43663 classify true and 144664 classify false. In other words 43663 positions are not-won-for-white because of a delay to pawn promotion dealt with by this subproblem.

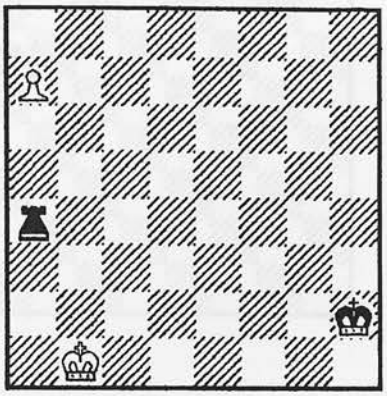


Figure 7/7. The "queening square". Here the pawn promotion (queening) square (marked) is controlled by the black rook.

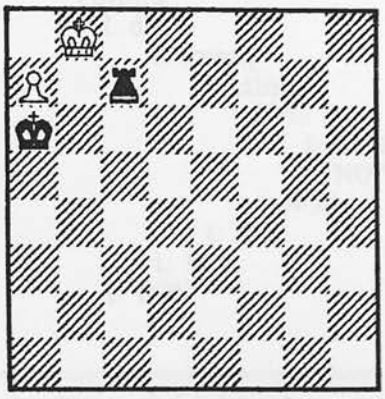


Figure 7/8. Can the BR be captured safely? In this position it would not be considered safe for the white king to capture the black rook because the black king would then take the white pawn.

No of examples in final working-set = 6.
No of nodes in final rule = 11.
Time taken to generate rule = 0.33 CPU seconds.

```
rimmx
  f: dq
      f: bxqsq
          f: stlmt
              f: ds
                  f: WON
                  t: NOT
              t: NOT
          t: NOT
      t: NOT
  t: WON
```

Run-time behaviour for each test.		
test	number decided	leaving
rimmx	21391 (WON)	188327 undecided
dq	43663 (NOT)	144664 undecided
bxqsq	36081 (NOT)	108583 undecided
stlmt	48 (NOT)	108535 undecided
ds	101 (NOT)	108434 (WON)

Figure 7/9. pa7 (top-level) decision tree.

The final dq attribute set was:

(1) thrmt O. Is there a good delay for Black to White's promoting the pawn because there is a mate threat.

(2) wka8d O. Is there a good delay for Black to White's promoting the pawn because the white king is on square a8.

(3) wkchk O. Is there a good delay for Black to White's promoting the pawn because the white king is in check.

(4) dblat O. Is there a good delay for Black to White's promoting the pawn because of a double attack threat.

(5) hdchk (12)*. Is there a good delay for Black to White's promoting the pawn because there is a hidden check (see Figure 7/10).

The machine-generated decision rule is shown in Figure 7/11.

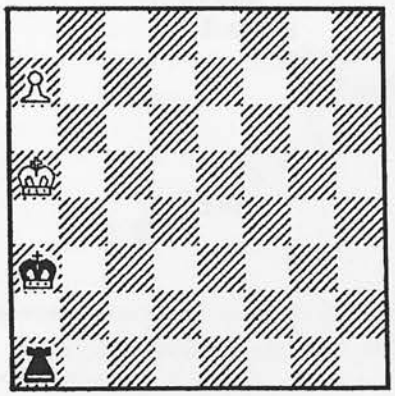


Figure 7/10. A "hidden check". If WP->a8 then BK->b[2-3] which is a good immediate skewer and not-won-for-white. If White does not promote the pawn then Black makes the same move and threatens to take the pawn (a move that will still result in a not-won-for-white value).

No of examples in final working-set = 6.
No of nodes in final rule = 11.
Time taken to generate rule = 0.12 CPU seconds.

```
thrmt
  f: wka8d
    f: wkchk
      f: dblat
        f: hdchk
          f: FALSE
          t: TRUE
        t: TRUE
      t: TRUE
    t: TRUE
  t: TRUE
```

Run-time behaviour for each test.		
test	number decided	leaving
thrmt	821 (TRUE)	187506 undecided
wka8d	3152 (TRUE)	184354 undecided
wkchk	32698 (TRUE)	151656 undecided
dblat	6962 (TRUE)	144694 undecided
hdchk	30 (TRUE)	144664 (FALSE)

Figure 7/11. dq decision tree.

7.7. Subproblem (level 2.2): ds

As described above, this problem deals with whether the threat of a delayed skewer prevents White from promoting the pawn and results in a not-won-for-white result. Of the 108535 positions to pass through this subproblem, 101 classify true and 108434 classify false.

The final ds attribute set was:

(1) wtoeg (6)*. Is the white king one away from the relevant edge. The relevant edge is defined to be the edge that the black rook will eventually use to attack the white king and promoted pawn.

(2) dsopp (1)*. Are the kings in normal opposition. Normal opposition exists when the two kings are separated by one empty square in a line perpendicular to the "relevant edge" with the black king closer to the center of the board than the white king (see Figure 7/12).

(3) dwipd (3)*. Is the white king distance to intersect point too great. The intersect point is the square that the black rook will use on the "relevant edge". This attribute test whether or not the white king will be able to attack that square thus preventing the final skewer.

(4) skewr (5)*. Is there a potential skewer as opposed to fork. This attribute determines whether the intersect point is between the promoted pawn and the white king or not. If it is between (fork) then the final check at the intersect point will not work since the newly promoted pawn will simply take the black rook. The only way for a fork to work is if the intersect point is defended by the black king. This can not be true within "ds" because of the king opposition requirements (the black king can not both be in opposition with the white king and defend the intersect point).

(5) bkxbq (1)*. Is the black king attacked in some way by the promoted white pawn.

(6) spcop (1)*. Is there a special opposition pattern present (see Figure 7/13).

The machine-generated decision rule is shown in Figure 7/14.

7.8. Subproblem (level 3.1): thrmt

As described above, this problem deals with whether the threat of checkmate prevents White from promoting the pawn and results in a not-won-for-white result. Of the 188327 positions to pass through this subproblem, 821 classify true and 187506 classify false.

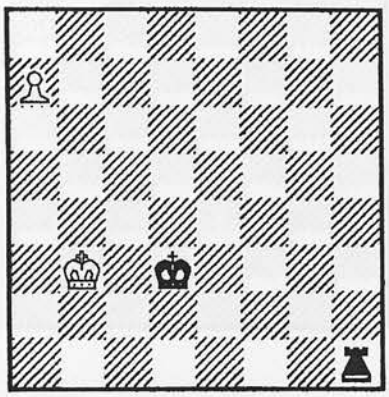


Figure 7/12. Normal opposition.

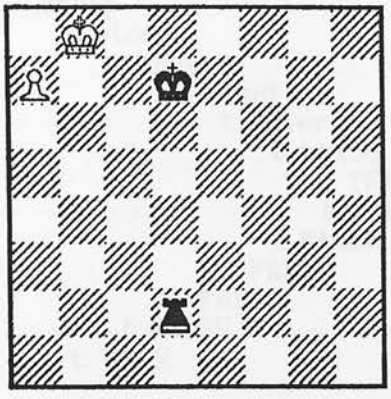


Figure 7/13. Special opposition. Here normal opposition is not present but due to the white king being on an edge, the only move that allows escape from check (after pawn promotion and BR->b2) is WK->a7. Now BR->b1 is a skewer.

No of examples in final working-set = 7.
No of nodes in final rule = 13.
Time taken to generate rule = 0.31 CPU seconds.

```
spcop
  f: wtoeg
      t: dsopp
          t: dwipd
              t: skewr
                  t: bkxbq
                      f: TRUE
                      t: FALSE
                  f: FALSE
              f: FALSE
          f: FALSE
      f: FALSE
  t: TRUE
```

Run-time behaviour for each test.		
test	number decided	leaving
spcop	5 (TRUE)	108530 undecided
wtoeg	84783 (FALSE)	23747 undecided
dsopp	23182 (FALSE)	565 undecided
dwipd	325 (FALSE)	240 undecided
skewr	120 (FALSE)	120 undecided
bkxbq	24 (FALSE)	96 (TRUE)

Figure 7/14. ds decision tree.

The final thrmt attribute set was:

(1) rxmsq (41)*. Does the black rook attack a mating square safely. The mating square is the square that the black rook will use to checkmate the white king. This attribute also makes sure that there is a mating square in the first place.

(2) bkxbq (1)*. Is the black king attacked in some way by the promoted white pawn.

(3) qxmsq (1)*. Is the mating square attacked in some way by the promoted white pawn.

(4) r2ar8 (31)*. Does the black rook have safe access to file A or rank 8. If it does then if there is a delay to promoting the white pawn, Black can move the rook to the safe square on rank 8 or file A and always capture the pawn next move. This will always result in a not-won-for-white game value (but not necessarily a won-for-black game value).

(5) bkxwp (1)*. Can the black king attack the white pawn. If it can then the mate threat has prevented White moving the pawn out of the way (because White had to move its king out of check) thus resulting in the safe capture (for Black) of the pawn.

The machine-generated decision rule is shown in Figure 7/15.

7.9. Subproblem (level 3.2): wka8d

As described above, this problem deals with delays to the promotion of the pawn because the white king is on the promotion square. If Black can make use of this delay the value of the subproblem is true and the top-level tree will return the result not-won-for-white. Of the 187506 positions to pass through this

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No of examples in final working-set = 6.
No of nodes in final rule = 11.
Time taken to generate rule = 0.27 CPU seconds.

```
rxmsq
  t: bkxwp
       f: bkxbq
            f: qxmsq
                 f: r2ar8
                      f: FALSE
                      t: TRUE
                 t: FALSE
            t: FALSE
       t: TRUE
  f: FALSE
```

Run-time behaviour for each test.		
test	number decided	leaving
rxmsq	186691 (FALSE)	1636 undecided
bkxwp	136 (TRUE)	1500 undecided
bkxbq	250 (FALSE)	1250 undecided
qxmsq	556 (FALSE)	694 undecided
r2ar8	685 (TRUE)	9 (FALSE)

Figure 7/15. thrmt decision tree.

---

subproblem, 3152 classify true and 184354 classify false.

The final wka8d attribute set was:

(1) r2ar8 (31)*. Does the black rook have safe access to file A or rank 8.

(2) wkna8 (1)*. Is the white king on square a8.

(3) blxwp (2)*. Does Black attack the white pawn (black rook in direction x<0, y=0 only).

(4) dblch (9)*. Can the white king be checked away from safety (see Figure 7/16).

(5) simpl (5). Does a very simple pattern apply. Deals with a stalemate (WK:a8 WP:a7 BK:a5 BR:b5), a delayed skewer (WK:a8 WP:a7 BK:d8 BR:d6) and a perpetual check (WK:a8 WP:a7 BK:a5 BR:c5). This is the only non-CLIP attribute in the KPKR solution, its code is given in Figure 7/17 to indicate how difficult to understand a simple *non-CLIP* chess-pattern match can be.

The machine-generated decision rule is shown in Figure 7/17.

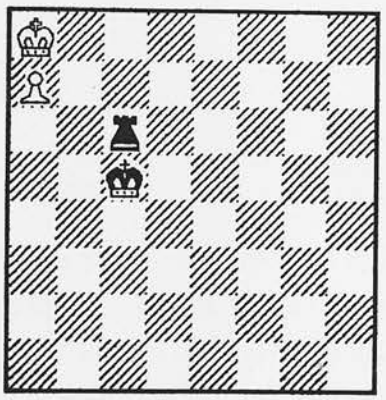

Figure 7/16. Can the white king be checked away from safety? Here WK->b7 is forced (prevents r2ar8) but BR->b6 (check) forces the white king to either retreat to a8 or c8 where BR->a6 makes sure of a not-won-for-white value. This attribute was found by Interactive ID3 to be unnecessary.

```
simpl() {
   if((bf == rf && rf == 4) || (br == rr && rr == 5))
      return(comment(TRUE, FALSE, "", "", "", LEVEL));
   if((bf == rf && br != 7) || (br == rr && bf != 2)) {
      return(comment(!(rr == 6 && rf == 3 && (br == rr && bf > 4) ||
                  (bf == rf && br <= 4)), FALSE, "", "", "", LEVEL));
   }
   return(comment(FALSE, FALSE, "", "", "", LEVEL));
}
```

Figure 7/17. A "simpl" definition.

No of examples in final working-set = 5.
No of nodes in final rule = 9.
Time taken to generate rule = 0.23 CPU seconds.

```
wkna8
  t: r2ar8
      f: blxwp
          f: simpl
              f: FALSE
              t: TRUE
          t: TRUE
      t: TRUE
  f: FALSE
```

Run-time behaviour for each test.		
test	number decided	leaving
wkna8	184153 (FALSE)	3353 undecided
r2ar8	3055 (TRUE)	298 undecided
blxwp	63 (TRUE)	235 undecided
simpl	34 (TRUE)	201 (FALSE)

Figure 7/18. wka8d decision tree.

7.10. Subproblem (level 3.3): wkchk

As previously described, this problem deals with delays to the promotion of the pawn because the white king is in check. If Black can make use of this delay the value of the subproblem is true and the top-level tree will return the result not-won-for-white. Of the 184354 positions to pass through this subproblem, 32698 classify true and 151656 classify false. The call to the "rimmx" attribute is completely redundant in this subproblem. No positions are classified as a result of applying that test in this subproblem. This situation arises because if the subproblem is taken in isolation, the question of the rook's safe capture is very important. However *no* positions where the black rook can be safely captured actually reach this subproblem because they have been filtered out by the call to "rimmx" in the top-level tree.

The final wkchk attribute set was:

(1) btoqs O. Can Black attack the queening square soon. This was a 55 line attribute it was turned into a subproblem as described at the beginning of this chapter.

(2) wknck (6)*. Is the white king in check.

(3) rimmx (15)*. Can the black rook be captured safely.

(4) bkxcr (3)*. Can the black king attack the critical square (b7).

(5) rkxwp (2)*. Does the black rook bear on the WP (direction x<0, y=0 only).

(6) thrsk (31)*. Is there a skewer threat lurking (see Figure 7/19).

(7) mulch (18)*. Can Black renew the check to good advantage (see Figure 7/20).

The machine-generated decision rule is shown in Figure 7/22.

7.11. Subproblem (level 3.4): dblat

This subproblem deals with delays to the promotion of the pawn by reason of double attack. If Black can make use of this delay the value of the subproblem is true and the top-level tree will return the result not-won-for-white. This subproblem fell foul of the number-of-attributes-per-tree criterion but was very easy to split into two subproblems; the first (dblat) establishes context for an double attack, the second (okskr) decides whether or not Black can make use of the potential delay. A successful double attack usually involves a skewer but it can also involve a fork where the black king supports the black rook (see Figure 7/22).

Of the 151656 positions to pass through this subproblem, 6962 classify true and 144694 classify false.

The final dblat attribute set was:

(1) cntxt (1)*. Is the white king on an edge and not on a8.

(2) katri (13)*. Does any king control intersect point. If so, which? This is the only three valued attribute in the KPa7KR WTM solution. If any one king controls the square that the black rook is going to use to skewer White's pieces then that kings's side will prevail. If both or neither king controls this square then further information is needed.

(3) bkblk (6)*. Is the black king in the way of the black rook.

(4) wkpos (3)*. Is the white king in a potential fork position as opposed to a skewer position. i.e. Is the black rook intersection point in between White's pieces or not.

(5) bkxbq (1)*. Is the black king attacked in some way by the promoted white pawn.

(6) okskr O. Can Black take advantage of this potential skewer.

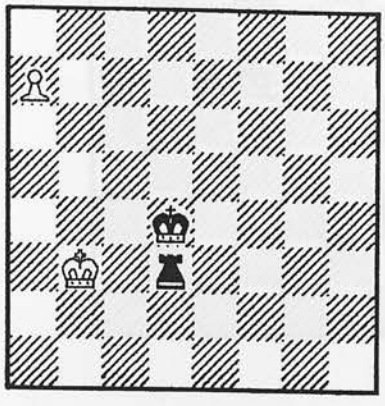

Figure 7/19. Is there a skewer threat lurking? Here the white king must move out of check and can move to one of four squares (a2, a4, b2 or c2) but there are two main features of this position as follows:

- WK->a4, BR->d1 now if WP->a8 (promotes) then BR->a1 (skewer).
- WK->a2, BR->d2 (check) if WK->a3 to threaten to capture the black rook then BR->a2 and will capture the white pawn next move. In fact this theme works for any white king move except WK->a4 i.e. keep checking the white king with the rook along c[1-3]. If the white king strays into the C file then BR->a? will win the pawn. The white king will eventually have to move to a4 or draw by repetition. Either way is not-won-for-white.

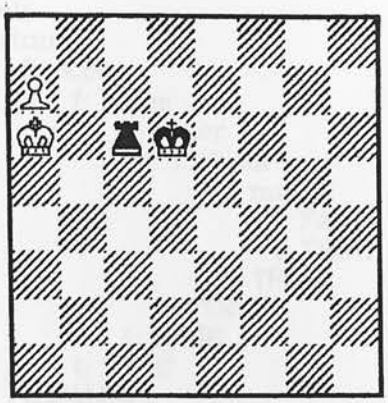

Figure 7/20. Can Black renew the check to good advantage? Here the white king must move to b7 to prevent BR->c8 or BR->a8. But BR->c7 (check) ensures a not-won-for-white value because the white king must now move out of check exposing the white pawn to the black rook.

No of examples in final working-set = 8.
No of nodes in final rule = 15.
Time taken to generate rule = 0.50 CPU seconds.

```
wknck
  t: rimmx
      f: rkxwp
          f: btoqs
              f: bkxcr
                  f: thrsk
                      f: mulch
                          f: FALSE
                          t: TRUE
                      t: TRUE
                  t: TRUE
              t: TRUE
          t: TRUE
      t: FALSE
  f: FALSE
```

Run-time behaviour for each test.		
test	number decided	leaving
wknck	150292 (FALSE)	34062 undecided
rimmx	0 (FALSE)	34062 undecided
rkxwp	3348 (TRUE)	30714 undecided
btoqs	27677 (TRUE)	3047 undecided
bkxcr	1057 (TRUE)	1990 undecided
thrsk	64 (TRUE)	1926 undecided
mulch	552 (TRUE)	1364 (FALSE)

Figure 7/21. wkchk decision tree. Note the rimmx test does not contribute towards the classification of any positions. It has already been called in the top-level (pa7) subtree so no positions with rimmx=true reach this part of the tree. The reason that this test is included is because this subproblem was induced in isolation of the rest of the solution hierarchy. Thus rook-immediate-capture is an important feature but, for this particular solution, it has already been tested.

The machine-generated decision rule is shown in Figure 7/23.

7.12. Subproblem (level 4.1): okskr

This subproblem deals with whether a potential skewer is of value to Black. It is called from the dblat subproblem (described above). If this subproblem

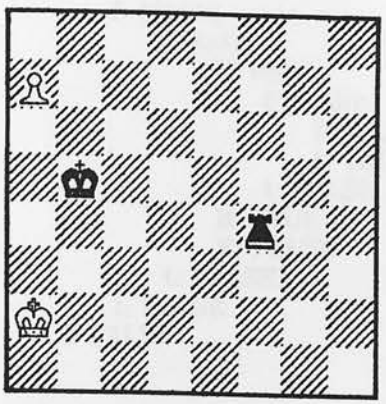

Figure 7/22. A supported fork. Here, if WP->a8 (promotes) then BR->a4 (check), the white king must move out of check or the promoted pawn (queen or rook) must take the black rook. If the former then the black rook takes the newly promoted pawn (not-won-for-white). If the latter then the black king takes the white queen or rook (not-won-for-white). If the white pawn does not promote then the black rook can get to file A or rank 8 (again not-won-for-white).

No of examples in final working-set = 12.
No of nodes in final rule = 14.
Time taken to generate rule = 0.55 CPU seconds.

```
cntxt
  t: bkblk
      f: bkxbq
          f: katri
              N: wkpos
                  f: okskr
                      f: FALSE
                      t: TRUE
                  t: FALSE
              B: TRUE
              W: FALSE
          t: FALSE
      t: FALSE
  f: FALSE
```

Run-time behaviour for each test.		
test	number decided	leaving
cntxt	117052 (FALSE)	34604 undecided
bkblk	3425 (FALSE)	31179 undecided
bkxbq	7783 (FALSE)	23396 undecided
katri	868 (TRUE)	
	5476 (FALSE)	17052 undecided
wkpos	10468 (FALSE)	6584 undecided
okskr	6094 (TRUE)	490 (FALSE)

Figure 7/23. dblat decision tree.

returns true then dblat and dq will return true and then the top-level tree will return the result not-won-for-white. Of the 6584 positions to pass through this subproblem, 6094 classify true and 490 classify false.

The final okskr attribute set was:

(1) r2ar8 (31)*. Does the black rook have safe access to file A or rank 8.

(2) wkcti (33)*. Can the white king control the intersect point. The intersect point is the square on an edge that the black rook will use to skewer White.

(3) bkspr (6)*. Can the black king support the black rook. This matters in the case of a "fork" type double attack where the black rook will end up between the white king and pawn.

(4) reskr (14)*. Can the black rook alone renew the double attack threat (see Figure 7/24).

(5) skach (10)*. Can the white king be skewered after one or more checks (see Figure 7/25).

(6) reskd (7)*. Can the white king be reskewered via a delayed skewer (see Figure 7/26).

The machine-generated decision rule is shown in Figure 7/27.

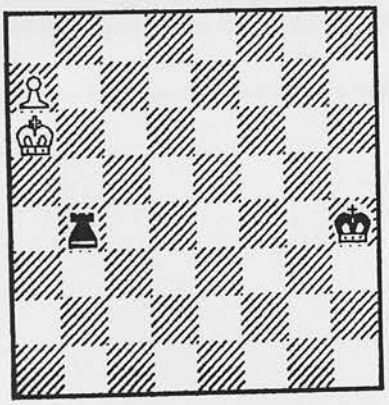

Figure 7/24. Can the black rook alone renew the double attack threat? In this position WK->a5 prevents BR->a4 (skewer), but BR->b1 again threatens to skewer on a1 after the next White move. White can not prevent this hence the term "renewed skewer".

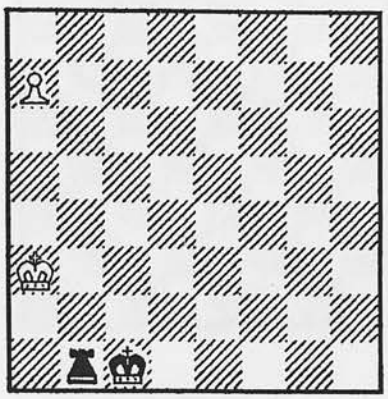

Figure 7/25. Can the white king be skewered after one or more checks? Here WK->a2 is forced to prevent BR->a1 (skewer). BR->b2 (check) is the reply, now if WK->a1 then BR->b1 (check), WK->a2, BR->b2 (check). Eventually the white king is forced to move to a3 or draw by repetition. When this move is made then BR->b1 is back to the original position. Again either draw by repetition or WK->a4 in which case BR->a1 (skewer). In all cases the value is not-won-for-white.

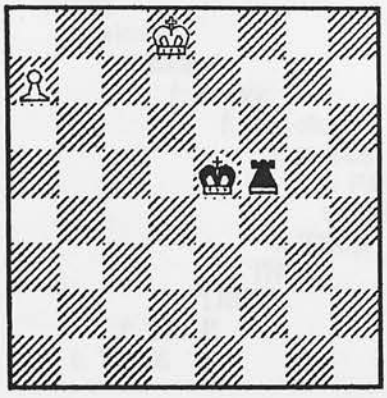

Figure 7/26. Can the white king be reskewered via a delayed skewer? In this position WK->e7 prevents BR->f8 which would prevent the white pawn promoting and eventually lead to pawn capture. After WK->e7 then BR->h5 threatens BR->h8 next move. WP->a8 (promote to queen) prevents the BR->h8 move but Black replies with BR->h7 (check). This is now a delayed skewer of the kind previously described.

No of examples in final working-set = 7.
No of nodes in final rule = 13.
Time taken to generate rule = 0.42 CPU seconds.

```
r2ar8
  f: wkcti
    t: bkspr
      f: reskr
        f: skach
          f: reskd
            f: FALSE
            t: TRUE
          t: TRUE
        t: TRUE
      t: TRUE
  f: TRUE
t: TRUE
```

Run-time behaviour for each test.		
test	number decided	leaving
r2ar8	5956 (TRUE)	628 undecided
wkcti	31 (TRUE)	597 undecided
bkspr	4 (TRUE)	593 undecided
reskr	73 (TRUE)	520 undecided
skach	6 (TRUE)	514 undecided
reskd	24 (TRUE)	490 (FALSE)

Figure 7/27. okskr decision tree.

7.13. Subproblem (level 4.2): btoqs

This subproblem deals with whether Black can get to the promotion square and make use of a delay caused by the white king being in check. It is called from the "wkchk" subproblem. Of the 30714 positions to pass through this subproblem, 27677 classify true and 3037 classify false.

The final btoqs attribute set was:

(1) skrxp (7)*. Can the black rook achieve a skewer or black king attack the white pawn (see Figure 7/28).

(2) bkona (21)*. Is the black king on rank A in a position to aid the black rook (see Figure 7/29).

(3) bkon8 (13)*. Is the black king on file 8 in a position to aid the black rook. The same kind of position as "bkona" but with the black king on rank 8.

(4) bknwy (11)*. Is the black king in the black rook's way (see Figure 7/30).

(5) wkovl (9)*. Is the white king overloaded and not able to cover all the black rook's intersect points. The black rook's intersect points are those safe edge squares that the black rook can use to attack the pawn promotion square (see Figure 7/31).

The machine-generated decision rule is shown in Figure 7/32.

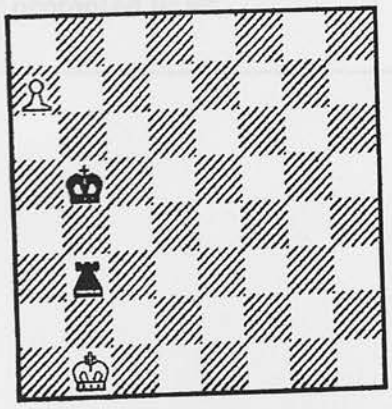

Figure 7/28. Can the BR achieve a skewer or BK attack the WP? In this position WK->a2 is forced (to prevent BR->a3) but BR->b4 renews the threat for the black rook to get to the A file. Now whatever White does (even if White promotes the pawn) then BR->a4 (fork type skewer supported by the black king).

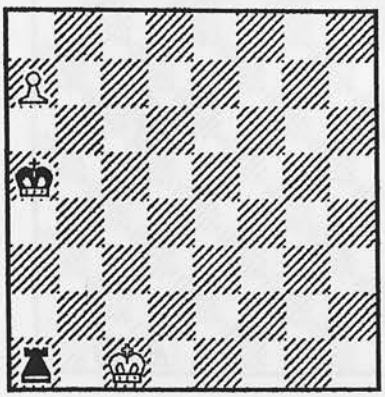

Figure 7/29. Is the BK on rank A in a position to aid the BR? Here WK->b2 attacks the black rook. The black king is too far away to aid the black rook however BK->b6 threatens to capture the pawn if the white king takes the black rook. WP->a8 (promotes) is White's only escape from pawn capture however the black rook can now take the newly promoted pawn.

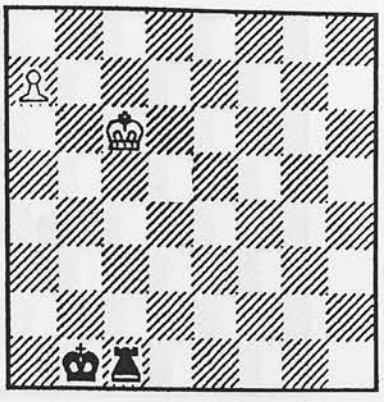

Figure 7/30. Is the black king in the black rook's way? In this position WK->b7 prevents BR->c8. Black would normally be able to move the black rook to a1 but the black king is in the way.

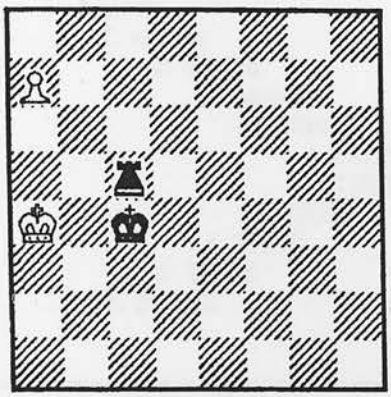

Figure 7/31. Is the white king overloaded? In this position WK->b6 prevents BR->a5 but can not prevent BR->c8 and eventual capture of the white pawn.

No of examples in final working-set = 6.
No of nodes in final rule = 11.
Time taken to generate rule = 0.23 CPU seconds.

```
bknwy
  f: skrxp
      f: bkona
          f: bkon8
              f: wkovl
                  f: FALSE
                  t: TRUE
              t: TRUE
          t: TRUE
      t: TRUE
  t: FALSE
```

Run-time behaviour for each test.		
test	number decided	leaving
bknwy	434 (FALSE)	30280 undecided
skrxp	998 (TRUE)	29282 undecided
bkona	291 (TRUE)	28991 undecided
bkon8	155 (TRUE)	28836 undecided
wkovl	26233 (TRUE)	2603 (FALSE)

Figure 7/32. btoqs decision tree.

7.14. Unstructured induction of decision rule

As with the KPK experiment, unstructured trees were generated from the base level attributes and the entire set of decision-vectors. 3196 vectors result from the 209,718 legal KPa7KR positions with WTM.

The 36 attributes used were

- bxqsq stlmt rimmx
  from the PA7 (top level) set of attributes.
- hdchk
  from the DQ set of attributes.
- wtoeg dsopp dwipd skewr bkqbq spcop
  from the DS set of attributes.
- rxmsq r2ar8 bkxwp
  from the THRMT set of attributes.

- wkna8 blxwp dblch simpl
  from the WKA8D set of attributes.
- wknck bkxcr rkxwp thrsk mulch
  from the WKCHK set of attributes.
- cntxt katri bkblk wkpos
  from the IS set of attributes.
- wkcti bkspr reskr skach reskd
  from the OKSKR set of attributes.
- skrxp bkona bkon8 bknwy wkovl from the BTOQS set of attributes.

By again varying the user-supplied parameters to Quinlan's ID3 another series of trees (50 in all) were generated, each equivalent to the structured tree in function though not in understandability. Appendix G shows the best (smallest) unstructured tree generated in this manner. The rules were run over the entire set of legal KPa7KR WTM positions and the statistics shown in Figure 7/33 were gathered.

7.15. Generation of unstructured decision rules using structured training set

Of the 3196 legal decision-vectors only 61 were necessary (minimal or near-minimal reconstruction set) to generate the structured solution (corresponding to 61 examples supplied at different levels in the decomposition hierarchy). About 170 were needed to generate the unstructured solutions. In the context of the structured decomposition hierarchy the 61 decision-vectors are sufficient to completely specify the KPa7KR WTM problem. However, removed from this context these vectors are not sufficient to completely classify the problem. It could

	Cost of classifying all legal (209718) positions (kbs)	Number of nodes
mean	24768.1	95.6
standard deviation	2720.0	7.9

Figure 7/33. Vital statistics for 50 (100% correct) unstructured trees. Each rule was run once over all the legal KPa7KR WTM positions.

be said that the decompositions hierarchy is worth about 110 examples in this case.

Another way to look at the value of the decomposition hierarchy is to find out what percentage of positions are incorrectly classified by decision trees trained from these 61 vectors. 66 unstructured trees were generated using the structured training set of 61 examples. Each tree was tested against the database for accuracy. The results are shown in Figure 7/34. Note that the observed rate of about 30% represents very poor reliability, since a random classifier (knowing only the frequency of won positions in the domain) would have about a 38% error rate. Implications are further discussed in 8.4.

7.16. Discussion of results

First a word on the accuracy of timing results. One particular unstructured tree was run 25 times in order to establish estimation error. The results of these runs are given in Figure 7/35.

Further features could confuse the timing figures. Some of the runs included checks for rule accuracy. This entailed a database lookup for every position. All the runs included a legal-move generator so that all legal positions could be tested. To determine how much of an overhead these two features entailed 50 tree timing runs were performed with a null decision tree as the

	% correct	Cost of classifying all legal (209718) positions (kbs)	Number of nodes
mean	72.1	14604.5	153.73
standard deviation	5.2	1003.7	9.10

Figure 7/34. Statistics for 66 unstructured trees trained using structured training set. These decision trees were trained using only the 61 decision-vectors that were used to induce the structured solution. The number of examples necessary to generate completely correct unstructured trees in the absence of expert supplied structure was 175 in one experiment.

Time to classify all legal (209718) positions (seconds)	
mean	35315.65
standard deviation	348.58

Figure 7/35. System timing tests using an unstructured decision rule. The same unstructured decision rule was run 25 times over all the legal KPa7KR WTM positions. Note the small standard deviation which shows that system load fluctuations do not play a significant role in timing results.

function to be evaluated and database checking turned off. A further 13 runs of the same null function were performed with database checking turned on. The results are shown in Figure 7/36 and Figure 7/37.

It would appear from these results that database lookup does not add to the user-execute time associated with a Unix process. Further testing will be performed using real-time analysis and the machine in single-user mode.

Time to generate 209718 legal positions database checks off (seconds)	
mean	21094.95
standard deviation	300.66

Figure 7/36. Timing results for 50 legal-position generation runs. These figures will be used to establish the non-rule related overhead of the previous timing runs.

Time to generate 209718 legal positions database checks on (seconds)	
mean	20830.82
standard deviation	397.44

Figure 7/37. Timing results for 13 legal-position generation runs. These figures will be used to establish the database lookup related overhead of previous timing runs.

Combining the above legal-move generation timing results and multiplying by the fraction of a kilo-byte that is taken up by a null decision rule gives the figures shown in Figure 7/38. If this non-rule related overhead is subtracted from the above sets of figures and the result divided by the number of legal KPa7KR WTM positions figures shown in Figure 7/39 are obtained.

Cost of generating 209718 legal positions (kbs)	
mean	731.8
standard deviation	27.2

Figure 7/38. Timing results for the 63 null-function runs. These figures will be used as the non-rule related overhead associated with classifying all the legal KPa7KR positions.

	Cost of classifying one position (kbs)	approximate standard deviation
structured tree	0.135	0.003
unstructured tree	0.114	0.12

Figure 7/39. Mean classification cost of a KPa7KR WTM position. The standard deviation is approximate because the standard deviation of the overhead has not been taken into account (0.00013 in all).

For the KPa7KR WTM experiment the unstructured trees were on average 15.5% cheaper to run than the structured tree. This may be thought a small price to pay for human understandability.

Recall that during the development phase of the structured hierarchy 4 stages were saved for later evaluation against the classification database. The results are shown in Figure 7/40.

7.17. Programmer costs

Excluding the time taken to develop the tools to generate and test the structured rules, stage 1 (initial top-down decomposition plus shallow training) took 4 man weeks to complete, stages 2-4 (further training by US National Master Kopec and refinement) took a further 6 man weeks. It is interesting to note that the structured rule was 99.085% correct after 4 man weeks but to find and correct the remaining 0.915% of the errors took a further 6 man weeks. Skill in the new "structured induction" style of program construction evidently accumulates and to some degree can be systematized with experience. The author was able, after the above work was completed, to instruct Mr. Robert Reinke, a graduate student at the University of Illinois, in the methods, with the result that his continuation of this work to determine WFB/not WFB took less than 2 man weeks (see Appendix C). As shown in his report his experience with D. Fraisl, on the WFW/not WFW for pb7 was fully confirmatory.

Stage no.	no. of new examples	total examples so far	positions wrong out of 209718
1	101	101	1918
2	80	181	227
3	80	261	69
4	103	364	15

Figure 7/40. Accuracy of rule development stages. Stage 1 (initial top-down decomposition plus shallow training) was completed in the USA with about 4 man-weeks of effort. Stages 2 to 4 (further training by Danny Kopec and refinement) were completed in a further 6 man-weeks. The database only became available *after* the completion of stage 4. Using self-commenting as a diagnostic aid all 15 errors present after stage 4 were corrected in under 2 hours.

CHAPTER 8

Discussion

8.1. Database as oracle versus expert as oracle

Of the two modes of induction (structured and unstructured) investigated in this thesis, unstructured induction of either chess endgame would have been very difficult without a database to supply decision classes for attribute vectors. Likewise without the availabililty of a domain expert to partition the problem space and supply relevant training examples, the structured approach would also have been very difficult. It follows that the non-availability of one or other of these resources (i.e. database or domain expert) largely predetermines the inductive mode that can be applied. Because of the initial non-availability of a KPa7KR database an unstructured approach was not possible until later when a database had been generated.

Given an attribute set there are four different ways that induction can be performed. The four-fold table of Figure 8/1 shows the two modes of induction (structured and unstructured) *versus* the two sources of classificatory information (database and expert). The result tabulated within this matrix is the performance of the product when measured against the criterion of brain compatibility. Three of the entries in this table are confirmed by work reported in this thesis. It is expected that the fourth (the product of induction generated using unstructured induction and an expert to supply examples) would be more brain-compatible than that generated using unstructured induction and the database to supply examples. This prediction should be tested.

There are two distinct modes of use for an oracle within the induction cycle;

- "induction oracle"
- "checking oracle".

Quinlan's ID3 uses the same oracle for rule induction and rule checking i.e. the database of decision vectors for both rule induction and to determine (by finding no exceptions to the current rule) whether rule induction is complete. Interactive ID3 also uses the same oracle for both rule induction and rule checking - the expert (with a possible final appeal to a database for "clean-up" to perfect accuracy). The examples supplied to date are used to induce a rule and further

Oracle \ Mode	unstructured	structured
database	poor	good
expert	—	excellent

Figure 8/1. The effectiveness of computer induction. Ranked scores for "brain-compatibility" of the products of four induction regimes. All but the square marked "—" have been tested in this research.

examples supplied by the expert are used to check it.

Given that both database and expert are freely available it is tempting to use the expert to supply the structure decomposition and the database to supply an exhaustive set of examples for each subproblem (as in the case of the KPK experiment). The advantage of this approach is that incorrectly classified examples will not be introduced. Incorrectly classified expert-supplied examples were a major source of annoyance in stages 1 to 3 of the KPa7KR experiment. These examples were not too difficult to detect because they eventually clashed with other expert-supplied examples. However there is no guarantee that this would be the case with other problems and for more complex problems the incidence of this nuisance can be expected to increase. There is however an overriding disadvantage to the use of databases as induction oracles, namely as concerns the human-like quality, or lack of it, in the generated rule. The KPK subtrees though linear are not as understandable as those generated for KPa7KR. One reason for this could be that since in KPa7KR we had no access to a classification database and thus no access to an exhaustive and correct source of decision vectors, each attribute had to be developed with the expert in mind. Each attribute had to follow his specifications as closely as possible since he would in the end have to advise on any failing in their classificatory power. This was not the case with the attributes generated for the KPK subproblems. These attributes could be (and often were) altered to correct single failures to classify correctly. The resulting attributes were cumbersome and internally opaque. The

alternative as shown in the KPa7KR experiment was that extra attributes should have been designed and possibly new subproblems introduced.

8.2. Meta-knowledge (old wives tales and rules of thumb)

The need for some kind of expert guidance in the way the rules are formed first came to light while debugging the top-level subproblem "pa7". Although a checkmate attribute was supplied, two of the examples in which White was checkmated:

(1) WK:a8 WP:a7 BK:c7 BR:h8 and
(2) WK:a1 WP:a7 BK:a3 BR:h1

were correctly classified as not WFW by other attributes (the first by wka8d and the second by wkchk). At first this was not disturbing to the experts because they do not immediately look for such rare occurrences as checkmate when classifying an endgame of this complexity. However if all the advantages of self-commenting are to be utilized it is vital that positions are classified for the correct reasons. This problem could be overcome if a facility existed in Interactive ID3 for the expert to suggest a list of constraints that the rule being generated must obey e.g.

- chkmt must appear in the rule
- bxqsq must always be followed by dq
- chkmt if used must be followed immediately by stlmt
- ds if used may only appear last in a rule

Guidelines like those shown above are known as "meta-rules" they contain information not explicitly stated in the examples or the decomposition structure supplied by the expert. These meta-rules are information gathered by the expert in the form of rules of thumb accumulated from experience. Interactive ID3 should build a solution satisfying as many of these rules as possible. The application of such meta-rules has been manually simulated with success in a school algebra domain (Paterson, 1983).

Meta-rules are also useful for implementing efficiency considerations; recall the following table from the last chapter.

Run-time behaviour for each test.		
test	number decided	leaving
thrmt	821 (TRUE)	187506 undecided
wka8d	3152 (TRUE)	184354 undecided
wkchk	32698 (TRUE)	151656 undecided
dblat	6962 (TRUE)	144694 undecided
hdchk	30 (TRUE)	144664 (FALSE)

Figure 7/11 dq decision tree (from Chapter 7).

Within a *linear* decision tree it is possible to permute the order of sequential tests that make identical decision discriminations. Another, functionally equivalent, order for this decision tree is shown in Figure 8/2. Note the order of tests allows the fastest filtering of positions through the rule.

Another factor that can be taken into account when building a rule is the execution cost of tests. Some tests will be more resource expensive than others to run. Such information could be included with the set of meta-rules and applied by Interactive ID3 in combination with position filtering information like that shown in Figure 7/11.

```
wkchk
  f: dblat
      f: wka8d
          f: thrmt
              f: hdchk
                  f: FALSE
                  t: TRUE
              t: TRUE
          t: TRUE
      t: TRUE
  t: TRUE
```

Figure 8/2. Execution optimised "dq" decision rule. Assuming a random set of input positions this rule will be more efficient at run-time than the rule induced by Interactive ID3.

A learning system that makes use of this kind of meta knowledge in the form of rule templates is being developed at MIRU (Niblett, 1983).

8.3. Structured induction versus unstructured induction

The type of classification source available (i.e. either database or domain expert) influences the choice of mode of induction (structured or unstructured) to be applied. There is also the question of whether for sufficiently complex problems an unstructured solution could or should even be attempted. In both of the experimental chess domains, structured solutions were attempted first. Once these had been successful, complete and correct unstructured solutions were guaranteed by making use of the primitive attributes which were by now known to be able to discriminate between *all* legal positions in the problem domains. The author does not know how he would have attempted unstructured solutions without the benefit of this previous structured work. Without the prior structured pass over the problem, there would seem little hope of developing a set of attributes adequate for a complete solution of any kind. We thus arrive at the following conclusion.

> ***A complex problem first needs to be tackled by the structured technique. But when it has yielded, no practical motivation remains for re-working it in unstructured mode. The evidence is that the product will disqualify itself on brain-compatibility while failing to offer compensating advantages of run-time efficiency.***

8.3.1. Quality of training examples

There is a striking difference between the examples left in the Quinlan's ID3 "working-set" following the generation of an unstructured rule compared to the examples left by Interactive ID3 following the generation of a structured rule. The examples left by ID3 after an unstructured run comprise a larger number altogether but far fewer examples that are meaningful to an expert. This is undoubtedly due to the experts' ability to supply meaningful examples and ID3's ability to filter still further and remove examples that do not contain unique features. The experts occasionally supplied redundant (sometimes duplicate) examples but all the examples were directed towards a preconceived solution framework. On the other hand in unstructured mode ID3, without any guidance, picked examples statistically and the resulting incomprehensibility of its rule reflected the arbitrariness of these examples. Could unstructured rule generation benefit from the use of the structured rule training set?

The answer to this question is found in the results section of the last chapter which shows that these 61 examples though sufficient to produce a correct structured solution, were not sufficient to produce a correct unstructured solution. Thus supplying a structure actually increases the usefulness of a given set of relevant examples to an induction system. Put another way it could be said that the expert-supplied structure contains information. The next section seeks to place this statement on a quantitative basis.

8.3.2. Information content of expert-supplied structure

By a slight extension of finite-message information theory (Michie, 1982a), entropy measures were calculated. We follow Attneave (1959) in using "entropy" and the technical denotation of "information" interchangeably.

(1) *Entropy per symbol without classification rule.*

We construct a message using an *alphabet of two symbols*, {w, n} (WFW, not WFW), by sampling M times from a set of 209,718 objects (KPa7KR WTM positions) each of which is labelled with one or other of the two symbols in the proportions P_w and P_n respectively. Sampling is with replacement and the final message has length M. The message is conceived as emanating from a *source* and being absorbed by a *receiver*. The receipt of each symbol incrementally conveys some information to the latter. The entropy per symbol is

$$-(P_w \times \log_2(P_w) + P_n \times \log_2(P_n)).$$

where:

P_w is the frequency already known to the receiver of w objects in the set, here equal to the prediction probability of "the next symbol is a w".

$P_n = (1 - P_w)$.

We depart from the classical treatment by allowing the receiver to be at given points in time in different *knowledge-states*. We further distinguish *static* and *dynamic* receivers. A static receiver may change its state between messages but not from symbol to symbol during the receipt of a given message. A dynamic receiver may change its state in the light of information contained not only in past messages but also in the symbols received to date of a message whose receipt is not yet finished. Let us, as a bare-bones illustration, consider the information-content of a static receiver's possession of the value of P_w.

Case 1: Static receiver - no knowledge of the value of P_w.
By the principle of insufficient reason, P_w is set at ½. Evaluation of the previous formula gives one bit per symbol, i.e. M bits in the total message.

Case 2: Static receiver - knowledge that P_w = 129,825/209,718.
Evaluation of the above formula gives 0.959 bits per symbol, i.e. 0.959×M bits in the total message.

The information contained in the specification of P_w is the difference between these two cases and is equal to 0.041 bits per symbol, i.e. 0.041×M bits for a given message.

So much for the information-content of a piece of knowledge (change of receiver's knowledge-state) when this is simply a number. Now we shall consider the information-content of a structure or rule.

(2) *Entropy per symbol after filtering each object through a rule.*

Objects that have been classified by a rule fall into four groups:

W_c - those objects that classify as w correctly: object is w, rule says w.
N_i - those objects that classify as w incorrectly: object is n, rule says w.
N_c - those objects that classify as n correctly: object is n, rule says n.
W_i - those objects that classify as n incorrectly: object is w, rule says n.

Figure 8/3 shows this more graphically. We now define the following probabilities:

P_w - described above but now specialized to the probability that an object picked at random is a w, equivalent to $(W_c+W_i)/(W_c+W_i+N_c+N_i)$. Note that since we are now dealing with a possibly faulty rule this quantity is no longer equal to the prediction probability.

P_n - As before this equals $(1-P_w)$ which can now be described as $(N_c+N_i)/(W_c+W_i+N_c+N_i)$.

P^*_w - the probability that an object picked at random is classified as a w.

P^*_n - the probability that an object picked at random is classified as a n.

R_w - the conditional probability that if an object is w the rule says w. This is equivalent to Wc/(Wc+Wi) (see Figure 8/3).

R_n - the conditional probability that if an object is n the rule says n. This is equivalent to $N_c/(N_c+N_i)$.

P'_w - the conditional probability that if the rule says w the object is w. This is equivalent to $W_c/(W_c+N_i)$.

P'_n - the conditional probability that if the rule says n the object is n. This is equivalent to $N_c/(N_c+W_i)$.

R_w and R_n can be thought of as measuring how "representative" a rule is of

an already known actuality. A general "representivity" measure for a rule is as follows:

$$R = P_w \times R_w + P_n \times R_n$$

which means: the probability, for an object sampled at random, that the rule assigns it correctly to its class. Having defined these quantities Figure 8/3 is re-cast as a probability matrix in Figure 8/4.

Let us follow the same sequence of steps for P'_w and P'_n that we took with R_w and R_n. P'_w and P'_n can be thought of as measuring how "predictive" a rule is of an actuality not yet known. A general "predictivity" measure of a rule is:

$$P' = P^*_w \times P'_w + P^*_n \times P'_n$$

Note that working from Figure 8/3 we can quickly shown that

$$R = P' = (W_c + N_c)/(W_c + W_i + N_i + N_c)$$

So a rule's representivity (which is an *a priori* way of looking at its reliability) is equal to its predictivity (which expresses the *a posteriori* view of its reliability).

Actual \ Rule	w	n
w	W_c	W_i
n	N_i	N_c

Figure 8/3. Confusion matrix for object classification through a 2 valued decision tree.

Actual \ Rule	w	n
w	$R_w \times P_w$	$(1-R_w) \times P_w$
n	$(1-R_n) \times P_n$	$R_n \times P_n$

Figure 8/4. Probability matrix for object classification through a 2 valued decision tree.

A *surprisal* (Samson 1951) is derived from a *prediction probability* of a symbol as

$-\log_2(p)$, where p is the prediction probability.

In (1) above (knowledge of value of P_w, but no rule) there are two cases as follows:

Case α: object is w, surprisal = $-\log_2 P_w$
Case β: object is n, surprisal = $-\log_2 P_n$

The average surprisal per symbol is

$$P_w \times -\log_2 P_w + P_n \times -\log_2 P_n$$

which agrees with the entropy measure given in (1).

In (2) above (after the application of a rule) the symbols have surprisals as follows:

Case α: object is w.
Case A: rule says w, surprisal = $-\log_2 P'_w$
Case B: rule says n, surprisal = $-\log_2(1-P'_n)$
Case β: object is n
Case A: rule says w, surprisal = $-\log_2(1-P'_w)$
Case B: rule says n, surprisal = $-\log_2 P'_n$

We need the average surprisal per symbol, as shown in Figure 8/5.

The information gain (entropy difference) resulting from use of a "knowledge-filter" is the average surprisal to a receiver in a state lacking the knowledge embedded in the filter *minus* the average surprisal to a receiver possessing the knowledge. The results obtained by the application of these formulæ to the 66 unstructured trees generated using the structured training set are given in Figure 8/6. It should be noted that no effort has been made to take into account any difference in the efficiency of the ID3 induction algorithm when it is supplied with attribute sets of different sizes. In structured induction the algorithm only has to cope at any one time with a subset - namely those set aside as relevant to the given sub-problem. Hence the failure of ID3 to generate more correct decision trees when structure is removed may be due in part to the nature of the ID3 induction algorithm. However the results show that, using the available tools, the expert-supplied structure accounts for an estimated 84% of the classificatory power of the structured solution. Since it has been shown that it is possible to generate a completely correct *unstructured* KPa7KR WTM decision rule using 175 examples, expert supplied structure is equivalent to about 114 examples (structured induction required 61 examples, unstructured 175, and the difference is 114). The information per tutorial example in the two cases is given in Figure 8/7. These results show that the ID3 algorithm does a good job of extracting examples relevant to the rule being formed.

Case	Frequency	Term
αA	$R_w \times P_w$	$-R_w \times P_w \times \log_2 P'_w$
αB	$(1-R_w) \times P_w$	$-(1-R_w) \times P_w \times \log_2 (1-P'_n)$
βA	$(1-R_n) \times P_n$	$-(1-R_n) \times P_n \times \log_2 (1-P'_w)$
βB	$R_n \times P_n$	$-R_n \times P_n \times \log_2 P'_n$

Figure 8/5. Average surprisal per symbol as a sum of terms over four cases.

Unfortunately, in the case of the unstructured rule, these examples are not necessarily meaningful to an expert. An experiment that would show the degree of relevancy these 175 examples have to the structured rule (and therefore perhaps to an expert) would attempt to reconstruct the structured rule using these 175 examples.

	Item	Information (bits)
A	Initial information per symbol associated with the KPa7KR WTM chess endgame	0.959
B	Information per symbol associated with the structured solution and correct unstructured solutions	0.959
C	Mean information associated with the 66 unstructured decision trees trained with the structured training set	0.151
D	Standard deviation for above mean	0.109
E	Information associated with the *best* (most accurate) unstructured decision tree trained with the structured training set	0.410
F	Estimate of effective information per symbol associated with the structuring of the domain (by subtraction of C from B)	0.808

Figure 8/6. Information measures for the KPa7KR WTM problem and decomposition structure. A is the information per symbol associated with the KPa7KR WTM problem. B is the mean information per symbol associated with 66 decision rules that were generated without any expert-supplied structure but with the same examples that were sufficient to generate a complete and correct structured solution. The entropy associated with a correct decision rule is zero. So with the tools at hand, the loss of classificatory power and corresponding information are attributable to the removal of the expert supplied-structure.

Training set	Information per tutorial example (bits)
61 examples from the structured training set produced a rule with 0.151 bits of information per symbol.	0.0025
An unstructured rule was generated using 175 examples. The rule classified perfectly i.e. 0.959 bits of information per symbol.	0.0055

Figure 8/7. Information per tutorial example.

8.3.3. Further classification-database compression

Recall in chapter 2 when database compression was being discussed the following table was printed:

Domain	Database size (space of legal positions)	Number of decision vectors classifying the total space	Size of training set required to induce a complete and correct unstructured classification rule
KPK	83622	2412	304
KPa7KR	209718	3196	175

The same table is reproduced in Figure 8/8 showing the size of the training set required to induce a complete and correct rule by structured induction.

As before, these examples (together with formal definitions of the attributes and induction system) form a complete replacement for the original classification database in the sense that the latter can be reconstructed on the machine from the former. They also represent a powerful set of tutorial examples to explain *most* of the features of the structured rules (they may not explain features like attribute ordering which can sometimes be immaterial).

Domain	Database size (space of legal positions)	Number of decision vectors classifying the total space	Size of training set required to induce a complete and correct structured classification rule
KPK	83622	2412	60
KPa7KR	209718	3196	61

Figure 8/8. Further training-set reductions.

8.3.4. Human understandability of rules

The most important result to come out of this work is the discovery of the enhanced human understandability of rules generated by structured induction. This greatly increases the usefulness of induction in expert systems which (by definition) demand that users should be able to question the machine's rationale. There are 2 major questions that an expert system must be able to answer at run-time. These are:

(1) How did the system arrive at this conclusion? and
(2) Why did the system ask this question or otherwise seek to evaluate this attribute?

Humanly intelligible rules coupled with the self-commenting explanatory feature make it possible to answer the "How?" question while Bratko's condensed form of postfix self-commenting (described in Chapter 4) with slight changes provides an answer to the "Why?" question. As an example of an answer to a Why? question, imagine that the attribute values to the KPa7KR problem are obtained by questioning the user (and not by programmed attribute evaluation) and that the user has just been asked "Is the BK attacked in some way by the promoted WP?". If the user's responded with "why?" the system response could be as shown in Figure 8/9. While this post-processing of self-commentary has not been implemented it does indicate a direction for future work.

if the BK controls the intersect point
    and the BK is not attacked in some way by the promoted WP
    seeing that the BK is not in the way
    and the WK is on an edge and not on a8
it will follow that there is a good delay because of a double attack threat
    and that there is a simple delay to white's queening the pawn
    and that this position is not won for white

Figure 8/9. Proposed form of answer to the "Why?" question. System response after the user responds "why" to the question "Is the BK attacked in some way by the promoted WP".

8.4. Nature of rule languages for computer induction

In Chapter 1 the need was indicated for constraints to be developed that would keep the products of induction within the bounds of brain-compatibility. Working from the results of these experiments which show that the human "brain machine" when looked at as a computer can process decision trees providing that they are both a) linear and b) no more than about 7 tests in length, a new type of computer language has been specified. CDL-1 (Concept Description Language 1, Michie, 1982b & 1983) accepts and processes all and only those expressions that are "concept expressions". The form of such expressions, based on findings with structured induction such as those reported in this thesis, is intended to ensure that they can be both understood and mentally applied. A program that generates only valid expressions of the CDL-1 language has been developed in PROLOG by Dr Ivan Bratko (visiting from the Josef Stefan Institute, Yugoslavia) together with an interpreter that executes these expressions. Appendix H shows the syntax of CDL-1 as implemented by my colleague Mr. Steven Muggleton in the form of a compiler written in "C".

8.5. Conclusions

Recall the question posed in Chapter 1 namely:

> Can techniques for the use of induction be developed which, for the complex domain of KPa7KR WTM, make possible the generation of a complete and correct rule of solution which is both machine-executable and brain-compatible. When translated into English text can such a rule constitute a training aid?

No such manual exists for the domain in question whether humanly or semi-automatically generated. Moreover no humanly generated manual exists for any domain of comparable complexity in chess: this work in part has sought to shed light on the conjecture that at this level of task-complexity the required codification is beyond unaided human power, but not beyond human power given the use of the new inductive tools.

Use of structured induction with self-commenting for the purposes of knowledge gathering and refining for KPa7KR WTM has led to the machine-aided generation of a solution that is both completely correct and literally self-explanatory. The complete rule-base produced by rearranging the self-commenting text and adding a few keywords, is presented in Appendix I. Plainly it could have value as a tutorial aid. The techniques used are themselves not complicated and are easily transferable to other workers in the field.

8.6. Further directions

The work presented in this thesis has shown that using the techniques of structured induction and self-commenting, it is possible to extract from one or more experts knowledge that was previously inaccessible on a conscious level. The question now arises as to whether these techniques could be used to "bootstrap" knowledge into an expert that previously was not available or even present in either man or machine? In other words could these techniques be used to aid experts on a course of investigation that would proceed not (as they do now) through introspection but through a guided man-machine partnership - each guiding the other through problem spaces that alone neither could penetrate?

This sort of investigation might proceed by the induction system proposing examples to the expert for validation. In the case of chess, the system could propose that the expert try and find positions that correspond to certain interesting attribute vectors not yet instantiated. As the search for these positions progresses other information may come to light that would indicate structural or attribute coding changes. If the system had better access to the primitive

attributes it might be able to correlate their internal functionality and suggest which attributes might best be expanded to include others and which might best be split into further primitive attributes or possibly subproblems. The first step in this direction is being taken by my colleague Mr. A. Paterson (1983a) who is investigating the possibility of a clustering algorithm due to Michalski and Stepp (1982) supplying the hierarchical structure. This work is under the direction of Dr. R. S. Michalski of the University of Illinois.

To a limited extent a process of "knowledge creation" has already taken place. The author had no knowledge of the KPa7KR problem at the start of this work. Yet by the conclusion of stage 1 (before having contact with specialists in the field) a solution had been constructed that was accurate in all but 1% of the legal positions. This amount of knowledge was previously not available or present in any member of the stage-1 implementation team. Furthermore throughout stages 2-4 the author found he was able to discuss the complexities of KPa7KR on an equal basis with US National Master Kopec. KPa7KR is still too simple a problem to exceed present human capabilities. A major question that needs answering concerns how well these techniques can function when the class values that an expert associates with situations are speculative or at best unverifiable.

To answer this question more development work must be done by way of languages and interactive environments for inductive systems. In order to perform this development work a suitably complex but well defined problem will need to be found. The problem will need to be so complex that even the world's best experts disagree on classifications. Successful classification of the chess endgame KRPKR using the successors to the techniques presented in this thesis, would powerfully assist towards resolving the matter. We have been assured by A. J. Roycroft, one of the world's leading endgame scholars, that the KRPKR is not only beyond the articulacy horizon of the greatest experts but also far beyond their performance horizon in terms both of dependably correct classification and of choice of move.

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APPENDIX A

Interactive ID3 user manual

Interactive ID3

USER MANUAL

NAME

xmp - Inductive programing by example

SYNOPSIS

xmp attfile

DESCRIPTION

Xmp is an interactive inductive learning program based on Quinlans ID3 (Iterative Dichotomizer 3) [Quinlan 79] which in turn uses Hunts CLS (Concept Learning System) [Hunt 66]. A working knowledge of these is not essential for the correct running of this program but would be advantageous.

Attfile is a file containing textual data about the attributes to be induced over. The first line of this file contains the number of attributes for this problem. Then for each attribute there is a line containing a five character attribute name separated by any number of spaces and/or tabs from a number representing the number of values the attribute can take, separated by any number of spaces and/or tabs from the single character symbols that are to represent these values in the decision trees produced by xmp. The last but one line of this file contains an integer representing the number of class values for this problem. The last line of this file contains five character strings one for each class value. These strings can be separated by any number of spaces and/or tabs. An example of a legal attfile that describes a problem with five attributes and three class values is as follows:

```
5
wethr 3 w d b    /* wet dry and blustery */
insde 2 ft       /* These comments should not be seen by xmp */
soakd 2 ft       /* The comment delimeters are not necessary */
inbed 2 ft       /* xmp demands one attribute per line, the rest */
incar 2 f t      /* of the line is ignored */
3
NTUSE   USEIT   RUBSH
```

Interactive ID3 maintains two example storage areas refered to as primary and secondary store. Examples in the primary store are used to form a rule when the induction process takes place.

The secondary store is used as a convenient place to keep examples that are not of immediate use but may become useful later. Examples are read in (function readexamples) from the terminal. Every example contradicting the current rule is automatically added to the primary store. Examples that do not contradict the current rule and have not been encountered before, are added to the secondary store which is a data store (of the same form as the primary store) that is ignored by the induction process. After every induction the secondary example store is checked for examples that contradict the newly formed rule. If any are found the user is notified. The user has complete control over these two example store areas. A description of the implemented commands follows:

1. R Prints the latest decision rule on the current output. Initially the rule is 'null'. If a developed rule contains 'search' this means that the examples supplied are inconsistent. ie. At least one example has been supplied that has the same attribute values as another example but a different class value.

2. C Prints the latest decision rule on the current output in the form of a 'C' function.

3. W Prints the indices of the items in the window that were used to form the latest rule. After the induction process has taken place the window contains pointers to the examples in the primary store that were used to form the rule; not all primary store examples need have been used. It is doubtful if this command is of any use in its present form since no attempt to map pointer to real example number has yet been made.

4. S This command switches the current output between the files /dev/tty and xmp.out. The latter is buffered so beware of ^C ing the program you may lose up to 512 characters of this file.

5. Q Quits the program in a tidy manner flushing buffers etc.

6. I This command causes a rule to be induced from the examples in the primary store. Each iteration of this process results in a rule which is checked against the remaining primary store items (those not yet used to form the rule). If no exceptions are found then the rule is complete otherwise a selection of exceptions are added to the window and a new rule is formed. The initial number of items in the window and the number added with each iteration can be changed, see the 'T' command. Each iteration of this process results in a line of output on the current output as follows:

- The iteration number for this line.

- The number of items in the window used to form the current (possibly intermediate) rule.

- The number of items added by the function query - not very useful.

- The next column (rule size) gives the size of the rule in nodes, every element of the tree counts as one node.

- The next two columns (fail node and fail data) are greater than zero only when clashes have been discovered in the input data.

- The 'wrong' column indicates how many exceptions there are to the current rule in the rest of the examples supplied.

- The 'search' column indicates the number of search nodes that have been added to the tree.

- Time refers to the CPU time in milliseconds since the induction process began.

7. E Prints out a range of examples from the example data stores in readable form. The default format is suitable for less than 9 attributes (if your terminal width is 80 characters). If you have more than 8 attributes the format for this command can be changed by using the 'T' (tweak) command. The range is prompted for (default all).

8. M Moves a range of examples from primary to secondary example store. The range of examples to be moved (as supplied by the 'E' command) is prompted for (default none).

9. D Same format as the 'M'ove command except that the fingered item/s are deleted.

10. A Same format as the 'M' and 'X' commands except that the range supplied by the user refers to an example in the secondary example store which are to be 'A'dded to the primary store.

11. F Reads in examples in user input format from file 'xmp.d'. The class value for each example should appear after the last attribute value for that example. Comments may take up the rest of the line and are ignored.

12. X This command checks the secondary example store for exceptions to the current rule. Any exceptions found are reported on the current output. The message is of the form:

example 5. now contradicts rule

The example number corresponds to the identically numbered example as reported by the 'E' command.

13. T This command allows the tweaking of certain system parameers. The three parameters that are, at present, tweakable are:

    - <start>. This is the number of examples in the window that the first iteration of rule formation is performed over. Initially zero.

    - <inc>. This is the number of examples that are included in the rule formation with each iteration. Initially one.

    - E command format. If the answer is 'Y' then long format is selected, if the answer is 'N' then short fromat is selected. Otherwise format remains unchanged. Initially long.

    A blank line in answer to any of the questions asked by this command results in no change being made to the variable in question.

ROUTINES

A description of the procedures and functions used in this program follows.

Unix Specific

```
procedure flush(var t : text); extern;
          Flush pending output for pascal file <t>

function clock : integer; extern;
          Return current CPU time in msecs.
```

Induction Specific

```
function min(a,b : integer) : integer;
```
Returns min of <a> and <b>.

```
function random : real;
```
Return a real number in the range 0..1. Successive calls deliver a series of random numbers which have a period of 16384 and a correlation of 0.008 +/- 0.007. See KNUTH vol 2. Uses <randseed> as seed (set to zero in main so that runs can be repeated).

```
procedure setuparms;
```
Initialise window pointers before induction.

```
procedure getdata;
```
Read attfile in format described above.

```
procedure swap(f,t : integer);
```
Swap pointer to data item <f>, with <t>.

```
procedure zerotree;
```
Initialise decision tree.

```
procedure formtree(fp,lp : integer);
```
Append to the current decision tree a rule adequate to explain data items p[fp] through p[lp].

```
procedure addnode(b : boolean; i : integer);
```
Append to the current decision tree a new node with <test> = <b> and <node> = <i>.

```
procedure findcost;
```
Find the cost of the current decision tree.

```
function sameclass : boolean;
```
Find whether all instances here belong to one class.

function costfn(c1,c2 : integer) : real;
Returns entropy of <c1> relative to <c2>. This function takes no account of desirability of attributes and should be modified to include a scaling factor supplied with each attribute in the attfile (future extension).

function query : integer;
A mostly redundant function that, when <start> is zero, loads the initial example.

procedure extract(from : integer);
Sort window by attribute starting at window item p[from].

procedure select(n,lp : integer);
Select <n> items from p[bar+1]..p[lp] placing them after p[bar] and increasing <bar> by <n>.

function decision(i : integer) : integer;
Returns the classification of data item <i> according to the current rule.

procedure findexceptions;
Find all exceptions in the data to the current rule and leave their item numbers in p[bar+1] through p[endexceptions].

procedure induce;
Perform induction on current primary store.

Interactive Specific

procedure printhelp;
Print out one line of help text for each command.

procedure printwindow;
Print the indices of the items in the window that were used to form the current rule. p[1]..p[bar].

procedure printexamples;
Print out the stored encoded examples in a readable form. The two sections correspond to 1) the primary and 2) the secondary example store. If long listing is selected headings are taken from the attfile otherwise they are suppressed and the inter-attribute printing gap is reduced to one space.

procedure printout;
Check if rule formed, if not, print error message, else print the decision tree.

procedure printtree(indent : integer);
Recursive tree printing routine starting at <tp>, indenting by <indent> places to the right.

procedure cprintout;
Check if rule formed, if not, print error message, else print the headers expected by the portable 'C' compiler and function head then call cprinttree to print the body of the function.

procedure cprinttree(indent : integer);
Recursive tree printing routine starting at <tp>, indenting by <indent> places to the right. Print tree as a 'C' function handling 3+ value attributes in 'case' statement.

procedure addtowindow(example : integer);
Move item <example> from <data> array to end of secondary example store, closing up any resultant gap in the primary store.

procedure subfromwindow(no : integer; save : boolean);
Remove item <no> from primary store. If <save> then put it in the secondary example store, else lose it.

procedure addexample(add : integer);
Move last example supplied into the secondary example store.

function contradicts(example : integer) : boolean;
Returns true if item <example> contradicts current rule, else false.

function checkrule : boolean;
Check the secondary example store for exceptions to the current rule. If any are found they are reported and the function returns true, else false.

function getchar(readeln : boolean) : char;
Returns the next non space or tab character typed by the user. Returns space if the user types a blank line. This function eats eoln if <readeln> is true.

function readexample : boolean;
General purpose attribute read routine which returns true if an example has been successfully read in. The newly read example is stored in data[tempd]. It puts out a prompt (constructed from attfile) and then waits for the user to type the attribute value symbols (again gleaned from attfile). If an incorrect symbol is typed or the line is terminated prematurely, the whole input line is ignored. The class value can be any character in the numeric range 1-<n> where <n> is the number of classes. A prompt is output to avoid confusion. It is recognised that the user may wish to interpose his own function between read and store. The user is at liberty to exchange his read function for this one. The only requirement is that he places the calculated attribute values in data[tempd,1..nav[attribute]], where <attribute> is the number of the attribute being decoded. (nav[attribute] is the number of values that this attribute can take). The class value must be stored in data[tempd,0]. It is intended to make this function reside in a library module and thus it is completely self contained. This is the only procedure that should have to contain any domain specific knowledge not available from attfile.

function testexample : integer;

Returns <tempd> if the last example is not a duplicate of one previously supplied, else -1.

function getno : integer;

Returns the value of the next legal decimal number read from the standard input ignoring leading spaces and tabs. Space and tab are treated as number terminators. The rest of the line including the eoln are eaten.

function command : boolean;

Command decoder. Commands are single letters in the set ['U', 'u', 'R', 'r', 'W', 'w', 'S', 's', 'Q', 'q', 'I', 'i', 'E', 'e', 'M', 'm', 'X', 'x', 'A', 'a', 'C', 'c', 'T', 't', 'H', 'h', 'F', 'f']. Spaces tabs and blank lines are ignored, as many commands as desired can be input on one line up to the first command requiring numerical input (after such a command the rest of the line after the number is swallowed up). Returns true until command 'Q' is typed, then false.

FUTURE EXTENSIONS

It is recognised that this version is deficient in certain respects. It is intended that the followiwg extensions be made in the not too distant future. Suggestions for further extensions are welcome (address given below).

- Facility for storing users input strings to be printed out by the P command. This would be useful when the user is supplying input that does not directly correspond to the attribute listing of the E command.

- Make a library module of the function readexamples so that this can be compiled seperately from the main program and linked in. This function is the only one that may need to embody problem dependant information, a library module would make the process much neater and would, depending on the type of Pascal compiler used, allow modules written in some other language to be linked (eg. 'C', CLIP etc.).

- Attribute weighting proportional to their cost of execution. Numbers can be supplied in attfile.

- Attribute vector weighting proportional to number of real world situations that map to the same vector. Would result in a more cost effective solution being

generated. Numbers can be supplied on example entry.

- Change 'W' command to reflect primary store example numbers.

BUGS

None known, please report any bugs found to the author

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Appendix 1

Non-standard features

Line	Feature
477	Zero widths are non-standard
608	clock is a nonstandard function
619	clock is a nonstandard function
1232	Zero widths are non-standard
1246	Zero widths are non-standard
1272	Zero widths are non-standard
1282	Zero widths are non-standard
1408	Zero widths are non-standard
1547	Two argument forms of reset and rewrite are non-standard
1552	Two argument forms of reset and rewrite are non-standard
1566	Two argument forms of reset and rewrite are non-standard
1584	Two argument forms of reset and rewrite are non-standard
1593	Two argument forms of reset and rewrite are non-standard
1712	Two argument forms of reset and rewrite are non-standard
1713	Type clash: unequal length strings
1714	Type clash: unequal length strings
1724	flush is a nonstandard procedure

APPENDIX B

Gordon and Schoppers project report

STRUCTURED INDUCTION

IN AN

INTERACTIVE ENVIRONMENT

Rob Gordon
Marcel Schoppers

CS397DM
December 18, 1981

TABLE OF CONTENTS

ABSTRACT

To date, computer induction has been carried out only for problem spaces describable by automatically generated, complete databases of problem instances. If continued, this will restrict the applicability of computer induction to problems which can be described by an algorithm, and which can be contained within the space and time bounds of practical databases. Our work aimed to show that it is possible to maintain expert performance levels for computer induction, without the use of a comprehensive database. We sought to classify all positions of the King-Pawn-King-Rook (KPKR) chess endgame, with the pawn on A7, as WON or NOT WON for White, using only our own knowledge of the problem. With the use of Interactive ID3 and decision tree structuring, we were able to surpass the performance of humans more knowledgeable than ourselves in this endgame. Although in the absence of an exhaustive database we cannot claim that our decision trees completely span the problem space (just as human experts are not usually able to express all their knowledge), this is a modest price to pay for the attainment of expert computer induction in problem areas too large or too informal for an enumeration of problem instances.

BACKGROUND

The research reported in this paper extends the relevance of automated induction to problem areas for which no complete information can be obtained. It is based especially on two earlier research efforts. Quinlan [QUIN79] developed the ID3 algorithm, which generated, from a database of exemplary chess positions, a decision tree to classify all positions of the KRKN game as won-for-white, drawn, or won-for-black. However, the decision tree generated by ID3 was found to be inscrutable even for experts, in the sense that it was too large and disorganised to be readily committed to memory. It was immediately clear that such a decision tree did not correspond well with the thinking of human experts. Shapiro and Niblett [SHAP81] attacked this divergence by examining the constraint that the decision tree should be 'linear'. More precisely, a decision tree (or subtree) was to be a list of subtrees. It was found that their approach resulted in decision trees that were not only well received by expert chess players, but were no more expensive to generate than Quinlan's trees.

In developing their linear decision trees, Shapiro and Niblett introduced several ideas which are of central significance to the work reported here. To achieve linearity it was necessary to decompose the problem space before generating a decision tree. That is, they identified, out of their chosen chess endgame, several themes which could be

considered as subproblems. For each of these subproblems a decision tree was generated, and the resulting trees were then combined as subtrees of the tree for the complete problem. By first identifying such subproblems Shapiro and Niblett gained control over the structure of the decision tree.

An outgrowth of their approach was the development of Interactive ID3, a modification of Quinlan's ID3. Interactive ID3 was designed to facilitate the task of the expert in selecting subproblems for which to generate decision subtrees, to provide him with a capability for entering problem examples on-line, and to let him interrogate ID3 at any stage in the process of decision tree synthesis. This gives the expert access to far more knowledge than the final decision tree alone. It allows him to observe patterns or counter-intuitive developments in the synthesis of a decision tree, and thus invites him to modify the tree. Hence, Shapiro and Niblett introduced into ID3's automated induction the possibility for human engineering of the decision tree's structure.

This has an important consequence. Since the expert can monitor the synthesis of the decision tree, he can select examples to contradict the decision tree at any stage of its synthesis. He is thus in an excellent position to build a training set of examples for Interactive ID3. By finding an instance from the problem space (e.g. a board position from a chess endgame) that contradicts the current decision tree, he invalidates that tree and adds to the training set. Interactive ID3 then

induces a better-informed decision tree, and the number of instances that can be classified correctly is thereby augmented.

For our purposes, the key aspect of this capability is that an adequate training set can be assembled by humans. It is at this point that our research departs from earlier work. To date, decision trees for chess endgames have been induced by supplying to ID3 a complete database of all possible endgame positions, along with their classifications. Of necessity, the endgames have been of limited complexity. But what of problem spaces for which it is infeasible to build a database? A database may be infeasible either because the number of problem instances is too large, or because there is no algorithm to exhaustively generate them. Unless human expertise can supplement or replace the database, automated induction will be restricted to relatively small and enumerable applications.

It was decided at the outset of this research that structured decision trees were preferred for the sake of intelligibility. This should not be construed as a necessary condition for this research, since we know of no evidence that humans cannot understand more general trees. However, the process of isolating, solving and combining subproblems into linear decision trees yields a memory-efficient recoding of the decision tree. While linearity is not our primary goal, its positive influence on the intelligibility of decision trees is fundamental to the success of our induction method.

The primary goal of our work has been to demonstrate the possibility of achieving expert performance for a chess endgame, without the use of a comprehensive database of board positions. We have shown that it is indeed possible to dispense with such a database while maintaining decision trees at expert level. Moreover, our use of Interactive ID3 required only modest chess experience, yet spurred consideration of a subproblem that had been overlooked by highly skilled players working with unstructured trees. This success implies that human knowledge aided by computer induction can achieve levels of performance comparable with that allowed by exhaustive enumeration of problem instances, where such enumeration is feasible. The reasonable expectation is that expert induction may also be achievable for problem domains in which only humans can supply information. Interactive ID3 may therefore be a useful tool for the extraction and codification of human expertise.

METHOD

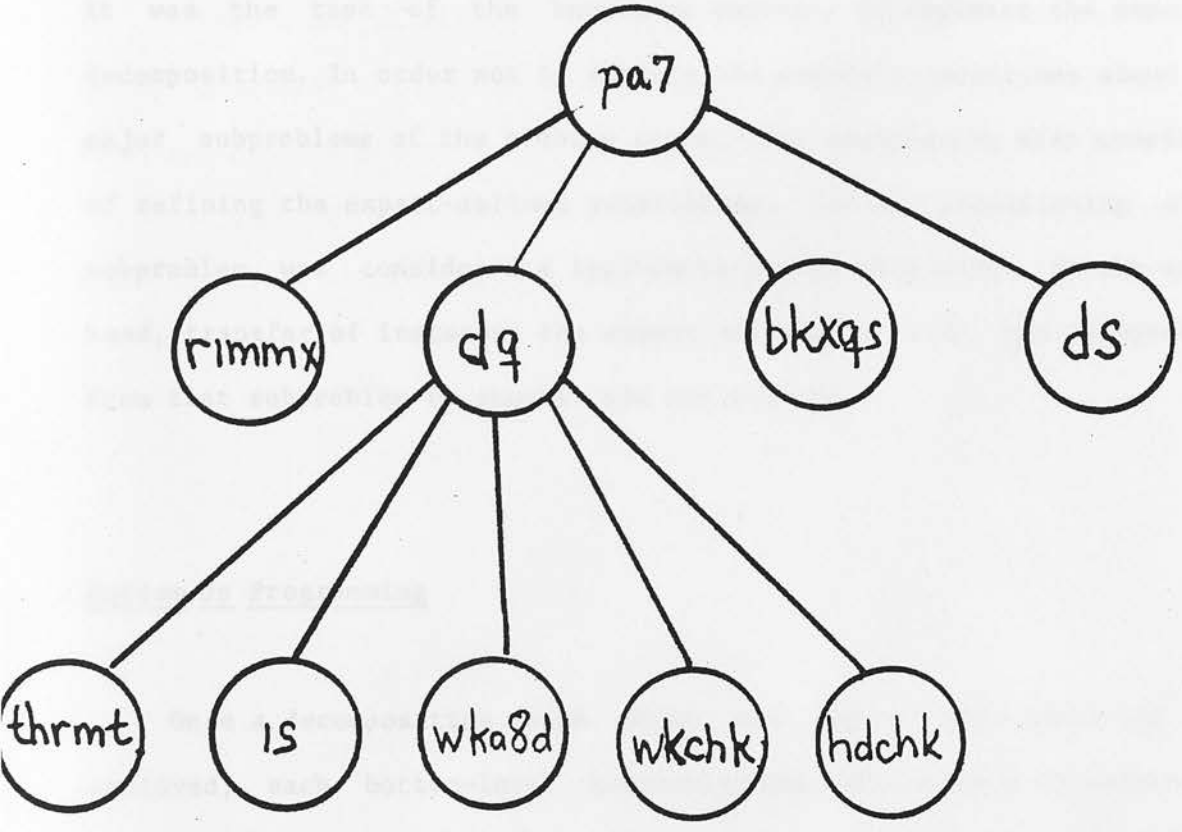
Top-Down Design Goals

Our basic methodology consisted of a top-down structuring of the problem space followed by iterative refinement of subproblems and attributes. Employing the techniques reported by Shapiro and Niblett [SHAP81], our first step was to partition the KPKR pa7 problem space into distinct subproblems. A group of attributes which captured the salient features of each subproblem was then defined. The subproblems and attributes were massaged and honed to satisfy two design goals.

The first design goal was to decompose the problem space in a manner consistent with an expert's perception of its structure. In an effort to achieve this goal, expert advice was employed. Using expert advice to decompose the problem space guarantees subproblems will be intelligible and compact. The final decomposition arrived at is represented as a tree in Figure 1.

Construction of linear decision trees was our second design goal. Linear decision trees are guaranteed if subproblems are chosen such that the evaluation of all instances of a given class value leads to an immediate classification of those instances one level up in the decomposition tree.

Figure 1: The problem decomposition tree



Tension between these design goals imposed the strictest constraint upon our solution. If the expert decomposition of the problem space did not satisfy the constraint necessary to generate linear decision trees, it was the task of the knowledge engineer to engineer the expert's decomposition. In order not to violate the expert's intuitions about the major subproblems of the problem space, this engineering step consisted of refining the expert-defined subproblems. Further partitioning of a subproblem was considered a legitimate engineering step. On the other hand, transfer of instances the expert associated with one subproblem from that subproblem to another was not allowed.

Bottom-Up Programming

Once a decomposition with which the expert was satisfied was achieved, each bottom-level subproblem was defined as a collection of simple attributes. A simple attribute is a feature of the problem space evaluated by hand-coded C functions and the CLIP emulator. Simple attributes correspond to leaf nodes of the decomposition tree. A partial list of simple attributes and their English descriptions appears in Appendix 1.

Attributes were chosen to correspond to questions the expert would ask when evaluating board positions. From the standpoint of human understandibility of the final decision trees, it is essential that the

C functions associated which each attribute reflect exactly the evaluation implied by these questions. This will become clear when the testing and refinement of decision trees is discussed.

Given a set of simple attributes that described a subproblem, we then generated a decision tree to classify instances from this subproblem space. The generation of the decision tree was an iterative procedure and is described in the next section. The final decision tree for the wkchk subproblem is shown in Figure 2.

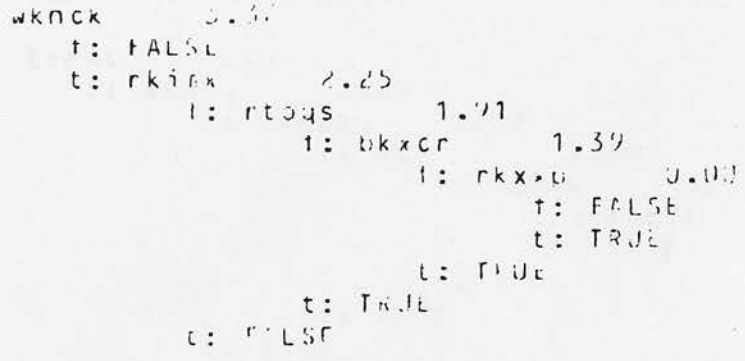


Figure 2: The WKCHK decision tree.

Each low-level subproblem was treated in this way. We then moved one level up in the decomposition tree and solved the subproblems at that level. Instead of defining non-leaf node subproblems in terms of simple attributes, they were described in terms of their descendant subproblems. An attribute which represents the decision tree solution to a lower level subproblem is called a composite attribute. Composite attributes are evaluated by Interactive ID3 generated C programs. These

C programs are a translation of the standard ID3 generated decision trees. dq is an example of a subproblem described in terms of composite attributes. Figure 3 shows the decision tree for this subproblem. Note the appearance of wkchk as an attribute. In the process of generating this tree, Interactive ID3 used the C function generated as a solution to the wkchk subproblem to evaluate instances of the dq problem space. The same is true for the other attributes of dq decision tree except for hdchk.

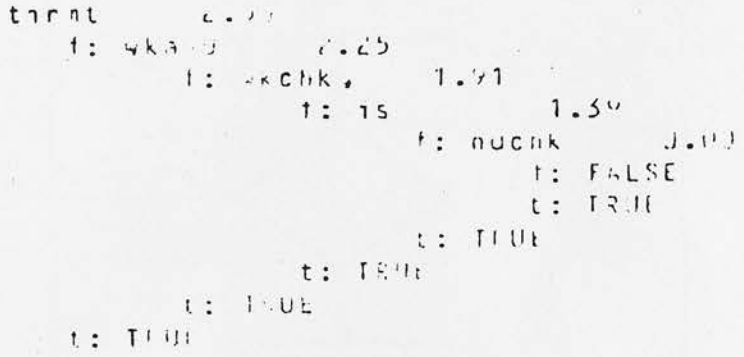


Figure 3: The DQ (delayed queening) subproblem.

Iterative Refinement: Engineering Loop

Generation of a final tree at each level of the decomposition tree was accomplished through a series of steps we call the engineering loop.

At the heart of this process is a generate and test strategy. Given a set a attributes and a set of example board positions, Interactive ID3 generates a decision tree which the expert then tests by supplying further examples and observing how the current decision tree behaves on these examples.

This procedure is illustrated in Figure 4. Two decision points are contained in the engineering loop. At these points the expert or the knowledge engineer is able to interact with Interactive ID3 and make decisions that influence the final structure of the tree.

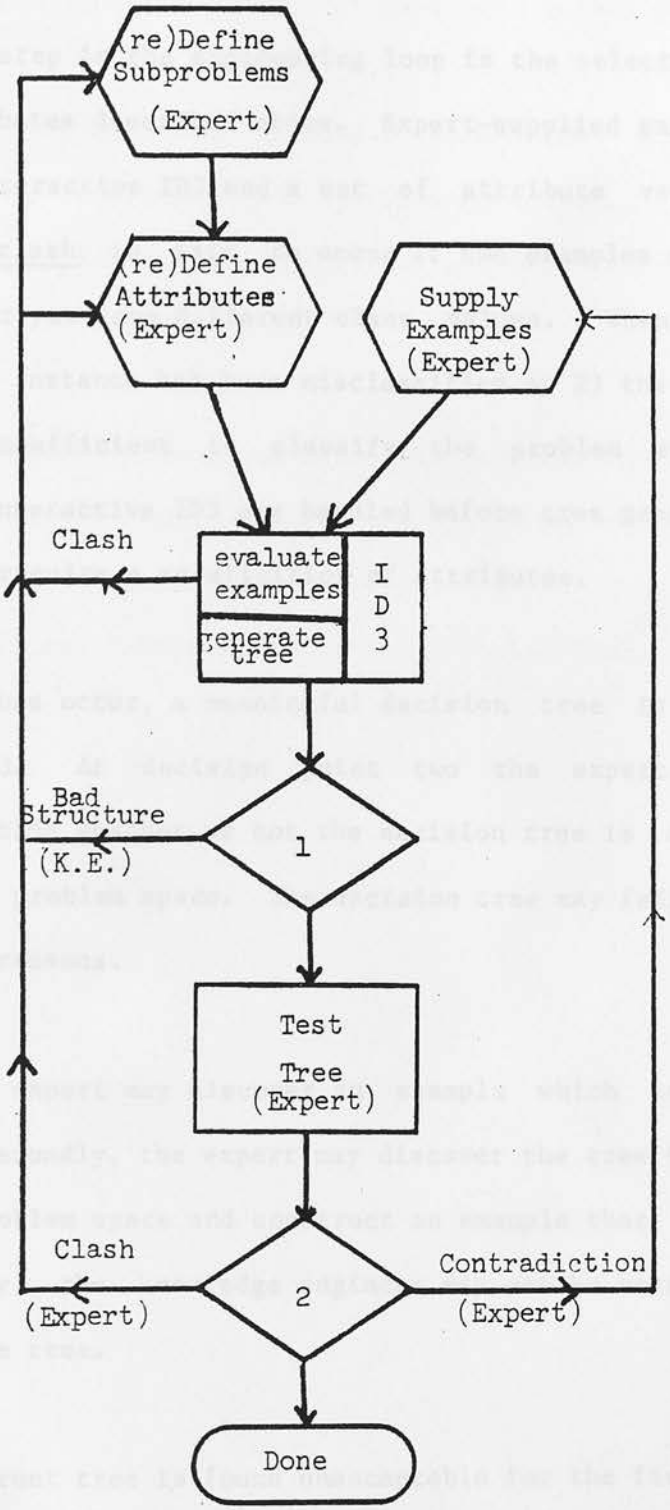


Figure 4: Engineering Loop

The first step in the engineering loop is the selection and definition of attributes described above. Expert-supplied examples are then evaluated by Interactive ID3 and a set of attribute vectors is constructed. A clash is said to occur if two examples map to the same attribute vector yet have different class values. When this occurs, either 1) an instance has been misclassified or 2) the current set of attributes is insufficient to classify the problem space. Clashes discovered by Interactive ID3 are handled before tree generation. Typically, clashes require a redefinition of attributes.

If no clashes occur, a meaningful decision tree is generated by Interactive ID3. At decision point two the expert and knowledge engineer may decide whether or not the decision tree is acceptable for the particular problem space. The decision tree may fail to be acceptable for three reasons.

First, the expert may discover an example which contradicts the current tree. Secondly, the expert may discover the tree is insufficient to cover the problem space and construct an example that results in a clash. Finally, the knowledge engineer may not be satisfied with the structure of the tree.

If the current tree is found unacceptable for the first reason, the example that contradicts the tree is added to the example set and a new tree is generated. This corresponds to the right arc leaving decision

point two and terminating in the block labelled 'Supply Examples'.

If a clash is discovered by the expert after tree generation, attributes must be redefined and a new tree generated. This corresponds to the left arc leaving decision point two and terminating in the block labelled '(re)Define Attributes'.

If the knowledge engineer is not satisfied with the structure of the tree, attributes must be redefined, or in the extreme case, a new subproblem generated. This case corresponds to the arc leaving decision point one and terminating in the block labelled '(re)Define Subproblems'.

RESULTS

Approximately four man-weeks were spent on the work reported here. An example set of decision trees generated in the solution of the KPKR pa7 problem appears in Appendix I. All decision trees are linear and each was constructed from a small number of attributes, satisfying the design goal of intelligibility.

These results can be viewed as a verification of Shapiro and Niblett's work [SHAP81]. However, our work goes a step further and shows that not only does structured induction make a solution more humanly understandable, but in the case where no classification data base exists, structured induction makes a solution feasible. Further, since a human expert's advice will be necessary in the case that no data base exists, the interactive environment provided by Interactive ID3 supplies the setting required to take advantage of this structuring.

We have isolated two tools, structured induction and the interactive environment provided by Interactive ID3, and have shown that along with expert knowledge these tools are sufficient to produce a feasible solution to some classification problems for which no data base exists. Our results also suggest that these to tools may be vital to the economical construction of expert systems that, for a given problem space, compete with human experts for accuracy. A close look at the engineer-

ing loop bears this out.

What role does structured induction play? Consider the point in the engineering loop at which the expert must test a decision tree. As long as the expert can concentrate on a small decision tree constructed from a set of conceptually clean attributes his task is manageable. To put this in a more familiar setting, consider the relative ease with which one can debug a highly modular program with no side-effects or global data. Compare this with the difficulty involved in debugging a loosely constructed program with arbitrary side-effects and global data. In debugging a modular program, a programmer is able to troubleshoot each procedure independently, assuming correct input and ignoring global features of the program. Similarly, with structured induction, the expert is able to concentrate on the correctness of small trees without regard to global features of the problem space. The fact that null nodes appear in the unstructured solution (see Appendix II) of KPKR pa7 is evidence that having to view the problem space at only the top-level limits the expert's ability to supply relevant examples and thereby limits his ability to test the correctness of decision trees.

What role does an interactive environment play? The solution to a problem of the class we are discussing is highly dependent on an expert to verify the correctness of results. This type of problem can be viewed as a knowledge acquisition problem: the more knowledge that can be extracted from the expert, the closer to an absolutely correct solu-

tion one can come. What type of environment provides the best means for extracting expert knowledge? Our results suggest that an interactive environment is that type of environment. An interactive environment allows the expert to operate in an environmemt to which he is accustomed. Interaction provides immediate feedback, backtracking in problem solving and trial and error testing and refinement techniques, all of which are common elements of an expert's approach to problem-solving. The importance of interaction in developing our solution is clear from the key role the expert plays at every step in the engineering loop.

The results of our work have immediate consequences in the field of expert systems. The knowledge acquisition problem mentioned earlier has been called the "bottleneck" [FEIG77] in the construction of reliable expert systems. Given that experts solve problems using an informal and intuitive approach, how is the knowledge they employ to be extracted and formalized so that it can be incorporated into rule bases used to drive expert systems? Assuming this question is answered adequately, what guarantee is there that the rules extracted will be transparent to humans? That expert systems operate on reliable rule bases is not sufficient. Their operation must be understandable to human beings who are to interact with them.

Our solution to the KPKR pa7 chess endgame is a partial answer to both these questions. First, knowledge acquisition is facilitated since, instead of answering questions of the sort "What rules do you use

to solve this problem?", the expert need only answer questions of the sort "What features of this problem are important?" Quinlan makes this point in his paper [QUIN79] on the KRKN endgame but does not discuss its implications for solving classification problems for which no data base exists. Secondly, rule transparency is improved through exploiting problem structure and engineering linear trees.

Our results are not absolutely correct but show that automatically induced rules can achieve expert performance in classifying some problem spaces. Finally, in domains for which no data base exists, the synthesis of a complete and consistent rule to classify problem instances can not be handed over to an induction algorithm as Quinlan [QUIN79] implies is possible. Verification of completeness is not possible without a data base and the best one can hope for is an expert-correct solution. The tools described in this paper offer a means of augmenting an inductive algorithm as a step toward expertly classifying complex problem domains.

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ACKNOWLEDGEMENT

We are indebted to Alen Shapiro for supplying many of the ideas and resources that motivated our research, and for the congenial setting he provided for our evening team efforts.

APPENDIX I

As a limitation inherent in computer induction without the use of a database or other means of guaranteeing completeness, it is not possible to claim that we have classified the entire space of KPKR pa7 positions. At best we can say that, for the subproblems dealt with so far, twenty simple positional attributes have proved sufficient. As an indication of the complexity of the problem this number may be too high (some of our attributes may be definable in terms of others) or too low (there may be subproblems which we have not yet recognized).

Subject to the same reservation, the problem space is presently partioned into eight subproblems. Seven of these are defined solely in terms of simple attributes, and the other one is defined as a collection of several subproblems. Below we describe three of the subproblems in detail, but it will be of interest to present the other five briefly.

rimmx: Can the rook be captured by White's first move? This subproblem is actually a simple attribute.

bkxqs: Does the Black king attack the promotion square A8? This is also a simple attribute.

hdchk: Is there a hidden check on White? Again a simple attribute.

thrmt: Is there a mate threat on White? This subproblem requires five simple attributes.

is: Can Black create a skewer as soon as White promotes the pawn? This subproblem requires seven simple attributes.

dq: Is pawn promotion delayed in such a way that Black can use the delay to prevent successful promotion altogether? This subproblem is a combination of the hdchk, thrmt, is, wka8d and wkchk subproblems.

The subproblems ds (delayed skewer), wkchk (White king in check) and wka8d (White king on A8) are described below.

Subproblem: Delayed Skewer

Description: After White's pawn promotion, is Black able to force the White king onto the same rank or file as the promoted pawn? This would achieve a skewer, with subsequent capture of the promoted pawn.

Attributes:

wtoeg - Is the White king one move away from rank 8 or file A?

dsopp - Are the kings in opposition, so that a check on the White king will force it to the edge?

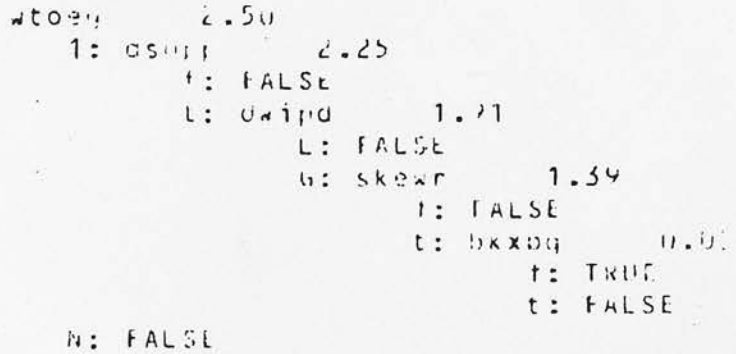
dwipd - Is the White king's distance to the rook's intersection point marked X below greater than two? If not, the White king can prevent the skewer by moving toward the intersection point.

skewr - Are the pawn and intersection point on opposite sides of the White king? If not, the intersection point can be defended against the rook by the promoted pawn.

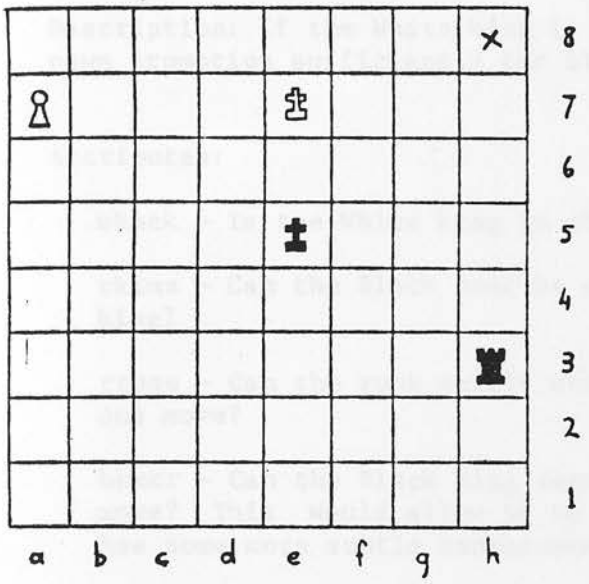
bkxbq - Will the Black king be in check after pawn promotion? This would also forestall the skewer.

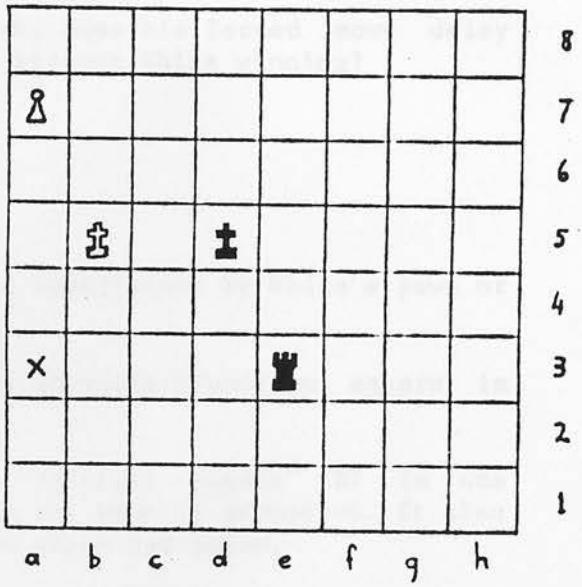
Training Set Size: 6 examples.

Decision Subtree:



Case Studies:-





White to move. The left board is NOT WON for White, since the delayed skewer results in Black's rook capturing the pawn. The right board is WON for White, because the Black king can be checked on pawn promotion, and because the White king can move to attack the rook's intersection point.

Subproblem: White in Check

Description: If the White king is in check, does his forced move delay pawn promotion sufficiently for Black to prevent White winning?

Attributes:

wknck - Is the White king in check?

rkimx - Can the Black rook be captured immediately by White's pawn or king?

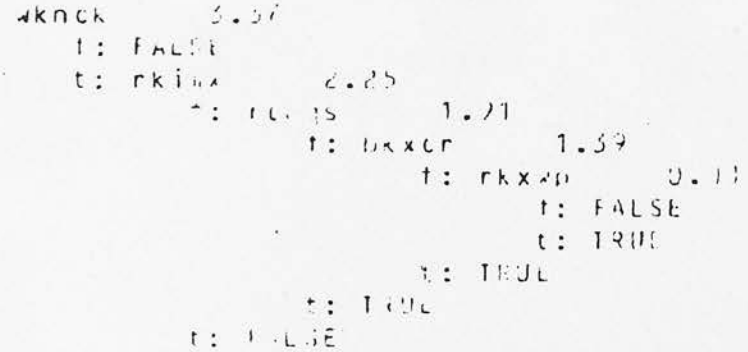
rtoqs - Can the rook safely attack the queening/promotion square in one move?

bkxcr - Can the Black king reach the "critical square" B7 in one move? This would allow it to capture the pawn on promotion. It also has some more subtle consequences, for which see below.

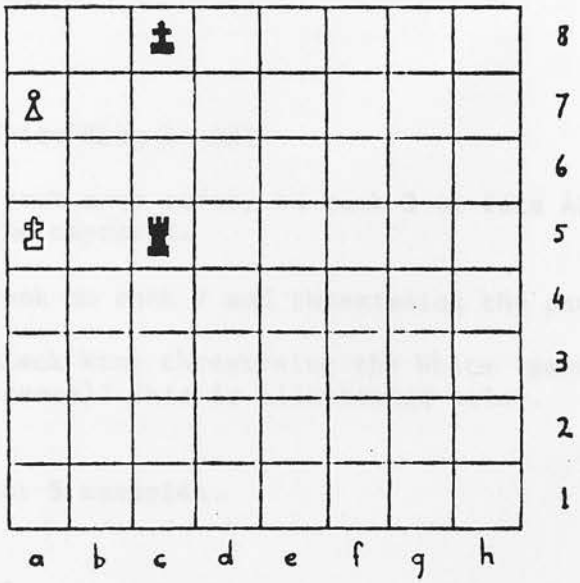
rkxwp - Is the rook on rank 7 and threatening the pawn?

Training Set Size: 6 examples.

Decision Subtree:



Case Study:



White to move. The board demonstrates the significance of the critical square. The best play is 1. K-B6, defending the pawn against the Black king, and A5 against the rook. But then 1. ... R-A5 and when the sacrifice is accepted, 2. KxR, K-B7 and the game is NOT WON for White.

Subproblem: White King on A8

Description: Does the White king's being on A8 give Black an opportunity to stop the pawn promotion?

Attributes:

wkna8 - Is the White King on A8?

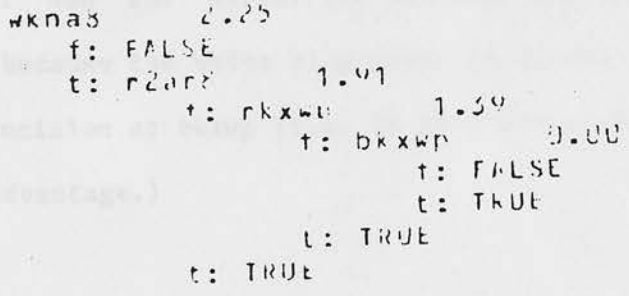
r2ar8 - Can the rook move safely to rank 8 or file A? If so the pawn will eventually be captured.

rkxwp - Is the rook on rank 7 and threatening the pawn? If so, RxP.

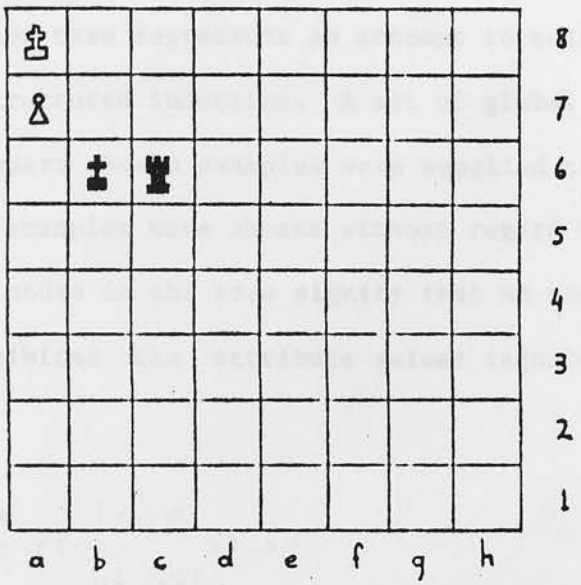
bkxwp - Is the Black king threatening the White pawn (ignoring the White king's presence)? This is illustrated below.

Training Set Size: 5 examples.

Decision Subtree:



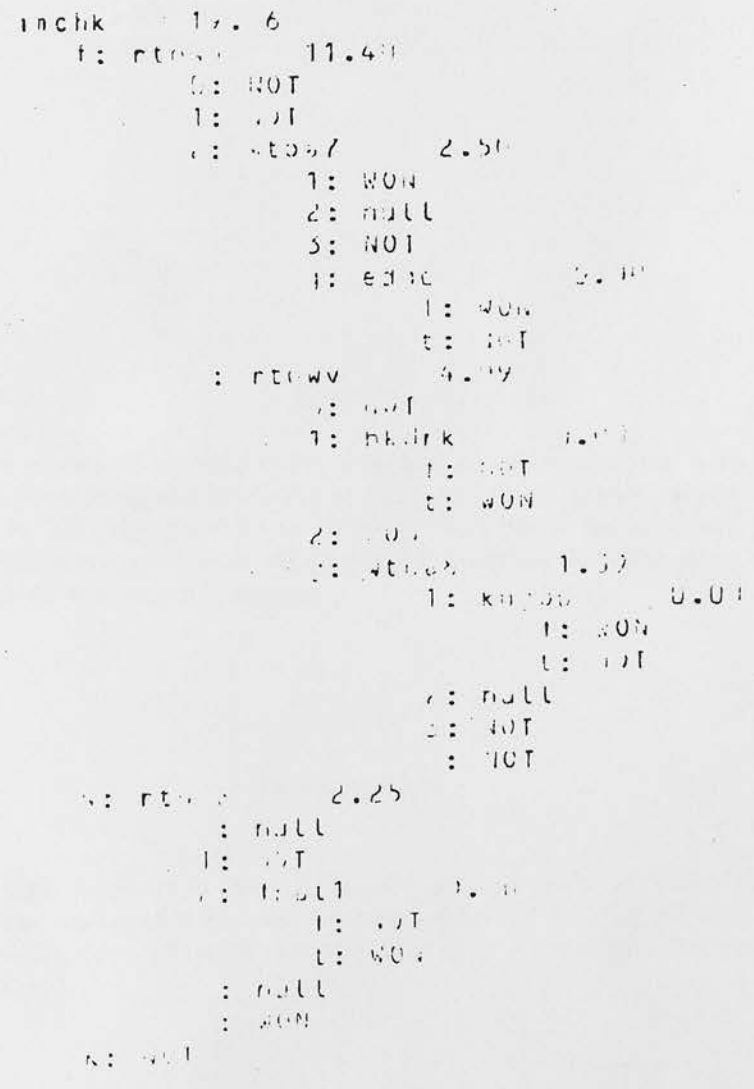
Case Study:



White to move. The play goes 1. K-B8, R-C8ch; 2. KxR, KxP and the game is NOT WON for White. The decision tree correctly evaluates r2ar8=false because the White king moves to defend C8, and reaches the TRUE decision at bkxwp (i.e. it is a useful delay because it is to Black's advantage.)

APPENDIX II

This decision tree represents an attempt to solve the KPKR pa7 without using structured induction. A set of global attributes along with a set of expert chosen examples were supplied to ID3. Both the attributes and examples were chosen without regard to problem structure. The null nodes in the tree signify that no examples were supplied that exhibited the attribute values required to reach those nodes.



APPENDIX C

Reinke project report

Structured Decision Trees for the Chess Endgame KP vs. KR (pa7 and pb7)

Term Project
Computer Science 397
Professor D. Michie

Robert Reinke
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Abstract

Using an interactive version of Quinlan's ID3 induction algorithm, decision trees were built for two subsets of the chess endgame King and Pawn versus King and Rook, pawn's (white) side to move. One tree classified all legal KPKR(pa7) positions as WON or NOT WON for the black (rook's) side. The other classifies legal KPKR(pb7) positions as WON or NOT WON for the white side. The trees have not, at this time, been proven to be completely correct.

Introduction

There are 209,718 legal positions in the KPKR(pa7) problem space, of which 129,825 are won for white. Shapiro (1983) has developed a decision tree which correctly classifies all legal positions as WON or NOT WON for the white side. The work described here is an attempt to to the same for pawn on b7, and for the black side of pa7.

Quinlan (1979) showed that machine induction could be successfully and efficiently applied to a chess endgame problem. His ID3 algorithm, based on Hunt's Concept Learning System (Hunt 1966), was the basic induction tool used in deciding lost n-ply for the endgame KR vs. KN. An interactive version of the same algorithm was used in this work, so some comments on the algorithm, its strengths, and its weaknesses are warranted.

ID3 takes as input a set of examples (specified in terms of user supplied attributes) and their class values. It produces a decision tree that completely and correctly classifies the examples (and hopefully a large part of the event space) on the basis of their attribute values. The algorithm works in an incremental fashion, selecting a group of examples from the example set to be the 'window' or 'working set.' A decision tree is built from the window. ID3 tests this decision tree against the set of examples provided. Any examples which contradict the current decision tree are added to the window, the old tree is discarded, and a new tree is built. When no contradictions to the current tree are found within a user-specified amount of time, ID3 halts and accepts the current decision tree.

Building a decision tree from the window is a recursive process. At any given point, the program selects the most valuable attribute (value being based an on information-theoretic measure) and branches on that attribute's values. Examples in the window are assigned to the child node to which they belong. For every child of the current node, the algorithm either recurses, or, if every example in the node is of one class, halts and moves to a sibling node.

Programs synthesized by ID3 are fast and efficient as compared to mini-max search and programs written by humans (Quinlan 1979). Quinlan (1982) has also show that the degree of correctness of a tree produced by ID3 depends on the number of examples provided, and not on the size of the example space. These are strong advantages, especially in a domain as complex as chess.

On the negative side, ID3-induced rules are rarely intelligible to humans, although they are human-executable. At present, the only way around this difficulty is through a process called structured induction (Shapiro and Niblett 1981). However, the comprehensibility of trees built by structured induction is obtained only at the cost of much more work by the programmer and/or domain expert.

In structured induction, the programmer and domain expert break the problem down into subproblems. Subproblems may be broken down further, if this is necessary. ID3 is then applied to each of the low-level subproblems in turn, producing a linear decision tree at each level. At the higher levels, the trees already built are use as simple attribute-evaluatores, giving linear trees at these levels, also.

For a more thorough discussion of structured as compared to unstructured induction, see (Shapiro and Niblett 1981).

Programming Tools

Development of the decision tree was done on a VAX 11/780 running UNIX. Two major software tools were used. The first was an interactive version of ID3 developed at the University of Edinburgh. The second was a parallel aray processing system, CLIP.

attrs.h file

```
#define ATTRS 2

int goodp(), Cds();

int (* attr[]) () = {
        Cds, goodp
};
```

attval.Bds file

```
2
Cds    2    f t     Is context for ds established
goodp  2    f t     Is this a good delayed skewer for black
2
FALSE TRUE
Bds
Does B win via a delayed skewer threat
```

Bds.atts.c file

```
#include "ds.h"

extern int wr, wf, pr, pf, br, bf, rr, rf;

Cds(){
  return(comment(ds(), FALSE, "", "", "", LEVEL));
}

goodp(){
  return(comment((rr <= 6 && rf == 8 && wf == 5 && br == 5), FALSE, "", "", "", LEVEL));
}
```

Bds.d file

```
a3 a7 c6 b7   1   /* not ds or goodp */
e7 a7 e5 h3   2   /* good ds */
b8 a7 d7 d4   1   /* ds, not goodp */
e3 a7 e5 h3   1   /* not ds, goodp */
```

Figure 1. XMP specification files for the subtree Bds (delayed skewer)

```
rimmx
  f: dq *
      f: bxqsq
          f: stlmt
              f: ds *
                  f: WON
                  t: NOT
              t: NOT
          t: NOT
      t: NOT
  t: WON
```

* = subtree
rimmx = can white capture the rook immediately and safely?
dq = is there some delay to white's queening the pawn?
bxqsq = does black control the queening square(a8) ?
stlmt = is the white king in stalemate?
ds = is there a delayed skewer?

Figure 2. Alen Shapiro's top-level structured tree for the white-to-win side.

The version of interactive ID3 used here is the same as that employed in developing Shapiro's white-to-win tree for pa7. The induction machinery is contained in a pascal program, XMP. Interactive ID3 is dependent on a number of user defined data files. Four files are used to define a subproblem area. These are:

attrs.h
Contains "C" declarations of the attribute names and their types, as well as defining the number of attributes to be used.

attval.<treename>
Contains natural language definitions of attributes, the number of values each may take, and associates a question with each. Also lists the class values examples may take, the number of such class values, and associates a question with the tree.

<treename>.atts.c
Contains the "C" code for the low-level attributes.

<treename>.d
Contains the examples XMP will use in building the final decision tree.

A listing of these files for the subproblem Bds of pa7 (see Problem Breakdown) is shown in Figure 1.

During the development stage, ID3 is used in interactive mode. Examples may be presented to the algorithm interactively, or read in from the <treename>.d file. The user may add and delete examples, move them to a 'secondary' store, or examine them in attribute form. The primary goals at this stage are to remove redundant examples and to resolve *clashes* -- examples that have identical attribute vectors but different class values. This is usually done by adding additional attributes. Once an acceptable group of attributes have been developed, and a good set of examples obtained, XMP is used to build a "C" -routine and a library. These allow higher level trees to use the subtree as if it were an attribute.

Two types of library are built: commented and uncommented. The commented version, when executed, provides a running commentary on decision tree traversal using the questions provided in the attval file. The uncommented version simply returns a class value.

Unix Makefiles were used extensively to aid in the compilation, loading, and execution of system trees, libraries, and routines.

The second major tool, CLIP, is described extensively in (Shapiro and Niblett 1981) and (Bratko et. al. 1981). In this domain, CLIP provides a large number of 8 x 8 bit planes which are used to simulate the chess board. The "C"-coded attributes make calls to CLIP functions to determine various positional relationships. Each piece on the board is delegated a single plane to represent its position (the piece's position being the single "1" in the plane). Other important pieces of positional information are also given single plane representation, e.g. the plane called WPROHIB represents thos positions which are prohibited to the white king. CLIP allows the user to apply logical operations to entire planes and pairs of planes. This yields the information necessary to determine the values of programmed attributes.

Problem Breakdown

Since a correct tree had already been built for the white-to-win side of pa7, it seemed an unnecessarily complex and wasteful task to start either of the two new trees from scratch. For this reason (and because of time considerations), the new trees are split into essentially the same subproblem areas as Shapiro's tree. Each of the new trees has its own special features, however, and they will be discussed separately.

1. Pawn on a7, Black to Win.

Each level two and level three subtree of Shapiro's tree was treated as a class of problems for the black-to-win side. This allowed us to deal with only those areas of the problem that were known to be NOT WON for white (the NOT nodes in Shapiro's top level tree, Fig. 2),

The various subtrees of the white-to-win problem were used to establish the *context* for the black-to-win side. Within a context, the only consideration need be "given that the position falls within this category, can Black win?". If none of the contexts were satisfied, then it could be safely assumed that black would not win. This approach saved much duplication of effort, but did provide a few difficulties; these will be discussed in the Results section.

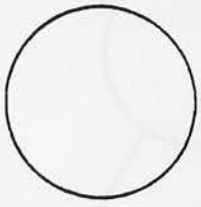
The final problem breakdown for KP vs. KR(pa7), black-to-win is show in Figure 3. Expert help was obtained from Danny Kopec in forming low level attributes for the various subtrees. Bds prove to be the simplest subproblem, requiring only one positional attribute; very few positions in this context were won for black. Bxqsq was also fairly simple. The delay to queening (Bdq) subproblems proved more challenging. Expert help was again available for the subproblem "white king on a8." However, no help was available for the other subproblems of dq; we were forced to develop our own attributes. Hopefully, debugging that took place after the trees were built made up for the lack of expert advice at this earlier stage.

The most important ideas for the black-to-win side of the KPKR(pa7) endgame of course center around white's ability to queen the pawn safely. For most positions there is nothing black can do to prevent queening. The interesting positions are those where black may delay queening or capture the pawn. One important consideration is whether the white king is close enough to defend the pawn. Another (related) idea is that of the black king restricting the movement of the white king. Almost every subtree contains attributes implementing these ideas.

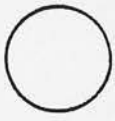
Another important idea is that of a safe check, i.e. can black check without being captured, thereby preventing the queening of the pawn. This idea is encoded in several of Shapiro's attributes (e.g. thrmt), several of those shown in Fig. 3 (e.g. Bwka8) and appears throughout the pb7 tree.

Figure 3. (next page) Problem Breakdown for the endgame KP vs. KR(pa7), black to win. See appendix for the definitions of the attributes and trees.

LEGEND



= black-to-win tree or subtree



= white-to-win subtree (Shapiro 1983).

= low-level ("C"-coded) attribute

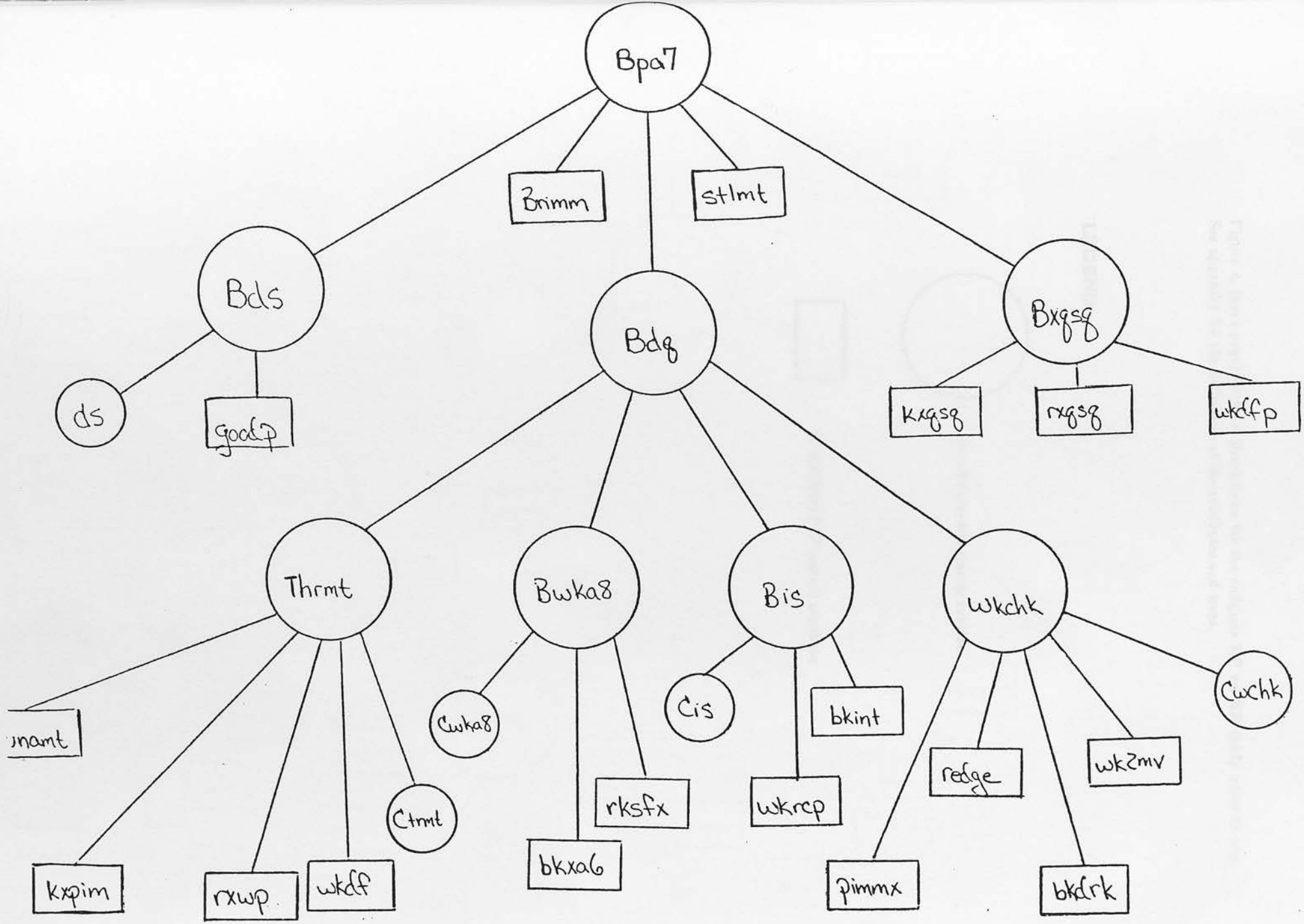
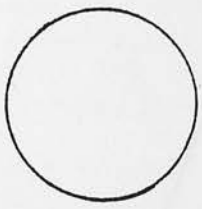

Bpa7
Brimm
stlmt
Bds
Bdq
Bxqsq
ds
goqfp
kxqsq
rxqsq
wkdfp
Thrmt
Bwka8
Bis
wkchk
rnamt
Cwka8
Cis
bkint
Cwchk
redge
wk2mv
Ctrmt
rksfx
wkrcp
kxpim
rxwp
wkdf
bkxa6
pimmx
bkdrk

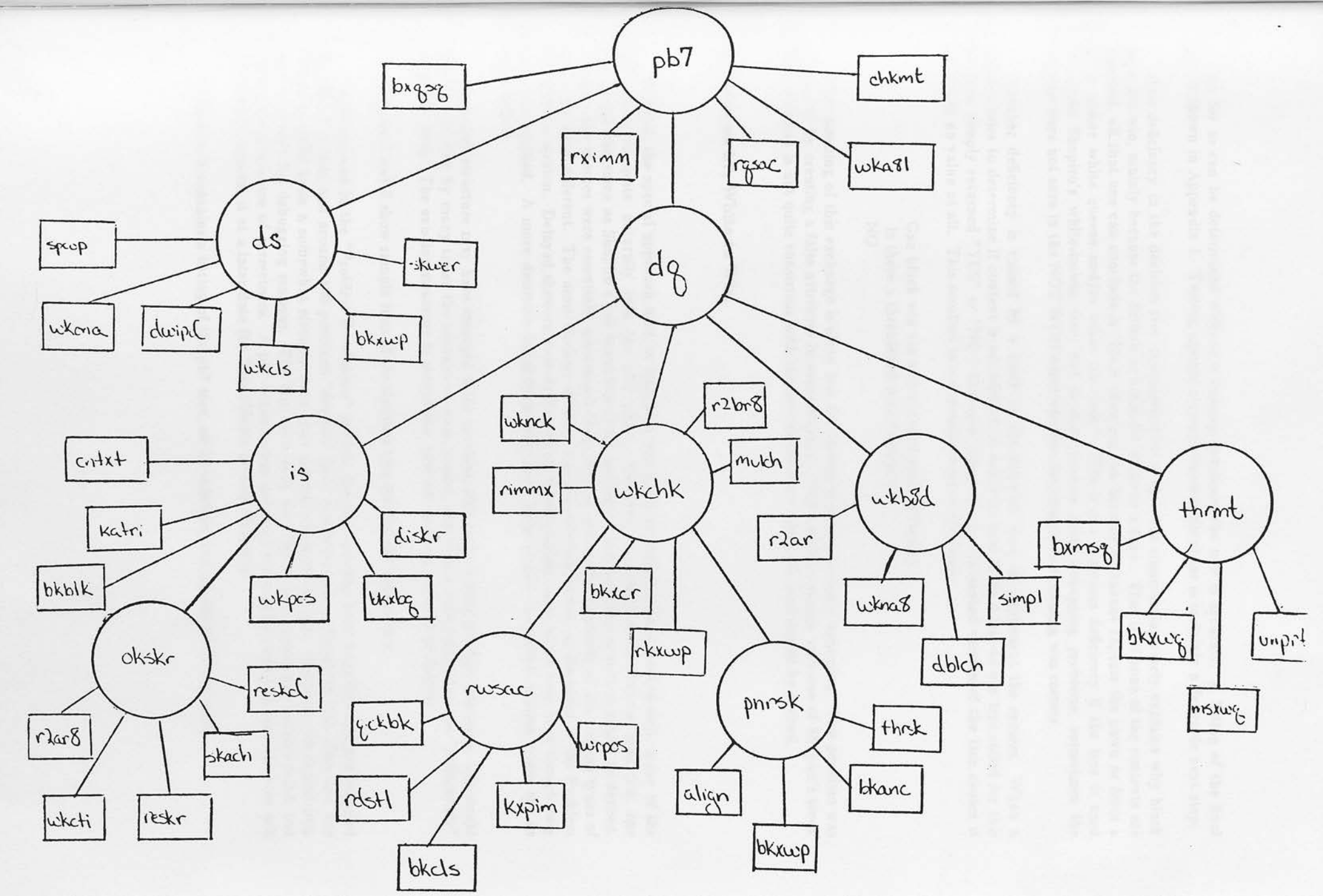
Figure 4. (next page) Problem Breakdown for the endgame KP vs. KR (pb7) white to win. See appendix for the definitions of the attributes and trees.

LEGEND



= white-to-win tree or subtree

= low-level ("C"-coded) attribute


pb7
bxqzg
chkmt
rximm
rqsac
wka8l
ds
sprop
skwer
wkcnq
dwipl
wkcls
bkxwp
dq
wknck
r2br8
mukh
rimmx
wkchk
r2ar
wkb8d
bxmsg
thrmt
cntxt
is
katri
diskr
bkblk
wkpcs
bkxbg
bkxcr
rkxwp
wkna8
simpl
dblch
bkxwg
unpkt
mrsxwg
okskr
reskd
r2ar8
skach
wkcti
reskr
rwsac
gckbk
wrpos
rdstl
kxpim
bkcls
phrsk
thrsk
align
bkxwp
bkanc

As for as can be determined without a look-up database, the tree is accurate. A listing of the final tree is shown in Appendix 1. Testing against a comprehensive database is certainly a desirable next step.

One deficiency in the decision tree developed here is that the commenting rarely explains why black does not win, mainly because the default is a win for white or a draw. That is, if none of the contexts are satisfied, all that one can conclude is "black does not win because he cannot capture the pawn or force a mate before white queens and/or takes the rook." This is not a serious deficiency if the tree is used alongside Shapiro's white-to-win tree, but it does provide some debugging problems; sometimes the experts were not sure in the NOT WON cases whether the program's reasoning was correct.

Another deficiency is caused by a quirk in the software used to implement the system. When a subtree tries to determine if context is established, it would be best if the white-to-win tree called for this purpose simply returned "YES" or "NO." However, using the uncommented version of the tree causes it to return no value at all. This resulted in commented runs of the form:

```
Can black win via a threatened mate? (Thrmt)
 Is there a threatened mate? (thrmt)
NO
```

The meaning of this exchange is clear, but the format is not particularly appealing. This problem was alleviated by creating a false attribute in every subtree. This attribute simply calls one of Shapiro's trees. This solution is not quite consistent with the overall software structure, and should be revised.

2. Pawn on b7, White to Win.

Overall, the special approach used in attacking this problem seems to have worked well; many of the pa7 ideas fit quite naturally into the pb7 game. However, the problem breakdown used (i.e. one essentially the same as Shapiro's pa7 breakdown) did not match pb7 problem as well as might be desired. Though the themes were essentially unchanged from the pa7 game, the frequency of the various types of positions were different. The most obvious of these was the delayed skewer, as discussed in the Problem Breakdown section. Delayed skewers were dealt with in three separate areas of the tree, even though only one idea applied. A more desirable breakdown may have been to put the wkchk subtree under the ds subproblem.

A cleaner structure may have emerged if the problem were tackled from scratch. However, this would have multiplied by many times the amount of work needed, and would have resulted in the "rediscovery" of many ideas. The existing structure is acceptable, just not as clean as could be desired.

Figures 7 and 8 show sample runs of the decision tree for the pb7 subproblem.

As mentioned in the "Problem Breakdown" section, the pb7 tree was built from 60 examples supplied by Danny Kopec and around 100 positions "derived" from examples used in Shapiro's tree. This tree was therefore built upon a somewhat stronger base than the pa7 black-to-win tree. However, no expert help was available for debugging sessions. The tree does work well for every example the authors tried, but that is no guarantee of correctness. A pb7 white-to-win database is available, however, and the tree will be checked against it at a later date (by Tim Niblett or Alen Shapiro).

Appendix 2 contains a listing of the pb7 tree, all its subtrees, and all the attributes used.

```
WK WP BK BR
a8 a7 b5 d8

Is this position won for black? (Bpa7)
  Can W capture the BR safely? (Brimm)
  NO
  Is the WK in stalemate? (stlmt)
  NO
  Does B win because he controls the queening square? (Bxqsq)
    Is the BK controlling the queening square? (kxqsq)
    NO
    Is the BR controlling the queening square? (rxqsq)
    NO
  NO
  Does B win via a delayed skewer threat? (Bds)
    Is context for ds established? (Cds)
    NO
  NO
  Does B win because there is a delay to queening? (Bdq)
    Does black win because there is a mate threat? (Thrmt)
      Is there a threatened mate? (Ctrmt)
      NO
    NO
    Does black win because the white king is on a8? (Bwka8)
      Can the BK control the WK's acces to a6? (bkxa6)
      YES
      Is there a delay because the white king is on a8? (Cwka8)
      YES
      Can the rook attack safely? (rksfx)
      YES
    YES
  YES
YES
```

Figure 5. An example of the execution of the Black-to-Win tree for the endgame KP vs. KR (pa7).

```
wk wp bk br
b5 a7 d5 b2

Is this position won for black? (Bpa7)
  Can W capture the BR safely? (Brimm)
  NO
  Is the WK in stalemate? (stlmt)
  NO
  Does B win because he controls the queening square? (Bxqsq)
    Is the BK controlling the queening square? (kxqsq)
    NO
    Is the BR controlling the queening square? (rxqsq)
    NO
  NO
  Does B win via a delayed skewer threat? (Bds)
    Is context for ds established? (Cds)
    NO
  NO
  Does B win because there is a delay to queening? (Bdq)
    Does B win because there is a mate threat? (Thrmt)
      Is there a threatened mate? (Ctrmt)
      NO
    NO
    Does black win because the WK is on a8? (Bwka8)
      Can the BK control the WK's access to a6? (bkxa6)
      NO
    NO
    Does black win because the WK is in check? (Wkchk)
      Is the white king in check? (Cwchk)
      YES
      Can the BR reach file a or rank 8 safely? (redge)
      YES
      Can the WP be captured on B's first move? (pimmx)
      NO
      Can the WK reach the WP in 2 moves? (wk2mv)
      YES
      Can the BK defend the rook's attack? (bkdrk)
      NO
    NO
    Does black win because there is an immediate skewer? (Bis)
      Is there an immediate skewer threat? (Cis)
      NO
    NO
  NO
NO
```

Figure 6. An example of the execution of the Black-to-Win tree for the endgame KP vs. KR (pa7).

```
WK WP BK BR
b6 b7 d6 a5

Is this position won for white? (pb7)
  Is there a good delayed skewer threat? (ds)
    Can R force WK off A into ds or df? (wkona)
    NO, because the WK in not on A
    Is there a special opposition pattern present? (spcop)
    NO
    Is the WK distance to intersect point too great? (dwipd)
    YES
  NO
  Is there a delay to white's queening the pawn? (dq)
    Is there a good delay because there is a mate threat? (thrmt)
      Is BK attacked by promoted WP? (bkxwq)
      YES
    NO
    Is there a good delay because the WK is on square a8? (wkb8d)
      Is the WK on square a8? (wkna8)
      NO
    NO
    Is there a good delay because the WK is in check? (wkchk)
      Is the WK in check? (wknck)
      NO
    NO
    Is there a good delay because of a skewer threat? (is)
      Will BK move and uncover a skewer? (diskr)
      NO
      Is the WK on an edge and not on a8? (cntxt)
      YES
      Is the BK in the way? (bkblk)
      NO
      Is the BK attacked in some way by the promoted WP? (bkxbq)
      YES
    NO
  NO
  Can white capture the black rook safely? (rximm)
  YES, but he will promote instead
YES
```

Figure 7. An example of the execution of the White-to-Win tree for the endgame KP vs. KR (pb7).

Figure 8 (next page). An example of the execution of the White-to-Win tree for the endgame KP vs. KR (pb7).

```
WK WP BK BR
a1 b7 b3 a2

Is this position won for white? (pb7)
  Is there a good delayed skewer threat? (ds)
    Can R force WK off A into ds or df? (wkona)
    NO
    Is there a special opposition pattern present? (spcop)
    NO
    Is the WK distance to intersect point too great? (dwipd)
    YES
  NO
  Is there a good delay to white's queening the pawn? (dq)
    Is there a good delay because there is a mate threat? (thrmt)
      Is BK attacked by promoted WP? (bkxwq)
      YES
    NO
    Is there a good delay because the WK is on square a8? (wkb8d)
      Is the WK on square a8? (wkna8)
      NO
    NO
    Is there a good delay because the WK is in check? (wkchk)
      Is the WK in check? (wknck)
      YES
      Can the BK attack the critical square (b7)? (bkxcr)
      NO
      Can B renew the check to good advantage? (mulch)
      NO
      Can the R sacrifice to WK allowing BK to stalemate? (rwsac)
        Is stalemate one move away? (rdstl)
        NO
        Is the BK close enough to get to b8? (bkcls)
        NO
      NO
      Can the BR be captured safely? (rimmx)
      NO
      Does the BR bear on the WP (direction x = -1 only)? (rkxwp)
      NO
      Can R get to B or 8 safely? (r2br8)
      NO
      Can B play the check into a skewer or pin? (pnrsk)
        Can BK reach c7 safely? (bkxwp)
        NO
        Is there a skewer threat? (thrsk)
        NO
        Are the WK and BR in line for pin or skewer? (align)
        YES
        Is BK in position to anchor a skr, fork or pin? (bkanc)
        YES
      YES
    YES
  YES
NO
```

Discussion

Although there are no good estimates on the accuracy of the decision trees shown in the appendic es, Danny Kopec estimated the pa7 tree to be approximately 80% accurate before the first debugging session. That percentage should be improved by now. Hopefully the pb7 tree was more accurate from the start due to the higher number of expert-derived examples available for induction.

The structured induction method seems to yield good results fairly rapidly. Each decision tree took approximately two weeks to build and debug. However, the problem being dealt with here is small relative to other problems in the same domain. Also, decision classes and problem breakdown were clean cut. An existing problem breakdown further reduced the time spent on the trees. Despite all this, much of the induction work was still done by humans. In many cases, the authors could determine what the tree should look like before ID3 was run on the example set. In these cases, ID3 was used more as a debugging tool for attributes than as an induction algorithm. When larger problem spaces are considered, the man-hours needed to break the problem down correctly may become unacceptable.

None of this suggests that the trees could have been built so quickly or accurately by some standard means (e.g. mini-max or look- ahead). However, the methodology will need improvement before larger problems can be handled. The next obvious step is to mechanize the structuring process. One possibility is to use the Michalski/Stepp "conceptual clustering" algorithm (Michalski and Stepp 1981) to derive subproblem areas. Using a top down approach, CLUSTER would be used recursively on the attribute space to determine sub - areas. An induction algorithm could then be used to obtain descriptions of each area. It may also be possible to approach the problem bottom-up; i.e. induce an unstructured tree and try and determine subproblem structure from information contained in it (Marcel Schoppers suggested this approach, and is pursuing it further).

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APPENDIX 1

This appendix contains listings of all the pa7 black-to-win decision trees, their attribute definitions, and the examples used to induce the trees. Attribute definitions are of the form:

<attribute name> <number of values> <values> <attribute definition>

```
Problem Bpa7


5
Bxqsq   2       f t     Does B win because he controls the queening square
stlmt   2       f t     Is the WK in stalemate
Brimm   2       f t     Can W capture the BR safely
Bds     2       f t     Does B win via a delayed skewer threat
Bdq     2       f t     Does B win because there is a delay to queening
2
WON   NOT
Bpa7
Is this position won for black

Brimm
   f: stlmt
         f: Bxqsq
               f: Bds
                     f: Bdq
                           f: NOT
                           t: WON
                     t: WON
               t: WON
         t: NOT
   t: NOT
```

Bpa7.d file

```
a1 a7 f2 b2 2  /* rook captured immediately */
a1 a7 f2 b8 2
h1 a7 g3 b8 2
a1 a7 a3 b2 2
a8 a7 h3 b1 2  /* stalemate */
d7 a7 b7 b2 1  /* BK captures pawn */
e7 a7 e5 h3 1  /* delayed skewer threat, rook takes pawn on file */
h6 a7 f3 e1 2  /* W queens safely */
a8 a7 c7 b2 2  /* stalemate */
a8 a7 c7 a2 2  /* stalemate */
b7 a7 b5 e7 1  /* good wkchk */
c7 a7 c5 e7 1  /* the same */
d7 a7 c5 d3 2
```

```
Subproblem Bds


2
Cds     2       f t     Is context for ds established
goodp   2       f t     Is this a good delayed skewer for black
2
FALSE   TRUE
Bds
Does B win via delayed skewer threat


Cds
   f: FALSE
   t: goodp
         f: FALSE
         t: TRUE
```

Bds.d file

```
a3 a7 c6 g7 1  /*not ds or goodp*/
e7 a7 e5 h3 2  /*good ds*/
b8 a7 d7 d4 1  /*ds, not goodp*/
e3 a7 e5 h3 1  /*not ds, goodp*/
```

```
Subproblem Bxqsq

3
kxqsq   2       f t     Is the BK controlling the queening square
rxqsq   2       f t     Is the BR controlling the queening square
wkdfp   2       f t     Is the WK able to defend the pawn
2
FALSE   TRUE
Bxqsq
Does B win because he controls the queening square

kxqsq
   f: rxqsq
         f: FALSE
         t: wkdfp
               f: TRUE
               t: FALSE
   t: TRUE
```

Bxqsq.d file

```
d5 a7 b7 h8  2  /* BK controls queening square */
h8 a7 b7 a4  2
d5 a7 b7 f6  2
g4 a7 b7 f6  2
c6 a7 c3 f8  1  /* WK defends pawn */
f3 a7 c3 f8  2  /* BR takes pawn */
c8 a7 c3 h6  1  /* B doesn't control queening square */
f2 a7 c3 h6  1
```

```
Subproblem Bdq

5
Thrmt 2 f t  Does black win because there is a mate threat
Bwka8 2 f t  Does black win because the WK is on a8
Wkchk 2 f t  Does black win because the WK is in check
Bis   2 f t  Does black win because there is an immediate skewer
ghdck 2 f t  Does black win through a hidden check
2
NO    YES
Bdq
Does B win because there is a delay to queening

Thrmt
   f: Bwka8
         f: Wkchk
               f: Bis
                     f: NO
                     t: YES
               t: YES
         t: YES
   t: YES
```

```
b1 a7 g3 d5  2  /* B wins via unavoidable mate */
a8 a7 b6 b5  2  /* BK prevents WK from def pawn */
e4 a7 h2 a4  2  /* WK moves out of ck, BR x pawn */
e8 a7 c1 g2  2  /* good skewer for B */
e5 a7 g3 b1  1  /* no delay to queening */
```

```
Subproblem Thrmt

5
Ctrmt    2        f t      Is there a threatened mate
unamt    2        f t      Is this an unavoidable mate
kxpim    2        f t      Can BK capture WP immediatley
rxwp     2        f t      can rook capture WP safely
wkdf     2        f t      Can WK defend the pawn
2
NO      YES
Thrmt
Does black win because of a threatened mate

Ctrmt
   f: NO
   t: kxpim
         f: wkdf
               f: YES
               t: NO
         t: YES
```

Thrmt.d file

```
h1 a7 h5 d5   1   /*not thrmt*/
h1 a7 g3 d5   2   /* unavoidable mate */
b8 a7 b6 d6   2   /* BK captures WP */
b8 a7 b6 f3   2
a3 a7 c3 h1   2
d8 a7 d6 h6   1
h5 a7 f5 d3   2
```

```
Subproblem Bwka8

3
Cwka8  2   f t Is there a delay because the white king is on a8
bkxa6  2   f t Can the BK control the WK's access to a6
rksfx  2   f t Can the rook attack safely
2
NO    YES
Bwka8
Does black win because the WK is on a8

bkxa6
   f: NO
   t: Cwka8
         f: NO
         t: rksfx
               f: NO
               t: YES
```

```
e7 a7 a3 a4  1  /* white king not on a8       */
a8 a7 a6 e4  2  /* rook able to mate with help of BK */
a8 a7 b5 c1  1  /* rook unable to safely check */
a8 a7 b6 b3  2  /* BK blocks WK, rook takes pawn */
a8 a7 h5 f1  1  /* draw */
a8 a7 h7 b1  1  /* black rook not able to check */
e7 a7 a3 a4  1  /* rest are not wka8d */
g8 a7 g5 d4  1
g8 a7 a6 c4  1
g8 a7 a6 d4  1
a8 a7 b5 d6  2  /* BK blocks WK defends */
```

```
Subproblem Wkchk

5
Cwchk 2    f t     Is the white king in check
pimmx 2    f t     Can the WP be captured on B's first move
redge 2    f t     Can the BR reach file a or rank 8 safely
wk2mv 2    f t     Can WK reach WP in 2 moves
bkdrk 2    f t     Can the BK defend the rook's attack
2
NO      YES
Wkchk
Does B win because he has the WK in check

Cwchk
   f: NO
   t: redge
         f: NO
         t: pimmx
               f: wk2mv
                     f: YES
                     t: bkdrk
                           f: NO
                           t: YES
               t: YES
```

Wkchk.d file

```
b1 a7 d3 f3 1 /* not wkchk */
e4 a7 b2 a4 2 /* pawn captured immediately */
d5 a7 h2 a5 2 /* pawn captured immediately */
d8 a7 d2 d4 1 /* wk defends pawn */
f8 a7 d2 f4 2 /* rook captures pawn in 2 */
c8 a7 a4 c4 1 /* wk defends pawn, rook can't move */
c3 a7 h8 h3 1 /* rook blocked */
e6 a7 g7 a4 1 /* not wkchk */
f3 a7 g7 a4 1
b5 a7 d6 b2 2 /* black king defends rook */
b5 a7 d6 d5 1 /* rook trapped */
g2 a7 c6 b2 2 /* wk can't defend pawn */
b5 a7 d5 b2 1 /* bk can't defend rook */
g2 a7 g7 a2 2
d7 a7 c5 d3 1 /* bk can't interfere, even though he's close */
b7 a7 b5 e7 2 /* bk can defend rooks attack by thrmt */
```

```
Subproblem Bis

3
Cis    2     f t  Is there an immediate skewer threat
bkint  2     f t  Does the BK interfere with the rook
wkrcp  2     f t  Can the WK reach the pawn
2
NO     YES
Bis
Does B win because there is an immediate skewer threat


Cis
   f: NO
   t: bkint
         f: wkrcp
               f: YES
               t: NO
         t: NO
```

Bis.d file

```
e8 a7 g8 h8  1  /* not is */
d8 a7 c3 h6  1  /* wk defends pawn */
f8 a7 d4 h4  1  /* rook can't safely skewer */
e8 a7 e5 h5  1  /* BK in rook's way */
e8 a7 c1 g2  2  /* white controls intersect but black takes pawn */
e8 a7 c1 h2  2  /* good skewer */
d8 a7 c2 f2  1  /* nothing works for black */
d8 a7 c1 f2  1  /* draw */
h4 a7 c1 e3  1  /* not is */
c7 a7 c1 e3  1  /* not is */
c7 a7 c3 e3  1
```

APPENDIX 2

This appendix contains listings of all the pb7 white-to-win decision trees, their attribute definitions, and the examples used to induce the trees. Attribute definitions are of the form:

<attribute name> <number of values> <values> <attribute definition>

Problem pb7

```
8
chkmt   2   f t   Is the WK in checkmate
bxqsq   2   f t   Does one or more B piece control the queening square
stlmt   2   f t   Is the WK in stalemate
rximm   2   f t   Can the BR be captured safely
rqsac   2   f t   Can R sacrifice to queen and force stalemate
ds      2   f t   Is there a good delayed skewer threat
dq      2   f t   Is there a simple delay to white's queening the pawn
wka8l   2   f t   Is WK on a losing a8 position
2
WON   NOT
pb7
Is this position won for white?


ds       5.29
  f: dq       4.16
       f: rximm     2.25
            f: bxqsq     1.91
                 f: rqsac     1.39
                      f: wka8l     0.00
                           f: WON
                           t: NOT
                      t: NOT
                 t: NOT
            t: WON
       t: NOT
  t: NOT

No exceptions to rule found
```

pb7.d file

```
b1 b7 a3 c5   2   /* R sac to Q for stlmt */
h1 b7 a7 a3   2   /* BK takes pawn */
a8 b7 b4 a1    2     /* WK on a8 */
a8 b7 c5 c1    2
a8 b7 c6 c1    2
a8 b7 d8 e8    2
a8 b7 d1 c1    2
a8 b7 e5 c5    2
a7 b7 d7 a5    2   /* R goes to 1 for ds */
a6 b7 d6 c6    2   /* BK forces WK into pin to protect P */
a6 b7 c2 c6    2   /* BK anchors skewer thrt, forces pin */
a5 b7 e5 c5    1   /* Q with +, WK to b4 or b6 */
a6 b7 b4 a3    2   /* normal ds ensues */
a3 b7 a5 a1    2   /* delayed fork */
c3 b7 c1 a3    2   /* pin or ds */
a6 b7 d7 d6    2    /* normal ds */
b6 b7 d7 d6    2    /* ds or bkxwp */
a6 b7 e5 c6    1   /* promoted pawn checks BK */
a5 b7 d5 c5    1   /* checks run out */
d5 b7 b5 c5    1   /* checks run out */
b6 b7 e6 d6    2   /* ds */
c3 b7 e4 e3    2
a7 b7 d6 a5    2   /* Q thrt BK forces R sac and stlmt */
a7 b7 d8 a5    2   /* same thing */
c5 b7 a7 c8    1   /* P under promotes to R after taking BR */
c6 b7 a7 c8    1   /* same as above */
a6 b7 b8 b6    2   /* WK takes rook on first move */
c5 b7 a7 c6    2   /* two moves to stalemate */
c7 b7 a7 c6    2
c5 b7 a7 c6    2
a7 b7 d6 a5    2   /* Q thrt BK forces R sac and stlmt */
a6 b7 d6 c6    2   /* BK takes WP in two */
b3 b7 b5 b1    2   /* BK reaches pawn for draw */
a1 b7 b3 a2    2   /* double check */
a1 b7 b3 a3    2   /* double check and draw */
a1 b7 d3 d1    1   /* checks run out */
d6 b7 b3 d3    1   /* checks run out */
b8 b7 d7 c8    2
e5 b7 e2 e3    2   /* R to b takes P */
f7 b7 c6 f6    2   /* WK takes R, BK takes P */
b5 b7 c7 c5    2   /* ds or BK takes pawn */
b6 b7 d7 d6    2    /* ds or BK takes pawn */
c7 b7 f8 f7    2   /* As soon as WK moves, pawn is taken */
c8 b7 b6 d8    2   /* BK takes P */
b6 b7 c6 d6    2   /* ds */
b6 b7 d8 d6    2   /* WK moves, BK gets to c7 */
b6 b7 d5 d6    2
c3 b7 e4 e3    2
c7 b7 e8 f7    2
e7 b7 a7 h7    2
c8 b7 f5 c6    2
```

```
c5 b7 e5 c7   2
a7 b7 d8 a5   2   /* same thing */
c5 b7 a7 c8   1   /* P under promotes to R after taking BR */
c6 b7 a7 c8   1   /* same as above */
a6 b7 b8 b6   2   /* WK takes rook on first move */
c5 b7 a7 c6   2   /* two moves to stalemate */
c7 b7 a7 c6   2
c5 b7 a7 c6   2
c5 b7 e5 c6   1
c5 b7 e5 c6   1
c4 b7 d8 c6   2
a7 b7 d5 a5   2
a7 b7 d7 a5   2   /* R goes to 1 for ds */
a6 b7 d6 c6   2   /* BK forces WK into pin to protect P */
a6 b7 c2 c6   2   /* BK anchors skewer thrt, forces pin */
a5 b7 e5 c5   1   /* Q with +, WK to b4 or b6 */
a6 b7 b4 a3   2   /* normal ds ensues */
a3 b7 a5 a1   2   /* delayed fork */
c3 b7 c1 a3   2   /* pin or ds */
a6 b7 d7 d6   2   /* normal ds */
b6 b7 d7 d6   2   /* ds or bkxwp */
a6 b7 e5 c6   1   /* promoted pawn checks BK */
a5 b7 d5 c5   1   /* checks run out */
d5 b7 b5 c5   1   /* checks run out */
d4 b7 b4 c4   1
d3 b7 b3 c3   1
a1 b7 d3 d1   1
h2 b7 a5 a2   2   /* R to B takes P */
a4 b7 c4 c2   2 /*rook blocked by BK */
a4 b7 c4 h1   2 /*rook can reach WP in time*/
h4 b7 f4 f1   1 /* promoted pawn checks BK */
a7 b7 c6 c5   2 /* same */
a1 b7 c1 c3   2 /* fork */
a6 b7 c6 c5   1 /* too close for fork */
e8 b7 b4 h2   1   /*promoted pawn + BK */
e8 b7 a4 h2   2
g8 b7 a4 h2   1   /* WK controls IP */
d8 b7 f7 e2   2   /* both kings control IP */
g8 b7 e7 f2   1   /* BK blocks R */
g8 b7 d6 e2   1   /* WK control IP */
h8 b7 d7 e4   2
b6 b7 b4 b3   2   /* discovered skewer */
h8 b7 d7 f4   1
h7 b7 d7 f2   1
e8 b7 h8 h1   1
d8 b7 h2 f4   1   /* R pinned against BK by promoted WP */
d8 b7 f4 h2   1   /* BK checked by Q */
b3 b7 f1 c1   1
g8 b7 d7 e4   2   /* fork thrt allow r 2 b */
e8 b7 h1 g4   2   /* skwr thrt allow r 2 b */
b3 b7 g3 c1   1
b3 b7 g3 c1   1   /* WK covers B, P covers 8 */
b4 b7 g4 e1   2   /* unavoidable skewer */
```

```
b3 b7 c4 c1   2   /* thrt skwr allows R -> B */
b4 b7 d2 c2   2   /* perpetual check */
h8 b7 d7 f7   1   /* P -> Q immediatley */
f8 b7 d7 h7   2
d8 b7 h7 f7   2
f8 b7 d7 h7   2   /* BK on 7th thrt skwr cov., but fork */
b6 b7 d4 c4   2   /* WK and Bk cover IP */
b6 b7 d3 c3   2   /* straight skewer */
b4 b7 h3 c2   1   /* WK covers IP */
d8 b7 f3 f7   2   /* R access to B (skewer threat) */
c8 b7 e7 g7   2   /* R access to 8 ("") */
b6 b7 h4 c4   2   /* reskr w/o BK support */
f8 b7 c1 h7   2   /* WK cover IP, but r 2 b */
f8 b7 b1 h7   1   /* P -> Q, BK in check */
f8 b7 d7 h7   2   /* BK supports reskr */
b3 b7 d1 c1   2   /* repeated checks or BK -> int */
b4 b7 d3 c2   1
d7 b7 d5 h2   2   /* dir = -1, WK forces draw */
c4 b7 e4 e1   2   /* WK can't go right because of BK */
c8 b7 e7 e4   2   /* special opposition */
a7 b7 a5 b6   2   /* special opposition */
d7 b7 d5 h5   2   /* normal ds */
a6 b7 c5 c4   2   /* could go to thrmt if W moves 0,-1 */
a7 b7 d5 c1   2   /* straight ds */
a1 b7 c5 c4   2   /* delayed fork */
c5 b7 e5 e1   1   /* Q with check */
g7 b7 g5 d3   1   /* BK can't protect rook */
g7 b7 g5 h2   1   /* same */
f7 b7 f5 g5   1   /* WK too close to intersect */
e7 b7 e5 h5   1   /* Q with check */
a3 b7 c3 h2   1   /* WK too close to intersect */
a3 b7 f6 c2   1   /* rook taken immediatley */
b7 b1 a3 e5   2   /* rook sac for stlmt */
b6 b7 d6 a5   1   /* promote instead of x R */
d7 b7 d5 h2   2   /* dir = -1, WK forces draw */
c4 b7 e4 e1   2   /* WK can't go right because of BK */
c8 b7 e7 e4   2   /* special opposition */
a7 b7 a5 b6   2   /* special opposition */
d7 b7 d5 h5   2   /* normal ds */
a6 b7 c5 c4   2   /* could go to thrmt if W moves 0,-1 */
a7 b7 d5 c1   2   /* straight ds */
a1 b7 c5 c4   2   /* delayed fork */
```

Problem ds

```
7
spcop    2    f t    Is there a special opposition pattern present
wkona    2    f t    Can R force WK off A into ds or df
dwipd    2    L G    Is the WK distance to intersect point too great
wkcls    2    f t    Is WK close enough to B or 8
dsopp    2    f t    Are the kings in normal opposition
bkxbq    2    f t    Is the BK attacked in some way by the promoted WP
skwer    2    f t    Is this a potential skewer rather than fork
2
FALSE    TRUE
ds
Is there a good delayed skewer threat?


wkona     4.19
  f: spcop     2.70
       f: dwipd     2.25
            L: FALSE
            G: wkcls     1.91
                 f: FALSE
                 t: bkxbq     1.39
                      f: skwer     0.00
                           f: FALSE
                           t: TRUE
                      t: FALSE
       t: TRUE
  t: TRUE

No exceptions to rule found
```

ds.d file

```
d7 b7 d5 h2   2   /* dir = -1, WK forces draw */
c4 b7 e4 e1   2   /* WK can't go right because of BK */
c8 b7 e7 e4   2   /* special opposition */
a7 b7 a5 b6   2   /* special opposition */
d7 b7 d5 h5   2   /* normal ds */
a6 b7 c5 c4   2   /* could go to thrmt if W moves 0,-1 */
a7 b7 d5 c1   2   /* straight ds */
a1 b7 c5 c4   2   /* delayed fork */
c5 b7 e5 e1   1   /* Q with check */
g7 b7 g5 d3   1   /* BK can't protect rook */
g7 b7 g5 h2   1   /* same */
a3 b7 d3 f3   1   /* no skewer */
e5 b7 d3 f3   1   /* no skewer */
f7 b7 f5 g5   1   /* WK too close to intersect */
e7 b7 e4 g4   1   /* Kings not in opp */
e7 b7 e5 h5   1   /* Q with check */
a3 b7 c3 h2   1   /* WK too close to intersect */
```

Problem dq

5
thrmt 2 f t Is there a good delay because there is a mate threat
wkb8d 2 f t Is there a good delay because the WK is on square a8
wkchk 2 f t Is there a good delay because the WK is in check
is 2 f t Is there a good delay because of a skewer threat
hdchk 2 f t Is there a good delay because there is a hidden check
2
FALSE TRUE
dq
Is there a simple delay to white's queening the pawn

thrmt 2.25
f: wkb8d 1.91
f: wkchk 1.39
f: is 0.00
f: FALSE
t: TRUE
t: TRUE
t: TRUE
t: TRUE

No exceptions to rule found

```
a7 b7 d7 a5   2   /* R goes to 1 for ds */
a6 b7 d6 c6   2   /* BK forces WK into pin to protect P */
a6 b7 c2 c6   2   /* BK anchors skewer thrt, forces pin */
a5 b7 e5 c5   1   /* Q with +, WK to b4 or b6 */
a6 b7 b4 a3   2   /* normal ds ensues */
a3 b7 a5 a1   2   /* delayed fork */
c3 b7 c1 a3   2   /* pin or ds */
a6 b7 d7 d6   2   /* normal ds */
b6 b7 d7 d6   2   /* ds or bkxwp */
a6 b7 e5 c6   1   /* promoted pawn checks BK */
a5 b7 d5 c5   1   /* checks run out */
d5 b7 b5 c5   1   /* checks run out */
b6 b7 c6 d6   2   /* ds */
c3 b7 e4 e3   2
a7 b7 d6 a5   2   /* Q thrt BK forces R sac and stlmt */
a7 b7 d8 a5   2   /* same thing */
c5 b7 a7 c8   1   /* P under promotes to R after taking BR */
c6 b7 a7 c8   1   /* same as above */
a6 b7 b8 b6   2   /* WK takes rook on first move */
c5 b7 a7 c6   2   /* two moves to stalemate */
c7 b7 a7 c6   2
c5 b7 a7 c6   2
a7 b7 d6 a5   2   /* Q thrt BK forces R sac and stlmt */
a6 b7 d6 c6   2   /* BK takes WP in two */
b3 b7 b5 b1   2   /* BK reaches pawn for draw */
a1 b7 b3 a2   2   /* double check */
a1 b7 b3 a3   2   /* double check and draw */
a1 b7 d3 d1   1   /* checks run out */
d6 b7 b3 d3   1   /* checks run out */
b8 b7 d7 c8   2
e5 b7 e2 e3   2   /* R to b takes P */
f7 b7 c6 f6   2   /* WK takes R, BK takes P */
b5 b7 c7 c5   2   /* ds or BK takes pawn */
b6 b7 d7 d6   2   /* ds or BK takes pawn */
c7 b7 f8 f7   2   /* As soon as WK moves, pawn is taken */
c8 b7 b6 d8   2   /* BK takes P */
b6 b7 e6 d6   2   /* ds */
b6 b7 d8 d6   2   /* WK moves, BK gets to c7 */
b6 b7 d5 d6   2
c3 b7 c4 e3   2
c7 b7 e8 f7   2
e7 b7 a7 h7   2
c8 b7 f5 c6   2
c5 b7 e5 c7   2
a7 b7 d8 a5   2   /* same thing */
c5 b7 a7 c8   1   /* P under promotes to R after taking BR */
c6 b7 a7 c8   1   /* same as above */
a6 b7 b8 b6   2   /* WK takes rook on first move */
c5 b7 a7 c6   2   /* two moves to stalemate */
c7 b7 a7 c6   2
```

```
c5 b7 a7 c6   2
c5 b7 e5 c6   1
c5 b7 e5 c6   1
c4 b7 d8 c6   2
a7 b7 d5 a5   1
a7 b7 d7 a5   2   /* R goes to 1 for ds */
a6 b7 d6 c6   2   /* BK forces WK into pin to protect P */
a6 b7 c2 c6   2   /* BK anchors skewer thrt, forces pin */
a5 b7 e5 c5   1   /* Q with +, WK to b4 or b6 */
a6 b7 b4 a3   2   /* normal ds ensues */
a3 b7 a5 a1   2   /* delayed fork */
c3 b7 c1 a3   2   /* pin or ds */
a6 b7 d7 d6   2   /* normads */
b6 b7 d7 d6   2   /* ds or bkxwp */
a6 b7 e5 c6   1   /* promoted pawn checks BK */
a5 b7 d5 c5   1   /* checks run out */
d5 b7 b5 c5   1   /* checks run out */
d4 b7 b4 c4   1
d3 b7 b3 c3   1
a1 b7 d3 d1   1
h2 b7 a5 a2   2   /* R to B takes P */
a4 b7 c4 c2   2 /*rook blocked by BK */
a4 b7 c4 h1   2 /*rook can reach WP in time*/
h4 b7 f4 f1   1 /* promoted pawn checks BK */
a7 b7 c6 c5   2 /* same */
a1 b7 c1 c3   2 /* fork */
a6 b7 c6 c5   1 /* too close for fork */
e8 b7 b4 h2   1   /*promoted pawn + BK */
e8 b7 a4 h2   2
g8 b7 a4 h2   1   /* WK controls IP */
d8 b7 f7 e2   2   /* both kings control IP */
g8 b7 e7 f2   1   /* BK blocks R */
g8 b7 d6 e2   1   /* WK control IP */
h8 b7 d7 e4   2
b6 b7 b4 b3   2   /* discovered skewer */
h8 b7 d7 f4   1
h7 b7 d7 f2   1
e8 b7 h8 h1   1
d8 b7 h2 f4   1   /* R pinned against BK by promoted WP */
d8 b7 f4 h2   1   /* BK checked by Q */
b3 b7 f1 c1   1
g8 b7 d7 e4   2   /* fork thrt allow r 2 b */
e8 b7 h1 g4   2   /* skwr thrt allow r 2 b */
b3 b7 g3 c1   1
b3 b7 g3 c1   1   /* WK covers B, P covers 8 */
b4 b7 g4 e1   2   /* unavoidable skewer */
b3 b7 e4 e1   2   /* thrt skwr allows R -> B */
b4 b7 d2 c2   2   /* perpetual check */
h8 b7 d7 f7   1   /* P -> Q immediatley */
f8 b7 d7 h7   2
d8 b7 h7 f7   2
f8 b7 d7 h7   2   /* BK on 7th thrt skwr cov., but fork */
b6 b7 d4 c4   2   /* WK and Bk cover IP */
```

```
b6 b7 d3 c3   2   /* straight skewer */
b4 b7 h3 c2   1   /* WK covers IP */
d8 b7 f3 f7   2   /* R access to B (skewer threat) */
c8 b7 e7 g7   2   /* R access to 8 ("") */
b6 b7 h4 c4   2   /* reskr w/o BK support */
f8 b7 c1 h7   2   /* WK cover IP, but r 2 b */
f8 b7 b1 h7   1   /* P -> Q, BK in check */
f8 b7 d7 h7   2   /* BK supports reskr */
b3 b7 d1 c1   2   /* repeated checks or BK -> int */
b4 b7 d3 c2   1
b8 b7 c5 d6 2 /* WK can only attack 1 intersection pt cos BK covers 1 */
b8 b7 c3 d6 1 /* as above but BK offers no support */
b8 b7 c3 d3 1 /* BK blocks 1 int pt WK attacks other */
b8 b7 f2 d3 2 /* R has access to A */
b8 b7 e3 f3 2 /* R access to 8 */
b8 b7 d6 d5 2 /* R access to A (nice pattern) */
b8 b7 e8 e7 2 /* R attacks P */
a7 b7 g3 f7 1 /* WK not on b8...no context */
b8 b7 c6 d6 2 /* BK bears on P and R sacs to allow capture */
b8 b7 d5 d6 2 /* WK checked away from safety */
b8 b7 e7 e6 2 /* WK->b7 BR->c1 ... delayed skewer */
b8 b7 e8 c6 2 /* WK->b7 BR->d7+ skewers for P */
b8 b7 b5 d5 2 /* best for white is perp check */
b8 b7 b4 d4 2 /* same but different first move */
```

Problem thrmt

```
5
bxmsq   2   f t   Can BR attack a mating square safely
bkxwq   2   f t   Is BK attacked by promoted WP
msxwq   2   f t   Is the mating square attacked by promoted WP
rtbr8   2   f t   Can BR reach file b or rank 8 safely
unprt   2   f t   Is W forced to underpromote to prevent mate
2
FALSE   TRUE
thrmt
Is there a good delay because of a mate threat



bkxwq     3.37
  f: msxwq     2.25
       f: bxmsq     1.39
            f: unprt     0.00
                 f: FALSE
                 t: TRUE
            t: TRUE
       t: FALSE
  t: FALSE

No exceptions to rule found
```

thrmt.d file

```
a4 b7 c4 c2   2 /*rook blocked by BK */
a4 b7 c4 h1   2 /*rook can reach WP in time*/
h4 b7 f4 f1   1 /* promoted pawn checks BK */
h5 b7 f5 c2   1 /* promoted pawn covers mating square */
e1 b7 a4 a3   1 /* no threatened mate */
a7 b7 c6 c4   2 /* forced underpromotion for draw */
a7 b7 c6 c5   2 /* same */
a1 b7 c1 c3   2 /* fork */
a6 b7 c6 c5   1 /* too close for fork */
```

Problem wkb8d

5
r2ar 2 f t Does the BR have safe access to file A or rank 8
wkna8 2 f t Is the WK on square a8
blxwp 2 f t Does B attacks the WP (BR in direction x = -1 only)
dblch 2 f t Can the WK be checked away from safety
simpl 2 f t Does a very simple pattern apply
2
FALSE TRUE
wkb8d
Is this a good delay to queening because the WK is on square a8?

wkna8 2.70
f: FALSE
t: dblch 1.91
f: r2ar 1.39
f: simpl 0.00
f: FALSE
t: TRUE
t: TRUE
t: TRUE

No exceptions to rule found

wkb8d.d file

```
b8 b7 c5 d6 2 /* WK can only attack 1 intersection pt cos BK covers 1 */
b8 b7 c3 d6 1 /* as above but BK offers no support */
b8 b7 c3 d3 1 /* BK blocks 1 int pt WK attacks other */
b8 b7 f2 d3 2 /* R has access to A */
b8 b7 e3 f3 2 /* R access to 8 */
b8 b7 d6 d5 2 /* R access to A (nice pattern) */
b8 b7 e8 e7 2 /* R attacks P */
a7 b7 g3 f7 1 /* WK not on b8...no context */
b8 b7 c6 d6 2 /* BK bears on P and R sacs to allow capture */
b8 b7 d5 d6 2 /* WK checked away from safety */
b8 b7 e7 e6 2 /* WK->b7 BR->c1 ... delayed skewer */
b8 b7 e8 e6 2 /* WK->b7 BR->d7+ skewers for P */
b8 b7 b5 d5 2 /* best for white is perp check */
b8 b7 b4 d4 2 /* same but different first move */
```

Problem is

7
cntxt 2 f t Is the WK on an edge and not on a8
katri 3 B W N Does any king control intersect point. If so, which
bkblk 2 f t Is the BK in the way
wkpos 2 f t Is the WK in a potential skewer position
bkxbq 2 f t Is the BK attacked in some way by the promoted WP
diskr 2 f t Will BK move and uncover a skewer
okskr 2 f t Is this potential skewer good
2
FALSE TRUE
is
Is there a good delay because of an immediate skewer threat?

```
diskr    5.22
  f: cntxt    4.77
       f: FALSE
       t: bkblk    4.19
            f: bkxbq    3.37
                 f: katri    1.91
                      B: TRUE
                      W: FALSE
                      N: wkpos    1.39
                           f: okskr    0.00
                                f: FALSE
                                t: TRUE
                           t: FALSE
                 t: FALSE
            t: FALSE
  t: TRUE
```

No exceptions to rule found

ls.d file

```
e8 b7 b4 h2   1   /*promoted pawn + BK */
e8 b7 a4 h2   2
g8 b7 a4 h2   1   /* WK controls IP */
d8 b7 f7 e2   2   /* both kings control IP */
g8 b7 e7 f2   1   /* BK blocks R */
g8 b7 d6 e2   1   /* WK control IP */
h8 b7 d7 e4   2
b6 b7 b4 b3   2   /* discovered skewer */
h8 b7 d7 f4   1
c8 b7 e6 e4   1
h7 b7 d7 f2   1
b8 b7 g7 e2   1
e8 b7 h8 h1   1
h8 b7 c7 d3   1   /* black wins, but no skewer */
b8 b7 h7 g1   1
d8 b7 h2 f4   1   /* R pinned against BK by promoted WP */
d8 b7 f4 h2   1   /* BK checked by Q */
b3 b7 f1 c1   1
g8 b7 d7 e4   2   /* fork thrt allow r 2 b */
f8 b7 d7 e4   1   /* no threat */
e8 b7 h1 g4   2   /* skwr thrt allow r 2 b */
b3 b7 g3 c1   1
b3 b7 g3 c1   1   /* WK covers B, P covers 8 */
b4 b7 g4 e1   2   /* unavoidable skewer */
b3 b7 e4 e1   2   /* thrt skwr allows R -> B */
b4 b7 d2 c2   2   /* perpetual check */
h8 b7 d7 f7   1   /* P -> Q immediatley */
f7 b7 d7 e7   1   /* no fork, but B wins */
f8 b7 d7 h7   2
d8 b7 h7 f7   2
f8 b7 d7 h7   2   /* BK on 7th thrt skwr cov., but fork */
b6 b7 d4 c4   2   /* WK and Bk cover IP */
b6 b7 d3 c3   2   /* straight skewer */
b4 b7 h3 c2   1   /* WK covers IP */
d8 b7 f3 f7   2   /* R access to B (skewer threat) */
c8 b7 e7 g7   2   /* R access to 8 ("") */
b6 b7 h4 c4   2   /* reskr w/o BK support */
f8 b7 c1 h7   2   /* WK cover IP, but r 2 b */
f8 b7 b1 h7   1   /* P -> Q, BK in check */
f8 b7 d7 h7   2   /* BK supports reskr */
b3 b7 d1 c1   2   /* repeated checks or BK -> int */
b4 b7 d3 c2   1
```

```
problem okskr

6
r2ar8   2   f t   Does the BR have safe access to file A or rank 8
wkcti   2   f t    can the WK control the intersect point
bkspr   2   f t    can the BK support the BR
reskr   2   f t   can the BR alone renew the skewer threat
skach   2   f t    can the WK be skewered after one or more checks
reskd   2   f t    can the WK be reskewered via a delayed skewer
2
FALSE   TRUE
okskr
is this potential skewer good?


r2ar8     2.50
  f: wkcti     2.25
       f: TRUE
       t: reskr     1.91
            f: skach     1.39
                 f: reskd     0.00
                      f: FALSE
                      t: TRUE
                 t: TRUE
            t: TRUE
  t: TRUE

No exceptions to rule found
```

okskr.d file

```
f8 b7 b1 h7 2 /* WK moves to inter pt but r2ar8 true */
b6 b7 d4 c4 2 /* WK and BK cand cover intersect pt */
b6 b7 d3 c3 2 /* straight forward skewer */
b4 b7 h3 c2 1 /* WK can control intersect pt */
d8 b7 f3 f7 2 /* R access to A (skewer threat) */
c8 b7 e7 g7 2 /* R access to 8 ("") */
b6 b7 h4 c4 2 /* reskewer without BK support */
b3 b7 d1 c1 2 /* at best repeated check for WK else BK moves to ipt */
b4 b7 d1 c2 2 /* WK can't control intersect cos b gets there first */
b4 b7 d3 c2 1 /*  wrong...not  reskewer after check */
b3 b7 d2 c1 2 /* wrong ... as above ... just in case */
b4 b7 d1 c2 2 /* BK can control ipt */
e8 b7 f5 g5 2 /* reskewer via delayed skewer (6 moves deep) */
f8 b7 g5 h5 1 /*  wrong... not (as above but to close to edge) */
b5 b7 d5 c5 2 /* WK in check R reskewers */
b4 b7 d4 c4 1 /* WK in check reskewer doesn't work */
```

Problem wkchk

```
8
wknck   2   f t   Is the WK in check
rimmx   2   f t   Can the BR be captured safely
bkxcr   2   f t   Can the BK attack the critical square (b7)
rkxwp   2   f t   Does the BR bear on the WP (direction x = -1 only)
mulch   2   f t   Can B renew the check to good advantage
r2br8   2   f t   Can R get to B or 8 safely
rwsac   2   f t   Can the R sacrifice to WK allowing BK to stalemate
pnrsk   2   f t   Can B play the check into a skewer or pin
2
FALSE   TRUE
wkchk
Is there a significant delay to queening because the WK is in check?


wknck   6.75
  f: FALSE
  t: bkxcr   6.11
       f: mulch   5.29
            f: rwsac   4.16
                 f: rimmx   2.25
                      f: rkxwp   1.91
                           f: r2br8   1.39
                                f: pnrsk   0.00
                                     f: FALSE
                                     t: TRUE
                                t: TRUE
                           t: TRUE
                      t: FALSE
                 t: TRUE
            t: TRUE
       t: TRUE

No exceptions to rule found
```

```
a7 b7 d6 a5   2   /* Q thrt BK forces R sac and stlmt */
a6 b7 d6 c6   2   /* BK takes WP in two */
b3 b7 b5 b1   2   /* BK reaches pawn for draw */
a1 b7 b3 a2   2   /* double check */
a1 b7 b3 a3   2   /* double check and draw */
a1 b7 d3 d1   1   /* checks run out */
d6 b7 b3 d3   1
b8 b7 d7 c8   2
e5 b7 c2 e3   2   /* R to b takes P */
f7 a7 b3 e3   1
f7 b7 c6 f6   2   /* WK takes R, BK takes P */
f7 b7 c6 g6   1   /* no check */
d5 b7 b2 f2   1   /* no check */
b5 b7 c7 c5   2   /* ds or BK takes pawn */
b6 b7 d7 d6   2   /* ds or BK takes pawn */
c7 b7 f8 f7   2   /* As soon as WK moves, pawn is taken */
c8 b7 b6 d8   2   /* BK takes P */
b6 b7 e6 d6   2   /* ds */
b6 b7 d8 d6   2   /* WK moves, BK gets to c7 */
b6 b7 d5 d6   2
c3 b7 e4 c3   2
c7 b7 e8 f7   2
e7 b7 a7 h7   2
c8 b7 f5 c6   2
c5 b7 e5 c7   2
a7 b7 d8 a5   2   /* same thing */
c5 b7 a7 c8   1   /* P under promotes to R after taking BR */
c6 b7 a7 c8   1   /* same as above */
a6 b7 b8 b6   2   /* WK takes rook on first move */
c5 b7 a7 c6   2   /* two moves to stalemate */
c7 b7 a7 c6   2
c5 b7 a7 c6   2
c5 b7 e5 c6   1
c5 b7 e5 c6   1
c4 b7 d8 c6   2
a7 b7 d5 a5   1
c5 b7 c2 b5   1
a7 b7 d7 a5   2   /* R goes to 1 for ds */
a6 b7 d6 c6   2   /* BK forces WK into pin to protect P */
a6 b7 c2 c6   2   /* BK anchors skewer thrt, forces pin */
a5 b7 e5 c5   1   /* Q with +, WK to b4 or b6 */
a6 b7 b4 a3   2   /* normal ds ensues */
a3 b7 a5 a1   2   /* delayed fork */
c3 b7 c1 a3   2   /* pin or ds */
a6 b7 d7 d6   2   /* normal ds */
b6 b7 d7 d6   2   /* ds or bkxwp */
a6 b7 e4 c2   1   /* checks run out */
a6 b7 e5 c6   1   /* promoted pawn checks BK */
a5 b7 d5 c5   1   /* checks run out */
a7 b7 a3 a2   1   /* no check */
d5 b7 b5 c5   1   /* checks run out */
d4 b7 b4 c4   1
```

```
d3 b7 b3 c3   1
a1 b7 d3 d1   1
d4 b7 a5 a2   1   /* no check */
c4 b7 a5 a2   1   /* no check */
b2 b7 a5 a2   2   /* R to B takes P */
```

Problem rwsac

```
5
qckbk    2    f t    Will promoted pawn ck BK
rdstl    2    f t    Is stalemate one move away
bkcls    2    f t    Is the BK close enough to get to b8
kxpim    2    f t    Will WK take the BR immediatley
wrpos    2    f t    Are WK and BR close enough to allow stlmt
2
FALSE    TRUE
rwsac
Can BR sacrifice and allow BK to get to b8 for stlmt


rdstl     4.50
  f: bkcls     3.37
       f: FALSE
       t: kxpim     2.25
            f: qckbk     1.39
                 f: FALSE
                 t: wrpos     0.00
                      f: FALSE
                      t: TRUE
            t: TRUE
  t: TRUE
```

No exceptions to rule found

rwsac.d file

```
a7 b7 d6 a5   2   /* Q thrt BK forces R sac and stlmt */
a7 b7 d8 a5   2   /* same thing */
c5 b7 a7 c8   1   /* P under promotes to R after taking BR */
c6 b7 a7 c8   1   /* same as above */
a6 b7 b8 b6   2   /* WK takes rook on first move */
c5 b7 a7 c6   2   /* two moves to stalemate */
c7 b7 a7 c6   2
c5 b7 a7 c6   2
c5 b7 e5 c6   1
c5 b7 e5 c6   1
c4 b7 d8 c6   1
a7 b7 d5 a5   1
c5 b7 c2 b5   1
a7 b7 d7 a5   1
```

Problem pnrsk

4
align 2 f t Are the WK and BR in line for pin or skewer
bkxwp 2 f t Can BK reach c7 safely
bkanc 2 f t Is BK in position to anchor a skr, fork or pin
thrsk 2 f t Is there a skewer threat
2
FALSE TRUE
pnrsk
Can B play the check into a pin or skewer or fork

bkxwp 2.77
f: thrsk 1.91
f: align 1.39
f: FALSE
t: bkanc 0.00
f: FALSE
t: TRUE
t: TRUE
t: TRUE

No exceptions to rule found

pnrsk.d file

```
a7 b7 d7 a5   2   /* R goes to 1 for ds */
a6 b7 d6 c6   2   /* BK forces WK into pin to protect P */
a6 b7 c2 c6   2   /* BK anchors skewer thrt, forces pin */
a5 b7 e5 c5   1   /* Q with +, WK to b4 or b6 */
a6 b7 b4 a3   2   /* normal ds ensues */
a3 b7 a5 a1   2   /* delayed fork */
c3 b7 c1 a3   2   /* pin or ds */
a6 b7 d7 d6   2   /* normal ds */
b6 b7 d7 d6   2   /* ds or bkxwp */
a6 b7 e4 c2   1   /* checks run out */
a6 b7 e5 c6   1   /* promoted pawn checks BK */
a5 b7 d5 c5   1   /* checks run out */
a7 b7 a3 a2   1   /* no check */
d5 b7 b5 c5   1   /* checks run out */
d4 b7 b4 c4   1
d3 b7 b3 c3   1
a1 b7 d3 d1   1
d4 b7 a5 a2   1   /* no check */
c4 b7 a5 a2   1   /* no check */
h2 b7 a5 a2   1   /* no skwr threat */
b6 b7 c6 d6   2   /* ds */
c3 b7 e4 e3   2
```

APPENDIX D

An unstructured KPK BTM WFW/not WFW decision tree

No of examples in final working-set = 304.
No of nodes in final tree = 79.
Time taken to generate rule = 243 CPU seconds.

```
imcap
  f: psquare
      f: wkahd
          f: LOST
          t: nearc8
              f: LOST
              t: DRAWN
      t: rookpawn_relevant
          f: pmove
              f: mp1
                  f: r6patt
                      f: dirop56
                          f: mp56
                              f: btop1
                                  f: rank56_relevant
                                      f: DRAWN
                                      t: LOST
                                  t: interfere
                                      f: plus2
                                          f: ahead
                                              f: LOST
                                              t: DRAWN
                                          t: DRAWN
                                      t: ahead
                                          f: DRAWN
                                          t: LOST
                              t: rank56_relevant
                                  f: rank7_relevant
                                      f: mp2
                                          f: diropw
                                              f: interfere
                                                  f: DRAWN
                                                  t: LOST
                                              t: LOST
                                          t: LOST
                                      t: stlmt
                                          f: plus1
                                              f: LOST
                                              t: dropb
                                                  f: DRAWN
                                                  t: LOST
                                          t: DRAWN
                                  t: LOST
                          t: LOST
                      t: LOST
                  t: stlmt
                      f: LOST
                      t: DRAWN
              t: stlmt
```

```
                f: LOST
                t: mp1
                    f: DRAWN
                    t: LOST
        t: nearc8
            f: rp1
                f: near2p
                    f: stalemate
                        f: nearp
                            f: rp2
                                f: neara8
                                    f: LOST
                                    t: DRAWN
                                t: DRAWN
                            t: ahead
                                f: plus2
                                    f: LOST
                                    t: DRAWN
                                t: DRAWN
                        t: DRAWN
                    t: DRAWN
                t: LOST
            t: crit
                f: DRAWN
                t: plus1
                    f: wkahd
                        f: LOST
                        t: DRAWN
                    t: DRAWN
t: DRAWN
```

APPENDIX E

List of chess books consulted by the author for KPa7KR

Ban, J. (1972) **Tactics in the endgame.** Budapest: Athenæum.

Barden, L. (1975) **How to Play the Endgame in Chess** New York: Bobbs-Merrill Co. Ltd.

Fine, R. (1969) **Basic chess endings.** New York: McKay.

Keres, P. (1974). **Practical chess endings.** London: Batsford. pp. 89-93.

Levenfish, G. and Smyslov, V. (1971) **Rook endings.** London: Batsford.

APPENDIX F

Listing of btoqs prior to its being made into a subproblem

```
btoqs() { /* should really be a sub problem */
   if(bf < 3 && bf == wf && bf == rf && br > rr && rr > wr)
      return(comment(TRUE, FALSE, "", "", "", LEVEL));
   if(br > 6 && br == wr && br == rr && bf < rf && rf < wf)
      return(comment(TRUE, FALSE, "", "", "", LEVEL));
   if(wf == 2 && rf == 2 && ((wr > 4 && rr < wr) ||
         (wr > 2 && rr > wr+1 && !bitset(1, rr, BKING))) &&
            !(br+bf == 9 || br == 8))
      return(comment(TRUE, "YES because W can't avoid being skewered", "", "", "",
                        LEVEL));
   if(bf == 1) {
      if(br == 5 && rr == 1 && rf == 1 && wr == 1 && wf == 3)
         return(comment(TRUE, FALSE, "", "", "", LEVEL));
      if(rf == bf && rr < br && !(rr == 1 && wr == rr && wf == 3 && br != 3))
         if(!bitset(bf+2, br, WATT))
            return(comment(TRUE, FALSE, "", "", "", LEVEL));
         if(bitset(1, 4, ROOK) && bitset(1, 5, BKING) && wr == 4 && (wf == 3 || wf == 4))
            return(comment(FALSE,
                        "NO  because the WK can keep the BK between the BR and WP",
                        "", "", "", LEVEL));
      if(rf == 2 && rr <= br) {
         if(wr > br && wf == rf)
            return(comment(TRUE, FALSE, "", "", "", LEVEL));
         if((wf == 2 && wr < rr-2) || br <= 2)
            return(comment(TRUE, FALSE, "", "", "", LEVEL));
      }
      if(rf == 2 && bf < 3 && br < wr)
         return(comment(TRUE, "YES the BK aids the BR to skewer", "", "", "", LEVEL));
      if(rf == 1) {
         expand(PAWN, 20);
         and(BATT, 20, 21);
         if(!zero(21))
            return(comment(TRUE, "YES the BK can sneak up on the WP", "", "", "",
                        LEVEL));
      }
   }
   else if(br == 8) {
      if(bf == 3 && rf == 8 && rr == 8 && wf == 8 && wr == 6)
         return(comment(TRUE, FALSE, "", "", "", LEVEL));
      if(rr == br && rf > bf && !(rf == 8 && wf == rf && wr == 6 && bf != 6))
         if(!bitset(bf, br-2, WATT))
            return(comment(TRUE, FALSE, "", "", "", LEVEL));
      if(rr == 8) {
         expand(PAWN, 20);
         and(BATT, 20, 21);
         if(!zero(21))
            return(comment(TRUE, "YES the BK can sneak up on the WP", "", "", "",
                        LEVEL));
      }
   }
   cand(RATT, WATT, 21);
   cand(PAWN, 21, 21);
   cand(BATT, 21, 21);
```

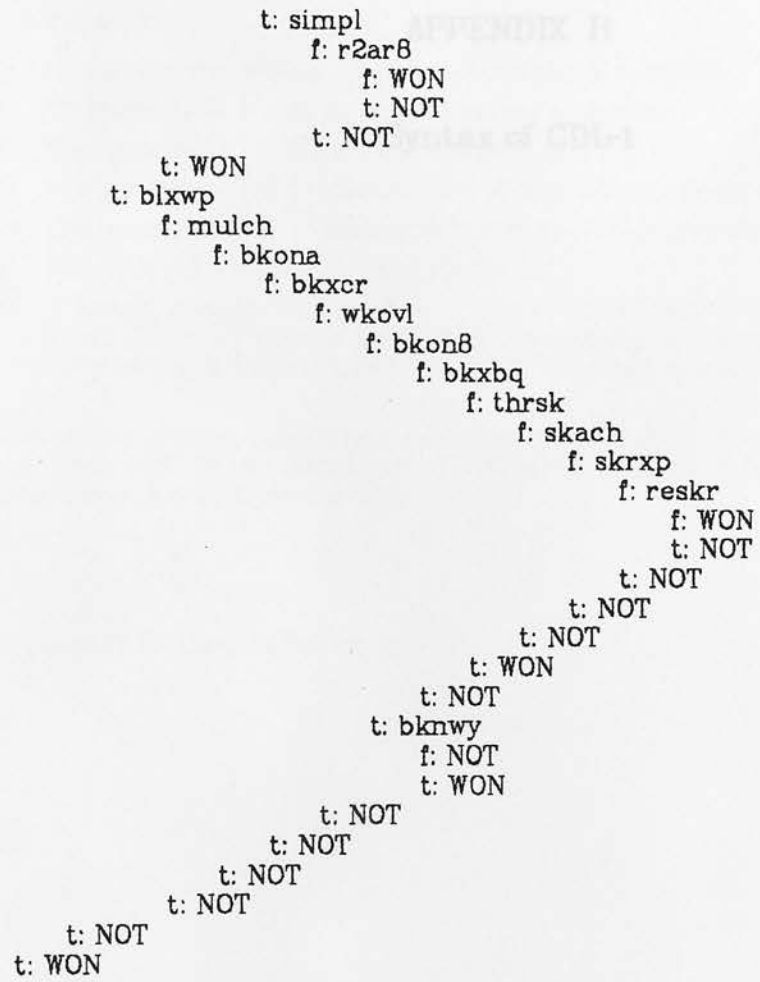
```
  /* WK in check and rook on edge with BK blocking posns */

  if(bitset(2, 7, 21)) {
     init(30);
     setbit(2, 7, 30);
     expand(30, 31);
     cand(31, DQ_INTERSECT, 31);
     if(zero(31))
        return(comment(FALSE, "NO  because the BK is in the BR's way", "", "", "",
              LEVEL));
  }
  expand(21, 21);
  cand(BATT, 21, 21);
  and(DQ_INTERSECT, 21, 21);
  if(bitcount(21) == 2)
     return(comment(TRUE, "YES because the WK can't cover both BR intersect points",
           "", "", "", LEVEL));
  return(comment(bitcount(DQ_INTERSECT) > bitcount(21), FALSE, "", "", "", LEVEL));
}
```

APPENDIX G

An unstructured KPa7KR WTM WFW/not WFW decision tree

No of examples in final working-set = 175.
No of nodes in final tree = 82.
Time taken to generate rule = 963 CPU seconds.

```
rimmx
  f: bxqsq
       f: wknck
            f: bkxbq
                 f: wkna8
                      f: bkblk
                           f: katri
                                B: NOT
                                W: WON
                                N: wkpos
                                     f: reskr
                                          f: wkcti
                                               f: NOT
                                               t: r2ar8
                                                    f: dsopp
                                                         f: thrsk
                                                              f: bkspr
                                                                   f: skach
                                                                        f: WON
                                                                        t: NOT
                                                                   t: NOT
                                                              t: NOT
                                                         t: WON
                                                    t: NOT
                                          t: NOT
                                     t: rxmsq
                                          f: wtoeg
                                               1: dsopp
                                                    f: WON
                                                    t: dwipd
                                                         L: WON
                                                         G: skewr
                                                              f: WON
                                                              t: NOT
                                               N: WON
                                          t: qxmsq
                                               f: NOT
                                               t: WON
                           t: hdchk
                                f: spcop
                                     f: rxmsq
                                          f: WON
                                          t: qxmsq
                                               f: NOT
                                               t: WON
                                     t: NOT
                                t: NOT
                      t: bknwy
                           f: NOT
```


t: simpl
f: r2ar8
f: WON
t: NOT
t: NOT
t: WON
t: blxwp
f: mulch
f: bkona
f: bkxcr
f: wkovl
f: bkon8
f: bkxbq
f: thrsk
f: skach
f: skrxp
f: reskr
f: WON
t: NOT
t: NOT
t: NOT
t: WON
t: NOT
t: bknwy
f: NOT
t: WON
t: NOT
t: NOT
t: NOT
t: NOT
t: NOT
t: WON

APPENDIX H

Syntax of CDL-1

Nomenclature:

(1) All upper-case words denote non-terminal symbols.

(2) All lower-case words denote terminal symbols.

(3) The construct [X|Y] means either X or Y.

(4) The construct [X]+ means that X may occur one or more times.

(5) The construct [X]* means that X may occur zero or more times.

(6) The symbol "->" means "is defined as".

(7) A space means "followed by". Thus "PROGRAM -> primitive [DEC]* PART [PROC]*" means "PROGRAM" is defined as; primitive followed by zero or more DECs followed by PART followed by zero or more PROCs.

Note that anything in capitals can have its definition looked up in some lower line. Thus DEC in the definition of PROGRAM is itself defined on the following three lines. A construction like:

```
X     ->    a
X     ->    b
```

means X is defined as "a" *or* "b".

Productions:

PROGRAM	->	primitive [DEC]* PART [PROC]*
DEC	->	int DECLIST
DEC	->	test DECLIST
DEC	->	NAME has cases { CASELIST }
DECLIST	->	NAME [DECLREST]+
DECLREST	->	, DECLIST
CASELIST	->	NAME [CASELREST]+
CASELREST	->	, CASELIST
PART	->	to partition NAME P_COND_LIST OTHER_COND
PROC	->	to decide NAME COND_LIST OTHER_COND
P_COND_LIST	->	[P_COND]+
P_COND	->	if PRED then CLASS
P_COND	->	P_CASE_STAT
COND_LIST	->	[COND]+
COND	->	if PRED then CLASS
COND	->	CASE_STAT
P_CASE_STAT	->	in case NAME is P_CASE_REST
P_CASE_REST	->	P_LOG_CASE
P_CASE_REST	->	P_NUM_CASE
P_LOG_CASE	->	PRED then TERM [P_NUM_C_TEST]*
P_LOG_C_CASE	->	or if PRED then TERM
P_NUM_CASE	->	< NUMBER then TERM [P_NUM_C_TEST]*
P_NUM_C_TEST	->	or if < NUMBER then TERM
CASE_STAT	->	in case NAME is CASE_REST
CASE_REST	->	LOG_CASE
CASE_REST	->	NUM_CASE
LOG_CASE	->	PRED then TERM [LOG_C_TEST]*
LOG_C_TEST	->	or if PRED then TERM
NUM_CASE	->	< NUMBER then TERM [NUM_C_TEST]*

```
NUM_C_TEST      ->    or if < NUMBER then TERM

OTHER_COND      ->    otherwise CLASS

TERM            ->    CLASS
TERM            ->    if PRED then CLASS

PRED            ->    NAME
PRED            ->    not NAME

CLASS           ->    NAME

NAME            ->    ALPHA [ALPHANUM]*

ALPHANUM        ->    ALPHA
ALPHANUM        ->    NUMBER
ALPHANUM        ->    _

ALPHA           ->    [a|b|c|d|e|f|g|h|i|j|k|l|m|n|o|p|q|r|s|t|u|v|w|x|y|z]
ALPHA           ->    [A|B|C|D|E|F|G|H|I|J|K|L|M|N|O|P|Q|R|S|T|U|V|W|X|Y|Z]

NUMBER          ->    - [NUM]+
NUMBER          ->    [NUM]+

NUM             ->    [0|1|2|3|4|5|6|7|8|9]
```

APPENDIX I

WFW/not WFW tutorial manual for KPa7KR WTM

The text which follows can be regarded as a fairly faithful free-hand translation of the "C" program generated by use of the structured induction interactive ID3 algorithm in the manner described in the body of this thesis. Conjunctions or disjunctions are chosen so as to minimize the number of negations while preserving meaning. The names of subproblems are capitalized for ease of identification. The numbers denote subproblem level as described in Chapter 7.

List of rules.

PA7, Top-level rule. This rule is used to decide if a KPa7KR position with White-to-move is won-for-white or not.

KPa7KR is won for White (PA7, 1) IFF
the BR can be captured safely (rimmx)

OR none of the following is true:

there is a simple delay to White's queening the pawn (DQ, 2.1)
OR one or more Black pieces control the queening square (bxqsq)
OR there is a good delayed skewer threat (DS, 2.2).

DQ, Level 2.1. This rule is used in the PA7 (top-level) rule to decide if Black can successfully delay White from queening its pawn.

There is a simple delay to White's queening the pawn (DQ, 2.1) IFF
there is a good delay because there is a mate threat (THRMT, 3.1)
OR there is a good delay because the WK is on square a8 (WKA8D, 3.2)
OR there is a good delay because the WK is in check (WKCHK, 3.3)
OR there is a good delay because of a double attack threat (DBLAT, 3.4)
OR there is a good delay because there is a hidden check (hdchk)

DS, Level 2.2. This rule is used in the PA7 (top-level) rule to decide if Black can force a double attack of type "skewer"

There is a good delayed skewer threat (DS, 2.2) IFF
there is a special opposition pattern present (spcop)

OR all of the following are true:

the WK is one away from the relevant edge (wtoeg)
AND the kings are in normal opposition (dsopp)
AND the WK distance to intersect point is too great (dwipd)
AND there is a potential skewer as opposed to fork (skewr)
AND the BK is not attacked in some way by the promoted WP (bkxbq)

THRMT, Level 3.1. This rule is used in the DQ (level 2.1) rule to decide if Black can successfully delay White from queening its pawn because of an initial checkmate threat.

There is a good delay because there is a mate threat (THRMT, 3.1) IFF
the BR attacks a mating square safely (rxmsq)

AND

EITHER the BK can attack the WP (bkxwp)

OR none of the following is true:

the BK is attacked in some way by the promoted WP (bkxbq)
OR the mating square is attacked in some way by the promoted WP (qxmsq)
OR the BR does not have safe access to file A or rank 8 (r2ar8)

WKA8D, Level 3.2. This rule is used in the DQ (level 2.1) rule to decide if Black can successfully delay White from queening its pawn because the White king is initially on the pawn promotion square (a8).

There is a good delay because the WK is on square a8 (WKA8D, 3.2) IFF
the WK is on square a8 (wkna8)

AND any of the following is true:

the BR has safe access to file A or rank 8 (r2ar8)
OR B attacks the WP (BR in direction x = -1 only) (blxwp)
OR a very simple pattern applies (simpl)

WKCHK, Level 3.3. This rule is used in the DQ (level 2.1) rule to decide if Black can successfully delay White from queening its pawn because the White king is initially in check.

There is a good delay because the WK is in check (WKCHK, 3.3) IFF
the WK is in check (wknck)
AND the BR cannot be captured safely (rimmx)

AND any of the following is true:

B can attack the queening square soon (BTOQS, 4.2)
OR the BK can attack the critical square (b7) (bkxcr)
OR the BR bears on the WP (direction x = -1 only) (rkxwp)
OR there is a skewer threat lurking (thrsk)
OR B can renew the check to good advantage (mulch)

DBLAT, Level 3.4. This rule is used in the DQ (level 2.1) rule to decide if Black can successfully delay White from queening its pawn because of an initial double-attack threat.

There is a good delay because of a double attack threat (DBLAT, 3.4) IFF
the WK is on an edge and not on a8 (cntxt)
AND the BK is not in the way (bkblk)
AND the BK s not attacked in some way by the promoted WP (bkxbq)

AND

EITHER the BK controls the intersect point (katri)

OR

the WK is in a potential skewer position (wkpos)
AND the potential double attack is good (OKSKR, 4.1)

OKSKR, Level 4.1. This rule is used in the DBLAT (level 3.4) rule to decide if Black can successfully capitalize on a potential double-attack threat. White king is initially on the pawn promotion square (a8).

The potential double attack is good (OKSKR, 4.1) IFF
- the BR has safe access to file A or rank 8 (r2ar8)
- OR the WK cannot control the intersect point (wkcti)
- OR the BK can support the BR (bkspr)
- OR the BR alone can renew the skewer threat (reskr)
- OR the WK can be skewered after one or more checks (skach)
- OR the WK can be reskewered via a delayed skewer (reskd)

BTOQS, Level 4.2. This rule is used in the WKCHK (level 3.3) rule to decide if any Black piece will soon be able to control the queening square as a result of the White king having initially been in check.

B can attack the queening square soon (BTOQS, 4.2) IFF
- the BK is not in the BR's way (bknwy)
- AND the BR can achieve a skewer or the BK attacks the WP (skrxp)
- AND the BK is on file A in a position to aid the BR (bkona)
- AND the BK is on rank 8 in a position to aid the BR (bkon8)
- AND the WK is overloaded (wkovl)

List of unique primitives attributes invoked in the above text.

the BR can be captured safely (rimmx)
one or more Black pieces control the queening square (bxqsq)
there is a good delay because there is a hidden check (hdchk)
there is a special opposition pattern present (spcop)
the WK is one away from the relevant edge (wtoeg)
the kings are in normal opposition (dsopp)
the WK distance to intersect point is too great (dwipd)
there is a potential skewer as opposed to fork (skewr)
the BK is not attacked in some way by the promoted WP (bkxbq)
the BR attacks a mating square safely (rxmsq)
the BK can attack the WP (bkxwp)
the mating square is attacked in some way by the promoted WP (qxmsq)
the BR does not have safe access to file A or rank 8 (r2ar8)
the WK is on square a8 (wkna8)
B attacks the WP (BR in direction x = -1 only) (blxwp)
a very simple pattern applies (simpl)
the WK is in check (wknck)
the BK can attack the critical square (b7) (bkxcr)
the BR bears on the WP (direction x = -1 only) (rkxwp)
there is a skewer threat lurking (thrsk)
B can renew the check to good advantage (mulch)
the WK is on an edge and not on a8 (cntxt)
the BK is not in the way (bkblk)
the BK controls the intersect point (katri)
the WK is in a potential skewer position (wkpos)
the WK cannot control the intersect point (wkcti)
the BK can support the BR (bkspr)
the BR alone can renew the skewer threat (reskr)
the WK can be skewered after one or more checks (skach)
the WK can be reskewered via a delayed skewer (reskd)
the BK is not in the BR's way (bknwy)
the BR can achieve a skewer or the BK attacks the WP (skrxp)
the BK is on rank A in a position to aid the BR (bkona)
the BK is on file 8 in a position to aid the BR (bkon8)
the WK is overloaded (wkovl)